Title: 13/14 PSI - Computational Methods in Physics - Lecture 3

Date: Sep 05, 2013 09:00 AM

URL: http://pirsa.org/13090054

Abstract:

Solving PDEs numerically

Discretization

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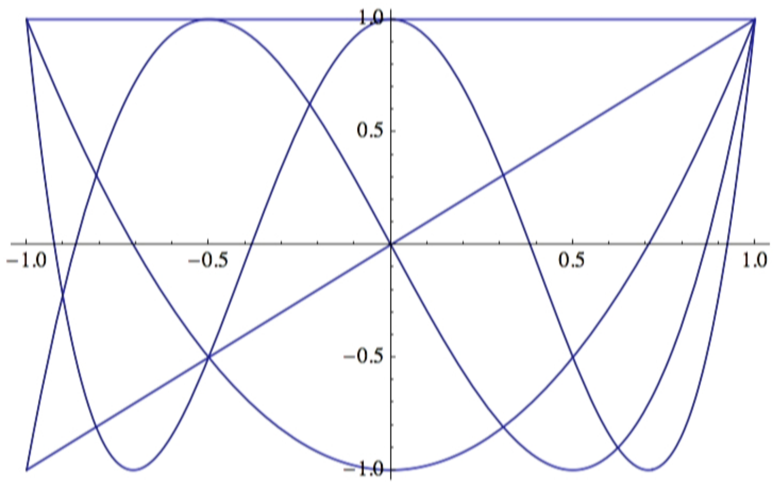
Discretization

In[12]:= `n = 4`

Out[12]= 4

In[13]:= `Plot[Table[ChebyshevT[i, x], {i, 0, n}], {x, -1, +1}]`

Out[13]=



`chebInt[f`

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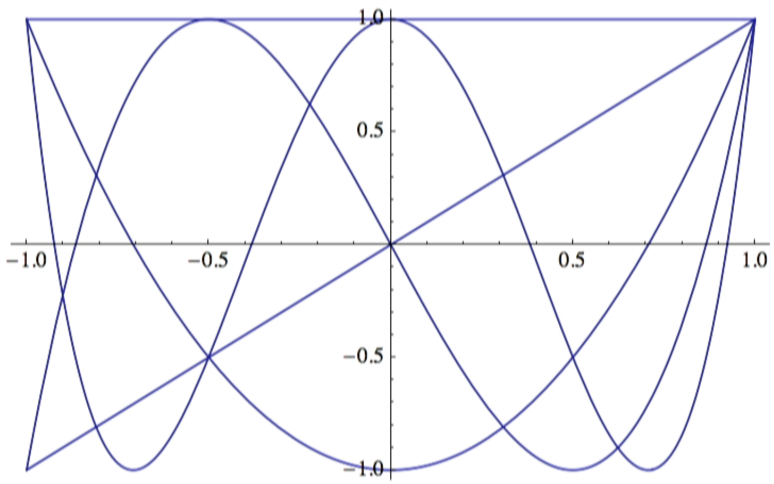
Untitled-7

Discretization

In[12]:=
```
n = 4
```
Out[12]= 4

In[13]:=
```
Plot[Table[ChebyshevT[i, x], {i, 0, n}], {x, -1, +1}]
```
Out[13]=



In[14]:=
$$\text{chebInt}[f\_, g\_] := \int_{-1}^{+1} \frac{f[x]\, g[x]}{\sqrt{1 - x^2}}\, dx$$

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Running...Untitled-7

Discretization

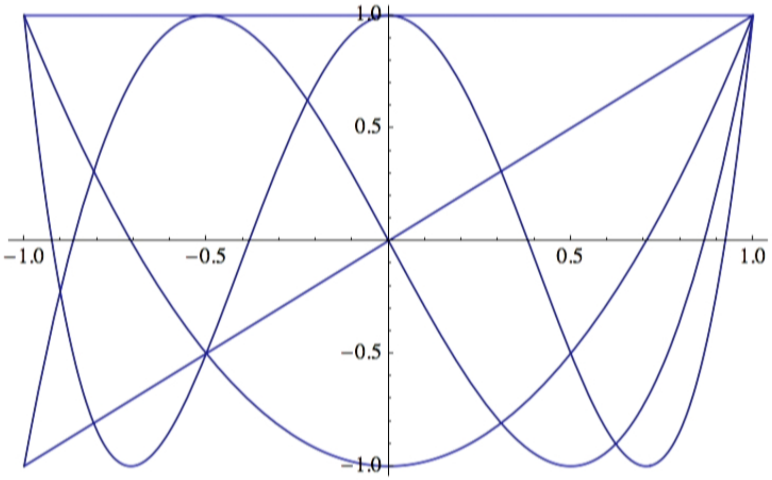
In[12]:= `n = 4`

Out[12]=

4

In[13]:= `Plot[Table[ChebyshevT[i, x], {i, 0, n}], {x, -1, +1}]`

Out[13]=



In[14]:= `chebInt[f_, g_] :=` $\int_{-1}^{+1} \frac{f[x]\, g[x]}{\sqrt{1 - x^2}}\, dx$

In[15]:= `Table[chebInt[ChebyshevT[i, #] &, ChebyshevT[j, #] &], {i, 0, n}, {j, 0, n}]`

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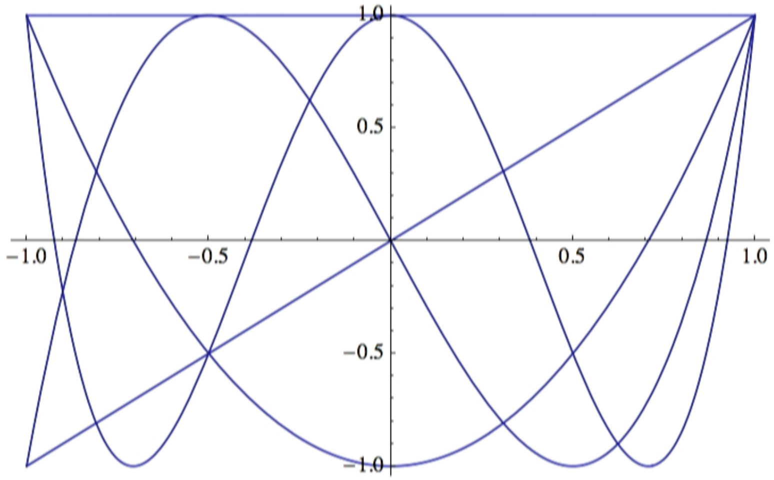
4

In[13]:=
```
Plot[Table[ChebyshevT[i, x], {i, 0, n}], {x, -1, +1}]
```

Out[13]=



In[14]:= chebInt[f_, g_] := $\int_{-1}^{+1} \frac{f[x]\, g[x]}{\sqrt{1 - x^2}}\, dx$

In[16]:=
```
Table[chebInt[ChebyshevT[i, #] &, ChebyshevT[j, #] &], {i, 0, n}, {j, 0, n}] // MatrixForm
```

Out[16]//MatrixForm=

$$\begin{pmatrix} \pi & 0 & 0 & 0 & 0 \\ 0 & \frac{\pi}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{\pi}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{\pi}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{\pi}{2} \end{pmatrix}$$

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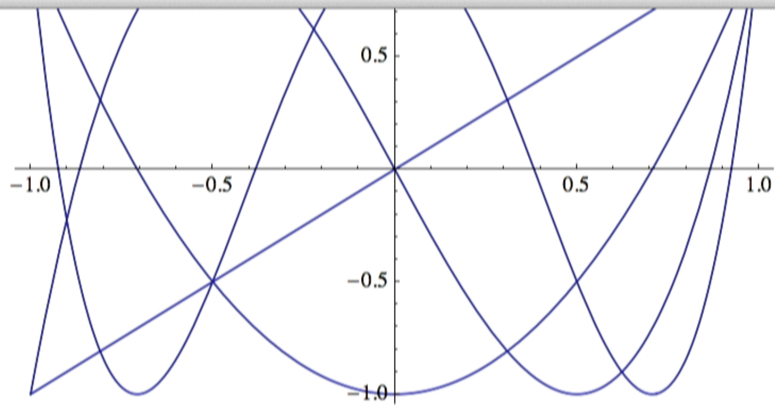

In[18]:=

$$\text{chebInt}[f\_,\ g\_] := \frac{2\int_{-1}^{+1}\frac{f[x]\,g[x]}{\sqrt{1-x^2}}\,dx}{\pi}$$

In[19]:=

```
Table[chebInt[ChebyshevT[i, #] &, ChebyshevT[j, #] &], {i, 0, n}, {j, 0, n}] // MatrixForm
```

Out[19]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Assuming a matrix | Use as *a list of lists* instead

determinant | inverse | matrix plot | eigenvalues | *more...*

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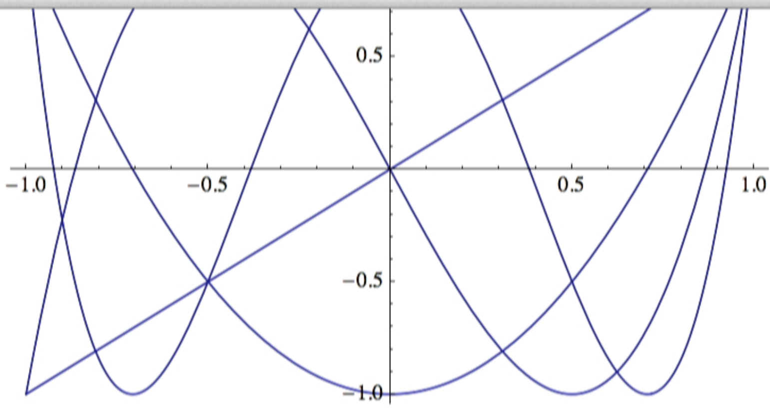

In[18]:= chebInt[f_, g_] :=

$$\frac{2\int_{-1}^{+1}\frac{f[x]\,g[x]}{\sqrt{1-x^2}}\,dx}{\pi}$$

```
In[19]:= Table[chebInt[ChebyshevT[i, #] &, ChebyshevT[j, #] &], {i, 0, n}, {j, 0, n}] // MatrixForm
```

Out[19]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
f[x_] := Sum[cs[[i]] ChebyshevT[i, x], {i, 0, n}]
```

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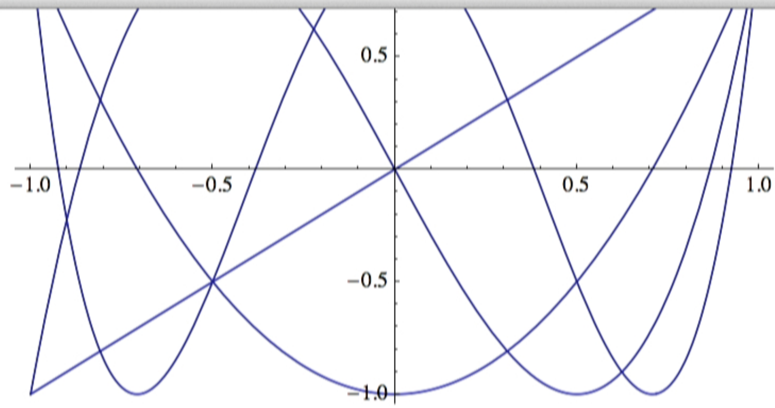

In[18]:= chebInt[f_, g_] := $\dfrac{2 \int_{-1}^{+1} \frac{f[x]\, g[x]}{\sqrt{1-x^2}}\, dx}{\pi}$

```
In[19]:= Table[chebInt[ChebyshevT[i, #] &, ChebyshevT[j, #] &], {i, 0, n}, {j, 0, n}] // MatrixForm
```

Out[19]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[20]:= f[x_] := $\sum_{i=0}^{n}$ cs[[i + 1]] ChebyshevT[i, x]

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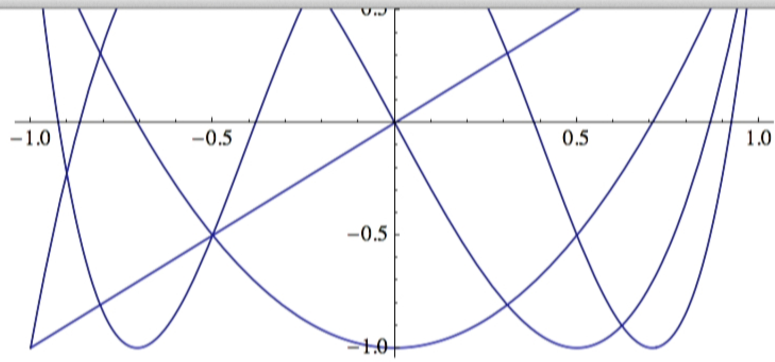

In[18]:=

$$\text{chebInt}[f\_, g\_] := \frac{2 \int_{-1}^{+1} \frac{f[x]\, g[x]}{\sqrt{1-x^2}}\, dx}{\pi}$$

In[19]:=

```
Table[chebInt[ChebyshevT[i, #] &, ChebyshevT[j, #] &], {i, 0, n}, {j, 0, n}] // MatrixForm
```

Out[19]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[21]:=

$$\text{c2f}[cs\_][x\_] := \sum_{i=0}^{n} cs[\![i+1]\!]\, \text{ChebyshevT}[i, x]$$

In[24]:=

```
c2f[{1, 2, 3, 4, 5}][x] // Simplify
```

Out[24]=

$$3 - 10\,x - 34\,x^2 + 16\,x^3 + 40\,x^4$$

plot factor x derivative x integral more...

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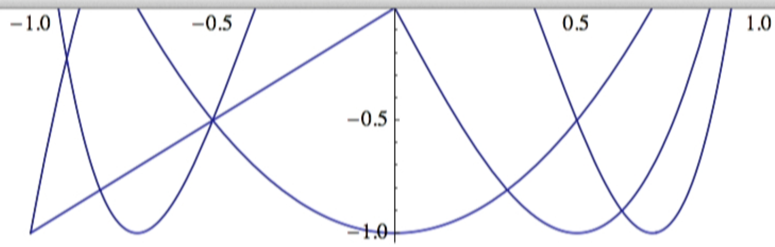

In[18]:=

```
chebInt[f_, g_] := (2 ∫_{-1}^{+1} f[x] g[x]/Sqrt[1-x^2] dx)/π
```

$$\text{chebInt}[f\_, g\_] := \frac{2 \int_{-1}^{+1} \frac{f[x]\, g[x]}{\sqrt{1-x^2}}\, dx}{\pi}$$

In[19]:=

```
Table[chebInt[ChebyshevT[i, #] &, ChebyshevT[j, #] &], {i, 0, n}, {j, 0, n}] // MatrixForm
```

Out[19]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[25]:=

$$\text{c2f}[cs\_, x\_] := \sum_{i=0}^{n} cs[\![i+1]\!]\ \text{ChebyshevT}[i, x]$$

In[26]:=

```
c2f[{1, 2, 3, 4, 5}, x] // Simplify
```

Out[26]=

$$3 - 10\,x - 34\,x^2 + 16\,x^3 + 40\,x^4$$

plot factor x derivative x integral more...

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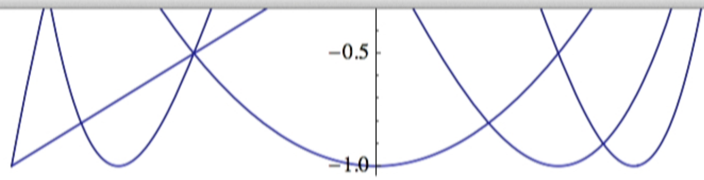

In[18]:= chebInt[f_, g_] := $\dfrac{2\int_{-1}^{+1}\frac{f[x]\,g[x]}{\sqrt{1-x^2}}\,dx}{\pi}$

In[19]:= `Table[chebInt[ChebyshevT[i, #] &, ChebyshevT[j, #] &], {i, 0, n}, {j, 0, n}] // MatrixForm`

Out[19]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[25]:= c2f[cs_, x_] := $\sum_{i=0}^{n}$ cs[[i + 1]] ChebyshevT[i, x]

In[26]:= `c2f[{1, 2, 3, 4, 5}, x] // Simplify`

Out[26]=

$$3 - 10\,x - 34\,x^2 + 16\,x^3 + 40\,x^4$$

`f2c[f_] := chebInt[f, ChebyshevT[i, #] &]`

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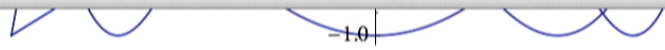

In[18]:=
```
chebInt[f_, g_] := (2 ∫_{-1}^{+1} (f[x] g[x])/Sqrt[1 - x^2] dx) / π
```

$$\text{chebInt}[f\_,\ g\_] := \frac{2 \int_{-1}^{+1} \frac{f[x]\, g[x]}{\sqrt{1-x^2}}\, dx}{\pi}$$

In[19]:=
```
Table[chebInt[ChebyshevT[i, #] &, ChebyshevT[j, #] &], {i, 0, n}, {j, 0, n}] // MatrixForm
```

Out[19]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[25]:=
```
c2f[cs_, x_] := Sum[cs[[i + 1]] ChebyshevT[i, x], {i, 0, n}]
```

$$\text{c2f}[cs\_,\ x\_] := \sum_{i=0}^{n} cs[\![i+1]\!]\ \text{ChebyshevT}[i,\ x]$$

In[26]:=
```
c2f[{1, 2, 3, 4, 5}, x] // Simplify
```

Out[26]=

$$3 - 10\,x - 34\,x^2 + 16\,x^3 + 40\,x^4$$

In[27]:=
```
f2c[f_] := Table[chebInt[f, ChebyshevT[i, #] &], {i, 0, n}]
```

In[28]:=
```
f2c[Sin]
```

Out[28]=
```
{0, 2 BesselJ[1, 1], 0, -2 BesselJ[3, 1], 0}
```

total sort mean delete duplicates more...

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Untitled-7

In[18]:=
```
chebInt[f_, g_] := (2 ∫_{-1}^{+1} f[x] g[x]/Sqrt[1-x^2] dx)/π
```

In[19]:=
```
Table[chebInt[ChebyshevT[i, #] &, ChebyshevT[j, #] &], {i, 0, n}, {j, 0, n}] // MatrixForm
```

Out[19]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[25]:=
```
c2f[cs_, x_] := Sum_{i=0}^{n} cs[[i + 1]] ChebyshevT[i, x]
```

In[26]:=
```
c2f[{1, 2, 3, 4, 5}, x] // Simplify
```

Out[26]=

$$3 - 10 x - 34 x^2 + 16 x^3 + 40 x^4$$

In[27]:=
```
f2c[f_] := Table[chebInt[f, ChebyshevT[i, #] &], {i, 0, n}]
```

In[28]:=
```
f2c[Sin]
```

Out[28]=

```
{0, 2 BesselJ[1, 1], 0, -2 BesselJ[3, 1], 0}
```

In[29]:=
```
cs = Table[c_i, {i, 0, n}]
```

Out[29]=

$$\{c_0, c_1, c_2, c_3, c_4\}$$

reverse   all subsets   permutations   rotate right   more...

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Untitled-7

```
In[19]:= Table[chebInt[ChebyshevT[i, #] &, ChebyshevT[j, #] &], {i, 0, n}, {j, 0, n}] // MatrixForm
```

Out[19]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[25]:= $\texttt{c2f[cs\_, x\_]} := \sum_{i=0}^{n} \texttt{cs[[i + 1]] ChebyshevT[i, x]}$

```
In[26]:= c2f[{1, 2, 3, 4, 5}, x] // Simplify
```

Out[26]=

$$3 - 10\,x - 34\,x^2 + 16\,x^3 + 40\,x^4$$

```
In[27]:= f2c[f_] := Table[chebInt[f, ChebyshevT[i, #] &], {i, 0, n}]

In[28]:= f2c[Sin]

Out[28]=
{0, 2 BesselJ[1, 1], 0, -2 BesselJ[3, 1], 0}
```

In[29]:= cs = Table[c_i, {i, 0, n}]

Out[29]=

$\{c_0, c_1, c_2, c_3, c_4\}$

```
In[30]:= f2c[c2f[cs, #] &]
```

Out[30]=

$\{2\,c_0, c_1, c_2, c_3, c_4\}$

reverse | all subsets | permutations | rotate right | more...

200%

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Untitled-7

In[25]:= $c2f[cs\_, x\_] := \sum_{i=0}^{n} cs[[i+1]]\, ChebyshevT[i, x]$

```
In[26]:= c2f[{1, 2, 3, 4, 5}, x] // Simplify
```

Out[26]=

$3 - 10\,x - 34\,x^2 + 16\,x^3 + 40\,x^4$

```
In[27]:= f2c[f_] := Table[chebInt[f, ChebyshevT[i, #] &], {i, 0, n}]

In[28]:= f2c[Sin]

Out[28]=
{0, 2 BesselJ[1, 1], 0, -2 BesselJ[3, 1], 0}
```

In[29]:= cs = Table[c_i, {i, 0, n}]

Out[29]=

$\{c_0, c_1, c_2, c_3, c_4\}$

```
In[30]:= f2c[c2f[cs, #] &]
```

Out[30]=

$\{2\,c_0, c_1, c_2, c_3, c_4\}$

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Untitled-7

```
In[25]:= c2f[cs_, x_] := Sum[cs[[i + 1]] ChebyshevT[i, x], {i, 0, n}]
```

$$\texttt{c2f[cs\_, x\_]} := \sum_{i=0}^{n} cs[[i+1]]\ \texttt{ChebyshevT}[i, x]$$

```
In[26]:= c2f[{1, 2, 3, 4, 5}, x] // Simplify
```

Out[26]=

$$3 - 10 x - 34 x^2 + 16 x^3 + 40 x^4$$

```
In[35]:= f2c[f_] := Table[If[i == 0, 1/2, 1] chebInt[f, ChebyshevT[i, #1] &], {i, 0, n}]

In[36]:= f2c[Sin]
```

Out[36]=

{0, 2 BesselJ[1, 1], 0, -2 BesselJ[3, 1], 0}

```
In[29]:= cs = Table[c_i, {i, 0, n}]
```

Out[29]=

$\{c_0, c_1, c_2, c_3, c_4\}$

```
In[37]:= f2c[c2f[cs, #] &]
```

Out[37]=

$\{c_0, c_1, c_2, c_3, c_4\}$

reverse all subsets permutations rotate right more...

200%

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Untitled-7

```
In[25]:= c2f[cs_, x_] := Sum[cs[[i + 1]] ChebyshevT[i, x], {i, 0, ...}]

In[26]:= c2f[{1, 2, 3, 4, 5}, x] // Simplify
```

Out[26]= $3 - 10\,x - 34\,x^2 + 16\,x^3 + 40\,x^4$

```
In[35]:= f2c[f_] := Table[If[i == 0, 1/2, 1] chebInt[f, ChebyshevT[i, #1] &], {i, 0, n}]

In[36]:= f2c[Sin]
```

Out[36]= {0, 2 BesselJ[1, 1], 0, -2 BesselJ[3, 1], 0}

```
In[29]:= cs = Table[c_i, {i, 0, n}]
```

Out[29]= $\{c_0, c_1, c_2, c_3, c_4\}$

```
In[37]:= f2c[c2f[cs, #] &]
```

Out[37]= $\{c_0, c_1, c_2, c_3, c_4\}$

Derivative Operator

```
In[38]:= D[c2f[cs, x], x]
```

Out[38]= $c_1 + 4\,x\,c_2 + \left(-3 + 12\,x^2\right) c_3 + \left(-16\,x + 32\,x^3\right) c_4$

plot simplify expand factor more...

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Untitled-7

```
In[35]:= f2c[f_] := Table[If[i == 0, 1/2, 1] ChebInt[f, ChebyshevT[i, #1] &], {i, 0, n}]

In[36]:= f2c[Sin]
Out[36]=
         {0, 2 BesselJ[1, 1], 0, -2 BesselJ[3, 1], 0}

In[29]:= cs = Table[c_i, {i, 0, n}]
Out[29]=
         {c_0, c_1, c_2, c_3, c_4}

In[37]:= f2c[c2f[cs, #] &]
Out[37]=
         {c_0, c_1, c_2, c_3, c_4}
```

Derivative Operator

```
In[40]:= f2c[Function[x, D[c2f[cs, x], x]]]
Out[40]=
         {c_1 + 3 c_3, 4 (c_2 + 2 c_4), 6 c_3, 8 c_4, 0}
```

Groebner basis | polynomial reduce | polynomial GCD | polynomial LCM | more...

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Untitled-7

In[35]:= f2c[f_] := Table[If[i == 0, 1/2, 1] ChebInt[f, ChebyshevT[i, #1] &], {i, 0, n}]

In[36]:= f2c[Sin]

Out[36]= {0, 2 BesselJ[1, 1], 0, -2 BesselJ[3, 1], 0}

In[29]:= cs = Table[c_i, {i, 0, n}]

Out[29]= {c_0, c_1, c_2, c_3, c_4}

In[37]:= f2c[c2f[cs, #] &]

Out[37]= {c_0, c_1, c_2, c_3, c_4}

Derivative Operator

In[41]:= dcoeffs = f2c[Function[x, D[c2f[cs, x], x]]]

Out[41]= {$c_1 + 3 c_3$, $4 (c_2 + 2 c_4)$, $6 c_3$, $8 c_4$, 0}

dd

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Untitled-7

In[36]:= `f2c[Sin]`

Out[36]=

$\{0,\ 2\,\mathrm{BesselJ}[1,1],\ 0,\ -2\,\mathrm{BesselJ}[3,1],\ 0\}$

In[29]:= `cs = Table[c_i, {i, 0, n}]`

Out[29]=

$\{c_0,\ c_1,\ c_2,\ c_3,\ c_4\}$

In[37]:= `f2c[c2f[cs, #] &]`

Out[37]=

$\{c_0,\ c_1,\ c_2,\ c_3,\ c_4\}$

Derivative Operator

In[41]:= `dcoeffs = f2c[Function[x, D[c2f[cs, x], x]]]`

Out[41]=

$\{c_1 + 3\,c_3,\ 4\,(c_2 + 2\,c_4),\ 6\,c_3,\ 8\,c_4,\ 0\}$

In[43]:= `dd = Table[Coefficient[dcoeffs[[j]], c_i], {i, 0, n}, {j, 0, n}];`

In[44]:= `dd // MatrixForm`

Out[44]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 3 & 0 & 6 & 0 \\ 0 & 0 & 8 & 0 & 8 \end{pmatrix}$$

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Untitled-7

```
In[36]:= f2c[Sin]
Out[36]=
{0, 2 BesselJ[1, 1], 0, -2 BesselJ[3, 1], 0}

In[29]:= cs = Table[c_i, {i, 0, n}]
Out[29]=
{c_0, c_1, c_2, c_3, c_4}

In[37]:= f2c[c2f[cs, #] &]
Out[37]=
{c_0, c_1, c_2, c_3, c_4}
```

Derivative Operator

```
In[41]:= dcoeffs = f2c[Function[x, D[c2f[cs, x], x]]]
Out[41]=
{c_1 + 3 c_3, 4 (c_2 + 2 c_4), 6 c_3, 8 c_4, 0}

In[47]:= dd = Table[Coefficient[dcoeffs[[j]], c_i], {i, 0, n}, {j, 0, n}];

In[48]:= dd // MatrixForm
Out[48]//MatrixForm=
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 3 & 0 & 6 & 0 \\ 0 & 0 & 8 & 0 & 8 \end{pmatrix}$$

Assuming a matrix | Use as *a list of lists* instead

200%

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In[37]:=
```
f2c[c2f[cs, #] &]
```
Out[37]=

$\{c_0, c_1, c_2, c_3, c_4\}$

Derivative Operator

In[41]:=
```
dcoeffs = f2c[Function[x, ∂x c2f[cs, x]]]
```
Out[41]=

$\{c_1 + 3\,c_3,\ 4\,(c_2 + 2\,c_4),\ 6\,c_3,\ 8\,c_4,\ 0\}$

In[51]:=
```
dd = Table[Coefficient[dcoeffs[[i + 1]], c_j], {i, 0, n}, {j, 0, n}];
```

In[52]:=
```
dd // MatrixForm
```
Out[52]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 4 & 0 & 8 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[53]:=
```
dd.cs // MatrixForm
```
Out[53]//MatrixForm=

$$\begin{pmatrix} c_1 + 3\,c_3 \\ 4\,c_2 + 8\,c_4 \\ 6\,c_3 \\ 8\,c_4 \\ 0 \end{pmatrix}$$

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Untitled-7

In[41]:= `dcoeffs = f2c[Function[x, ∂x c2f[cs, x]]]`

Out[41]=

$\{c_1 + 3\,c_3,\ 4\,(c_2 + 2\,c_4),\ 6\,c_3,\ 8\,c_4,\ 0\}$

In[51]:= `dd = Table[Coefficient[dcoeffs[[i + 1]], c_j], {i, 0, n}, {j, 0, n}];`

In[52]:= `dd // MatrixForm`

Out[52]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 4 & 0 & 8 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[53]:= `dd.cs // MatrixForm`

Out[53]//MatrixForm=

$$\begin{pmatrix} c_1 + 3\,c_3 \\ 4\,c_2 + 8\,c_4 \\ 6\,c_3 \\ 8\,c_4 \\ 0 \end{pmatrix}$$

In[54]:= `dd.dd // MatrixForm`

Out[54]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 32 \\ 0 & 0 & 0 & 24 & 0 \\ 0 & 0 & 0 & 0 & 48 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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Untitled-7

Out[53]//MatrixForm=

$$\begin{pmatrix} c_1 + 3\, c_3 \\ 4\, c_2 + 8\, c_4 \\ 6\, c_3 \\ 8\, c_4 \\ 0 \end{pmatrix}$$

In[54]:= `dd.dd // MatrixForm`

Out[54]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 32 \\ 0 & 0 & 0 & 24 & 0 \\ 0 & 0 & 0 & 0 & 48 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Heat Equation

In[55]:= `rhs[x_] = Sin[Pi x]`

In[60]:= `deq = f''[x] == rhs[x]`

Out[60]=

`f''[x] == Sin[π x]`

solve ode

200%

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Untitled-7

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 8 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[53]:= dd.cs // MatrixForm
```

Out[53]//MatrixForm=

$$\begin{pmatrix} c_1 + 3\,c_3 \\ 4\,c_2 + 8\,c_4 \\ 6\,c_3 \\ 8\,c_4 \\ 0 \end{pmatrix}$$

```
In[54]:= dd.dd // MatrixForm
```

Out[54]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 32 \\ 0 & 0 & 0 & 24 & 0 \\ 0 & 0 & 0 & 0 & 48 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Example: Sin

```
In[61]:= f2c[Sin]

Out[61]=
 {0, 2 BesselJ[1, 1], 0, -2 BesselJ[3, 1], 0}
```

Heat Equation

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Untitled-7

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 8 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[53]:= `dd.cs // MatrixForm`

Out[53]//MatrixForm=

$$\begin{pmatrix} c_1 + 3\, c_3 \\ 4\, c_2 + 8\, c_4 \\ 6\, c_3 \\ 8\, c_4 \\ 0 \end{pmatrix}$$

In[54]:= `dd.dd // MatrixForm`

Out[54]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 32 \\ 0 & 0 & 0 & 24 & 0 \\ 0 & 0 & 0 & 0 & 48 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Example: Sin

In[61]:= `f2c[Sin]`

Out[61]=

```
{0, 2 BesselJ[1, 1], 0, -2 BesselJ[3, 1], 0}
```

total   sort   mean   delete duplicates   more...

200%

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Untitled-7

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 8 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[53]:=
```
dd.cs // MatrixForm
```

Out[53]//MatrixForm=

$$\begin{pmatrix} c_1 + 3\,c_3 \\ 4\,c_2 + 8\,c_4 \\ 6\,c_3 \\ 8\,c_4 \\ 0 \end{pmatrix}$$

In[54]:=
```
dd.dd // MatrixForm
```

Out[54]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 32 \\ 0 & 0 & 0 & 24 & 0 \\ 0 & 0 & 0 & 0 & 48 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Example: Sin

In[62]:=
```
f2c[Sin] // N
```

Out[62]=
```
{0., 0.880101, 0., -0.0391267, 0.}
```

total sort mean delete duplicates more...

200%

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Untitled-7

Out[53]//MatrixForm=

$$\begin{pmatrix} c_1 + 3\,c_3 \\ 4\,c_2 + 8\,c_4 \\ 6\,c_3 \\ 8\,c_4 \\ 0 \end{pmatrix}$$

In[54]:= **dd.dd // MatrixForm**

Out[54]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 32 \\ 0 & 0 & 0 & 24 & 0 \\ 0 & 0 & 0 & 0 & 48 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Example: Sin

In[63]:= **dsin = f2c[Sin] // N**

Out[63]=

```
{0., 0.880101, 0., -0.0391267, 0.}
```

In[64]:= **dd.dsin**

Out[64]=

```
{0.762721, 0., -0.23476, 0., 0.}
```

f2c[Sin'] // N

Heat Equation

200%

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Untitled-7

Out[53]//MatrixForm=

$$\begin{pmatrix} c_1 + 3 c_3 \\ 4 c_2 + 8 c_4 \\ 6 c_3 \\ 8 c_4 \\ 0 \end{pmatrix}$$

In[54]:= `dd.dd // MatrixForm`

Out[54]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 32 \\ 0 & 0 & 0 & 24 & 0 \\ 0 & 0 & 0 & 0 & 48 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Example: Sin

In[63]:= `dsin = f2c[Sin] // N`

Out[63]=

{0., 0.880101, 0., -0.0391267, 0.}

In[64]:= `dd.dsin`

Out[64]=

{0.762721, 0., -0.23476, 0., 0.}

In[65]:= `f2c[Sin'] // N`

Out[65]=

{0.765198, 0., -0.229807, 0., 0.00495328}

total sort mean delete duplicates more...

200%

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Untitled-7

0

In[54]:=
```
dd.dd // MatrixForm
```

Out[54]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 32 \\ 0 & 0 & 0 & 24 & 0 \\ 0 & 0 & 0 & 0 & 48 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Example: Sin

In[63]:=
```
dsin = f2c[Sin] // N
```

Out[63]=
```
{0., 0.880101, 0., -0.0391267, 0.}
```

In[64]:=
```
dd.dsin
```

Out[64]=
```
{0.762721, 0., -0.23476, 0., 0.}
```

In[65]:=
```
f2c[Sin'] // N
```

Out[65]=
```
{0.765198, 0., -0.229807, 0., 0.00495328}
```

```
Plot[{Sin'[x], c2f[dd.dsin, x]}, {x, -1, +1}]
```

Heat Equation

200%

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Untitled-7

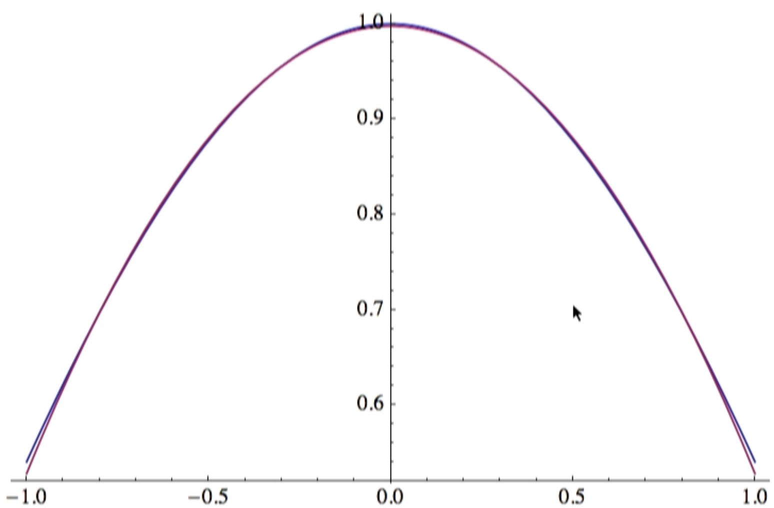
Example: Sin

In[63]:=
```
dsin = f2c[Sin] // N
```
Out[63]=
```
{0., 0.880101, 0., -0.0391267, 0.}
```

In[64]:=
```
dd.dsin
```
Out[64]=
```
{0.762721, 0., -0.23476, 0., 0.}
```

In[65]:=
```
f2c[Sin'] // N
```
Out[65]=
```
{0.765198, 0., -0.229807, 0., 0.00495328}
```

In[66]:=
```
Plot[{Sin'[x], c2f[dd.dsin, x]}, {x, -1, +1}]
```
Out[66]=



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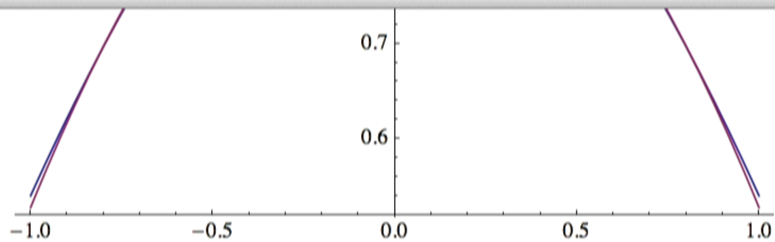
Untitled-7



Heat Equation

In[20]:= `rhs[x_] = Sin[Pi x]`

Out[20]= `Sin[π x]`

In[21]:= `deq = f''[x] == rhs[x]`

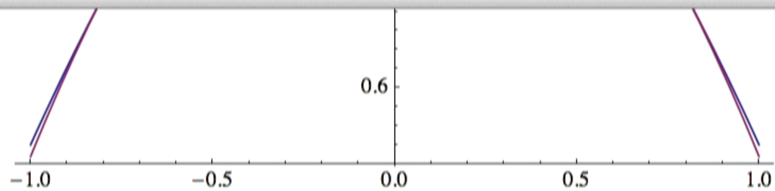
Out[21]= `f''[x] == Sin[π x]`

`bcs = {f[-1] ==`

200%

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-7



Heat Equation

In[20]:= rhs[x_] = Sin[Pi x]

Out[20]= Sin[π x]

In[21]:= deq = f''[x] == rhs[x]

Out[21]= f''[x] == Sin[π x]

In[22]:= bcs = {f[-1] == 1/10, f[+1] == 0}

Out[22]= {f[-1] == 1/10, f[1] == 0}

In[23]:= DSolve[Join[{deq}, bcs], f, x]

Out[23]= $\left\{\left\{f \to \text{Function}\left[\{x\}, \frac{\pi^2 - \pi^2 x - 20\,\text{Sin}[\pi x]}{20\,\pi^2}\right]\right\}\right\}$

Assuming a list of rules | Use as a two-dimensional array instead

get solution | apply rules to variable | apply rules to expr... | convert rules to lists

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Untitled-7

```
In[21]:= deq = f''[x] == rhs[x]

Out[21]=
```

$$f''[x] == \mathrm{Sin}[\pi\, x]$$

```
In[22]:= bcs = {f[-1] == 1/10, f[+1] == 0}

Out[22]=
```

$$\left\{f[-1] == \frac{1}{10},\ f[1] == 0\right\}$$

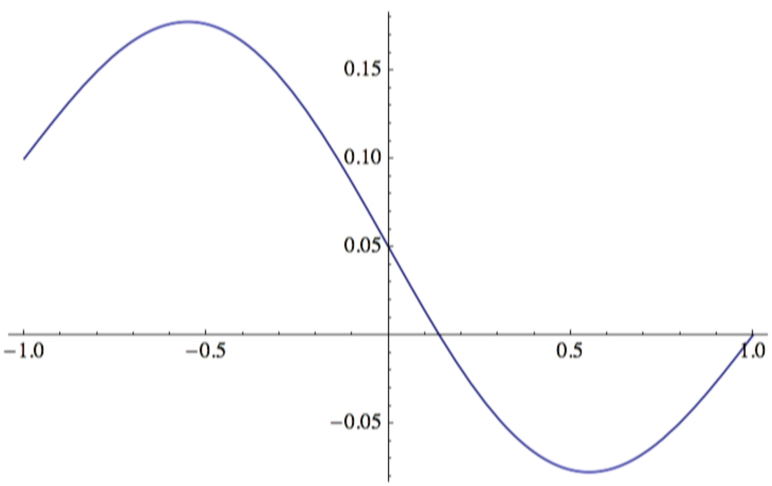
```
In[24]:= asol = DSolve[Join[{deq}, bcs], f, x]

Out[24]=
```

$$\left\{\left\{f \to \mathrm{Function}\left[\{x\}, \frac{\pi^2 - \pi^2\, x - 20\, \mathrm{Sin}[\pi\, x]}{20\, \pi^2}\right]\right\}\right\}$$

```
In[26]:= Plot[f[x] /. asol[[1]], {x, -1, +1}]

Out[26]=
```



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Out[16]=

```
{0., 0.880101, 0., -0.0391267, 0.}
```

In[17]:=

```
dd.dsin
```

Out[17]=

```
{0.762721, 0., -0.23476, 0., 0.}
```

In[18]:=

```
f2c[Sin'] // N
```

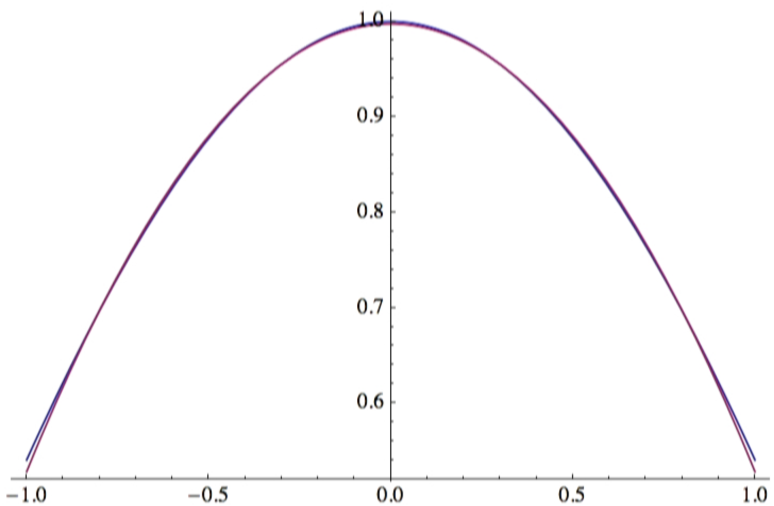
Out[18]=

```
{0.765198, 0., -0.229807, 0., 0.00495328}
```

In[19]:=

```
Plot[{Sin'[x], c2f[dd.dsin, x]}, {x, -1, +1}]
```

Out[19]=



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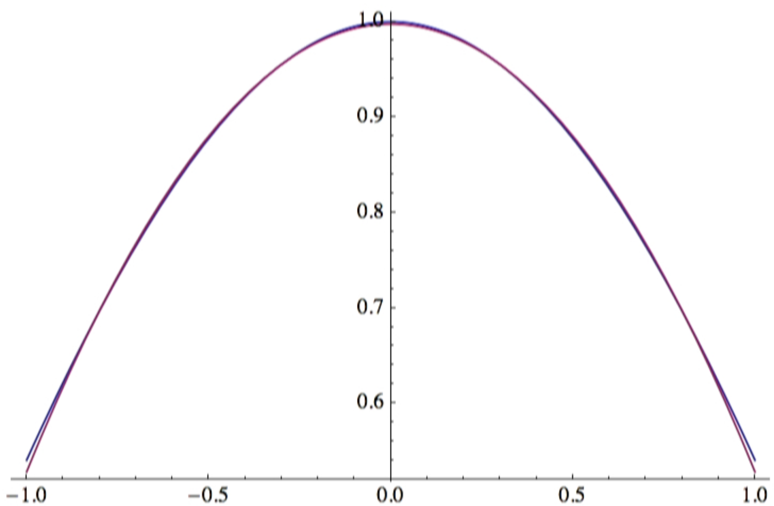
Untitled-7

{0.765198, 0., -0.229807, 0., 0.00495328}

In[19]:= `Plot[{Sin'[x], c2f[dd.dsin, x]}, {x, -1, +1}]`

Out[19]=



Boundary conditions

```
bcoeffs =
```

```
dcoeffs = f2c[Function[x, ∂x c2f[cs, x]]]
```

Heat Equation

In[20]:= `rhs[x_] = Sin[Pi x]`

200%

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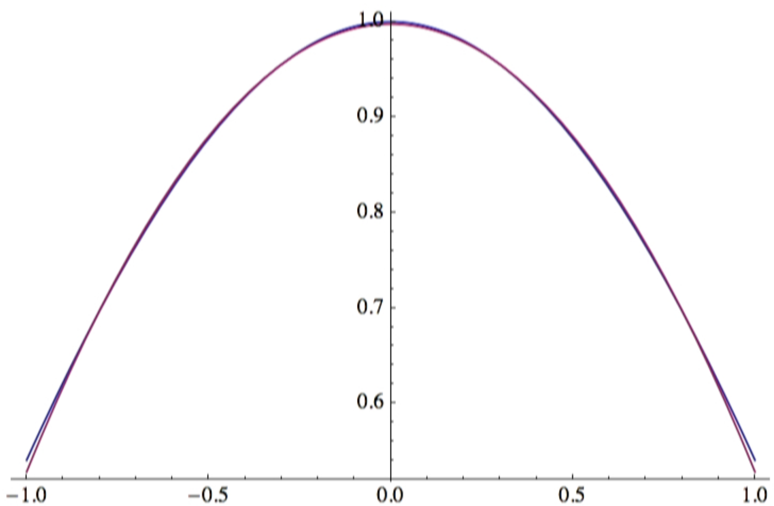
Untitled-7

```
{0.765198, 0., -0.229807, 0., 0.00495328}
```

```
In[19]:= Plot[{Sin'[x], c2f[dd.dsin, x]}, {x, -1, +1}]
```

Out[19]=



Boundary conditions

```
bcoeffLeft =
```

```
dcoeffs = f2c[Function[x, ∂x c2f[cs, x]]]
```

Heat Equation

```
In[20]:= rhs[x_] = Sin[Pi x]
```

200%

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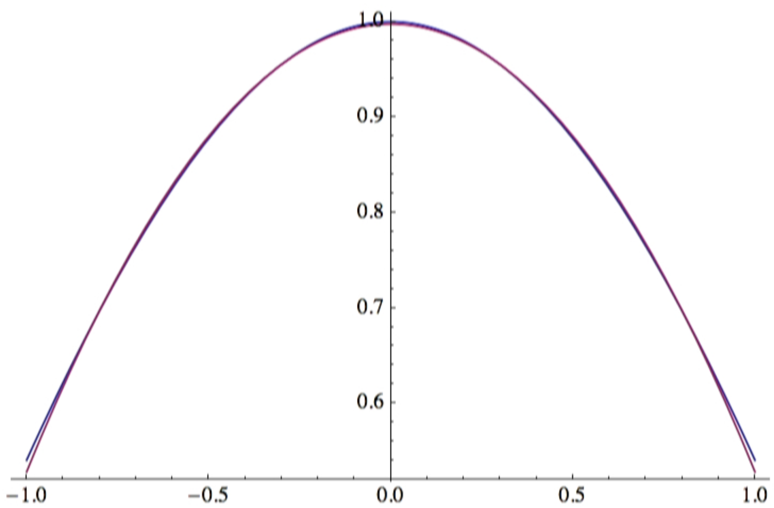
Untitled-7

```
{0.765198, 0., -0.229807, 0., 0.00495328}
```

In[19]:= `Plot[{Sin'[x], c2f[dd.dsin, x]}, {x, -1, +1}]`

Out[19]=



Boundary conditions

```
bcoeffLeft = Function[x, c2f[cs, x]]
```

```
dcoeffs = f2c[Function[x, ∂x c2f[cs, x]]]
```

Heat Equation

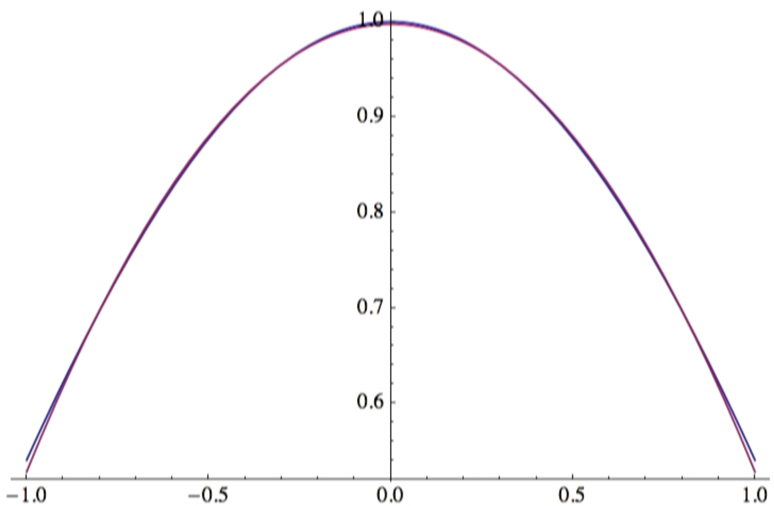
In[20]:= `rhs[x_] = Sin[Pi x]`

200%

{0.765198, 0., -0.229807, 0., 0.00495328}

In[19]:=
```
Plot[{Sin'[x], c2f[dd.dsin, x]}, {x, -1, +1}]
```

Out[19]=



Boundary conditions

In[27]:=
```
bcoeffLeft = c2f[cs, 0]
```

Out[27]=

$c_0 - c_2 + c_4$

```
dcoeffs = f2c[Function[x, ∂x c2f[cs, x]]]
```

Heat Equation

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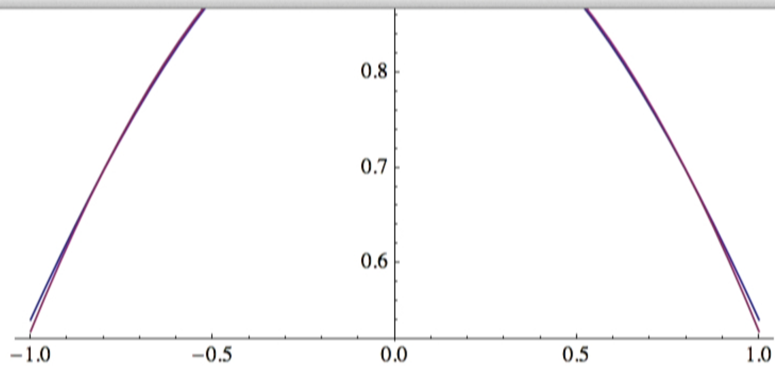
Untitled-7



Boundary conditions

In[28]:=
```
bcoeffLeft = c2f[cs, -1]
```

Out[28]=

$c_0 - c_1 + c_2 - c_3 + c_4$

In[29]:=
```
bcoeffRight = c2f[cs, +1]
```

Out[29]=

$c_0 + c_1 + c_2 + c_3 + c_4$

c_0 derivative | c_0 integral | grad | series around $c_0 = 0$ | more...

```
dcoeffs = f2c[Function[x, ∂x c2f[cs, x]]]
```

Heat Equation

In[20]:=
```
rhs[x_] = Sin[Pi x]
```

200%

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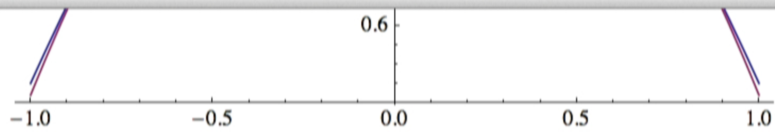
Untitled-7



Boundary conditions

In[28]:= `bcoeffLeft = c2f[cs, -1]`

Out[28]=

$c_0 - c_1 + c_2 - c_3 + c_4$

In[29]:= `bcoeffRight = c2f[cs, +1]`

Out[29]=

$c_0 + c_1 + c_2 + c_3 + c_4$

In[32]:= `bbLeft = Table[Coefficient[bcoeffLeft, c_j], {j, 0, n}];`

In[33]:= `bbLeft // MatrixForm`

Out[33]//MatrixForm=

$$\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$$

In[34]:= `bbRight = Table[Coefficient[bcoeffRight, c_j], {j, 0, n}];`

Heat Equation

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Untitled-7

$$\begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$$

In[34]:=
```
bbRight = Table[Coefficient[bcoeffRight, c_j], {j, 0, n}];
```

In[35]:=
```
bbRight // MatrixForm
```

Out[35]//MatrixForm=

$$\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

```
bb1 = Array[0 &, {n - 2 + 1, n + 1}]
```

Heat Equation

In[20]:=
```
rhs[x_] = Sin[Pi x]
```

Out[20]=

```
Sin[π x]
```

In[21]:=
```
deq = f''[x] == rhs[x]
```

Out[21]=

```
f''[x] == Sin[π x]
```

In[22]:=
```
bcs = {f[-1] == 1/.., f[+1] == 0}
```

200%

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Untitled-7

$$\begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$$

In[34]:=
```
bbRight = Table[Coefficient[bcoeffRight, c_j], {j, 0, n}];
```

In[35]:=
```
bbRight // MatrixForm
```

Out[35]//MatrixForm=

$$\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

In[36]:=
```
bb1 = Join[Array[0 &, {n - 2 + 1, n + 1}], {bbLeft, bbRight}]
```

Out[36]=
```
{{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {1, -1, 1, -1, 1}, {1, 1, 1, 1, 1}}
```

In[37]:=
```
bb1 // MatrixForm
```

Out[37]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Assuming a matrix | Use as *a list of lists* instead

determinant | matrix plot | matrix rank | eigenvalues | *more...*

200%

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Untitled-7

```
In[34]:= bbRight = Table[Coefficient[bcoeffRight, c_j], {j, 0, n}];

In[35]:= bbRight // MatrixForm
```

Out[35]//MatrixForm=

$$\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

```
In[36]:= bb1 = Join[Array[0 &, {n - 2 + 1, n + 1}], {bbLeft, bbRight}]

Out[36]=
{{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {1, -1, 1, -1, 1}, {1, 1, 1, 1, 1}}

In[37]:= bb1 // MatrixForm
```

Out[37]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

```
In[38]:= dd.dd // MatrixForm
```

Out[38]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 32 \\ 0 & 0 & 0 & 24 & 0 \\ 0 & 0 & 0 & 0 & 48 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

200%

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Untitled-7

In[34]:= **bbRight = Table[Coefficient[bcoeffRight, c_j], {j, 0, n}];**

In[35]:= **bbRight // MatrixForm**

Out[35]//MatrixForm=

$$\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

In[39]:= **bb1 = Join[Array[0, {n - 2 + 1, n + 1}], {bbLeft, bbRight}]**

Out[39]=

```
{{0[1, 1], 0[1, 2], 0[1, 3], 0[1, 4], 0[1, 5]}, {0[2, 1], 0[2, 2], 0[2, 3], 0[2, 4], 0[2, 5]},
 {0[3, 1], 0[3, 2], 0[3, 3], 0[3, 4], 0[3, 5]}, {1, -1, 1, -1, 1}, {1, 1, 1, 1, 1}}
```

Assuming a two-dimensional array | Use as a *list of lists* instead

dimensions transpose flatten reverse sublists *more...*

In[37]:= **bb1 // MatrixForm**

Out[37]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

In[38]:= **dd.dd // MatrixForm**

Out[38]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 32 \\ 0 & 0 & 0 & 24 & 0 \\ 0 & 0 & 0 & 0 & 48 \end{pmatrix}$$

200%

Out[35]//MatrixForm=

$$\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

```
In[41]:= bb1 = Join[Table[0, {i, 0, n - 2}, {j, 0, n}], {bbLeft, bbRight}]

Out[41]= {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {1, -1, 1, -1, 1}, {1, 1, 1, 1, 1}}

In[42]:= bb1 // MatrixForm
```

Out[42]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Assuming a matrix | Use as *a list of lists* instead

determinant | matrix plot | matrix rank | eigenvalues | *more...*

```
In[38]:= dd.dd // MatrixForm
```

Out[38]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 32 \\ 0 & 0 & 0 & 24 & 0 \\ 0 & 0 & 0 & 0 & 48 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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Untitled-7

Heat Equation

In[20]:=
```
rhs[x_] = Sin[Pi x]
```
Out[20]=

$\text{Sin}[\pi\, x]$

In[21]:=
```
deq = f''[x] == rhs[x]
```
Out[21]=

$f''[x] == \text{Sin}[\pi\, x]$

In[22]:=
```
bcs = {f[-1] == 1/10, f[+1] == 0}
```
Out[22]=

$\left\{f[-1] == \frac{1}{10},\ f[1] == 0\right\}$

Analytic Solution

In[24]:=
```
asol = DSolve[Join[{deq}, bcs], f, x]
```
Out[24]=

$\left\{\left\{f \to \text{Function}\left[\{x\},\ \frac{\pi^2 - \pi^2 x - 20\,\text{Sin}[\pi\, x]}{20\,\pi^2}\right]\right\}\right\}$

In[26]:=
```
Plot[f[x] /. asol[[1]], {x, -1, +1}]
```
Out[26]=

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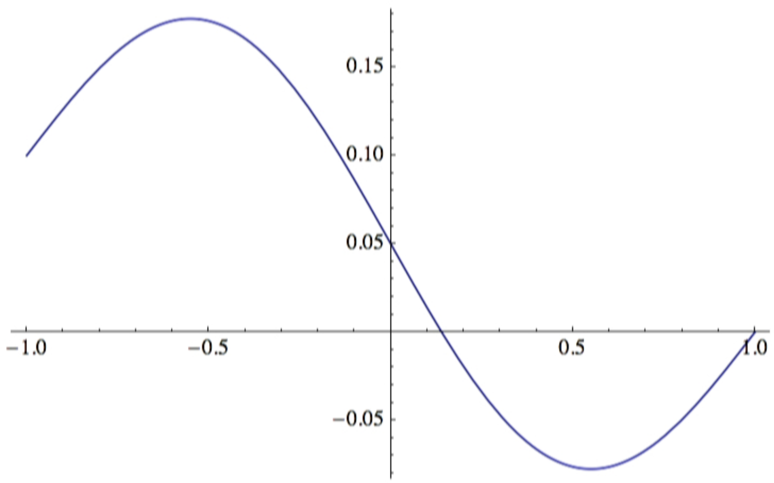
Untitled-7

In[26]:=
```
Plot[f[x] /. asol[[1]], {x, -1, +1}]
```

Out[26]=



Numeric Solution

In[43]:=
```
drhs = f2c[rhs]
```

Out[43]=
```
{0, 2 BesselJ[1, π], 0, -2 BesselJ[3, π], 0}
```

total sort mean delete duplicates more...

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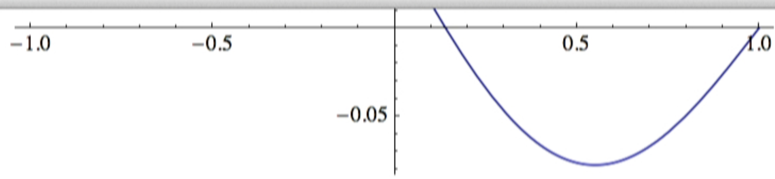
Untitled-7



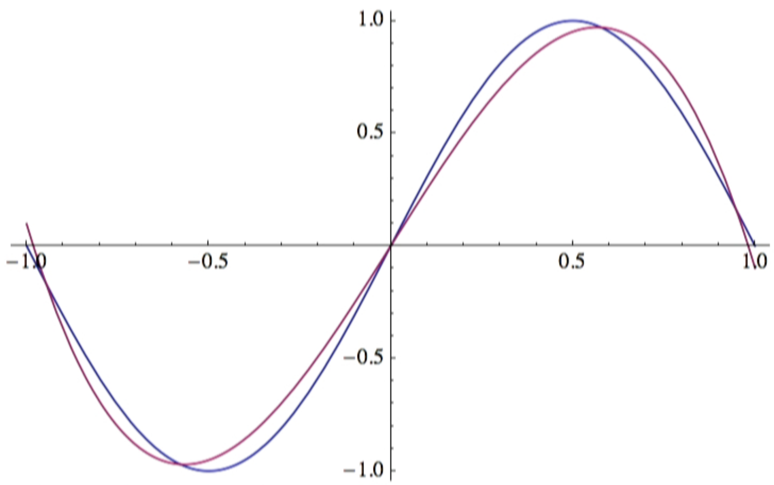
Numeric Solution

In[43]:=
```
drhs = f2c[rhs]
```

Out[43]=
```
{0, 2 BesselJ[1, π], 0, -2 BesselJ[3, π], 0}
```

In[44]:=
```
Plot[{rhs[x], c2f[drhs, x]}, {x, -1, +1}]
```

Out[44]=



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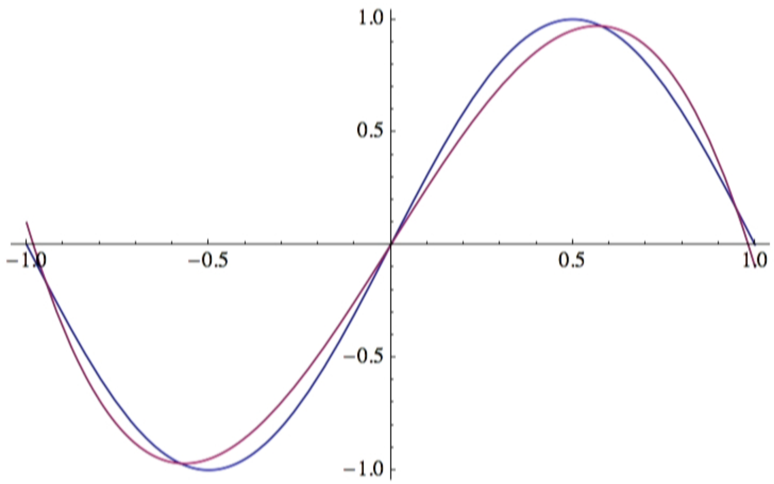
Untitled-7

Numeric Solution

In[43]:=
```
drhs = f2c[rhs]
```
Out[43]=
```
{0, 2 BesselJ[1, π], 0, -2 BesselJ[3, π], 0}
```

In[44]:=
```
Plot[{rhs[x], c2f[drhs, x]}, {x, -1, +1}]
```
Out[44]=




```
LinearSolve[dd.dd + bb1,
```

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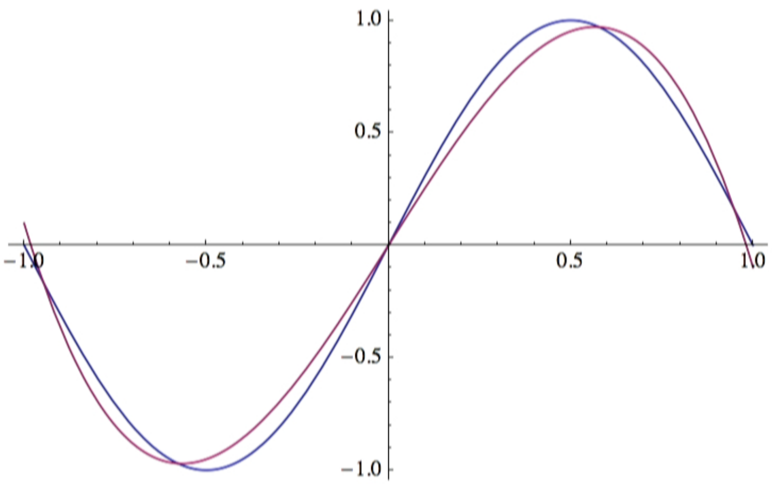
Untitled-7

```
{0, 2 BesselJ[1, π], 0, -2 BesselJ[3, π], 0}

In[44]:= Plot[{rhs[x], c2f[drhs, x]}, {x, -1, +1}]

Out[44]=
```




```
In[47]:= drhs1 = Join[Take[drhs, n - 2 + 1], {-1 / 10, 0}]

Out[47]=
```

$$\left\{0,\ 2\,\text{BesselJ}[1,\pi],\ 0,\ -\frac{1}{10},\ 0\right\}$$

```
In[48]:= LinearSolve[dd.dd + bb1, drhs1]

Out[48]=
```

$$\left\{-\frac{1}{20},\ \frac{1}{20}-\frac{1}{12}\,\text{BesselJ}[1,\pi],\ 0,\ \frac{1}{12}\,\text{BesselJ}[1,\pi],\ 0\right\}$$

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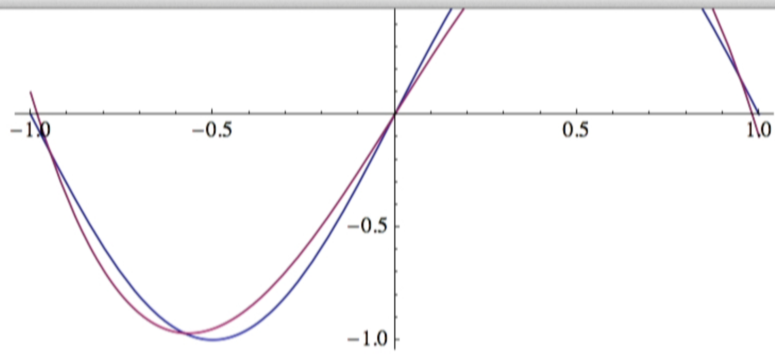
Untitled-7



In[47]:=
```
drhs1 = Join[Take[drhs, n - 2 + 1], {-1 / 10, 0}]
```

Out[47]=

$$\left\{0,\ 2\,\text{BesselJ}[1,\pi],\ 0,\ -\frac{1}{10},\ 0\right\}$$

In[49]:=
```
nsol = LinearSolve[dd.dd + bb1, drhs1]
```

Out[49]=

$$\left\{-\frac{1}{20},\ \frac{1}{20}-\frac{1}{12}\,\text{BesselJ}[1,\pi],\ 0,\ \frac{1}{12}\,\text{BesselJ}[1,\pi],\ 0\right\}$$

In[50]:=
```
nsol // N // MatrixForm
```

Out[50]//MatrixForm=

$$\begin{pmatrix} -0.05 \\ 0.0262821 \\ 0. \\ 0.0237179 \\ 0. \end{pmatrix}$$

total sort mean delete duplicates more...

200%

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Untitled-7

In[53]:= **nsol = LinearSolve[dd.dd + bb1, drhs1]**

Out[53]=

$$\left\{\frac{1}{20}, -\frac{1}{20} - \frac{1}{12}\,\text{BesselJ}[1, \pi], 0, \frac{1}{12}\,\text{BesselJ}[1, \pi], 0\right\}$$

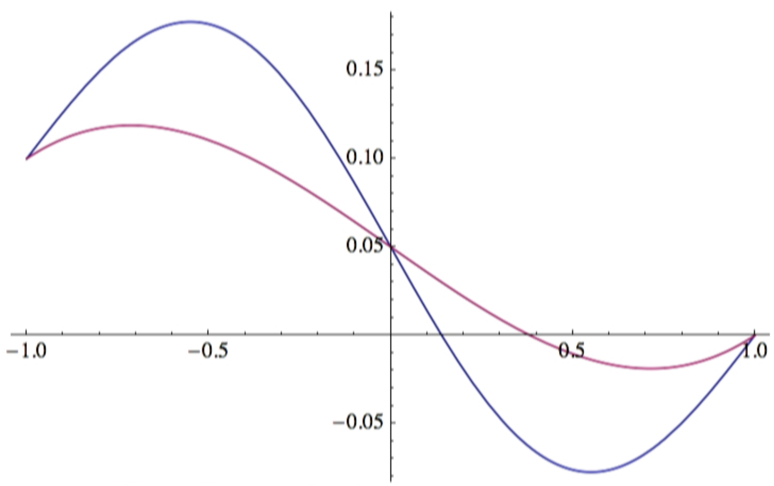
In[54]:= **nsol // N // MatrixForm**

Out[54]//MatrixForm=

$$\begin{pmatrix} 0.05 \\ -0.0737179 \\ 0. \\ 0.0237179 \\ 0. \end{pmatrix}$$

In[55]:= **Plot[{f[x] /. asol⟦1⟧, c2f[nsol, x]}, {x, -1, +1}]**

Out[55]=



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Untitled-7

```
In[20]:= rhs[x_] = Sin[Pi x]
Out[20]= Sin[π x]

In[56]:= rhs[x_] = If[x ≥ 0 && x ≤ 1 / 2, 1, 0]
Out[56]= If[x ≥ 0 && x ≤ 1/2, 1, 0]

In[57]:= deq = f''[x] == rhs[x]
Out[57]= f''[x] == If[x ≥ 0 && x ≤ 1/2, 1, 0]

In[58]:= bcs = {f[-1] == 1/10, f[+1] == 0}
Out[58]= {f[-1] == 1/10, f[1] == 0}
```

Analytic Solution

```
In[24]:= asol = DSolve[Join[{deq}, bcs], f, x]
```

Out[24]=

$$\left\{\left\{f \to \text{Function}\left[\{x\}, \frac{\pi^2 - \pi^2 x - 20\,\text{Sin}[\pi x]}{20\,\pi^2}\right]\right\}\right\}$$

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Untitled-7

$$\left\{f[-1] == \frac{1}{10},\ f[1] == 0\right\}$$

Analytic Solution

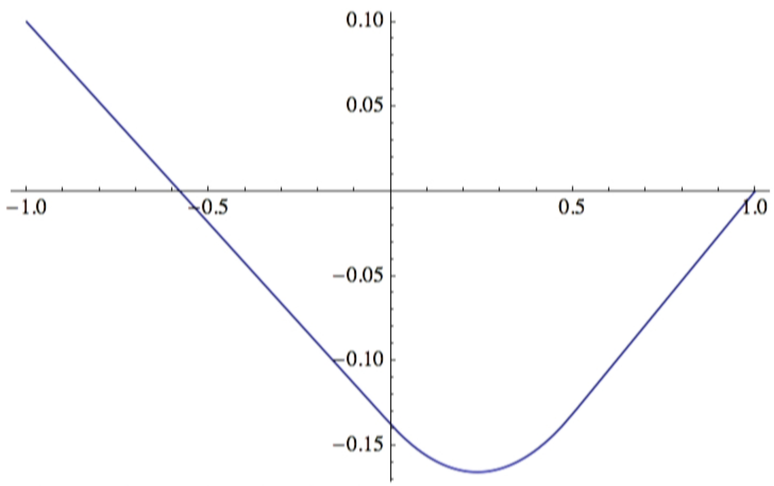
In[59]:= `asol = DSolve[Join[{deq}, bcs], f, x]`

Out[59]=

$$\left\{\left\{f \to \text{Function}\left[\{x\},\ \frac{1}{80}\left(-11 - 19\,x + 80\left(\begin{cases} 0 & x \le 0 \\ \frac{x^2}{2} & 0 < x \le \frac{1}{2} \\ -\frac{1}{8} + \frac{x}{2} & \text{True} \end{cases}\right)\right)\right]\right\}\right\}$$

In[60]:= `Plot[f[x] /. asol[[1]], {x, -1, +1}]`

Out[60]=



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Untitled-7

Numeric Solution

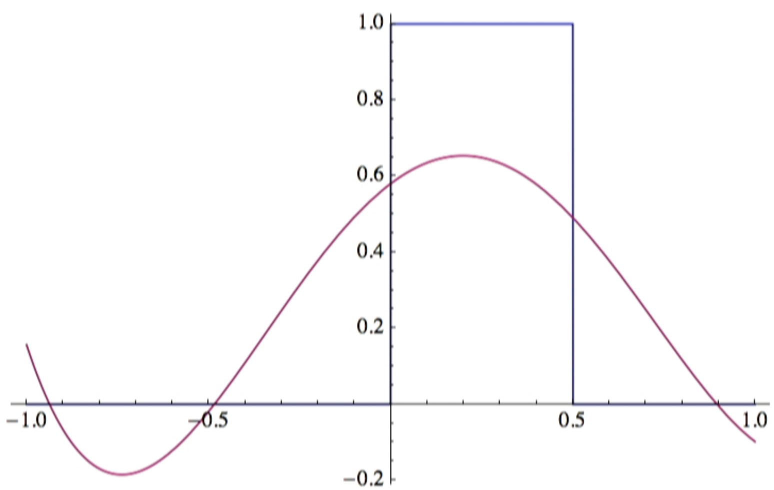
In[61]:= `drhs = f2c[rhs]`

Out[61]=

$$\left\{\frac{1}{6}, \frac{2\left(1-\frac{\sqrt{3}}{2}\right)}{\pi}, -\frac{\sqrt{3}}{2\,\pi}, -\frac{2}{3\,\pi}, \frac{\sqrt{3}}{4\,\pi}\right\}$$

In[62]:= `Plot[{rhs[x], c2f[drhs, x]}, {x, -1, +1}]`

Out[62]=



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Untitled-7

In[64]:= `nsol = LinearSolve[dd.dd + bb1, drhs1]`

Out[64]=

$$\left\{\frac{1}{120}-\frac{7}{32\sqrt{3}\,\pi},\ -\frac{1}{20}-\frac{1}{12\,\pi}+\frac{1}{8\sqrt{3}\,\pi},\ \frac{1}{24}+\frac{1}{4\sqrt{3}\,\pi},\ \frac{2-\sqrt{3}}{24\,\pi},\ -\frac{1}{32\sqrt{3}\,\pi}\right\}$$

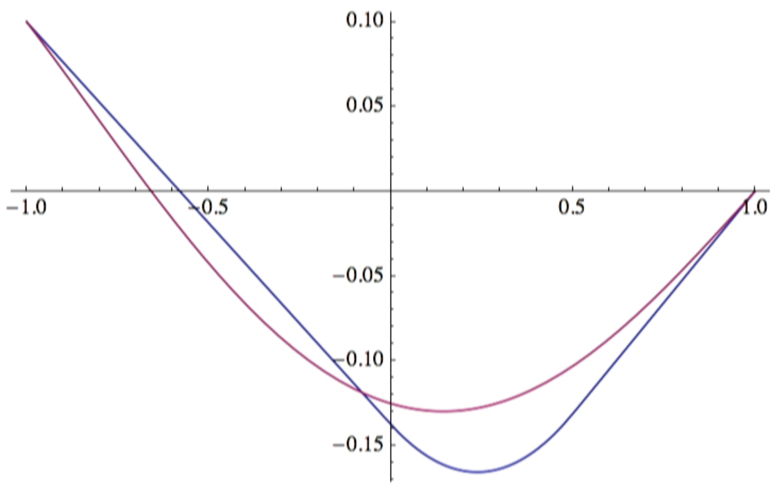
In[65]:= `nsol // N // MatrixForm`

Out[65]//MatrixForm=

$$\begin{pmatrix} -0.0318677 \\ -0.0535538 \\ 0.0876107 \\ 0.00355379 \\ -0.00574301 \end{pmatrix}$$

In[66]:= `Plot[{f[x] /. asol[[1]], c2f[nsol, x]}, {x, -1, +1}]`

Out[66]=



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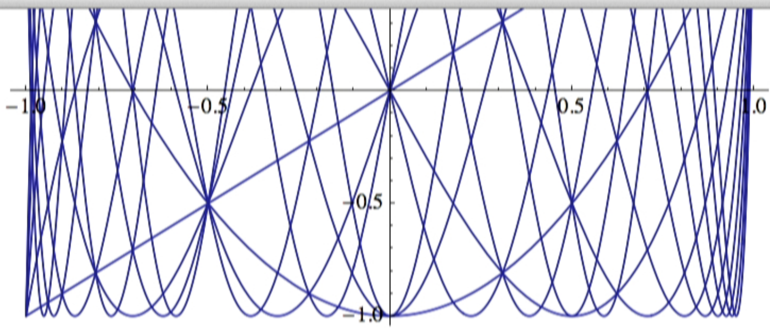
Untitled-7



In[3]:=
$$\texttt{chebInt}[f\_,\ g\_] := \frac{2 \int_{-1}^{+1} \frac{f[x]\, g[x]}{\sqrt{1-x^2}}\, dx}{\pi}$$

In[4]:= `Table[chebInt[ChebyshevT[i, #] &, ChebyshevT[j, #] &], {i, 0, n}, {j, 0, n}] // MatrixForm`

Out[4]//MatrixForm=
$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[5]:=
$$\texttt{c2f}[cs\_,\ x\_] := \sum_{i=0}^{n} cs[[i+1]]\ \texttt{ChebyshevT}[i,\ x]$$

In[6]:= `c2f[{1, 2, 3, 4, 5}, x] // Simplify`

Out[6]= $3 - 10\, x - 34\, x^2 + 16\, x^3 + 40\, x^4$

In[7]:=
$$\texttt{f2c}[f\_] := \texttt{Table}\left[\texttt{If}\left[i == 0,\ \frac{1}{2},\ 1\right] \texttt{chebInt}[f,\ \texttt{ChebyshevT}[i,\ \#1] \&],\ \{i,\ 0,\ n\}\right]$$

200%

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In[70]:= `dcoeffs = f2c[Function[x, ∂_x c2f[cs, x]]]`

Out[70]=

$\{c_1 + 3 c_3 + 5 c_5 + 7 c_7 + 9 c_9,\ 4 (c_2 + 2 c_4 + 3 c_6 + 4 c_8 + 5 c_{10}),\ 2 (3 c_3 + 5 c_5 + 7 c_7 + 9 c_9),\ 4 (2 c_4 + 3 c_6 + 4 c_8 + 5 c_{10}),\ 2 (5 c_5 + 7 c_7 + 9 c_9),\ 4 (3 c_6 + 4 c_8 + 5 c_{10}),\ 2 (7 c_7 + 9 c_9),\ 4 (4 c_8 + 5 c_{10}),\ 18 c_9,\ 20 c_{10},\ 0\}$

In[71]:= `dd = Table[Coefficient[dcoeffs[[i + 1]], c_j], {i, 0, n}, {j, 0, n}];`

In[72]:= `dd // MatrixForm`

Out[72]//MatrixForm=

$$\begin{pmatrix}
0 & 1 & 0 & 3 & 0 & 5 & 0 & 7 & 0 & 9 & 0\\
0 & 0 & 4 & 0 & 8 & 0 & 12 & 0 & 16 & 0 & 20\\
0 & 0 & 0 & 6 & 0 & 10 & 0 & 14 & 0 & 18 & 0\\
0 & 0 & 0 & 0 & 8 & 0 & 12 & 0 & 16 & 0 & 20\\
0 & 0 & 0 & 0 & 0 & 10 & 0 & 14 & 0 & 18 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 16 & 0 & 20\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 14 & 0 & 18 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 20\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 18 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Assuming a matrix | Use as *a list of lists* instead

determinant matrix rank eigenvalues dimensions *more...*

In[14]:= `dd.cs // MatrixForm`

Out[14]//MatrixForm=

$$\begin{pmatrix}
c_1 + 3 c_3\\
4 c_2 + 8 c_4\\
6 c_3\\
8 c_4\\
0
\end{pmatrix}$$

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Untitled-7

Numeric Solution

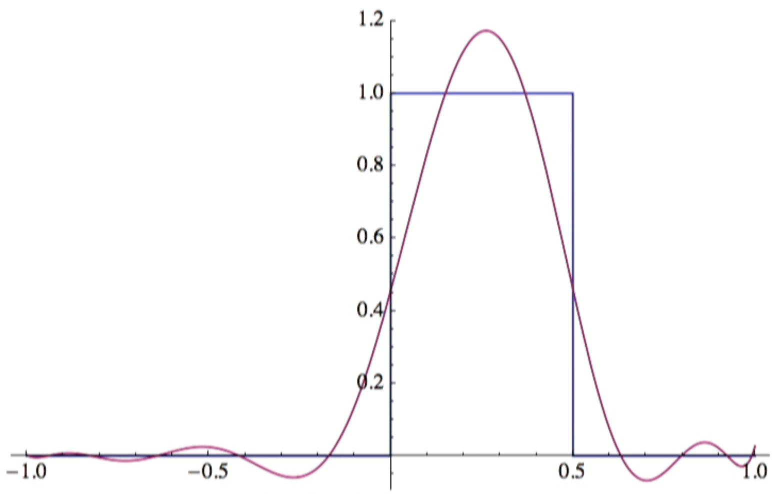
In[82]:=
```
drhs = f2c[rhs]
```

Out[82]=

$$\left\{\frac{1}{6}, \frac{2\left(1-\frac{\sqrt{3}}{2}\right)}{\pi}, -\frac{\sqrt{3}}{2\,\pi}, -\frac{2}{3\,\pi}, \frac{\sqrt{3}}{4\,\pi}, \frac{2+\sqrt{3}}{5\,\pi}, 0, \frac{-2-\sqrt{3}}{7\,\pi}, -\frac{\sqrt{3}}{8\,\pi}, \frac{2}{9\,\pi}, \frac{\sqrt{3}}{10\,\pi}\right\}$$

In[83]:=
```
Plot[{rhs[x], c2f[drhs, x]}, {x, -1, +1}]
```

Out[83]=



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Untitled-7

In[86]:= `nsol // N // MatrixForm`

Out[86]//MatrixForm=

$$\begin{pmatrix} -0.0341102 \\ -0.0653426 \\ 0.0933538 \\ 0.0217665 \\ -0.0103374 \\ -0.00901659 \\ 0.000738387 \\ 0.00318201 \\ 0.000546953 \\ -0.00058926 \\ -0.000191434 \end{pmatrix}$$

In[87]:= `Plot[{f[x] /. asol[[1]], c2f[nsol, x]}, {x, -1, +1}]`

Out[87]=