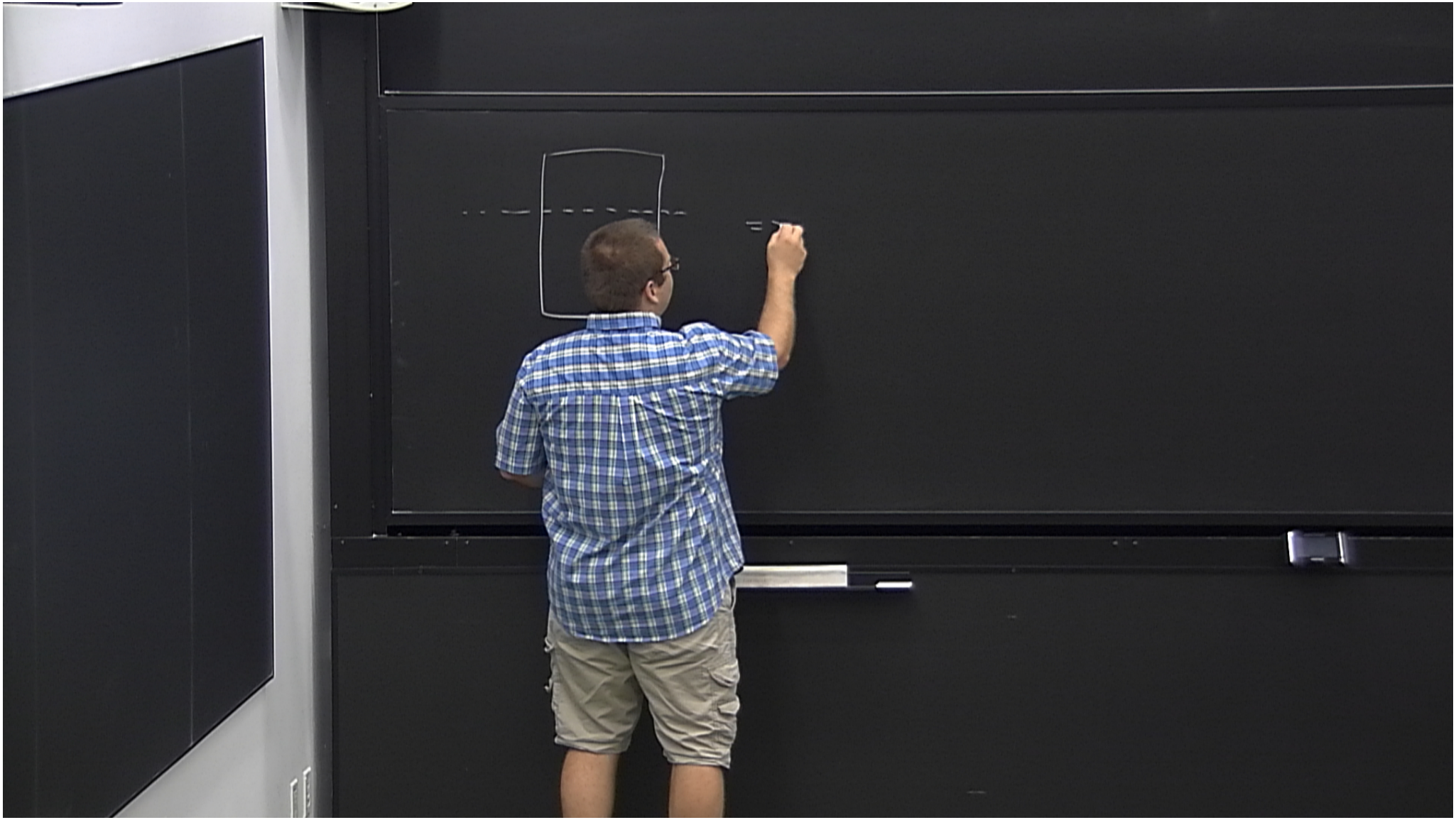
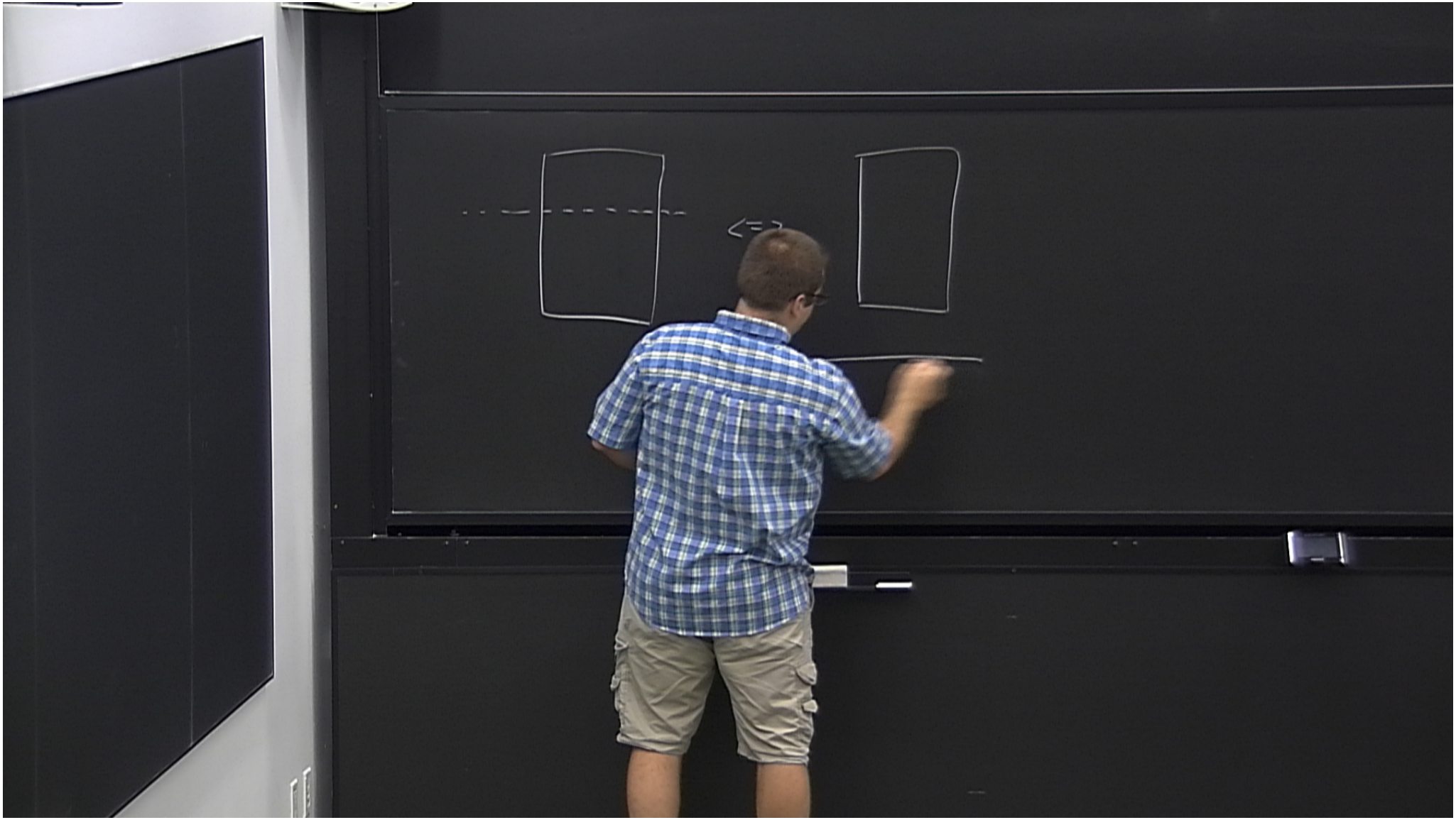
Title: 13/14 PSI - Relativity - Lecture 4

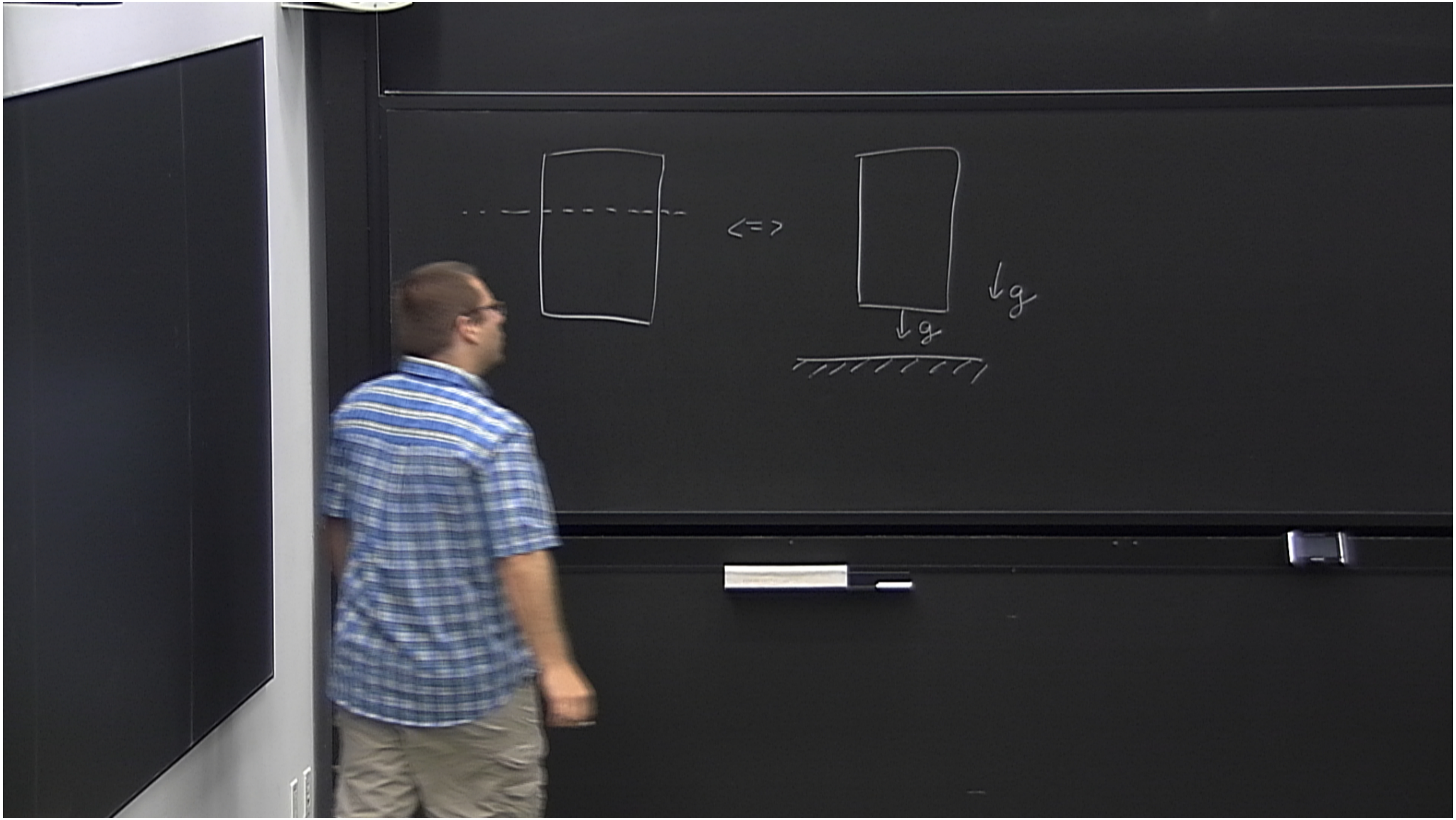
Date: Sep 12, 2013 09:00 AM

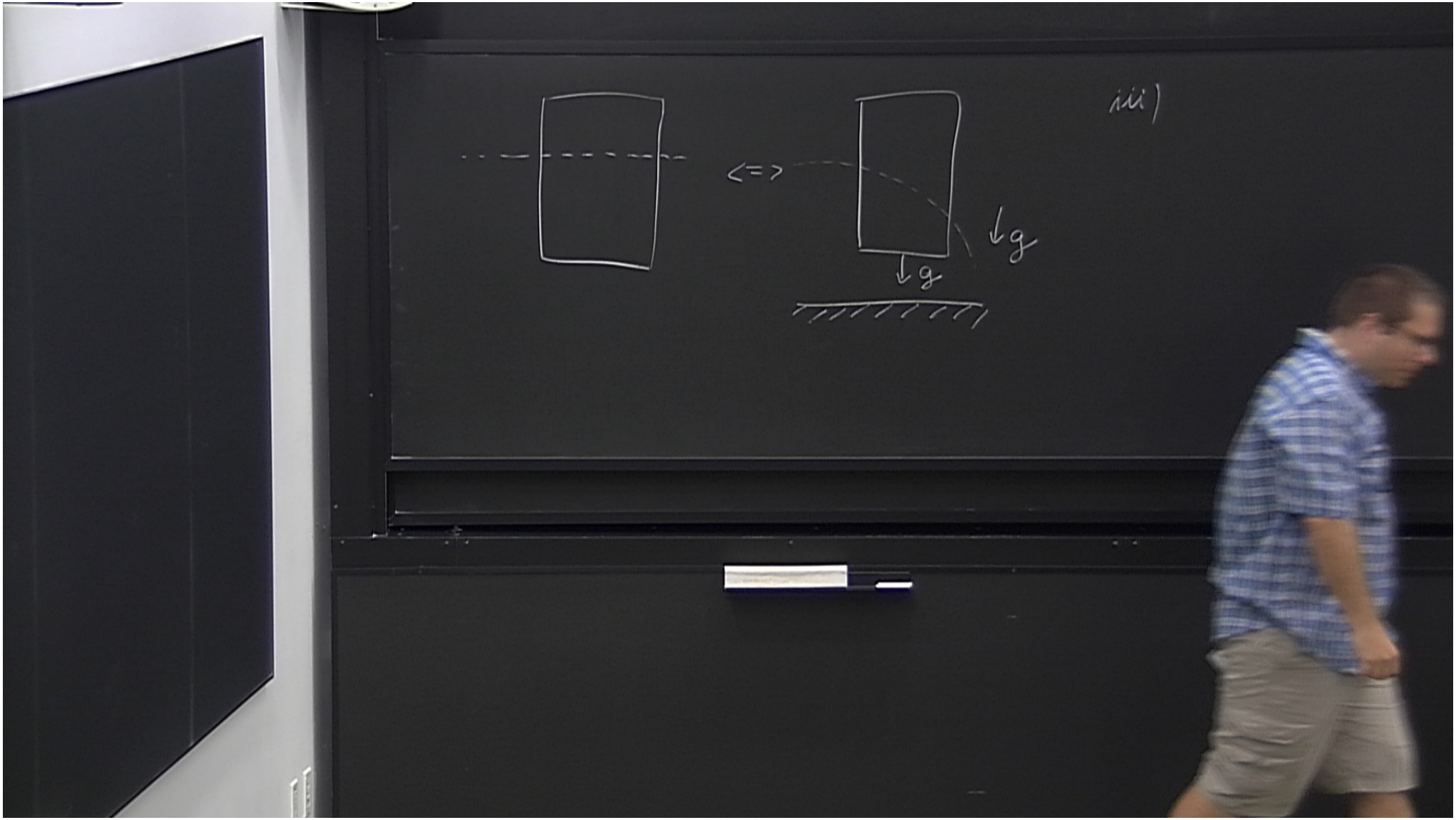
URL: http://pirsa.org/13090014

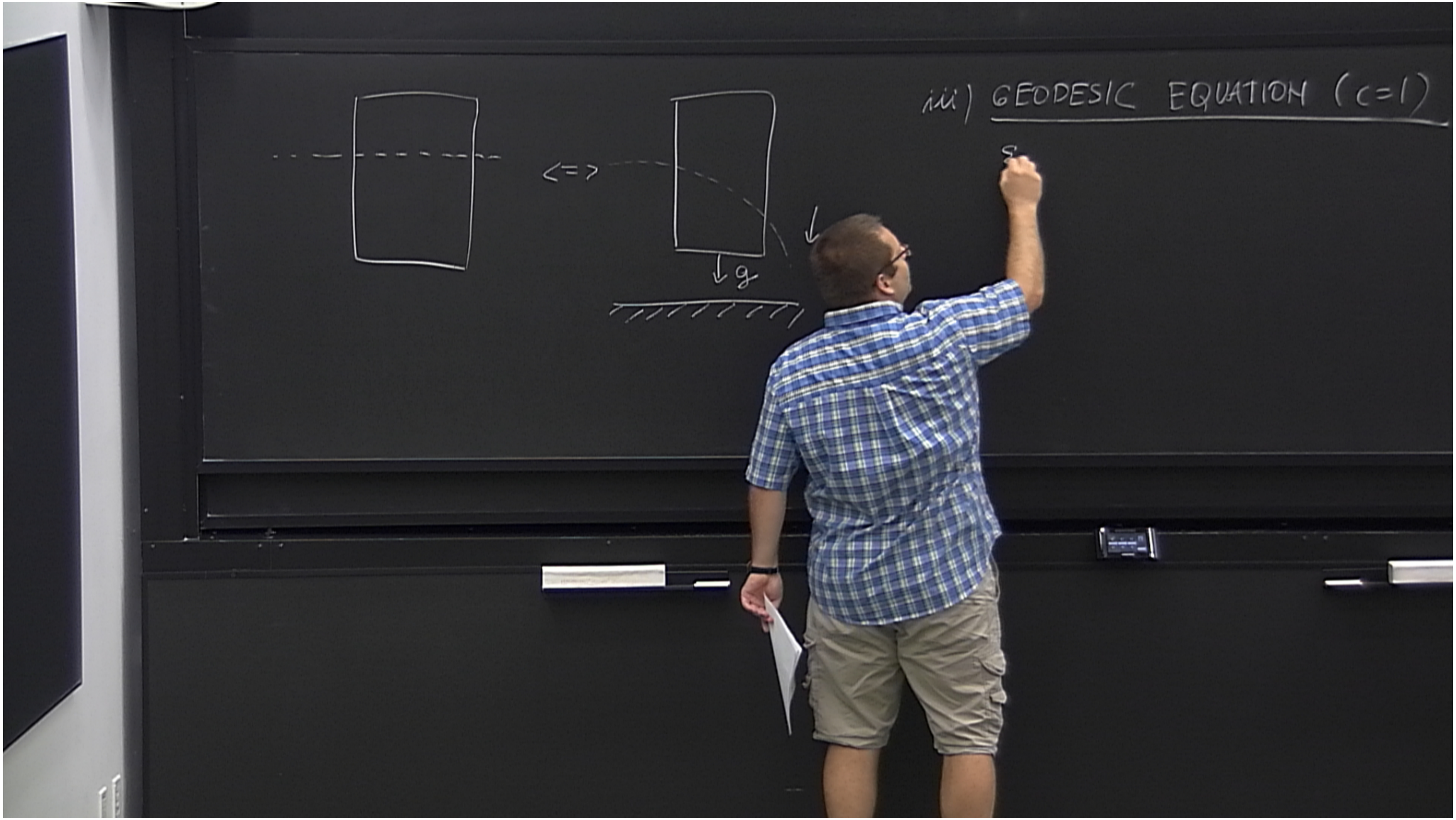
Abstract:

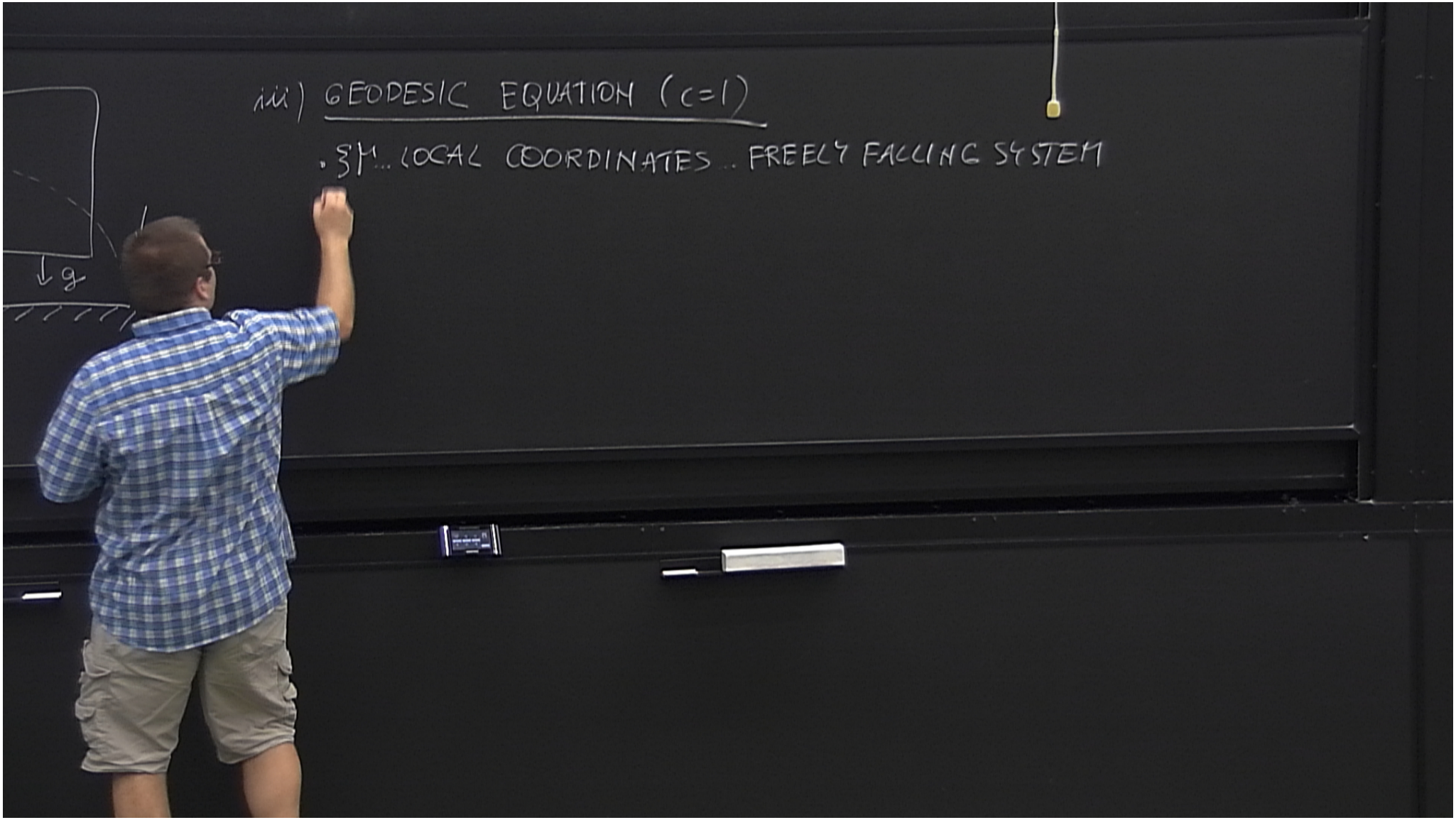


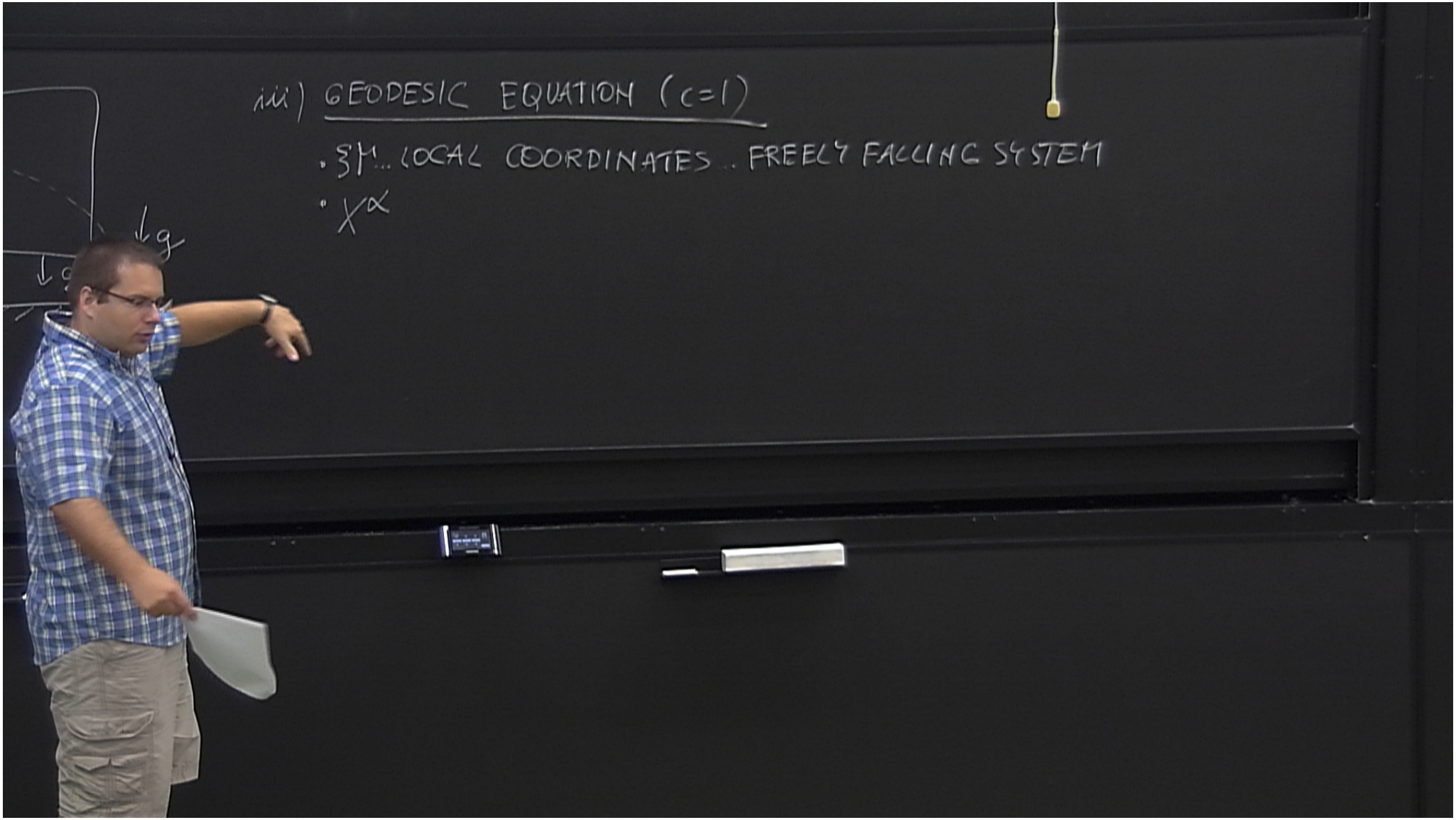


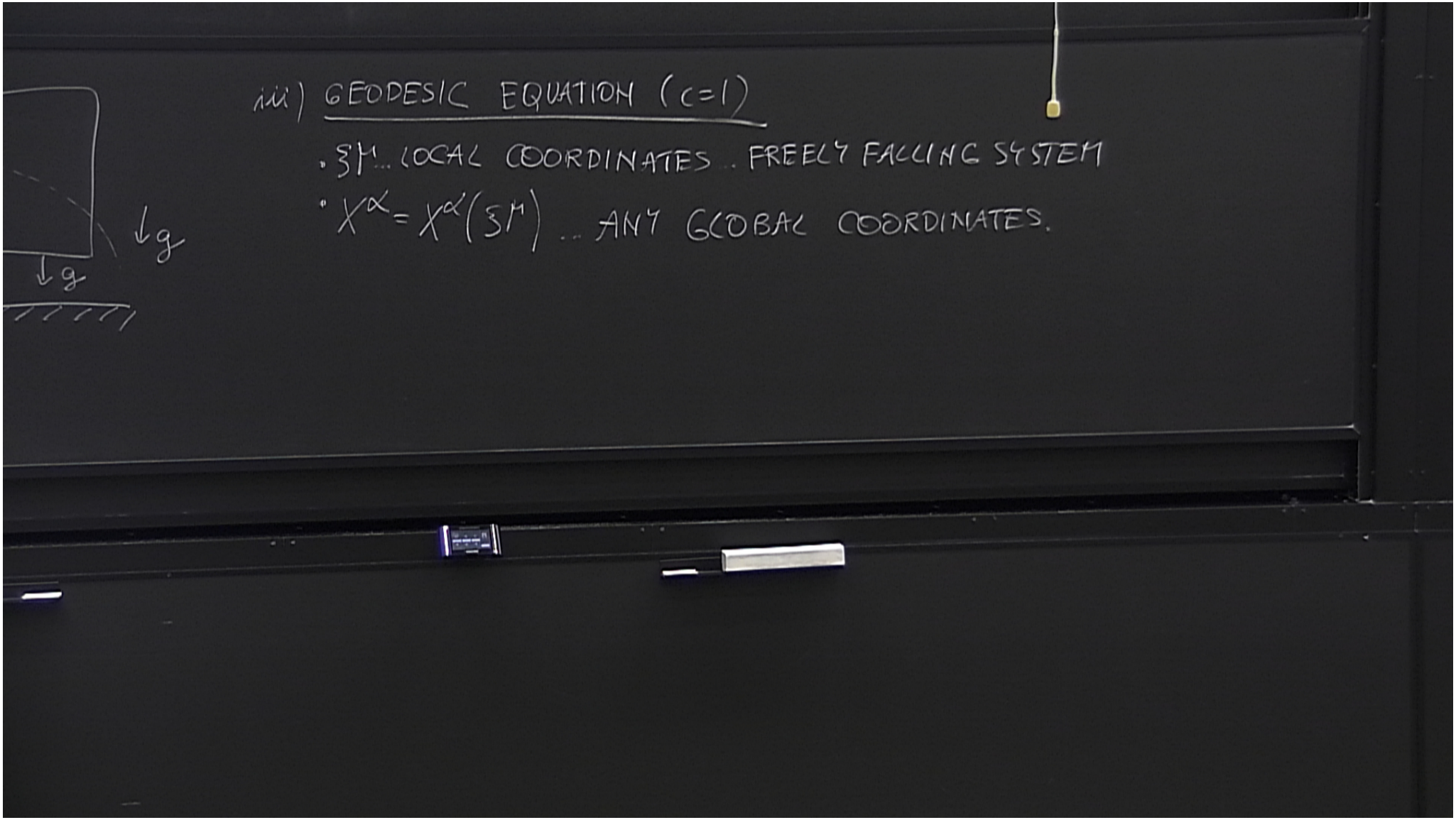

g
g

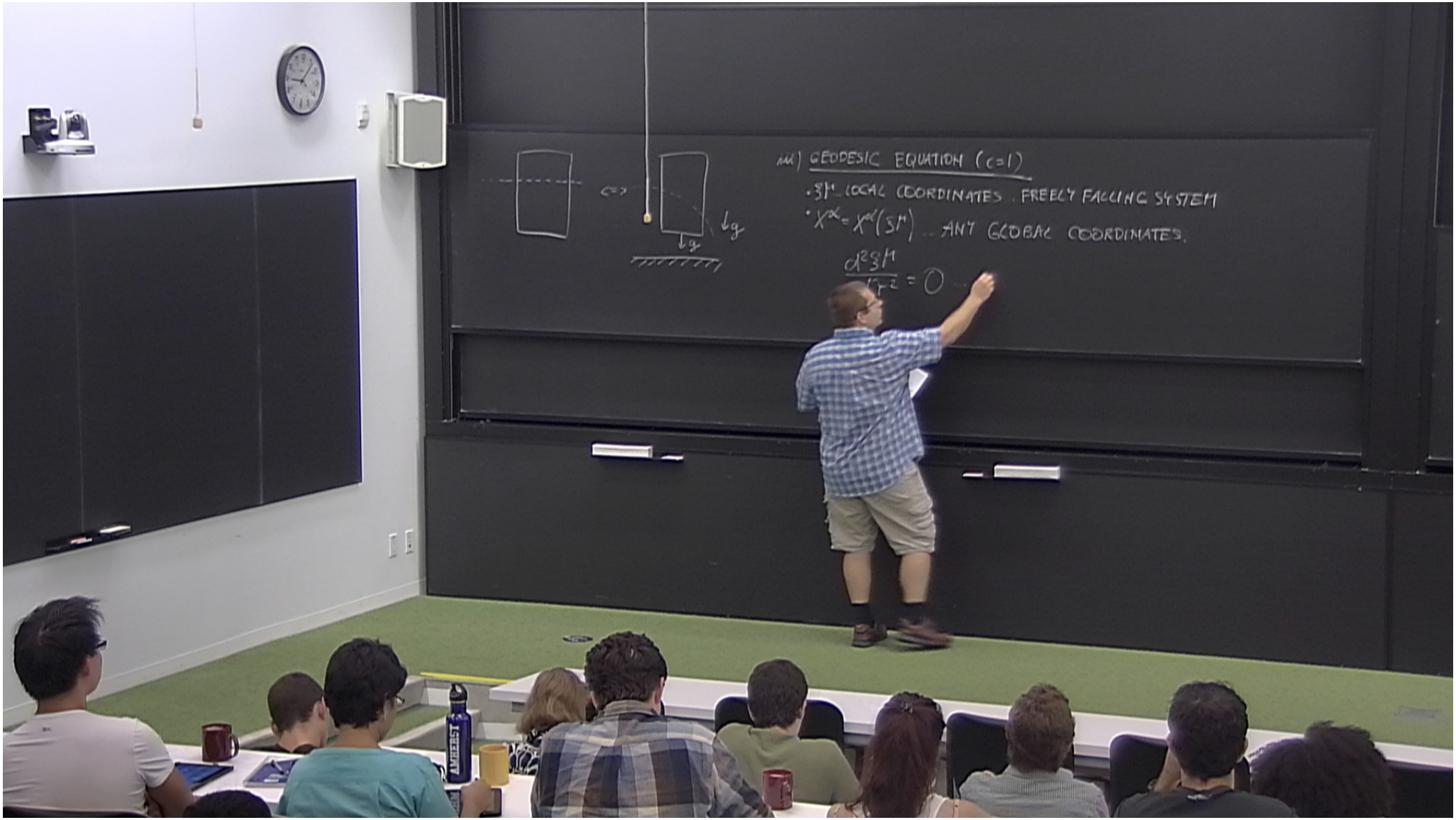

iii)
<=>
g
g

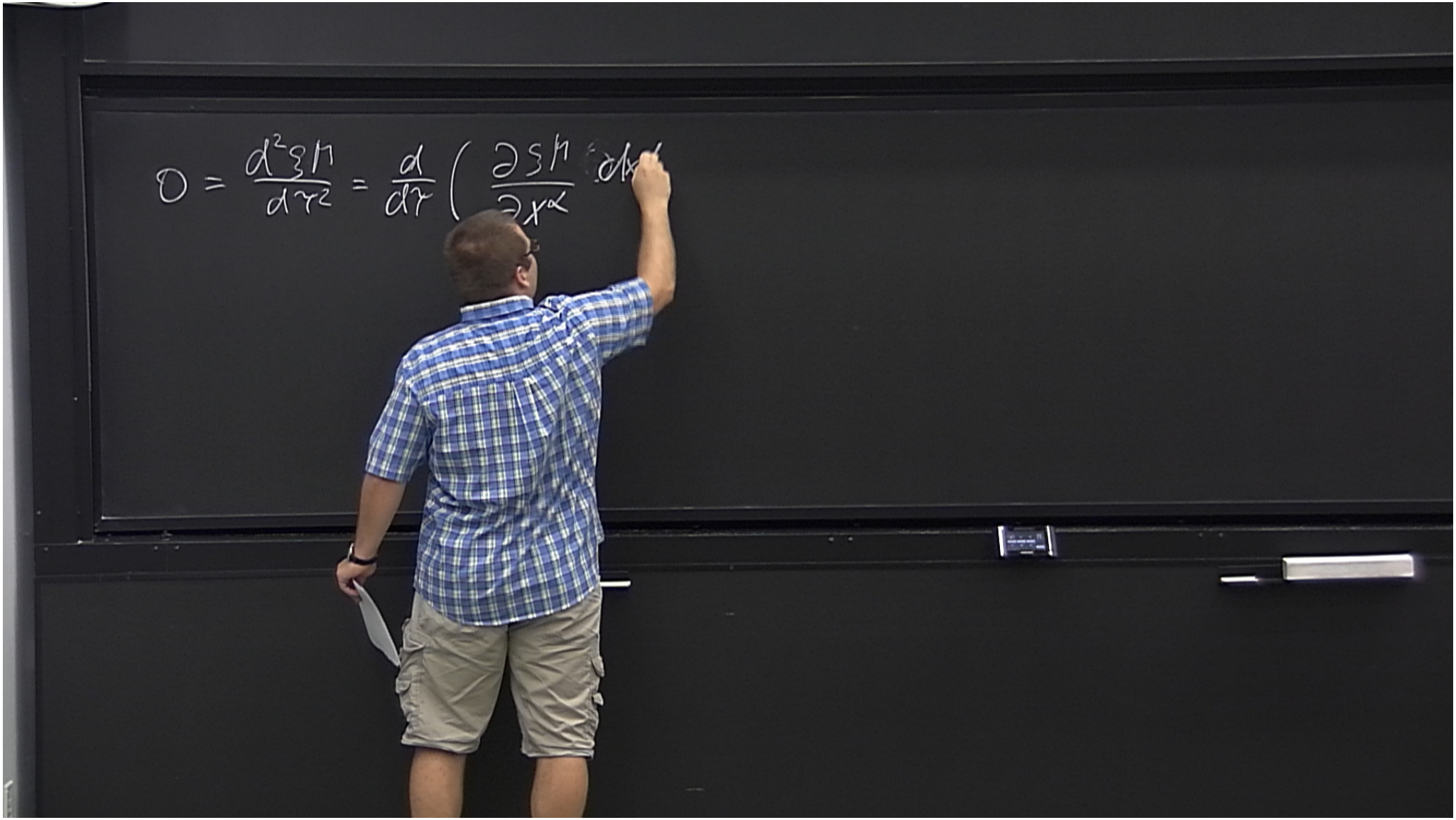

iii) GEODESIC EQUATION (c=1)
<=>
g

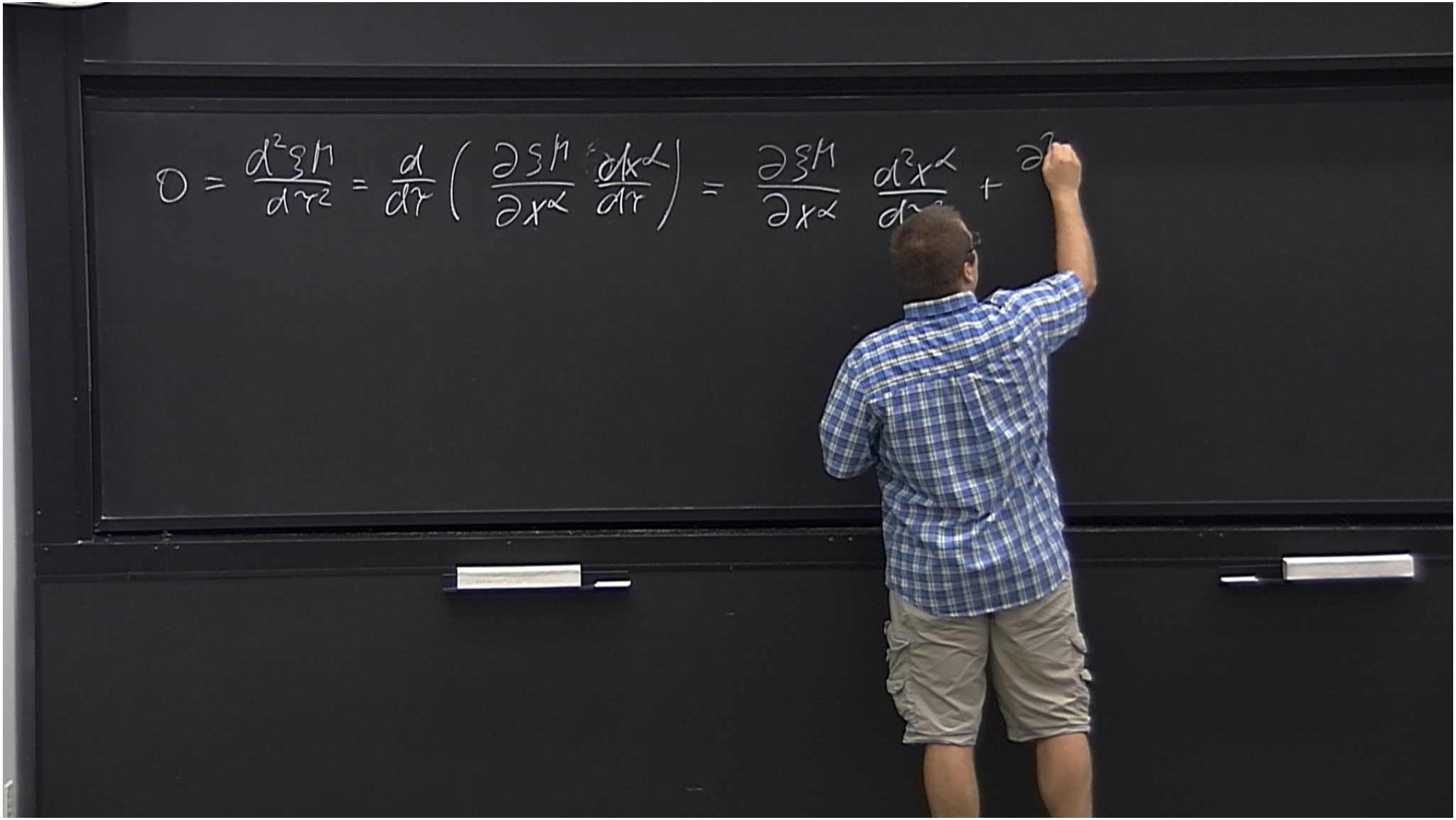

iii) GEODESIC EQUATION (c=1)
. ξ^μ .. LOCAL COORDINATES .. FREELY FALLING SYSTEM
g


iii) GEODESIC EQUATION (c=1)
• ξ^μ .. LOCAL COORDINATES .. FREELY FALLING SYSTEM
• X^α
g
g

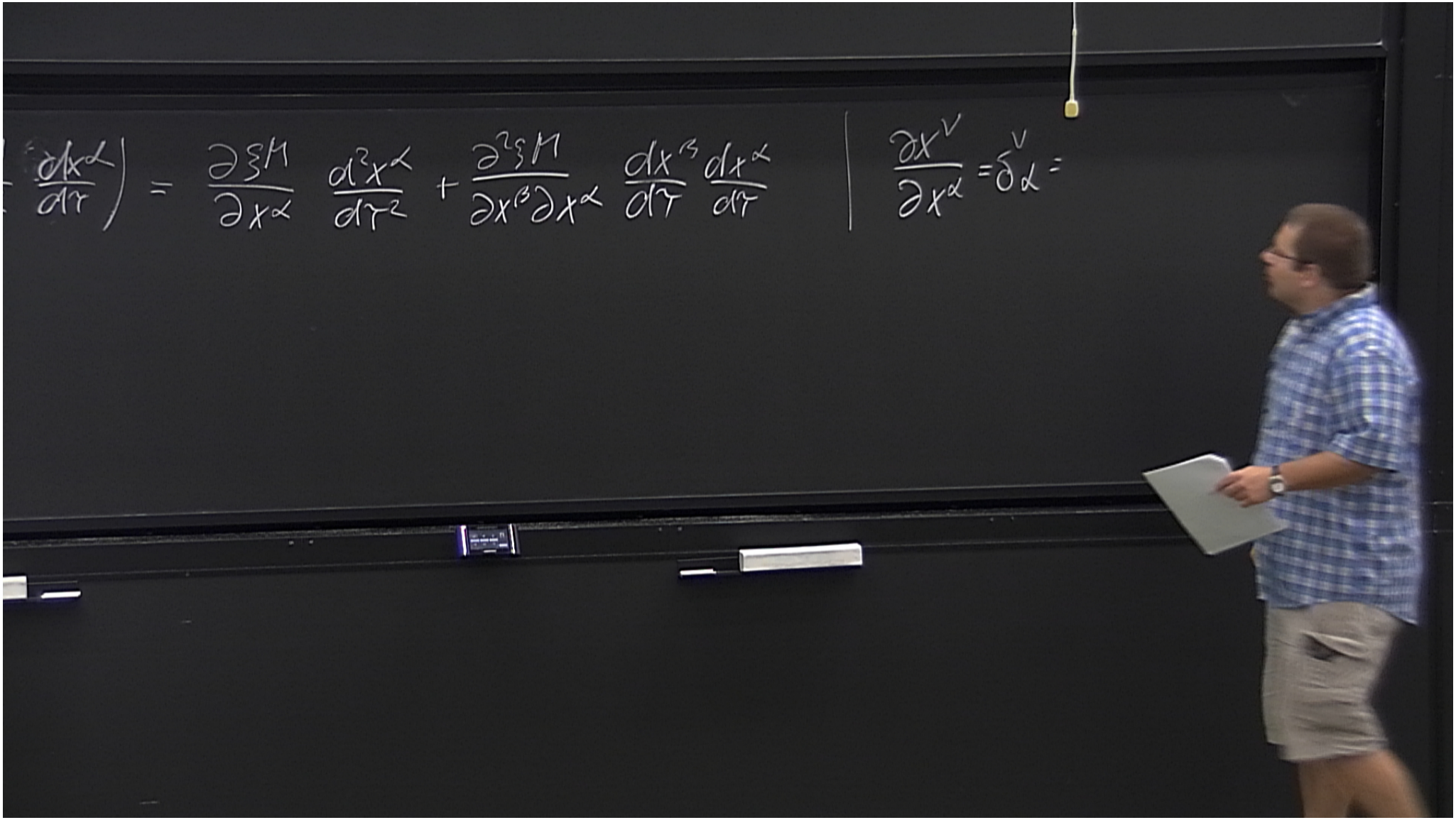

iii) GEODESIC EQUATION (c=1)
• ξ^μ .. LOCAL COORDINATES .. FREELY FALLING SYSTEM
• $X^\alpha = X^\alpha(\xi^\mu)$ .. ANY GLOBAL COORDINATES.
g
g

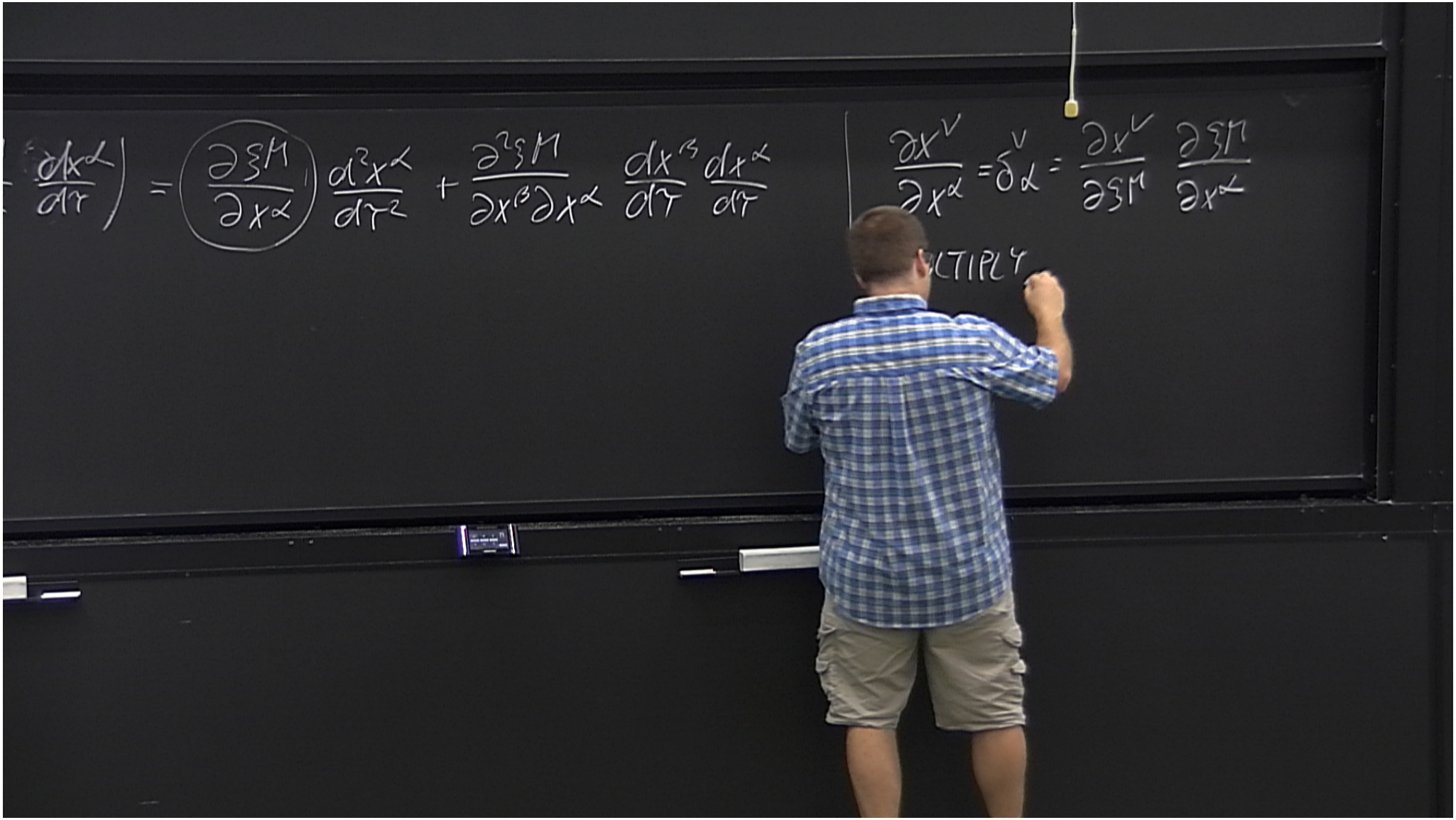

III) GEODESIC EQUATION (c=1)
• ξ^μ LOCAL COORDINATES . FREELY FALLING SYSTEM
• $x^\alpha = x^\alpha(\xi^\mu)$ .. ANY GLOBAL COORDINATES.
$\frac{d^2\xi^\mu}{d\tau^2} = 0$
g
g

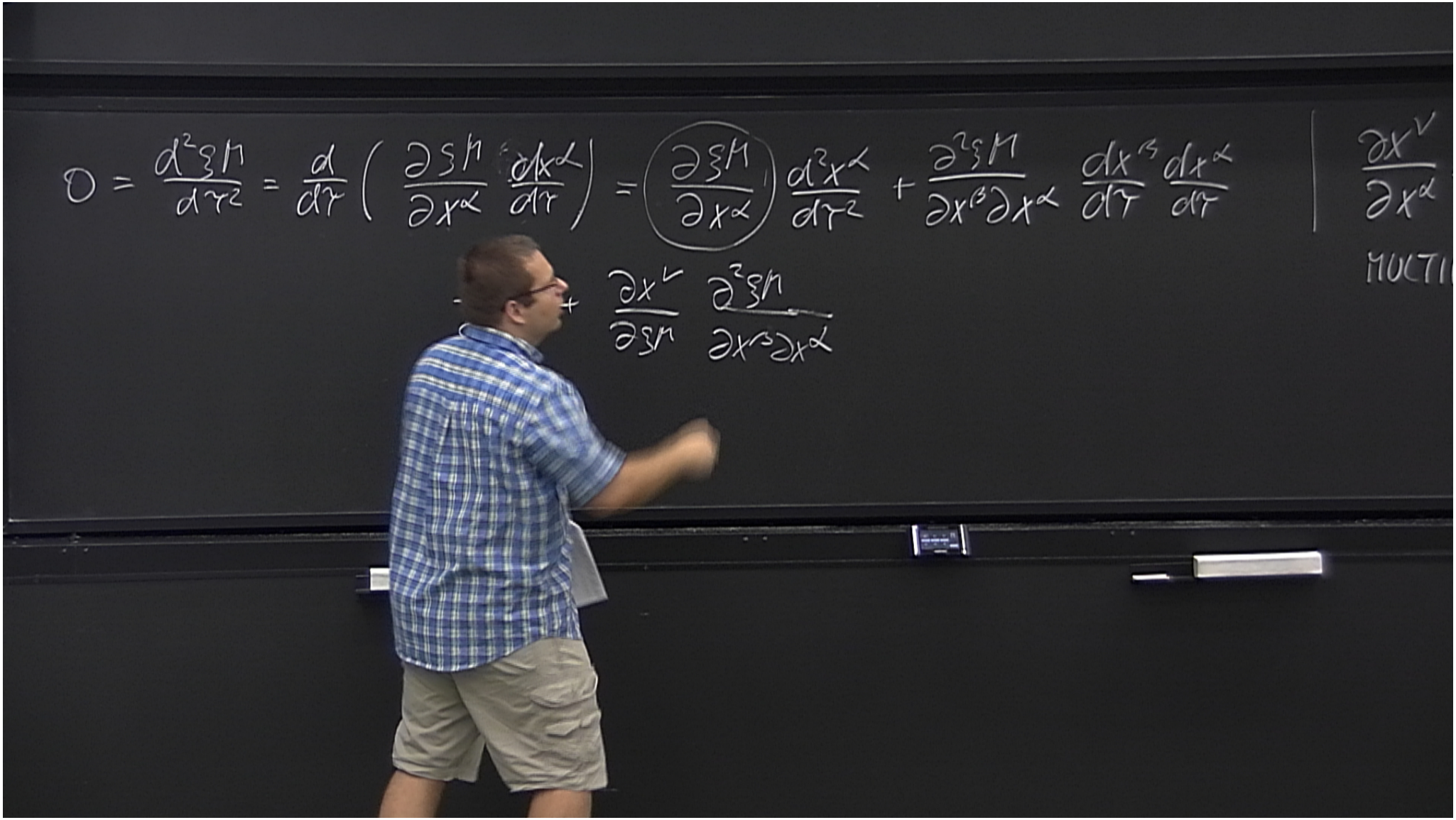

$0 = \frac{d^2 s^\mu}{d\tau^2} = \frac{d}{d\tau}\left(\frac{\partial s^\mu}{\partial x^\alpha}\right.$

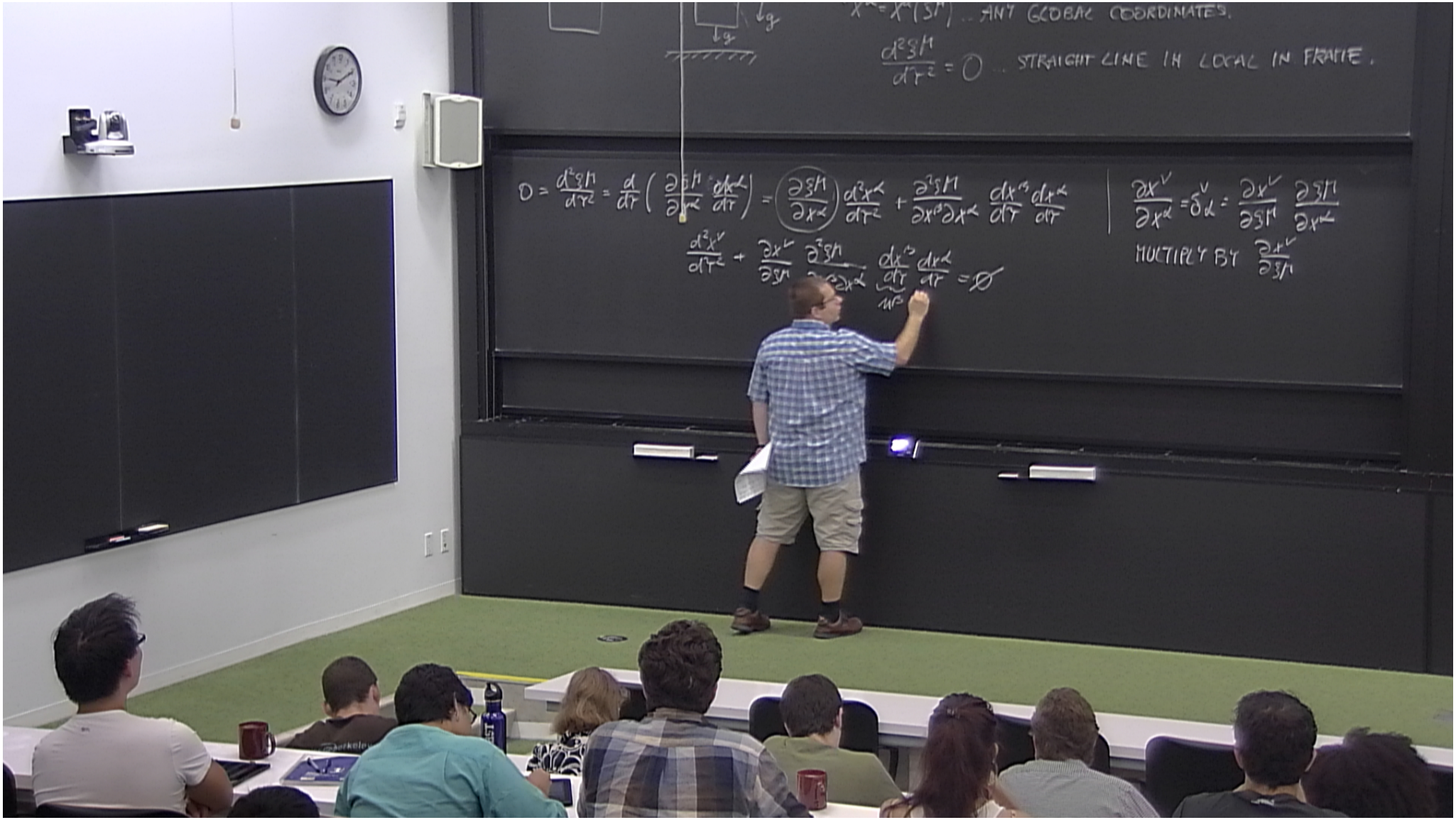

$0 = \frac{d^2\xi^\mu}{d\tau^2} = \frac{d}{d\tau}\left(\frac{\partial \xi^\mu}{\partial x^\alpha}\frac{dx^\alpha}{d\tau}\right) = \frac{\partial \xi^\mu}{\partial x^\alpha}\frac{d^2x^\alpha}{d\tau^2} + \partial^2$

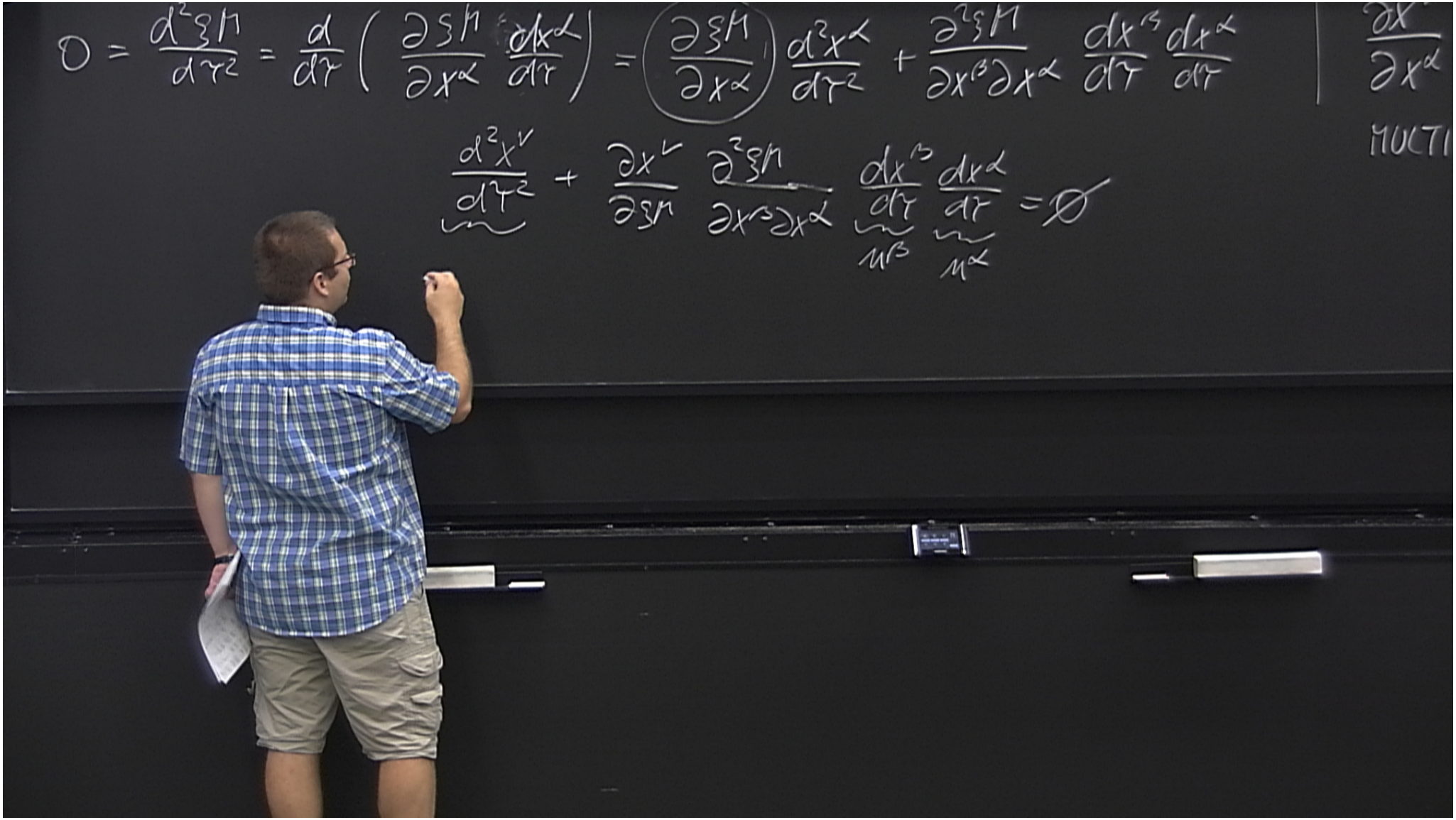
$$0 = \frac{d^2\xi^\mu}{d\tau^2} = \frac{d}{d\tau}\left(\frac{\partial \xi^\mu}{\partial x^\alpha}\frac{dx^\alpha}{d\tau}\right) = \frac{\partial \xi^\mu}{\partial x^\alpha}\frac{d^2x^\alpha}{d\tau^2} + \frac{\partial^2 \xi^\mu}{\partial x^\beta \partial x^\alpha}\frac{dx^\beta}{d\tau}\frac{dx^\alpha}{d\tau}$$

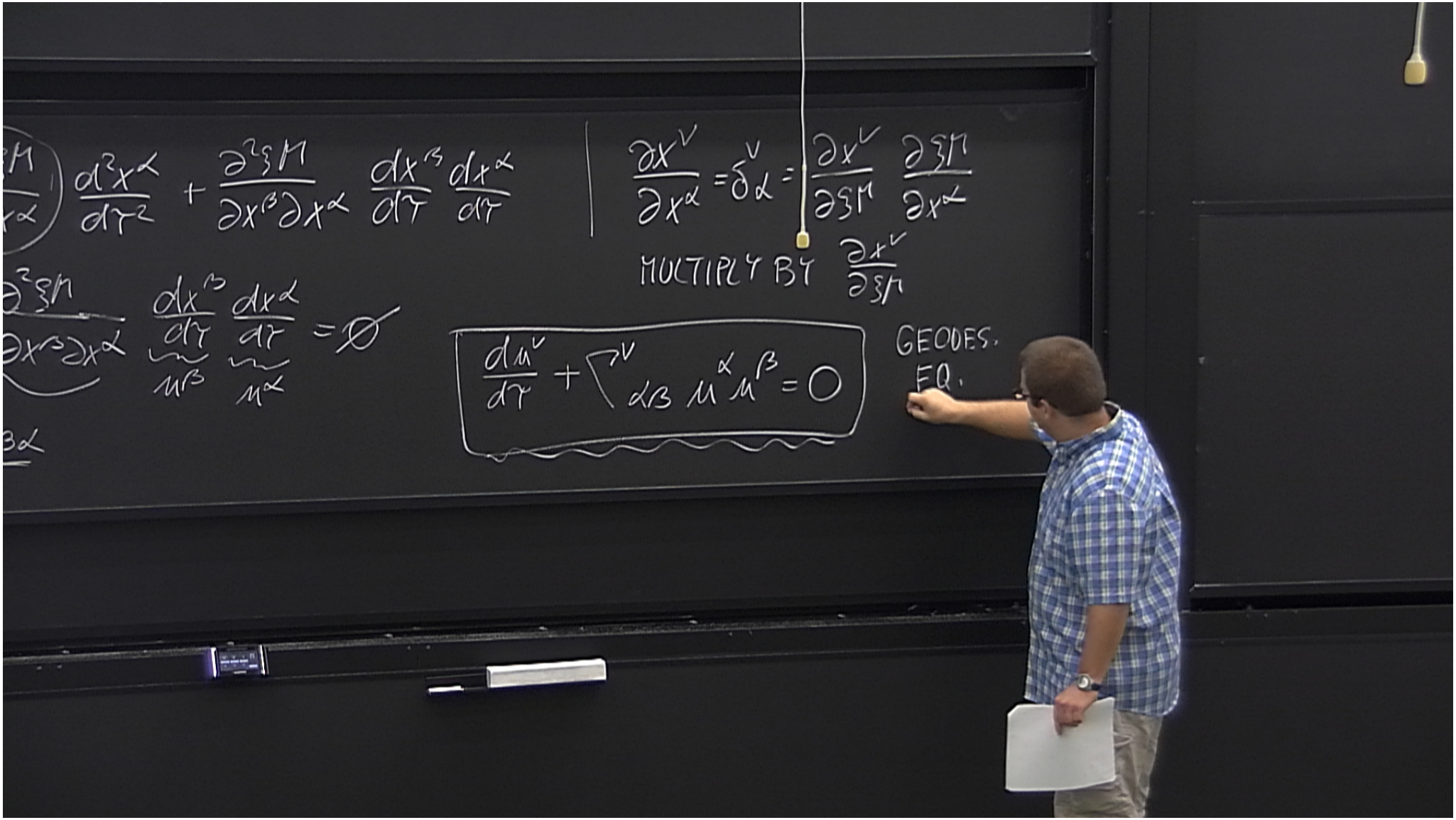

$\frac{dx^\alpha}{d\tau}\Big) = \frac{\partial \xi^\mu}{\partial x^\alpha}\frac{d^2x^\alpha}{d\tau^2} + \frac{\partial^2 \xi^\mu}{\partial x^\beta \partial x^\alpha}\frac{dx^\beta}{d\tau}\frac{dx^\alpha}{d\tau}$
$\frac{\partial x^\nu}{\partial x^\alpha} = \delta^\nu_\alpha =$

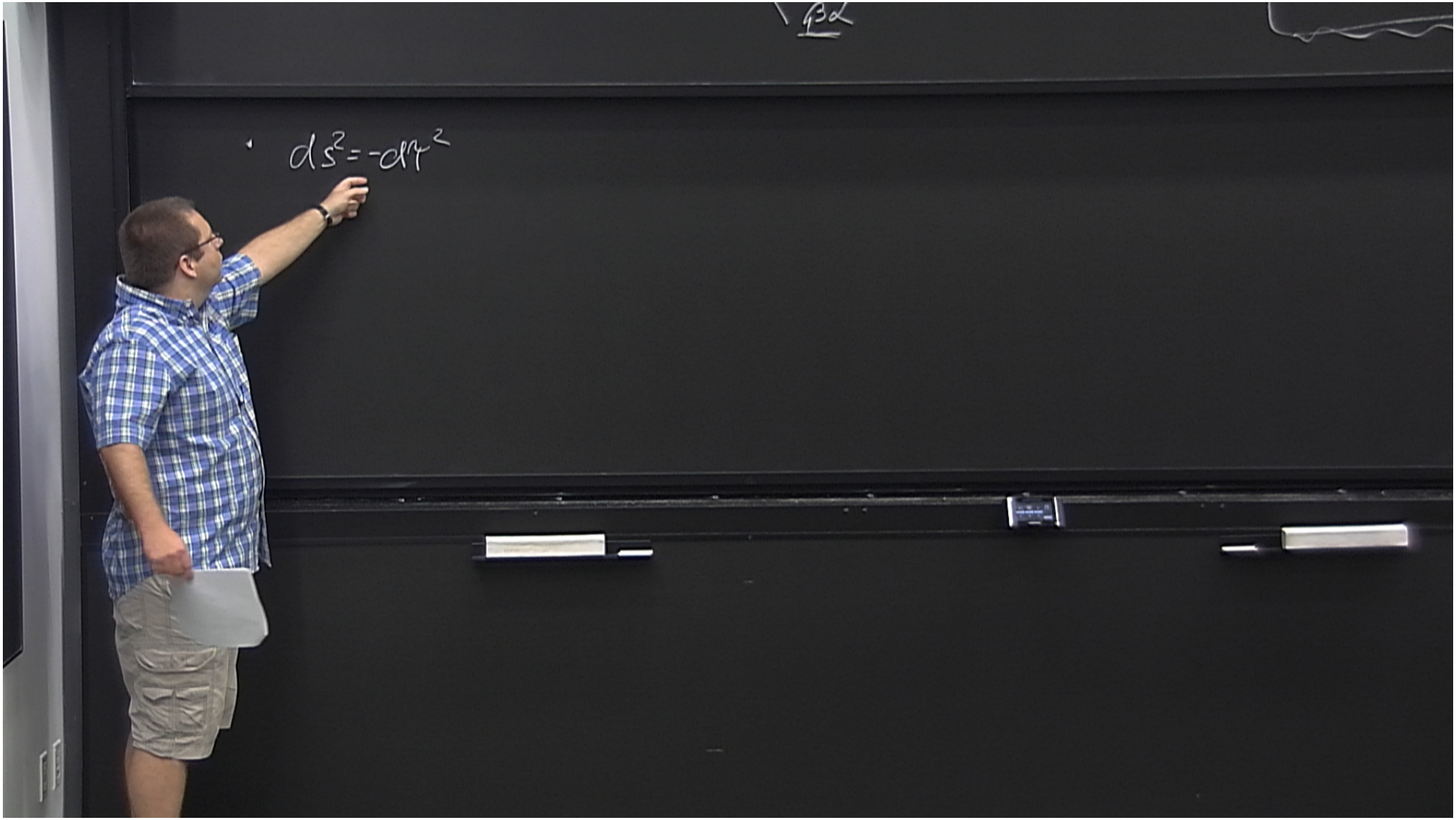

$\frac{dx^\alpha}{d\tau}\Big) = \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{d^2x^\alpha}{d\tau^2} + \frac{\partial^2 \xi^\mu}{\partial x^\beta \partial x^\alpha} \frac{dx^\beta}{d\tau} \frac{dx^\alpha}{d\tau}$
$\frac{\partial x^\nu}{\partial x^\alpha} = \delta^\nu_\alpha = \frac{\partial x^\nu}{\partial \xi^\mu} \frac{\partial \xi^\mu}{\partial x^\alpha}$
LTIPLY

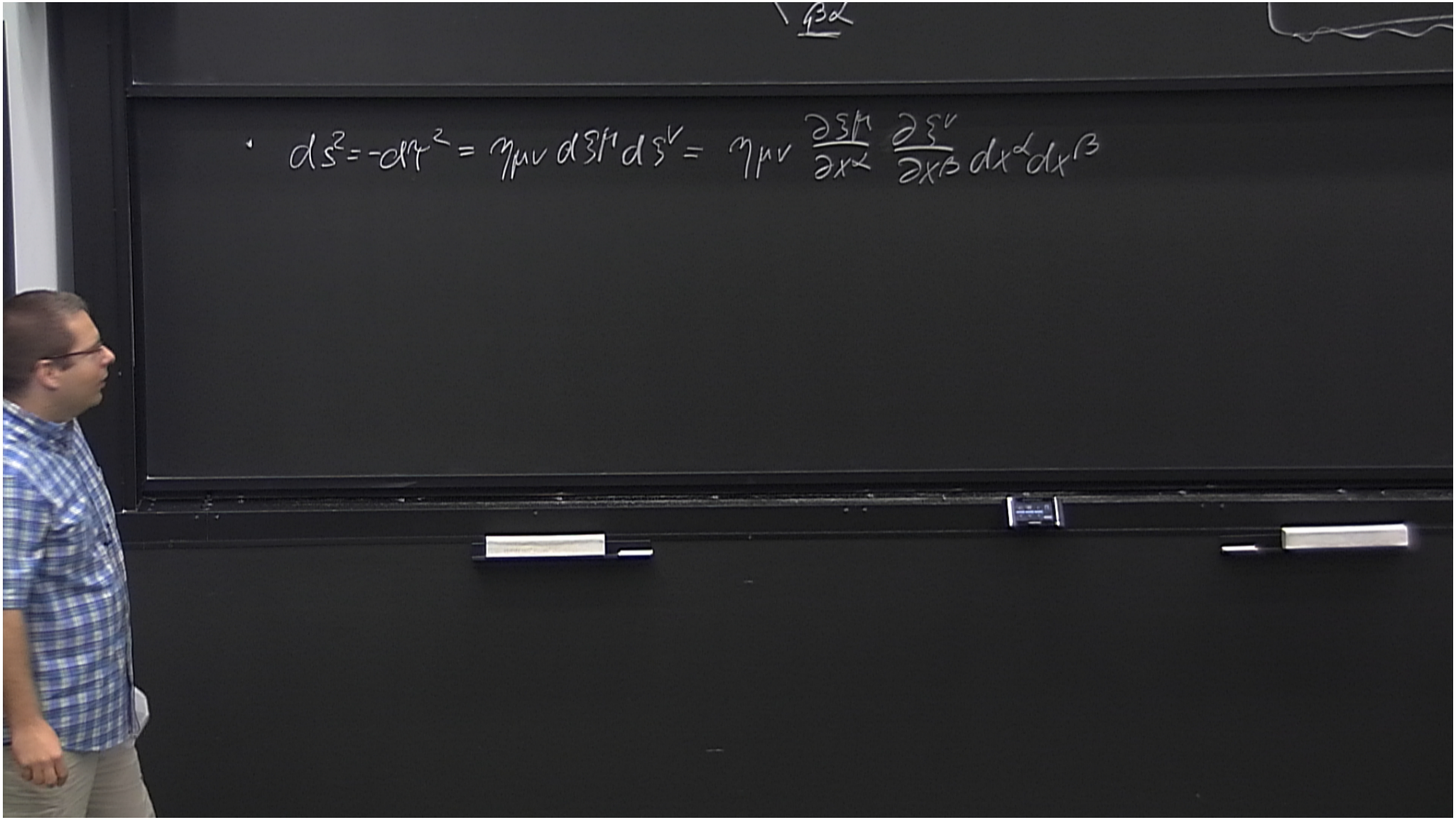

$0 = \frac{d^2\xi^\mu}{d\tau^2} = \frac{d}{d\tau}\left(\frac{\partial \xi^\mu}{\partial x^\alpha}\frac{dx^\alpha}{d\tau}\right) = \left(\frac{\partial \xi^\mu}{\partial x^\alpha}\right)\frac{d^2x^\alpha}{d\tau^2} + \frac{\partial^2\xi^\mu}{\partial x^\beta \partial x^\alpha}\frac{dx^\beta}{d\tau}\frac{dx^\alpha}{d\tau}$
$+ \frac{\partial x^\nu}{\partial \xi^\mu}\frac{\partial^2 \xi^\mu}{\partial x^\nu \partial x^\alpha}$
$\frac{\partial x^\nu}{\partial x^\alpha}$
MULTI

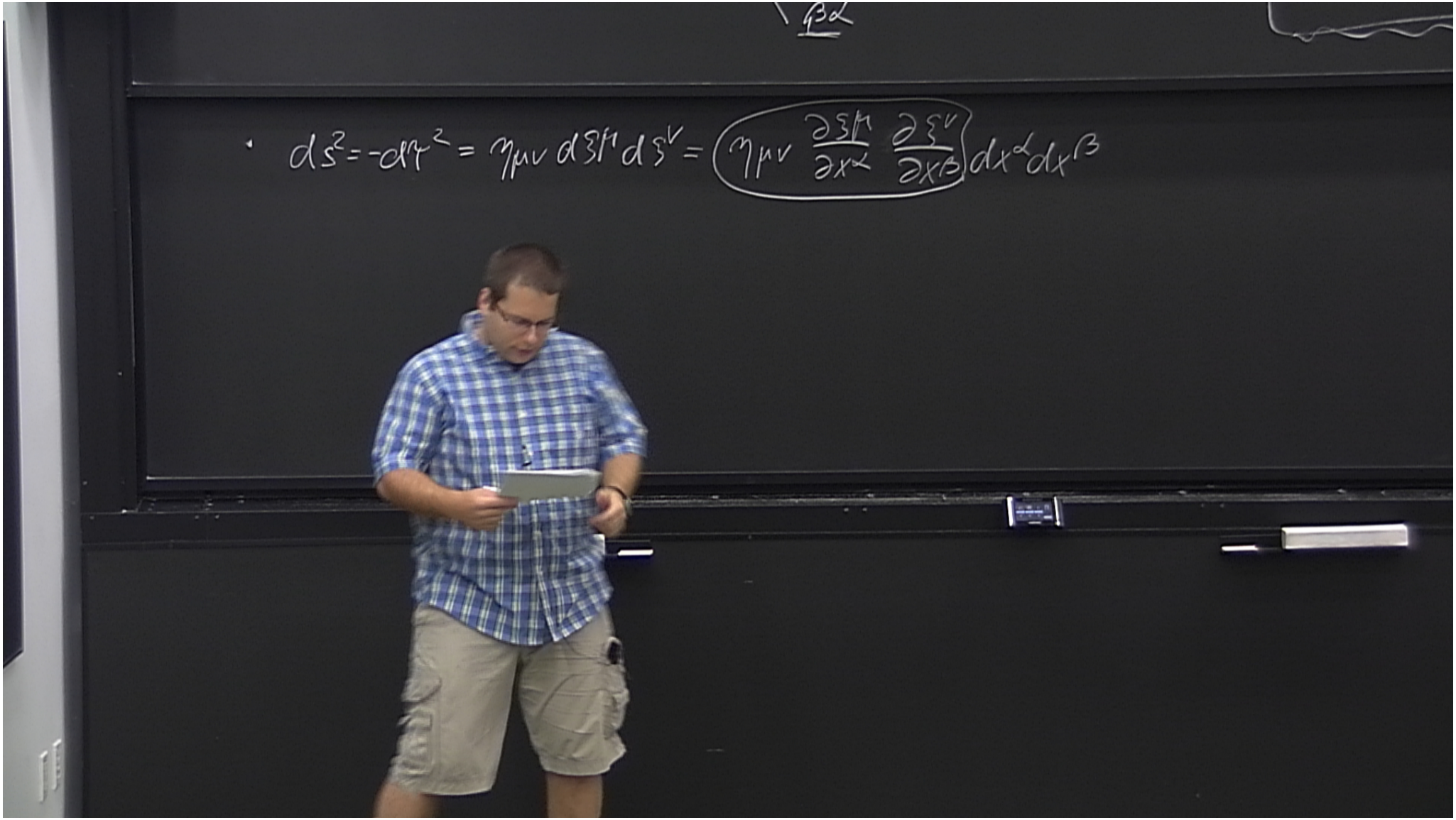

ANY GLOBAL COORDINATES.
STRAIGHT LINE IN LOCAL IN FRAME.
MULTIPLY BY

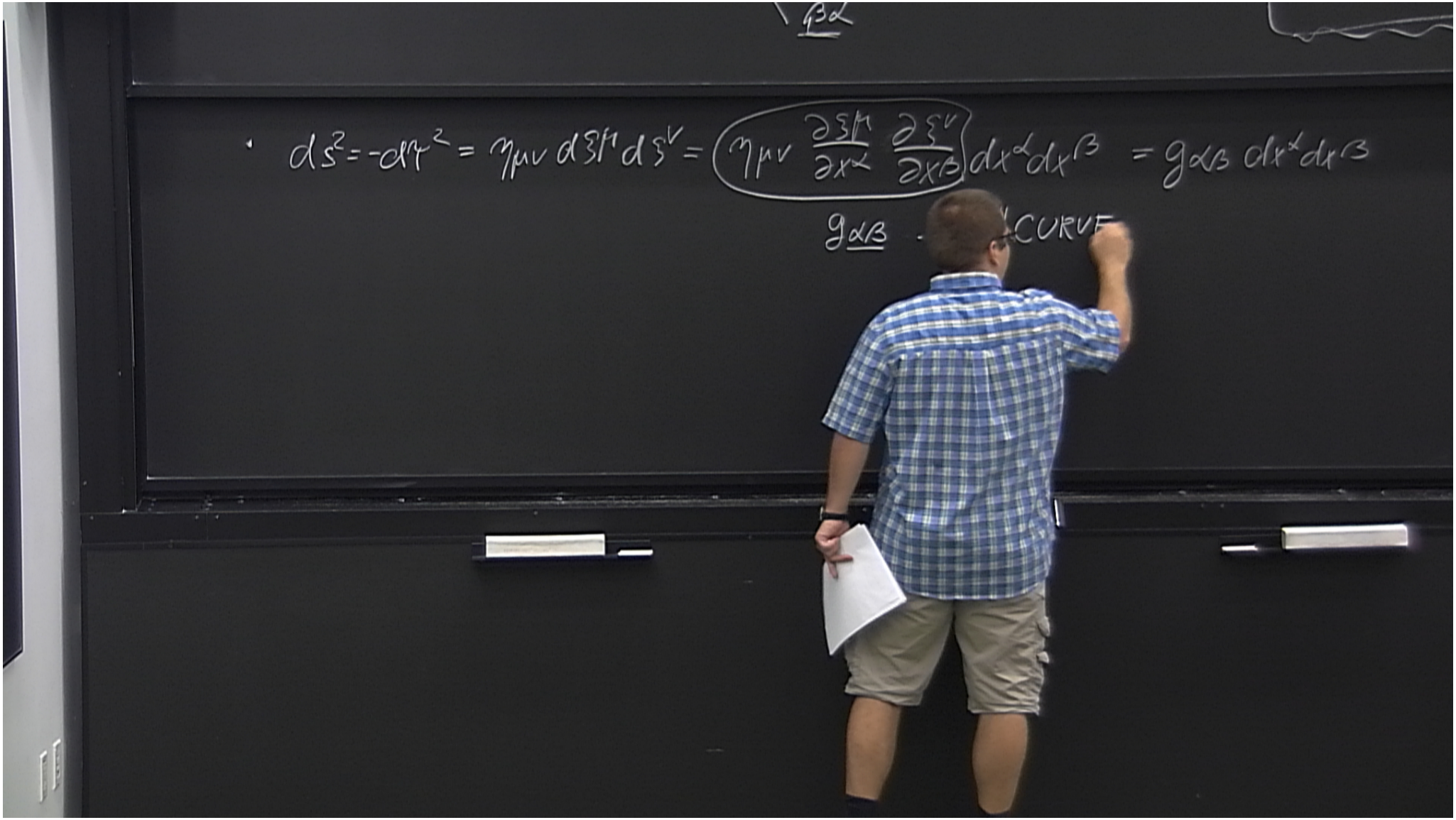

$0 = \frac{d^2\xi^\mu}{d\tau^2} = \frac{d}{d\tau}\left(\frac{\partial \xi^\mu}{\partial x^\alpha}\frac{dx^\alpha}{d\tau}\right) = \left(\frac{\partial \xi^\mu}{\partial x^\alpha}\right)\frac{d^2x^\alpha}{d\tau^2} + \frac{\partial^2 \xi^\mu}{\partial x^\beta \partial x^\alpha}\frac{dx^\beta}{d\tau}\frac{dx^\alpha}{d\tau}$
$\frac{d^2x^\nu}{d\tau^2} + \frac{\partial x^\nu}{\partial \xi^\mu}\frac{\partial^2 \xi^\mu}{\partial x^\beta \partial x^\alpha}\frac{dx^\beta}{d\tau}\frac{dx^\alpha}{d\tau} = 0$
u^β
u^α
$\frac{\partial x^\nu}{\partial x^\alpha}$
MULTI

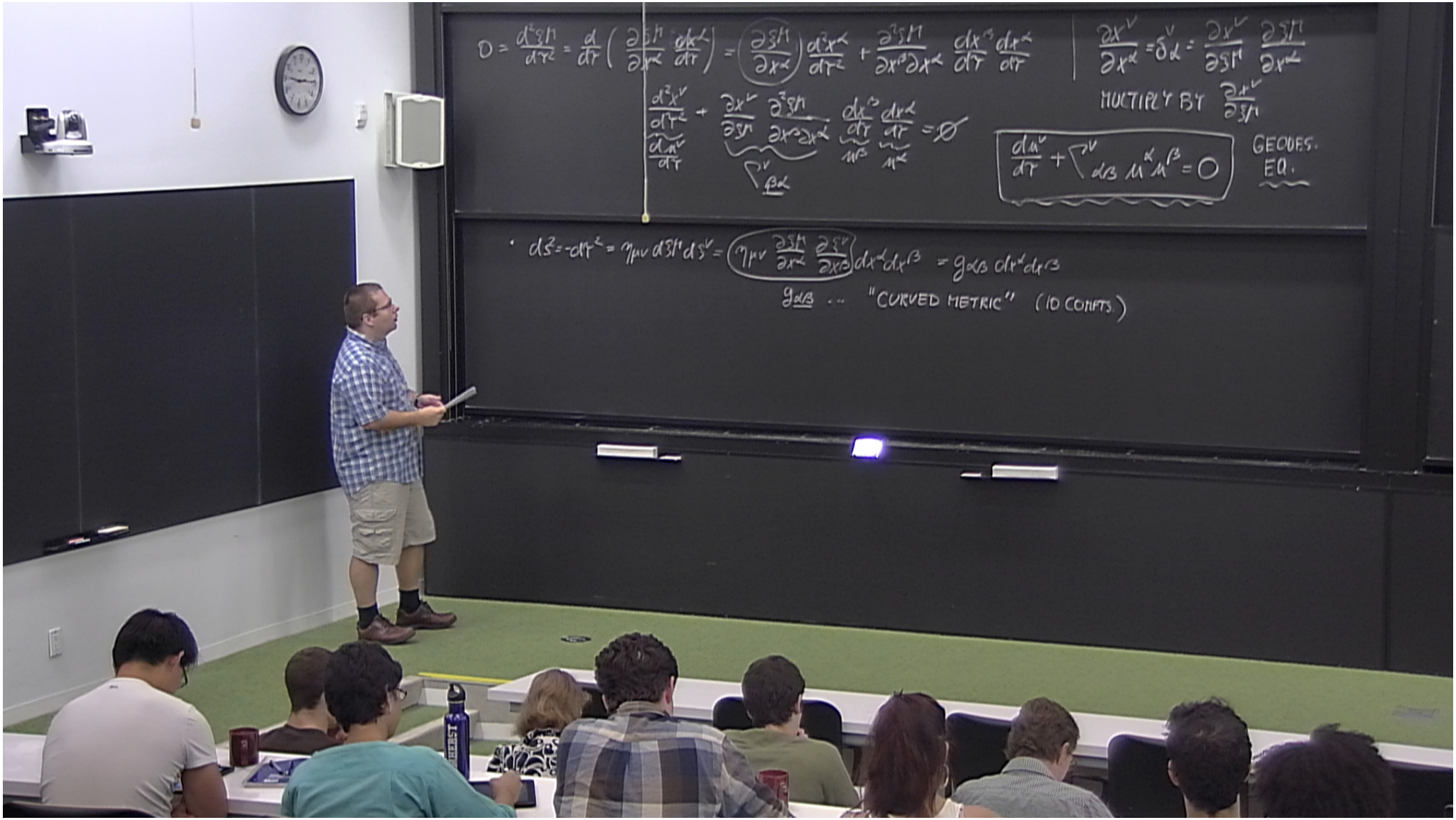

$\frac{d^2x^\alpha}{d\tau^2} + \frac{\partial^2 \xi^\mu}{\partial x^\beta \partial x^\alpha} \frac{dx^\beta}{d\tau} \frac{dx^\alpha}{d\tau}$
$\frac{\partial x^\nu}{\partial x^\alpha} = \delta^\nu_\alpha = \frac{\partial x^\nu}{\partial \xi^\mu} \frac{\partial \xi^\mu}{\partial x^\alpha}$
MULTIPLY BY $\frac{\partial x^\nu}{\partial \xi^\mu}$
$\frac{dx^\beta}{d\tau} \frac{dx^\alpha}{d\tau} = 0$
u^β u^α
$\frac{du^\nu}{d\tau} + \Gamma^\nu_{\alpha\beta} u^\alpha u^\beta = 0$
GEODES. EQ.

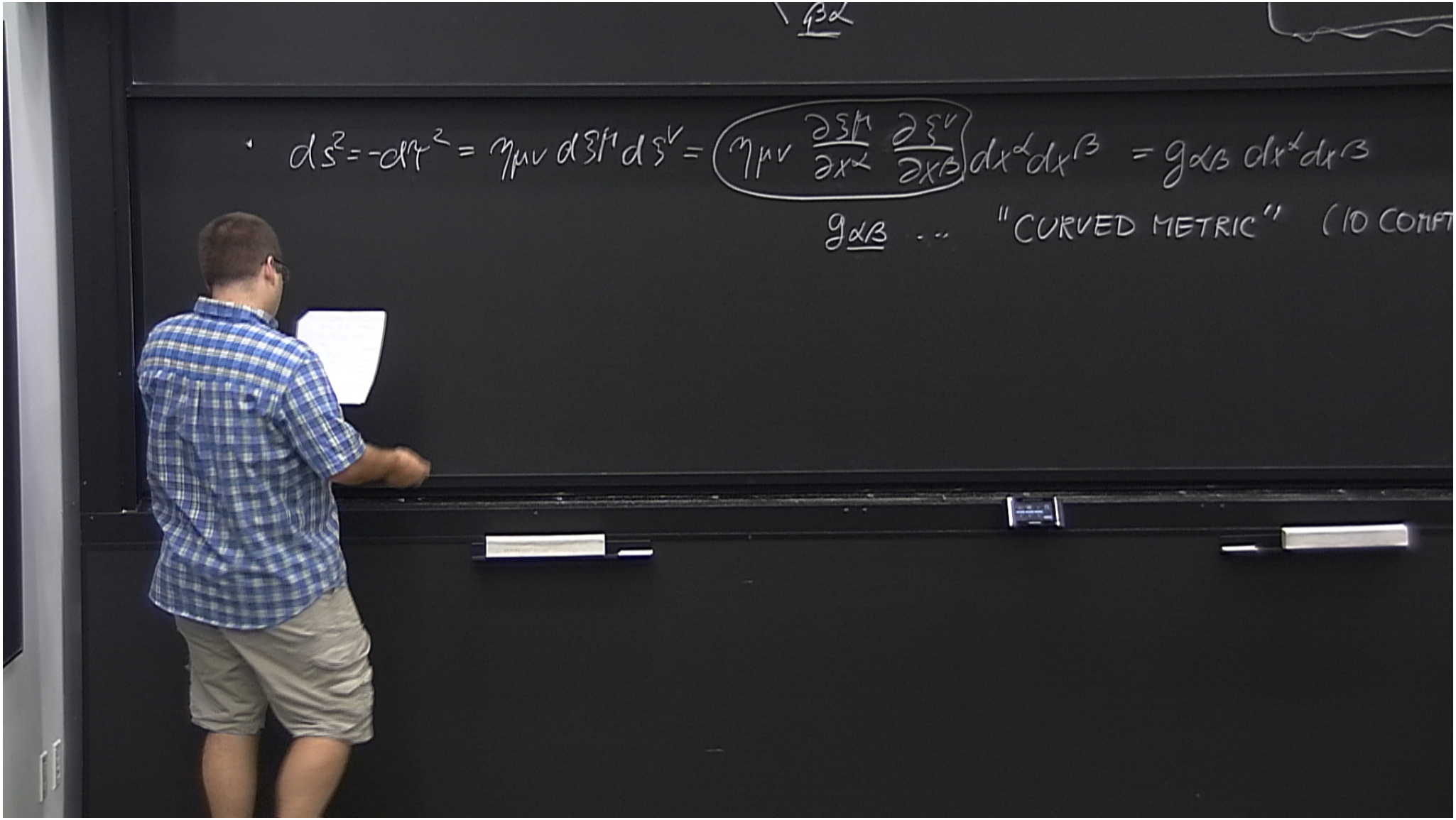

$ds^2 = -dt^2$

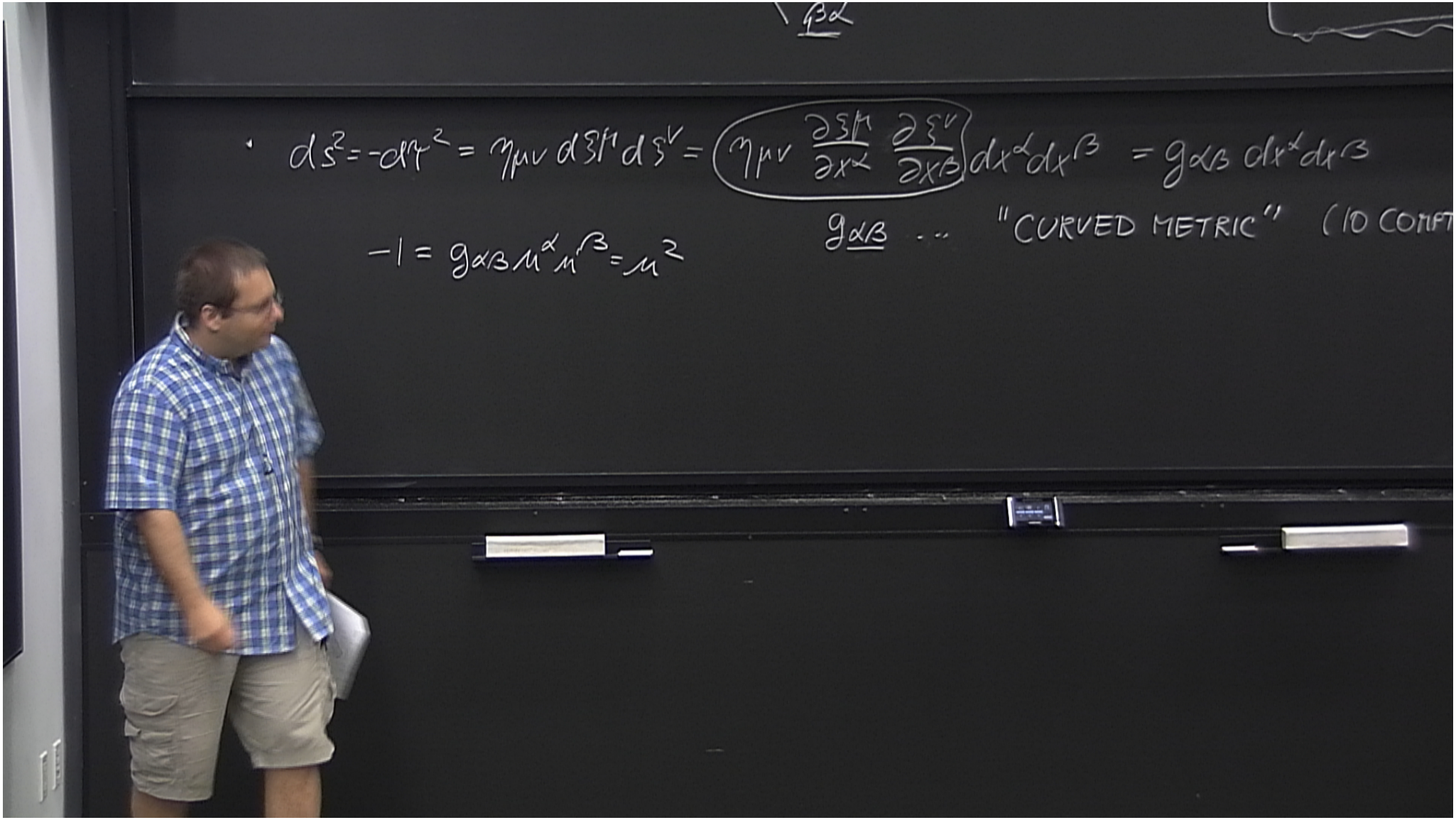

$ds^2 = -d\tau^2 = \eta_{\mu\nu} d\xi^\mu d\xi^\nu = \eta_{\mu\nu} \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{\partial \xi^\nu}{\partial x^\beta} dx^\alpha dx^\beta$

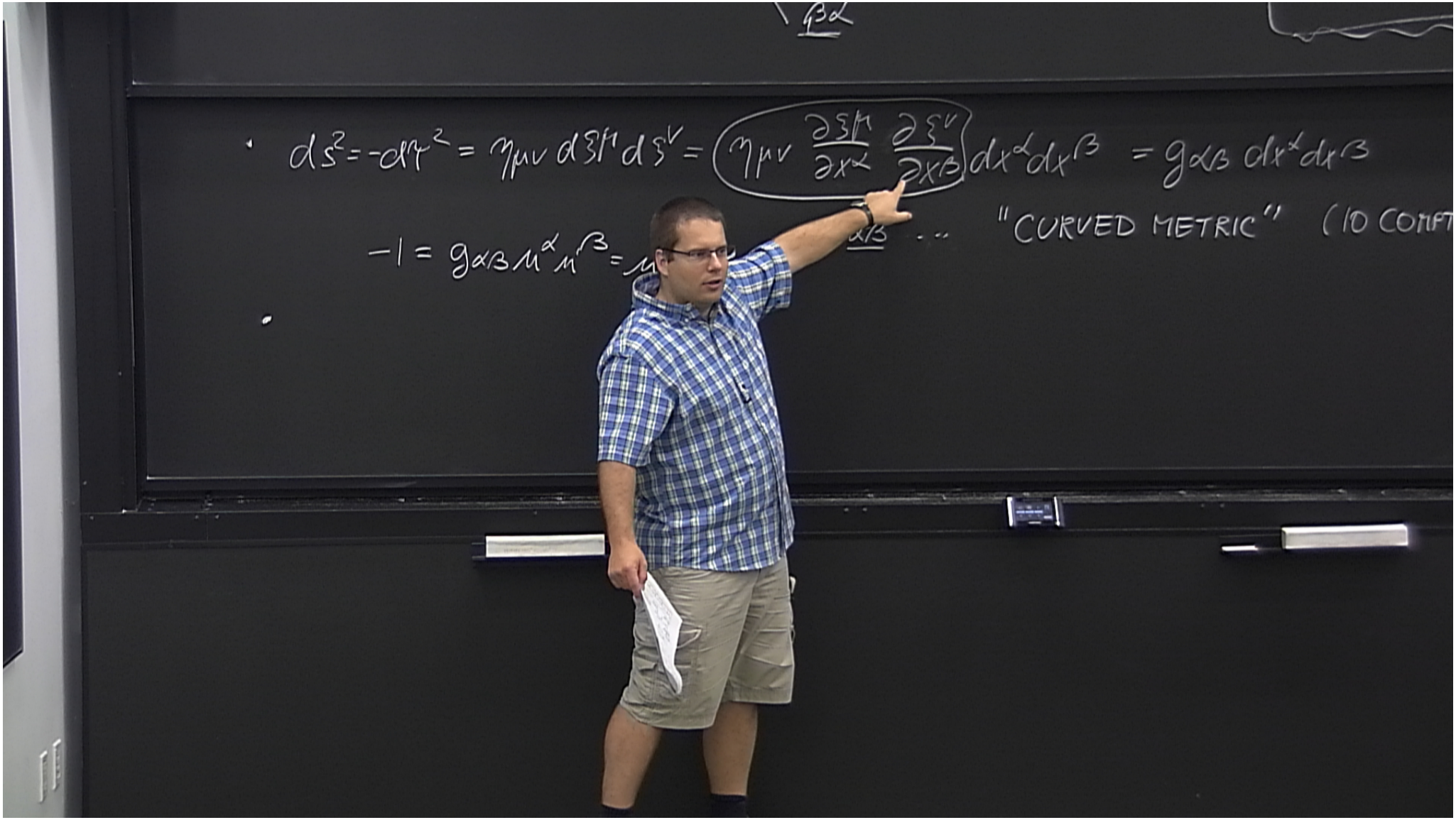

$ds^2 = -d\tau^2 = \eta_{\mu\nu} d\xi^\mu d\xi^\nu = \eta_{\mu\nu} \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{\partial \xi^\nu}{\partial x^\beta} dx^\alpha dx^\beta$

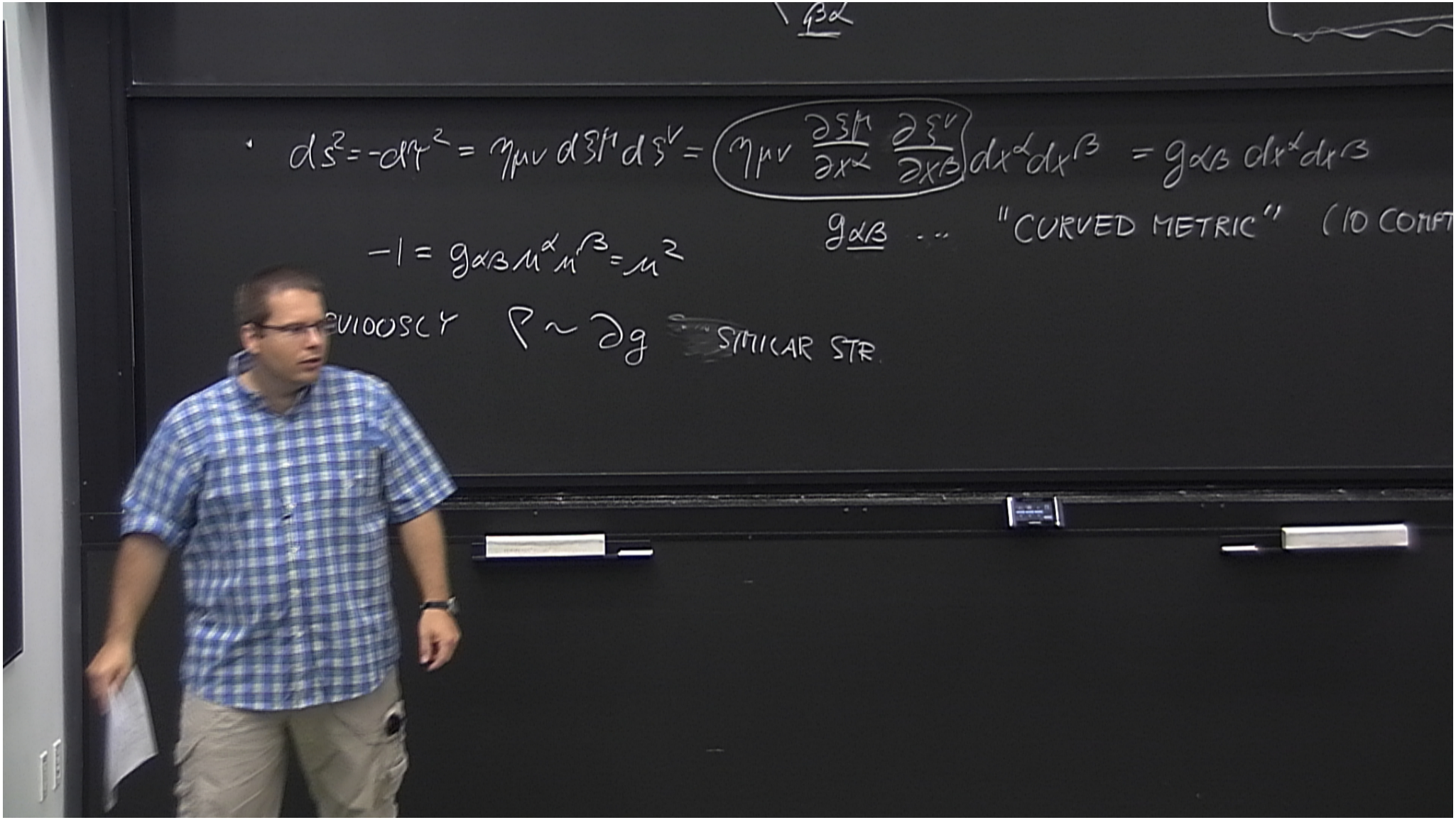

$ds^2 = -d\tau^2 = \eta_{\mu\nu} d\xi^\mu d\xi^\nu = \eta_{\mu\nu} \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{\partial \xi^\nu}{\partial x^\beta} dx^\alpha dx^\beta = g_{\alpha\beta} dx^\alpha dx^\beta$
$g_{\alpha\beta}$

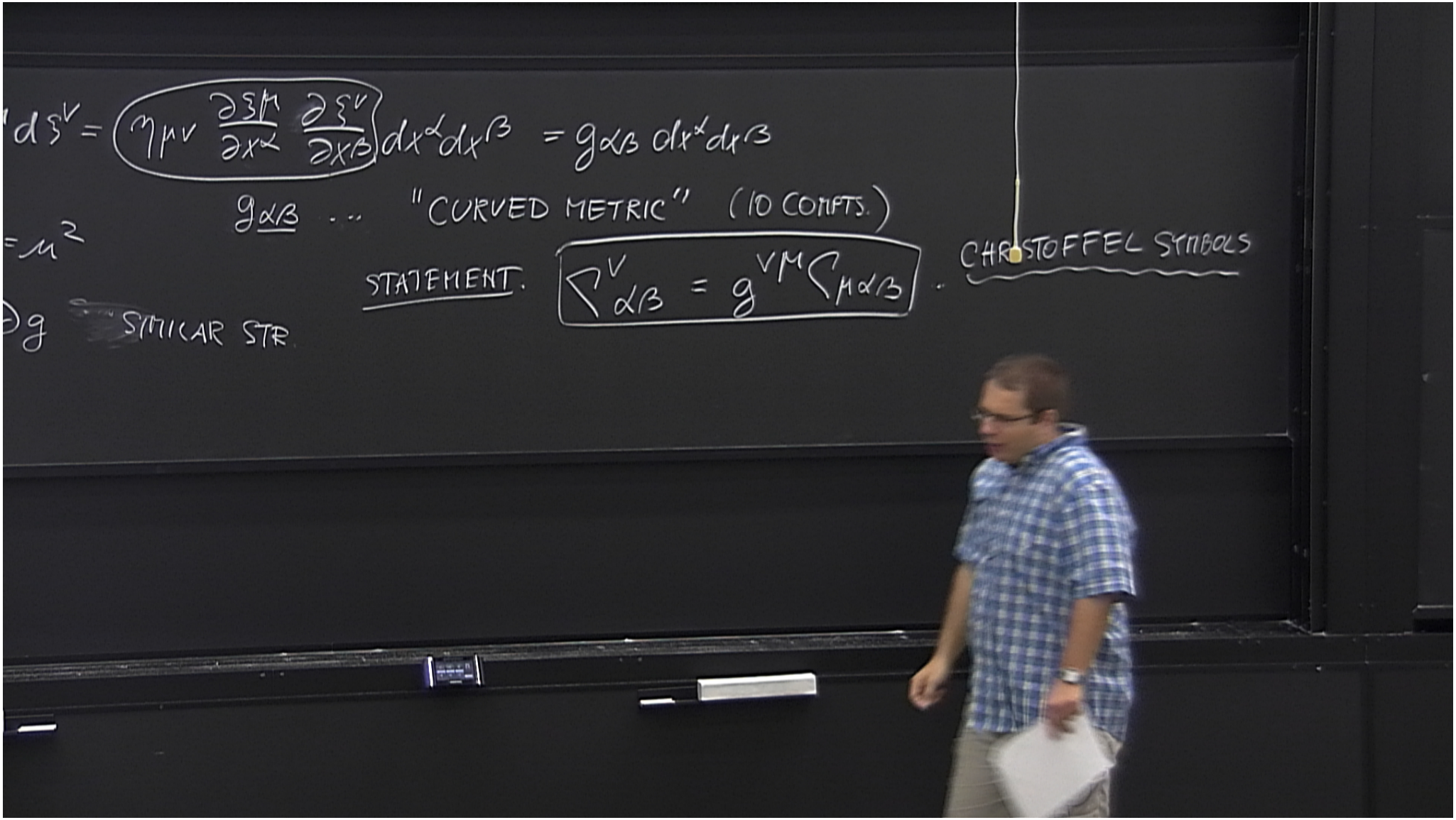

MULTIPLY BY
GEODES. EQ.
"CURVED METRIC" (10 COMPTS.)

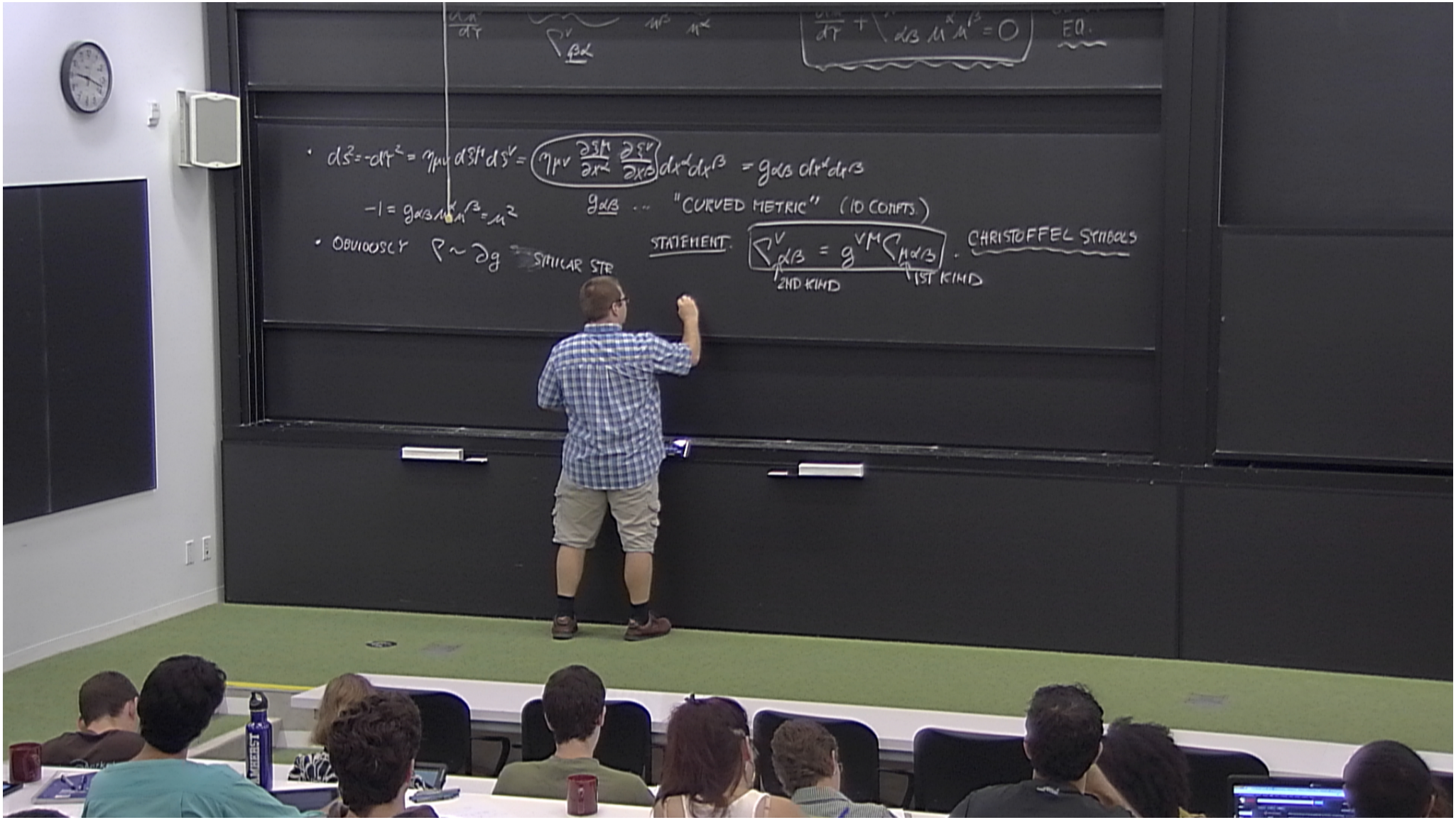

$ds^2 = -d\tau^2 = \eta_{\mu\nu} d\xi^\mu d\xi^\nu = \eta_{\mu\nu} \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{\partial \xi^\nu}{\partial x^\beta} dx^\alpha dx^\beta = g_{\alpha\beta} dx^\alpha dx^\beta$
$g_{\alpha\beta}$ ... "CURVED METRIC"


$ds^2 = -d\tau^2 = \eta_{\mu\nu} d\xi^\mu d\xi^\nu = \eta_{\mu\nu} \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{\partial \xi^\nu}{\partial x^\beta} dx^\alpha dx^\beta = g_{\alpha\beta} dx^\alpha dx^\beta$
$-1 = g_{\alpha\beta} u^\alpha u^\beta = u^2$
$g_{\alpha\beta}$ ... "CURVED METRIC"


$ds^2 = -d\tau^2 = \eta_{\mu\nu} d\xi^\mu d\xi^\nu = \eta_{\mu\nu} \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{\partial \xi^\nu}{\partial x^\beta} dx^\alpha dx^\beta = g_{\alpha\beta} dx^\alpha dx^\beta$
$-1 = g_{\alpha\beta} u^\alpha u^\beta$
"CURVED METRIC"


$ds^2 = -d\tau^2 = \eta_{\mu\nu} d\xi^\mu d\xi^\nu = \eta_{\mu\nu} \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{\partial \xi^\nu}{\partial x^\beta} dx^\alpha dx^\beta = g_{\alpha\beta} dx^\alpha dx^\beta$
$g_{\alpha\beta}$ ... "CURVED METRIC" (10 COMPT
$-1 = g_{\alpha\beta} u^\alpha u^\beta = u^2$
$\rho \sim \partial g$
SIMILAR STR


$ds^2 = \eta_{\mu\nu} \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{\partial \xi^\nu}{\partial x^\beta} dx^\alpha dx^\beta = g_{\alpha\beta} dx^\alpha dx^\beta$
$g_{\alpha\beta}$ ... "CURVED METRIC" (10 COMPTS.)
STATEMENT. $\Gamma^\nu_{\alpha\beta} = g^{\nu\mu} \zeta_{\mu\alpha\beta}$. CHRISTOFFEL SYMBOLS
$= m^2$
g SIMILAR STR


EQ.
"CURVED METRIC" (10 COMPTS.)
STATEMENT:
CHRISTOFFEL SYMBOLS
2ND KIND
1ST KIND

$= \eta_{\mu\nu} d\xi^\mu d\xi^\nu = \eta_{\mu\nu} \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{\partial \xi^\nu}{\partial x^\beta} dx^\alpha dx^\beta = g_{\alpha\beta} dx^\alpha dx^\beta$

$g_{\alpha\beta}$ ... "CURVED METRIC" (10 COMPTS.)

$g_{\alpha\beta} u^\alpha u^\beta = u^2$

$\rho \sim \partial g$ SIMILAR STR.

STATEMENT. $\Gamma^\nu_{\alpha\beta} = g^{\nu\mu} \Gamma_{\mu\alpha\beta}$. CHRISTOFFEL SYMBOLS

2ND KIND 1ST KIND

$\Gamma_{\mu\alpha\beta} = \frac{1}{2} \left(g_{\mu\alpha,\beta} + g_{\mu\beta,\alpha} - g_{\alpha\beta,\mu} \right)$

$= \eta_{\mu\nu} d\xi^\mu d\xi^\nu = \left(\eta_{\mu\nu} \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{\partial \xi^\nu}{\partial x^\beta}\right) dx^\alpha dx^\beta = g_{\alpha\beta}\, dx^\alpha dx^\beta$

$g_{\alpha\beta}$ ... "CURVED METRIC" (10 COMPTS.)

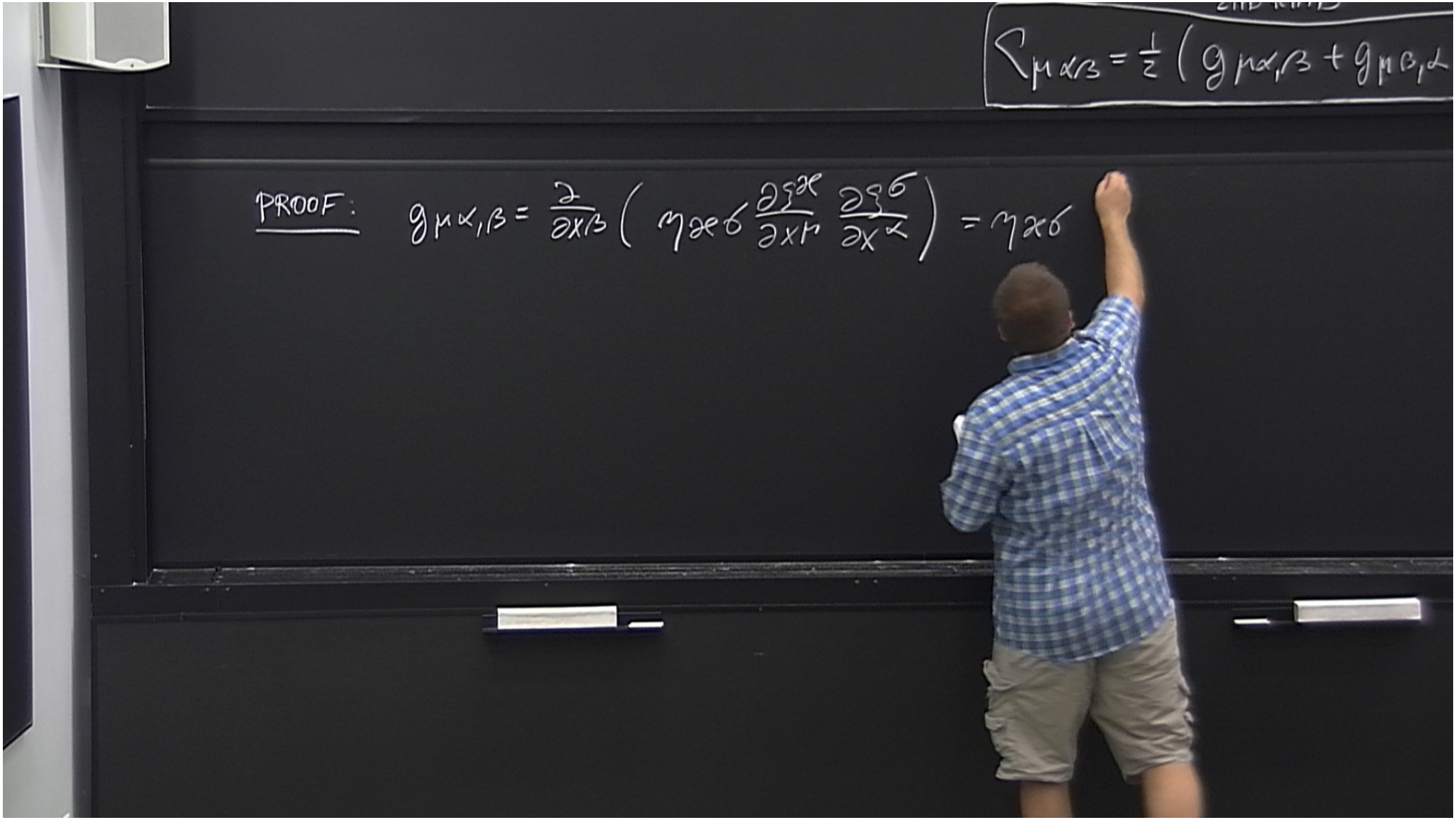
$g_{\alpha\beta} u^\alpha u^\beta = u^2$

$\rho \sim \partial g$ SIMILAR STR.

STATEMENT. $\boxed{\Gamma^\nu_{\alpha\beta} = g^{\nu\mu} \Gamma_{\mu\alpha\beta}}$ .. CHRISTOFFEL SYMBOLS

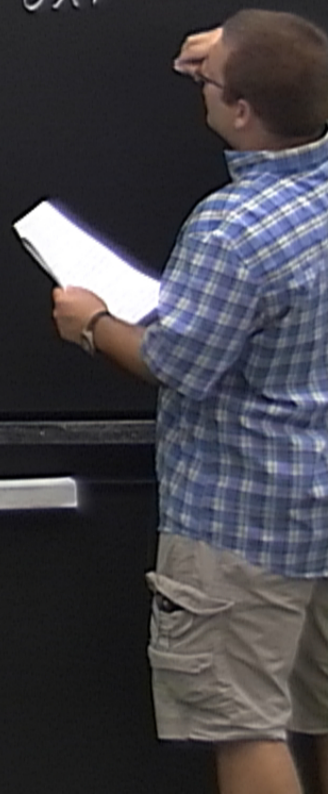
2ND KIND ↑ ↑ 1ST KIND

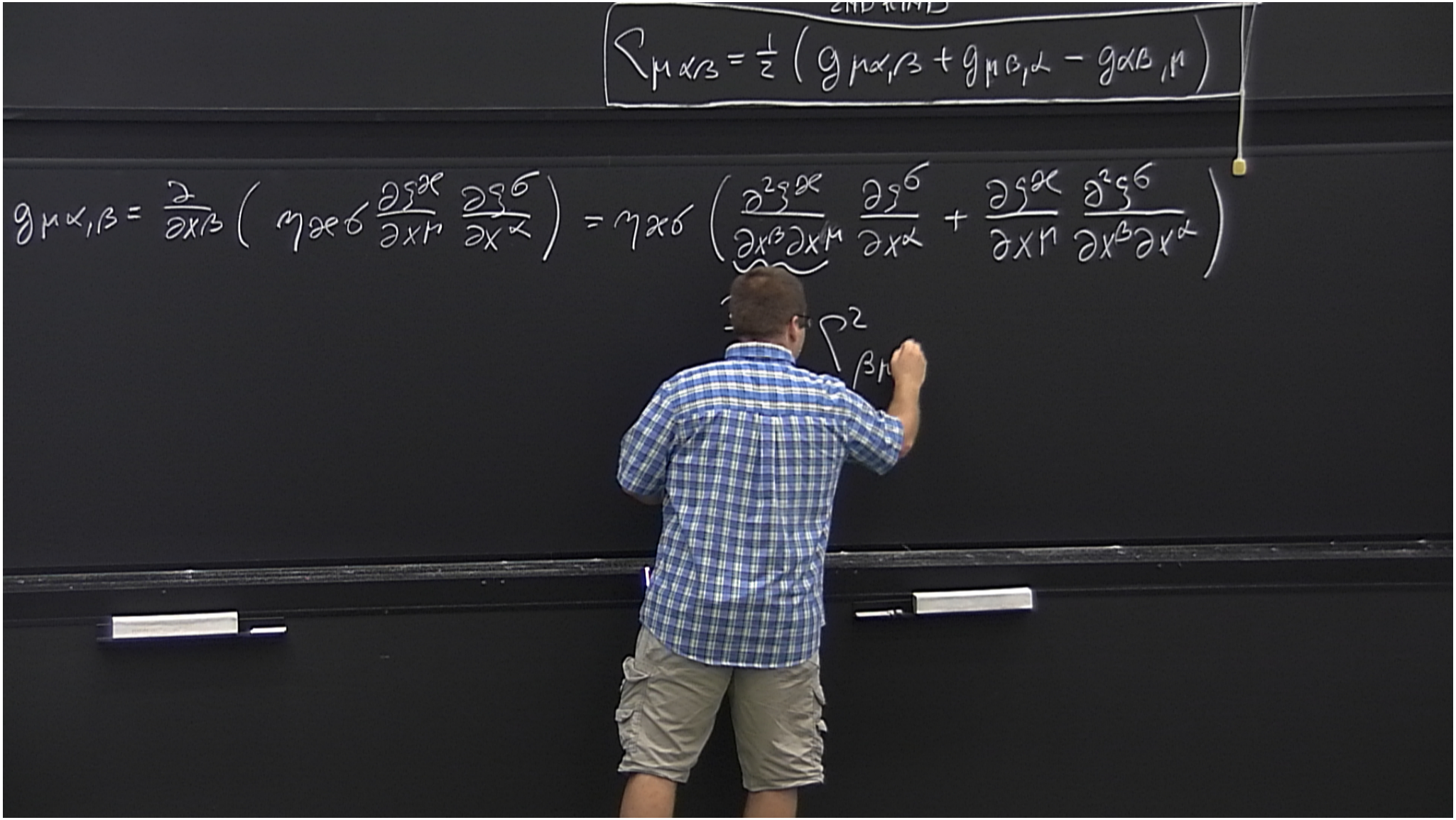
$$\boxed{\Gamma_{\mu\alpha\beta} = \frac{1}{2}\left(g_{\mu\alpha,\beta} + g_{\mu\beta,\alpha} - g_{\alpha\beta,\mu}\right)}$$

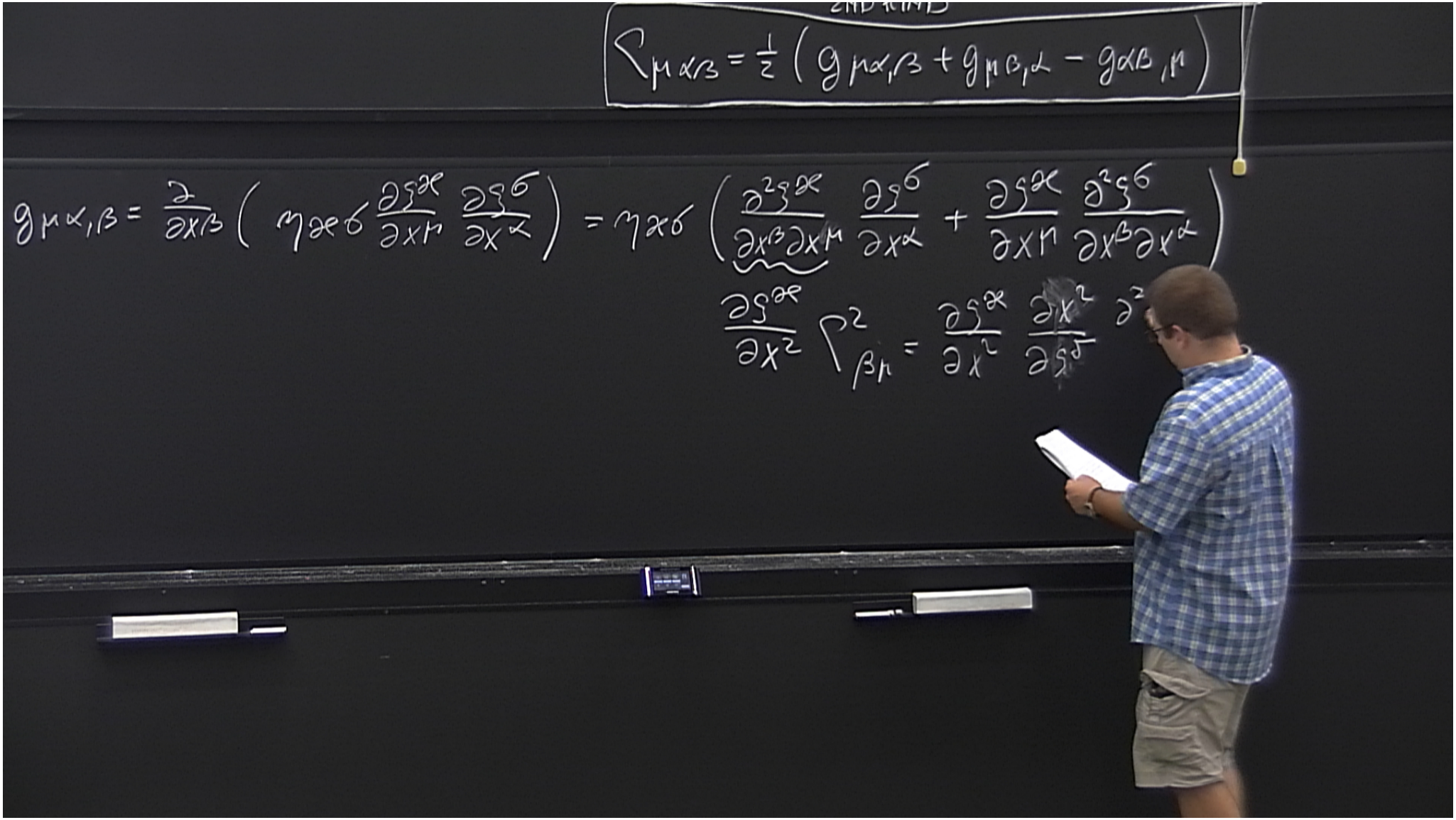

$\Gamma_{\mu\alpha\beta} = \frac{1}{2}\left(g_{\mu\alpha,\beta} + g_{\mu\beta,\alpha}\right.$
PROOF: $g_{\mu\alpha,\beta} = \frac{\partial}{\partial x^\beta}\left(\eta_{\rho\sigma}\frac{\partial \xi^\rho}{\partial x^\mu}\frac{\partial \xi^\sigma}{\partial x^\alpha}\right) = \eta_{\rho\sigma}$

$$\Gamma_{\mu\alpha\beta} = \frac{1}{2}\left(g_{\mu\alpha,\beta} + g_{\mu\beta,\alpha} - g_{\alpha\beta,\mu}\right)$$

PROOF: $$g_{\mu\alpha,\beta} = \frac{\partial}{\partial x^\beta}\left(\eta_{\rho\sigma}\frac{\partial \xi^\rho}{\partial x^\mu}\frac{\partial \xi^\sigma}{\partial x^\alpha}\right) = \eta_{\rho\sigma}\left(\frac{\partial^2 \xi^\rho}{\partial x^\beta \partial x^\mu}\frac{\partial \xi^\sigma}{\partial x^\alpha} + \frac{\partial \xi^\rho}{\partial x^\mu}\frac{\partial^2 \xi^\sigma}{\partial x^\beta \partial x^\alpha}\right)$$



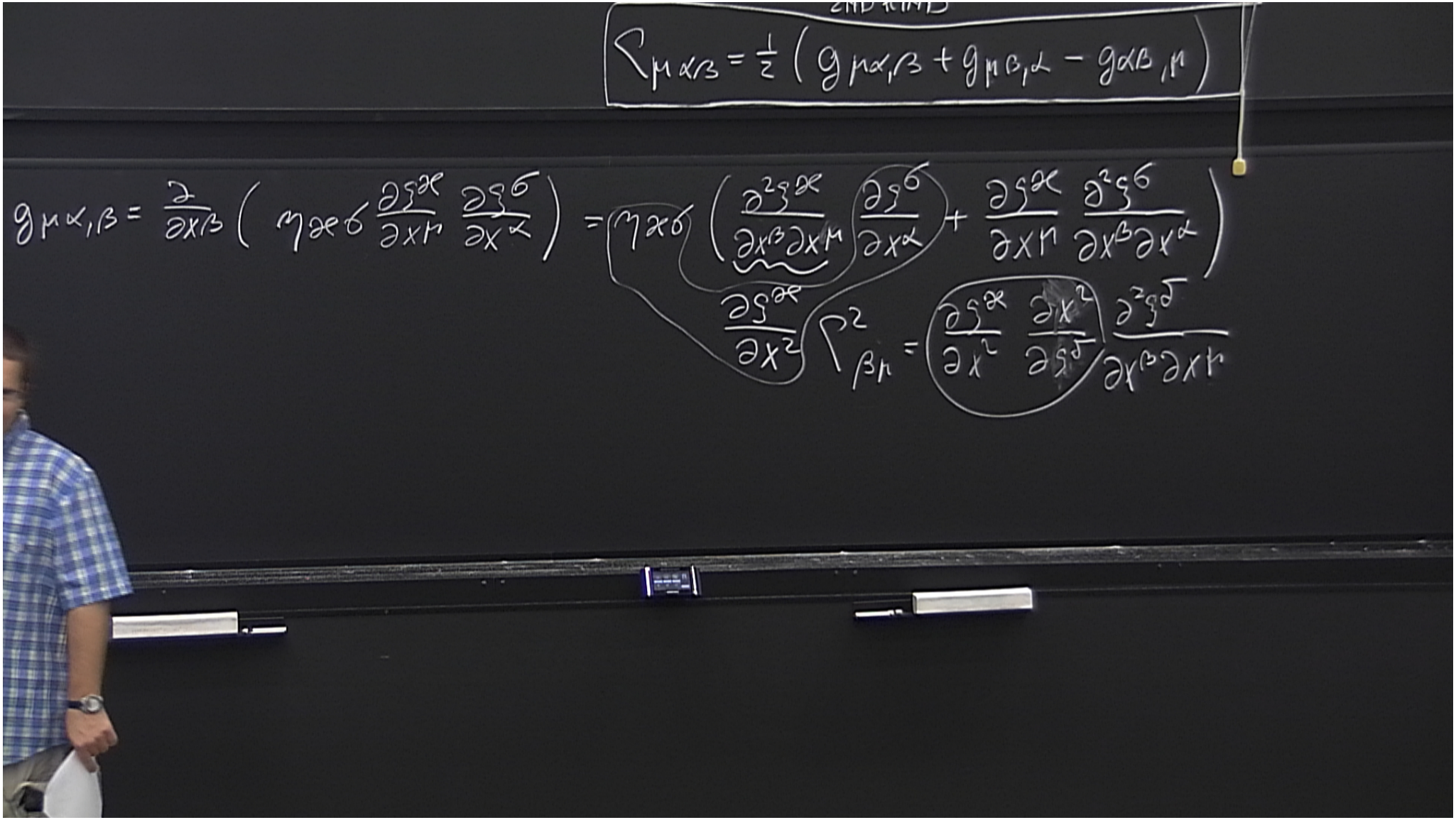

$\Gamma_{\mu\alpha\beta} = \frac{1}{2}\left(g_{\mu\alpha,\beta} + g_{\mu\beta,\alpha} - g_{\alpha\beta,\mu}\right)$
$g_{\mu\alpha,\beta} = \frac{\partial}{\partial x^\beta}\left(\eta_{\rho\sigma}\frac{\partial \xi^\rho}{\partial x^\mu}\frac{\partial \xi^\sigma}{\partial x^\alpha}\right) = \eta_{\rho\sigma}\left(\frac{\partial^2 \xi^\rho}{\partial x^\beta \partial x^\mu}\frac{\partial \xi^\sigma}{\partial x^\alpha} + \frac{\partial \xi^\rho}{\partial x^\mu}\frac{\partial^2 \xi^\sigma}{\partial x^\beta \partial x^\alpha}\right)$

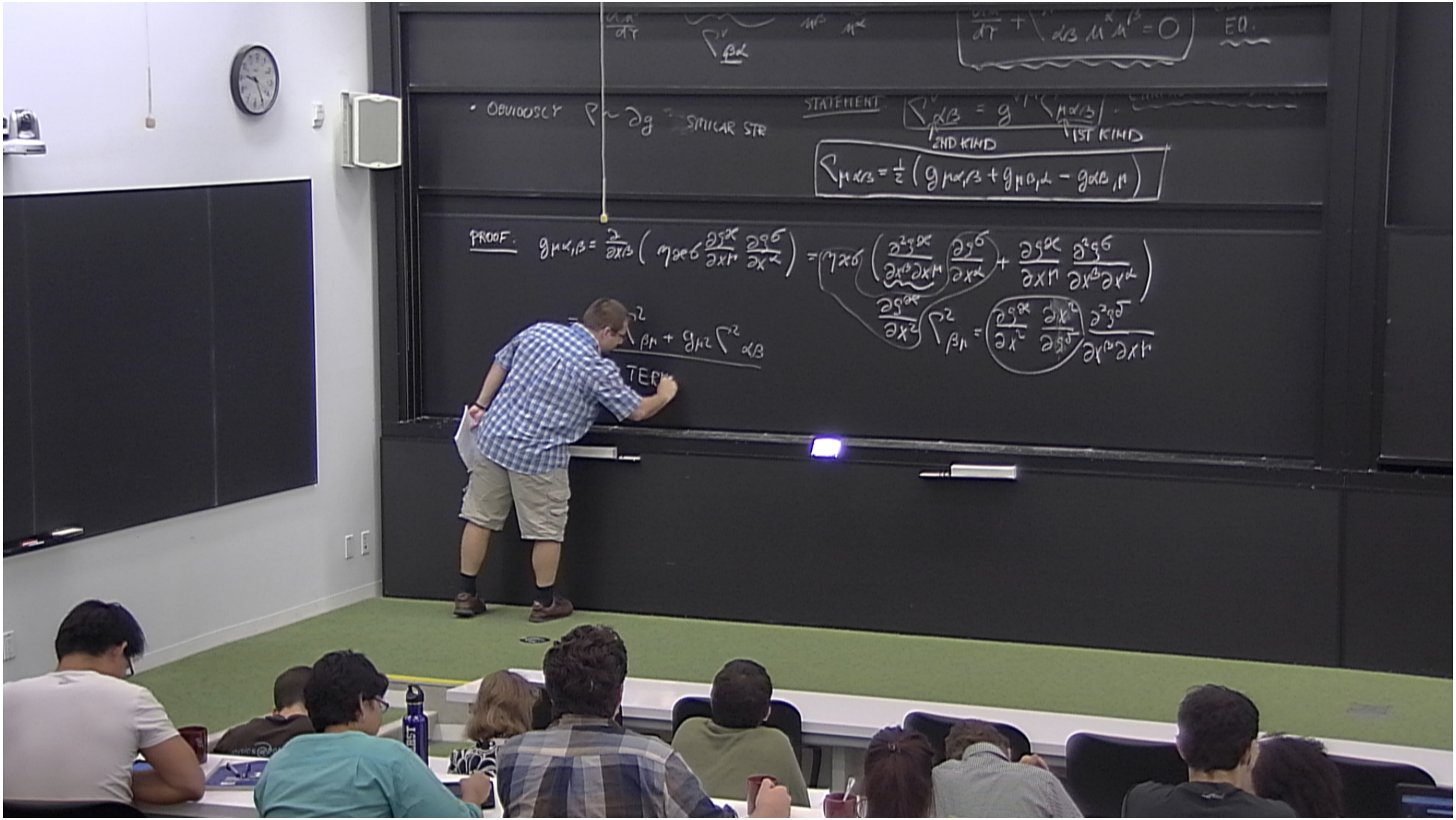

$\Gamma_{\mu\alpha\beta} = \frac{1}{2}\left(g_{\mu\alpha,\beta} + g_{\mu\beta,\alpha} - g_{\alpha\beta,\mu}\right)$
$g_{\mu\alpha,\beta} = \frac{\partial}{\partial x^\beta}\left(\eta_{\rho\sigma}\frac{\partial \xi^\rho}{\partial x^\mu}\frac{\partial \xi^\sigma}{\partial x^\alpha}\right) = \eta_{\rho\sigma}\left(\frac{\partial^2 \xi^\rho}{\partial x^\beta \partial x^\mu}\frac{\partial \xi^\sigma}{\partial x^\alpha} + \frac{\partial \xi^\rho}{\partial x^\mu}\frac{\partial^2 \xi^\sigma}{\partial x^\beta \partial x^\alpha}\right)$
$\frac{\partial \xi^\rho}{\partial x^\lambda}\Gamma^\lambda_{\beta\mu} = \frac{\partial \xi^\rho}{\partial x^\lambda}\frac{\partial x^\lambda}{\partial \xi^\sigma}\partial^2$

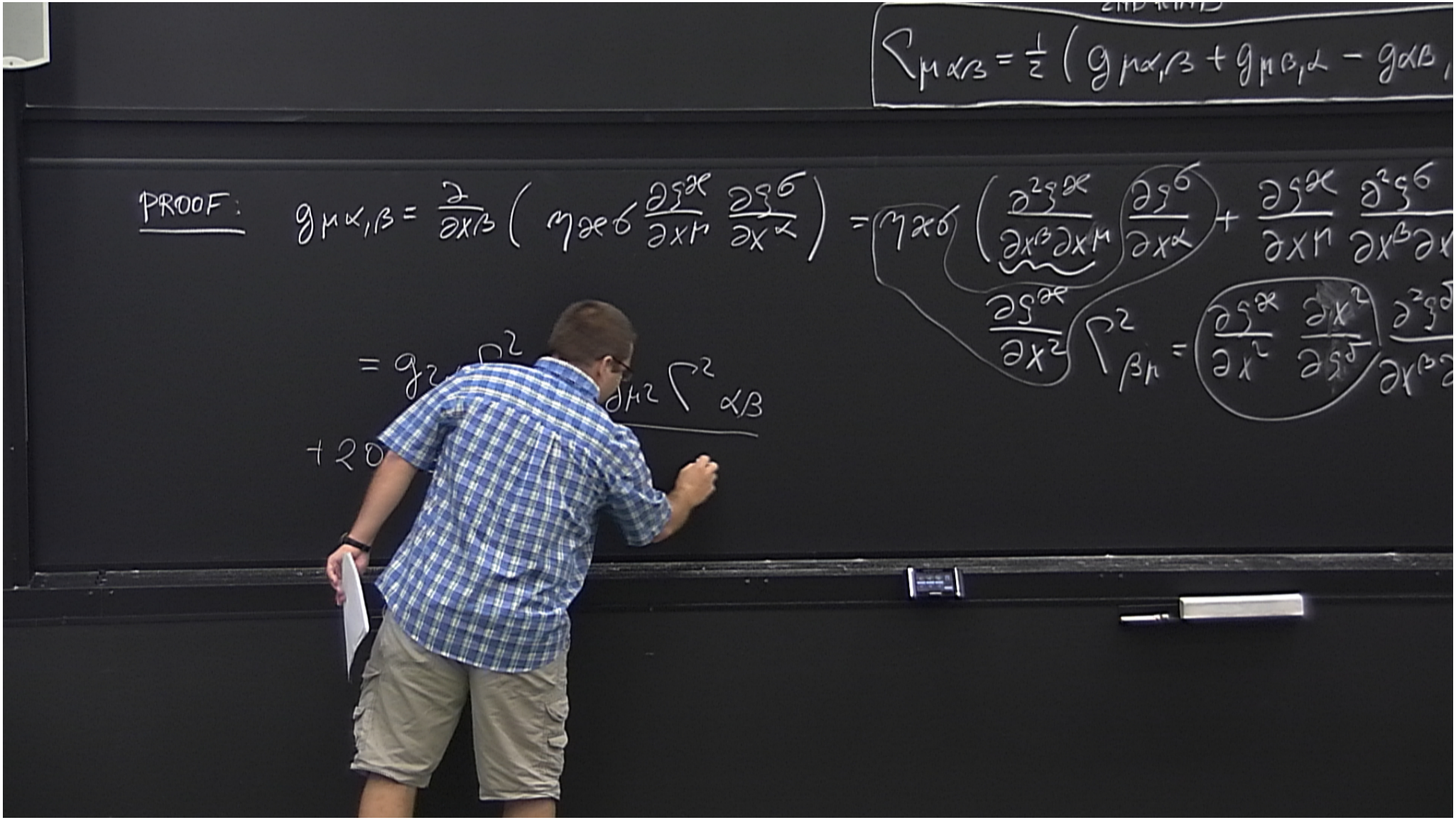
$$\Gamma_{\mu\alpha\beta} = \frac{1}{2}\left(g_{\mu\alpha,\beta} + g_{\mu\beta,\alpha} - g_{\alpha\beta,\mu}\right)$$

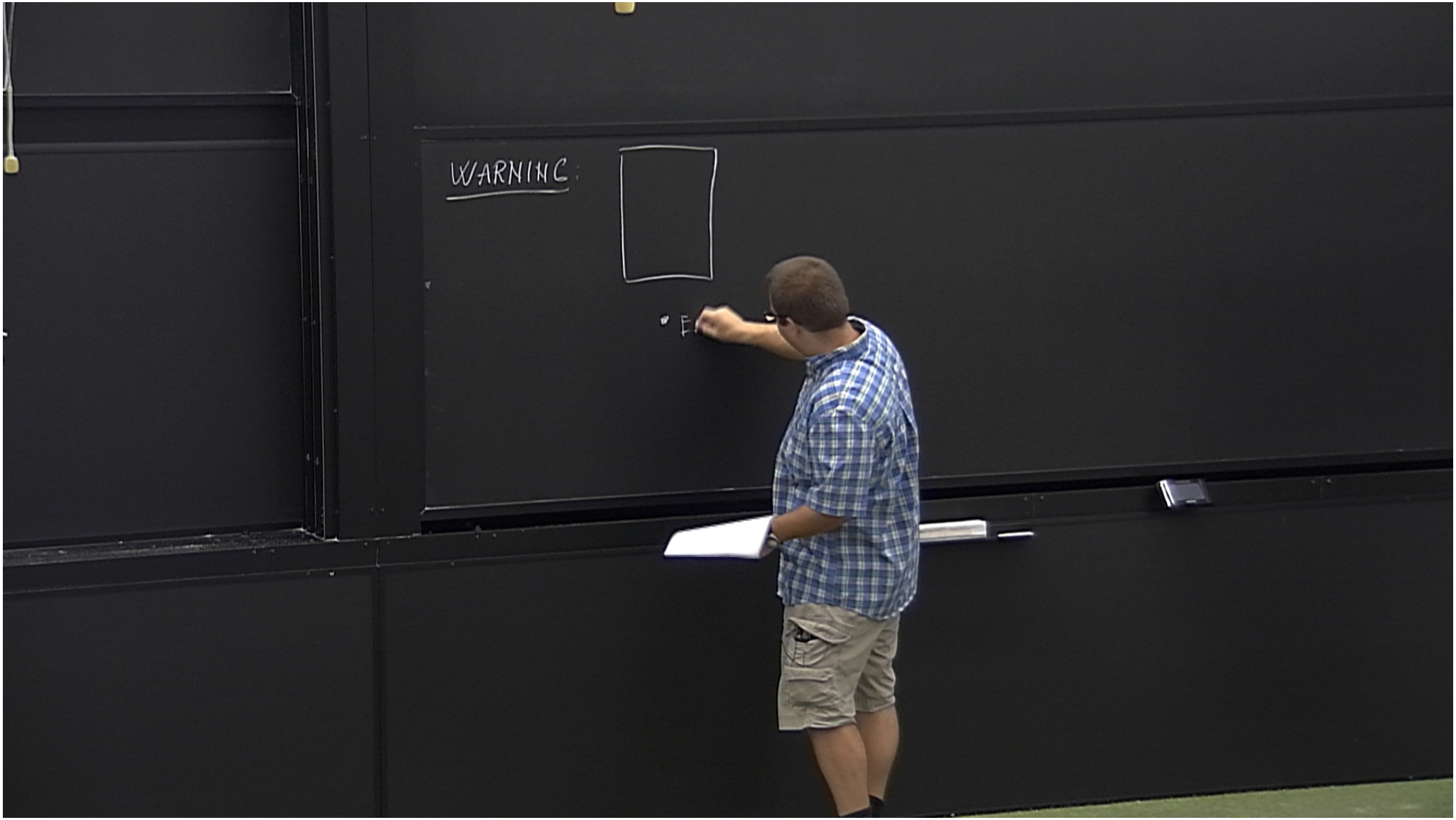
$$g_{\mu\alpha,\beta} = \frac{\partial}{\partial x^\beta}\left(\eta_{\rho\sigma}\frac{\partial \xi^\rho}{\partial x^\mu}\frac{\partial \xi^\sigma}{\partial x^\alpha}\right) = \eta_{\rho\sigma}\left(\frac{\partial^2 \xi^\rho}{\partial x^\beta \partial x^\mu}\frac{\partial \xi^\sigma}{\partial x^\alpha} + \frac{\partial \xi^\rho}{\partial x^\mu}\frac{\partial^2 \xi^\sigma}{\partial x^\beta \partial x^\alpha}\right)$$

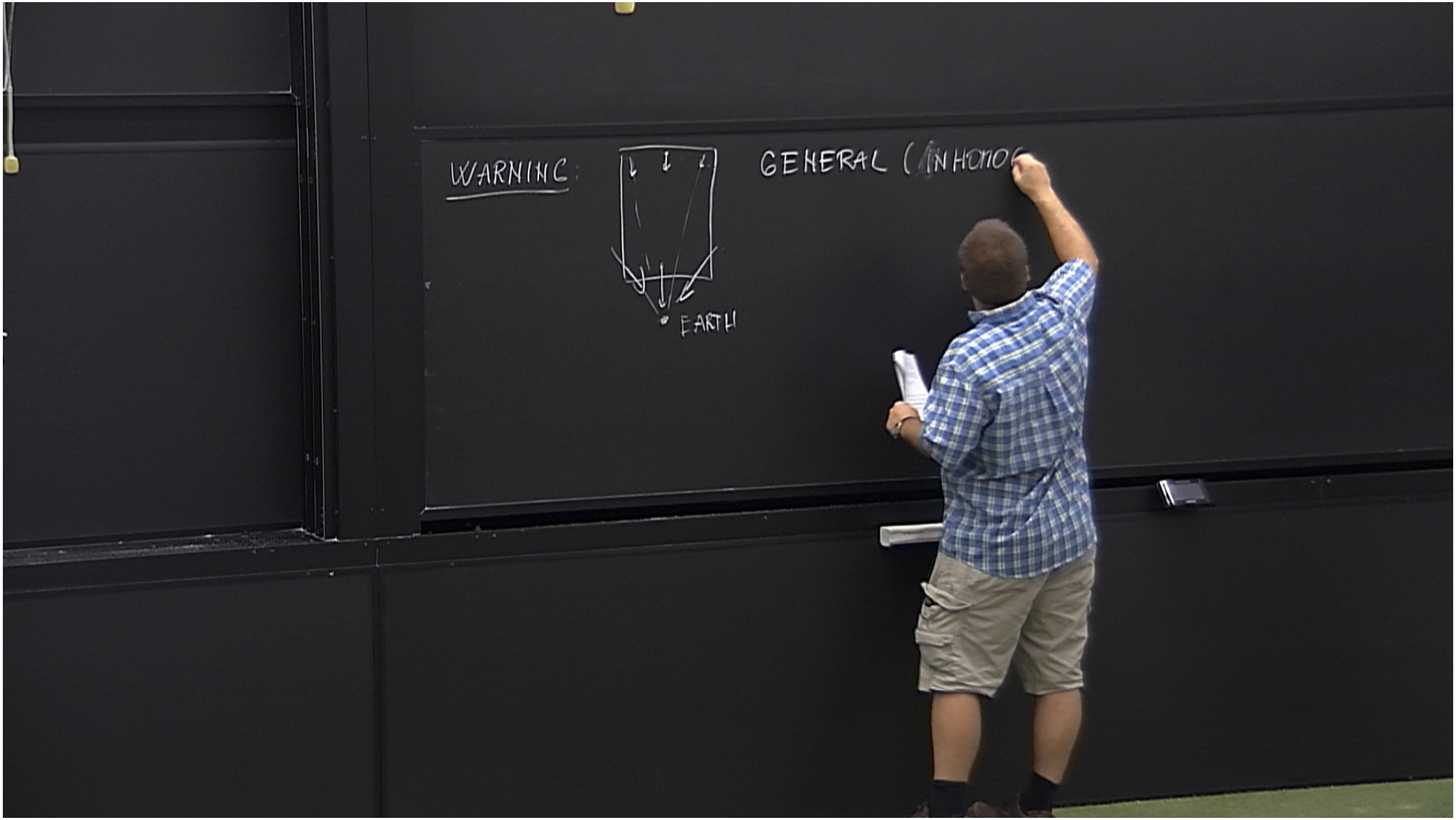
$$\frac{\partial \xi^\rho}{\partial x^\lambda}\Gamma^\lambda_{\beta\mu} = \frac{\partial \xi^\rho}{\partial x^\lambda}\frac{\partial x^\lambda}{\partial \xi^\sigma}\frac{\partial^2 \xi^\sigma}{\partial x^\beta \partial x^\mu}$$

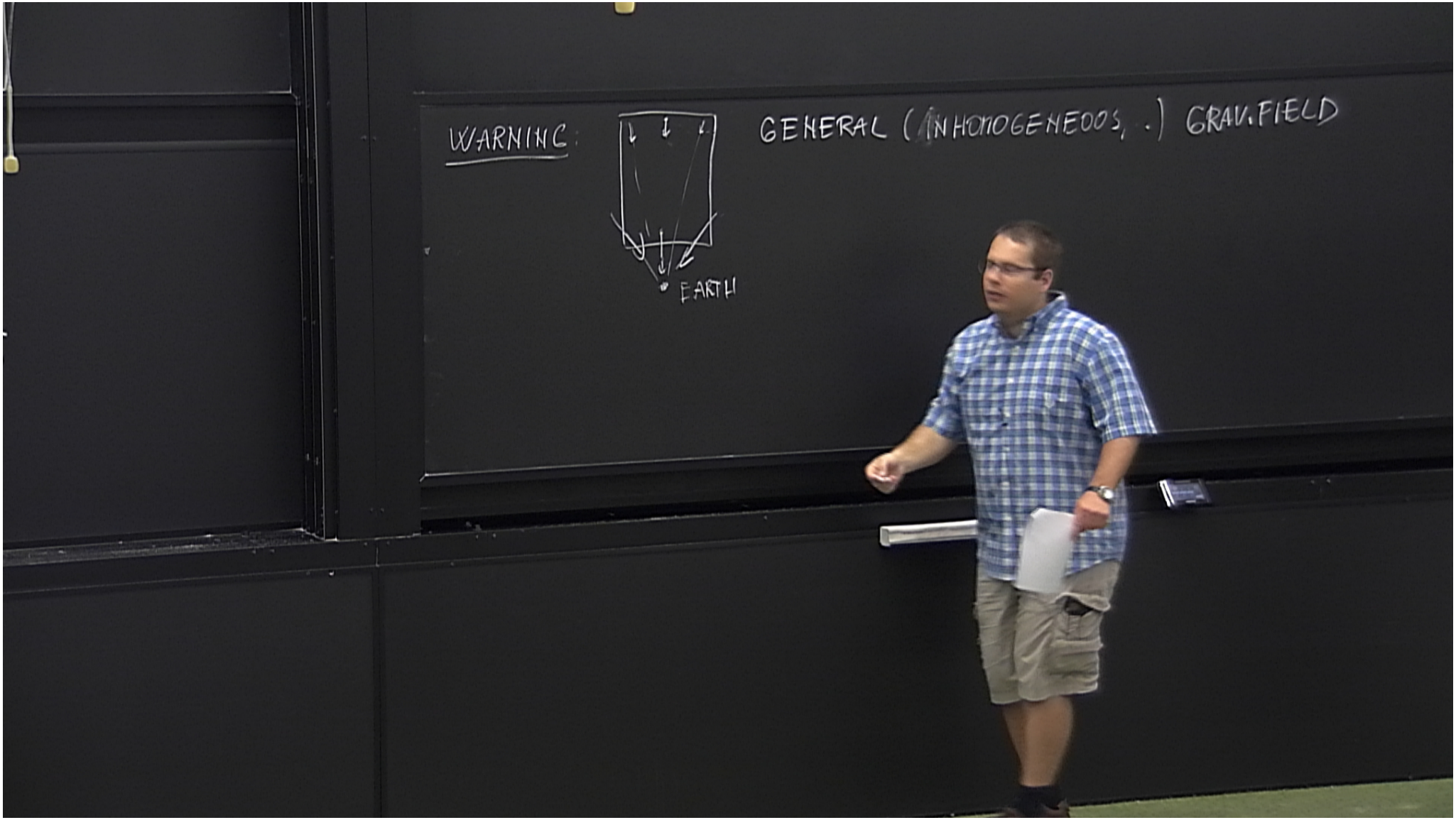

$\Gamma_{\mu\alpha\beta} = \frac{1}{2}\left(g_{\mu\alpha,\beta} + g_{\mu\beta,\alpha} - g_{\alpha\beta,\mu}\right)$
$g_{\mu\alpha,\beta} = \frac{\partial}{\partial x^\beta}\left(\eta_{\rho\sigma}\frac{\partial \xi^\rho}{\partial x^\mu}\frac{\partial \xi^\sigma}{\partial x^\alpha}\right) = \eta_{\rho\sigma}\left(\frac{\partial^2 \xi^\rho}{\partial x^\beta \partial x^\mu}\frac{\partial \xi^\sigma}{\partial x^\alpha} + \frac{\partial \xi^\rho}{\partial x^\mu}\frac{\partial^2 \xi^\sigma}{\partial x^\beta \partial x^\alpha}\right)$
$\frac{\partial \xi^\rho}{\partial x^\lambda}\Gamma^\lambda_{\beta\mu} = \frac{\partial \xi^\rho}{\partial x^\lambda}\frac{\partial x^\lambda}{\partial \xi^\sigma}\frac{\partial^2 \xi^\sigma}{\partial x^\beta \partial x^\mu}$

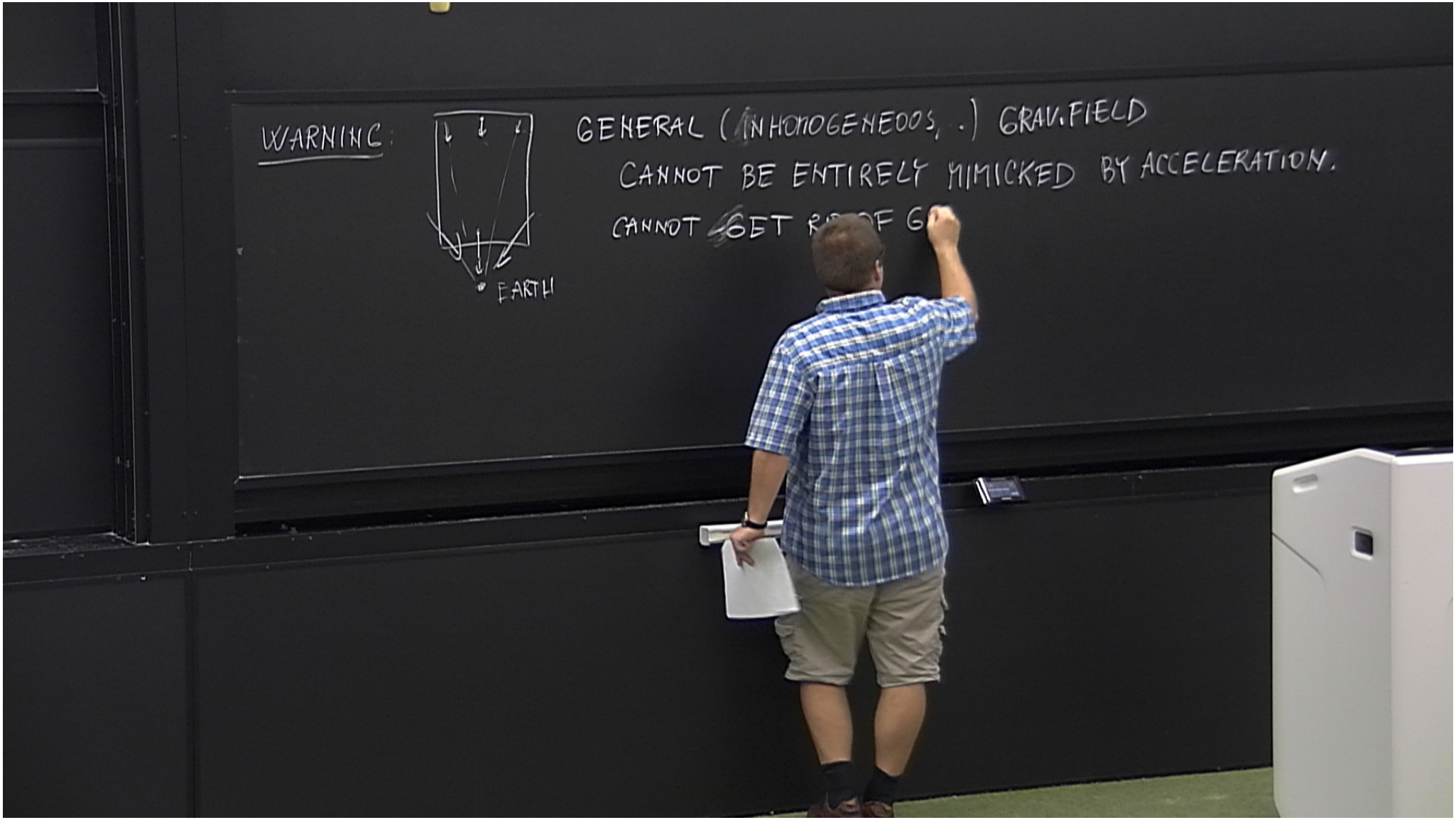

STATEMENT
2ND KIND
1ST KIND
PROOF
TERM

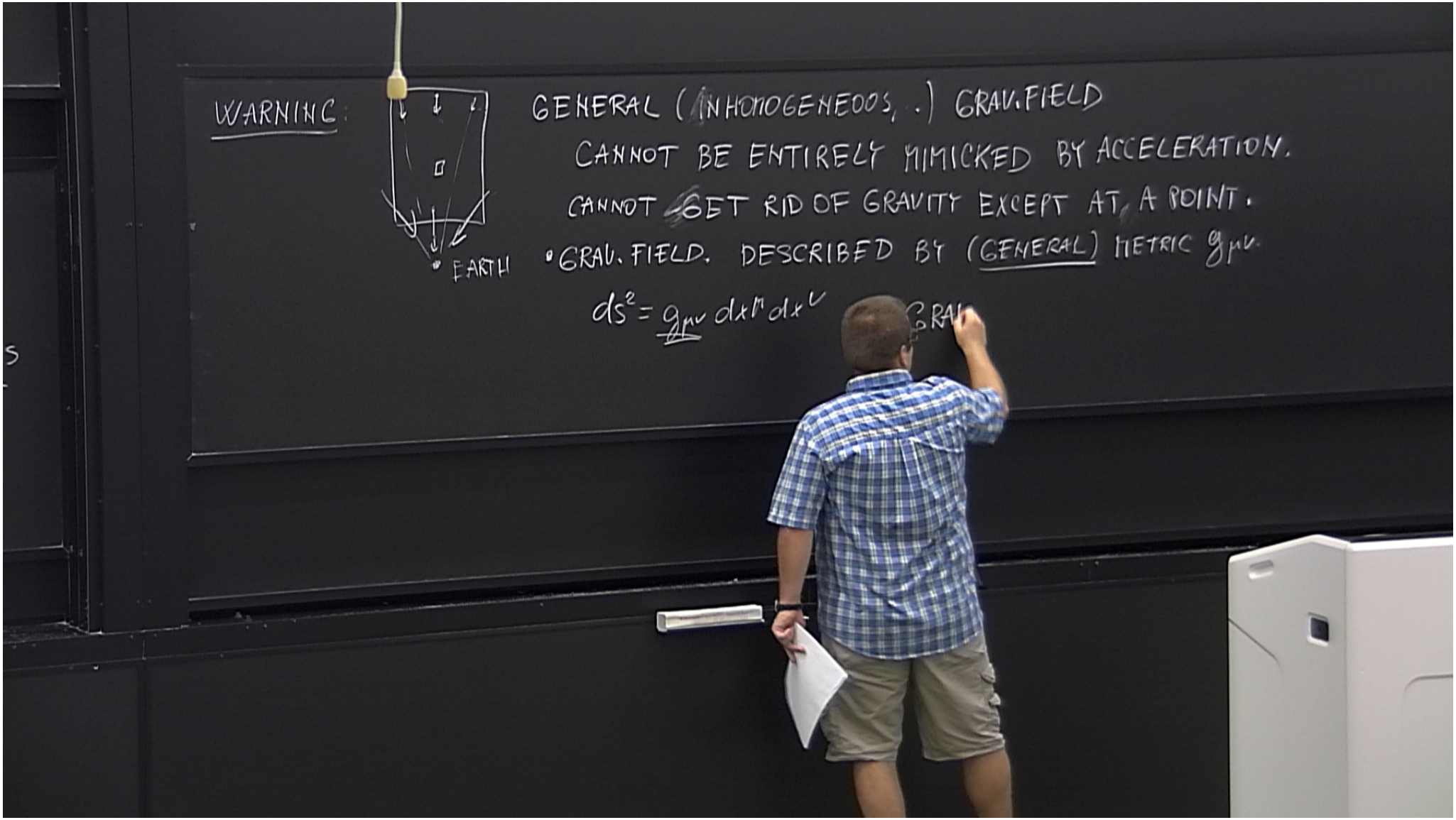

$\Gamma_{\mu\alpha\beta} = \frac{1}{2}\left(g_{\mu\alpha,\beta} + g_{\mu\beta,\alpha} - g_{\alpha\beta,}\right.$
PROOF:
$g_{\mu\alpha,\beta} = \frac{\partial}{\partial x^\beta}\left(\eta_{\sigma\rho}\frac{\partial \xi^\sigma}{\partial x^\mu}\frac{\partial \xi^\rho}{\partial x^\alpha}\right) = \eta_{\sigma\rho}\left(\frac{\partial^2 \xi^\sigma}{\partial x^\beta \partial x^\mu}\frac{\partial \xi^\rho}{\partial x^\alpha} + \frac{\partial \xi^\sigma}{\partial x^\mu}\frac{\partial^2 \xi^\rho}{\partial x^\beta \partial x^\alpha}\right.$
$\frac{\partial \xi^\sigma}{\partial x^\lambda}\Gamma^\lambda_{\beta\mu} = \frac{\partial^2 \xi^\sigma}{\partial x^\beta \partial x^\mu}$
$= g_{\lambda\alpha}\Gamma^\lambda_{\beta\mu} + g_{\lambda\beta}\Gamma^\lambda_{\alpha\mu}$
$+ 2\Gamma_{\mu\alpha\beta}$


WARNING

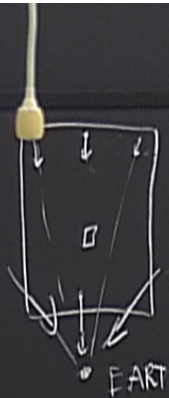

WARNING:
GENERAL (IN HONO
EARTH


WARNING:
EARTH
GENERAL (INHOMOGENEOUS,.) GRAV. FIELD


WARNING:
EARTH
GENERAL (INHOMOGENEOUS,.) GRAV. FIELD
CANNOT BE ENTIRELY MIMICKED BY ACCELERATION.
CANNOT GET


WARNING:
EARTH
GENERAL (INHOMOGENEOUS,..) GRAV. FIELD
CANNOT BE ENTIRELY MIMICKED BY ACCELERATION.
CANNOT GET RID OF GRAVITY EXCEPT AT A POINT.
• GRAV. FIELD. DESCRIBED BY (GENERAL) METRIC $g_{\mu\nu}$.
$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

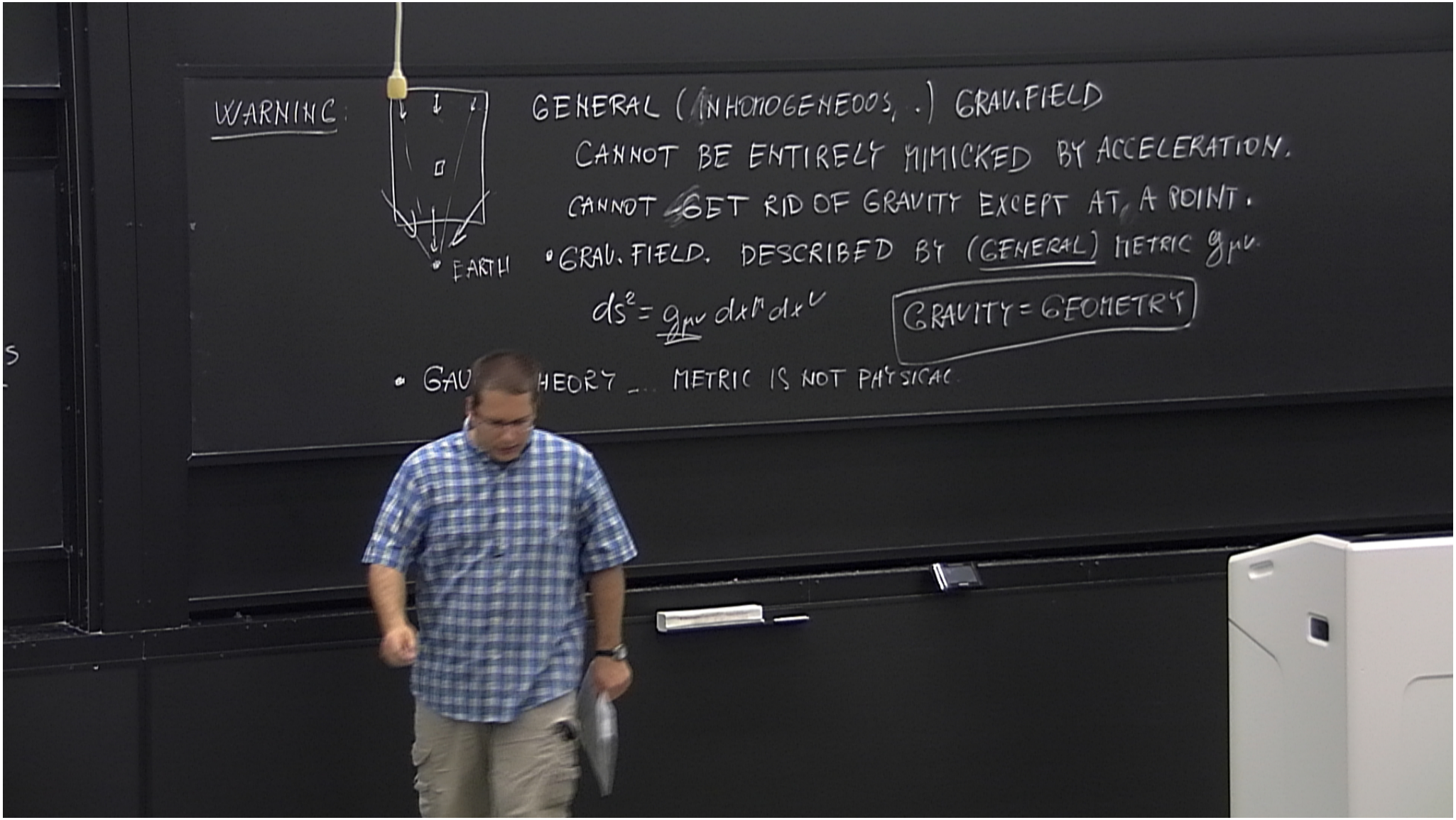
WARNING:



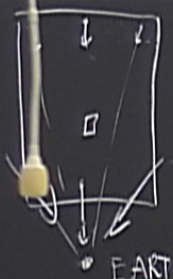
GENERAL (INHOMOGENEOUS, ...) GRAV. FIELD
CANNOT BE ENTIRELY MIMICKED BY ACCELERATION.
CANNOT GET RID OF GRAVITY EXCEPT AT A POINT.
• GRAV. FIELD DESCRIBED BY (GENERAL) METRIC $g_{\mu\nu}$.

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

GRAVITY = GEOMETRY


WARNING:
EARTH
GENERAL (INHOMOGENEOUS,..) GRAV. FIELD
CANNOT BE ENTIRELY MIMICKED BY ACCELERATION.
CANNOT GET RID OF GRAVITY EXCEPT AT A POINT.
• GRAV. FIELD. DESCRIBED BY (GENERAL) METRIC $g_{\mu\nu}$.
$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$
GRAVITY = GEOMETRY
• GAU HEORY ... METRIC IS NOT PHYSICAL

ARNING:



GENERAL (INHOMOGENEOUS, .) GRAV. FIELD

CANNOT BE ENTIRELY MIMICKED BY ACCELERATION.

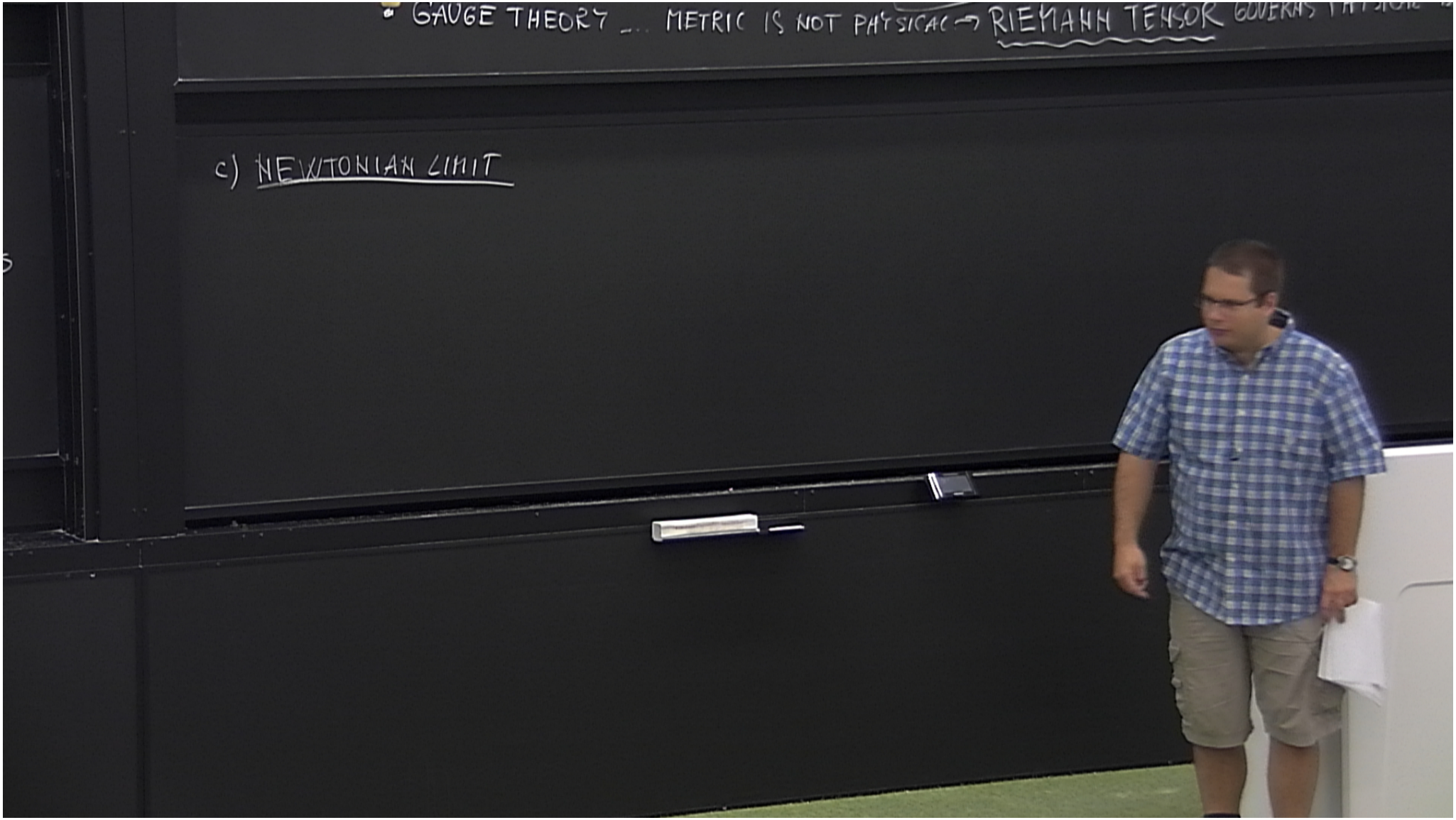
CANNOT GET RID OF GRAVITY EXCEPT AT A POINT.

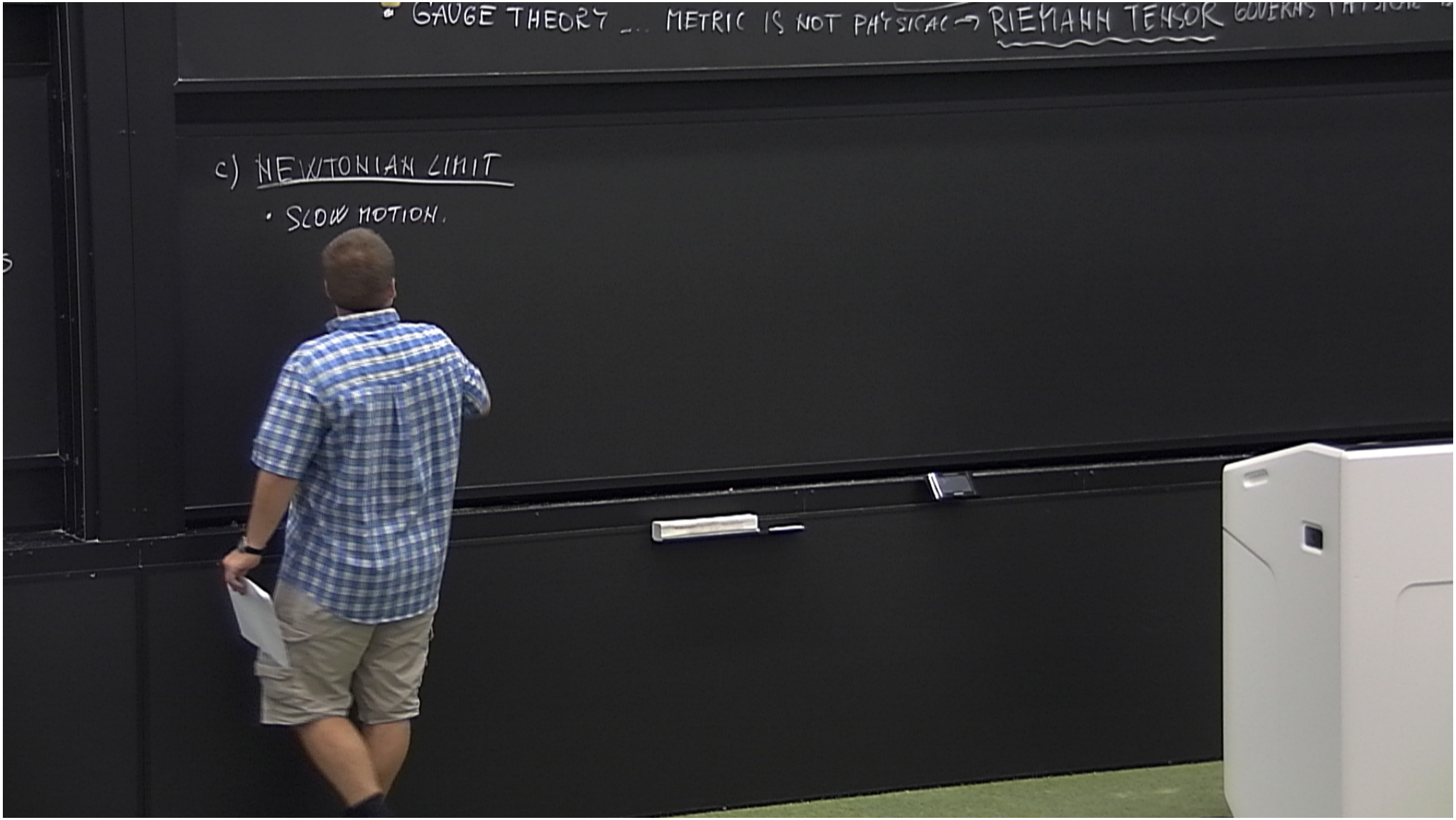
• GRAV. FIELD. DESCRIBED BY (GENERAL) METRIC $g_{\mu\nu}$.

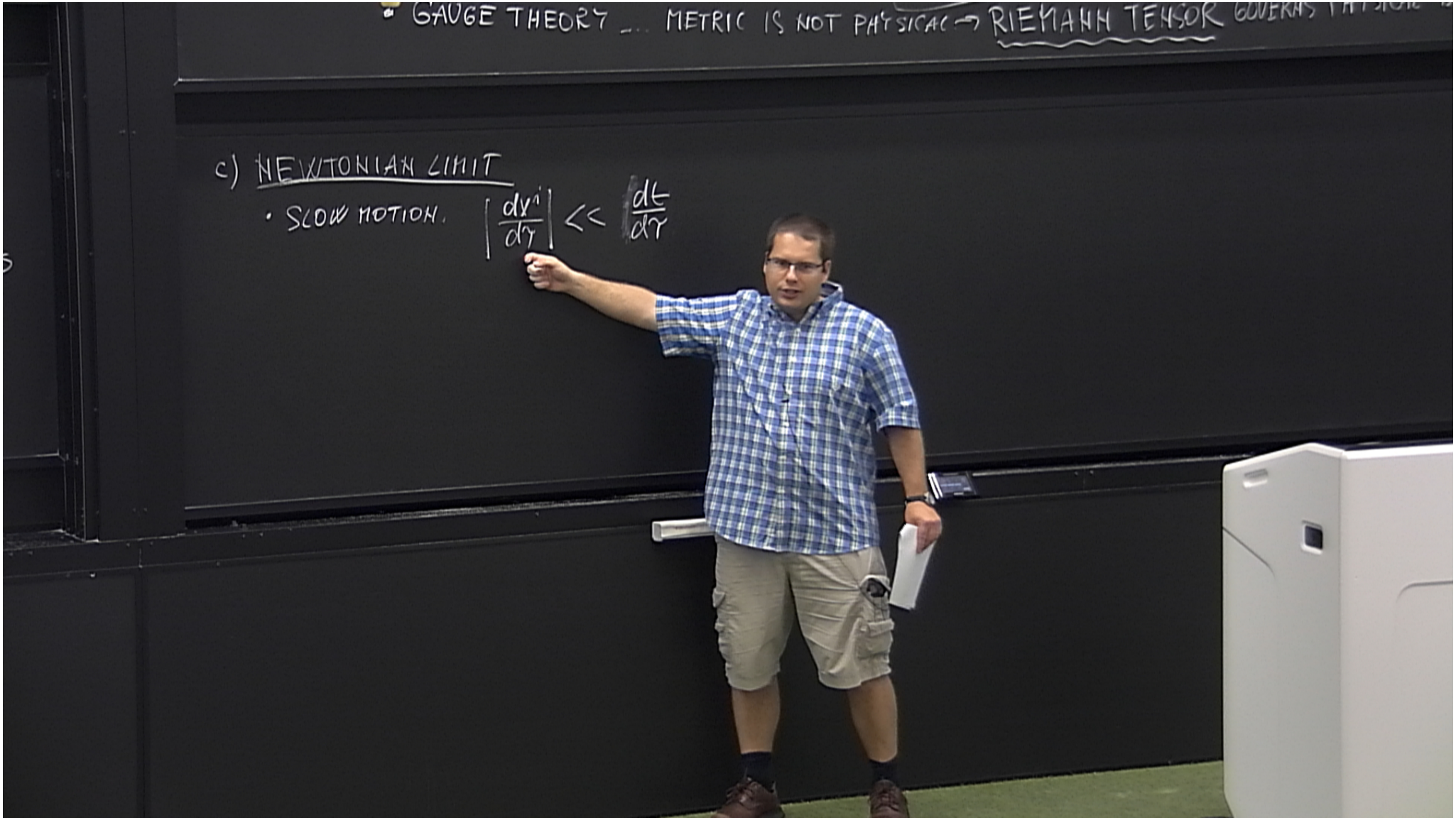
$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

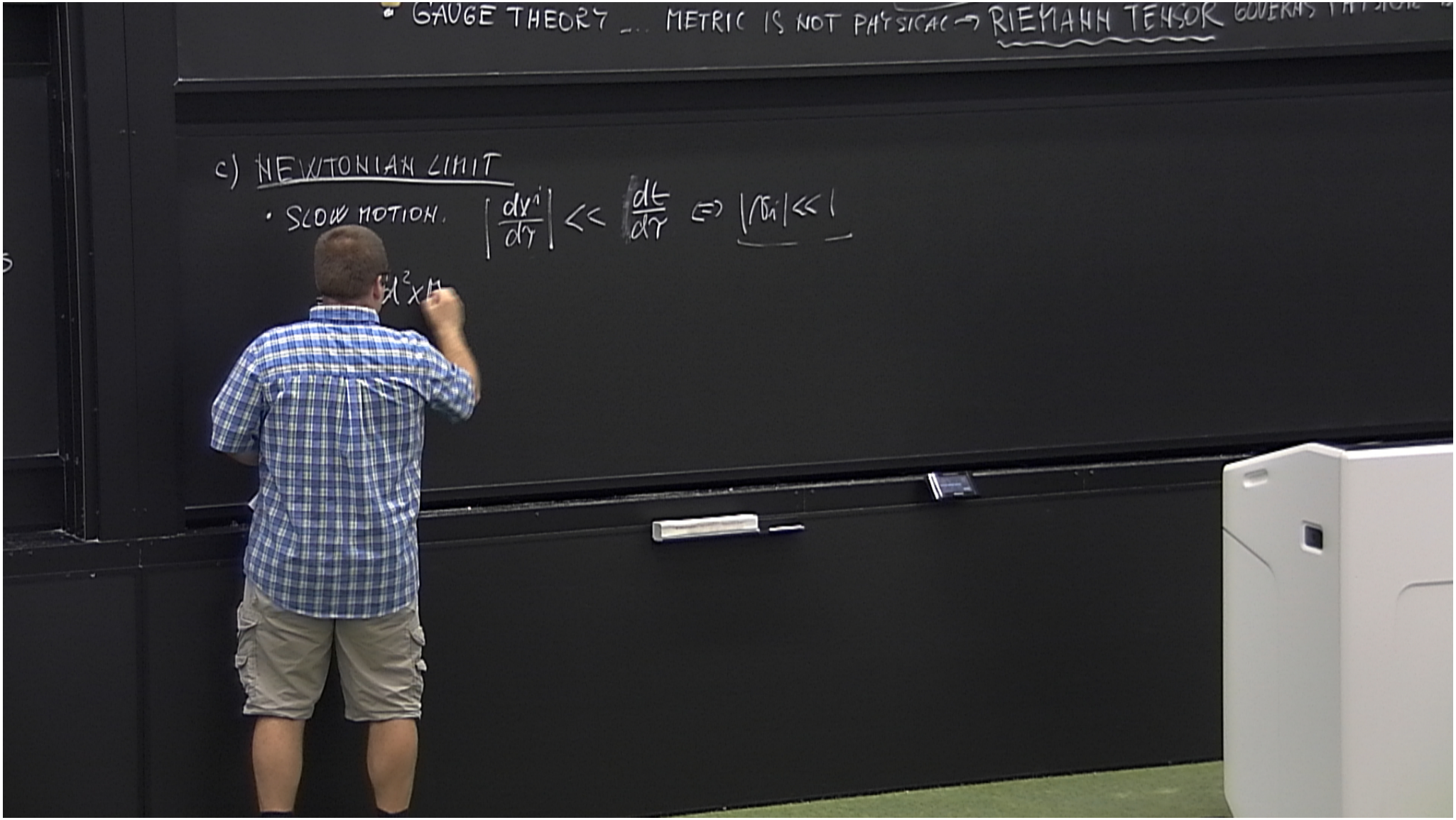
GRAVITY = GEOMETRY

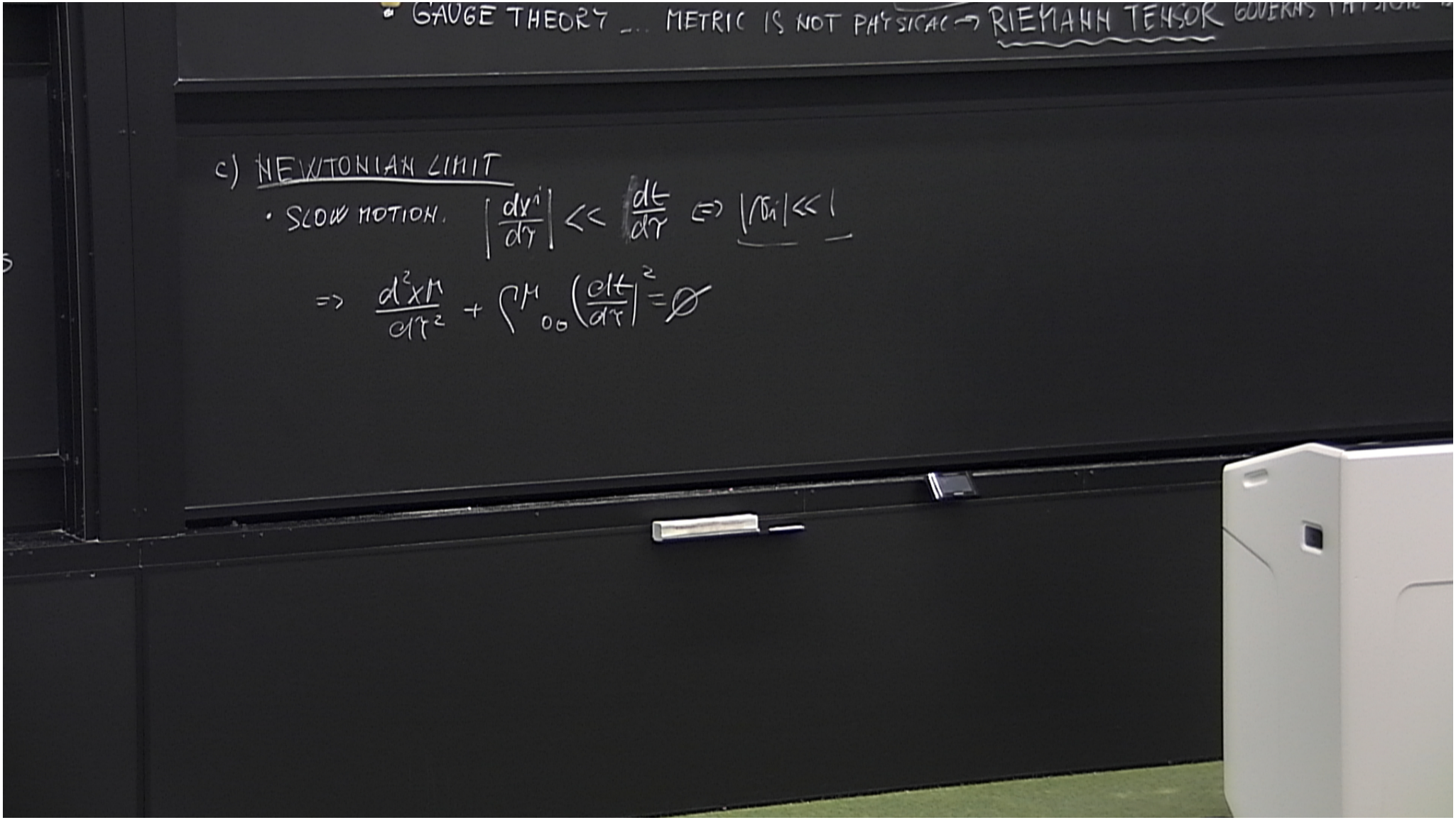
• GAUGE THEORY _ METRIC IS NOT PHYSICAL → RIEMANN TENSOR GOVERNS PHYSICAL EFFECTS.

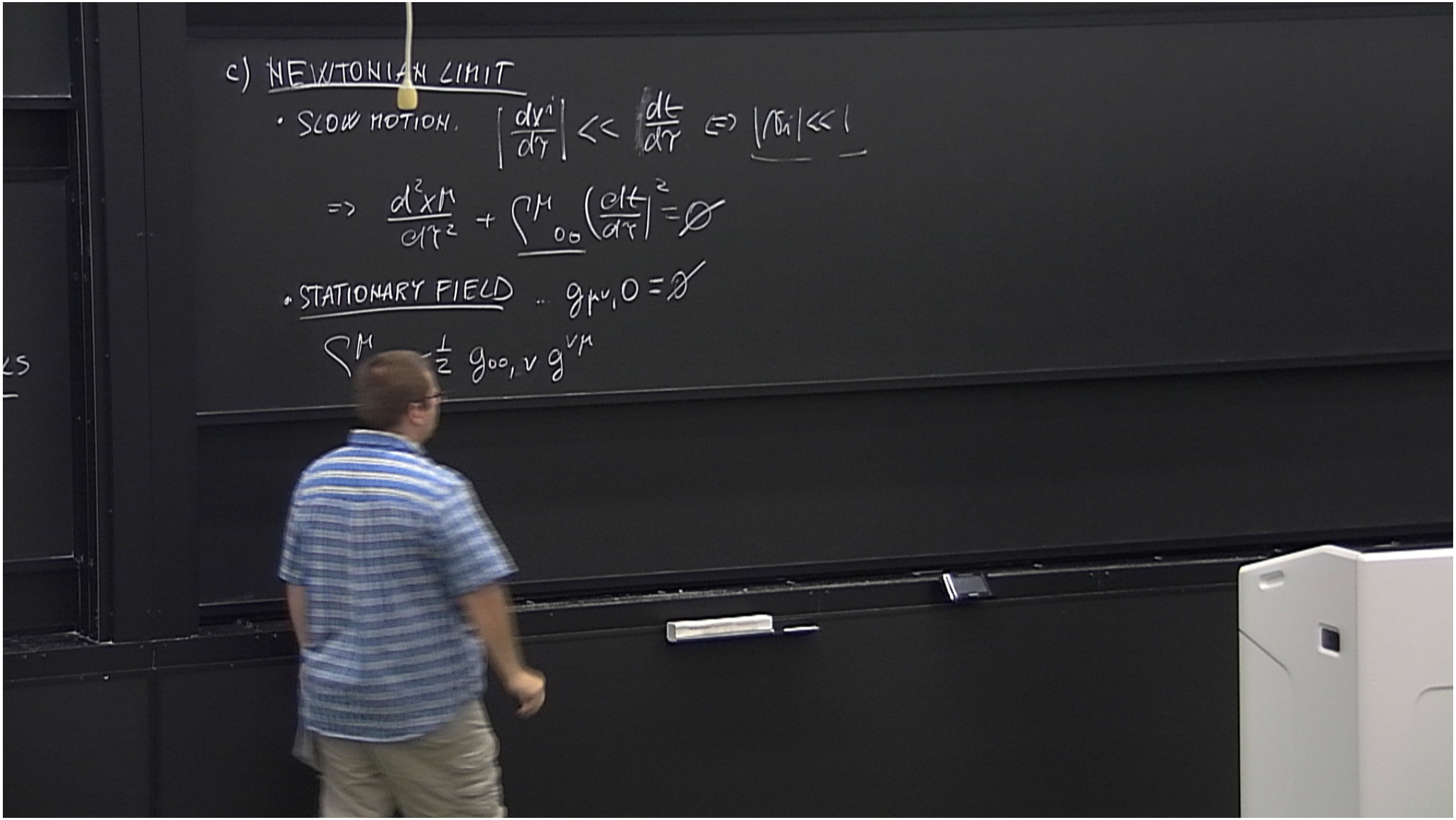

GAUGE THEORY ... METRIC IS NOT PHYSICAL → RIEMANN TENSOR
c) NEWTONIAN LIMIT

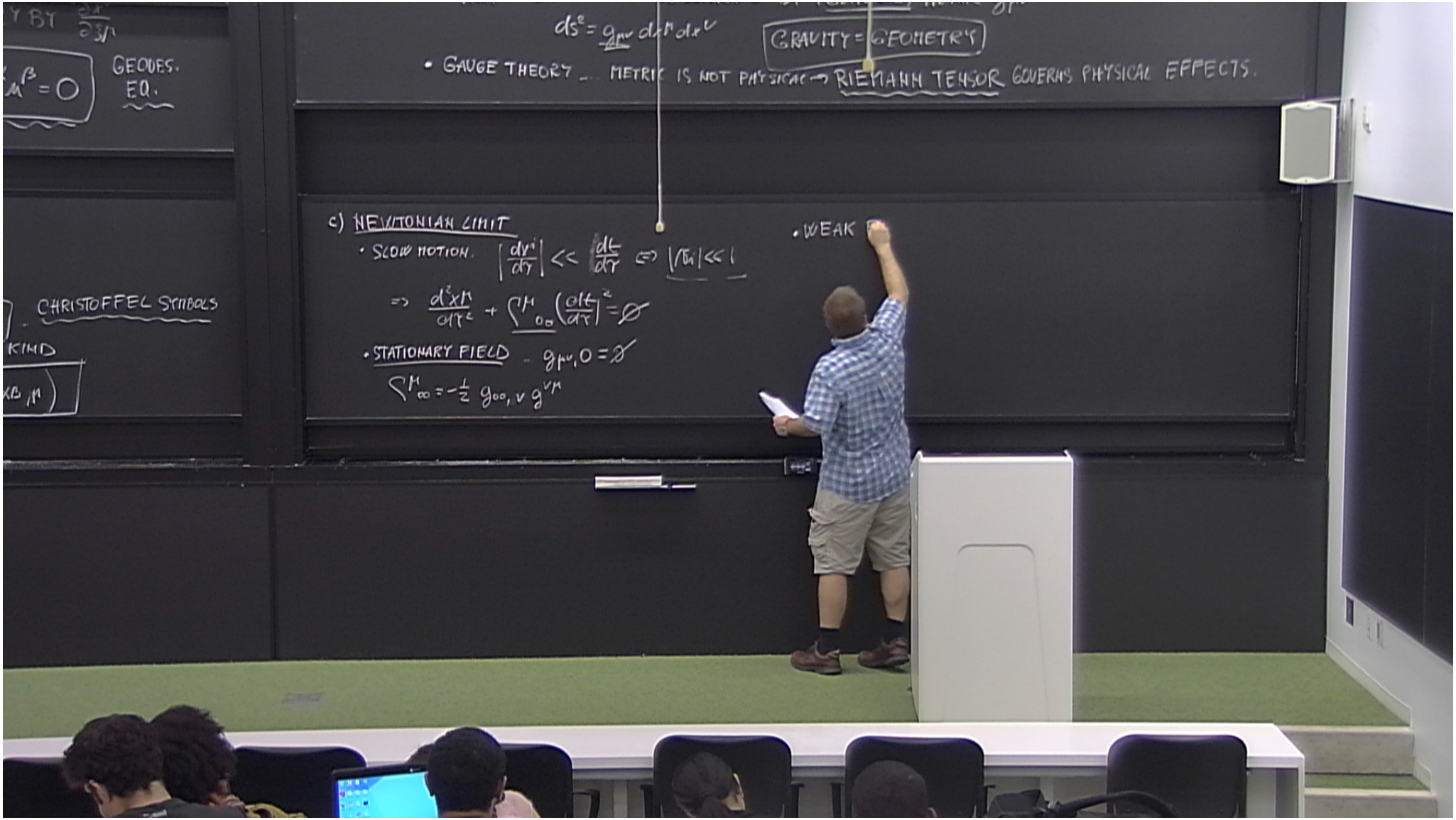

GAUGE THEORY ... METRIC IS NOT PHYSICAL → RIEMANN TENSOR
c) NEWTONIAN LIMIT
• SLOW MOTION.

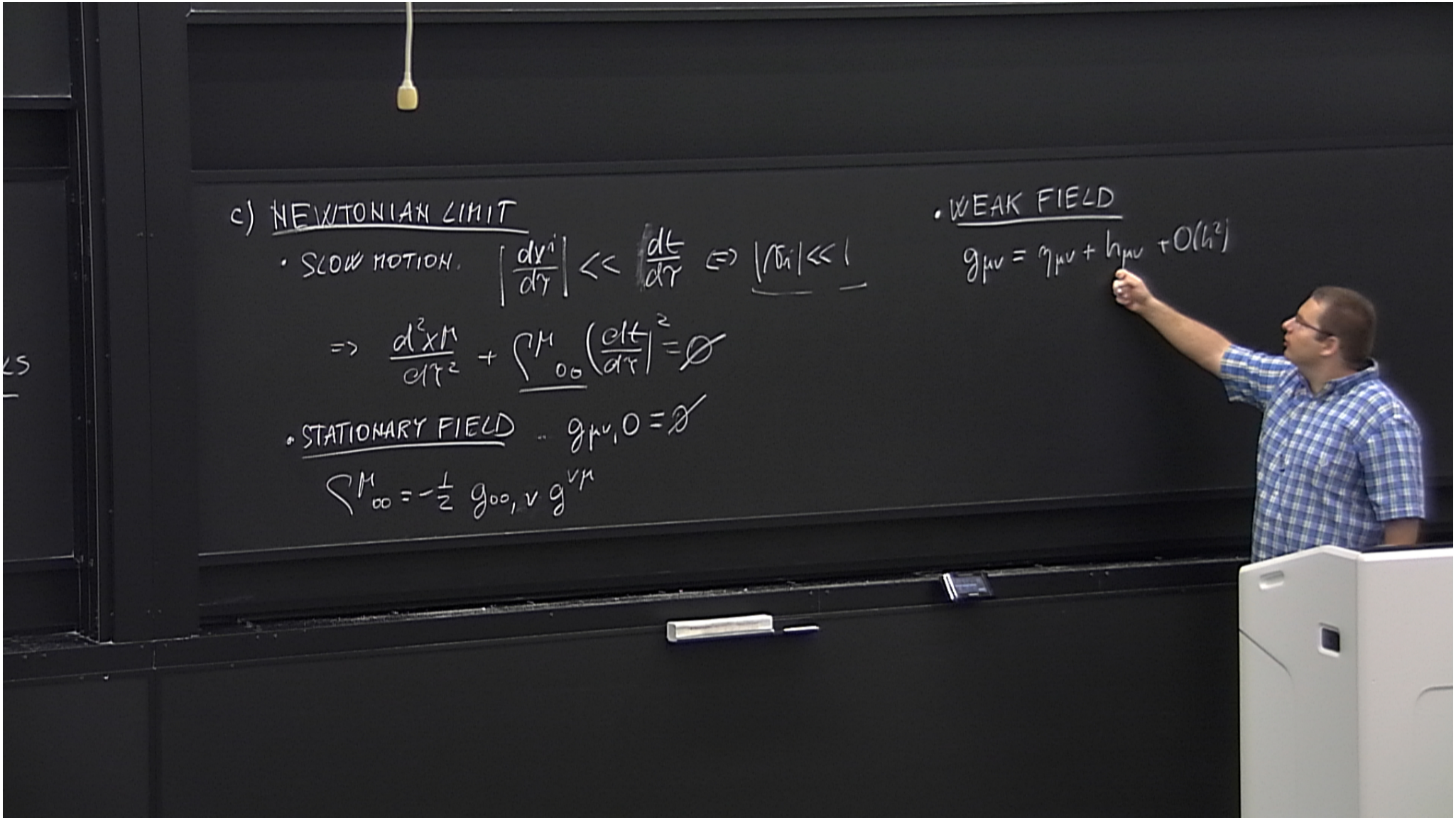

GAUGE THEORY ... METRIC IS NOT PHYSICAL → RIEMANN TENSOR
c) NEWTONIAN LIMIT
• SLOW MOTION. $\left|\frac{dx^i}{d\tau}\right| << \left|\frac{dt}{d\tau}\right|$

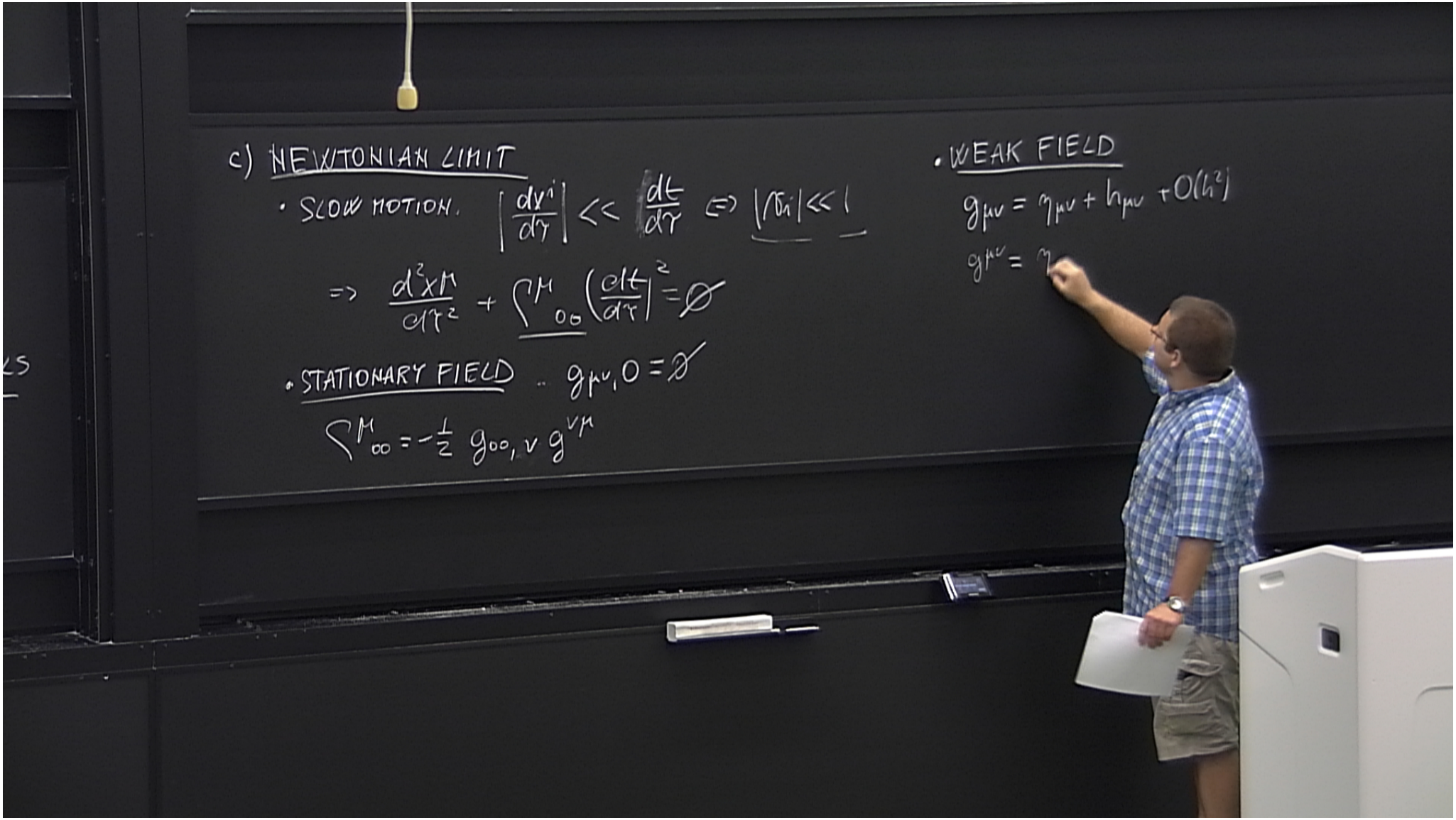

GAUGE THEORY ... METRIC IS NOT PHYSICAL → RIEMANN TENSOR
c) NEWTONIAN LIMIT
• SLOW MOTION. $\left|\frac{dx^i}{d\tau}\right| \ll \left|\frac{dt}{d\tau}\right| \Leftrightarrow |\Gamma^i_{00}| \ll 1$

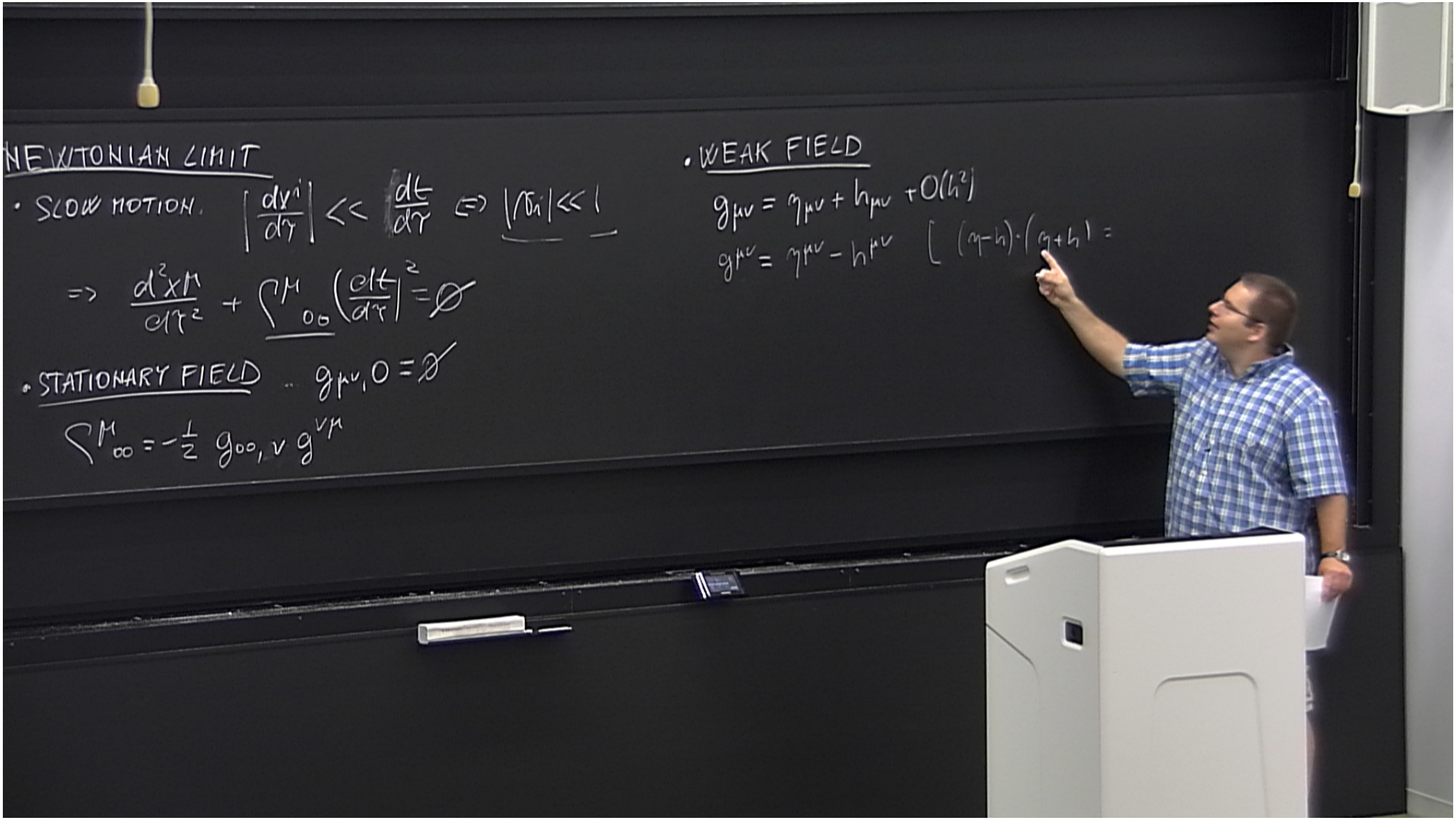

GAUGE THEORY ... METRIC IS NOT PHYSICAL → RIEMANN TENSOR
c) NEWTONIAN LIMIT
• SLOW MOTION. $\left|\frac{dx^i}{d\tau}\right| \ll \left|\frac{dt}{d\tau}\right| \Leftrightarrow |v_i| \ll 1$
$\Rightarrow \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$

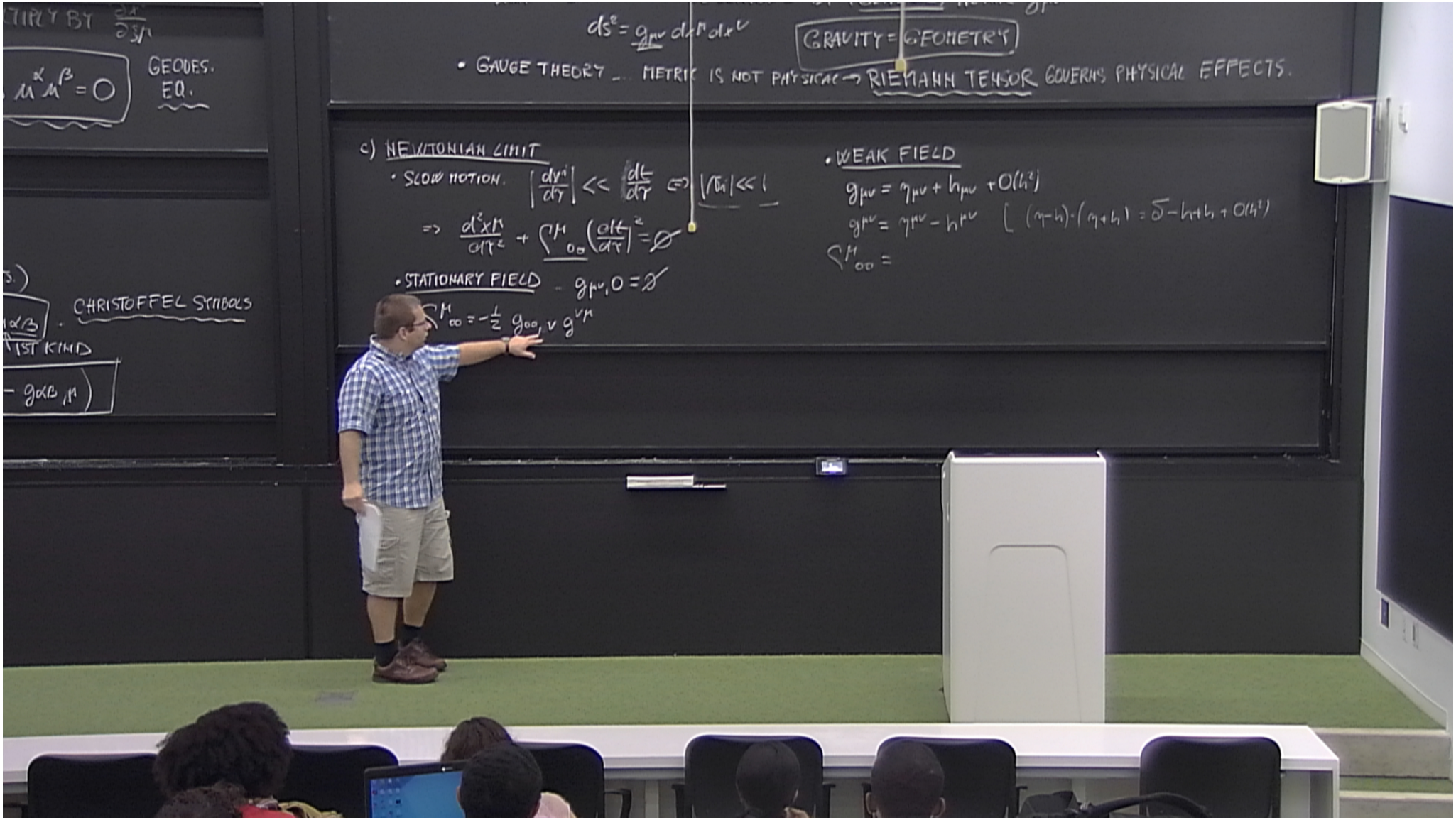

c) NEWTONIAN LIMIT
• SLOW MOTION.
• STATIONARY FIELD

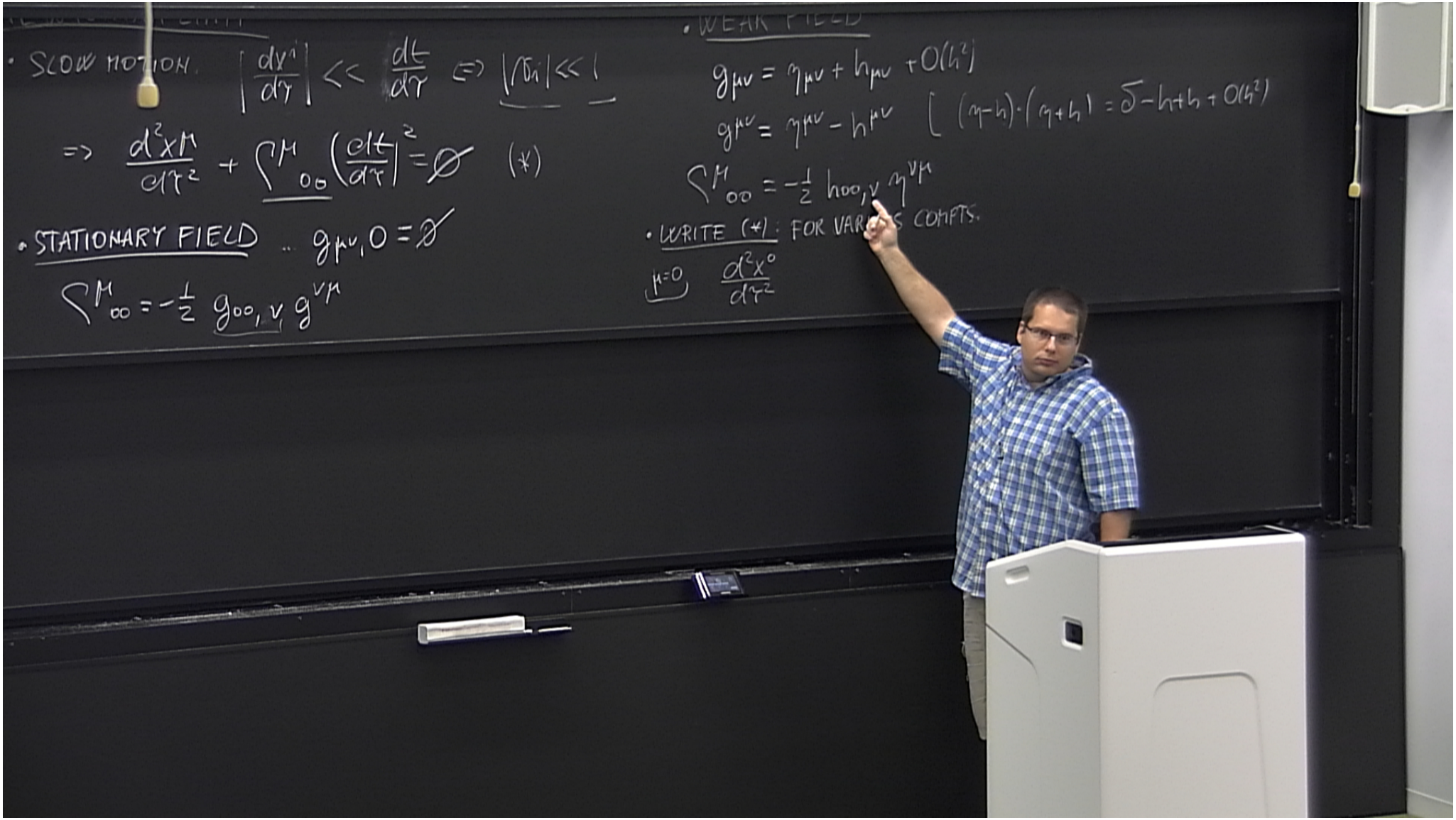

GEODES. EQ.
CHRISTOFFEL SYMBOLS
GRAVITY = GEOMETRY
• GAUGE THEORY ... METRIC IS NOT PHYSICAL → RIEMANN TENSOR GOVERNS PHYSICAL EFFECTS.
c) NEWTONIAN LIMIT
• SLOW MOTION.
• STATIONARY FIELD

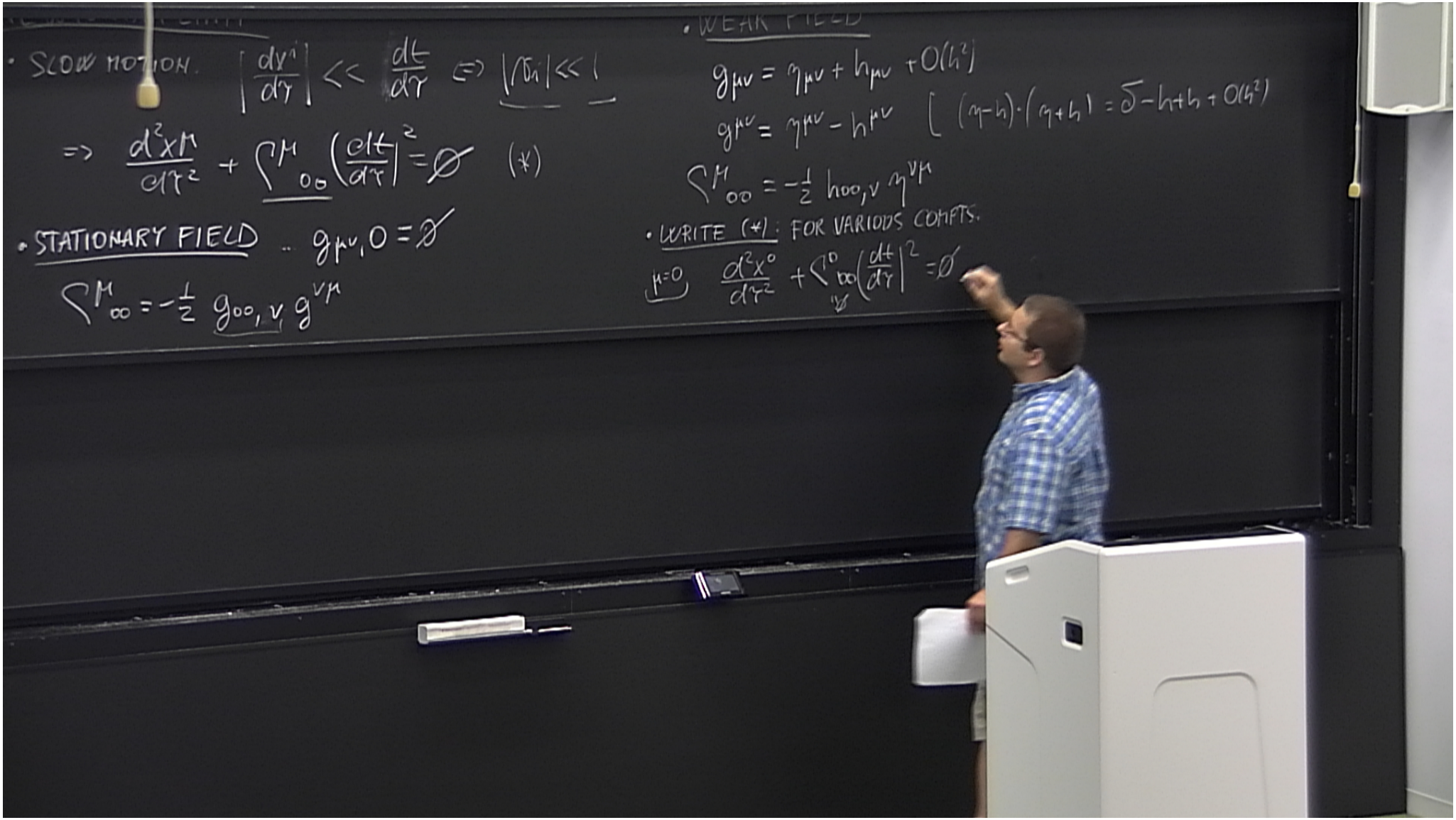

c) NEWTONIAN LIMIT
• SLOW MOTION. $\left|\frac{dx^i}{d\tau}\right| \ll \frac{dt}{d\tau} \Leftrightarrow |v_i| \ll 1$
$\Rightarrow \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$
• STATIONARY FIELD $g_{\mu\nu,0} = 0$
$\Gamma^\mu_{00} = -\frac{1}{2} g_{00,\nu} g^{\nu\mu}$
• WEAK FIELD
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$

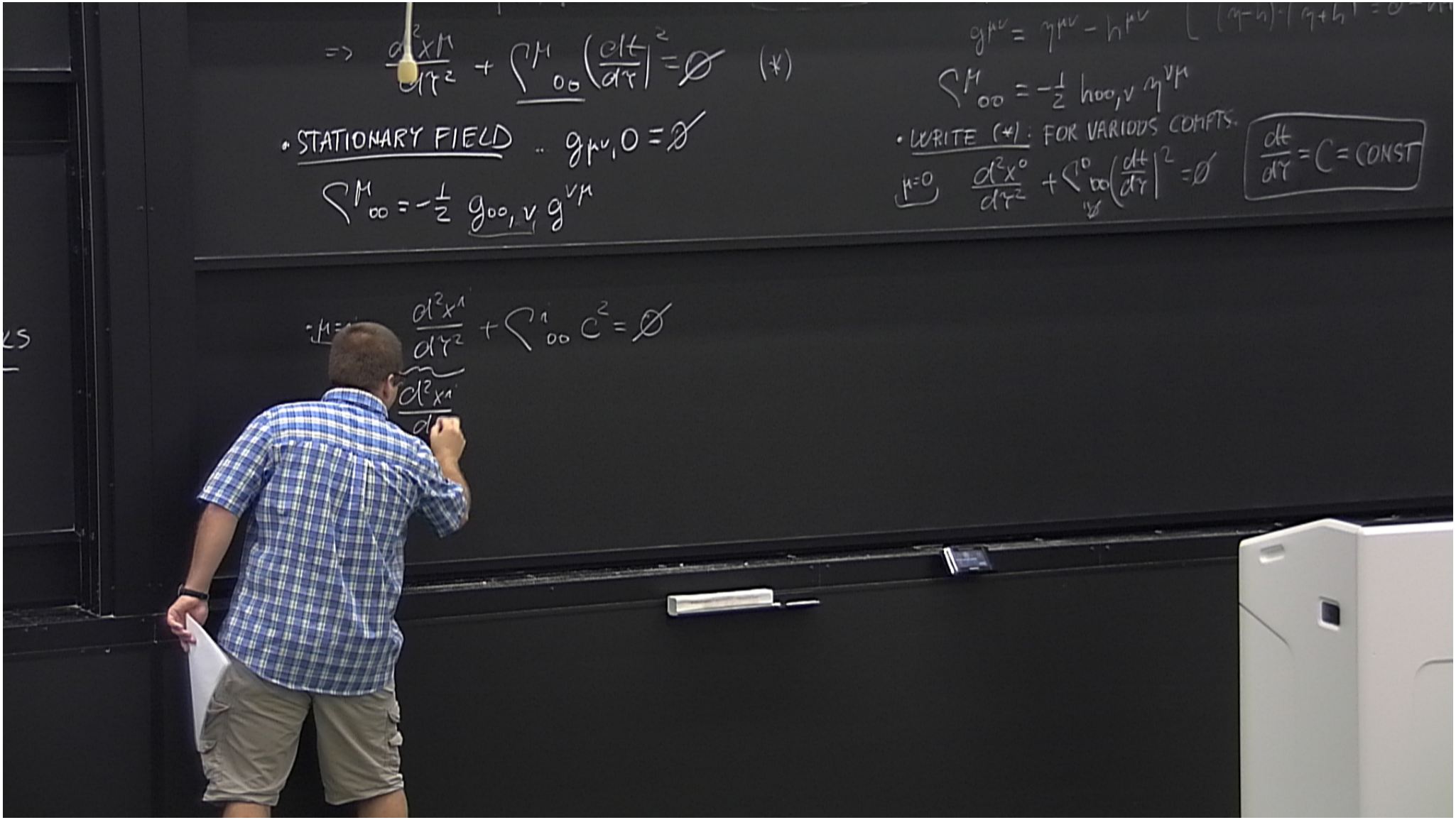

c) NEWTONIAN LIMIT
• SLOW MOTION. $\left|\frac{dx^i}{d\tau}\right| \ll \frac{dt}{d\tau} \Leftrightarrow |v_i| \ll 1$
$\Rightarrow \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$
• STATIONARY FIELD .. $g_{\mu\nu,0} = 0$
$\Gamma^\mu_{00} = -\frac{1}{2} g_{00,\nu} g^{\nu\mu}$
• WEAK FIELD
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$
$g^{\mu\nu} = \eta$

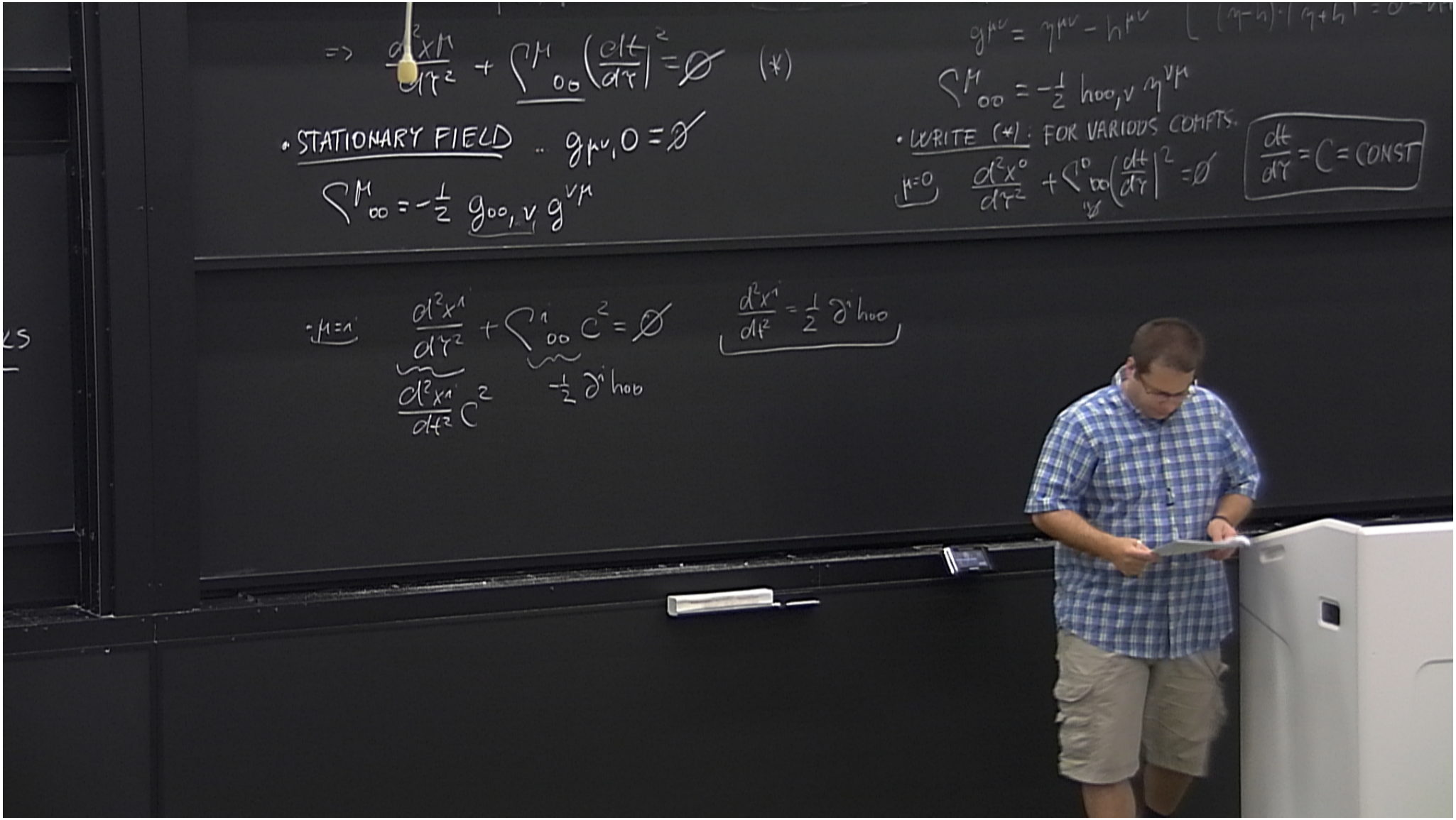

NEWTONIAN LIMIT
• SLOW MOTION. $\left|\frac{dx^i}{d\tau}\right| \ll \left|\frac{dt}{d\tau}\right| \Leftrightarrow |v_i| \ll 1$
$\Rightarrow \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$
• STATIONARY FIELD $g_{\mu\nu,0} = 0$
$\Gamma^\mu_{00} = -\frac{1}{2} g_{00,\nu} g^{\nu\mu}$
• WEAK FIELD
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$
$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$ [$(\eta - h)\cdot(\eta + h) =$

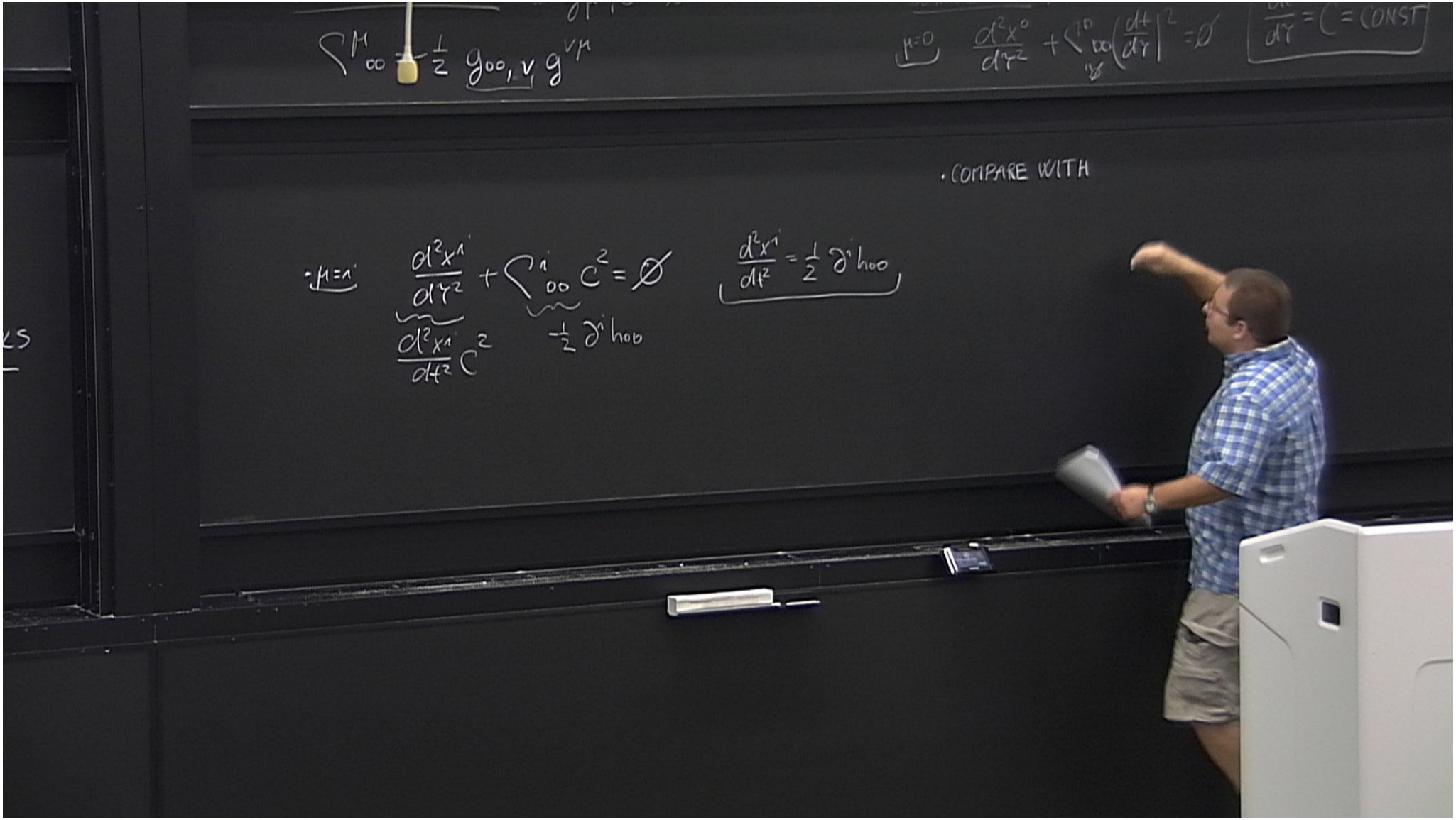

GEODES. EQ.
CHRISTOFFEL SYMBOLS
GAUGE THEORY
METRIC IS NOT PHYSICAL
RIEMANN TENSOR GOVERNS PHYSICAL EFFECTS.
GRAVITY = GEOMETRY
c) NEWTONIAN LIMIT
SLOW MOTION.
STATIONARY FIELD
WEAK FIELD

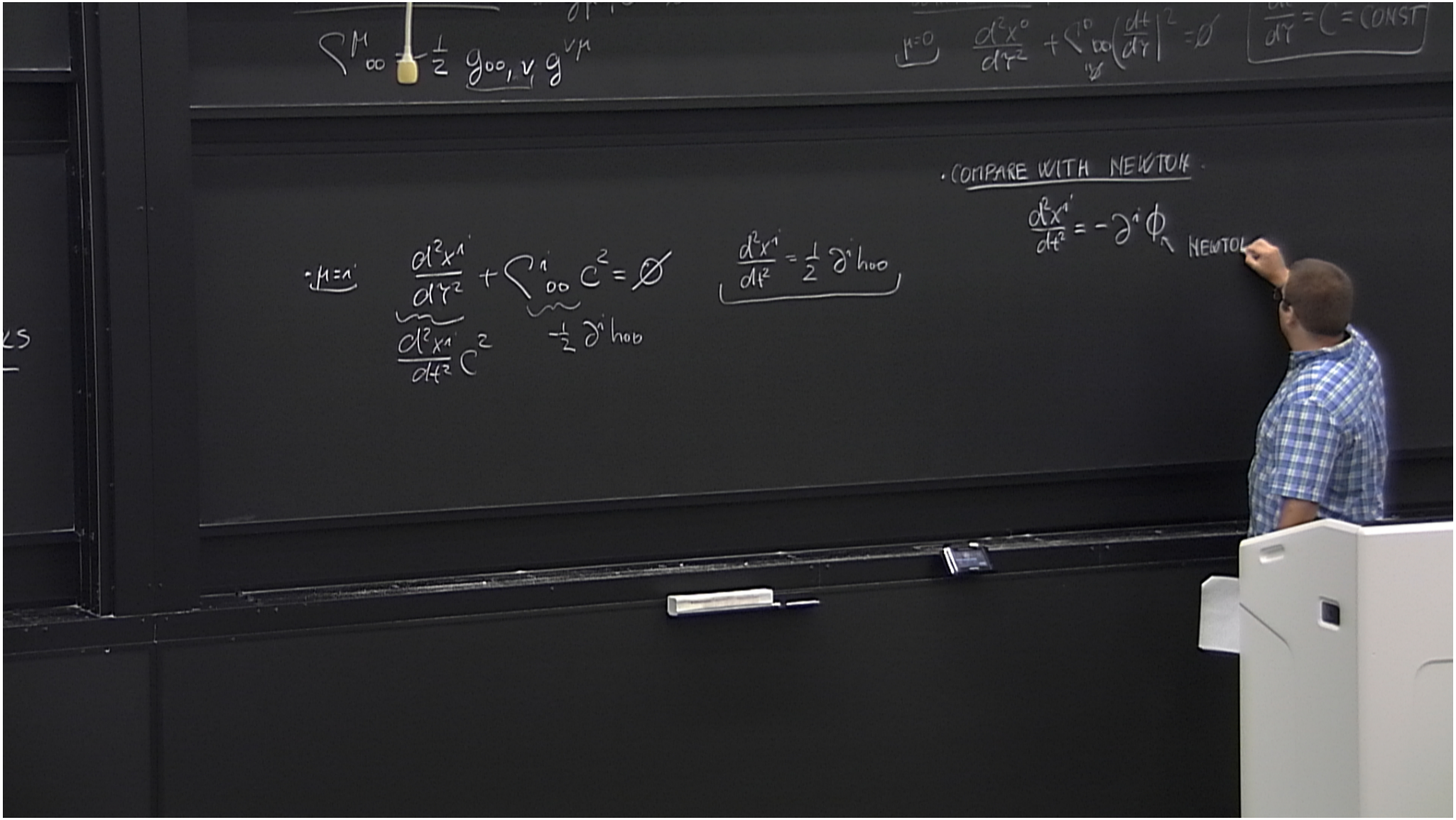

• SLOW MOTION. $\left|\frac{dx^i}{d\tau}\right| \ll \frac{dt}{d\tau} \Leftrightarrow |v_i| \ll 1$
$\Rightarrow \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$ (*)
• STATIONARY FIELD .. $g_{\mu\nu,0} = 0$
$\Gamma^\mu_{00} = -\frac{1}{2} g_{00,\nu} g^{\nu\mu}$
• WEAK FIELD
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$
$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$ $[(\eta-h)\cdot(\eta+h) = \delta - h + h + O(h^2)$
$\Gamma^\mu_{00} = -\frac{1}{2} h_{00,\nu} \eta^{\nu\mu}$
• WRITE (*) FOR VARIOUS COMPTS.
$\mu=0$ $\frac{d^2x^0}{d\tau^2}$

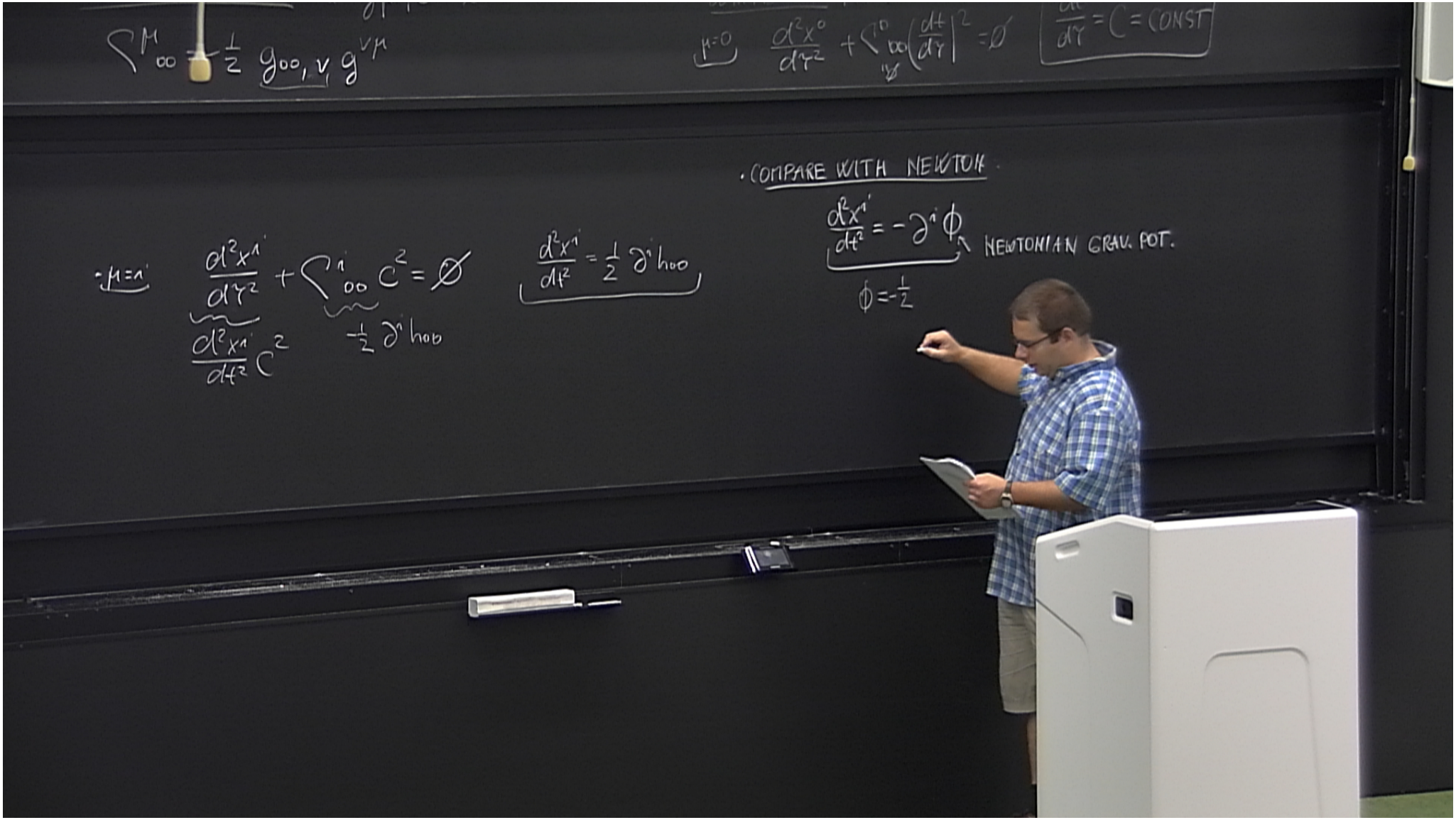

• SLOW MOTION. $\left|\frac{dx^i}{d\tau}\right| \ll \frac{dt}{d\tau} \Leftrightarrow |v_i| \ll 1$
$\Rightarrow \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$ (*)
• STATIONARY FIELD .. $g_{\mu\nu,0} = 0$
$\Gamma^\mu_{00} = -\frac{1}{2} g_{00,\nu} g^{\nu\mu}$
• WEAK FIELD
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$
$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$ $[(\eta-h)\cdot(\eta+h) = \delta - h + h + O(h^2)$
$\Gamma^\mu_{00} = -\frac{1}{2} h_{00,\nu} \eta^{\nu\mu}$
• WRITE (*): FOR VARIOUS COMPTS.
$\mu=0$ $\frac{d^2x^0}{d\tau^2} + \Gamma^0_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$

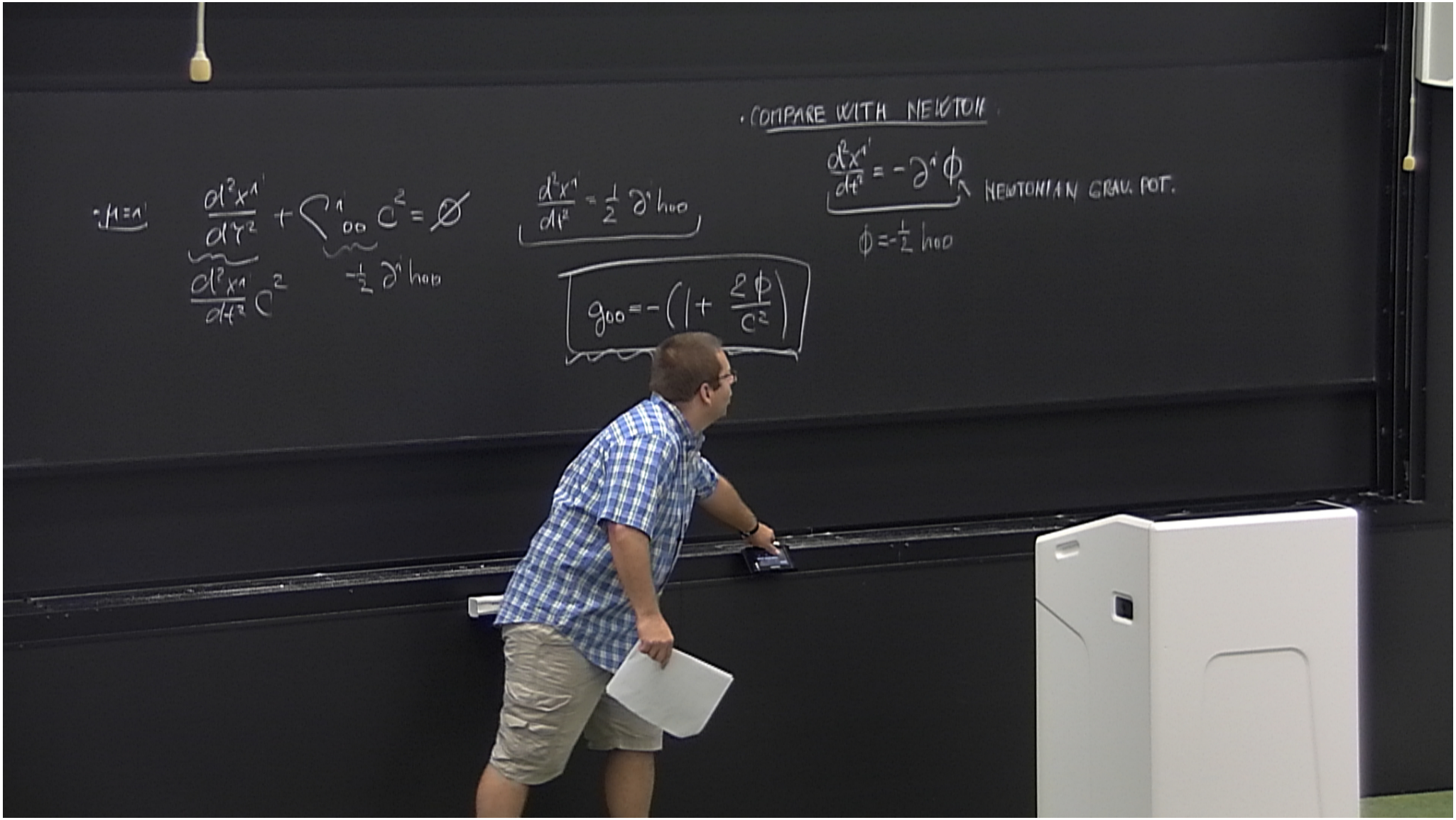

• STATIONARY FIELD
• WRITE (*): FOR VARIOUS COMPTS.
dt/dτ = C = CONST

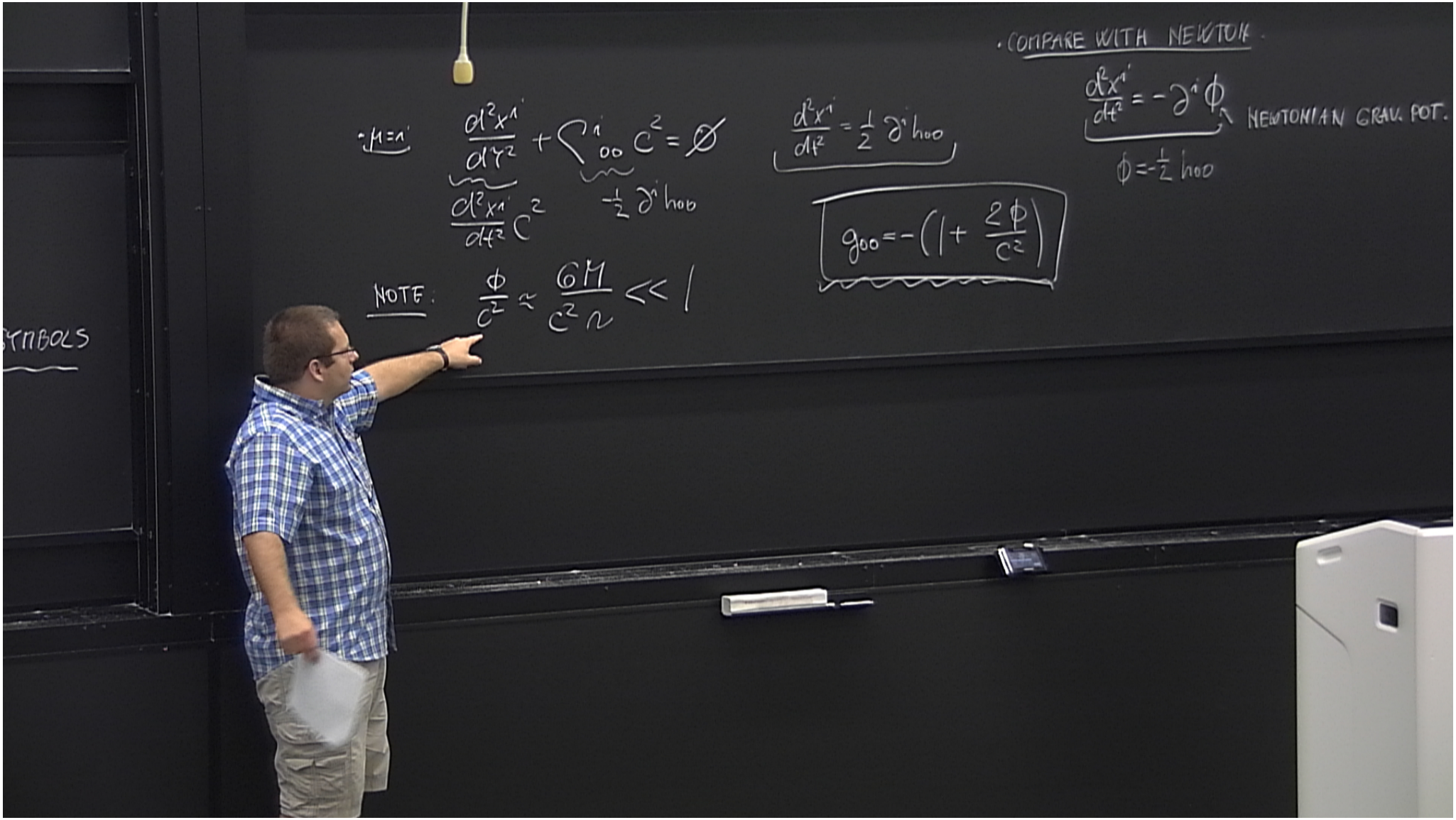

$\Rightarrow \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$ (*)
• STATIONARY FIELD .. $g_{\mu\nu,0} = 0$
$\Gamma^\mu_{00} = -\frac{1}{2} g_{00,\nu} g^{\nu\mu}$
$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$
$\Gamma^\mu_{00} = -\frac{1}{2} h_{00,\nu} \eta^{\nu\mu}$
• WRITE (*): FOR VARIOUS COMPTS.
$\mu=0$ $\frac{d^2x^0}{d\tau^2} + \Gamma^0_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$
$\frac{dt}{d\tau} = C = \text{CONST}$
$\mu=i$ $\frac{d^2x^i}{d\tau^2} + \Gamma^i_{00} C^2 = 0$
$\frac{d^2x^i}{dt^2} C^2$ $-\frac{1}{2}\partial^i h_{00}$
$\frac{d^2x^i}{dt^2} = \frac{1}{2}\partial^i h_{00}$

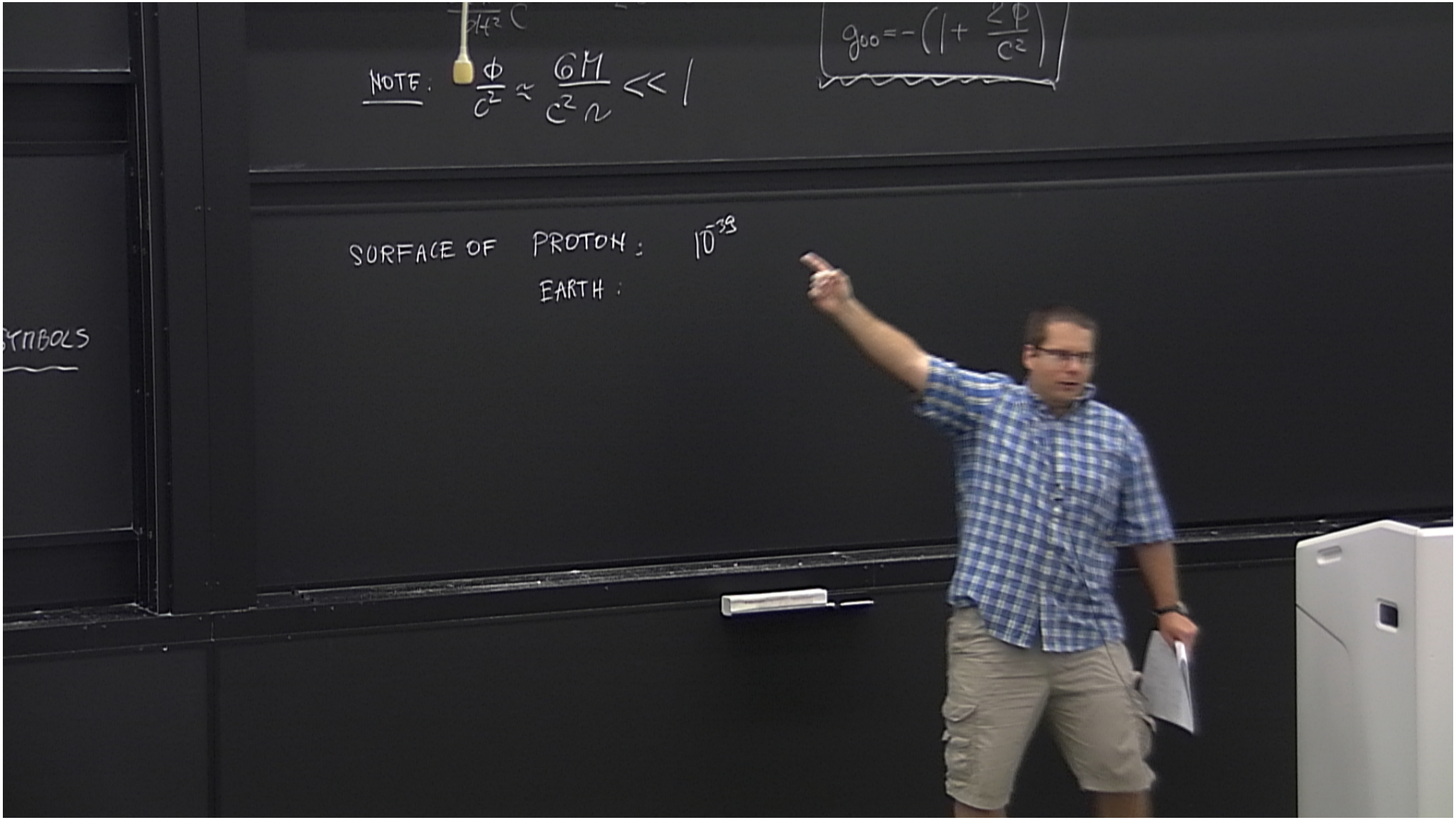

$\Gamma^{\mu}_{00} = -\frac{1}{2} g_{00,\nu} g^{\nu\mu}$
$\mu=0$ $\frac{d^2x^0}{d\tau^2} + \Gamma^{0}_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$
$\frac{dt}{d\tau} = C = \text{CONST}$
• COMPARE WITH
$\mu=i$ $\frac{d^2x^i}{d\tau^2} + \Gamma^{i}_{00} C^2 = 0$
$\frac{d^2x^i}{dt^2} C^2$ $-\frac{1}{2}\partial^i h_{00}$
$\frac{d^2x^i}{dt^2} = \frac{1}{2}\partial^i h_{00}$

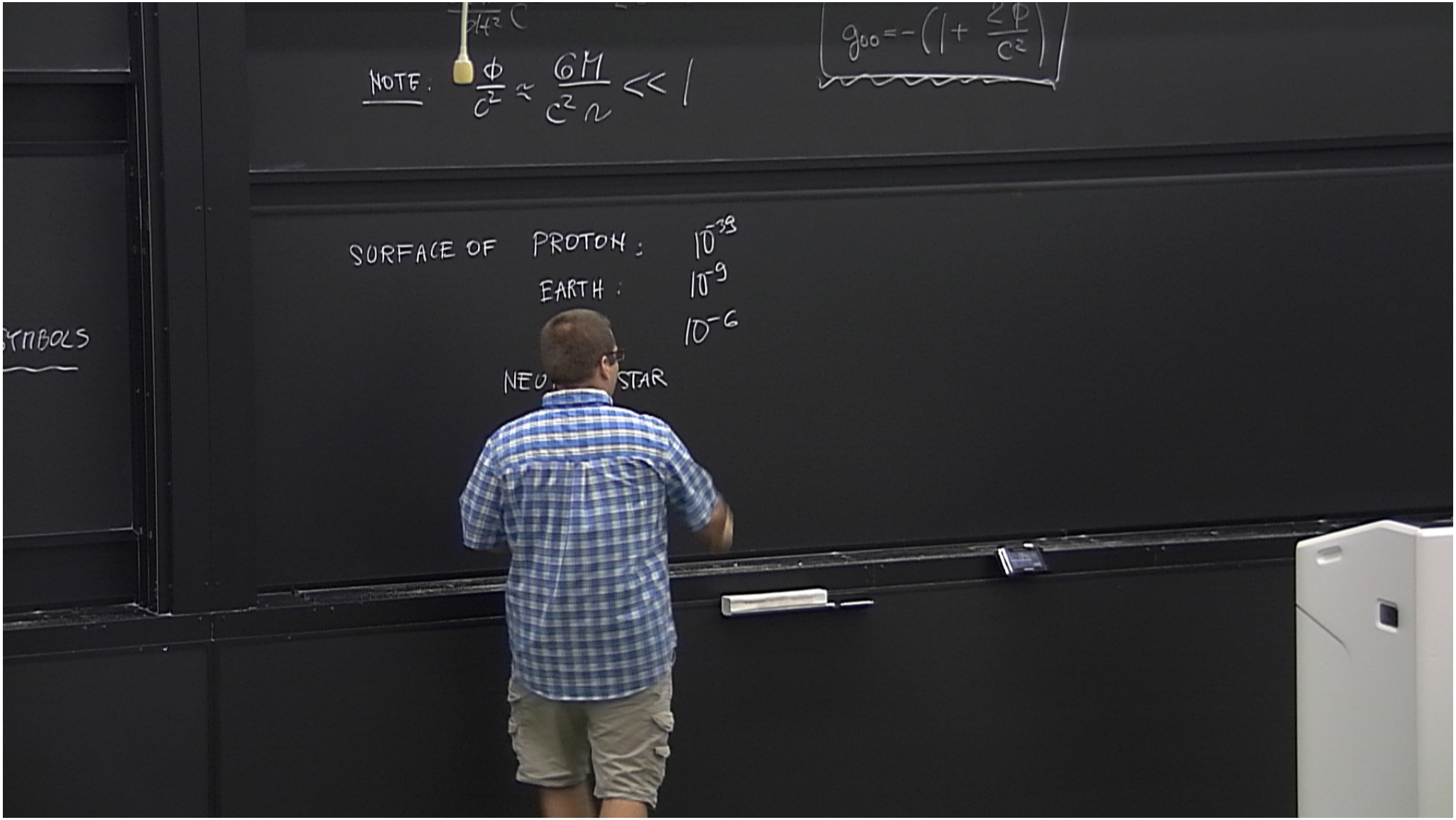

$\Gamma^{\mu}_{00} = -\frac{1}{2} g_{00,\nu} g^{\nu\mu}$
$\mu=0$ $\frac{d^2x^0}{d\tau^2} + \Gamma^0_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$
$\frac{dt}{d\tau} = C = \text{CONST}$
• COMPARE WITH NEWTON
$\frac{d^2x^i}{dt^2} = -\partial^i \phi$ NEWTO
$\mu = i$ $\frac{d^2x^i}{d\tau^2} + \Gamma^i_{00} C^2 = 0$
$\frac{d^2x^i}{dt^2} C^2$ $-\frac{1}{2}\partial^i h_{00}$
$\frac{d^2x^i}{dt^2} = \frac{1}{2}\partial^i h_{00}$

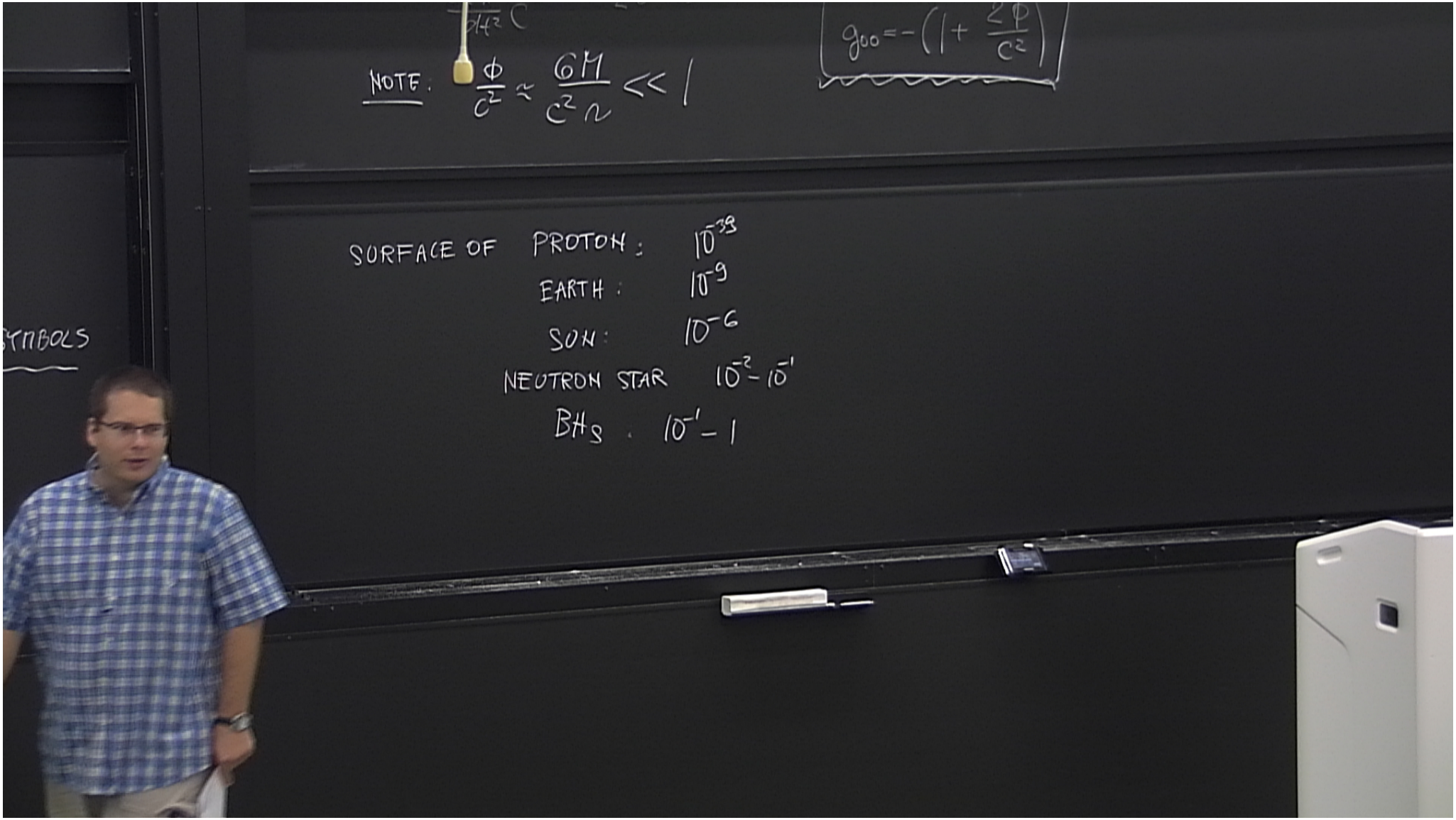

$\Gamma^{\mu}_{00} = -\frac{1}{2} g_{00,\nu} g^{\nu\mu}$
$\mu=0$ $\frac{d^2x^0}{d\tau^2} + \Gamma^0_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$
$\frac{dt}{d\tau} = C = \text{CONST}$
• COMPARE WITH NEWTON
$\mu=i$ $\frac{d^2x^i}{dt^2} + \Gamma^i_{00} C^2 = 0$
$\frac{d^2x^i}{dt^2} C^2$ $-\frac{1}{2}\partial^i h_{00}$
$\frac{d^2x^i}{dt^2} = \frac{1}{2}\partial^i h_{00}$
$\frac{d^2x^i}{dt^2} = -\partial^i \phi$ NEWTONIAN GRAV. POT.
$\phi = -\frac{1}{2}$

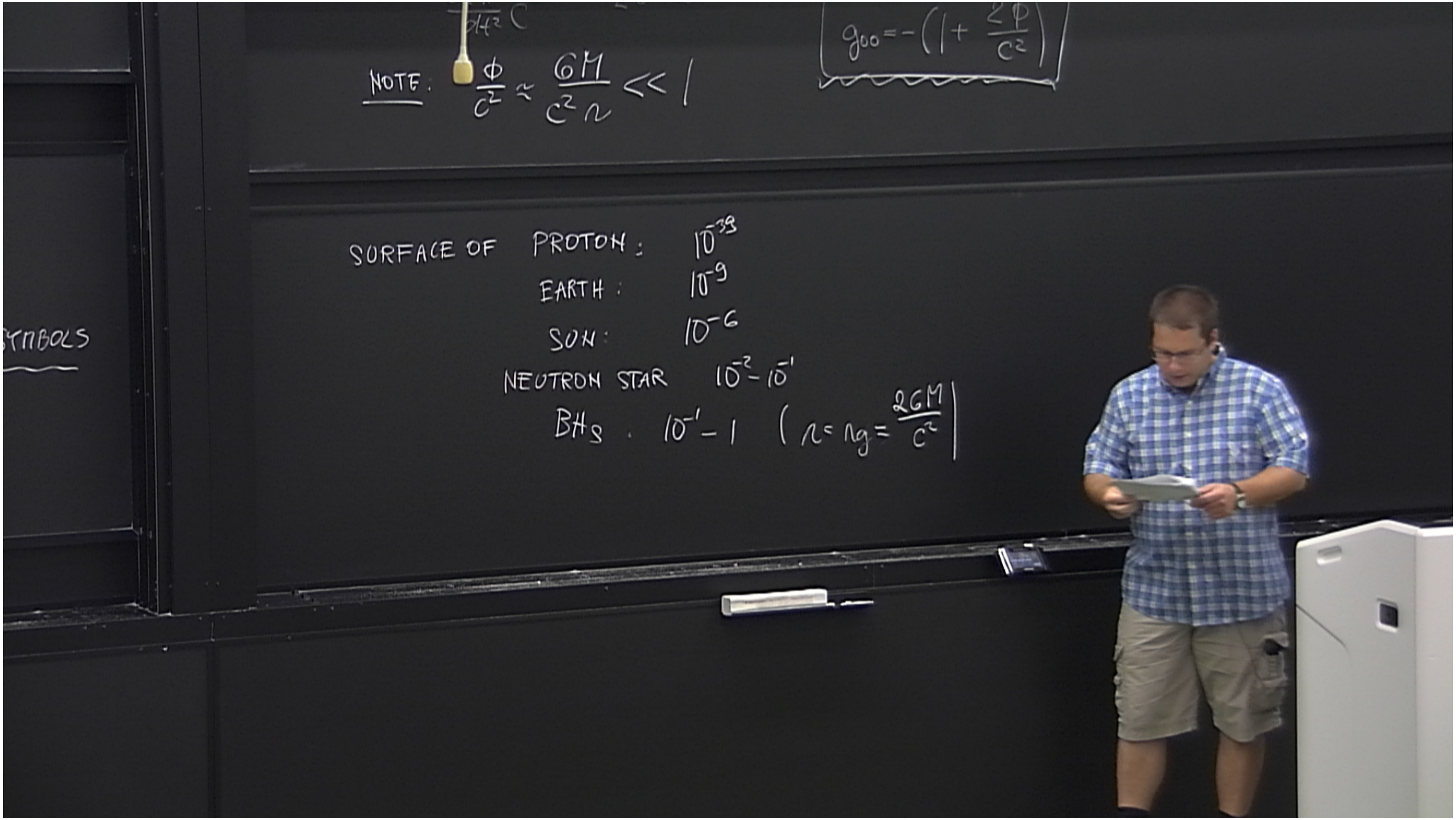

· COMPARE WITH NEWTON
$\frac{d^2x^i}{dt^2} = -\partial^i\phi$
NEWTONIAN GRAV. POT.
$\phi = -\frac{1}{2}h_{00}$
$\mu = i$
$\frac{d^2x^i}{dt^2} + \Gamma^i_{00}c^2 = 0$
$\frac{d^2x^i}{dt^2}c^2$
$-\frac{1}{2}\partial^i h_{00}$
$\frac{d^2x^i}{dt^2} = \frac{1}{2}\partial^i h_{00}$
$g_{00} = -\left(1 + \frac{2\phi}{c^2}\right)$

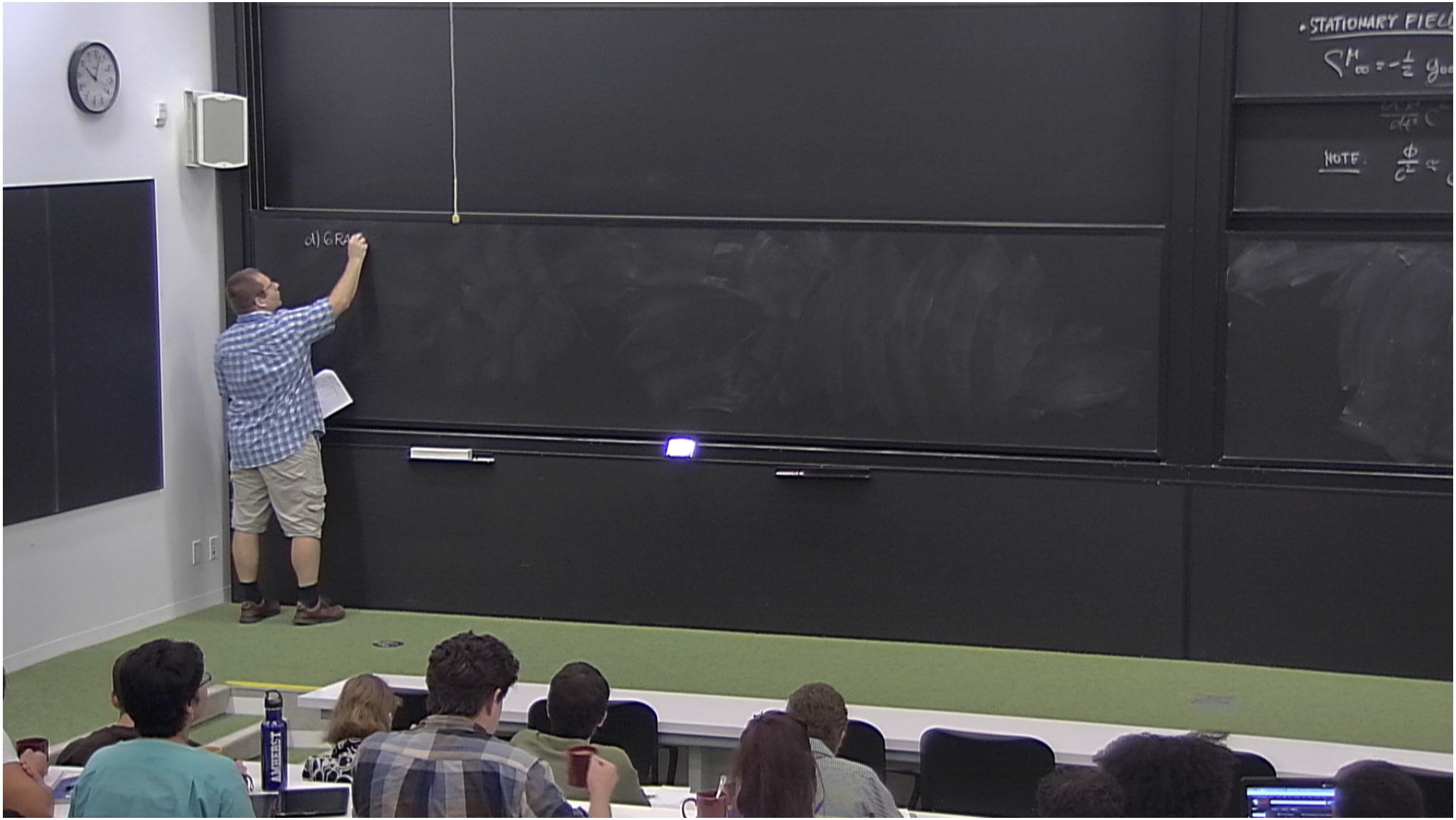

· COMPARE WITH NEWTON
$\frac{d^2x^i}{dt^2} = -\partial^i\phi$
NEWTONIAN GRAV. POT.
$\phi = -\frac{1}{2}h_{00}$
$\mu = i$
$\frac{d^2x^i}{dt^2} + \Gamma^i_{00}c^2 = 0$
$\frac{d^2x^i}{dt^2}c^2$
$-\frac{1}{2}\partial^i h_{00}$
$\frac{d^2x^i}{dt^2} = \frac{1}{2}\partial^i h_{00}$
$g_{00} = -\left(1 + \frac{2\phi}{c^2}\right)$
NOTE:
$\frac{\phi}{c^2} \approx \frac{GM}{c^2 r} \ll 1$
SYMBOLS

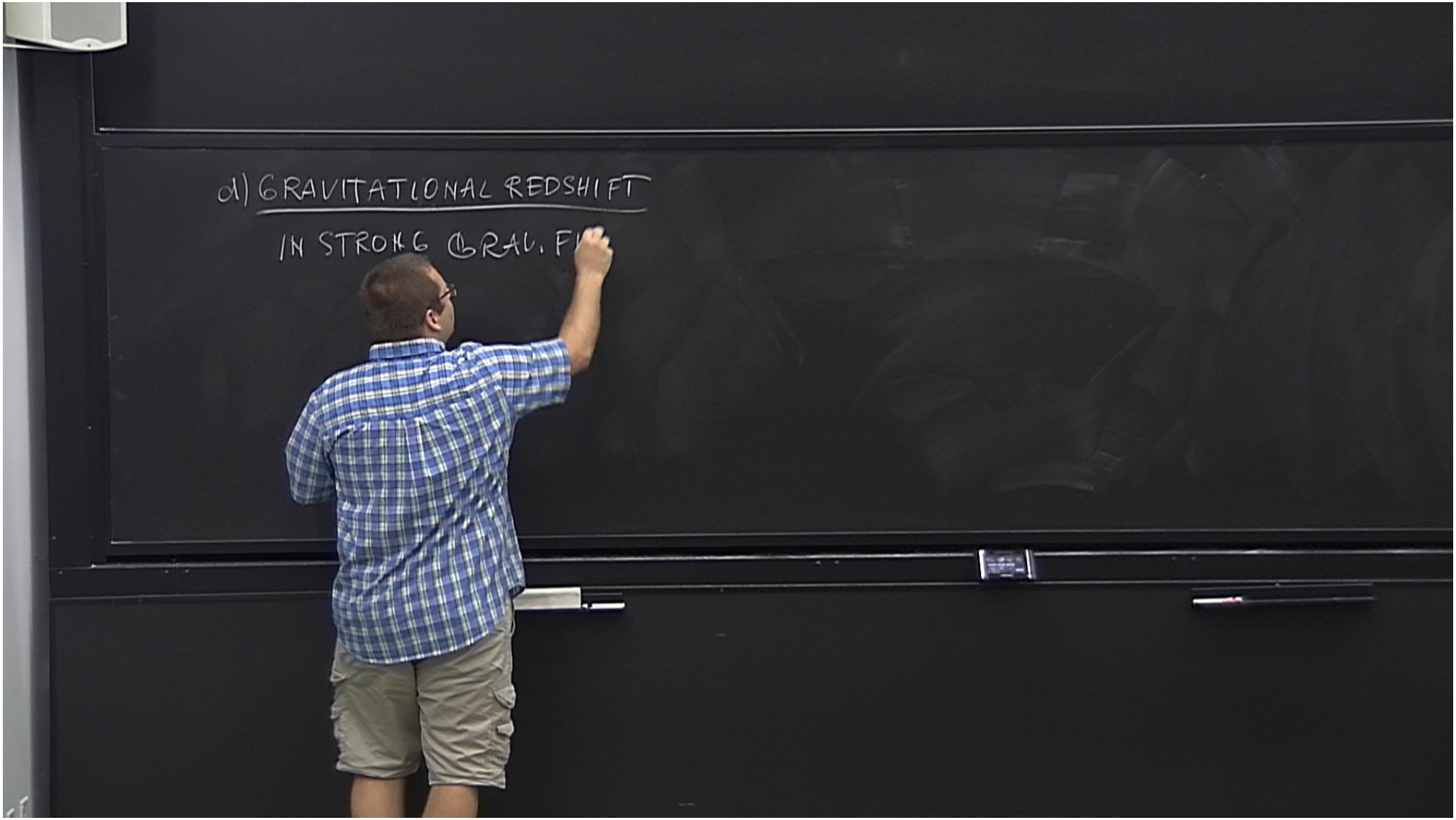

NOTE: $\frac{\Phi}{c^2} \simeq \frac{GM}{c^2 r} \ll 1$
$g_{00} = -\left(1 + \frac{2\Phi}{c^2}\right)$
SURFACE OF PROTON: 10^{-38}
EARTH:
SYMBOLS

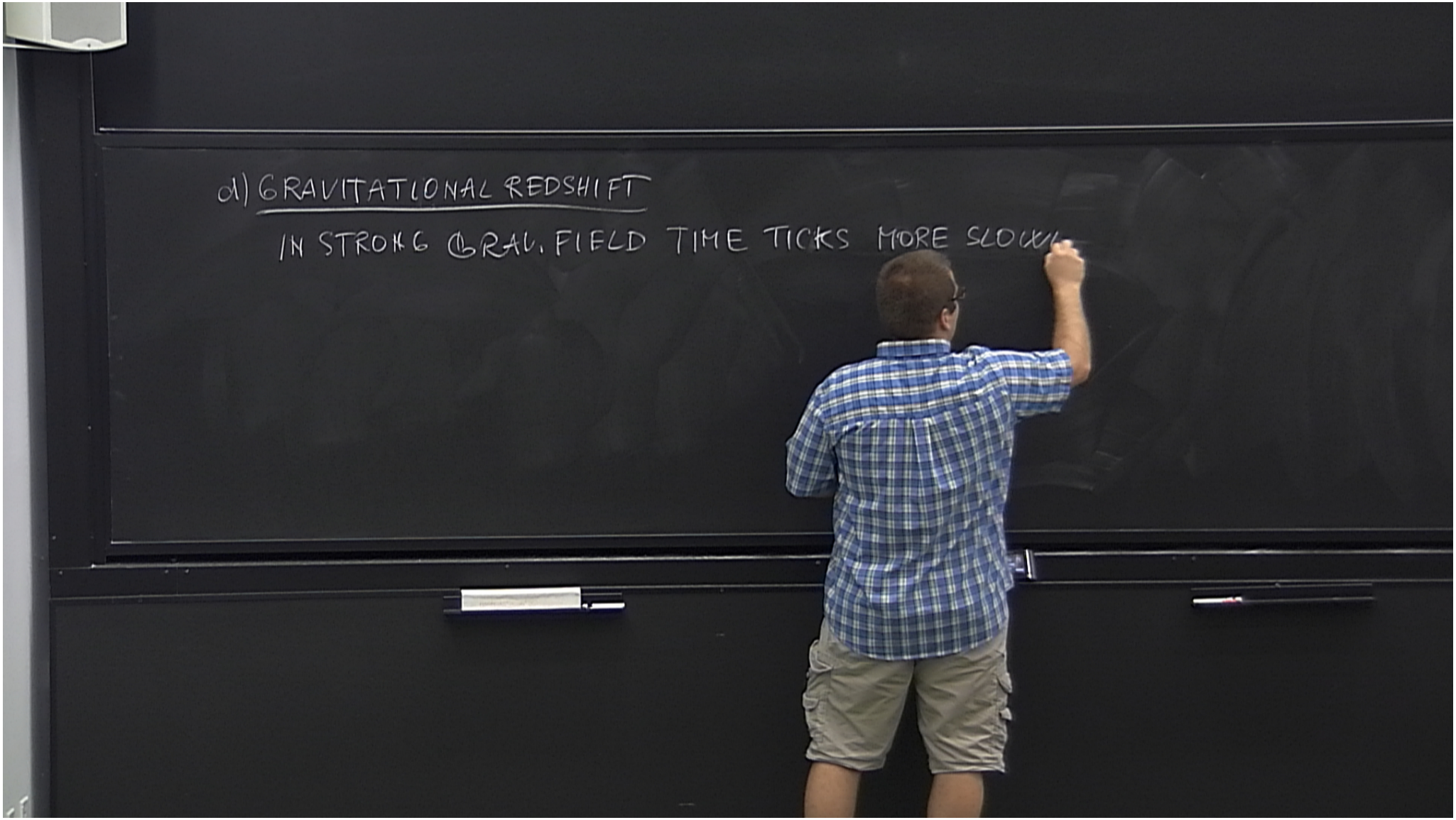

NOTE: $\frac{\Phi}{c^2} \simeq \frac{GM}{c^2 r} \ll 1$
$g_{00} = -\left(1 + \frac{2\Phi}{c^2}\right)$
SURFACE OF PROTON: 10^{-38}
EARTH: 10^{-9}
10^{-6}
NEU. STAR
SYMBOLS

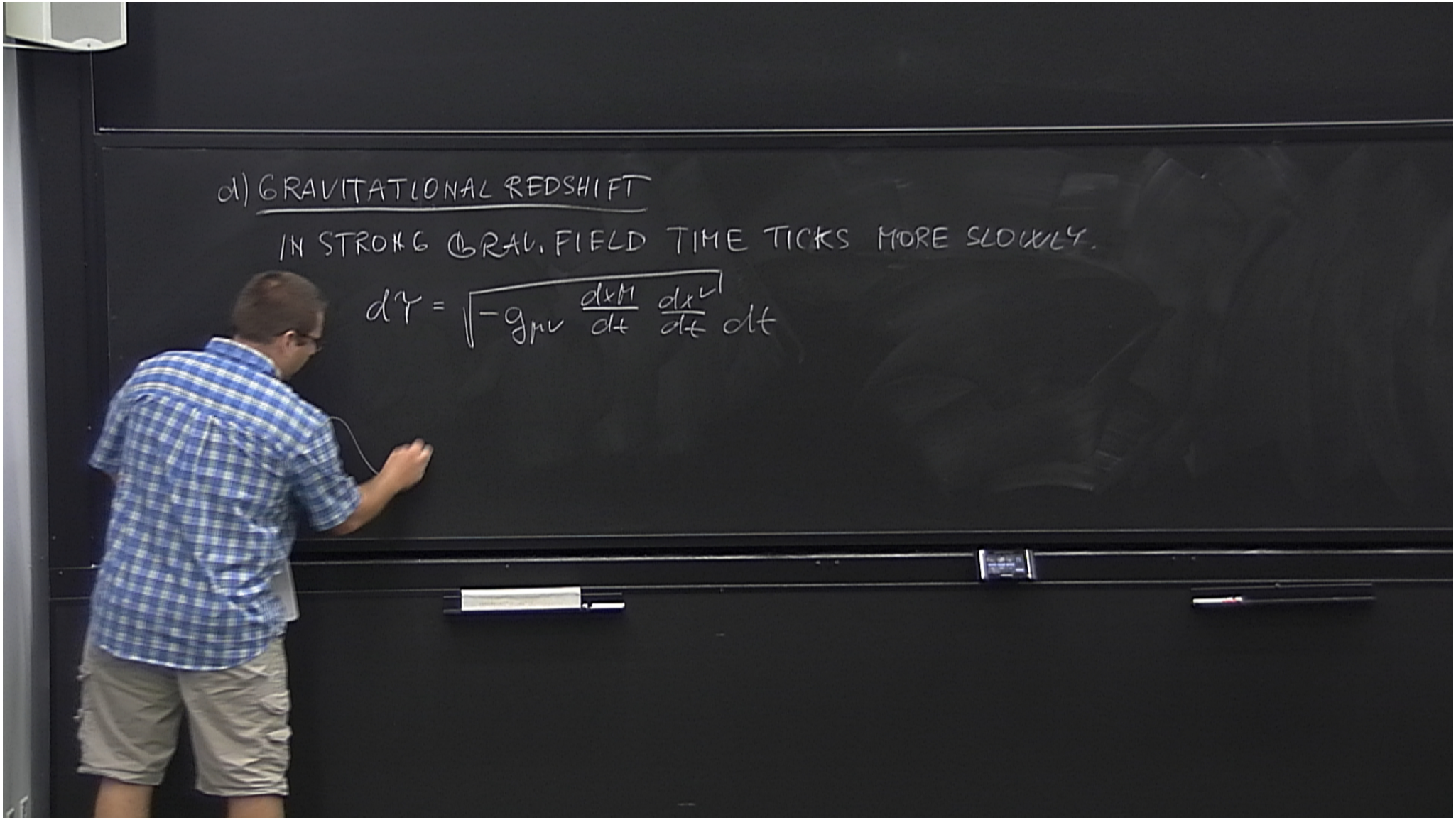

NOTE: $\frac{\Phi}{c^2} \simeq \frac{GM}{c^2 r} \ll 1$
$g_{00} = -\left(1 + \frac{2\Phi}{c^2}\right)$
SURFACE OF PROTON: 10^{-39}
EARTH: 10^{-9}
SUN: 10^{-6}
NEUTRON STAR $10^{-2} - 10^{-1}$
BH_S . $10^{-1} - 1$
SYMBOLS

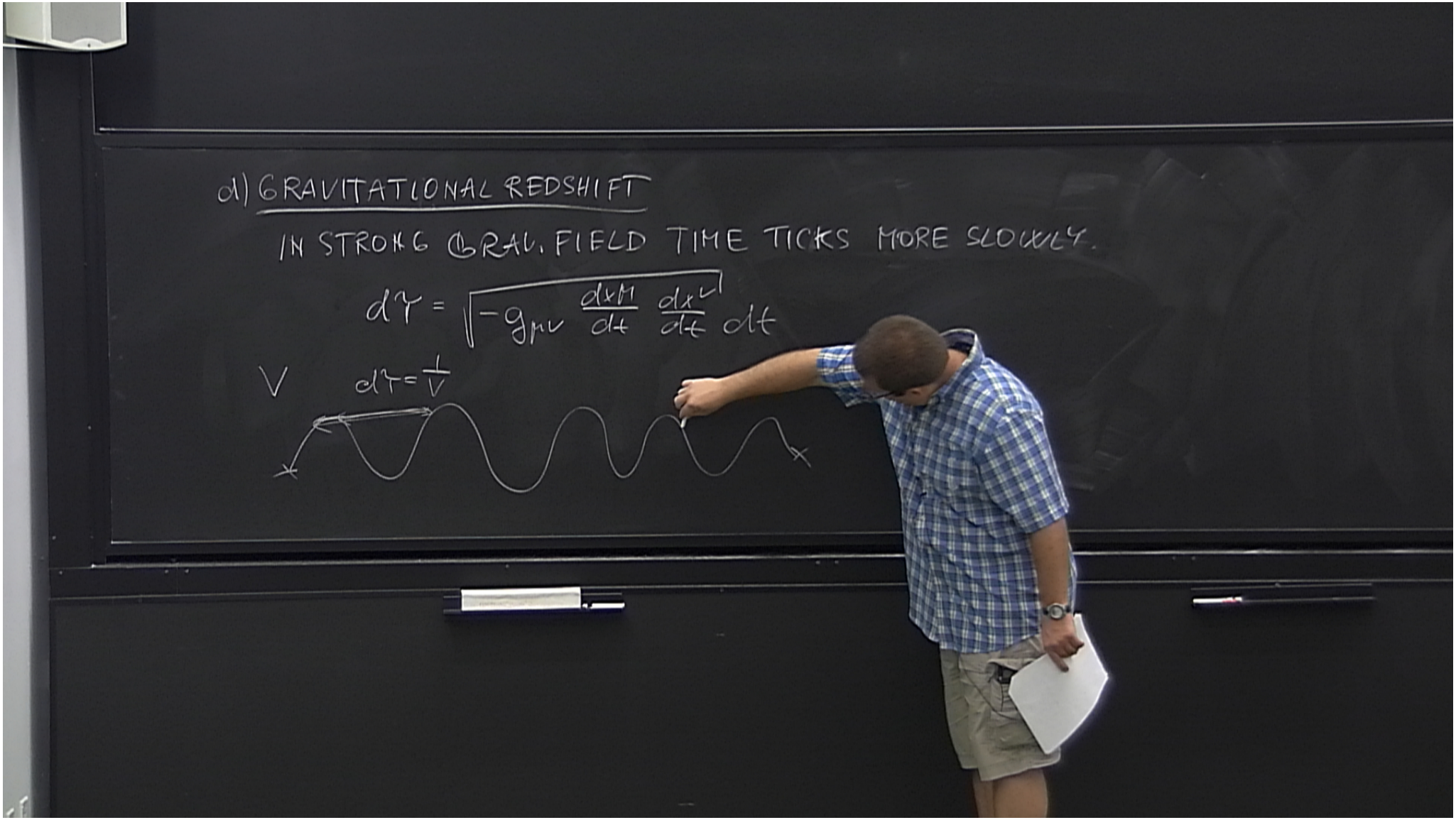

NOTE: $\frac{\Phi}{c^2} \simeq \frac{GM}{c^2 r} \ll 1$
$g_{00} = -\left(1 + \frac{2\Phi}{c^2}\right)$
SURFACE OF PROTON: 10^{-39}
EARTH: 10^{-9}
SUN: 10^{-6}
NEUTRON STAR $10^{-2} - 10^{-1}$
BHs $10^{-1} - 1$ $\left(r = r_g = \frac{2GM}{c^2}\right)$
SYMBOLS

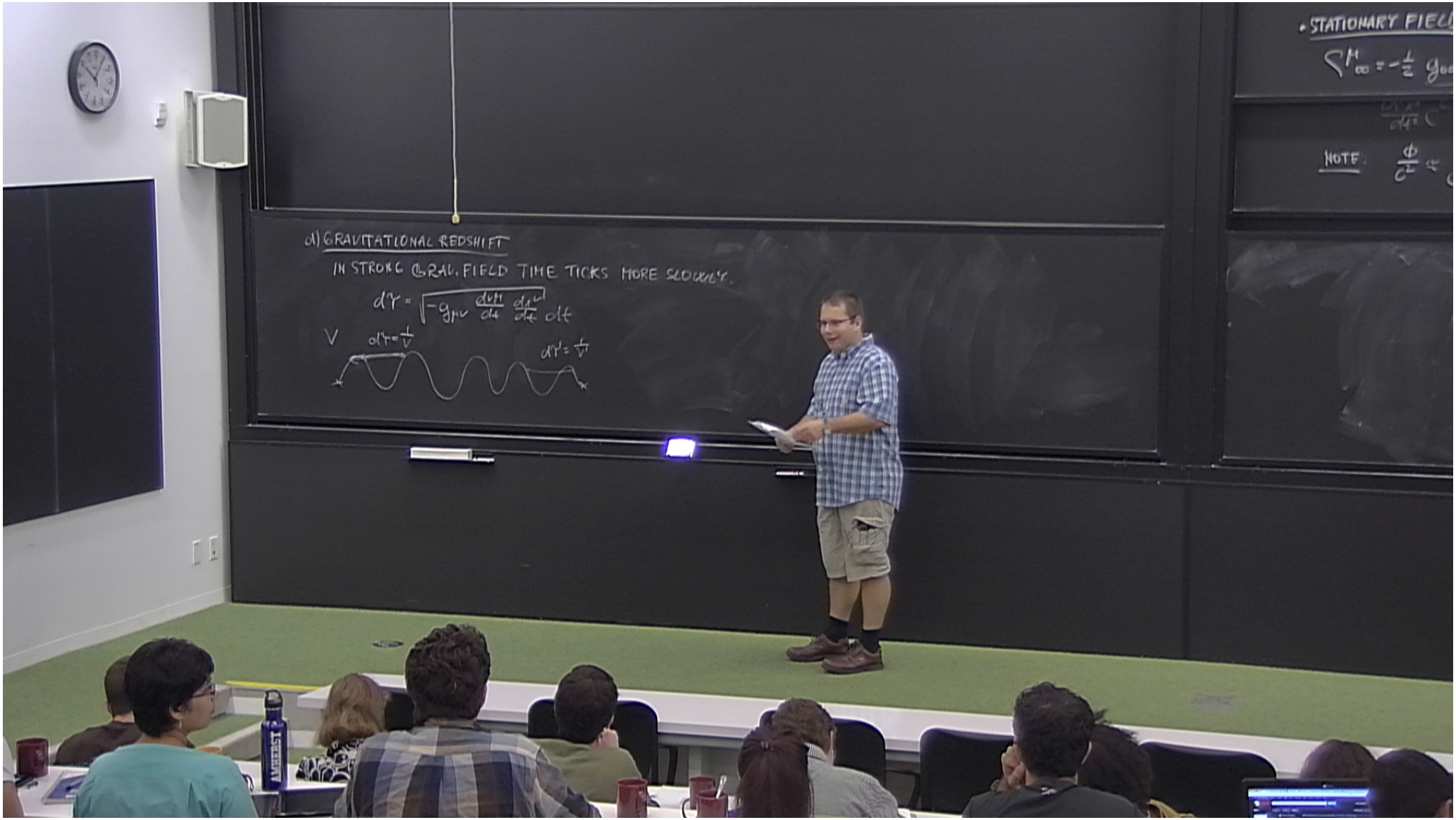

• STATIONARY FIEL
NOTE:

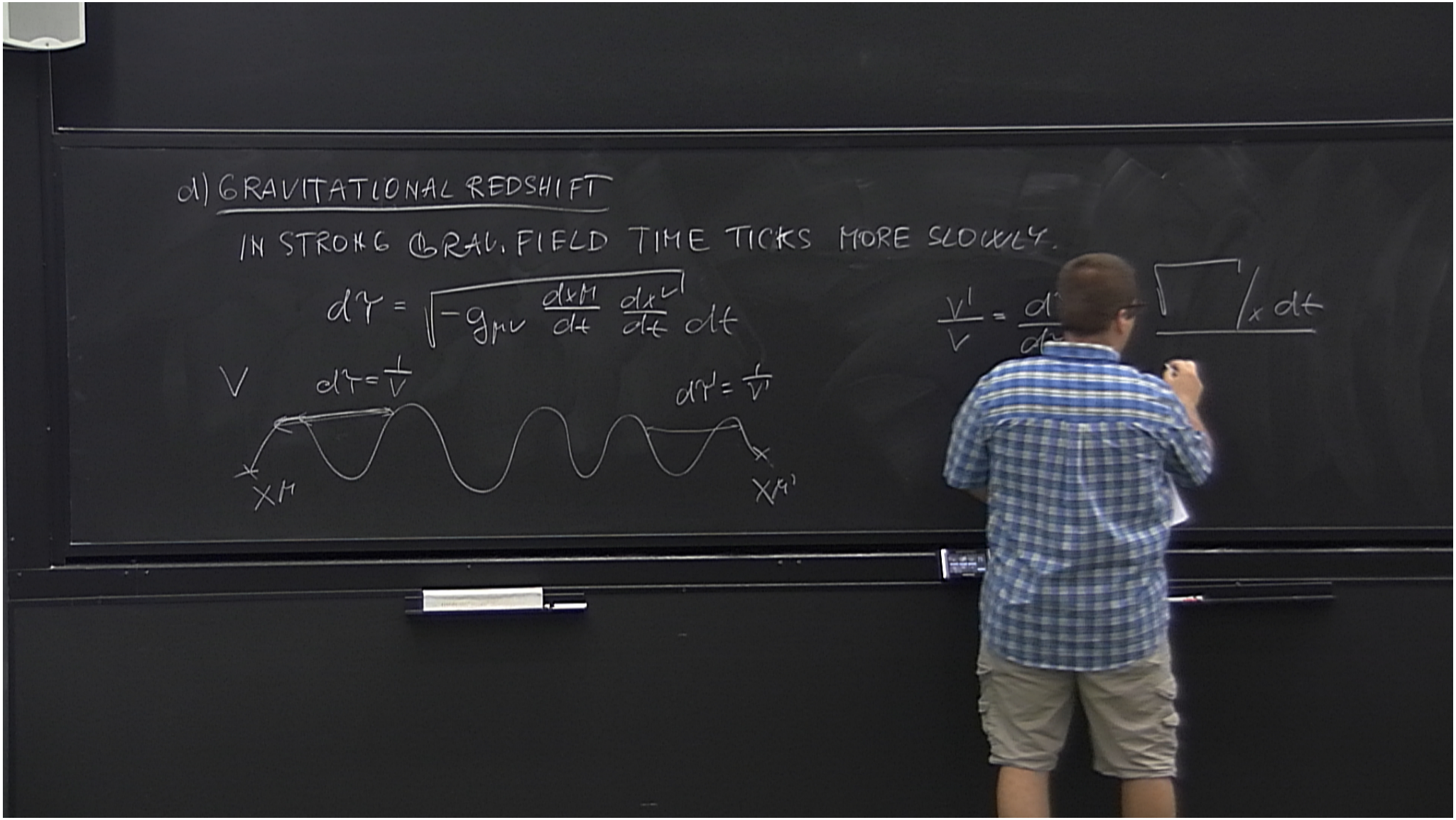

a) GRAVITATIONAL REDSHIFT
IN STRONG GRAV. FI

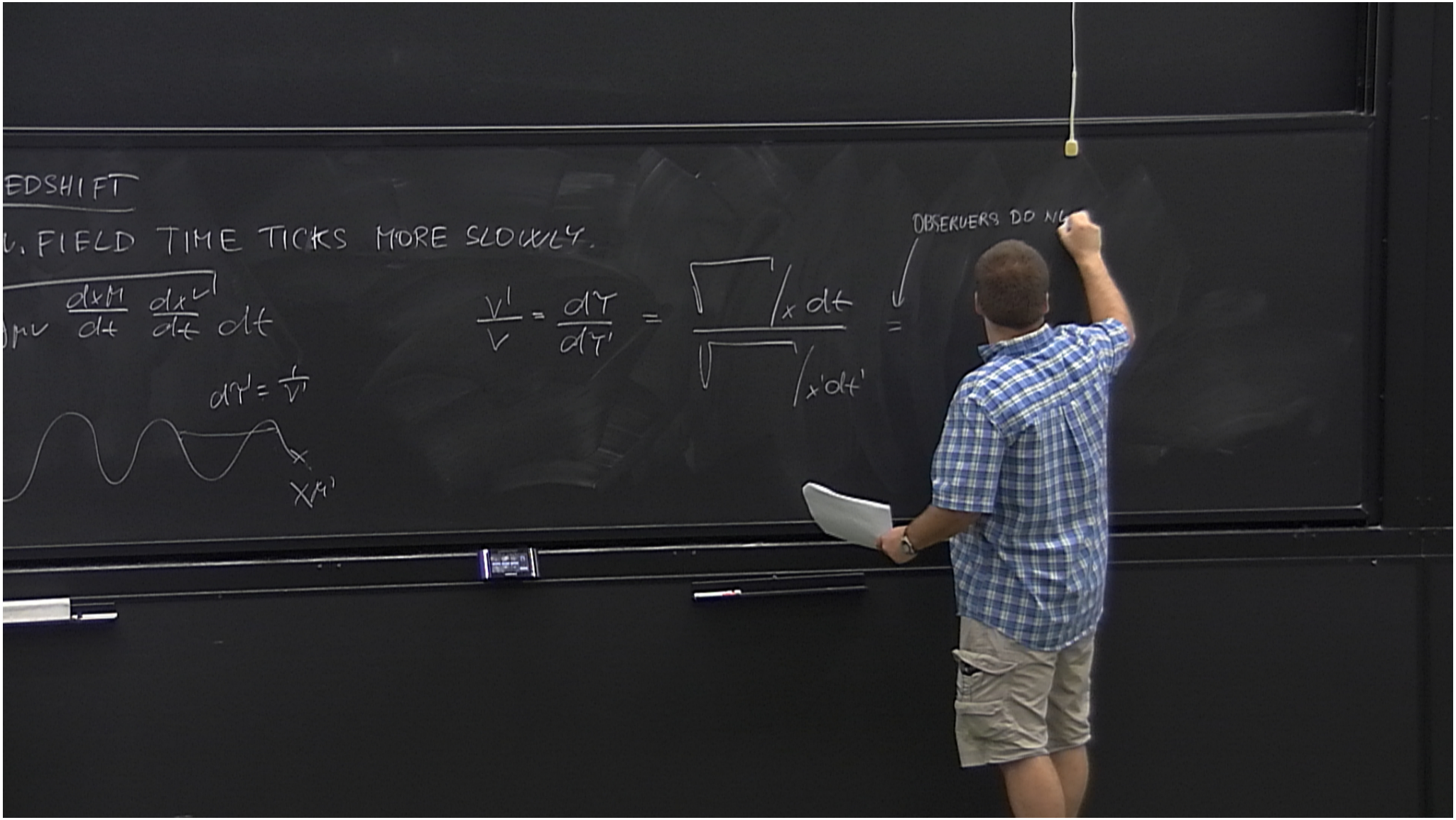

d) GRAVITATIONAL REDSHIFT
IN STRONG GRAV. FIELD TIME TICKS MORE SLOW

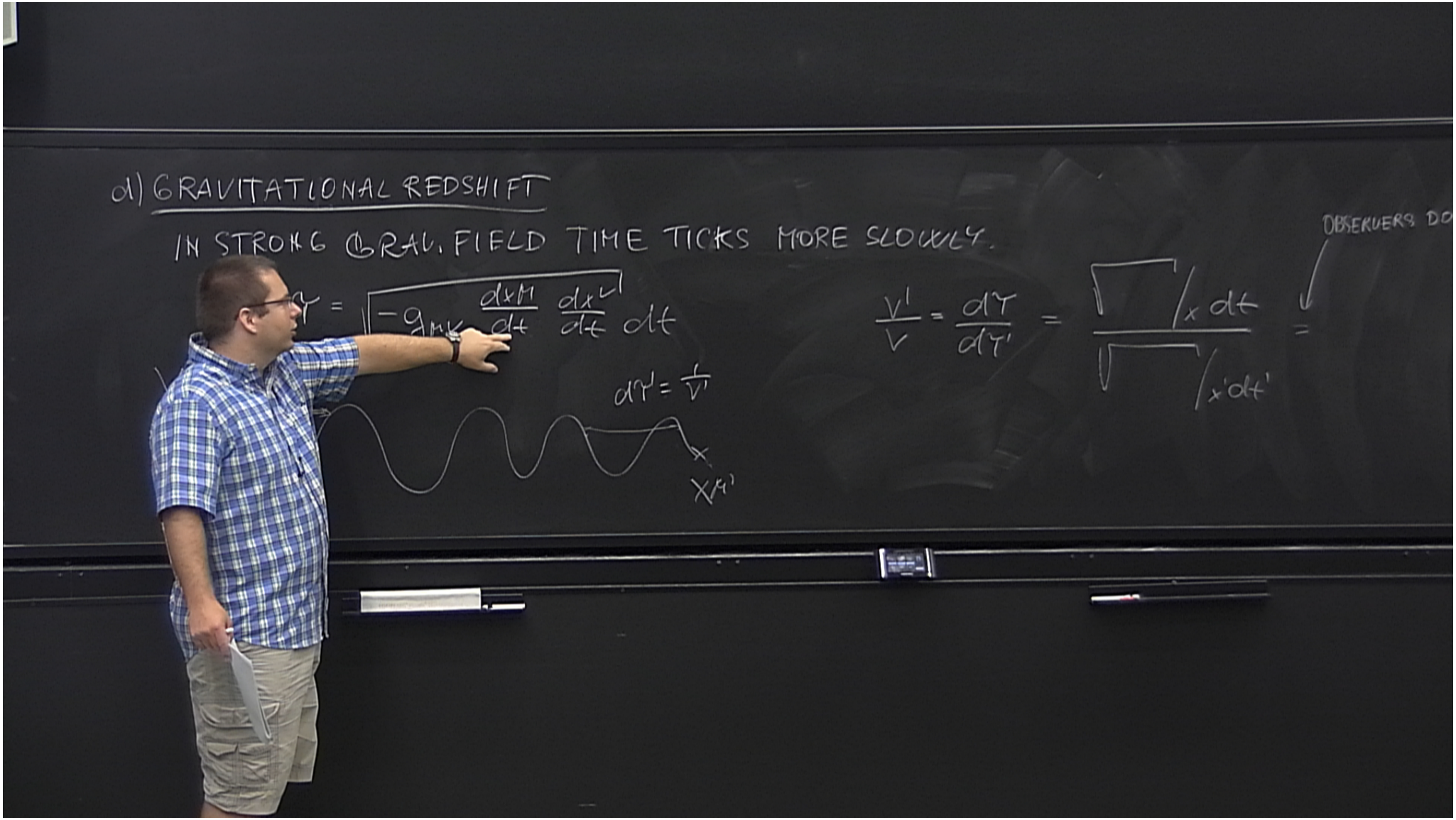

d) GRAVITATIONAL REDSHIFT
IN STRONG GRAV. FIELD TIME TICKS MORE SLOWLY.
$d\tau = \sqrt{-g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt}}\, dt$

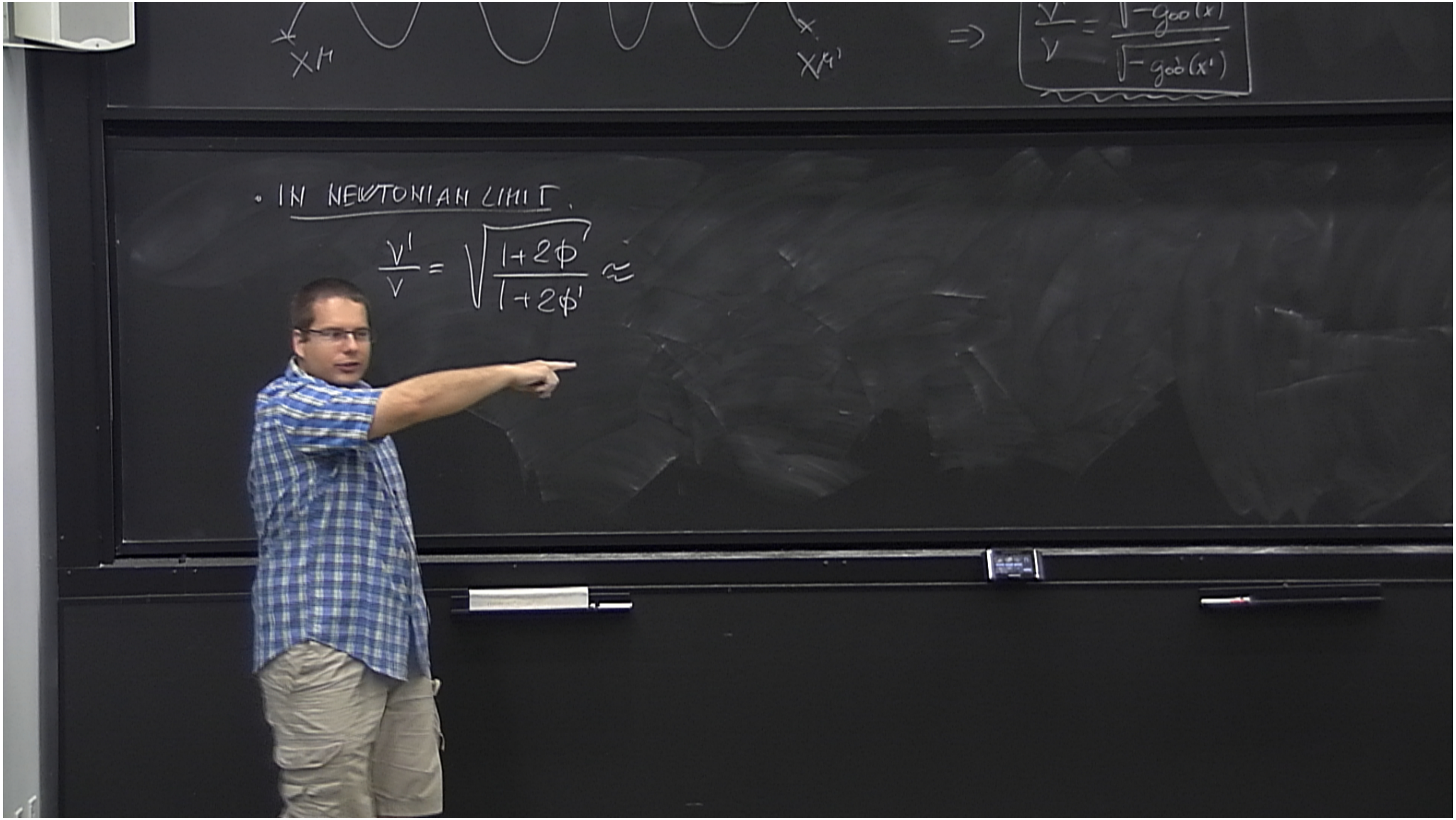

d) GRAVITATIONAL REDSHIFT
IN STRONG GRAV. FIELD TIME TICKS MORE SLOWLY.
$d\tau = \sqrt{-g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt}}\, dt$
V
$d\tau = \frac{1}{\nu}$

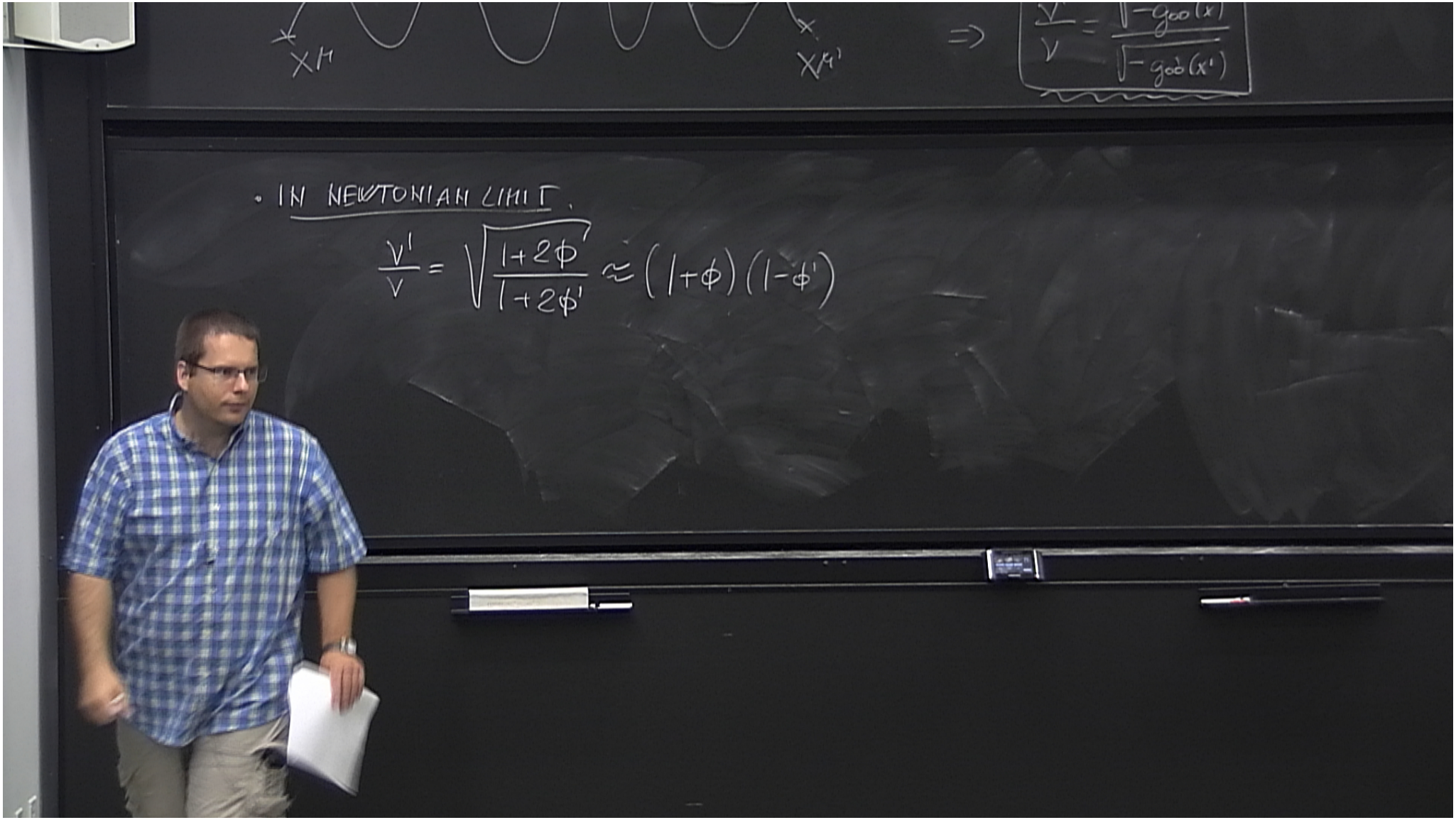

• STATIONARY FIELD
NOTE:
a) GRAVITATIONAL REDSHIFT
IN STRONG GRAV. FIELD TIME TICKS MORE SLOWLY.
$d\tau = \sqrt{-g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}\,dt$
$d\tau = \frac{1}{\nu}$
$d\tau = \frac{1}{\nu}$

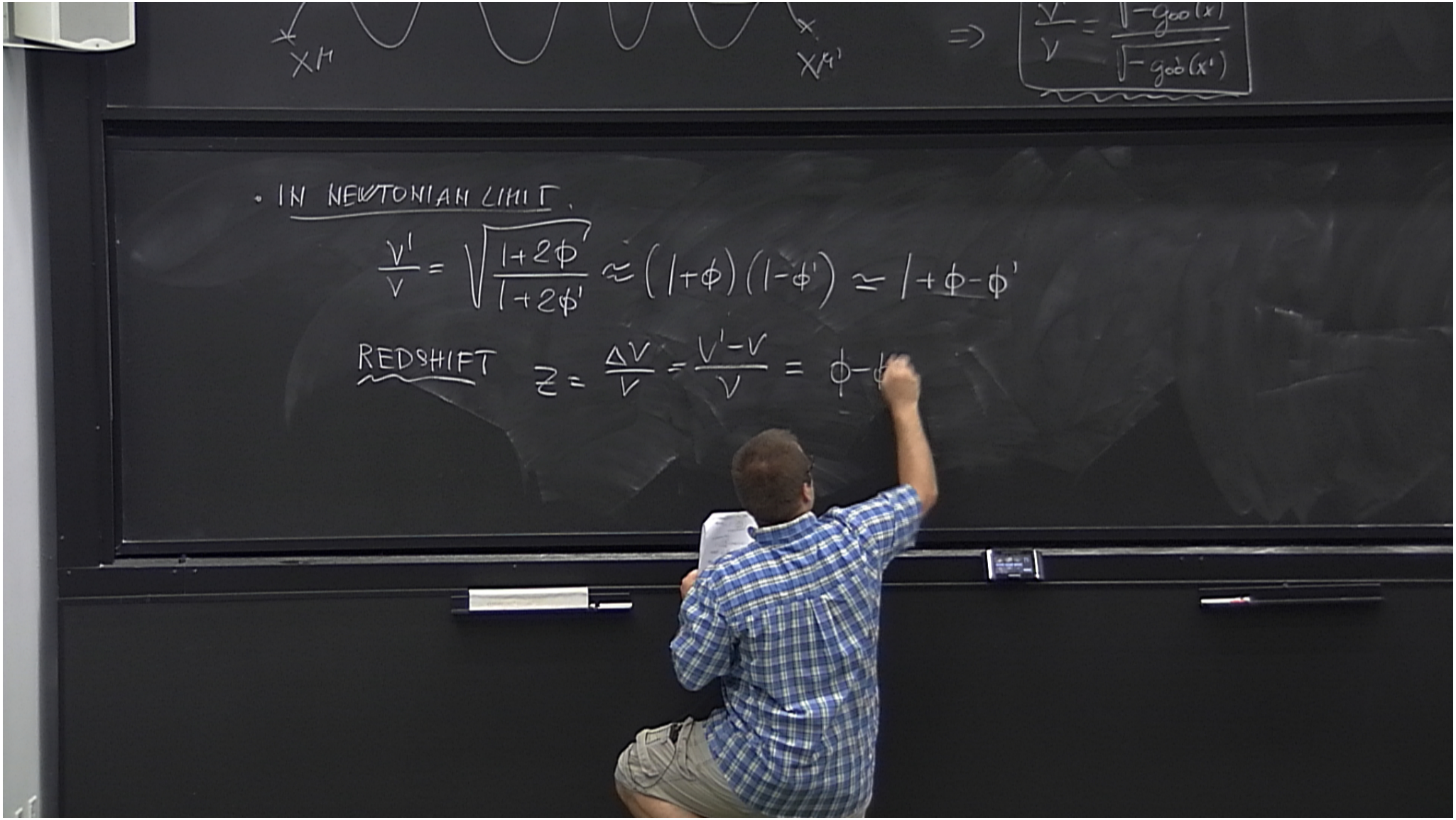

d) GRAVITATIONAL REDSHIFT
IN STRONG GRAV. FIELD TIME TICKS MORE SLOWLY.
$d\tau = \sqrt{-g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}\,dt$
$\frac{\nu'}{\nu} = \frac{d}{dt}$
$\sqrt{\quad}/_x\,dt$
V
$d\tau = \frac{1}{\nu}$
$d\tau' = \frac{1}{\nu'}$
x^μ
$x^{\mu'}$

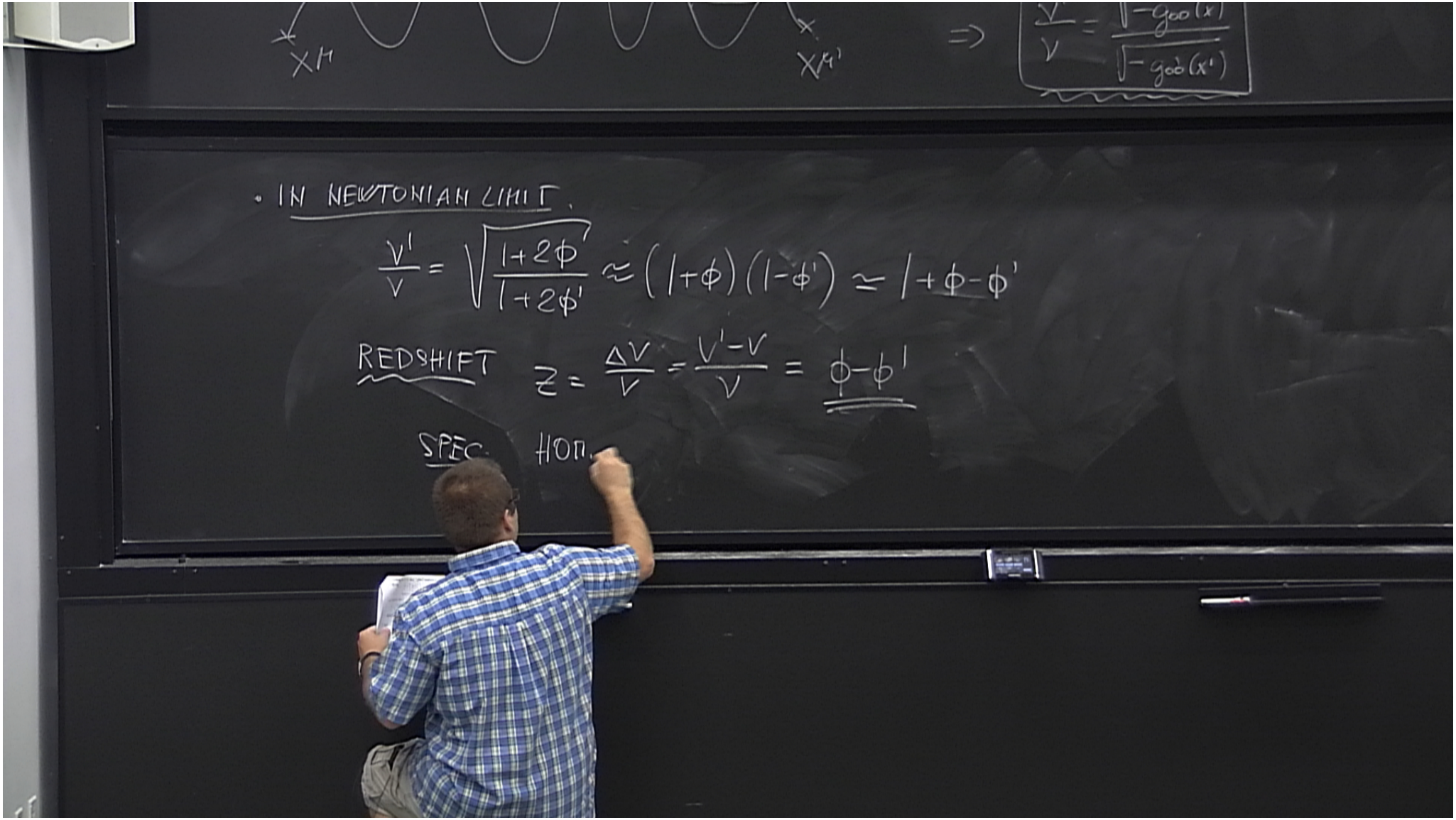

EDSHIFT
. FIELD TIME TICKS MORE SLOWLY.
OBSERVERS DO NO
dτ = 1/ν'
x
x'

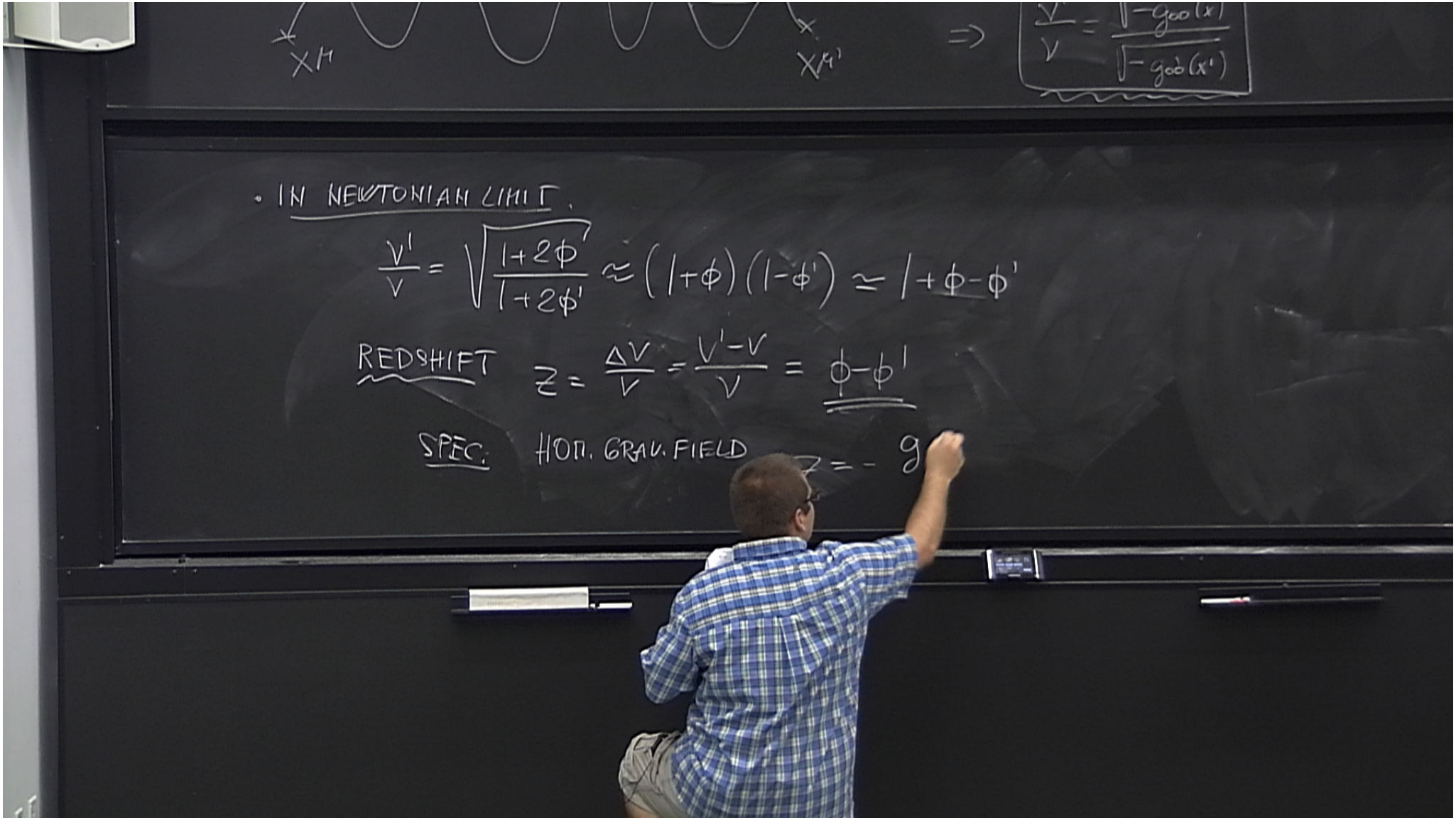

d) GRAVITATIONAL REDSHIFT
IN STRONG GRAV. FIELD TIME TICKS MORE SLOWLY.
OBSERVERS DO
dτ = 1/ν'
x
x(τ')

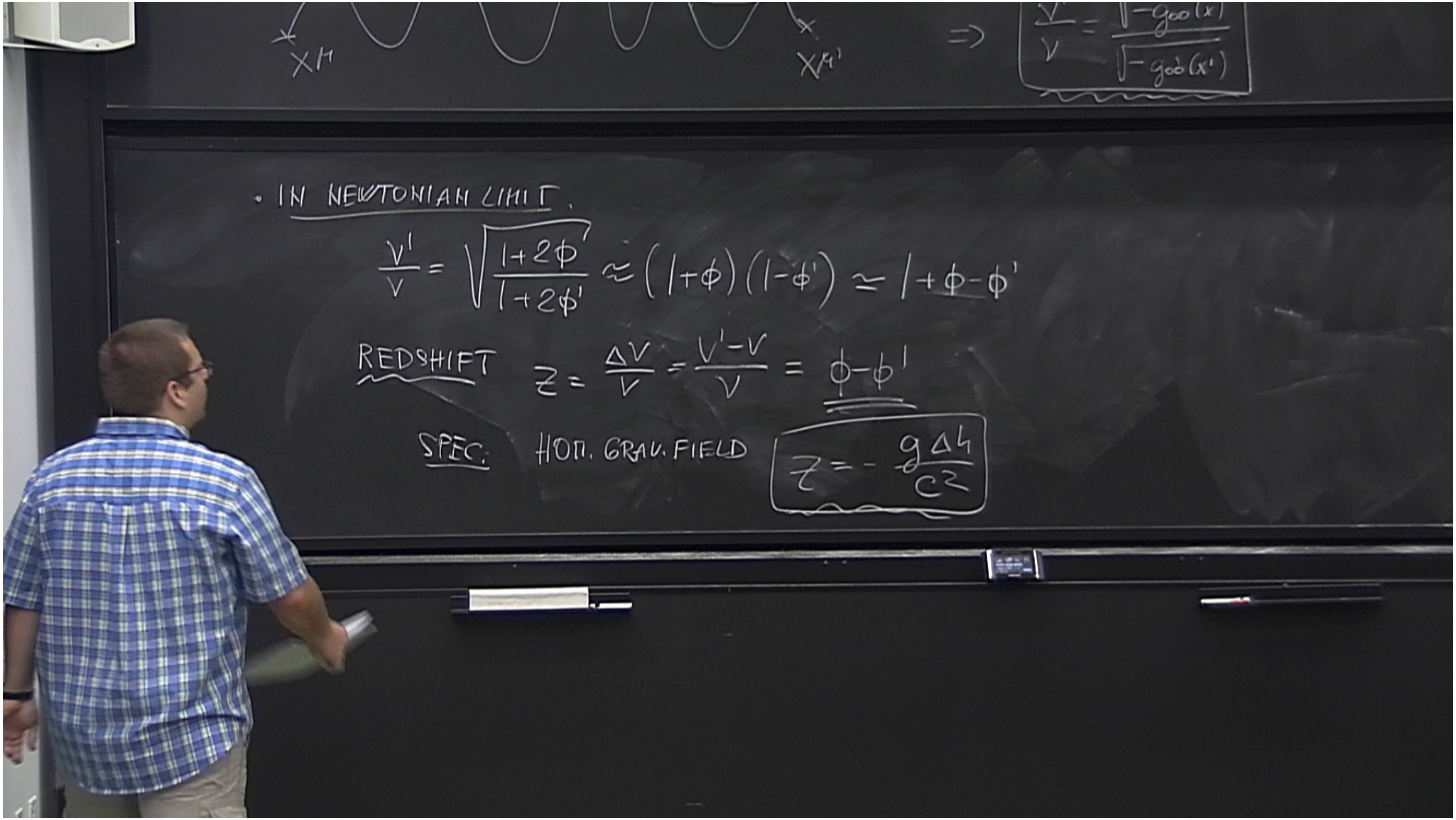

x^{μ}
$x^{\mu'}$
$\Rightarrow$
$\frac{\nu'}{\nu} = \frac{\sqrt{-g_{00}(x)}}{\sqrt{-g_{00}(x')}}$
• IN NEWTONIAN LIMIT.
$\frac{\nu'}{\nu} = \sqrt{\frac{1+2\phi}{1+2\phi'}} \simeq$

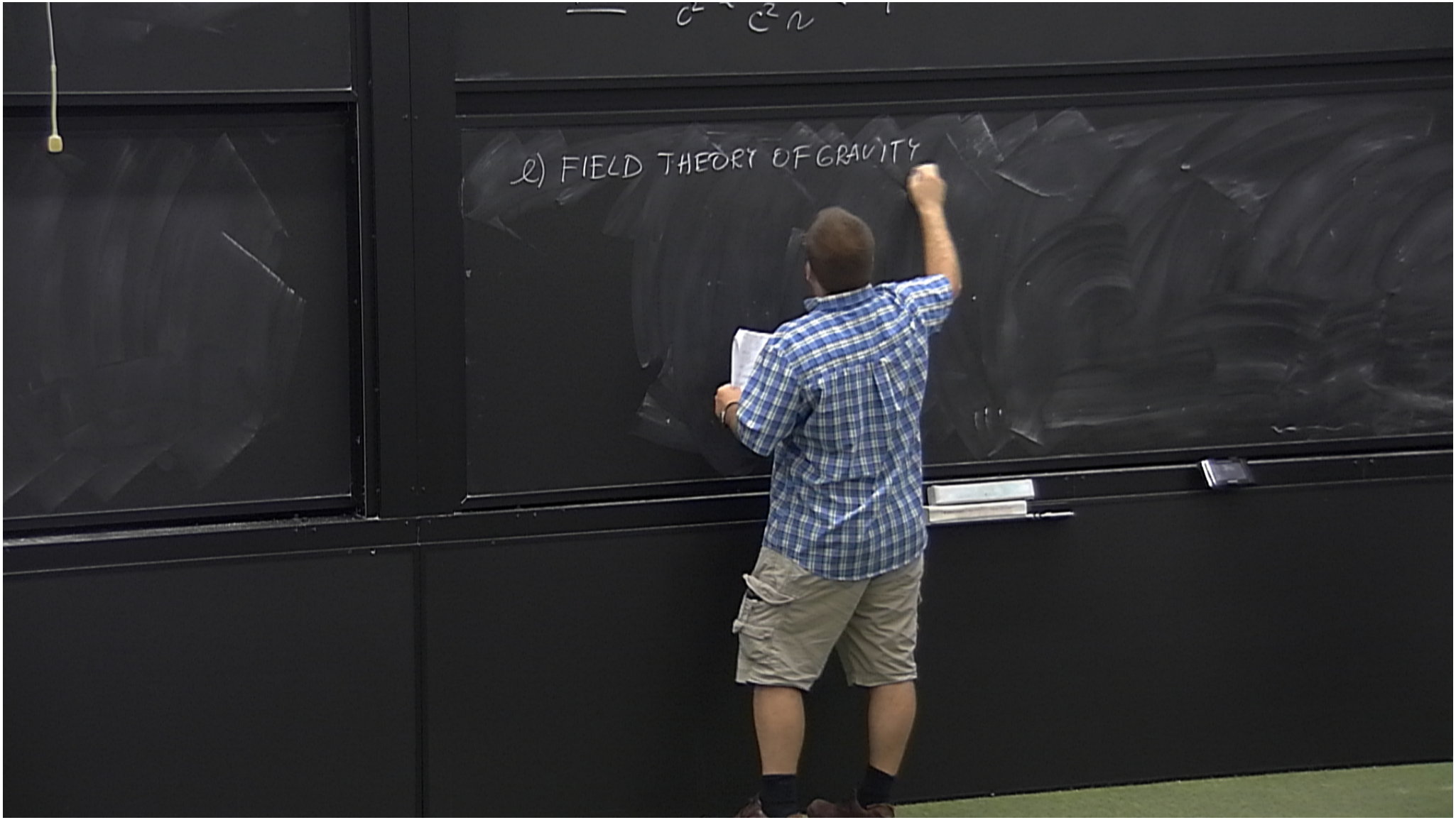

x^μ
$x^{\mu'}$
$\Rightarrow$
$\frac{\nu'}{\nu} = \frac{\sqrt{-g_{00}(x)}}{\sqrt{-g_{00}(x')}}$
• IN NEWTONIAN LIMIT.
$\frac{\nu'}{\nu} = \sqrt{\frac{1+2\phi}{1+2\phi'}} \simeq (1+\phi)(1-\phi')$


x^μ
$x^{\mu'}$
$\Rightarrow$
$\frac{V'}{V} = \frac{\sqrt{-g_{00}(x)}}{\sqrt{-g_{00}(x')}}$
• IN NEWTONIAN LIMIT.
$\frac{V'}{V} = \sqrt{\frac{1+2\Phi}{1+2\Phi'}} \simeq (1+\Phi)(1-\Phi') \simeq 1+\Phi-\Phi'$
REDSHIFT $z = \frac{\Delta V}{V} = \frac{V'-V}{V} = \Phi-$

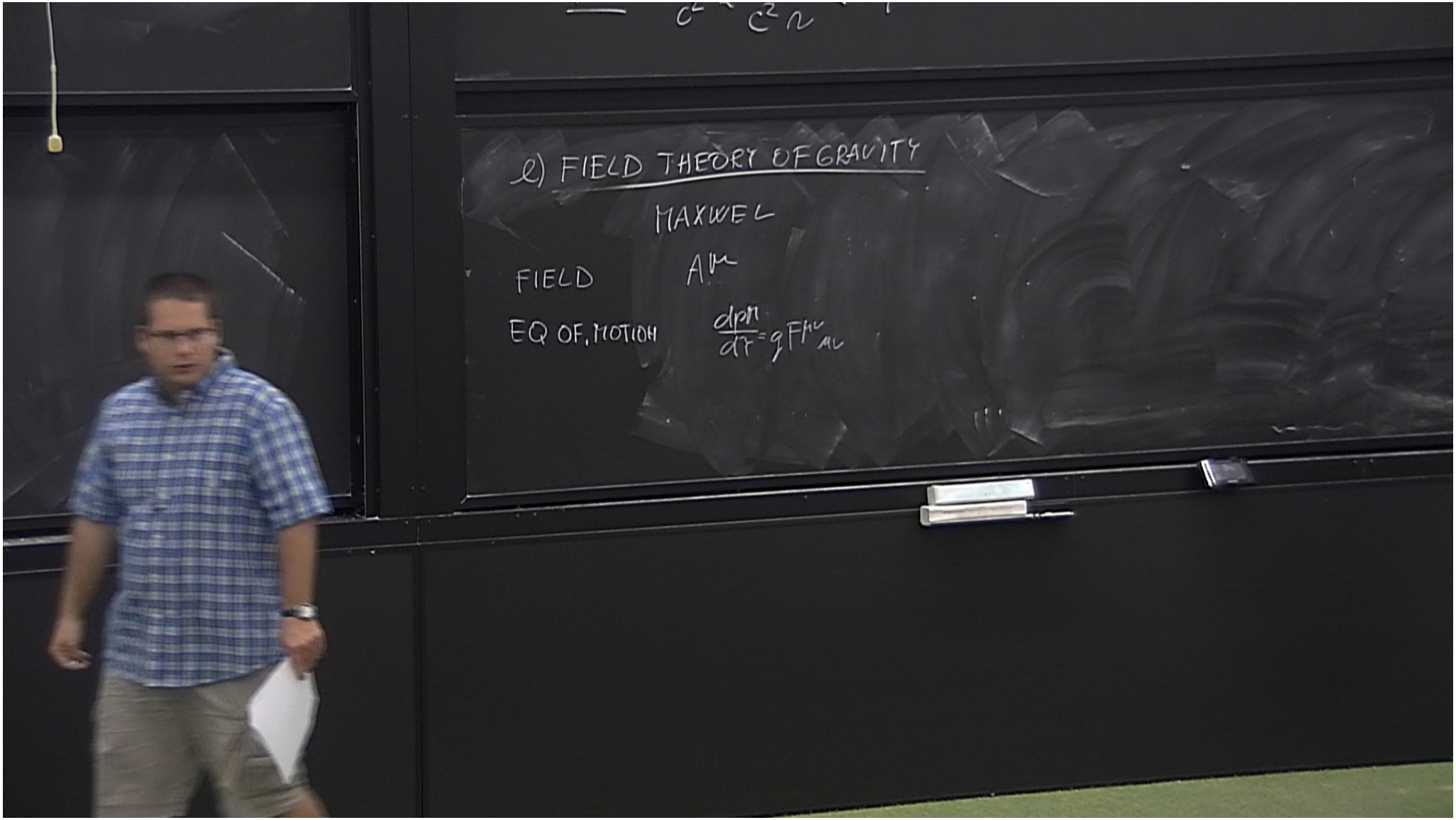

x^μ
$x^{\mu'}$
$\Rightarrow$
$\frac{\nu'}{\nu} = \frac{\sqrt{-g_{00}(x)}}{\sqrt{-g_{00}(x')}}$
• IN NEWTONIAN LIMIT.
$\frac{\nu'}{\nu} = \sqrt{\frac{1+2\phi}{1+2\phi'}} \simeq (1+\phi)(1-\phi') \simeq 1+\phi-\phi'$
REDSHIFT $z = \frac{\Delta\nu}{\nu} = \frac{\nu'-\nu}{\nu} = \phi-\phi'$
SPEC HOM.

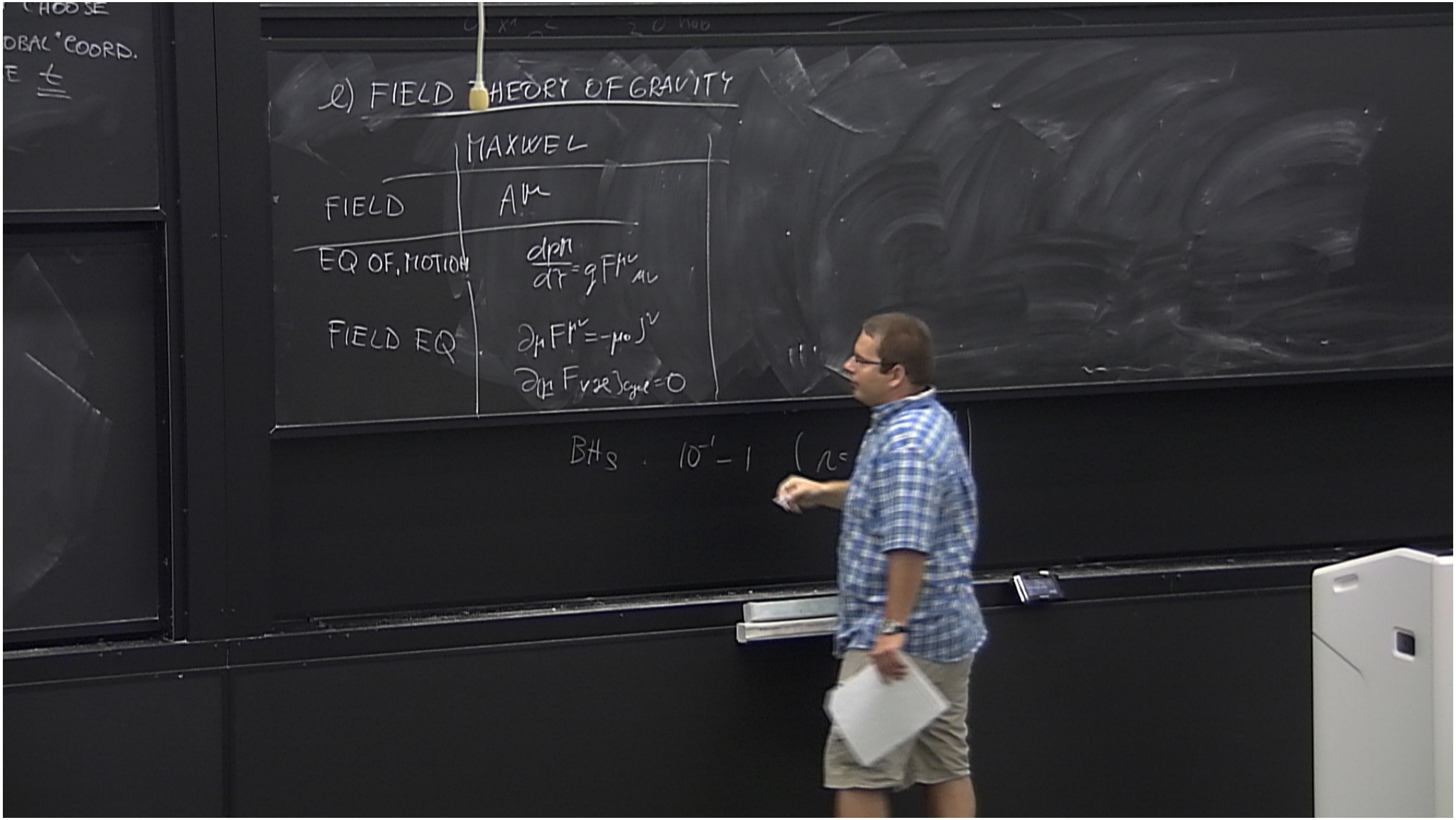

x^μ
$x^{\mu'}$
$\Rightarrow$
$\frac{\nu'}{\nu} = \frac{\sqrt{-g_{00}(x)}}{\sqrt{-g_{00}(x')}}$
• IN NEWTONIAN LIMIT.
$\frac{\nu'}{\nu} = \sqrt{\frac{1+2\phi}{1+2\phi'}} \simeq (1+\phi)(1-\phi') \simeq 1+\phi-\phi'$
REDSHIFT $z = \frac{\Delta\nu}{\nu} = \frac{\nu'-\nu}{\nu} = \phi-\phi'$
SPEC: HOM. GRAV. FIELD $z = -g$

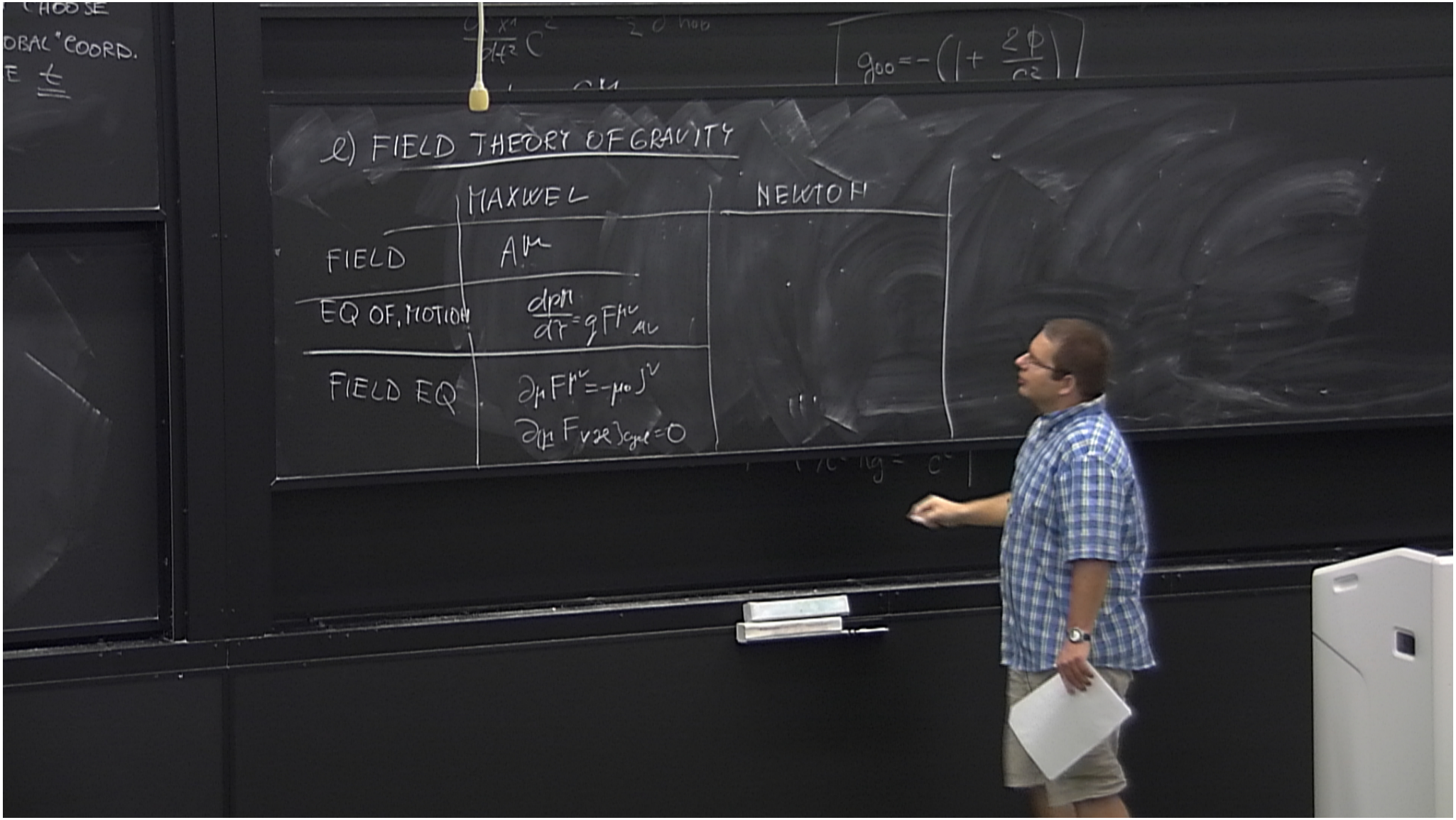

$\frac{\nu'}{\nu} = \frac{\sqrt{-g_{00}(x)}}{\sqrt{-g_{00}(x')}}$
• IN NEWTONIAN LIMIT.
$\frac{\nu'}{\nu} = \sqrt{\frac{1+2\phi}{1+2\phi'}} \simeq (1+\phi)(1-\phi') \simeq 1+\phi-\phi'$
REDSHIFT $z = \frac{\Delta\nu}{\nu} = \frac{\nu'-\nu}{\nu} = \phi-\phi'$
SPEC: HOM. GRAV. FIELD $z = -\frac{g\Delta h}{c^2}$

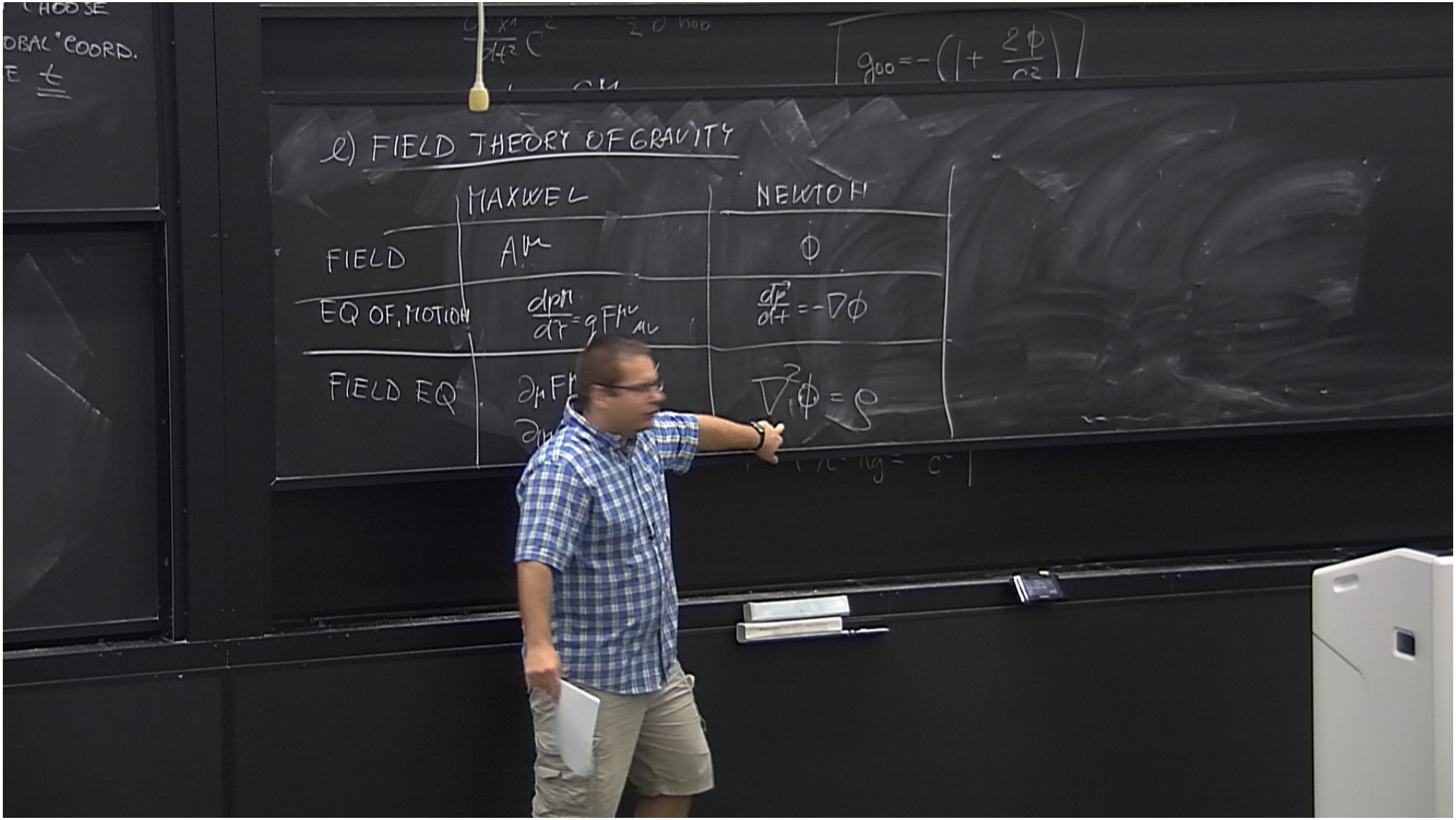

2) FIELD THEORY OF GRAVITY

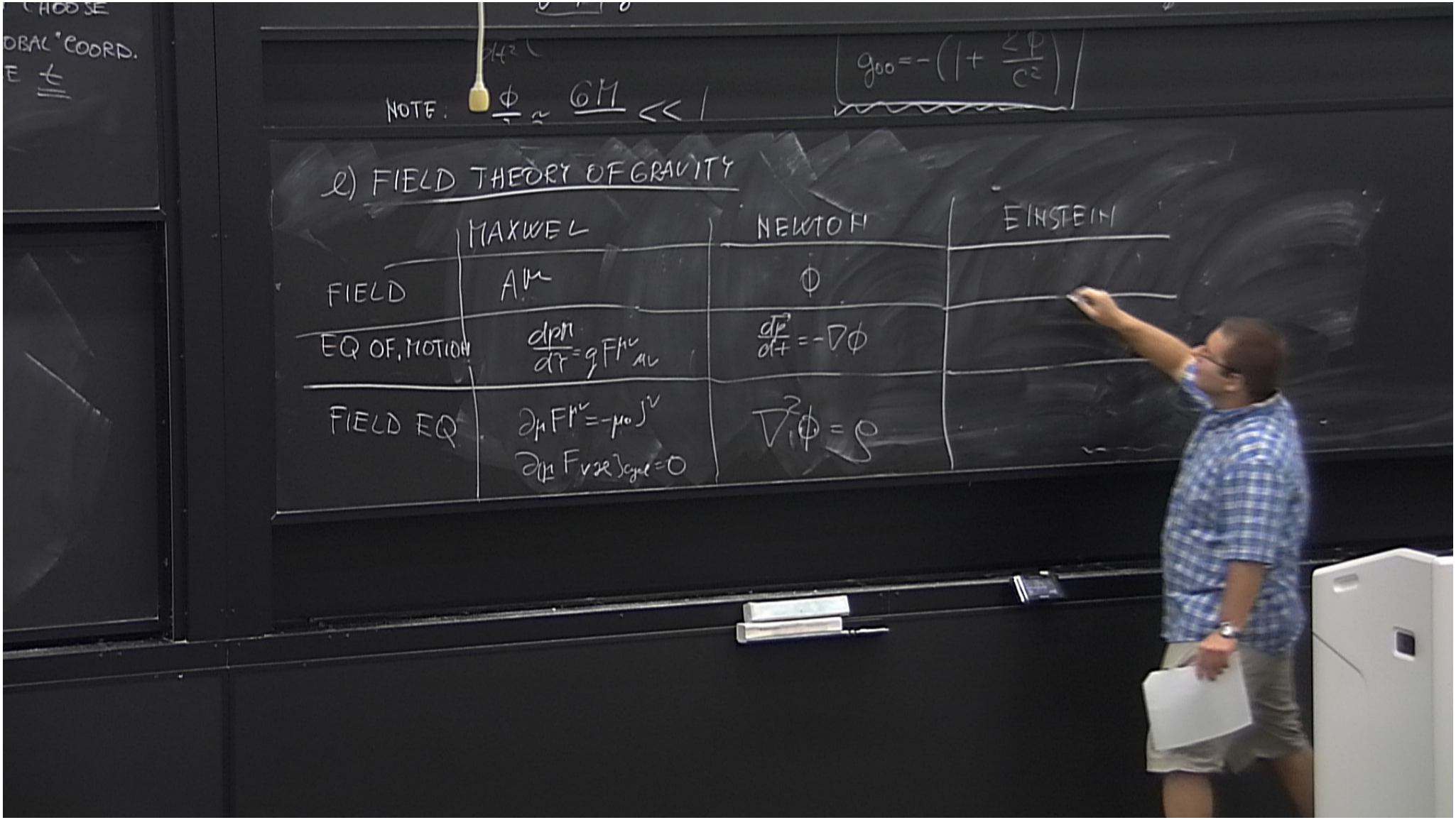

2) FIELD THEORY OF GRAVITY
MAXWEL

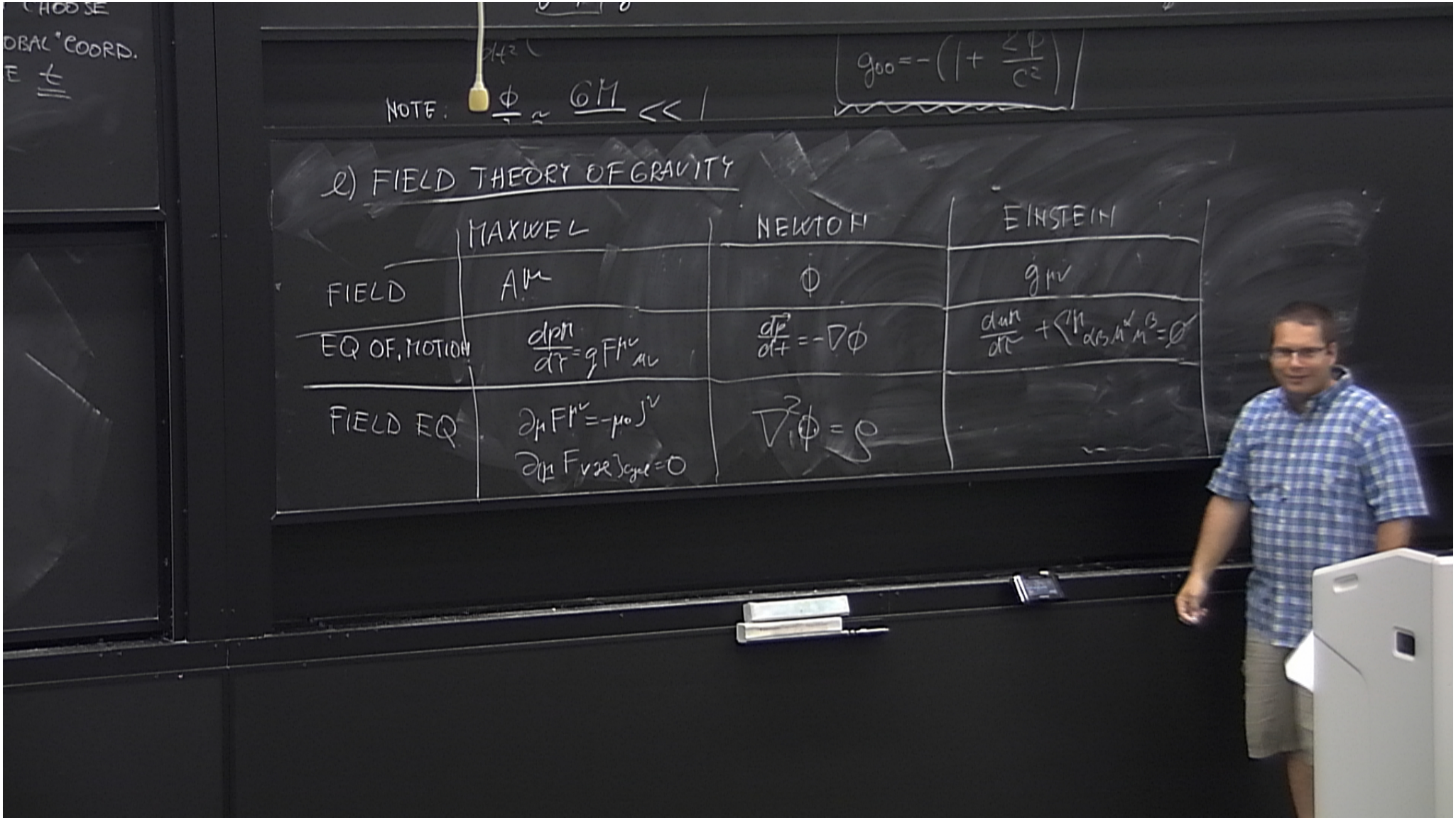

2) FIELD THEORY OF GRAVITY
MAXWEL
FIELD
A
EQ OF. MOTION

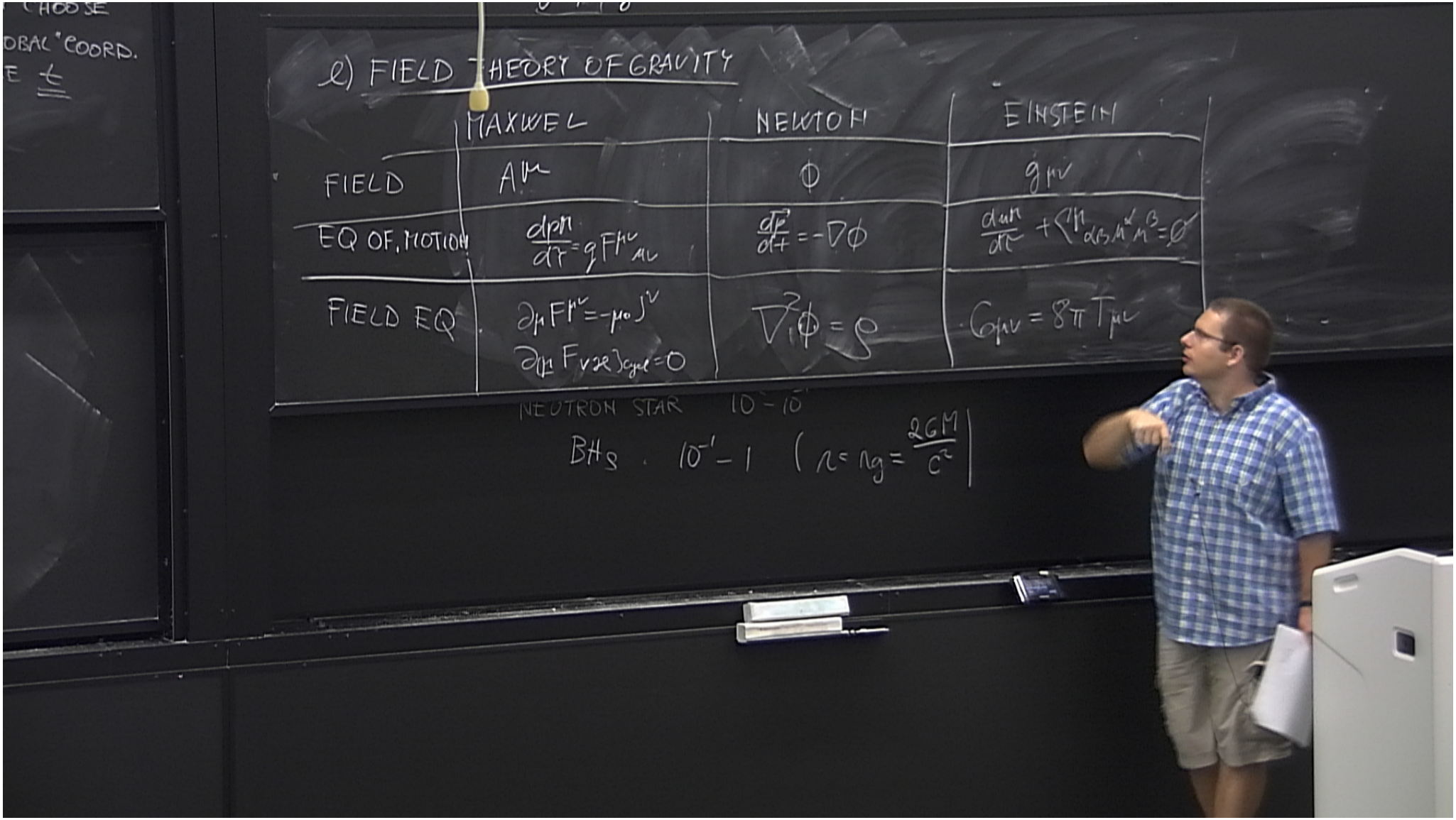

OBAL "COORD.
E ≡
ℓ) FIELD THEORY OF GRAVITY
MAXWEL
FIELD
A^μ
EQ OF. MOTION
$\frac{dp^\mu}{d\tau} = qF^{\mu\nu}$
FIELD EQ
$\partial_\mu F^{\mu\nu} = -\mu_0 J^\nu$
$\partial_\mu F_{\nu\rho]} = 0$
BHs . $10^{-1} - 1$

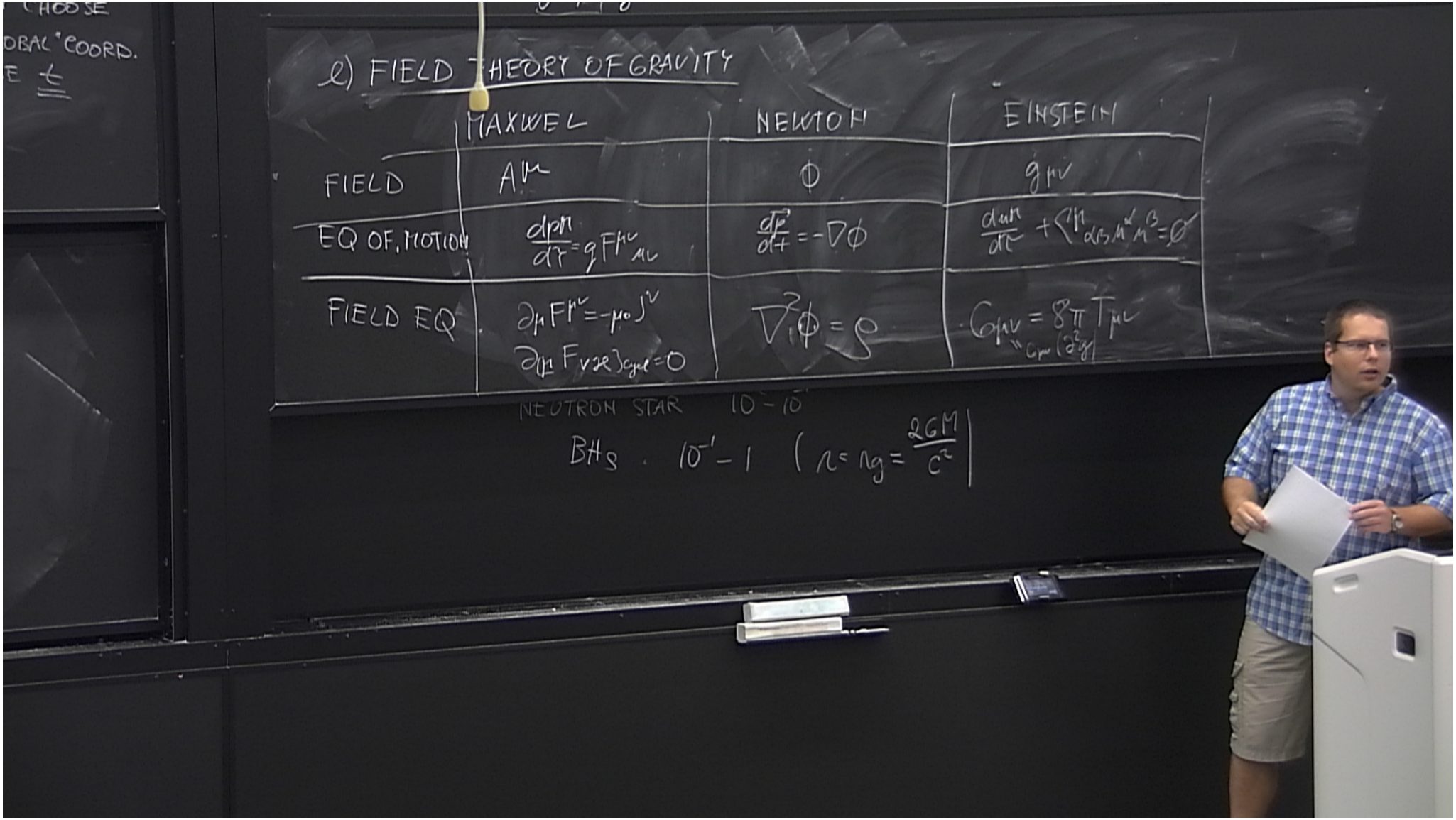

OBAL "COORD.
$g_{00} = -\left(1 + \frac{2\phi}{c^2}\right)$
e) FIELD THEORY OF GRAVITY
MAXWEL
NEWTON
FIELD
A^μ
EQ OF. MOTION
$\frac{dp^\mu}{d\tau} = q F^{\mu\nu} u_\nu$
FIELD EQ
$\partial_\mu F^{\mu\nu} = -\mu_0 J^\nu$
$\partial_\mu F_{\nu\rho]} = 0$

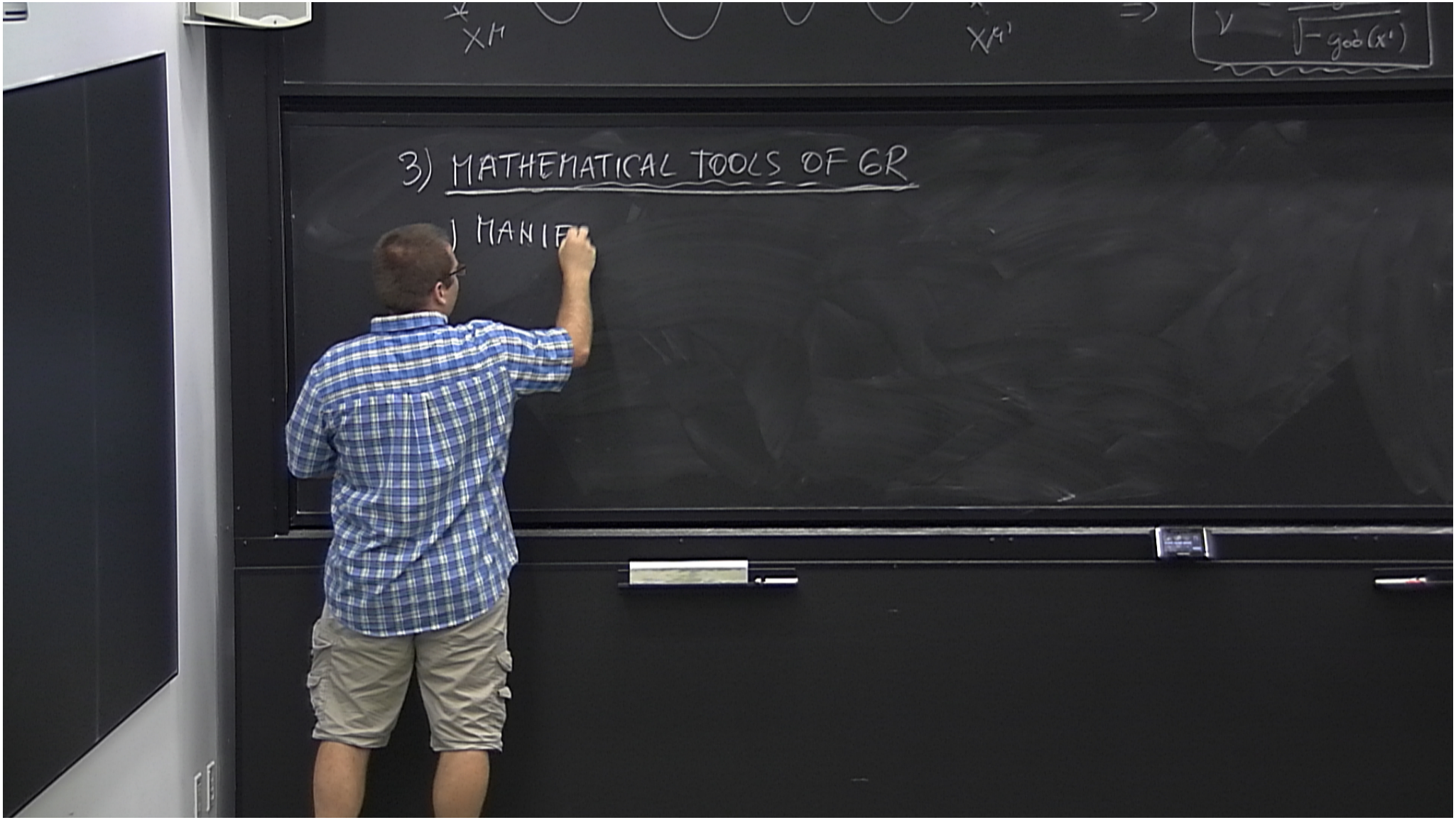

OBAL "COORD.
$g_{00} = -\left(1 + \frac{2\phi}{c^2}\right)$
e) FIELD THEORY OF GRAVITY
MAXWELL
NEWTON
FIELD
A_μ
ϕ
EQ OF. MOTION
$\frac{dp_\mu}{d\tau} = q F^{\mu\nu} u_\nu$
$\frac{d\vec{p}}{dt} = -\nabla\phi$
FIELD EQ
$\partial_\mu F^{\mu\nu}$
$\nabla^2\phi = \rho$

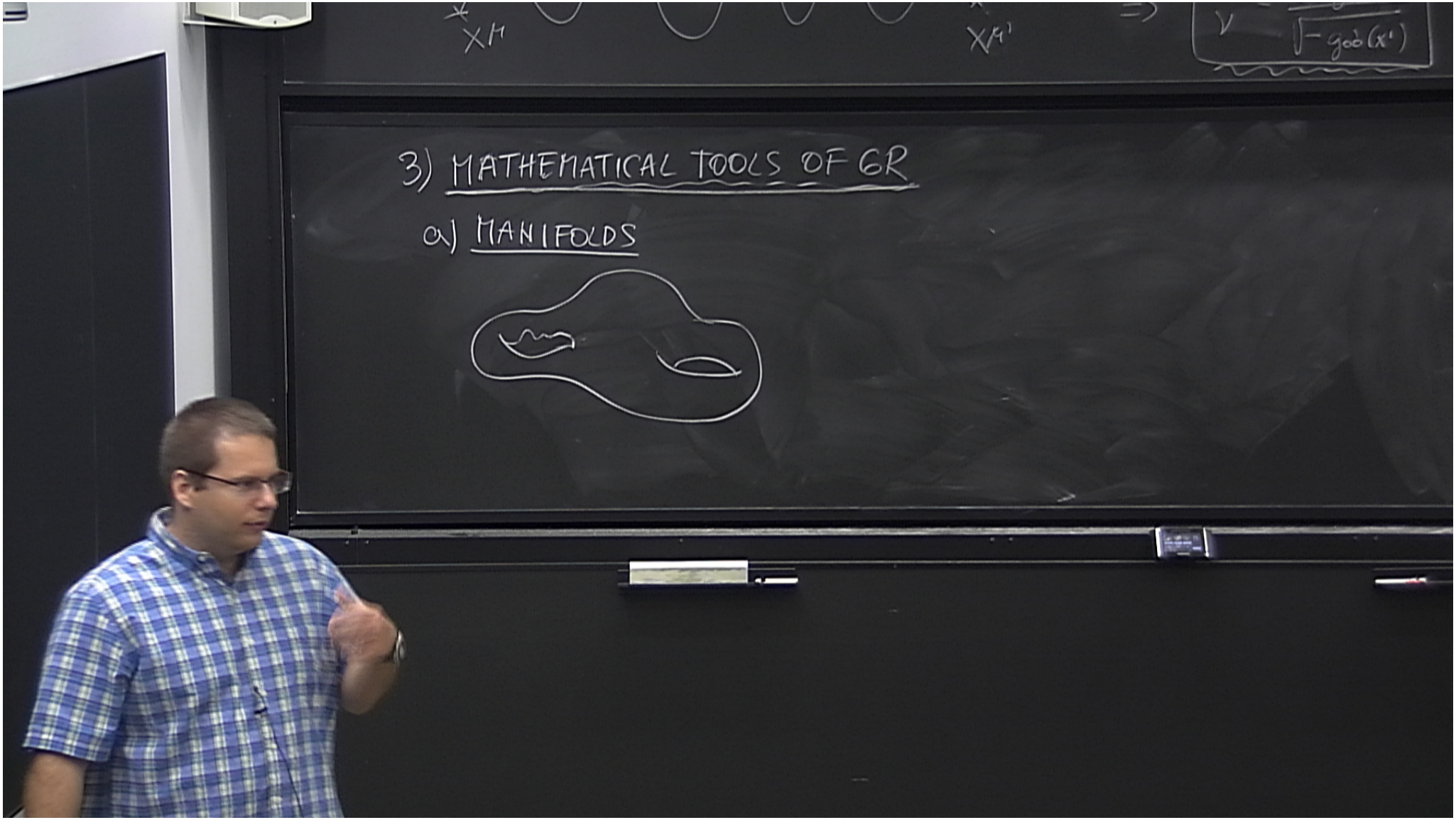

NOTE: $\frac{\Phi}{c^2} \simeq \frac{GM}{} << 1$
$g_{00} = -\left(1 + \frac{2\Phi}{c^2}\right)$
ℓ) FIELD THEORY OF GRAVITY
MAXWELL
NEWTON
EINSTEIN
FIELD
A^μ
Φ
EQ OF MOTION
$\frac{dp^\mu}{d\tau} = qF^{\mu\nu}$
$\frac{d\vec{p}}{dt} = -\nabla\Phi$
FIELD EQ
$\partial_\mu F^{\mu\nu} = -\mu_0 J^\nu$
$\partial_{[\mu} F_{\nu\rho]} = 0$
$\nabla^2\Phi = \rho$

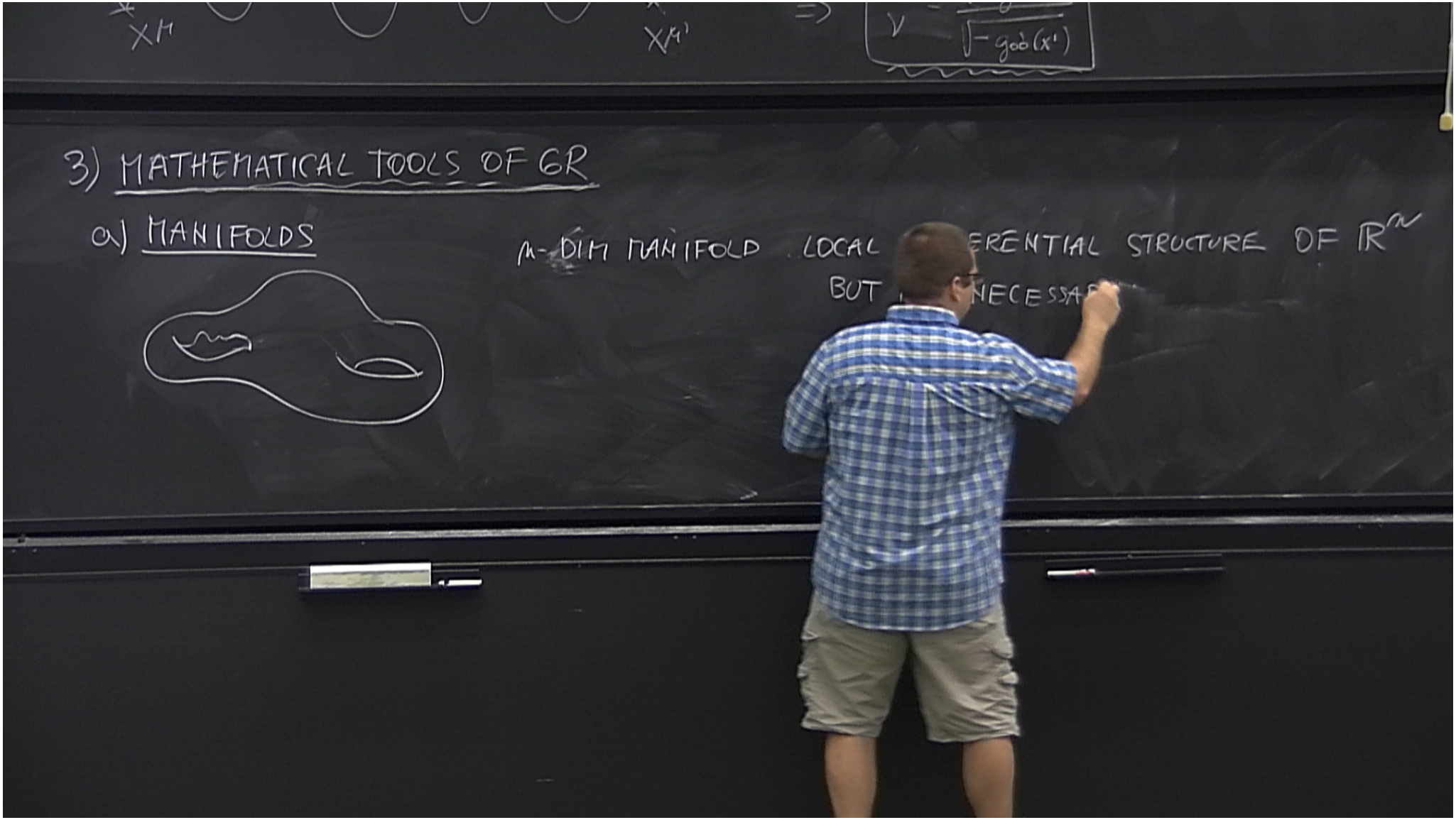

NOTE: $\frac{\Phi}{c^2} \simeq \frac{GM}{rc^2} \ll 1$
$g_{00} = -\left(1 + \frac{2\Phi}{c^2}\right)$
e) FIELD THEORY OF GRAVITY
MAXWELL
NEWTON
EINSTEIN
FIELD
A^μ
Φ
$g_{\mu\nu}$
EQ OF MOTION
$\frac{dp^\mu}{d\tau} = qF^\mu{}_\nu$
$\frac{d^2\vec{r}}{dt^2} = -\nabla\Phi$
$\frac{du^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} u^\alpha u^\beta = 0$
FIELD EQ
$\partial_\mu F^{\mu\nu} = -\mu_0 J^\nu$
$\partial_{[\mu} F_{\nu\rho]} = 0$
$\nabla^2\Phi = \rho$

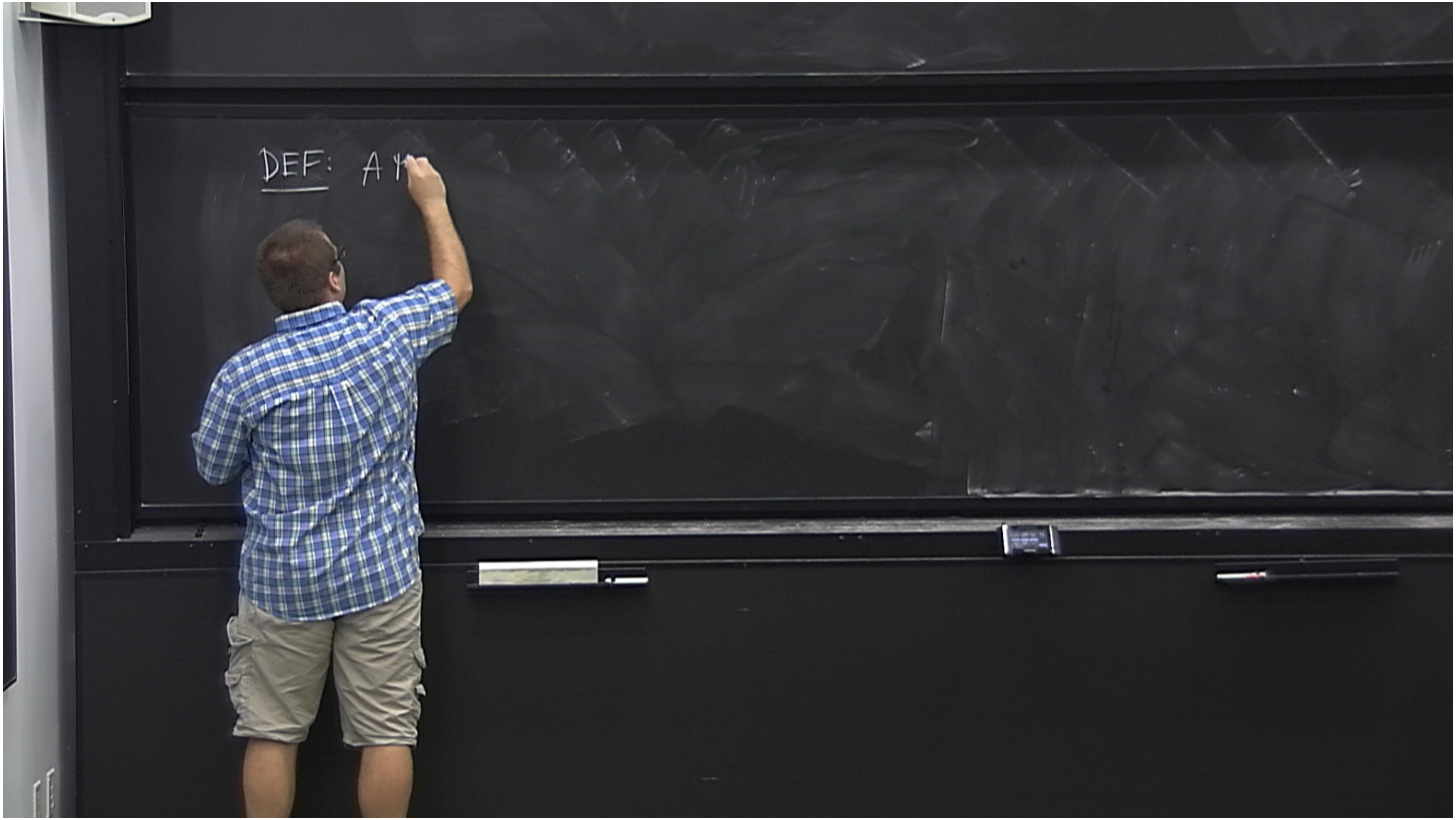

OBAL "COORD.
E t
e) FIELD THEORY OF GRAVITY
MAXWELL
NEWTON
EINSTEIN
FIELD
EQ OF. MOTION
FIELD EQ
NEUTRON STAR 10 - 10
BHs . 10 - 1

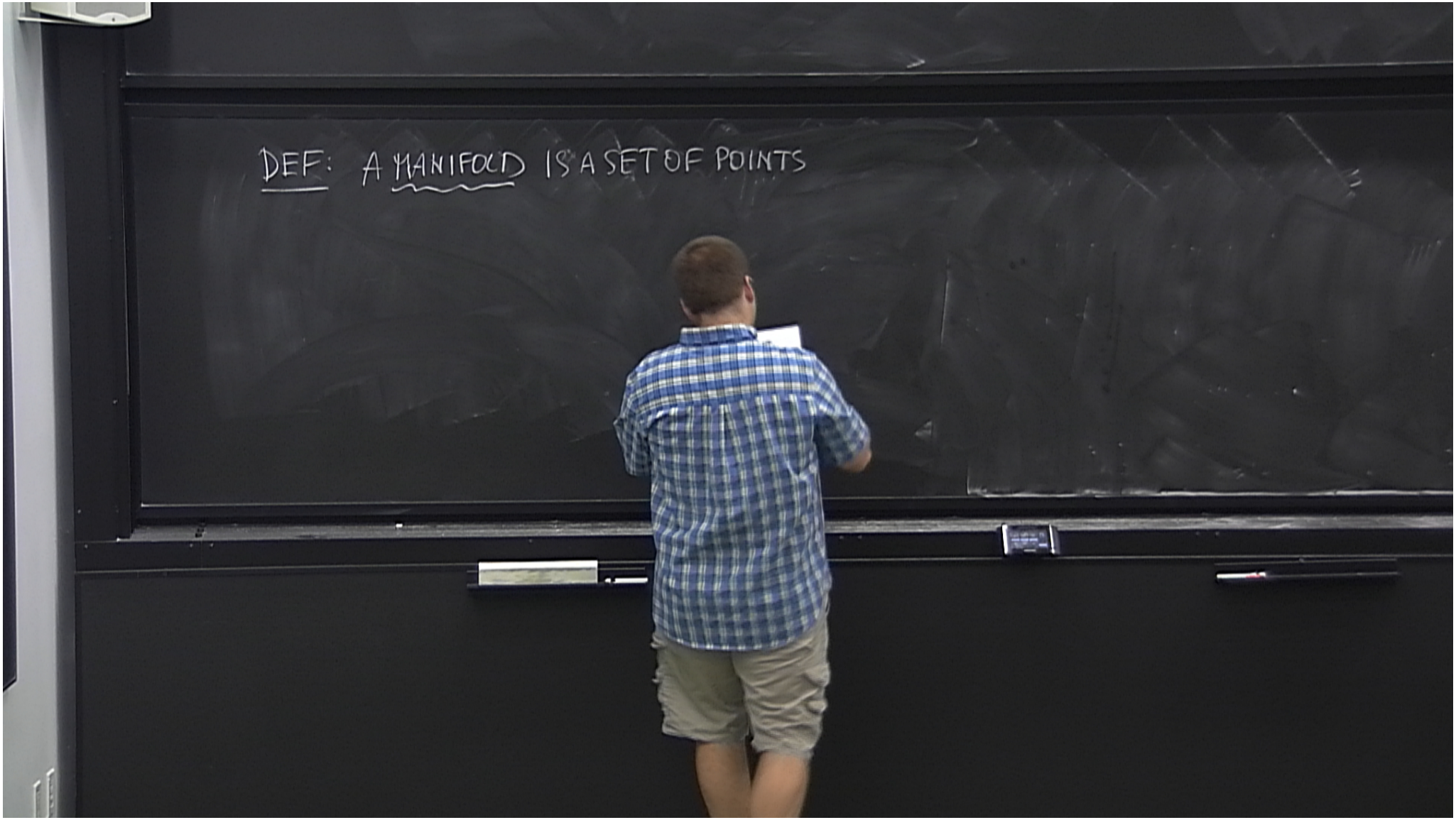

OBAL "COORD.
e) FIELD THEORY OF GRAVITY
MAXWELL
NEWTON
EINSTEIN
FIELD
EQ OF. MOTION
FIELD EQ
NEUTRON STAR
BHs


3) MATHEMATICAL TOOLS OF GR
) MANIF


3) MATHEMATICAL TOOLS OF GR
a) MANIFOLDS


3) MATHEMATICAL TOOLS OF GR
a) MANIFOLDS
n-DIM MANIFOLD
LOCAL
ERENTIAL STRUCTURE OF $\mathbb{R}^n$
BUT
NECESSA


DEF:


DEF: A MANIFOLD IS A SET OF POINTS