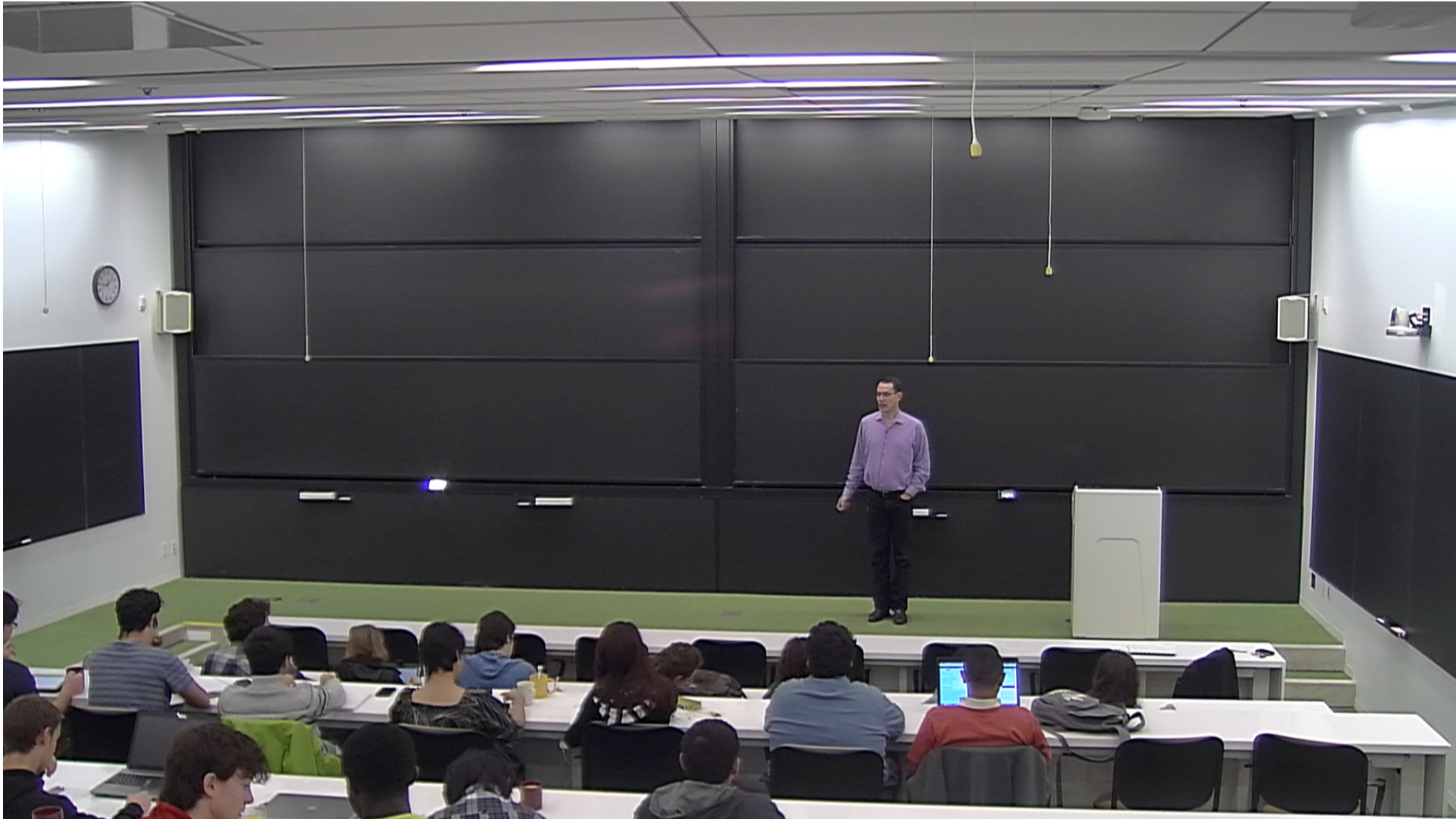


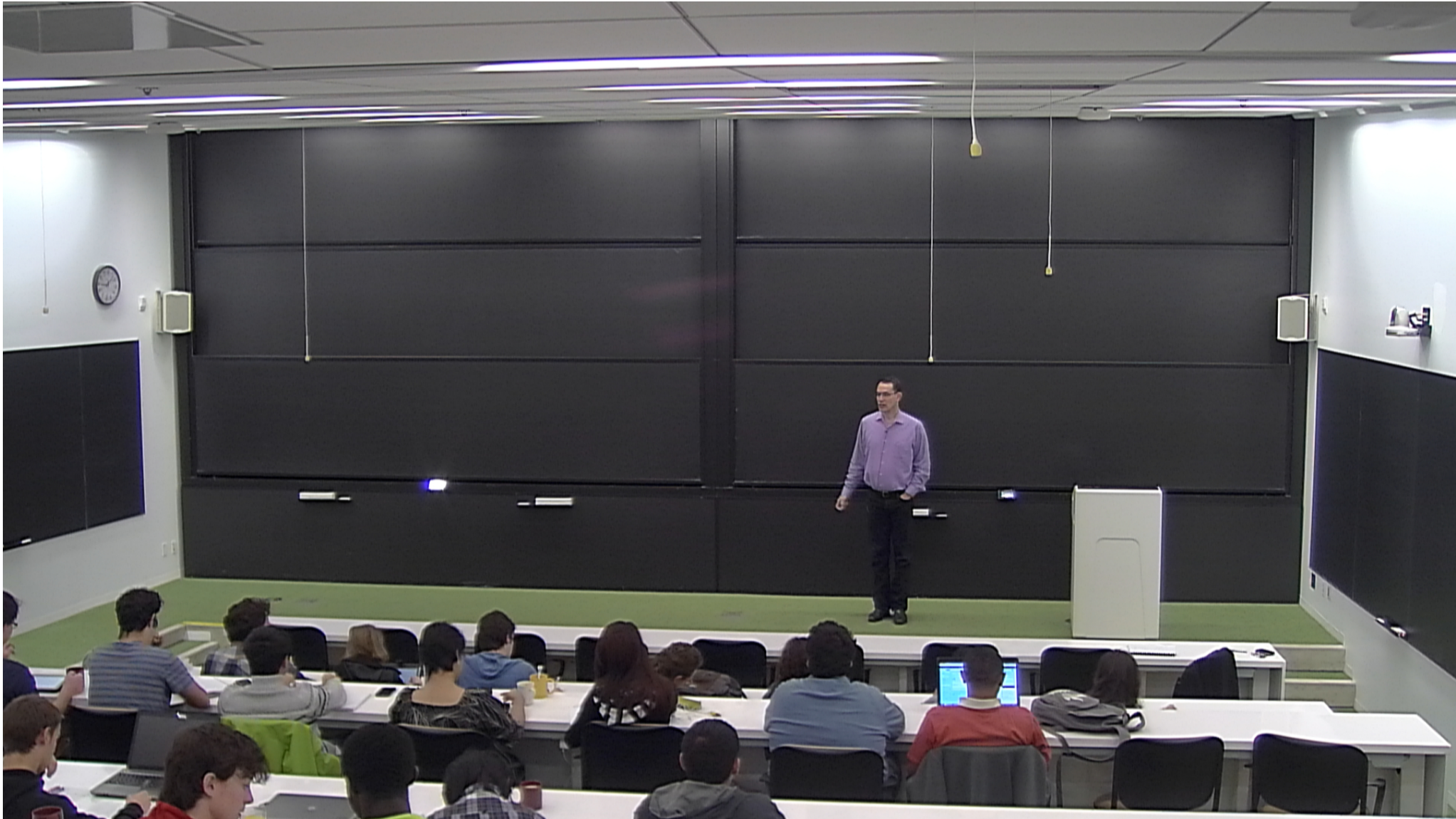
Title: 13/14 PSI - Relativity - Lecture 13

Date: Sep 25, 2013 09:00 AM

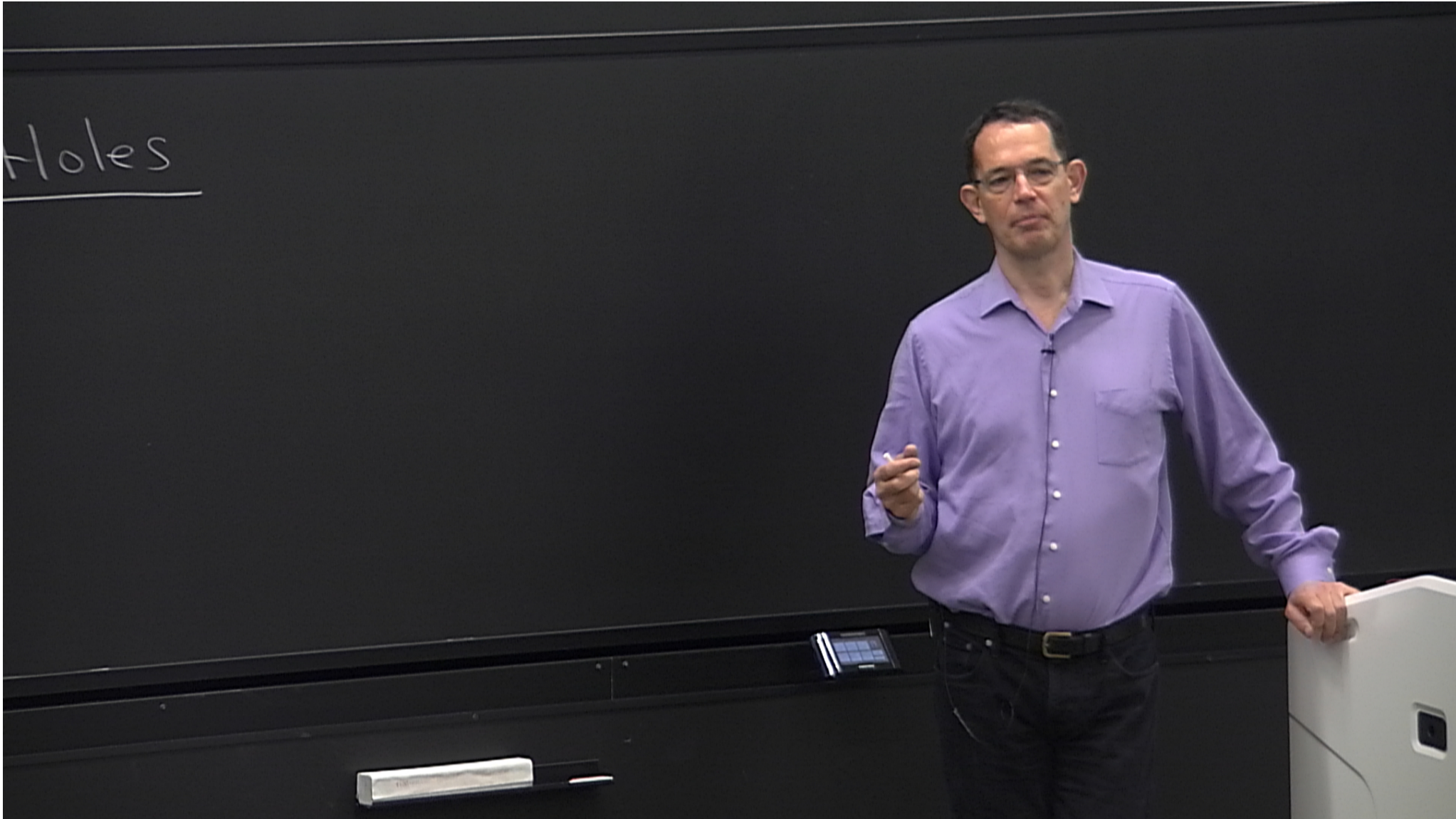
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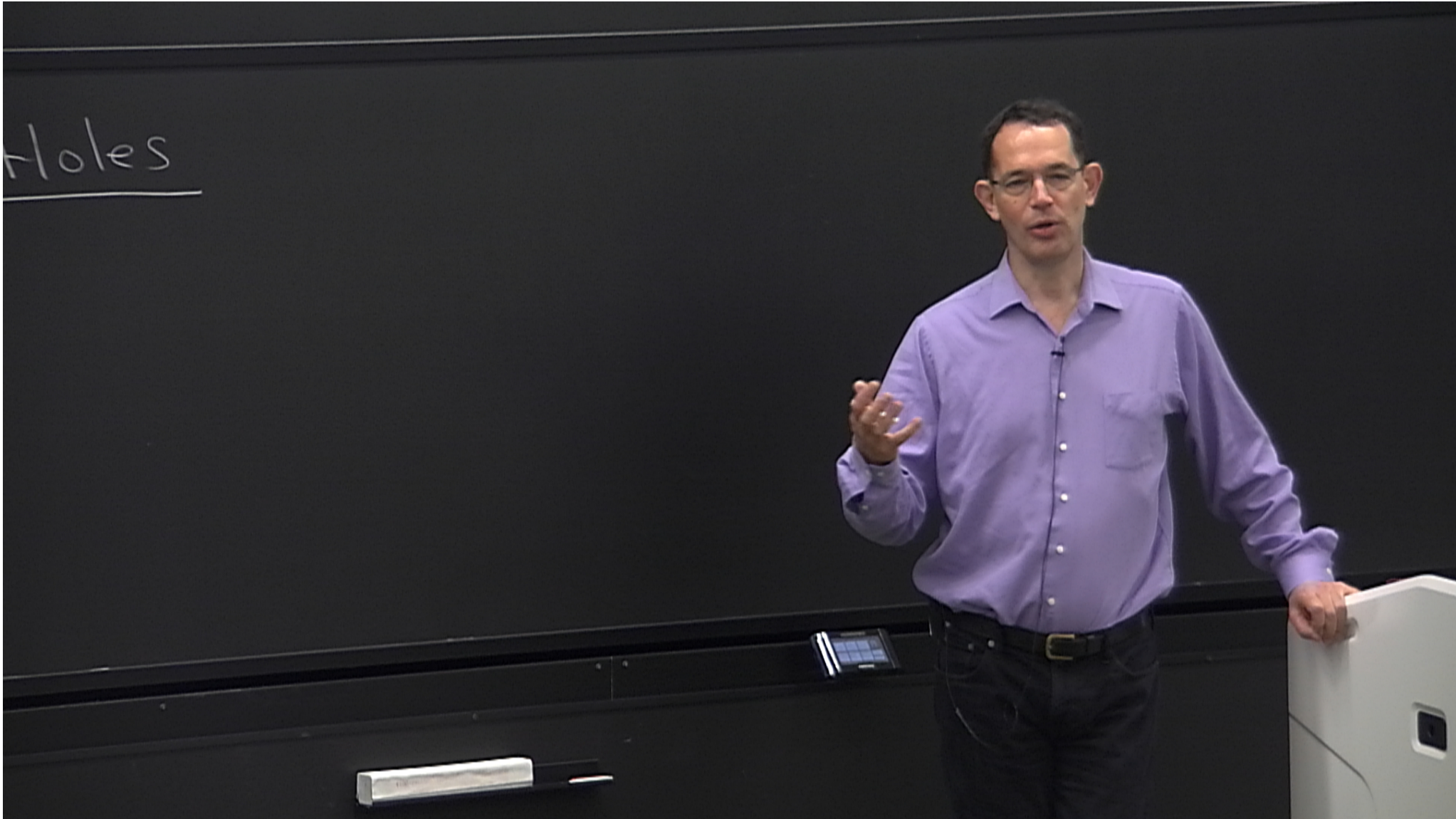
Abstract:





Black Holes



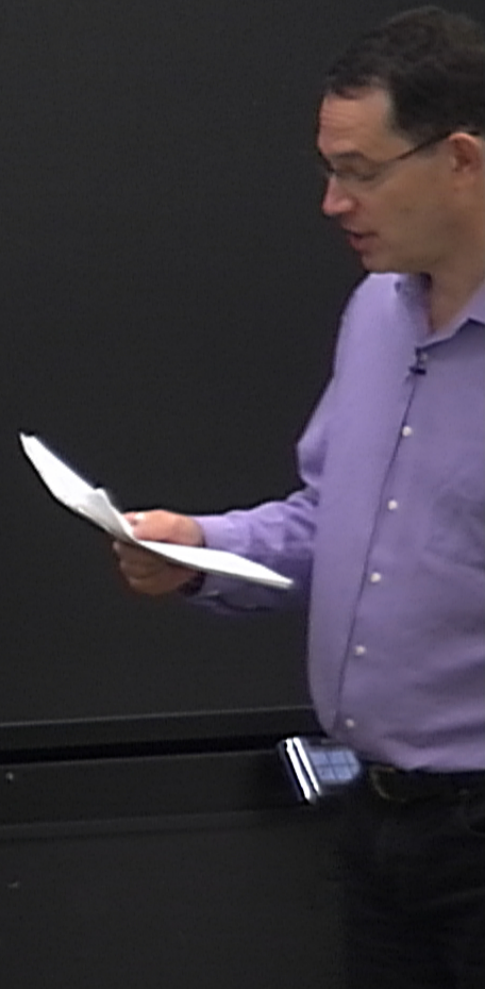


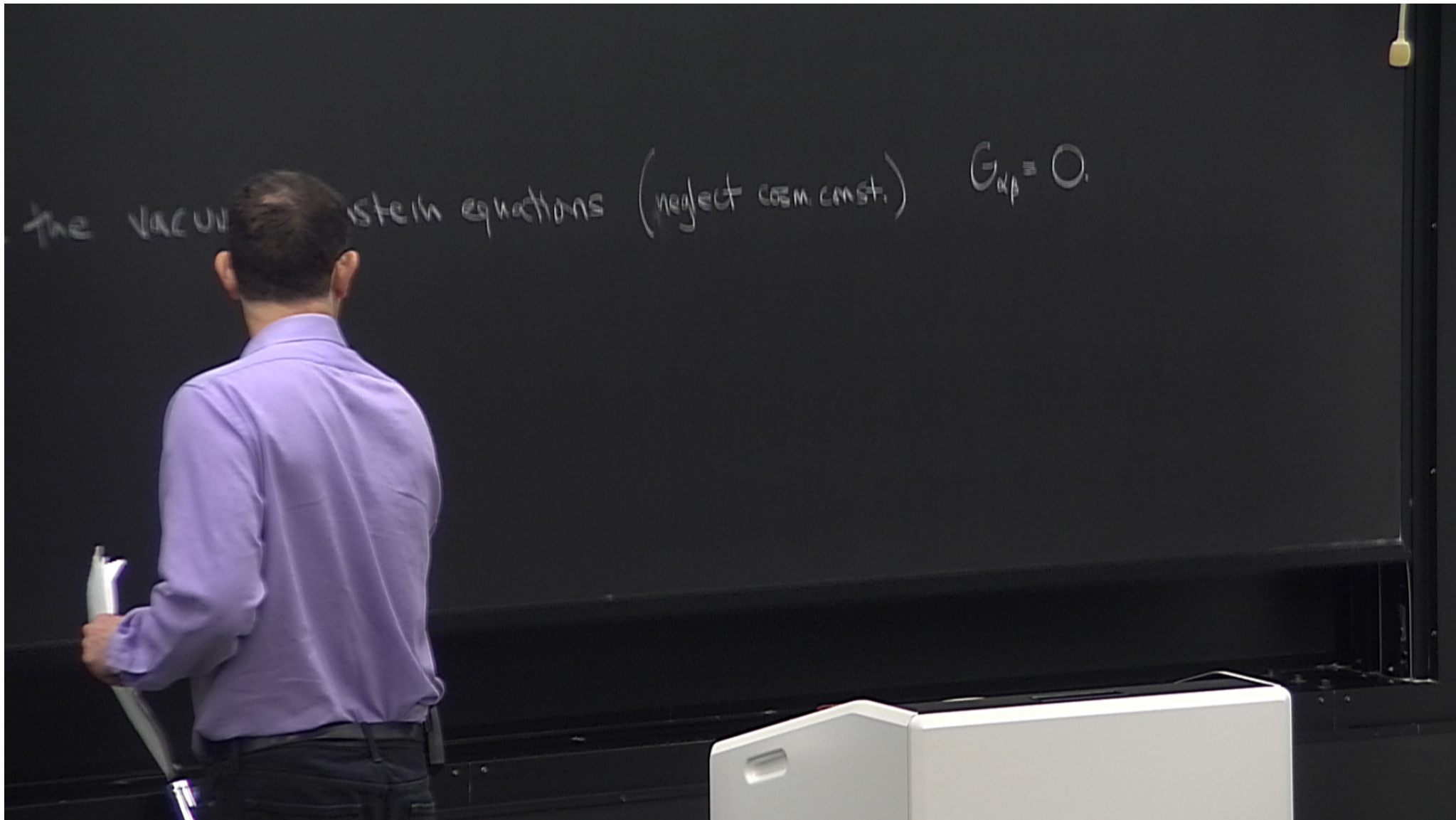
Black Holes

Want to study spherically symmetric

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Want to study spherically symmetric solutions of the vacuum Einstein equations (neglect cosm. const.) $G_{\alpha\beta} = 0$.

ie. $g_{\mu\nu}(t, \underline{x})$: symmetric under $\underline{x}' = \underline{O}\underline{x}$, $\underline{O}^T \underline{O} = \underline{1}$, $\underline{O}^T = -\underline{O}$

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line element should depend only on rotational invariants, e.g. dt , t , $r = \sqrt{\underline{x}^2}$

The vacuum Einstein equations (1913)

$$\nabla^2 O = 1, \quad O = \text{rotation}(O(3))$$

$$\underline{x} = \begin{pmatrix} r \sin \theta \cos \varphi \\ r \sin \theta \sin \varphi \\ r \cos \theta \end{pmatrix}$$

units, e.g. $dt, t, r = \sqrt{\underline{x}^2}$ $\underline{x} \cdot d\underline{x} = r dr$ $d\underline{x}^2 = dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$

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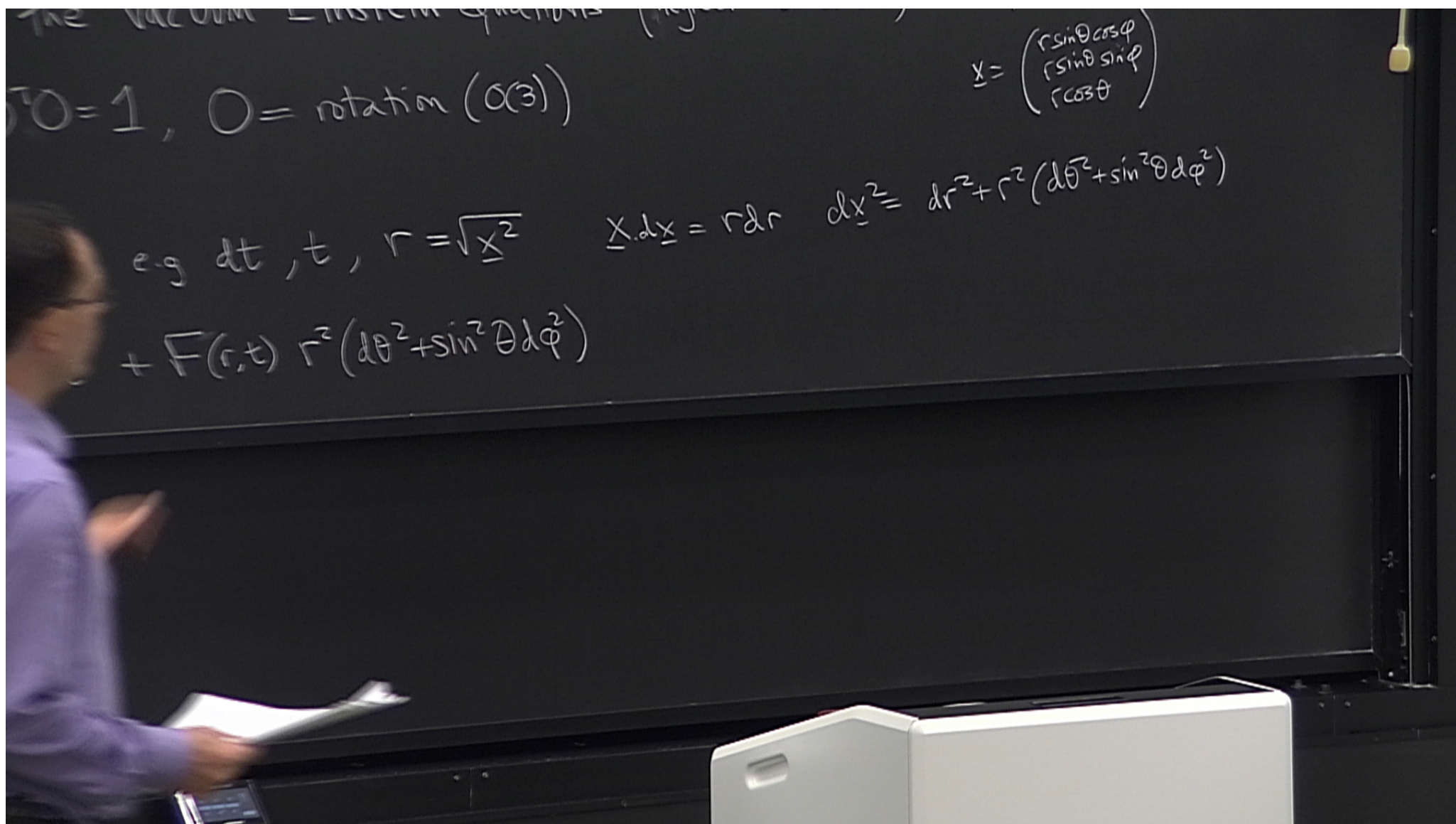
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line element should depend only on rotational invariants, t, r

$$d\tau^2 = -C(r, t) dt^2 + D(r, t) dr^2 + 2E(r, t) dr dt +$$



The vacuum Einstein equations (in the

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$$\underline{x} \cdot d\underline{x} = r dr \quad d\underline{x}^2 = dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

$$r dt + F(r, t) r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

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$$g_{\mu\nu} dx^\mu dx^\nu \equiv ds^2 = -C(r,t) dt^2 + D(r,t) dr^2 + 2E(r,t) dr dt + F(r,t) d\Omega^2$$

Black Notes

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$$t) dr^2 + 2E(r,t) dr dt + \underbrace{F(r,t)}_{r^{1/2}} \underbrace{r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)}$$

symmetric solutions of the vacuum Einstein equations (neglect cosm. const.)

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$$D(r,t) dr^2 + 2 E(r,t) \underset{\downarrow dr'}{dr} dt + \underbrace{F(r,t)}_{r^{1/2}, \text{ drop time}} r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

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E: define

$$dt' = \eta(r,t) [C(r,t) dt - E(r,t) dr]$$

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$$(-E)^2 \sim \underbrace{C^2 - 2CE + E^2}_{C(C-2E)}$$

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"Integrating Factor": designed to ensure the coord. transform

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remove E: define "Integrating Factor": designed to ensure the coord transformation exists

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$\leftarrow -\eta E$

$$dx^\mu dx^\nu \equiv ds^2 = -C(r,t)dt^2 + D(r,t)dr^2 + 2E(r,t)drdt + r^2 d\Omega_2^2$$

r^{12} , drop prime

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Given $C(r,t)$ and $F(r,t)$, this is a differential equation for $\eta(r,t)$

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 for any $\eta(r, t_0)$ _{initial time}, eqⁿ determines $\eta(r,t)$ for $t > t_0$.

$$ds^2 = -\frac{dt'^2}{\eta^2 C} + (D + \bar{C}'E)dr^2 + r^2 d\Omega_2^2$$

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$$ds^2 = -\frac{dt'^2}{\eta^2 C} + (D + \bar{C}^{-1} \bar{E}) dr^2 + r^2 d\Omega_2^2$$

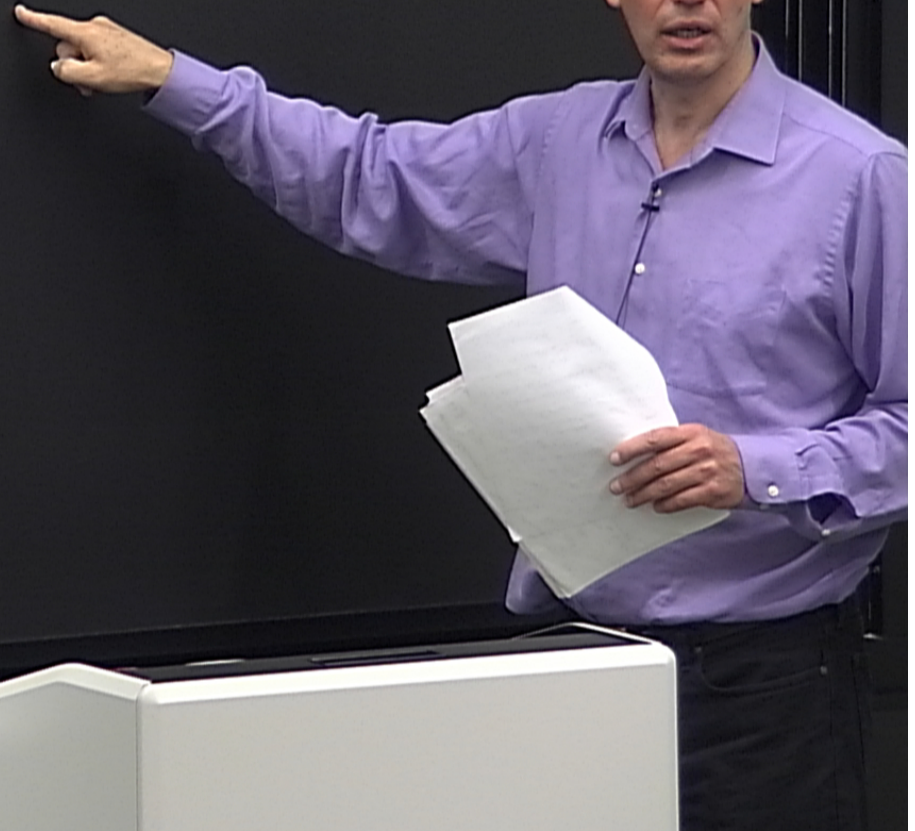
$$\equiv -B(r,t) dt^2 + A(r,t) dr^2 + r^2 d\Omega_2^2$$

partial equation for $\eta(r, t)$
determines $\eta(r, t)$ for $t > t_0$.

$$r' = F^{\vee}(r, t)$$

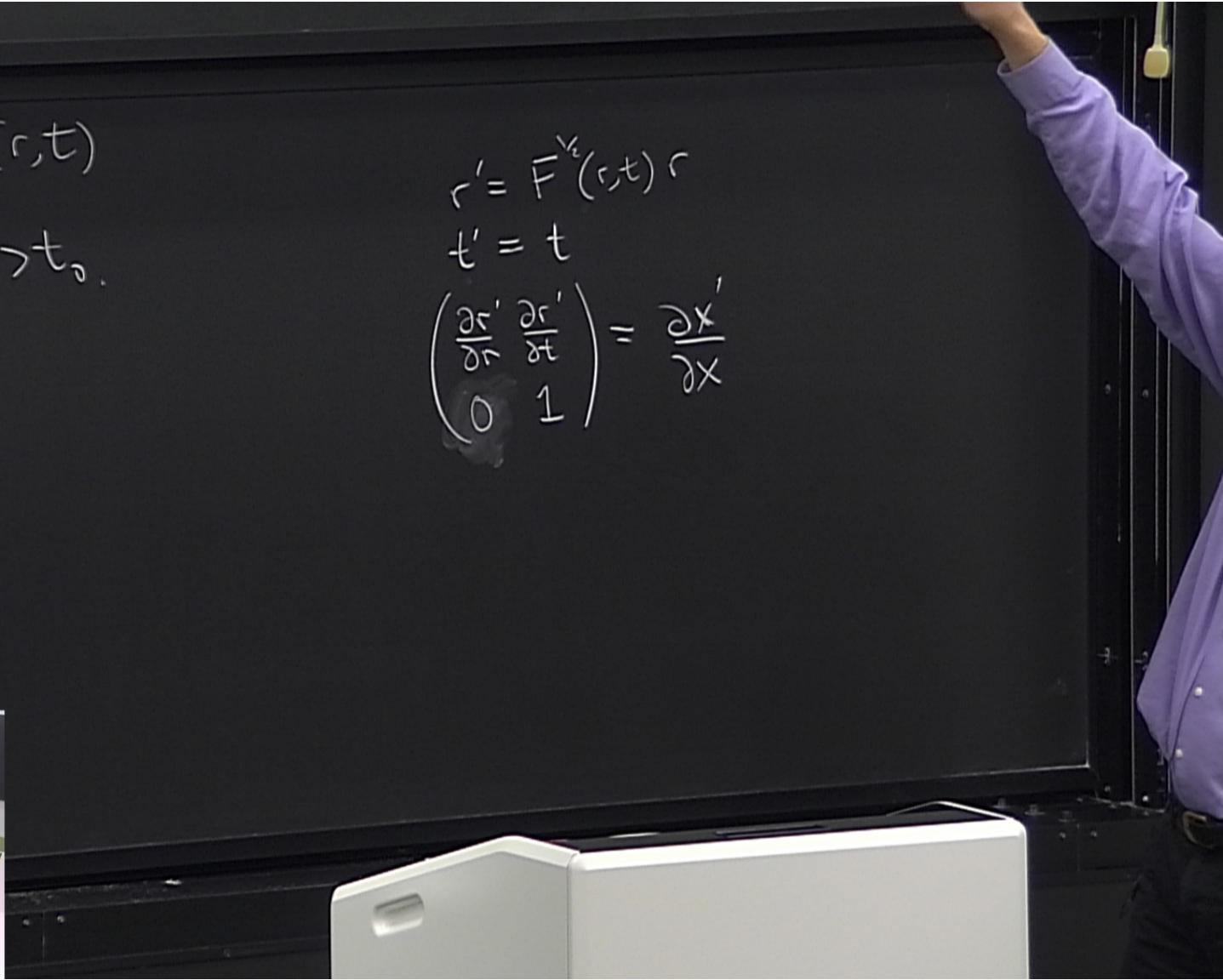
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$$r' = F^{\mathbf{v}}(r, t) r$$
$$t' = t$$



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$$\begin{aligned} r' &= F^{\chi}(r, t) r \\ t' &= t \\ \begin{pmatrix} \frac{\partial r'}{\partial r} & \frac{\partial r'}{\partial t} \\ 0 & 1 \end{pmatrix} &= \frac{\partial x'}{\partial x} \end{aligned}$$



partial equation for $\eta(r, t)$
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$$r' = F^{\frac{1}{2}}(r, t) r$$

$$t' = t$$

$$\begin{pmatrix} \frac{\partial r'}{\partial r} & \frac{\partial r'}{\partial t} \\ 0 & 1 \end{pmatrix} = \frac{\partial x'}{\partial x}$$

$$\Rightarrow \begin{pmatrix} \frac{\partial x'}{\partial x} \\ \frac{\partial x'}{\partial t} \end{pmatrix} = \frac{1}{\det} \begin{pmatrix} 1 & 0 \\ -\frac{\partial r'}{\partial t} & \frac{\partial r'}{\partial r} \end{pmatrix}$$

$$g' = \frac{\partial x'}{\partial x} \frac{\partial x'}{\partial t}$$

partial equation for $\eta(r, t)$
mines $\eta(r, t)$ for $t > t_0$.

$$\begin{aligned} r' &= F^{\frac{1}{2}}(r, t) r & g' &= \frac{\partial x}{\partial x'} \frac{\partial x'}{\partial x} g \\ t' &= t \\ \begin{pmatrix} \frac{\partial r'}{\partial r} & \frac{\partial r'}{\partial t} \\ 0 & 1 \end{pmatrix} &= \frac{\partial x'}{\partial x} \\ \Rightarrow \begin{pmatrix} \frac{\partial x}{\partial x'} \end{pmatrix} &= \frac{1}{\det} \begin{pmatrix} 1 & 0 \\ -\frac{\partial r'}{\partial t} & \frac{\partial r'}{\partial r} \end{pmatrix} \end{aligned}$$

partial equation for $\eta(r, t)$
 determines $\eta(r, t)$ for $t > t_0$.

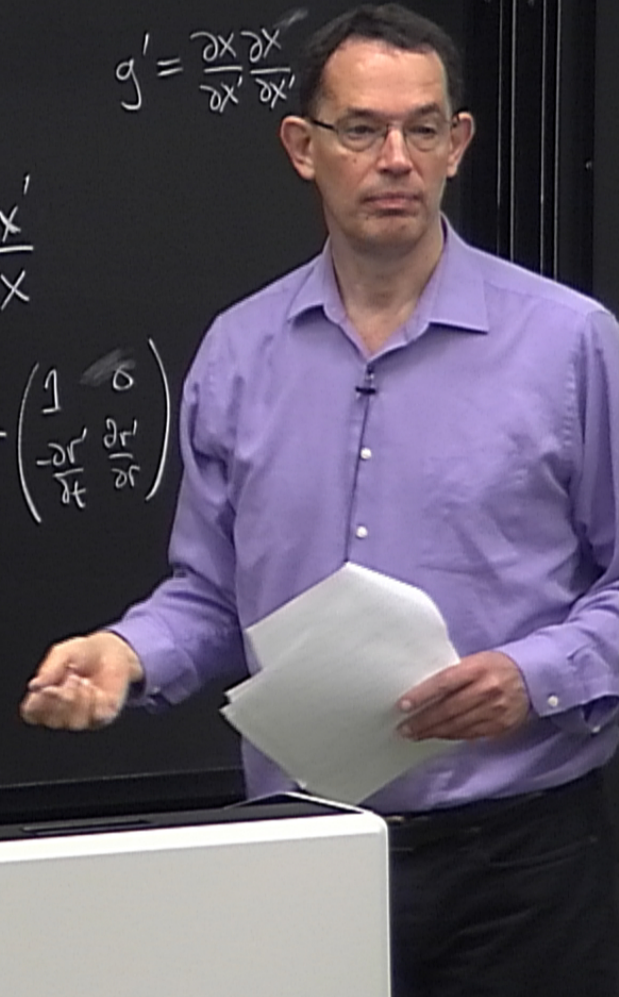
$$\begin{aligned} r &= r(r', t') \\ t &= t(r', t') \end{aligned} \quad \begin{cases} r' = F^{\vee}_r(r, t) \\ t' = t \end{cases}$$

$$dr = \frac{\partial r}{\partial r'} dr' + \frac{\partial r}{\partial t'} dt'$$

$$\begin{pmatrix} \frac{\partial r'}{\partial r} & \frac{\partial r'}{\partial t} \\ 0 & 1 \end{pmatrix} = \frac{\partial x'}{\partial x}$$

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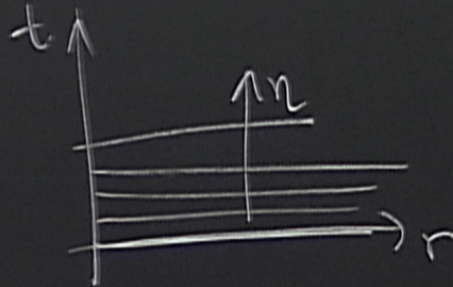
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Initial time

$$t + r^2(d\Omega_2^2)$$

the coord transformation exists.

$$\underline{\underline{(\eta(r,t)E(r,t))}}$$



$$\frac{\partial r}{\partial r'} \frac{\partial r}{\partial t'} = \frac{\partial x}{\partial x'}$$

$$\Rightarrow \begin{pmatrix} \frac{\partial x}{\partial x'} \\ \frac{\partial x}{\partial t'} \end{pmatrix} = \frac{1}{\det} \begin{pmatrix} 1 & 0 \\ -\frac{\partial r'}{\partial t} & \frac{\partial r'}{\partial r} \end{pmatrix}$$

Above is the most general time-dependent, spherically symmetric metric.

Above is the most general time-dependent, sph

$$G_{\alpha\beta} = 0$$
$$\Rightarrow R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 0$$

Above is the most general time-dependent, sph

$$\begin{aligned} G_{\alpha\beta} &= 0 \quad (1) \\ \Rightarrow R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R &= 0 \\ \times g^{\alpha\beta} \Rightarrow R - 2R &= 0 \\ \Rightarrow R &= 0 \\ (1) \Rightarrow R_{\alpha\beta} &= 0 \end{aligned}$$

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta}$$

Above is the most general time-dependent, sph

$$g^{\alpha\beta} G_{\alpha\beta} = -R$$

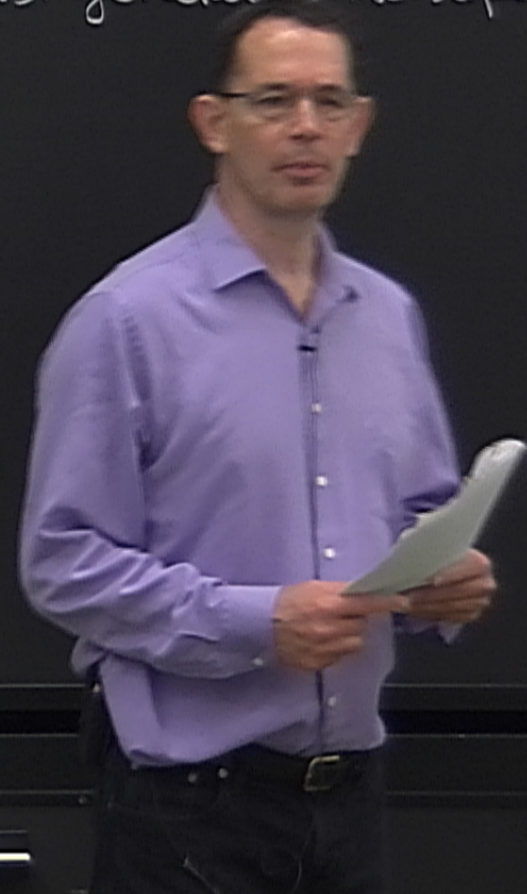
$$\boxed{G_{\alpha\beta} = 0} \quad (1)$$

$$\Rightarrow R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 0$$

$$\times g^{\alpha\beta} \Rightarrow R - 2R = 0$$

$$\Rightarrow \boxed{R = 0}$$

$$(1) \Rightarrow R_{\alpha\beta} = 0$$



re is the most general time-dependent, spherically symmetric metric.

$$\Gamma_{tt}^t = \frac{1}{2} g^{00} \left(g_{00,0} \right) = \frac{1}{2} \frac{\dot{B}}{B} \quad \left(\cdot = \frac{\partial}{\partial t}, \quad ' = \frac{\partial}{\partial r} \right)$$

$$\Gamma_{tr}^t = \Gamma_{rt}^t$$

re is the most general time-dependent, spherically symmetric metric.

$$\Gamma_{tt}^t = \frac{1}{2} g^{00} \left(g_{00,0} \right) = \frac{1}{2} \frac{\dot{B}}{B} \quad \left(\cdot = \frac{\partial}{\partial t}, \quad ' = \frac{\partial}{\partial r} \right)$$

$$\Gamma_{tr}^t = \Gamma_{rt}^t = \frac{1}{2} g^{00} \left(g_{r0,0} + g_{00,r} - g_{r0,0} \right)$$

re is the most general time-dependent, spherically symmetric metric.

$$\Gamma_{\frac{1}{B}}^{00} \left(g_{\frac{-B}{00,0}} \right) = \frac{1}{2} \frac{\dot{B}}{B} \quad \left(\cdot = \frac{\partial}{\partial t}, \quad ' = \frac{\partial}{\partial r} \right)$$

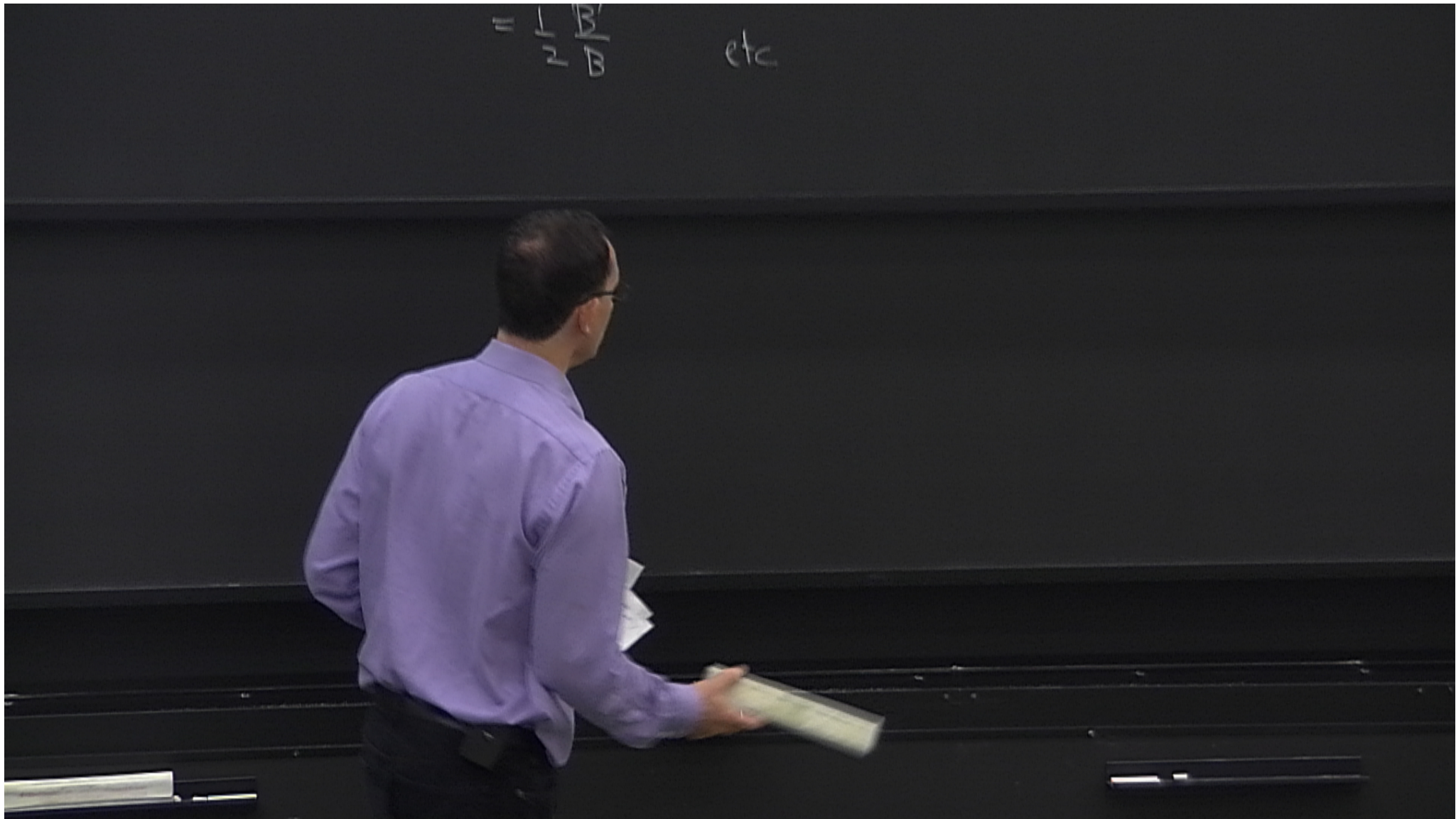
$$\Gamma_{rt}^t = \frac{1}{2} g^{00} \left(g_{r0,0} + g_{00,r} - g_{r0,0} \right)$$

re is the most general time-dependent, spherically symmetric metric.

$$\Gamma_{tt}^t = \frac{1}{2} g^{00} \left(g_{00,0} \right) = \frac{1}{2} \frac{\dot{B}}{B} \quad \left(\cdot = \frac{\partial}{\partial t}, \quad ' = \frac{\partial}{\partial r} \right)$$

$$\Gamma_{tr}^t = \Gamma_{rt}^t = \frac{1}{2} g^{00} \left(g_{r0,0} + g_{00,r} - g_{r0,0} \right) = \frac{1}{2} \frac{B'}{B}$$

both zero



$$g^{\mu\nu} G_{\mu\nu} = R$$

$$\Rightarrow R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 0$$

$$\Rightarrow R - 2R = 0$$

$$\Rightarrow \boxed{R=0}$$

$$(1) \Rightarrow R_{\alpha\beta} = 0$$

$$\Gamma_{tt}^t = \frac{1}{2} g^{tt} (g_{tt,t} + g_{tt,t} - g_{tt,t}) = \frac{1}{2} \frac{\dot{B}}{B} \quad \left(\dot{} = \frac{\partial}{\partial t}, \quad ' = \frac{\partial}{\partial r} \right)$$

$$\Gamma_{tr}^t = \Gamma_{rt}^t = \frac{1}{2} g^{tt} (g_{tr,t} + g_{tr,t} - g_{tt,r})$$

both zero

$$= \frac{1}{2} \frac{B'}{B} \quad \text{etc}$$

$$\Gamma_{rr}^t$$

$$\Gamma_{rr}^t = \frac{1}{2} \frac{\dot{A}}{B} \quad \Gamma_{rt}^r = \Gamma_{tr}^r = \frac{\dot{A}}{2A} \quad \Gamma_{tt}^r = \frac{B'}{2A} \quad \Gamma_{rr}^r = \frac{A'}{2A} \quad \Gamma_{\theta\theta}^r$$

$$\Gamma_{tt}^r = \frac{B'}{2A} \quad \Gamma_{rr}^r = \frac{A'}{2A} \quad \Gamma_{\theta\theta}^r = -\frac{r}{A} \quad \Gamma_{\varphi\varphi}^r = -\frac{r \sin^2 \theta}{A}$$

$$\begin{aligned}
 g^{\mu\nu} G_{\mu\nu} &= -R \\
 \boxed{G_{\alpha\beta} = 0} \quad (1) \\
 \Rightarrow R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R &= 0 \\
 \times g^{\alpha\beta} \Rightarrow R - 2R &= 0 \\
 \Rightarrow \boxed{R=0} \\
 (1) \Rightarrow R_{\alpha\beta} &= 0
 \end{aligned}$$

$$\begin{aligned}
 \Gamma_{tt}^t &= \frac{1}{2} g^{tt} (g_{tt,t}) = \frac{1}{2} \frac{\dot{B}}{B} \quad \left(\dot{} = \frac{\partial}{\partial t}, \quad ' = \frac{\partial}{\partial r} \right) \\
 \Gamma_{tr}^t &= \Gamma_{rt}^t = \frac{1}{2} g^{tt} (g_{tr,t} + g_{tr,r} - g_{r,t,t}) \\
 &= \frac{1}{2} \frac{B'}{B} \quad \text{etc}
 \end{aligned}$$

$$\Gamma_{rr}^t = \frac{1}{2} \frac{\dot{A}}{B} \quad \Gamma_{tt}^r = \Gamma_{tr}^r = \frac{\dot{A}}{2A} \quad \Gamma_{tt}^r = \frac{B'}{2A} \quad \Gamma_{rr}^r = \frac{A'}{2A} \quad \Gamma_{\theta\theta}^r = -\frac{r}{A} \quad \Gamma_{\phi\phi}^r = -\frac{r \sin^2 \theta}{A}$$

$$\Gamma_{\theta\theta}^t$$

$$\begin{aligned}
 \Gamma_{rr}^t &= \frac{1}{2} \frac{\dot{A}}{B} & \Gamma_{rt}^r &= \Gamma_{tr}^r = \frac{\dot{A}}{2A} & \Gamma_{tt}^r &= \frac{B'}{2A} & \Gamma_{rr}^r &= \frac{A'}{2A} & \Gamma_{\theta\theta}^r &= -\frac{r}{A} \\
 \Gamma_{\theta r}^\theta &= \Gamma_{r\theta}^\theta = \frac{1}{r} & \Gamma_{\varphi\varphi}^\theta &= -\sin\theta\cos\theta & \Gamma_{\varphi r}^\varphi &= \Gamma_{r\varphi}^\varphi = \frac{1}{r} & \Gamma_{\varphi\theta}^\varphi &= \Gamma_{\theta\varphi}^\varphi = \cot\theta.
 \end{aligned}$$

$$\Gamma_{rr}^t = \frac{1}{2} \frac{\dot{A}}{B} \quad \Gamma_{rt}^r = \Gamma_{tr}^r = \frac{\dot{A}}{2A} \quad \Gamma_{tt}^r = \frac{B'}{2A} \quad \Gamma_{rr}^r = \frac{A'}{2A} \quad \Gamma_{\theta\theta}^r = -\frac{r}{A}$$

$$\Gamma_{\theta r}^\theta = \Gamma_{r\theta}^\theta = \frac{1}{r} \quad \Gamma_{\varphi\varphi}^\theta = -\sin\theta \cos\theta \quad \Gamma_{rr}^\varphi = \Gamma_{r\varphi}^\varphi = \frac{1}{r} \quad \Gamma_{\varphi\theta}^\varphi = \Gamma_{\theta\varphi}^\varphi = \cot\theta$$

$$R_{\mu\nu} - R^\lambda{}_{\mu\lambda\nu} = - \left(\Gamma_{\mu\lambda,\nu}^\lambda - \Gamma_{\mu\lambda,\nu}^\lambda - \Gamma_{\mu\nu,\lambda}^\lambda + \Gamma_{\mu\nu,\lambda}^\lambda \right)$$

$\Rightarrow$

$$\Gamma_{rr}^t = \frac{1}{2} \frac{\dot{A}}{B} \quad \Gamma_{rt}^r = \Gamma_{tr}^r = \frac{\dot{A}}{2A} \quad \Gamma_{tt}^r = \frac{B'}{2A} \quad \Gamma_{rr}^r = \frac{A'}{2A} \quad \Gamma_{\theta\theta}^r = -\frac{r}{A}$$

$$\Gamma_{\theta r}^{\theta} = \Gamma_{r\theta}^{\theta} = \frac{1}{r} \quad \Gamma_{\varphi\varphi}^{\theta} = -\sin\theta \cos\theta \quad \Gamma_{\varphi r}^{\varphi} = \Gamma_{r\varphi}^{\varphi} = \frac{1}{r} \quad \Gamma_{\varphi\theta}^{\varphi} = \Gamma_{\theta\varphi}^{\varphi} = \cot\theta$$

$$R_{\mu\nu} = R^{\lambda}_{\mu\nu\lambda} = - \left(\Gamma_{\mu\lambda,\kappa}^{\lambda} - \Gamma_{\mu\kappa,\lambda}^{\lambda} + \Gamma_{\mu\lambda}^{\eta} \Gamma_{\kappa\eta}^{\lambda} - \Gamma_{\mu\kappa}^{\eta} \Gamma_{\eta\lambda}^{\lambda} \right)$$

$$\Rightarrow$$

$$\Gamma_{rr}^t = \frac{1}{2} \frac{\dot{A}}{B} \quad \Gamma_{rt}^r = \Gamma_{tr}^r = \frac{\dot{A}}{2A} \quad \Gamma_{tt}^r = \frac{B'}{2A} \quad \Gamma_{rr}^r = \frac{A'}{2A} \quad \Gamma_{\theta\theta}^r = -\frac{r}{A}$$

$$\Gamma_{\theta r}^\theta = \Gamma_{r\theta}^\theta = \frac{1}{r} \quad \Gamma_{\varphi\varphi}^\theta = -\sin\theta \cos\theta \quad \Gamma_{\varphi r}^\varphi = \Gamma_{r\varphi}^\varphi = \frac{1}{r} \quad \Gamma_{\varphi\theta}^\varphi = \Gamma_{\theta\varphi}^\varphi = \cot\theta$$

$$R_{\mu\nu} = R_{\mu\lambda\nu}^\lambda = - \left(\Gamma_{\mu\lambda,\nu}^\lambda - \Gamma_{\mu\nu,\lambda}^\lambda + \Gamma_{\mu\lambda}^\eta \Gamma_{\nu\eta}^\lambda - \Gamma_{\mu\nu}^\eta \Gamma_{\eta\lambda}^\lambda \right)$$

$$\Rightarrow R_{tt} = \frac{B''}{2A} - \frac{B'A'}{4A^2} + \frac{B'}{Ar} - \frac{B'^2}{4A^2} + \frac{\dot{A}^2}{4A^2} + \frac{\dot{B}\dot{A}}{4AB}$$

$$R_{rr} = -\frac{B''}{2B} + \frac{B'^2}{4B^2} + A \frac{\ddot{A}}{A^2} + \frac{\dot{A}\dot{B}}{4B^2} + \frac{\dot{A}^2}{4AB}$$

$$\Gamma_{rr}^t = \frac{1}{2} \frac{\dot{A}}{B} \quad \Gamma_{rt}^r = \Gamma_{tr}^r = \frac{\dot{A}}{2A} \quad \Gamma_{tt}^r = \frac{B'}{2A} \quad \Gamma_{rr}^r = \frac{A'}{2A} \quad \Gamma_{\theta\theta}^r = -\frac{r}{A}$$

$$\Gamma_{\theta r}^{\theta} = \Gamma_{r\theta}^{\theta} = \frac{1}{r} \quad \Gamma_{\varphi\varphi}^{\theta} = -\sin\theta \cos\theta \quad \Gamma_{\varphi r}^{\varphi} = \Gamma_{r\varphi}^{\varphi} = \frac{1}{r} \quad \Gamma_{\varphi\theta}^{\varphi} = \Gamma_{\theta\varphi}^{\varphi} = \cot\theta$$

$$R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu} = - \left(\Gamma_{\mu\lambda,\nu}^{\lambda} - \Gamma_{\mu\nu,\lambda}^{\lambda} + \Gamma_{\mu\lambda}^{\eta} \Gamma_{\nu\eta}^{\lambda} - \Gamma_{\mu\nu}^{\eta} \Gamma_{\eta\lambda}^{\lambda} \right)$$

$$\Rightarrow R_{tt} = \frac{B''}{2A} - \frac{B'A'}{4A^2} + \frac{B'}{Ar} - \frac{B'^2}{4AB} - \frac{\ddot{A}}{2A} + \frac{\dot{A}^2}{4A^2} + \frac{\dot{B}\dot{A}}{4AB}$$

$$R_{rr} = -\frac{B''}{2B} + \frac{B'^2}{4B^2} + \frac{A'B'}{4AB} + \frac{A'}{Ar} - \frac{\ddot{A}}{2B} + \frac{\dot{A}\dot{B}}{4B^2} + \frac{\dot{A}^2}{4AB}$$

$$R_{rt} =$$

$$R_{rr} = -\frac{B''}{2B} + \frac{B'^2}{4B^2} + \frac{A'B'}{4AB} + \frac{A'}{Ar} - \frac{\ddot{A}}{2B} + \frac{\dot{A}\dot{B}}{4B^2} + \frac{\dot{A}^2}{4AB}$$

$$R_{rt} = \frac{\dot{A}}{Ar}$$

$$R_{\theta\theta} = 1 - \frac{1}{A} + r \frac{A'}{2A^2} - r \frac{B'}{2AB}$$

$$\Gamma_{rt}^t = \frac{1}{2} \frac{\dot{A}}{B} \quad \Gamma_{rt}^r = \Gamma_{tr}^r = \frac{\dot{A}}{2A} \quad \Gamma_{tt}^r = -\frac{B'}{2A} \quad \Gamma_{rr}^r = \frac{A'}{2A} \quad \Gamma_{\theta\theta}^r = -\frac{r}{A} \quad \Gamma_{\varphi\varphi}^r = -\frac{r \sin^2 \theta}{A}$$

$$\Gamma_{\theta r}^\theta = \Gamma_{r\theta}^\theta = \frac{1}{r} \quad \Gamma_{\varphi\varphi}^\theta = -\sin\theta \cos\theta \quad \Gamma_{\varphi r}^\varphi = \Gamma_{r\varphi}^\varphi = \frac{1}{r} \quad \Gamma_{\varphi\theta}^\varphi = \Gamma_{\theta\varphi}^\varphi = \cot\theta$$

$$R_{\mu\nu} = R_{\mu\lambda\nu}^\lambda = -\left(\Gamma_{\mu\lambda,\nu}^\lambda - \Gamma_{\mu\nu,\lambda}^\lambda + \Gamma_{\mu\lambda}^\lambda \Gamma_{\nu\lambda}^\lambda - \Gamma_{\mu\nu}^\lambda \Gamma_{\lambda\lambda}^\lambda\right)$$

$$\Rightarrow R_{tt} = \frac{B''}{2A} - \frac{B'A'}{4A^2} + \frac{B'}{Ar} - \frac{B'^2}{4AB} - \frac{\ddot{A}}{2A} + \frac{\dot{A}^2}{4A^2} + \frac{\dot{B}\dot{A}}{4AB}$$

$$R_{rr} = -\frac{B''}{2B} + \frac{B'^2}{4B^2} + \frac{A'B'}{4AB} + \frac{A'}{Ar} - \frac{\ddot{A}}{2B} + \frac{\dot{A}\dot{B}}{4B^2} + \frac{\dot{A}^2}{4AB}$$

$$R_{rt} = \frac{\dot{A}}{Ar}$$

$$R_{\theta\theta} = 1 - \frac{1}{A} + \frac{rA'}{2A^2} - \frac{rB'}{2AB}$$

(other components $R_{\theta\theta} = R_{\varphi\varphi} = R_{\theta\varphi} = R_{\theta t} = R_{\varphi t} = 0$ by spherical symmetry)

$$R_{rr} = -\frac{B''}{2B} + \frac{B'^2}{4B^2} + \frac{A'B'}{4AB} + \frac{A'}{Ar} - \frac{\ddot{A}}{2B} + \frac{\dot{A}\dot{B}}{4B^2} + \frac{\dot{A}^2}{4AB}$$

$$R_{rt} = \frac{\dot{A}}{Ar}$$

$$R_{\theta\theta} = 1 - \frac{1}{A} + r \frac{A'}{2A^2} - \frac{r B'}{2AB} \quad ; \quad R_{\varphi\varphi} = \sin^2\theta R_{\theta\theta}$$

(other components $R_{r\theta} = R_{r\varphi} = R_{\theta\varphi} = R_{\theta t} = R_{\varphi t} = 0$ by spherical symmetry)

$$R_{rr} = -\frac{B''}{2B} + \frac{B'^2}{4B^2} + \frac{A'B'}{4AB} + \frac{A'}{Ar} - \frac{\ddot{A}}{2B} + \frac{\dot{A}\dot{B}}{4B^2} + \frac{\dot{A}^2}{4AB}$$

$$R_{rt} = \frac{\dot{A}}{Ar}$$

$$R_{\theta\theta} = 1 - \frac{1}{A} + \frac{rA'}{2A^2} - \frac{rB'}{2AB} \quad ; \quad R_{\varphi\varphi} = \sin^2\theta R_{\theta\theta}$$

(other components $R_{r\theta} = R_{r\varphi} = R_{\theta\varphi} = R_{\theta t} = R_{\varphi t} = 0$ by spherical symmetry)

$\Rightarrow$ we have these four coupled nonlinear p.d.e.'s for $A(r,t)$ and $B(r,t)$.

$$R_{rr} = -\frac{B''}{2B} + \frac{B'^2}{4B^2} + \frac{A'B'}{4AB} + \frac{A'}{Ar} - \frac{\ddot{A}}{2B} + \frac{\dot{A}\dot{B}}{4B^2} + \frac{\dot{A}^2}{4AB}$$

$$R_{rt} = \frac{\dot{A}}{Ar}$$

$$R_{\theta\theta} = 1 - \frac{1}{A} + \frac{rA'}{2A^2} - \frac{rB'}{2AB} = \sin^2\theta R_{\phi\phi}$$

(other components $R_{r\theta} = R_{r\phi} = R_{\theta\phi} = R_{\theta t} = R_{\phi t} = 0$ by spherical symmetry)

$\Rightarrow$ we have to solve these four coupled nonlinear PDEs for $A(r,t)$ and $B(r,t)$

$$R_{rr} = -\frac{B''}{2B} + \frac{B'^2}{4B^2} + \frac{A'B'}{4AB} + \frac{A'}{Ar} - \frac{\ddot{A}}{2B} + \frac{\dot{A}\dot{B}}{4B^2} + \frac{\dot{A}^2}{4AB}$$

$$R_{rt} = \frac{\dot{A}}{Ar}$$

$$R_{\theta\theta} = 1 - \frac{1}{A} + \frac{rA'}{2A^2} - \frac{rB'}{2AB} \quad ; \quad R_{\varphi\varphi} = \sin^2\theta R_{\theta\theta}$$

(other components $R_{r\theta} = R_{r\varphi} = R_{\theta\varphi} = R_{\theta t} = R_{\varphi t} = 0$ by spherical symmetry)

$\Rightarrow$ we have to solve these four coupled nonlinear p.d.e.'s for $A(r,t)$ and $B(r,t)$.

amazingly simple:

$$R_{rr} = -\frac{B''}{2B} + \frac{B'^2}{4B^2} + \frac{A'B'}{4AB} + \frac{A'}{Ar} - \frac{\ddot{A}}{2B} + \frac{\dot{A}\dot{B}}{4B^2} + \frac{\dot{A}^2}{4AB}$$

$$R_{rt} = \frac{\dot{A}}{Ar}$$

$$R_{\theta\theta} = 1 - \frac{1}{A} + \frac{rA'}{2A^2} - \frac{rB'}{2AB} \quad ; \quad R_{\varphi\varphi} = \sin^2\theta R_{\theta\theta}$$

(other components $R_{r\theta} = R_{r\varphi} = R_{\theta\varphi} = R_{\theta t} = R_{\varphi t} = 0$ by spherical symmetry)

$\Rightarrow$ we have to solve these four coupled nonlinear p.d.e.'s for $A(r,t)$ and $B(r,t)$.

amazingly simple: $R_{rt} = 0 \Rightarrow \frac{\partial}{\partial t}(A(r,t)) = 0 \Rightarrow \underline{A = A(r)}$

$$\Gamma_{rt}^t = \frac{1}{2} \frac{\dot{A}}{B} \quad \Gamma_{rt}^r = \Gamma_{tr}^r = \frac{\dot{A}}{2A} \quad \Gamma_{tt}^r = \frac{B'}{2A} \quad \Gamma_{rr}^r = \frac{A'}{2A} \quad \Gamma_{\theta\theta}^r = -\frac{r}{A} \quad \Gamma_{\varphi\varphi}^r = -\frac{r \sin^2 \theta}{A}$$

$$\Gamma_{\theta r}^\theta = \Gamma_{r\theta}^\theta = \frac{1}{r} \quad \Gamma_{\varphi\varphi}^\theta = -\sin \theta \cos \theta \quad \Gamma_{\varphi r}^\varphi = \Gamma_{r\varphi}^\varphi = \frac{1}{r} \quad \Gamma_{\varphi\theta}^\varphi = \Gamma_{\theta\varphi}^\varphi = \cot \theta$$

$$R_{\mu\nu} = R^\lambda_{\mu\lambda\nu} = -(\Gamma_{\mu\lambda,\nu}^\lambda - \Gamma_{\mu\nu,\lambda}^\lambda + \Gamma_{\mu\lambda}^\lambda \Gamma_{\nu\lambda}^\lambda - \Gamma_{\mu\nu}^\lambda \Gamma_{\lambda\lambda}^\lambda)$$

$$\Rightarrow R_{tt} = \frac{B''}{2A} - \frac{B'A'}{4A^2} + \frac{B'}{Ar} - \frac{B'^2}{4AB} \left(-\frac{\ddot{A}}{2A} + \frac{\dot{A}^2}{4A^2} + \frac{\dot{B}\dot{A}}{4AB} \right)$$

$$R_{rr} = -\frac{B''}{2B} + \frac{B'^2}{4B^2} + \frac{A'B'}{4AB} + \frac{A'}{Ar} \left(-\frac{\ddot{A}}{2B} + \frac{\dot{A}\dot{B}}{4B^2} + \frac{\dot{A}^2}{4AB} \right)$$

$$R_{rt} = \frac{\dot{A}}{Ar}$$

$$R_{\theta\theta} = 1 - \frac{1}{A} + \frac{rA'}{2A^2} - \frac{rB'}{2AB} \quad R_{\varphi\varphi} = \sin^2 \theta R_{\theta\theta}$$

(other components $R_{r\theta} = R_{\theta r} = R_{r\varphi} = R_{\varphi r} = 0$ by spherical symmetry)

$\Rightarrow$ we have to solve these four coupled equations for $A(r,t)$ and $B(r,t)$.

amazingly simple: $R_{rt} = 0 \Rightarrow \frac{\partial}{\partial t} \left(\frac{\dot{A}}{r} \right) = 0$

$$\Rightarrow R_{tt} = \frac{B''}{2A} - \frac{B'A'}{4A^2} + \frac{B'}{Ar} - \frac{B'^2}{4AB} \left[-\frac{\ddot{A}}{2A} + \frac{\dot{A}^2}{4A^2} + \frac{\dot{B}\dot{A}}{4AB} \right]$$

$$R_{rr} = -\frac{B''}{2B} + \frac{B'^2}{4B^2} + \frac{A'B'}{4AB} + \frac{A'}{Ar} \left[-\frac{\ddot{A}}{2B} + \frac{\dot{A}\dot{B}}{4B^2} + \frac{\dot{A}^2}{4AB} \right]$$

$$R_{rt} = \frac{\dot{A}}{Ar}$$

$$R_{\theta\theta} = 1 - \frac{1}{A} + r \frac{A'}{2A^2} - r \frac{B'}{2AB} \quad ; \quad R_{\varphi\varphi} = \sin^2\theta R_{\theta\theta}$$

other components $R_{r\theta} = R_{r\varphi} = R_{\theta\varphi} = R_{\theta t} = R_{\varphi t} = 0$ by spherical symmetry)

have to solve these four coupled nonlinear p.d.e.'s for $A(r,t)$ and $B(r,t)$.

Amazingly simple: $R_{rt} = 0 \Rightarrow \frac{\partial}{\partial t}(A(r,t)) = 0 ! \Rightarrow \underline{A = A(r)} \Rightarrow$ all time derivatives disappear from equations!

$$\frac{R_{tt}}{B} + \frac{R_{rr}}{A}$$

$$\frac{R_{tt}}{B} + \frac{R_{rr}}{A} = \frac{A'}{A^2 r} + \frac{B'}{ABr} = 0$$

$$\Gamma_{rr}^t = \frac{1}{2} \frac{\dot{A}}{B} \quad \Gamma_{rt}^r = \Gamma_{tr}^r = \frac{\dot{A}}{2A} \quad \Gamma_{tt}^r = \frac{B'}{2A} \quad \Gamma_{rr}^r = \frac{A'}{2A} \quad \Gamma_{\theta\theta}^r = -\frac{r}{A}$$

$$\Gamma_{\theta r}^{\theta} = \Gamma_{r\theta}^{\theta} = \frac{1}{r} \quad \Gamma_{\varphi\varphi}^{\theta} = -\sin\theta \cos\theta \quad \Gamma_{\varphi r}^{\varphi} = \Gamma_{r\varphi}^{\varphi} = \frac{1}{r} \quad \Gamma_{\varphi\theta}^{\varphi} = \Gamma_{\theta\varphi}^{\varphi} = \cot\theta$$

$$R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu} = - \left(\Gamma_{\mu\lambda,\nu}^{\lambda} - \Gamma_{\mu\nu,\lambda}^{\lambda} + \Gamma_{\mu\lambda}^{\eta} \Gamma_{\nu\eta}^{\lambda} - \Gamma_{\mu\nu}^{\eta} \Gamma_{\eta\lambda}^{\lambda} \right)$$

$$\Rightarrow R_{tt} = \frac{B''}{2A} - \frac{B'A'}{4A^2} + \frac{B'}{Ar} - \frac{B'^2}{4AB} \left[-\frac{\ddot{A}}{2A} + \frac{\dot{A}^2}{4A^2} + \frac{\dot{B}\dot{A}}{4AB} \right]$$

$$R_{rr} = -\frac{B''}{2B} + \frac{B'^2}{4B^2} + \frac{A'B'}{4AB} + \frac{A'}{Ar} \left[-\frac{\ddot{A}}{2B} + \frac{\dot{A}\dot{B}}{4B^2} + \frac{\dot{A}^2}{4AB} \right]$$

$$R_{rt} = \frac{\dot{A}}{Ar}$$

$$R_{\theta\theta} = 1 - \frac{1}{A} + \frac{rA'}{2A^2} - \frac{rB'}{2AB} \quad : \quad R_{\varphi\varphi} = \sin^2\theta R_{\theta\theta}$$

$\partial_t \partial_r$ $\partial_r \partial_t$

$\frac{\partial}{\partial r \partial t} = \frac{\partial}{\partial r} \left(\frac{\partial}{\partial t} \right) = \frac{\partial}{\partial t} \frac{\partial}{\partial r}$

$$\frac{R_{tt}}{B} + \frac{R_{rr}}{A} = \frac{A'}{A^2 r} + \frac{B'}{ABr} = 0 \Rightarrow (AB)' = 0$$

$$\frac{R_{tt}}{B} + \frac{R_{rr}}{A} = \frac{A'}{A^2 r} + \frac{B'}{ABr} = 0 \Rightarrow (AB)' = 0 \Rightarrow AB = f(t)$$

$$\frac{R_{tt}}{B} + \frac{R_{rr}}{A} = \frac{A'}{A^2 r} + \frac{B'}{ABr} = 0 \Rightarrow (AB)' = 0 \Rightarrow AB = f(t)$$

Boundary conditions: want spacetime to be asymptotically Minkowski at

$$+ \frac{R_{rr}}{A} = \frac{A'}{A^2 r} + \frac{B'}{ABr} = 0 \Rightarrow (AB)' = 0 \Rightarrow AB = f(t)$$

boundary condition want spacetime to be asymptotically Minkowski at large r .

$$\frac{R_{tt}}{B} + \frac{R_{rr}}{A} = \frac{A'}{A^2 r} + \frac{B'}{ABr} = 0 \Rightarrow (AB)' = 0 \Rightarrow AB = f(t)$$

Boundary conditions: want spacetime to be asymptotically Minkowski at large

$$\Rightarrow \left. \begin{array}{l} A \rightarrow 1 \\ B \rightarrow 1 \end{array} \right\} \text{ as } r \rightarrow \infty$$

$$\frac{R_{tt}}{B} + \frac{R_{rr}}{A} = \frac{A'}{A^2 r} + \frac{B'}{ABr} = 0 \Rightarrow (AB)' = 0 \Rightarrow AB = f(t)$$

Boundary conditions: want spacetime to be asymptotically Minkowski at large

$$\Rightarrow \left. \begin{array}{l} A \rightarrow 1 \\ B \rightarrow 1 \end{array} \right\} \text{ as } r \rightarrow \infty \Rightarrow f(t) = 1.$$

$$\frac{R_{tt}}{B} + \frac{R_{rr}}{A} = \frac{A'}{A^2 r} + \frac{B'}{ABr} = 0 \Rightarrow (AB)' = 0 \Rightarrow AB = f$$

Boundary conditions: want spacetime to be asymptotically Minkowski.

$$ds^2 \rightarrow -dt^2 + dr^2 + r^2 d\Omega^2 \Rightarrow \left. \begin{array}{l} A \rightarrow 1 \\ B \rightarrow 1 \end{array} \right\} \text{ as } r \rightarrow \infty \Rightarrow f(t) = 1.$$

$\frac{\partial}{\partial r} \frac{\partial}{\partial t}$

$$\Rightarrow (AB)' = 0 \Rightarrow AB = f(t)$$

time to be asymptotically Minkowski at large r .

$$\Rightarrow f(t) = 1 \Rightarrow B = \frac{1}{A} \Rightarrow B = B(r)$$

$$\frac{R_{tt}}{B} + \frac{R_{rr}}{A} = \frac{A'}{A^2 r} + \frac{B'}{ABr} = 0 \Rightarrow (AB)' = 0 \Rightarrow AB = f$$

Boundary conditions: want spacetime to be asymptotically Minkowski

$$ds^2 \rightarrow -dt^2 + dr^2 + r^2 d\Omega^2 \Rightarrow \left. \begin{array}{l} A \rightarrow 1 \\ B \rightarrow 1 \end{array} \right\} \text{ as } r \rightarrow \infty \Rightarrow f(t) = 1 \Rightarrow B = \frac{1}{A}$$

now use $R_{\theta\theta} =$

$$\frac{R_{tt}}{B} + \frac{R_{rr}}{A} = \frac{A'}{A^2 r} + \frac{B'}{ABr} = 0 \Rightarrow (AB)' = 0 \Rightarrow AB = f$$

Boundary conditions: want spacetime to be asymptotically Minkowski

$$ds^2 \rightarrow -dt^2 + dr^2 + r^2 d\Omega^2 \Rightarrow \left. \begin{array}{l} A \rightarrow 1 \\ B \rightarrow 1 \end{array} \right\} \text{ as } r \rightarrow \infty \Rightarrow f(t) = 1 \Rightarrow B = \frac{1}{A}$$

now use $R_{\theta\theta} = 1 - B$

$$R_{rr} = -\frac{B}{2B} + \frac{B'^2}{4B^2} + \frac{A'B'}{4AB} + \frac{A'}{Ar} \left[-\frac{\ddot{A}}{2B} + \frac{\dot{A}\dot{B}}{4B^2} + \frac{\dot{A}^2}{4AB} \right]$$

$$R_{rt} = \frac{\dot{A}}{Ar}$$

$$R_{\theta\theta} = 1 - \frac{1}{A} + \frac{rA'}{2A^2} - \frac{rB'}{2AB} \quad ; \quad R_{\varphi\varphi} = \sin^2\theta R_{\theta\theta}$$

other components $R_{r\theta} = R_{r\varphi} = R_{\theta\varphi} = R_{\theta t} = R_{\varphi t} = 0$ by spherical symmetry

have to solve these four coupled nonlinear p.d.e.'s for $A(r,t)$ and $B(r,t)$.

Amazingly simple: $R_{rt} = 0 \Rightarrow \frac{\partial}{\partial t}(A(r,t)) = 0 ! \Rightarrow \underline{A = A(r)} \Rightarrow$ all time derivatives

$$\frac{R_{tt}}{B} + \frac{R_{rr}}{A} = \frac{A'}{A^2 r} + \frac{B'}{ABr} = 0 \Rightarrow (AB)' = 0 \Rightarrow AB = f(t)$$

Boundary conditions: want spacetime to be asymptotically flat

$$ds^2 \rightarrow -dt^2 + dr^2 + r^2 d\Omega^2 \Rightarrow \left. \begin{array}{l} A \rightarrow 1 \\ B \rightarrow 1 \end{array} \right\} \text{as } r \rightarrow \infty \Rightarrow f(t) = 1 \Rightarrow B = \frac{1}{A}$$

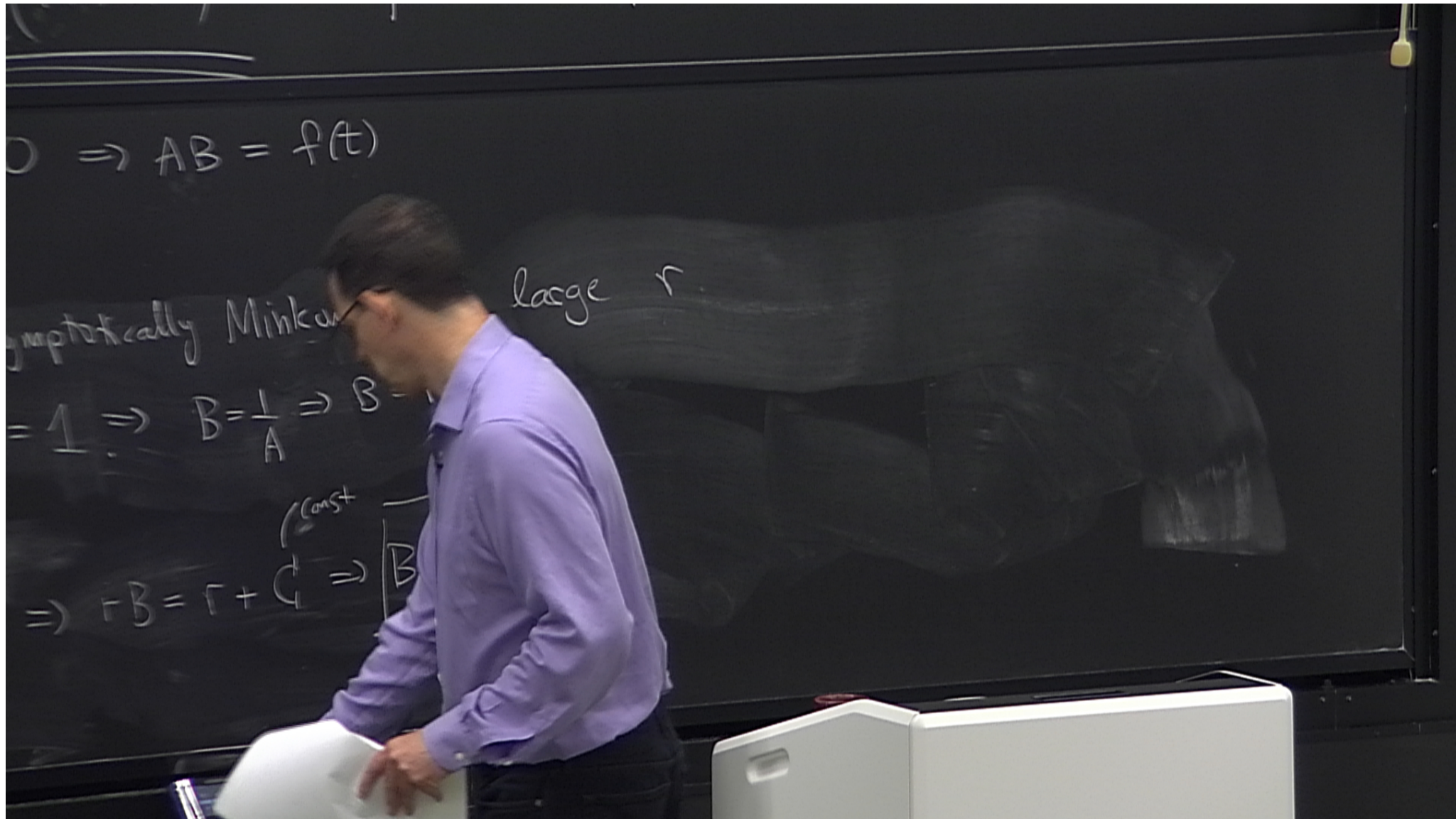
$$\text{now use } R_{\theta\theta} = 1 - B - rB' = 0 \Rightarrow (rB)' = 1 \Rightarrow rB = r + C$$

$$\frac{R_{tt}}{B} + \frac{R_{rr}}{A} = \frac{A'}{A^2 r} + \frac{B'}{ABr} = 0 \Rightarrow (AB)' = 0 \Rightarrow AB = f(t)$$

Boundary conditions: want spacetime to be asymptotically Minkowski

$$ds^2 \rightarrow -dt^2 + dr^2 + r^2 d\Omega^2 \Rightarrow \left. \begin{array}{l} A \rightarrow 1 \\ B \rightarrow 1 \end{array} \right\} \text{ as } r \rightarrow \infty \Rightarrow f(t) = 1 \Rightarrow B = \frac{1}{A} \Rightarrow B = B$$

$$\text{now use } R_{\theta\theta} = 1 - B - rB' = 0 \Rightarrow (rB)' = 1 \Rightarrow rB = r + C \Rightarrow B = \frac{r+C}{r}$$



asymptotically Minkowski at large r .

$$= 1 \Rightarrow B = \frac{1}{A} \Rightarrow B = B(r)$$

$$\Rightarrow rB = r + C \Rightarrow \boxed{B = 1 + \frac{C}{r}} \quad \Rightarrow \quad \boxed{A = \frac{1}{1 + \frac{C}{r}}}$$

— very simple and very general!

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consider a mass M (e.g. Sun or Earth)

at large r , $\phi = -\frac{GM}{r}$, recall from correspondence with Newton that

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with Newton that $h_{00} = -\frac{2\phi}{c^2} \Rightarrow g_{00} \approx -1 - \frac{2\phi}{c^2} = -\left(1 - \frac{2GM}{rc^2}\right)$

— very simple and very general!

(Birkhoff Theorem)

consider a mass M (e.g. Sun or Earth)

at large r , $\phi = -\frac{GM}{r}$, recall from correspondence with Newton

$$\Rightarrow \text{fixes } C, \quad B = 1 - \frac{2GM}{rc^2}, \quad A = B^{-1}$$

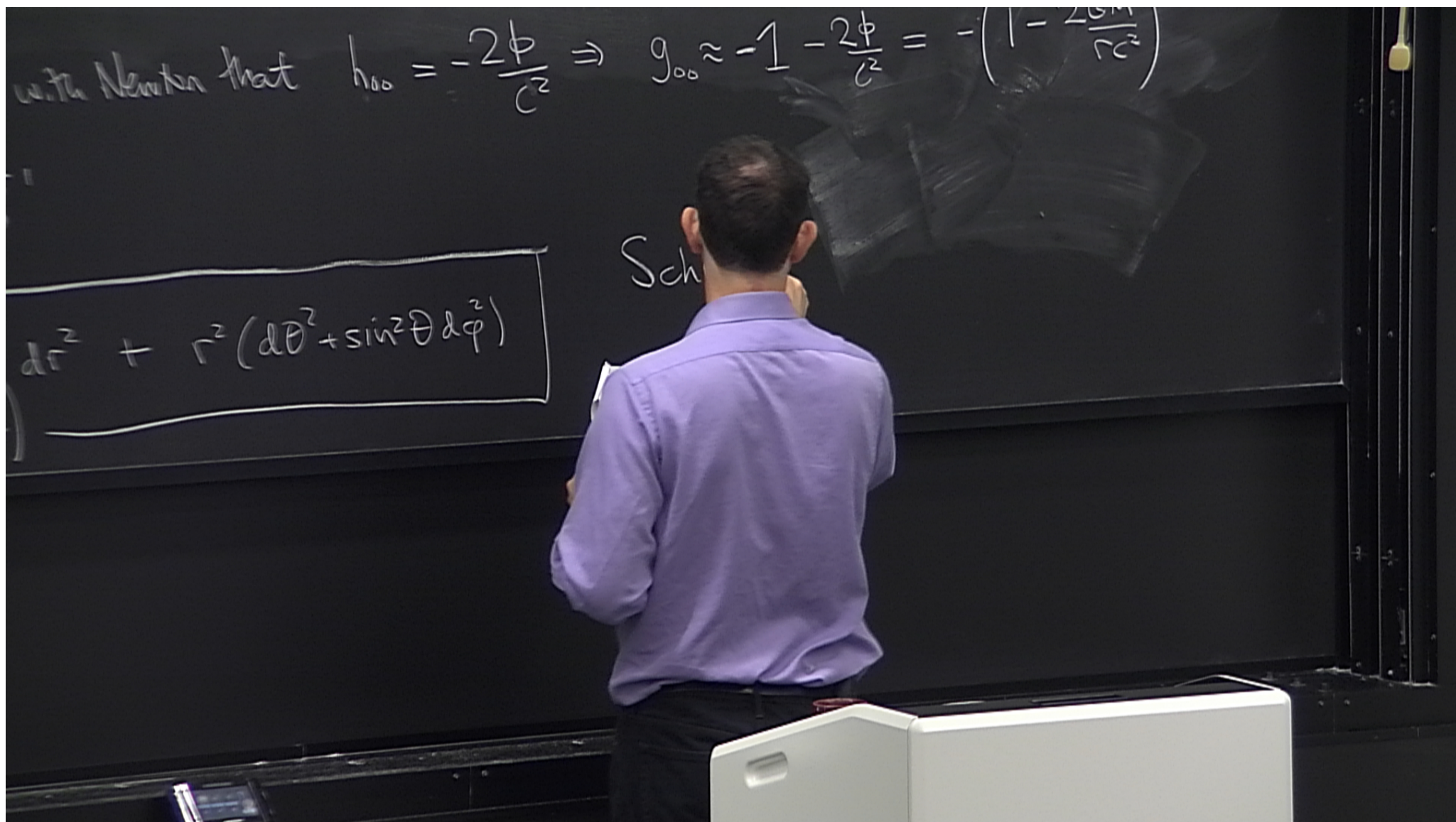
$$ds^2 =$$

Consider a mass M (spherically symmetric)

at large r , $\phi = -\frac{GM}{r}$, recall from correspondence with Newton that

$$\Rightarrow \text{fixes } C, \quad B = 1 - \frac{2GM}{rc^2}, \quad A = B^{-1}$$

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right) dt^2 + \frac{1}{\left(1 - \frac{2GM}{rc^2}\right)} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$



with Newton that $h_{00} = -\frac{2\phi}{c^2} \Rightarrow g_{00} \approx -1 - \frac{2\phi}{c^2} = -\left(1 - \frac{2GM}{rc^2}\right)$

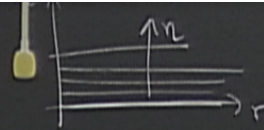
$$dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

Schwarzschild metric (1915)

$$\frac{\partial t'}{\partial t \partial r} = \frac{\partial t'}{\partial r \partial t}$$

$$= \frac{\partial t}{\partial t} \frac{dt}{dt} + \frac{\partial t}{\partial r} \frac{dr}{dt}$$

$$\frac{\partial t'}{\partial r \partial t} = \frac{\partial}{\partial r} \left(\eta(r,t) \Phi(r,t) \right) = \frac{\partial t'}{\partial t \partial r} = \frac{\partial}{\partial t} \left(\eta(r,t) E(r,t) \right)$$



— very simple and very general! (Birkhoff theorem)

consider a mass M (e.g. Sun or Earth)

at large r , $\phi = -\frac{GM}{r}$, recall from correspondence with Newton that $h_{00} = -\frac{2\phi}{c^2} \Rightarrow g_{00} \approx -1 - \frac{2\phi}{c^2} = -\left(1 - \frac{2GM}{rc^2}\right)$

$\Rightarrow$ fix C , $B = 1 - \frac{2GM}{rc^2}$, $A = B$

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right) dt^2 + \frac{1}{\left(1 - \frac{2GM}{rc^2}\right)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

Schwarzschild metric (1915)

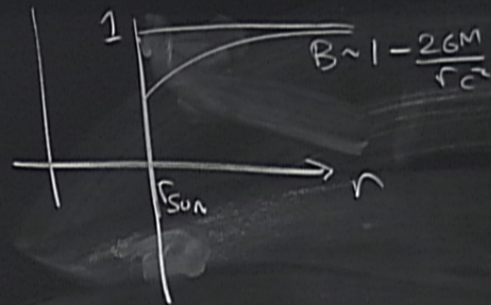
$$\frac{1}{2B} + \frac{D}{4B^2} + \frac{A/D}{4AB} + \frac{A}{Ar} \left[\frac{A}{2B} + \frac{AB}{4B^2} + \frac{A}{4AB} \right]$$

VACUUM

r_{sun} : Schwarzschild metric is valid outside the sun i.e. for $r > r_{\text{sun}}$



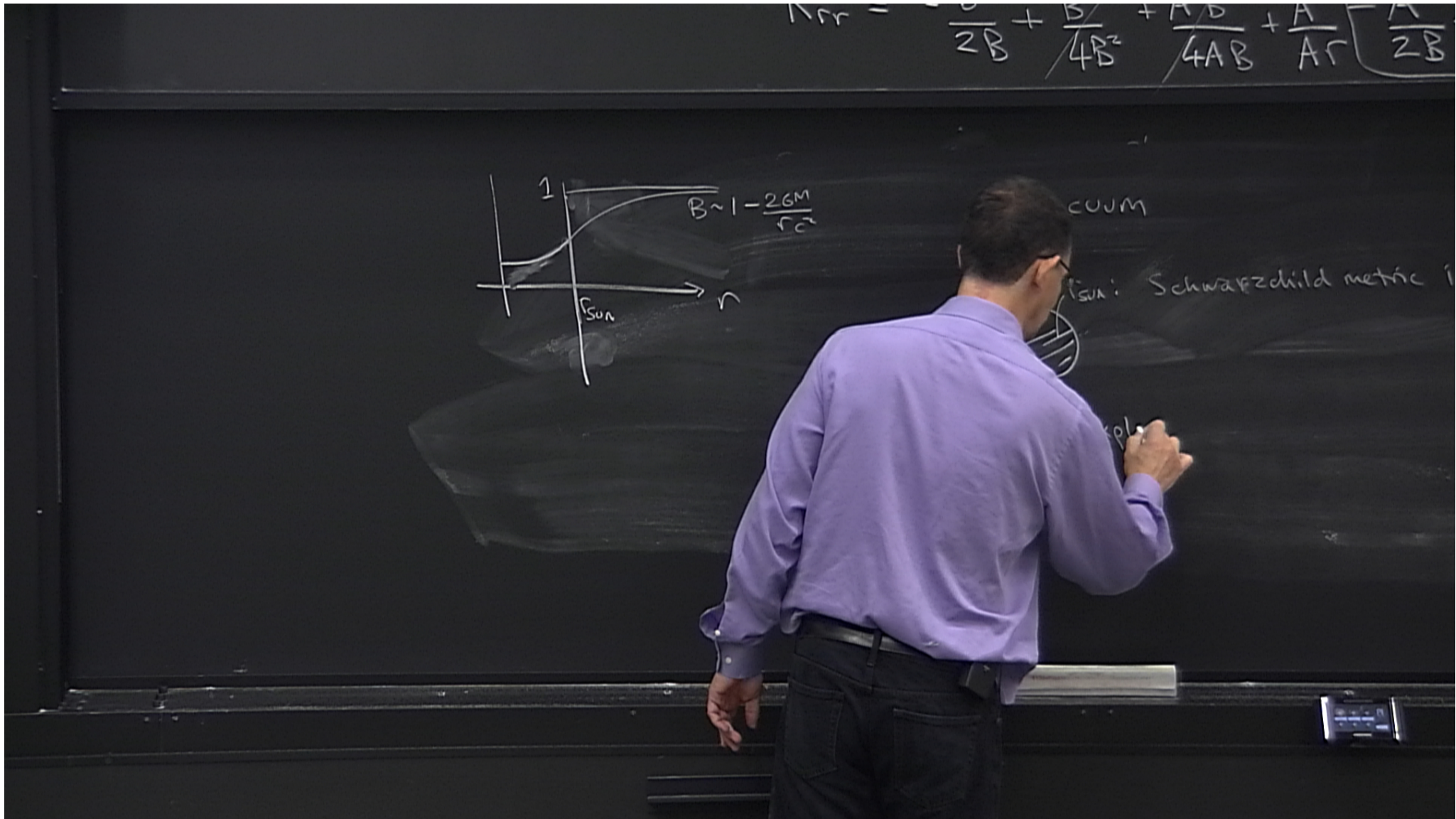
$$K_{rr} = -\frac{0}{2B} + \frac{B}{4B^2} + \frac{A/B}{4AB} + \frac{A}{Ar} \left[\frac{A}{2B} \right]$$



VACUUM

r_{sun} : Schwarzschild metric





with Newton that $h_{00} = -\frac{2\phi}{c^2} \Rightarrow g_{00} \approx -1 - \frac{2\phi}{c^2} = -\left(1 - \frac{2GM}{rc^2}\right)$

$$dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

Schwarzschild metric (1915)

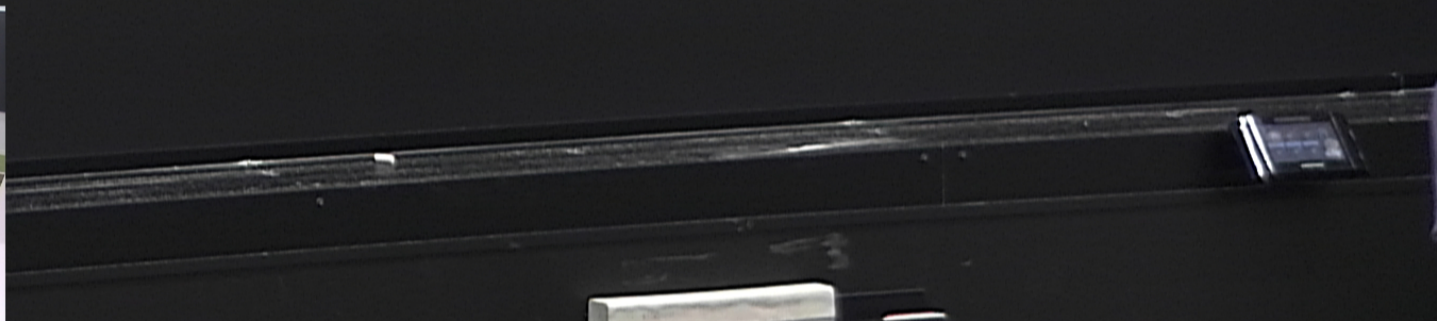
defined so that a const- r surface has area exactly $4\pi r^2$.

Consider a mass M (very small)

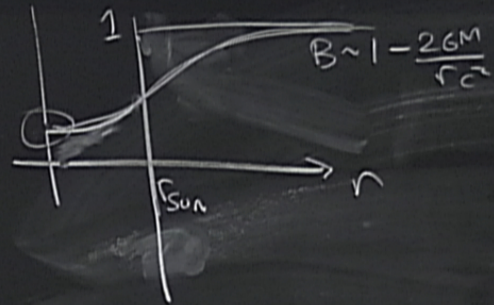
at large r , $\phi = -\frac{GM}{r}$, recall from correspondence with Newton that

$$\Rightarrow \text{fixes } C, \quad B = 1 - \frac{2GM}{rc^2}, \quad A = B^{-1}$$

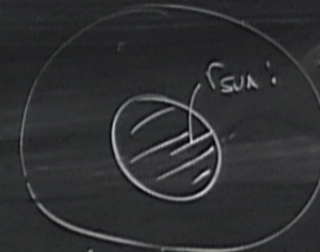
$$ds^2 = - \left(1 - \frac{2GM}{rc^2} \right) dt^2 + \frac{1}{\left(1 - \frac{2GM}{rc^2} \right)} dr^2 + r^2 (d\theta^2 + d\phi^2)$$



$$K_{rr} = -\frac{0}{2B} + \frac{B}{4B^2} + \frac{A}{4AB} + \frac{A}{Ar} \left[\frac{A}{2B} \right]$$



Vacuum



Imagine it explodes

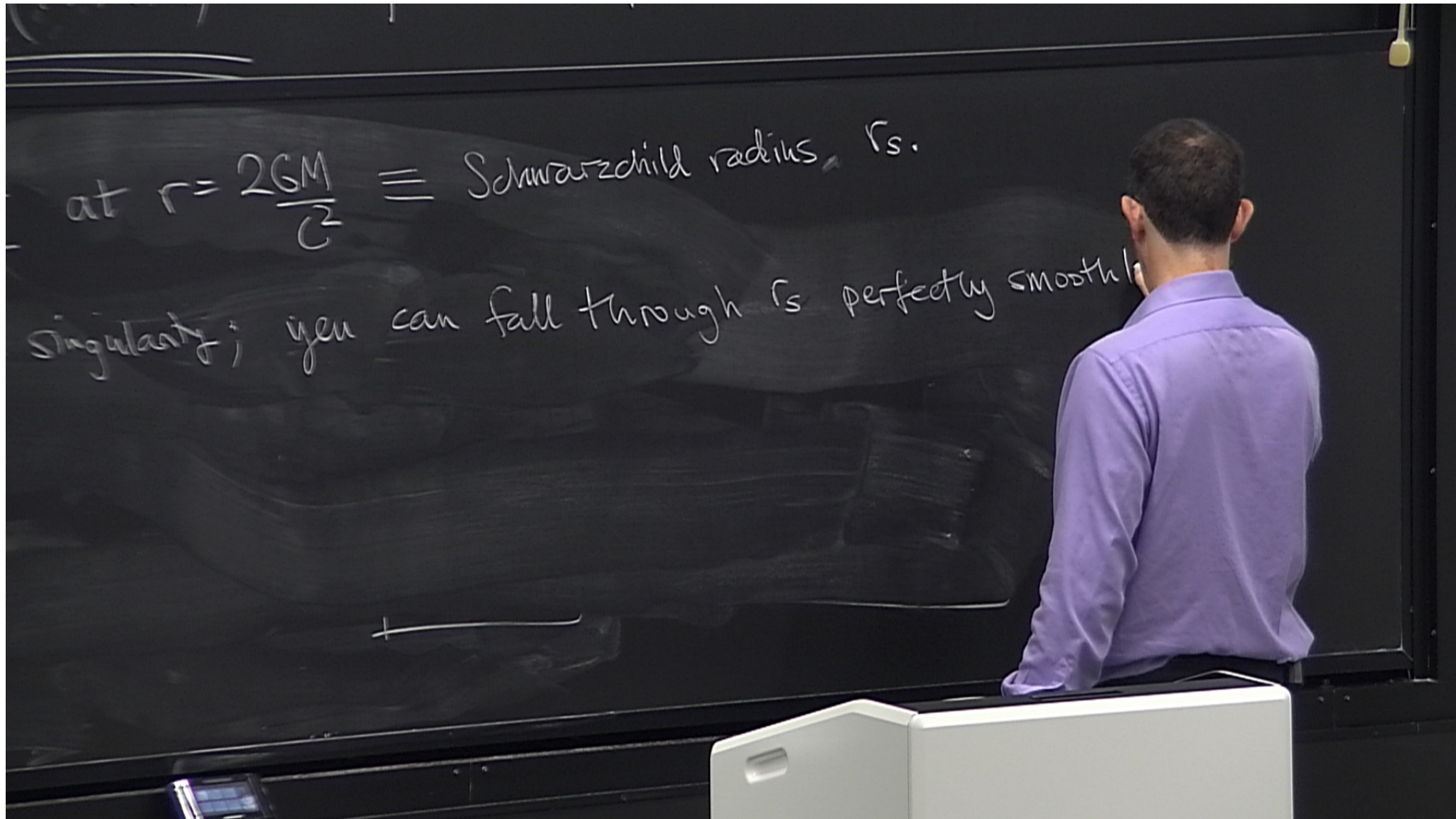
Note: Schwarzschild metric is singular

Note: Schwarzschild metric is singular at $r = 2G$

at $r = \frac{2GM}{c^2} \equiv$ Schwarzschild radius.

$\frac{\partial}{\partial t} = \frac{\partial}{\partial r} \left(\frac{dr}{dt} \right) \frac{\partial}{\partial r} + \frac{\partial}{\partial t} \frac{\partial}{\partial t}$

Note: Schwarzschild metric is singular at $r = \frac{2GM}{c^2}$
this is actually just a coordinate singularity; ye



$$(-E) \sim \frac{C - 2CE + E^2}{(C - 2E)}$$

$$= -C(r,t)dt^2 + D(r,t)dr^2 + 2E(r,t)drdt + r^2 d\Omega^2$$

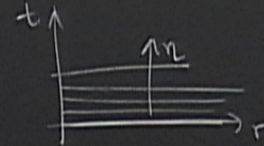
remove E: define "Integrating Factor": designed to ensure the coord transformation exists.

$$dt' = \eta(r,t) [C(r,t)dt - E(r,t)dr]$$

$$= \frac{\partial t'}{\partial t} dt + \frac{\partial t'}{\partial r} dr$$

$$\frac{\partial t'}{\partial t \partial r} = \frac{\partial t'}{\partial r \partial t}$$

$$\frac{\partial^2 t'}{\partial r \partial t} = \frac{\partial}{\partial r} \left(\eta(r,t) C(r,t) \right) = \frac{\partial t'}{\partial r \partial t} = - \frac{\partial}{\partial t} \left(\eta(r,t) E(r,t) \right)$$



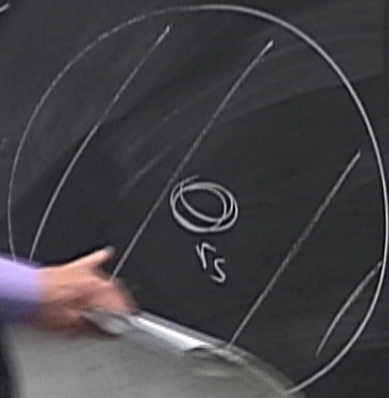
Note: Schwarzschild metric is singular at $r = \frac{2GM}{c^2} \equiv$ Schwarzschild radius, r_s .

this is actually just a coordinate singularity; you can fall through r_s perfectly smoothly.

$$G = 6.7 \times 10^{-11}$$

$$M_0 = 2 \times 10^{30}$$

$$\frac{2GM}{c^2} = 3 \text{ km}$$

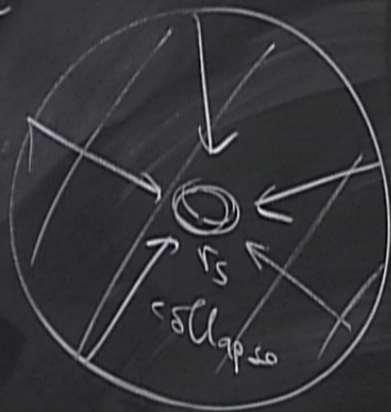


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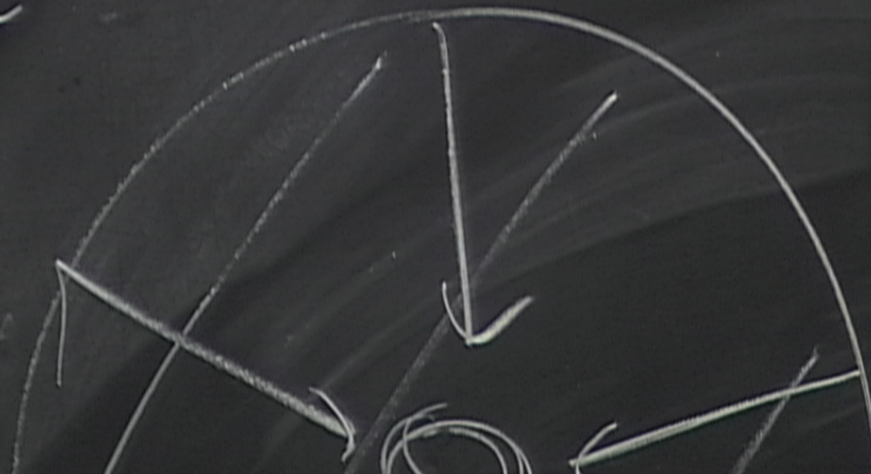
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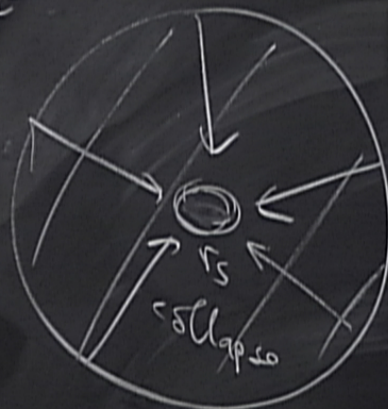
Note: S



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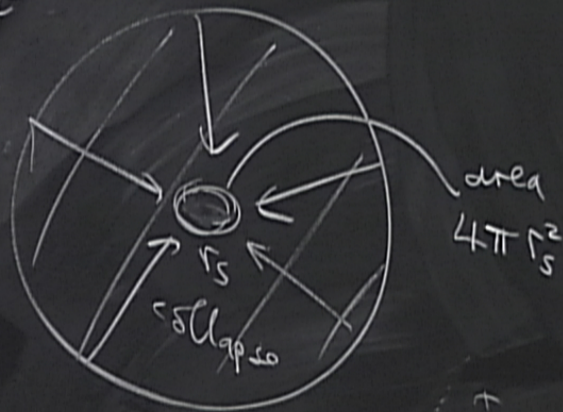


Note: Schwarzschild metric is singular at $r = 2G$
 this is only just a coordinate singularity;

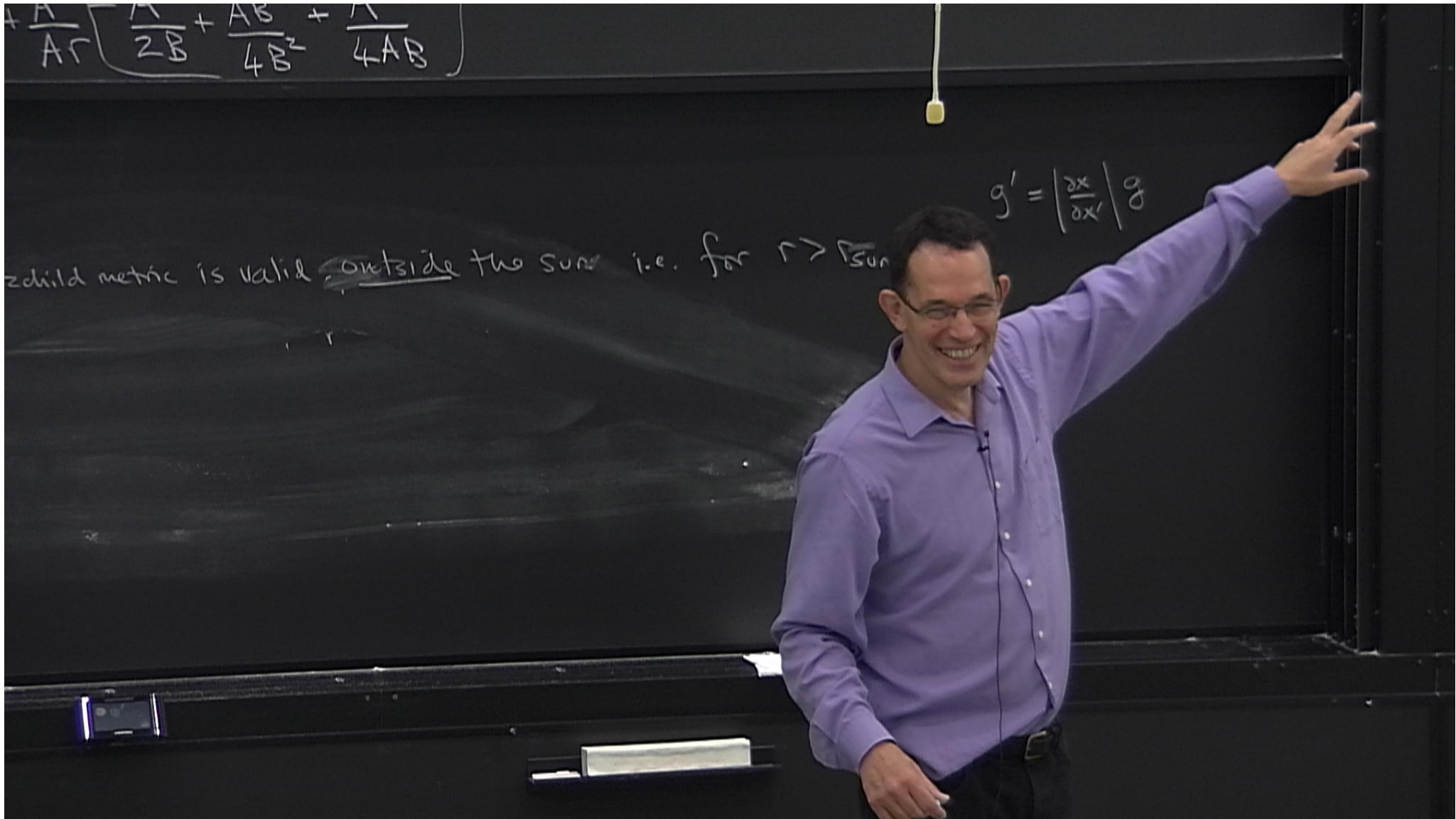
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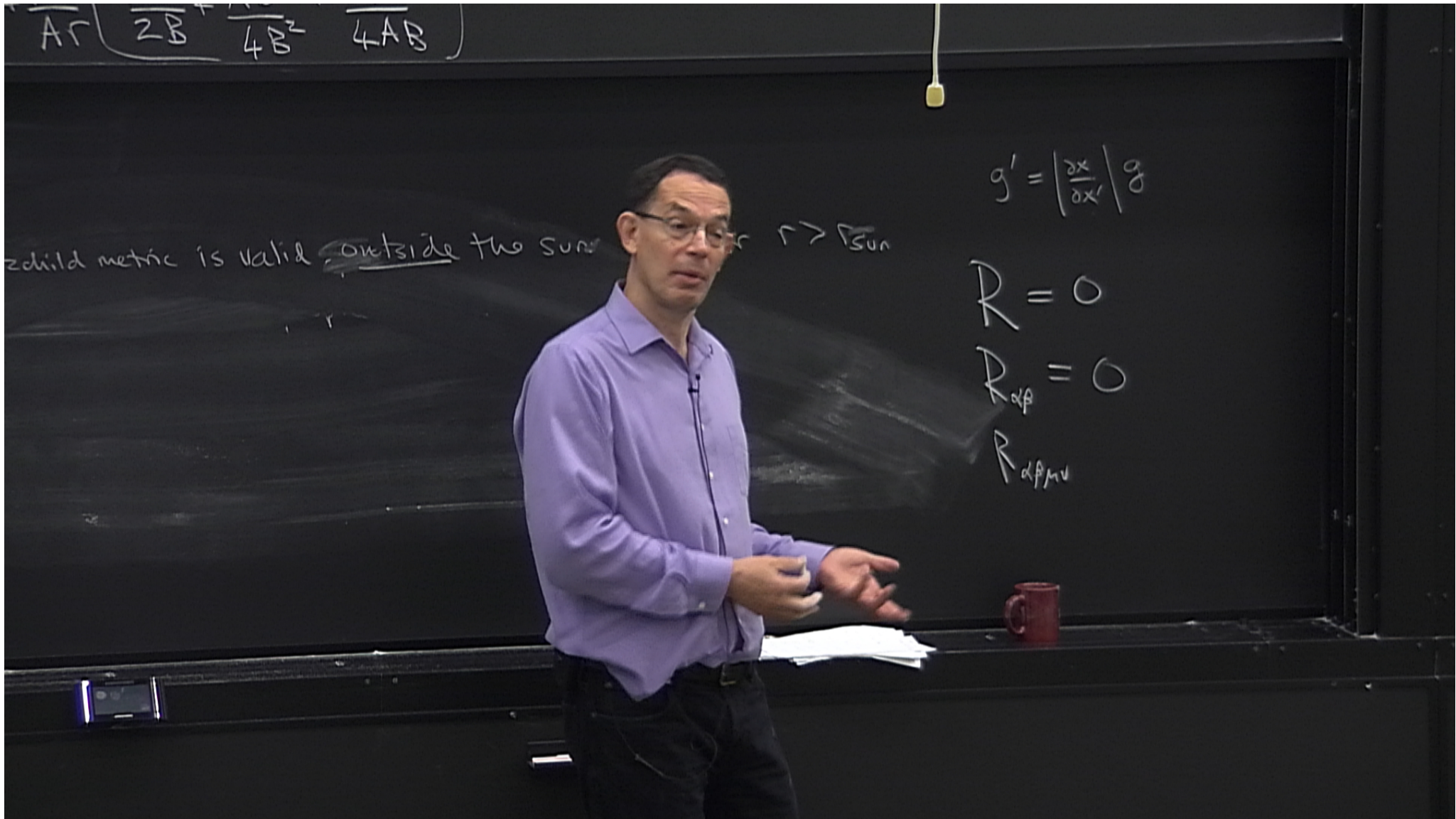
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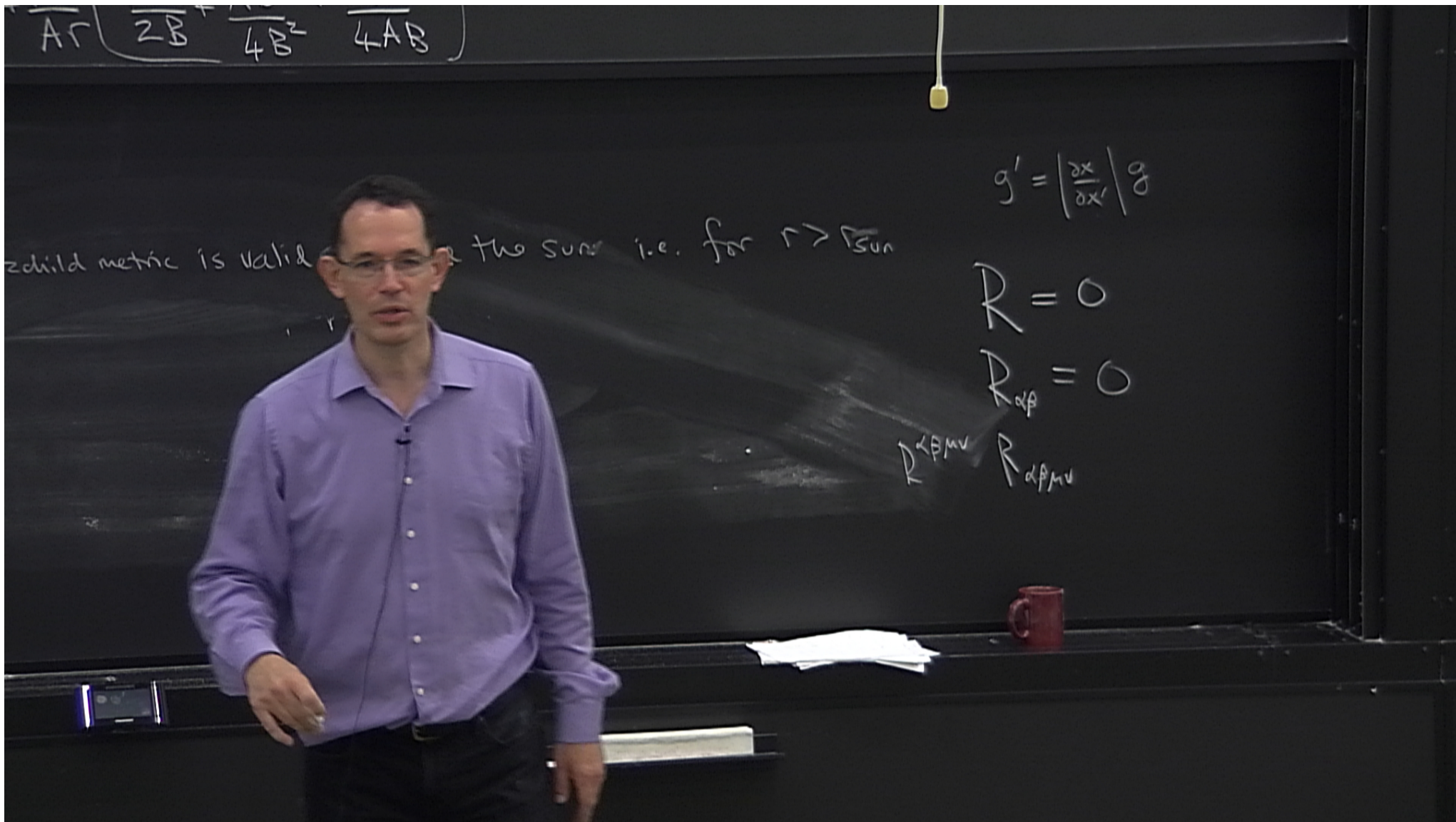


$$+ \frac{A}{Ar} \left(\frac{A}{2B} + \frac{AB}{4B^2} + \frac{A}{4AB} \right)$$

$$g' = \left| \frac{\partial x}{\partial x'} \right| g$$

Schild metric is valid outside the sun i $r > r_{\text{sun}}$





$$\overline{A^2} \overline{2B} + \overline{4B^2} \overline{4AB}$$

is valid outside the sun i.e. for $r > r_{\text{sun}}$

$$g' = \left| \frac{\partial x}{\partial x'} \right| g$$

$$R = 0$$

$$R_{\alpha\beta} = 0$$

$$\left(R^{\alpha\beta\mu\nu} R_{\alpha\beta\mu\nu} \right)$$

$$\overline{Ar} \left(\overline{2B} + \frac{\overline{1}}{4B^2} \overline{4AB} \right)$$

Schild metric is valid outside the sun i.e. for $r > r_{\text{sun}}$

$$g' = \left| \frac{\partial x}{\partial x'} \right| g$$

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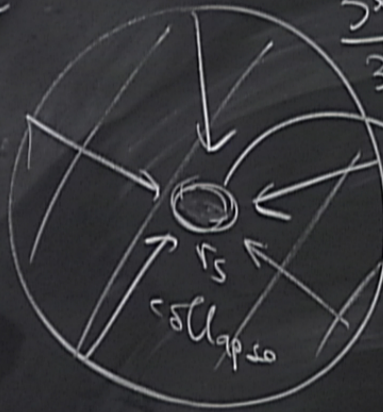
$$R_{\alpha\beta} = 0$$

$$\left(R^{\alpha\beta\mu\nu} R_{\alpha\beta\mu\nu} \right) = \frac{GM^2}{r^4}$$

$$G = 6.7 \times 10^{-11}$$

$$M_0 = 2 \times 10^{30}$$

$$\frac{2GM_0}{c^2} = 3 \text{ km}$$



$$\frac{3 \times 10^9 \text{ m}}{3 \times 10^8 \text{ m/s}} \sim 10 \text{ s}$$

this is actually just a coordinate singularity;

area

$$4\pi r_s^2$$