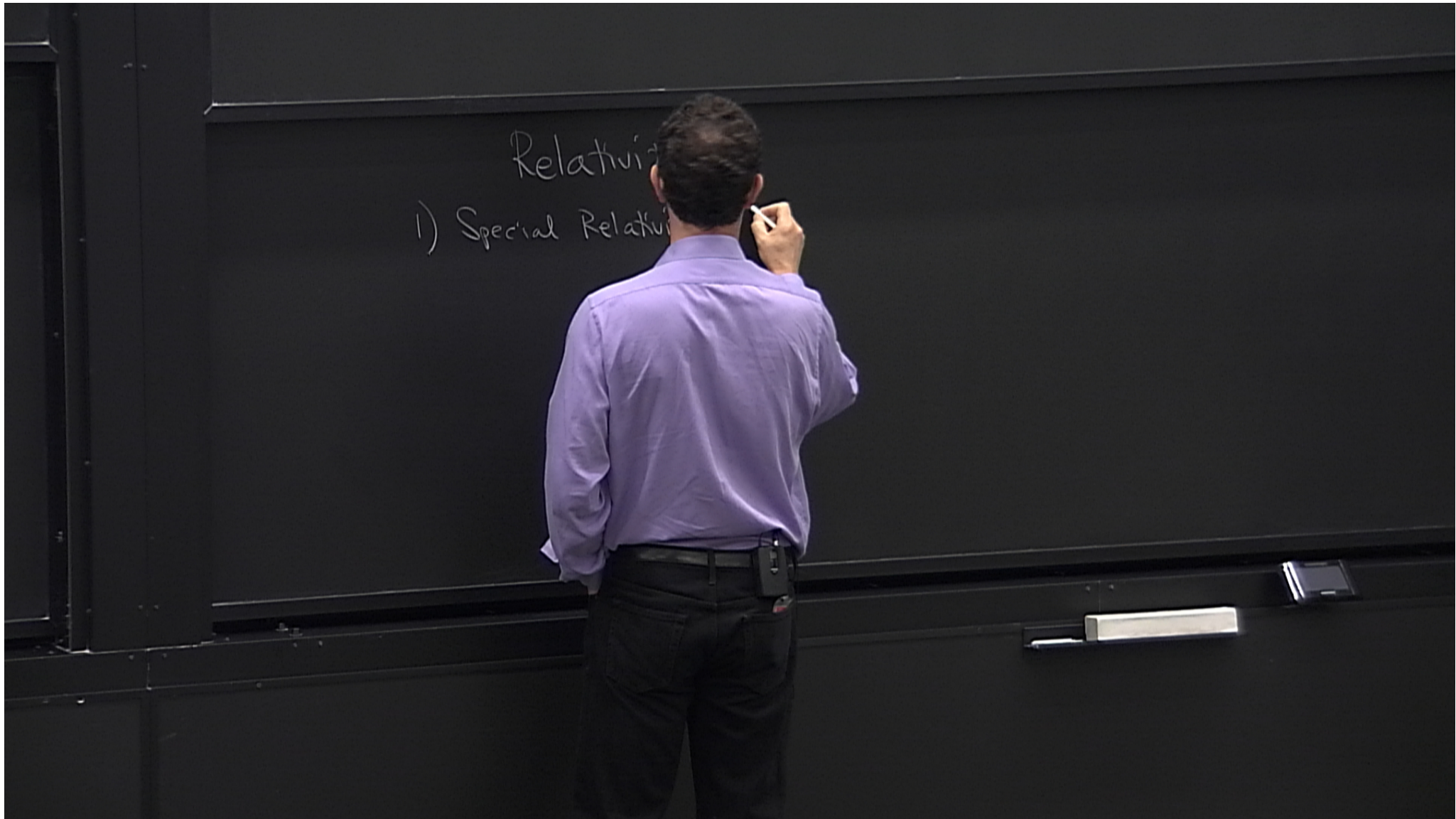


Title: 13/14 PSI - Relativity - Lecture 1

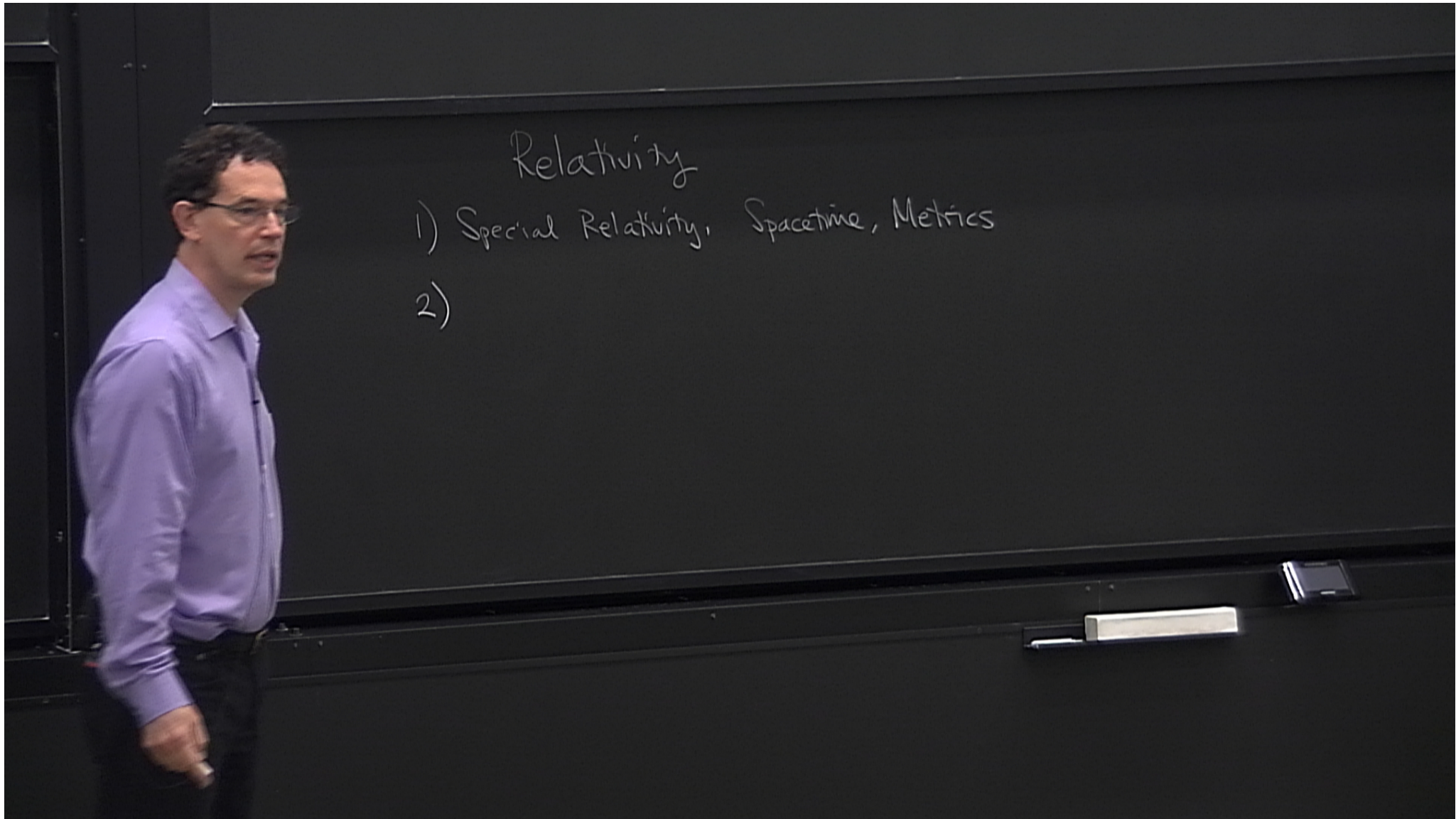
Date: Sep 09, 2013 09:00 AM

URL: <http://pirsa.org/13090011>

Abstract:

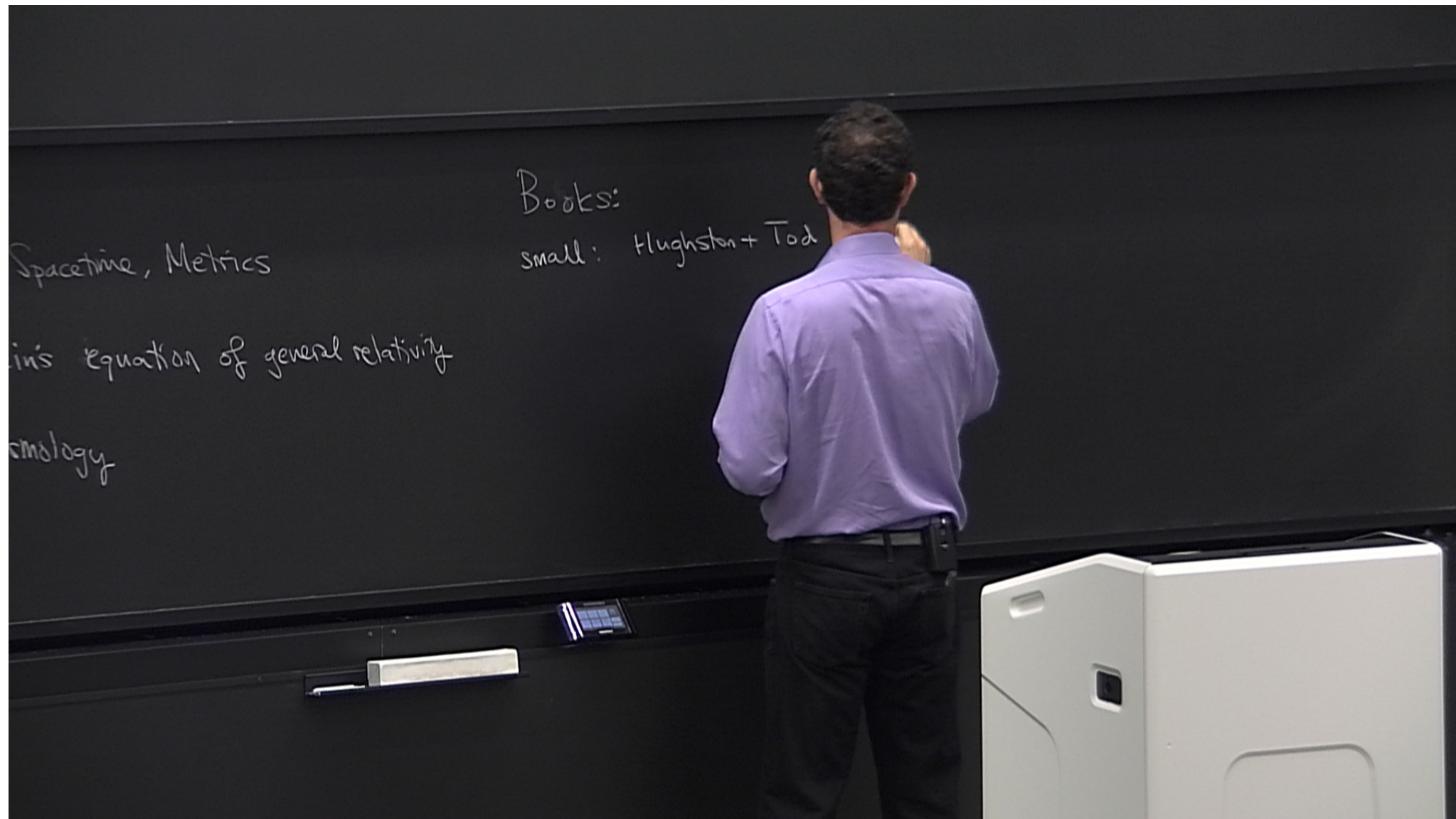


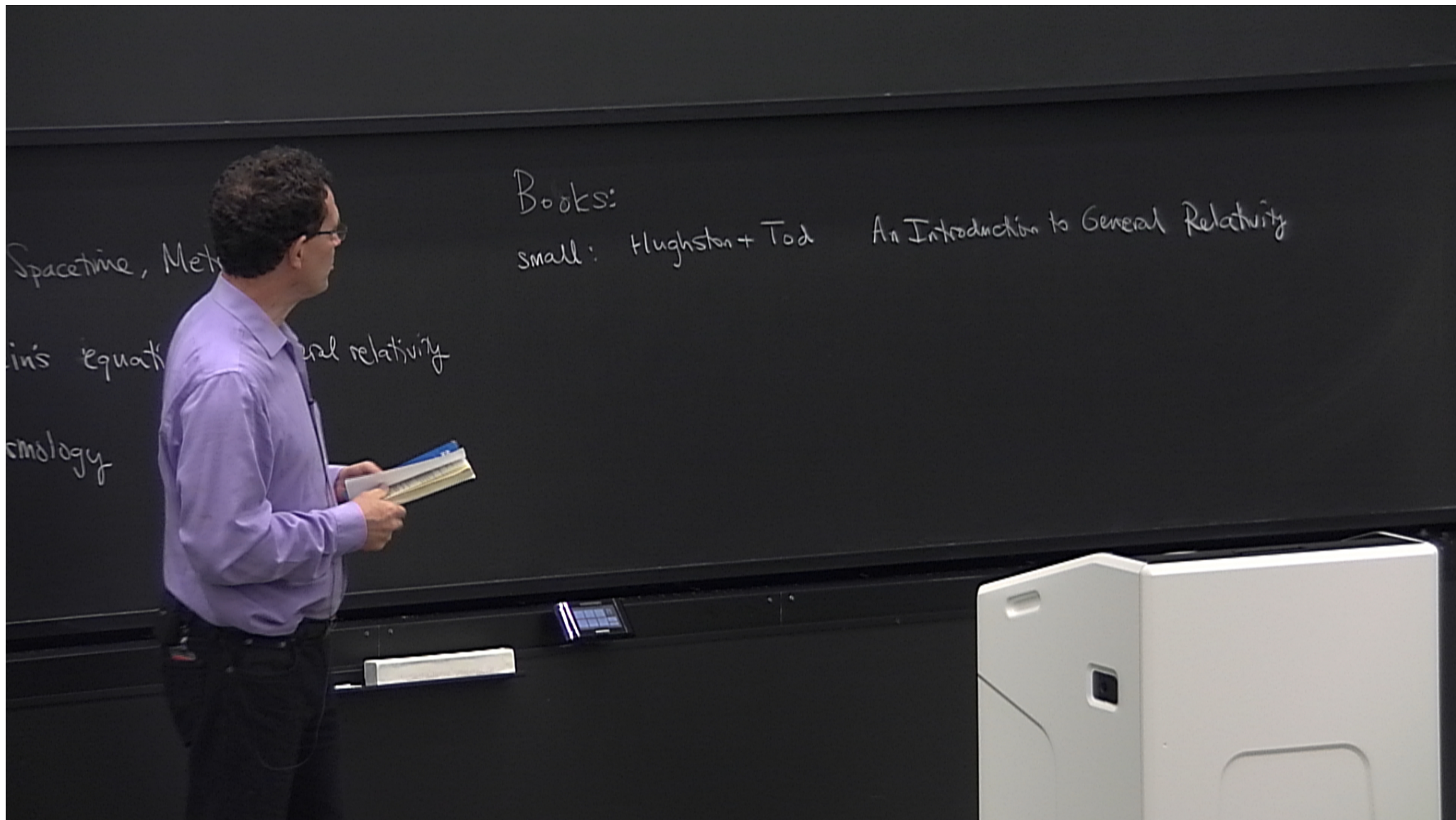
Relativity
1) Special Relativity, Spacetime, Metrics



Relativity

- 1) Special Relativity, Spacetime, Metrics
- 2) Curvature, Einstein's equation of general relativity





Spacetime, Metrics

Einstein's equation of general relativity

Cosmology

Books:

small: Hughston + Tod

medium: S Carroll

An Introduction to General Relativity

"

Spacetime, Metrics

Einstein's equation of general relativity

cosmology

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Spacetime + Geometry

Spacetime, Metrics

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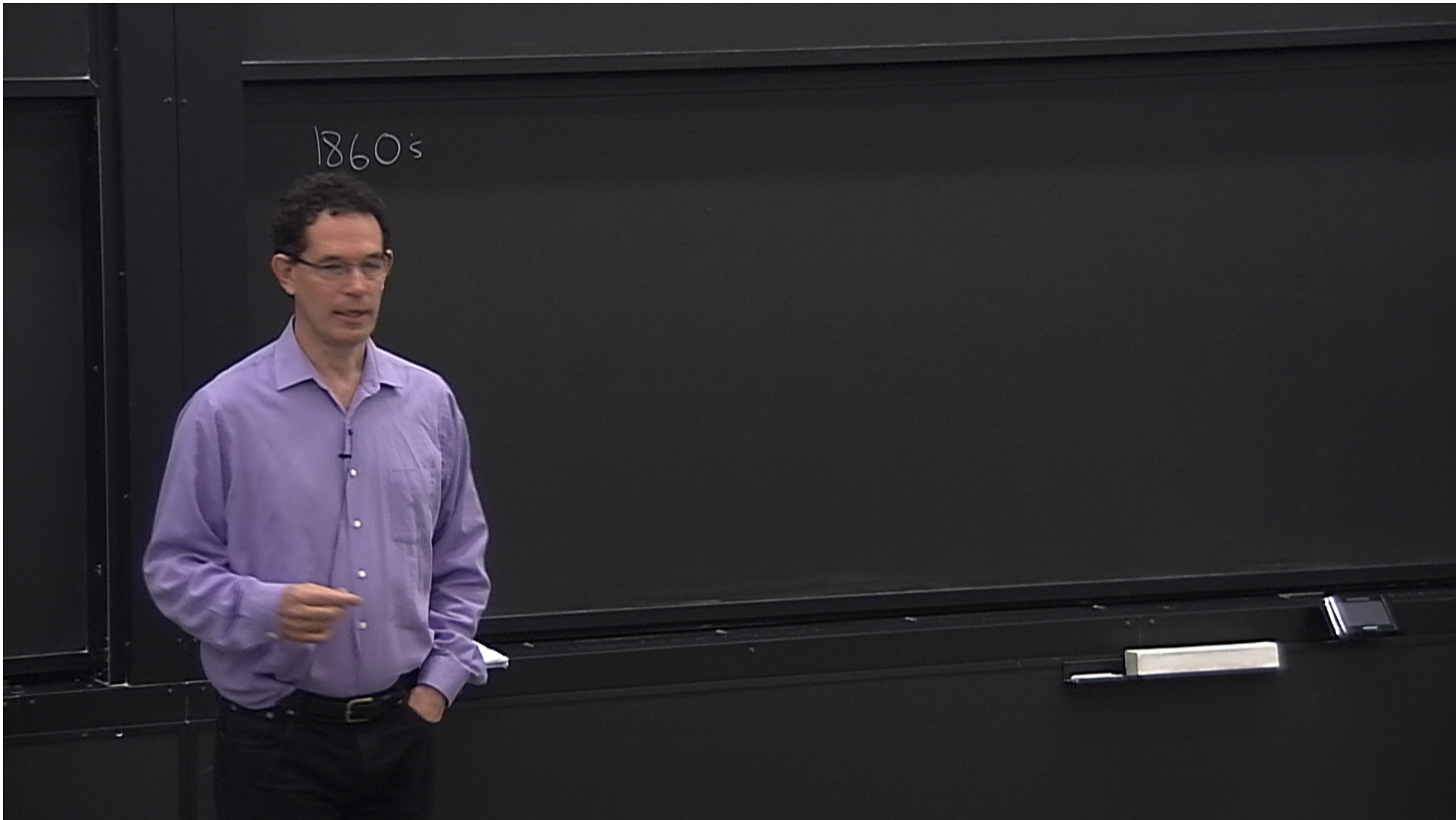
large: A. Zee

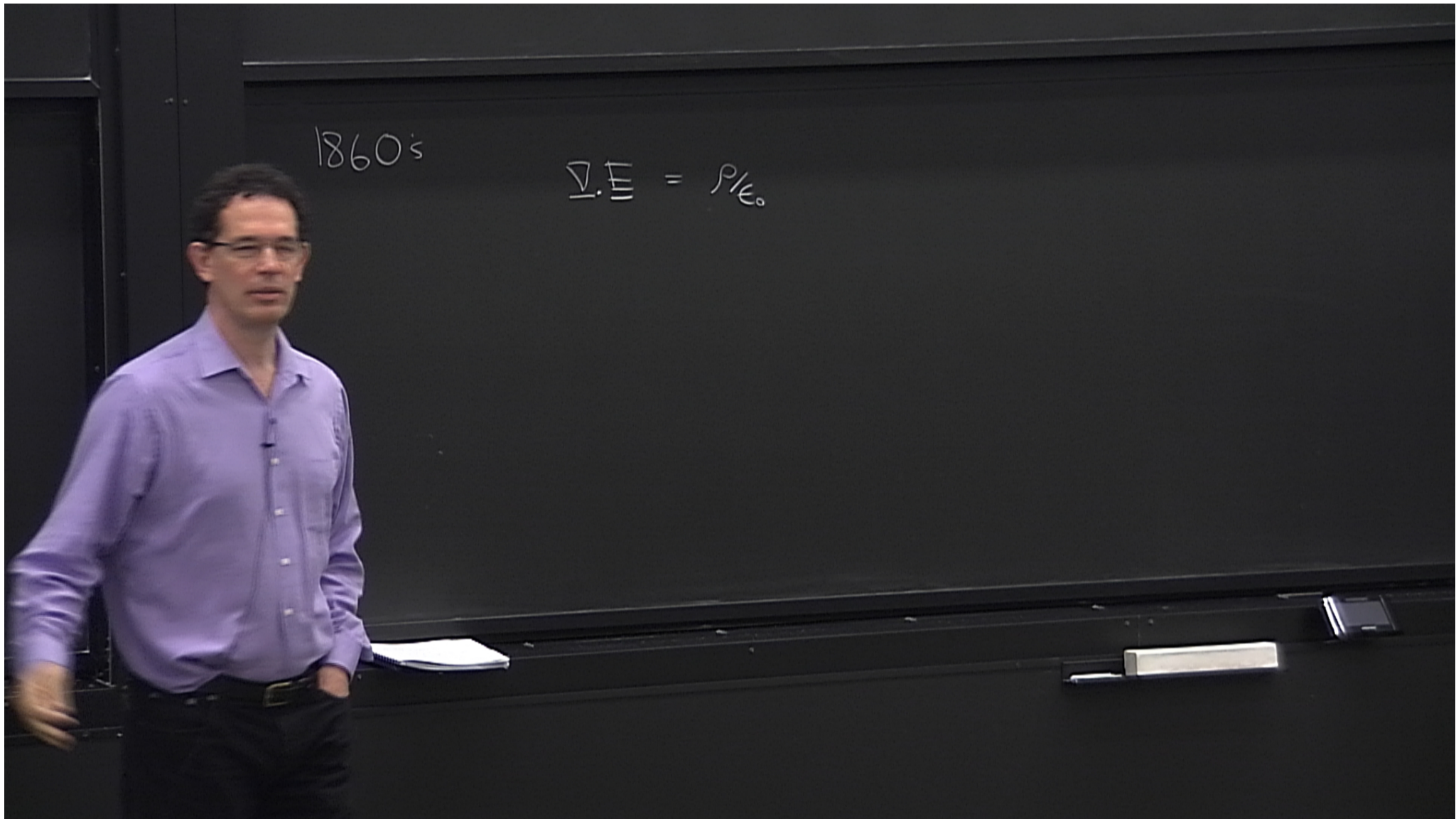
(Dirac: General Relativity)

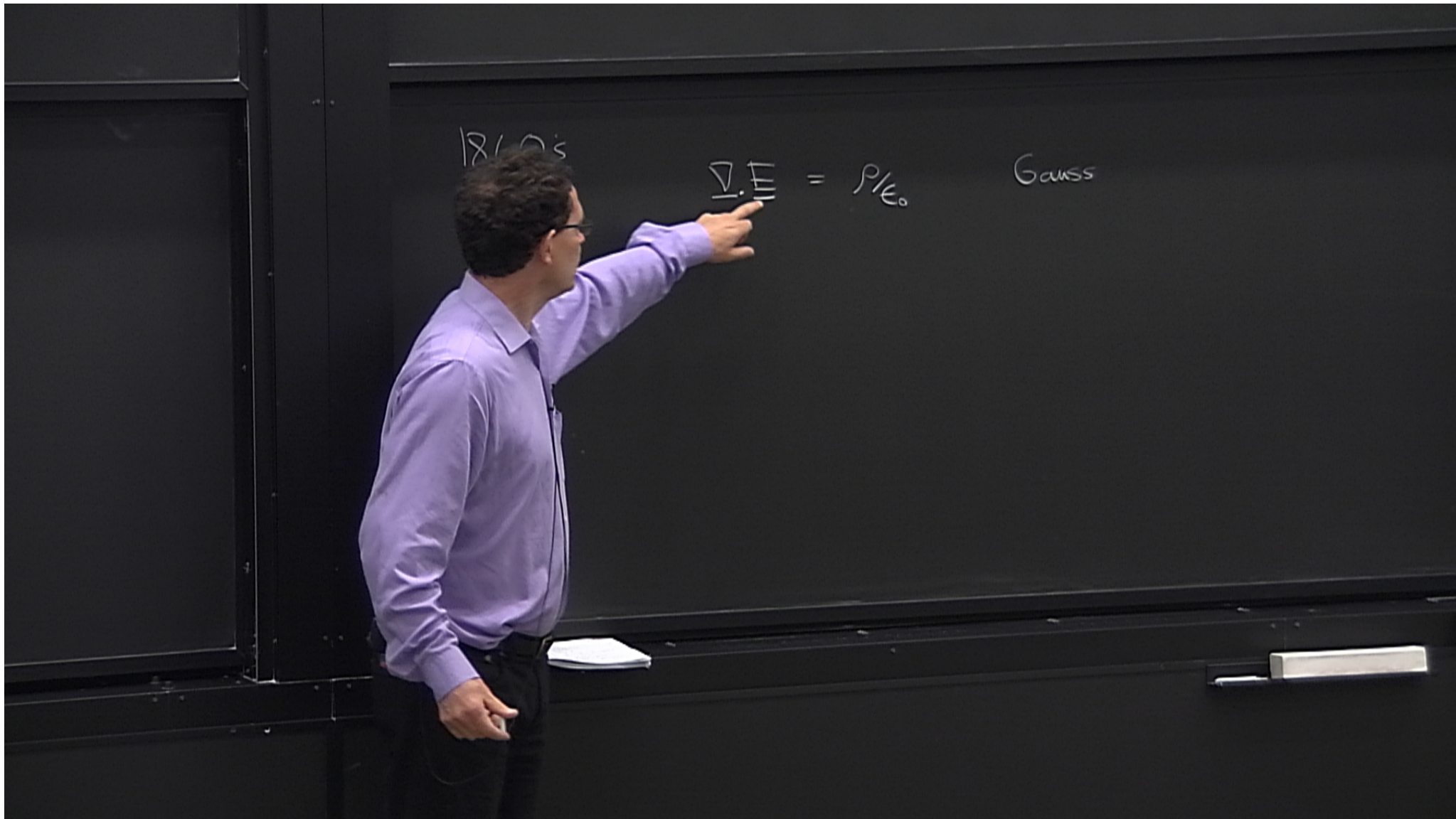
An Introduction to Relativity

"
Spacetime

Einstein (Mitschell)







1860's

$$\nabla \cdot \underline{E} = \rho / \epsilon_0$$

$$\nabla \cdot \underline{B} = 0$$

Gauss

1860's

$$\nabla \cdot \underline{E} = \rho / \epsilon_0$$

Gauss

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

1860's

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1860's

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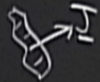
$$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

Faraday

$$\nabla \wedge \underline{B} = \mu_0 \underline{J}$$

Ampere

1860's



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Gauss

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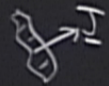
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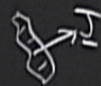
$$\nabla \wedge \underline{B} = \mu_0 \underline{J}$$

Amper's law

+ charge conservation

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \underline{J}$$

1860's



$$\frac{\partial}{\partial x^i} \epsilon_{ijk} \frac{\partial}{\partial x^j} B_k$$

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0$$

Gauss

$$\underline{\nabla} \cdot \underline{B} = 0$$

$$\underline{\nabla} \wedge \underline{E} = - \frac{\partial \underline{B}}{\partial t}$$

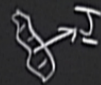
$$\underline{\nabla} \wedge \underline{B} = \mu_0 \underline{J} +$$

Ampere

+ charge con

$$\frac{\partial \rho}{\partial t} =$$

1860's



$$\frac{\partial}{\partial x^i} \epsilon_{ijk} \frac{\partial}{\partial x^j} B_k$$

$$\nabla \cdot \underline{E} = \rho / \epsilon_0$$

Gauss

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

Faraday

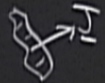
$$\nabla \wedge \underline{B} = \mu_0 \underline{J} + \epsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$$

Ampere

charge con

$$\frac{\partial \rho}{\partial t} =$$

1860's



$$\frac{\partial}{\partial x^i} \epsilon_{ijk} \frac{\partial}{\partial x^j} B_k$$

$$\nabla \cdot \underline{E} = \rho / \epsilon_0$$

Gauss

$$\nabla \cdot \underline{B} = 0$$

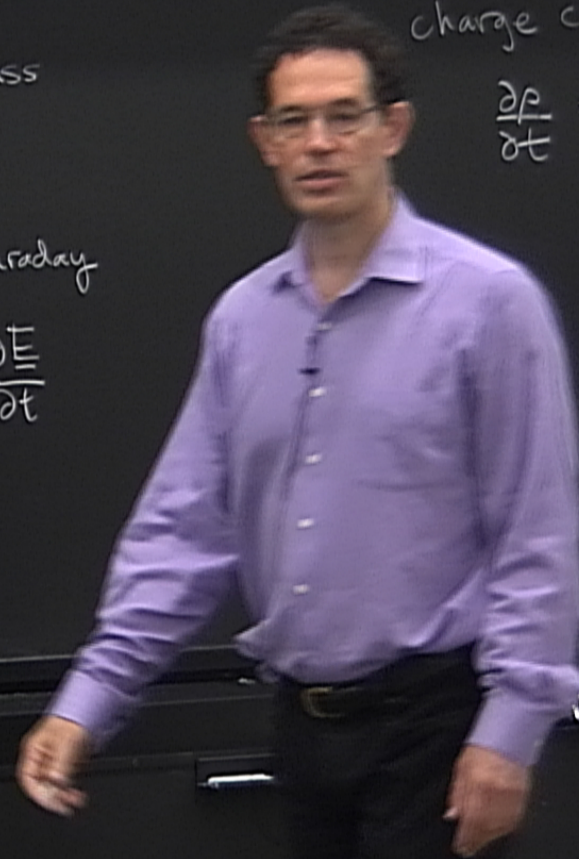
$$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

Faraday

$$\underbrace{\nabla \wedge \underline{B}}_{\text{Ampere}} = \mu_0 \underline{J} + \epsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$$

charge conservation

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \underline{J}$$



1860's

Maxwell Equations

$$\nabla \cdot \underline{E} = \rho / \epsilon_0$$

Gauss

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

Faraday

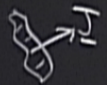
$$\underline{B} = \mu_0 \underline{J} + \epsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$$

Ampere

↑
Maxwell's
displacement
current

+ charge conservation

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \underline{J}$$



$\frac{\partial}{\partial t}$

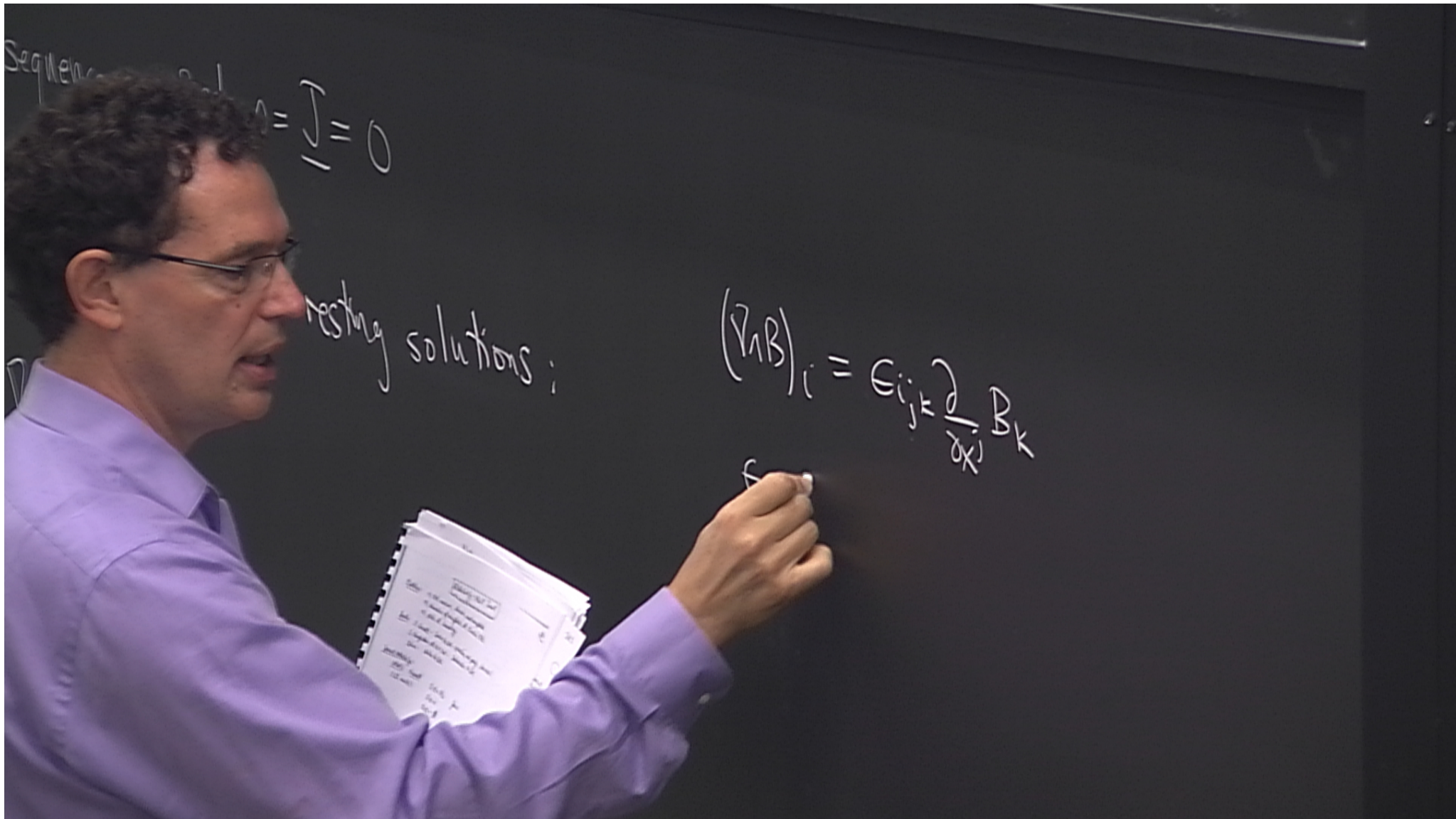
Amazing consequence: set $\rho = \underline{J} = 0$
till h

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still have interesting solutions:

Amazing consequence: set $\rho = \underline{J} = 0$

still have interesting solutions:



Sequence: Set $\rho = \underline{J} = 0$

interesting solutions:

$$(\nabla \cdot B)_i = \epsilon_{ijk} \frac{\partial}{\partial x_j} B_k$$

$$\epsilon_{ijk} \epsilon_{kmn} = \delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}$$

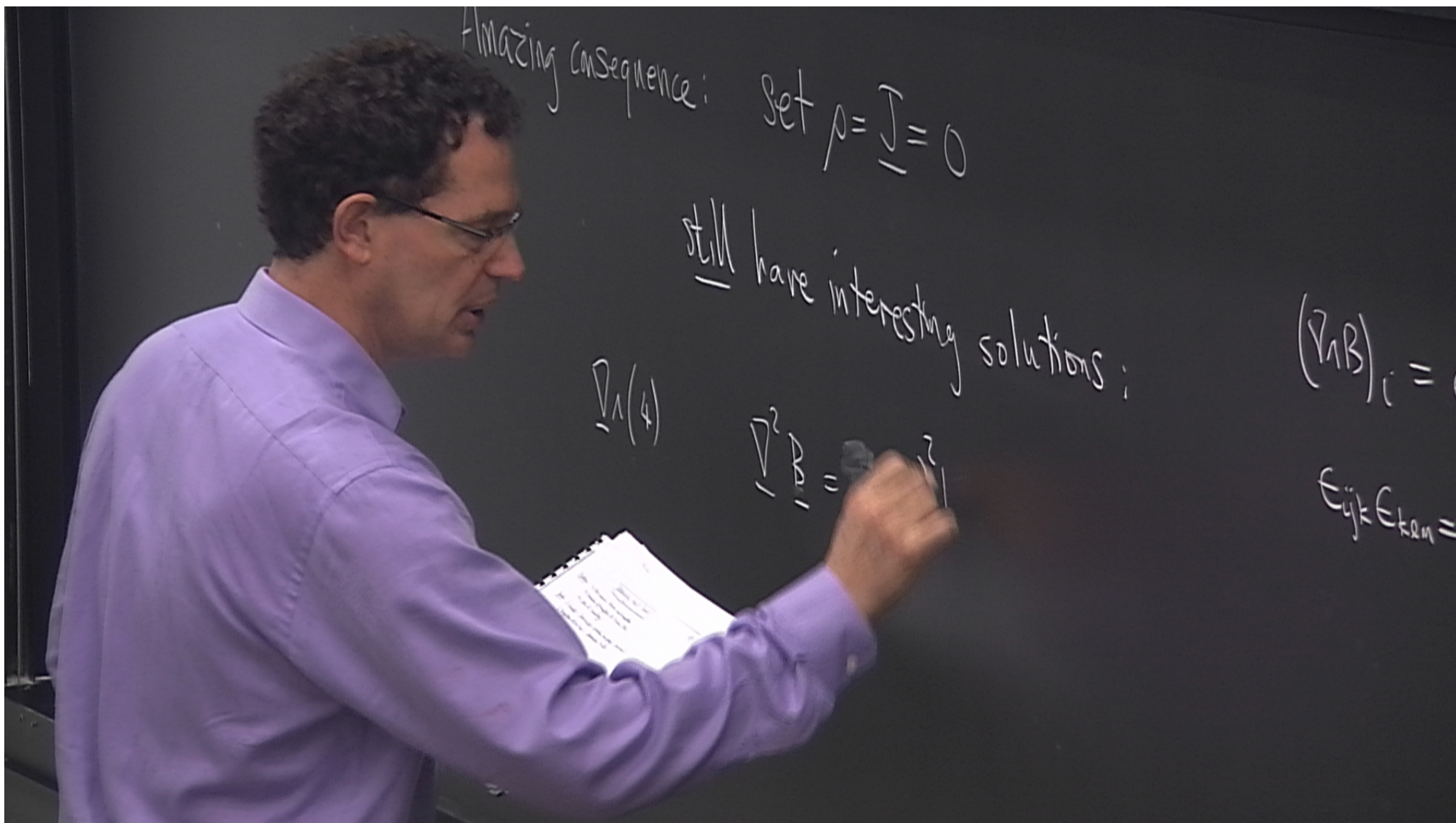
Sequence: Set $\rho = \underline{J} = 0$

still have interesting solutions:

$\underline{V}_1(4)$

$$(\nabla_1 B)_i = \epsilon_{ijk} \frac{\partial}{\partial x^j} B_k$$

$$\epsilon_{ijk} \epsilon_{ken} = \delta_{le} \delta_{jm} - \delta_{im} \delta_{je}$$



Amazing consequence: Set $\rho = \underline{J} = 0$

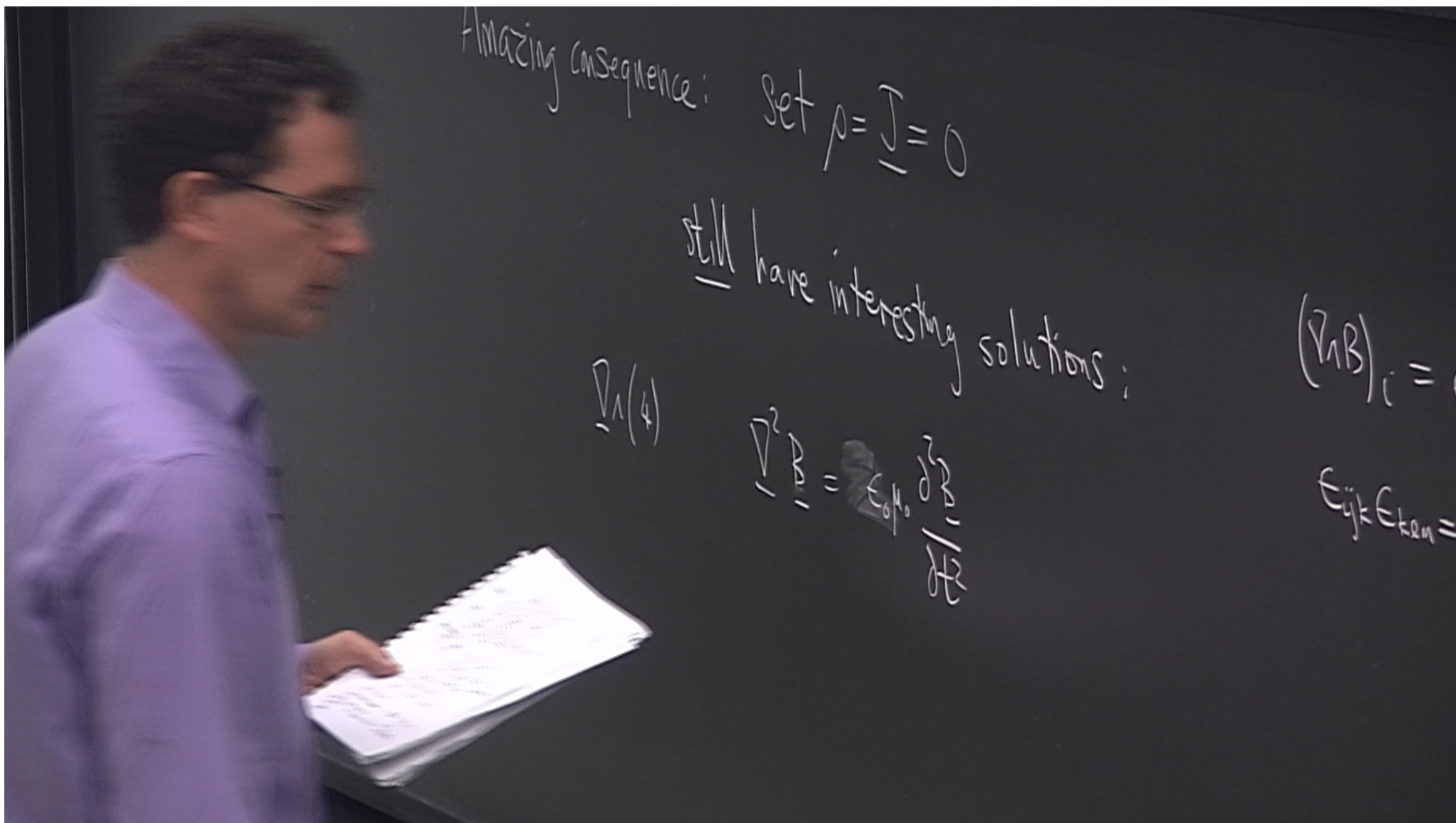
still have interesting solutions:

$$\underline{V}_1(4)$$

$$\underline{V}^2 \underline{B} = \underline{V}^2 \underline{1}$$

$$(\underline{V}_1 B)_i = e$$

$$\epsilon_{ij} \epsilon_{km} =$$



Amazing consequence: Set $\rho = \underline{J} = 0$

still have interesting solutions:

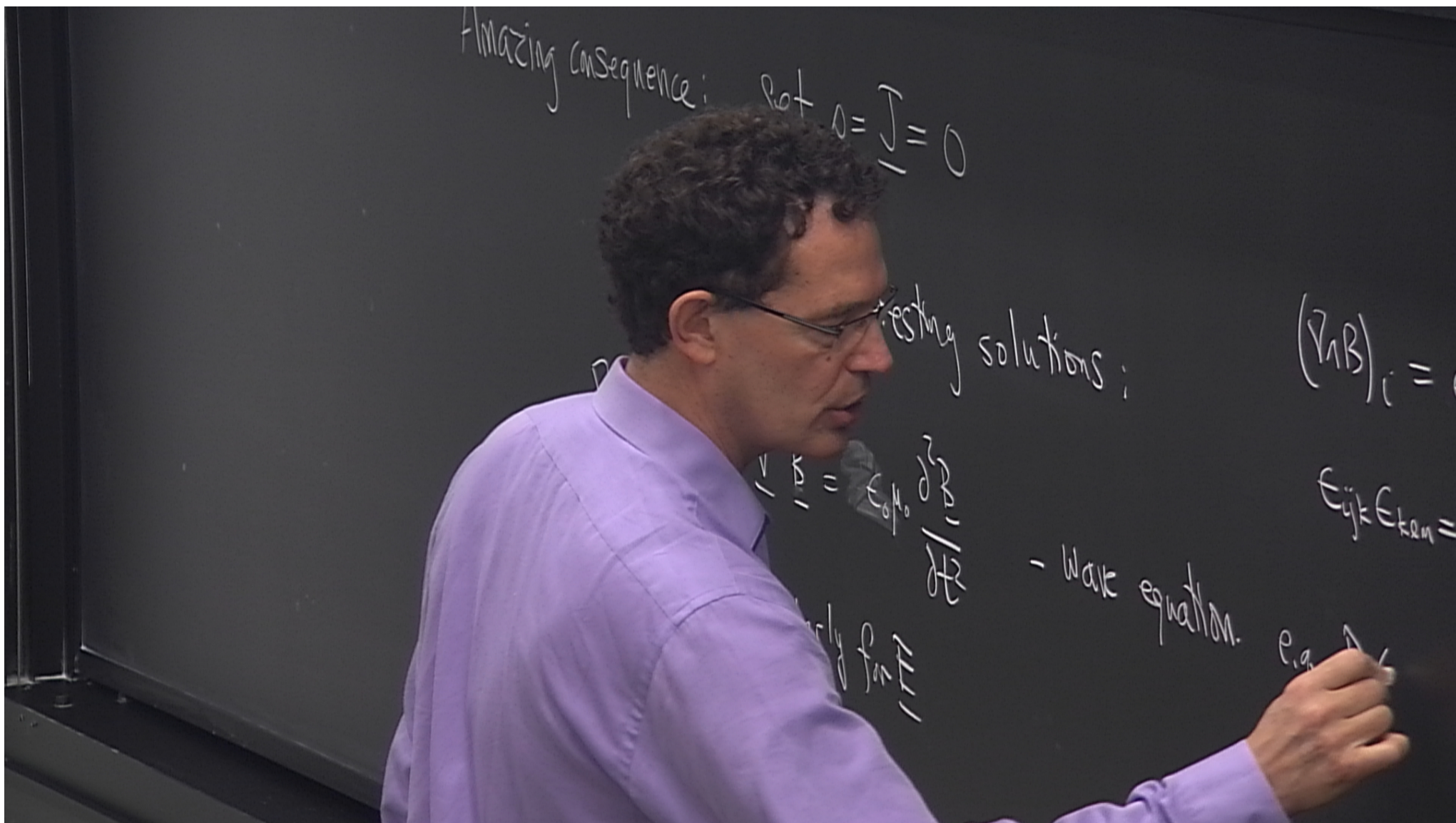
$$\underline{V}_1(4)$$

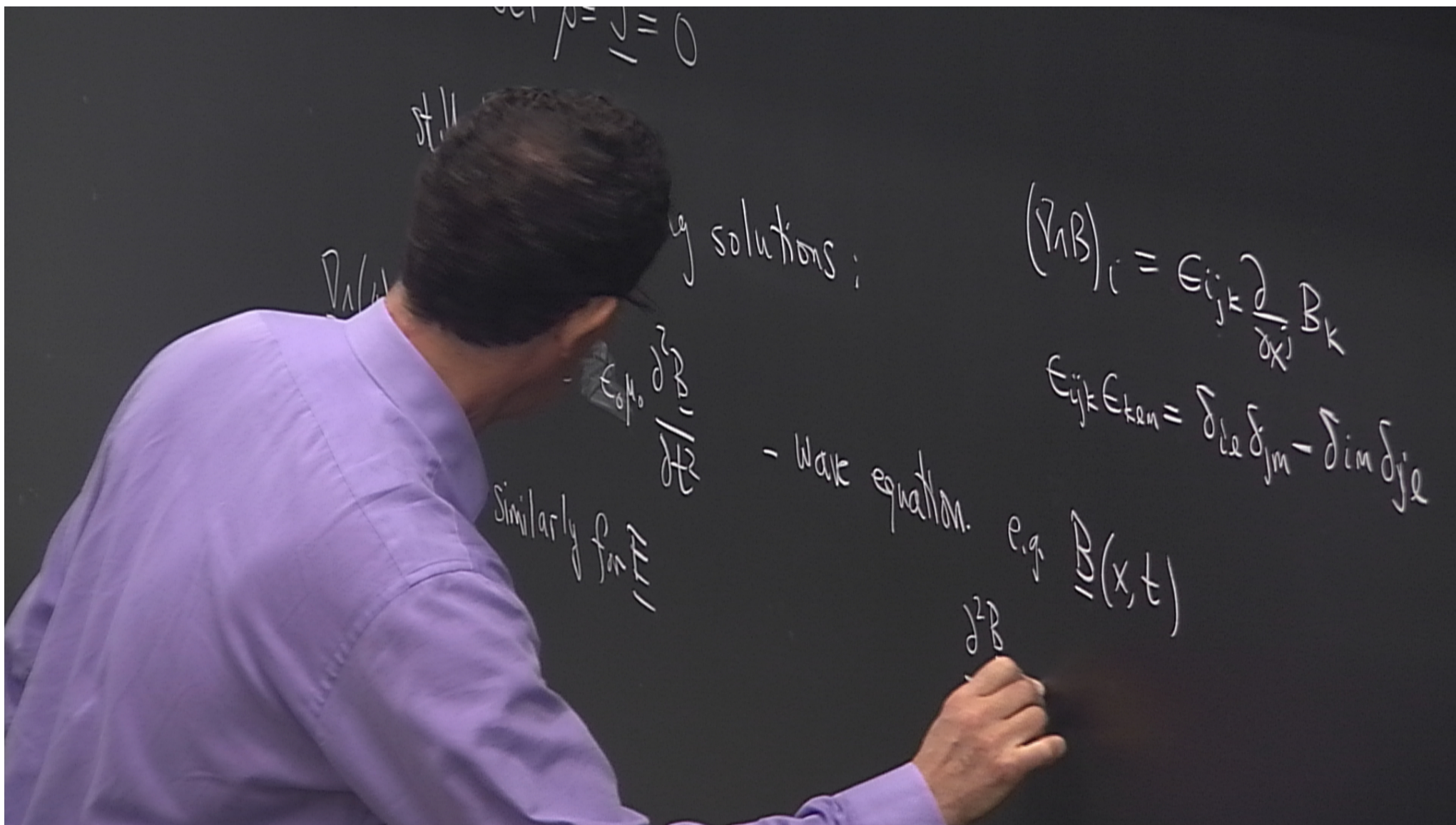
$$\underline{\nabla}^2 \underline{B} = \epsilon_0 \mu_0 \frac{\partial^2 \underline{B}}{\partial t^2}$$

Similarly for $\underline{E}$

$$(\underline{\nabla} \cdot \underline{B})_i = 0$$

$$\epsilon_{ijk} \epsilon_{klm} =$$





$$\rho = \underline{j} = 0$$

still have interesting solutions:

$$\underline{\nabla} \cdot (4)$$

$$\underline{\nabla}^2 \underline{B} = \epsilon_0 \mu_0 \frac{\partial^2 \underline{B}}{\partial t^2}$$

Similarly for $\underline{E}$

- wave equation.

$$(\underline{\nabla} \cdot \underline{B})_i = \epsilon_{ijk} \frac{\partial}{\partial x_j} B_k$$

$$\epsilon_{ijk} \epsilon_{kmn} = \delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}$$

e.g. $\underline{B}(x, t)$

$$\frac{\partial^2 \underline{B}}{\partial t^2} = \frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 \underline{B}}{\partial x^2}$$

solutions:

$$(\nabla \wedge \underline{B})_i = \epsilon_{ijk} \frac{\partial}{\partial x_j} B_k$$

$$\epsilon_{ijk} \epsilon_{kmn} = \delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}$$

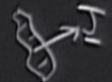
- wave equation. e.g. $\underline{B}(x, t)$

$$\frac{\partial^2 \underline{B}}{\partial t^2} = \frac{1}{\epsilon \mu_0} \frac{\partial^2 \underline{B}}{\partial x^2}$$

$$\Rightarrow \underline{B} = \underline{f}(x - ct) + \underline{g}(x + ct)$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

1860's



$$\frac{\partial}{\partial t} \epsilon_{ijk} \frac{\partial}{\partial x_j} B_k$$

solutions:

$$(\nabla \wedge \underline{B})_i = \epsilon_{ijk} \frac{\partial}{\partial x_j} B_k$$

$$\epsilon_{ijk} \epsilon_{kmn} = \delta_{in} \delta_{jm} - \delta_{im} \delta_{jn}$$

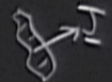
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$$\Rightarrow \underline{B} = \underline{f}(x - ct) + \underline{g}(x + ct)$$

1860's



$$\epsilon_{ijk} \frac{\partial}{\partial x_j} B_k$$

Amazing consequence: $v \rightarrow 0 \Rightarrow \underline{J} = 0$

still

interesting solutions:

$\nabla \cdot (\underline{A})$

$$= \epsilon_0 \mu_0 \frac{\partial^2 \underline{E}}{\partial t^2}$$

- wave equation

$$(\nabla^2 \underline{A})_i = \epsilon_0 \mu_0 \frac{\partial^2 \underline{A}_i}{\partial t^2}$$

$$\epsilon_0 \mu_0 \nabla^2 \underline{A} = \epsilon_0 \mu_0 \nabla^2 \underline{A} - \epsilon_0 \mu_0 \nabla^2 \underline{A}$$

$$\frac{\partial^2 \underline{B}}{\partial t^2} = \frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 \underline{B}}{\partial x^2} \Rightarrow \underline{B} = \underline{f}(x - ct) + \underline{g}(x + ct)$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Relativity

- 1) Special Relativity, Spacetime, Metrics
- 2) Curvature, Einstein's Equation of general relativity
- 3) Black holes and cosmology

1860s

Maxwell Equations

$$(1) \nabla \cdot \underline{E} = \rho / \epsilon_0 \quad \text{Gauss}$$

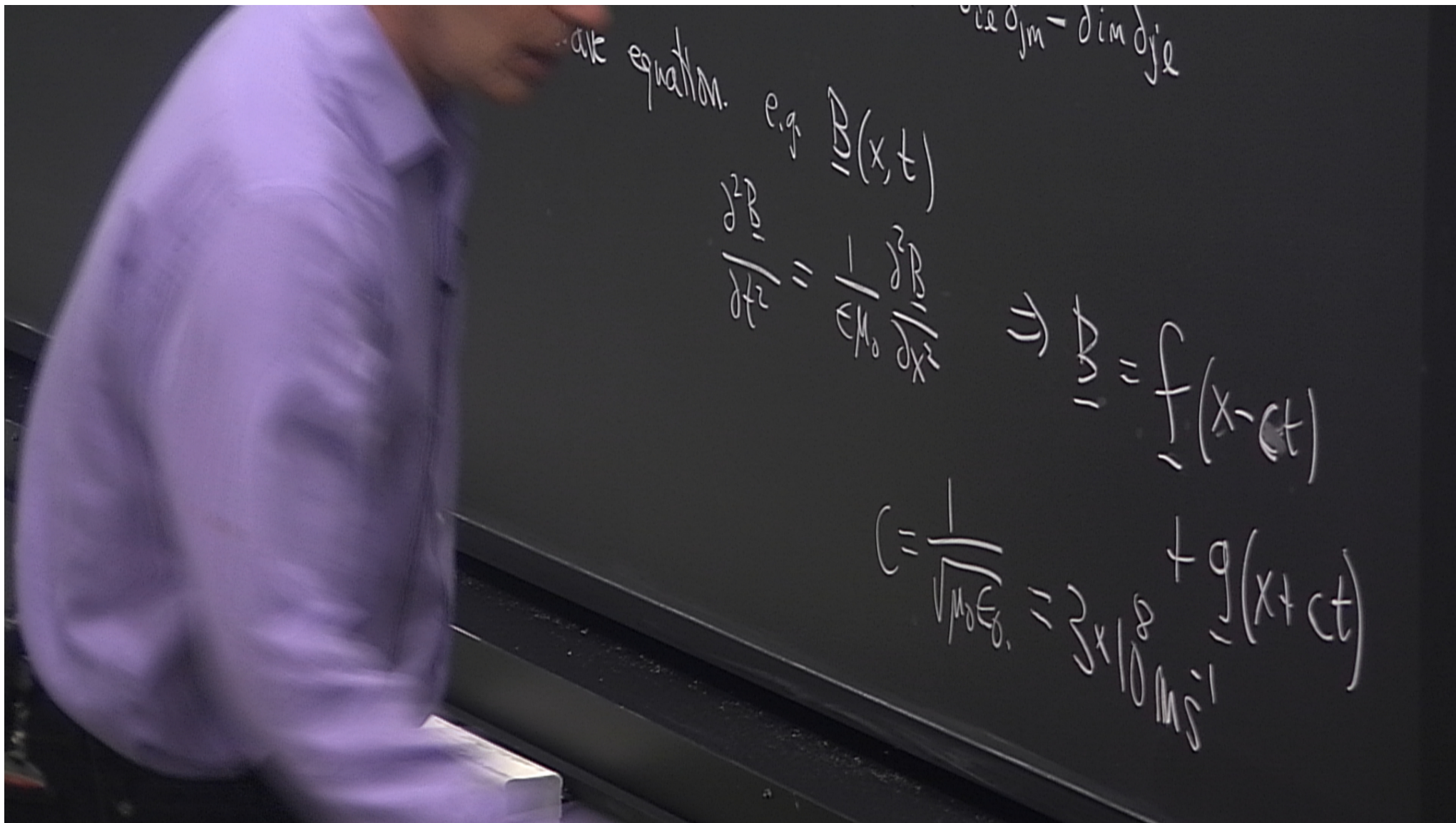
$$(2) \nabla \cdot \underline{B} = 0$$

$$(3) \nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t} \quad \text{Faraday}$$

$$(4) \nabla \wedge \underline{B} = \mu_0 \underline{J} + \epsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$$

Ampere

Maxwell's displacement current

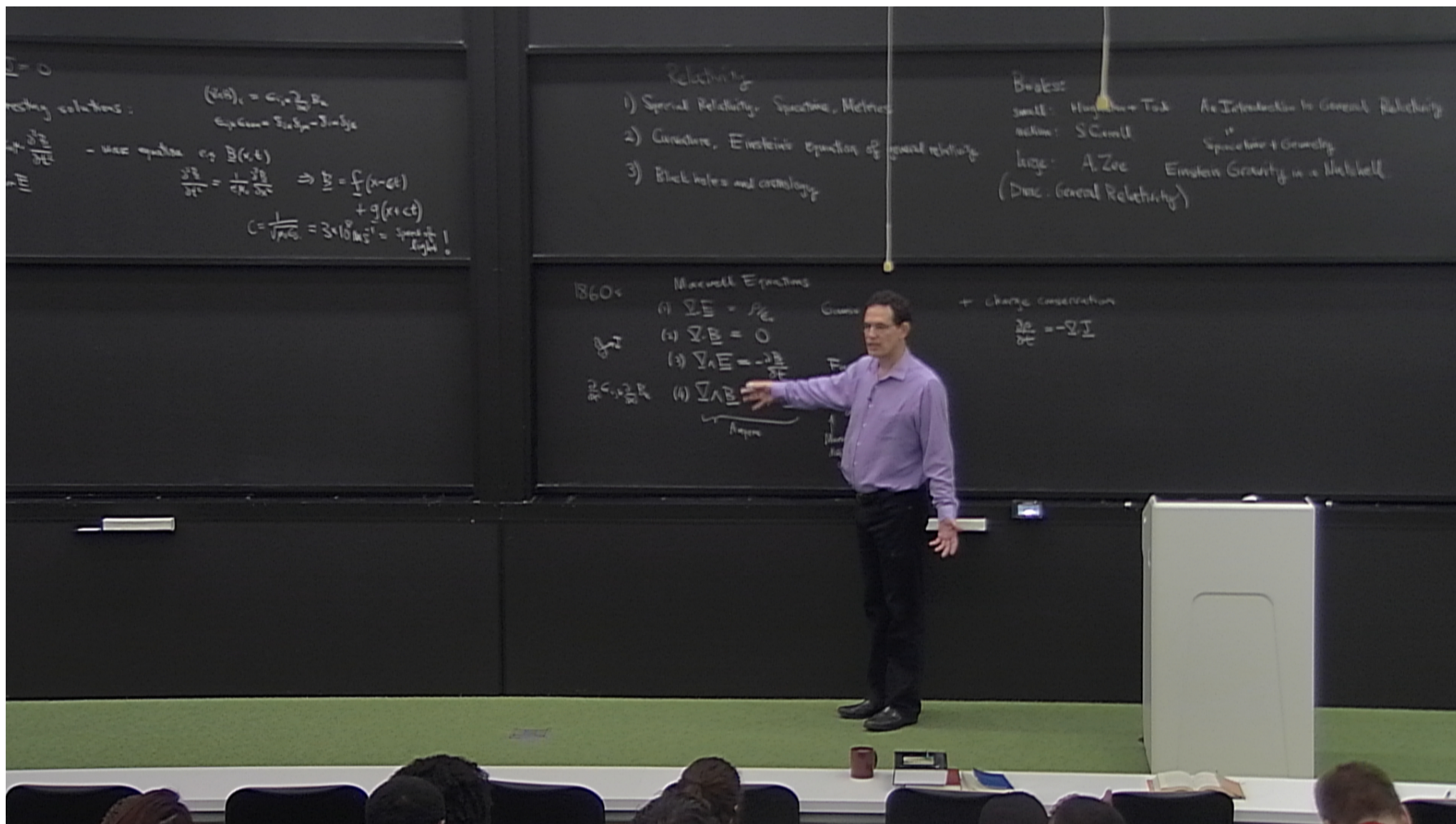


$$\frac{\partial^2 B}{\partial t^2} = \frac{1}{\epsilon \mu_0} \frac{\partial^2 B}{\partial x^2} \Rightarrow B = f(x - ct)$$

$$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m s}^{-1} = \text{Speed of light}$$

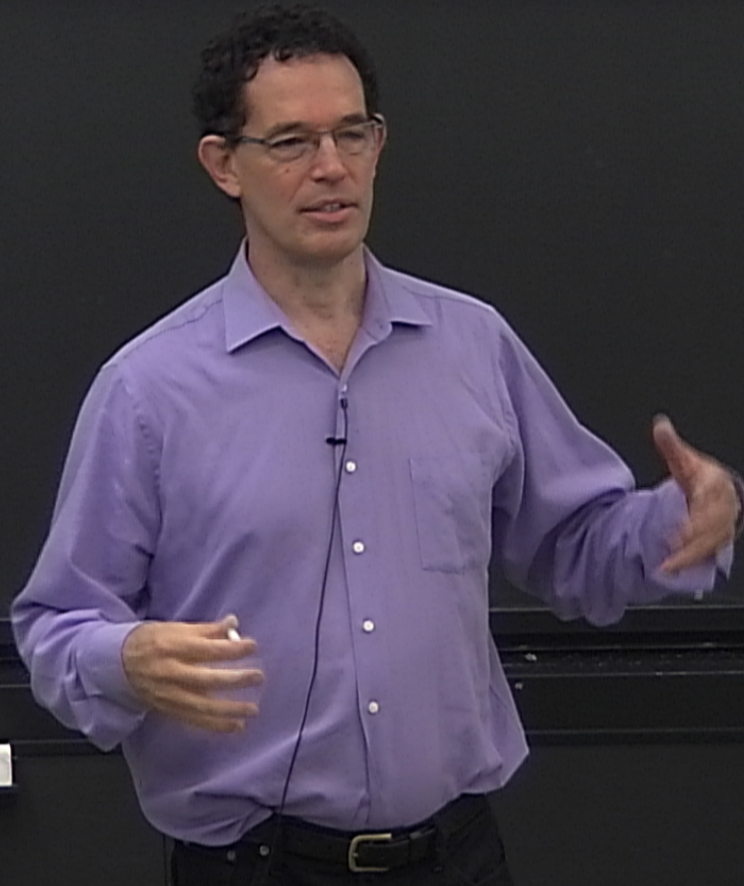
$$B = f(x - ct)$$

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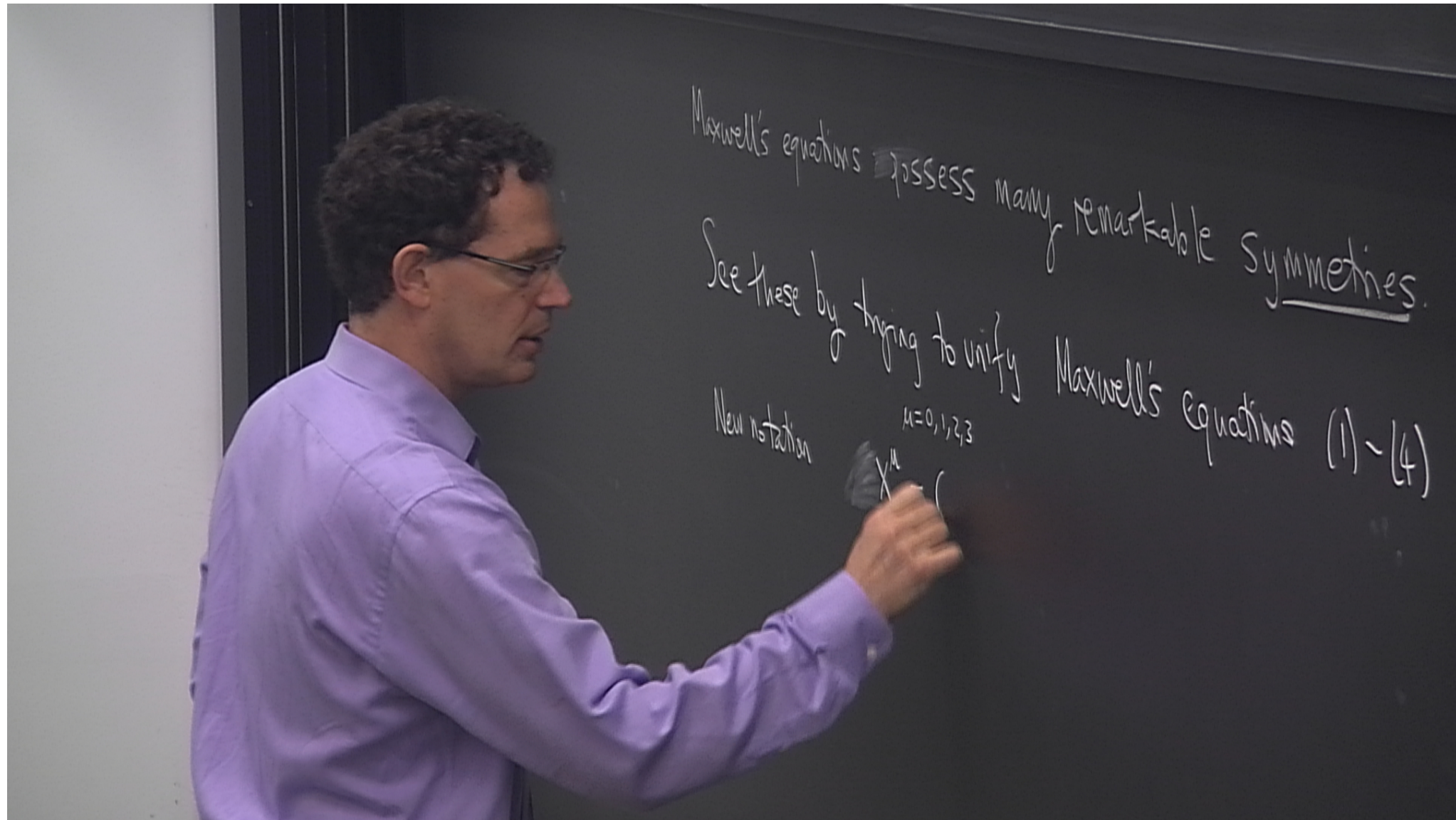
Maxwell's equations possess many remarkable

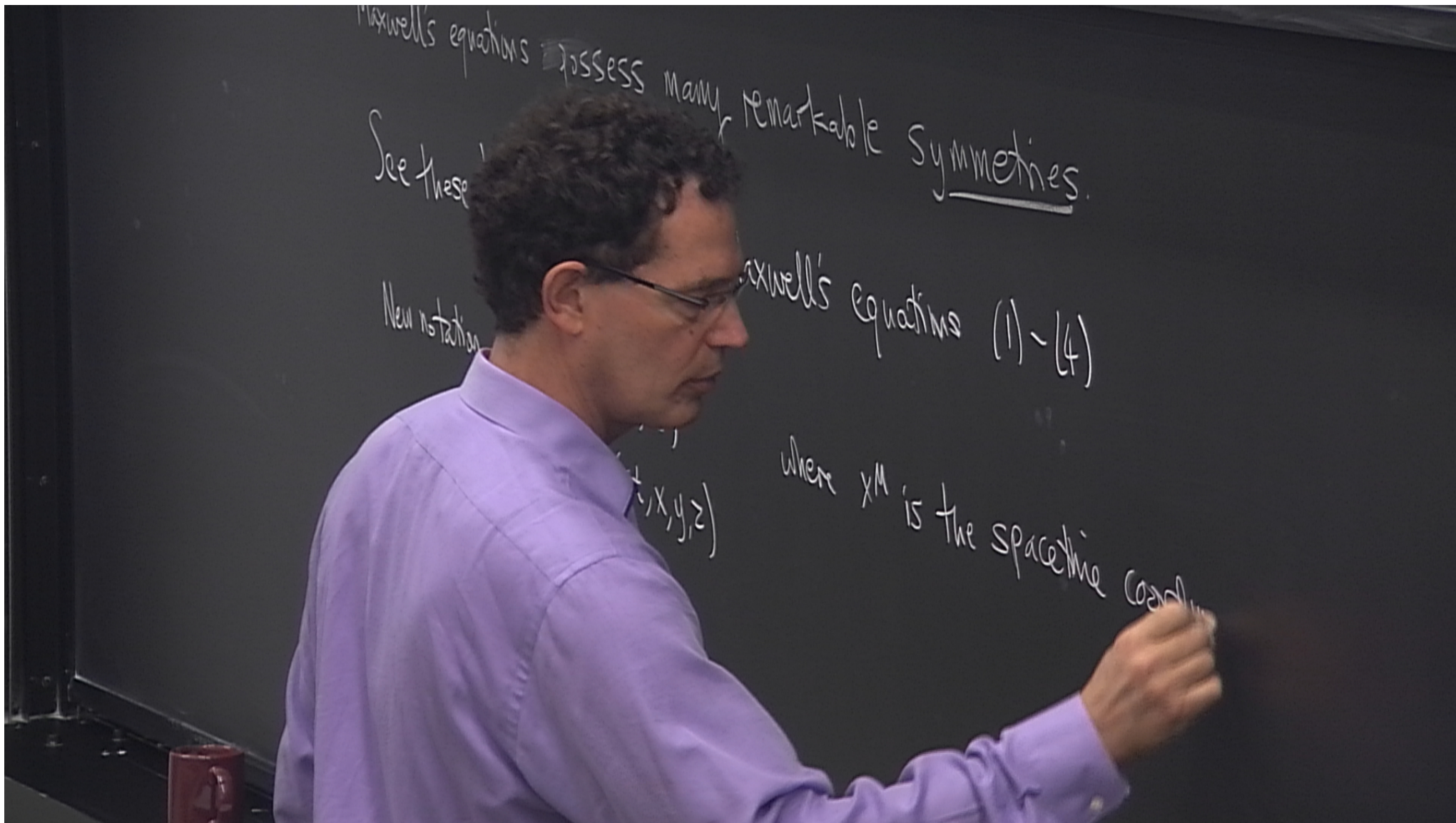
Maxwell's equations possess many remarkable symmetries.

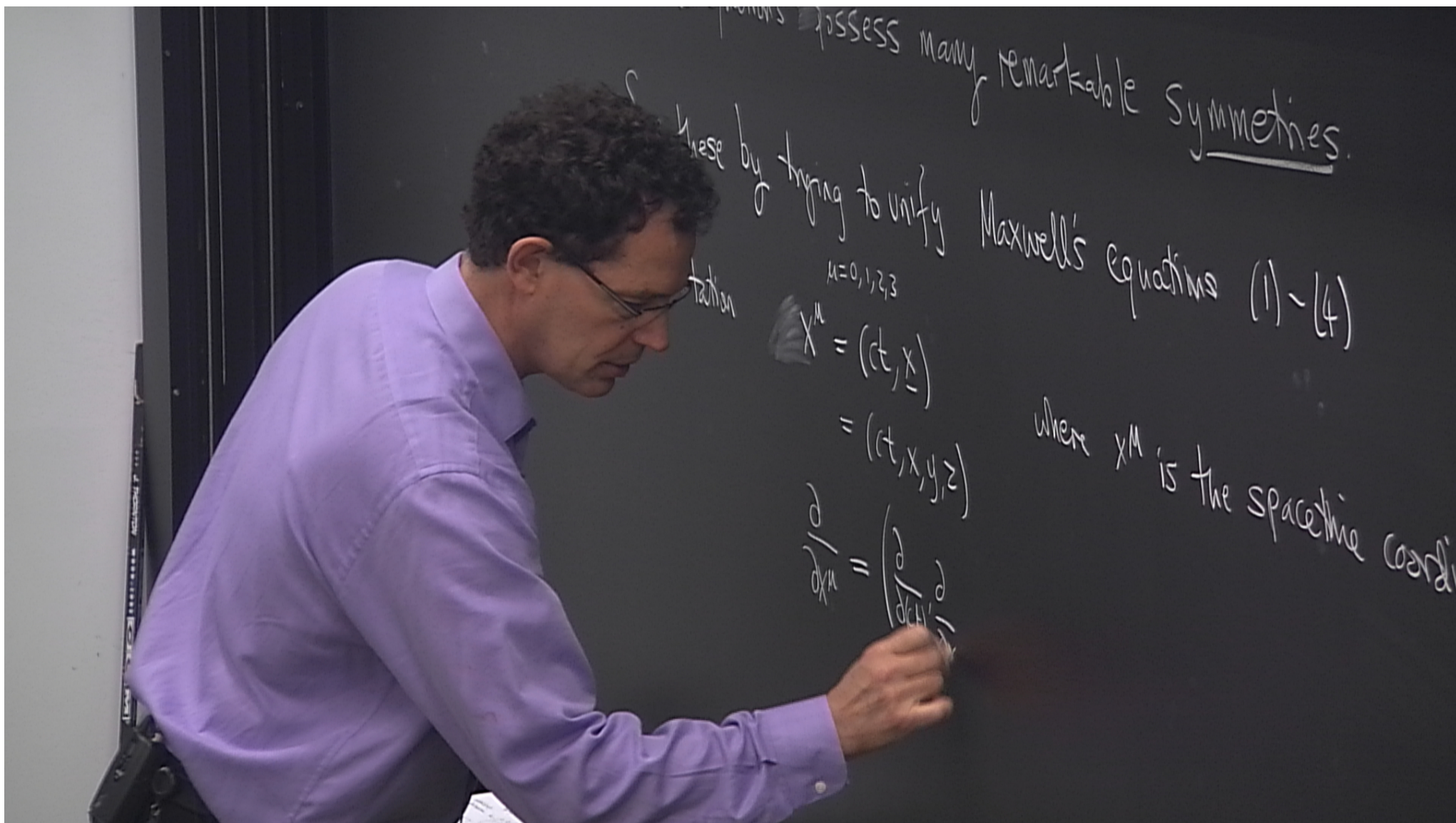


Maxwell's equations possess many remarkable symmetries.

See these by trying to unify Maxwell's equations (1)-(4)







Maxwell's equations possess many remarkable symmetries.

See these by trying to unify Maxwell's equations (1)-(4)

New notation

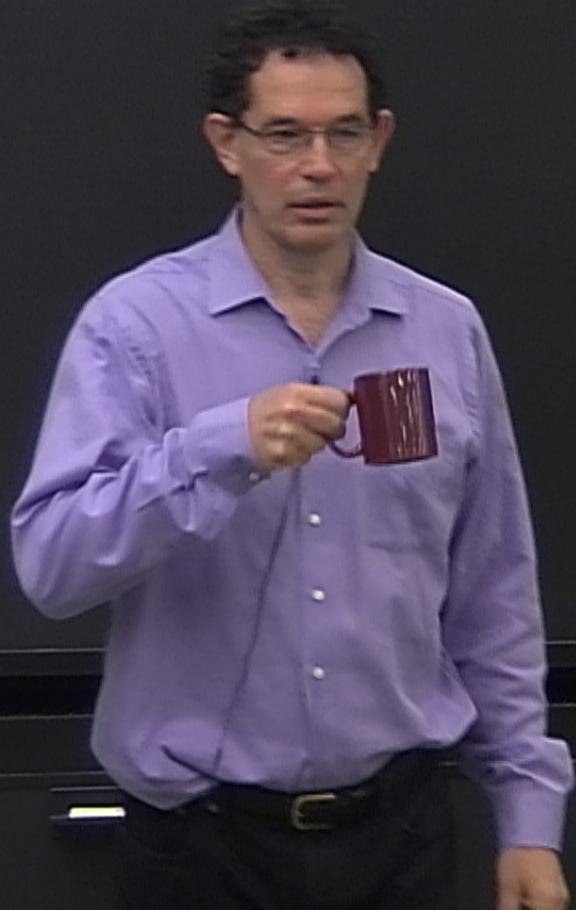
$$x^\mu = (ct, \underline{x})$$
$$= (ct, x, y, z)$$

where x^μ is the spacetime coord

$$\frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial ct}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

Need to introduce

$$\eta^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



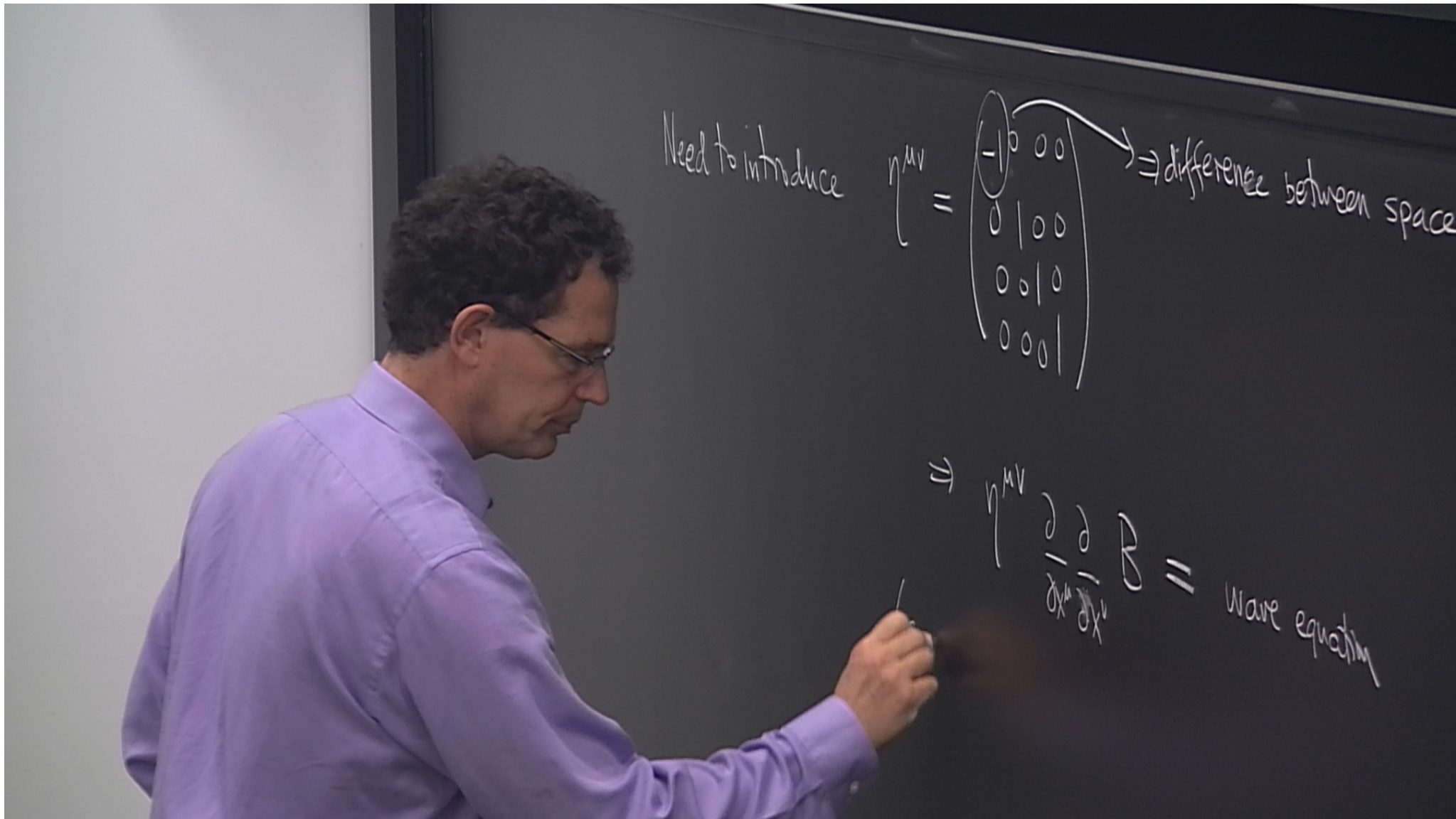
nce $\eta^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow \text{difference between space and time}$

Need to introduce

$$\eta^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow$ difference between space

$$\Rightarrow \eta^{\mu\nu}$$

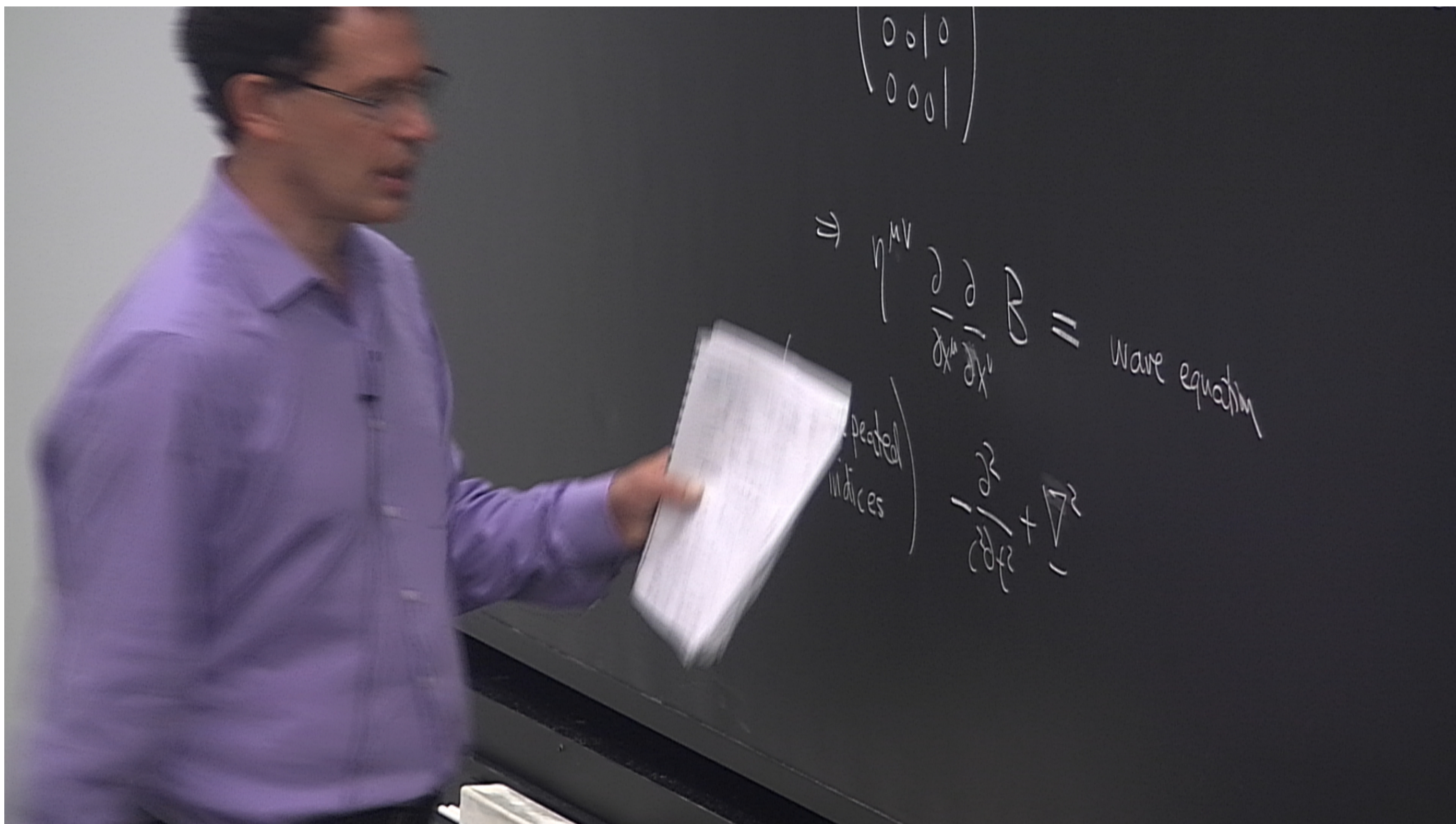


Need to introduce

$$\eta^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow \text{difference between space and time}$$

$$\Rightarrow \eta^{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x^\nu} B = \text{wave equation}$$

(Sum repeated indices)



Need to introduce

$$\eta^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow \text{difference between space and time}$$

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$$\left(\begin{array}{c} \text{Sum repeated} \\ \text{indices} \end{array} \right) - \frac{\partial^2}{c^2 \partial t^2} + \nabla^2$$

$$\Rightarrow \eta^{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x^\nu} B = \text{wave equation}$$

(Sum repeated indices)

$$\left(\frac{\partial^2}{c^2 \partial t^2} + \nabla^2 \right) B$$

Similarly, $j^\mu =$

Amazing consequence: set $\rho = \underline{J} = 0$

still have interesting solutions:

$$\nabla \cdot (\underline{E}) \quad \nabla^2 \underline{B} = -\epsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t} \quad \text{wave equation e.g. } \underline{B}(x,t)$$

Similarly $\nabla \cdot \underline{B} = 0$

$$(\nabla \times \underline{B})_i = \epsilon_{ijk} \frac{\partial}{\partial x_j} B_k$$

$$\epsilon_{ijk} \epsilon_{ilm} = \delta_{jl} \delta_{km} - \delta_{jm} \delta_{kl}$$

$$\frac{\partial \underline{B}}{\partial t} = \frac{1}{\epsilon_0 \mu_0} \nabla \times \underline{B} \Rightarrow \underline{B} = \underline{f}(x-ct) + \underline{g}(x+ct)$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m s}^{-1} = \text{Speed of light!}$$

Need to introduce $\eta^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow \text{difference between space and time}$

$$\Rightarrow \eta^{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x^\nu} \underline{B} = \text{wave equation}$$

(Sum repeated indices) $\left(-\frac{\partial^2}{\partial t^2} + \nabla^2 \right) \underline{B}$

Similarly, $J^\mu = (\rho, \underline{J})$

Relativity

- 1) Special Relativity, Spacetime,
- 2) Curvature, Einstein's equation
- 3) Black holes and cosmology

1860s

Maxwell Equations

$$(1) \nabla \cdot \underline{E} = \rho / \epsilon_0 \quad \text{Gauss}$$

$\nabla \times$

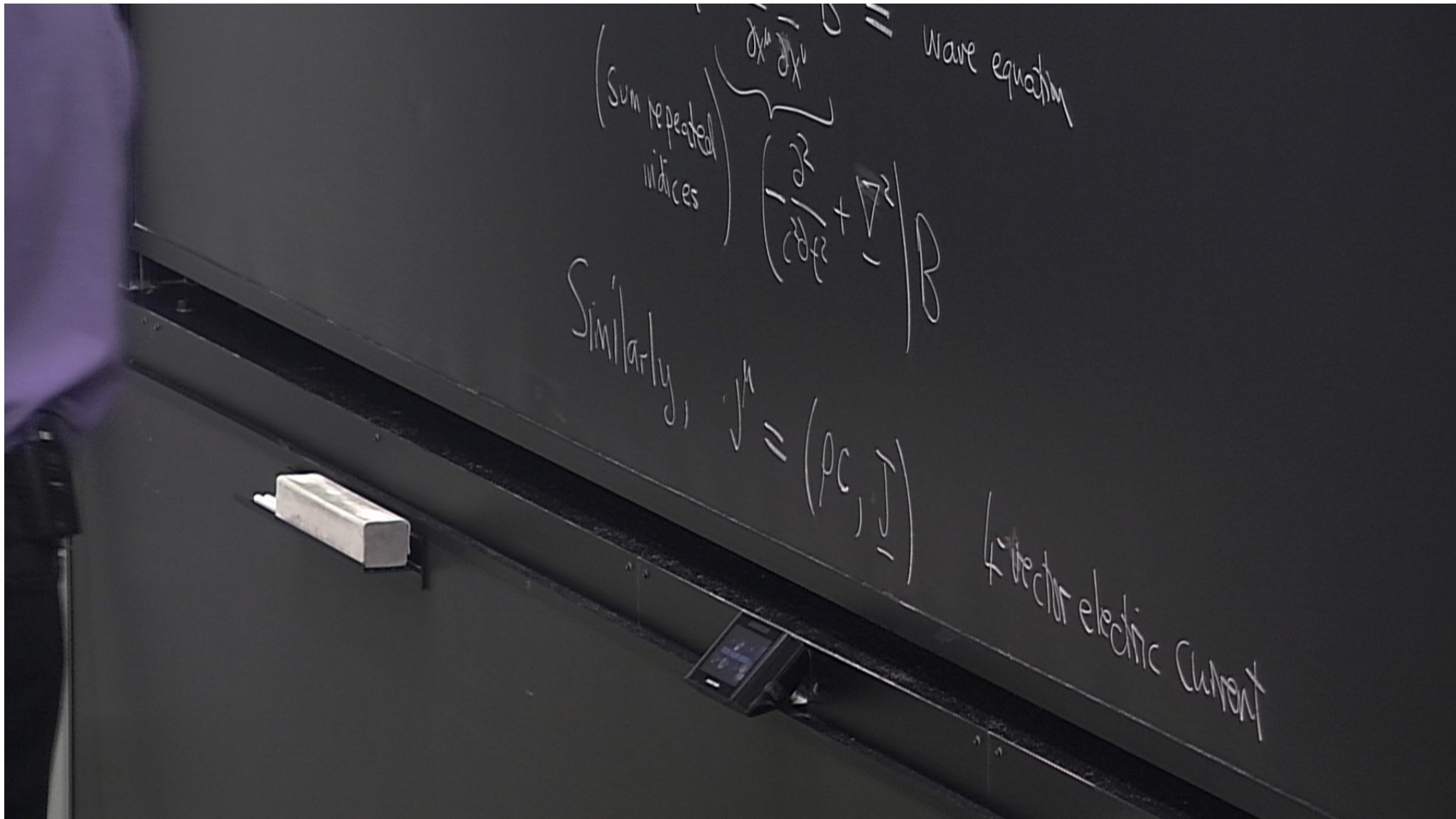
$$(2) \nabla \cdot \underline{B} = 0$$

$$(3) \nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} \quad \text{Faraday}$$

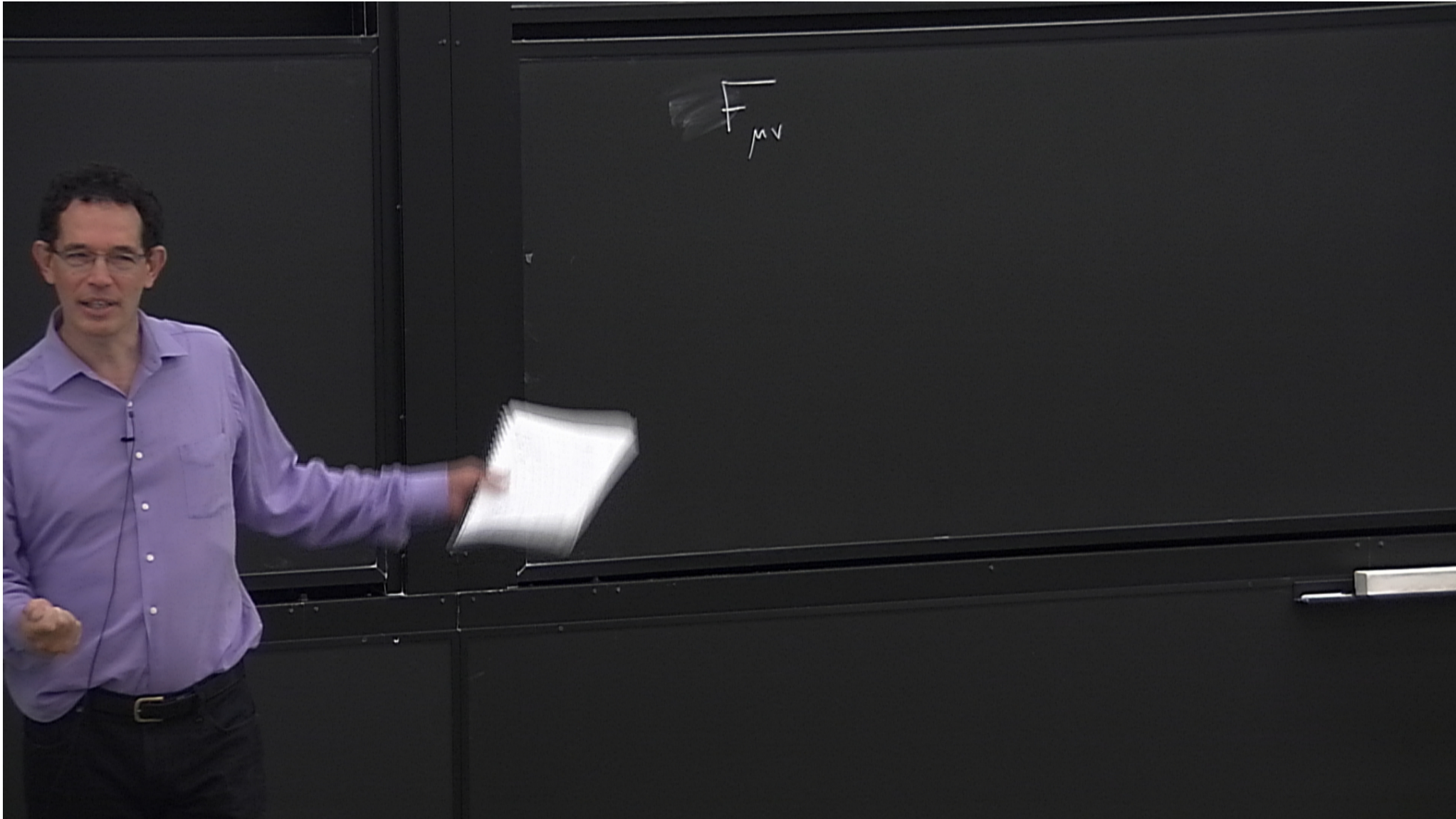
$$\frac{\partial}{\partial t} \epsilon_0 \mu_0 \underline{B} \quad (4) \nabla \times \underline{B} = \mu_0 \underline{J} + \epsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$$

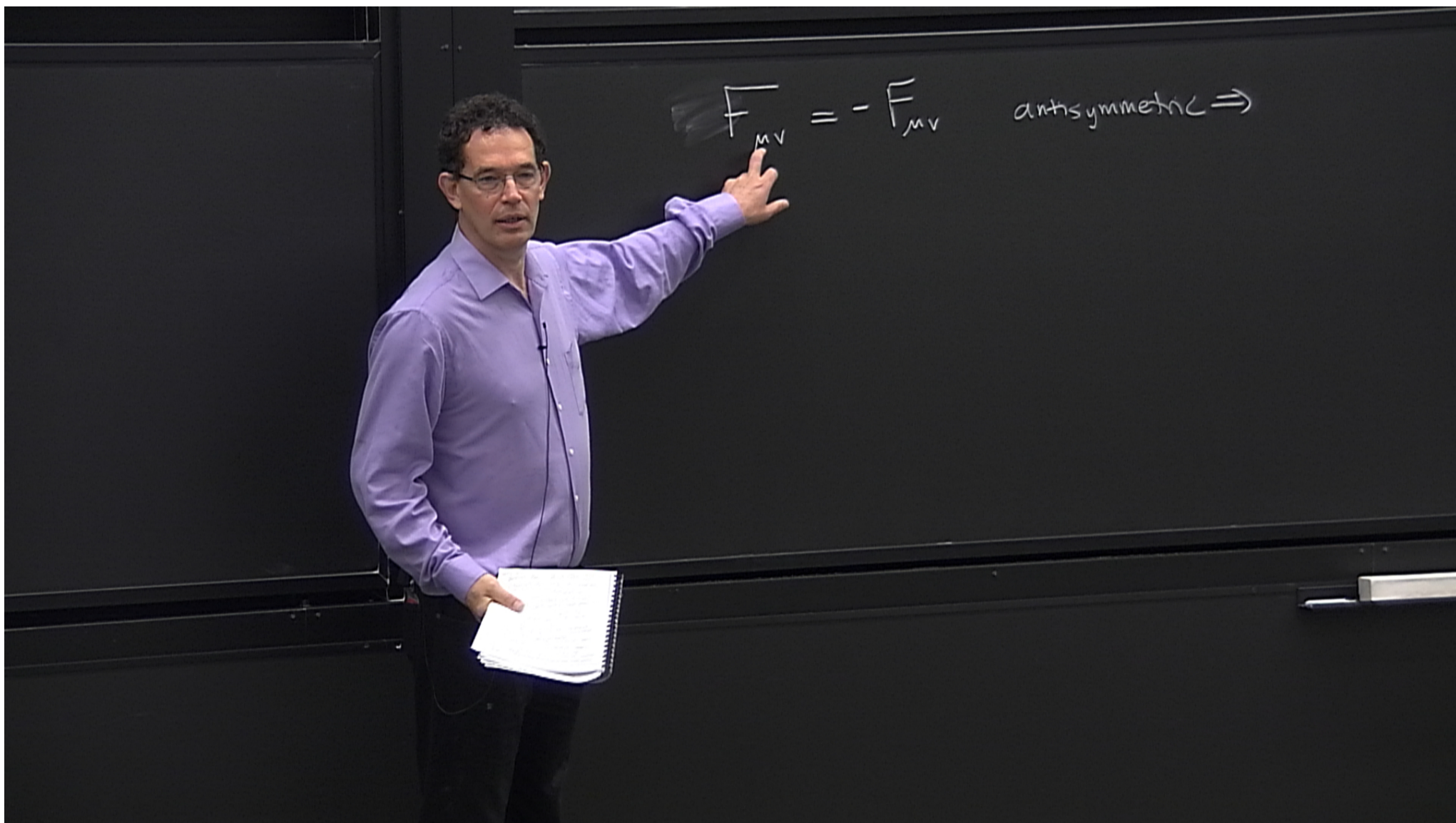
Ampere

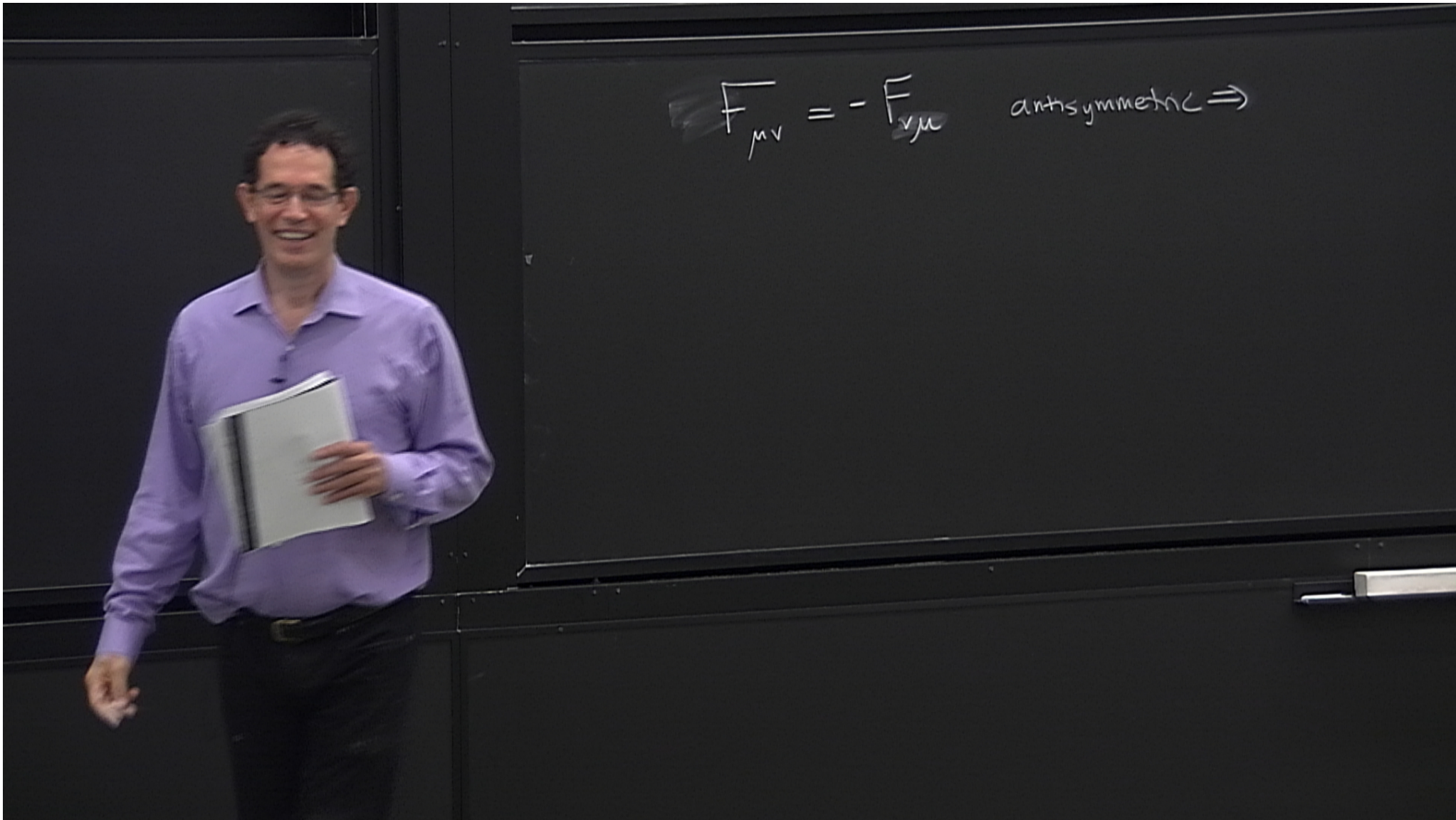
Maxwell
displacement
current

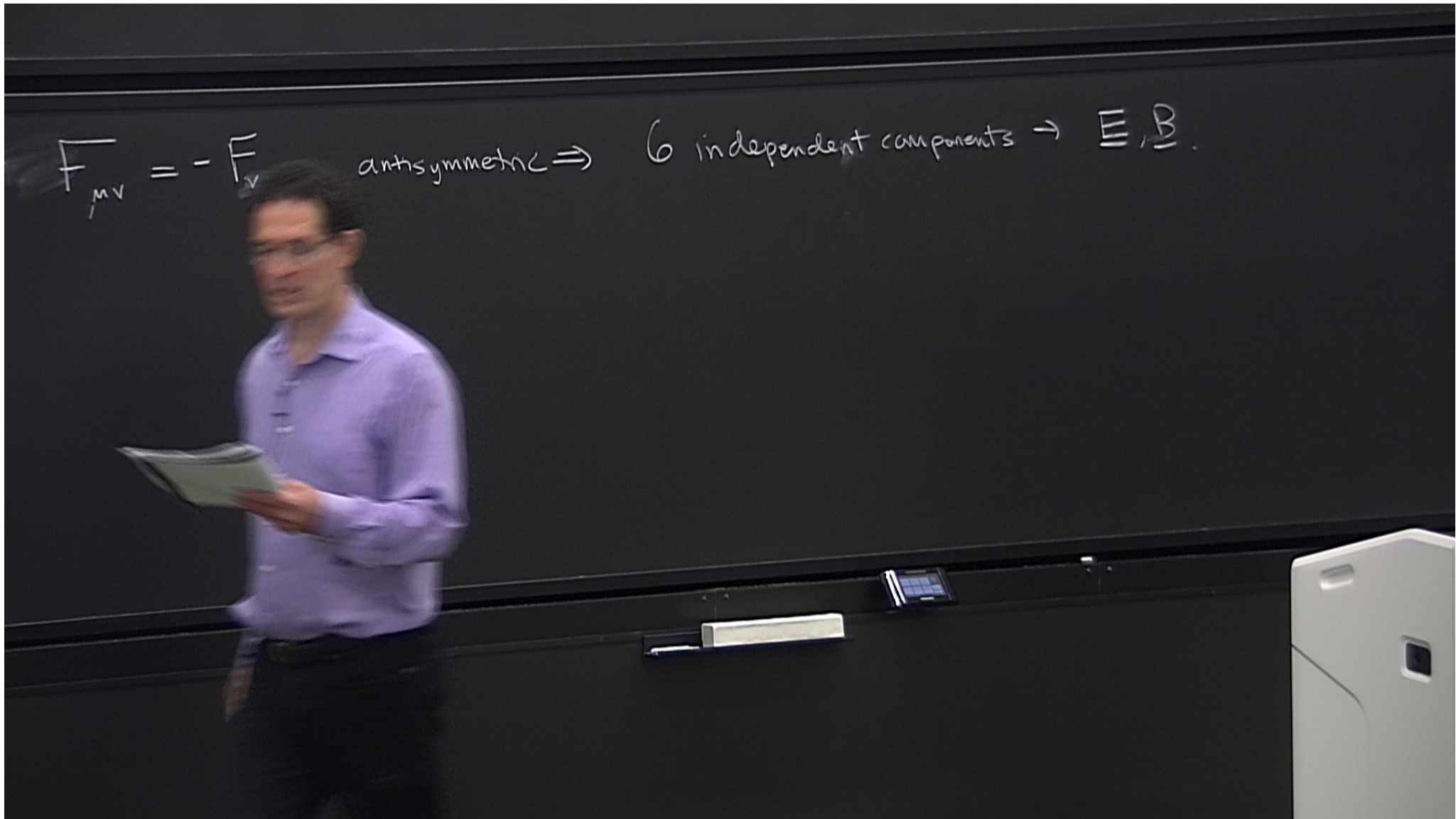












$$F_{\mu\nu} = -F_{\nu\mu} \quad \text{antisymmetric} \Rightarrow 6 \text{ independent components} \rightarrow \underline{E}, \underline{B}.$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ 0 & 0 & B_z & -B_y \\ 0 & 0 & 0 & B_x \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$F_{\mu\nu} = -F_{\nu\mu} \quad \text{antisymmetric} \Rightarrow 6 \text{ independent components} \rightarrow \underline{E}, \underline{B}.$$

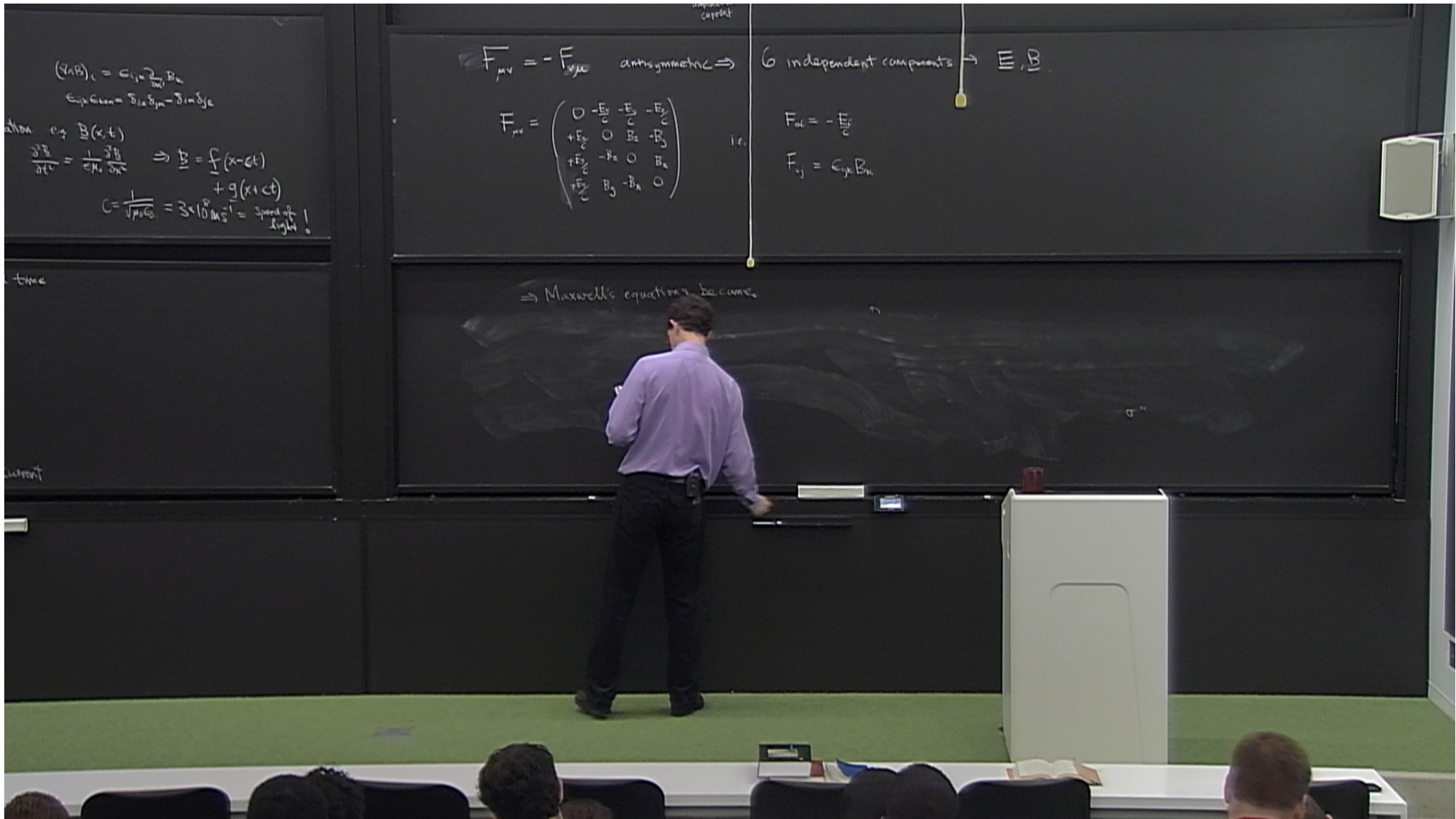
$$F_{\mu\nu} = \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ +\frac{E_x}{c} & 0 & B_z & -B_y \\ +\frac{E_y}{c} & -B_z & 0 & B_x \\ +\frac{E_z}{c} & B_y & -B_x & 0 \end{pmatrix}$$

$$F_{\mu\nu} = -F_{\nu\mu} \quad \text{antisymmetric} \Rightarrow 6 \text{ independent components} \rightarrow \underline{E}, \underline{B}.$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ +\frac{E_x}{c} & 0 & B_z & -B_y \\ +\frac{E_y}{c} & -B_z & 0 & B_x \\ +\frac{E_z}{c} & B_y & -B_x & 0 \end{pmatrix}$$

i.e.

$$F_{0i} = -\frac{E_i}{c}$$



$$(\nabla \times \mathbf{B})_i = \epsilon_{ijk} \frac{\partial B_k}{\partial x_j}$$
$$\epsilon_{ijk} \epsilon_{lmn} = \delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}$$

atom $\mathbf{B}(\mathbf{x}, t)$

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{1}{\epsilon_0 \mu_0} \nabla \times \mathbf{B} \Rightarrow \mathbf{B} = \frac{\mathbf{f}}{c}(\mathbf{x} - ct) + \mathbf{g}(\mathbf{x} + ct)$$
$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8 \text{ m s}^{-1} = \text{Speed of light!}$$

$$F_{\mu\nu} = -F_{\nu\mu} \text{ antisymmetric} \Rightarrow$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ +E_x/c & 0 & B_z & -B_y \\ +E_y/c & -B_z & 0 & B_x \\ +E_z/c & B_y & -B_x & 0 \end{pmatrix}$$

6 independent components $\rightarrow \mathbf{E}, \mathbf{B}$

$$F_{0i} = -E_i/c$$

$$F_{ij} = \epsilon_{ijk} B_k$$

$\Rightarrow$ Maxwell's equations become

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$$\partial_\mu A^\mu = -\mu_0 j^\nu$$

$$\partial_\mu F^{\mu\nu} = \mu_0 j^\nu$$

$\Rightarrow$ Maxwell equations become

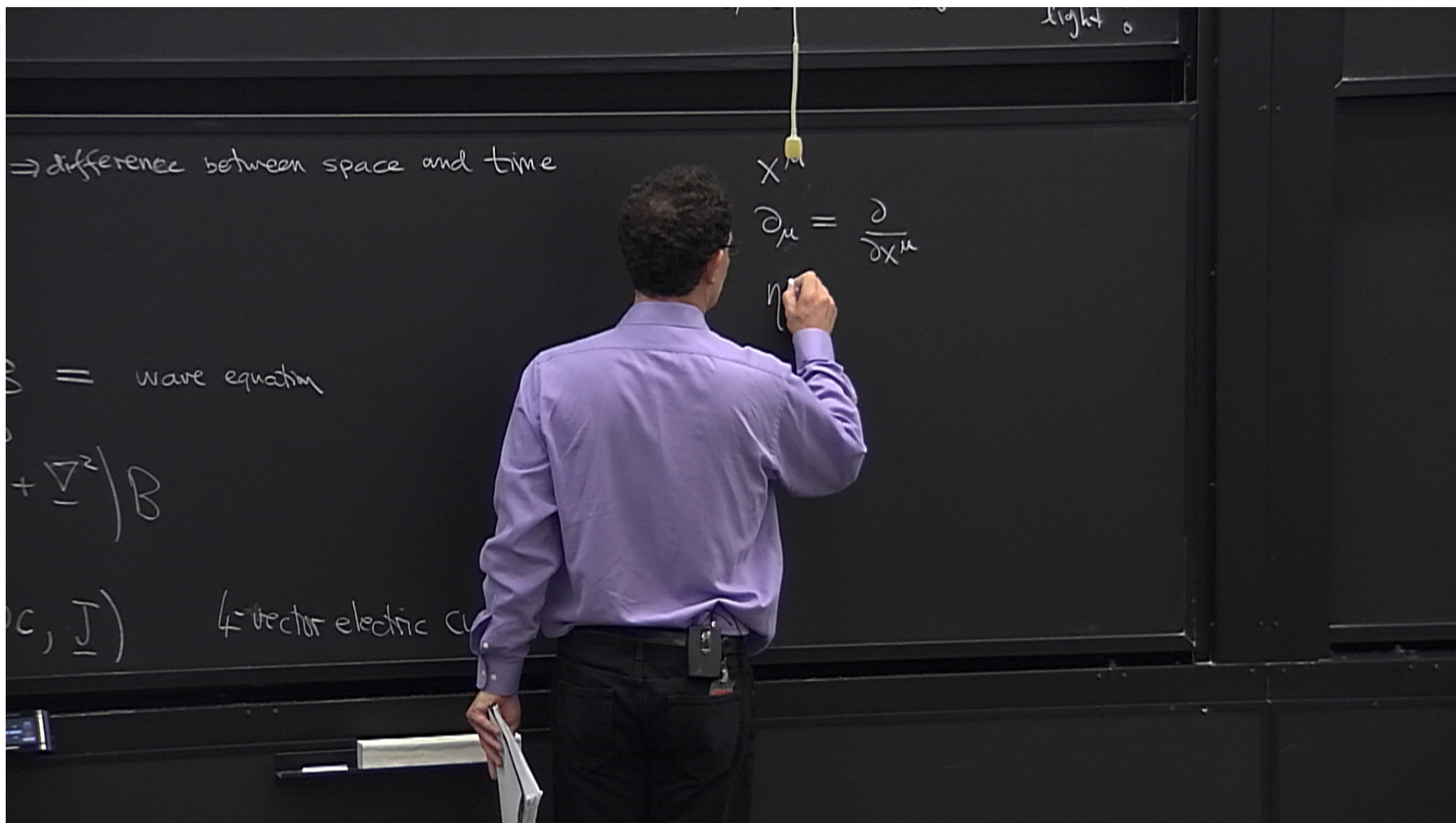
$$\partial_\mu F^{\mu\nu} = -\mu_0 j^\nu$$

$$\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0$$

$\Rightarrow$ Maxwell's equations become

$$(1),(4) \Rightarrow \partial_\mu F^{\mu\nu} = -\mu_0 j^\nu$$

$$(2),(3) \Rightarrow \partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0$$



$\Rightarrow$ difference between space and time

$\square =$ wave equation

$$+\nabla^2)B$$

$(c, \underline{J})$ 4-vector electric current

$$x^\mu$$

$$\partial_\mu =$$

$$\eta^{\mu\nu}$$

$\eta^{\mu\nu}$ defined to be
inverse of

$\Rightarrow$ difference between space and time

x^μ

$$\partial_\mu = \frac{\partial}{\partial x^\mu}$$

$\eta^{\mu\nu}$

$\eta_{\mu\nu}$ defined to be
inverse of
 $\eta^{\mu\nu}$

$$\eta_{\alpha\beta} = \delta_{\beta}^{\alpha}$$

$\square =$ wave equation

$$+\nabla^2)B$$

$(c, \underline{J})$

4-vector elect

$\Rightarrow$ difference between space and time

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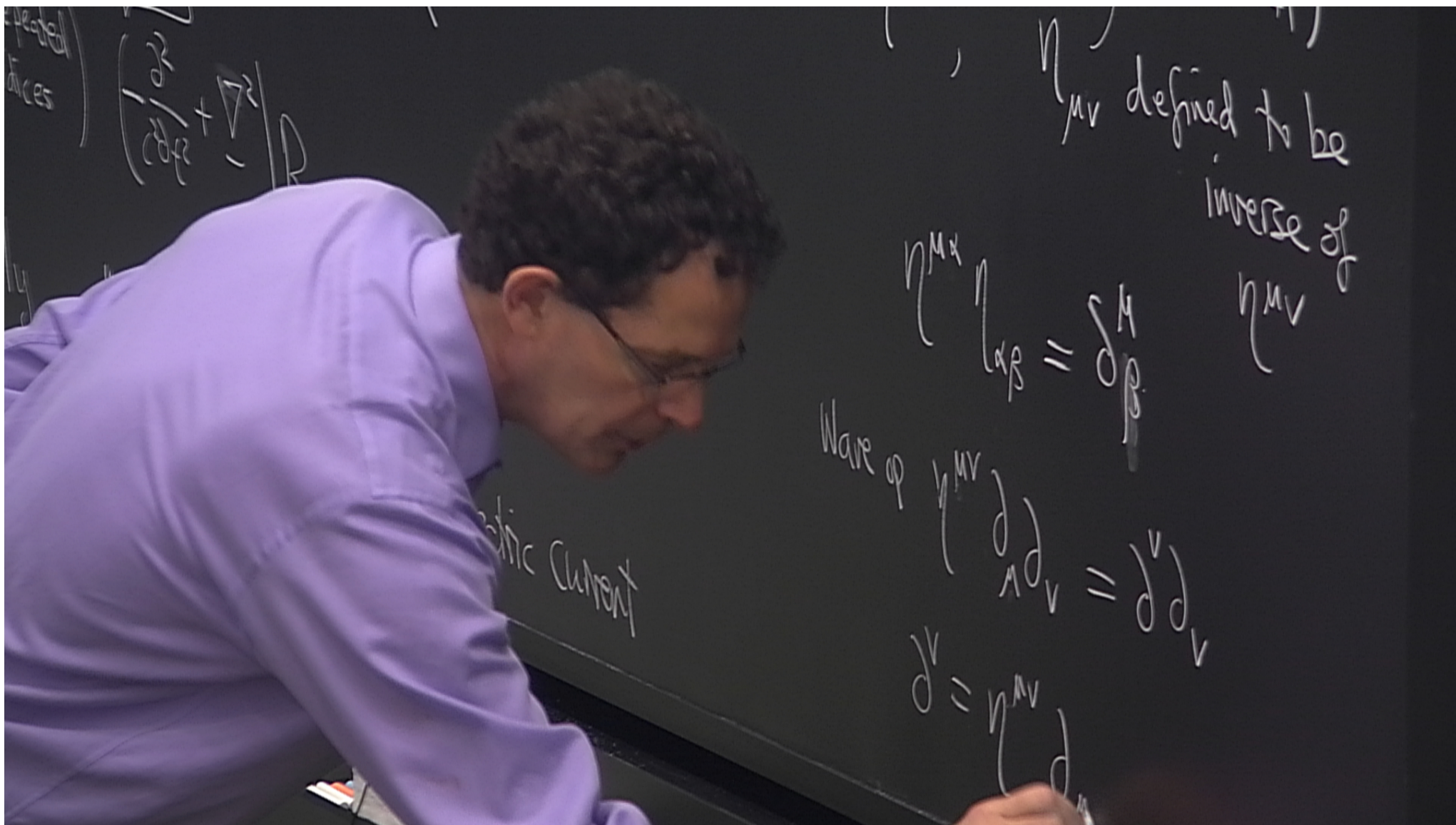
$$\partial_\mu = \frac{\partial}{\partial x^\mu}$$

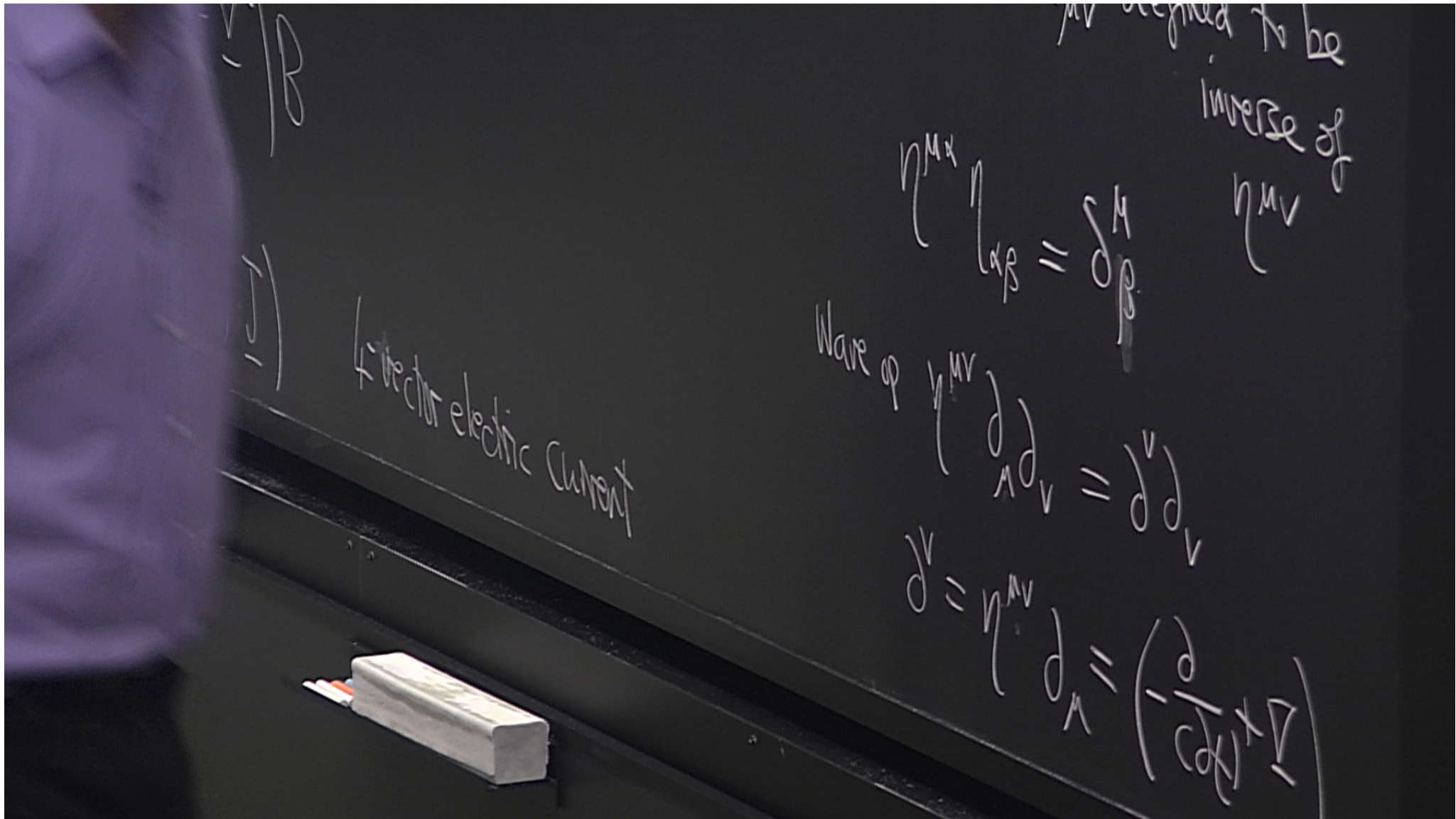
$\eta^{\mu\nu}$

$\eta_{\mu\nu}$ defined to be
inverse of
 $\eta^{\mu\nu}$

$$\eta^{\mu\alpha} \eta_{\alpha\beta} = \delta^\mu_\beta$$

light 0





ell's equations become

$$(1),(4) \Rightarrow \partial_\mu F^{\mu\nu} = -\mu_0 j^\nu$$

$$(2),(3) \Rightarrow \partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0$$

$$j^\mu = (\rho c, \mathbf{J})$$

$$J_\mu = (-\rho c, \mathbf{J})$$

$$F^\mu{}_\nu = \eta^{\mu\beta} F_{\beta\nu}$$

$\Rightarrow$ Maxwell's equations become

$$(1),(4) \Rightarrow \partial_\mu F^{\mu\nu} = -\mu_0 j^\nu$$

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e.g. (1), $\nu=0$ $\partial_i F^{i0}$

$$j^\mu = (\rho c, \mathbf{I})$$

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$$F^\mu{}_\nu = \eta^{\mu\beta} F_{\beta\nu}$$

$$F^{\mu\nu} = \eta^{\mu\alpha} \eta^{\nu\beta} F_{\alpha\beta}$$

$\Rightarrow$ Maxwell's equations become

$$(1),(4) \Rightarrow \partial_\mu F^{\mu\nu} = \dots \quad (A)$$

$$(2),(3) \Rightarrow \partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0 \quad (B)$$

e.g. (A), $\nu=0$ $\partial_i F^{i0}$

$$j^\mu = (\rho c, \mathbf{I})$$

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$\Rightarrow$ Maxwell's equations become

$$(1),(4) \Rightarrow \partial_\mu F^{\mu\nu} = -\mu_0 j^\nu \quad (A)$$

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e.g. (A), $\nu=0$ $\partial_i F^{i0} = +\nabla \cdot \frac{\mathbf{E}}{c}$

$$F^{i0} = -F_{i0}$$

$$j^\mu = (\rho c, \mathbf{I})$$

$$j_\mu = (-\rho c, \mathbf{I})$$

$$F^\mu{}_\nu = \eta^{\mu\beta} F_{\beta\nu}$$

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$\Rightarrow$ Maxwell's equations become

$$(1),(4) \Rightarrow \partial_\mu F^{\mu\nu} = -\mu_0 j^\nu \quad (A)$$

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e.g. (A), $\nu=0$ $\partial_i F^{i0} = +\frac{1}{c} \nabla \cdot \underline{E}$

$$F^{i0} = -F_{i0} \quad -\mu_0$$

$$j^\mu = (\rho c, \underline{I})$$

$$= (-\rho c, \underline{I})$$

$$= \eta^{\mu\beta} F_{\beta\alpha}$$

$$\eta^{\mu\alpha} \eta^{\nu\beta} F_{\alpha\beta}$$

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e.g. (A), $\nu=0$

$$\partial_i F^{i0} = +\frac{\nabla \cdot \mathbf{E}}{c}$$

$$F^{i0} = -F_{i0}$$

$$-\mu_0 j^0 = -\mu_0 \rho c$$

$$\left. \begin{array}{l} \partial_i F^{i0} = +\frac{\nabla \cdot \mathbf{E}}{c} \\ -\mu_0 j^0 = -\mu_0 \rho c \end{array} \right\} \nabla \cdot \mathbf{E}$$

$$j^\mu = (\rho c, \mathbf{I})$$

$$J_\mu = (-\rho c, \mathbf{I})$$

$$F^\mu{}_\nu = \eta^{\mu\beta} F_{\beta\nu}$$

$$F^{\mu\nu} = \eta^{\mu\alpha} \eta^{\nu\beta} F_{\alpha\beta}$$

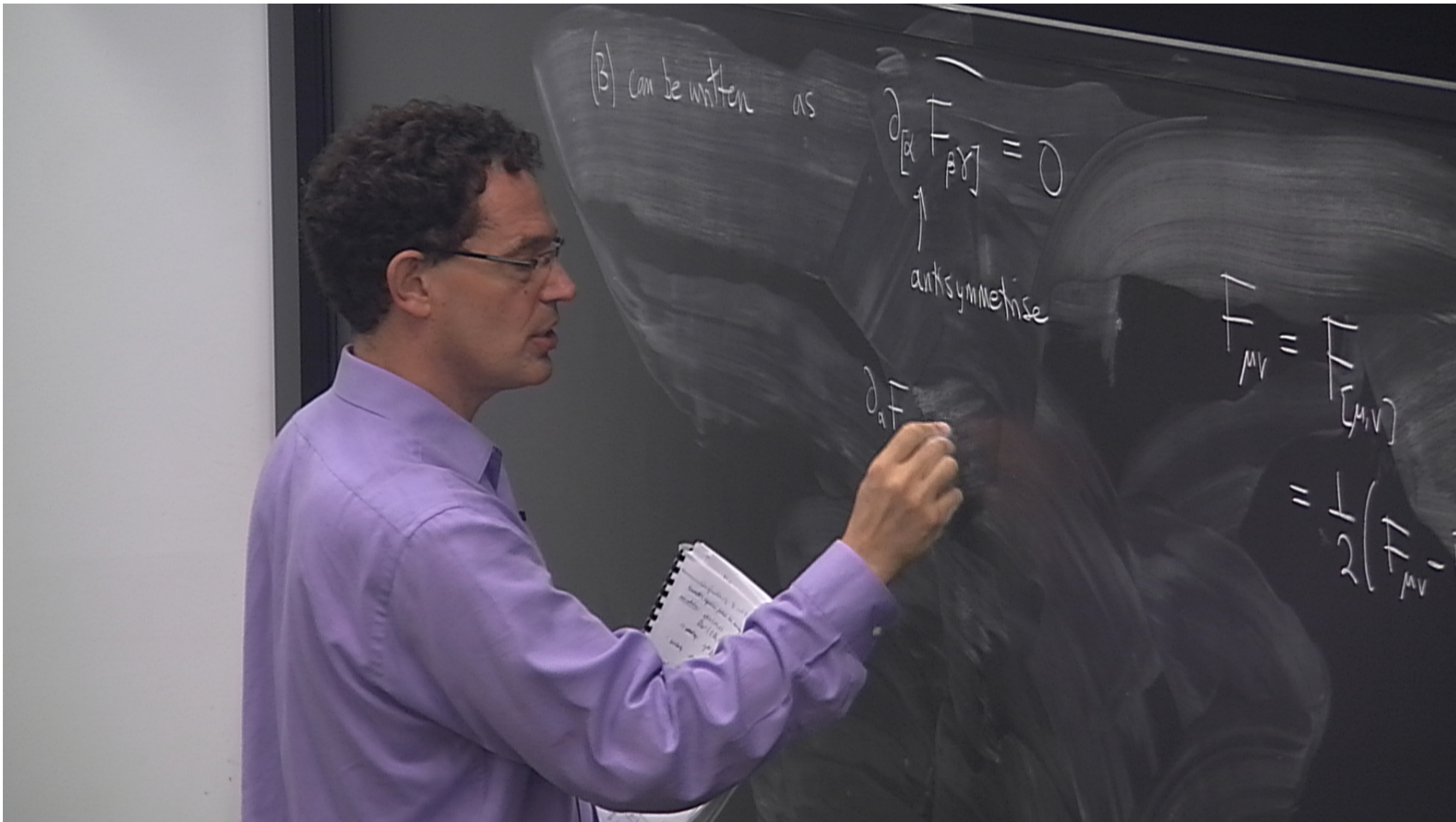
(B) can be written as $\partial_{[\alpha} \overline{F_{\beta\gamma]} =$

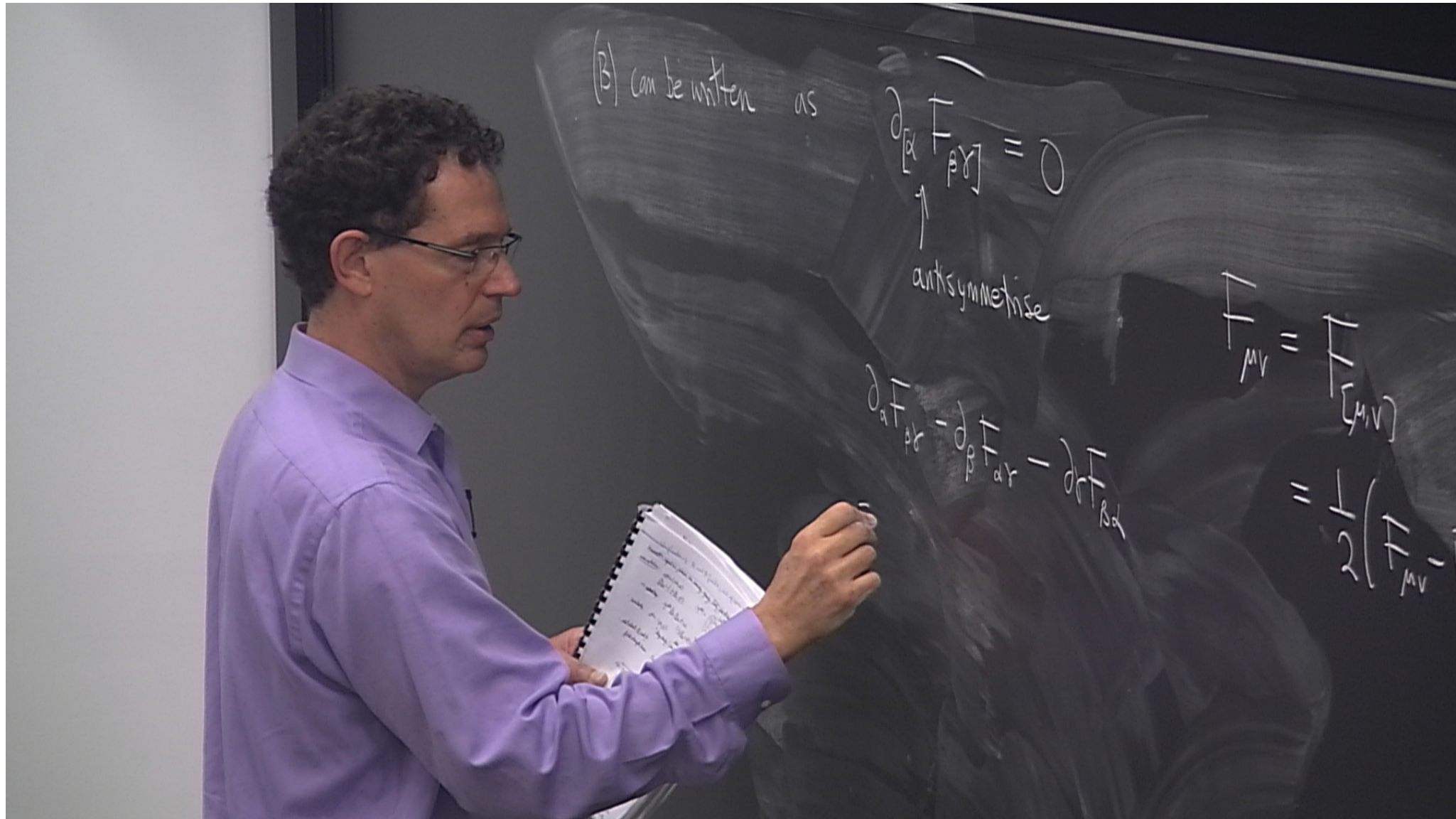
(B) can be written as

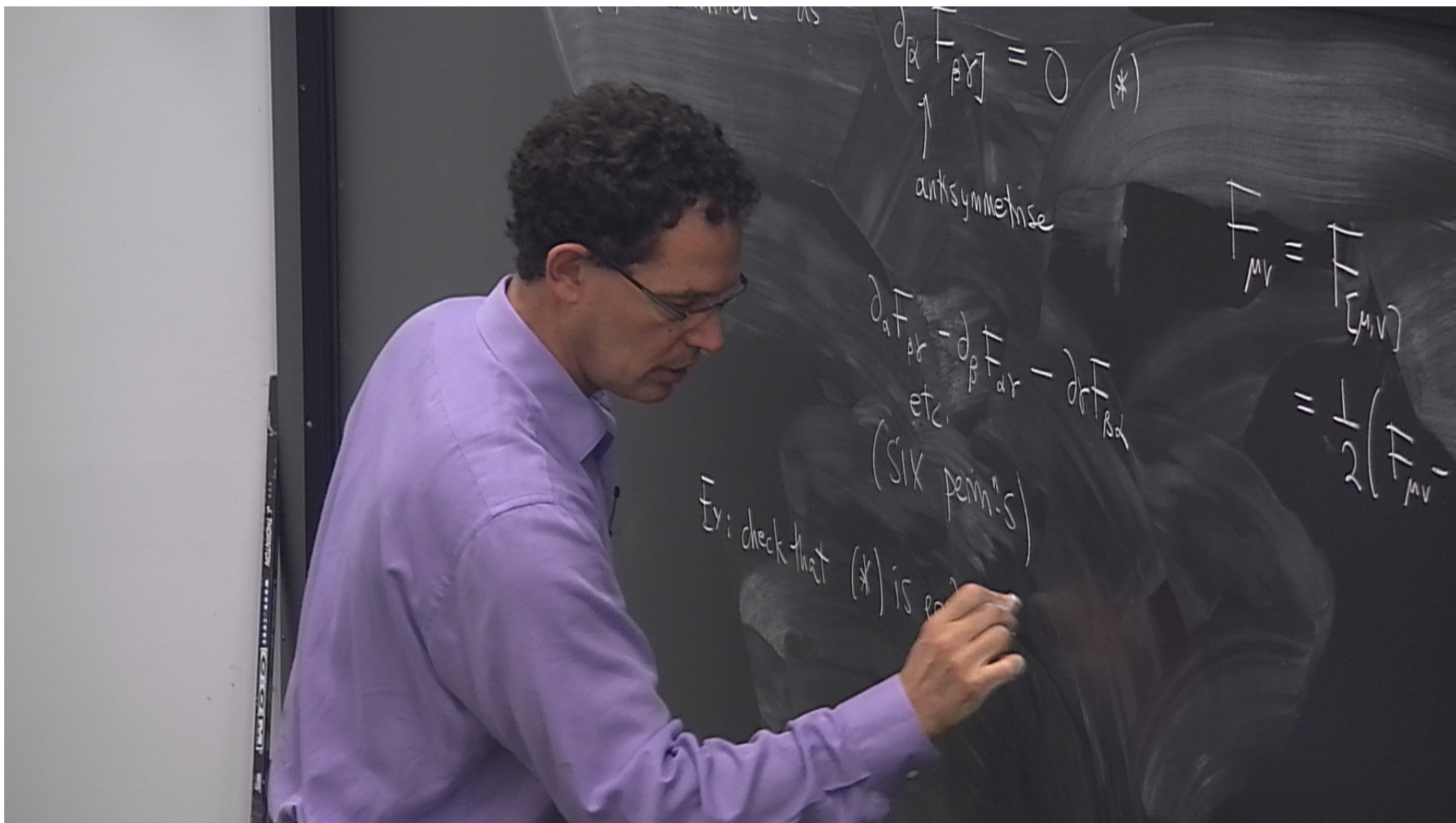
$$\partial_{[\alpha} \overline{F_{\beta\gamma]} = 0$$

↑
antisymmetrise

$$F_{\mu\nu} =$$







is written as

$$\partial_{[\alpha} \overline{F_{\beta\gamma]} = 0 \quad (*)$$

↑
antisymmetrise

$$\partial_\alpha F_{\beta\gamma} - \partial_\beta F_{\alpha\gamma} - \partial_\gamma F_{\alpha\beta}$$

etc.
(SIX perm's)

Ex: check that (*) is equivalent to (B).

$$F_{\mu\nu}$$

$$(F_{\mu\nu} - F_{\nu\mu})$$

written

$$\epsilon^{\mu\nu\rho\alpha} \partial_\nu F_{\rho\alpha} = 0$$

is written as

$$\partial_{[\alpha} \overline{F_{\beta\gamma]} = 0 \quad (*)$$

↑
antisymmetrise

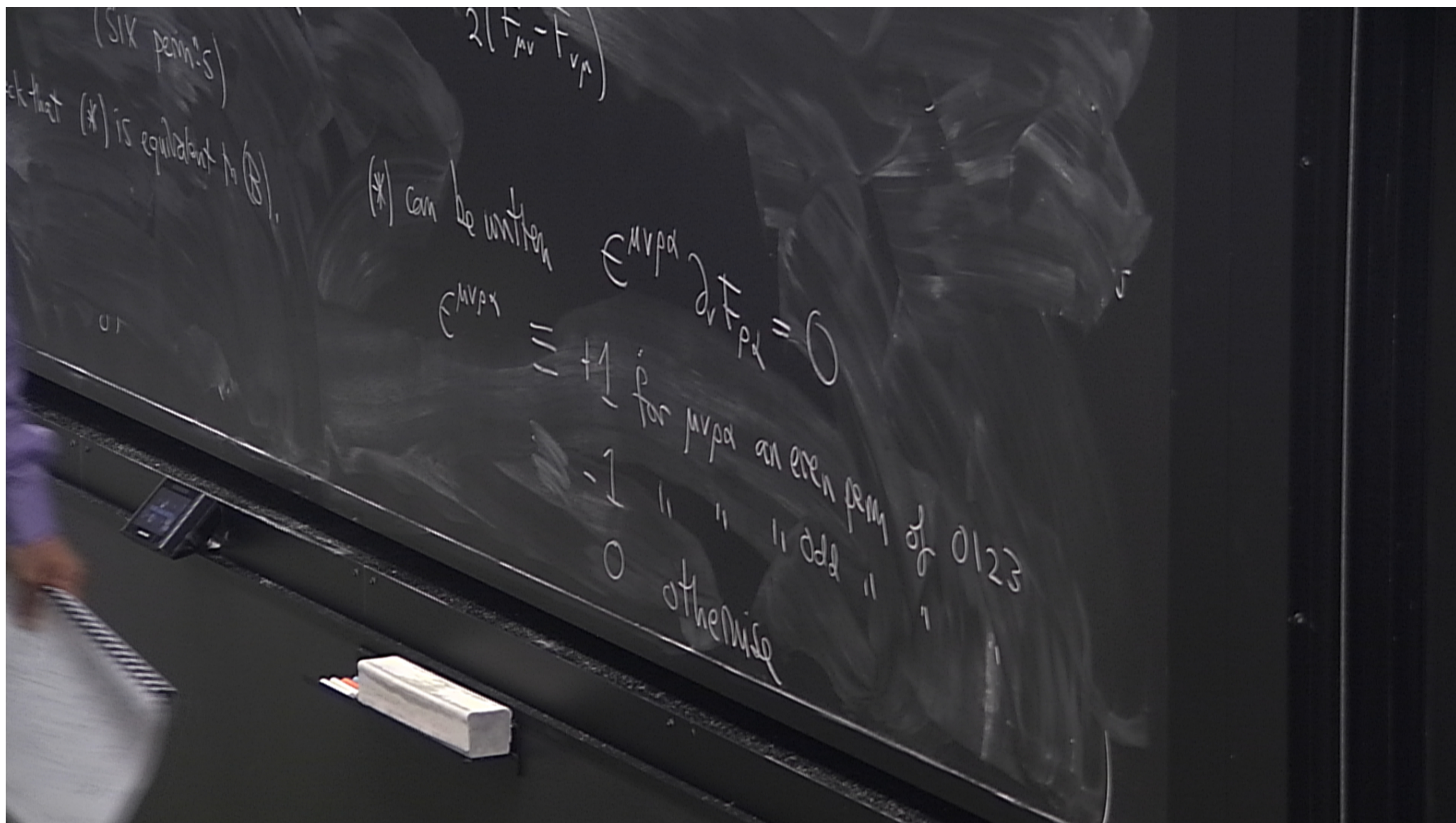
$$\partial_\alpha F_{\beta\gamma} - \partial_\beta F_{\alpha\gamma} - \partial_\gamma F_{\alpha\beta}$$

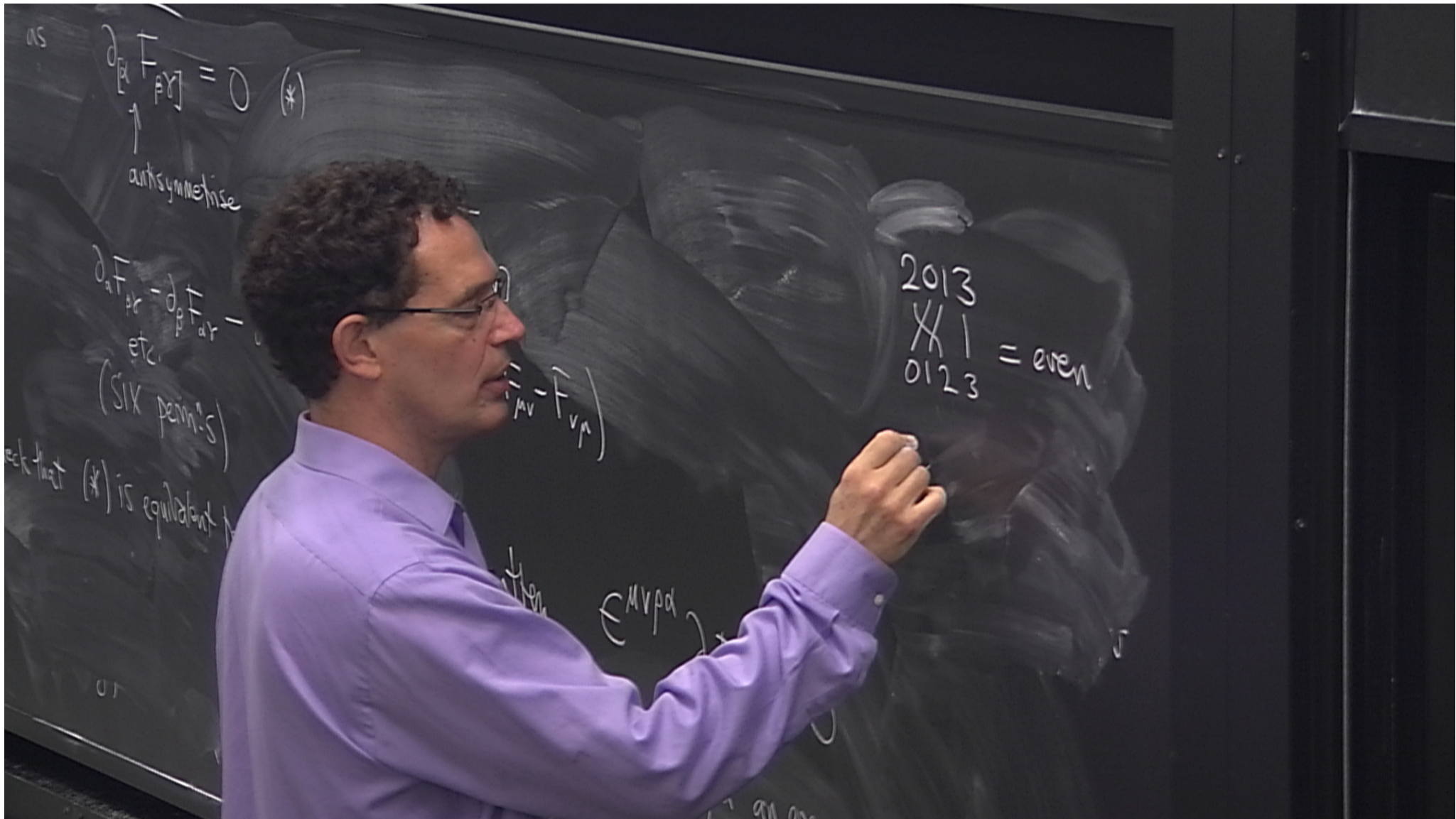
etc.
(SIX perm's)

Ex: check that (*) is equivalent to (B).

$$F_{\mu\nu} = F_{[\mu,\nu]} \\ = \frac{1}{2}(F_{\mu\nu} - F_{\nu\mu})$$

(*) can be written $\epsilon^{\mu\nu\rho\alpha} \partial_\nu F_{\rho\alpha} = 0$



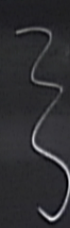


Electric potential ϕ $\underline{E} = -\underline{\nabla}\phi$

Magnetic vector potential $\underline{A}$, $\underline{B} = \underline{\nabla} \wedge \underline{A}$

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Magnetic vector potential $\underline{A}$, $\underline{B} = \nabla \wedge \underline{A}$

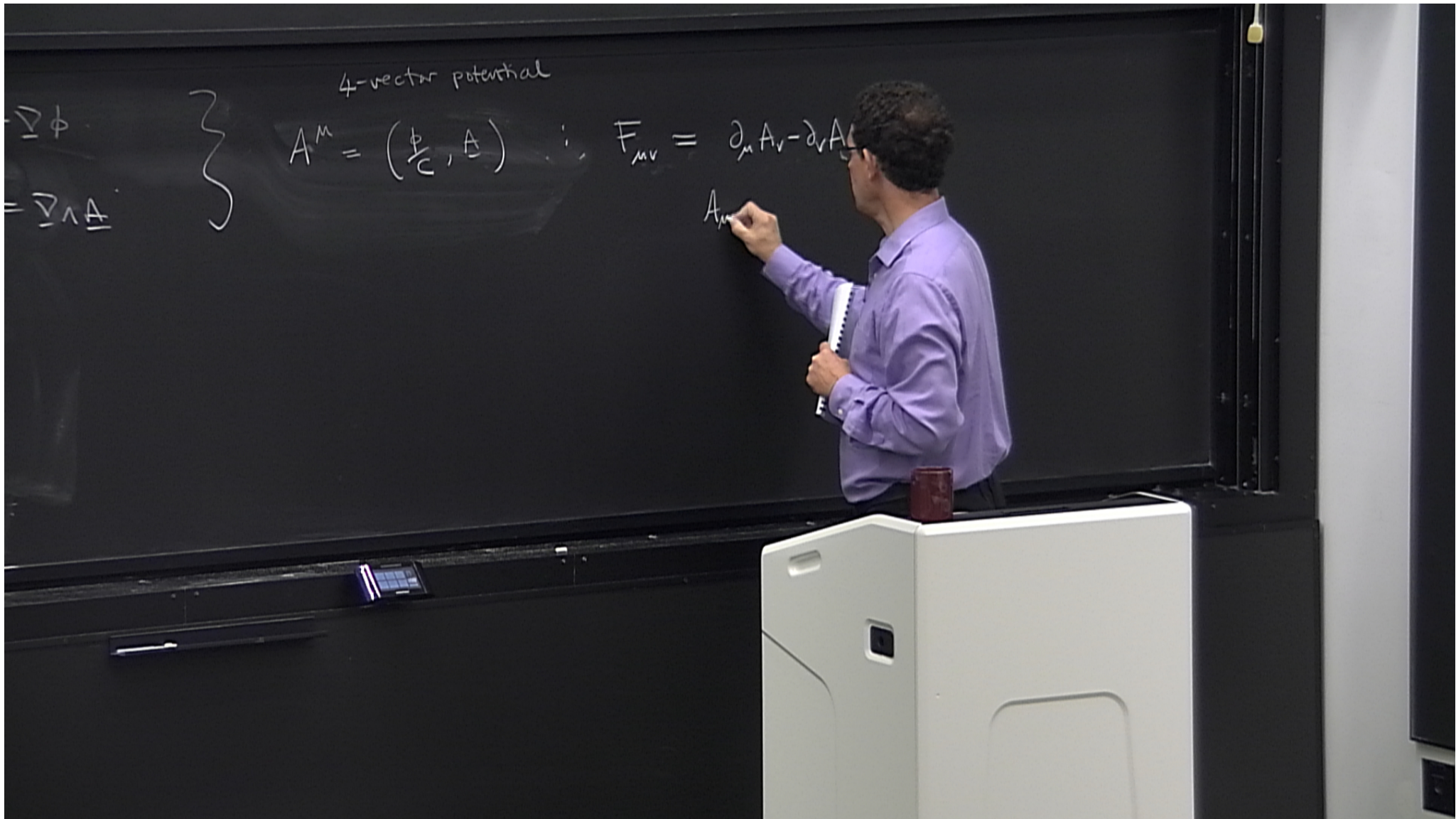


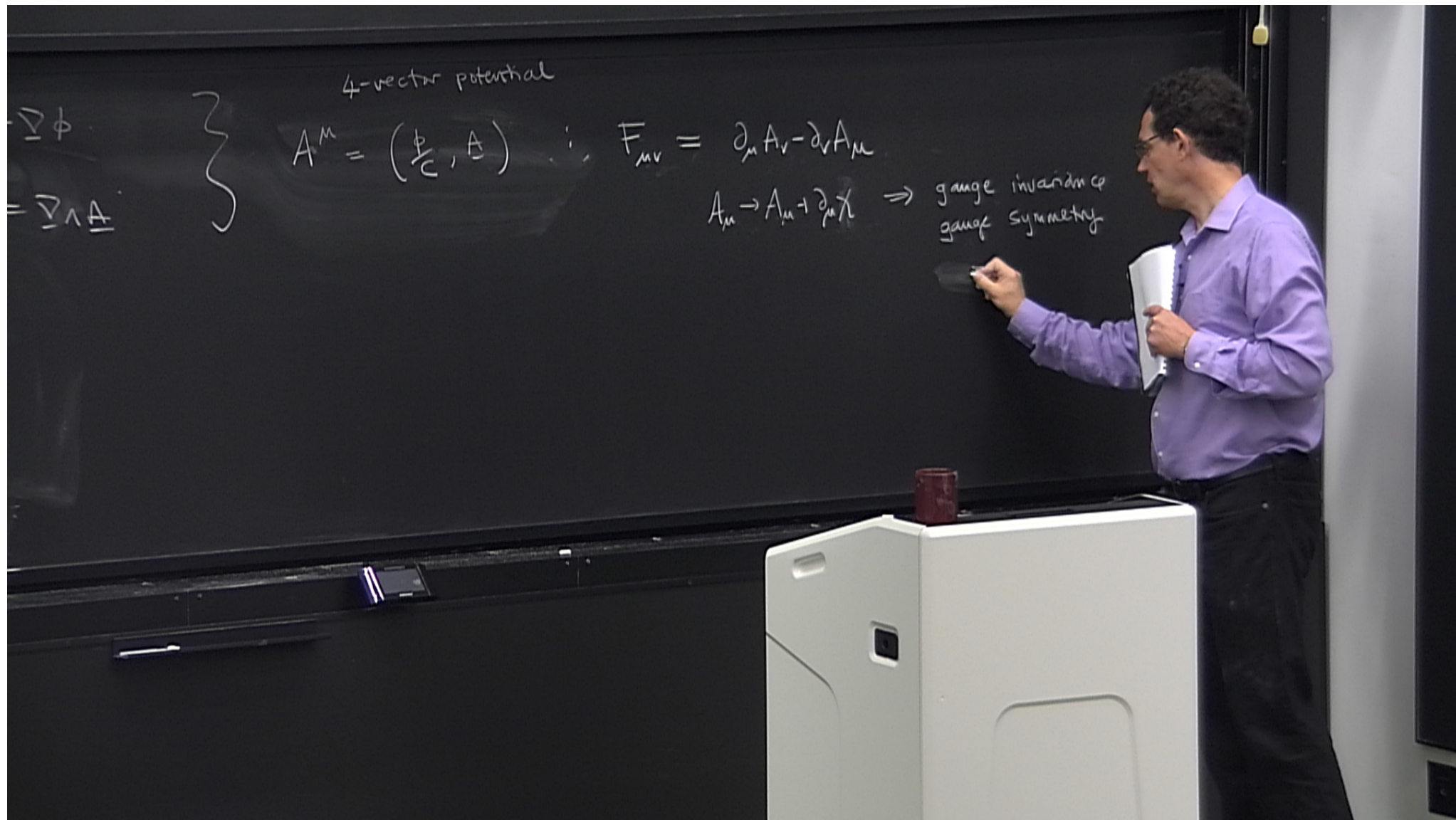
A^M

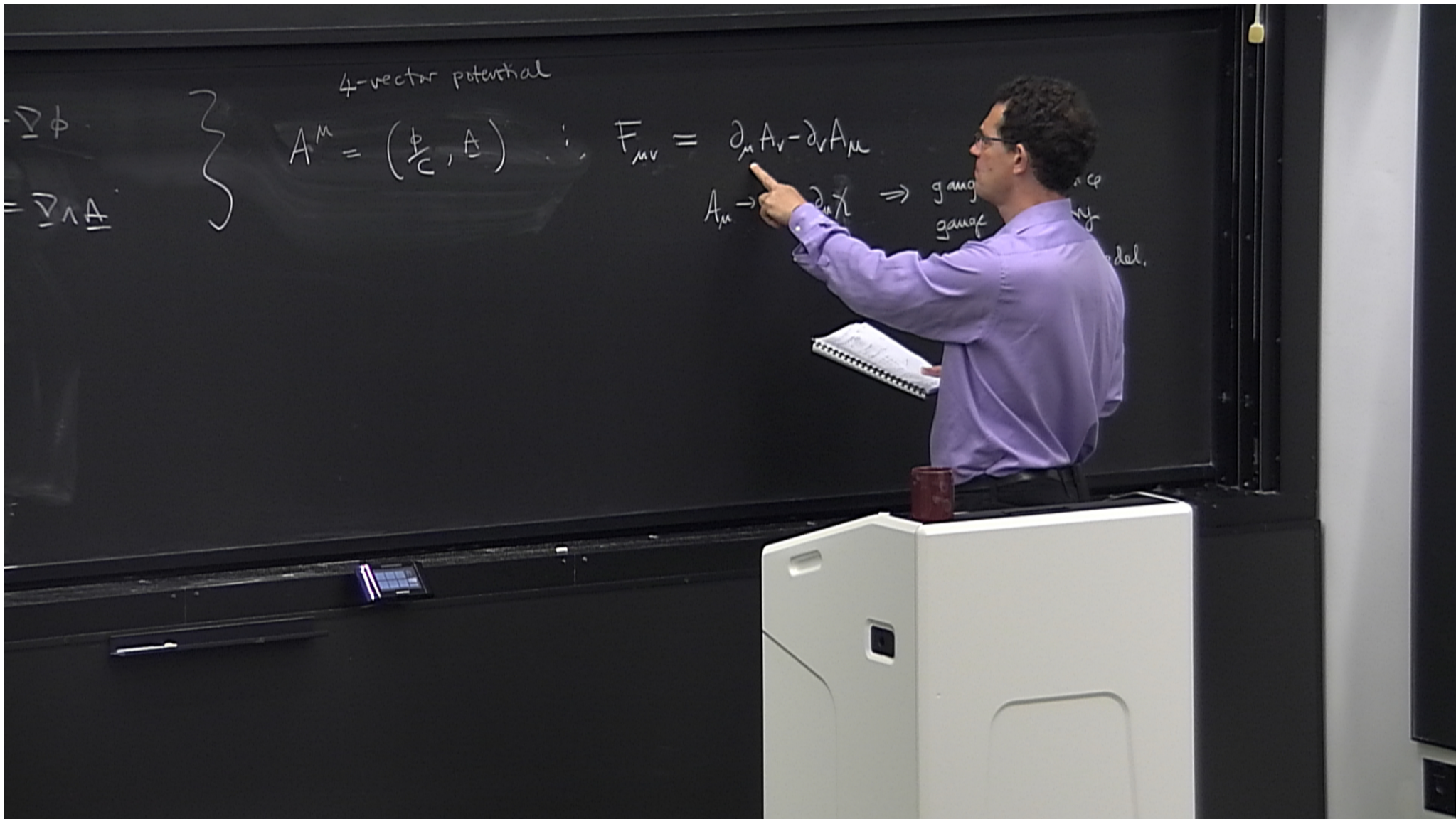
potential ϕ $\underline{E} = -\underline{\nabla}\phi$
 vector potential $\underline{A}$, $\underline{B} = \underline{\nabla} \wedge \underline{A}$

4-vector potential

$$A^M = \left(\frac{\phi}{c}, \underline{A} \right) \quad ; \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$







Electric potential ϕ

$$\underline{E} = -\underline{\nabla}\phi + \frac{\partial \underline{A}}{\partial t}$$

Magnetic vector potential $\underline{A}$, $\underline{B} = \underline{\nabla} \wedge \underline{A}$

Also, Lorentz force $\underline{F} = q(\underline{E} + \underline{v} \wedge \underline{B})$

4-vector pot

$$A^M = \left(\frac{\phi}{c}, \underline{A} \right)$$

Electric potential ϕ

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Also, Lorentz force $\underline{F} = q(\underline{E} + \underline{v} \times \underline{B})$

$$m \frac{d^2 \underline{x}}{dt^2}$$

4-vector potential

$$A^\mu = \left(\frac{\phi}{c}, \underline{A} \right)$$

Electric potential ϕ

$$\underline{E} = -\underline{\nabla}\phi$$

Magnetic vector potential $\underline{A}$, $\underline{B} = \underline{\nabla} \wedge \underline{A}$

Also, Lorentz force $\underline{F} = q(\underline{E} + \underline{v} \wedge \underline{B})$

$$m \frac{d^2 x^\mu}{dt^2} = q$$

4-vector potential

$$A^\mu = (\phi, \underline{A}) \quad \therefore F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

(B) can be written as

$$\eta_{\mu\nu} dx^\mu dx^\nu = -c^2 dt^2 + d\underline{x}^2$$

Ex: check that (*)

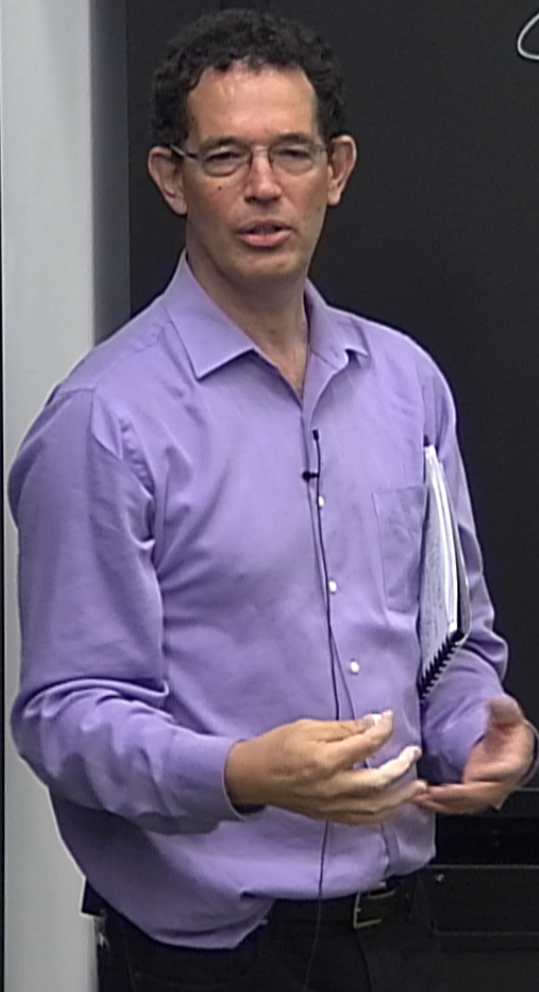
(B) can be written as

$$\eta_{\mu\nu} dx^\mu dx^\nu = -c^2 dt^2 + dx^2$$

$$t = i dt_E$$

$$-c^2 dt_E^2 + dx^2$$

Ex: check that (*)



$$\sum e^{-E_{\text{ET}}}$$

↓

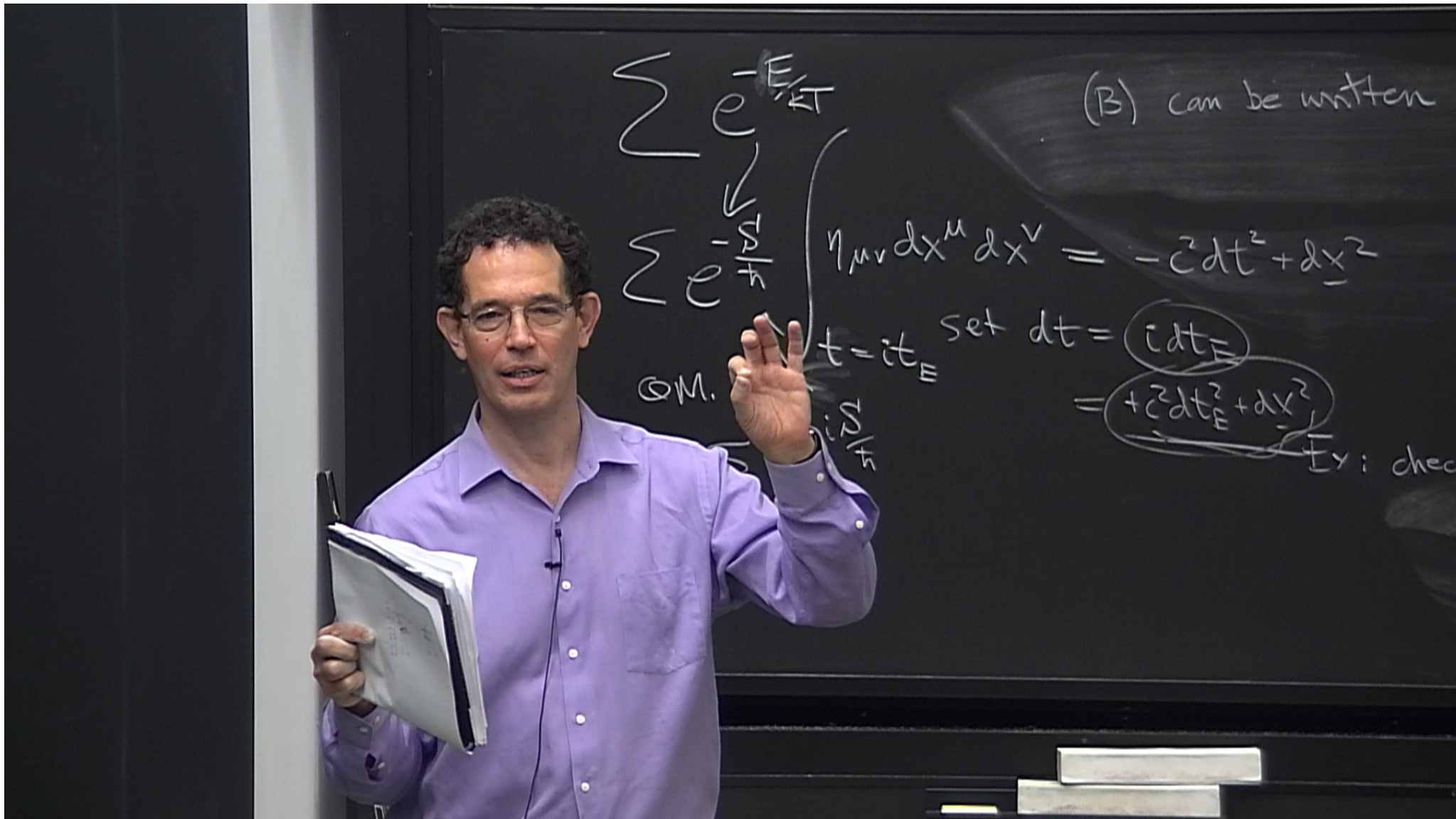
(B) can be written

$$\eta_{\mu\nu} dx^\mu dx^\nu = -c^2 dt^2 + dx^2$$

set $dt = i dt_E$

$$= +c^2 dt_E^2 + dx^2$$

✓ Γ_y : check



$$(2),(3) \Rightarrow \partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0 \quad (B)$$

$$\text{e.g. (1), } r=0 \quad \left. \begin{aligned} \partial_i F^{i0} &= \frac{\nabla \cdot \underline{E}}{c} \\ F^{i0} &= -F_{i0} \\ -M_0 j^0 &= -M_0 \rho c \end{aligned} \right\} \begin{aligned} \nabla \cdot \underline{E} &= \mu_0 \rho c^2 \\ &= \rho / \epsilon_0 \end{aligned}$$

$$(A) : \underbrace{\partial_\nu \partial_\mu F^{\mu\nu}}_0 = -\mu_0 j^\alpha$$

$$\left. \begin{aligned} \text{Electric potential } \phi \quad \underline{E} &= -\nabla \phi + \frac{\partial \underline{A}}{\partial t} \\ \text{Magnetic potential } \underline{A}, \quad \underline{B} &= \nabla \wedge \underline{A} \end{aligned} \right\}$$

$$\text{Also, } \underline{F} = q(\underline{E} + \underline{v} \wedge \underline{B})$$

$$m d^2 x^\mu = q F^\mu_\nu dx^\nu$$