

Title: Slippery Waves: Brilliance Brings Blind Spots

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Abstract: Superfluidity and superconductivity are two remarkable phenomena in which, at low temperatures, materials abruptly gain the ability to flow without friction. Microscopic quantum theories of these phases of matter were constructed in blockbuster papers of Lev Landau (1940) and John Bardeen, Leon Cooper, and J. Robert Schrieffer (1957). The actual explanation of the flow, however, is rooted in a Einstein paper of 1924 that introduces a condensate, a quantum configuration describing a finite fraction of the particles in the system.

Superfluidity can then be understood in terms of the wave function for this configuration, which necessarily extends over a finite fraction of the system. Neither blockbuster paper mentions Einstein or the crucial idea of a condensate wave function. The reasons for this omission are mooted.

Bose Particles, Bose-Einstein transition

In 1924, a completely unknown physicist, **Satyendra Nath Bose**, wrote to **Albert Einstein** pointing out one of the possible generic behaviors of quantum particles. Einstein himself translated Bose's paper into German and got it published. Then in 1925, Einstein pointed out a possible phase transition in a fluid composed of non-interacting quantum particles of the type Bose had proposed. The phase transition, now known as a Bose-Einstein transition, was caused by having a finite fraction of the entire number of particles in the system falling into a single quantum mode.

In 1925, **Schrödinger** invented the wave function. Einstein's macroscopically occupied mode could then be understood as being described by a single wave function, $\Psi(\mathbf{r})$. This wave function would necessarily be extended over the entire system.

Bose (1924). "Plancks Gesetz und Lichtquantenhypothese", Zeitschrift für Physik 26:178–181.
(Einstein's translation into German of Bose's paper on Planck's law).

Einstein, A. "Quantentheorie des einatomigen idealen Gases". Sitzungsberichte der Preussischen Akademie der Wissenschaften I: 3. (1925). (Einstein's paper on Bose-Einstein phase transition.)

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Einstein's phase transition idea was not believed, because

- The idea was too strange. No wave function could do that! Never??
- There was no obvious way that this transition could occur in nature, since most Bose liquids were believed to be too dilute for important quantum effects
- **George Uhlenbeck**, a young specialist on phase transitions, wrote a thesis in which he said that because this phase transition could only occur in an infinite system, it therefore could not occur*. Uhlenbeck was convincing but utterly wrong.

Thus everyone, including Einstein, ignored the idea of a Bose-Einstein transition.

* published in UEHLING E.A. and UHLENBECK G.E., Phys. Rev., 43 (1933) 552.

retracted in KAHN B. and UHLENBECK G.E., Physica, 5 (1938) 399.

Superconductivity: Waterloo, Ontario 9/15/13

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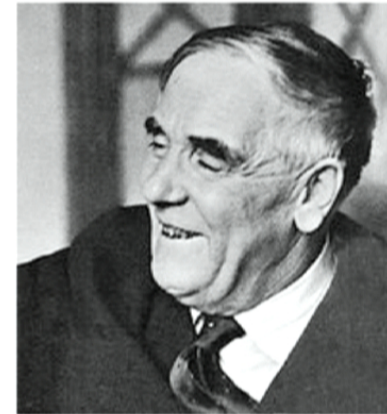
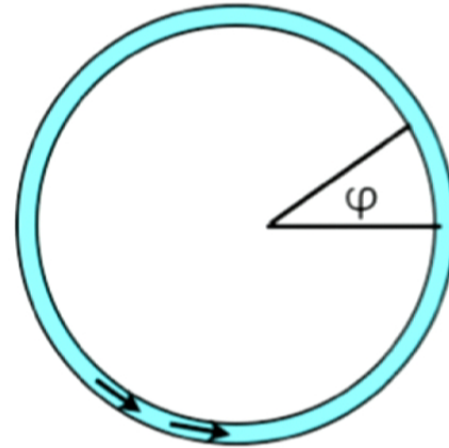
But then Superfluidity was discovered

In 1938, an amazing behavior of liquid Helium (mostly composed of the isotope, ^4He , a boson) was discovered. In that year, in the USSR and England, **Pyotr Kapitsa** and **John F. Allen, & Don Misener (Canadians)** experimentally showed that, when Helium was cooled below 2.17 degrees Kelvin, it undergoes a sudden change in behavior, a phase transition. Below this critical temperature, it would flow without friction through a small crack, as a thin film, or around a doughnut-shaped ring without any force pushing it. This new behavior was called superfluidity.

Kapitsa, P. (1938). "Viscosity of liquid helium below the λ -point". Nature 141 (3558): 74

Allen, J. F.; Misener, A. D. (1938). "Flow of Liquid Helium II". Nature 142 (3597): 643.

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Pyotr Kapitsa

Two-Fluids Approach

Fritz and Heinz London had already produced a model of flow and electrodynamics in materials called superconductors. Superconductors are a kind of superfluid with charged particles in motion. We'll come to them soon. The London brothers' model described the material in terms of the two-fluid approach originally used by Gorter and Casimir. These were a normal component that moved with friction and a frictionless superfluid component. Lazlo Tisza adopted and adapted their theory and said that the low temperature phase of Helium (called Helium II) was essentially similar to the condensed phase of Einstein's Bose gas. Following Einstein's description, Tisza said that Helium II moved as if it had two components,

- a condensed fluid (with density ρ_s --now "s" for super) producing the frictionless flow
- and a normal fluid (with density ρ_n -- "n" for normal) producing a frictional part of the flow.

C.J. Gorter and H.G.B. Casimir, *Physik Z.* **35** 963 (1934), *Z. techn. Physik.* **15** 539 (1934).

L. Tisza, *Comptes Rendu* **207**, 1035, 1186 (1938). *J. Phys. rad.* **1** 350 (1940).

London, F.; H. London. "The Electromagnetic Equations of the Supraconductor". *Proc. Roy. Soc. (London)* **A149** (866): 71 (1935)

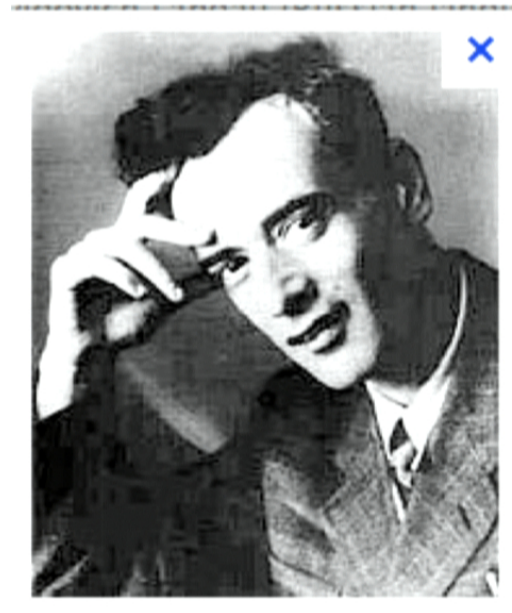
Part 7 Phase Transitions--MFT Physics 352 11/20/10

Lev Landau: 1940

Since 1937 Landau had been the head of the Theoretical Department of the Institute for Physical Problems of the Academy of Sciences of the U.S.S.R. in Moscow. He was a very important physicist.

We shall look at his celebrated paper (1940): “On the Theory of Superfluidity of Helium II”

published in L. N. Khlatnikov, *An Introduction to the Theory of Superfluidity*, translator Pierre C. Hohenberg, pp. 185-204, W.A. Benjamin, New York, (1965). In Russian in JETP **11** 592 (1941) and in English in J. Phys. U.S.S.R. **5** 71-90 (1941)



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Lev Landau: 1940

Important contents of paper:

quasiparticle methods

The word “quasiparticle,” now in wide use, was not invented yet. The idea is that excitations with momentum \mathbf{p} and energy $\epsilon(\mathbf{p})$ move through the system like particles and undergo occasional collisions. One can then give a complete description of what goes on in terms of the familiar concepts of the kinetic theory of gases.

Landau makes the first use of this idea for a liquid, and suggests the form of $\epsilon(\mathbf{p})$.

Landau's form of $\epsilon(p)$ in part follows the work of Tisza, who suggested a sound wave mode: $\epsilon(p) = cp$, c = sound velocity.

Landau 1940 contents, continued ...

Next this paper gives **Landau criterion** for superfluidity, based on how the quasiparticles might lose momentum to walls. He asserts that superfluidity requires a nonzero minimum-value of $\epsilon(p)/p$. Since Landau does not really describe the nature of superfluidity, the validity of this Landau criterion is problematical.

He also brings up an argument due to D'Alembert (1768) and Euler that if a fluid (or fluid component) moves with a velocity proportional to the gradient of a scalar it would move around obstacles without friction. But Landau offers no hint of what that scalar might be.

In final section Landau said that superfluids are much like superconductors. (I'll discuss superconductors in a few moments.) He repeats **Fritz London's** old argument that a superconductor is like a big atom.

References in 1940 paper

Landau does not mention Fritz London at all.

Landau vociferously rejects the idea of his one-time postdoc, Tisza, that Bose-Einstein phase transition is related to superfluidity and says, in a rather unpleasant fashion:

...[Tisza] suggested that the atoms found in the normal state (a state of zero energy) move through the fluid without friction. the explanation advanced by Tisza has no foundation in his suggestions but is in direct contradiction to them.

Landau does not accept the importance of the condensate. He hardly mentions that Helium II is composed of bose particles.

At present, we believe that the condensate and the bose nature of Helium are both crucial to superfluidity. So Landau's position might appear strange to us. That is because his paper, entitled, *On the Theory of Superfluidity of Helium II*, totally failed to explain the phenomenon of superfluidity

We shall return to Landau's failure to understand superfluidity. But now go back to 1904.....

Earlier work: The Discovery of Superconductivity

In 1904, **Heike Kamerlingh Onnes**, Professor of Physics at Leiden University, founded a lab that applied industrial techniques to physics at low temperatures. It was the start of what we know as “big science”.

The lab's first triumph (1908) was the liquefaction of Helium gas at 4.2 degrees Kelvin. This was an important step toward being able to do experimental science at low temperatures.

In 1911, the Leiden lab discovered that the behavior of a mercury loop of wire changed qualitatively when it was cooled below 4.15 degrees Kelvin. At that temperature, the electrical resistance of the wire abruptly dropped to zero.

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HKO



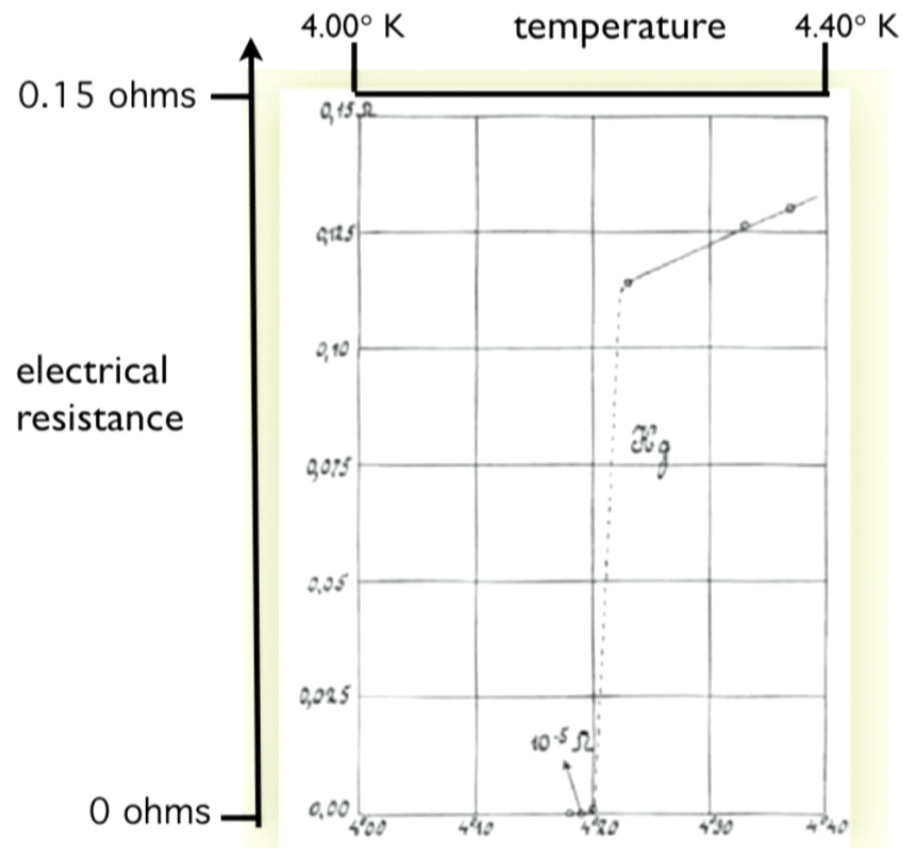
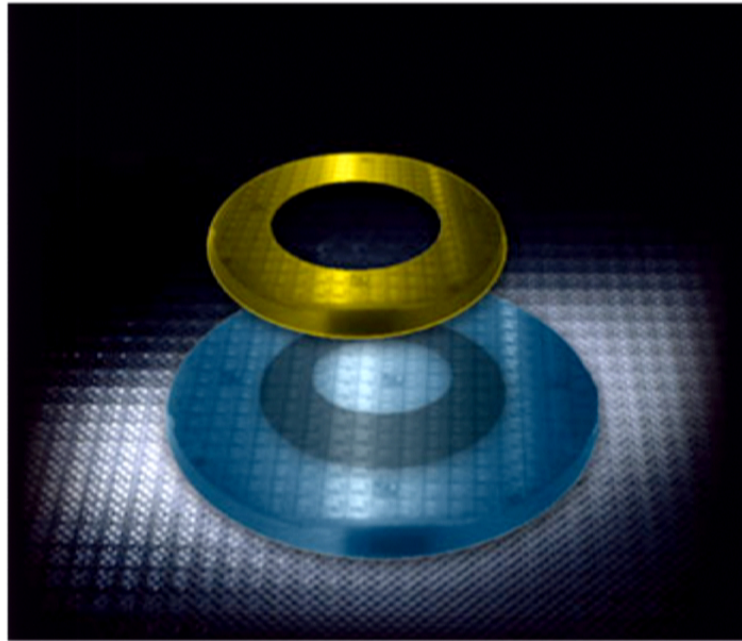


Figure 4. Historic plot of resistance (ohms) versus temperature (kelvin) for mercury from the 26 October 1911 experiment shows the superconducting transition at 4.20 K. Within 0.01 K, the resistance jumps from unmeasurably small (less than $10^{-6} \Omega$) to 0.1 Ω . (From ref. 9.)

Superconductivity observed



superconducting ring (gold color) with electrical current held up by field from magnet (blue color). The current will keep on going for years if the ring is held at low temperature. In a normal metal, the current will come to a stop in roughly 10^{-12} seconds.

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from Physics Today

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Superconductivity was not really explained until almost fifty years after this first observation.

Why so long?

- A theorem of **Felix Bloch** from the 1930s was often described by the statement that “All theories of superconductivity are wrong.”
- This theorem simply states that currents must vanish in the ground state of any material.
- But in superconductivity charged particles keep moving apparently forever. How can that be?

Flows, Frictional and otherwise:

usual current carrying wires show frictional behavior

friction is caused by particles bumping into one another or into obstacles.

Friction is always present in usual materials:

So force is necessary to keep currents in motion.

e.g. $\mathbf{j} = \sigma \mathbf{E}$ inside material
current is proportional to force

But friction is sometimes absent

atomic motion:

Bohr analysis: $m\mathbf{a} = m \, d\mathbf{v}/dt = \mathbf{Force}$

rate of change of current is proportional to force

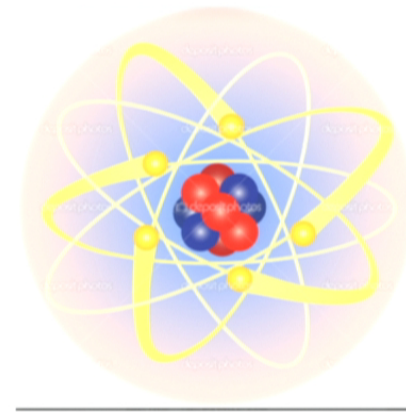
friction is absent in an atom (not understood in 1911).

friction is absent in superconductors (not understood in 1911)

friction is absent in superfluids (not known in 1911)

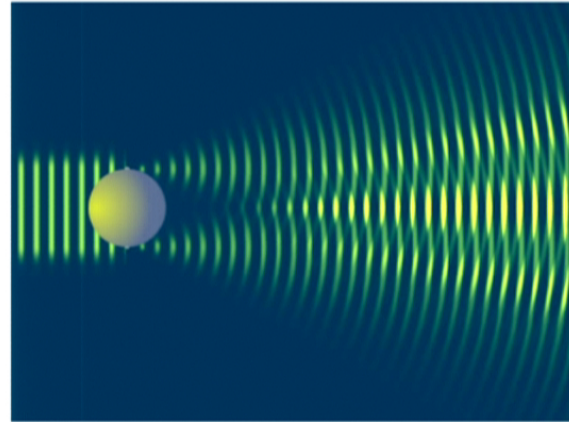
why don't particles bump into one another in these situations?

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In contrast to classical physics: Quantum motion occurs via wave functions

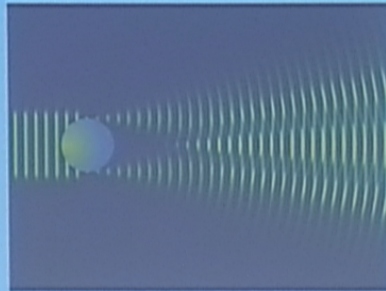
In 1926 **Schrödinger** suggested that, in quantum mechanics, an electron is described by a wave function, $\psi(\mathbf{r})$, a complex function of position, \mathbf{r} . Particles bump into things, but waves can go around them. One might guess that in an atom or superfluid or superconductor, the particles behave like waves and move around obstacles. This is a sort of spooky behavior. But as Einstein said, quantum theory shows aspects of spooky behavior.



This argument hints at a possible and partial explanation of super-flow. But it is just hand-waving. How can one make this into a theory?

In contrast to classical physics:
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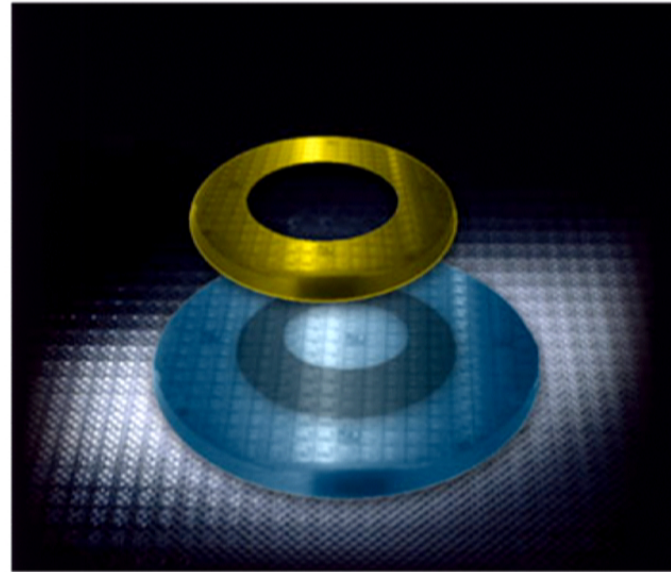
Lecture 10: Quantum Mechanics

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$$\frac{\psi(\mathbf{r})}{p}$$

Macroscopic theory of superconductors

1935 **Fritz and Heinz London** described the electrical and magnetic behavior of these superconductors by assuming electrons obey Newton's $F=ma$, but they never bump into anything so they feel no friction. They say superconductors behave like big atoms. But why is friction absent in superconductors but not in the usual metals?



So the London brothers can describe the macroscopic properties of superconductors but they don't know where how and why these properties arose.

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In Moscow theorists approached
superconductivity via the theory of phase
transitions

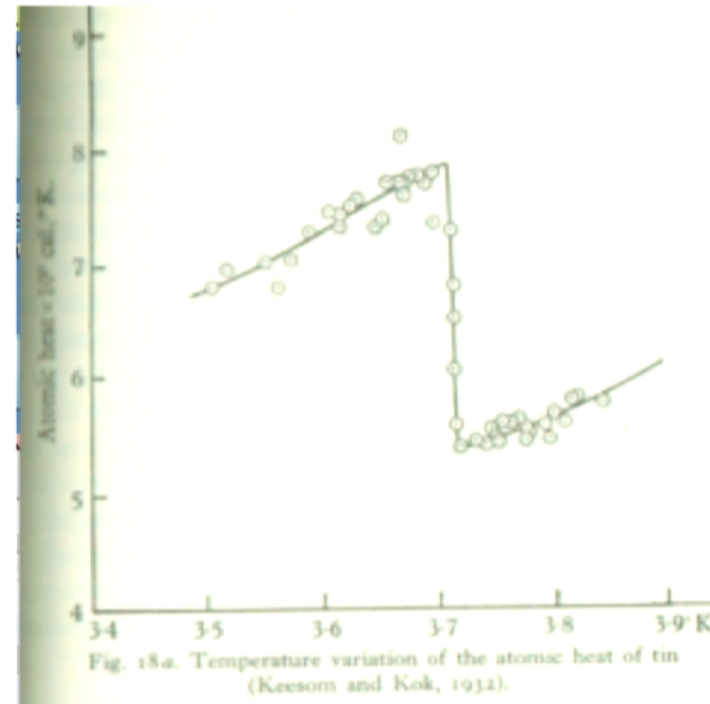
Landau-school 1935-1950

One clue to superconducting behavior is the jump in heat capacity as the temperature is lowered to the superconducting transition temperature.. The jump indicates that a new kind of process has suddenly arisen in the material. and is a signal that a phase transition has occurred.

In other cases, like the onset of ferromagnetism, this jump indicates a phase transition to a new behavior, a new “phase of matter”. Landau has a theory to describe such phase transitions.

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Heat Capacity of Tin vs. temperature



W. H. Keesom and J. A. Kok Comm. Lab. Univ of Leiden no. 231e (1932).

Landau's Phase Transition Theory

In Kharkov and Moscow, in 1935-1937, Lev Landau put all the previous phase transition theories together in one package by saying that in all these cases the order parameter, O , would obey a “universal” equation of the form

$$a (T - T_c) O + b O^3 = 0$$

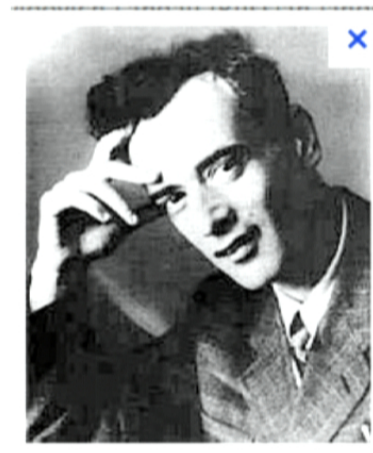
where a and b are parameters that depended upon the particular phase transition. He then got additional solutions and behaviors

$$O = \pm [a(T_c - T)/b]^{1/2} \text{ for } T < T_c.$$

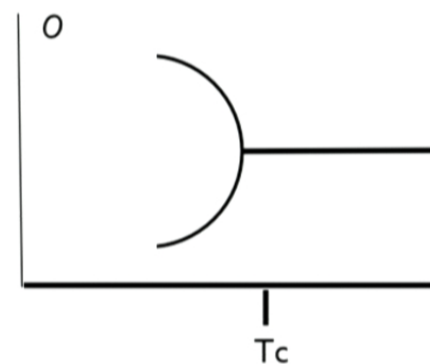
L. D. Landau, *Collected papers* (Oxford: Pergamon Press, 1965) p. 96-100 and 193-216.

For Landau, a term in prison and World War II intervened, but in 1950 or so Landau and his colleague Vitaly Ginzburg returned to the problem of describing phase transitions.

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Lev Landau



Breakthrough in 1950:

Vitaly Ginzburg and Landau wanted to describe the macroscopic behavior of the superconductor, including its ability to carry electrical current from place to place. They start from Landau's theory for the order parameter

$$a (T-T_c) \phi + b \phi^3 = 0$$

They had two great insights. The first was that they could describe situations in which there was a slow variation of the material properties in space by letting ϕ vary slowly in space. This then permits adding a term to the Landau theory which included a derivative in space, or rather $\nabla^2 \phi$. However, because they want to describe a superconductor they would really like to have something that looked roughly like the Schrödinger equation. So they replaced ϕ by a complex quantity $\psi(\mathbf{r})$. Pretty soon they had

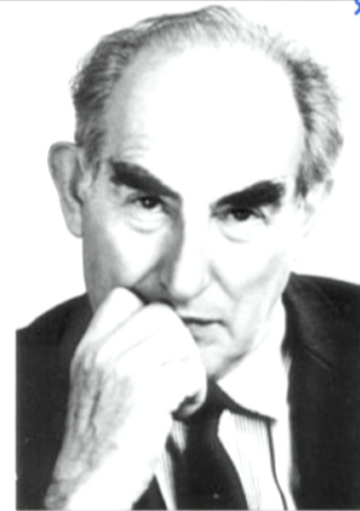
$$[a (T-T_c) + b |\psi(\mathbf{r})|^2 + d (-i\nabla)^2] \psi(\mathbf{r}) = 0$$

where d is another unknown parameter. They now had the structure now known as the Ginzburg-Landau equation.

It was a good-looking guess, but was it the right guess?

V.L. Ginzburg and L.D. Landau, *Zh. Eksp. Teor. Fiz.* **20**, 1064 (1950). English translation in: L. D. Landau, *Collected papers* (Oxford: Pergamon Press, 1965) p. 546

* Superconductivity 12/12/12 Leo Kadanoff



Vitaly Ginzburg

The Schrödinger equation:

$$E \psi(\mathbf{r}) = [(-i\hbar\nabla)^2/(2m) + V(\mathbf{r})] \psi(\mathbf{r})$$

One further step. The gauge invariance built into quantum theory tells us how to include the effect of a vector potential, $A(r)$, to represent a magnetic field.

$$a \psi(r) (T - T_c) + b \psi(r) |\psi(r)|^2 + d (-i\hbar \nabla - qA(r)/c)^2 \psi(r) = 0$$

However, the values of the parameters a , b , and d were still unknown. (c and \hbar are physical constants..) Lastly, q would be the charge on the particle being described by the wave function. The authors assumed that q would be the charge on the electron. They were wrong.

By the process of guesswork and scholarship, Ginzburg and Landau put together an equation that would perhaps be capable of describing the electromagnetic properties of superconductors in a much more accurate way than the London equations. In fact, during the years following 1950, science made very considerable use of this equation.

V.L. Ginzburg and L.D. Landau, *Zh. Eksp. Teor. Fiz.* **20**, 1064 (1950). English translation in: L. D. Landau, *Collected papers* (Oxford: Pergamon Press, 1965) p. 546

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An irony

As I have said, in 1940, Landau had done as much as he could to reject the idea that a condensate wave function played an important role in superfluidity. Now, ten years later, driven by the logic of his research, Landau played a role in developing a theory of a wave function which would describe superconductivity.

However, apparently Landau still refused to believe that the Ginzburg-Landau work is a broadly useful theory of superconductivity.

A further irony: G-L and condensate

Ginzburg and Landau have one paragraph in which they gave a microscopic definition of the condensate wave function in terms of the quantum mechanical density matrix. This definition agrees with the one introduced in parallel by **Oliver Penrose**, which then came to be named off-diagonal-long-range-order (ODLRO). ODLRO is considered today to be the defining characteristic of superfluids. The paragraph in G-L in which this definition is given is quite remarkable. It is marred by a crucial misprint in an equation that makes the entire paragraph hard to understand.

Moreover, in contrast with the usual confident language of the authors, it is filled with tentative words: ``consider'', ``suppose'', ``it might be thought'', ``it is reasonable to suppose''. It is as if the authors could not agree on simple declarative statements. Perhaps this tone is part of the reason why the community took ten years to pick up on the Ginzburg-Landau paper as a definition of important characteristics of a superconductor. Further the tone partially explains why Ginzburg-Landau is not generally cited as a source of ODLRO.

*This paragraph and this misprint was pointed out to me by Pierre Hohenberg.

Ginzburg-Landau proves itself:

The first use of the new work was **A.A.Abrikosov**'s two papers on application of the Ginzburg-Landau equation to the behavior of superconductors in a magnetic field. The first of these papers described the behavior of films. It introduced the idea that there were two kinds of superconductors, Type I and Type II, with the latter being a new kind introduced by Abrikosov. This kind of superconductor has novel properties because it tends to break up into normal and superconducting regions. This work was quite important because most superconductors actually fall into the Type II category.



A.A.Abrikosov

A.A.Abrikosov, Doklady Akademii Nauk SSSR **86**, 489 (1952).

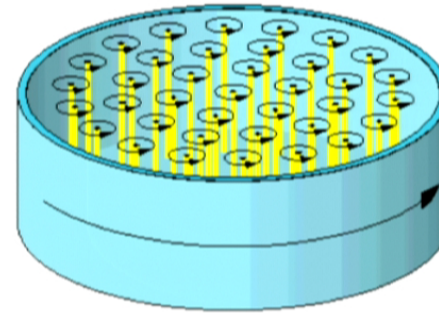
Landau's group in Moscow, 1956. Are sitting: L. Prozorova, A. Abrikosov, I. Khalatnikov, L. Landau, E. Lifshitz. Are standing: S. Gershtein, Lev Pitiaevskii, L. Vainshtein, R. Arkhipov, I. Dzyaloshinskii



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Ginzburg-Landau proves itself:

The second of the papers described bulk materials rather than films. It showed that in a magnetic field type II materials formed vortices, swirls of supercurrent surrounding normal regions. Landau termed this behavior "exotic" and at his suggestion this work remained unpublished for several years. After **Lars Onsager** and **Richard Feynman** had described vortices in Helium, Landau relented, Abrikosov finally published, and the work became a major contribution to superconductivity theory. **Once again, it would appear, Landau undervalued work involving the condensate wave function.**



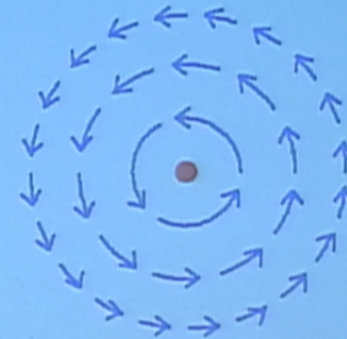
type II superconductor

A.A. Abrikosov, Zh. Eksp. Teor. Fiz. **32**, 1442 (1957) [Sov. Phys.—JETP **5**, 1174(1957)]

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Abrikosov's vortex

A single vortex has the phase angle growing as you go around the vortex. It grows and grows until you get all the way around when it has grown by 360 degrees, and you are back to your starting point. This produces a current and angular momentum which is quantized, i.e. has a fixed strength.



$$\Psi(r) \sim \exp(iM\phi) \quad M \text{ must be an integer}$$

This is one of the first descriptions of a topological excitation.

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A Different thread:
The development of a microscopic theory of
conduction in solids

From Metals to Transistors

Over the entire first half of the twentieth century there was a development of knowledge of the motion of electronic currents in various kinds of materials, including metals and semiconductors. The latter are materials with only a few electrons that are free to move. In this case, the electronic motion can be channeled and controlled with relative ease.

At Bell Labs, a part of the US telephone company, **John Bardeen** and **Walter Brattain** worked under the direction of **William Shockley** on the development of a device to control electrical currents based within a small hunk of semiconducting material. In 1947, these workers put together the first transistor and opened up the age of electronics.

Thereafter Bardeen wanted to work on superconductivity, as the primary and deepest puzzle in the physics of condensed matter. Shockley (and Bell) did not permit him to do so.



John Bardeen, William Shockley and Walter Brattain at Bell Labs, 1948.

From Bell to Urbana

In 1951, Bardeen left Bell, angry about Shockley's grab for fame in the publicity about the transistor and angry that Shockley and Bell would not let him work on problems of his choice. Further, despite the fact that it would earn him his first Nobel Prize, Bardeen did not believe that the transistor work represented the best work he could do. So he came to Urbana to work on superconductivity*.

After a group of preparatory papers by Bardeen and coworkers, **Leon Cooper** showed in 1956 that if you followed carefully two electrons in a metal you would see that they tended to interact so as to form a loosely bound pair. The existence of a huge number of such pairs is one of the crucial ideas that would solve the problem of superconductivity.



Cooper

Leon Cooper, Phys. Rev. **104**, 1189-1190 (1956).

* Lillian Hoddeson and Vicki Daitch, *True Genius*, Joseph Henry Press (2002). p.2

The ideas of London and Tisza enter the picture

In a quite separate thread of activity, **Schafroth** and **Blatt, Butler, and Schafroth** followed Fritz London and Tisza's old ideas and argued that superconductors were produced by a Bose Einstein condensation akin to the one described by Einstein. But, before Cooper's pair work this argument was not too convincing because electrons are fermions not bosons. And fermions cannot condense.

J. M. Blatt, S. T. Butler, M. R. Schafroth Phys. Rev. **100**, 481 (1955)
M. R. Schafroth Phys. Rev. **100**, 502 (1955)

After Cooper's paper, **Schafroth, Butler and Blatt** then argued that Cooper's pairs were effectively bosons and these bosons produced the Bose-Einstein transition at the heart of superconductivity.

Schafroth, M.R., Butler, S.T., Blatt, J.M.: Helv. Phys. Acta. 30, 93 (1957)

At Urbana excitement, excitement was growing

Bardeen, Cooper and **Robert Schrieffer** (BCS) began a push toward building an understanding of superconductivity based directly upon microscopic knowledge.

Beyond Cooper's pairs, the other crucial idea was the nature of the superconducting "vacuum", its basic state. That idea was supplied by **Robert Schrieffer** following up on an approach originally used by the Japanese field theorist **Sin-Itiro Tomonaga**. This vacuum turned out to be closely related to a 1947 study by the Soviet mathematician, **N.N. Bogoliubov***. BCS's "vacuum" was filled with weakly bound pairs of electrons, spinning in opposite directions. Each pair is required to have zero total momentum. This gave BCS a basis for constructing a theory of the quasiparticles producing the superconductivity.



Schrieffer

S. Tomonaga, Prog. Theor. Phys. (Kyoto) **2**, 6 (1947)

BCS published their results in a very celebrated paper:

BCS: Bardeen, J.; Cooper, L. N., Schrieffer, J. R., "Microscopic Theory of Superconductivity". *Physical Review* **106** (1): 162–164: (1957) "Theory of Superconductivity", Phys. Rev. 108, 1175 (1957)

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BCS described the behavior of superconductors in terms of quasiparticles that were a combination of particle-like and hole-like excitations

Things that were previously major mysteries were made evident. For example a new characteristic energy, denoted by the symbol Δ , emerged from the calculation as a descriptor of pair binding. This descriptor also appear as an energy gap in the theory.

This and other quantities had temperature dependences that fit experimental data in an admirable fashion.

A microscopic theory of superconductivity was here!
Superconductivity was completely solved!! Or was it?



Stamp commemorating the achievements of two-time Nobel Prize-winner John Bardeen

Anomalies in BCS

BCS does not explain why supercurrents can exist apparently forever. Despite the title of their paper, their work is not “[A Microscopic Theory of Superconductivity](#)” As we shall see, present day theory of supercurrents talk about bose-einstein condensation and condensate wave functions.

BCS does not mention either condensates or condensate wave functions. In talking about the supercurrents, they do reiterate F. London's old argument about a superconductor being like a big atom but take that argument no further than did Landau in 1940. In fact, this paper leaves out all description of work from the USSR, neither Landau, nor Bogoliubov, nor Ginzburg-Landau.

Anomalies in BCS, continued

BCS did not accept the Bose-Einstein arguments of Schafroth, Blatt, and Butler and its connection to condensate wave functions. On the contrary they say

Our picture differs from that of Schafroth, Butler, and Blatt, *Helv. Phys. Acta* **30**, 93 (1957) who suggest that pseudomolecules of pairs of electrons of opposite spin are formed. They show that if the size of the pseudomolecules is less than the average distance between them, and if other conditions are fulfilled, the system has properties similar to a charged Bose-Einstein gas, including a Meissner effect and a critical temperature of condensation. Our pairs are not localized in that sense, and our transition is not analogous to a Bose-Einstein transition.

On the contrary, the BCS transition is precisely analogous to a Bose-Einstein transition, and, as was shown by Lev Gor'kov in 1960, the BCS gap function, Δ , is, in fact, both the Bose condensate wave function and obeys the Ginzberg-Landau order parameter equation.

The biggest embarrassment for Bardeen's position is ...

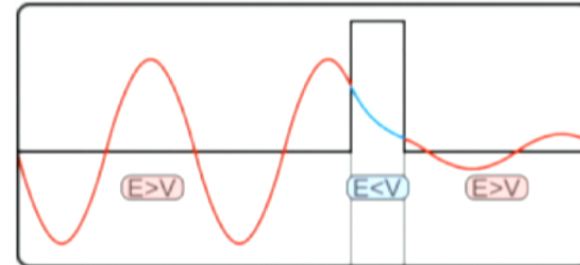
Between Micro and Macro

In 1960, **Ivar Giaever** found that he could see the structure of the quasiparticle spectrum of superconductors by using an electrical voltage to pull electrons from the superconductor, through a narrow gap of insulating materials, and thence into another metal. The quasiparticle spectrum could be inferred from a measurement of the current as a function of the voltage across the gap. This kind of device has been extensively used to study superconductors. But most workers in

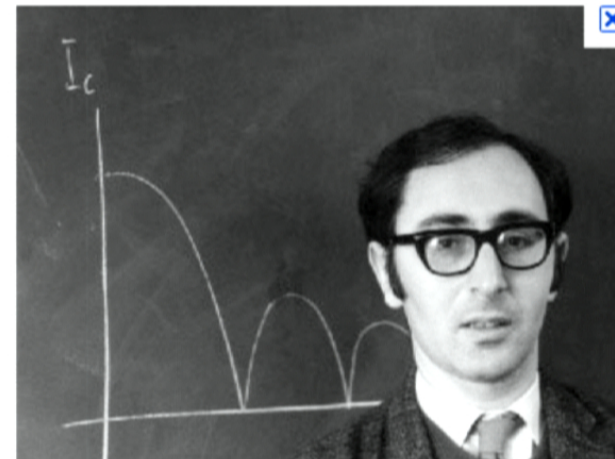
the period just after BCS did not think about tunneling produced by the pair wave function. However, **Brian Josephson** did point out in 1964 that such tunneling could occur, depending on the sine of the relative phase in the pair wave function on the two sides of the junctions.

$$J = J_0 \sin \Delta\alpha,$$

where $\Delta\alpha$ is the phase difference in phase angle between the condensate wave functions of the two superconductors.



Schematic representation of quantum tunnelling through a barrier. The energy of the tunneled particle is the same, only the quantum amplitude (and hence the probability of the process) is decreased.



Josephson ~ 1964

However, Bardeen did not believe Josephson's results and said so at one of the Triennial low temperature physics meetings. Experiment proved Bardeen wrong. He acknowledged his error in a talk at the next meeting. This must have been hard for John. He did not like to make errors.

Josephson's phase angle

and the behavior of the Ginzburg-Landau function $\Psi(\mathbf{r})$ has proven crucial to our developed understanding of superfluids and superconductors. The first step was L. Gor'kov's calculation which served to connect the Ginzburg-Landau theory and BCS. In fact he derived the G-L equation as a property of the BCS gap, Δ .

Much additional work has studied superfluids and superconductors in atoms and nuclei and all over. Much of the further work on these materials has emphasized the qualitative similarity of the different “supers”, in their macroscopics, rather than their differences, in microscopics. In fact, macroscopic understanding has grown and grown.

On the other hand, theoretical understanding of the microscopics of superconductors has hit a brick wall. In the years after BCS, B.T. Matthias pointed out that the theorists could not predict the transition temperature of new classes of superconducting compounds. This point was made in spades when in 1986, high temperature superconductors were discovered. Their microscopics remains a mystery.

Today's explanation of Superconductivity/Superfluidity

Two things to explain about superfluid flows:

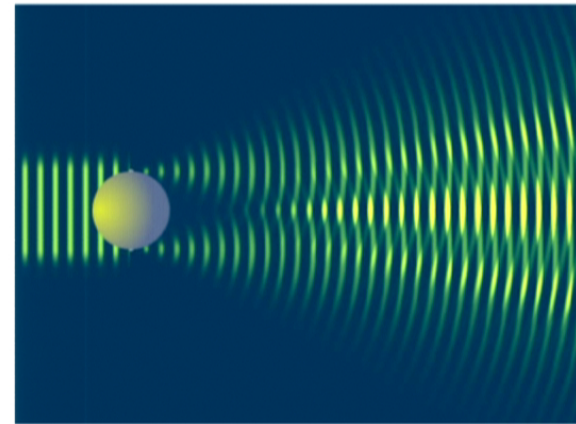
- i. These flows go around obstacles without producing a force on the obstacles.
- ii. These flows can continue around a ring essentially forever

Description: A finite proportion of all the particles in the system fall into a single quantum mode described by the complex wave function,

$$\Psi(\mathbf{r}) = |\Psi(\mathbf{r})| \exp [i\alpha(\mathbf{r})]$$

The squared magnitude $|\Psi(\mathbf{r})|^2$ is proportional to the probability of finding a condensate particle at the point \mathbf{r} , or to the density of such particles. The phase angle, $\alpha(\mathbf{r})$, determines the velocity of superflow by having that velocity be proportional to the gradient of the phase

$$m\mathbf{v}(\mathbf{r}) = \hbar \nabla \alpha(\mathbf{r})$$



explanation of i. intuition: particles bump into obstacles; wave can go around. One might guess that in an atom or superconductor, the particles behave like waves and move around obstacles. mathematics: **D'Alembert** paradox (1752) gradient flow produces no force or pressure on obstacle.

Josephson's α is the scalar Landau sought but never found

Today's explanation of Superconductivity/Superfluidity

explanation for ii. These flow can continue around a ring essentially forever

In a thin channel, a flow is produced by having the condensate wave function be of the form

$$\Psi(r) \sim \exp [i M \varphi]$$

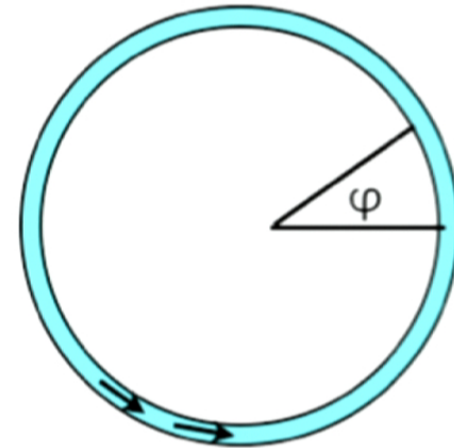
where M must be an integer to make this wave function meaningful. The flow is non-zero if M is not zero. The value of M cannot change by any simple quantum process. To change requires that one move an Abrikosov vortex from one side of the channel to the other.

Vortices are almost macroscopic flows. These vortices tend to get stuck on imperfections in the system. The process of vortex motion is very slow for a realistic channel. It can easily require a time longer than the lifetime of the universe.

The superflow is only metastable, not really stable. But the metastable state can decay very slowly.

James Langer and Michael Fisher, Phys. Rev. Letts. **19** 560 (1967)

J. S. Langer and V. Ambegaokar. Physical Review, **164**, 498 (1967).



The importance of the phase angle

has grown and grown. **P.W.Anderson** understood oscillations in that angle. **Allen Goldman** observed these oscillations. These oscillations then formed the major part of the basis for predicting and understanding the much-touted “**Higgs**” particle of the Large Hadron Collider.

But the importance of the microscopics has not disappeared

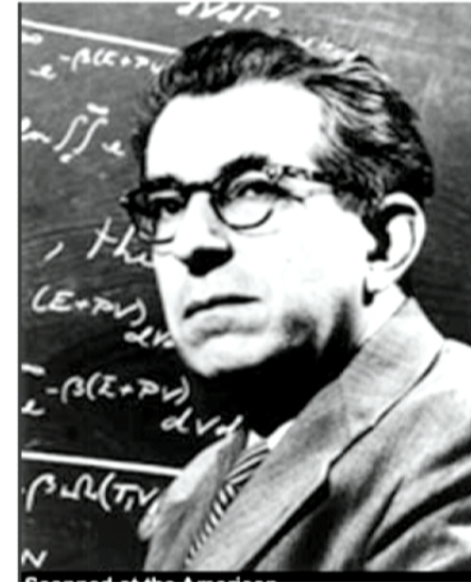
The van der Waals, Maxwell, Landau conception of order parameter and symmetry breaking has enriched nuclear physics, particle theory, condensed matter, atomic theory, and astrophysics most especially through the concepts of topological excitations that grew out of the Onsager, Feynmann, Abrikosov vortex.

Recognition, at last

The most significant recognition of the condensate concept came from John Bardeen. In 1972, John contributed monies to be used for the Award in Low Temperature Physics, which would thenceforth be the **Fritz London Award**. He also set up a lectureship in London's name at Duke University. In 1990, John wrote a piece, an afterward, for Kostas Gavroglu's *Fritz London, a scientific biography* [pp 267-272], saying, among other things,

By far the most important step in understanding the phenomena [of superconductivity] was the recognition by Fritz London that both superconductors and superfluid helium are macroscopic quantum systems [...] The key to understanding superfluidity is macroscopic occupation of a quantum state.

However, the contributions of Tisza, and of Blatt, Butler, and Schafroth remain largely unrecognized.



Fritz London

Why?

Why did Landau and Bardeen miss the importance of the condensate and its wave function. I see several reasons:

- **Too Radical:** The concept that a wave function could be seen macroscopically was too novel for them to readily accept it.
- **“Not Invented Here”:** They had played no part of the development of these ideas. They wanted to emphasize their own inventions. However, they hurt themselves quite substantially by not studying, recognizing, and understanding what others had done.
- **Little Guys Did It:** It was hard for these world-renowned physicists to believe that the very important problems of superconductivity and superfluidity had, in substantial measure, been solved by not-very-famous physicists like London, Tisza, Blatt, Butler, Schafroth, and Josephson.

pride goeth before a fall
hubris -----> nemesis

excellent general references:

A. Griffin, "A BRIEF HISTORY OF OUR
UNDERSTANDING OF BEC: FROM BOSE TO BELIAEV",
in Varenna Lecture Notes (1995).

Sebastien Balibar "The Discovery of Superfluidity." Journal of
Low Temp. Physics **146**, 441-470 (2007)



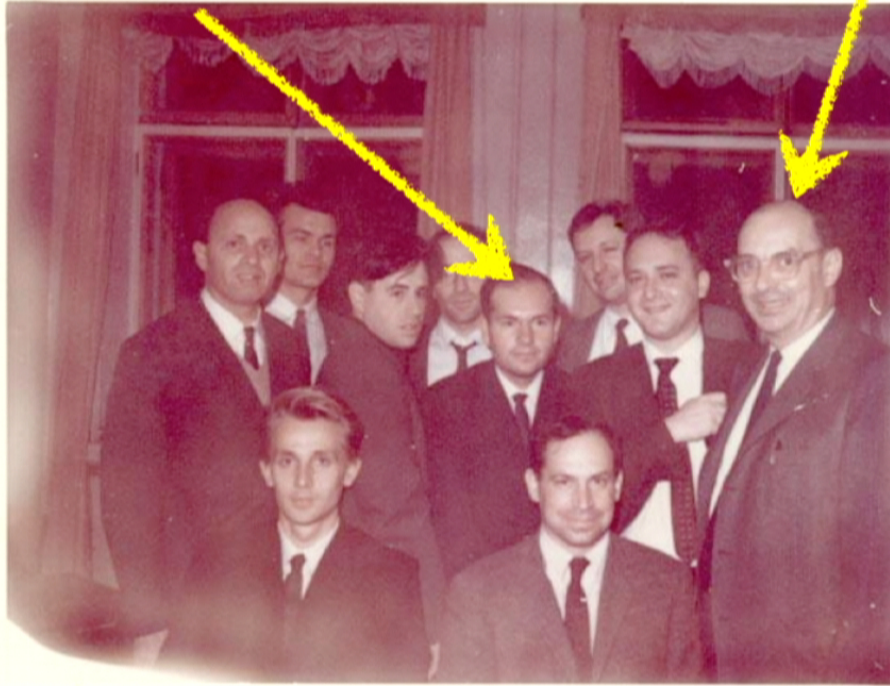
Lazlo Tisza 1907-2008



Max Robert Schafroth (1923-1959)

Superconductivity 12/12/12 Leo Kadanoff

Moscow 1965. USSR-USA Seminar.
Sitting: A. Andreev, D. Pines. Standing:
I. Khalatnikov, Rusinov, L. Pitaevskii, G. Eliashberg,
A. Abrikosov, P. Martin, L. Kadanoff, J. Bardeen



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