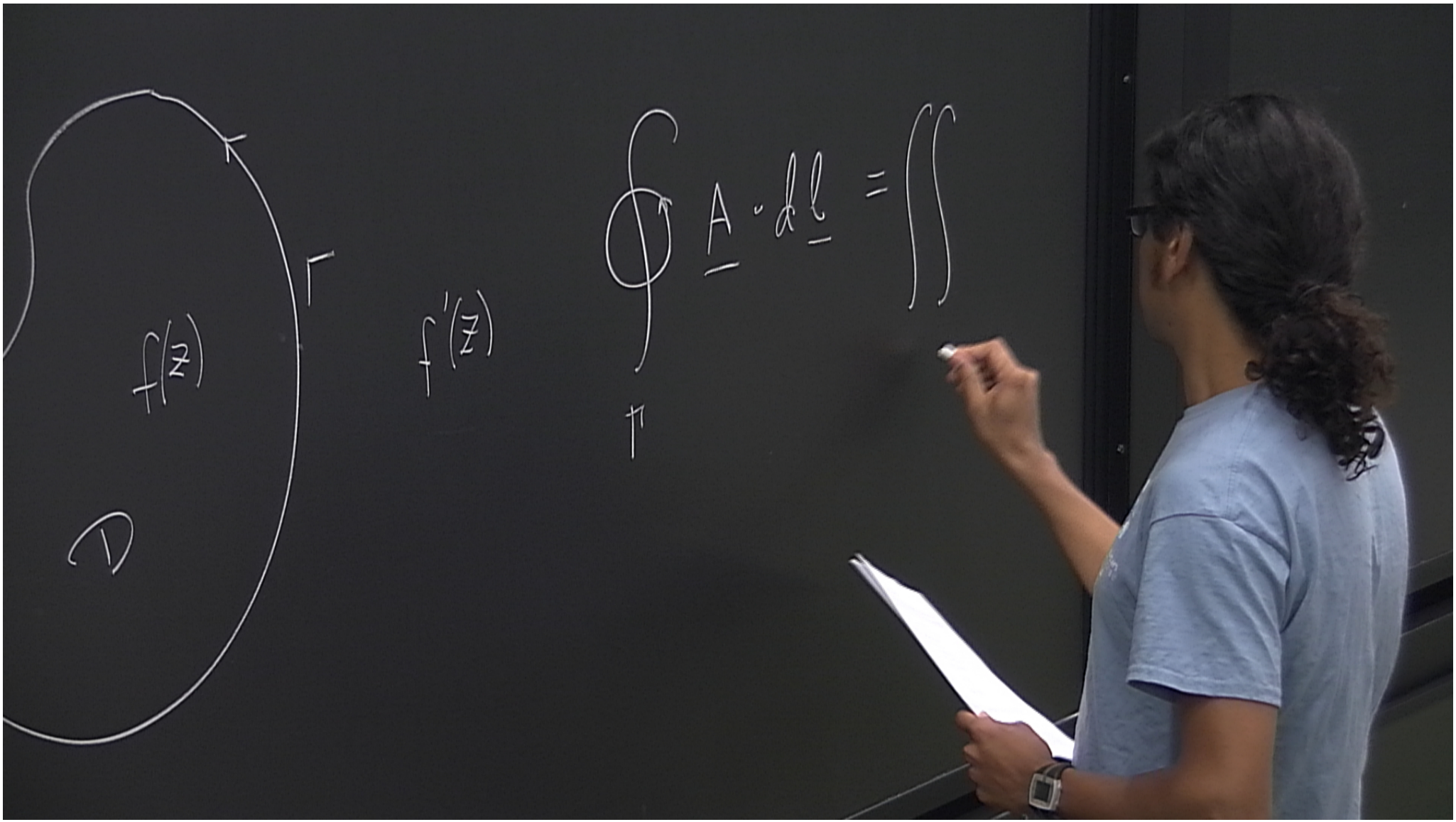
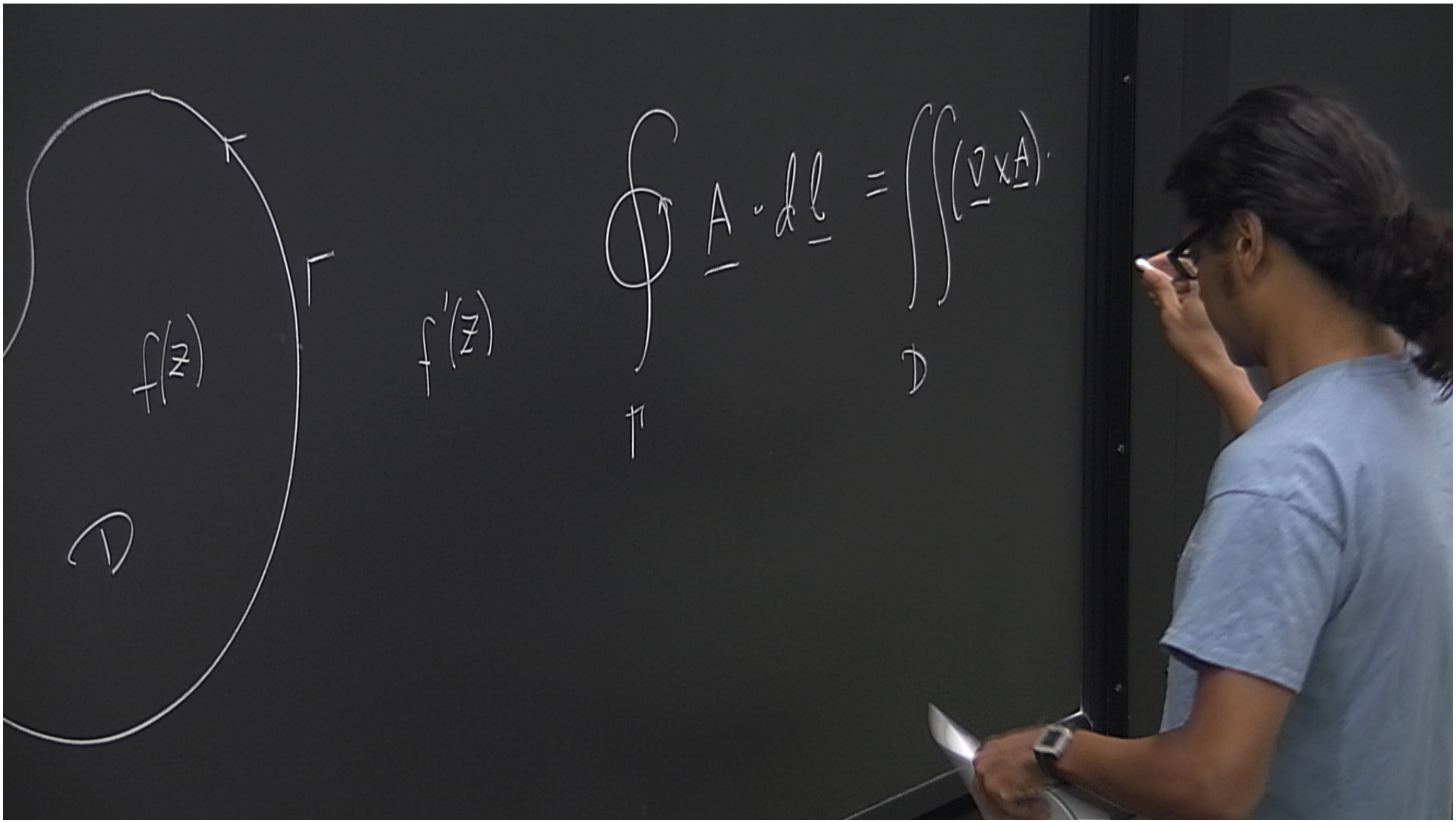
Title: 13/14 PSI- Complex Analysis - Lecture 3

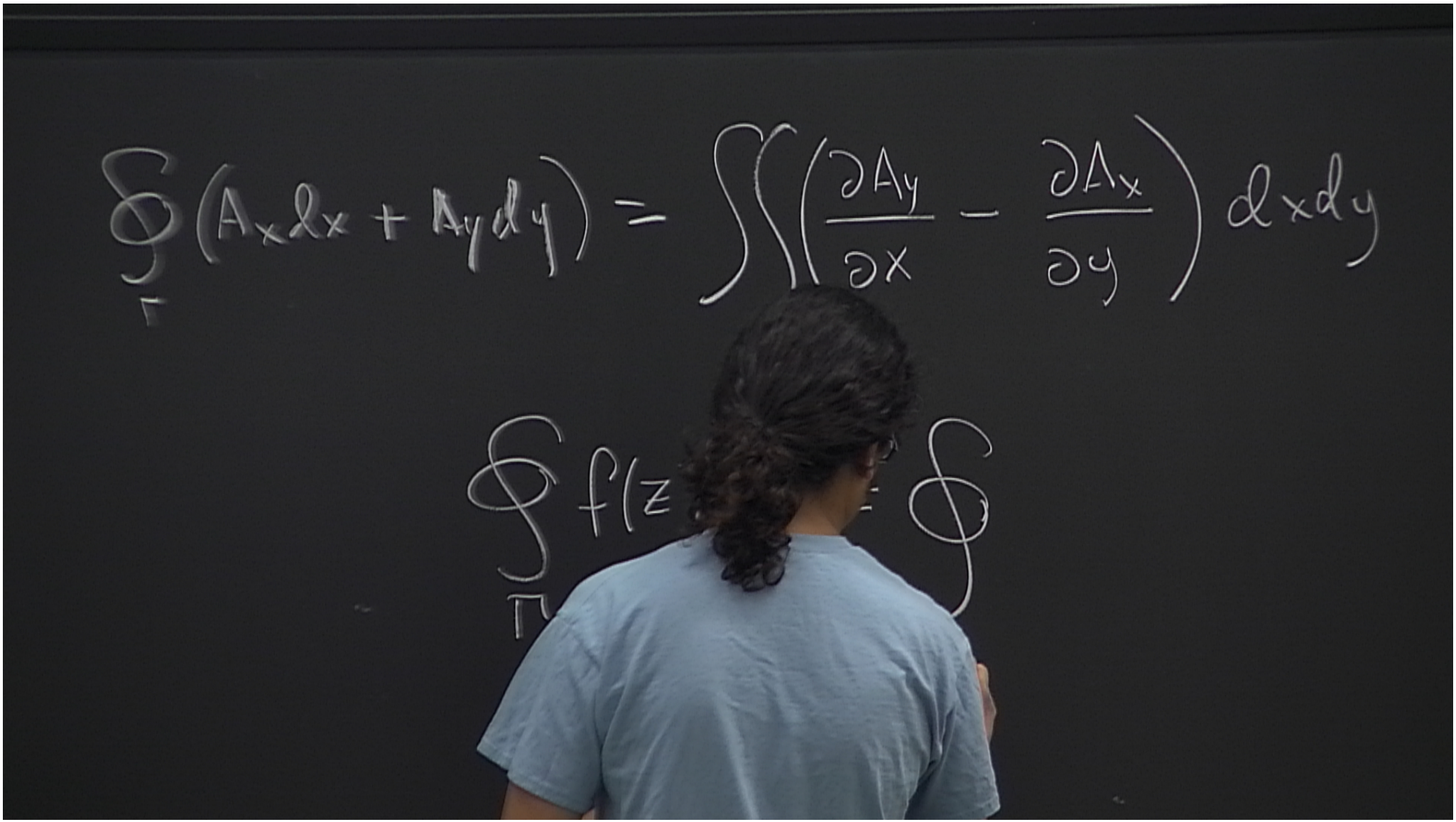
Date: Aug 19, 2013 09:00 AM

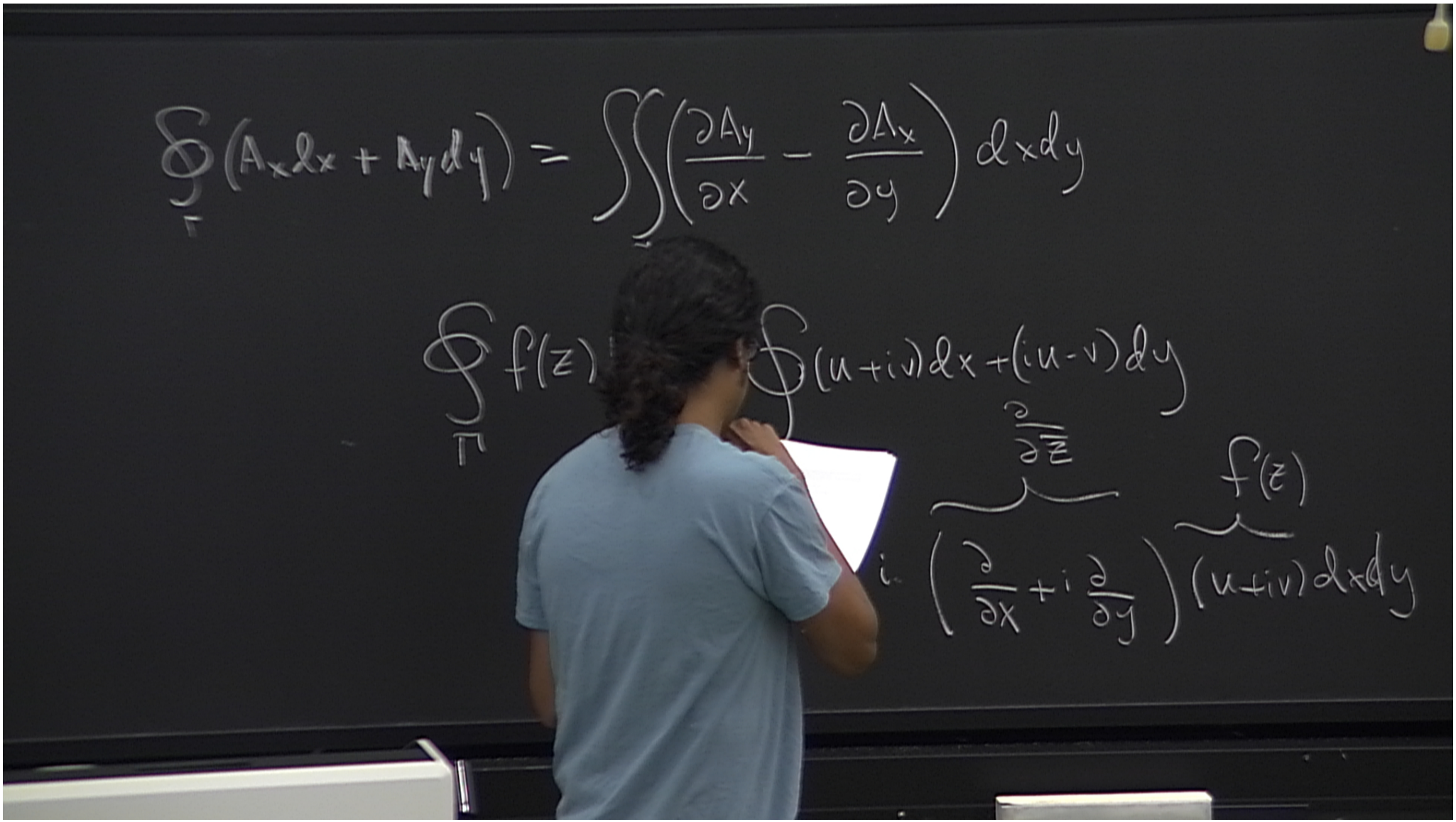
URL: http://www.pirsa.org/13080051

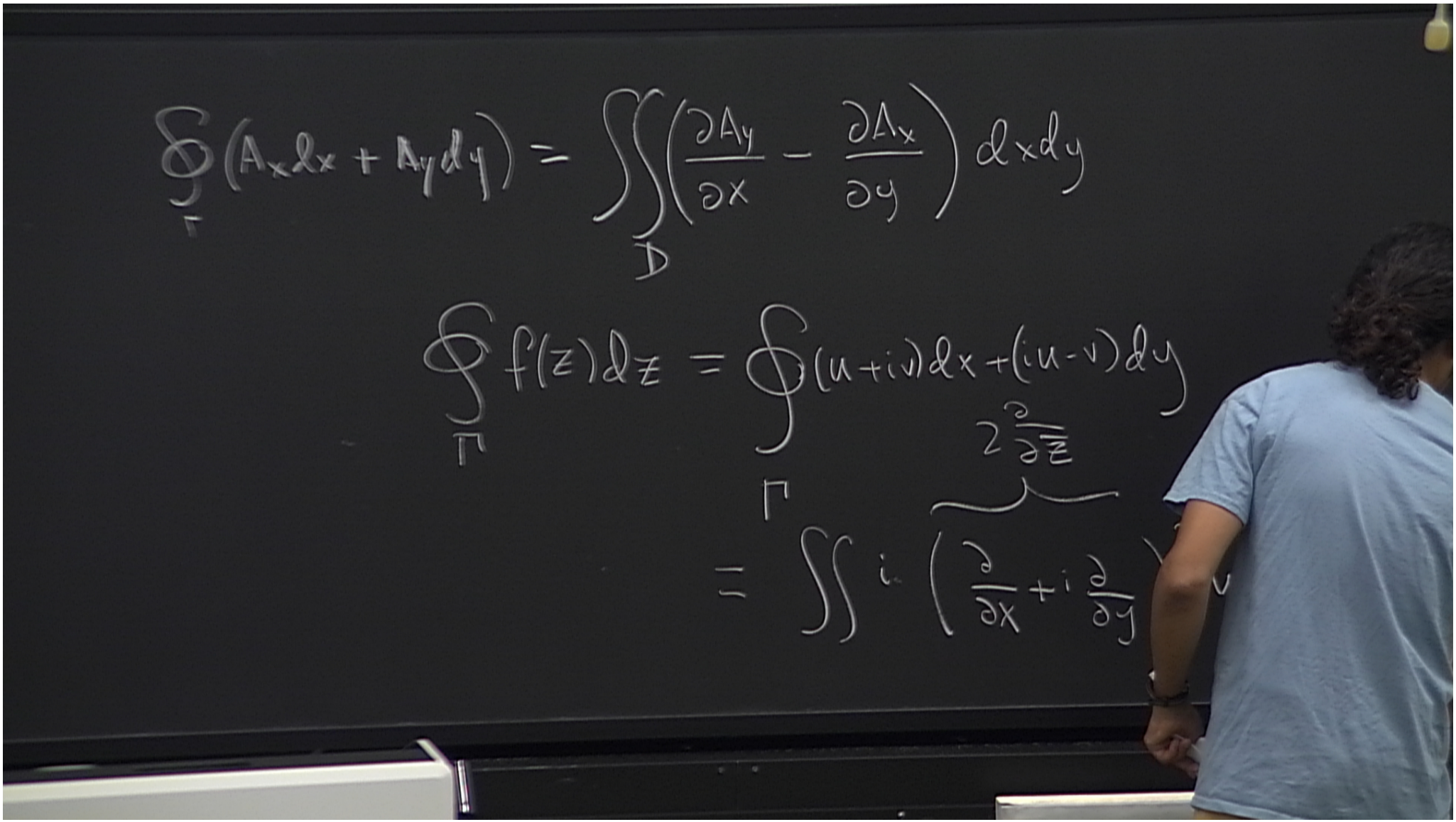
Abstract:

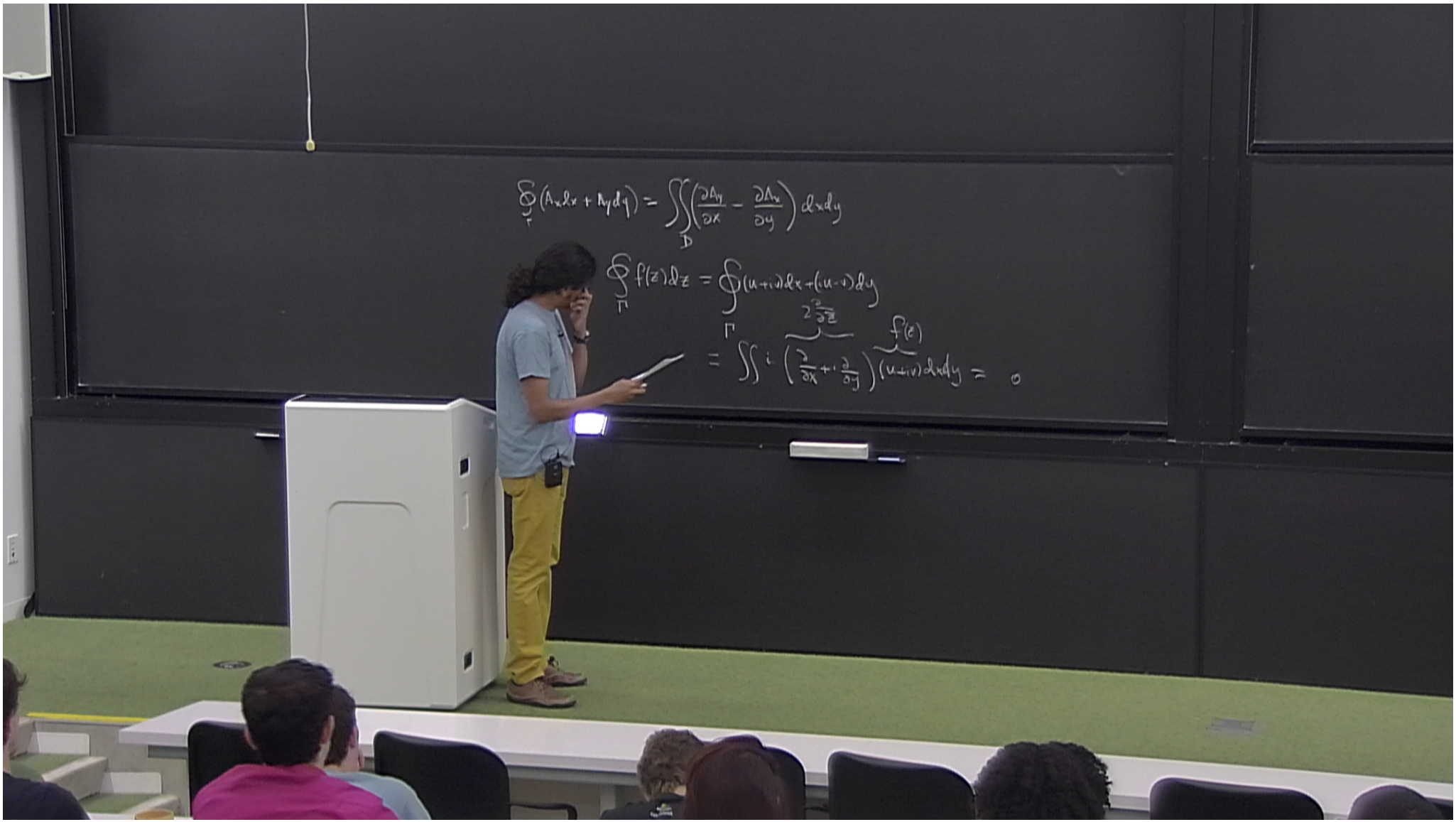

f(z)
D
f'(z)
A · dl =

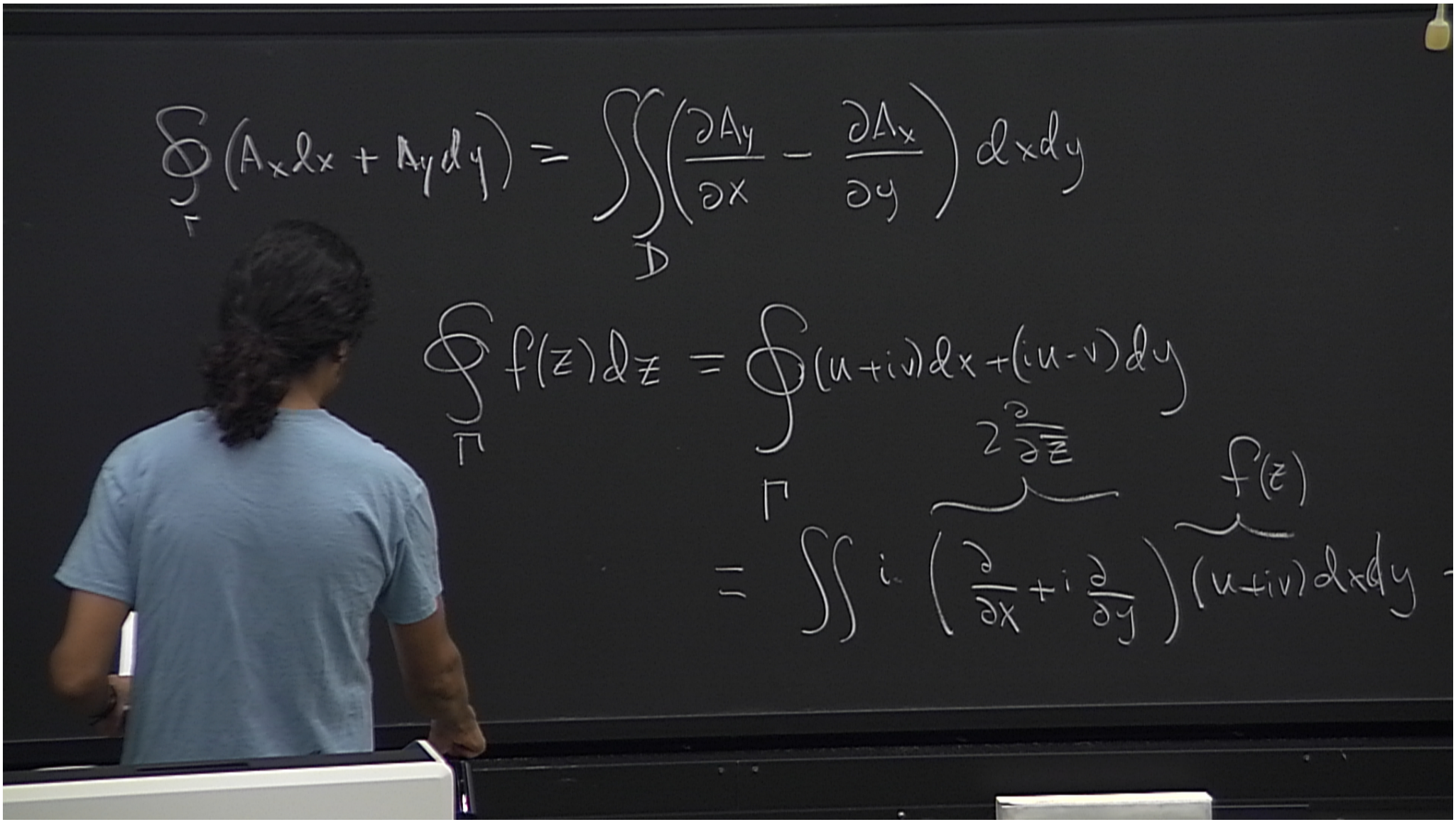

f(z)
D
Γ
f'(z)
∮ A · dl = ∬ (∇ × A)
Γ
D

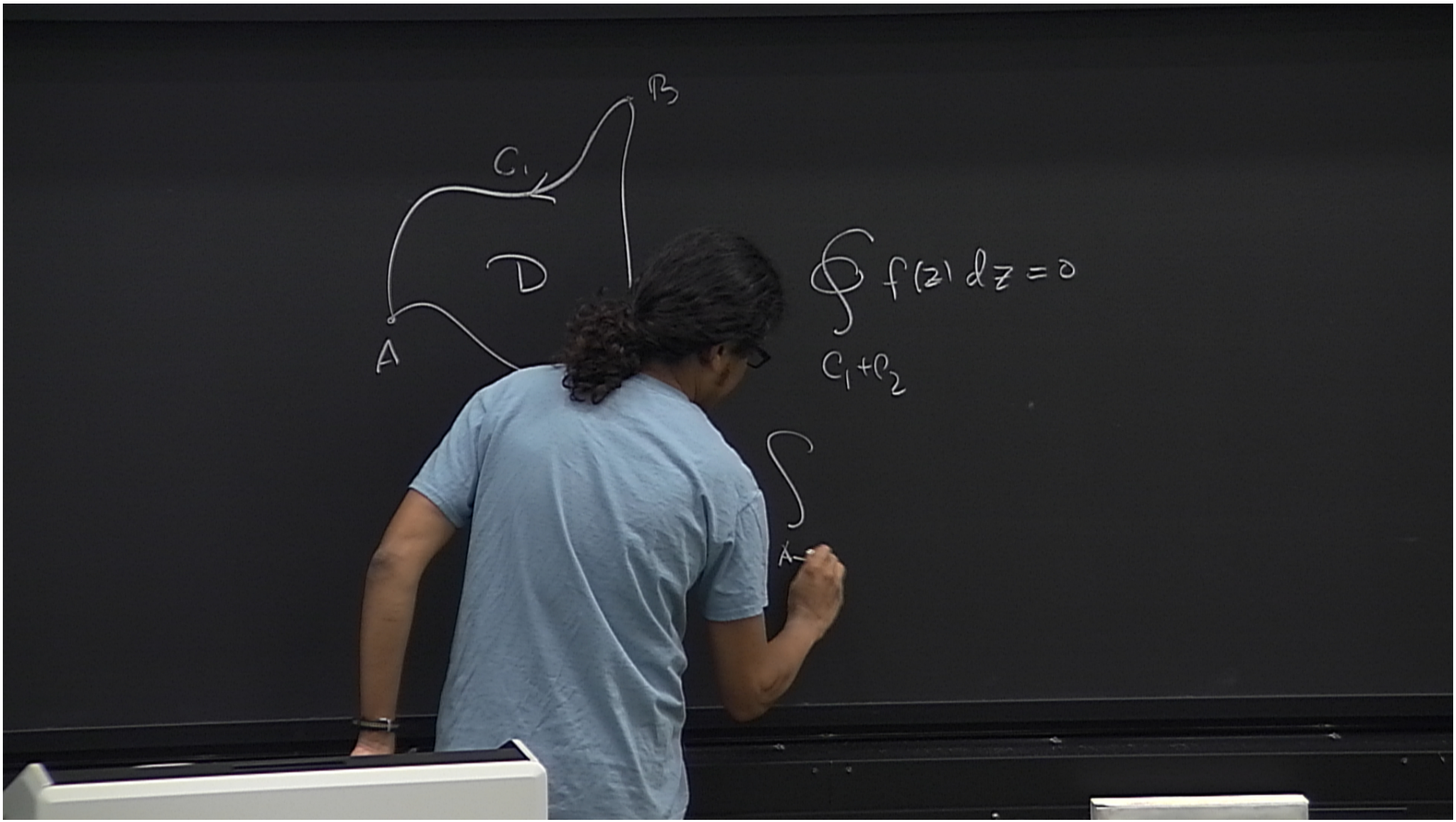

$\oint_\Gamma (A_x dx + A_y dy) = \iint \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) dx dy$

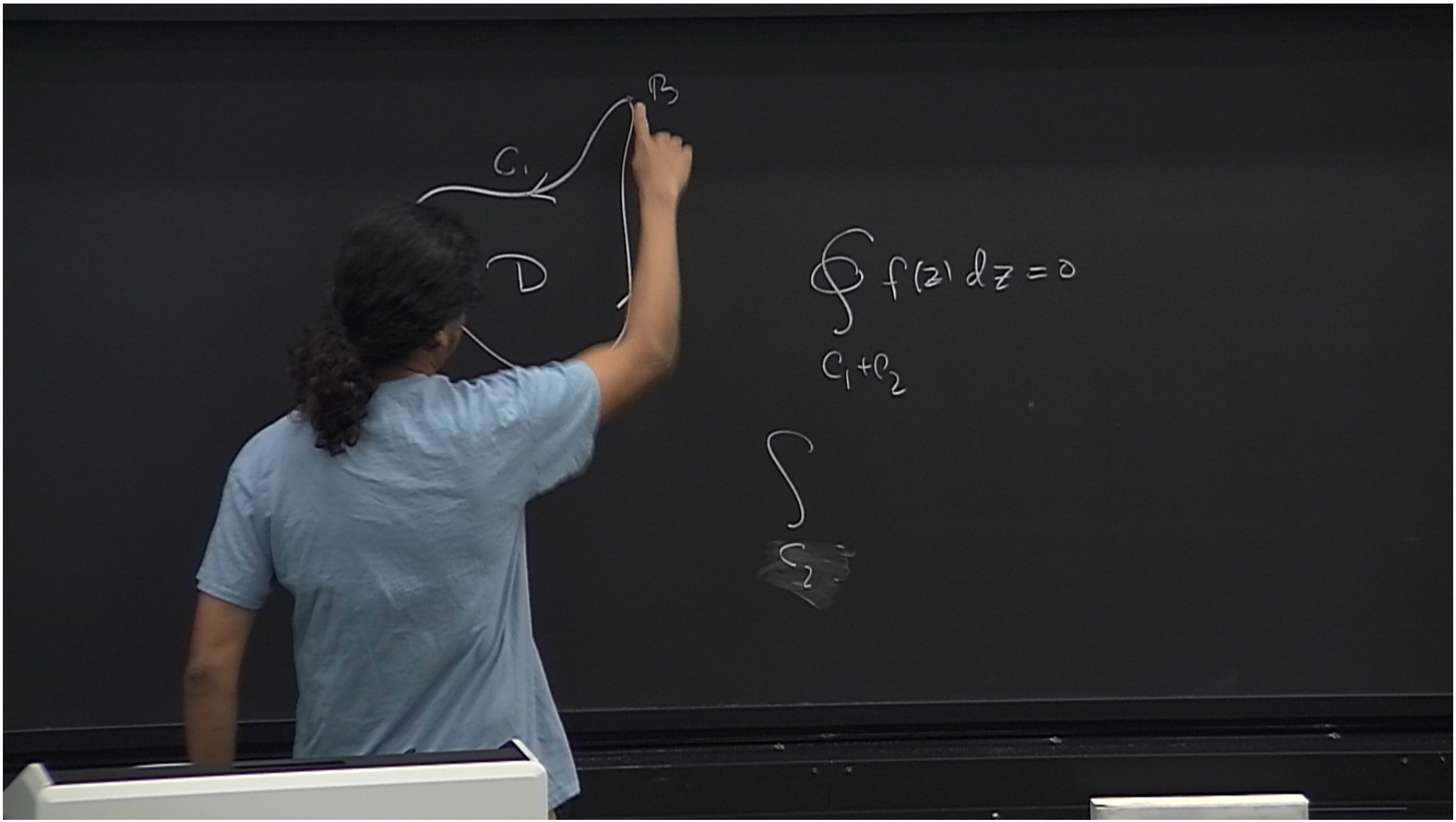

$\oint_\Gamma (A_x dx + A_y dy) = \iint \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) dx dy$
$\oint_\Gamma f(z)$
$\oint (u+iv)dx + (iu-v)dy$
$\frac{\partial}{\partial \bar{z}}$
$f(z)$
$i \left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)(u+iv)dxdy$

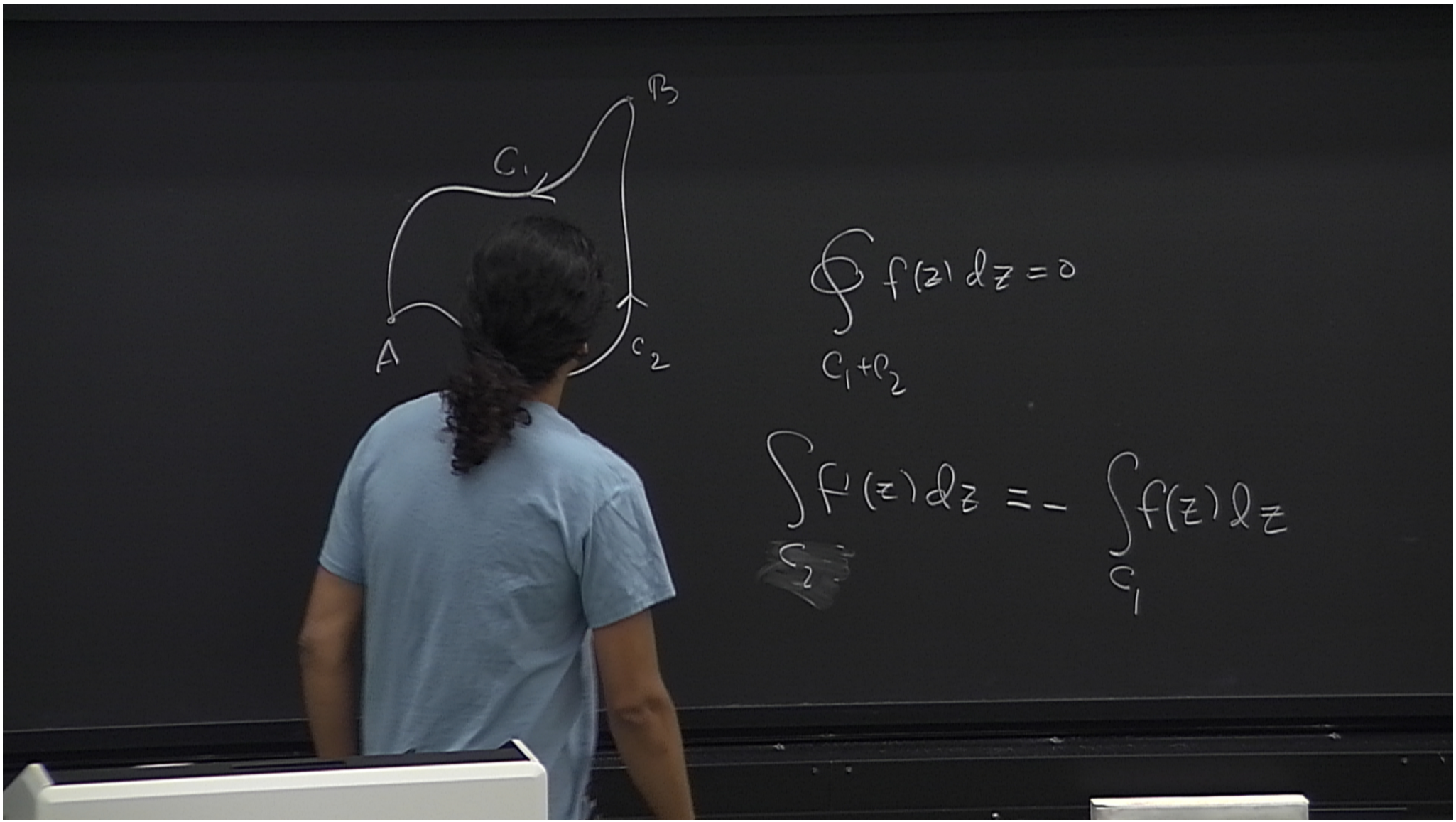

$\oint_\Gamma (A_x dx + A_y dy) = \iint_D \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) dx\,dy$
$\oint_\Gamma f(z)\,dz = \oint_\Gamma (u+iv)\,dx + (iu-v)\,dy$
$2\frac{\partial}{\partial \bar{z}}$
$= \iint i \left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)$

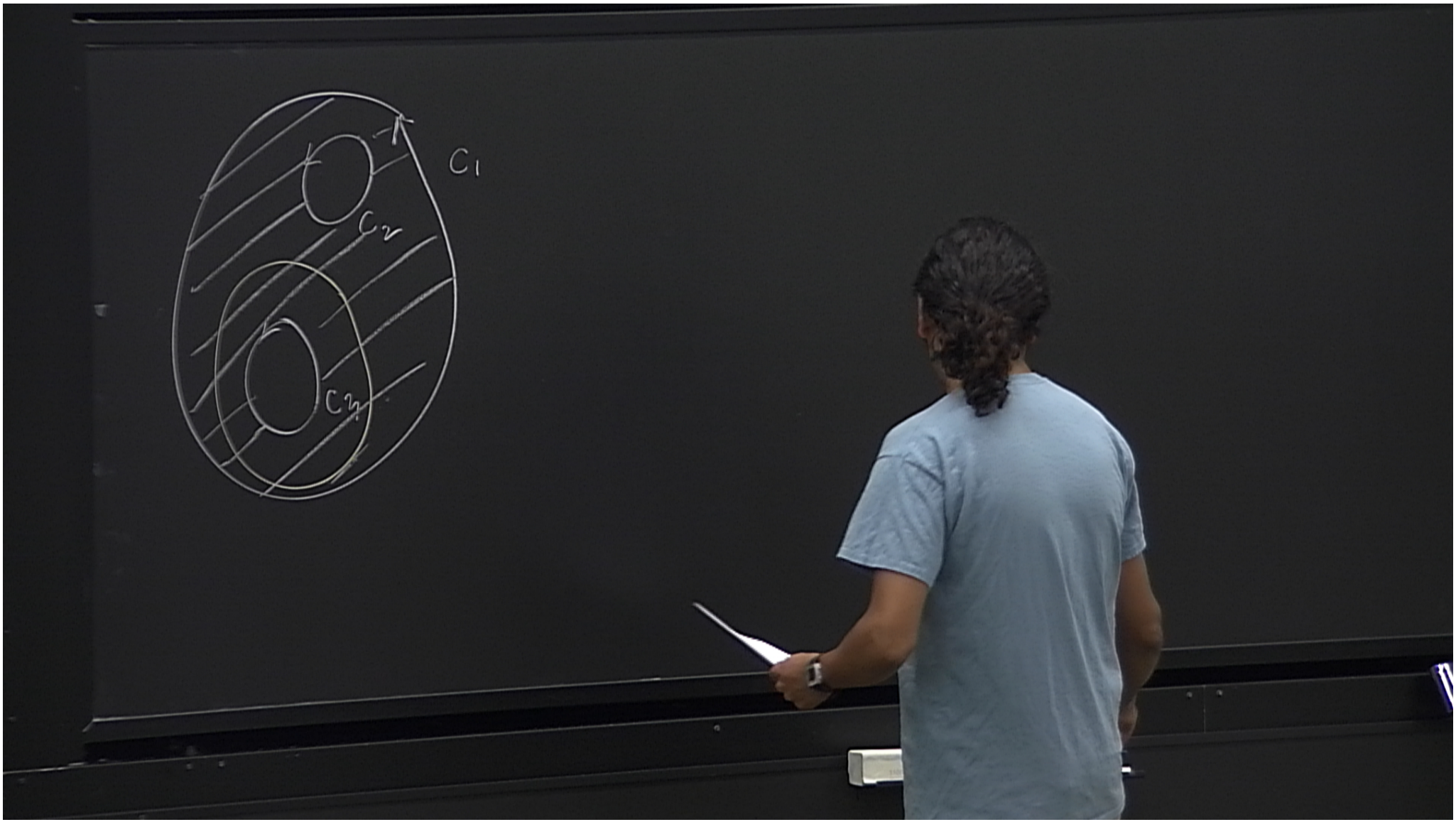

$\oint_\Gamma (A_x dx + A_y dy) = \iint_D \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) dx dy$
$\oint_\Gamma f(z) dz = \oint_\Gamma (u+iv) dx + (iu-v) dy$
$= \iint i \left(\frac{\partial}{\partial x} + i \frac{\partial}{\partial y}\right) (u+iv) dx dy = 0$
$2\frac{\partial}{\partial \bar{z}}$
$f(z)$

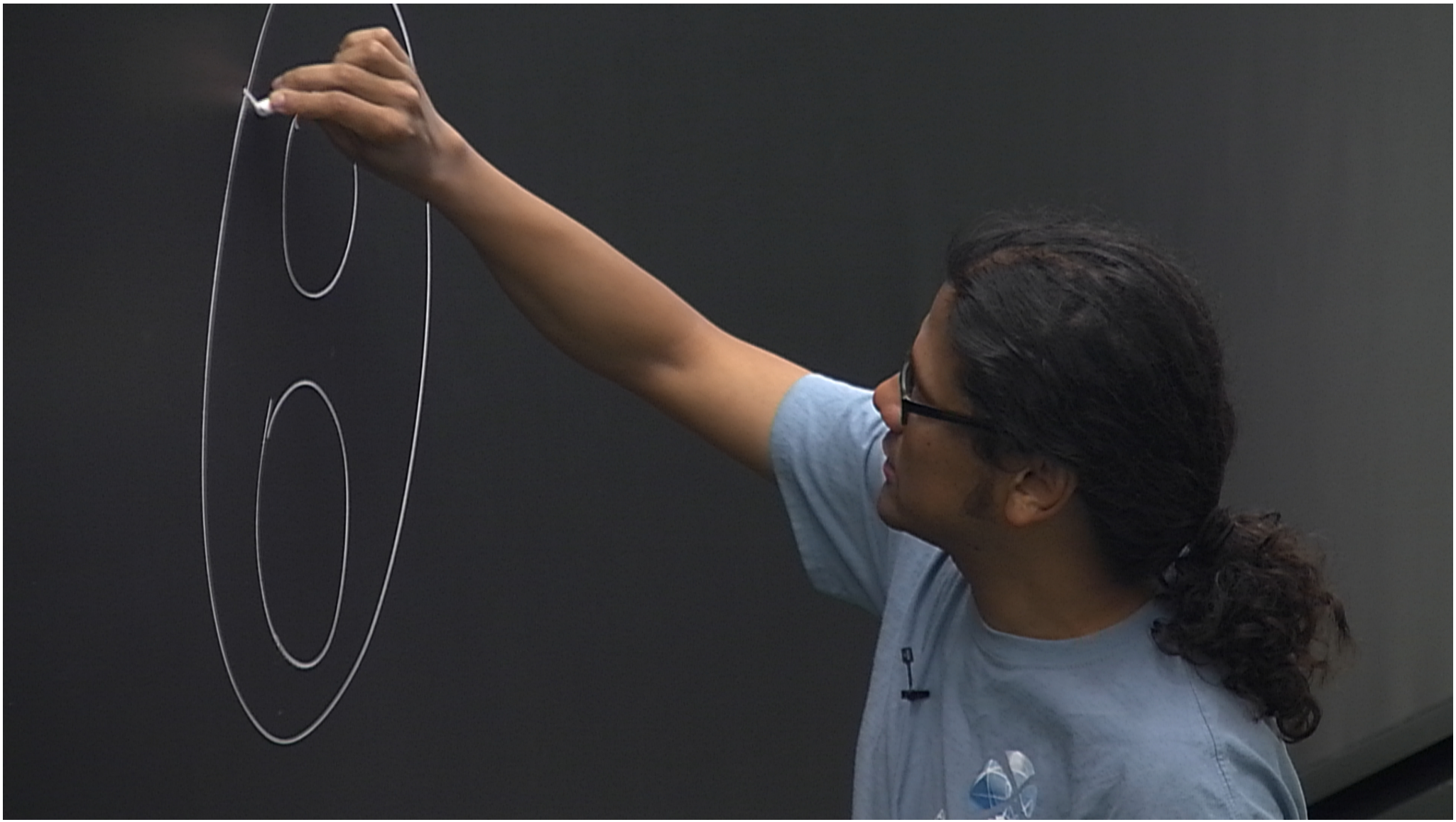

$\oint_\Gamma (A_x dx + A_y dy) = \iint_D \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) dx dy$
$\oint_\Gamma f(z) dz = \oint_\Gamma (u+iv) dx + (iu - v) dy$
$= \iint i \underbrace{\left(\frac{\partial}{\partial x} + i \frac{\partial}{\partial y}\right)}_{2\frac{\partial}{\partial \bar{z}}} \underbrace{(u+iv)}_{f(z)} dx dy$

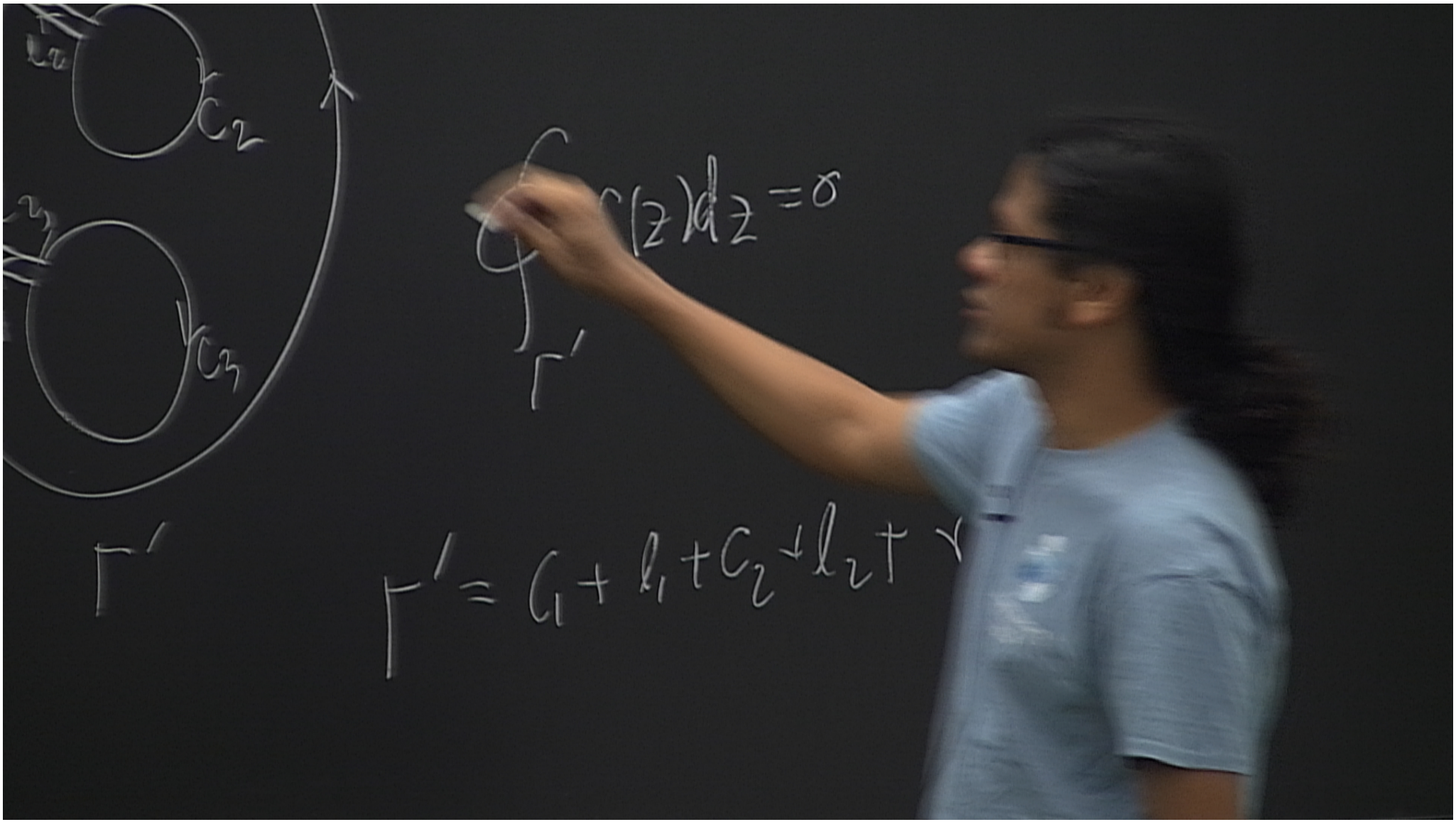

B
C1
D
A
∮ f(z) dz = 0
C1+C2
∫
A

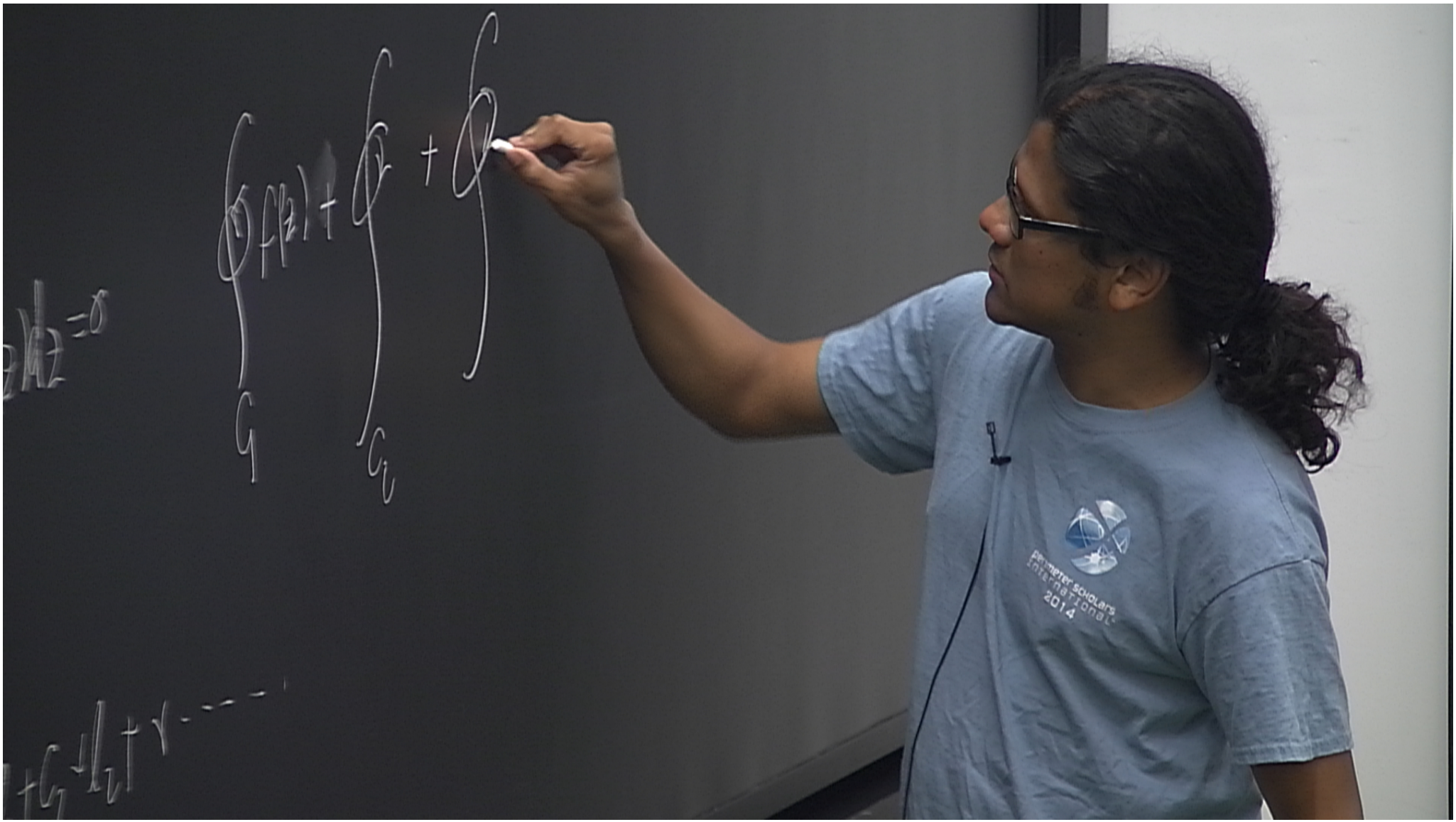

B
C1
D
∮ f(z) dz = 0
C1+C2
∫
C2

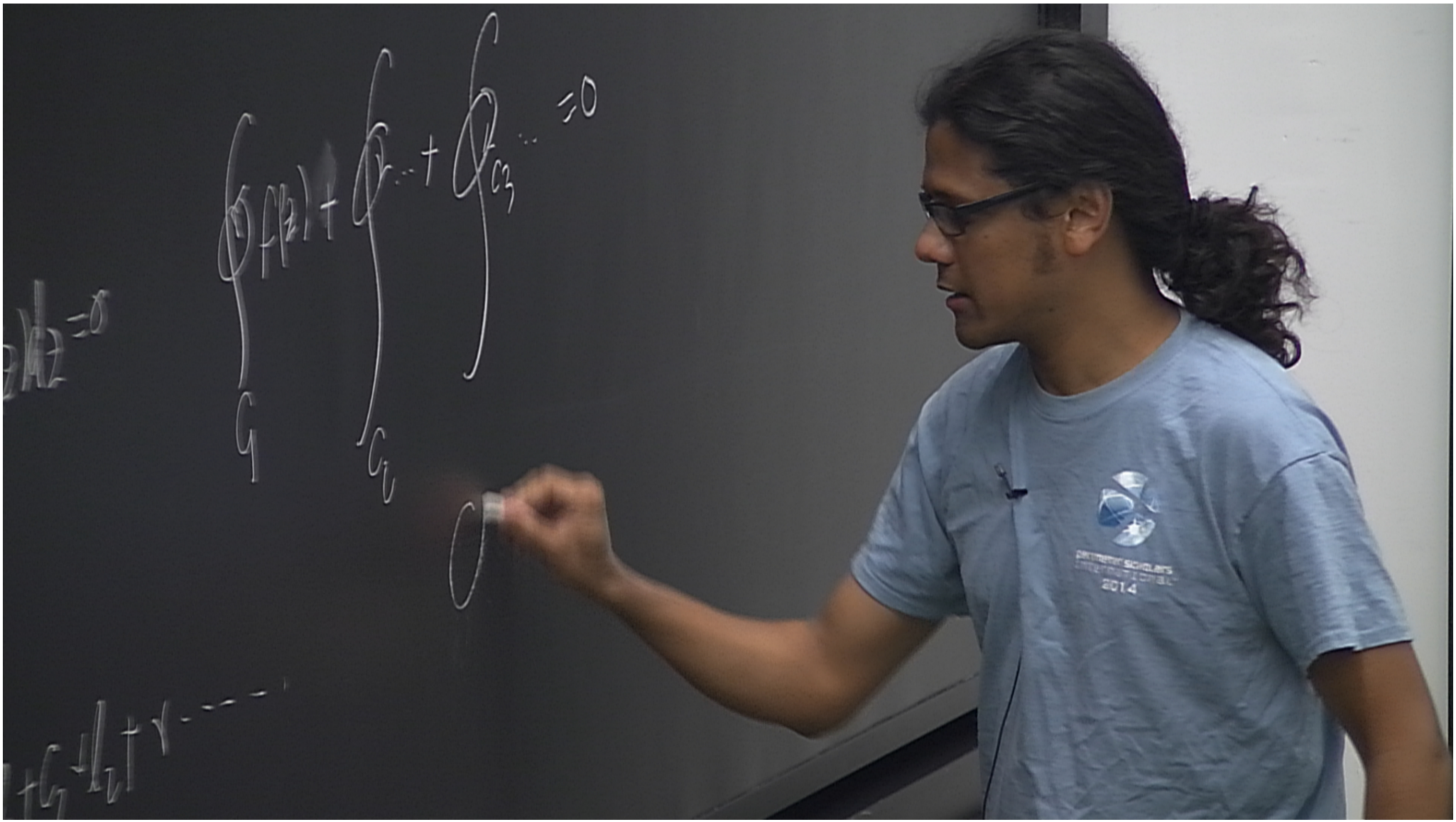

B
C1
A
C2
∮ f(z) dz = 0
C1+C2
∫ f(z) dz = − ∫ f(z) dz
C2
C1


C1
C2
C3



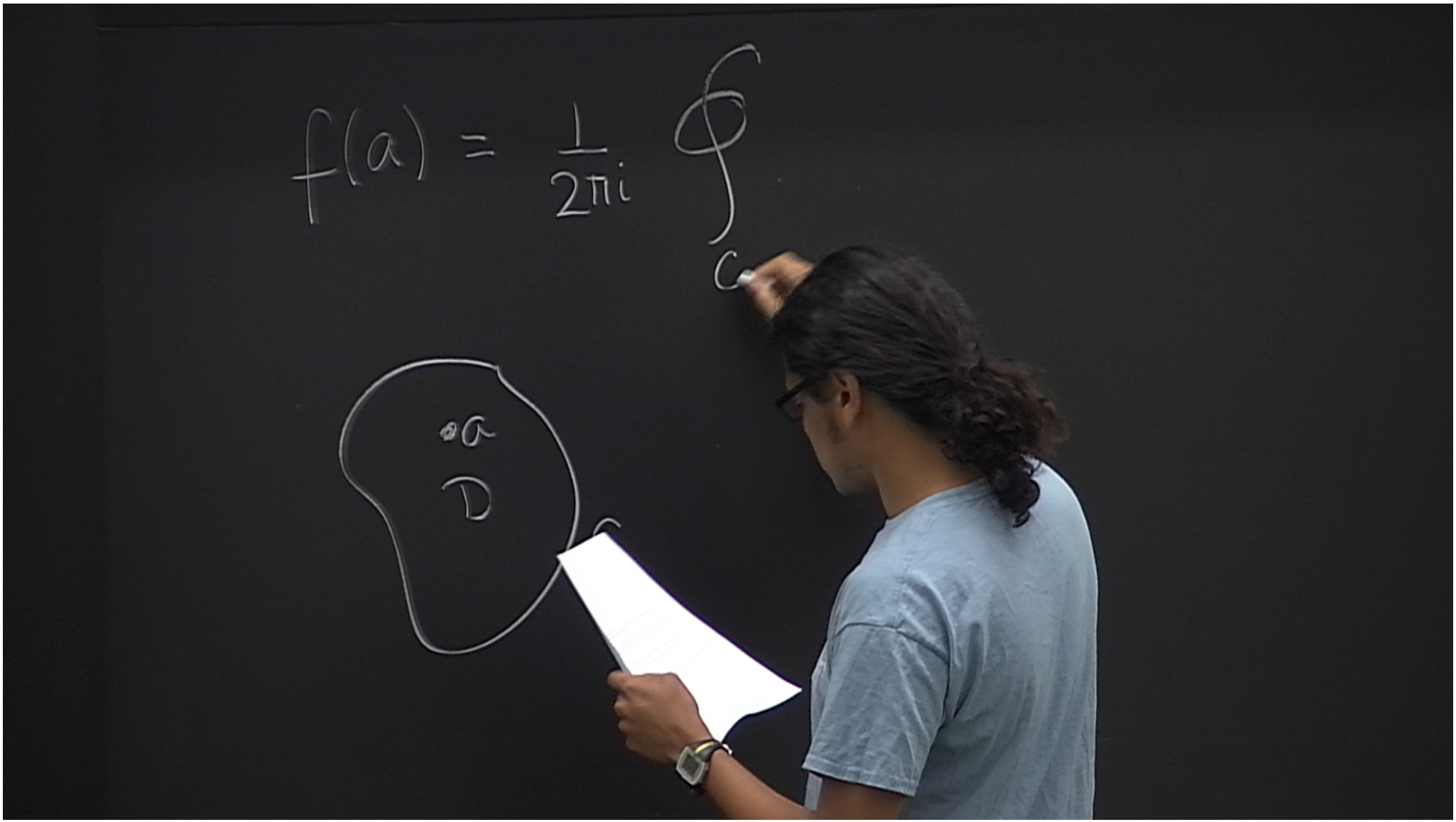

$C_2$
$C_3$
$\Gamma'$
$\oint_{\Gamma'} f(z)\,dz = 0$
$\Gamma' = C_1 + l_1 + C_2 + l_2 + \gamma$


Perimeter Scholars International
2014

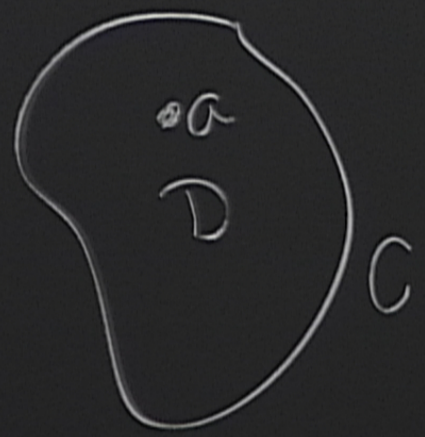

=0

$$\oint_{C_1} f(z) + \oint_{C_2} \cdots + \oint_{C_3} \cdots = 0$$

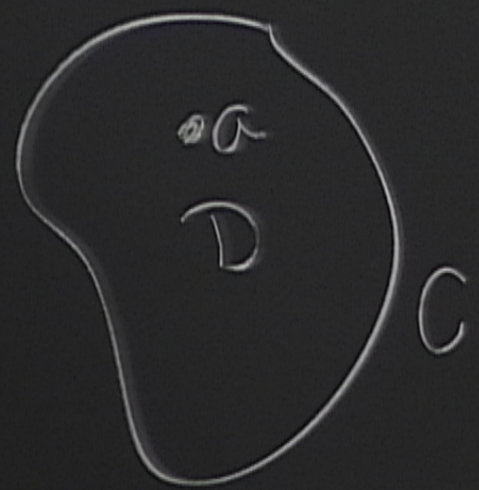
$$\oint_{C_1} f(z)\,dz = \oint_{C_2} f(z)\,dz + \oint_{C_3} f(z)\,dz$$

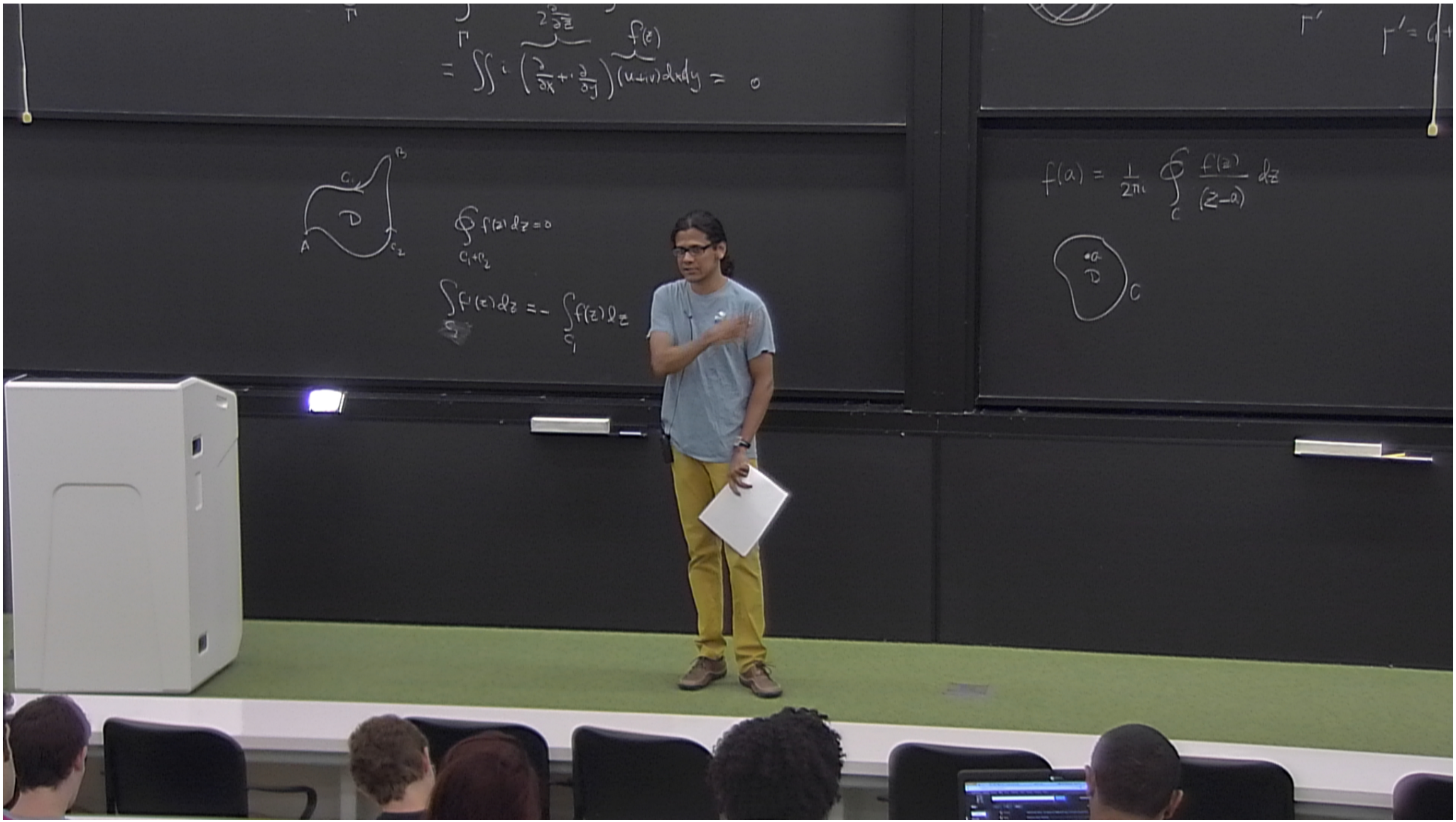

$f(a) = \frac{1}{2\pi i} \oint_C$
a
D
C

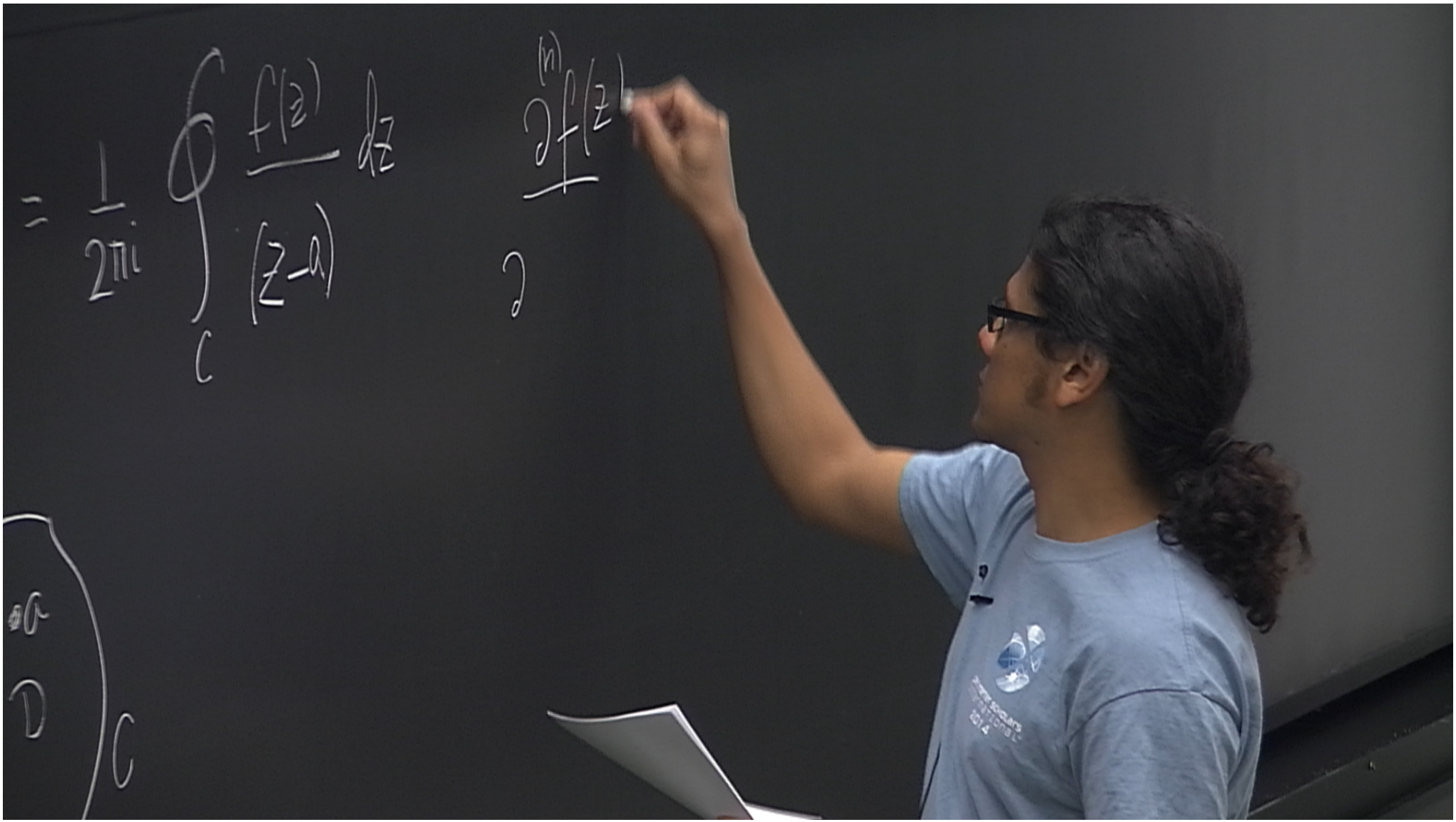
$$f(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-a)} \, dz$$

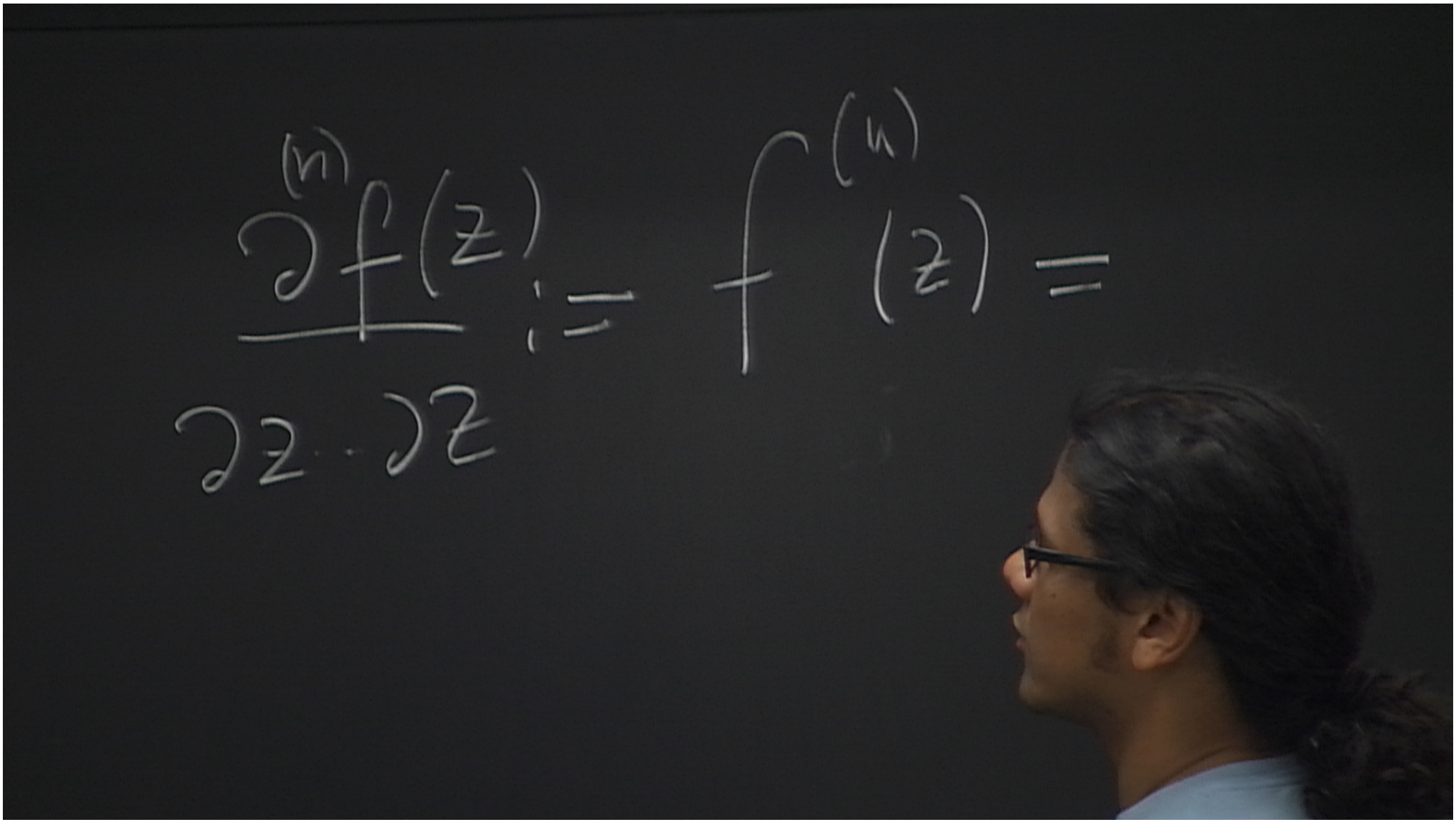


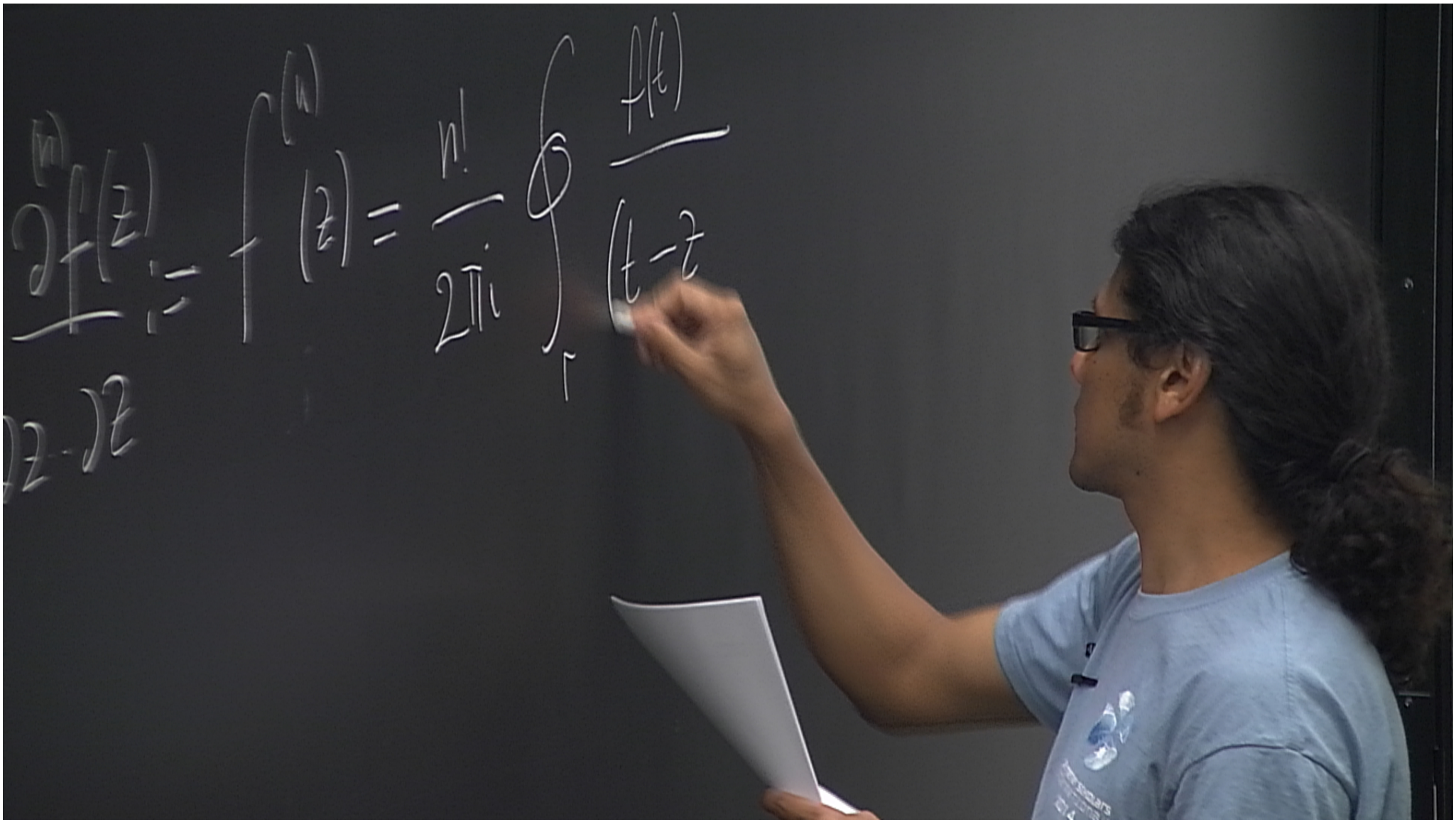
$$f(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-a)} \, dz$$

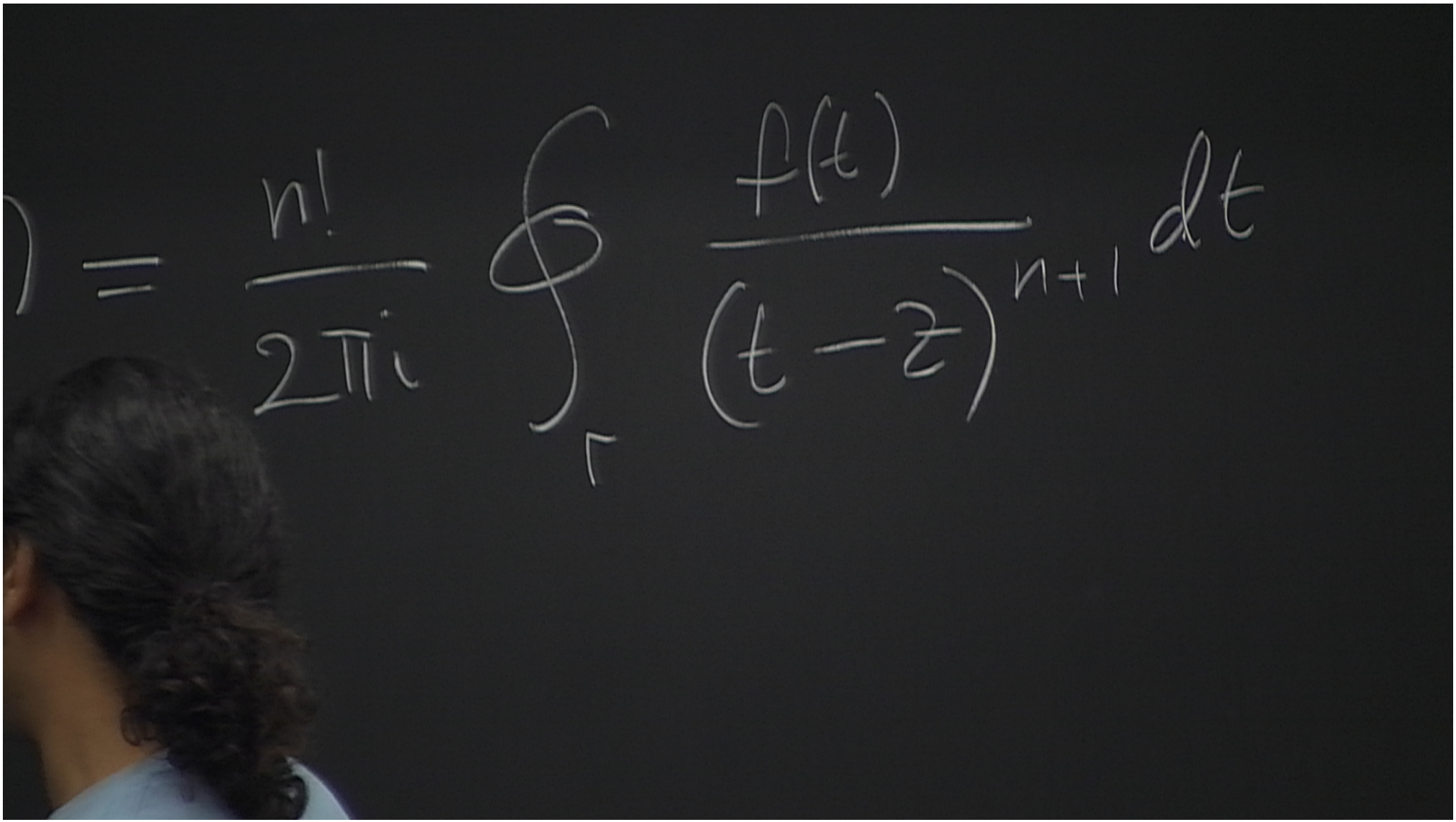



$= \iint i \left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)(u+iv)\,dx\,dy = 0$
$\oint_{C_1+C_2} f(z)\,dz = 0$
$\int_{C_2} f(z)\,dz = -\int_{C_1} f(z)\,dz$
$f(a) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{(z-a)}\,dz$

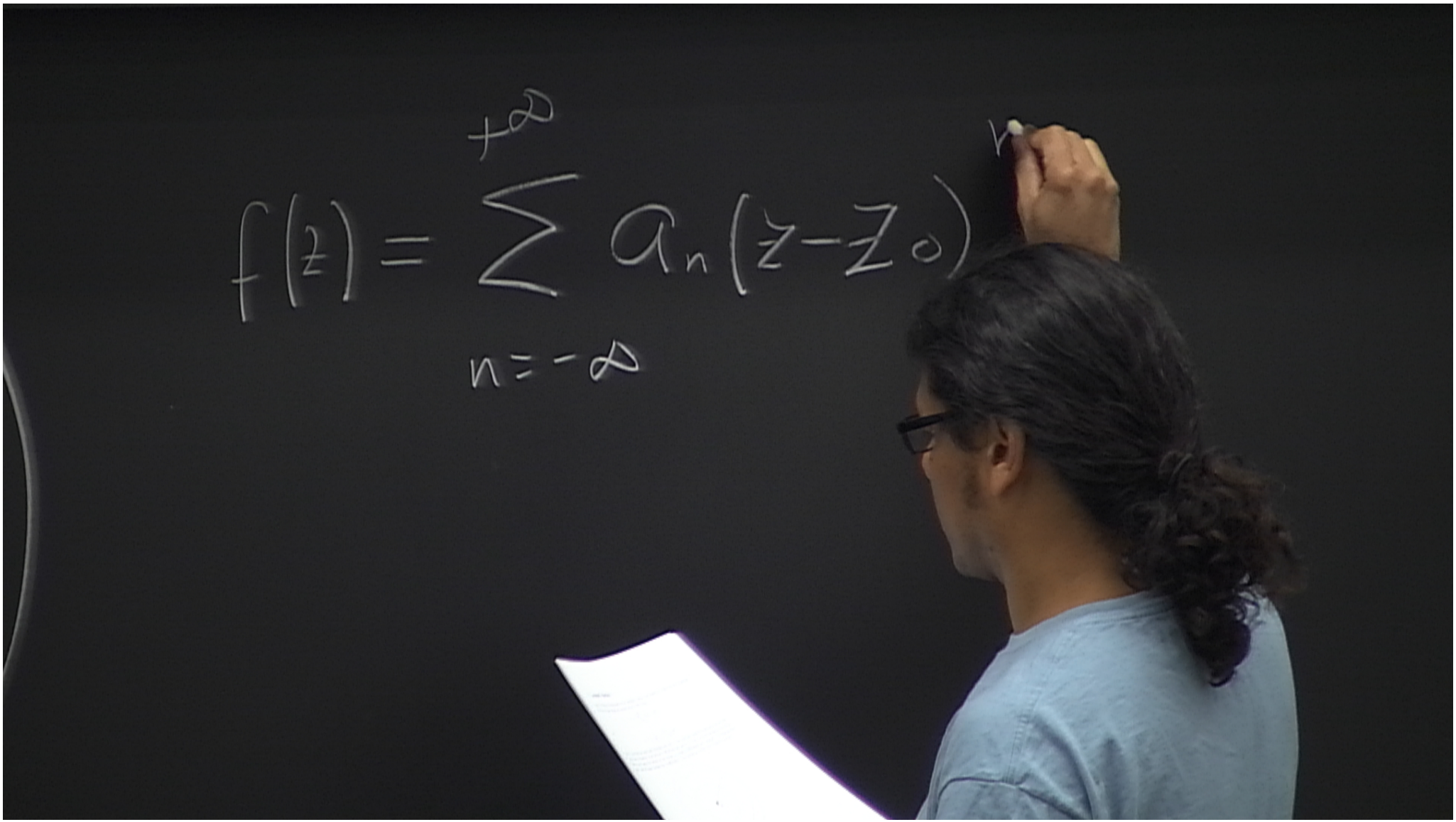

$= \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-a)} dz$
$\partial^{(n)} f(z)$
$\partial$
a
D
C


$\frac{\partial^{(n)} f(z)}{\partial z \cdot \partial z} := f^{(n)}(z) =$


$\frac{\partial^n f(z)}{\partial z \cdots \partial z} := f^{(n)}(z) = \frac{n!}{2\pi i} \oint_\Gamma \frac{f(t)}{(t-z}$

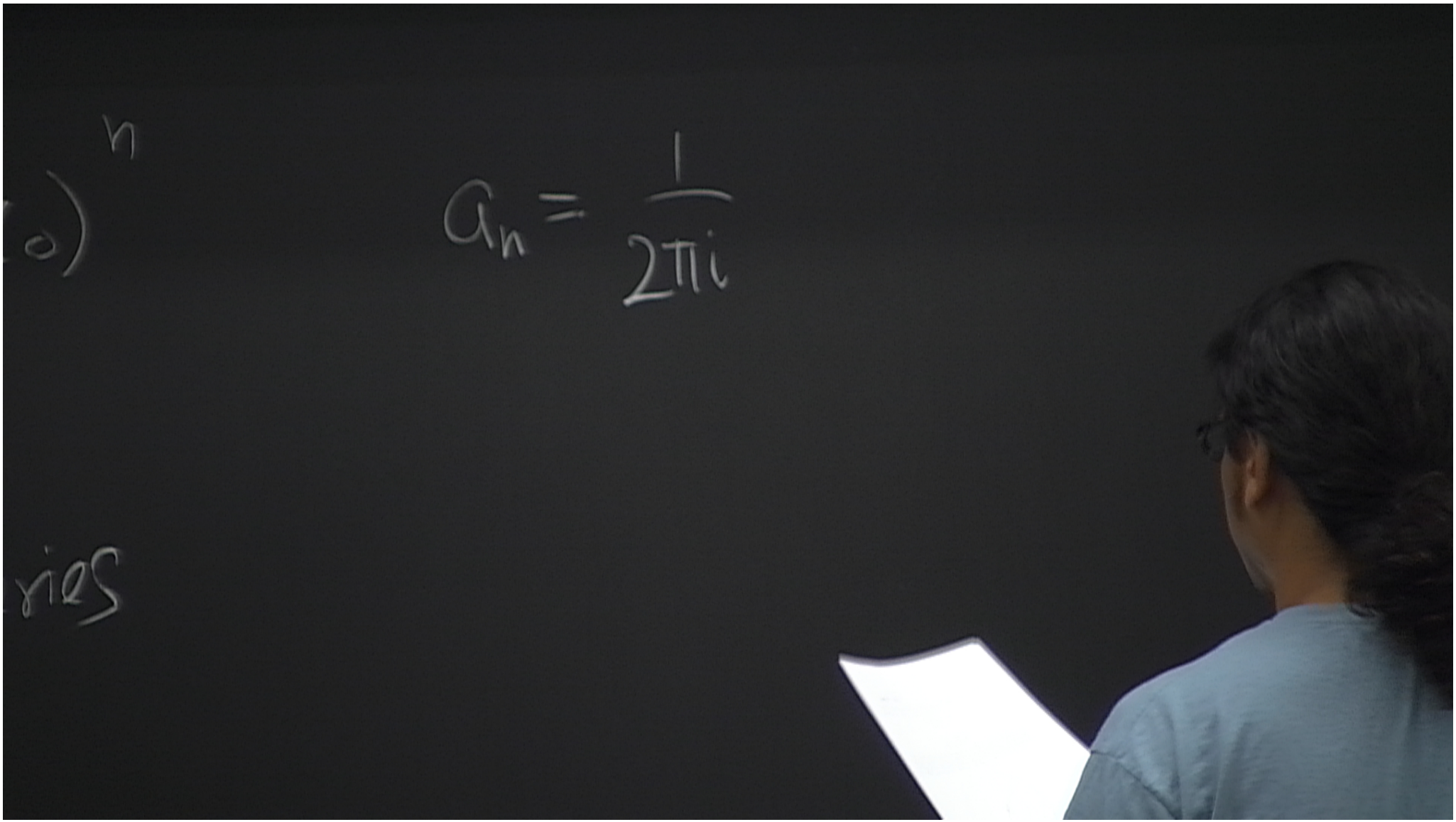

$) = \frac{n!}{2\pi i} \oint_{\Gamma} \frac{f(t)}{(t-z)^{n+1}} dt$

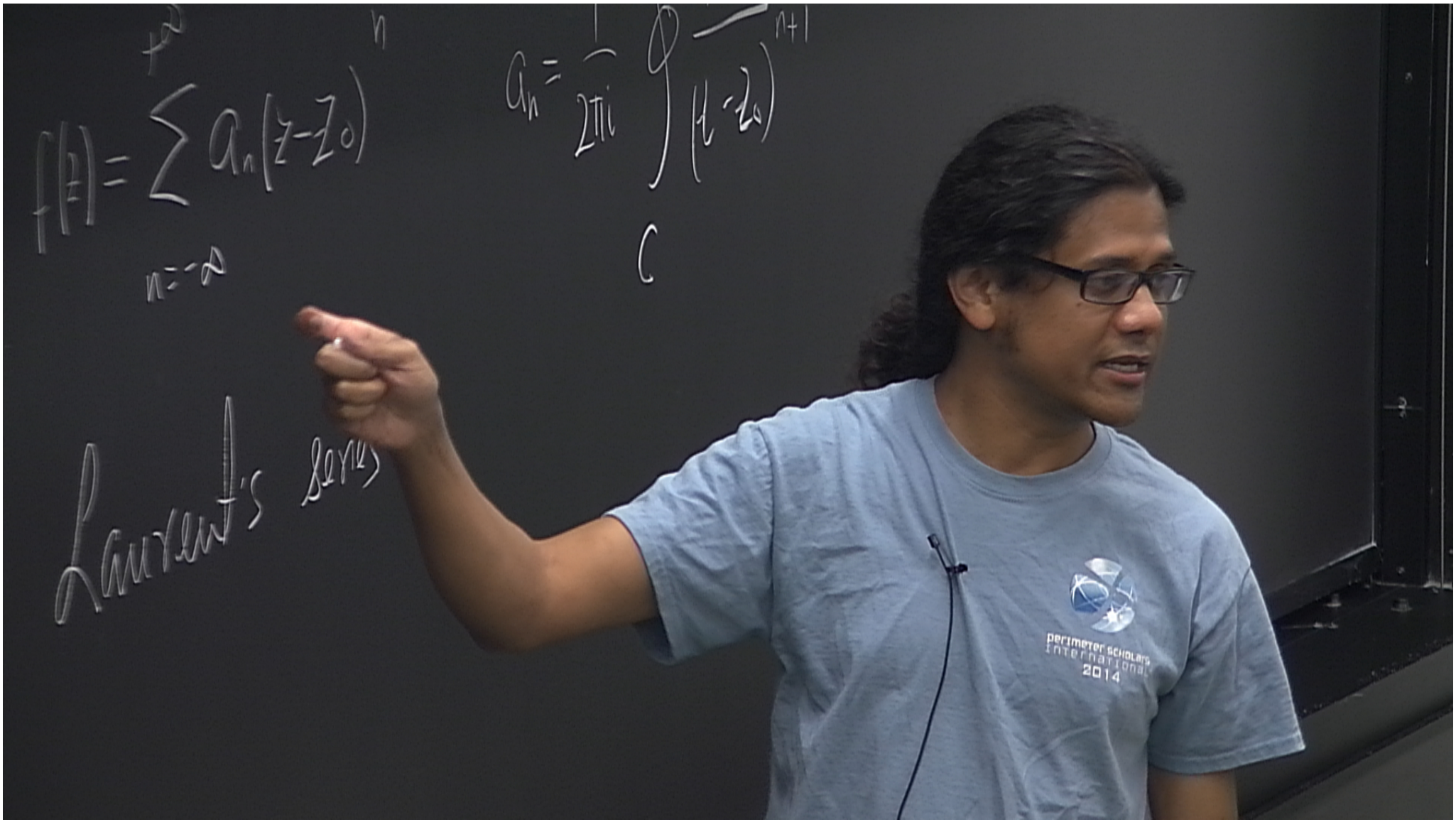
$$f(a+z) = \sum_{n=0}^{\infty} z^n f^{(n)}(a)$$


$f(z) = \sum_{n=-\infty}^{+\infty} a_n (z - z_0)^n$

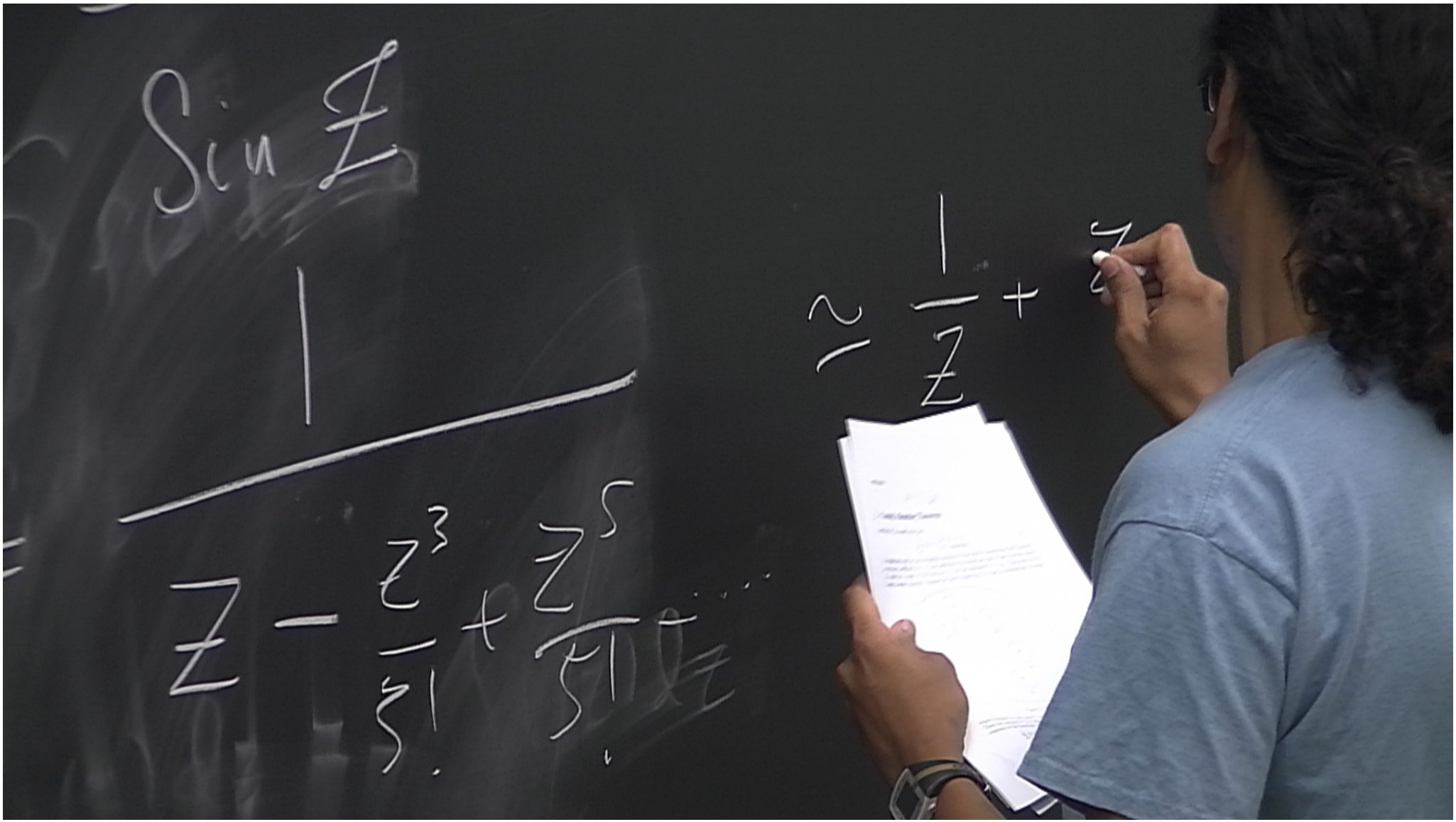
$$f(z) = \sum_{n=-\infty}^{+\infty} a_n (z - z_0)^n$$

Laurent's series


$a_n = \frac{1}{2\pi i}$

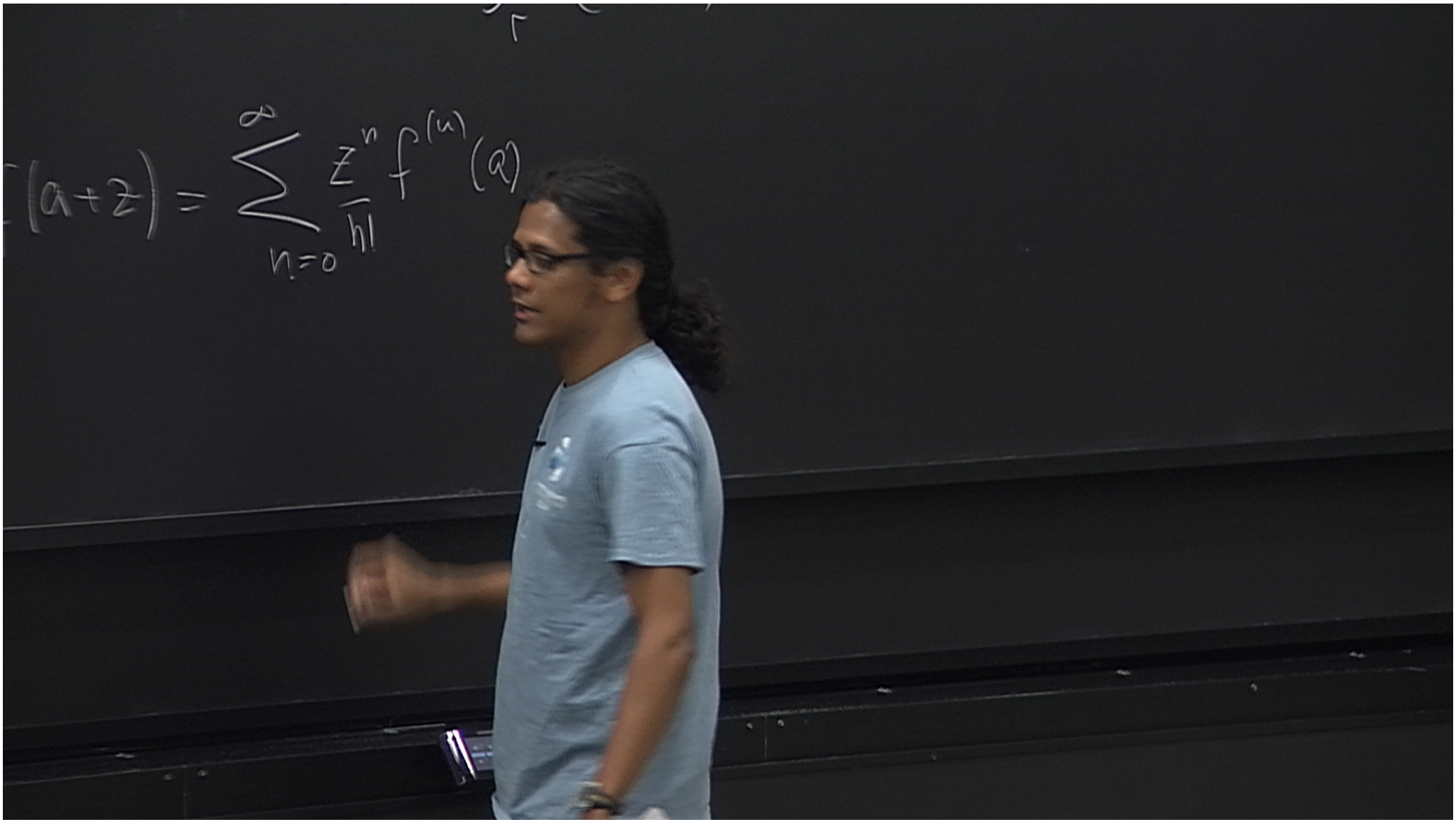

$f(z) = \sum_{n=-\infty}^{+\infty} a_n (z-z_0)^n$
$a_n = \frac{1}{2\pi i} \oint_C \frac{f}{(t-z_0)^{n+1}}$
Laurent's series
perimeter scholars
International
2014

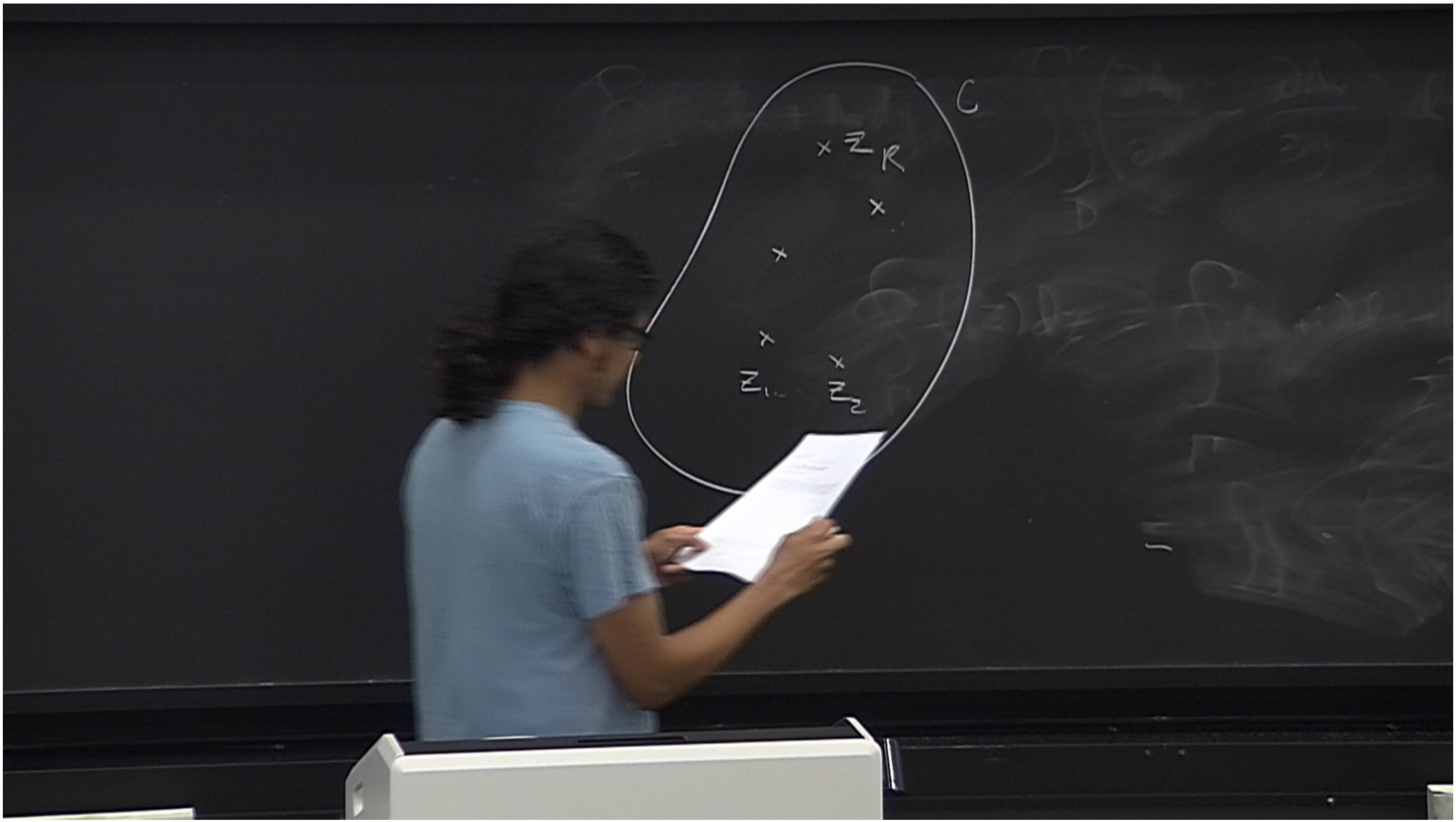
$$f(z) = \frac{1}{\sin z}$$

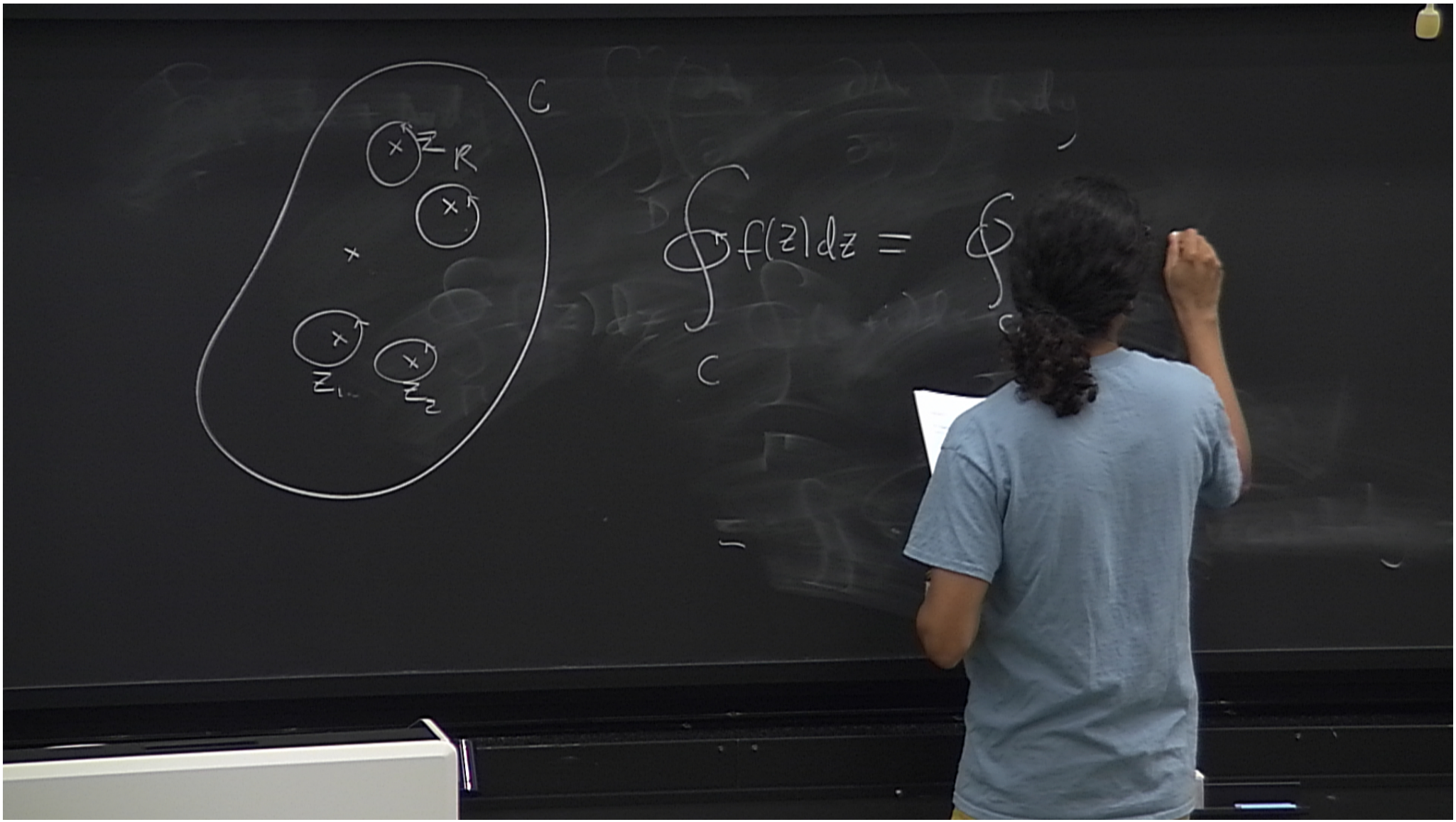

Sin Z
1
Z − Z^3/3! + Z^5/5!
≃ 1/Z +

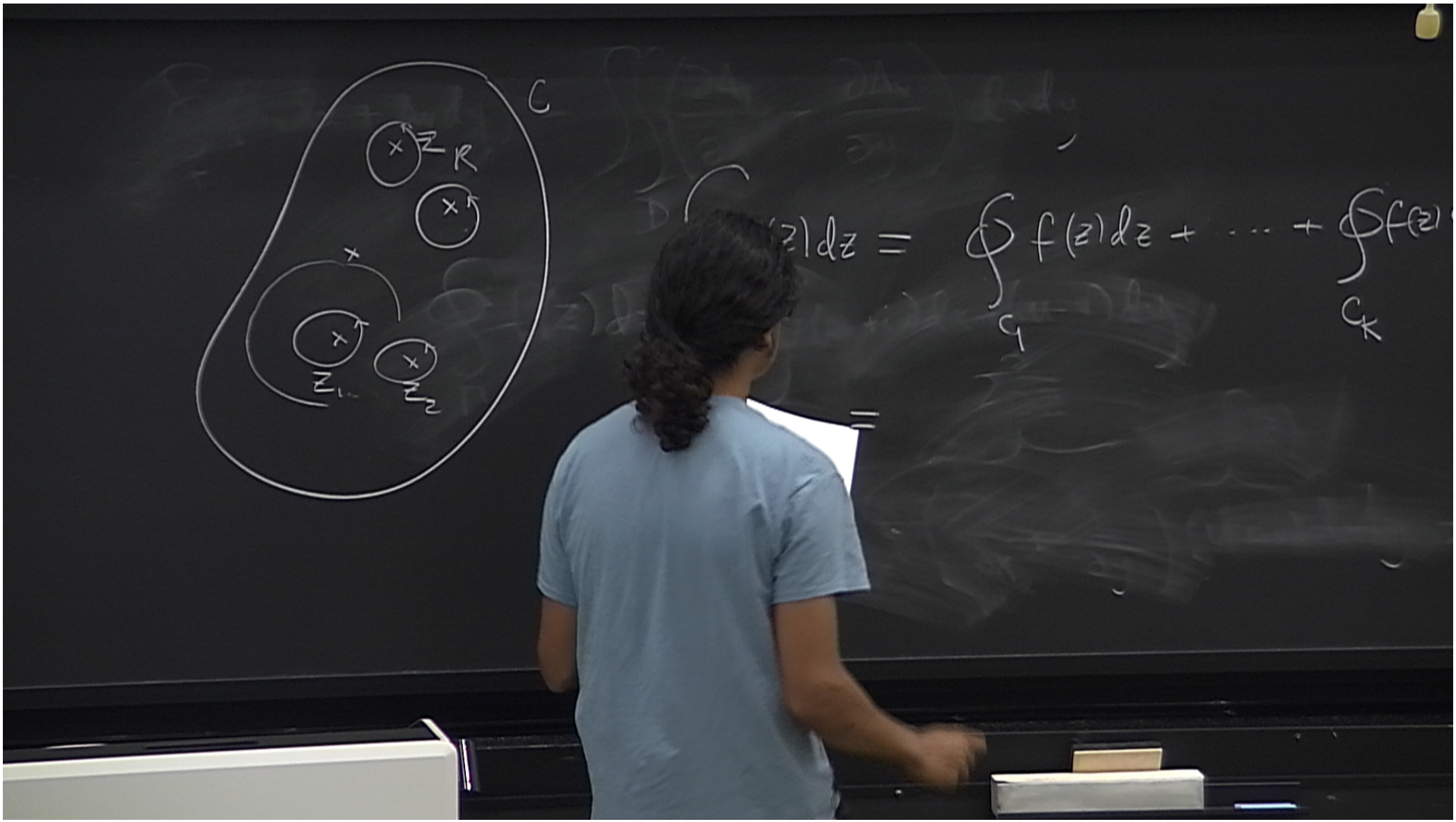
$$\simeq \frac{1}{z} + \frac{z}{6} + \frac{7}{360} z^3 + \cdots$$

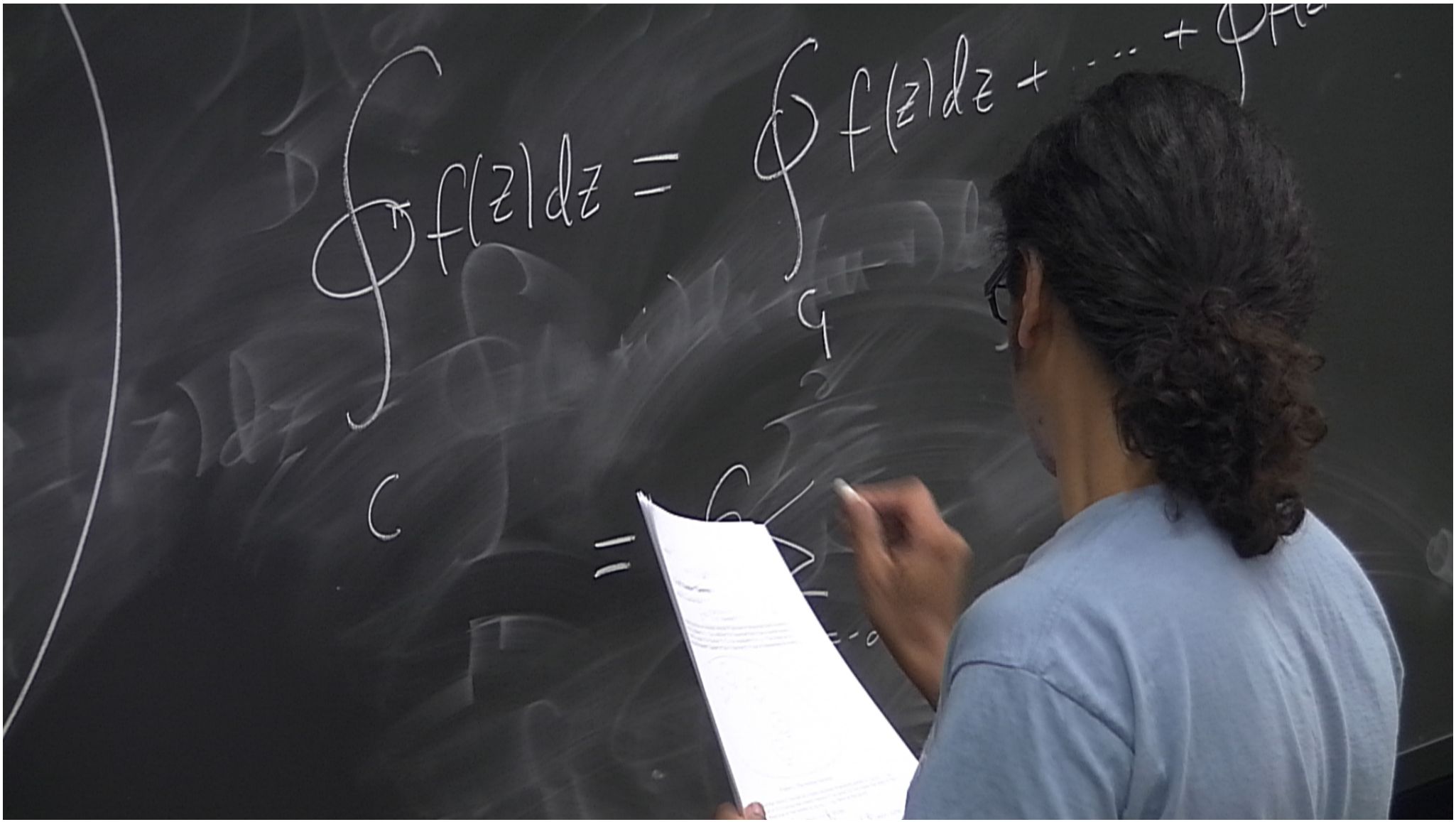
$$\oint_C z^n \, dz = \begin{cases} 2\pi i & n = -1 \\ 0 & \text{otherwise} \end{cases}$$

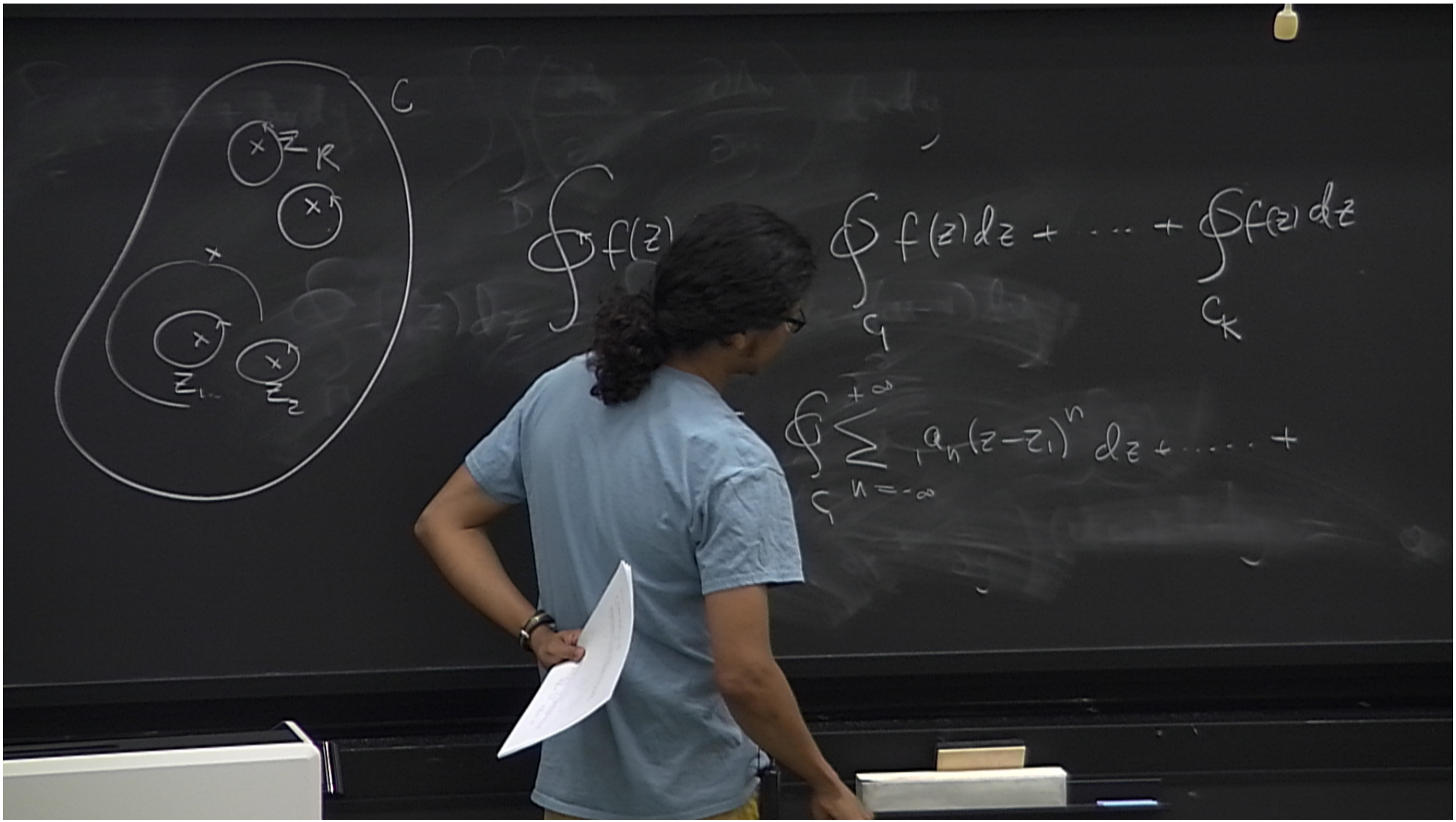

(a+z) = Σ_{n=0}^{∞} z^n/n! f^(n)(a)

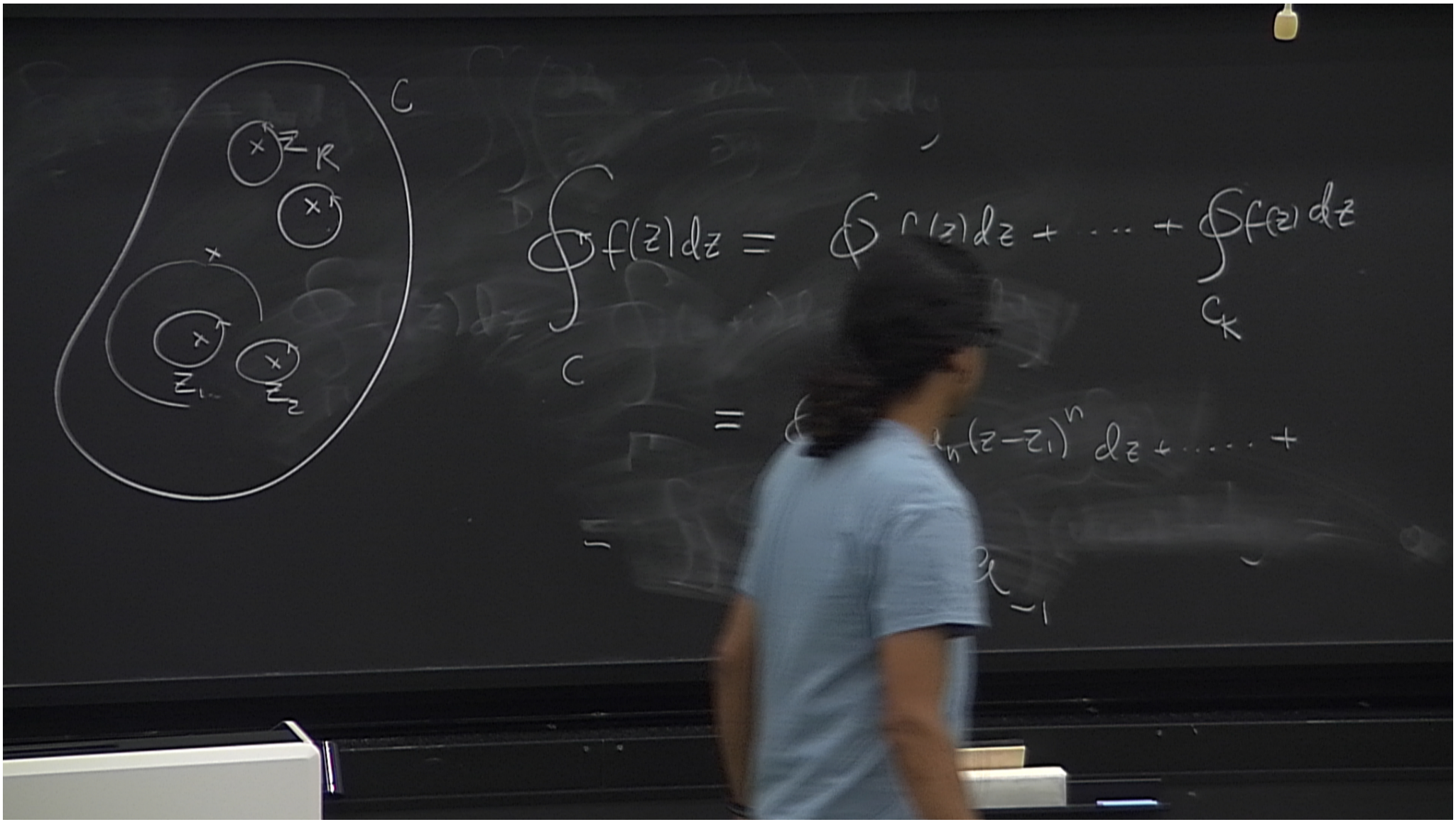

C
$z_R$
$z_1$
$z_2$

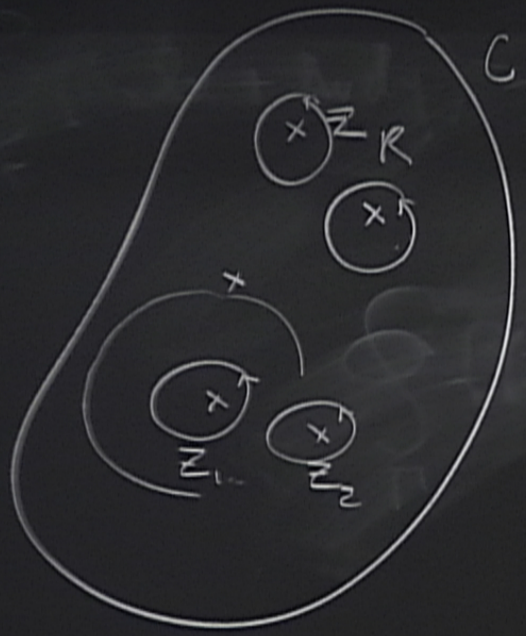

C
z
R
z1
z2
∮ f(z) dz =
C


C
R
$z_1$
$z_2$
$\oint_{C_1} f(z)dz + \cdots + \oint_{C_K} f(z)$


$\oint_C f(z)dz = \oint_{C_1} f(z)dz + \cdots +$


C
z
R
z1
z2
∮ f(z)
∮C1 f(z)dz + ... + ∮Ck f(z)dz
∮C1 Σ_{n=-∞}^{+∞} a_n (z-z1)^n dz + ... +


C
z
R
z1
z2
∮C f(z)dz = ∮ f(z)dz + ... + ∮CK f(z)dz
= ∮ an(z−z1)^n dz + ... +




$$\oint_C f(z)\,dz = \oint_{C_1} f(z)\,dz + \cdots + \oint_{C_K} f(z)\,dz$$

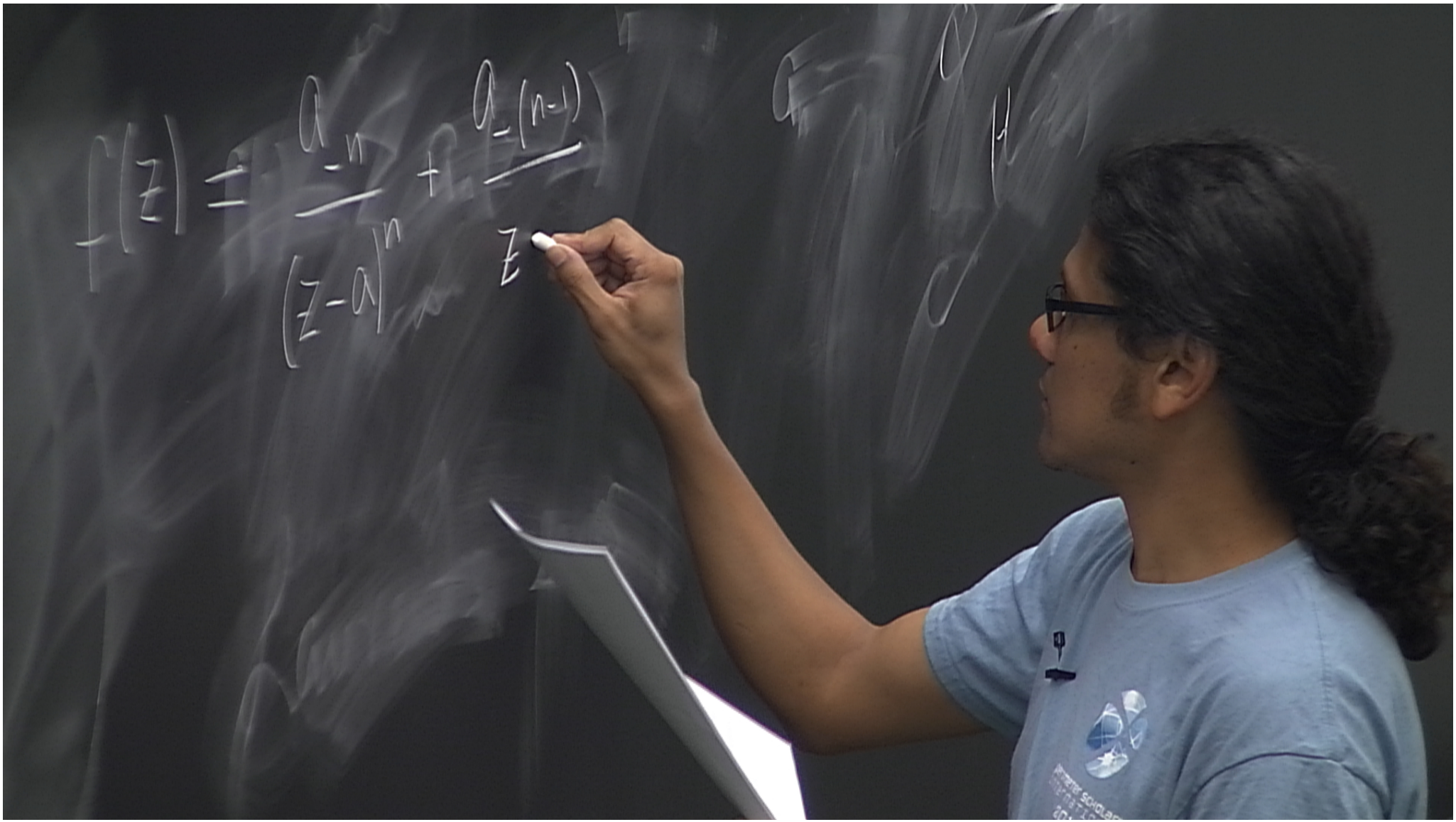
$$= \oint_{C_1} \sum_{n=-\infty}^{+\infty} a_n (z-z_1)^n\,dz + \cdots + $$

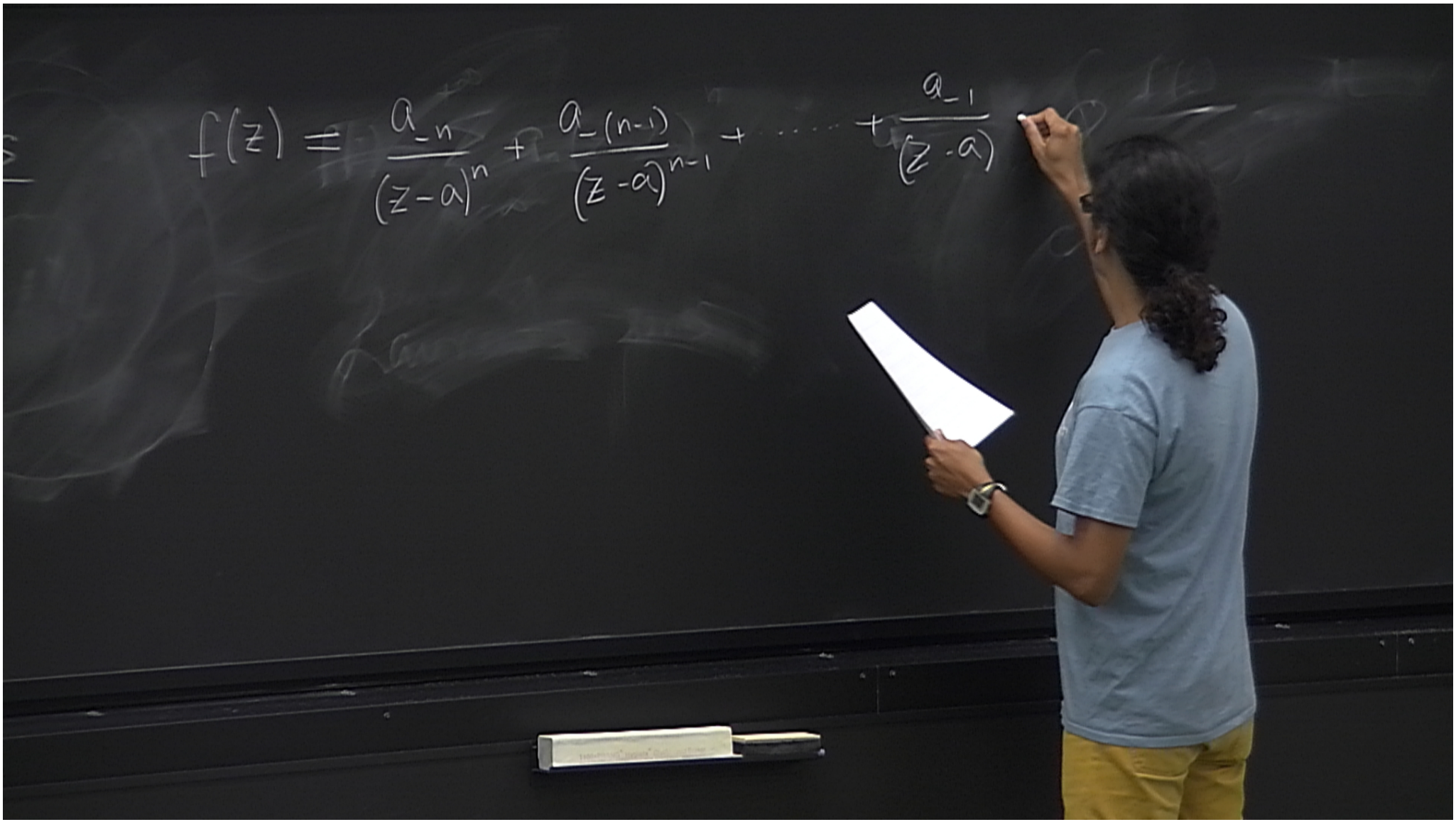
$$= 2\pi i \sum_{p=1}^{K} a_{-1}$$

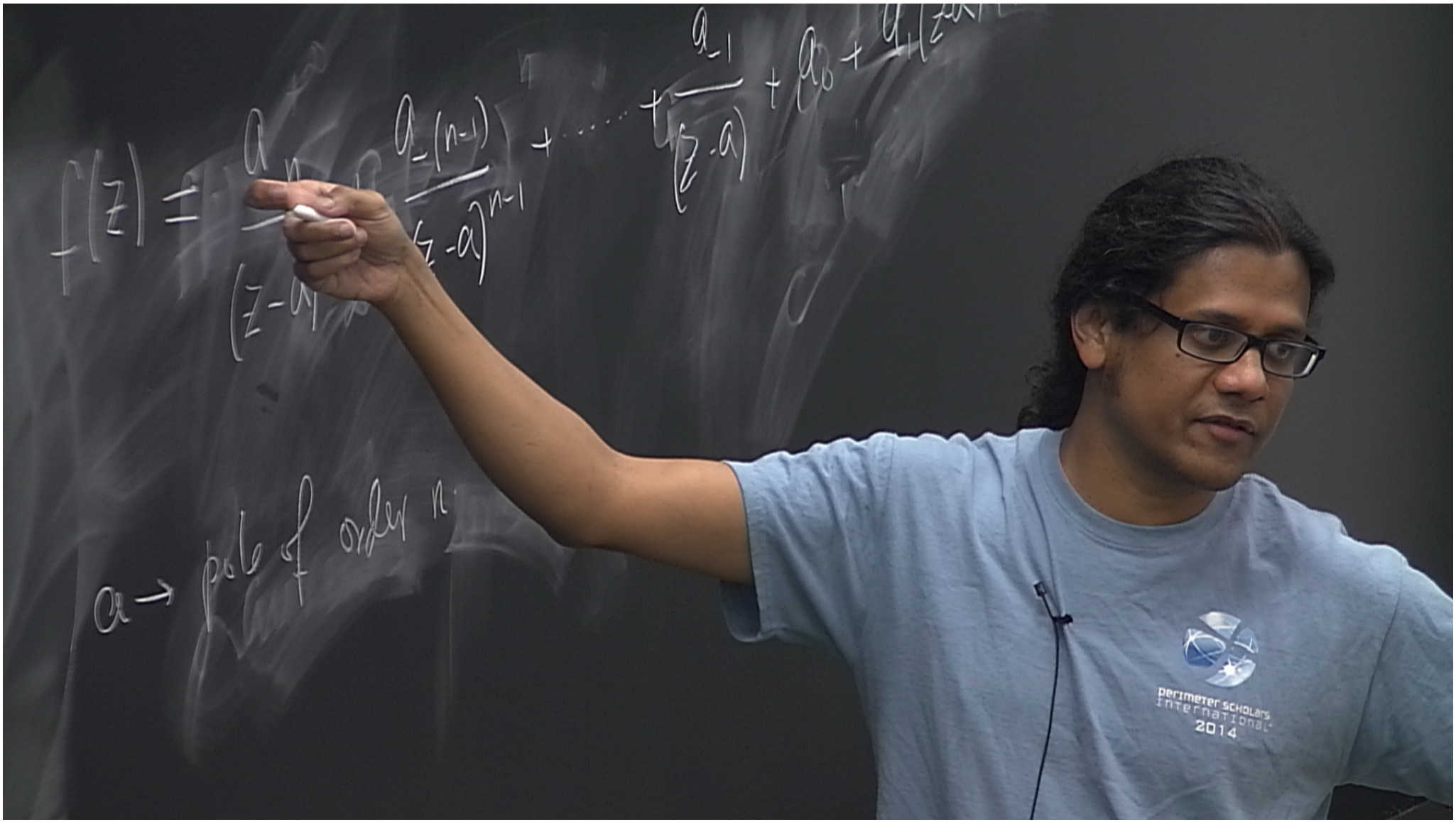
$$\int_C dz \; f(z) = 2\pi i \sum_{z_i} \operatorname{Res} f(z)$$

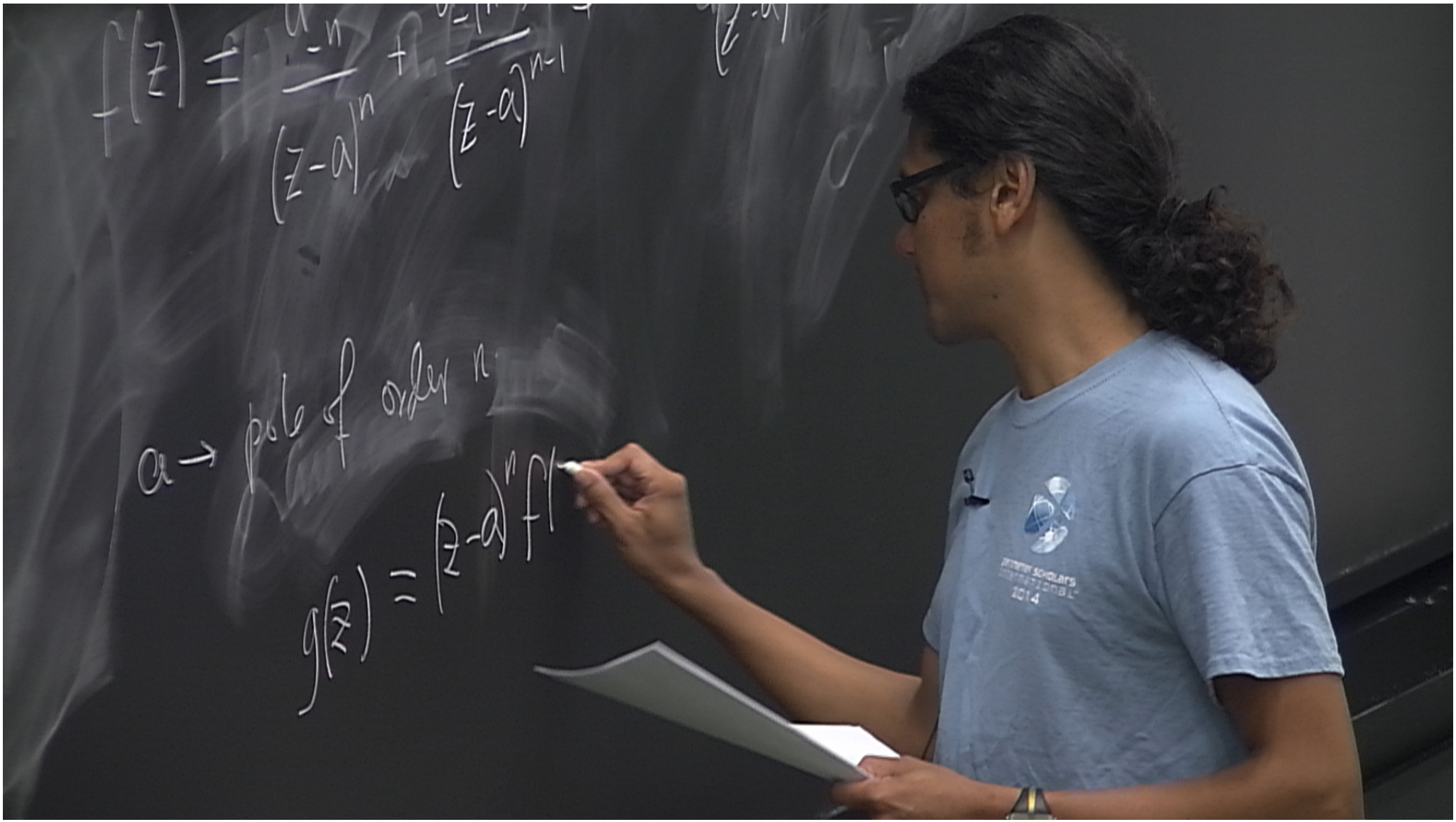
$$\int_C \cdots \, dz$$

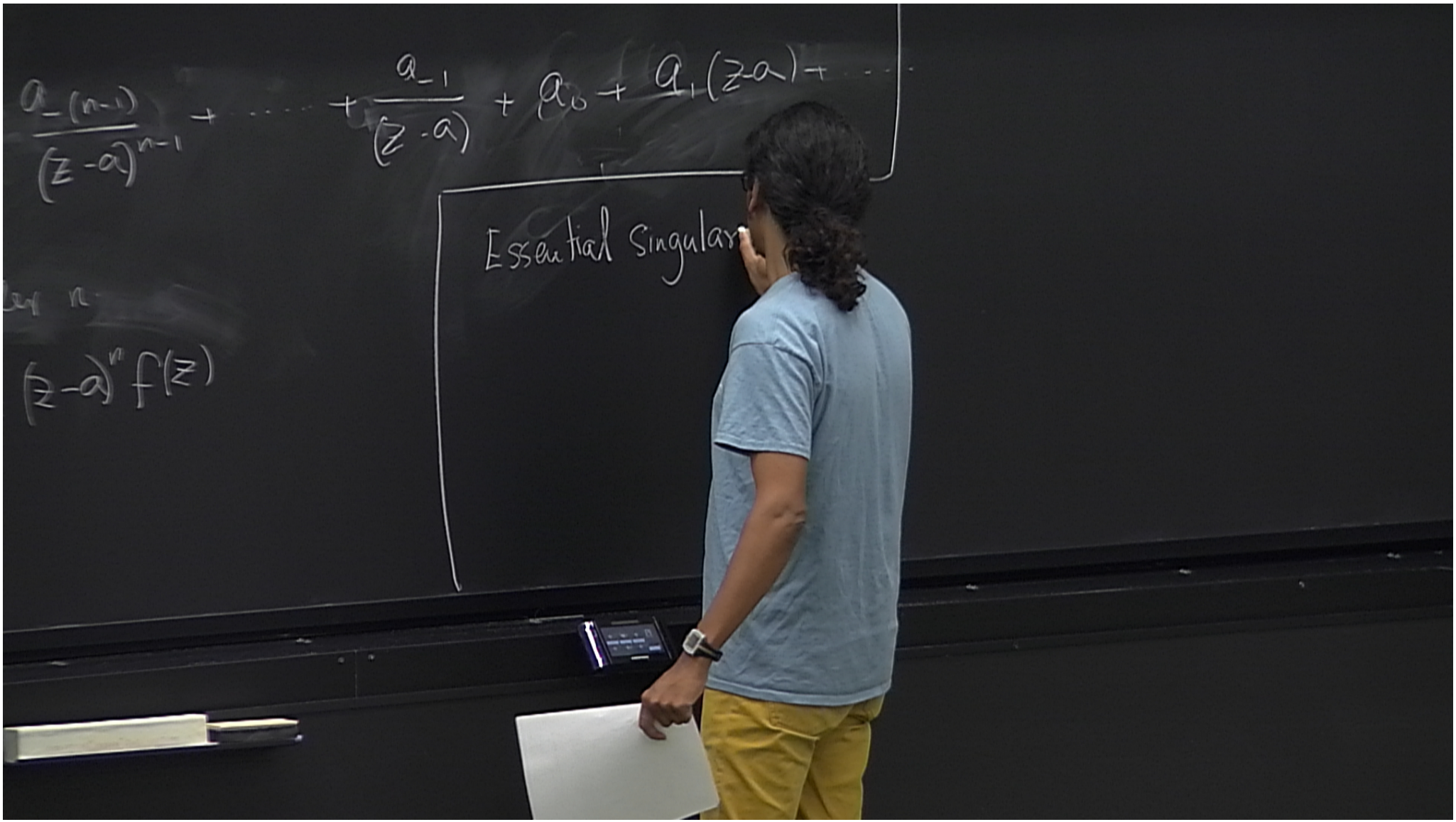
$$= 2\pi i \sum_{z_i} \operatorname{Res} f(z) \text{ at } z = z_i$$

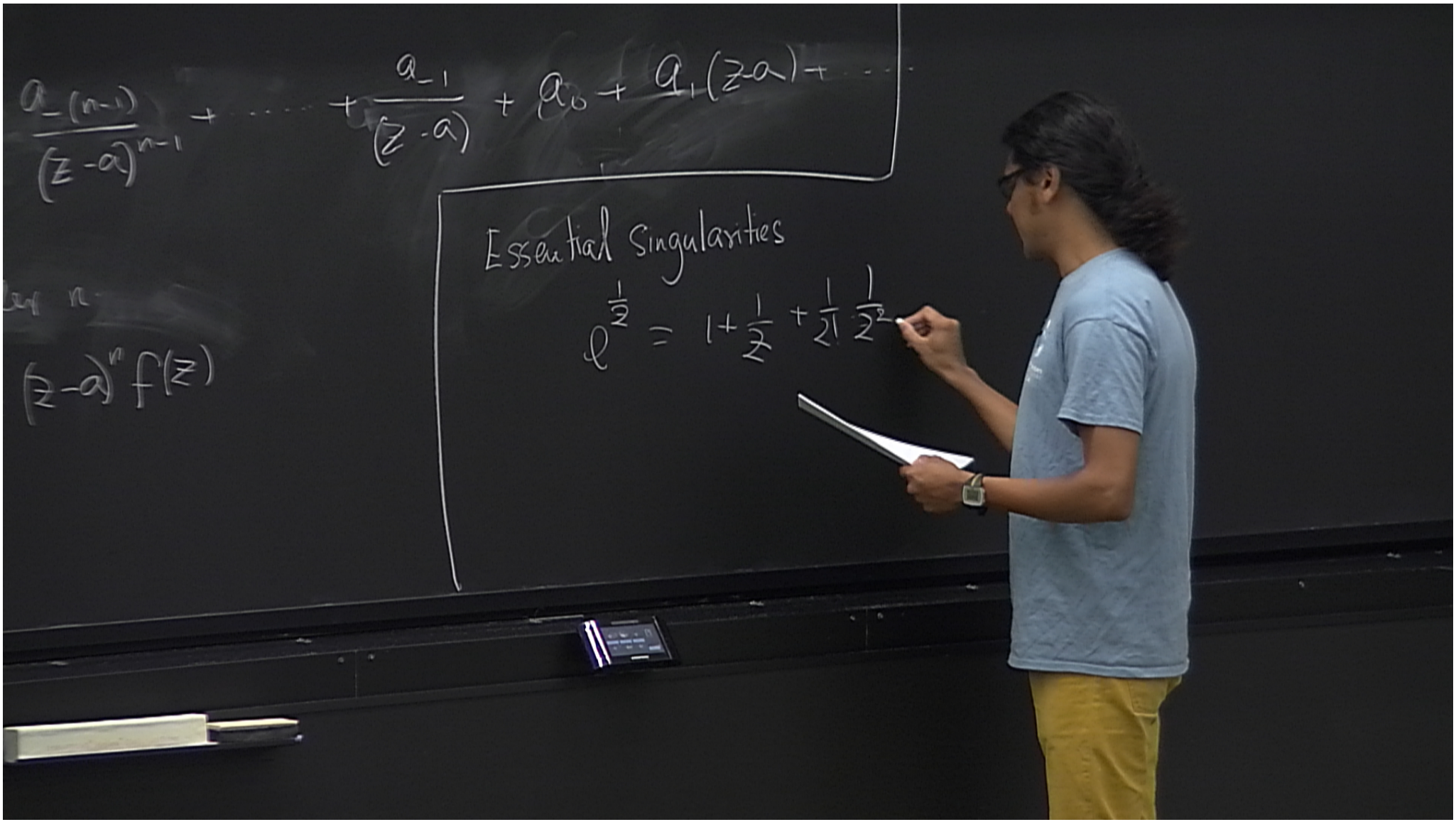

f(z) = a_-n / (z-a)^n + a_-(n-1) / z


$f(z) = \frac{a_{-n}}{(z-a)^n} + \frac{a_{-(n-1)}}{(z-a)^{n-1}} + \cdots + \frac{a_{-1}}{(z-a)}$

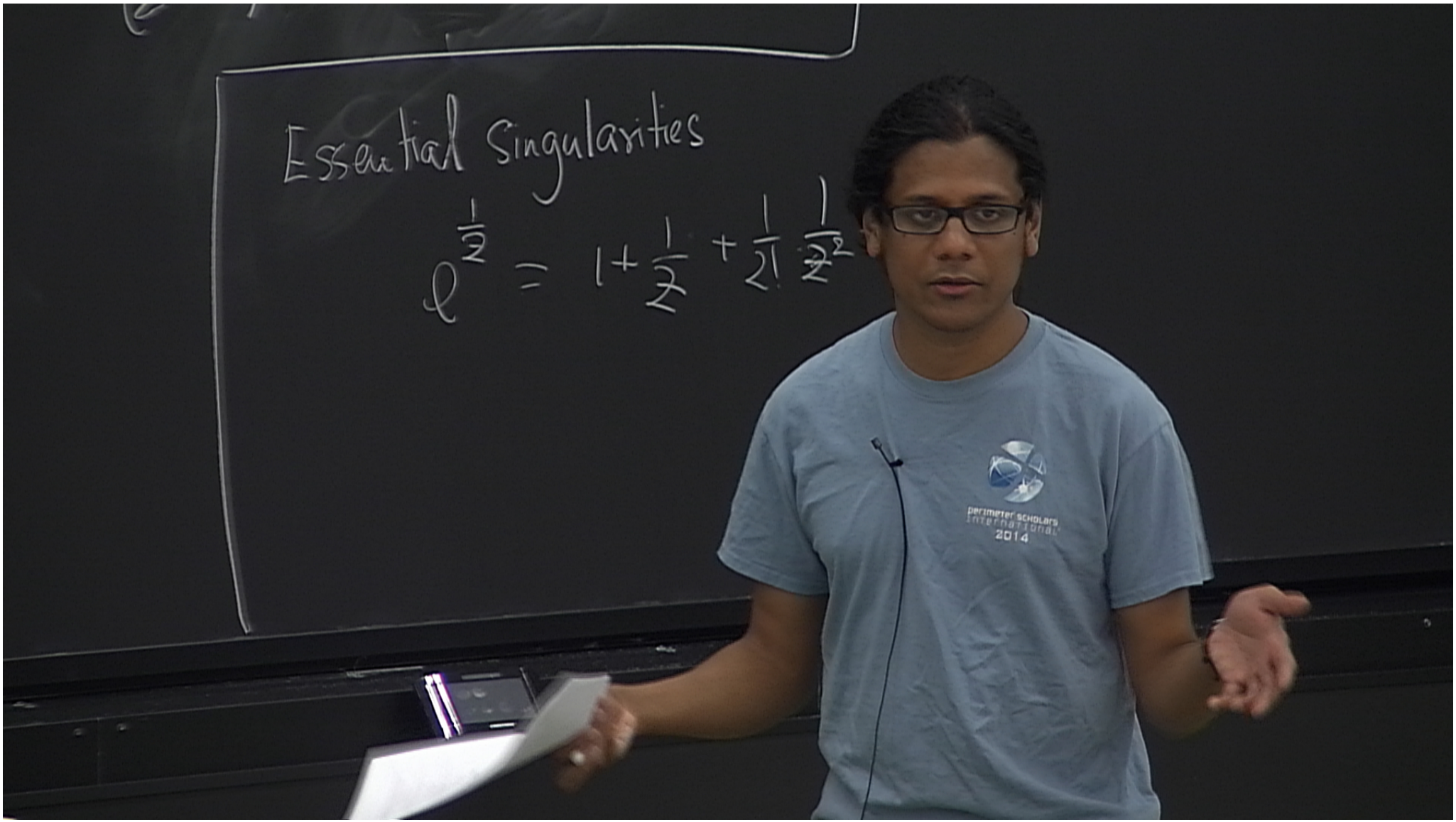

a → pole of order n
perimeter scholars
international
2014

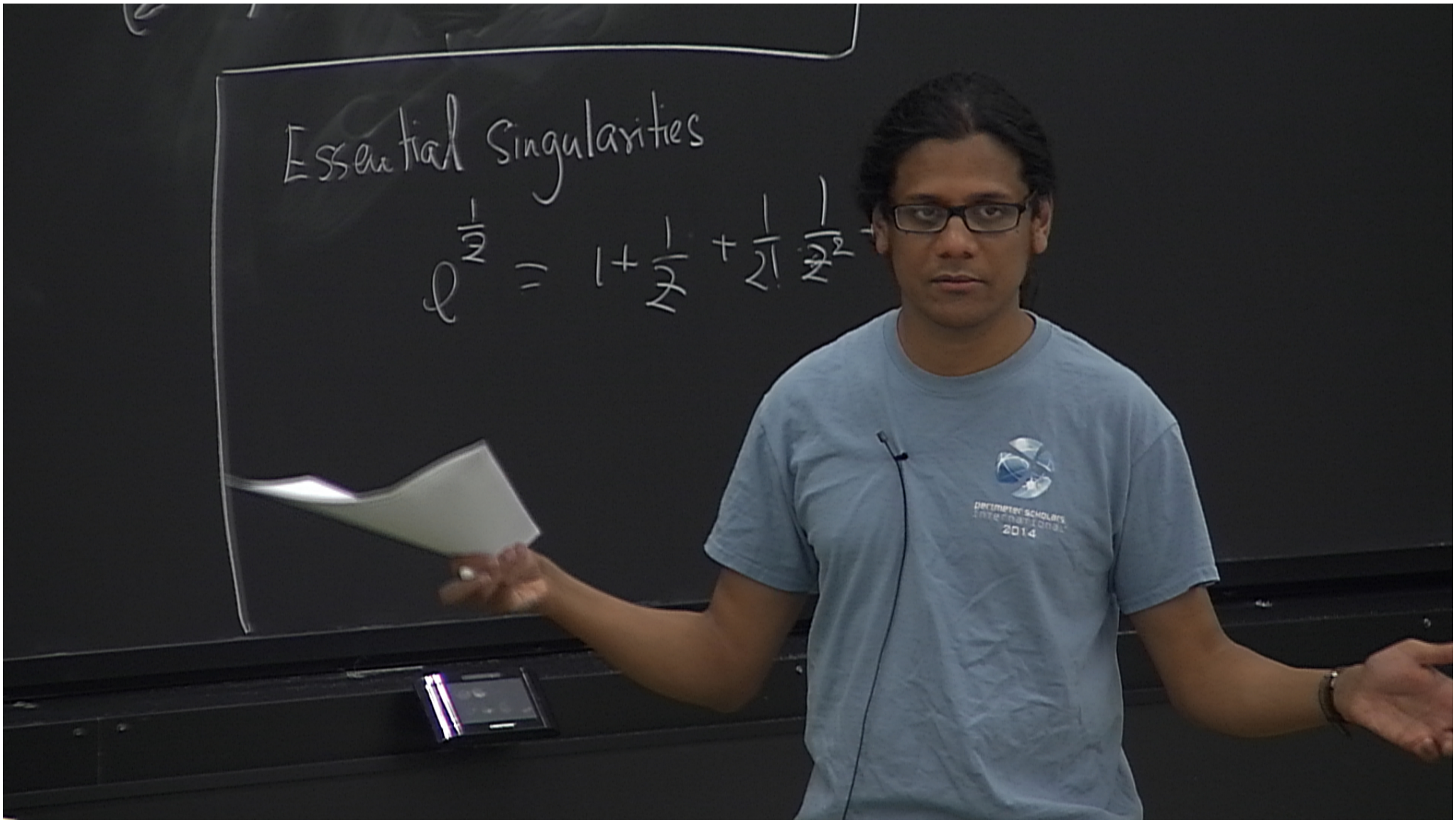

a → pole of order n
g(z) = (z-a)^n f(

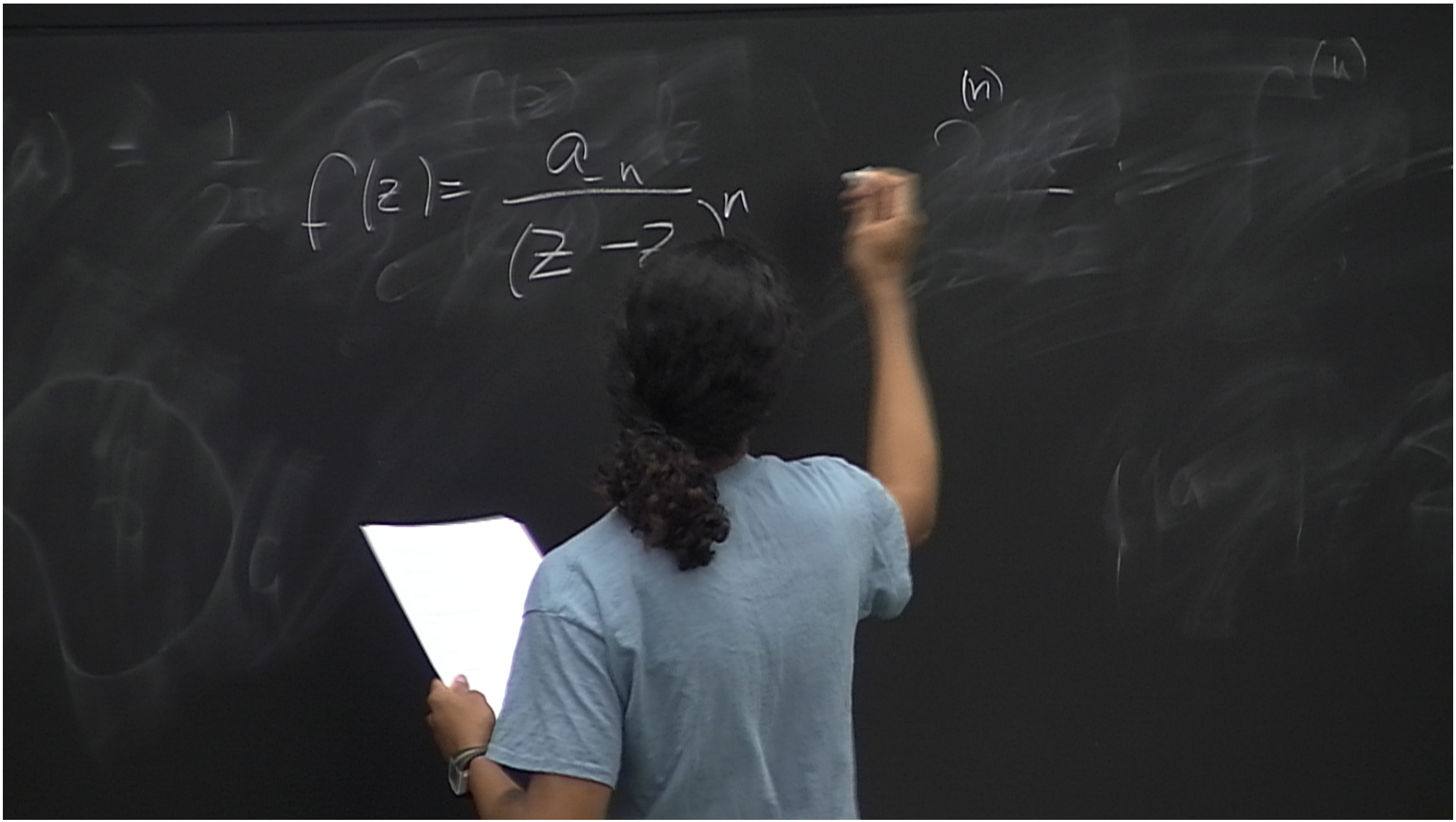

$\frac{a_{-(n-1)}}{(z-a)^{n-1}} + \cdots + \frac{a_{-1}}{(z-a)} + a_0 + a_1(z-a) + \cdots$
Essential Singular
$(z-a)^n f(z)$

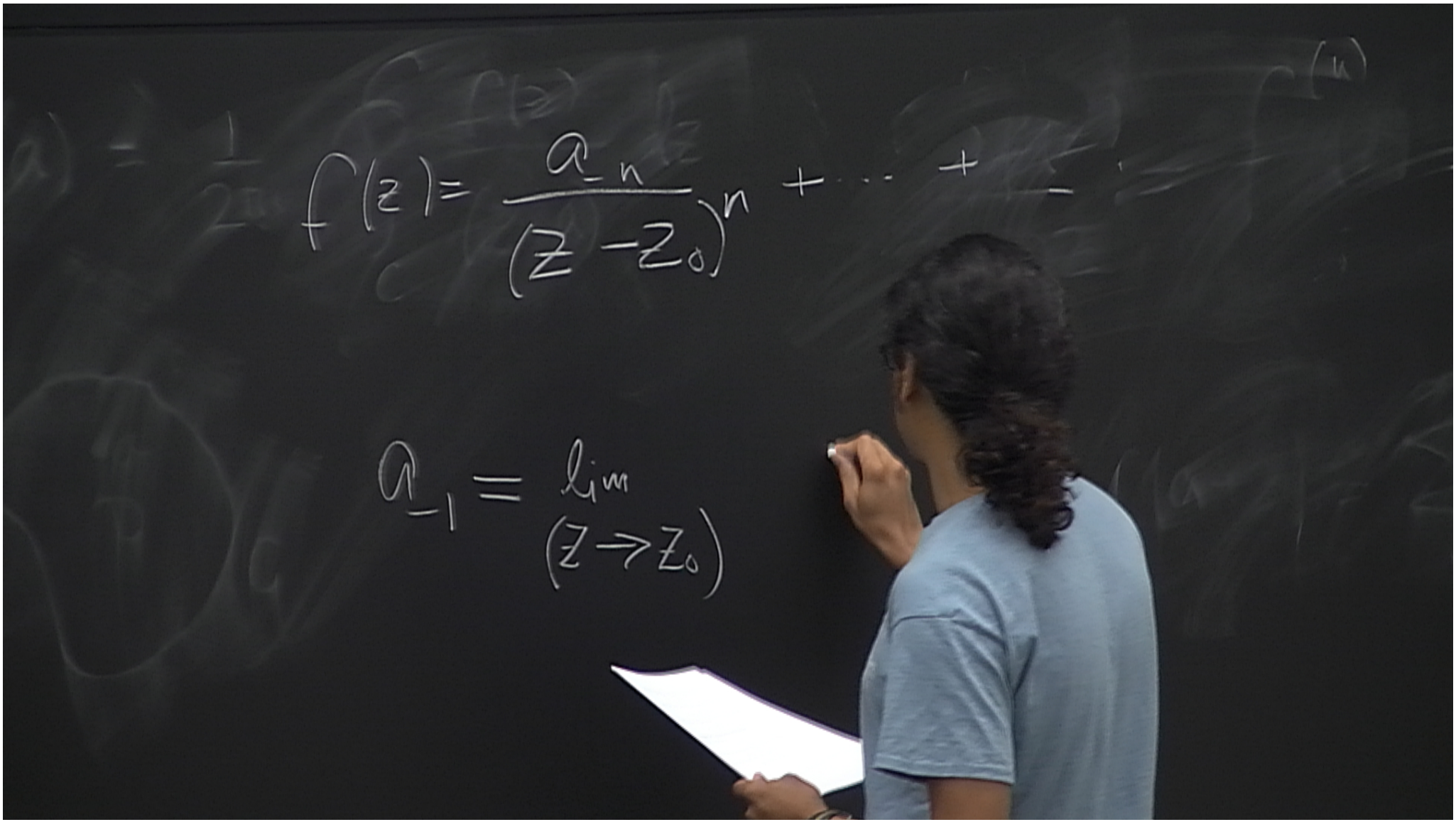

$\frac{a_{-(n-1)}}{(z-a)^{n-1}} + \cdots + \frac{a_{-1}}{(z-a)} + a_0 + a_1(z-a) + \cdots$
$(z-a)^n f(z)$
Essential Singularities
$e^{\frac{1}{z}} = 1 + \frac{1}{z} + \frac{1}{2!}\frac{1}{z^2}$

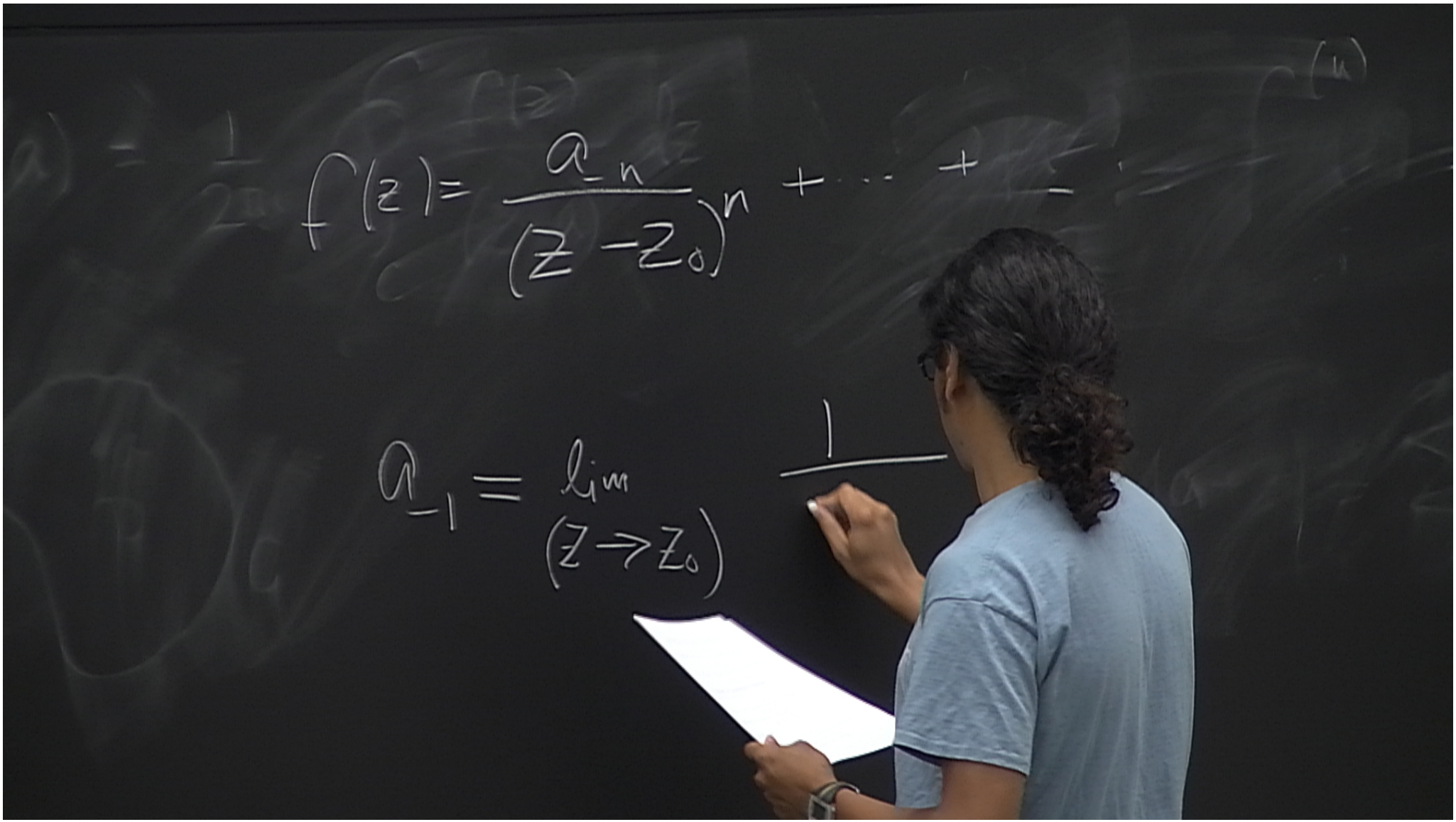
2014


Essential Singularities
$e^{\frac{1}{z}} = 1 + \frac{1}{z} + \frac{1}{2!}\frac{1}{z^2}$
perimeter scholars
international
2014


Essential Singularities
$e^{\frac{1}{z}} = 1 + \frac{1}{z} + \frac{1}{2!}\frac{1}{z^2} +$
2014


f(z) = a_-n / (z - z)^n


$f(z) = \frac{a_{-n}}{(z-z_0)^n} + \cdots +$
$a_{-1} = \lim_{(z \to z_0)}$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$a_{-1} = \lim_{(z \to z_0)}$

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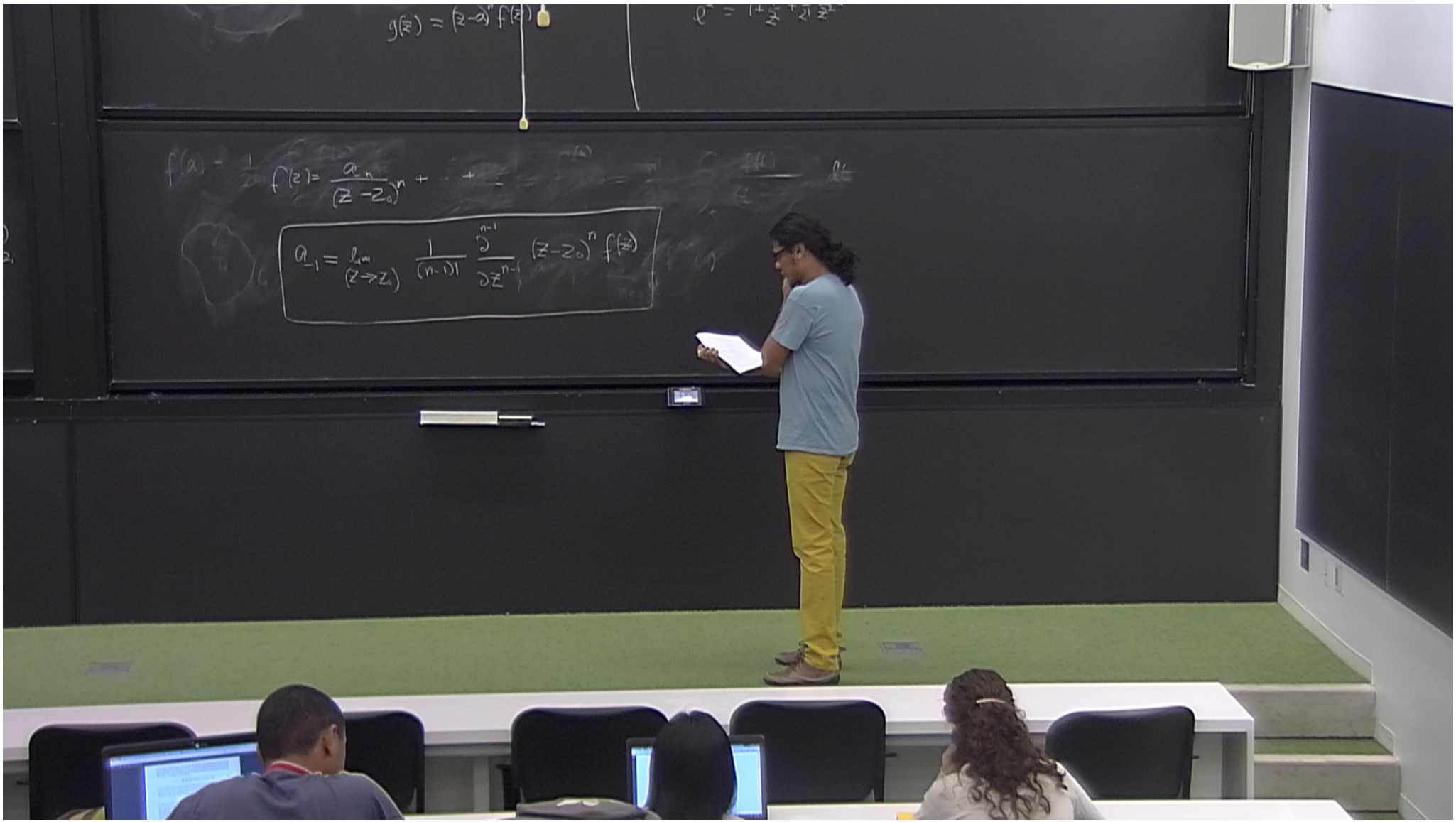
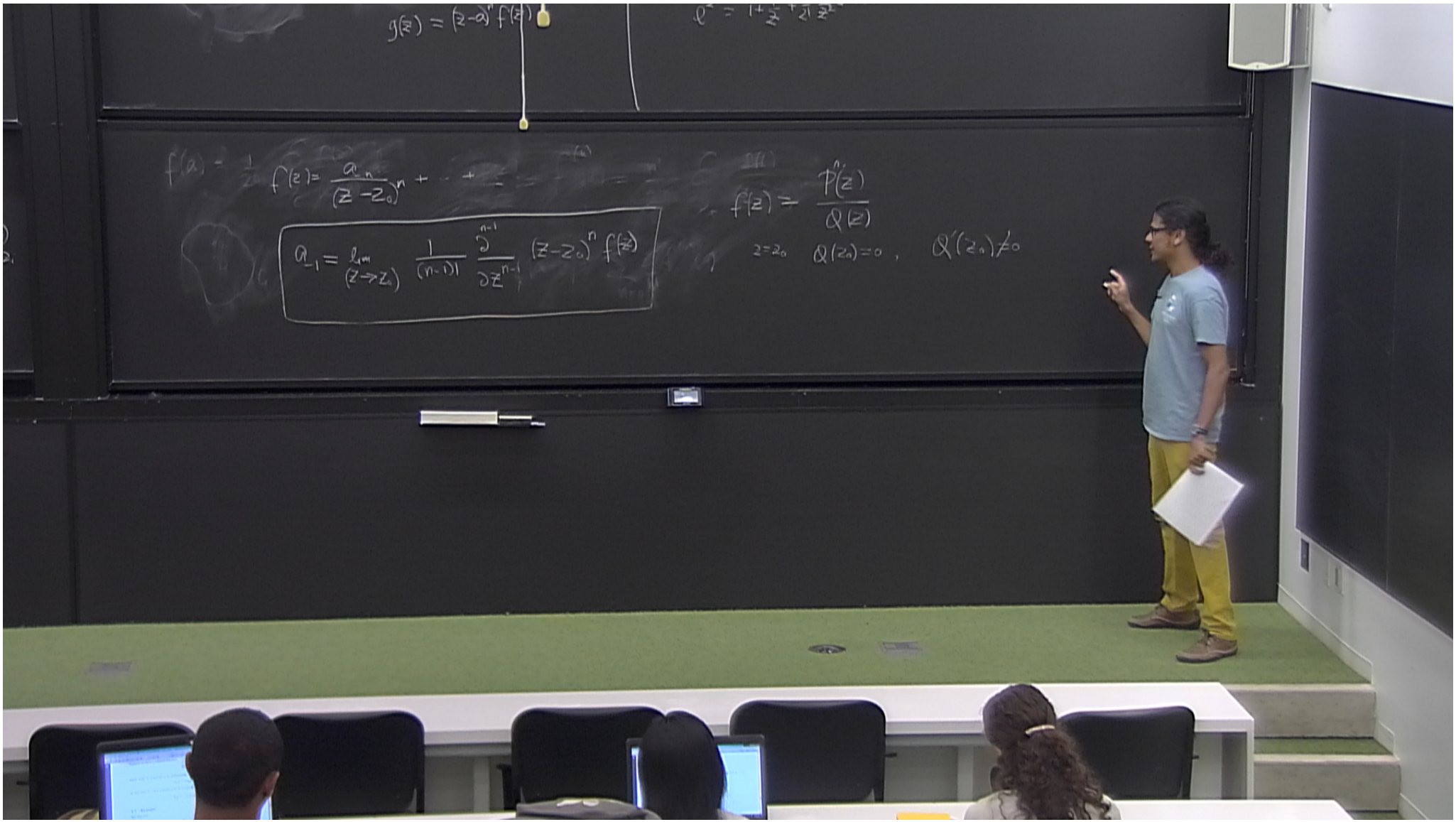
$$a_{-1} = \lim_{(z \to z_0)} \frac{1}{(n-1)!} \frac{\partial^{n-1}}{\partial z^{n-1}} (z-z_0)^n f(z)$$


$g(z) = (z-a)^n f(z)$
$f(z) = \frac{a_{-n}}{(z-z_0)^n} + \cdots$
$a_{-1} = \lim_{(z \to z_0)} \frac{1}{(n-1)!} \frac{\partial^{n-1}}{\partial z^{n-1}} (z-z_0)^n f(z)$


$g(z) = (z-a)^n f(z)$
$f(z) = \frac{a_{-n}}{(z-z_0)^n} + \cdots$
$a_{-1} = \lim_{(z \to z_0)} \frac{1}{(n-1)!} \frac{\partial^{n-1}}{\partial z^{n-1}} (z-z_0)^n f(z)$
$f(z) = \frac{P(z)}{Q(z)}$
$z = z_0 \quad Q(z_0) = 0, \quad Q'(z_0) \neq 0$

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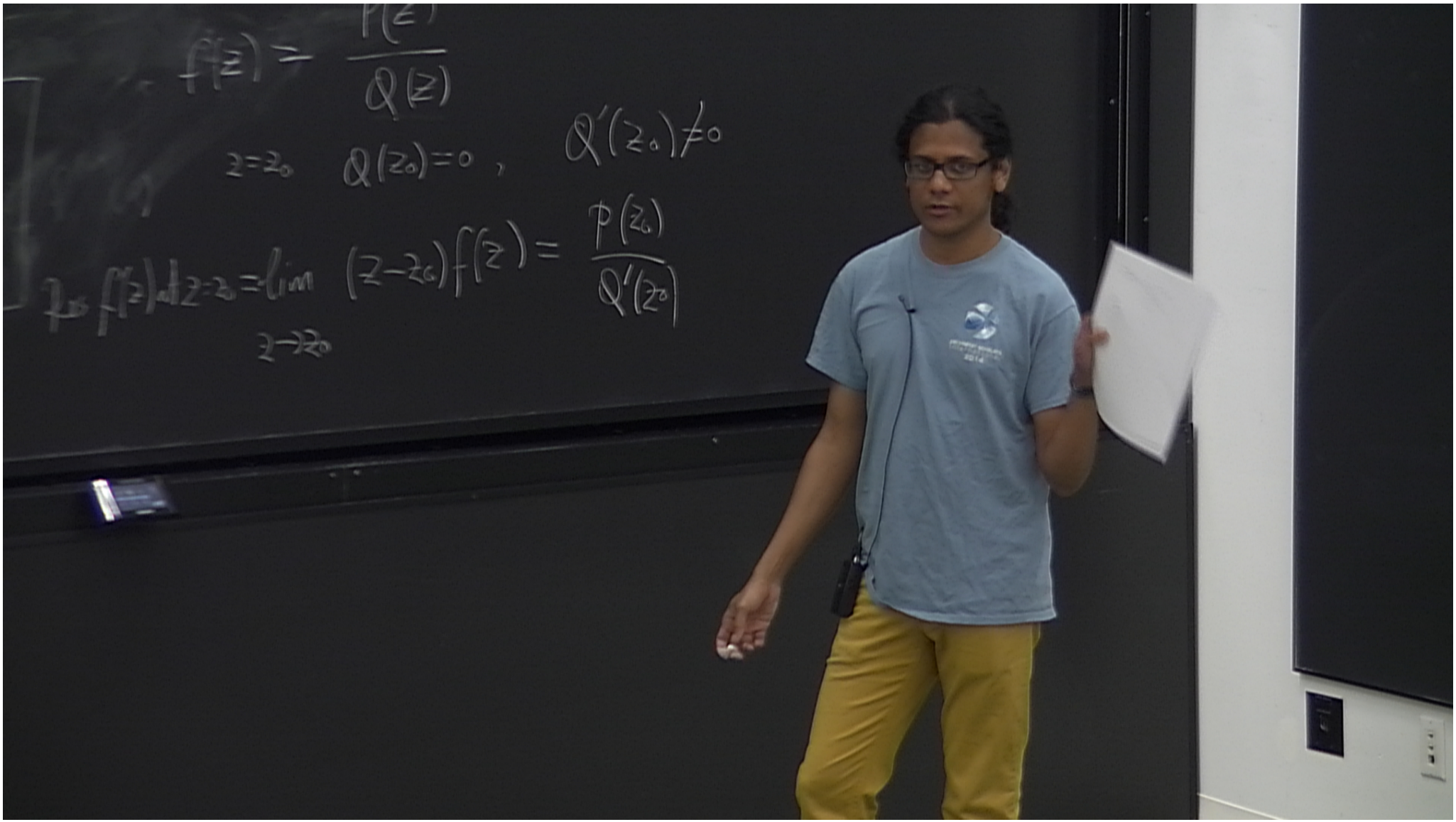
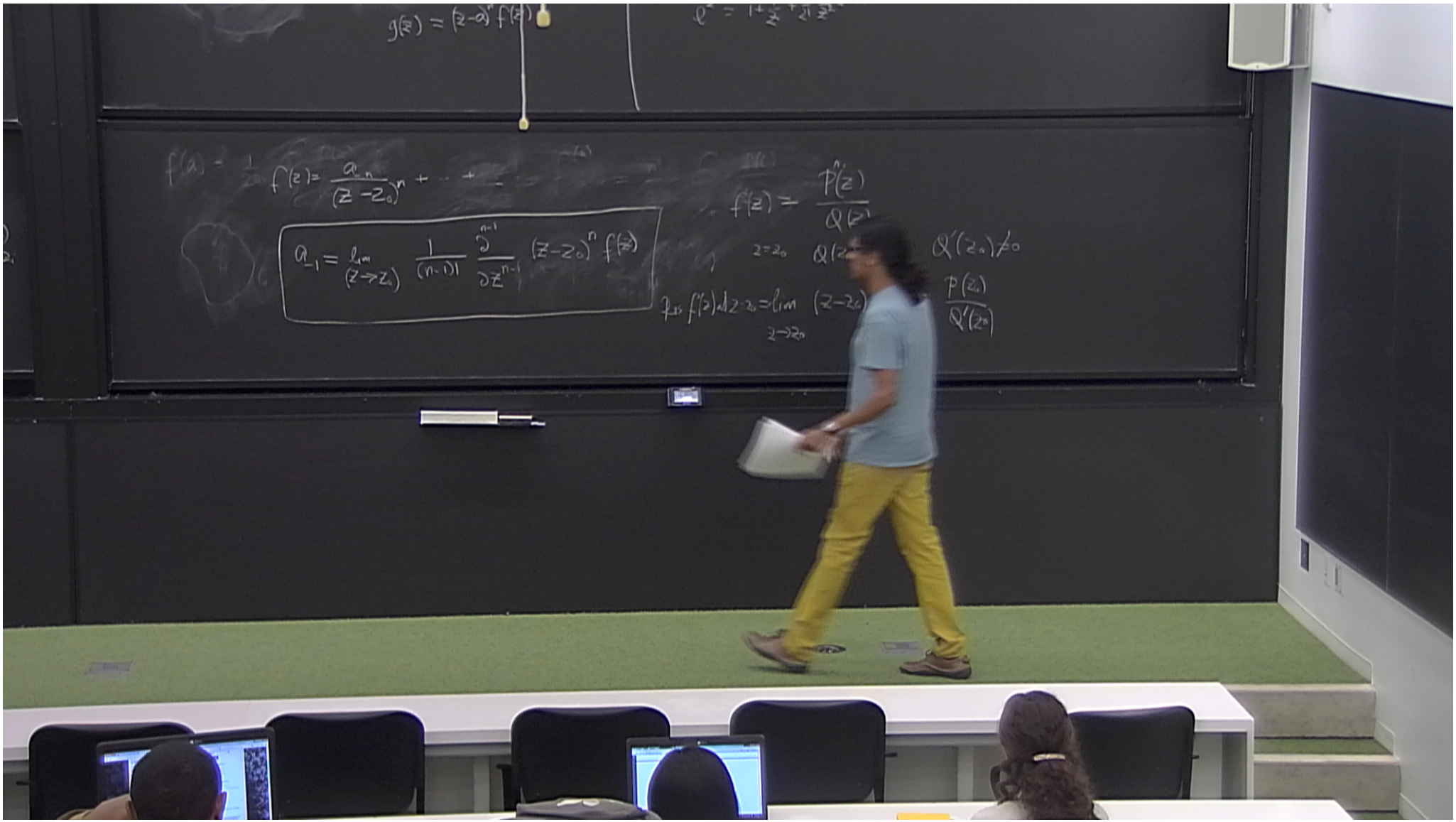
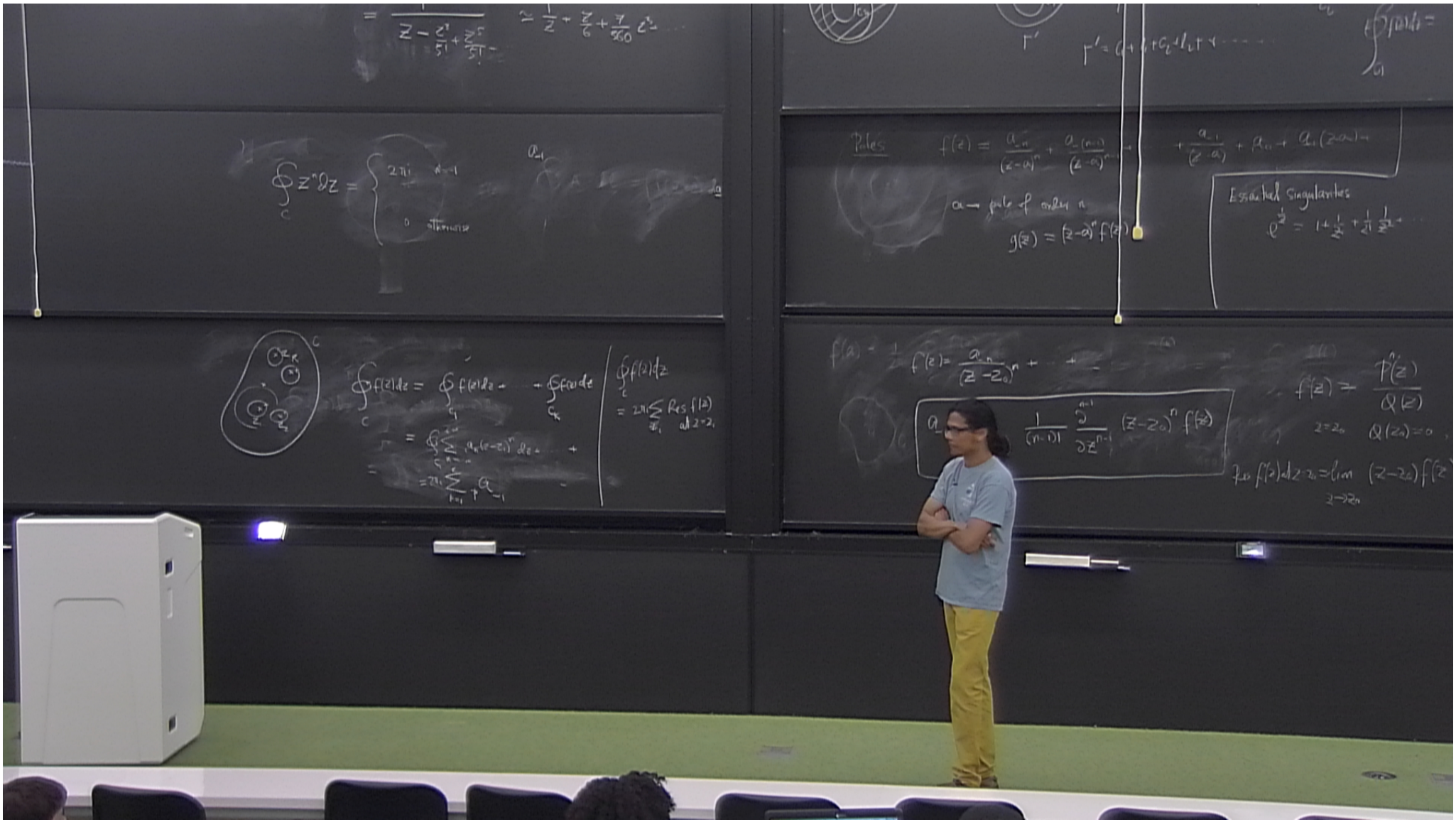
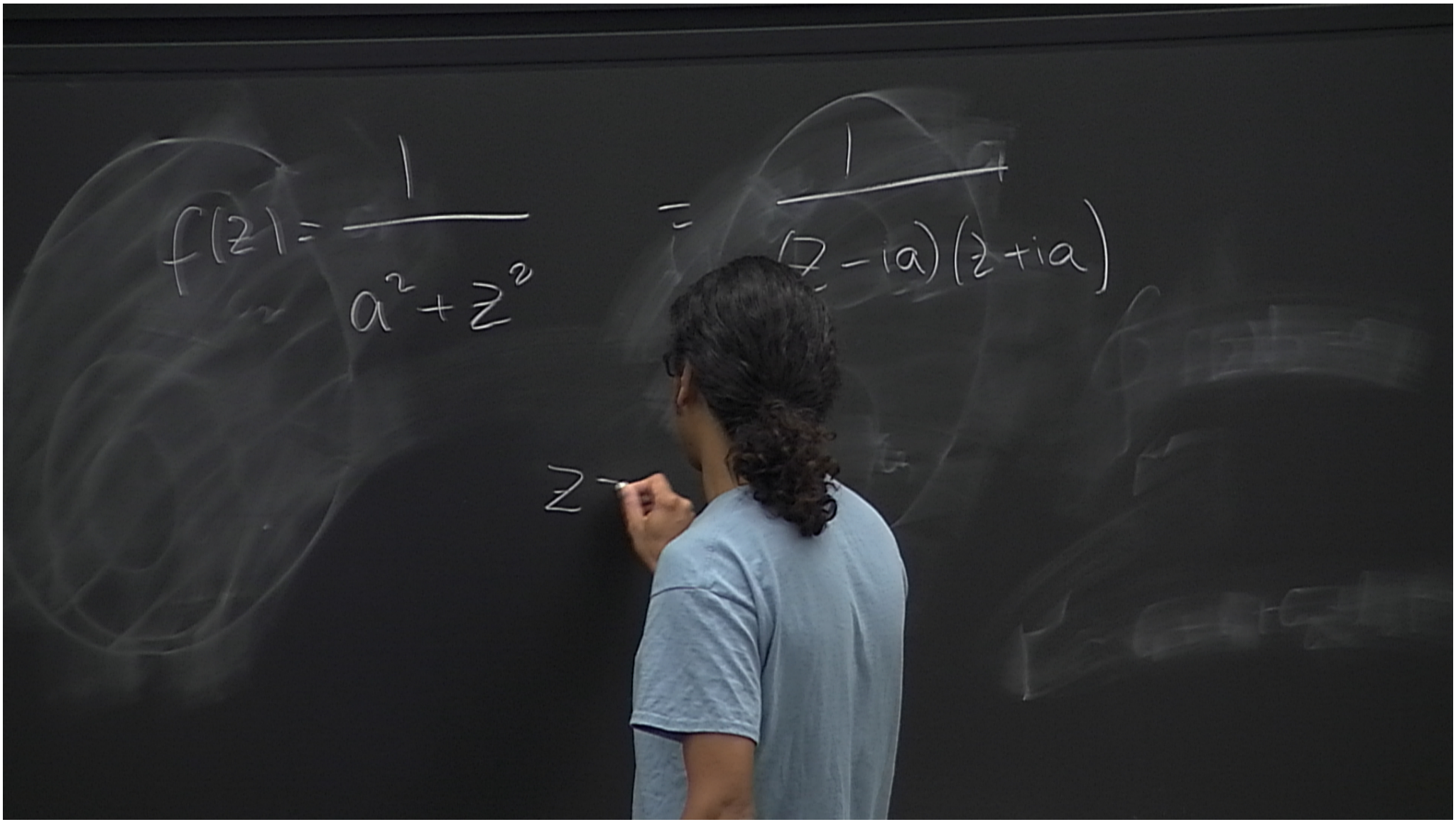
$z = z_0 \quad Q(z_0) = 0, \quad Q'(z_0) \neq 0$

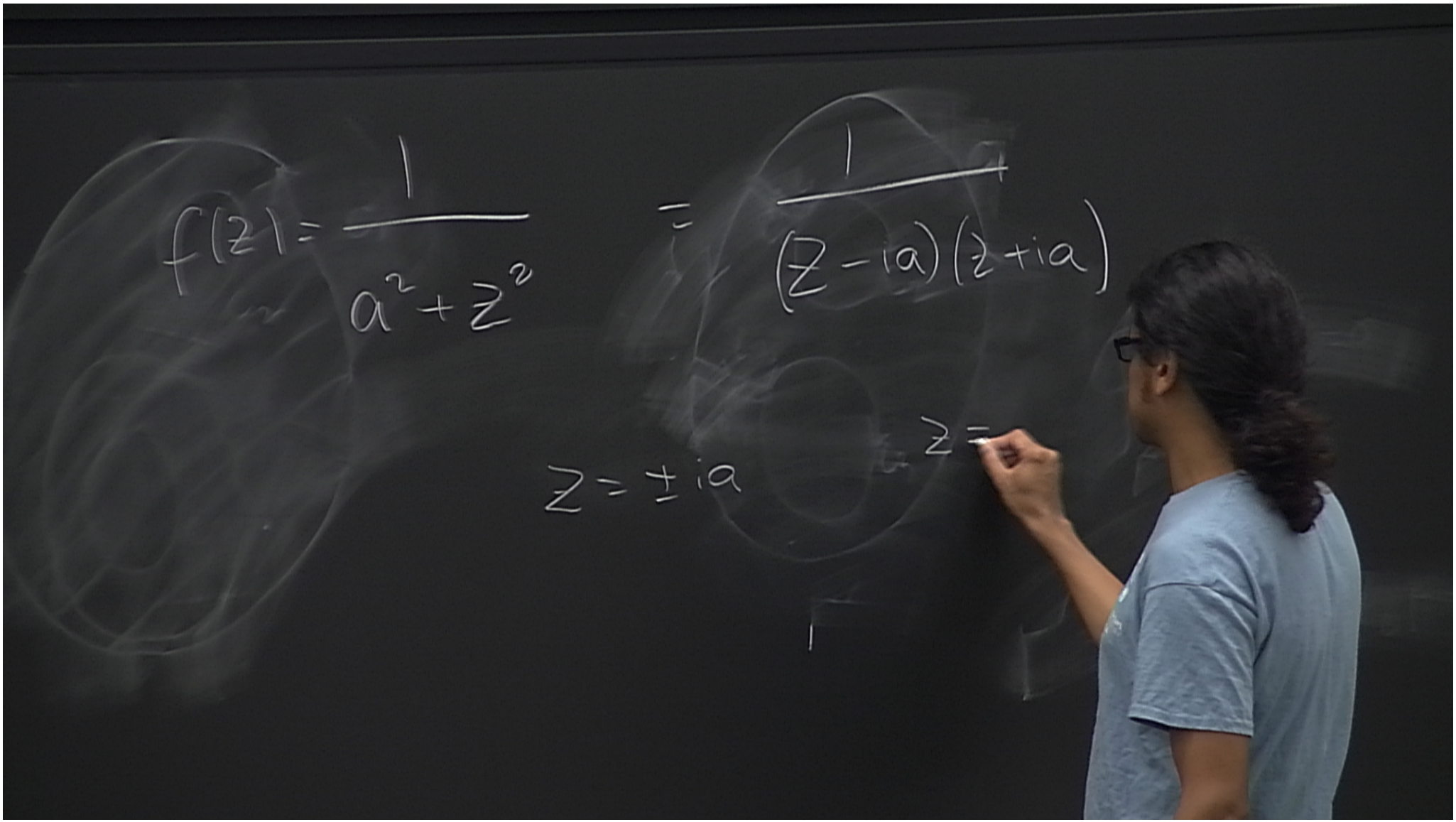
$$\text{Res } f(z) \text{ at } z = z_0 = \lim_{z \to z_0} (z - z_0) f(z) = \frac{P(z_0)}{Q'(z_0)}$$

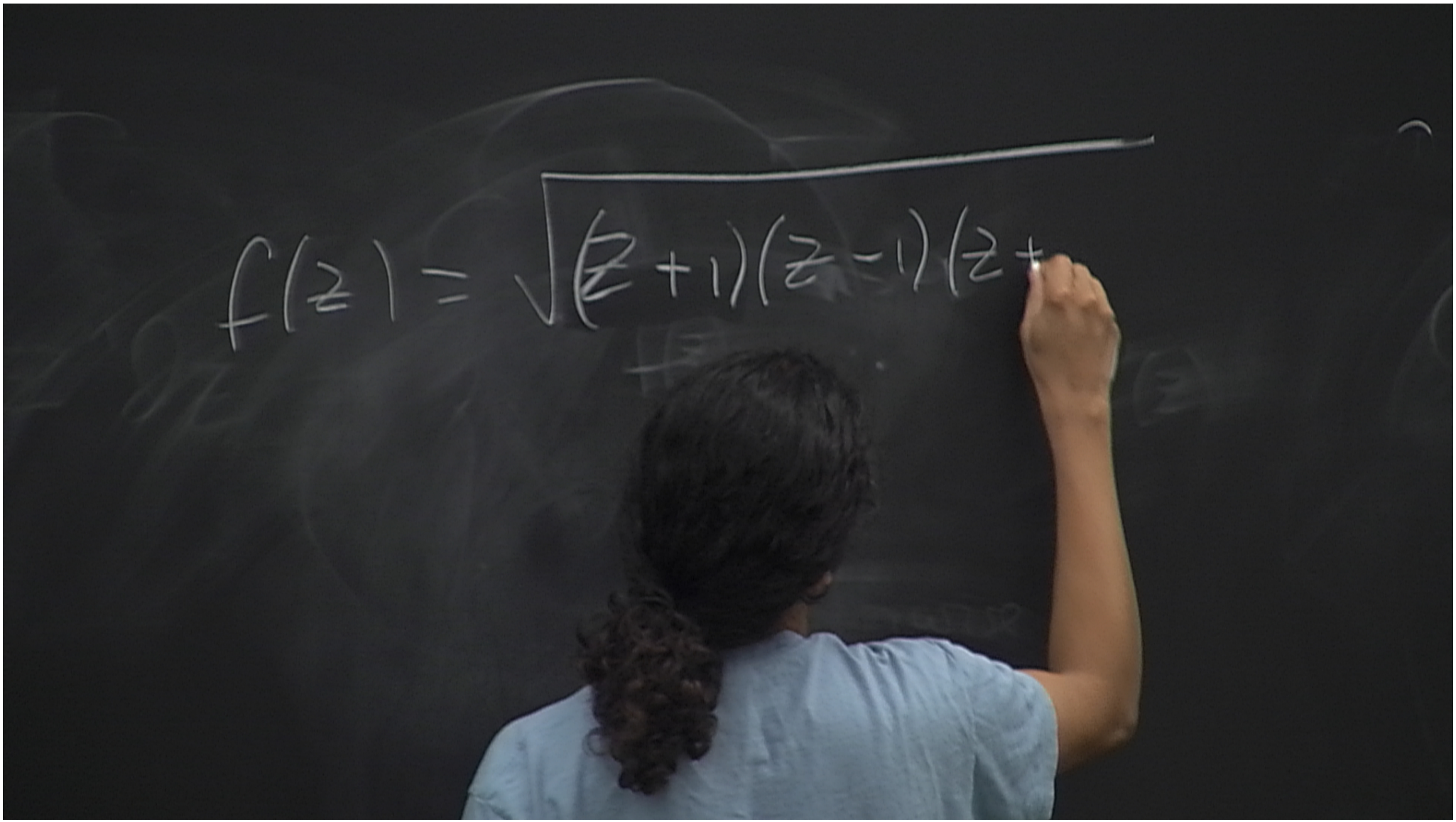

f(z) = P(z) / Q(z)
z = z0  Q(z0) = 0 ,  Q'(z0) ≠ 0
lim (z − z0) f(z) = P(z0) / Q'(z0)
z → z0

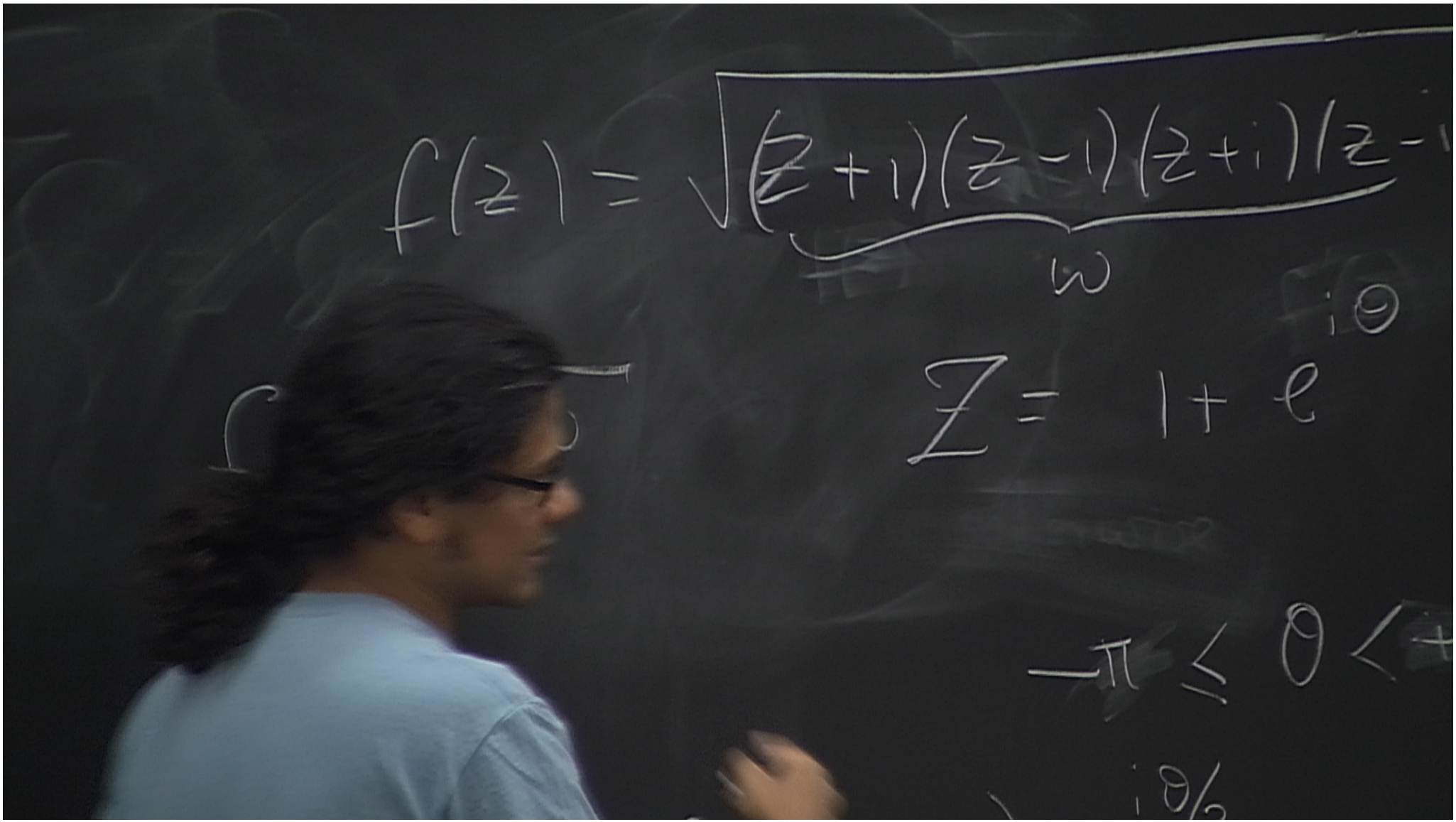

$g(z) = (z-a)^n f(z)$
$f(z) = \frac{a_{-n}}{(z-z_0)^n} + \cdots +$
$a_{-1} = \lim_{(z \to z_0)} \frac{1}{(n-1)!} \frac{\partial^{n-1}}{\partial z^{n-1}} (z-z_0)^n f(z)$
$f(z) = \frac{P(z)}{Q(z)}$
$z = z_0$
$Q'(z_0) \neq 0$
$\frac{P(z_0)}{Q'(z_0)}$
$z \to z_0$


Poles
Essential Singularities


$f(z) = \frac{1}{a^2 + z^2} = \frac{1}{(z - ia)(z + ia)}$
z


$f(z) = \frac{1}{a^2 + z^2} = \frac{1}{(z - ia)(z + ia)}$
$z = \pm ia$
$z =$


f(z) = √(z+1)(z-1)(z+


f(z) = √(z+1)(z−1)(z+i)(z−
w
Z = 1 + e^{iθ}
−π ≤ θ <
iθ/2

$$f(z) = \sqrt{\underbrace{(z+1)(z-1)(z+i)(z-}_{w}}$$

$$f(z) = \sqrt{w} \qquad Z = 1 + e^{i\theta}$$

$$-\pi \leq \theta <$$

$$i\theta/2$$

$$f(z) = \sqrt{1 + \sqrt{z}}$$

$$g(t) = \sqrt{1 + \frac{1}{\sqrt{t}}}$$

$$=$$

$$f(z) = \sqrt{1+\sqrt{z}}$$

$$g(t) = \sqrt{1+\frac{1}{\sqrt{t}}}$$

$$= \frac{\quad}{t^{1/4}}$$