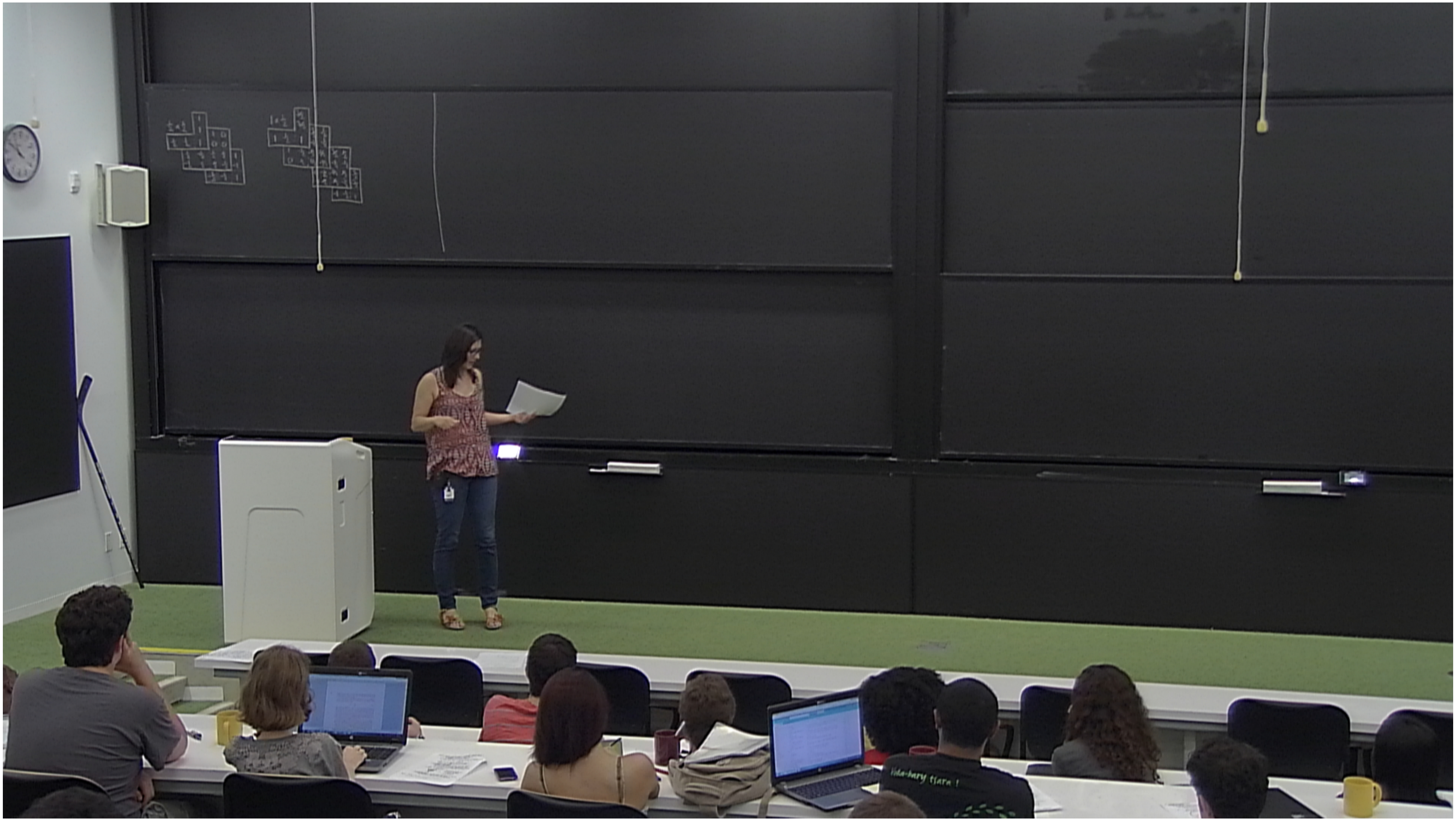
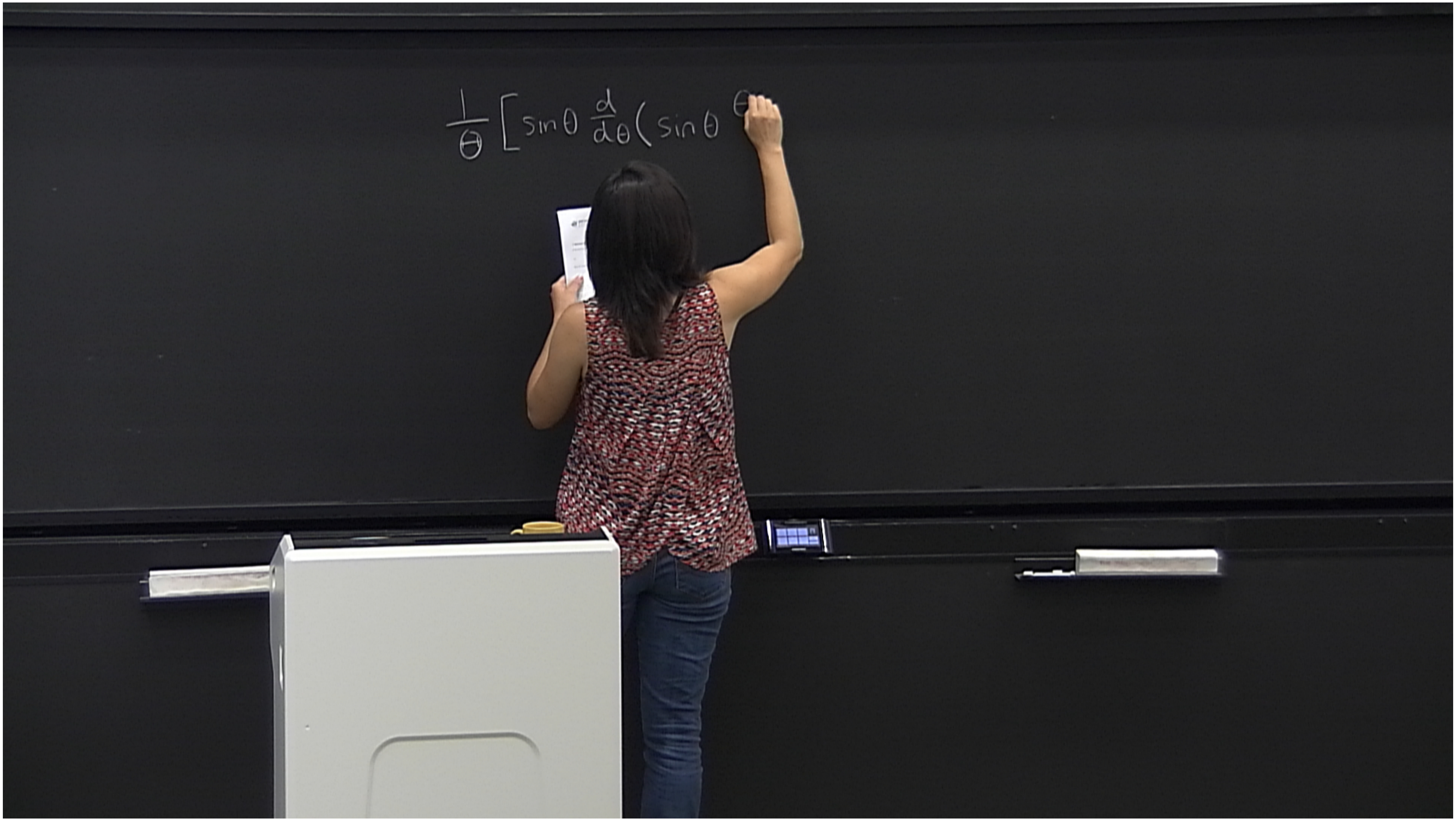
Title: 13/14 PSI - Quantum Mechanics - Lecture 3

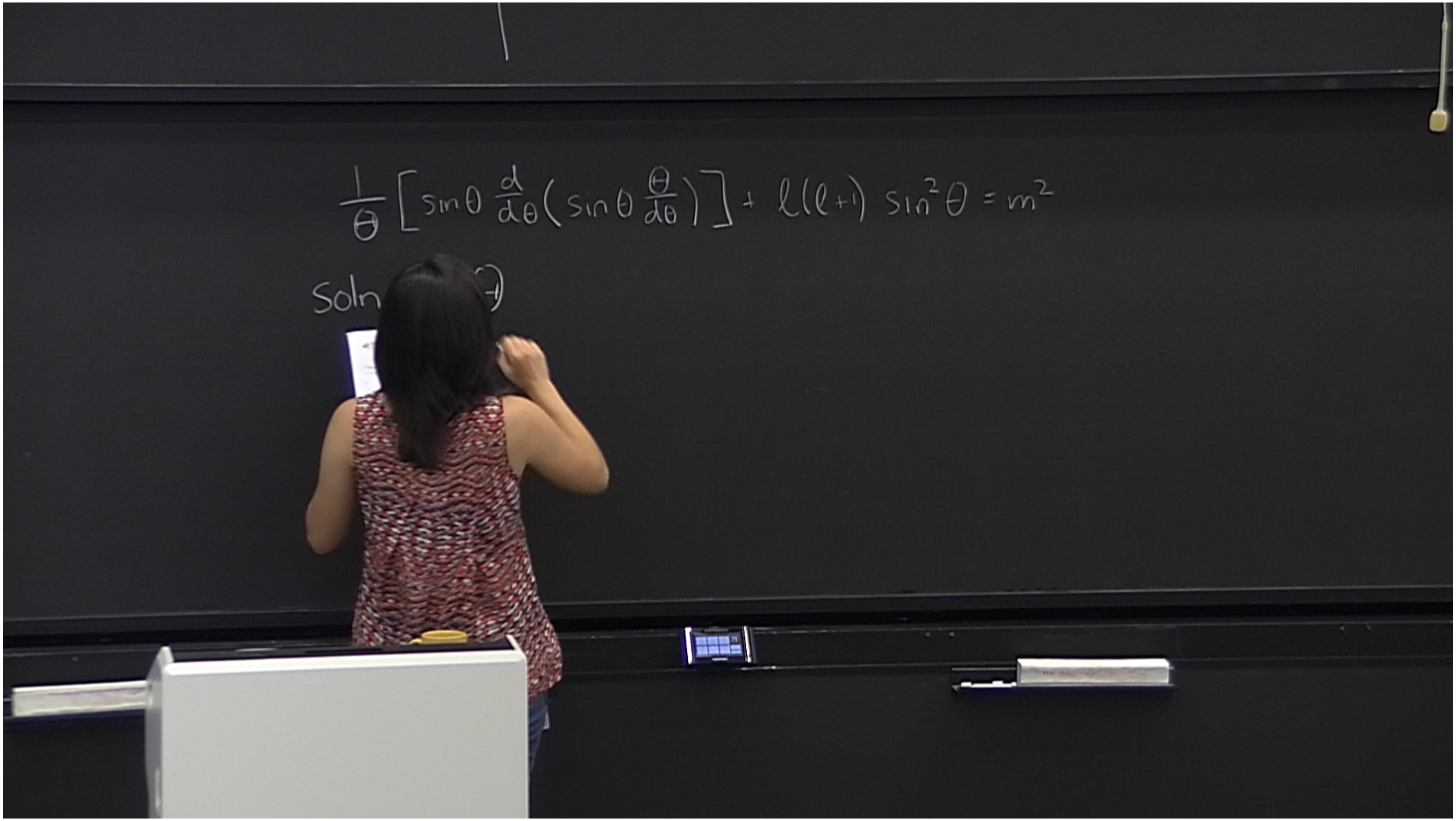
Date: Aug 29, 2013 10:30 AM

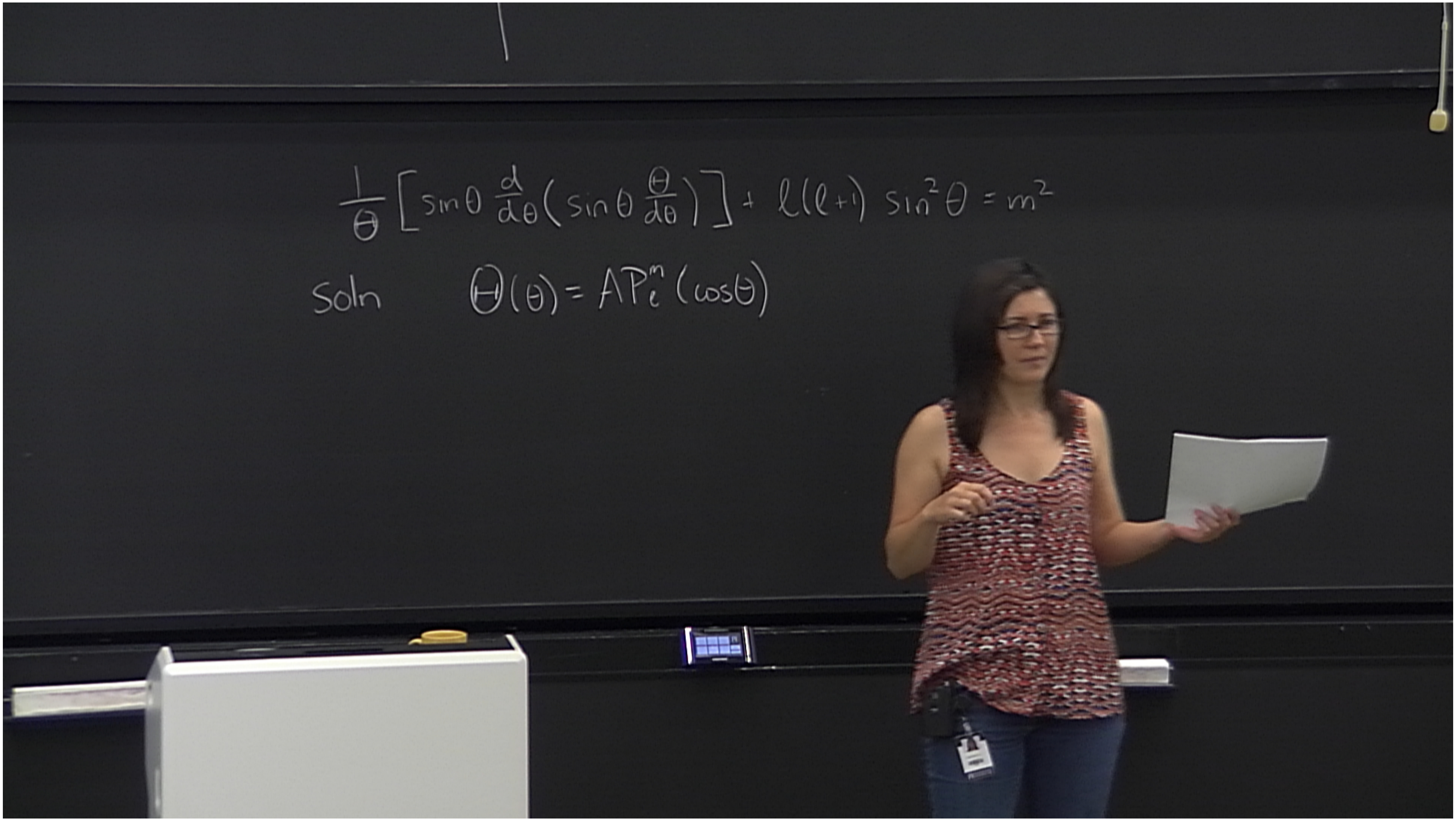
URL: http://pirsa.org/13080034

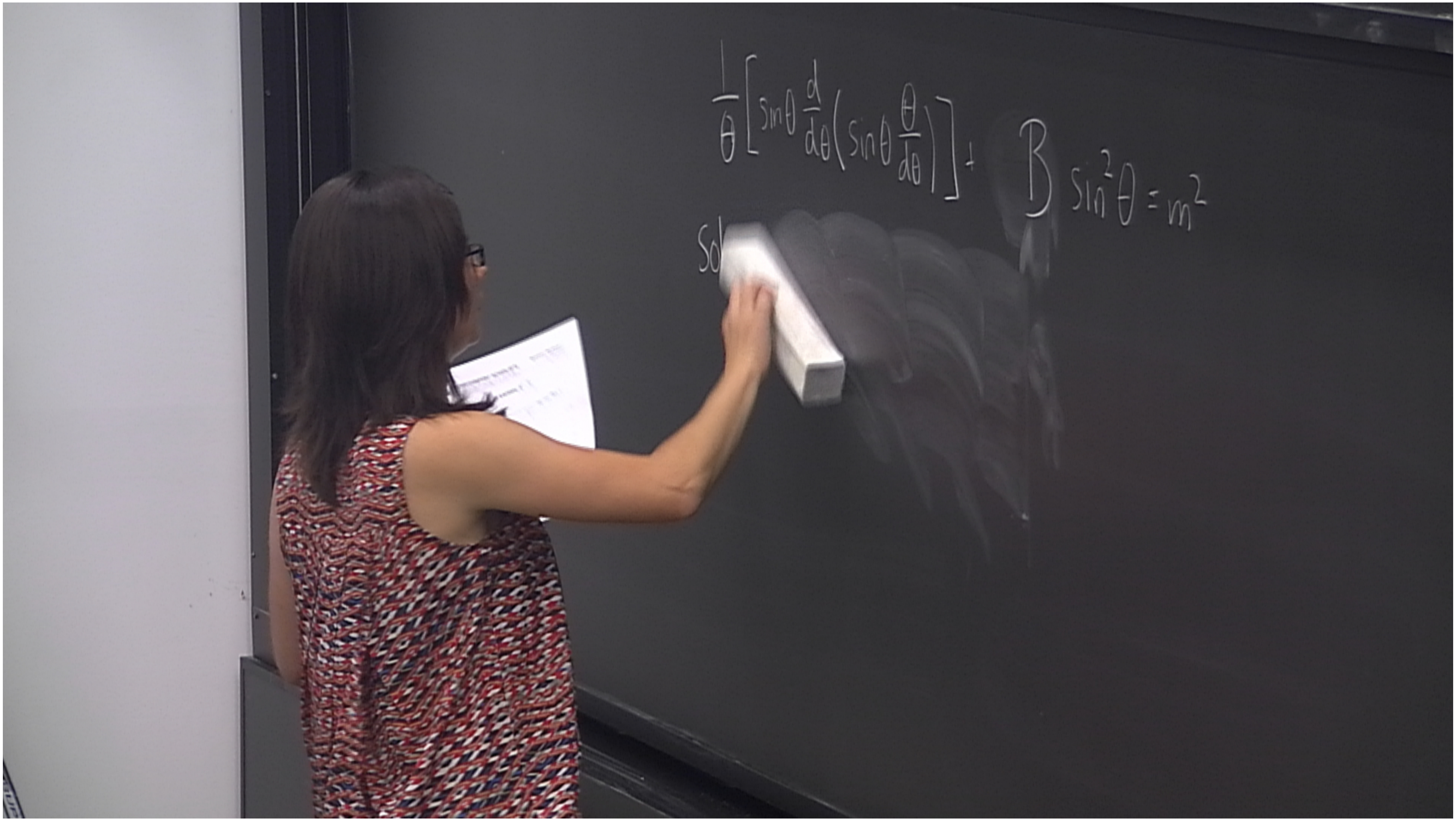
Abstract:

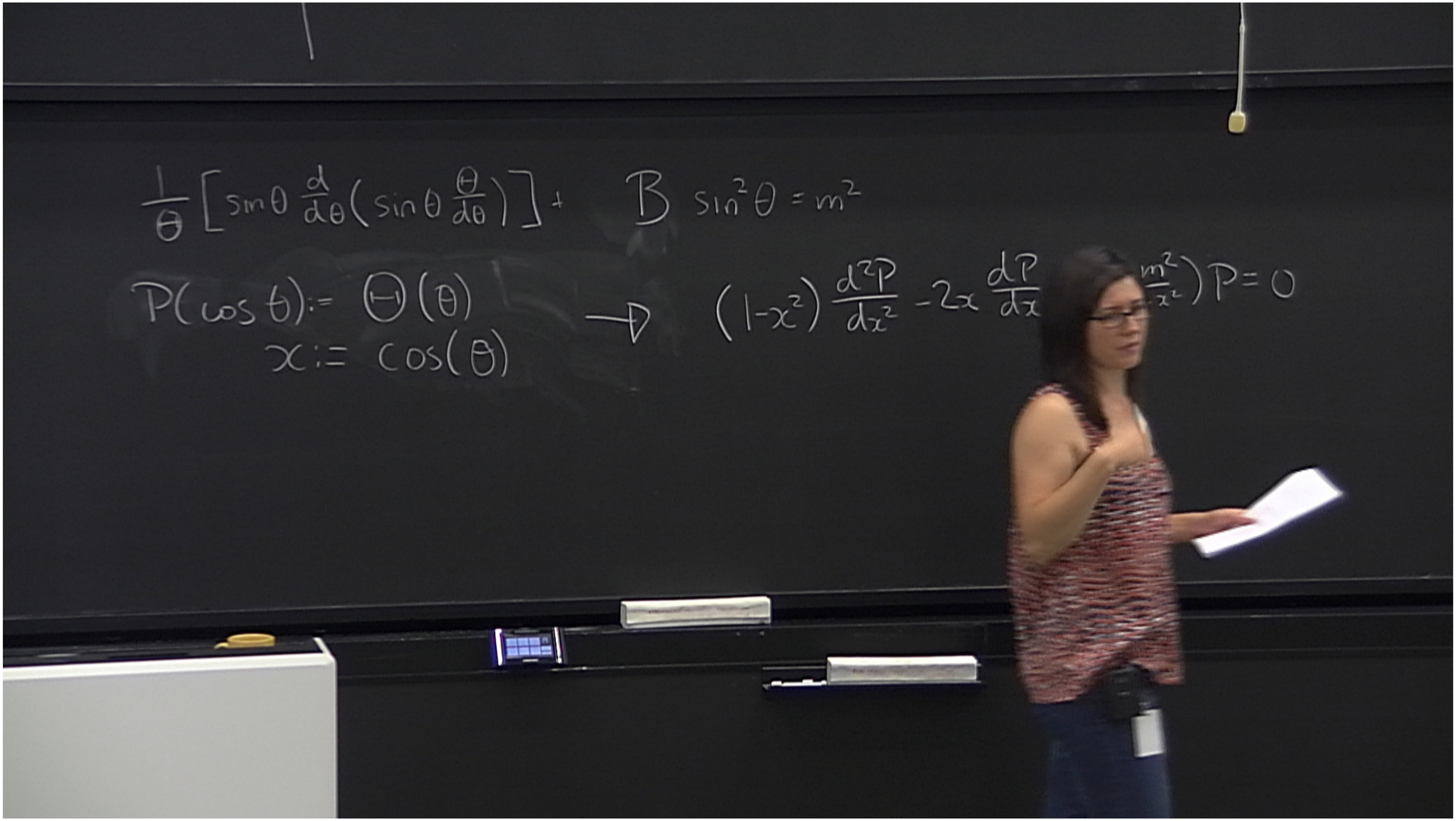


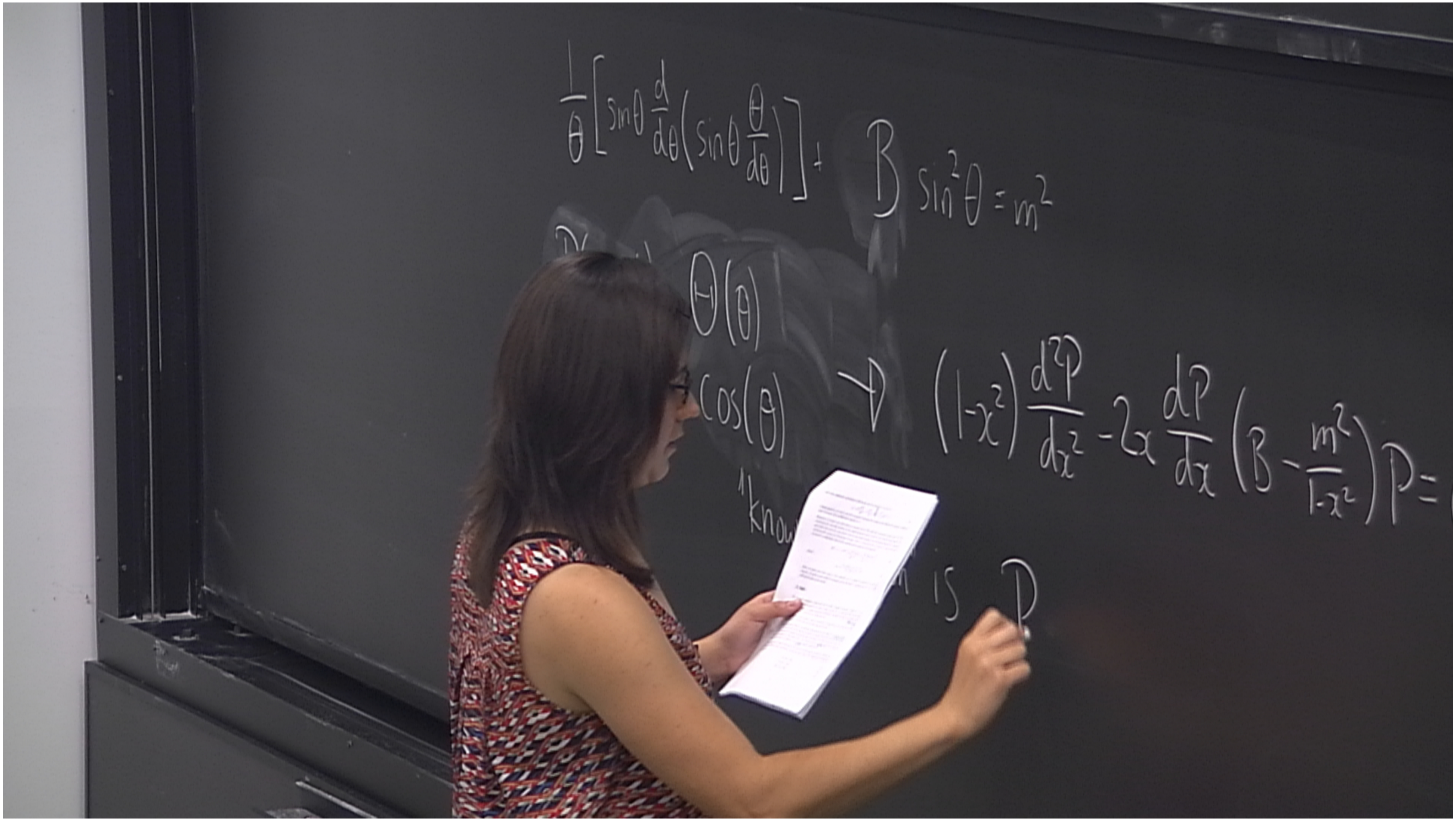

$\frac{1}{\Theta}\Big[\sin\theta \frac{d}{d\theta}\Big(\sin\theta$

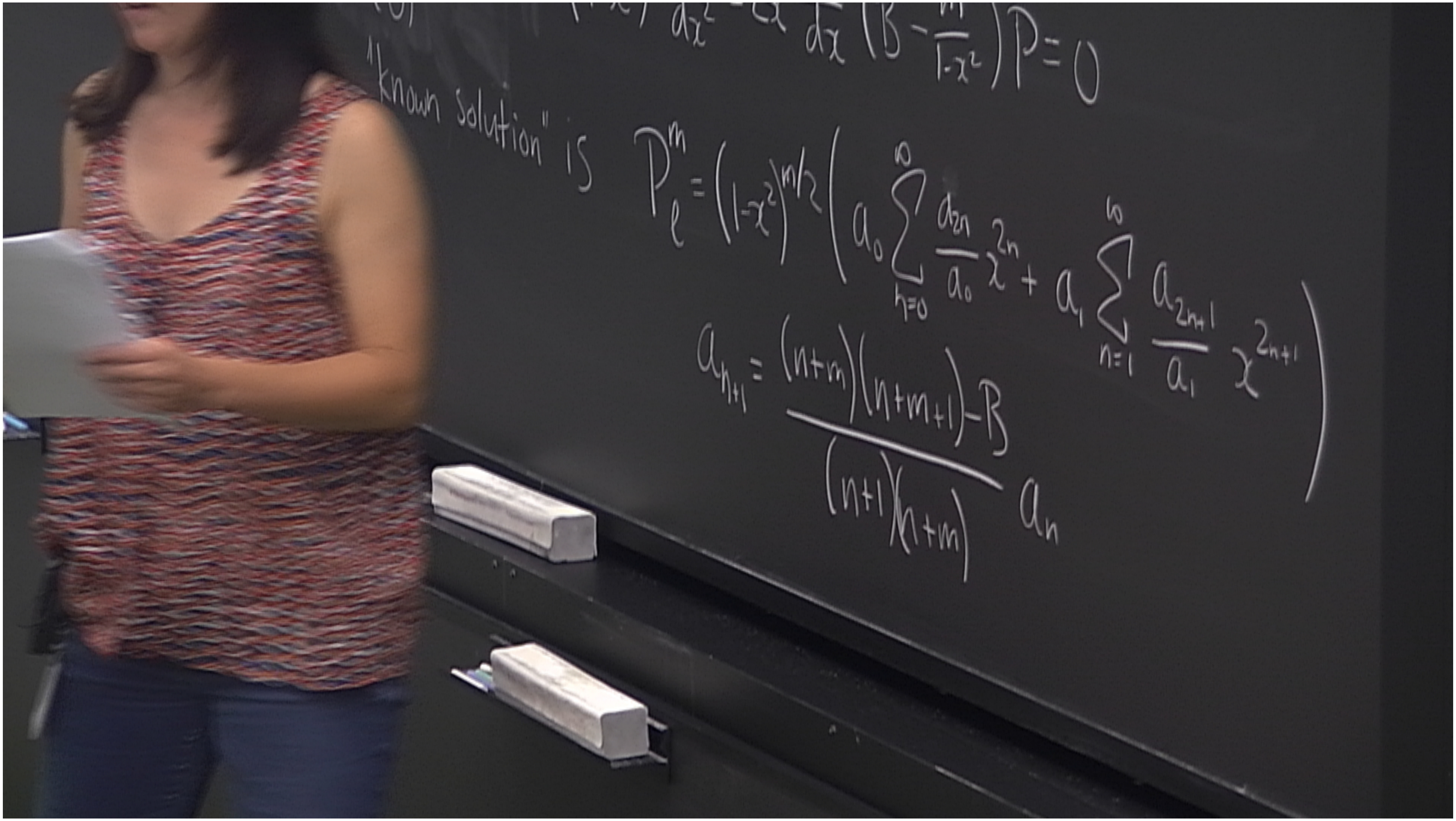

1/Θ [sinθ d/dθ (sinθ Θ/dθ)] + ℓ(ℓ+1) sin²θ = m²
Soln

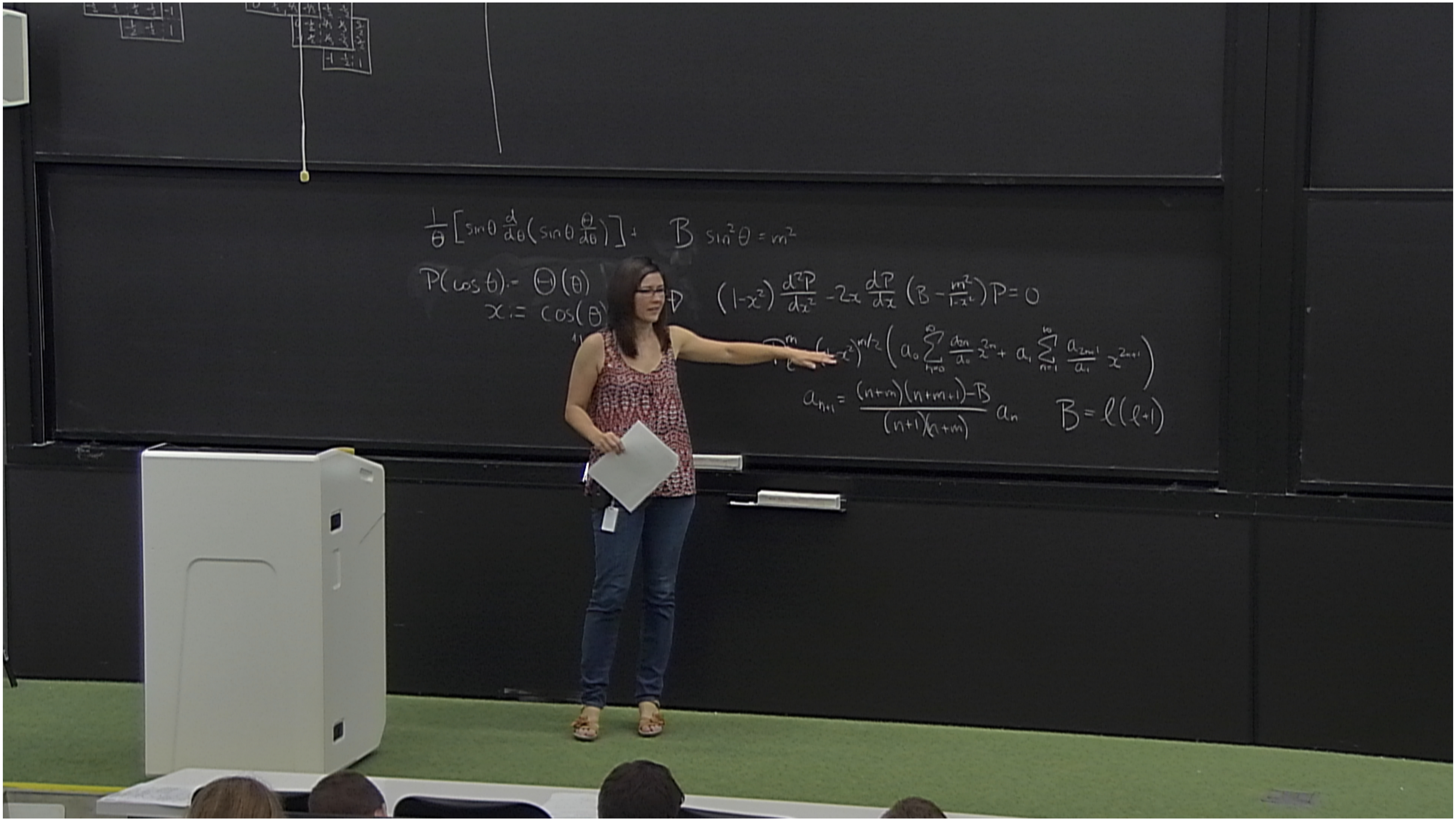

1/Θ [sinθ d/dθ (sinθ Θ/dθ)] + l(l+1) sin²θ = m²
Soln Θ(θ) = A P_l^m (cosθ)

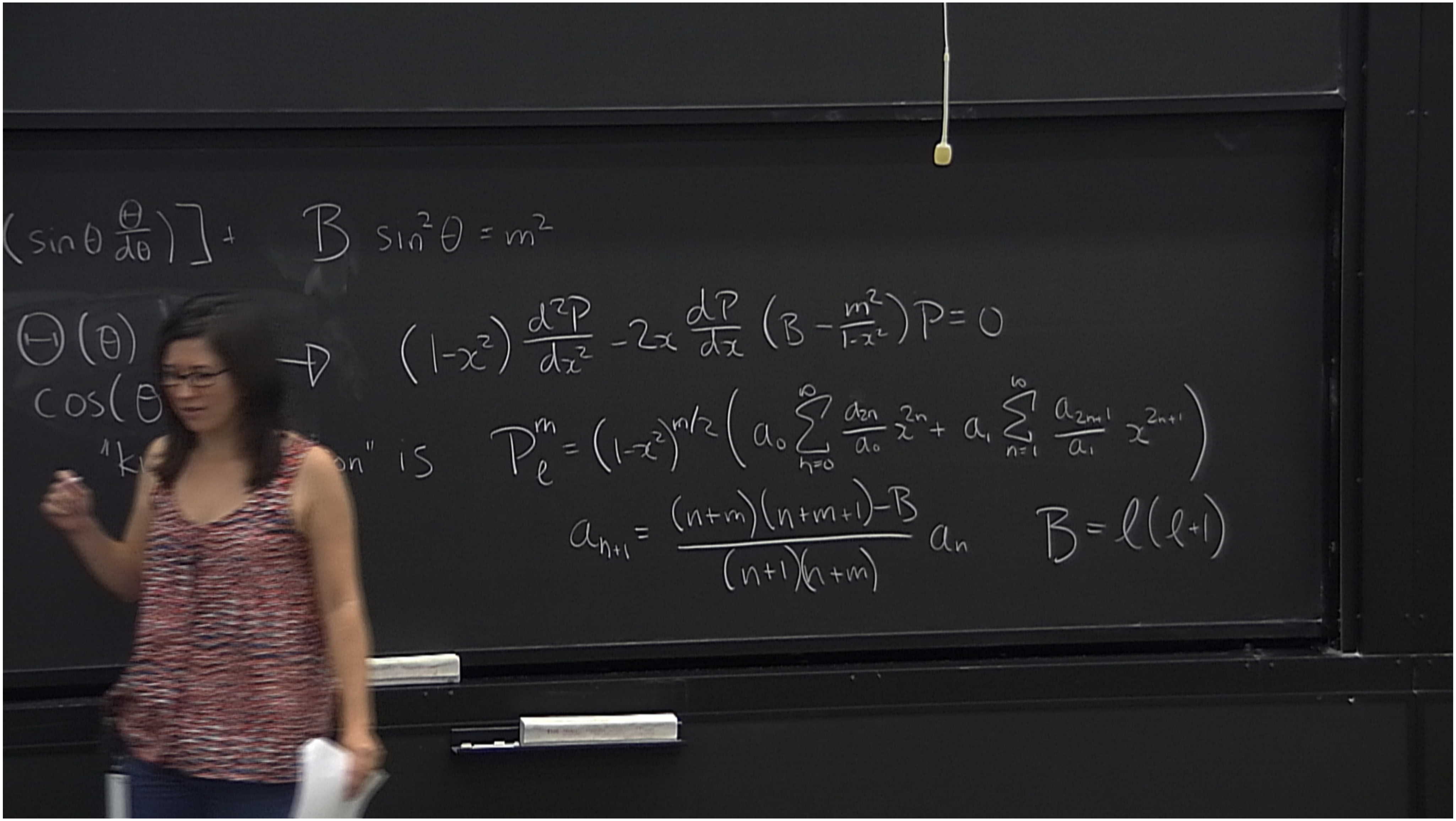

$\frac{1}{\Theta}\left[\sin\theta \frac{d}{d\theta}\left(\sin\theta \frac{\Theta}{d\theta}\right)\right] + B \sin^2\theta = m^2$

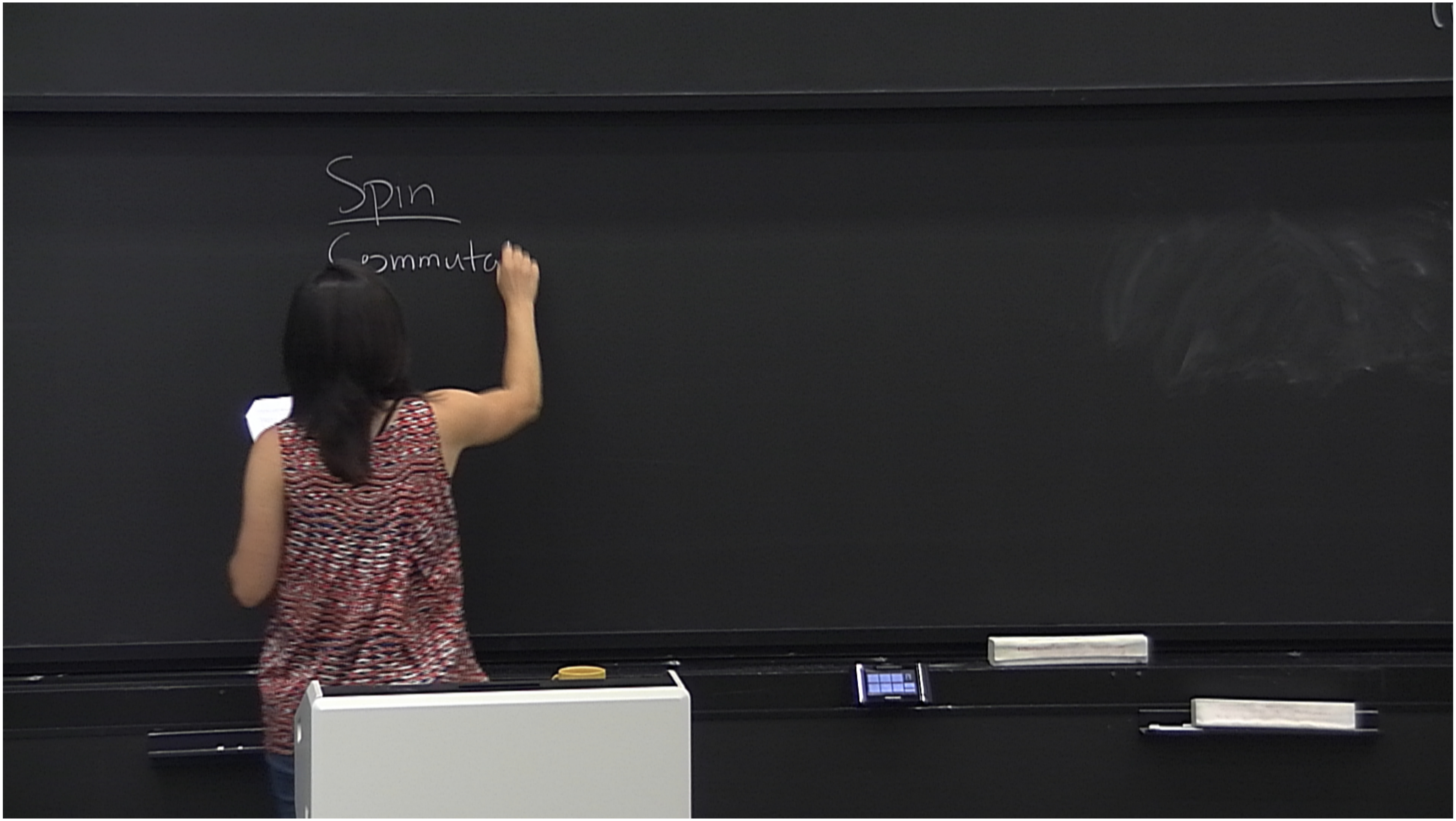

1/Θ [sinθ d/dθ (sinθ Θ/dθ)] + B sin²θ = m²
P(cos θ) := Θ(θ)
x := cos(θ)
(1-x²) d²P/dx² - 2x dP/dx
)P = 0

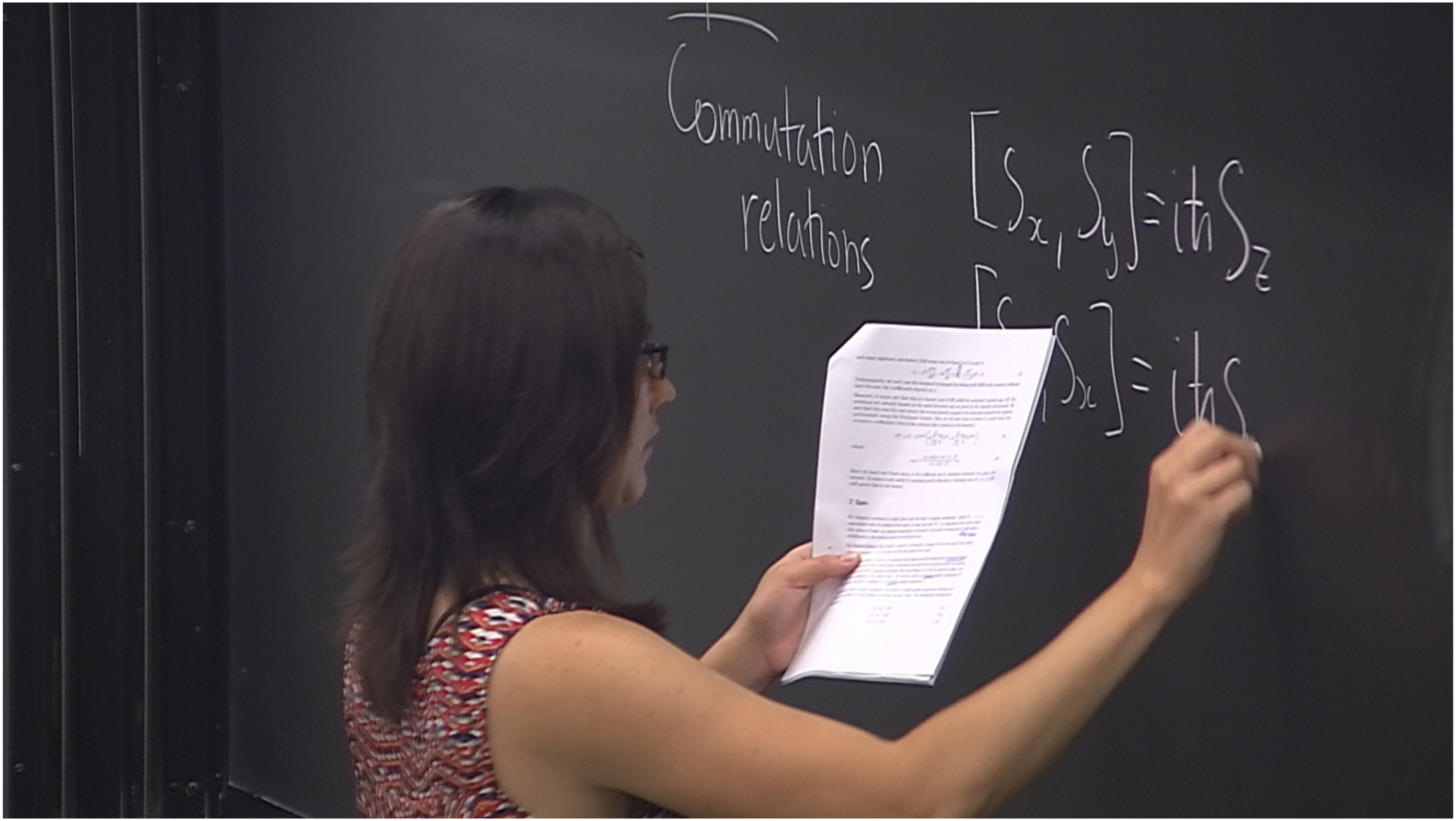

1/Θ [sinθ d/dθ (sinθ Θ/dθ)] + B sin²θ = m²
Θ(θ)
cos(θ)
(1-x²) d²P/dx² - 2x dP/dx (B - m²/1-x²) P =
is P

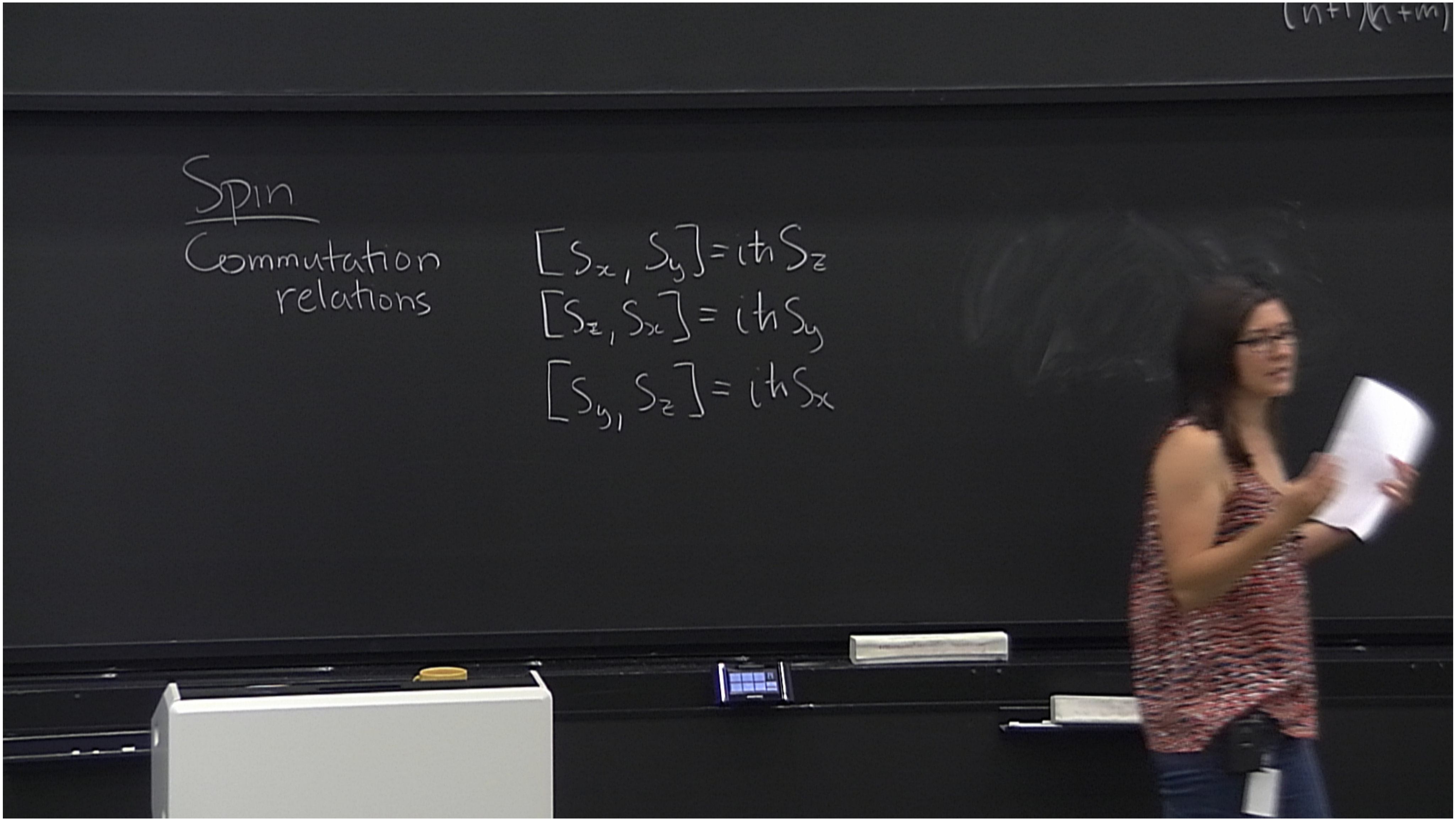

"known solution" is $P_\ell^m = (1-x^2)^{m/2}\left(a_0 \sum_{n=0}^{\infty} \frac{a_{2n}}{a_0} x^{2n} + a_1 \sum_{n=1}^{\infty} \frac{a_{2n+1}}{a_1} x^{2n+1}\right)$
$a_{n+1} = \frac{(n+m)(n+m+1)-B}{(n+1)(n+m)} a_n$

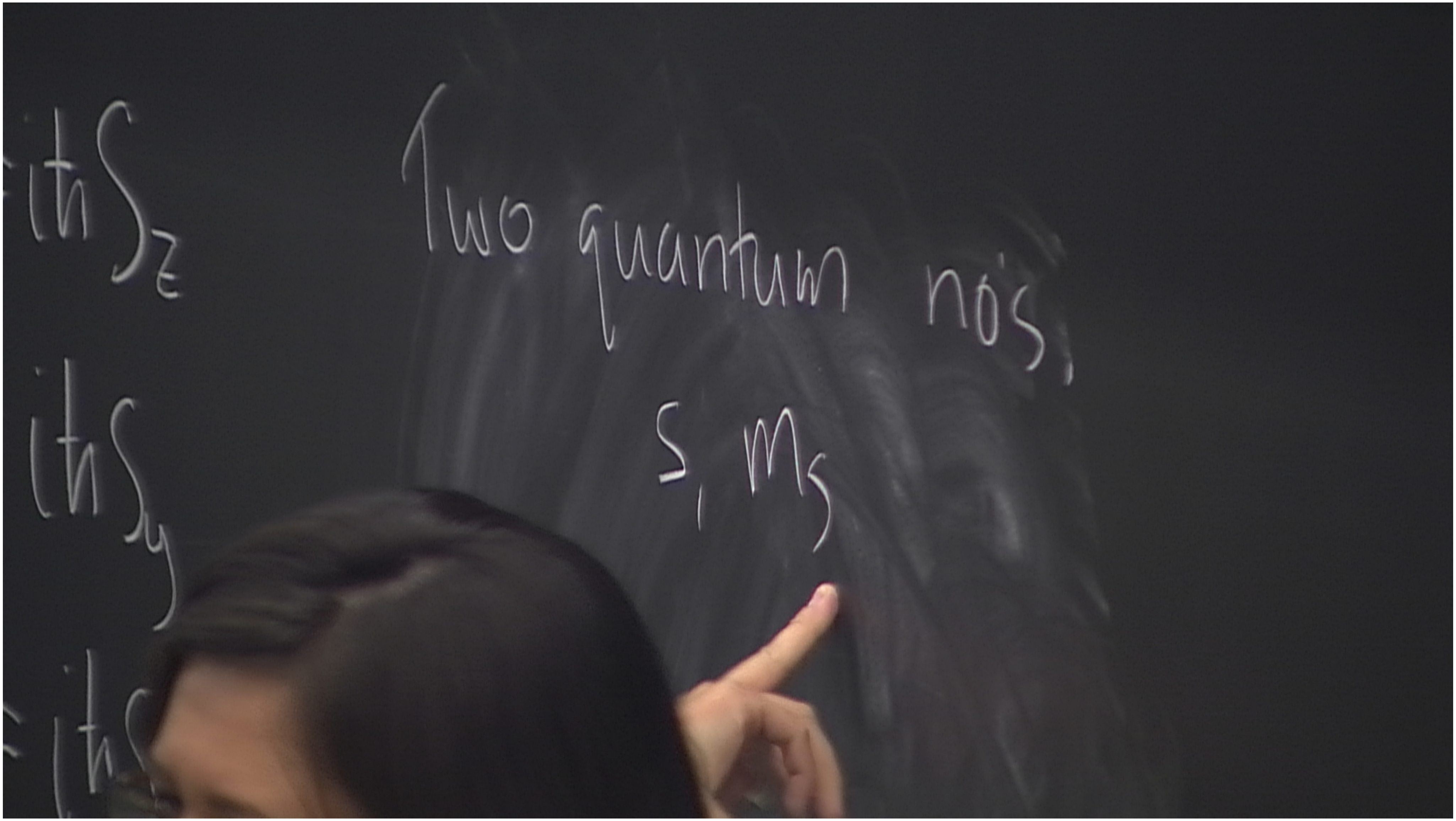

B sin²θ = m²
P(cos θ) := Θ(θ)
x := cos(θ)
(1-x²) d²P/dx² - 2x dP/dx (B - m²/(1-x²)) P = 0
B = ℓ(ℓ+1)

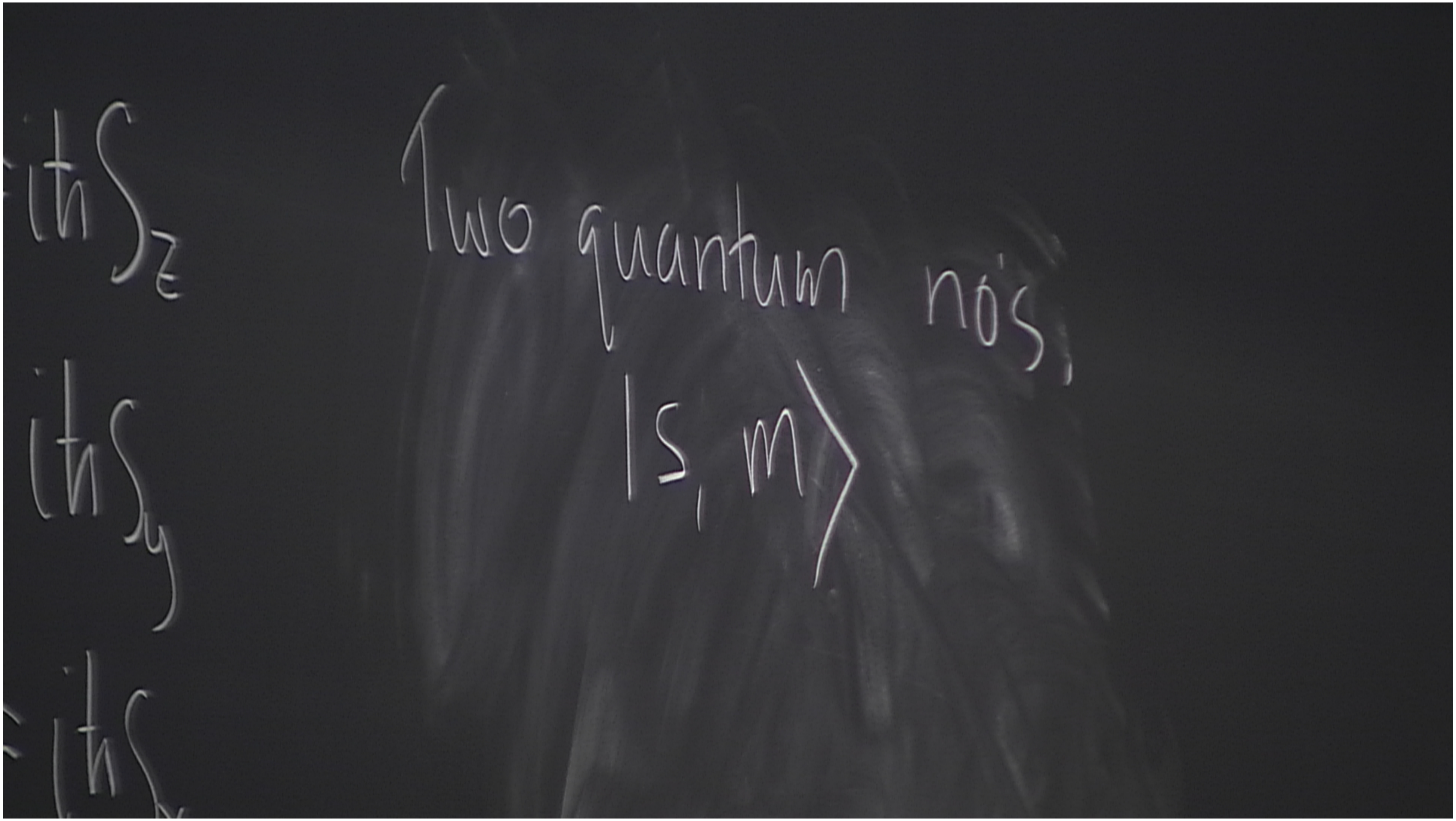

$(\sin\theta \frac{\Theta}{d\theta})] + B \sin^2\theta = m^2$
$\Theta(\theta)$
$\rightarrow (1-x^2)\frac{d^2P}{dx^2} - 2x\frac{dP}{dx}(B - \frac{m^2}{1-x^2})P = 0$
cos(θ
"k on" is $P_\ell^m = (1-x^2)^{m/2}\left(a_0 \sum_{n=0}^{\infty} \frac{a_{2n}}{a_0} x^{2n} + a_1 \sum_{n=1}^{\infty} \frac{a_{2n+1}}{a_1} x^{2n+1}\right)$
$a_{n+1} = \frac{(n+m)(n+m+1) - B}{(n+1)(n+m)} a_n$
$B = \ell(\ell+1)$

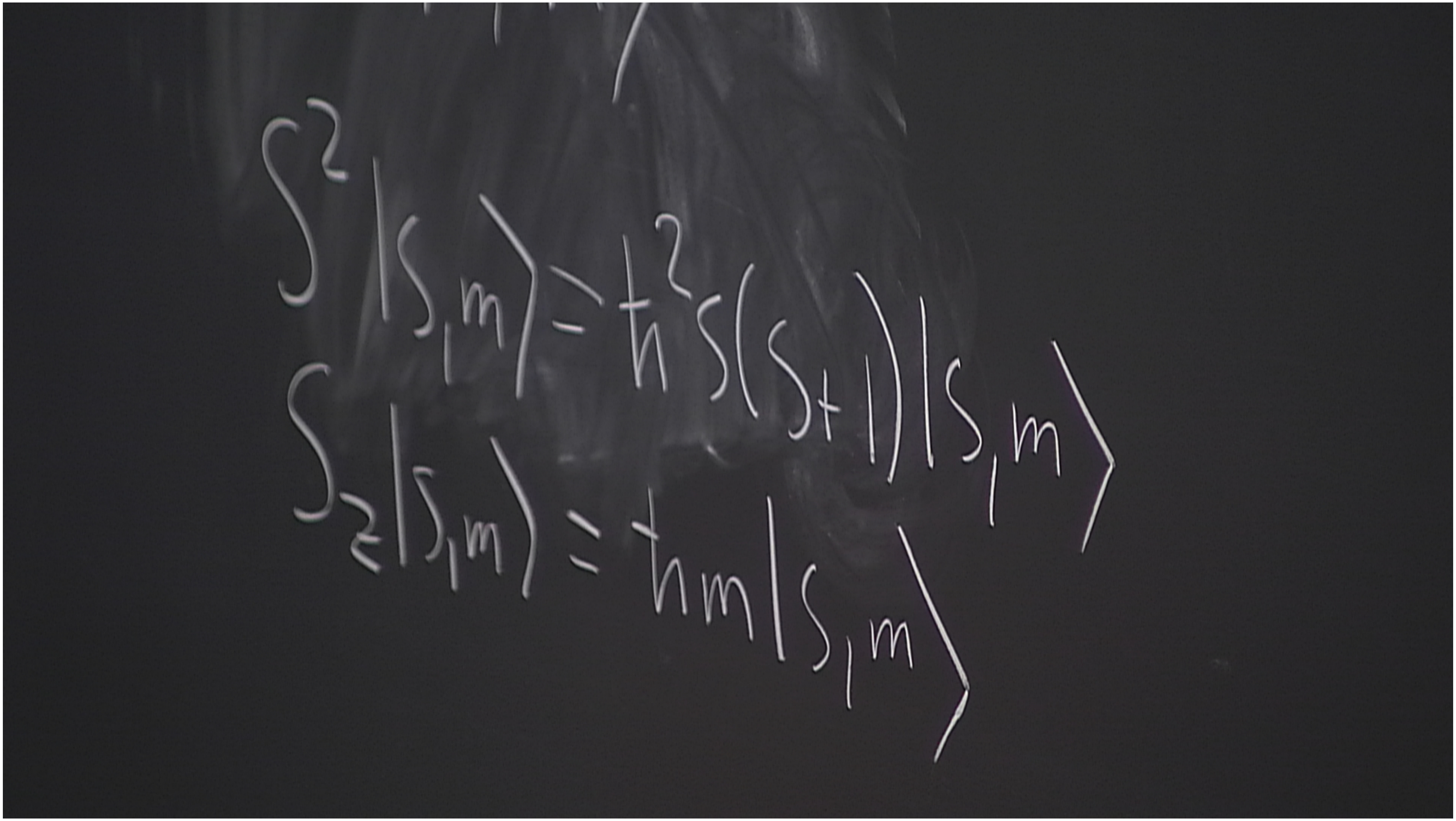

Spin
Commuta

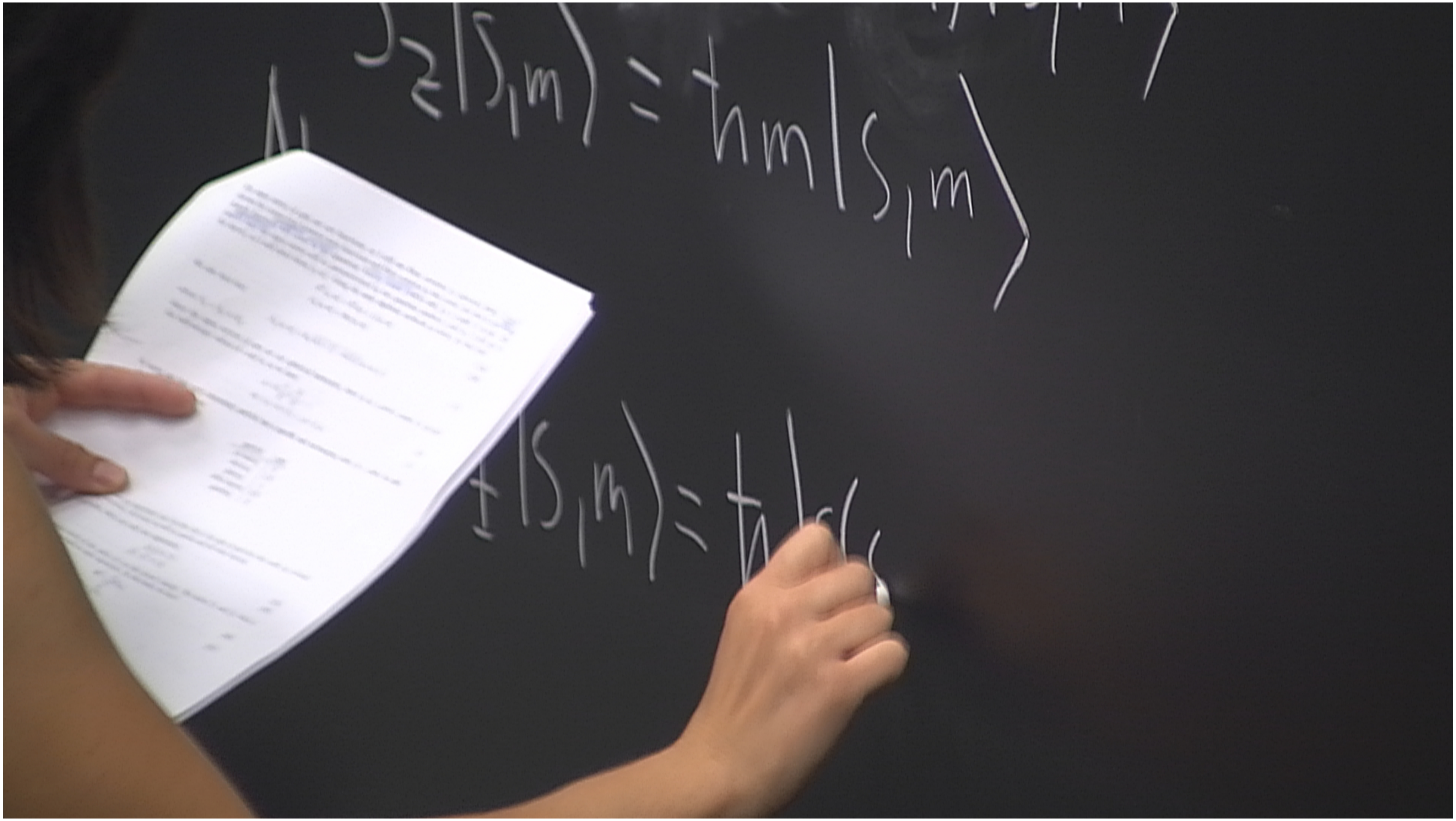

Commutation
relations
[Sx, Sy] = iħSz

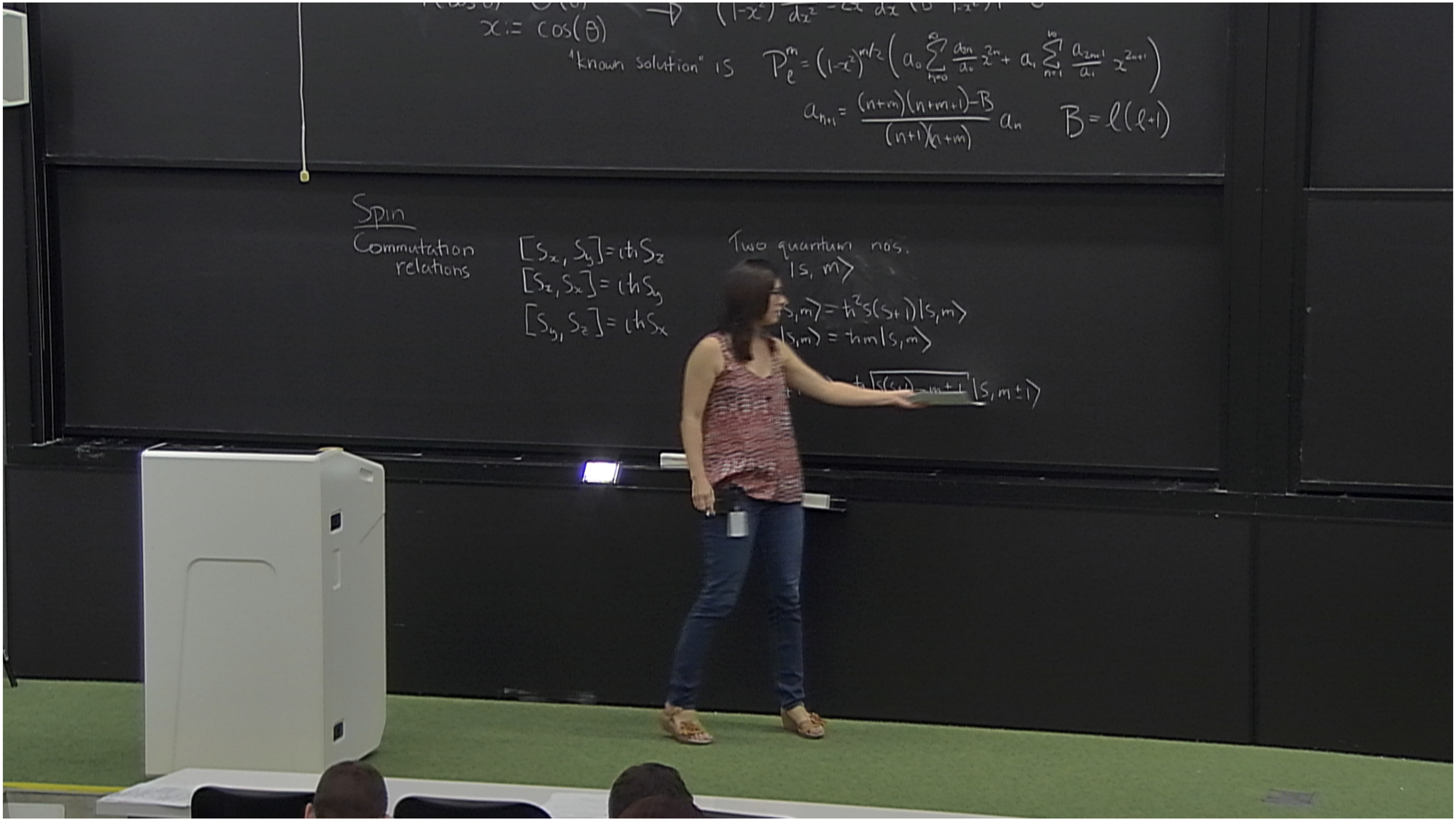

Spin
Commutation relations
$[S_x, S_y] = i\hbar S_z$
$[S_z, S_x] = i\hbar S_y$
$[S_y, S_z] = i\hbar S_x$

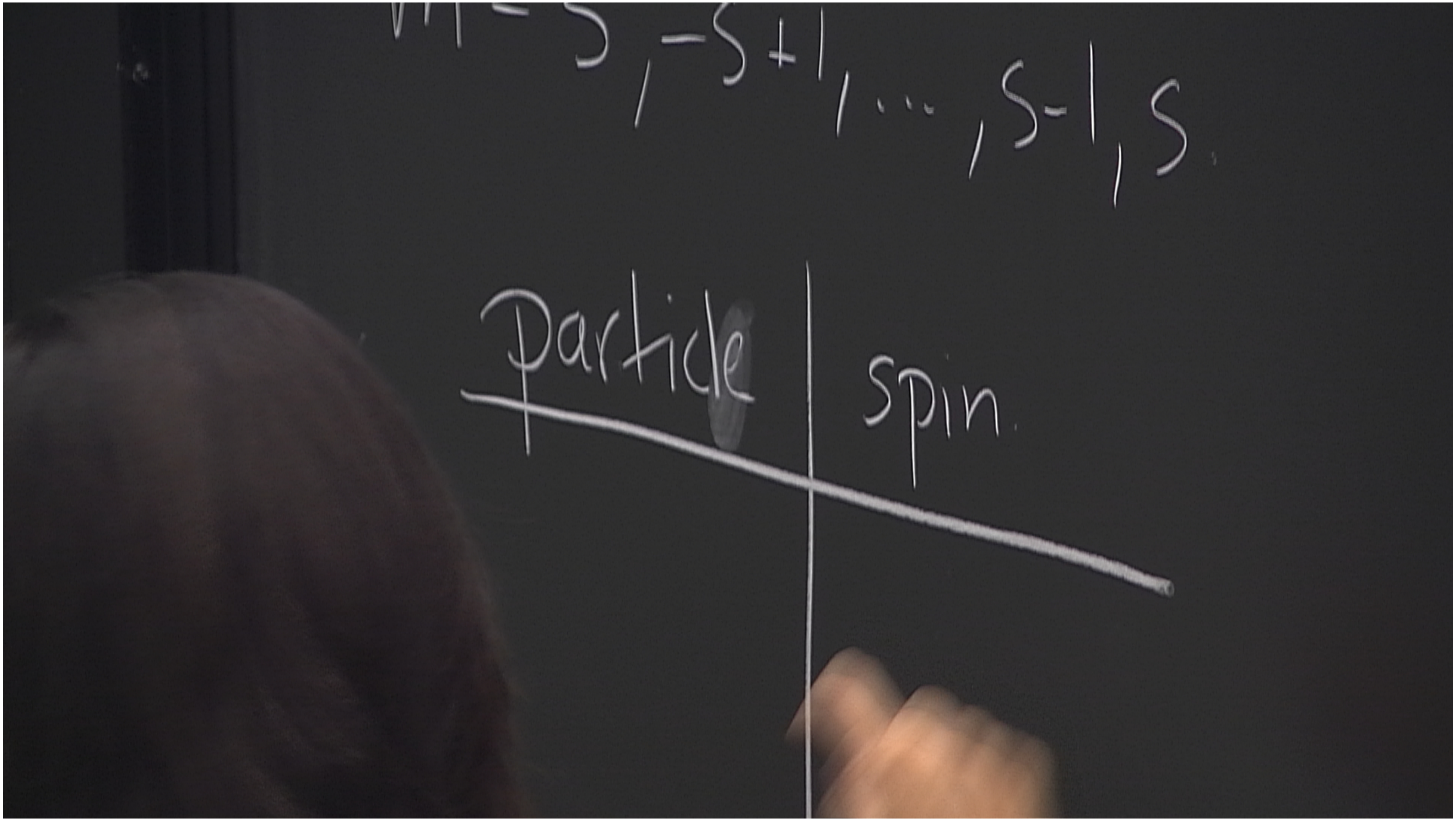

Two quantum no's;
s, m_s

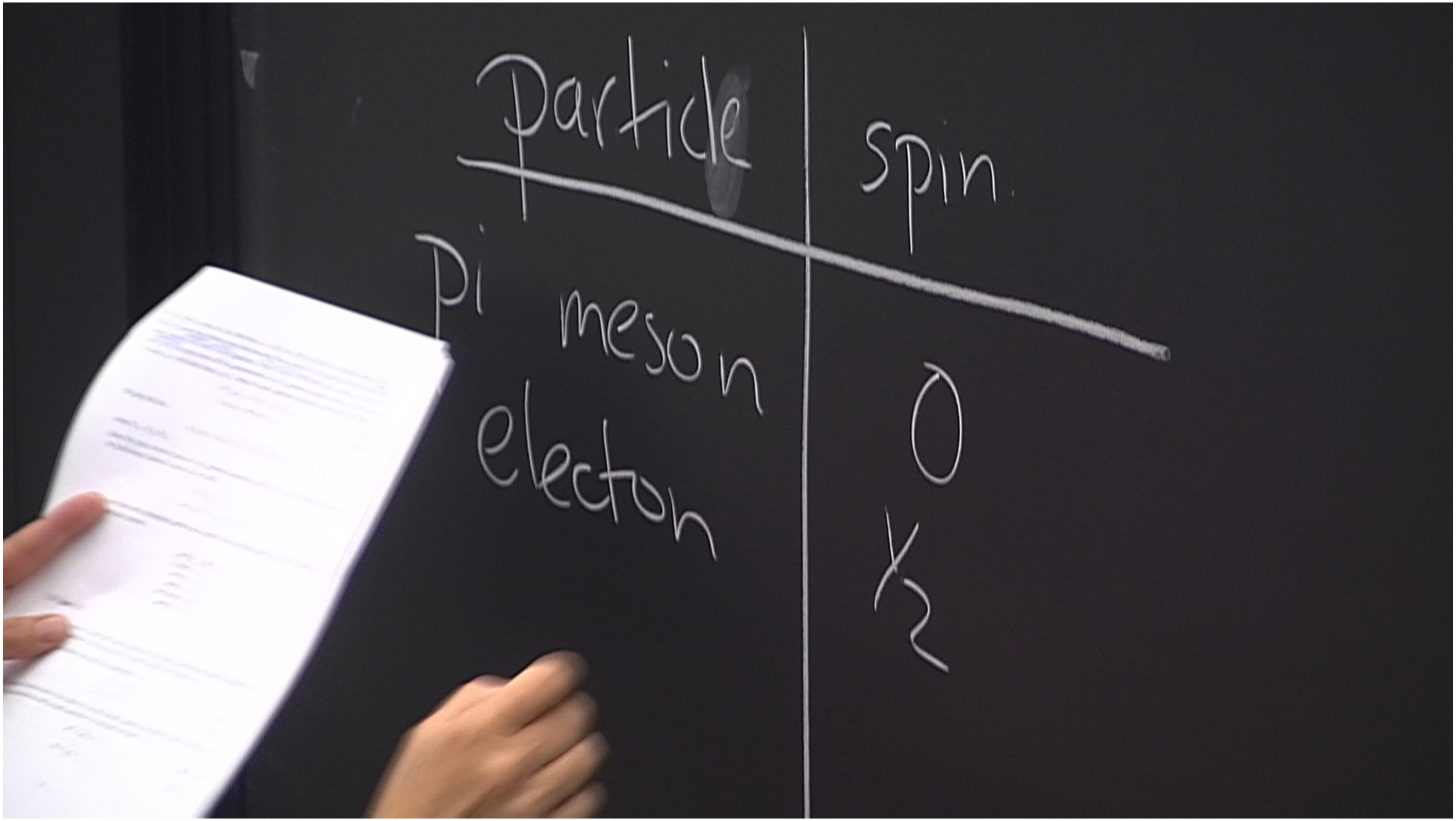

Two quantum no's
|s, m⟩

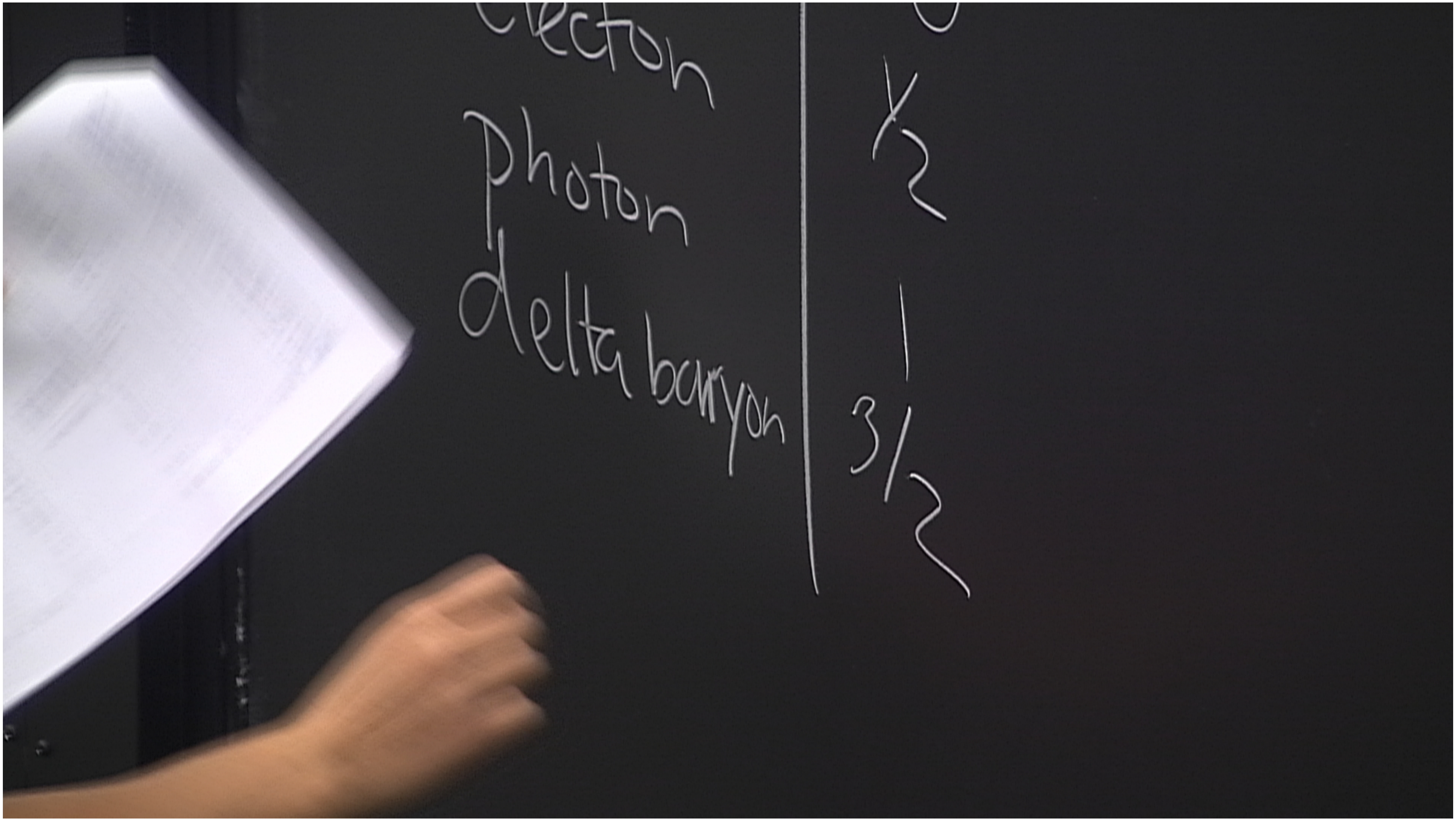

$S^2|s,m\rangle = \hbar^2 s(s+1)|s,m\rangle$
$S_z|s,m\rangle = \hbar m|s,m\rangle$

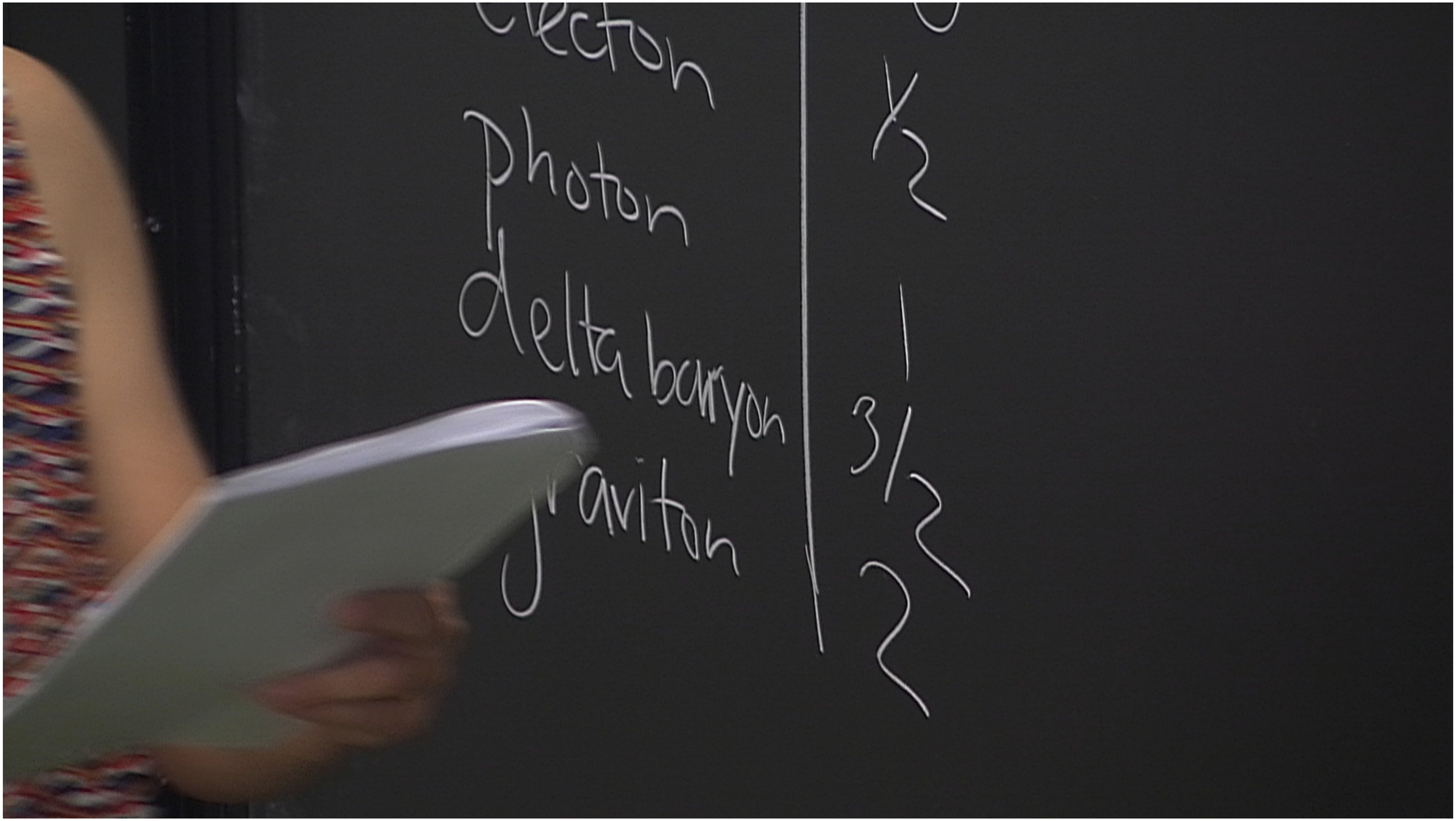

J_z|s,m⟩ = ħm|s,m⟩

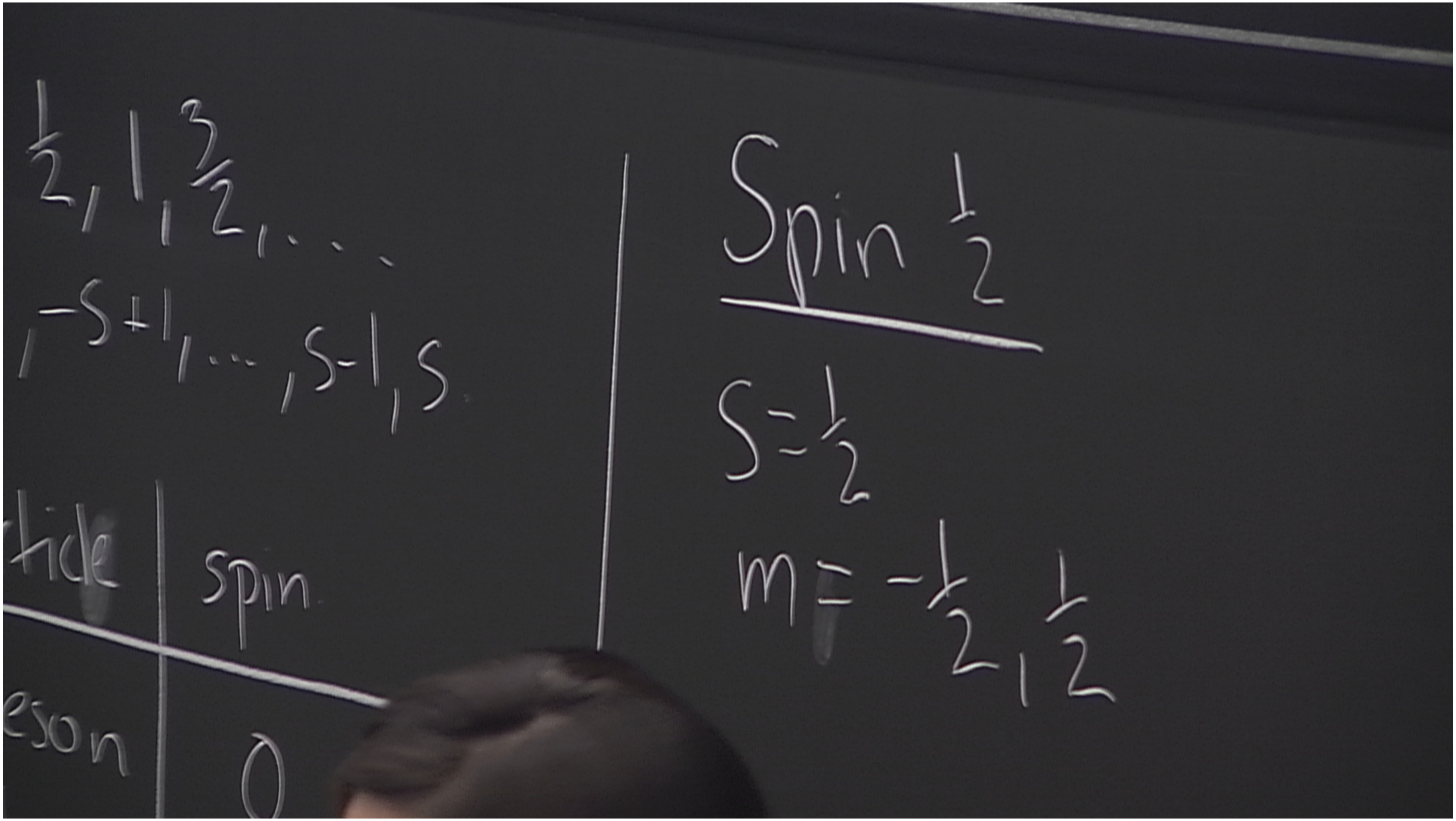

x := cos(θ)
"known solution" is
B = ℓ(ℓ+1)
Spin
Commutation relations
Two quantum nos.
|s, m⟩

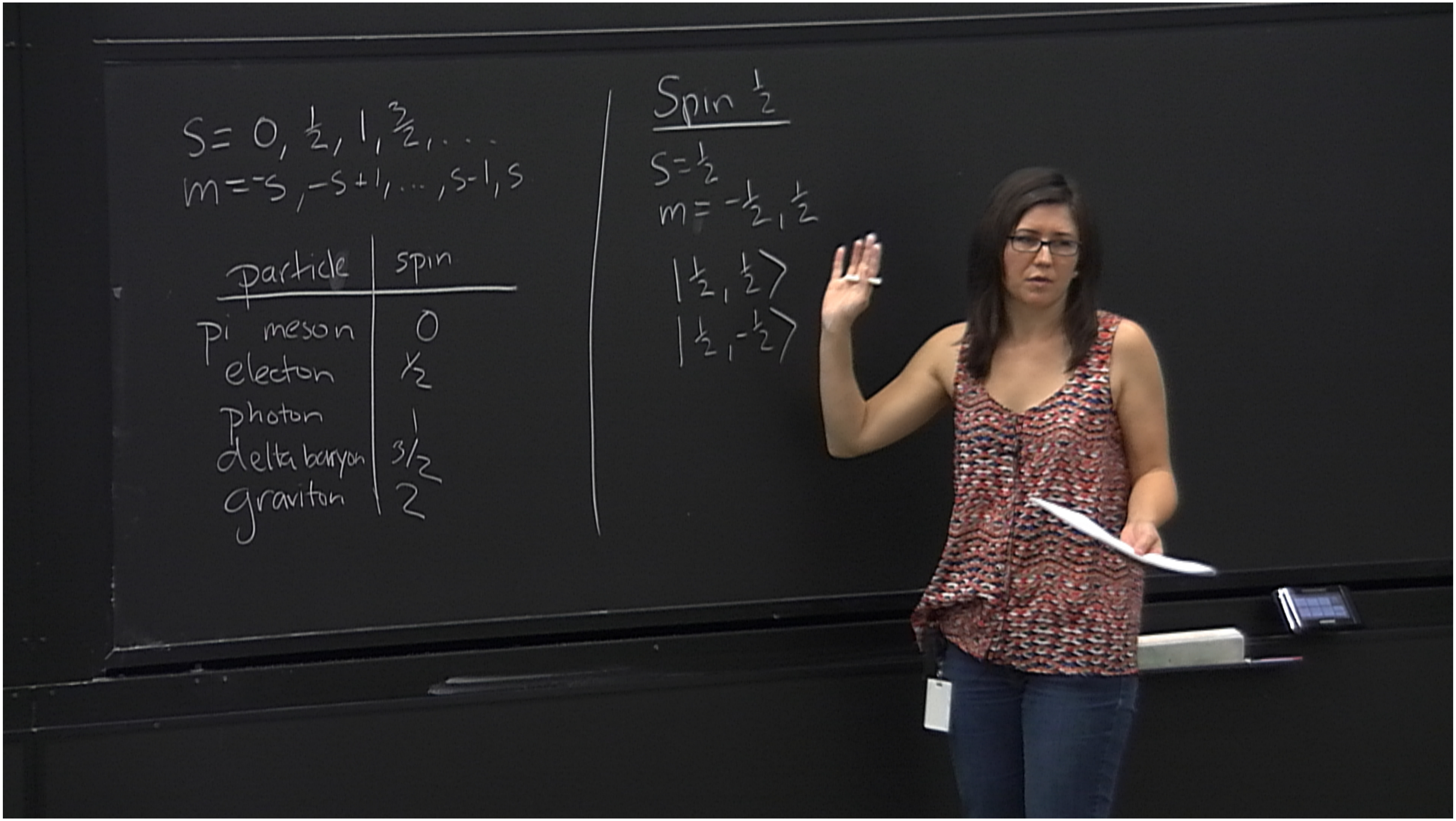

-s, -s+1, ..., s-1, s
particle | spin

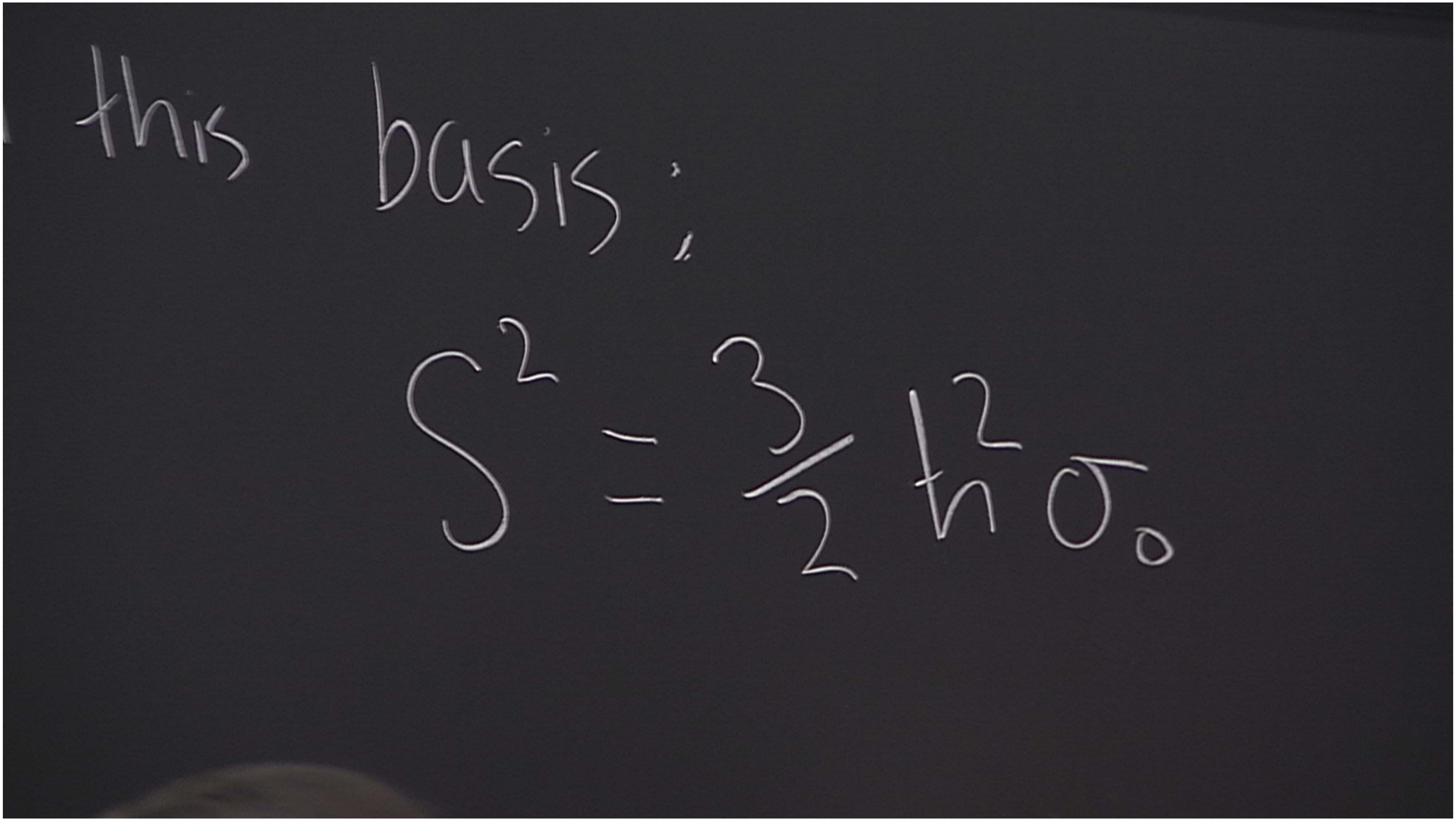

Particle
spin
pi meson
0
electon
½

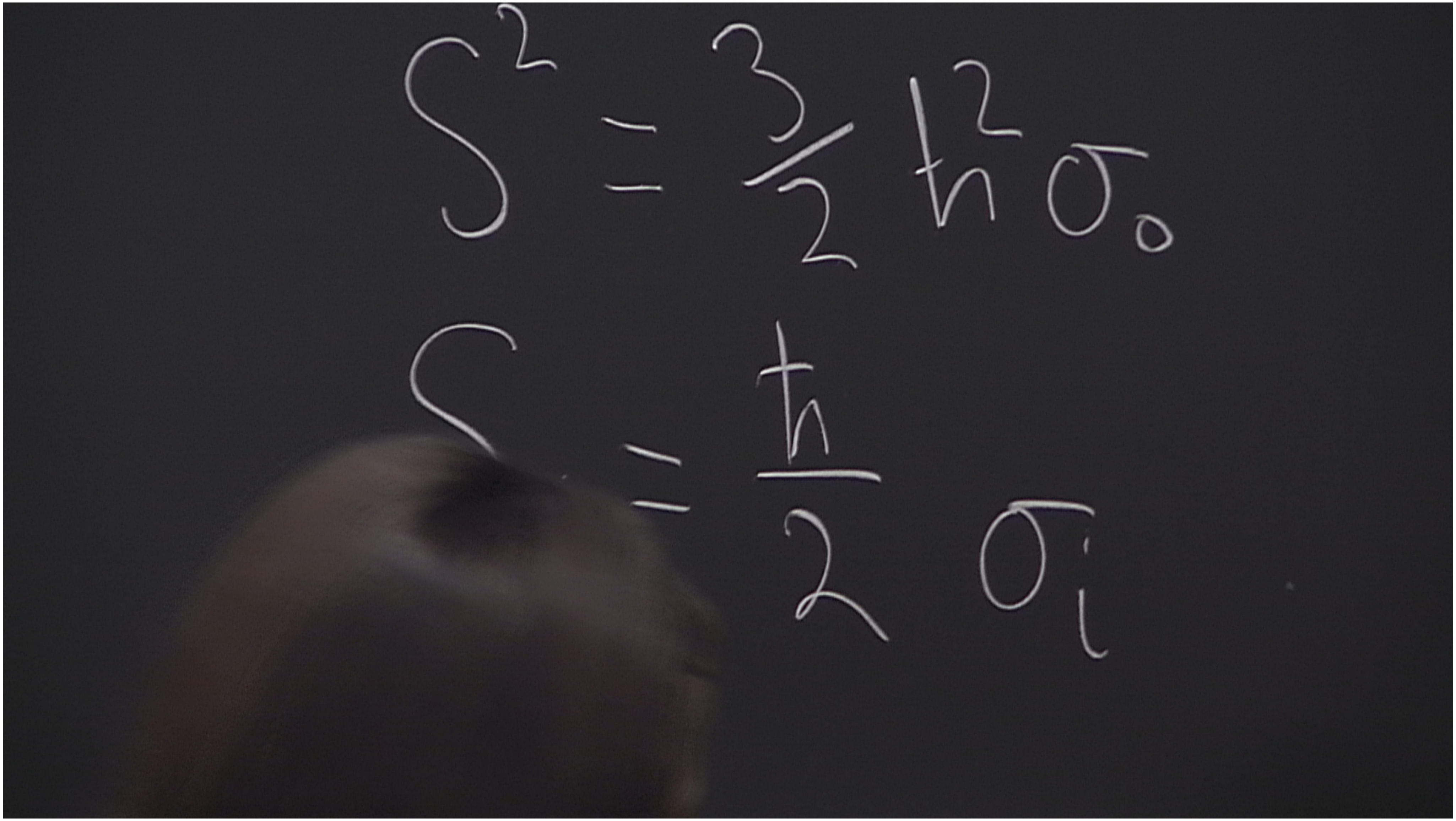

photon
1/2
delta baryon
1
3/2

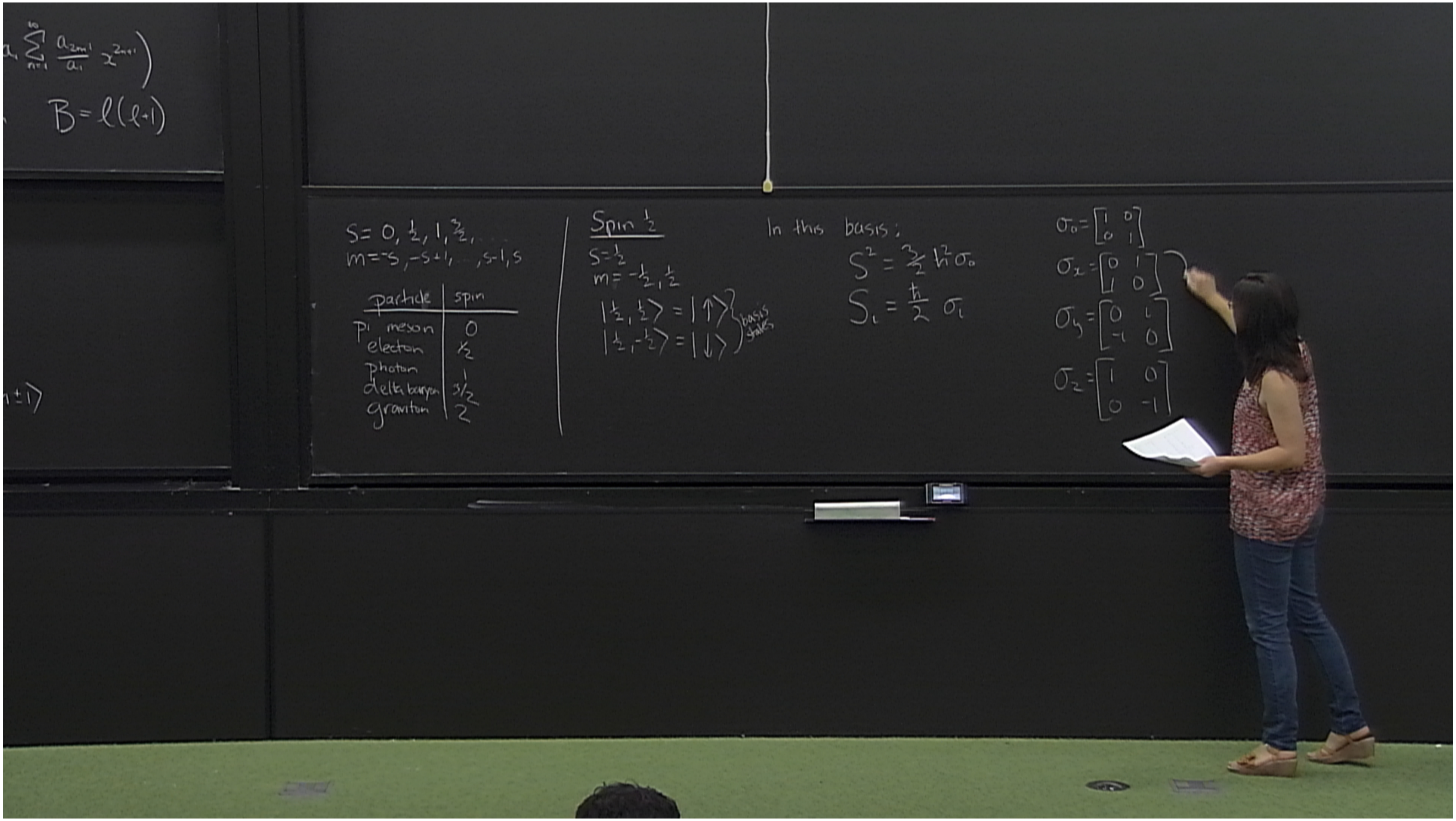

electron
photon
delta baryon
graviton
1/2
1
3/2
2

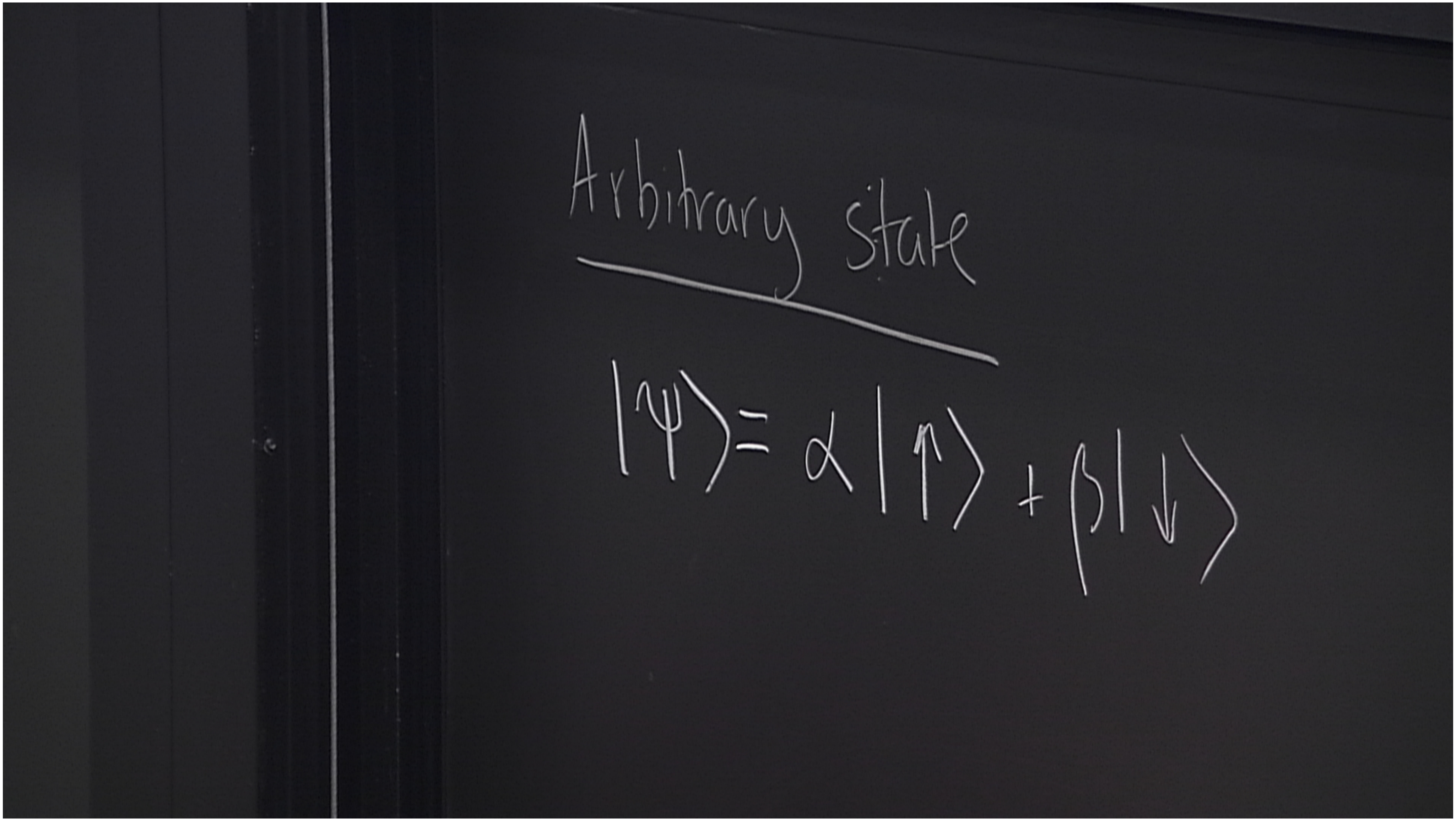

1/2, 1, 3/2, ...
-s+1, ..., s-1, s
spin
0
Spin 1/2
s = 1/2
m = -1/2, 1/2

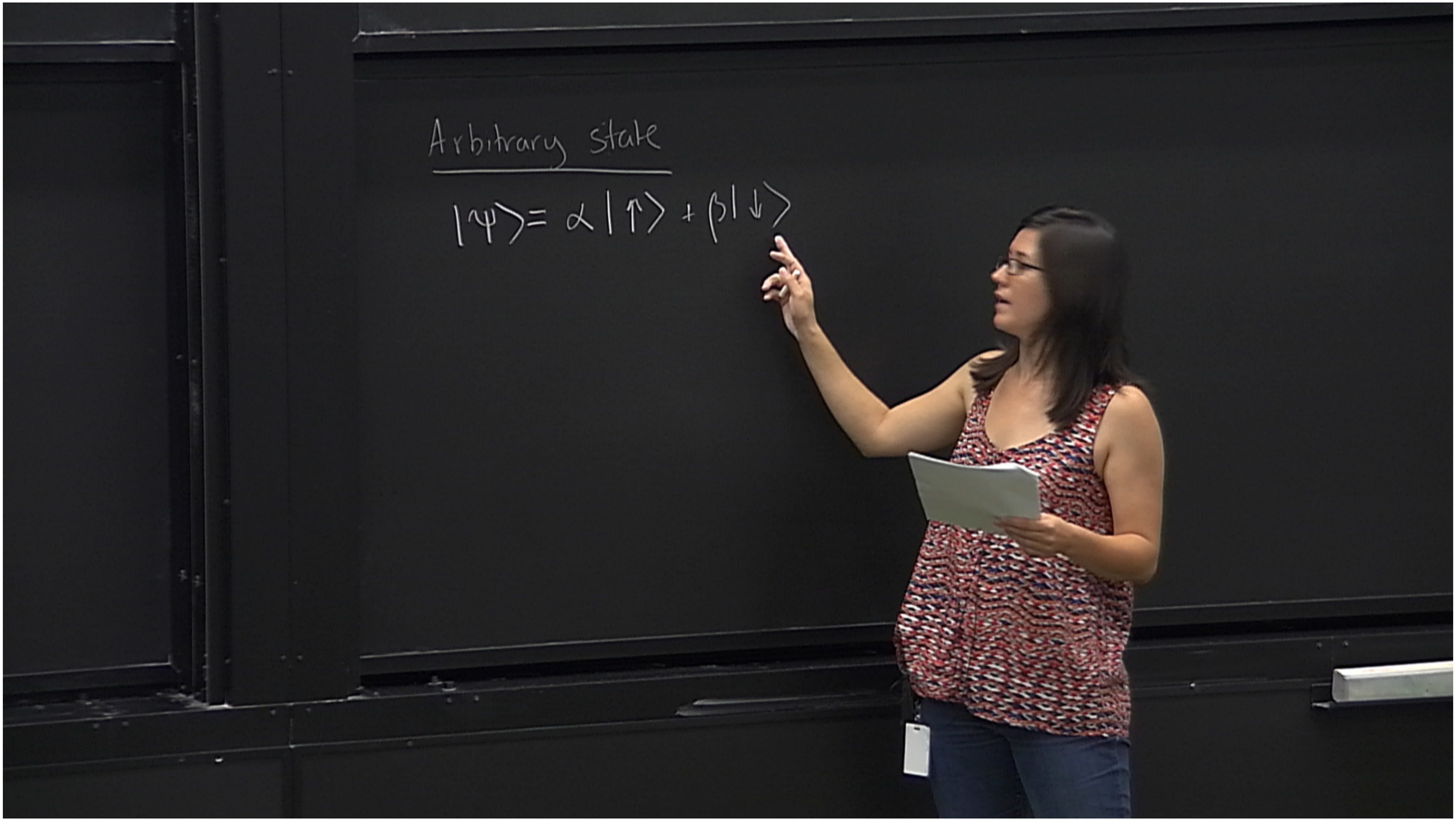

S= 0, 1/2, 1, 3/2, . . .
m=-s, -s+1, ..., s-1, s
particle | spin
pi meson | 0
electon | 1/2
photon | 1
delta baryon | 3/2
graviton | 2
Spin 1/2
S=1/2
m= -1/2, 1/2
|1/2, 1/2>
|1/2, -1/2>

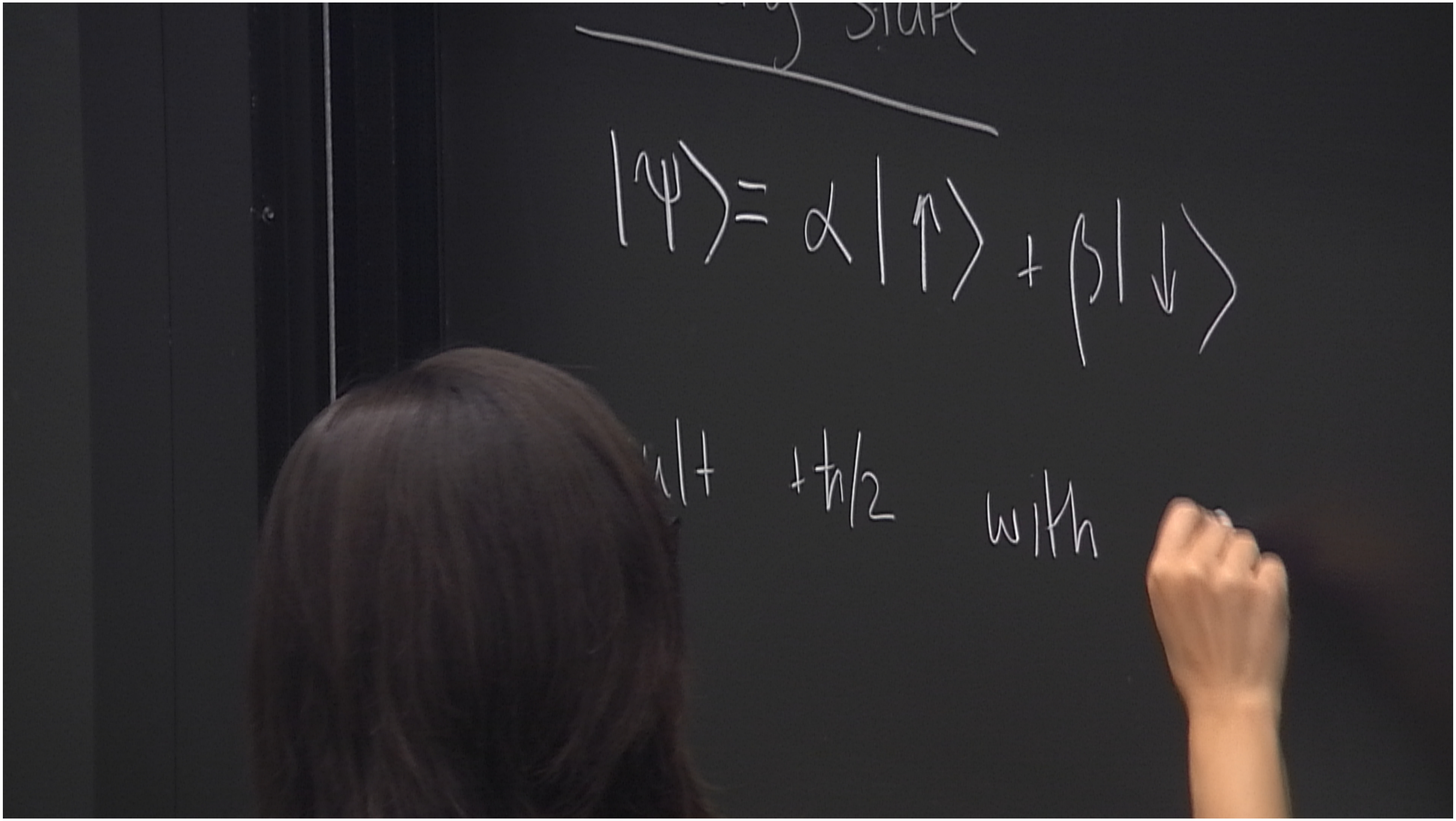

this basis;
$S^2 = \frac{3}{2}\hbar^2\sigma_0$

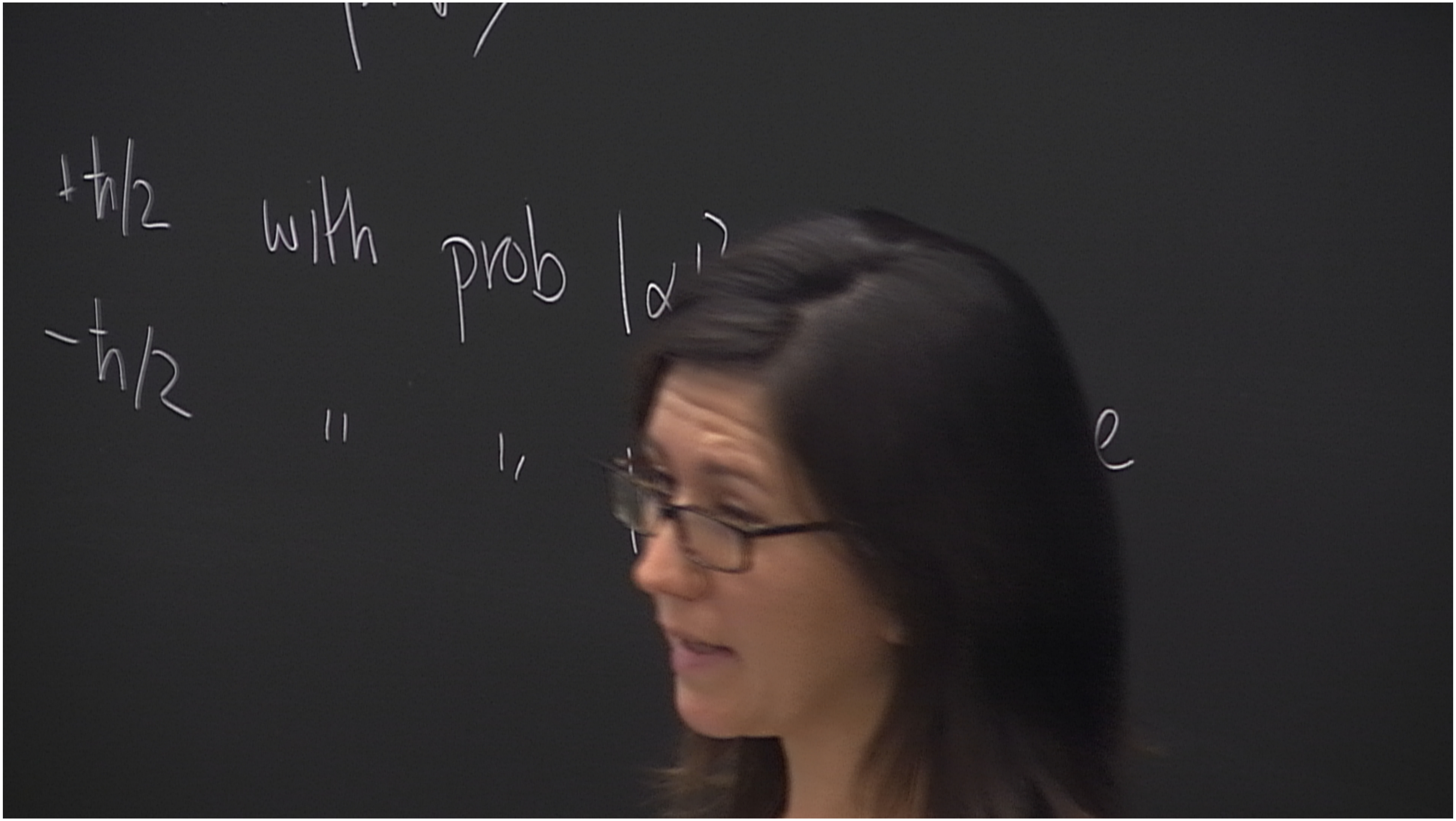

$S^2 = \frac{3}{2} \hbar^2 \sigma_0$
$S = \frac{\hbar}{2} \sigma_i$

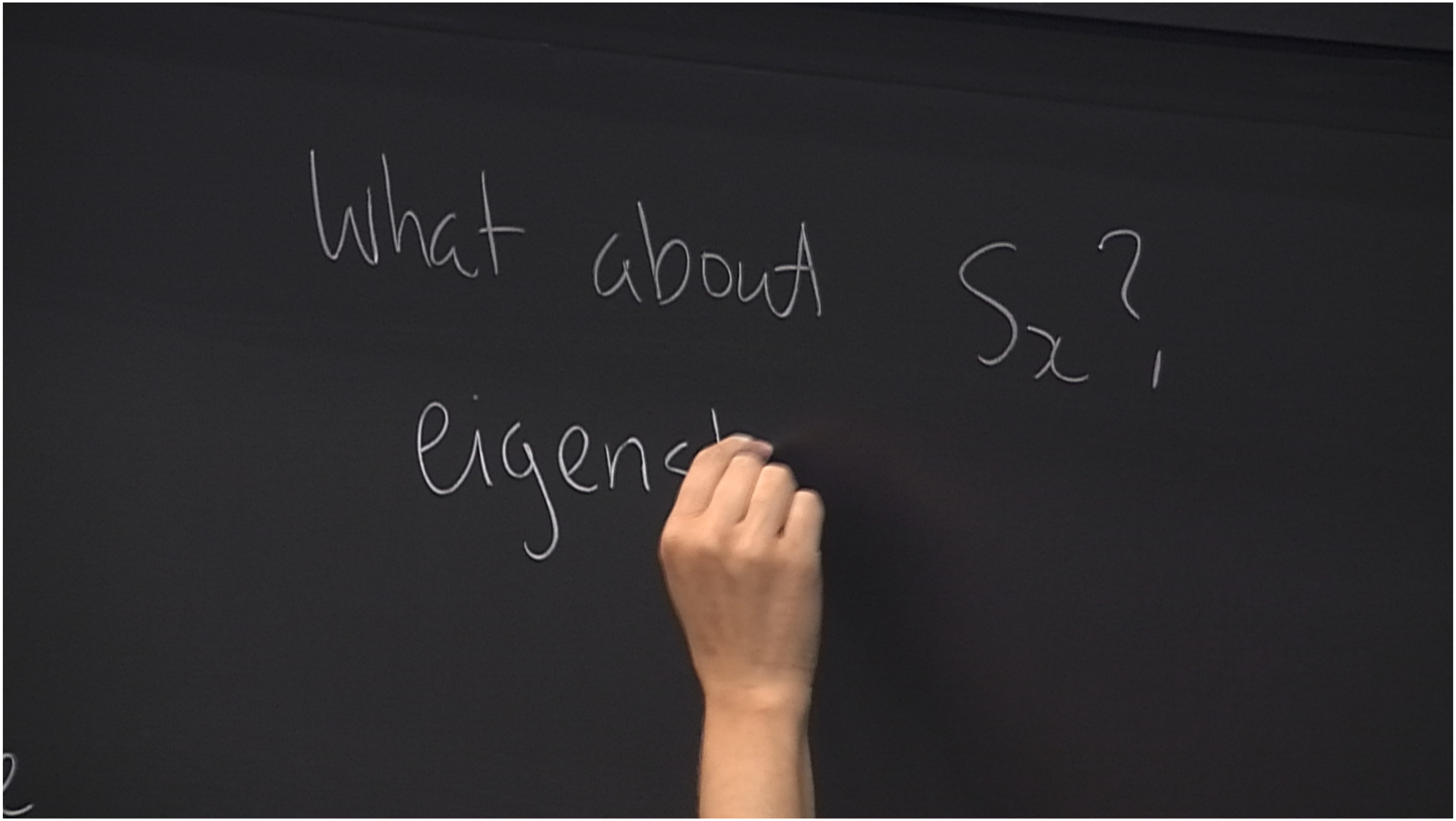

B = ℓ(ℓ+1)
S = 0, 1/2, 1, 3/2, ...
m = -s, -s+1, ..., s-1, s
particle | spin
pi meson | 0
electron | 1/2
photon | 1
delta baryon | 3/2
graviton | 2
Spin 1/2
s = 1/2
m = -1/2, 1/2
|1/2, 1/2> = |↑>
|1/2, -1/2> = |↓>
basis states
In this basis:
S² = 3/4 ħ² σ₀
Sᵢ = ħ/2 σᵢ
σ₀ = [1 0; 0 1]
σₓ = [0 1; 1 0]
σᵧ = [0 i; -i 0]
σ_z = [1 0; 0 -1]

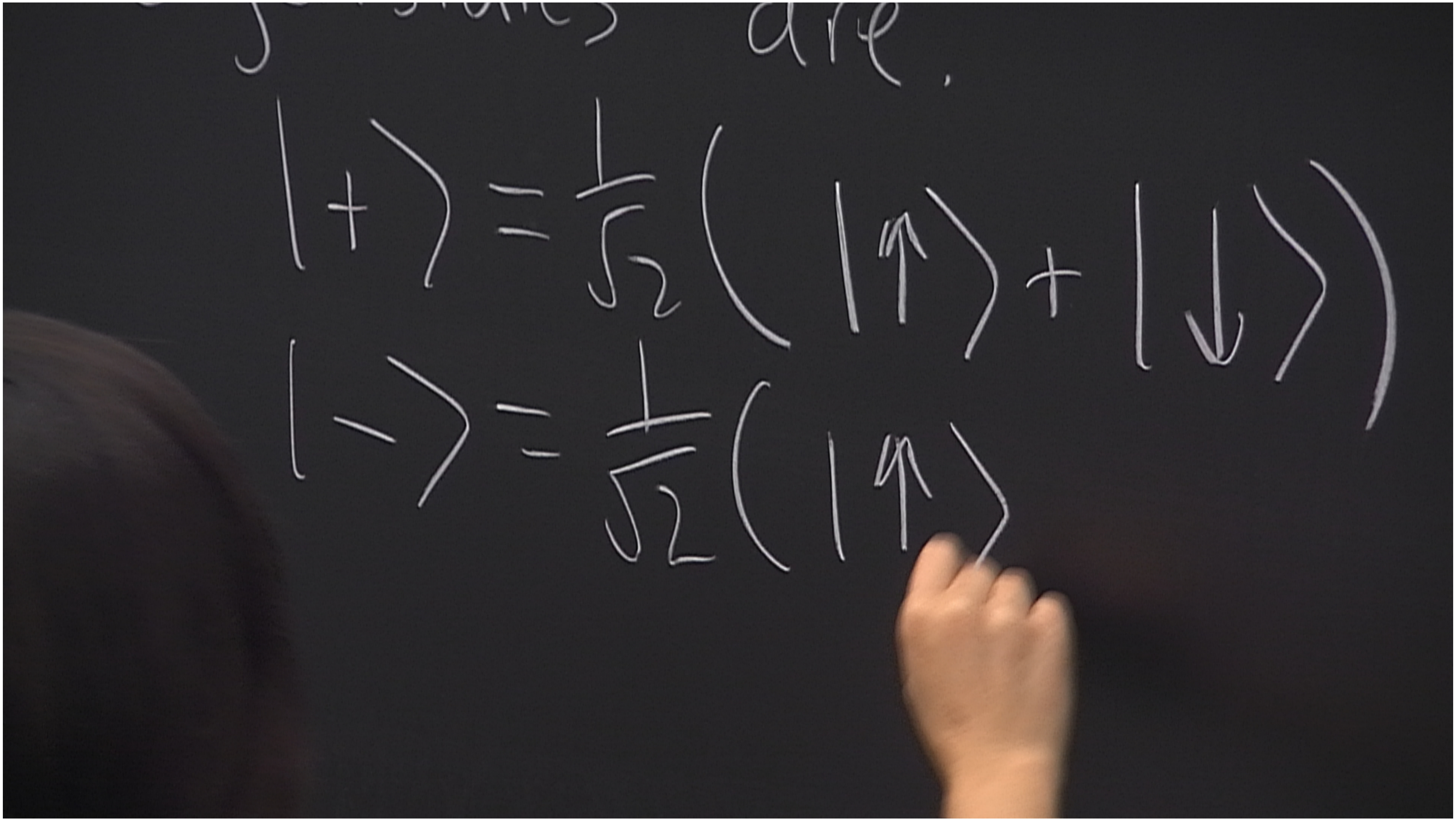

Arbitrary state
$|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$

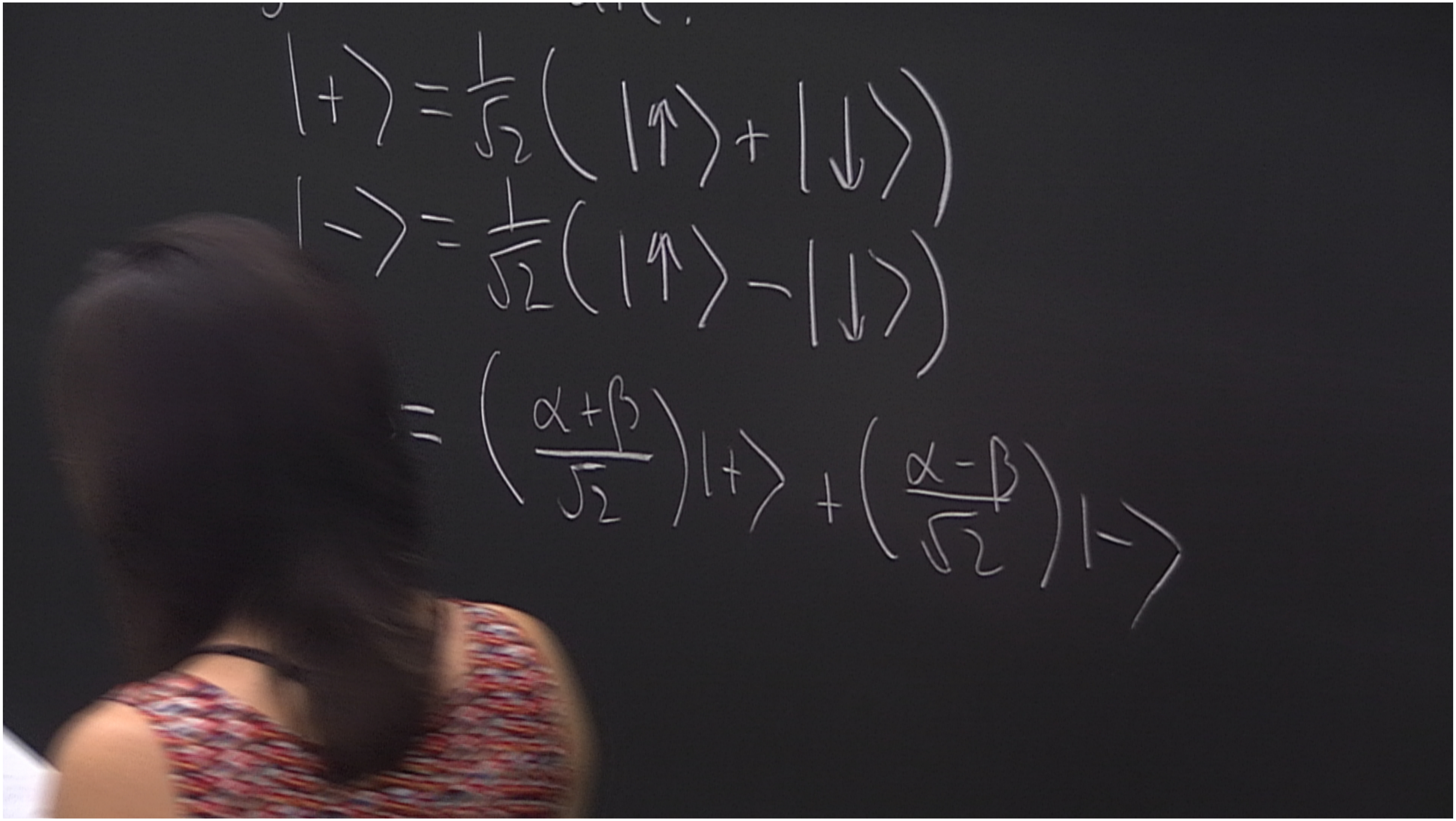

Arbitrary state
|ψ⟩ = α|↑⟩ + β|↓⟩

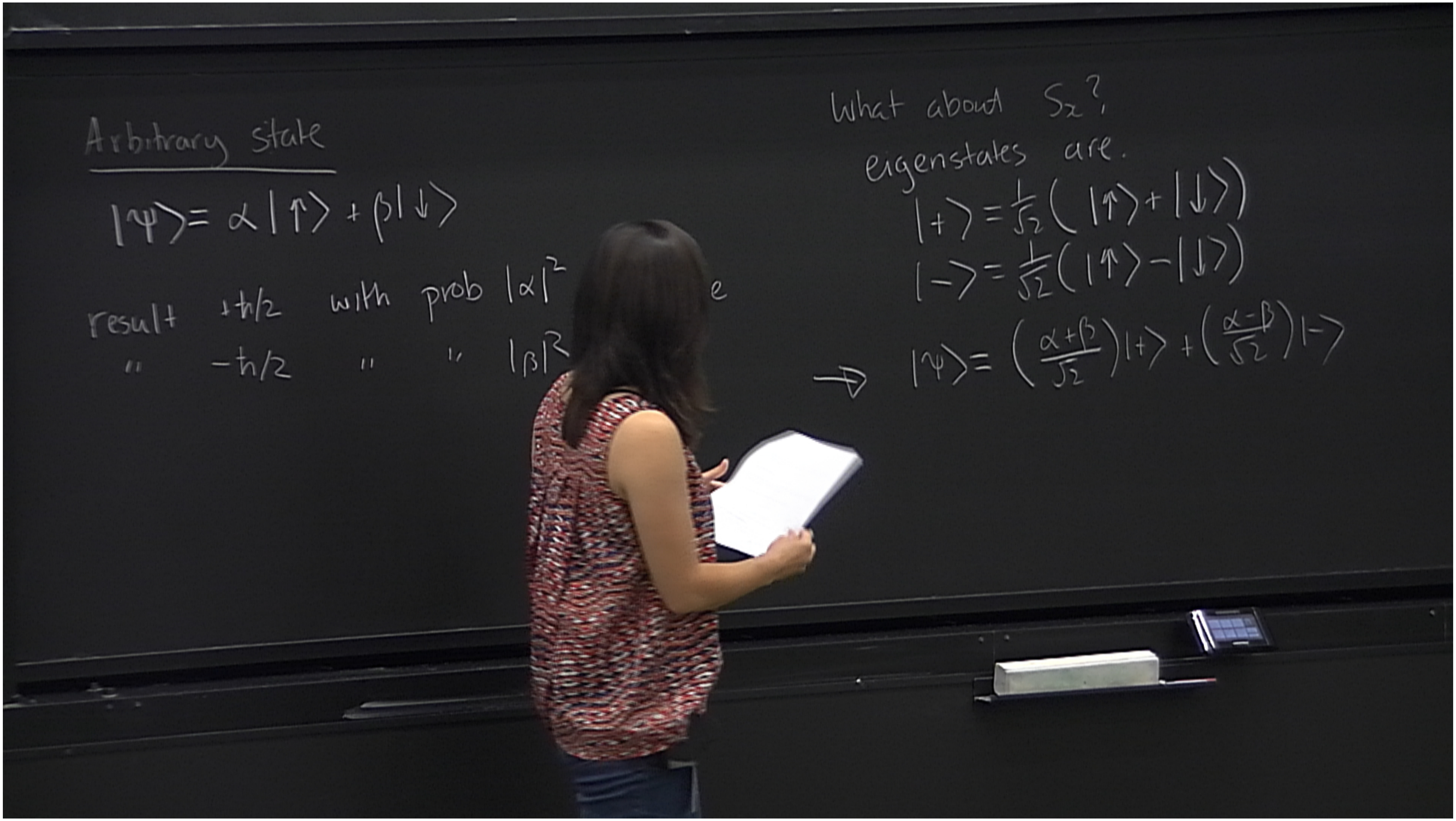

|ψ⟩ = α|↑⟩ + β|↓⟩
+ħ/2
with

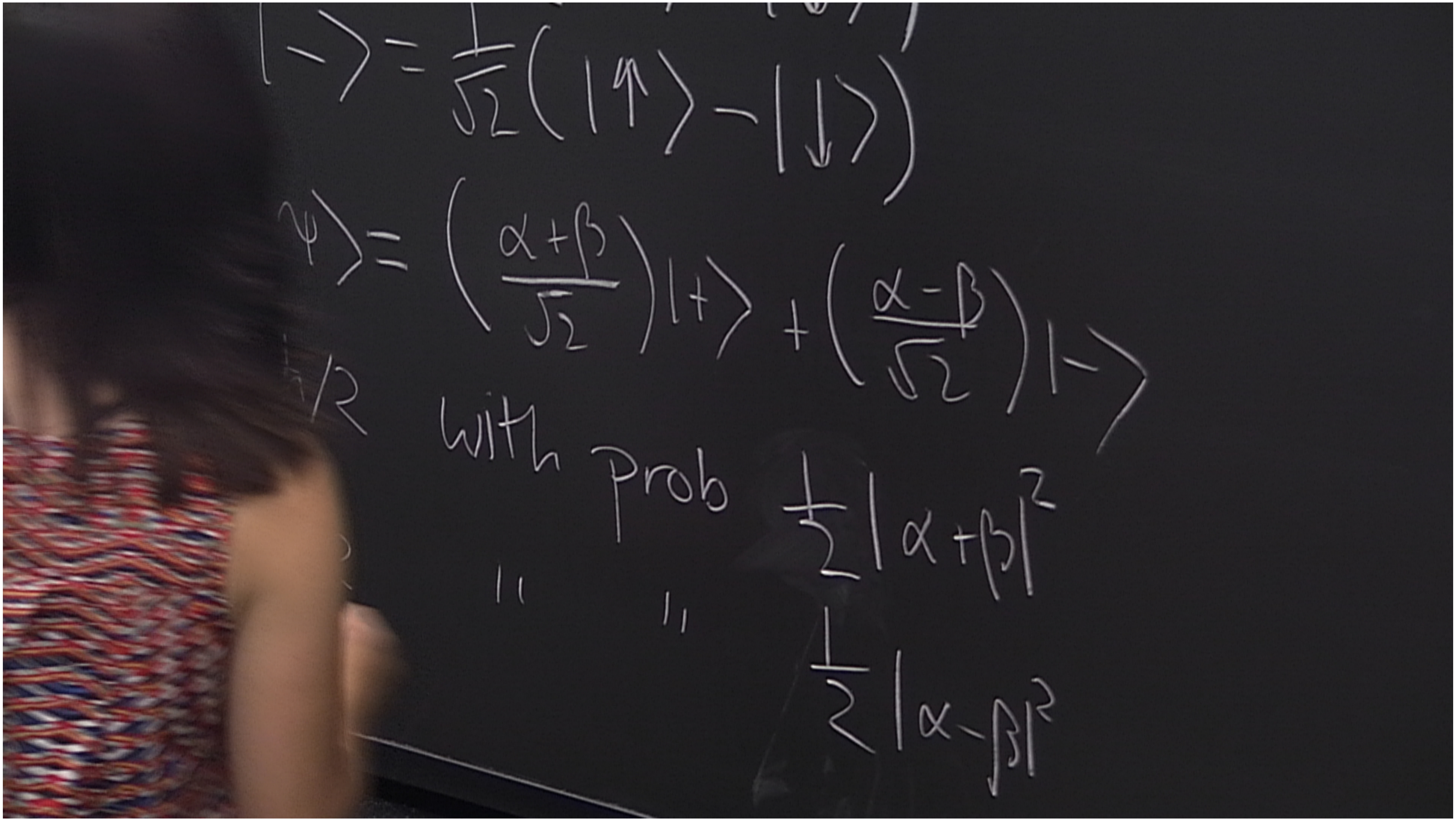

+ħ/2 with prob
-ħ/2

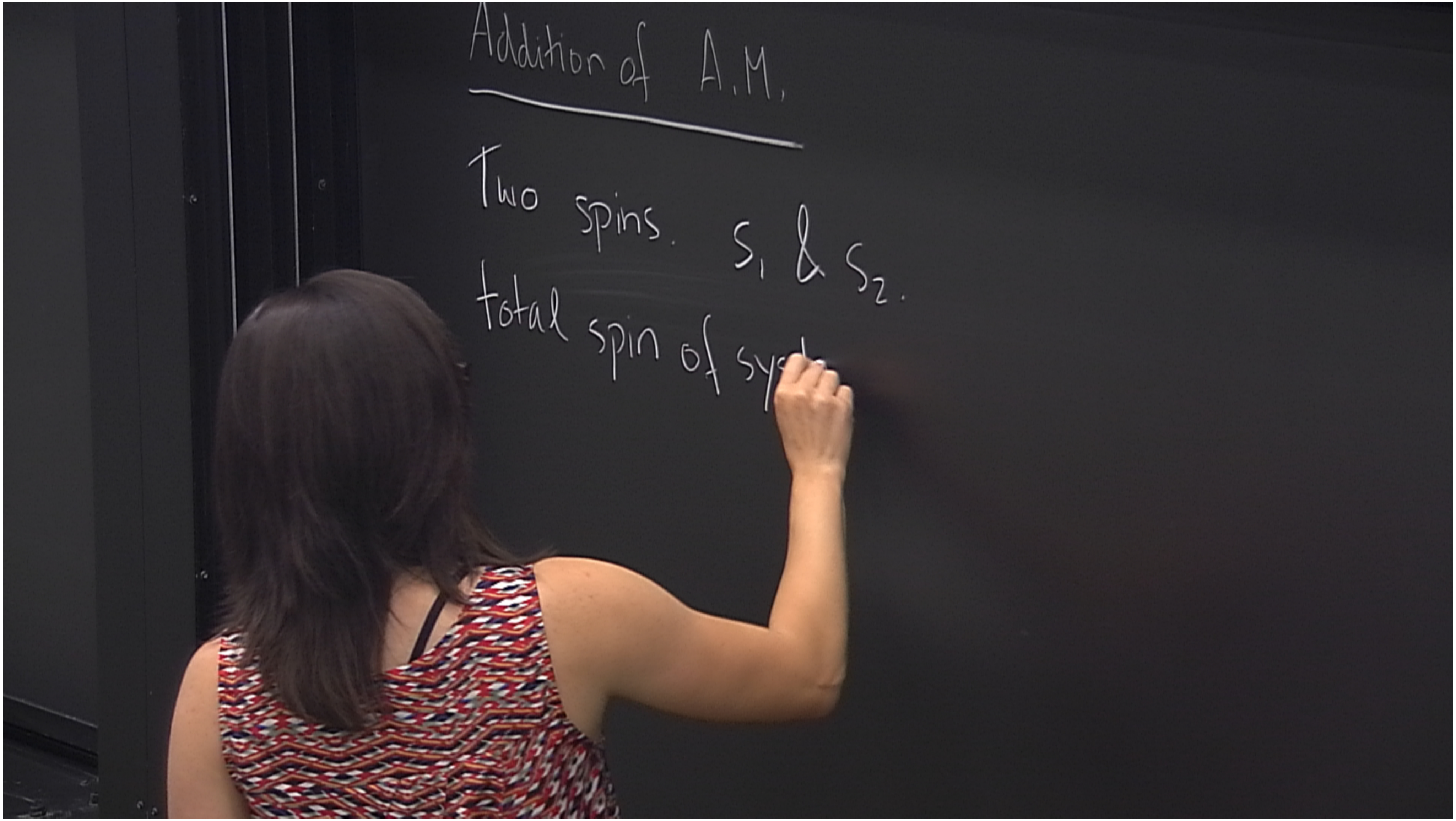

What about S_x?,
eigens

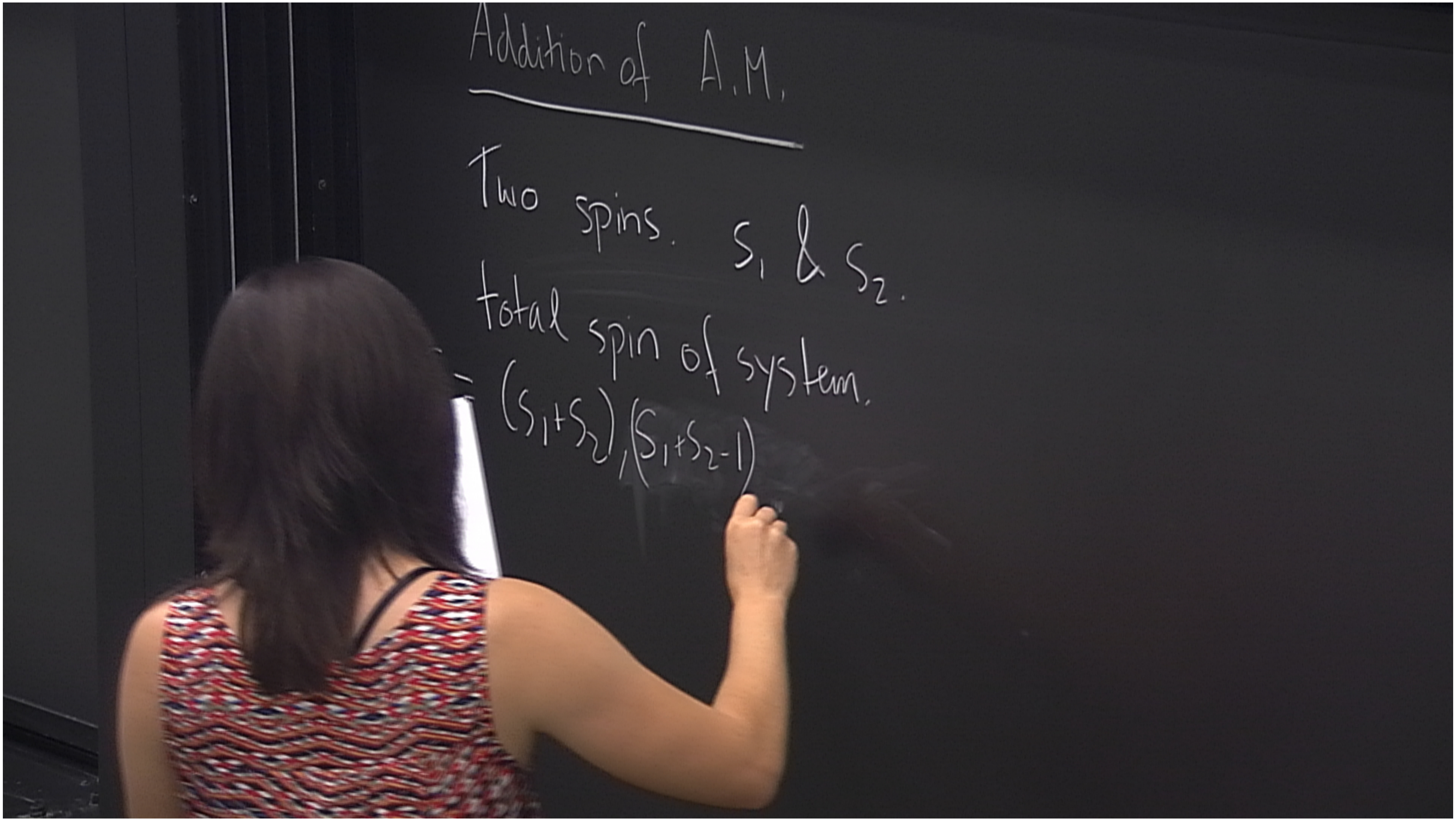

are.
|+⟩ = 1/√2 (|↑⟩ + |↓⟩)
|−⟩ = 1/√2 (|↑⟩

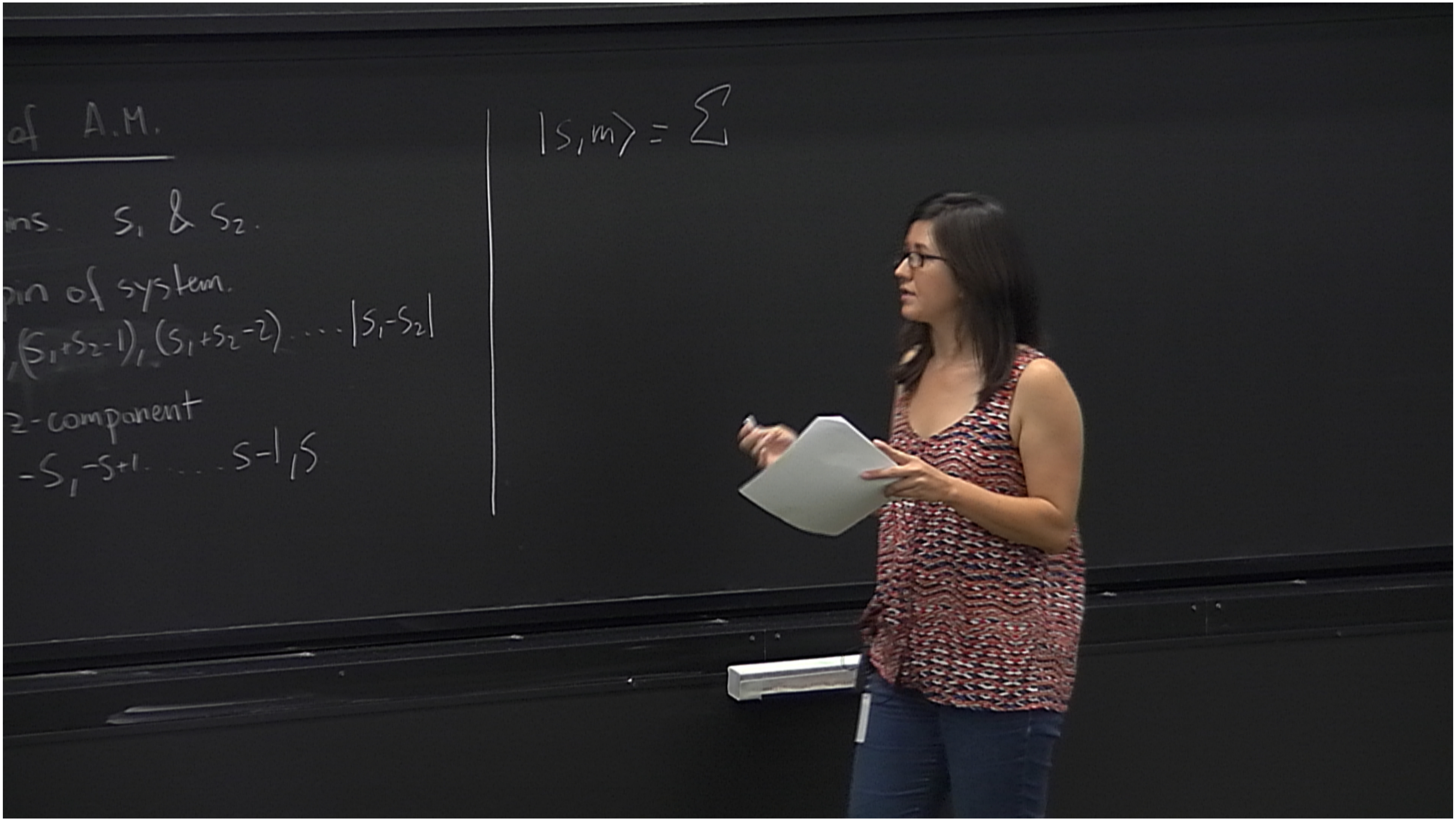

$|+\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$
$|-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle - |\downarrow\rangle\right)$
$= \left(\frac{\alpha+\beta}{\sqrt{2}}\right)|+\rangle + \left(\frac{\alpha-\beta}{\sqrt{2}}\right)|-\rangle$

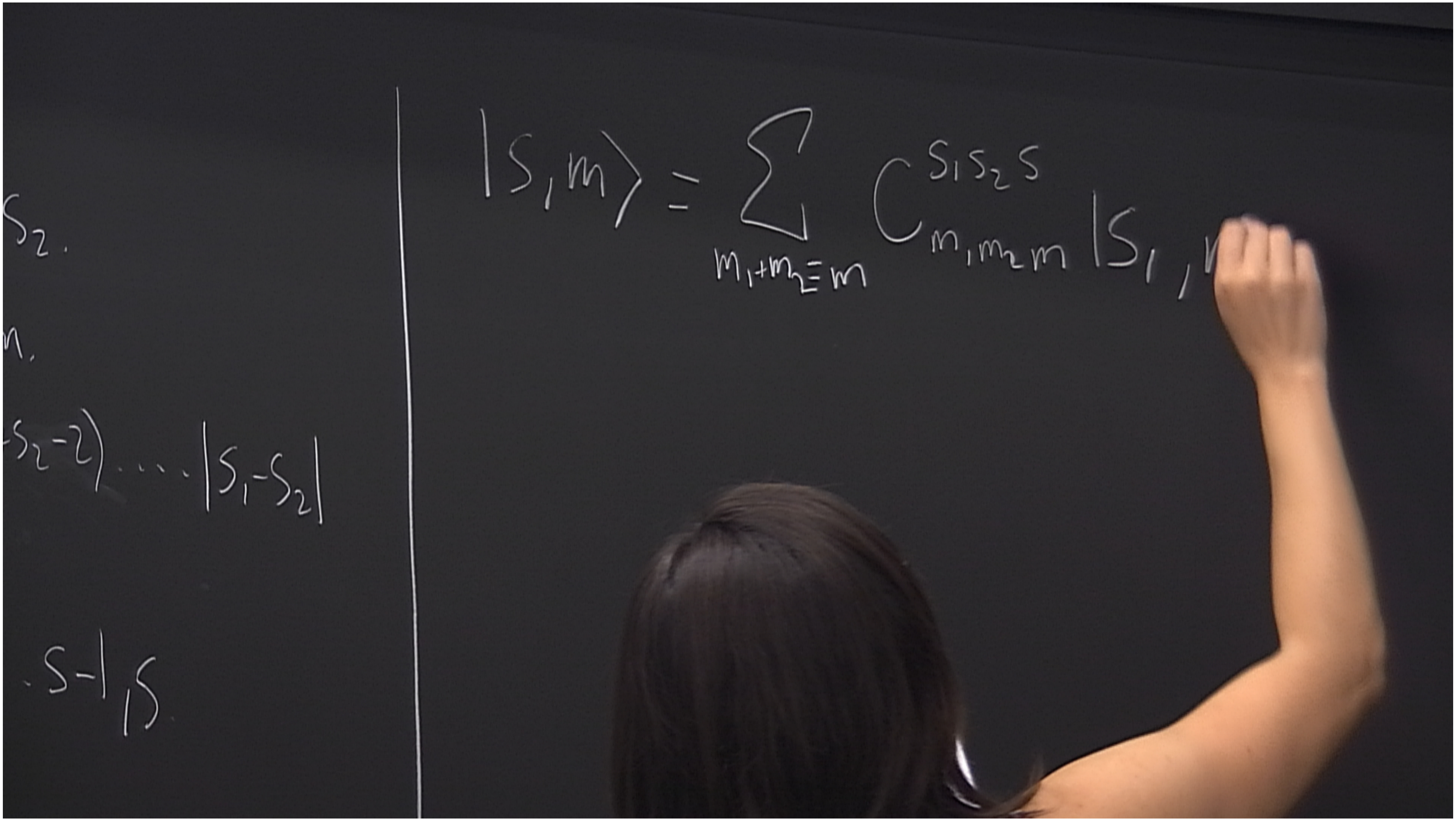

Arbitrary state
|ψ> = α|↑> + β|↓>
result +ħ/2 with prob |α|²
" −ħ/2 " " |β|²
What about Sx?
eigenstates are.
|+> = 1/√2 (|↑> + |↓>)
|−> = 1/√2 (|↑> − |↓>)
⇒ |ψ> = ((α+β)/√2) |+> + ((α−β)/√2) |−>

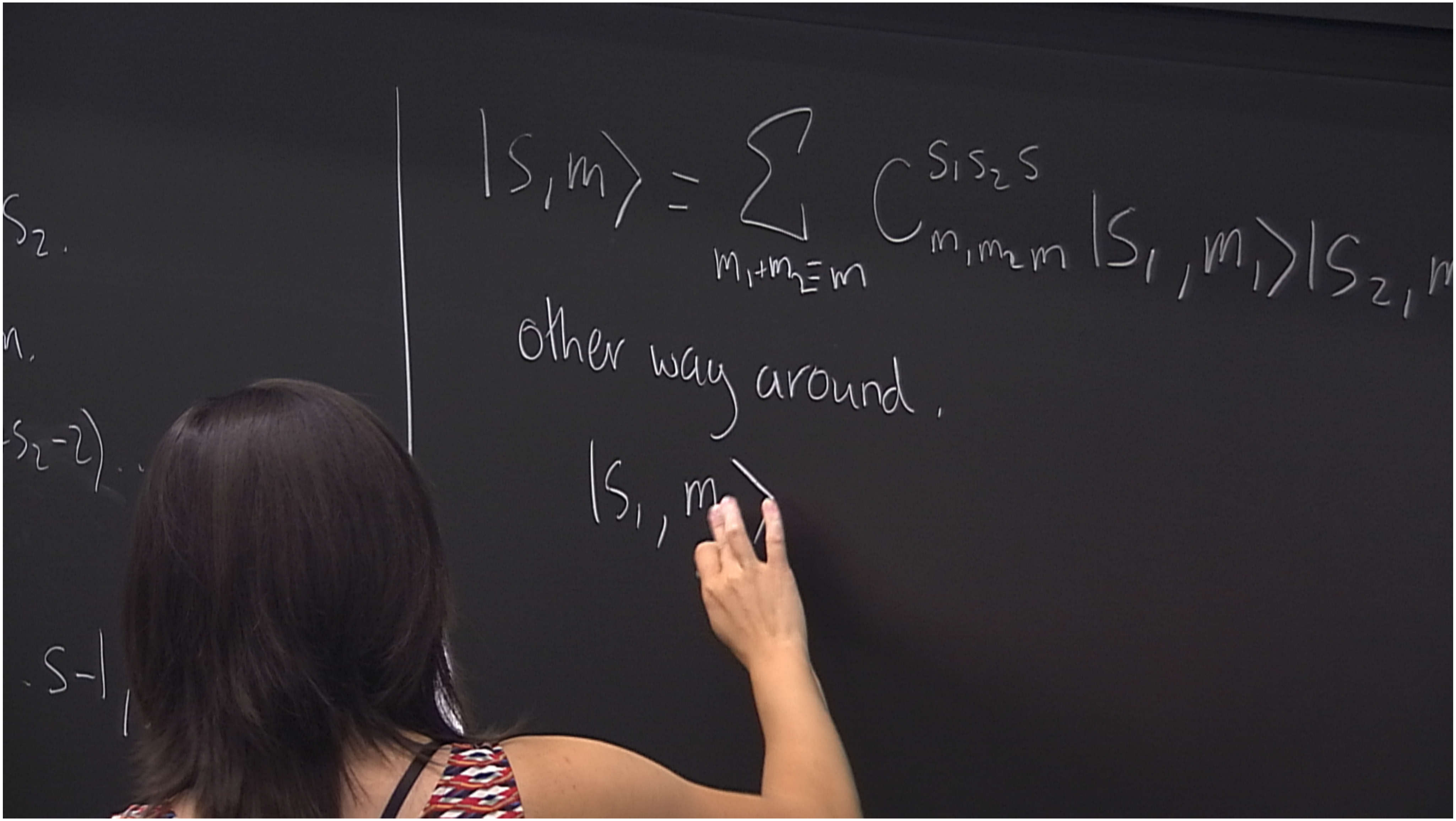

$|-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$
$\psi\rangle = \left(\frac{\alpha+\beta}{\sqrt{2}}\right)|+\rangle + \left(\frac{\alpha-\beta}{\sqrt{2}}\right)|-\rangle$
with prob $\frac{1}{2}|\alpha+\beta|^2$
" " $\frac{1}{2}|\alpha-\beta|^2$

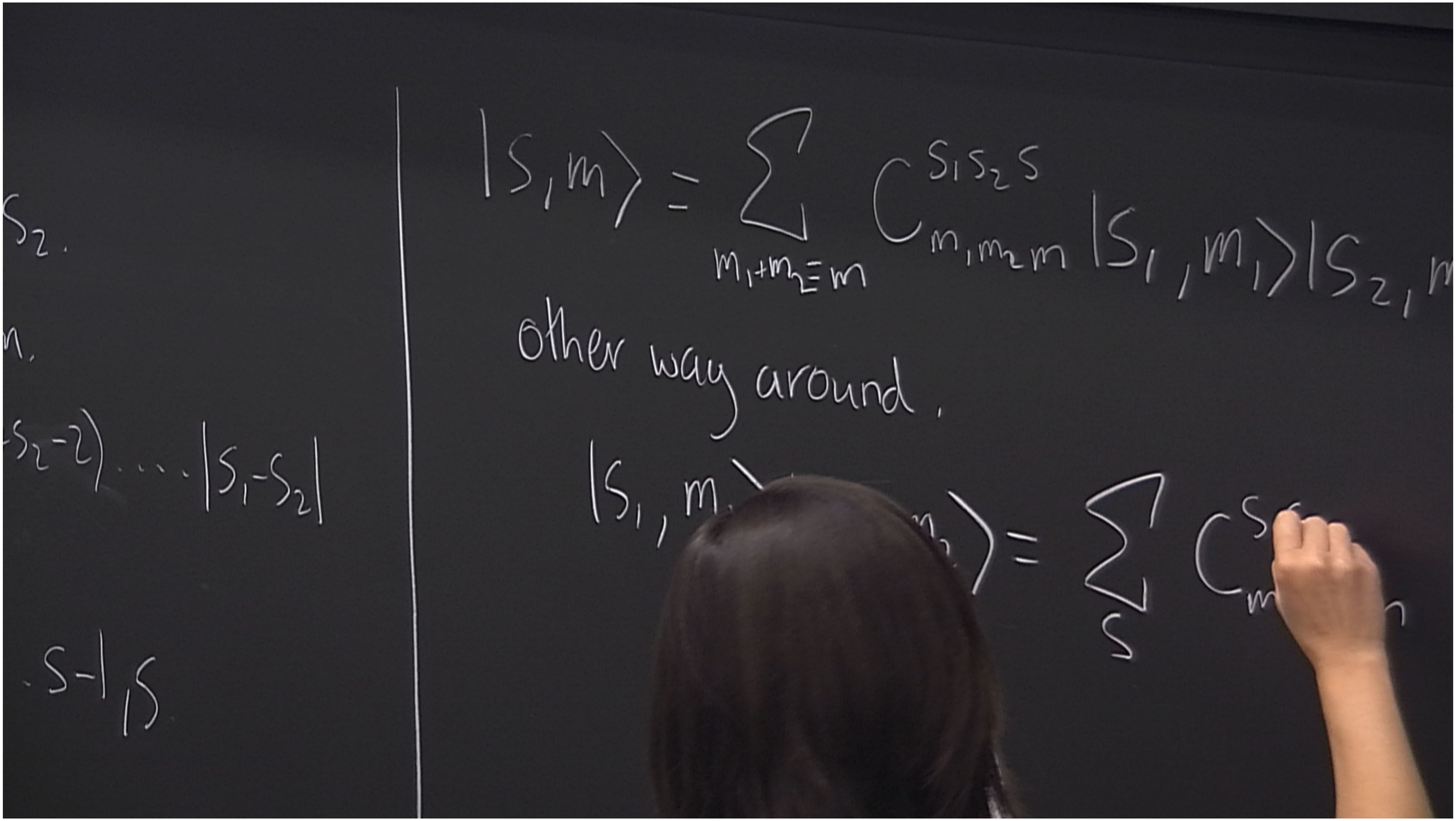

Addition of A.M.
Two spins. S1 & S2.
total spin of sy


Addition of A.M.
Two spins. S1 & S2.
total spin of system.
(S1+S2), (S1+S2-1)


of A.M.
ins. s₁ & s₂.
pin of system.
(s₁+s₂-1), (s₁+s₂-2) .... |s₁-s₂|
z-component
-s, -s+1 ..... s-1, s
|s,m⟩ = Σ


|s,m⟩ = Σ C^{s1 s2 s}_{m1 m2 m} |s1 ,
m1+m2=m
-s2-2) .... |s1-s2|
s-1, s

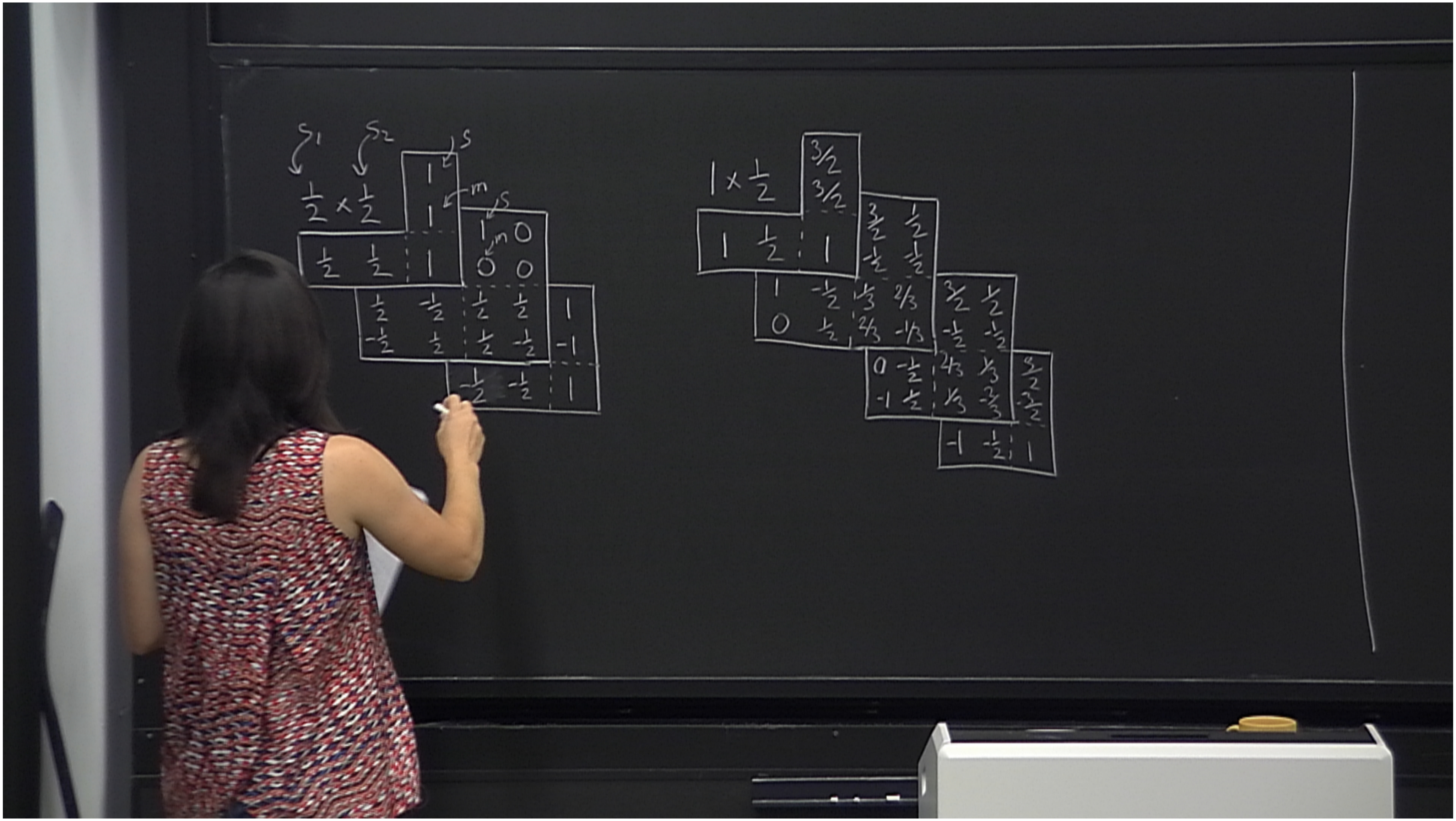

|s, m⟩ = Σ C^{s1 s2 s}_{m1 m2 m} |s1, m1⟩|s2, m
m1+m2=m
other way around.
|s1, m
s2.
s-1,

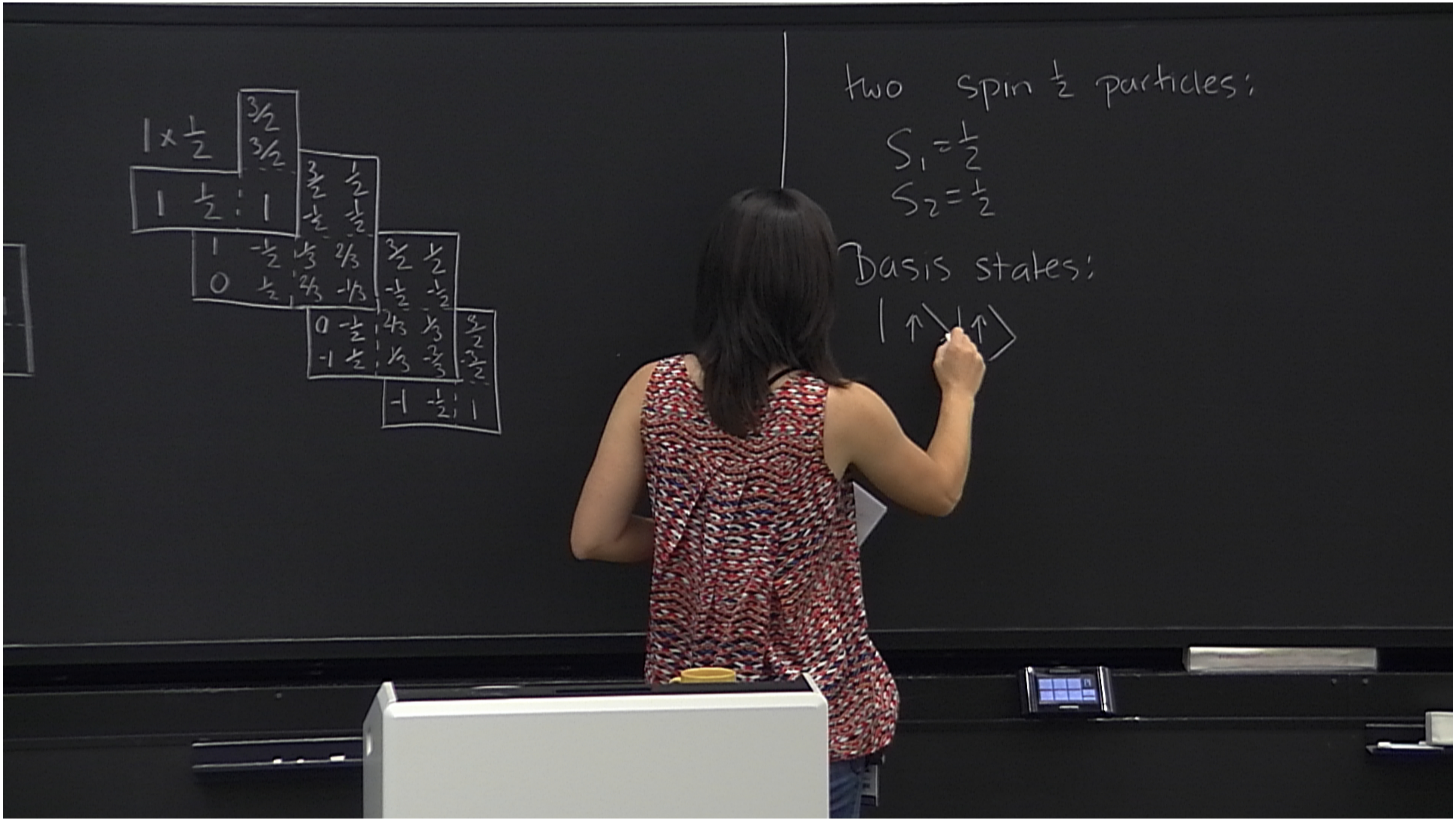

$|s, m\rangle = \sum_{m_1+m_2=m} C^{s_1 s_2 s}_{m_1 m_2 m} |s_1, m_1\rangle |s_2, m$
other way around.
$|s_1, m$
$\rangle = \sum_s C$
$\ldots |s_1 - s_2|$
$s-1, s$

$$|s,m\rangle = \sum_{m_1+m_2=m} C^{s_1 s_2 s}_{m_1 m_2 m} |s_1, m_1\rangle |s_2, m_2\rangle$$

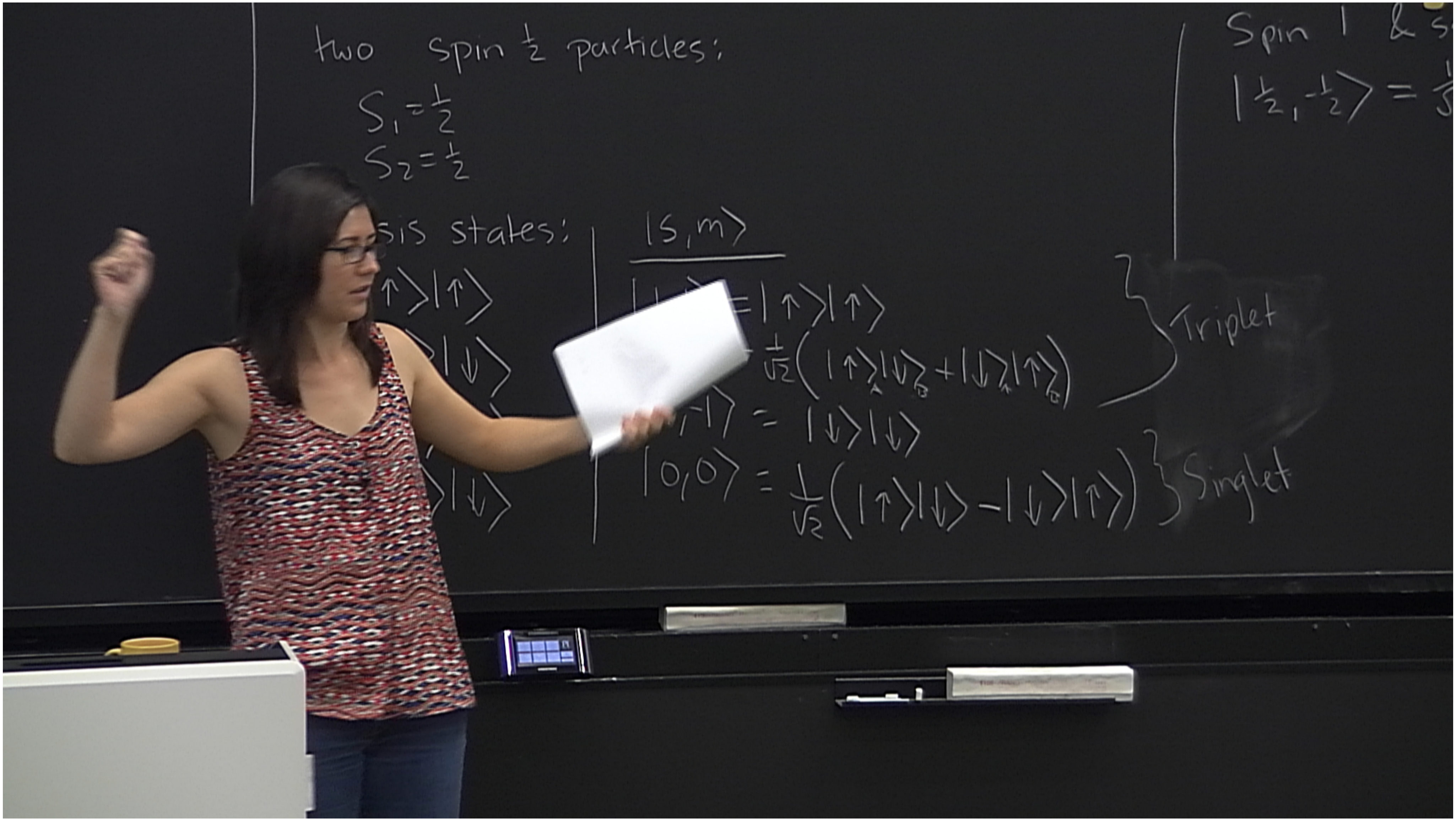
other way around.

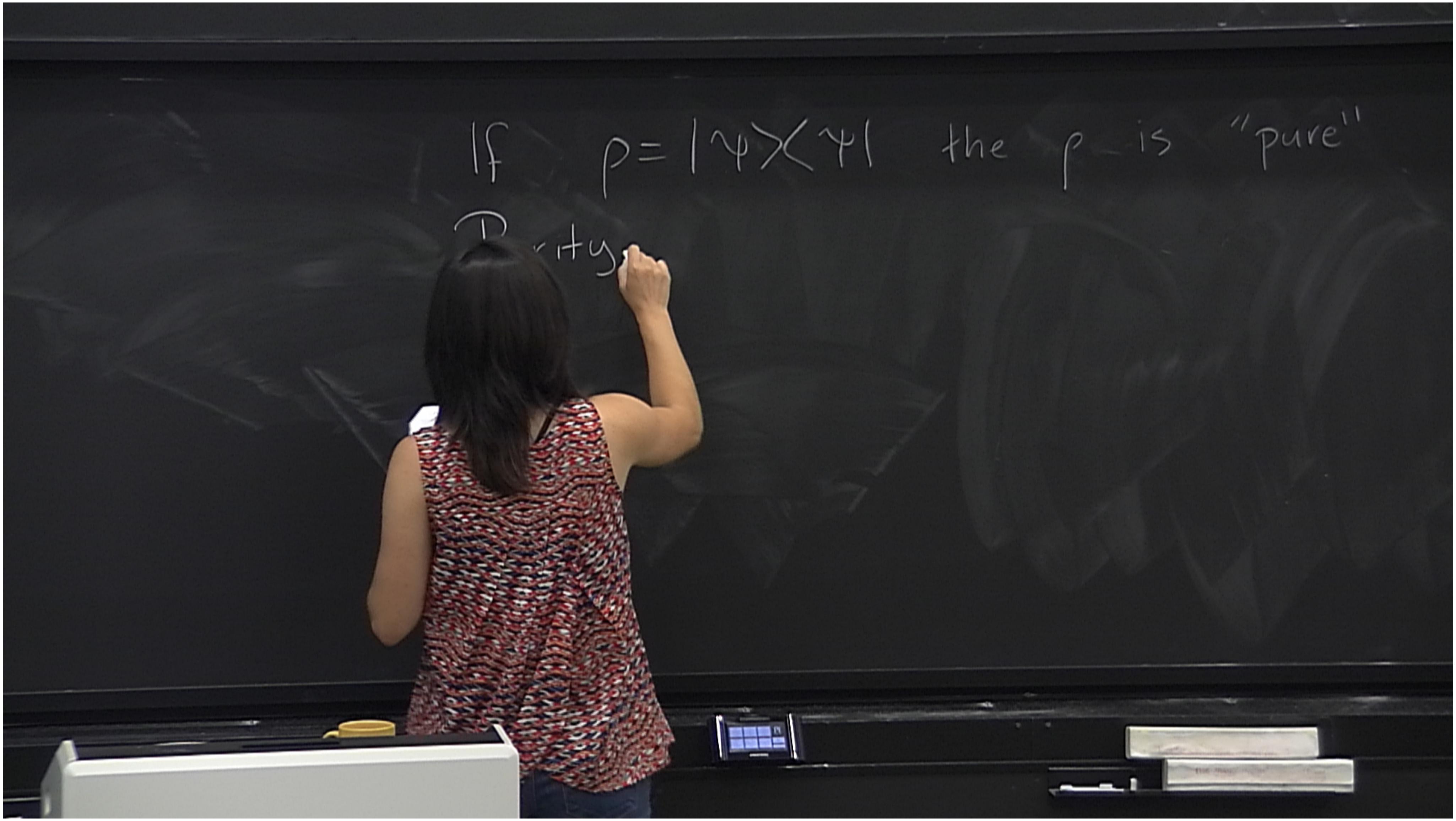
$$|s_1, m_1\rangle |s_2, m_2\rangle = \sum_{s} C^{s_1 s_2 s}_{m_1 m_2 m} |s, m\rangle$$

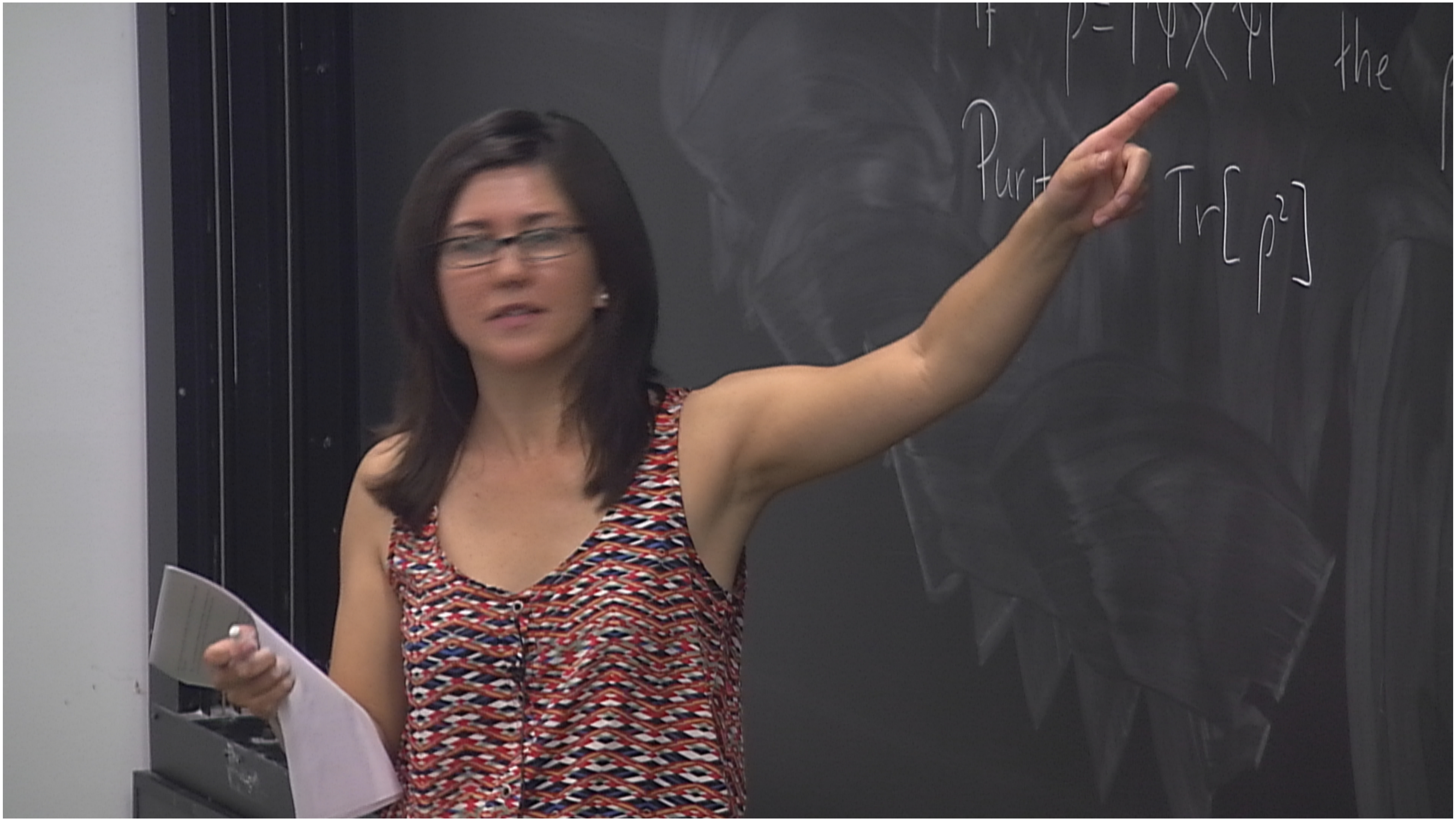

s_1
s_2
$\frac{1}{2} \times \frac{1}{2}$
$1 \times \frac{1}{2}$

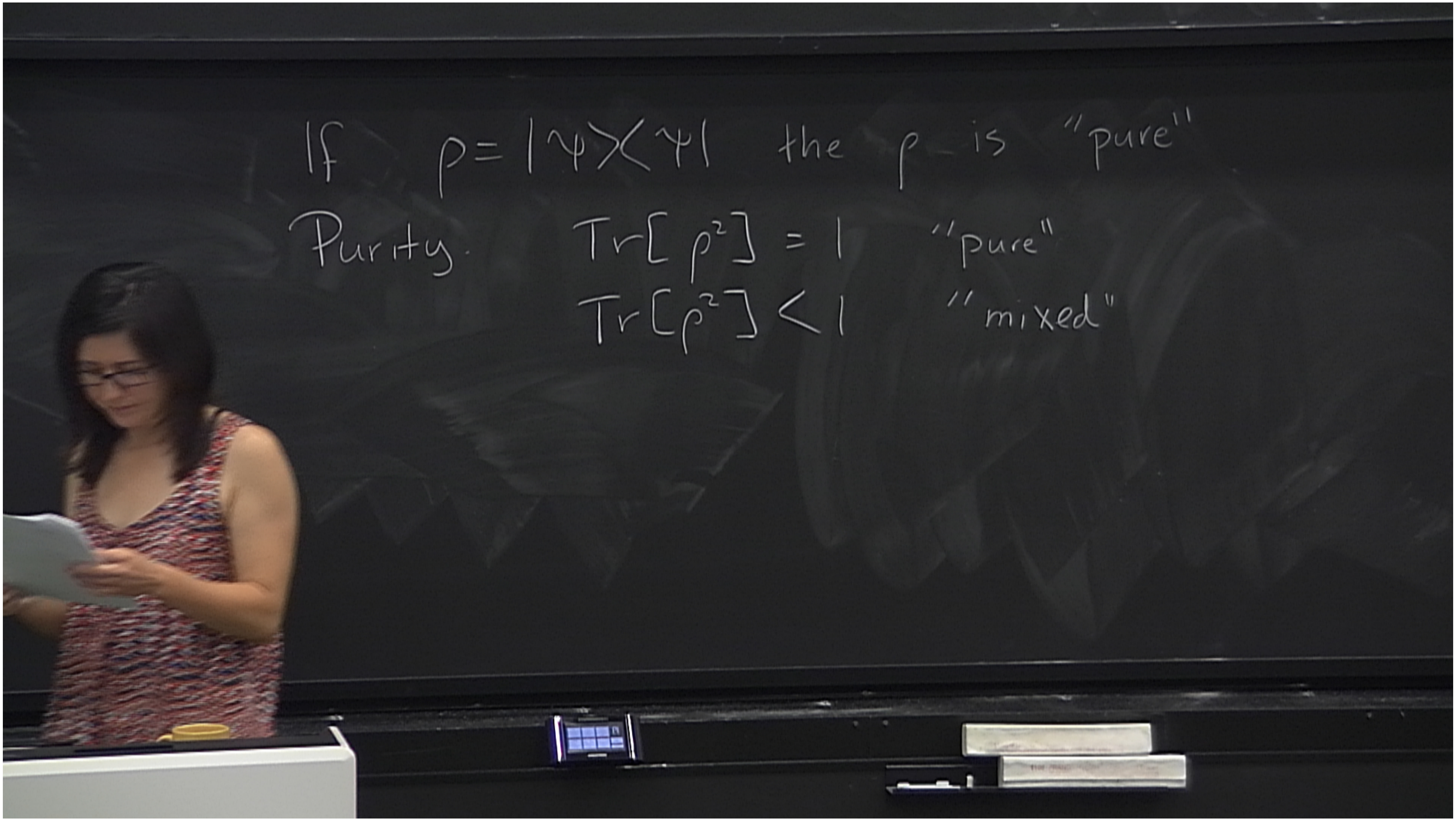


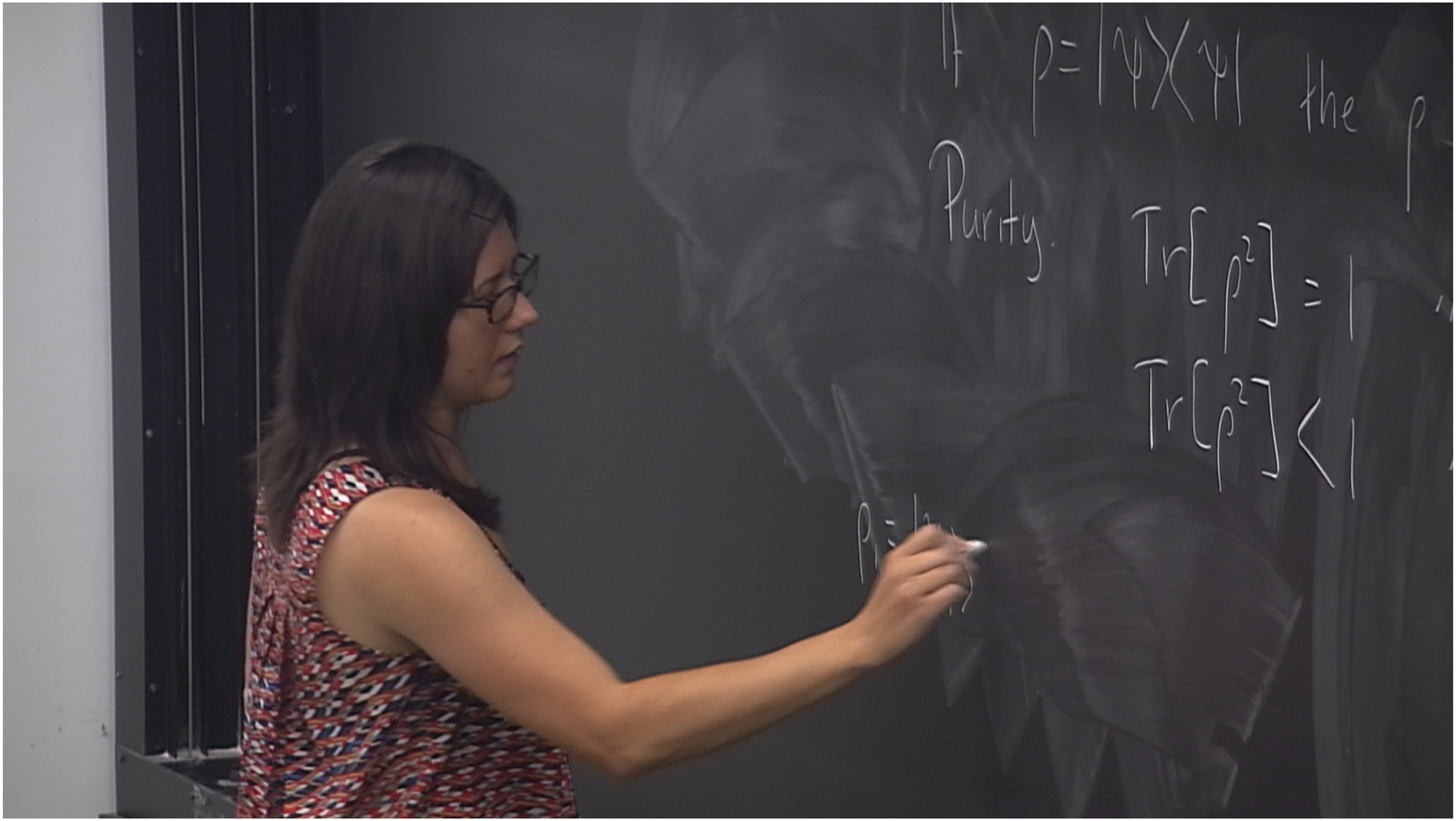
1 × ½
two spin ½ particles:
S1 = ½
S2 = ½
Basis states:
|↑⟩|↑⟩

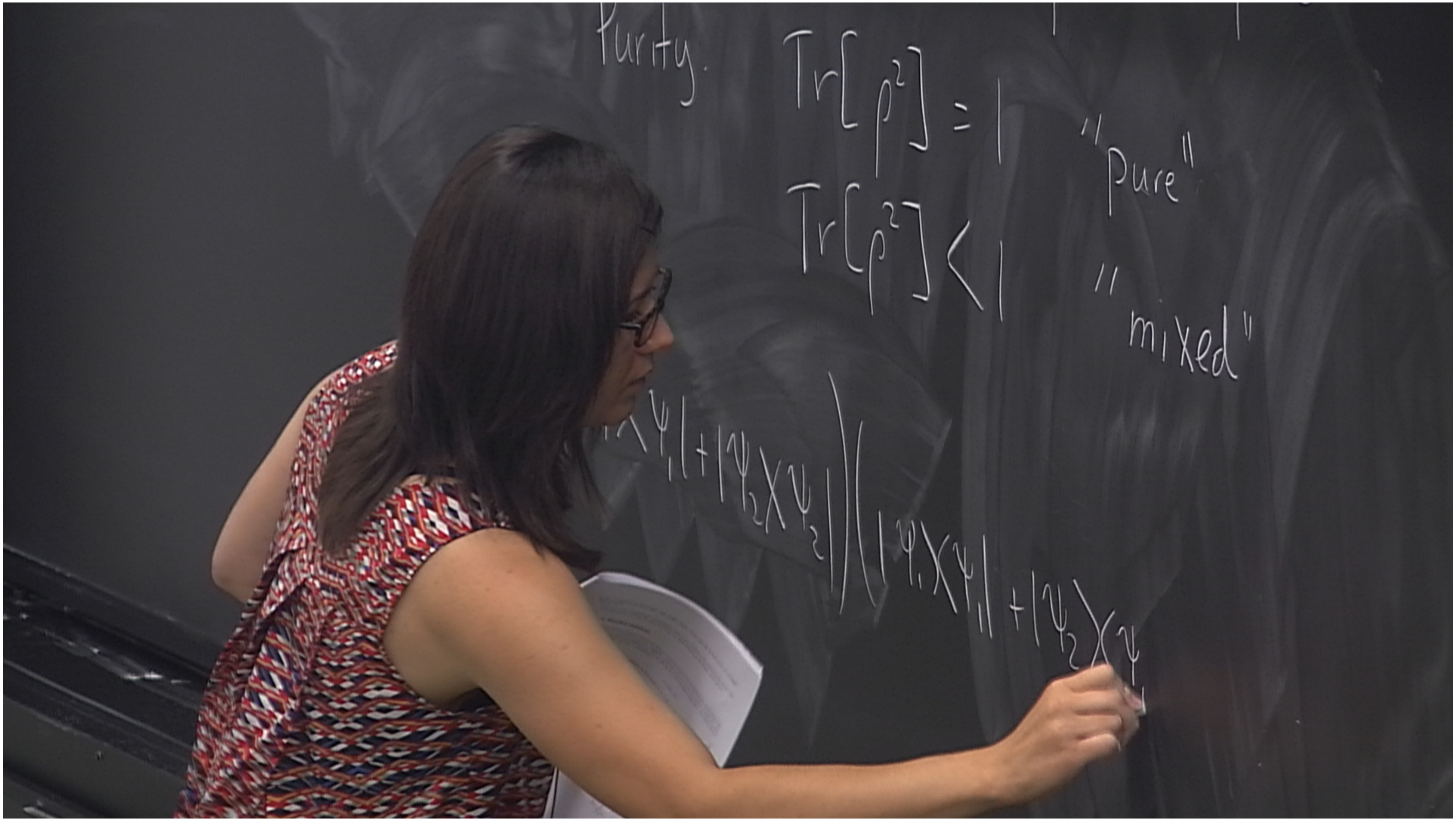

two spin ½ particles:
S₁ = ½
S₂ = ½
states:
|s,m⟩
= |↑⟩|↑⟩
Triplet
= |↓⟩|↓⟩
|0,0⟩ = 1/√2 (|↑⟩|↓⟩ − |↓⟩|↑⟩)
Singlet
Spin 1 &
|½, −½⟩ =

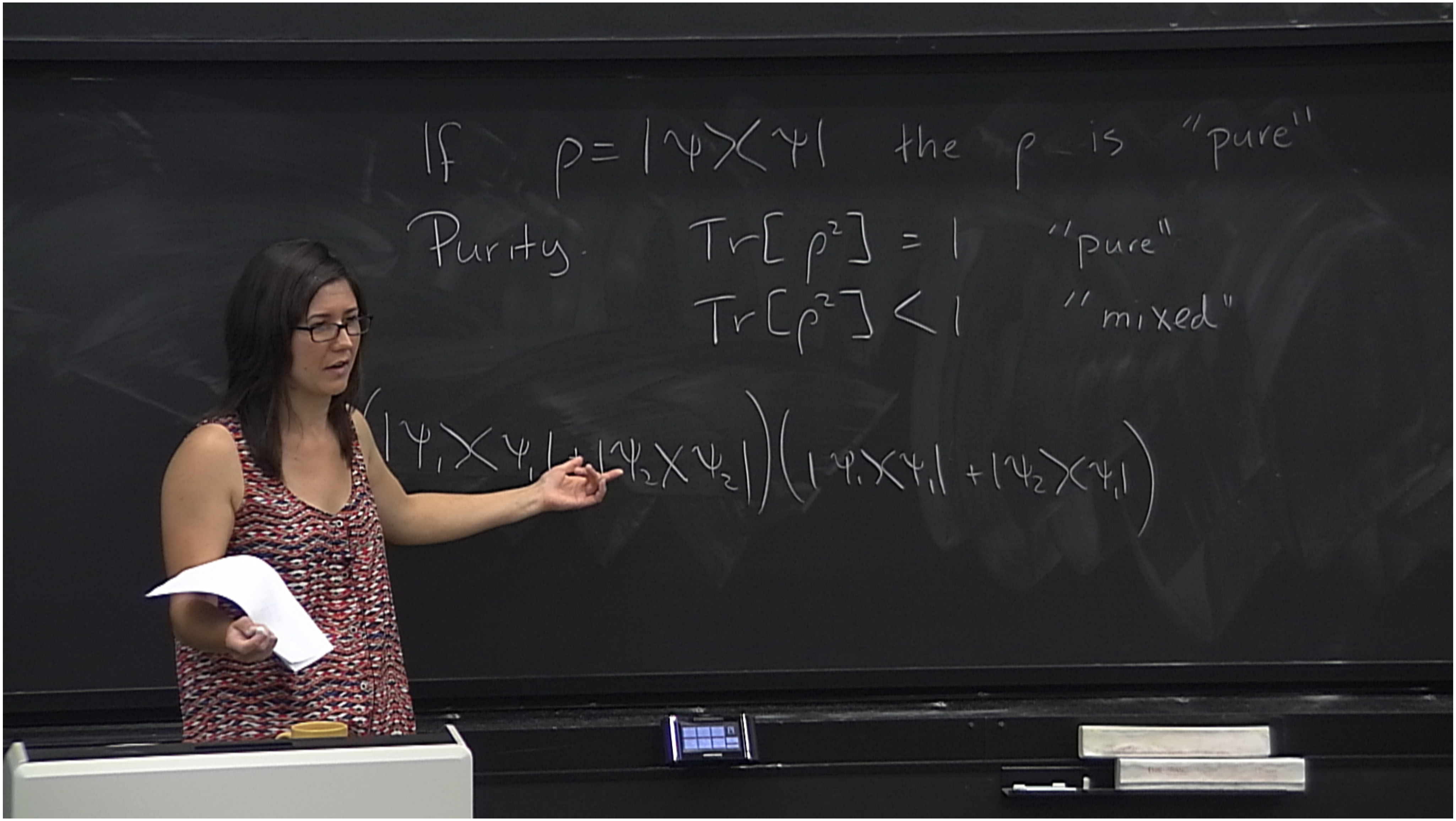

If ρ=|ψ⟩⟨ψ| the ρ is "pure"

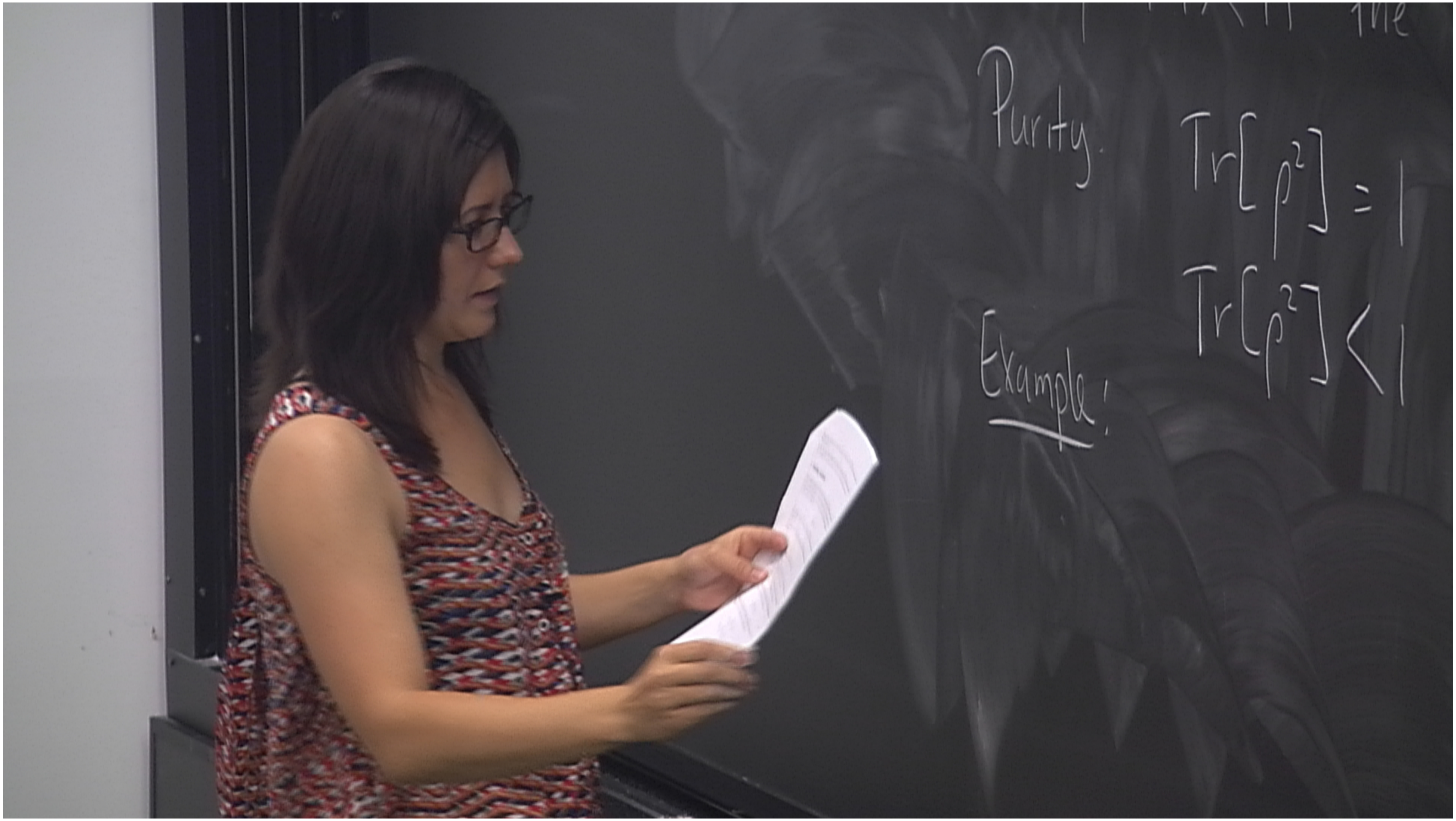

the
Tr[ρ²]

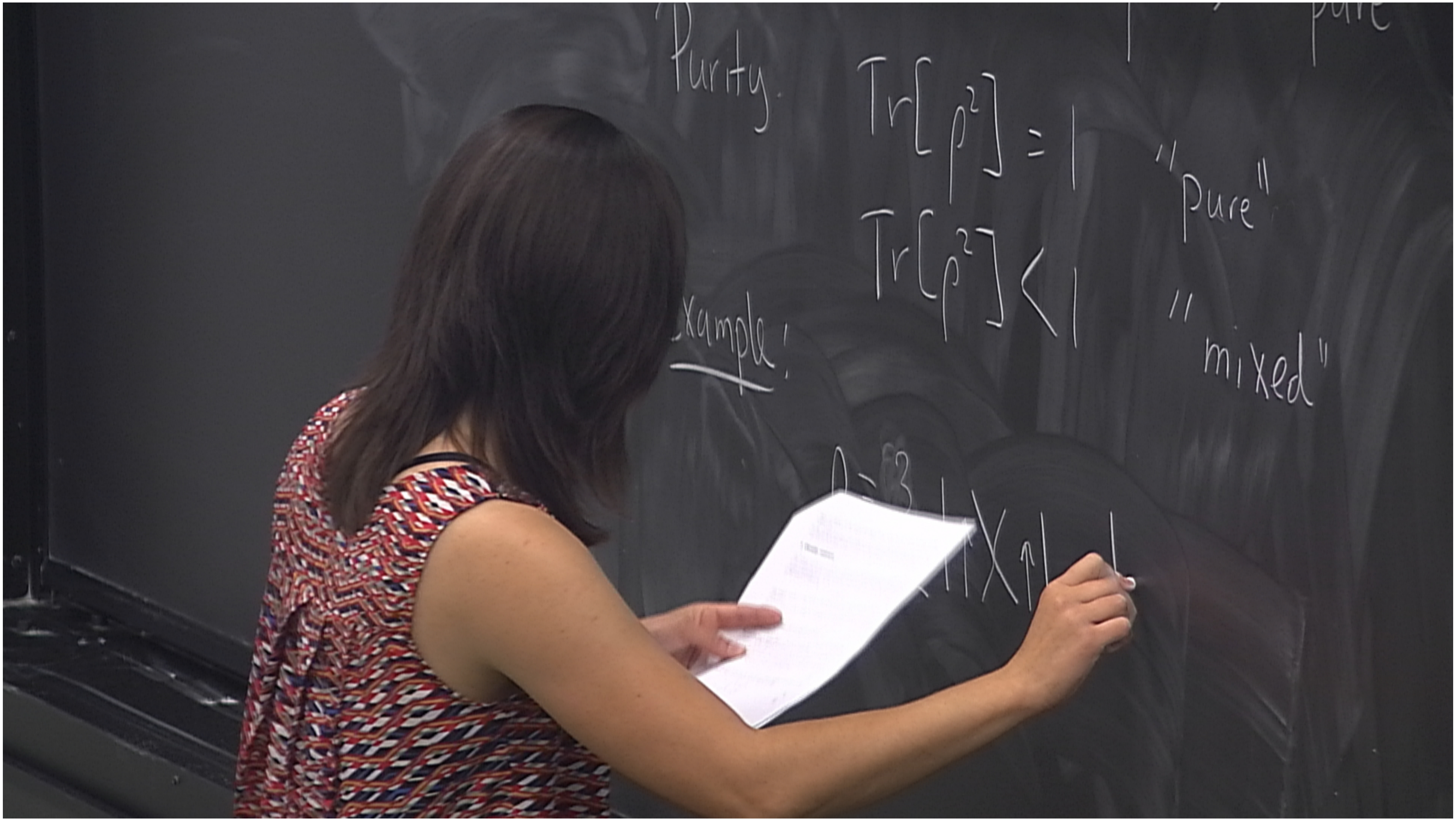

If ρ=|ψ⟩⟨ψ| the ρ is "pure"
Purity. Tr[ρ²] = 1 "pure"
Tr[ρ²] < 1 "mixed"

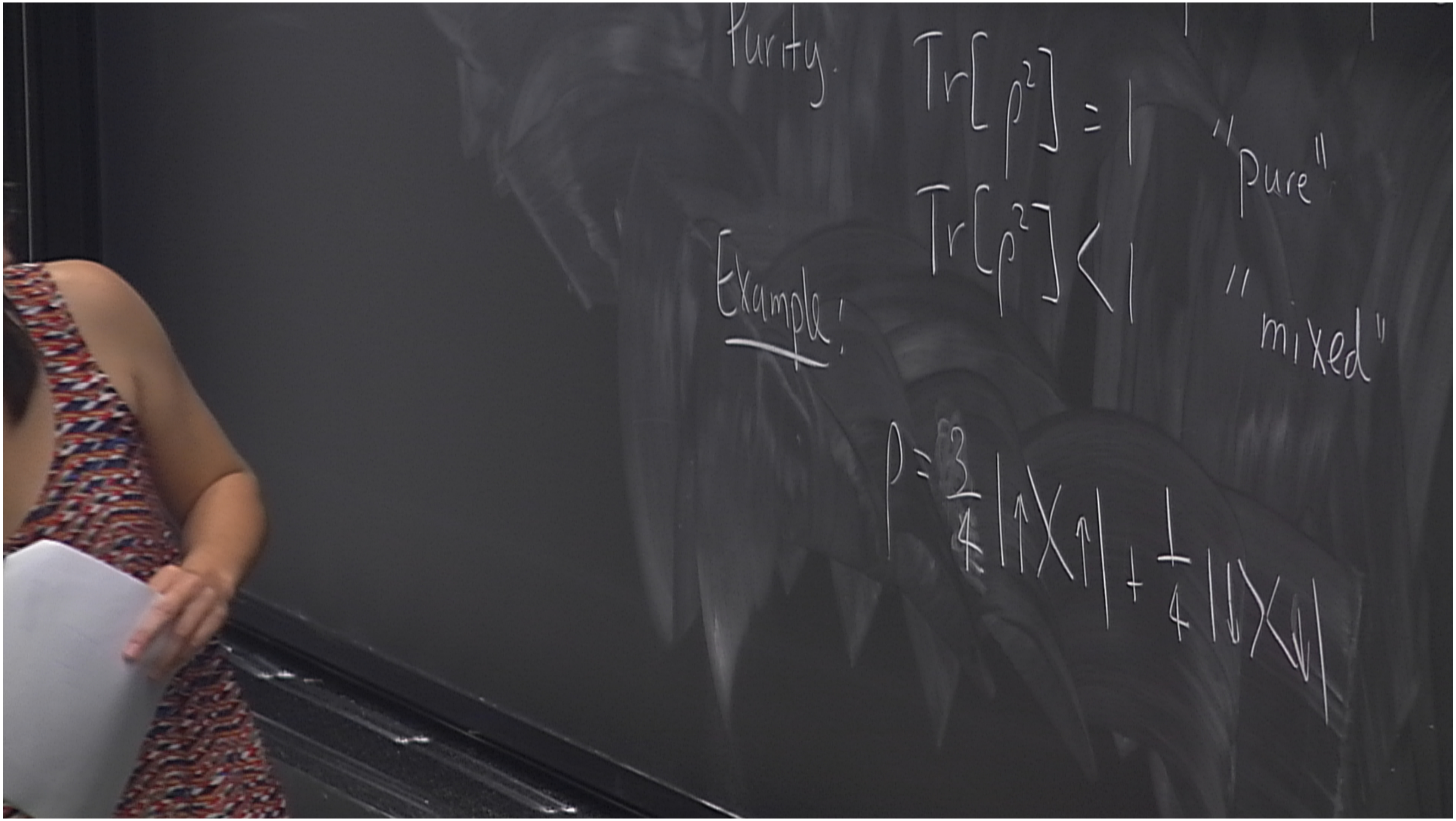

If ρ=|ψ⟩⟨ψ| the
Purity. Tr[ρ²] = 1
Tr[ρ²] < 1

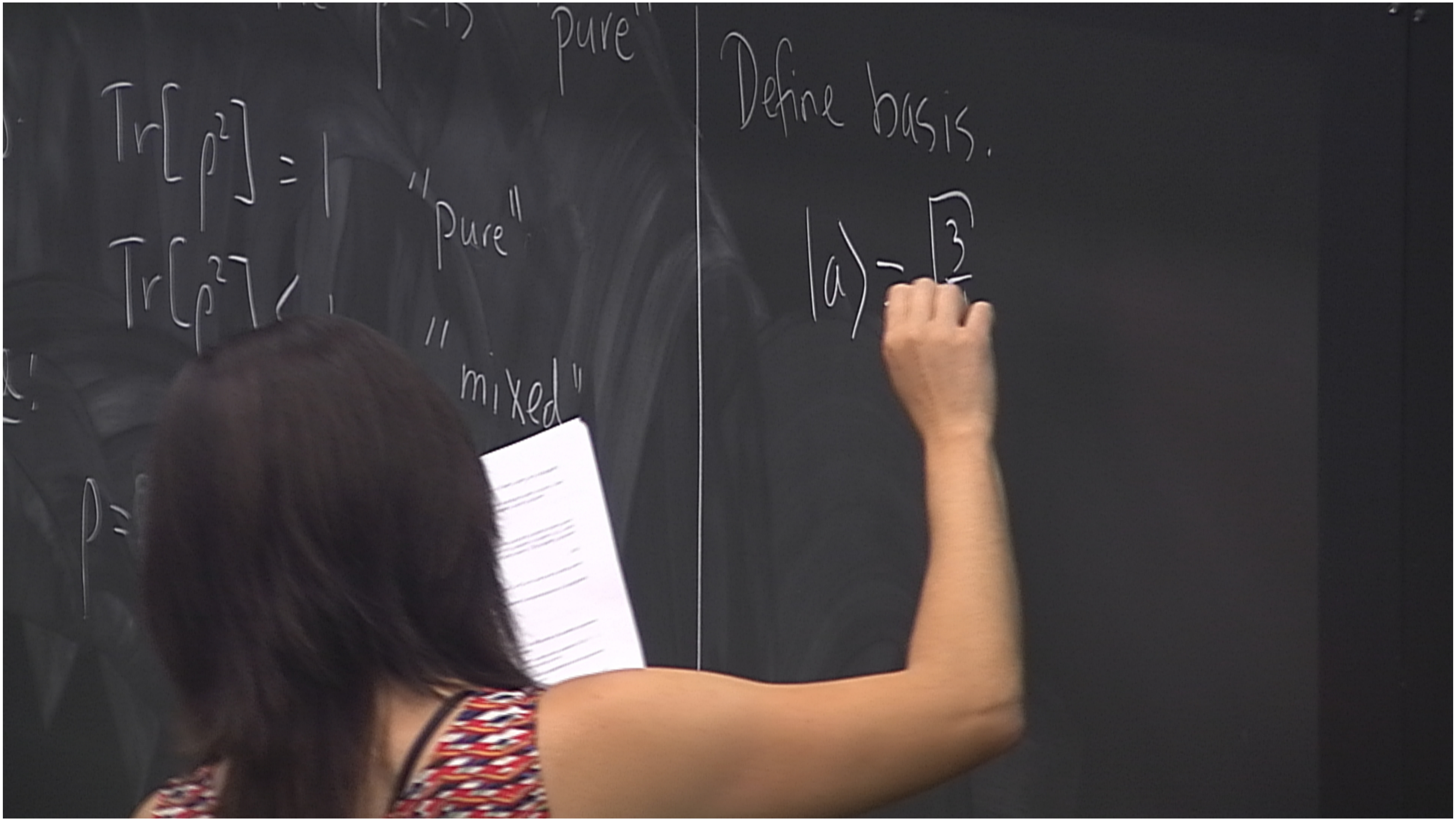

Purity:
Tr[ρ²] = 1 "pure"
Tr[ρ²] < 1 "mixed"

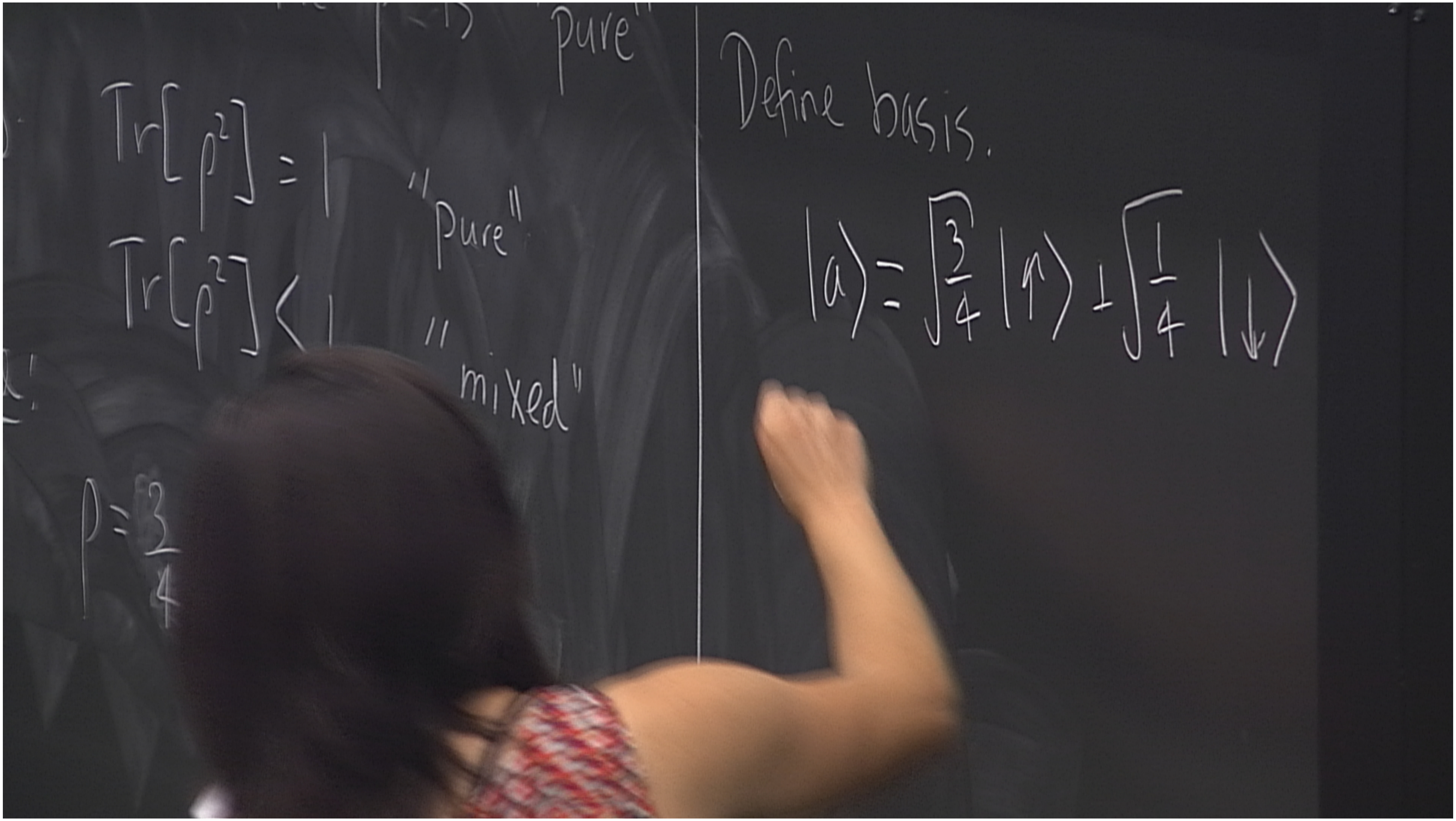

If $\rho = |\psi\rangle\langle\psi|$ the ρ is "pure"
Purity. $Tr[\rho^2] = 1$ "pure"
$Tr[\rho^2] < 1$ "mixed"
$(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|)(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_1|)$

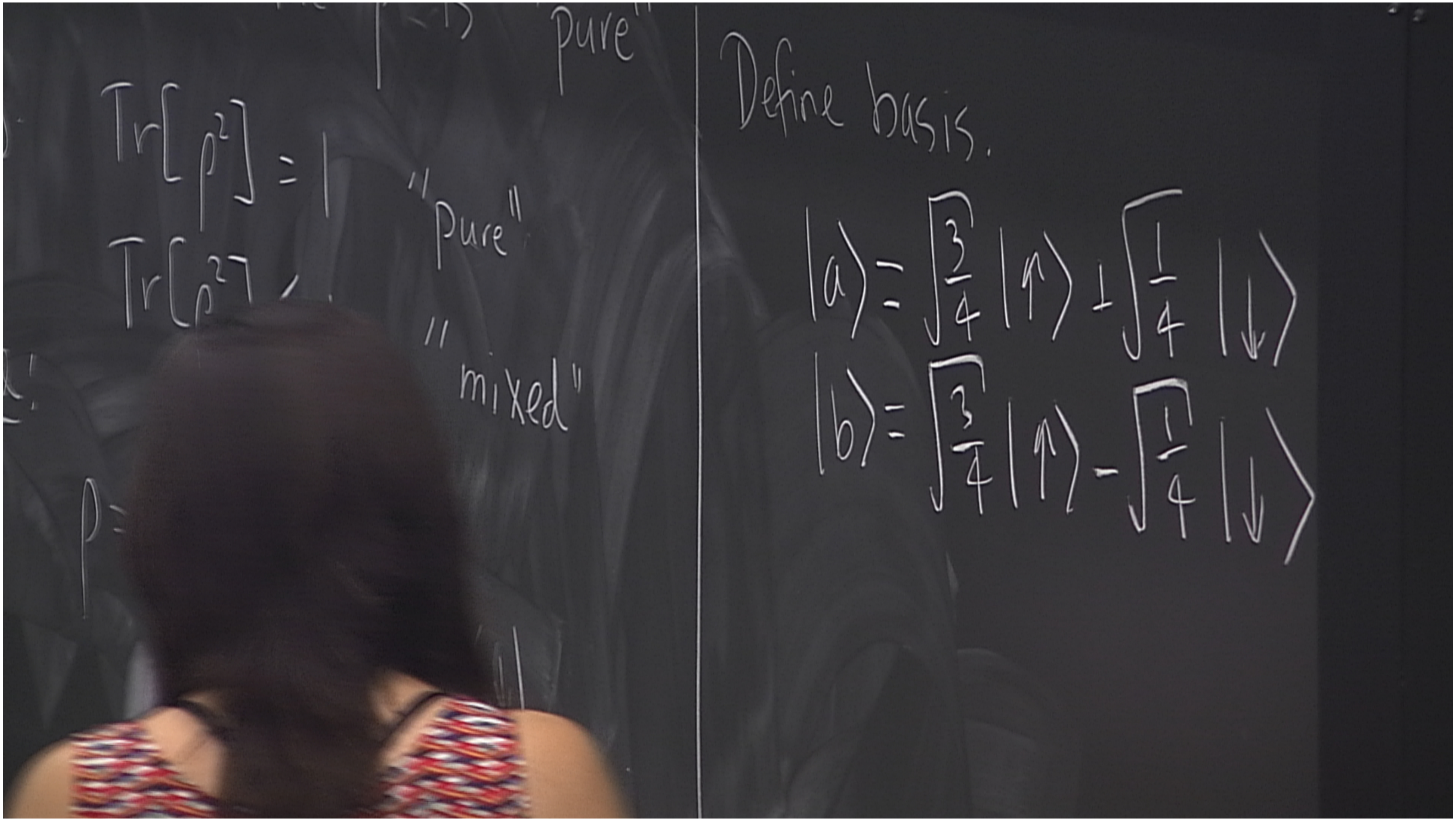

Purity:
Tr[ρ²] = 1
Tr[ρ²] < 1
Example:

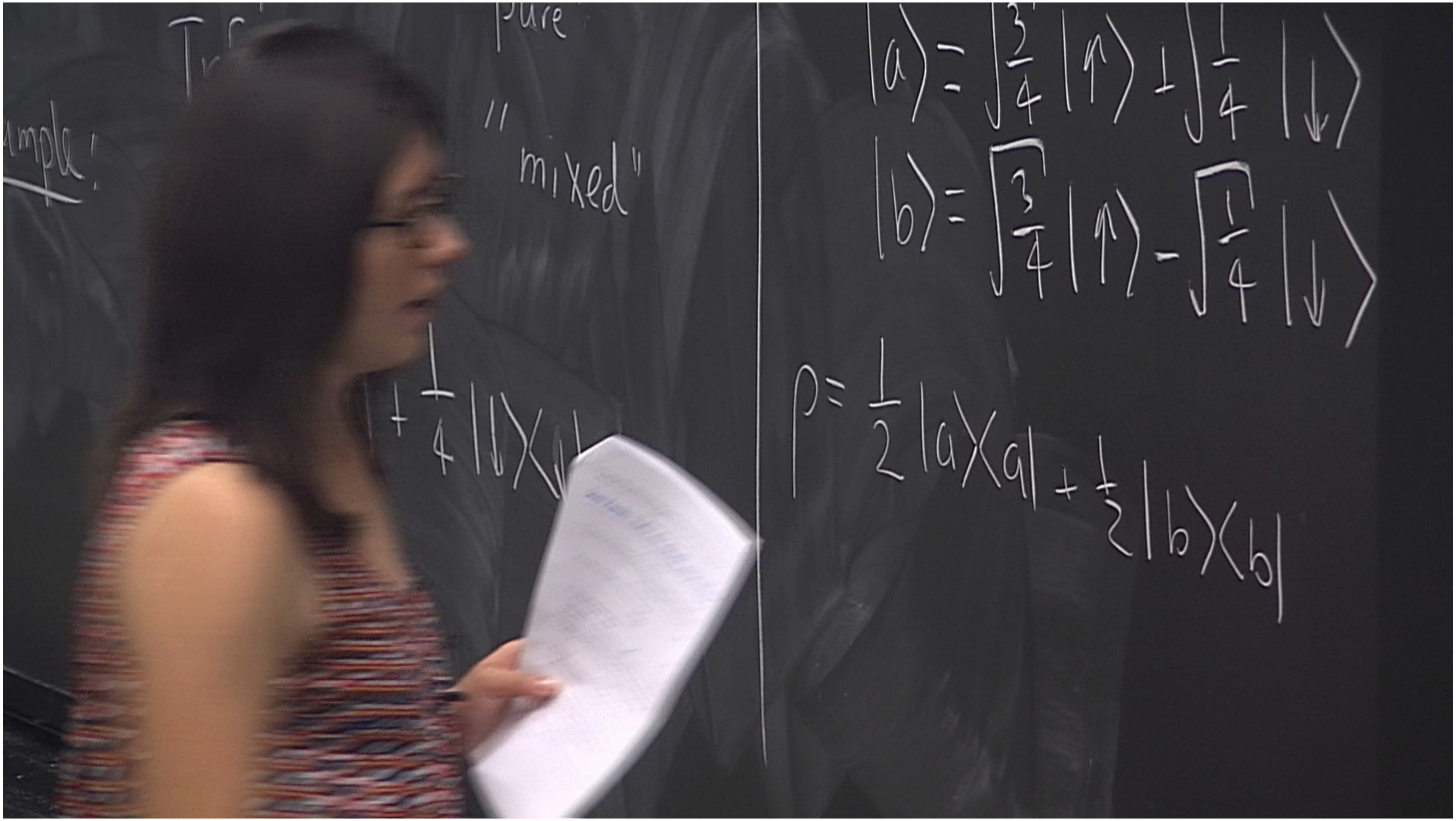

Purity:
Tr[ρ²] = 1 "pure"
Tr[ρ²] < 1 "mixed"
Example:


Purity:
$\mathrm{Tr}[\rho^2] = 1$ "pure"
$\mathrm{Tr}[\rho^2] < 1$ "mixed"
Example:
$\rho = \frac{3}{4}|\uparrow\rangle\langle\uparrow| + \frac{1}{4}|\downarrow\rangle\langle\downarrow|$

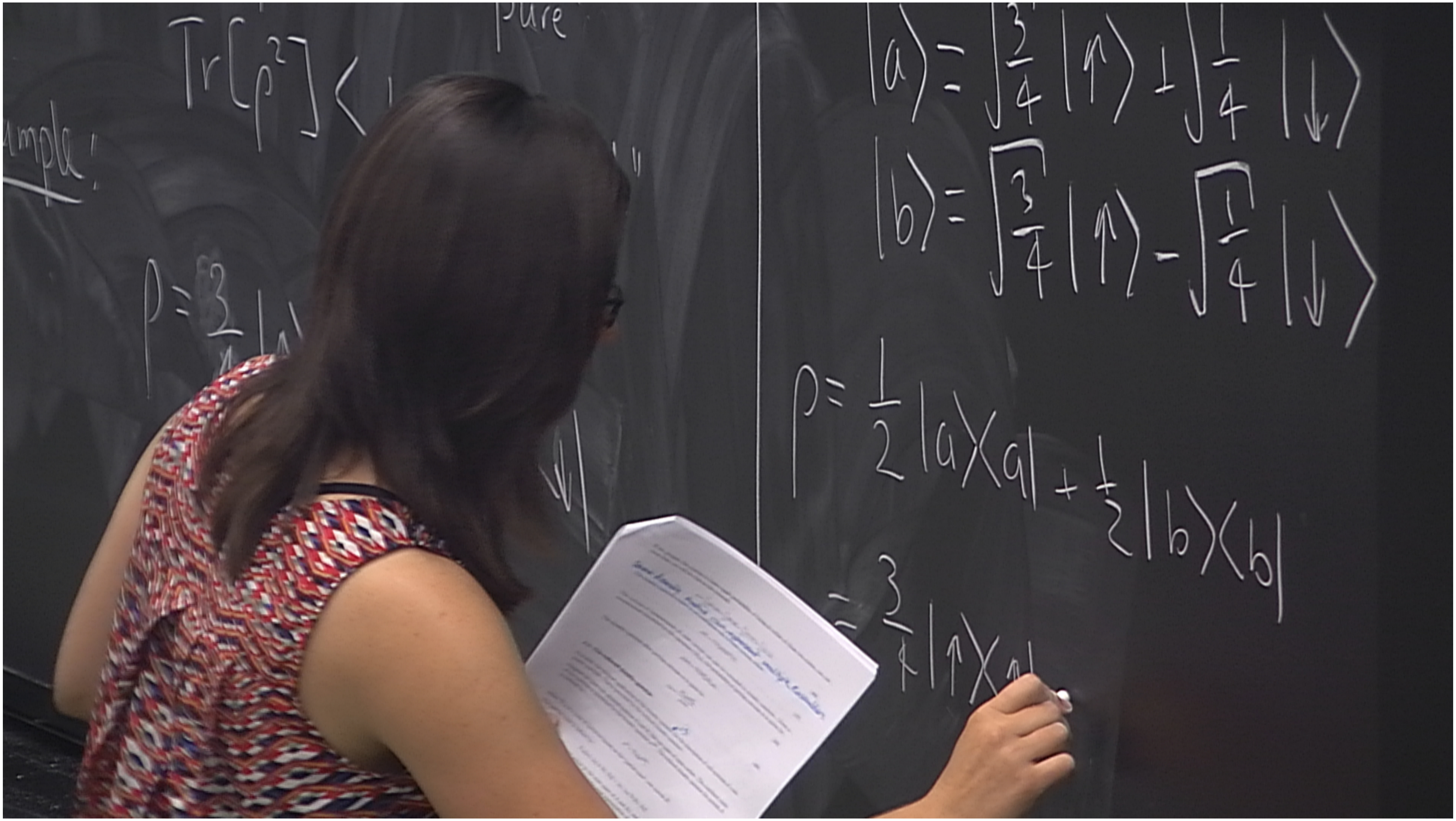

"pure"
Tr[ρ²] = 1 "pure"
Tr[ρ²] < 1 "mixed"
Define basis.

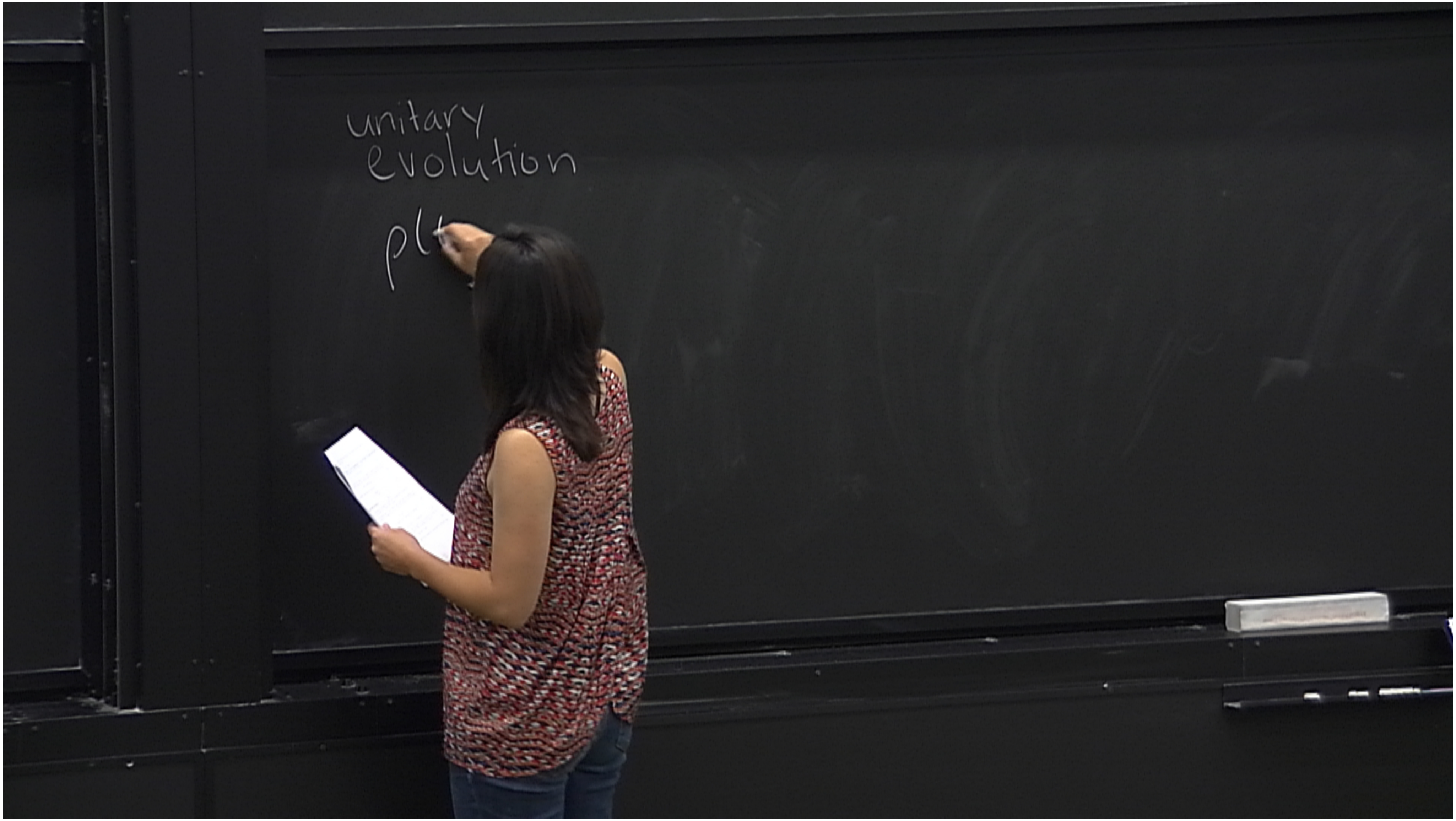

"pure"
Tr[ρ²] = 1 "pure"
Tr[ρ²] < 1 "mixed"
ρ = 3/4
Define basis.
|a⟩ = √(3/4) |↑⟩ ± √(1/4) |↓⟩

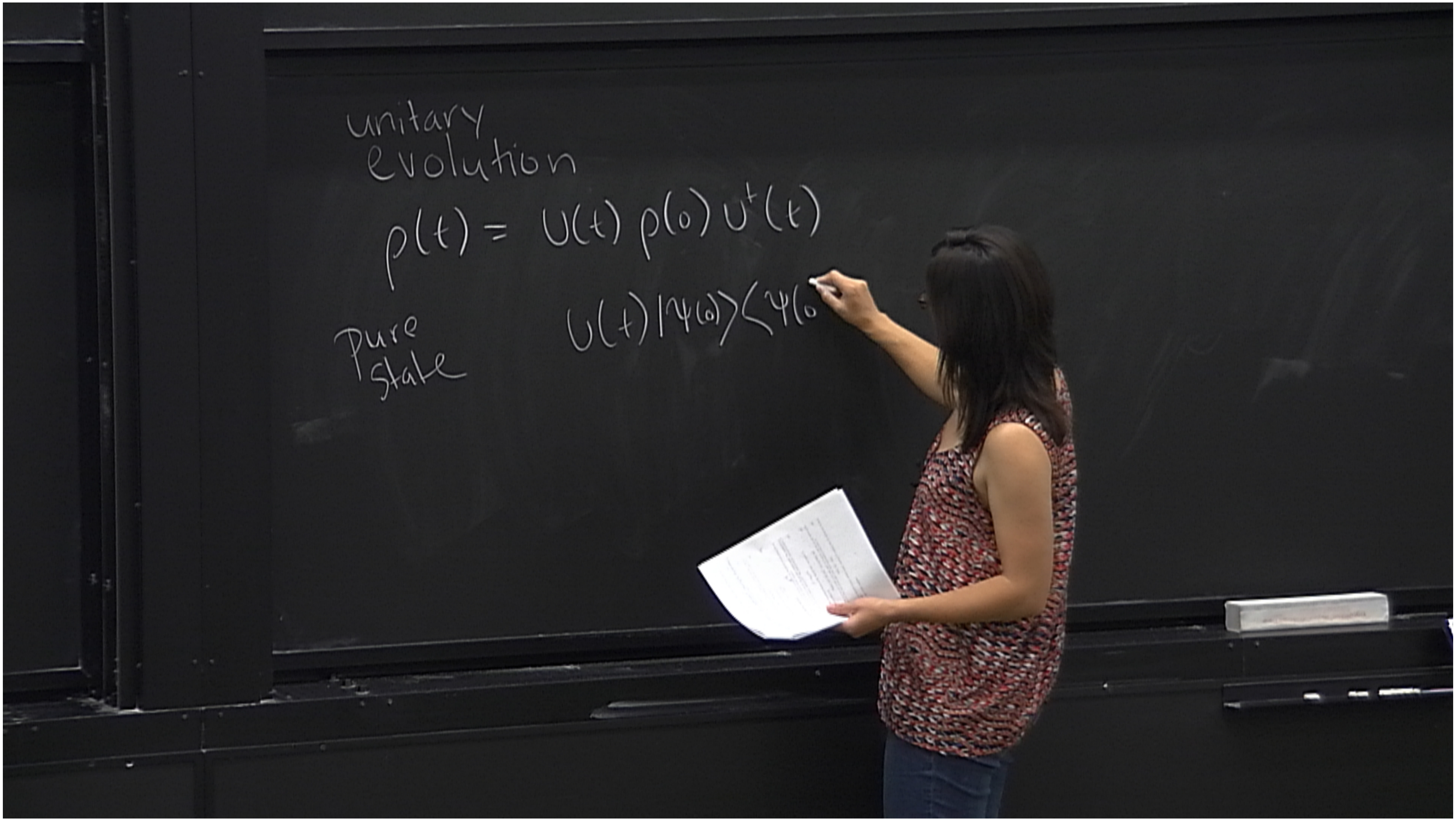

"pure"
$\mathrm{Tr}[\rho^2] = 1$ "pure"
"mixed"
Define basis.
$|a\rangle = \sqrt{\frac{3}{4}}\,|\uparrow\rangle + \sqrt{\frac{1}{4}}\,|\downarrow\rangle$
$|b\rangle = \sqrt{\frac{3}{4}}\,|\uparrow\rangle - \sqrt{\frac{1}{4}}\,|\downarrow\rangle$

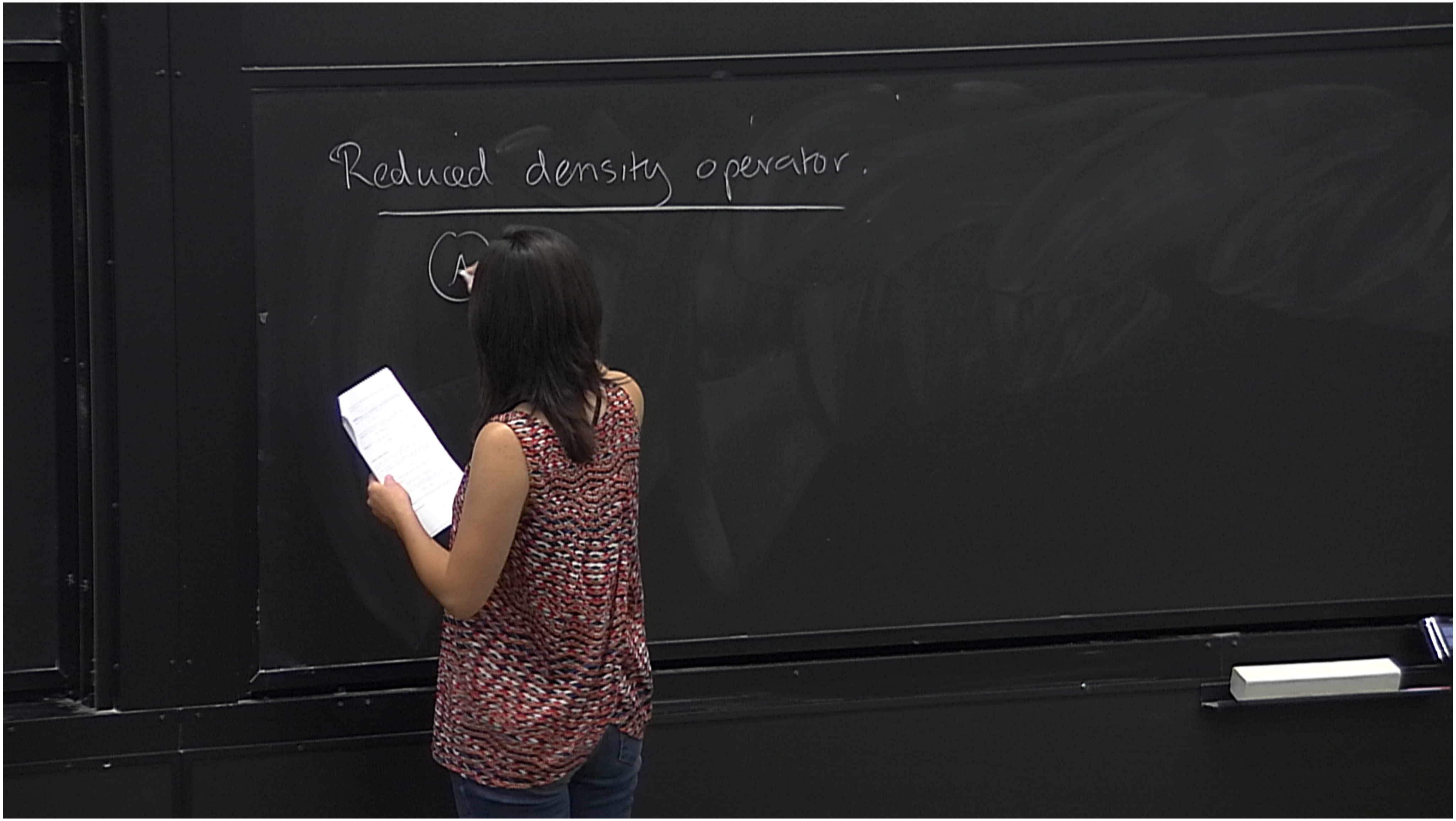


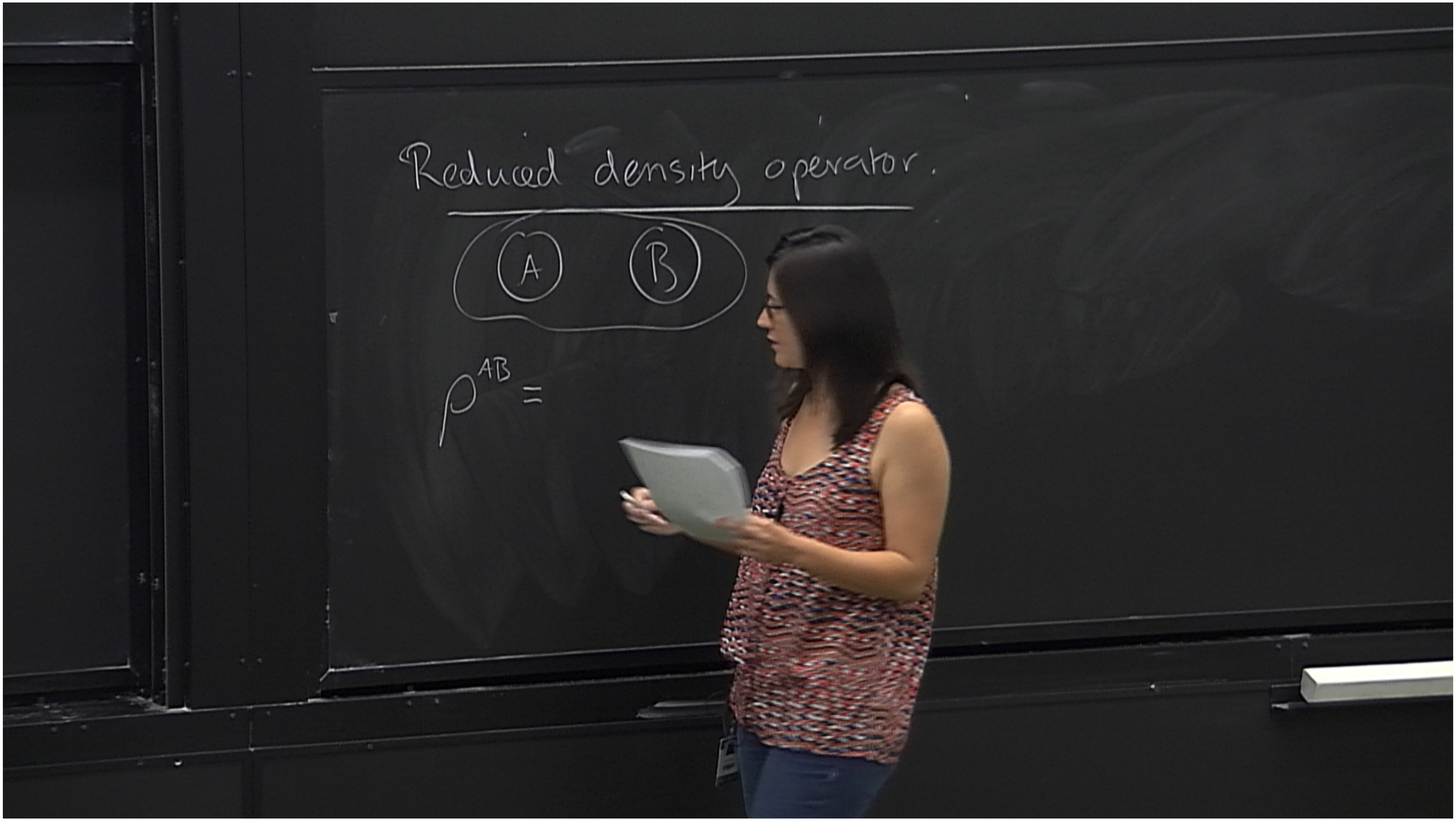
"pure"
"mixed"
$|a\rangle = \sqrt{\frac{3}{4}}|\uparrow\rangle + \sqrt{\frac{1}{4}}|\downarrow\rangle$
$|b\rangle = \sqrt{\frac{3}{4}}|\uparrow\rangle - \sqrt{\frac{1}{4}}|\downarrow\rangle$
$\rho = \frac{1}{2}|a\rangle\langle a| + \frac{1}{2}|b\rangle\langle b|$

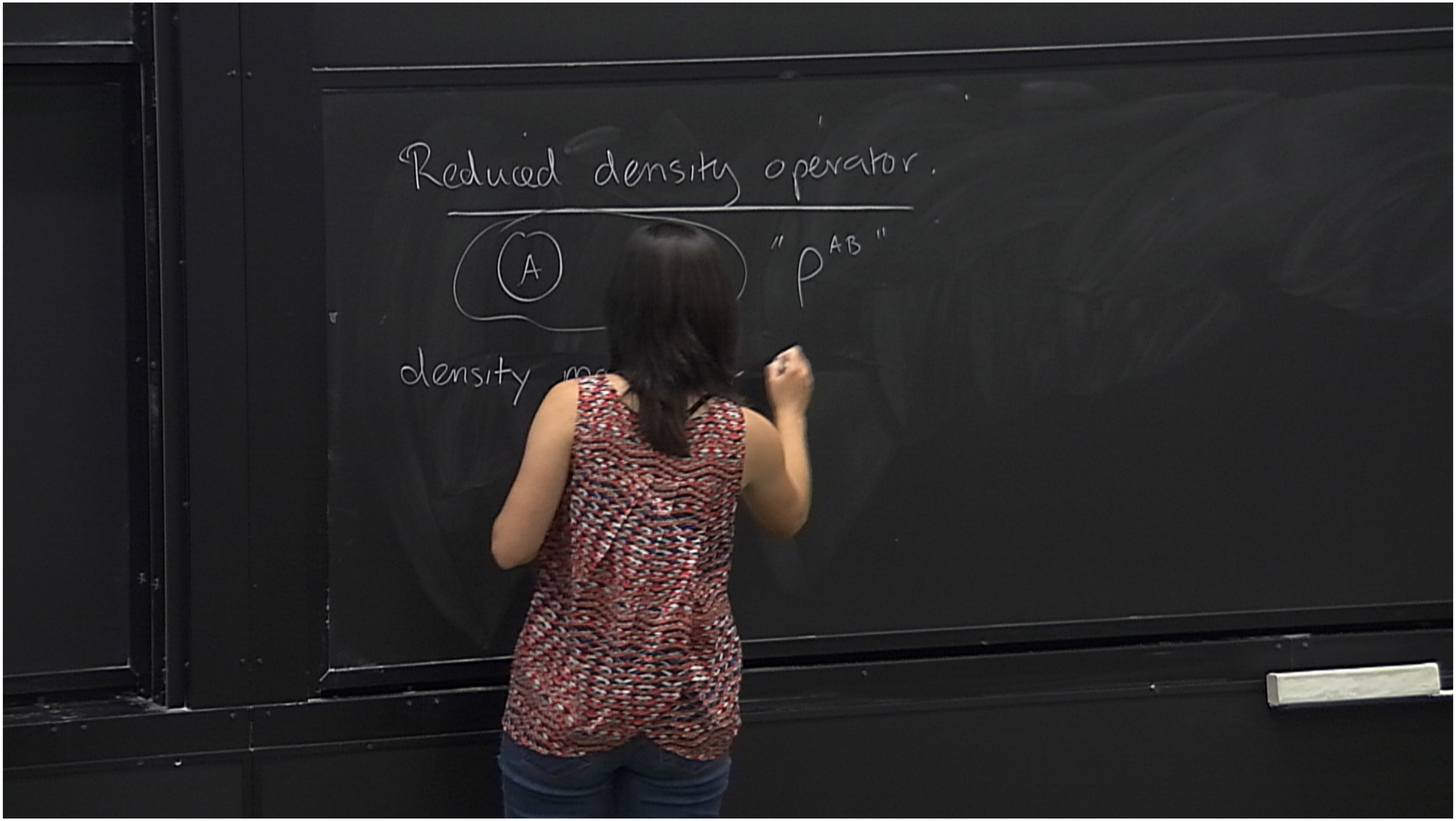

pure
Tr[ρ²] <
ample!
|a⟩ = √(3/4) |↑⟩ + √(1/4) |↓⟩
|b⟩ = √(3/4) |↑⟩ − √(1/4) |↓⟩
ρ = 1/2 |a⟩⟨a| + 1/2 |b⟩⟨b|
= 3/4 |↑⟩⟨↑|

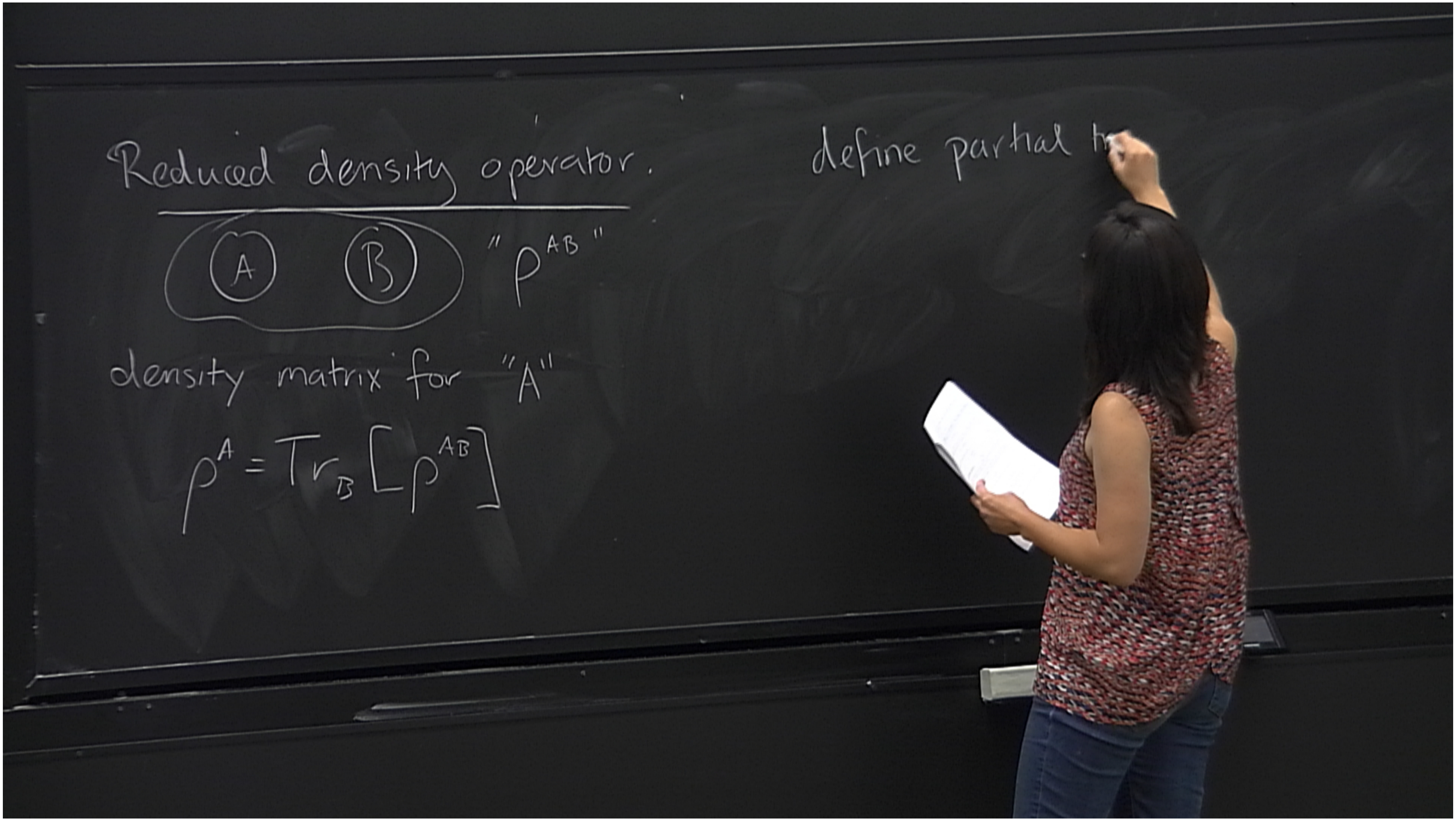

unitary
evolution


unitary evolution
ρ(t) = U(t) ρ(0) U†(t)
pure state
U(t)|ψ(0)⟩⟨ψ(0

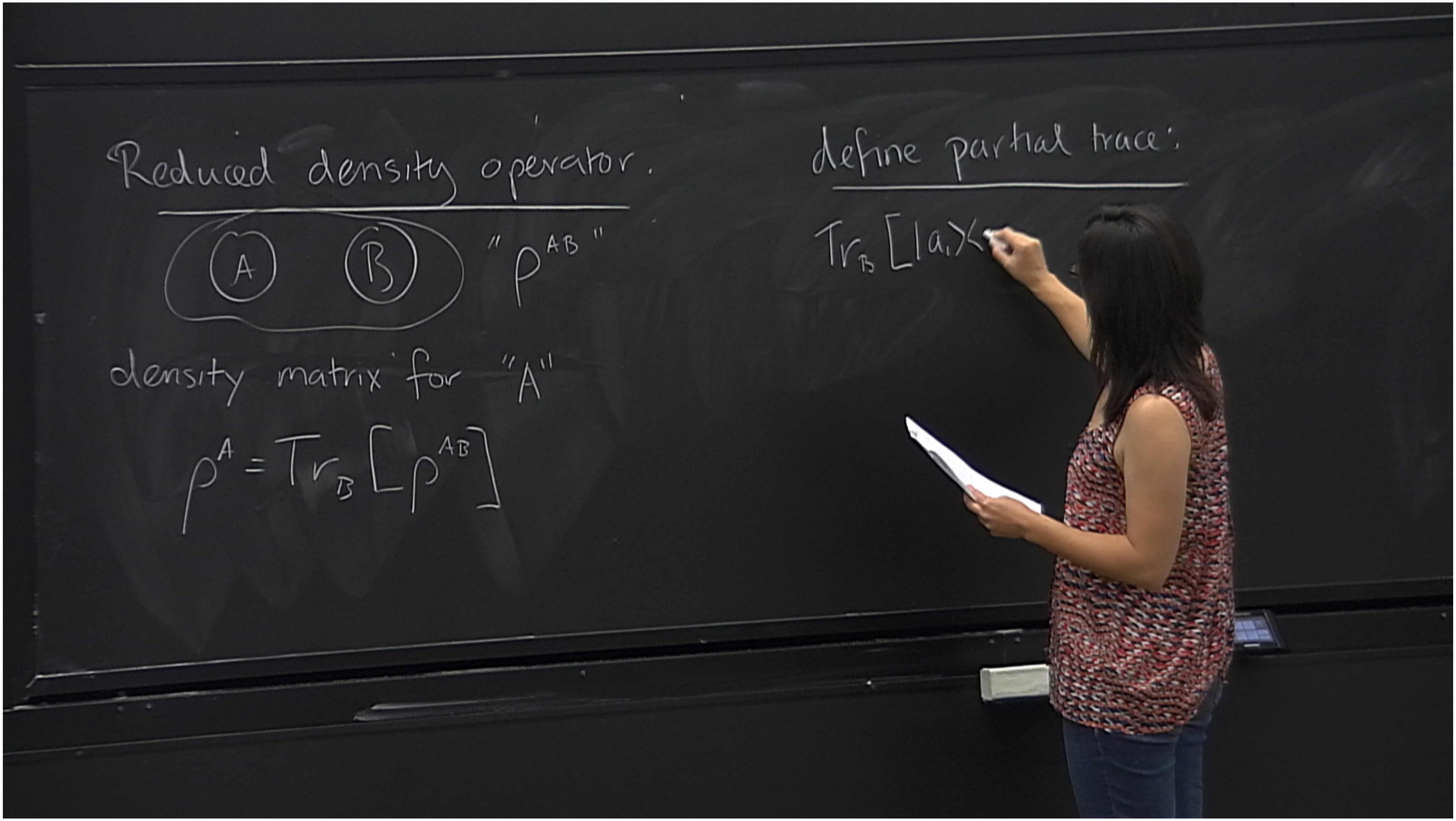

Reduced density operator.

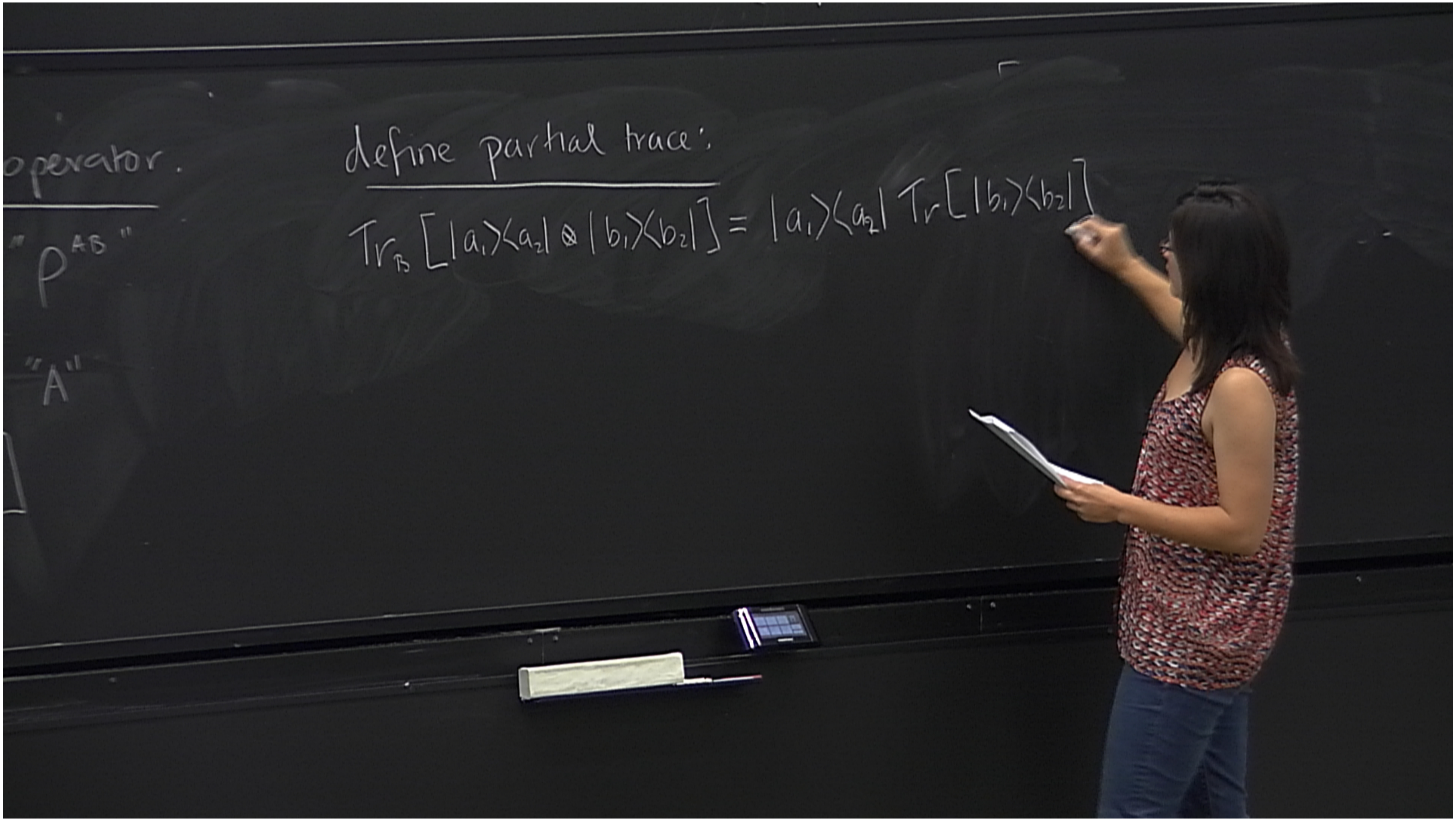

Reduced density operator.
A
B
ρ^AB =

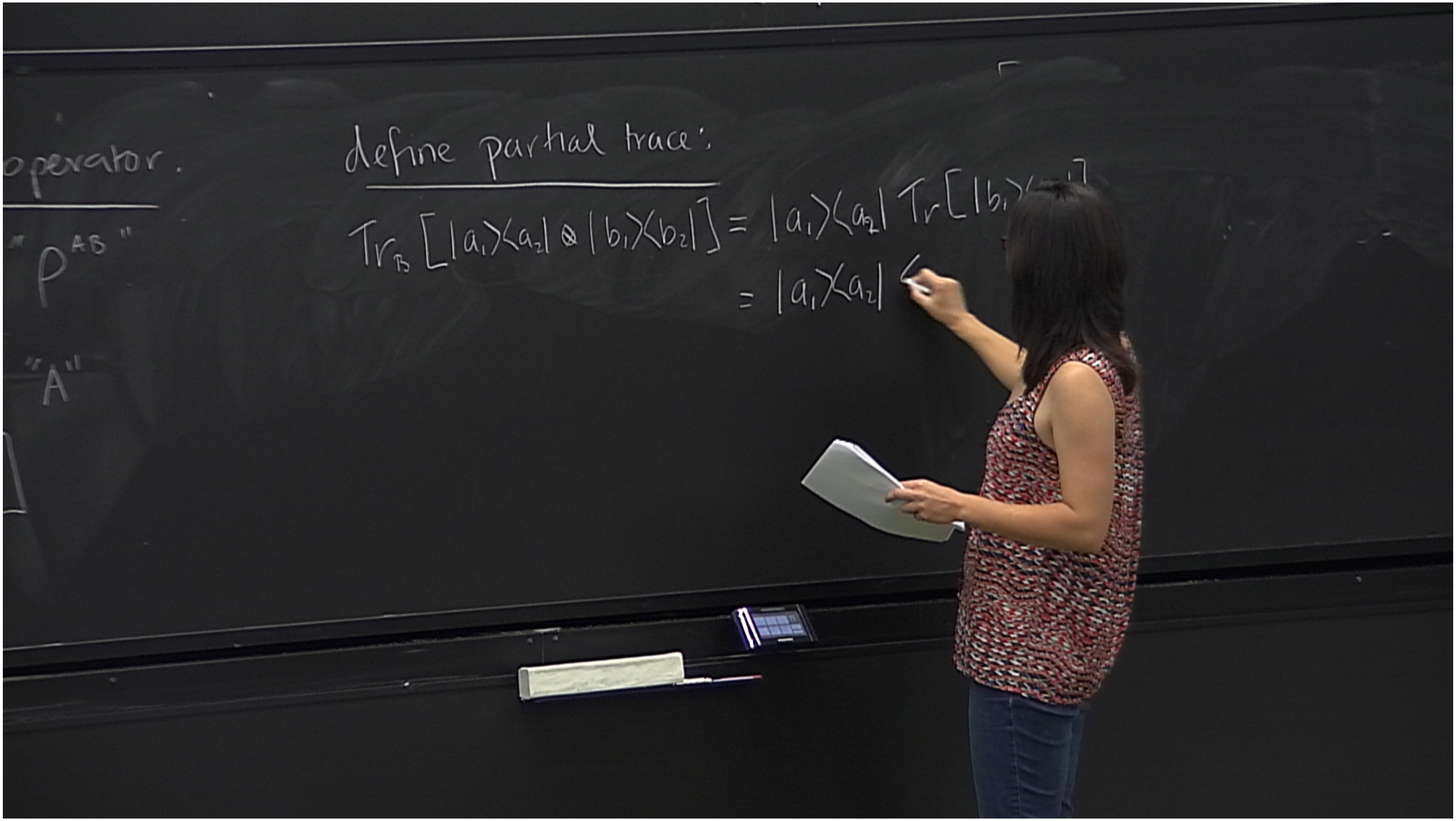

Reduced density operator.
A
" ρ^AB "
density

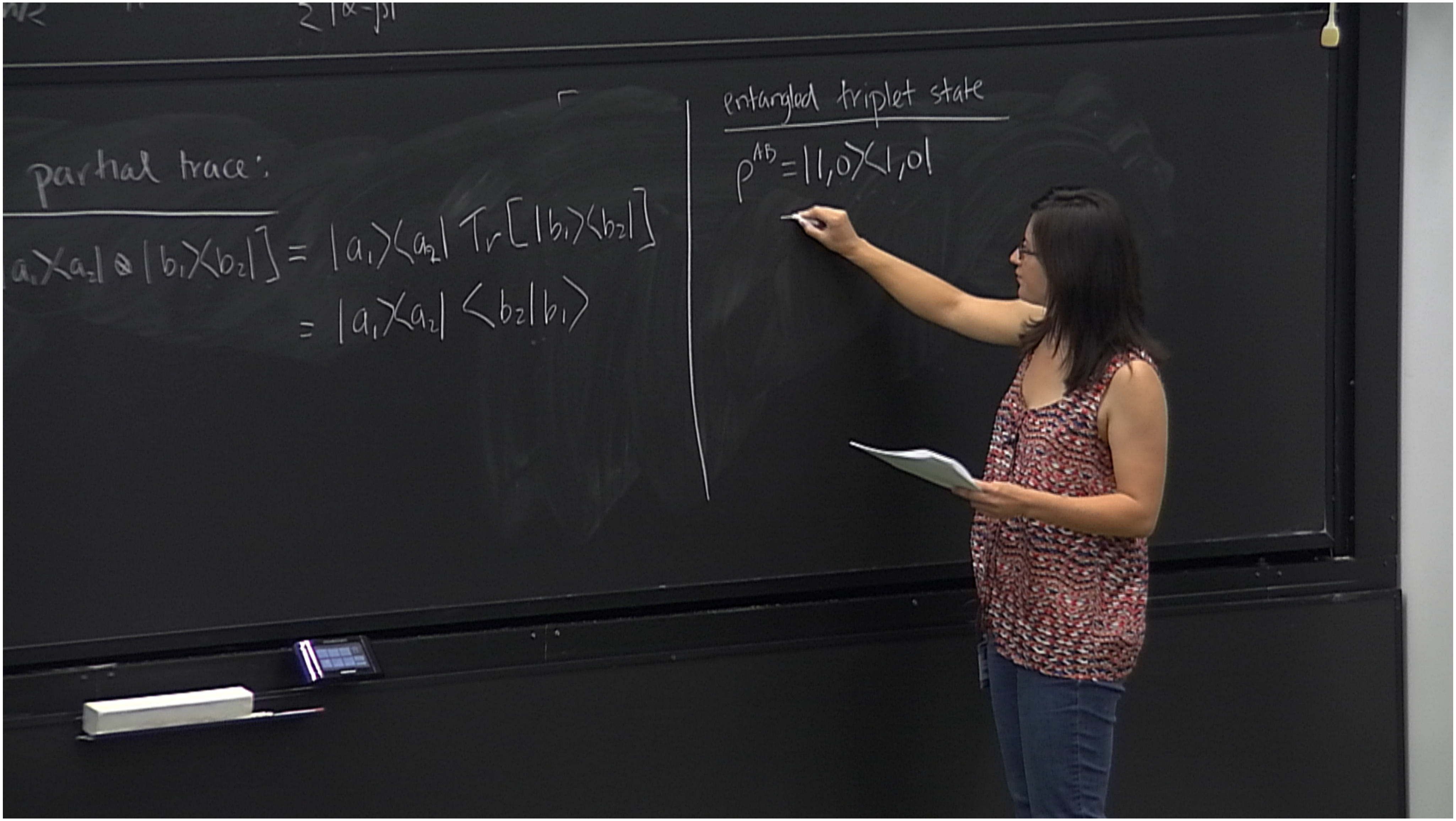


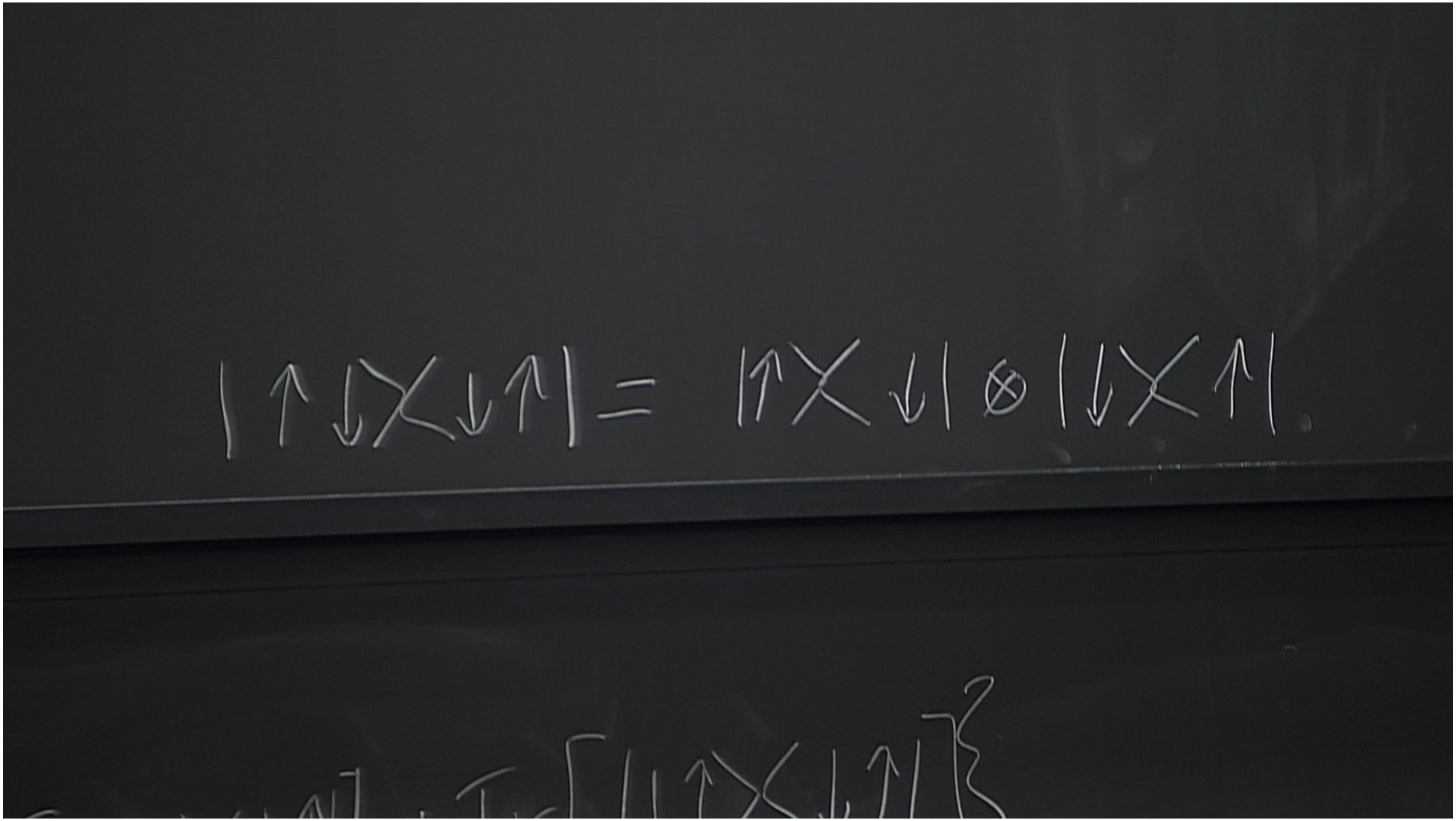
Reduced density operator.
A
B
"ρ^AB"
density matrix for "A"
ρ^A = Tr_B [ρ^AB]
define partial

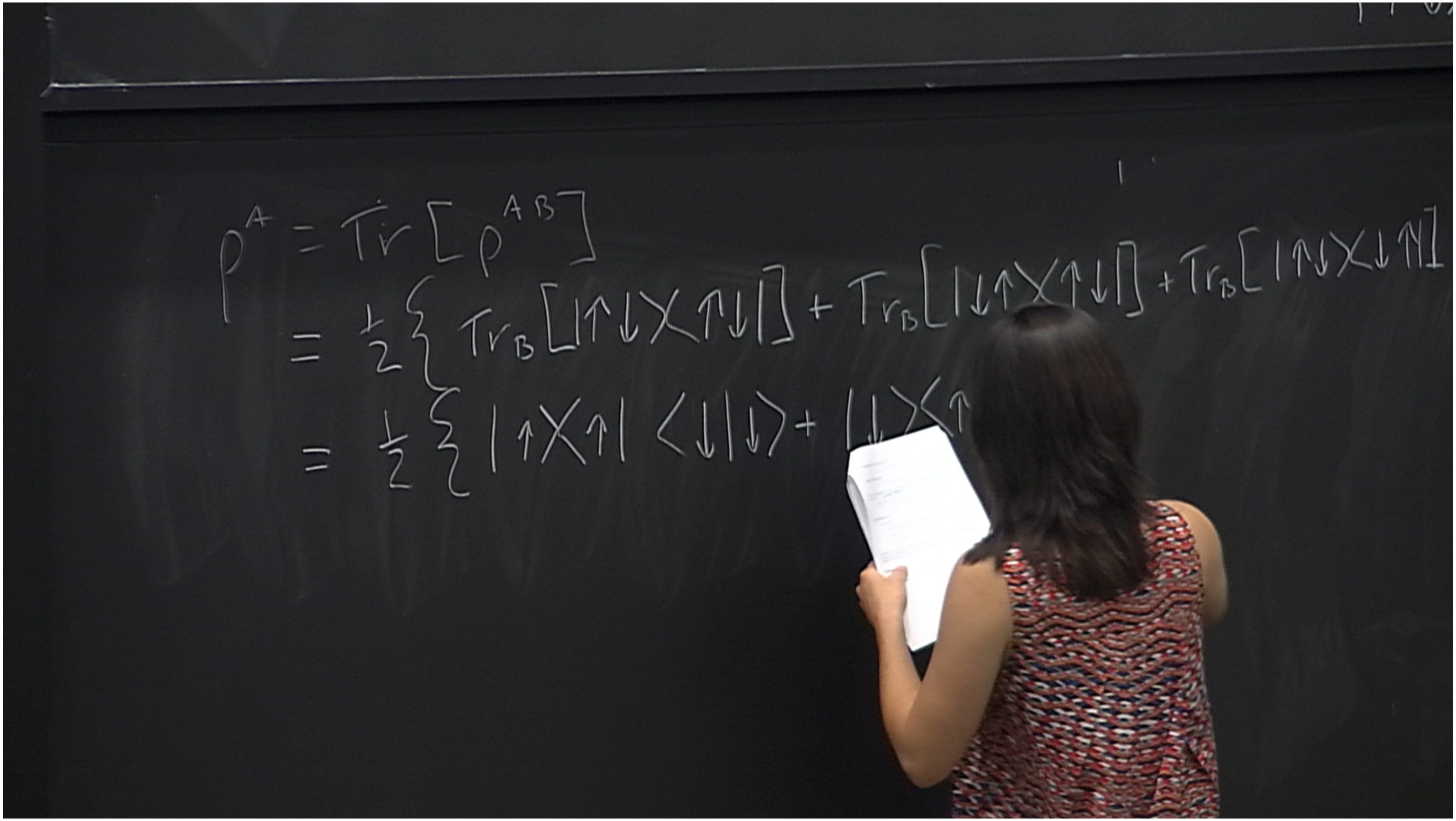

Reduced density operator.
A
B
"ρ^{AB}"
density matrix for "A"
$\rho^{A} = Tr_{B}[\rho^{AB}]$
define partial trace:
$Tr_{B}[|a_i\rangle\langle$

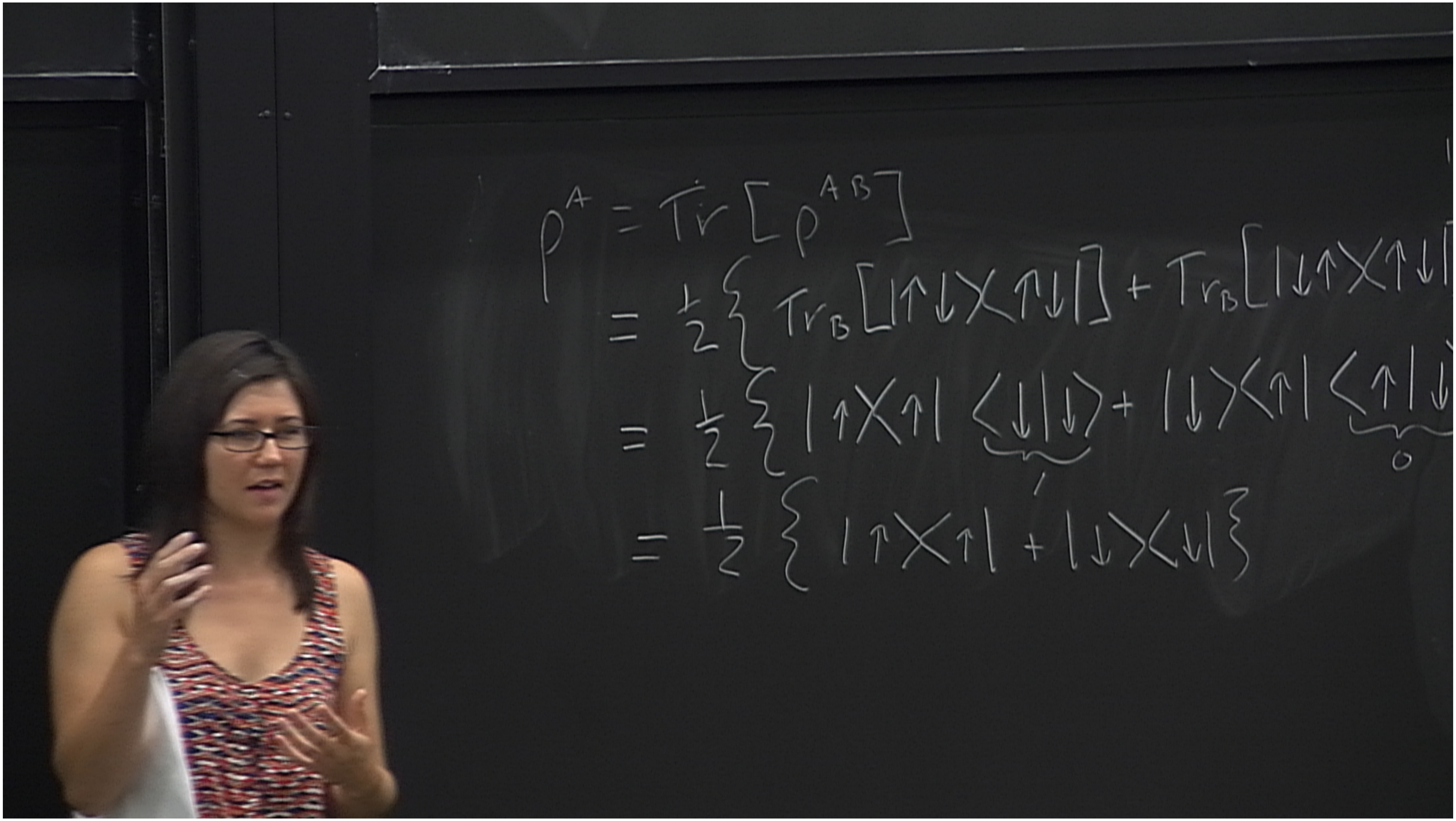

operator.
"ρ^AB"
"A"
define partial trace:
Tr_B [|a1⟩⟨a2| ⊗ |b1⟩⟨b2|] = |a1⟩⟨a2| Tr [|b1⟩⟨b2|]


operator.
"ρ^AB"
"A"
define partial trace:
Tr_B [|a₁⟩⟨a₂| ⊗ |b₁⟩⟨b₂|] = |a₁⟩⟨a₂| Tr[|b₁⟩
= |a₁⟩⟨a₂|


partial trace:
$a_1\rangle\langle a_2| \otimes |b_1\rangle\langle b_2|] = |a_1\rangle\langle a_2| \, Tr[|b_1\rangle\langle b_2|]$
$= |a_1\rangle\langle a_2| \langle b_2|b_1\rangle$
entangled triplet state
$\rho^{AB} = |1,0\rangle\langle 1,0|$




ρ^A = Tr[ρ^AB]
= 1/2 { Tr_B[|↑↓⟩⟨↑↓|] + Tr_B[|↓↑⟩⟨↑↓|] + Tr_B[|↑↓⟩⟨↓↑|]
= 1/2 { |↑⟩⟨↑| ⟨↓|↓⟩ + |↓⟩⟨↑



$\rho^A = Tr[\rho^{AB}]$
$= \frac{1}{2}\{ Tr_B[|\uparrow\downarrow\rangle\langle\uparrow\downarrow|] + Tr_B[|\downarrow\uparrow\rangle\langle\uparrow\downarrow|$
$= \frac{1}{2}\{ |\uparrow\rangle\langle\uparrow| \langle\downarrow|\downarrow\rangle + |\downarrow\rangle\langle\uparrow| \langle\uparrow|\downarrow$
$= \frac{1}{2}\{ |\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow| \}$