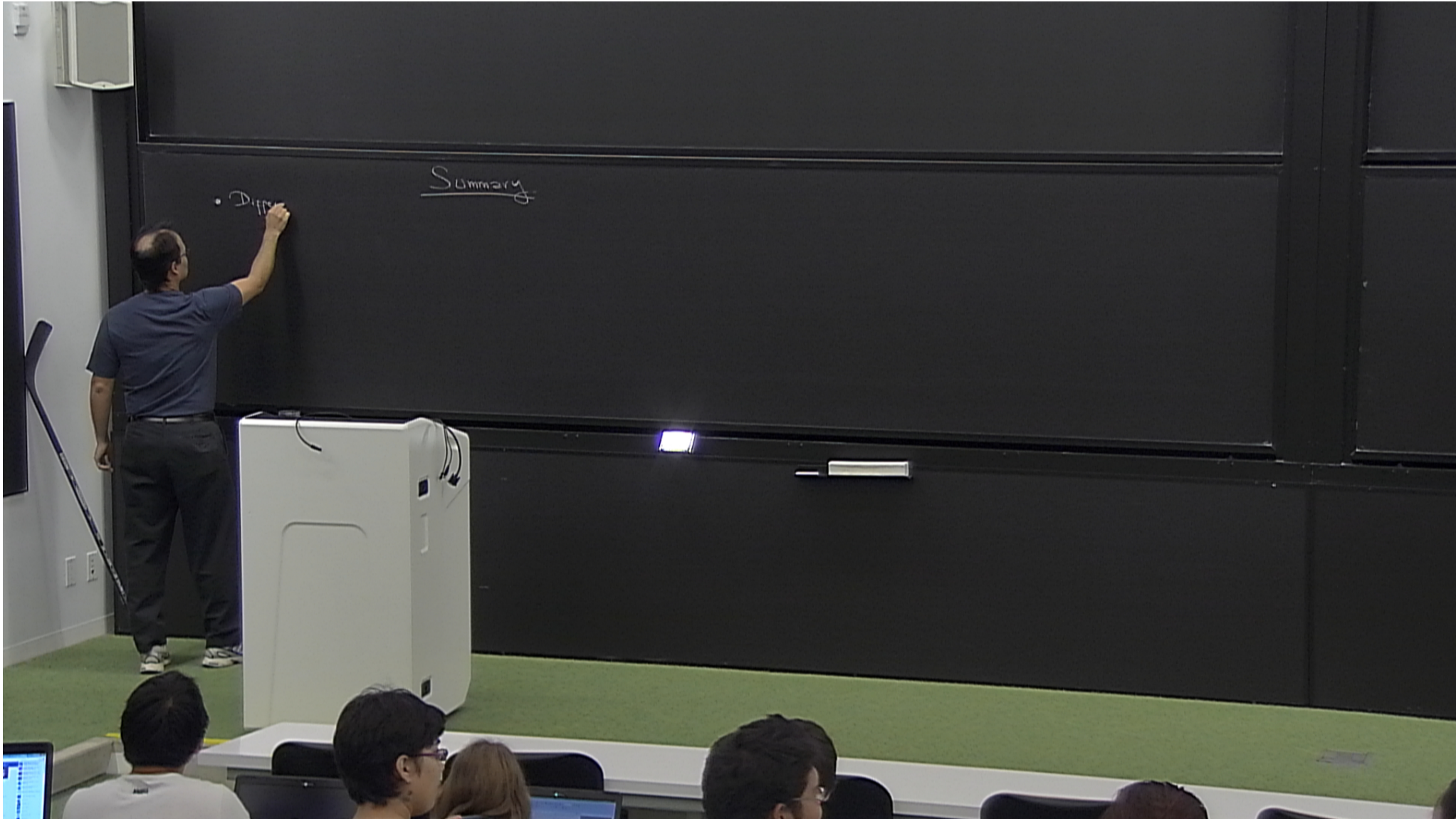


Title: 13/14 PSI - QFT 0 - Lecture 4

Date: Sep 06, 2013 10:30 AM

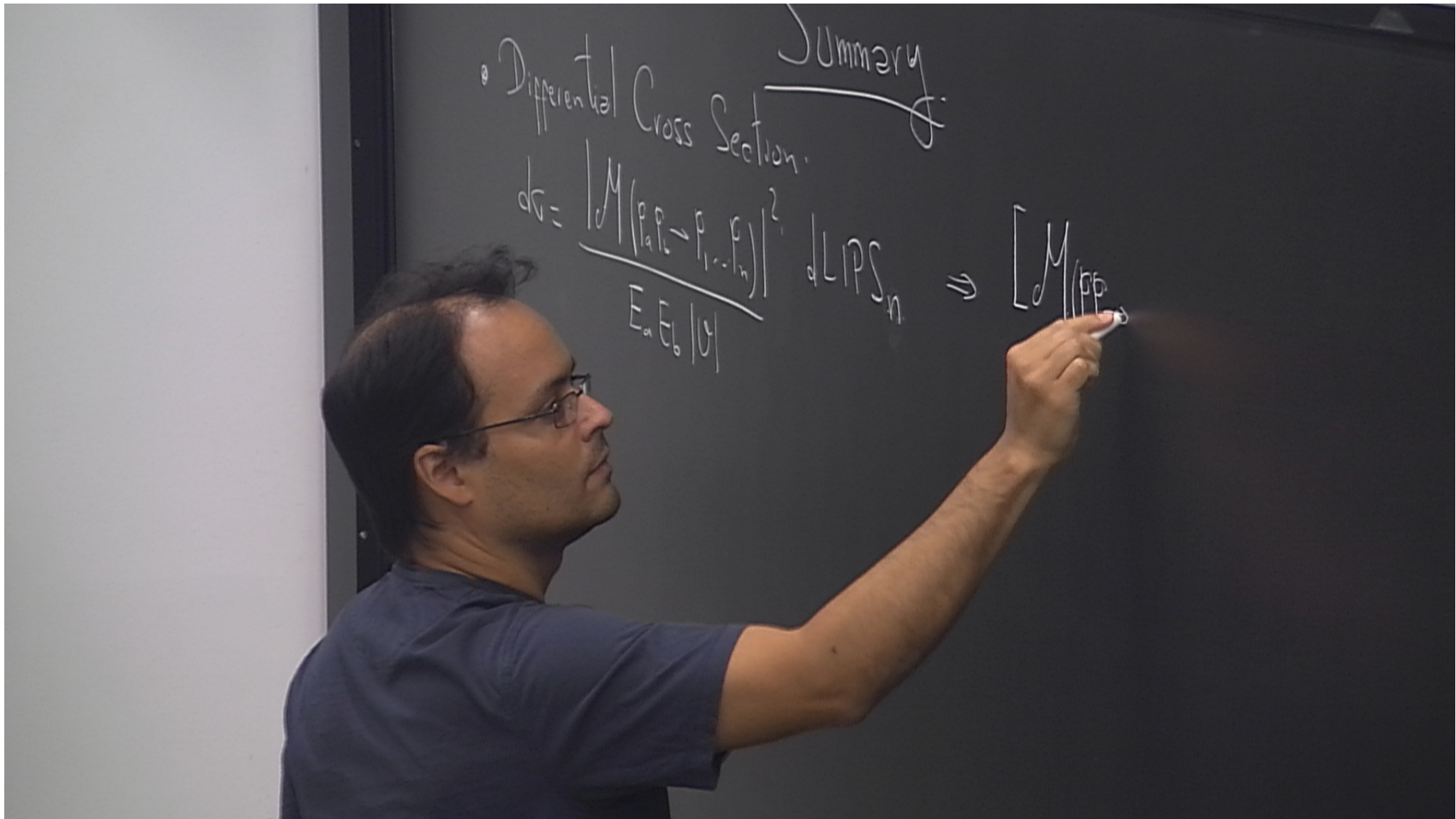
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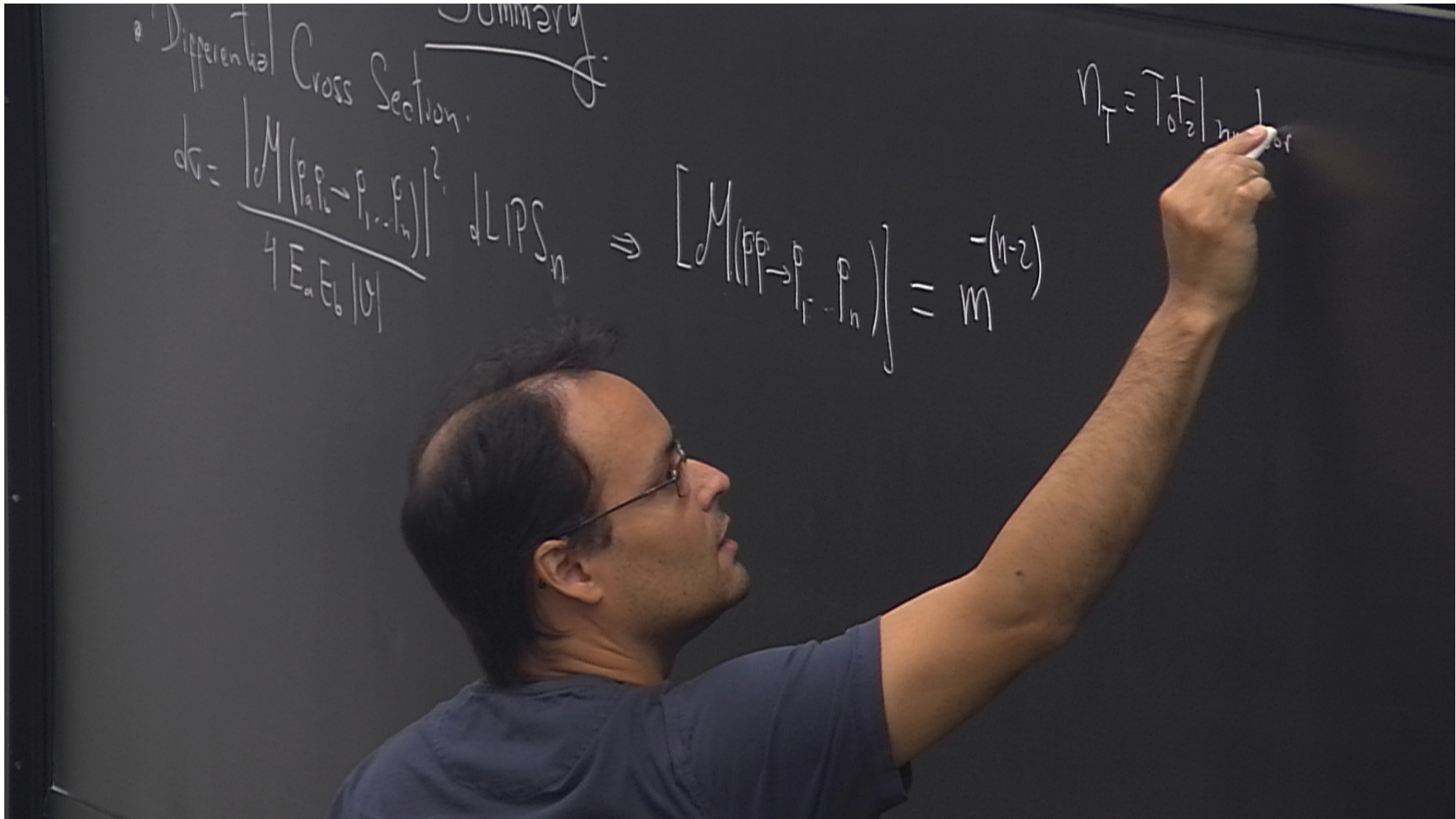
Abstract:

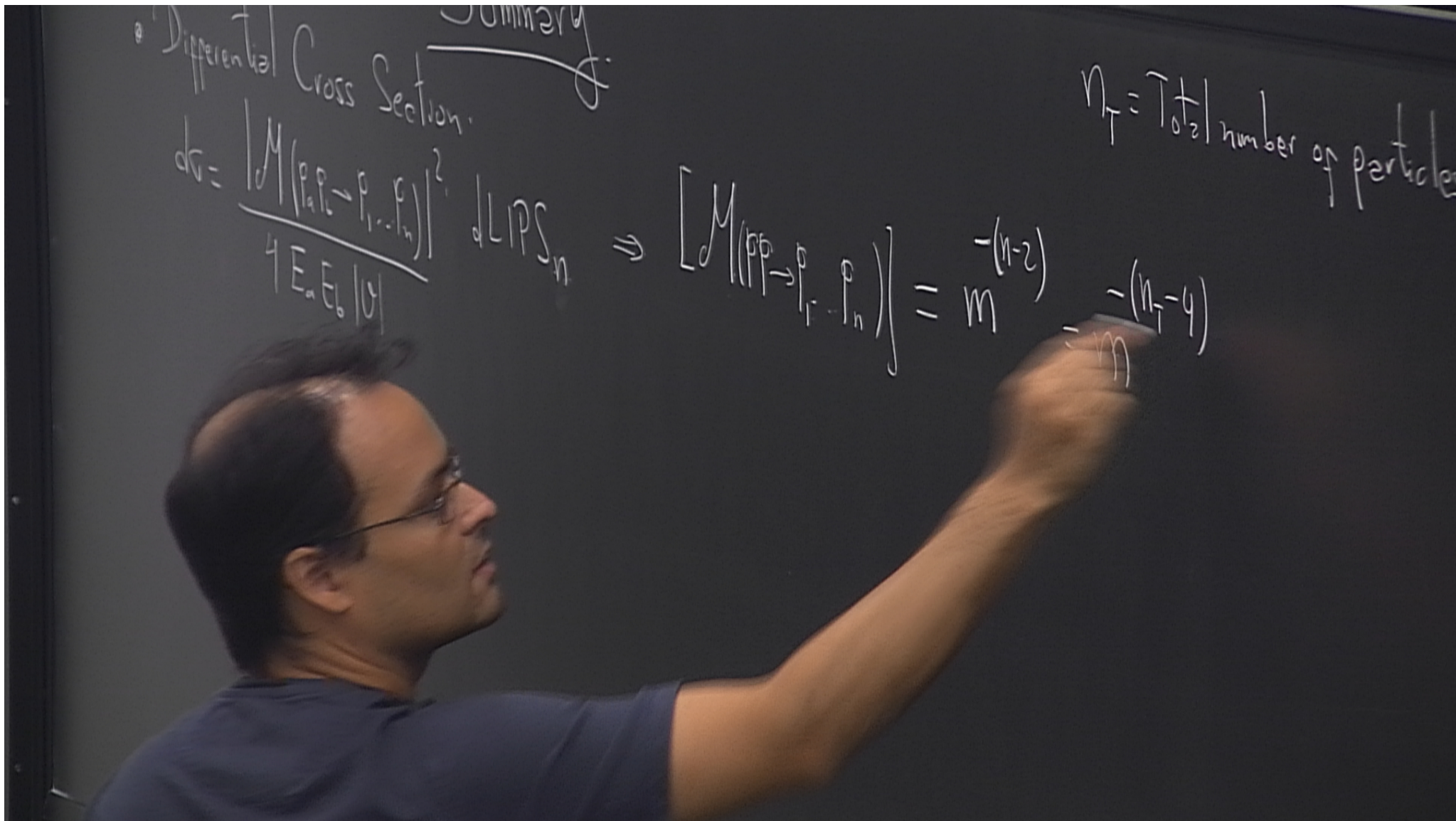


• Differential Cross S Summary.

$$[d\sigma] = m^{-2} \quad \left[\frac{1}{E_a E_b |\Theta|} \right] = m^{-2} \quad [dLIPS_n] = \underbrace{\left(\prod_{i=1}^n d^4 p_i \right)}_{m^{-4n}} S(p_i - m^2) \delta^4(\sum p - p_a - p_b)$$





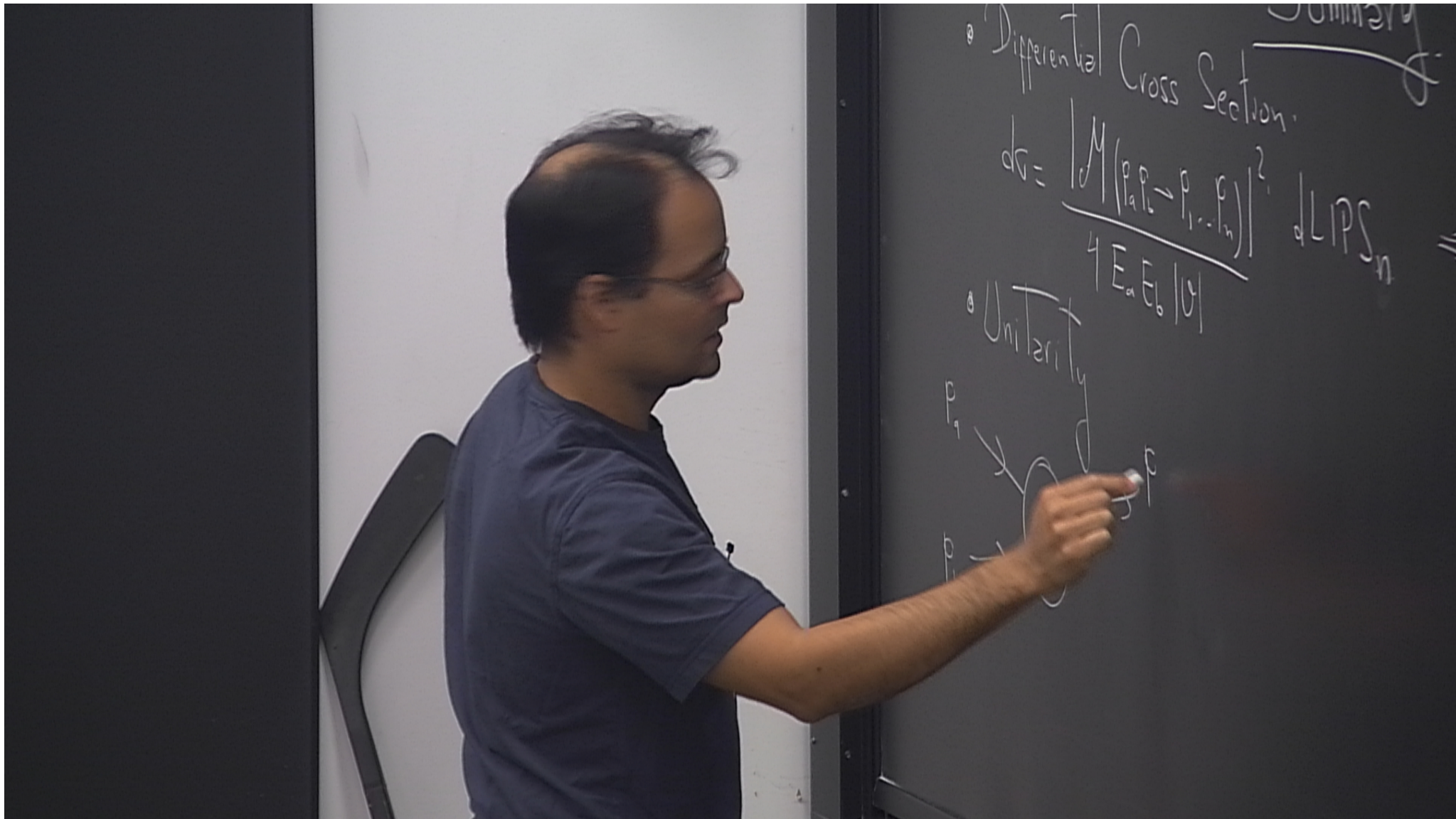


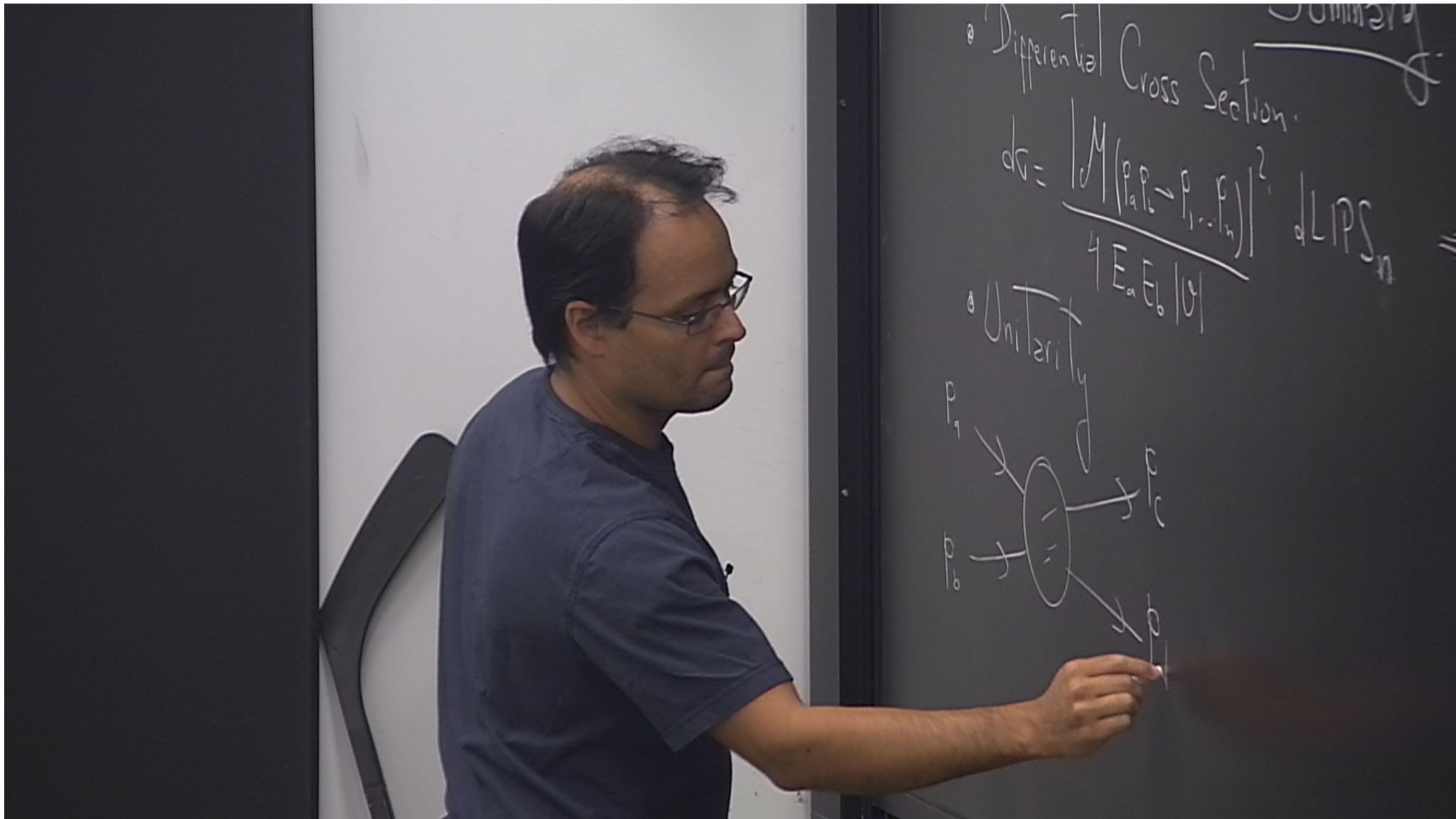
Summary

- Differential Cross Section.

$$d\sigma = \frac{|M(p_a p_b \rightarrow p_1 \dots p_n)|^2}{4 E_a E_b |v|} d\text{LIPS}_n \Rightarrow [M(p_a p_b \rightarrow p_1 \dots p_n)]^2 =$$

Unitarity



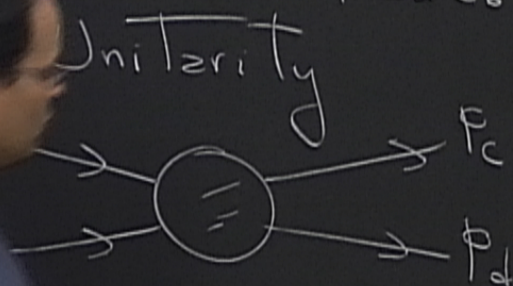


Summary

- Differential Cross Section.

$$d\sigma = \frac{|\mathcal{M}(p_a p_b \rightarrow p_1 \dots p_n)|^2}{4 E_a E_b |v|} d\text{LIPS}_n$$

$$\Rightarrow [\mathcal{M}(p_a p_b \rightarrow p_1 \dots p_n)] =$$



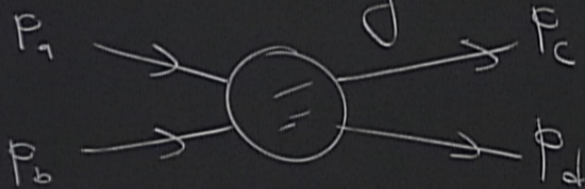
$$(p_a + p_b)^2 \rightarrow m^2$$

Summary

- Differential Cross Section.

$$d\sigma = \frac{|M(p_a p_b \rightarrow p_1 \dots p_n)|^2}{4 E_a E_b |v|} d\text{LIPS}_n \Rightarrow [M(p_a p_b \rightarrow p_1 \dots p_n)] =$$

- Unitarity



"Factorize"

$$\frac{1}{(p_a + p_b)^2 - m^2 + i\epsilon}$$

$$(p_a + p_b)^2 \rightarrow m^2$$

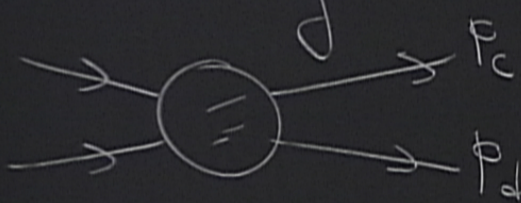
Summary

- Differential Cross Section.

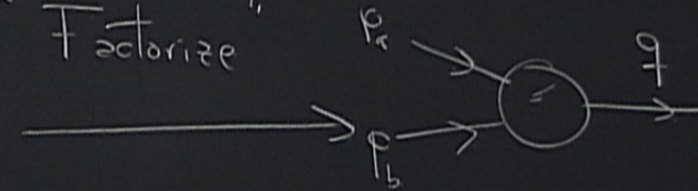
$$d\sigma = \frac{|\mathcal{M}(p_a p_b \rightarrow p_1 \dots p_n)|^2}{4 E_a E_b |v|} d\text{LIPS}_n$$

$$\Rightarrow [\mathcal{M}(p_a p_b \rightarrow p_1 \dots p_n)] = m^{-(n-2)} = m^{-1}$$

- Unitarity

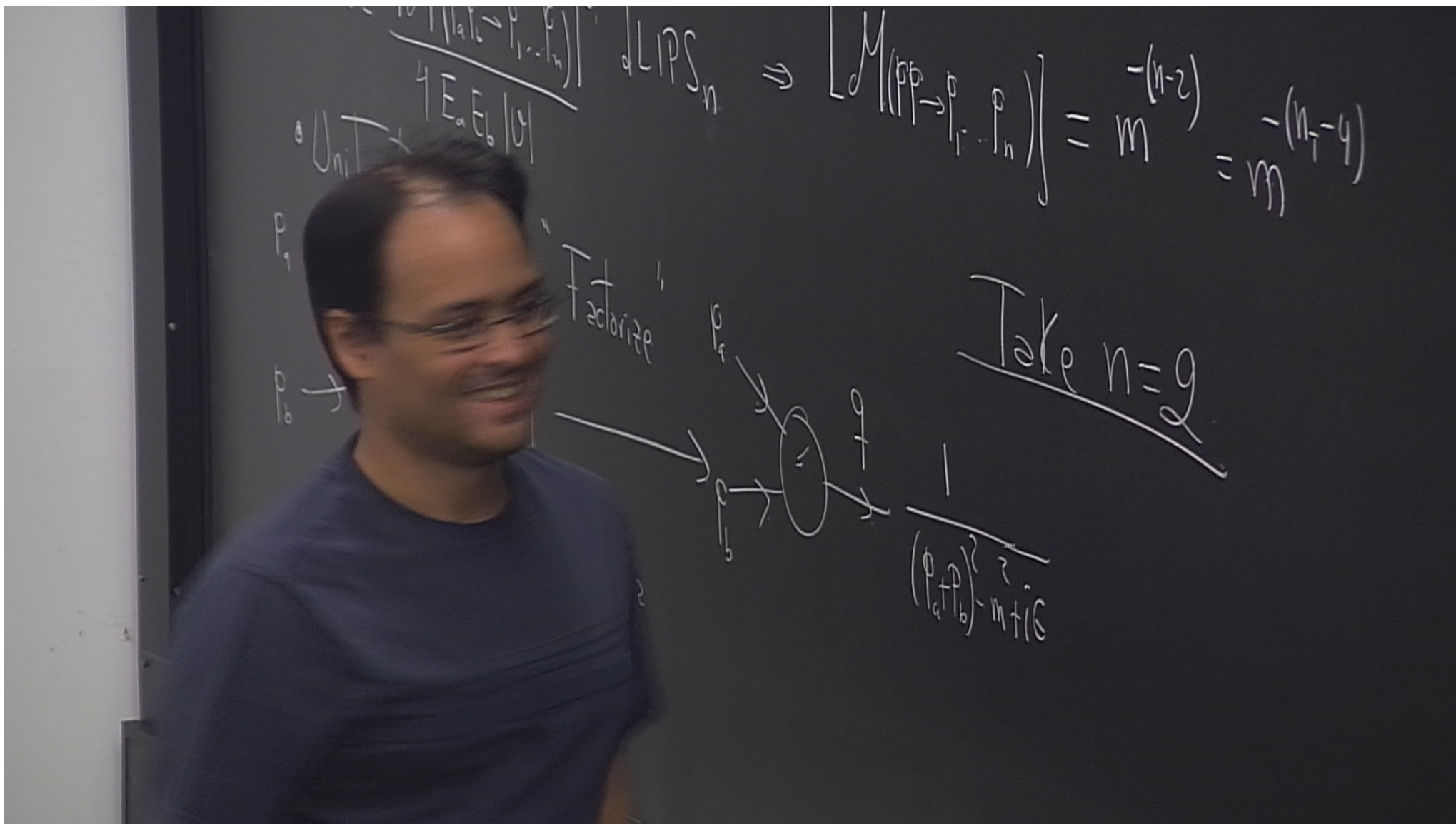


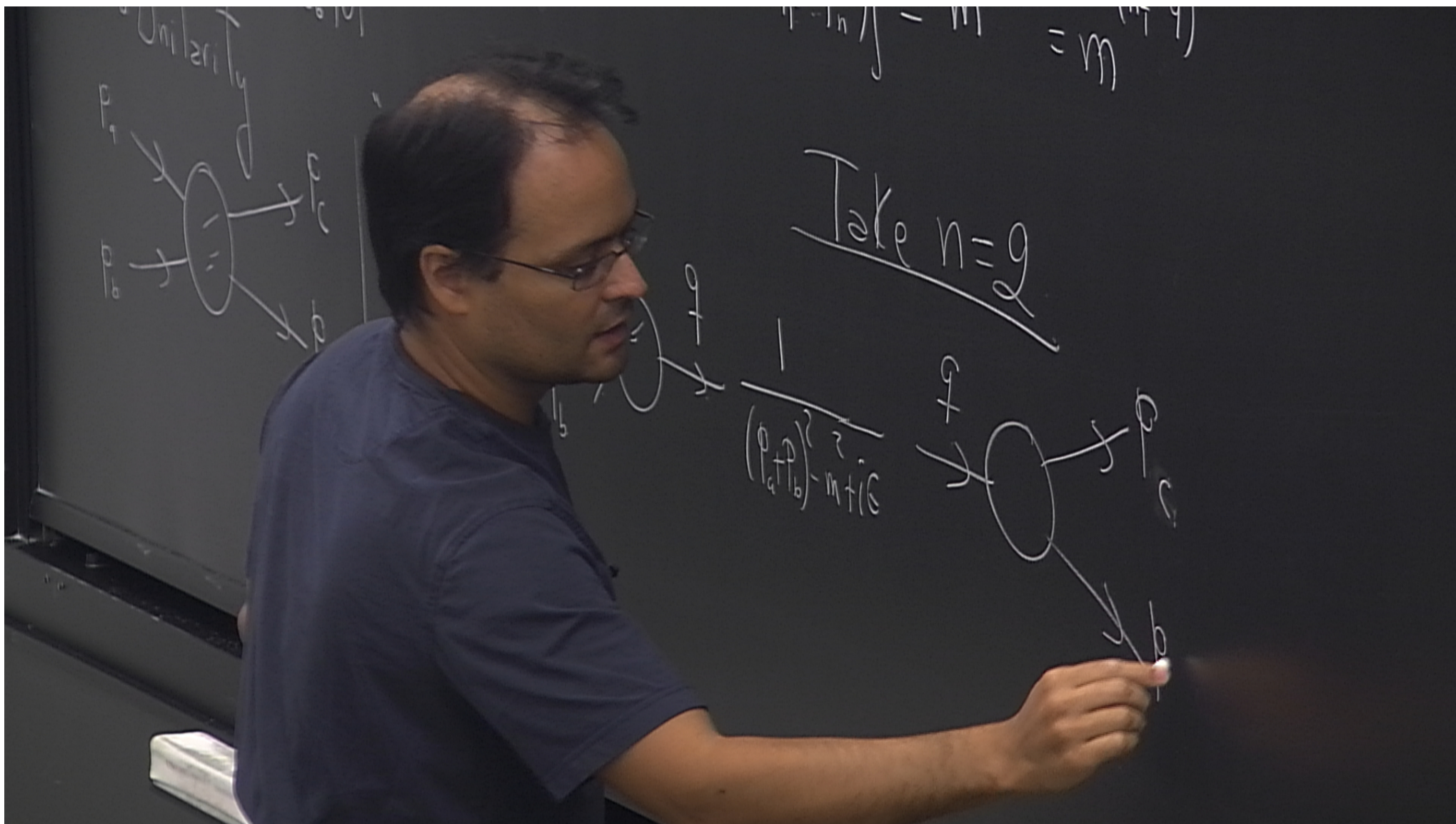
"Factorize"



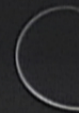
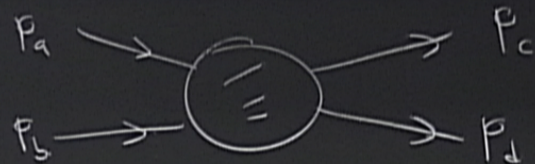
$$\frac{1}{(p_a + p_b)^2 - m^2 + i\epsilon}$$

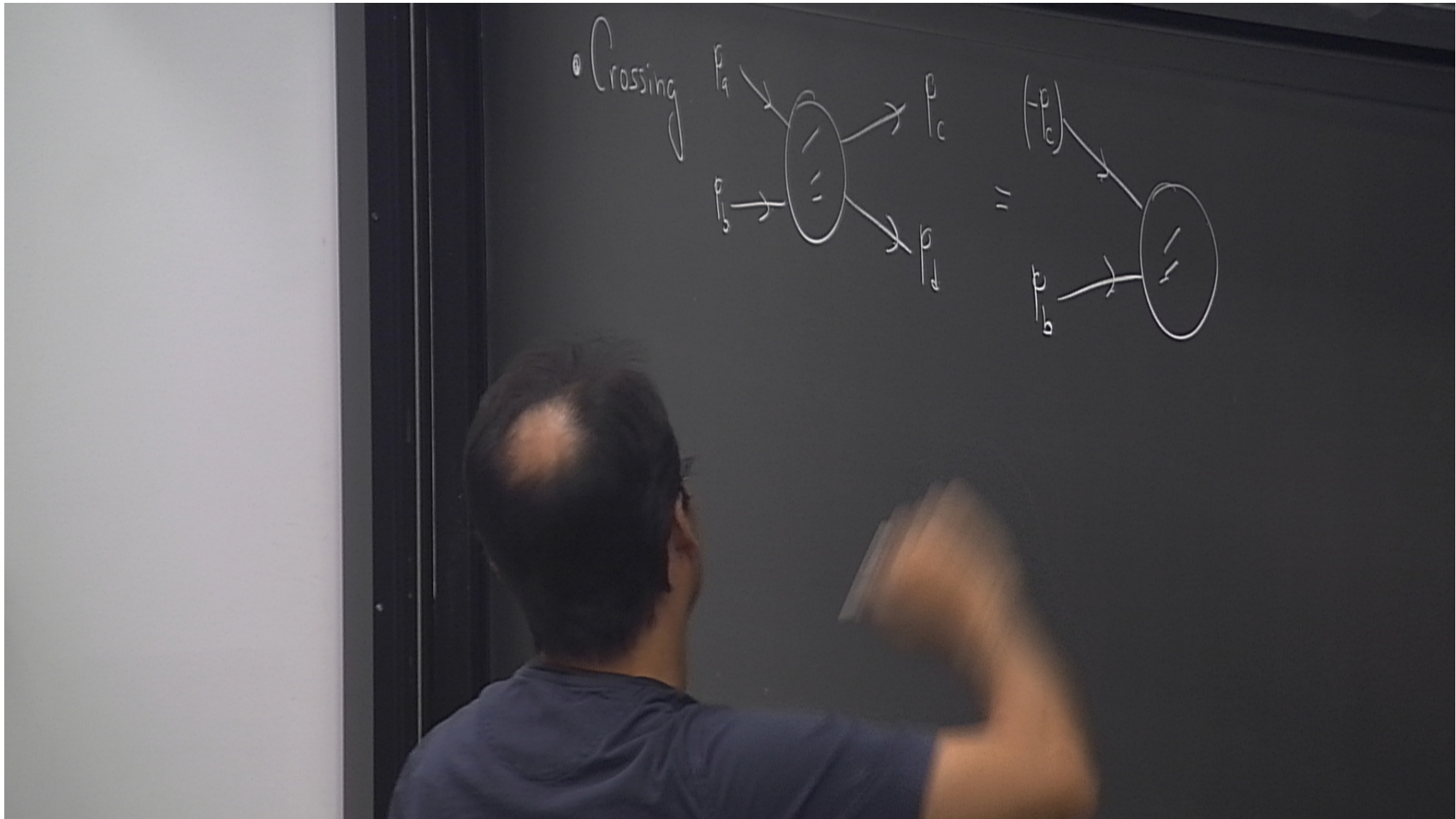
$$(p_a + p_b)^2 \rightarrow m^2$$

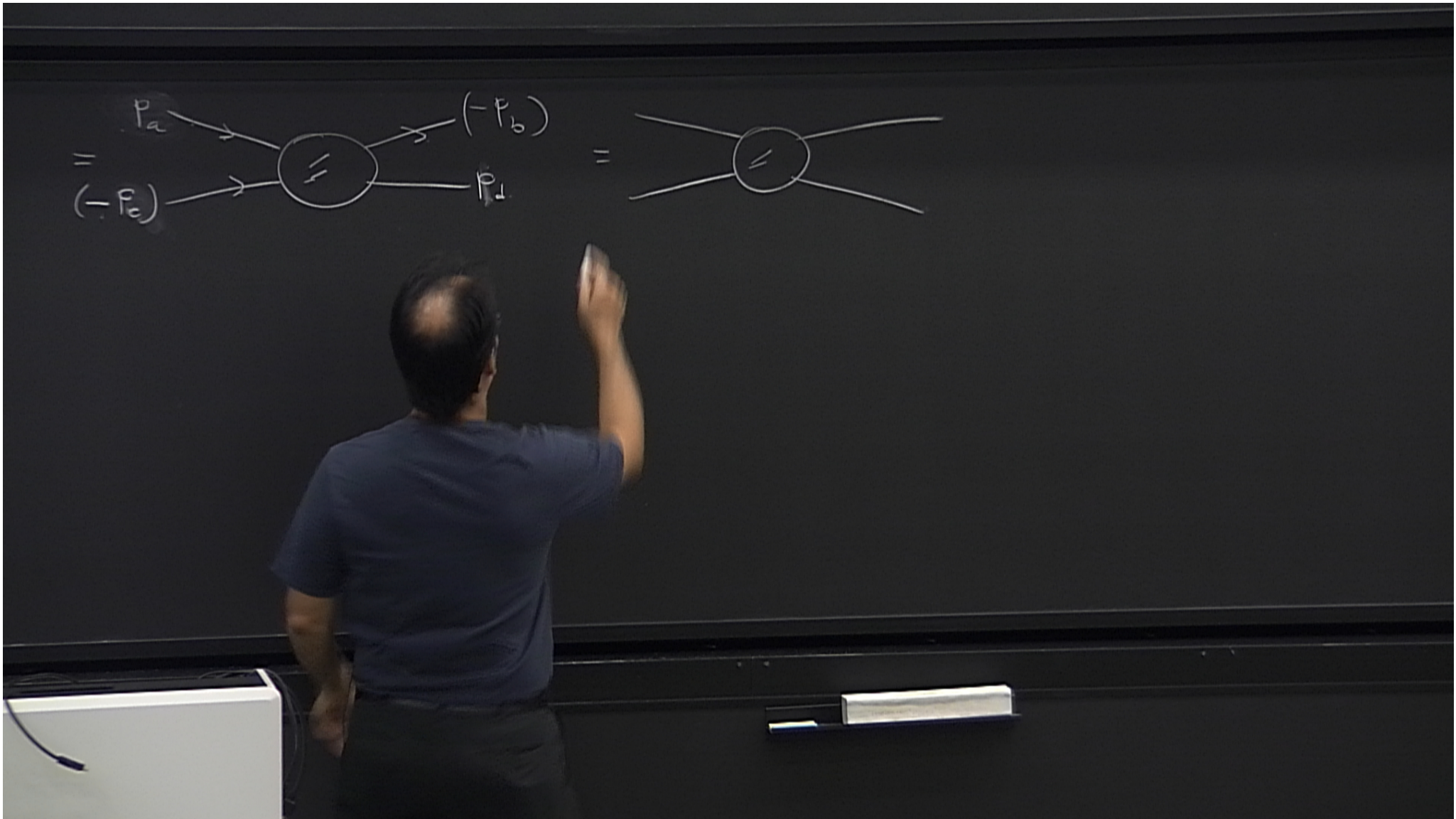


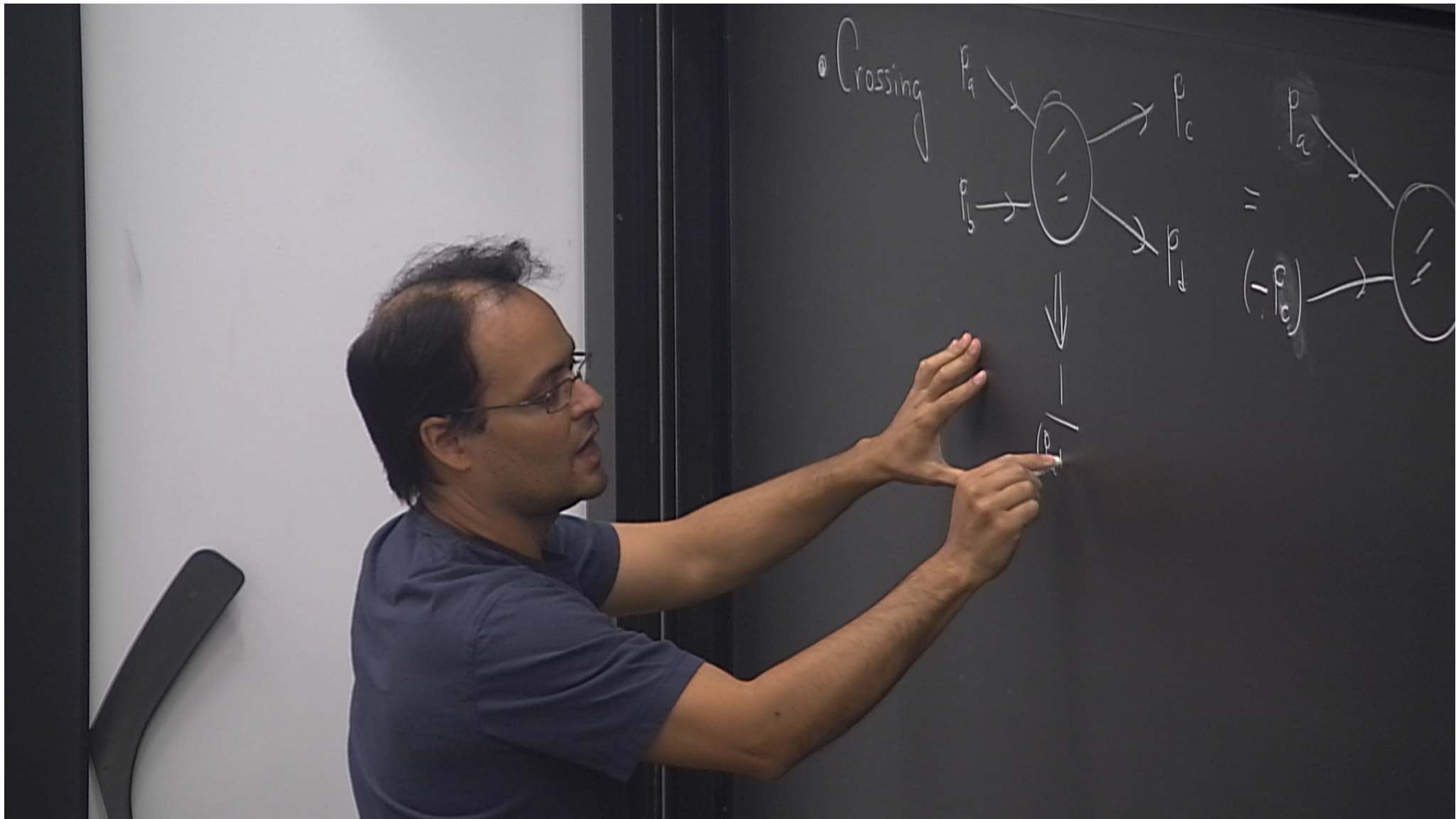


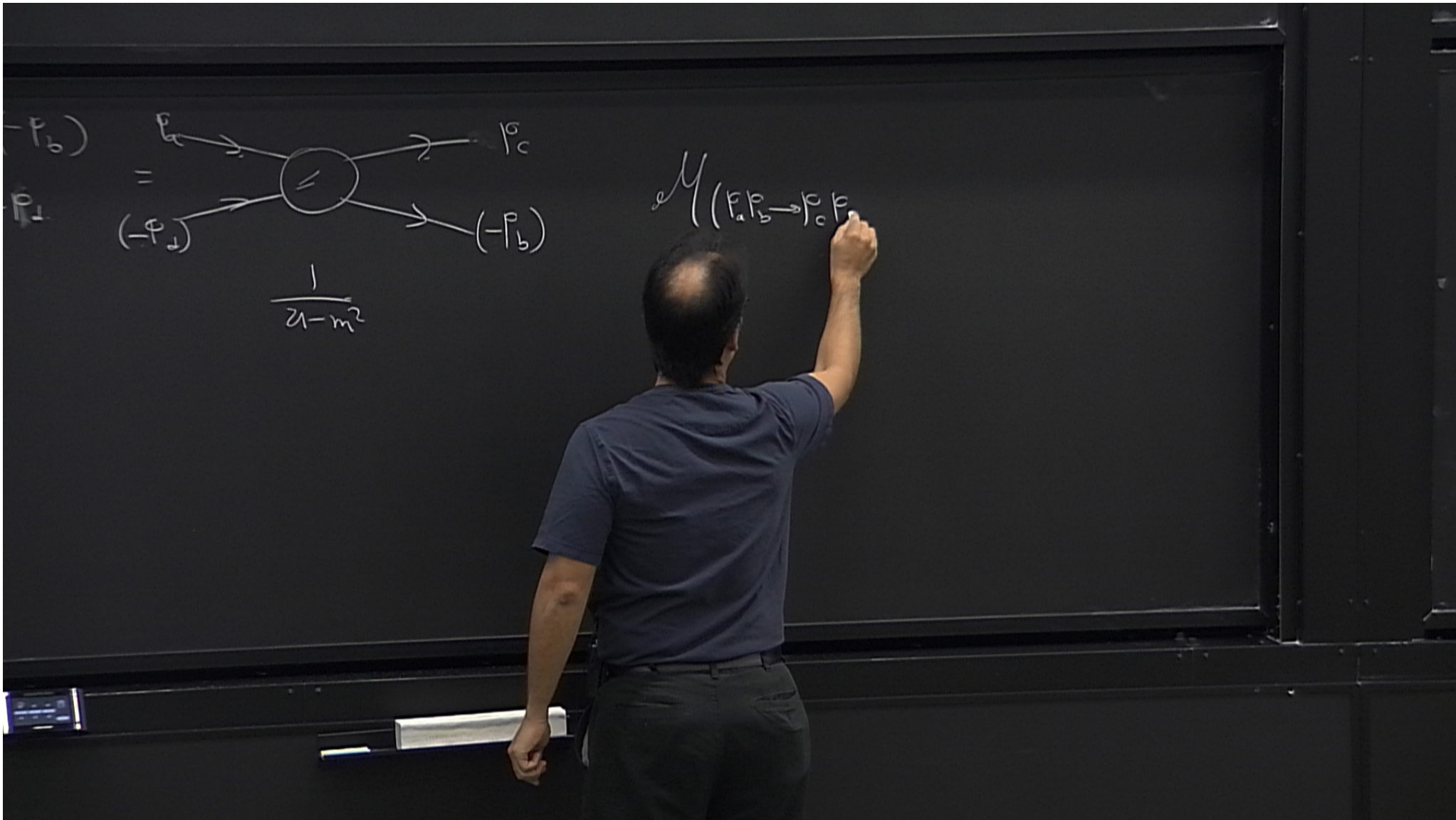
• Crossing

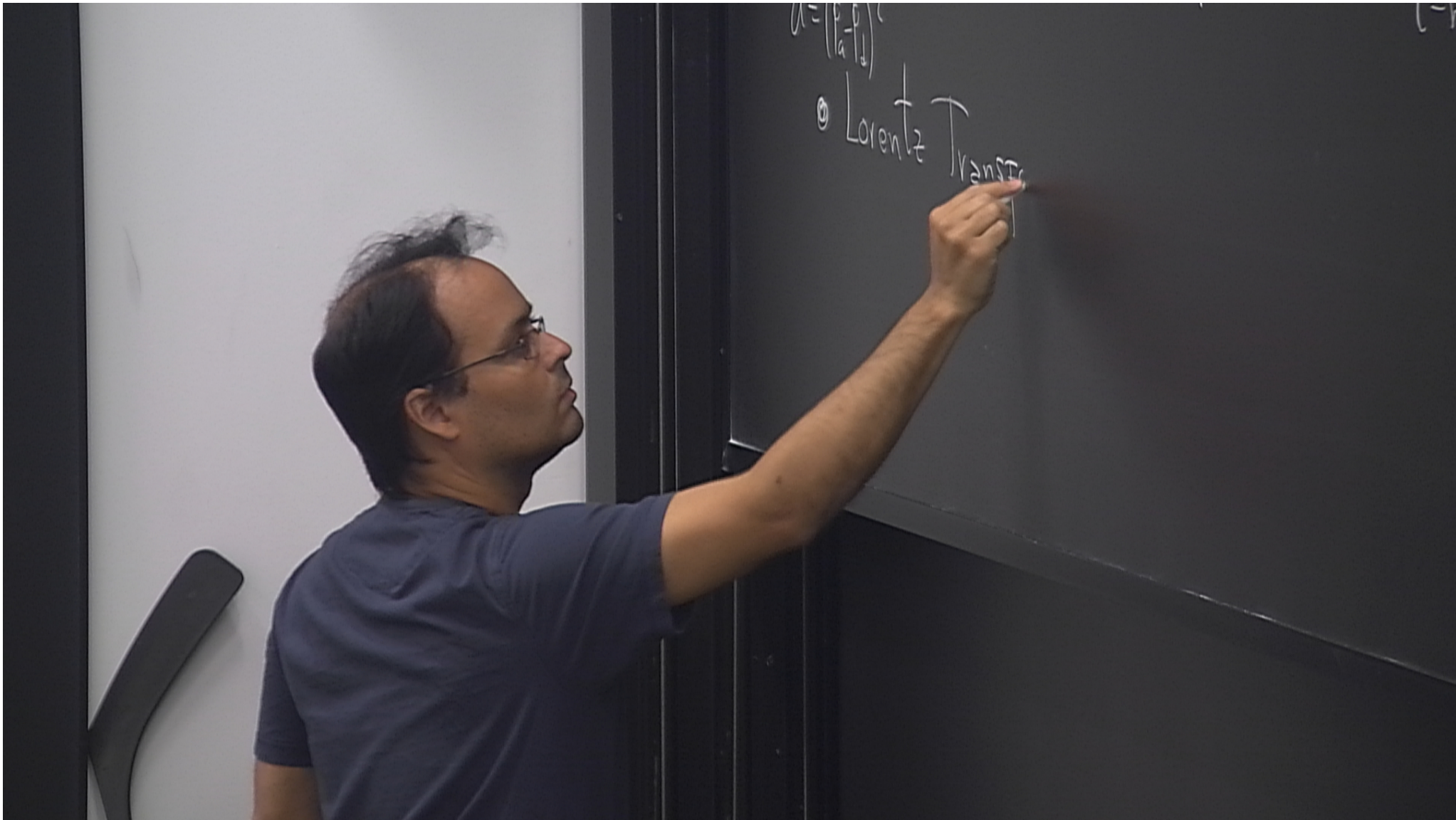












$$S = (p_a + p_b)$$

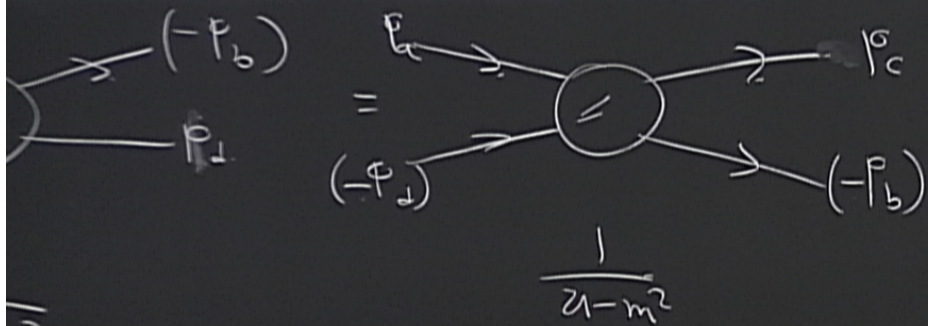
$$t = (p_a - p_c)^2$$

$$u = (p_a - p_d)^2$$

$$\frac{1}{(p_a + p_b)^2 - m^2} = \frac{1}{s - m^2}$$

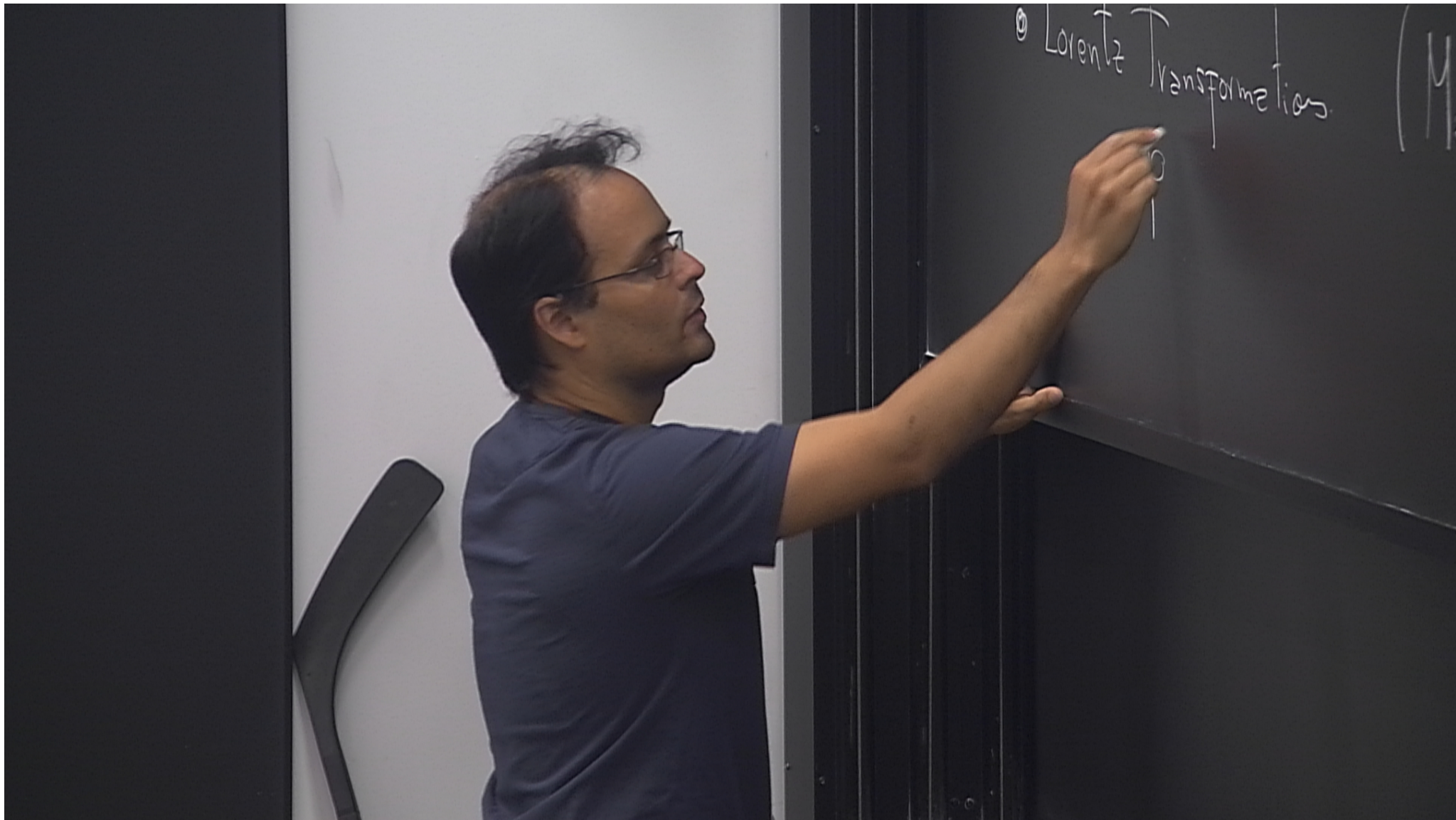
$$\frac{1}{t - m^2}$$

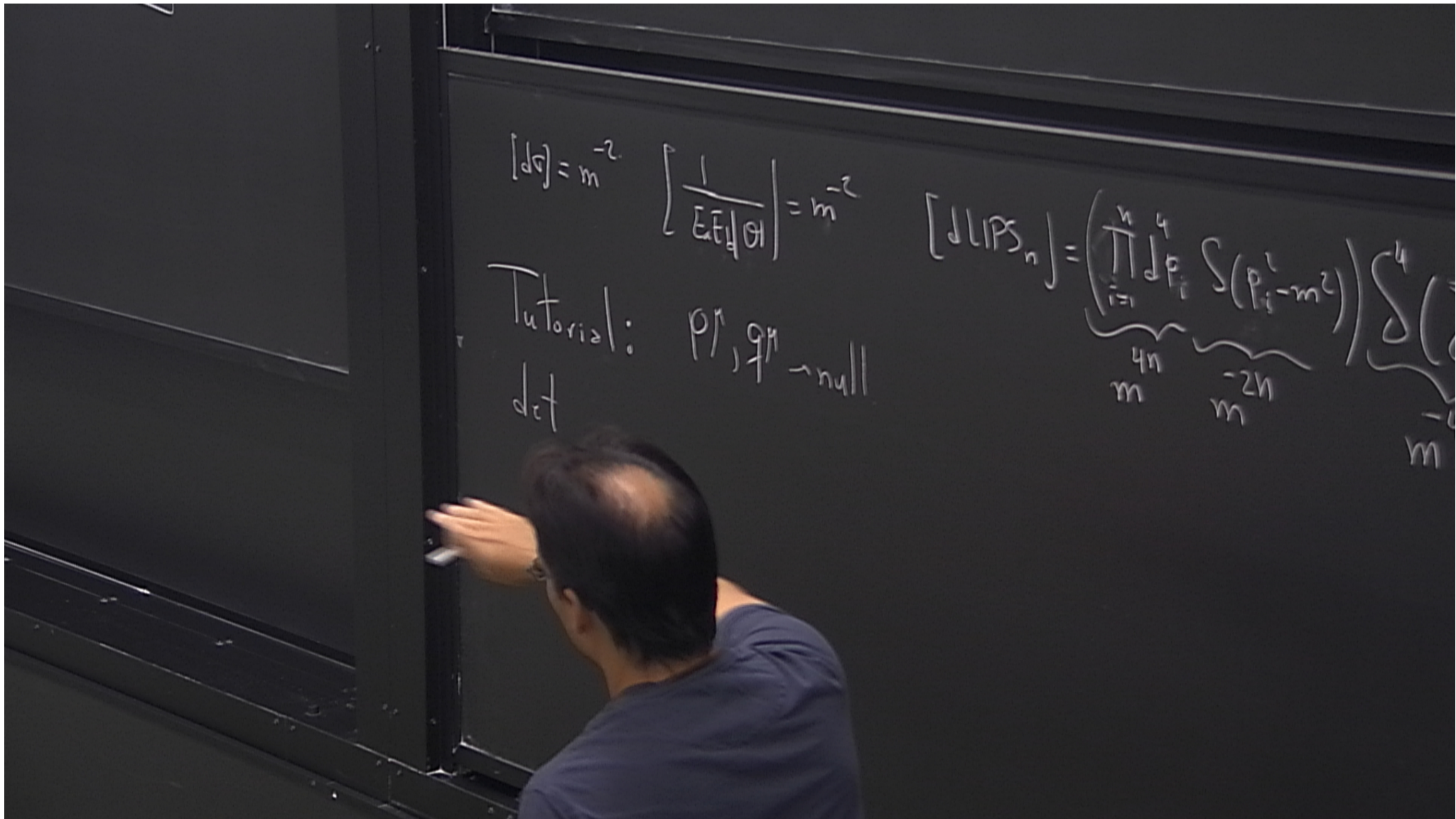
⊙ Lorentz Transformation (Massless)



$$\mathcal{M}(p_a p_b \rightarrow p_c p_d) = \frac{\mathcal{N}}{(s-m^2)(t-m^2)(u-m^2)}$$

$$p_a + p_b =$$





$$[d\sigma] = m^{-2} \quad \left[\frac{1}{E_F |\Theta|} \right] = m^{-2}$$

Tutorial: $p_i^\mu, q_i^\mu \sim \text{null}$
 $\det$

$$[\text{LIPS}_n] = \underbrace{\left(\prod_{i=1}^n \int \frac{d^4 p_i}{m^{4n}} \right)}_{m^{4n}} \underbrace{S(p_i^2 - m^2)}_{m^{-2n}} \underbrace{\delta(\dots)}_{m^{-1}}$$

$$[d\Omega] = m^{-2}$$

$$\left[\frac{1}{E_1 E_2 |\Theta|} \right] = m^{-2}$$

$$[d\text{LIPS}_n] = \underbrace{\left(\prod_{i=1}^n d^4 p_i \right)}_{m^{4n}} \underbrace{S(p_i - m^2)}_{m^{-2n}} \underbrace{\delta^4(\sum p - p_a - p_b)}_{m^{-4}} = m^{2n-4}$$

Tutorial 1: $p^\mu, q^\mu \rightarrow \text{null}$

$$\det \begin{pmatrix} \lambda_1^p & \lambda_1^q \\ \lambda_2^p & \lambda_2^q \end{pmatrix} =$$

$$[d\Omega] = m^{-2} \quad \left[\frac{1}{E_a E_b |\Theta|} \right] = m^{-2} \quad [d\text{LIPS}_n] = \underbrace{\left(\prod_{i=1}^n d^4 p_i \right)}_{m^{4n}} \underbrace{S(p_i^2 - m^2)}_{m^{-2n}} \underbrace{\delta\left(\sum p - p_a - p_b\right)}_{m^{-4}} = m^{2n-4}$$

Tutorial: $p^\mu, q^\mu \sim \text{null}$

$$\det \begin{pmatrix} \lambda_1^p & \lambda_1^q \\ \lambda_2^p & \lambda_2^q \end{pmatrix} = \langle p q \rangle = \lambda_\alpha^p \lambda_\beta^q \in \mathbb{C}$$

$$\frac{1}{t-m^2}$$

$$\frac{1}{2t-m^2}$$

$$p_a + p_b = p_c + p_d$$

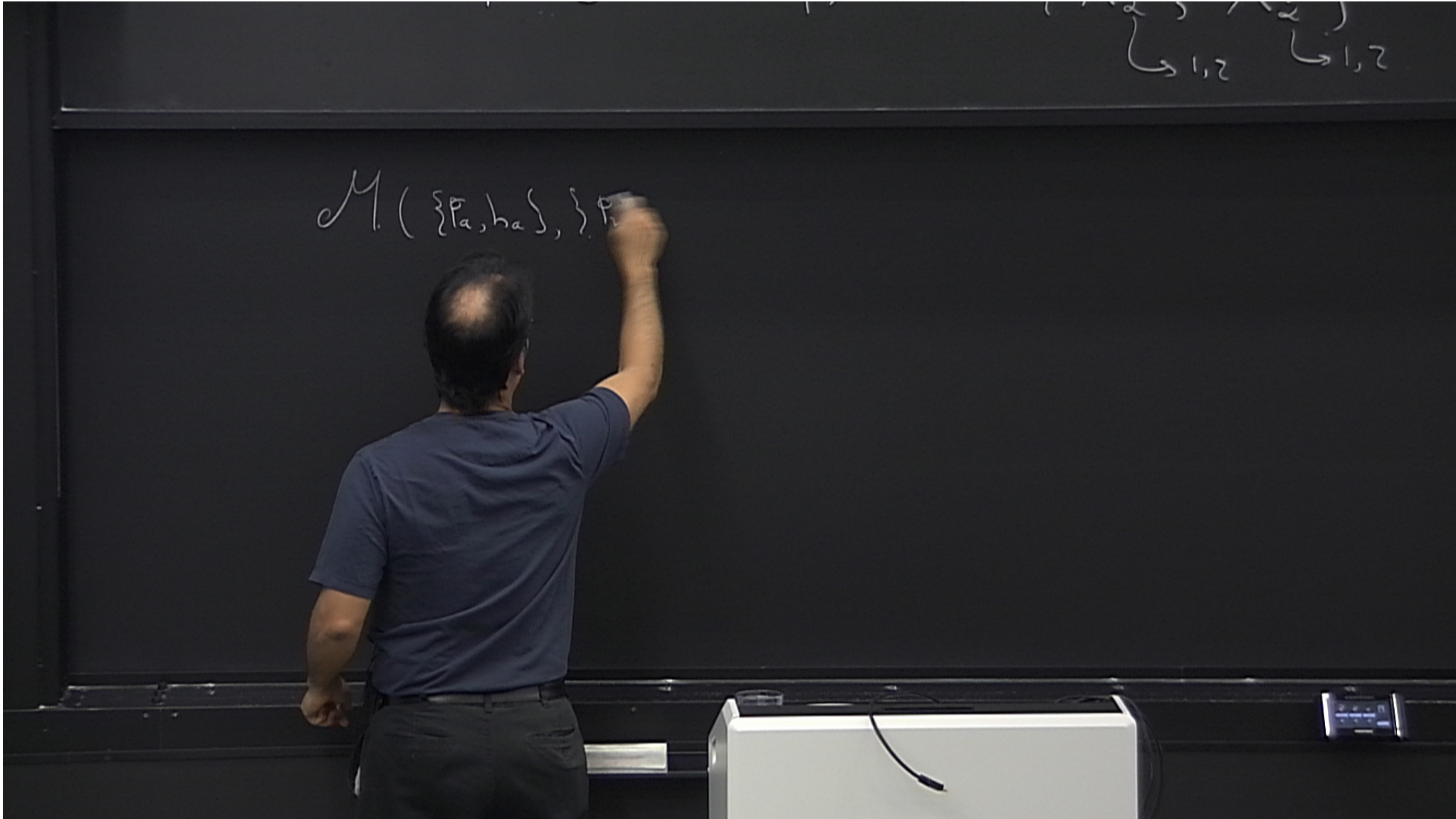
massless Particles)

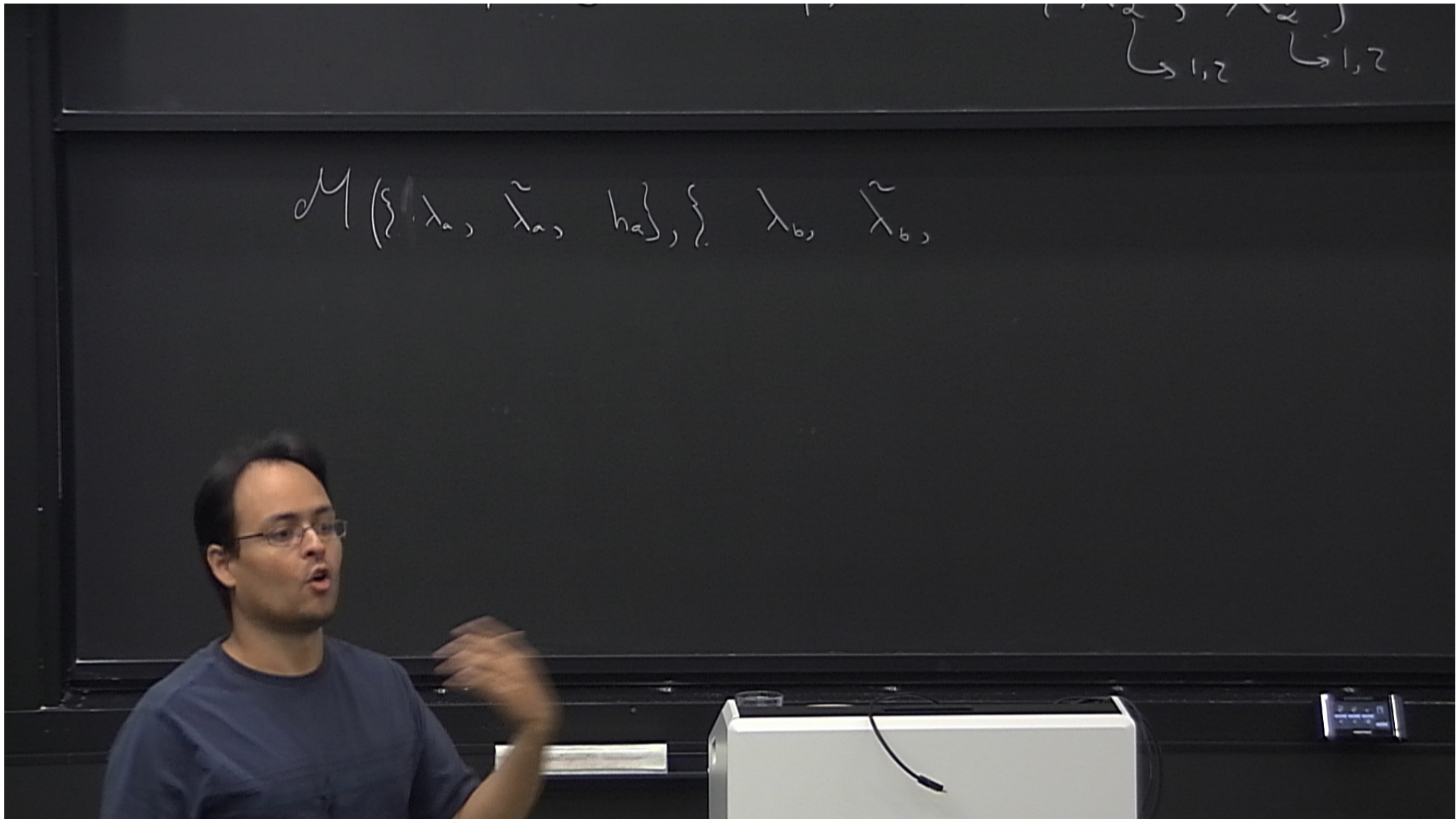
$$\text{If } p^\mu \text{ is real} \Rightarrow \lambda^{(p)} = \pm \tilde{\lambda}^{(p)*}$$

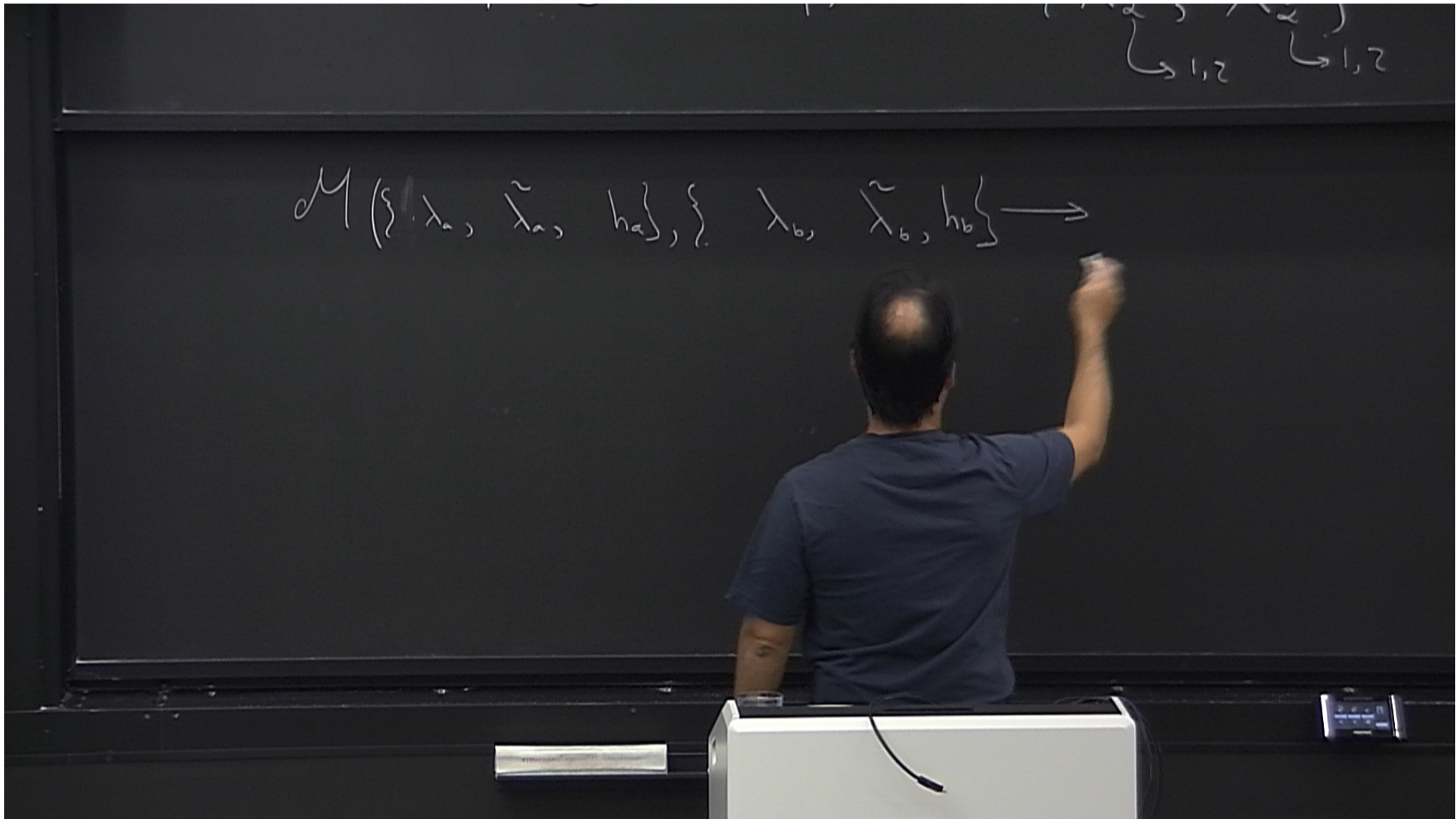
$$\left\{ \lambda_a^{(p)}, \tilde{\lambda}_i^{(p)} \right\}$$

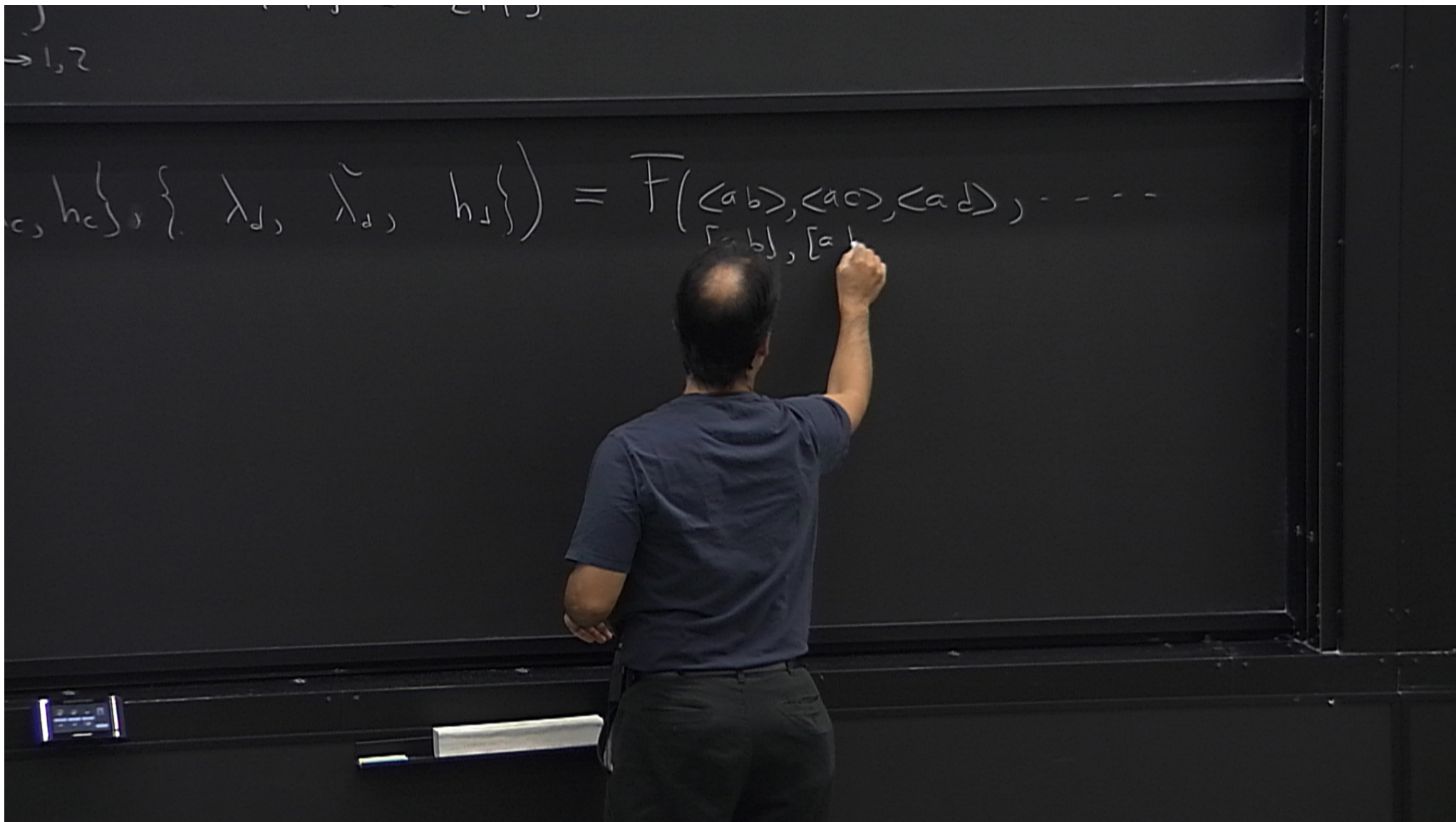
$\downarrow \quad \quad \downarrow$
 $1, 2 \quad \quad 1, 2$

$$\langle p q \rangle =$$

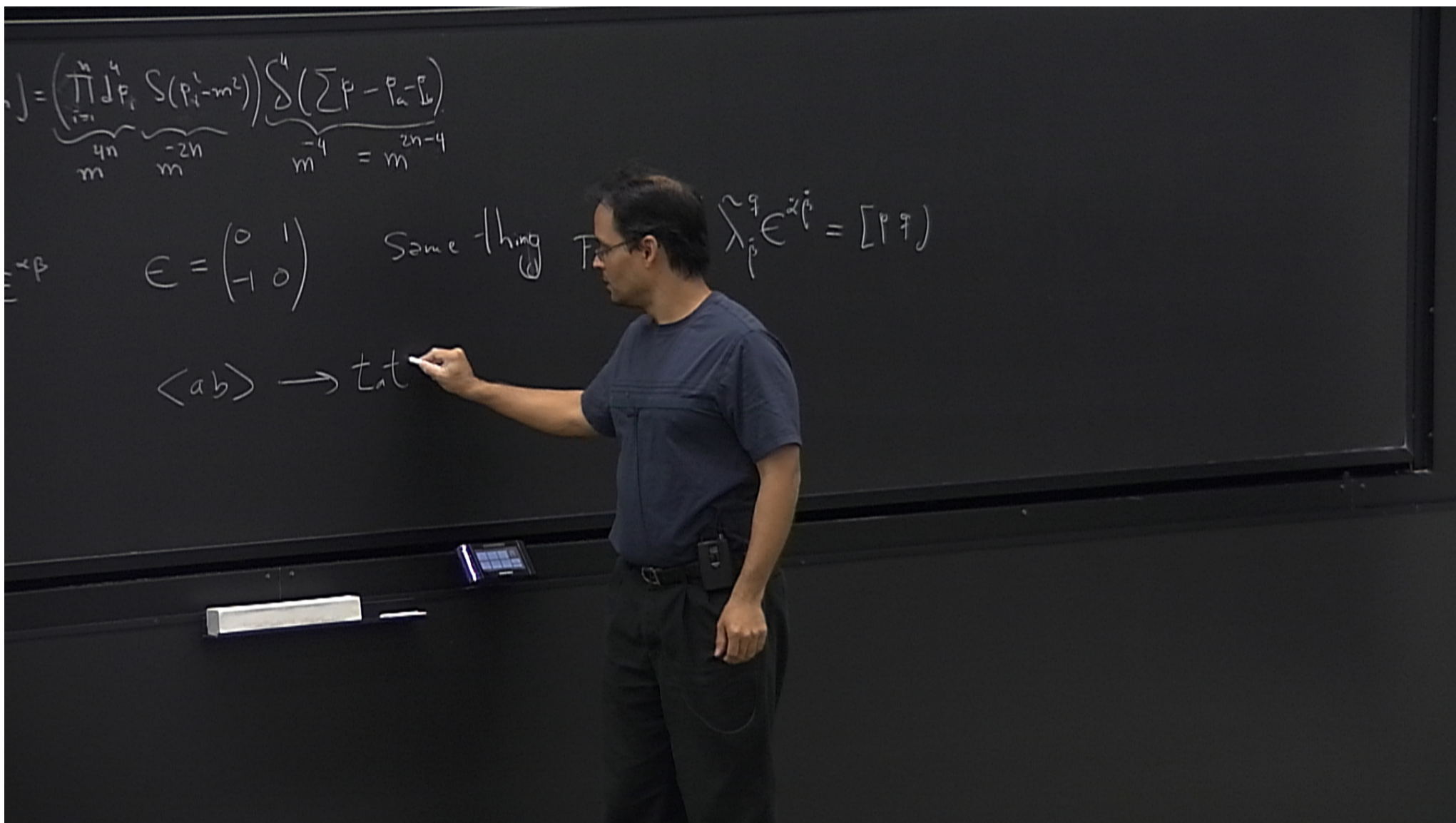








$$\mathcal{M}(\{t_a \lambda_a, t_a^{-1} \tilde{\lambda}_a, h_a\}, \{t_b \lambda_b, t_b^{-1} \tilde{\lambda}_b, h_b\}) \longrightarrow \{t_c \lambda_c, t_c^{-1} \tilde{\lambda}_c, h_c\}, \{$$



$$J = \left(\prod_{i=1}^n \underbrace{d_{p_i}}_{4n} \underbrace{S(p_i - m^2)}_{-2n} \right) \underbrace{\delta(\sum p - p_a - p_b)}_{m^4 = m^{2n-4}}$$

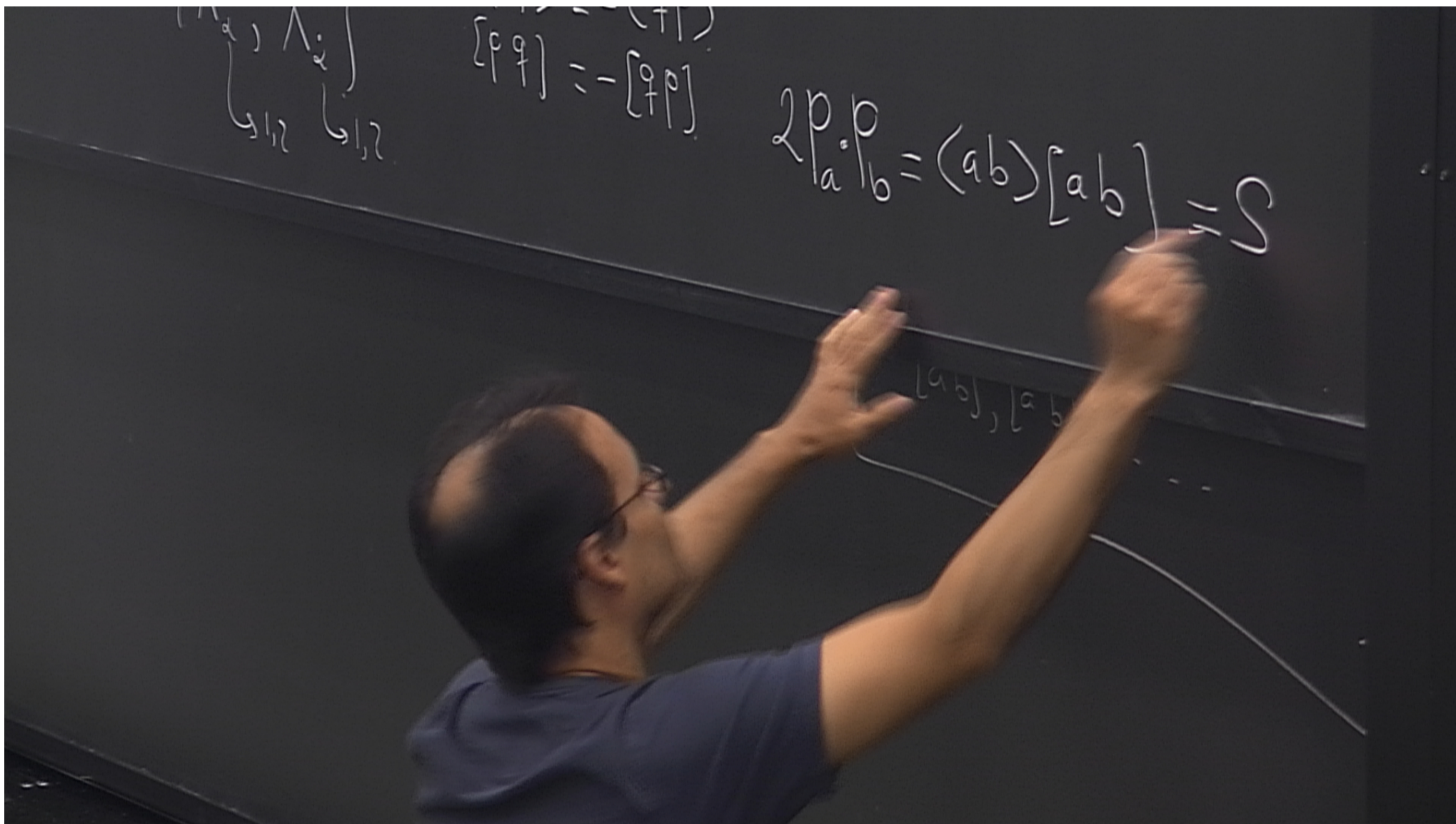
$$E = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

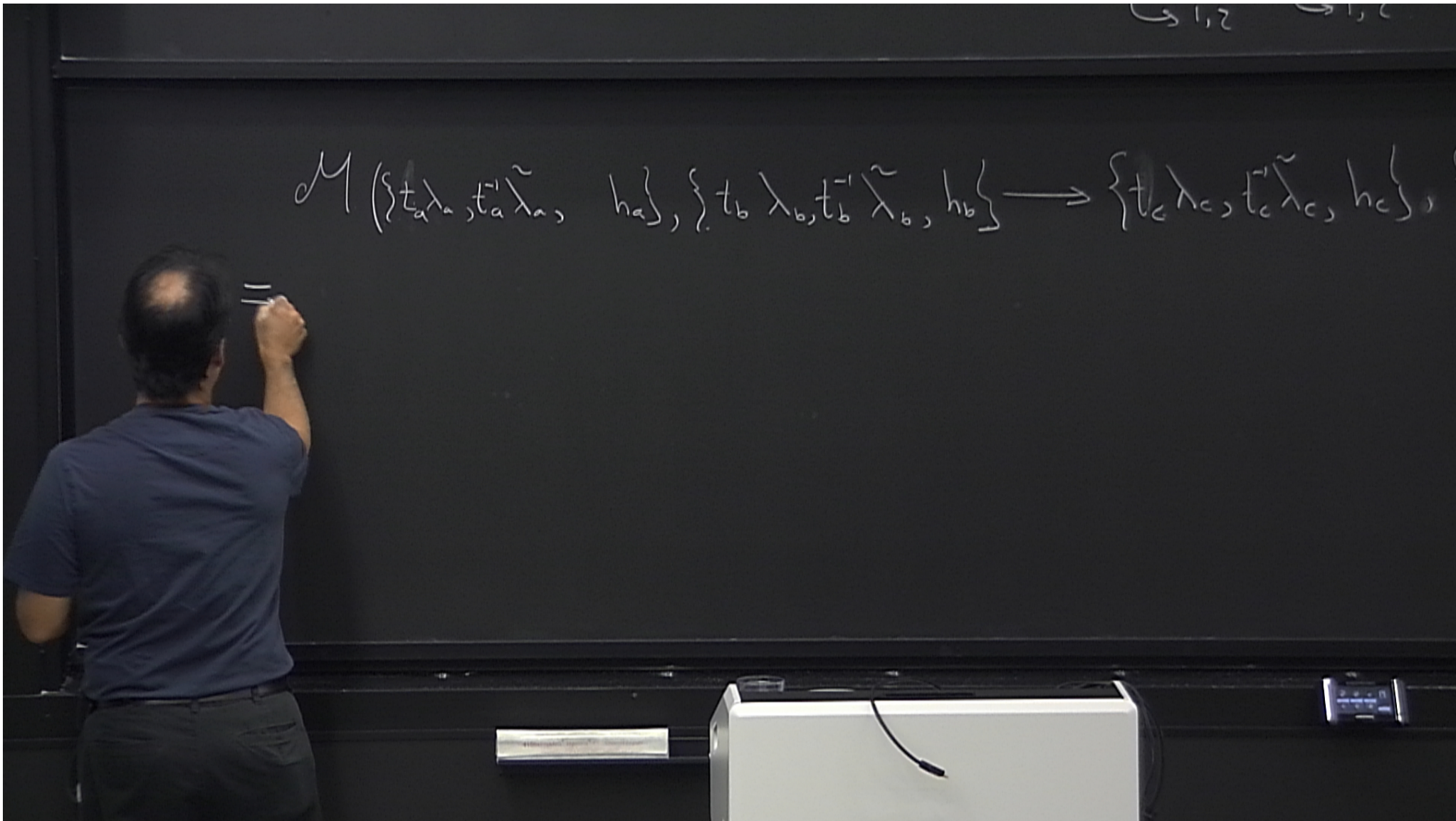
Some thing For $\sum_a^p \sum_b^q E^{ab} = [p, q]$

$$\langle ab \rangle \rightarrow t_a t_b \langle ab \rangle$$

$$[ab] \rightarrow t_a^{-1} t_b^{-1} [ab]$$

$$\langle ab \rangle [ab] \rightarrow \langle ab \rangle [ab]$$

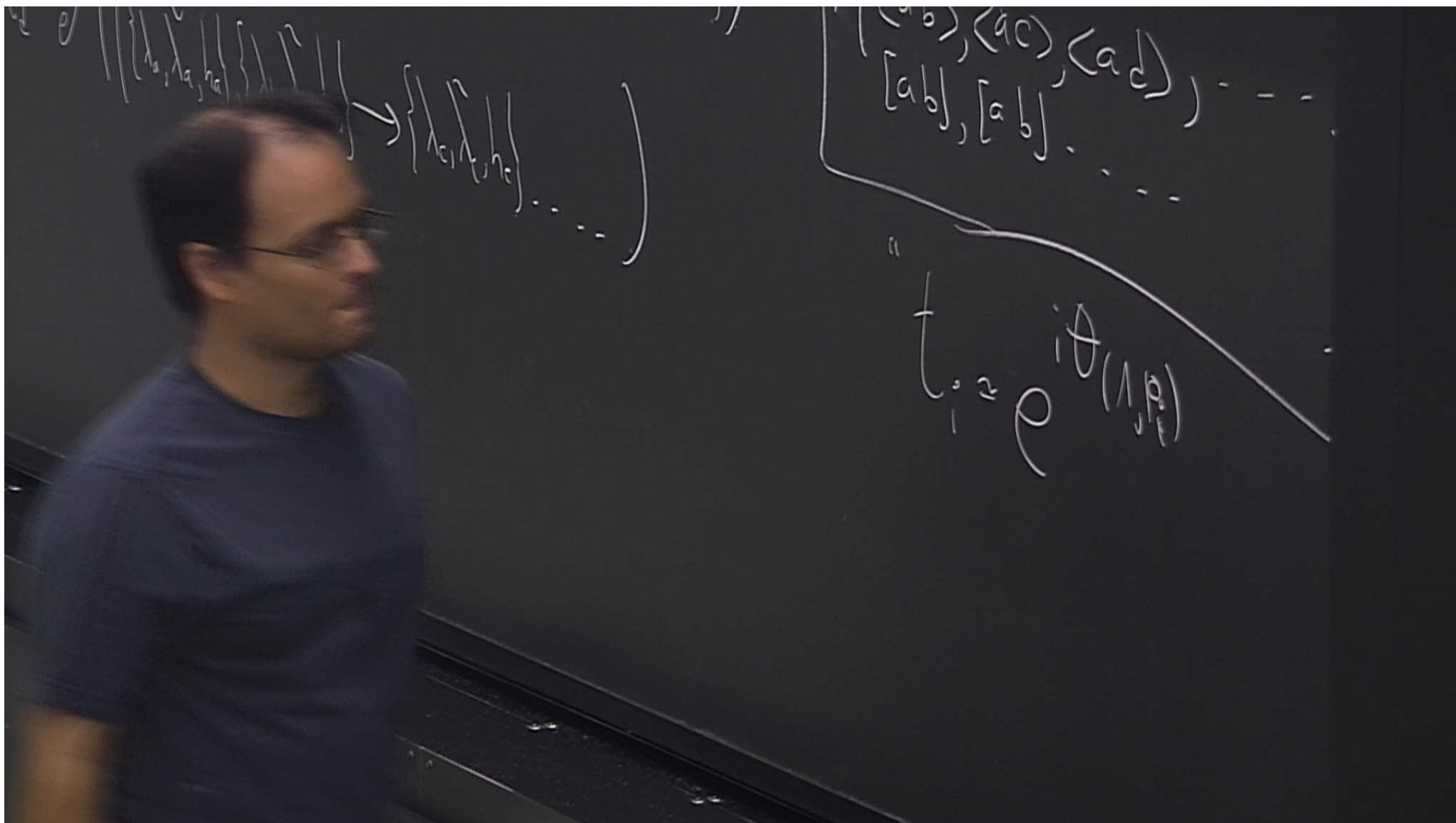




$$\mathcal{M}(\{t_a \lambda_a, t_a^{-1} \tilde{\lambda}_a, h_a\}, \{t_b \lambda_b, t_b^{-1} \tilde{\lambda}_b, h_b\}) \rightarrow \{t_c \lambda_c, t_c^{-1} \tilde{\lambda}_c, h_c\},$$

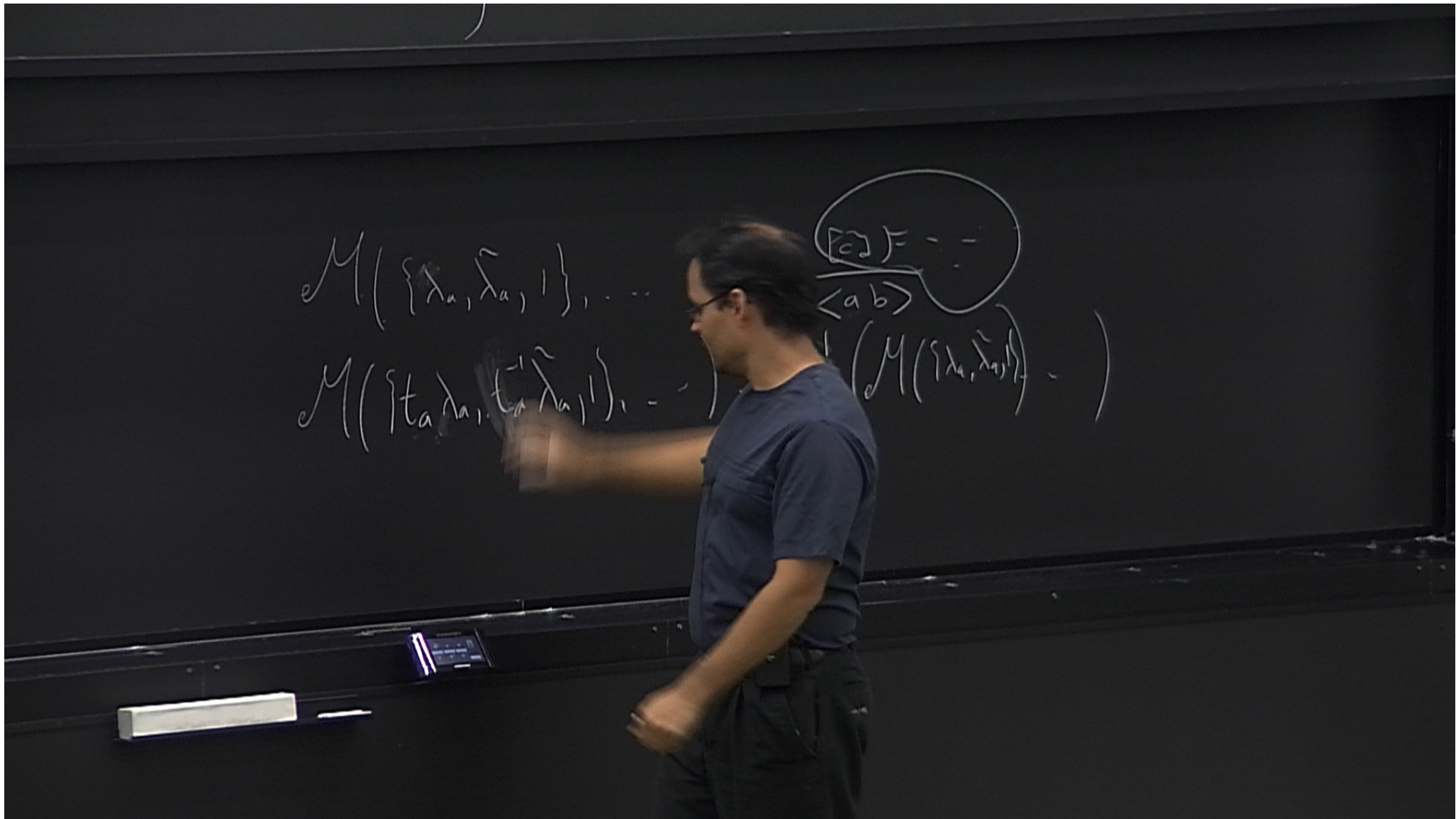
$$= t_a^{2h_a} t_c^{-2h_b} t_c^{-2h_c} t_d^{-2h_d}$$

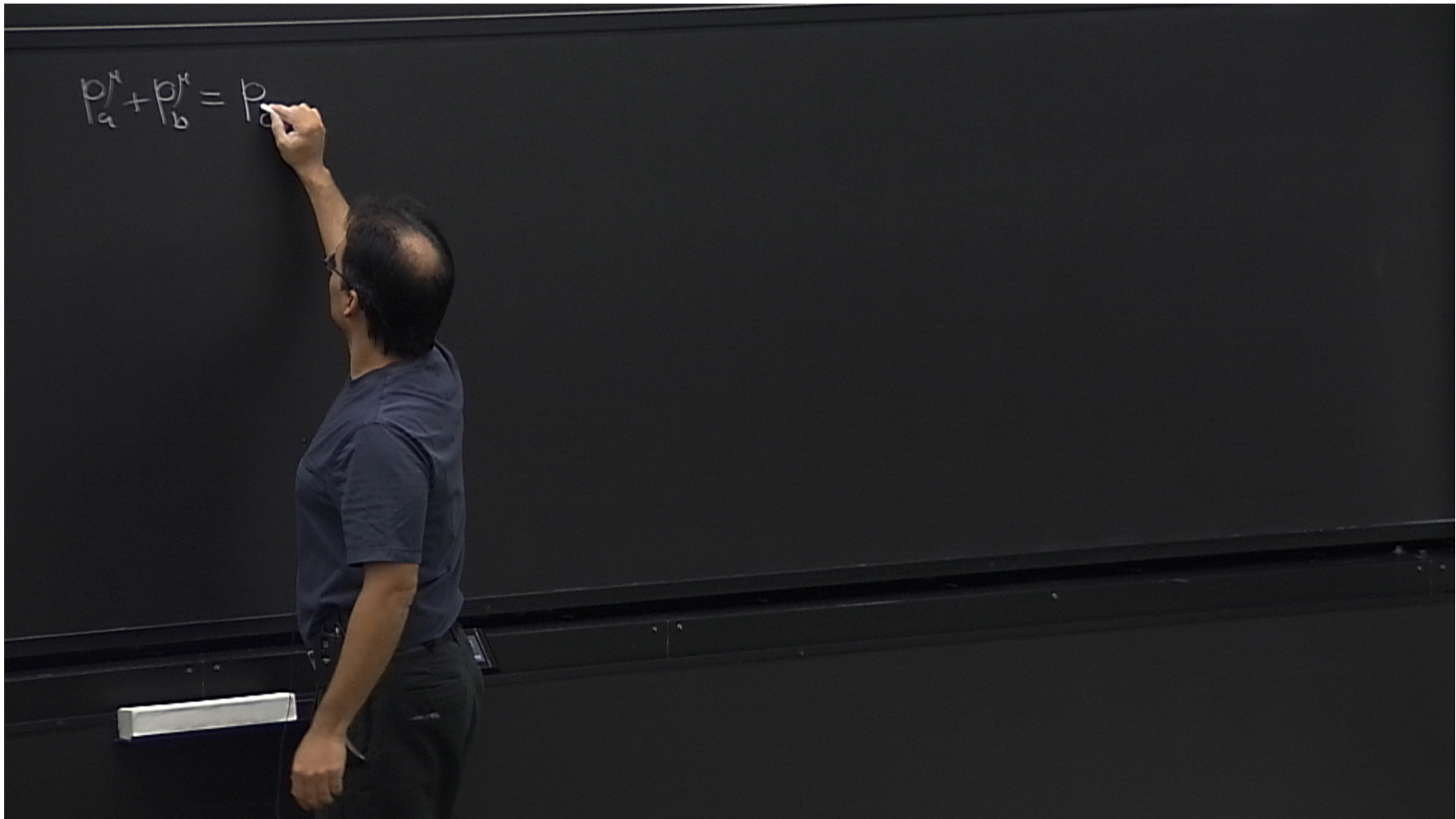
$$\begin{aligned}
 & \mathcal{M}(\{t_a \lambda_a, t_a^{-1} \tilde{\lambda}_a, h_a\}, \{t_b \lambda_b, t_b^{-1} \tilde{\lambda}_b, h_b\}) \rightarrow \{t_c \lambda_c, t_c^{-1} \tilde{\lambda}_c, h_c\}, \\
 & = t_a^{2h_a} t_b^{2h_b} t_c^{-2h_c} t_d^{-2h_d} e^{\mathcal{M}(\{\lambda_a\}, \{\lambda_b\})}
 \end{aligned}$$



$$\mathcal{M}(\{\lambda_a, \tilde{\lambda}_a, 1\}, \dots) = \frac{[c]}{\langle ab \rangle} \text{---}$$

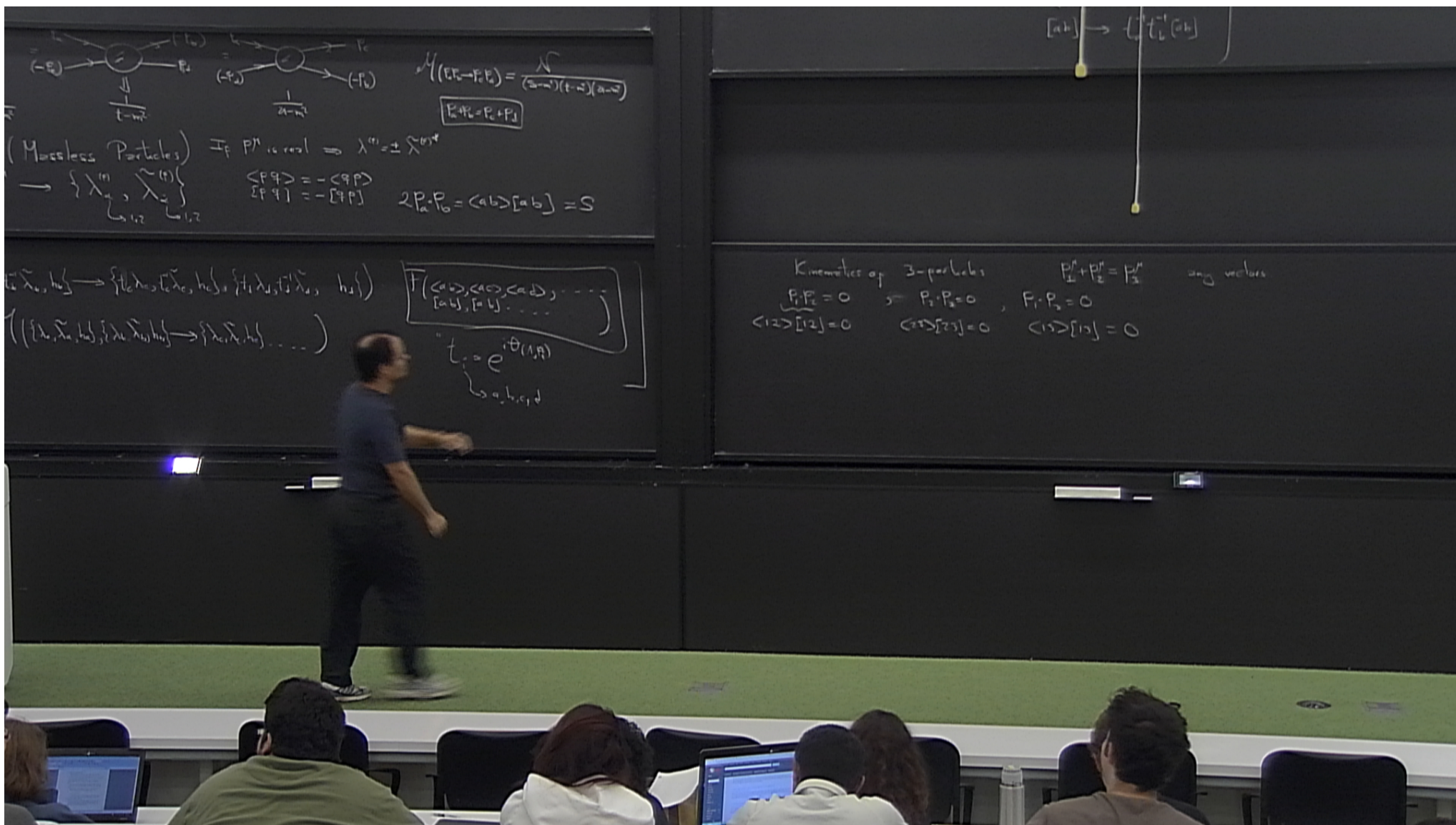
$$\mathcal{M}(\{t_a \lambda_a, t_a^{-1} \tilde{\lambda}_a, 1\}, \dots) =$$





$$p_1^M + p_2^M = p_3^M$$

any vectors.



any vectors.

Cplx moment₂.



any vectors.

Cplx moments.

$\Rightarrow$

$\lambda \neq$

λ^*

$\downarrow$ not necessarily

Free do

Kinematics of 3-particles

$$p_1^\mu + p_2^\mu = p_3^\mu$$

any vect

$$p_1 \cdot p_2 = 0$$

$$p_2 \cdot p_3 = 0, \quad p_1 \cdot p_3 = 0$$

$$\langle 12 \rangle [12] = 0$$

$$\langle 23 \rangle [23] = 0$$

$$\langle 13 \rangle [13] = 0$$

$$\langle 12 \rangle = 0$$

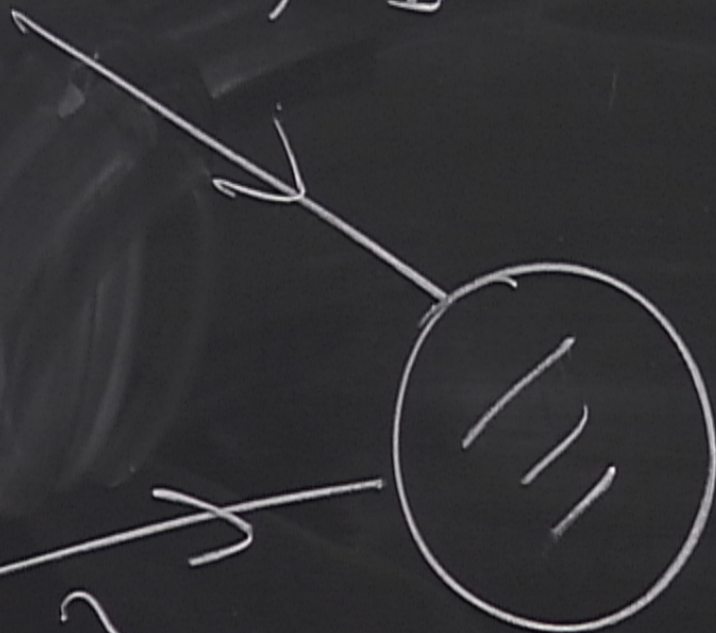
$$\langle 23 \rangle = 0$$

or

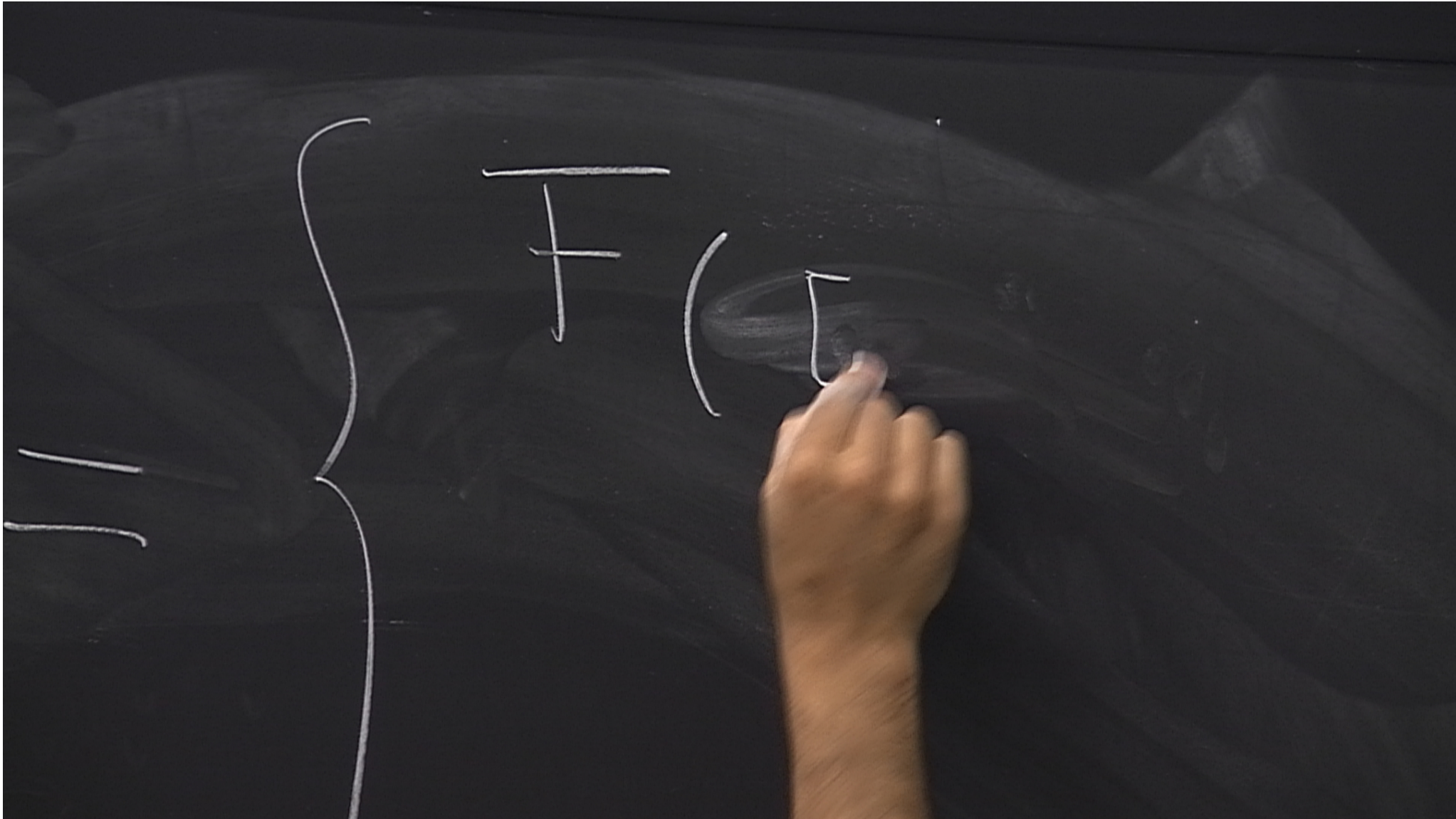
$$[12] = 0$$

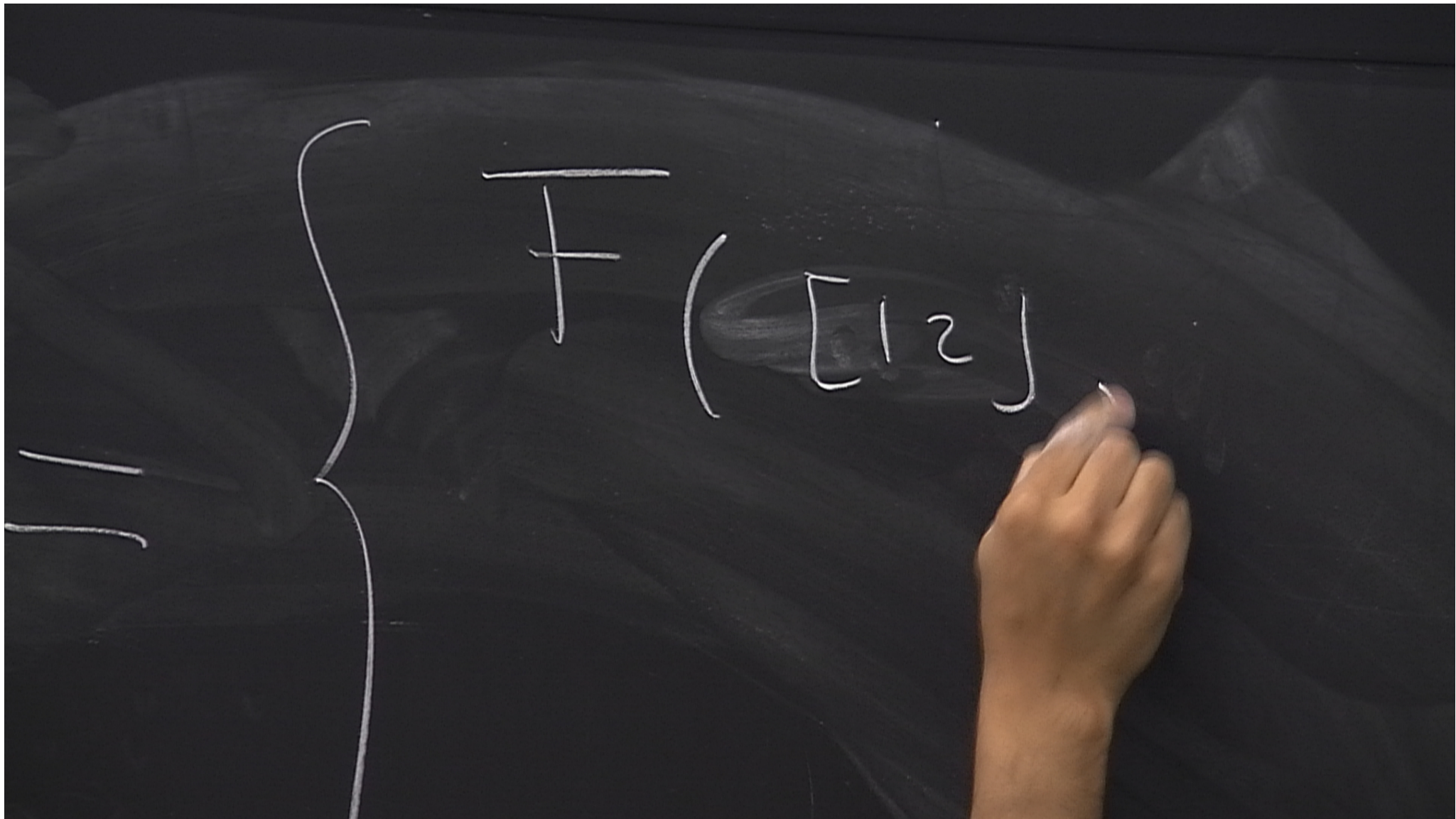
$$[23] \text{ or } [23] = 0$$

hb, hb, hb



ha, ha, ha





3-particles

$$p_1^\mu + p_2^\mu = p_3^\mu$$

$$p_2 \cdot p_3 = 0, \quad p_1 \cdot p_3 = 0$$

$$\langle 23 \rangle [23] = 0$$

$$\langle 13 \rangle [13] = 0$$

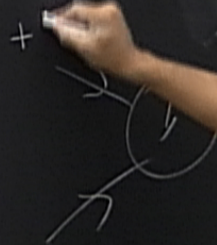
$$\langle 23 \rangle = 0 \checkmark$$

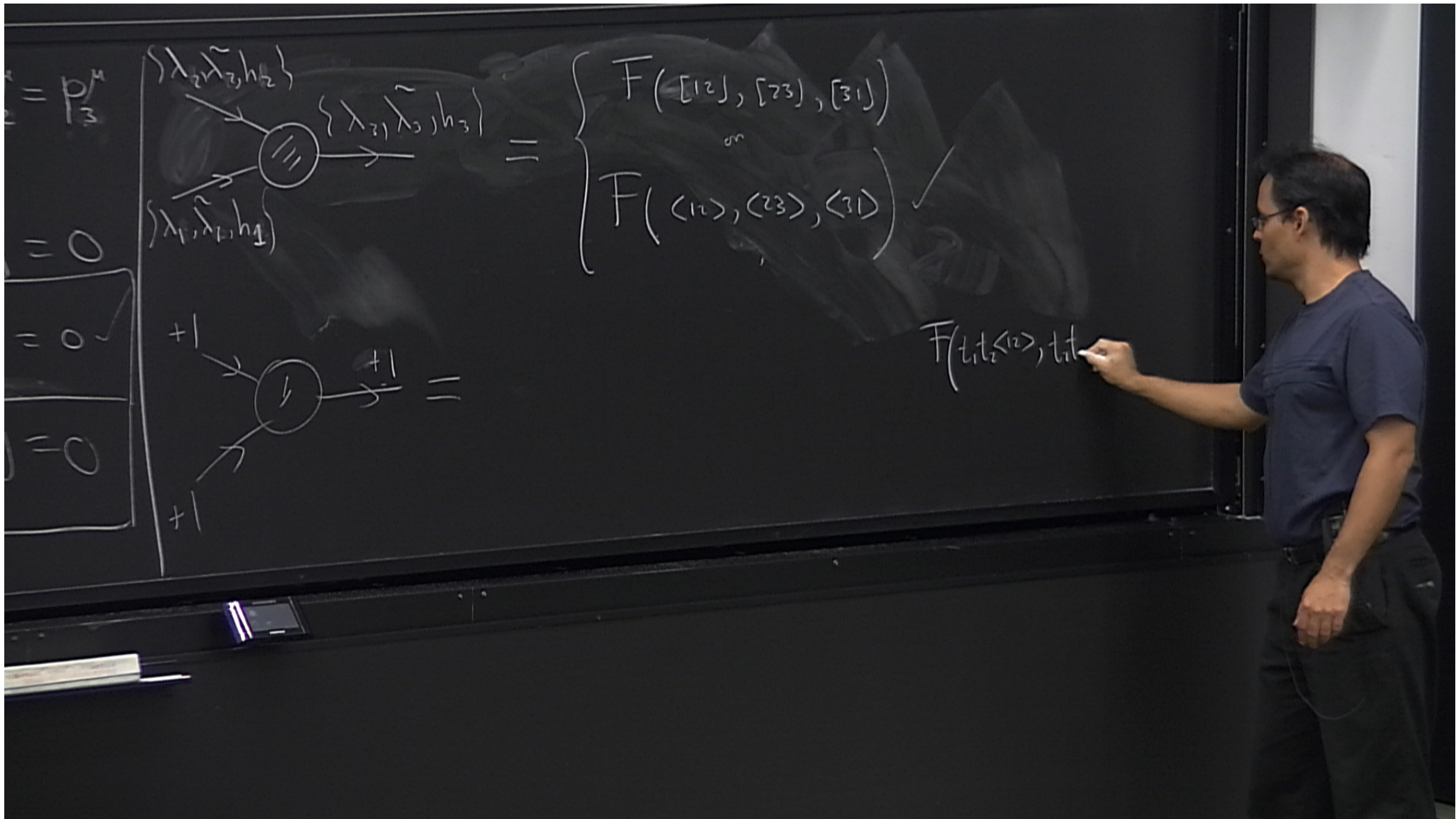
$$\langle 13 \rangle = 0 \checkmark$$

$$[23] = 0$$

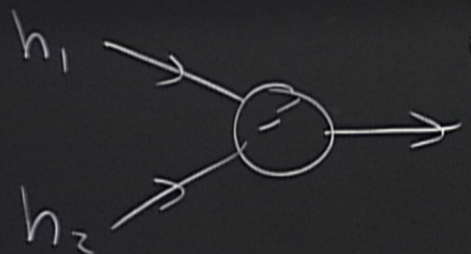
$$[13] = 0$$

$$\begin{aligned} & \{ \lambda_2, \tilde{\lambda}_3, h_2 \} \rightarrow \text{circle} \rightarrow \{ \lambda_2, \tilde{\lambda}_3, h_3 \} \\ & \{ \lambda_1, \tilde{\lambda}_1, h_1 \} \rightarrow \text{circle} \rightarrow \{ \lambda_2, \tilde{\lambda}_3, h_3 \} \\ & = \begin{cases} F([12], [23], [31]) \\ \text{or} \\ F(\langle 12 \rangle, \langle 23 \rangle, \langle 31 \rangle) \end{cases} \end{aligned}$$

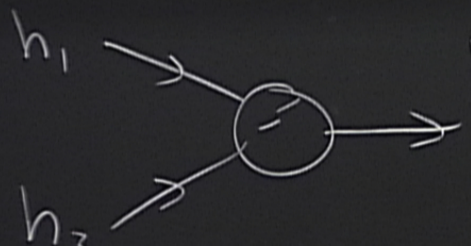




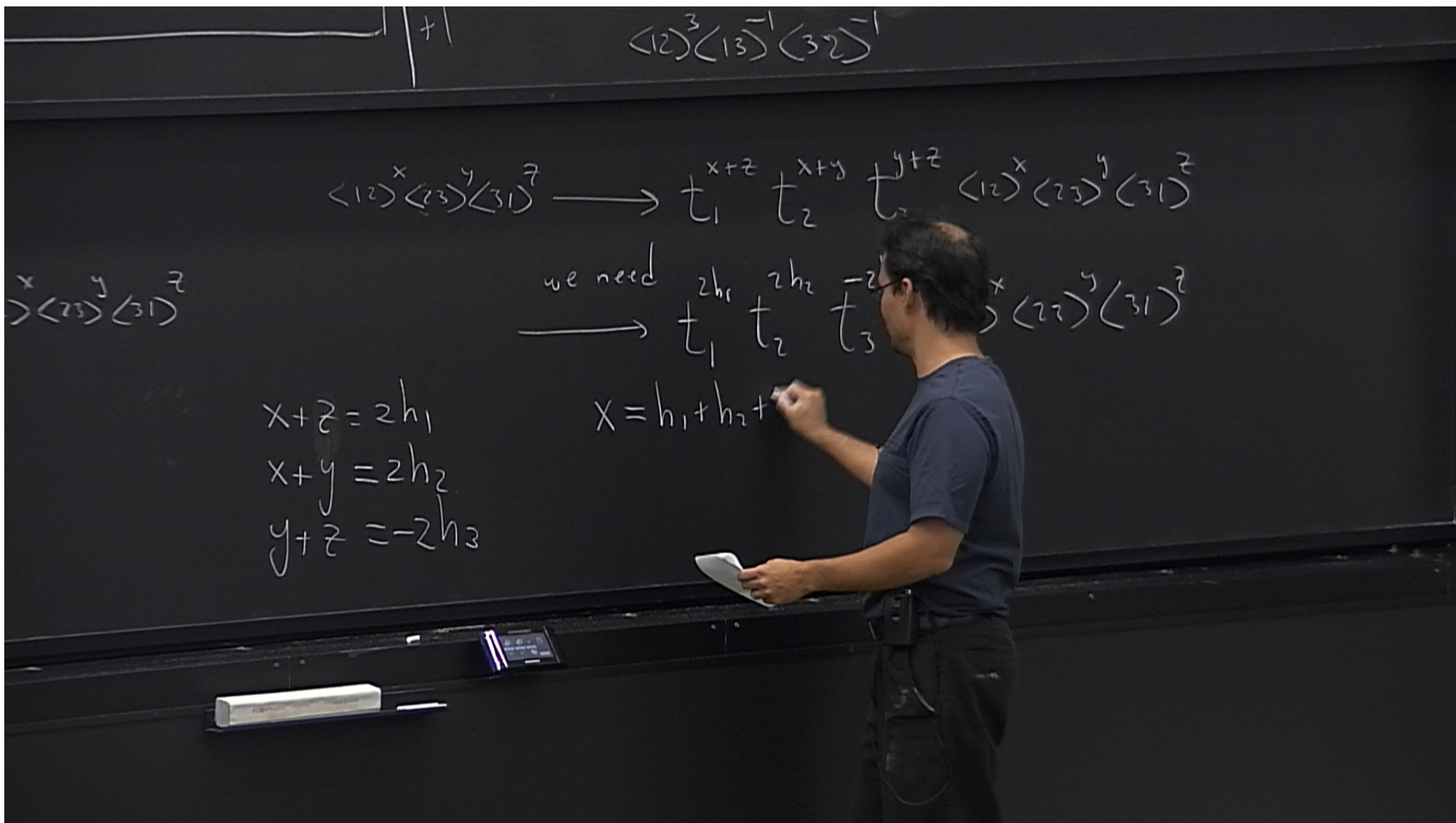
General Case


$$h_3 = \sum_{x,y,z=-\infty}^{\infty} g_{xyz} \langle 12 \rangle^x \langle 23 \rangle^y \langle 31 \rangle^z$$

General Case



$$h_3 = \sum_{x,y,z=-\infty}^{\infty} g_{xyz} \langle 12 \rangle^x \langle 23 \rangle^y \langle 31 \rangle^z$$



$$(12)^3 (13)^{-1} (32)^{-1}$$

$$(12)^x (23)^y (31)^z \longrightarrow t_1^{x+z} t_2^{x+y} t_3^{y+z} (12)^x (23)^y (31)^z$$

$$(12)^x (23)^y (31)^z$$

we need

$$\longrightarrow t_1^{2h_1} t_2^{2h_2} t_3^{-2h_3} (12)^x (23)^y (31)^z$$

$$\begin{aligned} x+z &= 2h_1 \\ x+y &= 2h_2 \\ y+z &= -2h_3 \end{aligned}$$

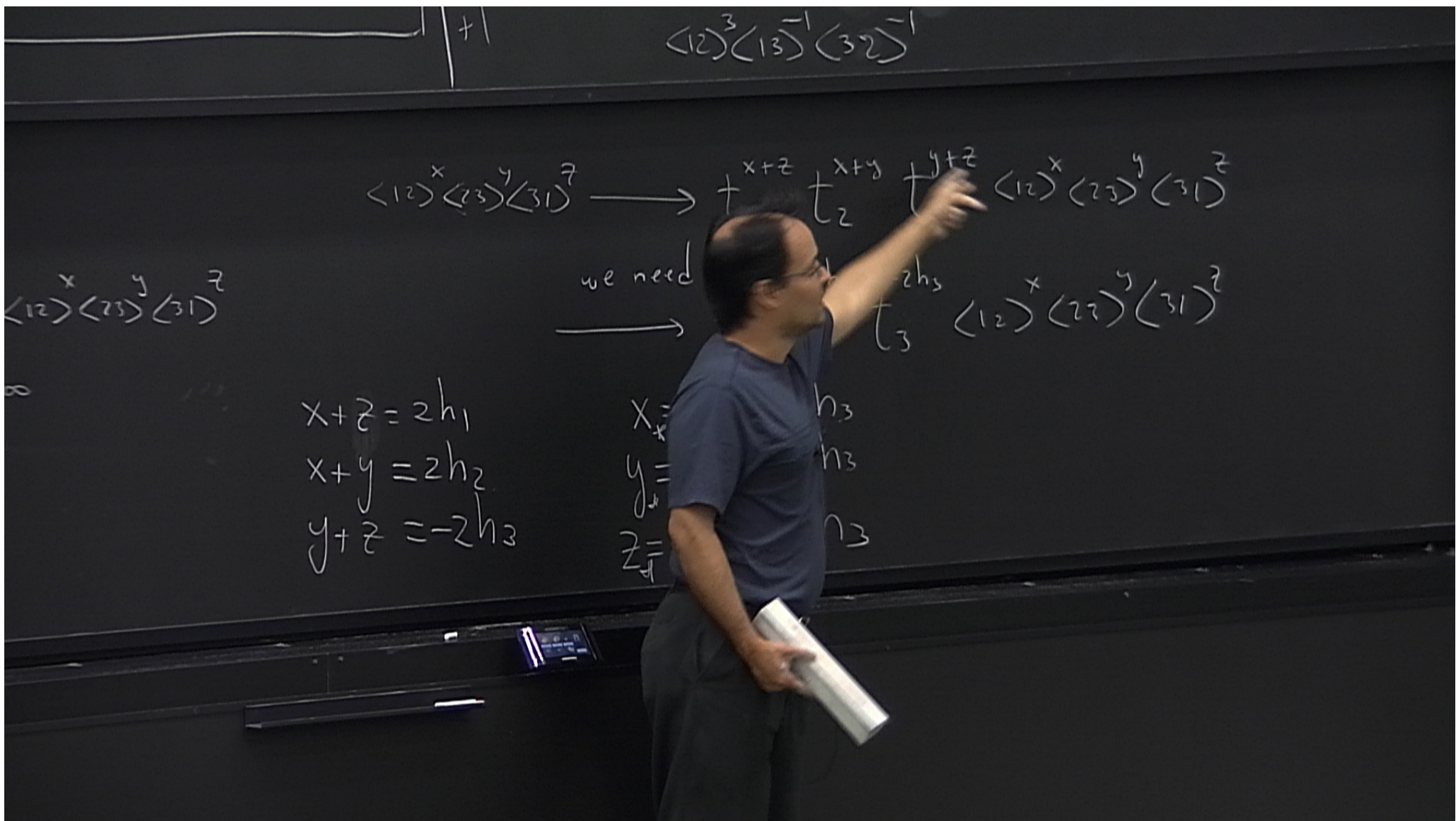
$$\begin{aligned} x &= h_1 + h_2 + h_3 \\ y &= -h_1 + h_2 - h_3 \\ z &= h_1 - h_2 - h_3 \end{aligned}$$

General Case

$$h \rightarrow \text{Sol's.} = \sum_{x,y,z=-\infty}^{\infty} g_{xyz} \langle 12 \rangle^x \langle 23 \rangle^y \langle 31 \rangle^z$$

x_* y_* z_*
 $g_{x_* y_* z_*} \langle 12 \rangle^{x_*} \langle 23 \rangle^{y_*} \langle 31 \rangle^{z_*}$

$$\begin{aligned} x+z &= 2h \\ x+y &= 2h \\ y+z &= -2h \end{aligned}$$



$\frac{1}{+1}$

$\langle 12 \rangle^3 \langle 13 \rangle^{-1} \langle 32 \rangle^{-1}$

$\langle 12 \rangle^x \langle 23 \rangle^y \langle 31 \rangle^z \longrightarrow t_1^{x+z} t_2^{x+y} t_3^{y+z} \langle 12 \rangle^x \langle 23 \rangle^y \langle 31 \rangle^z$

we need

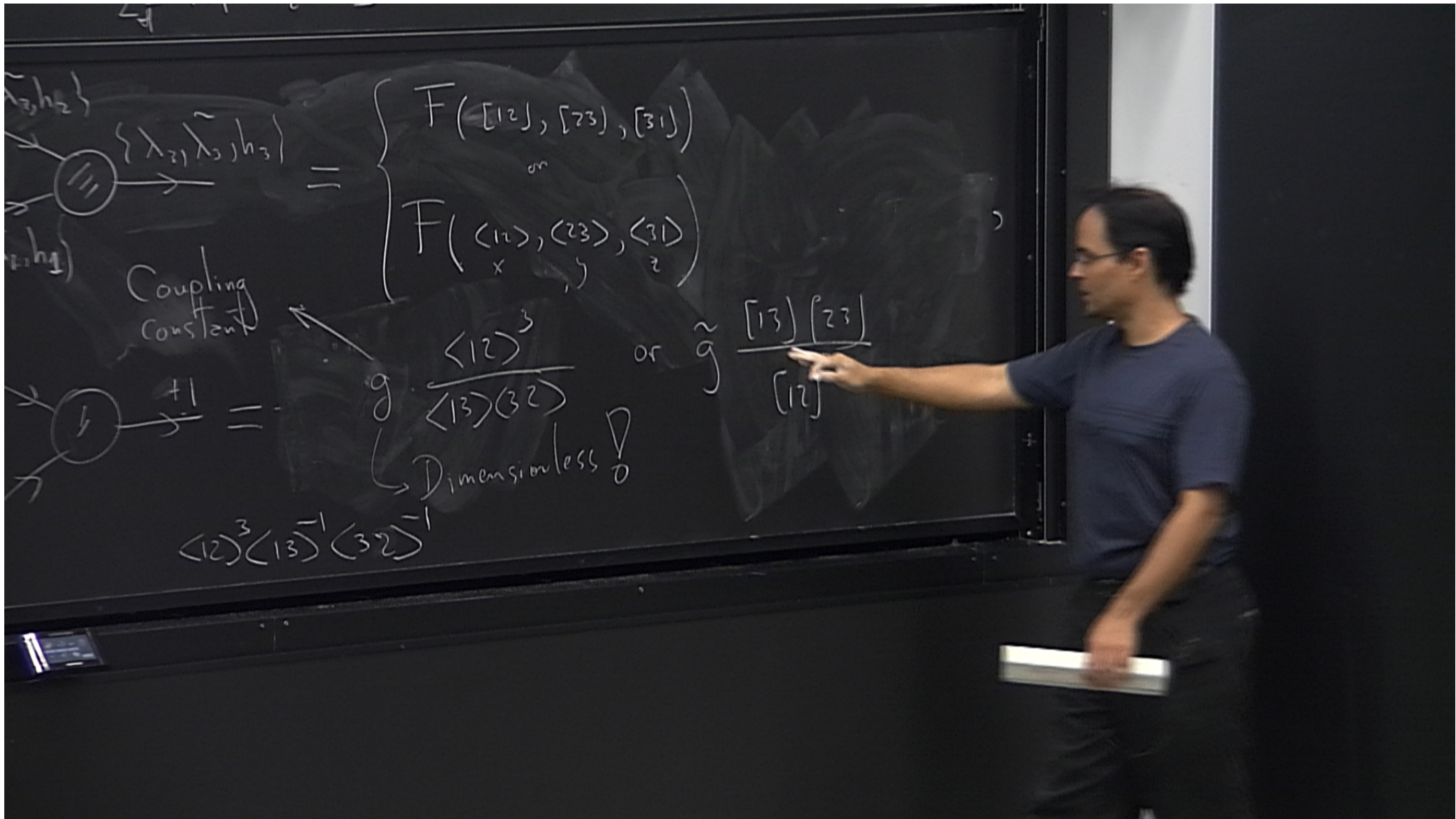
 $\longrightarrow t_1^{2h_1} t_2^{2h_2} t_3^{-2h_3} \langle 12 \rangle^x \langle 23 \rangle^y \langle 31 \rangle^z$

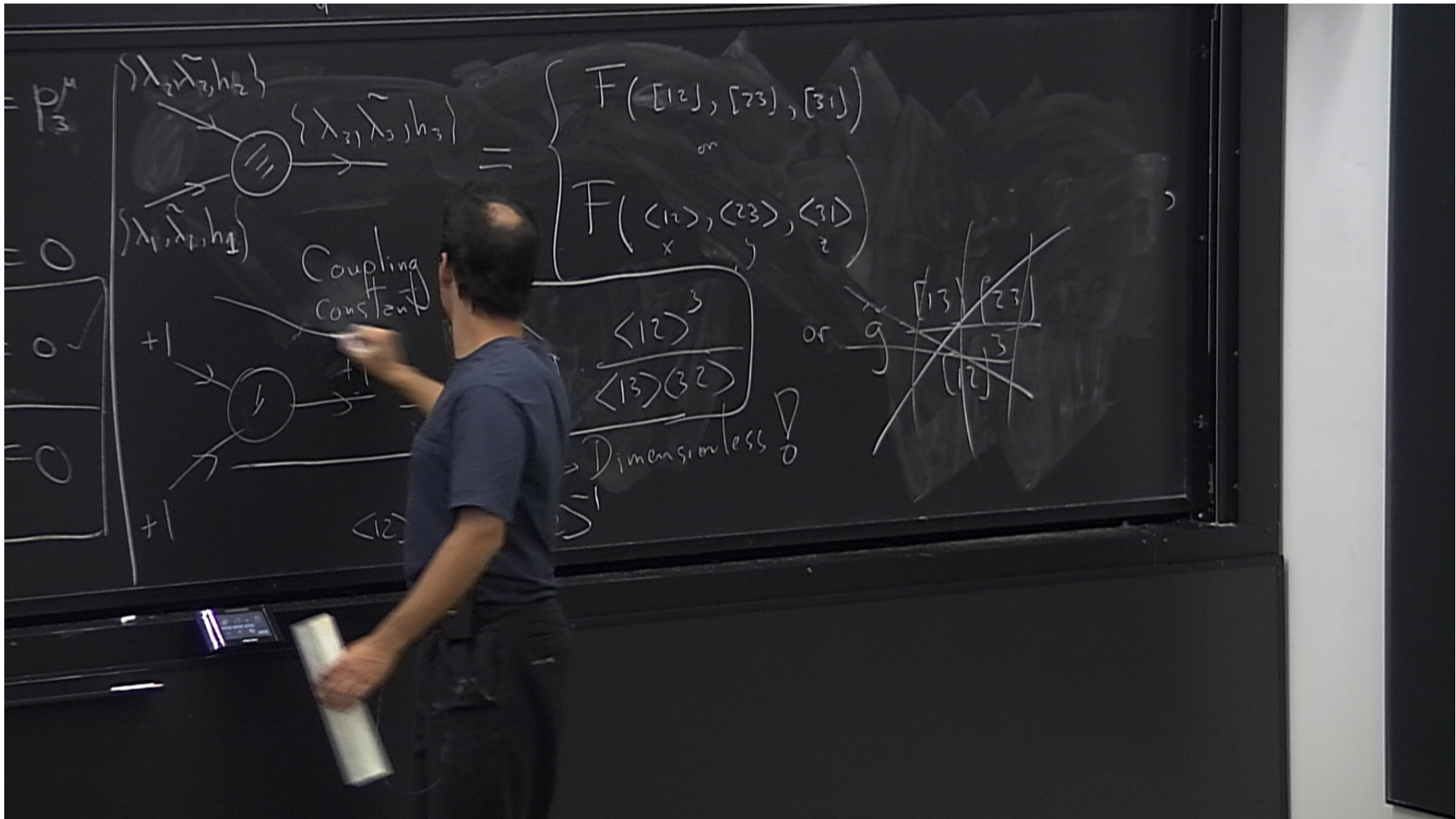
$x+z = 2h_1$
 $x+y = 2h_2$
 $z = -2h_3$

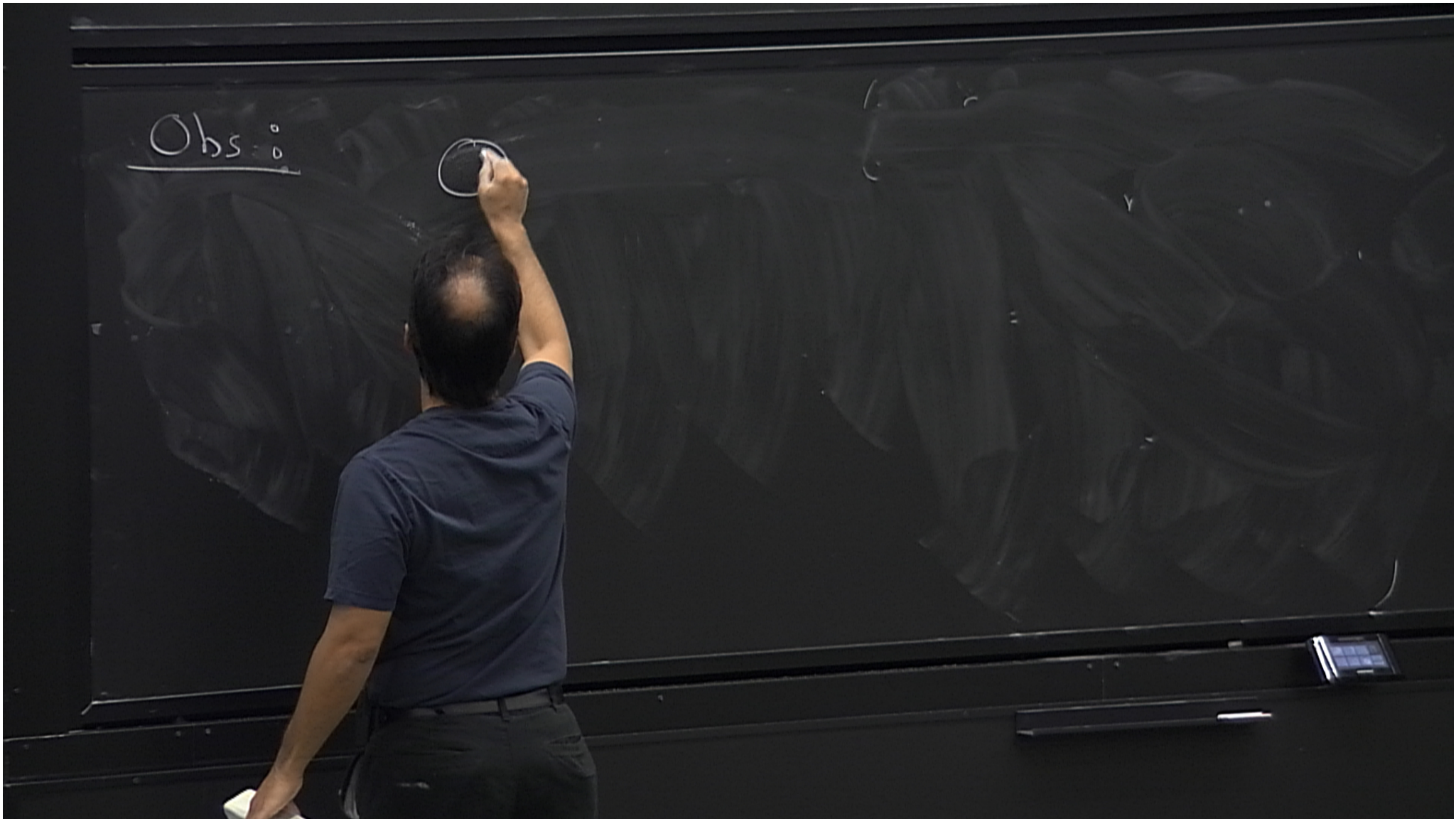
$x = h_1 + h_2 + h_3$
 $y = -h_1 + h_2 - h_3$
 $z = h_1 - h_2 - h_3$

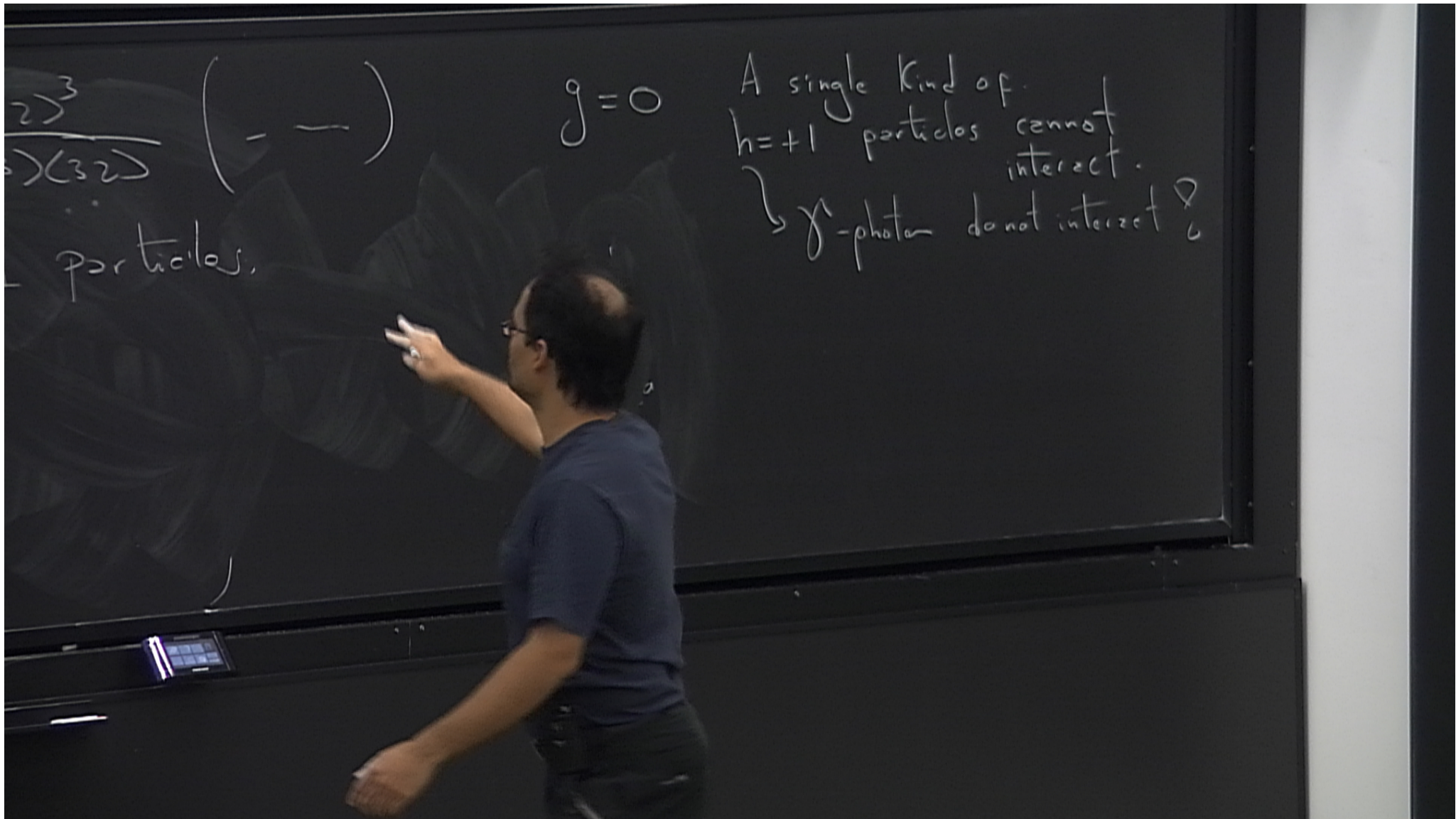
$\sum_{x,y,z=-\infty}^{\infty} q_{xyz} \langle 12 \rangle^x \langle 23 \rangle^y \langle 31 \rangle^z$

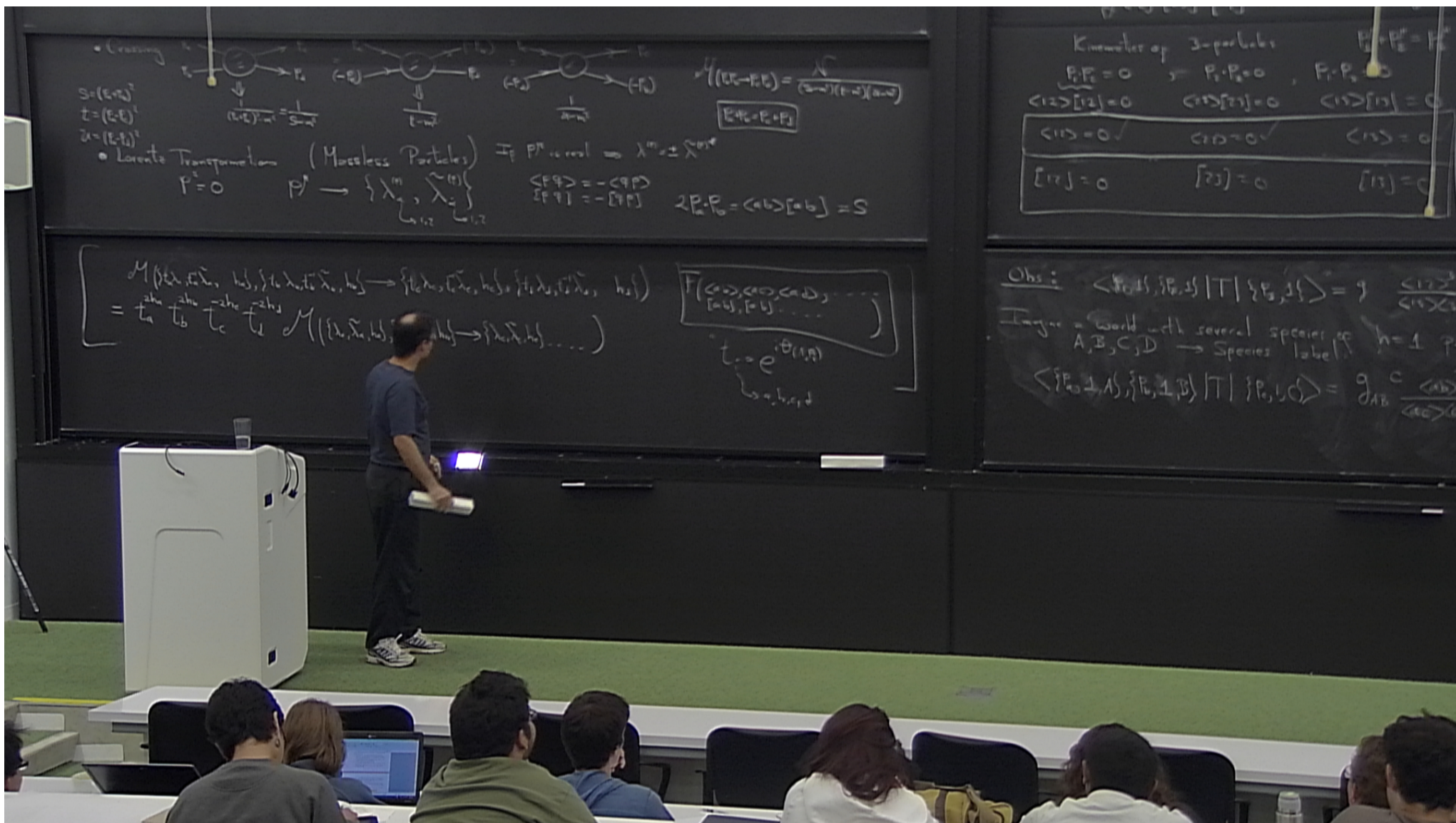
$\begin{matrix} z_+ \\ \langle 31 \rangle \\ -z_+ \\ [31] \end{matrix}$

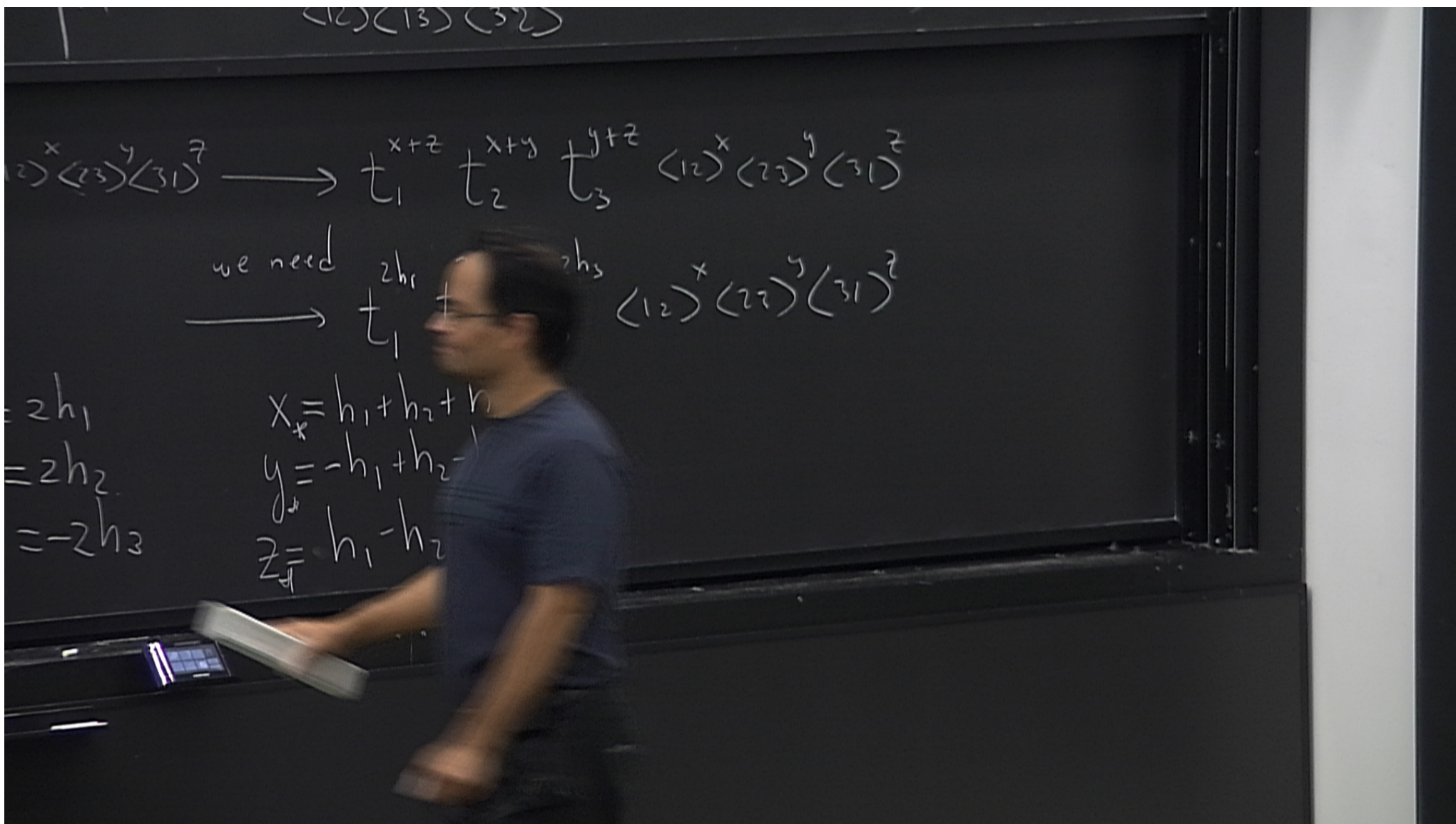


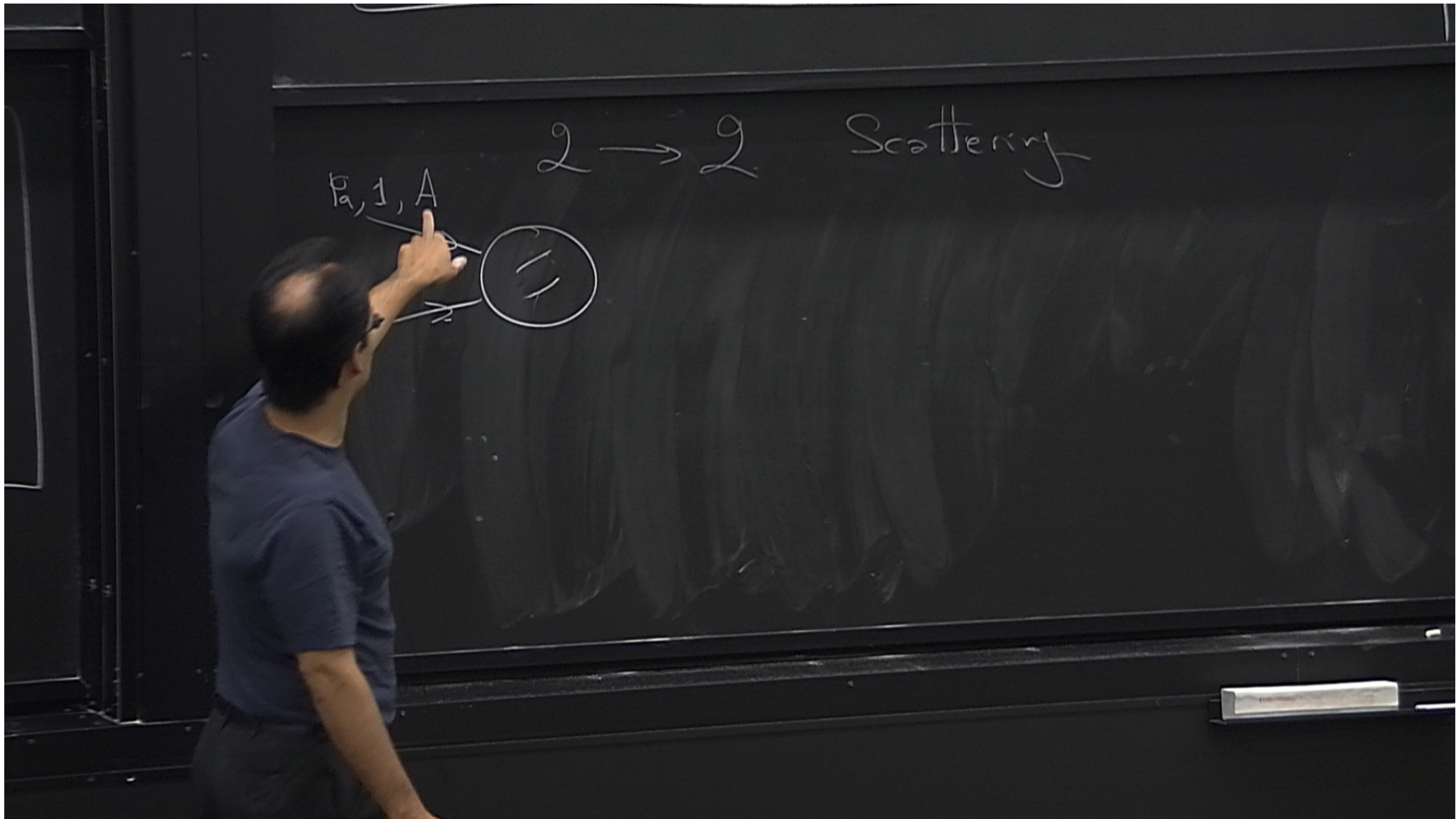


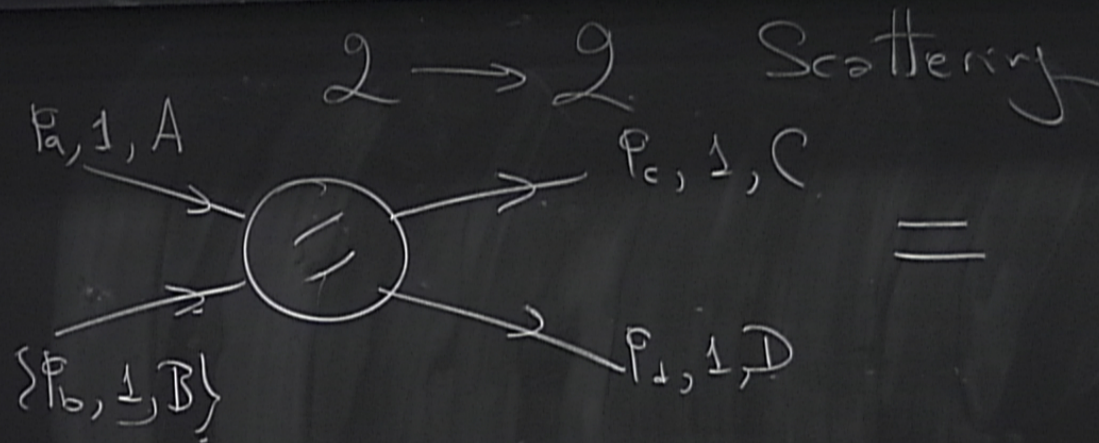




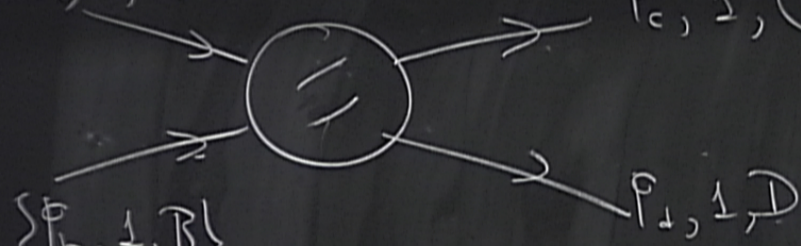








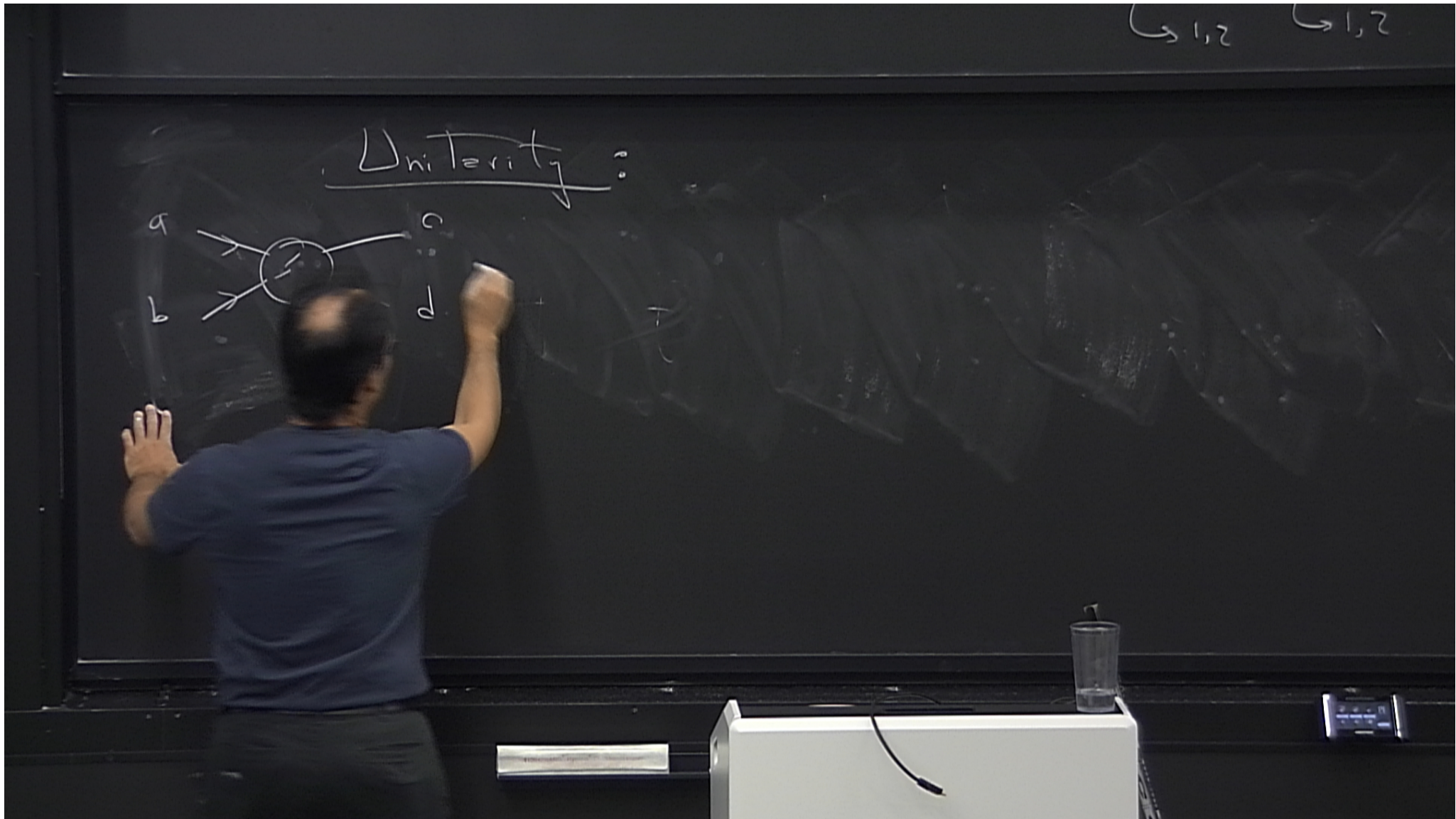
2 $\rightarrow$ 2 Scattering

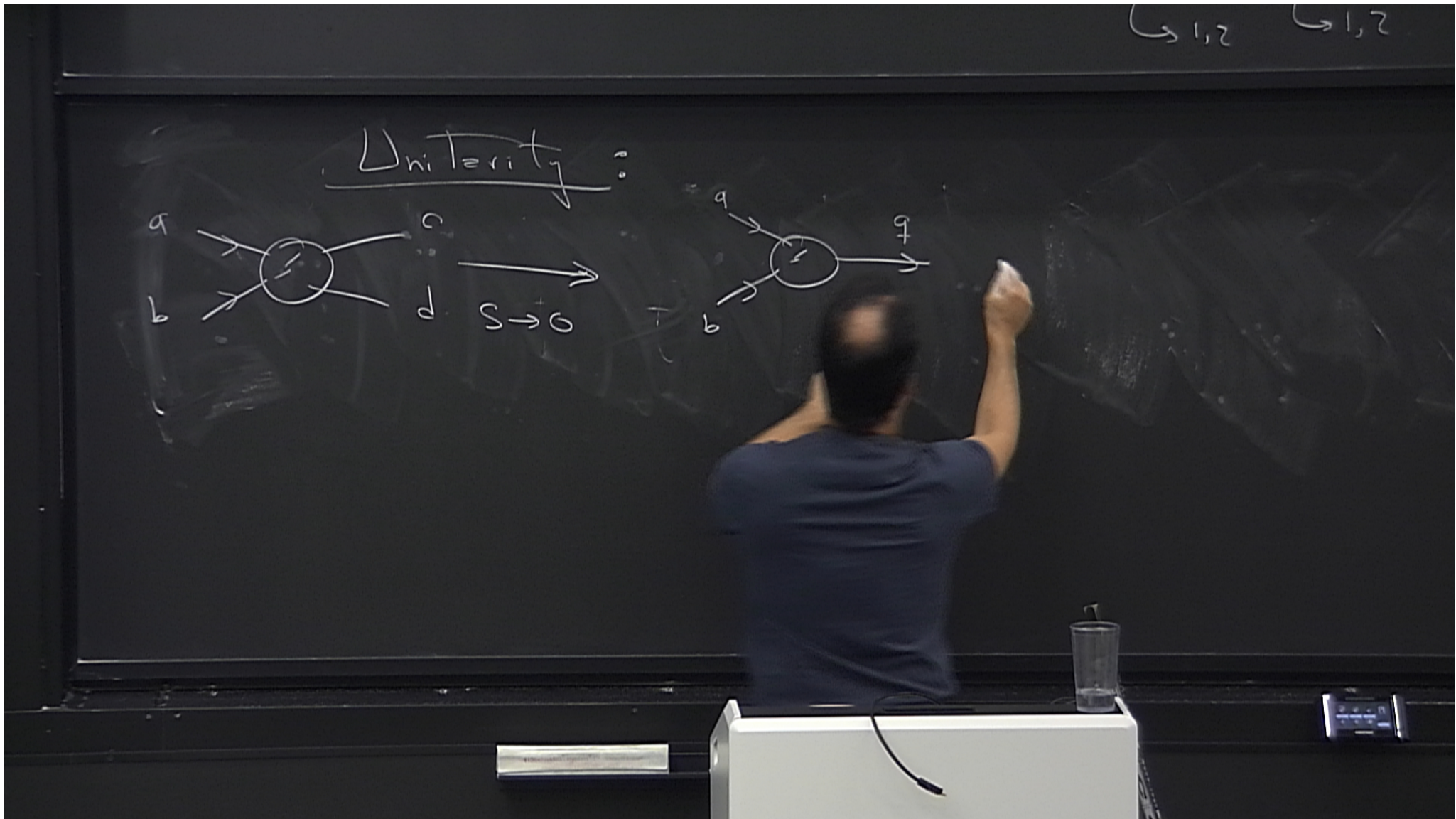


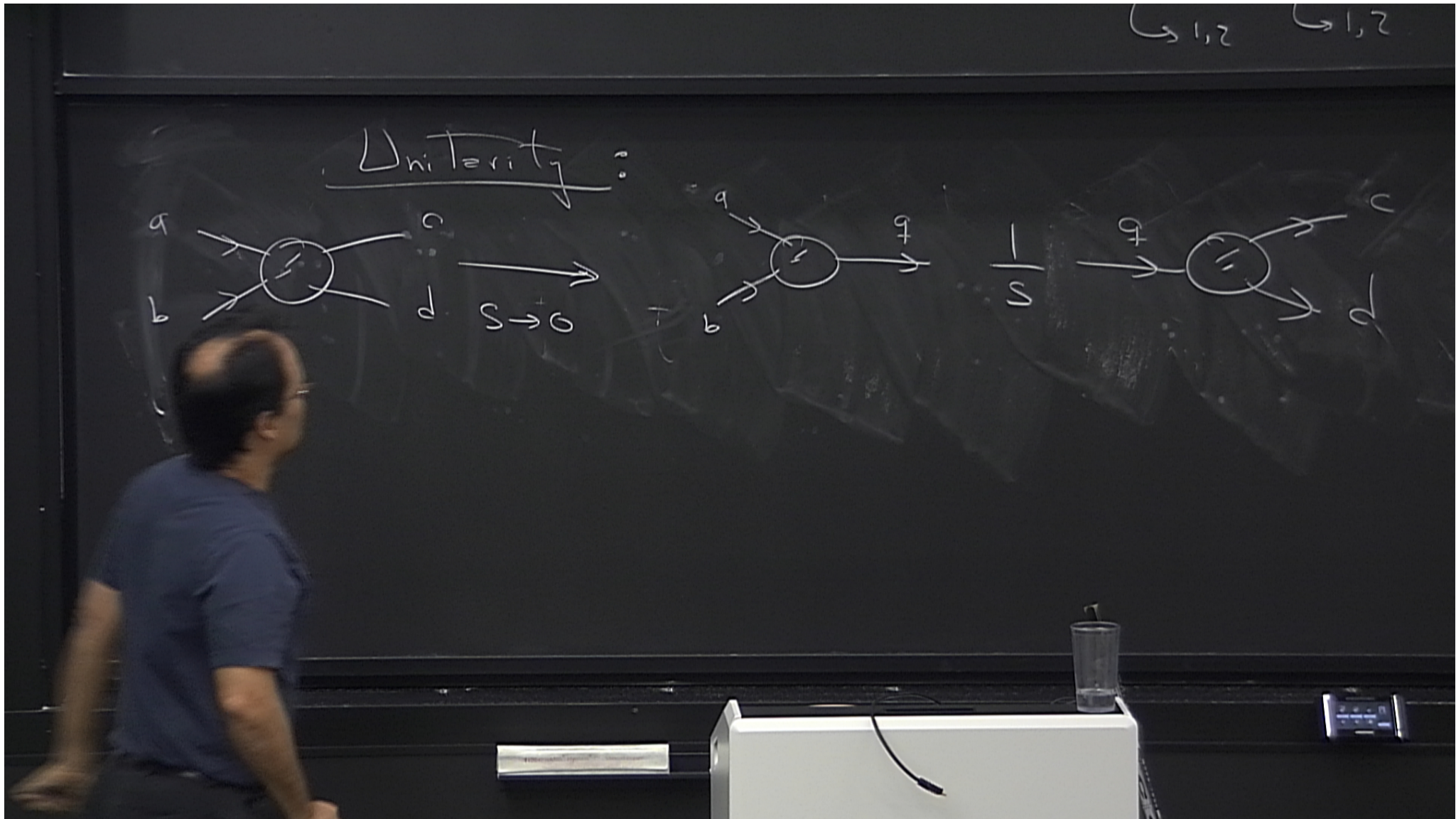
$$= \frac{\langle ab \rangle^2 [cd]^2}{stu}$$

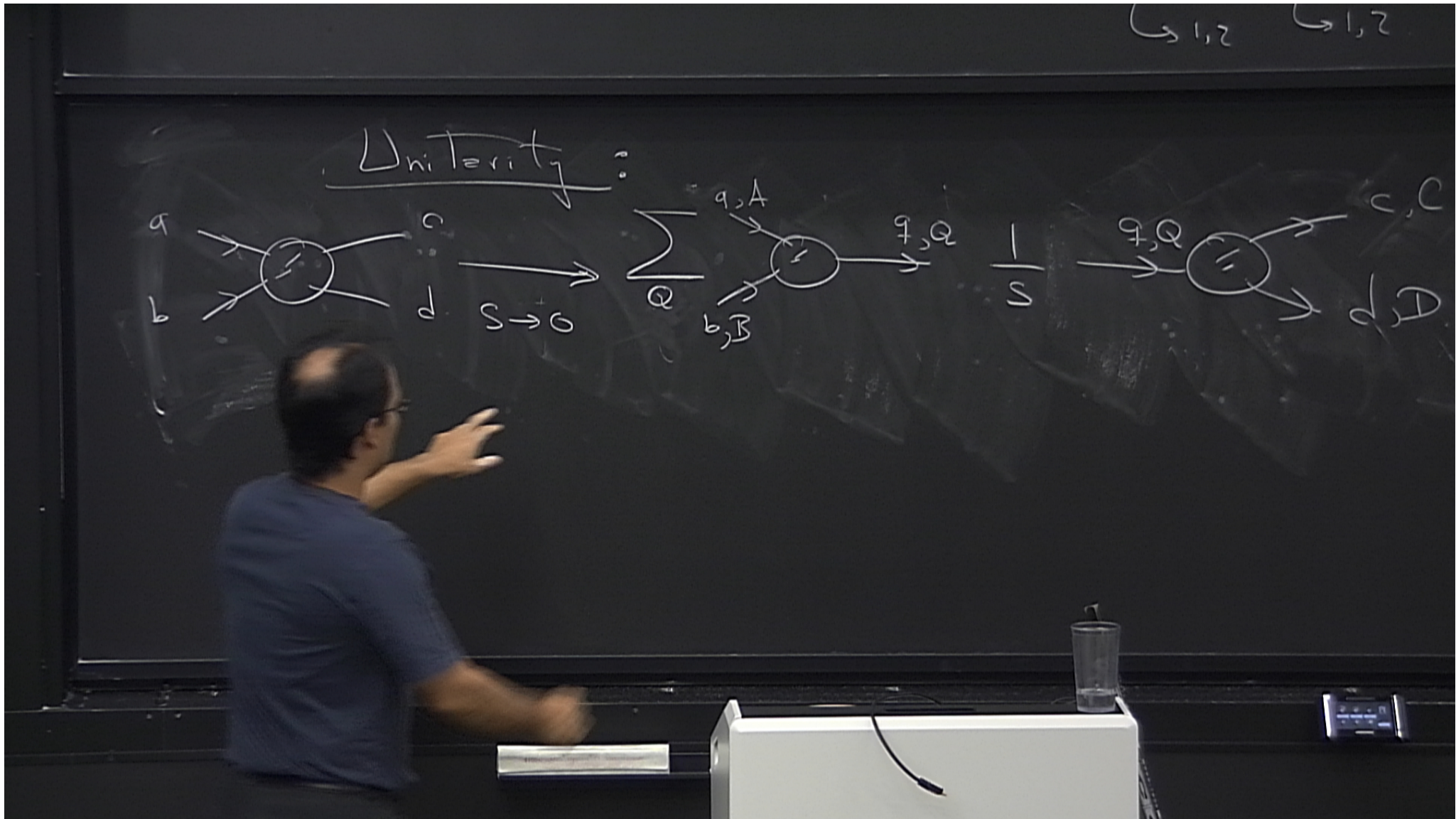
m^2

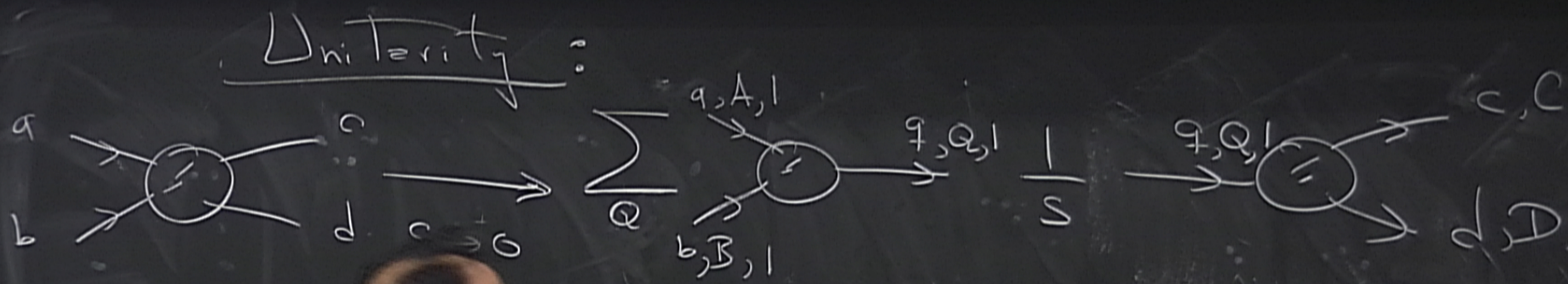
$$\rightarrow t_a^2 t_b^2 \bar{t}_c^2 \bar{t}_d^2 (\otimes)$$

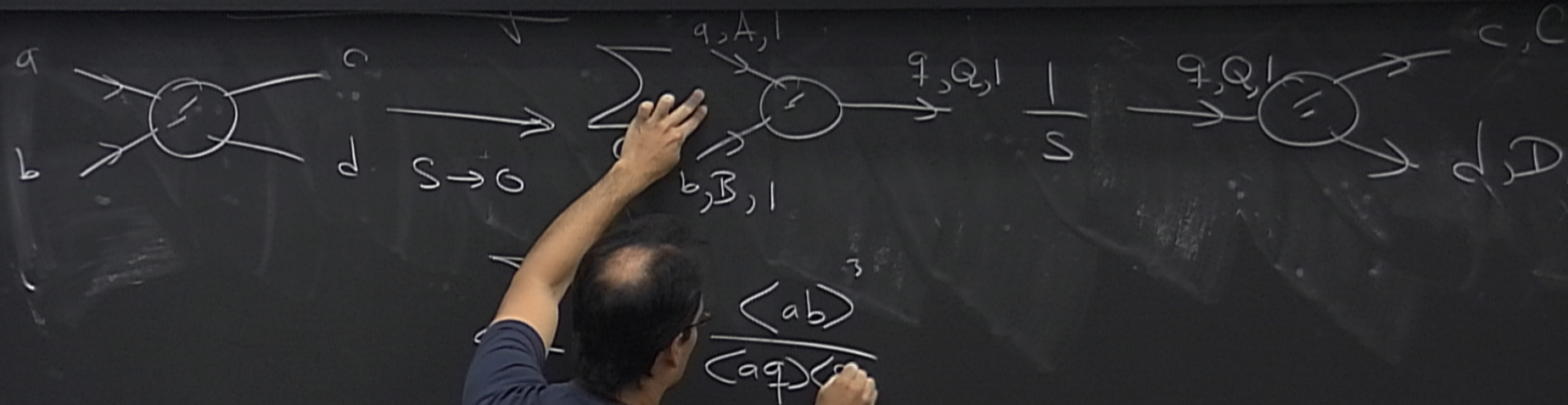


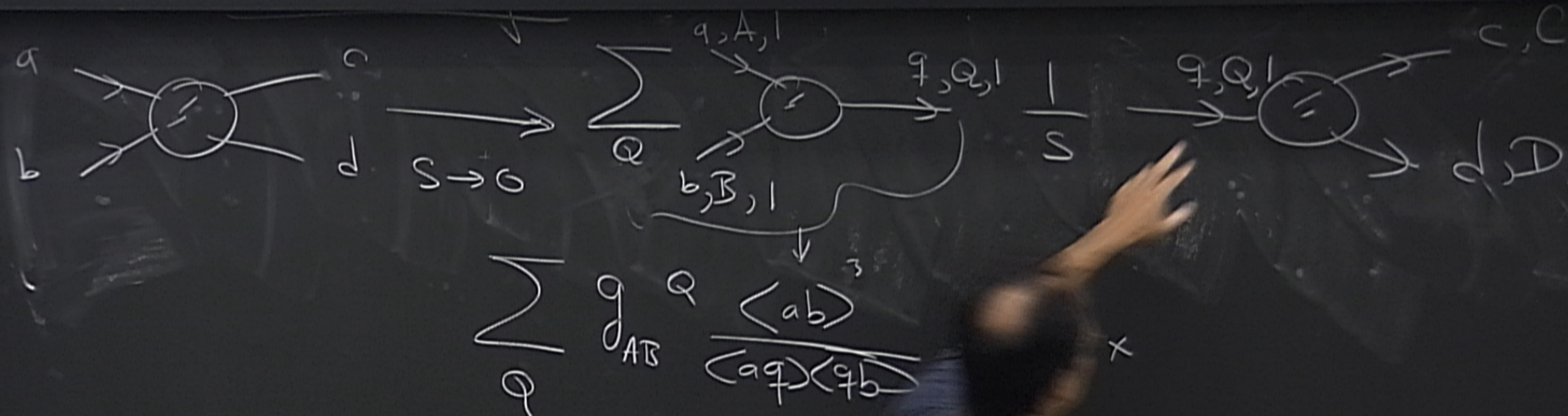


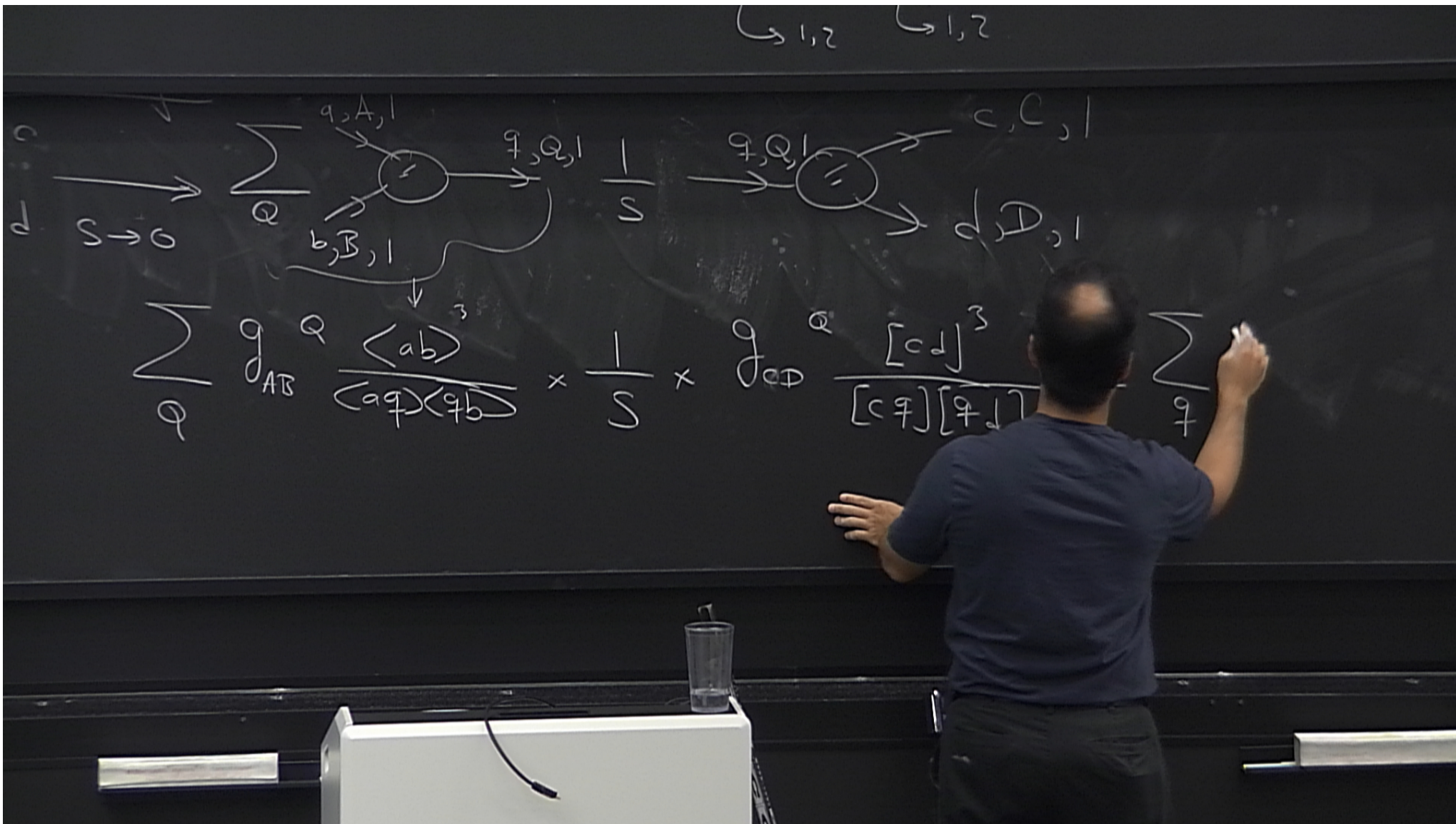


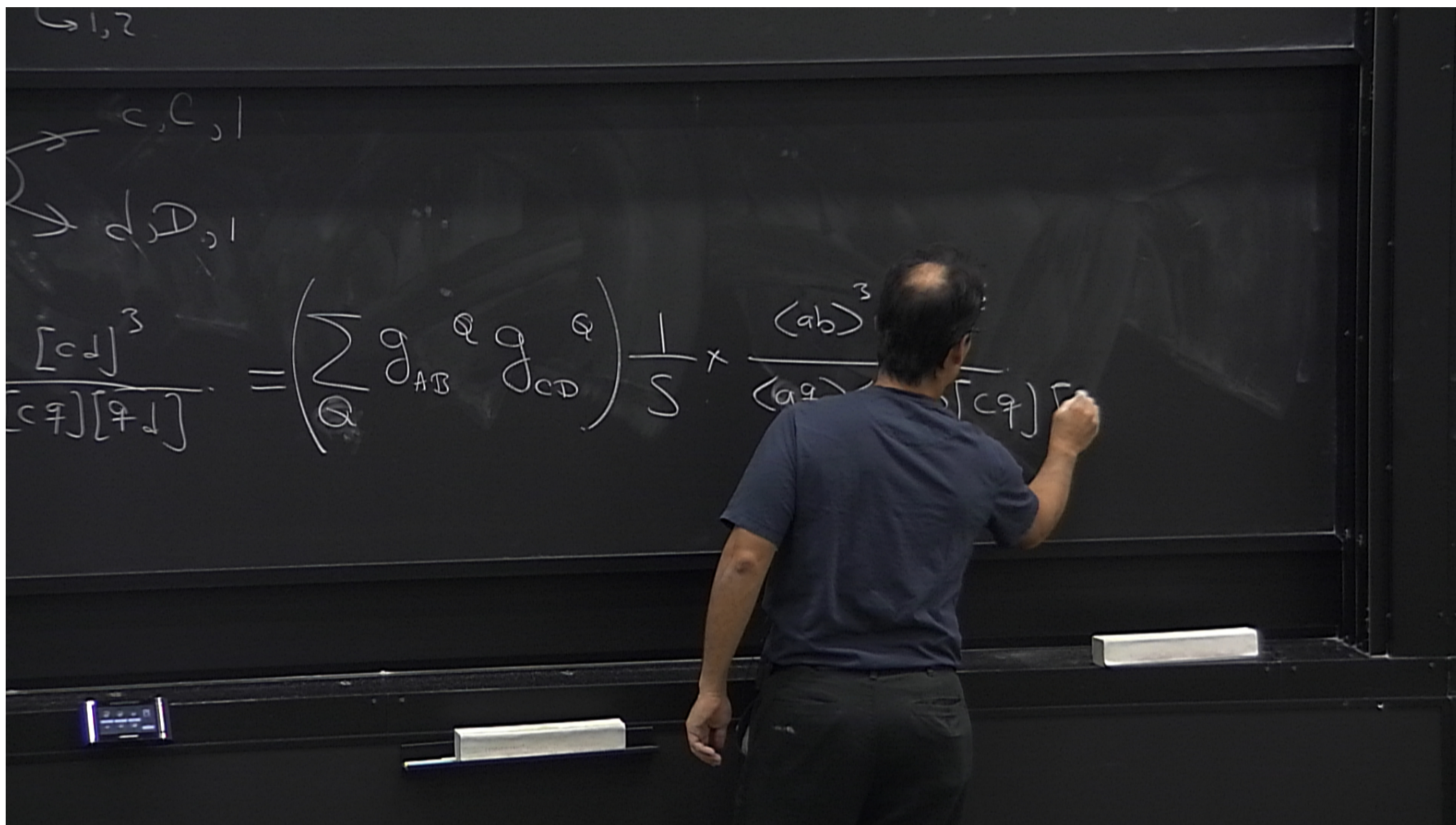


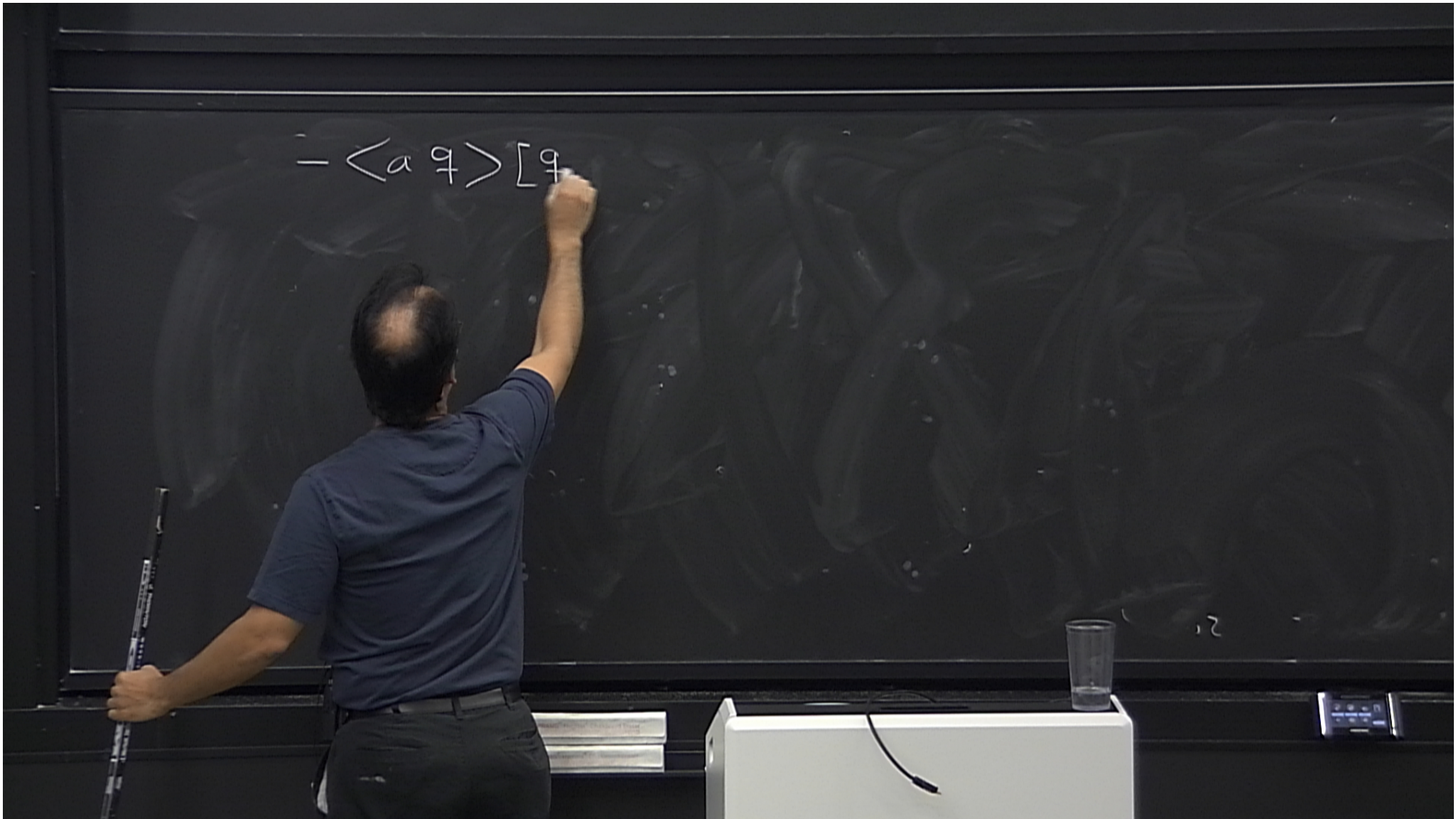






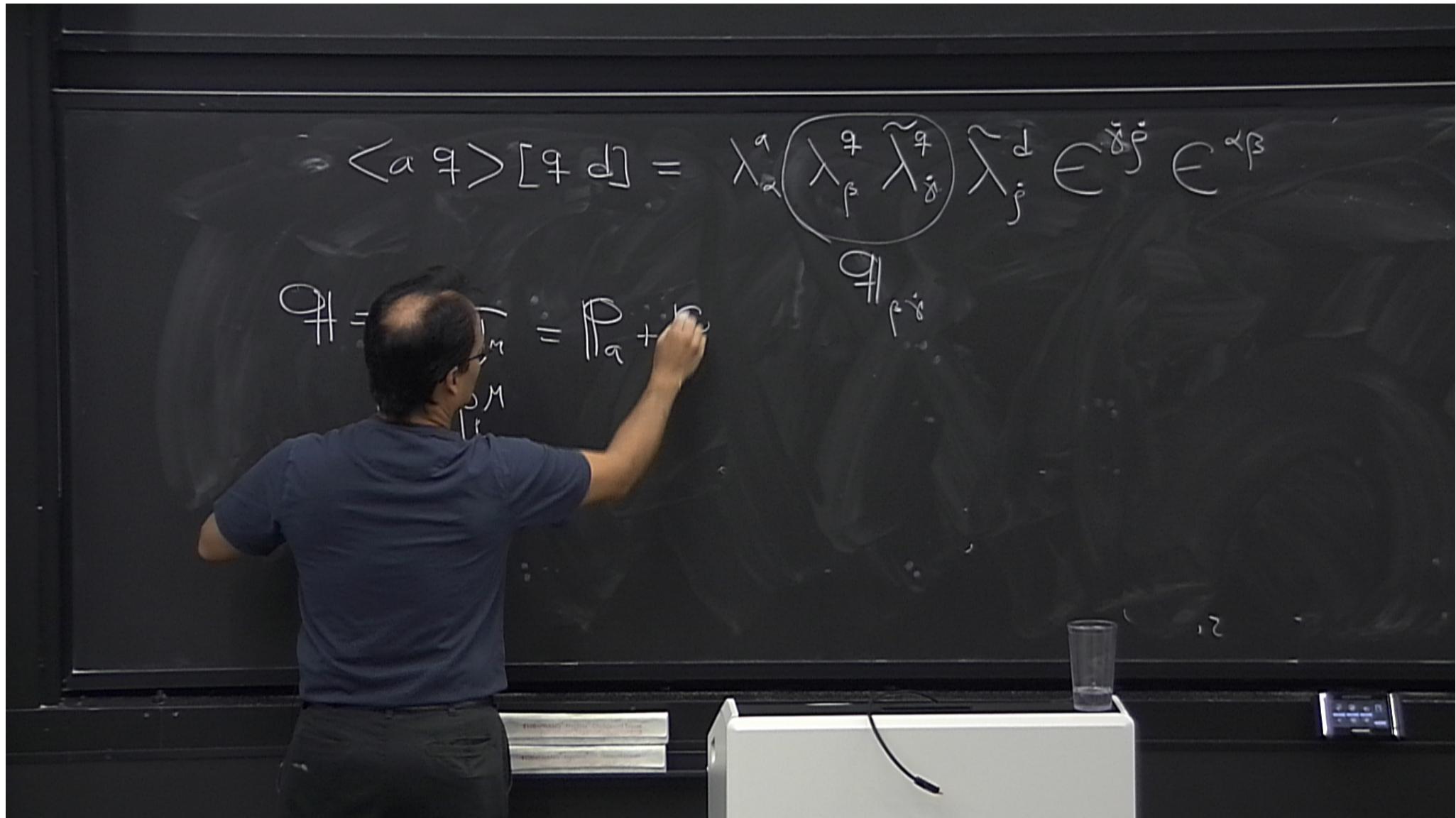






$$\langle a \, q \rangle [q \, d] = \lambda_{\alpha}^q \lambda_{\beta}^q \tilde{\lambda}_{\delta}^q \tilde{\lambda}_j^d \in^{\alpha} \in^{\beta}$$

$$\langle a \, q \rangle [q \, d] = \lambda_{\alpha}^q \lambda_{\beta}^q \tilde{\lambda}_{\delta}^q \tilde{\lambda}_j^d \in^{\delta \dot{\beta}} \in^{\alpha \beta}$$





$$n_u - n_t =$$



↓ Crossing

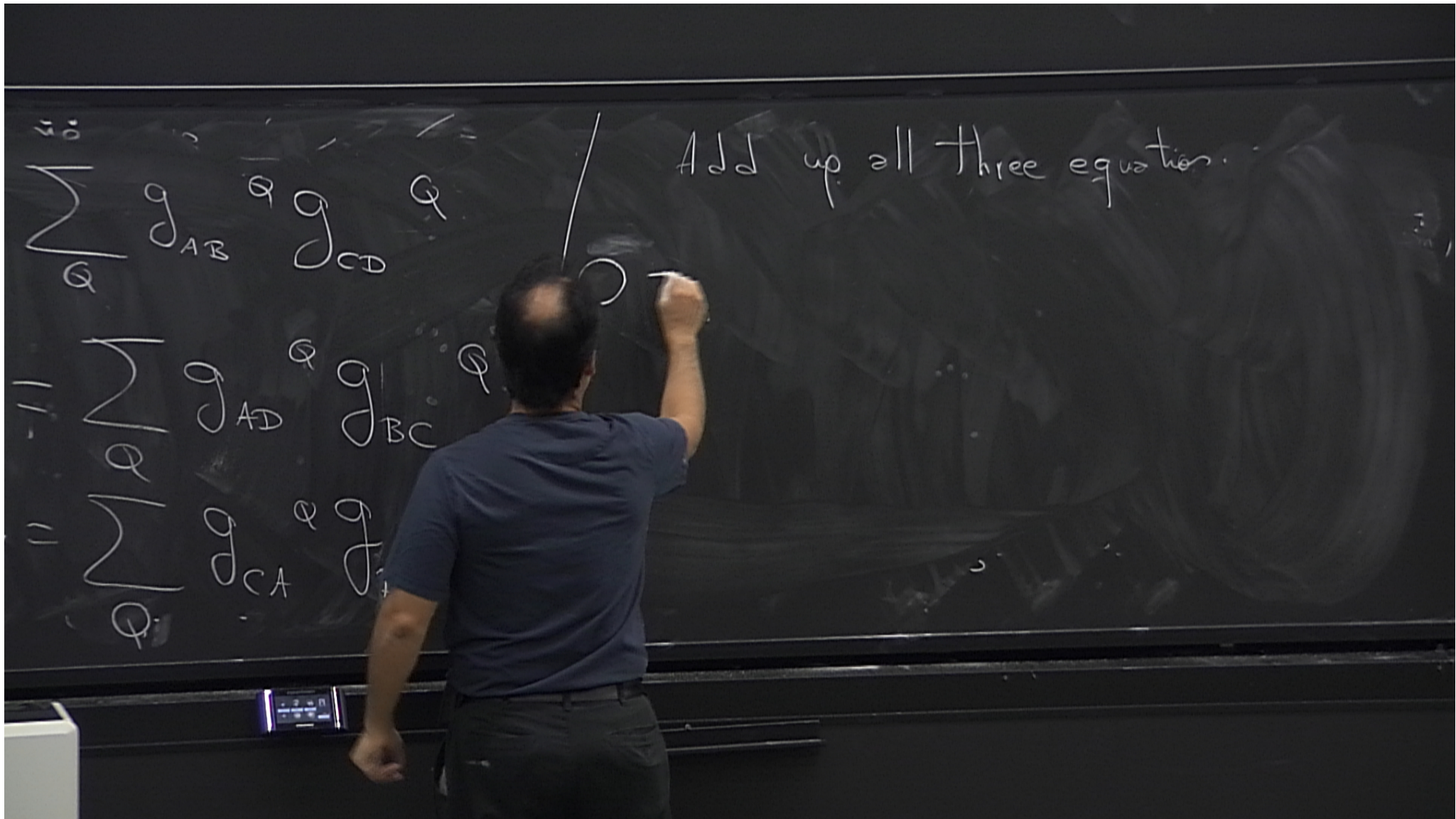
$$s \rightarrow 0 \Rightarrow n_u - n_t =$$

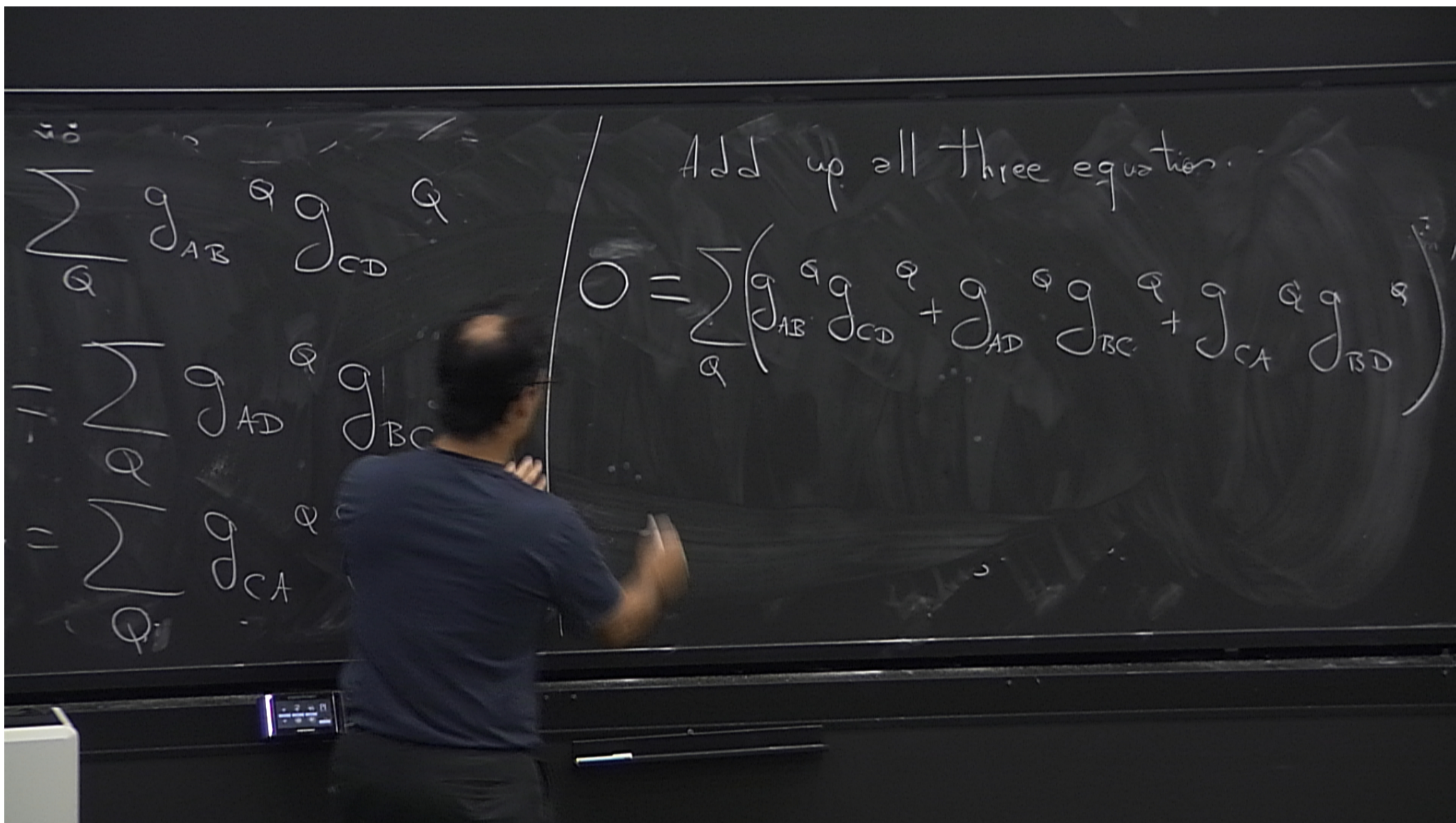
$$\sum_Q g_{AB}^Q g_{CD}^Q$$

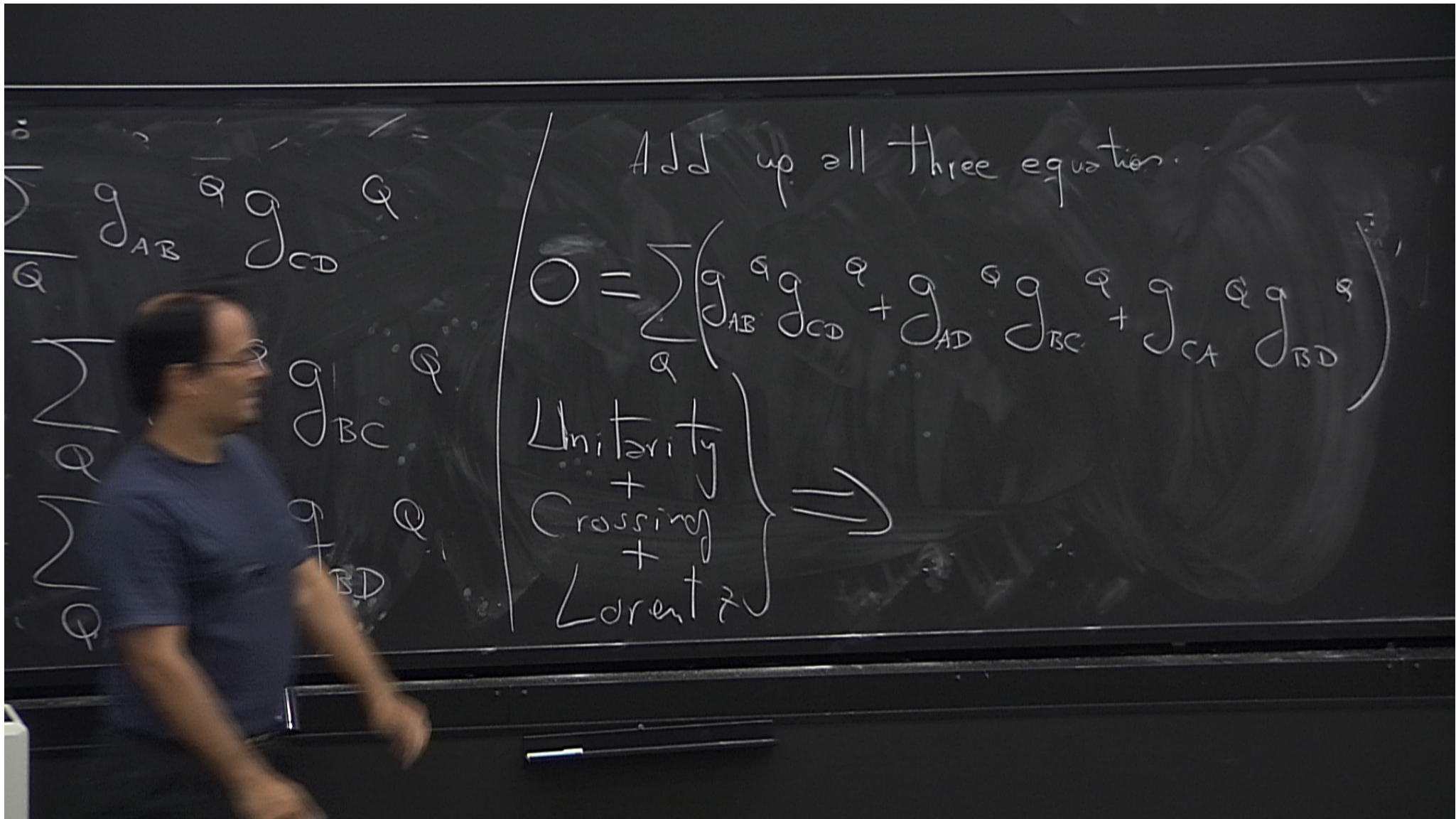
$$u \rightarrow 0 \Rightarrow n_t -$$

$$\sum_Q g_{AD}^Q g_{BC}^Q$$

$$t \rightarrow 0 =$$







Add up all three equations.

$$0 = \sum_Q \left(g_{AB}^Q g_{CD}^Q + g_{AD}^Q g_{BC}^Q + g_{CA}^Q g_{BD}^Q \right)$$

Unitarity
+
Crossing
+
Lorentz $\Rightarrow$