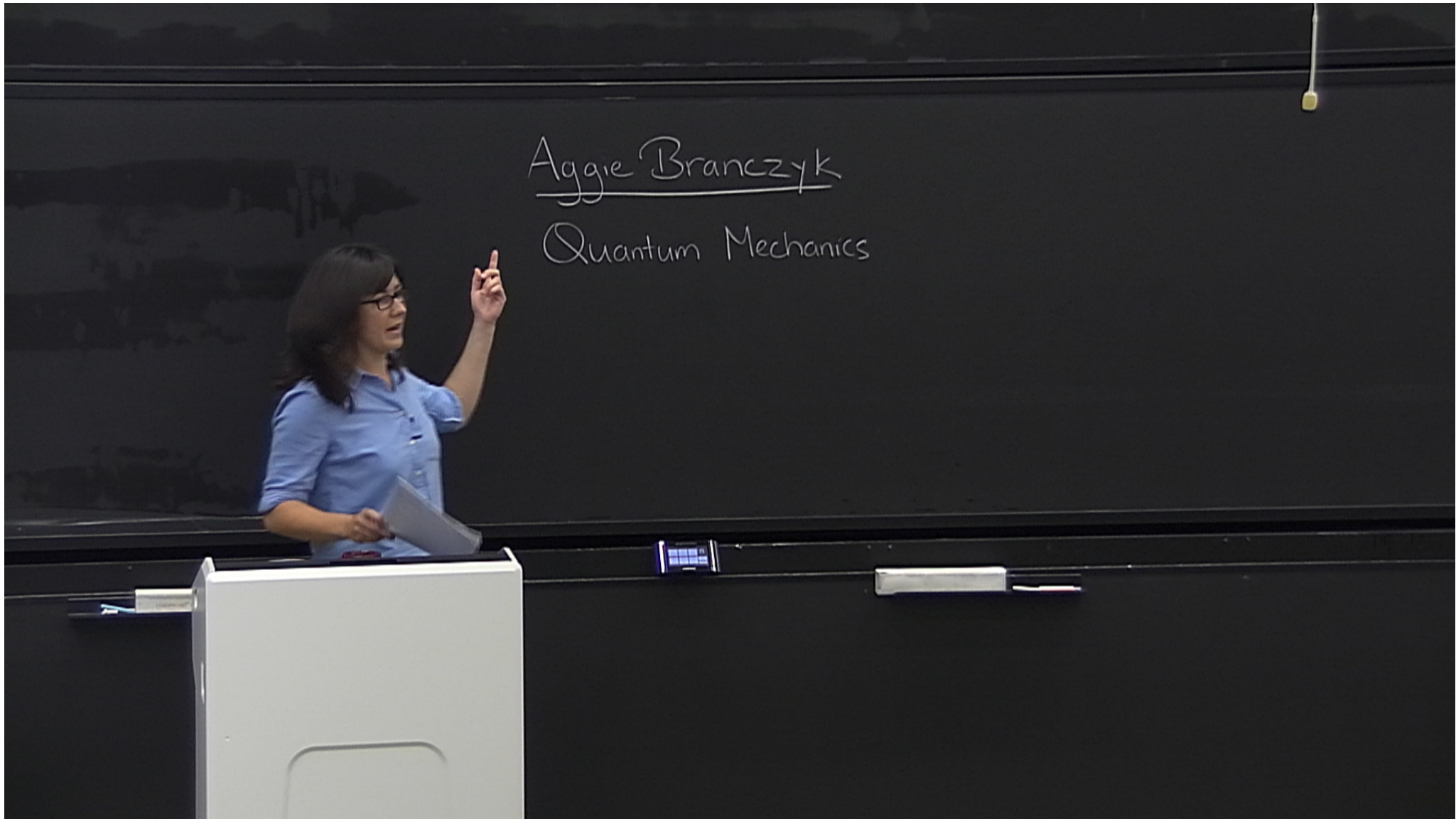


Title: 13/14 PSI - Quantum Mechanics - Lecture 1

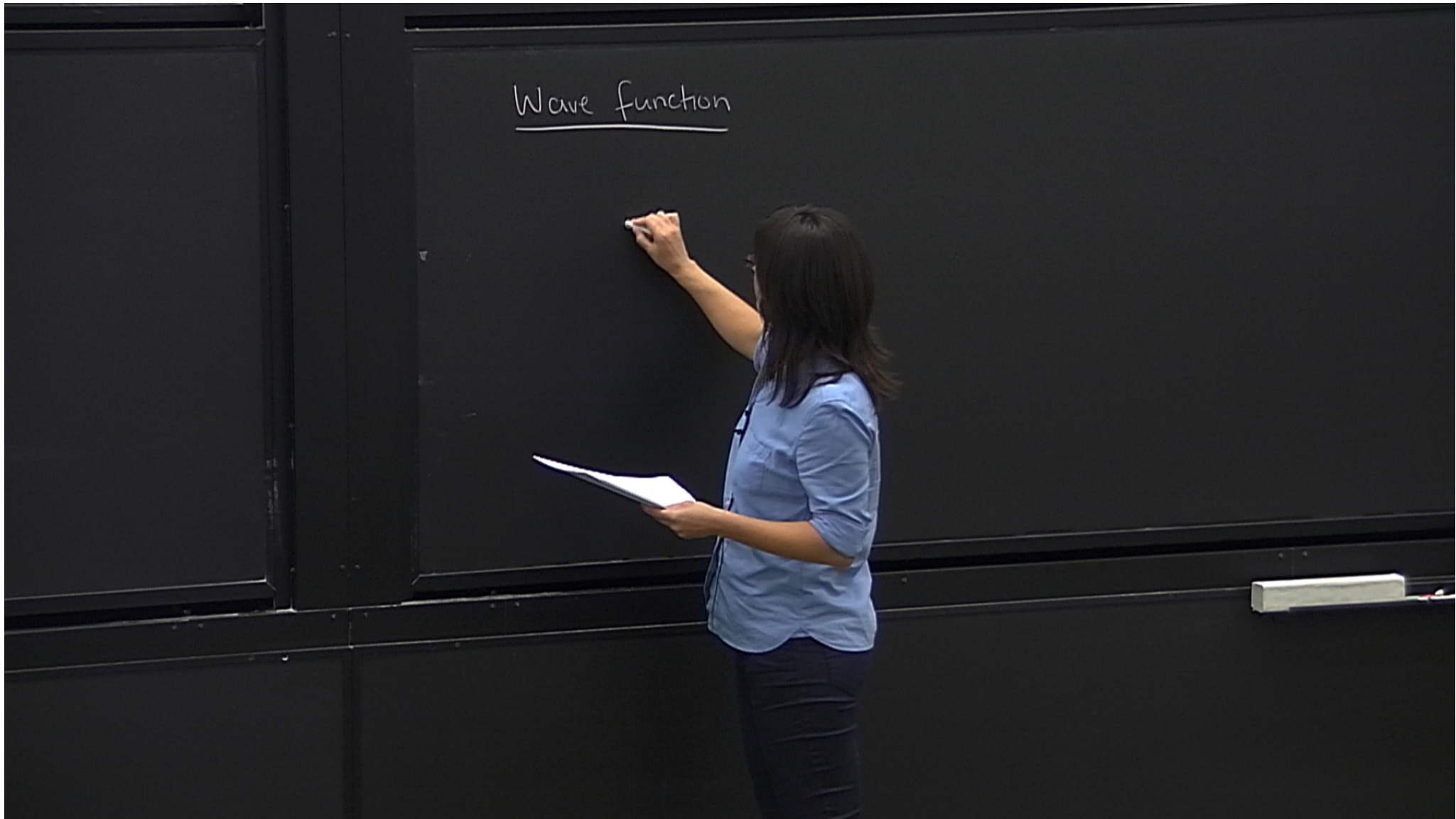
Date: Aug 27, 2013 10:30 AM

URL: <http://pirsa.org/13080032>

Abstract:

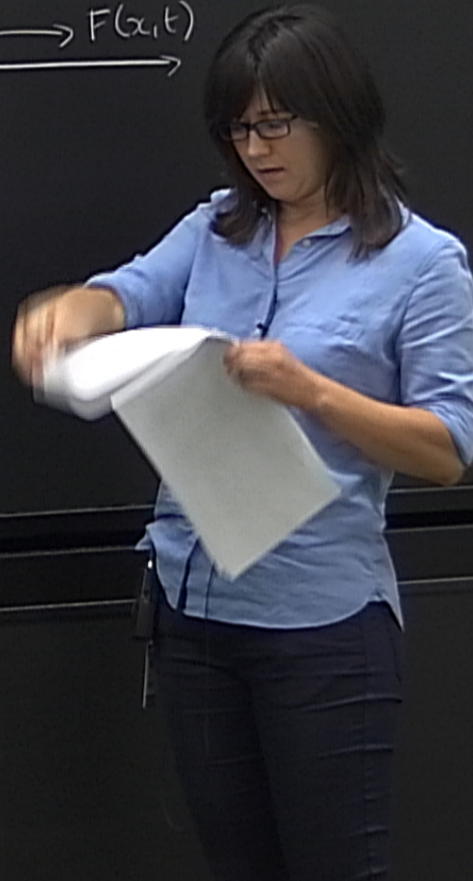


Wave function



Wave function

$$\begin{array}{c} m \\ \bullet \\ \hline x(t) \end{array} \longrightarrow F(x,t)$$



Wave function

Classical mechanics:

$$\begin{array}{c} m \\ \downarrow \\ x(t) \end{array} \rightarrow F(x,t)$$

$$v = \frac{dx}{dt}$$

$$p = mv$$

$$T = \frac{1}{2}mv^2$$

Newton's 2nd: $F = ma$

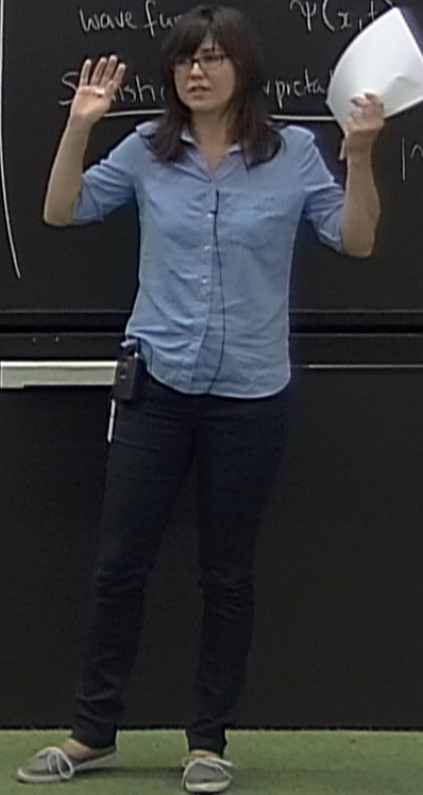
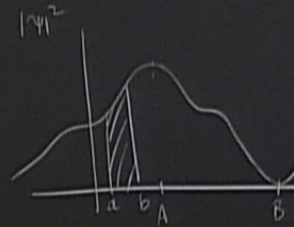
$$m \frac{d^2x}{dt^2} = -\frac{\partial V}{\partial x}$$

Quantum Mechanics:

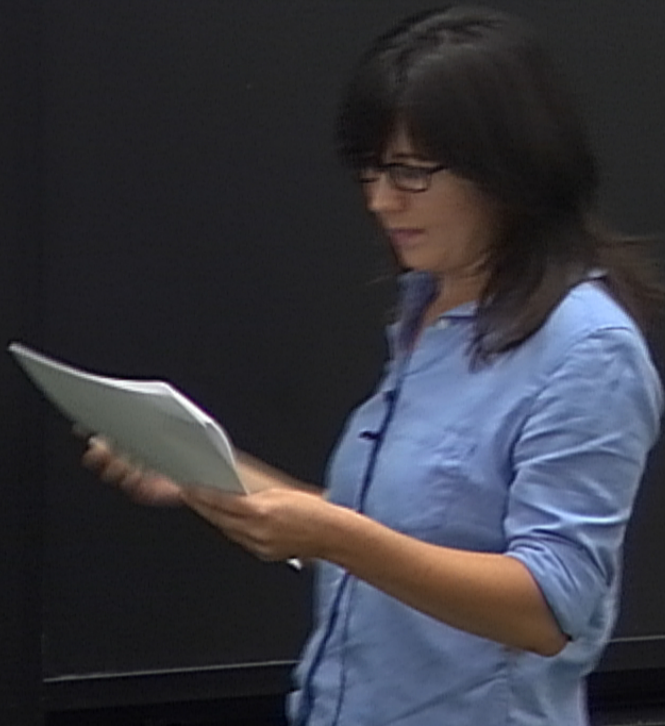
wave function $\Psi(x,t)$

Schrodinger equation

$$|\Psi(x,t)|^2 dx$$



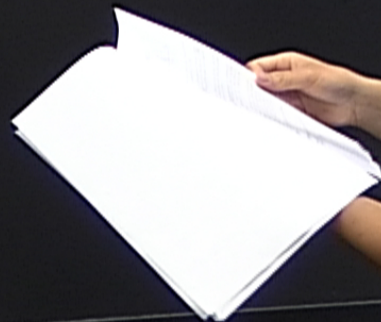
Normalization



Normalization

$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1$$

Schroedinger equation



$$p = mv$$
$$T = \frac{1}{2}mv^2$$

Probability $\int_0^1$

Normalization

$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1$$

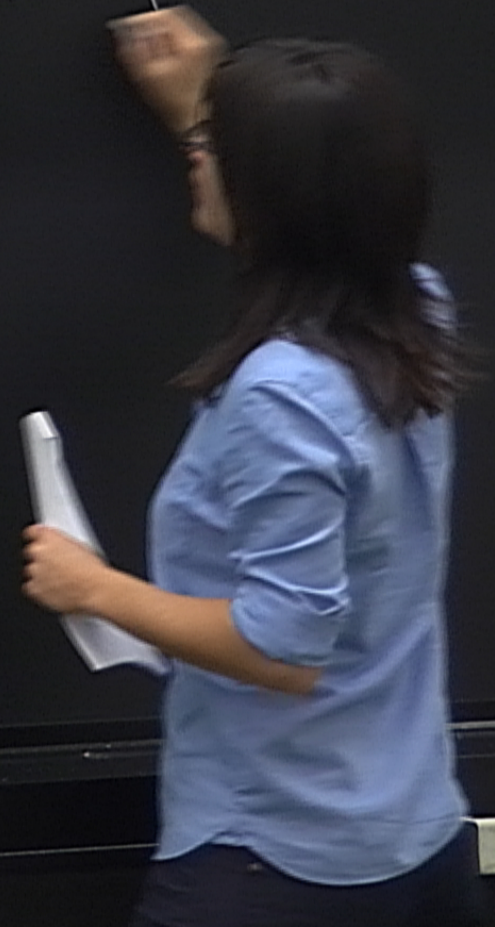
Schrodinger equation

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

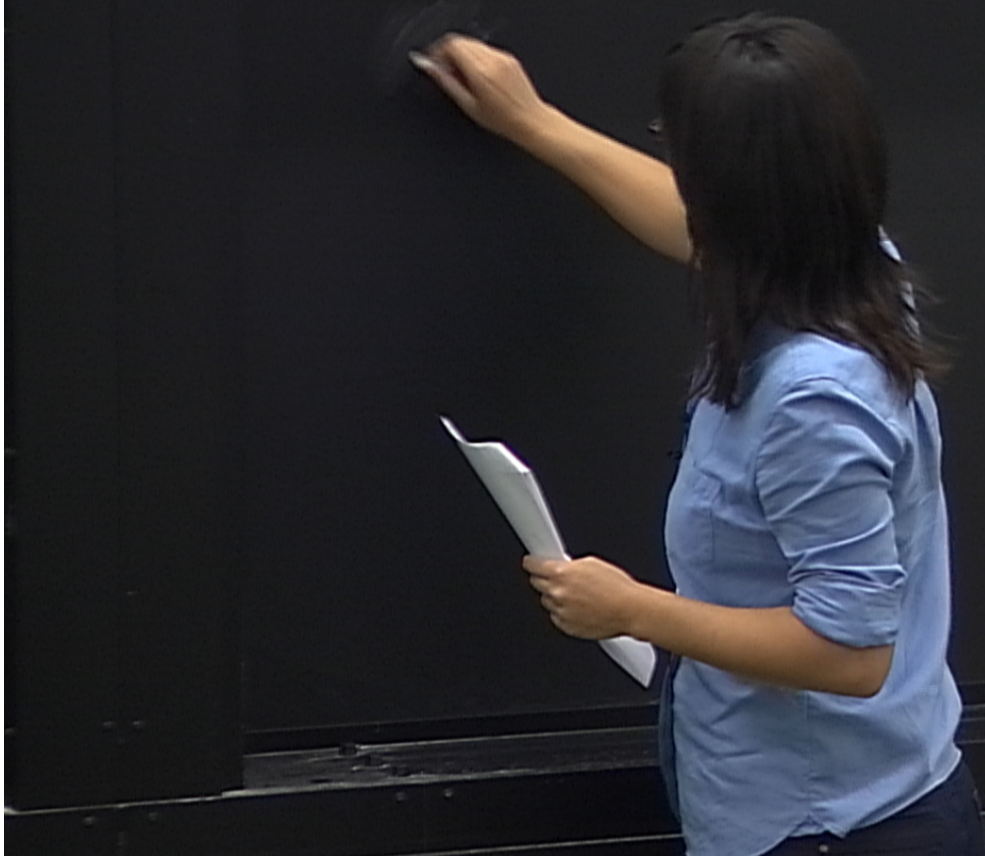
$A\Psi$ is a soln.

- if Ψ is a soln, then $A\Psi$ is a soln.
- Some solns $\int_{-\infty}^{\infty} |\Psi|^2 = \infty$; $\Psi =$

Momentum & expectation values of operators



Momentum & expectation values of operators



Momentum & expectation values of operators

particle in state ψ

$$\langle x \rangle = \int_{-\infty}^{\infty} x |\Psi(x,t)|^2$$

Momentum & expectation values of operators

particle in state Ψ

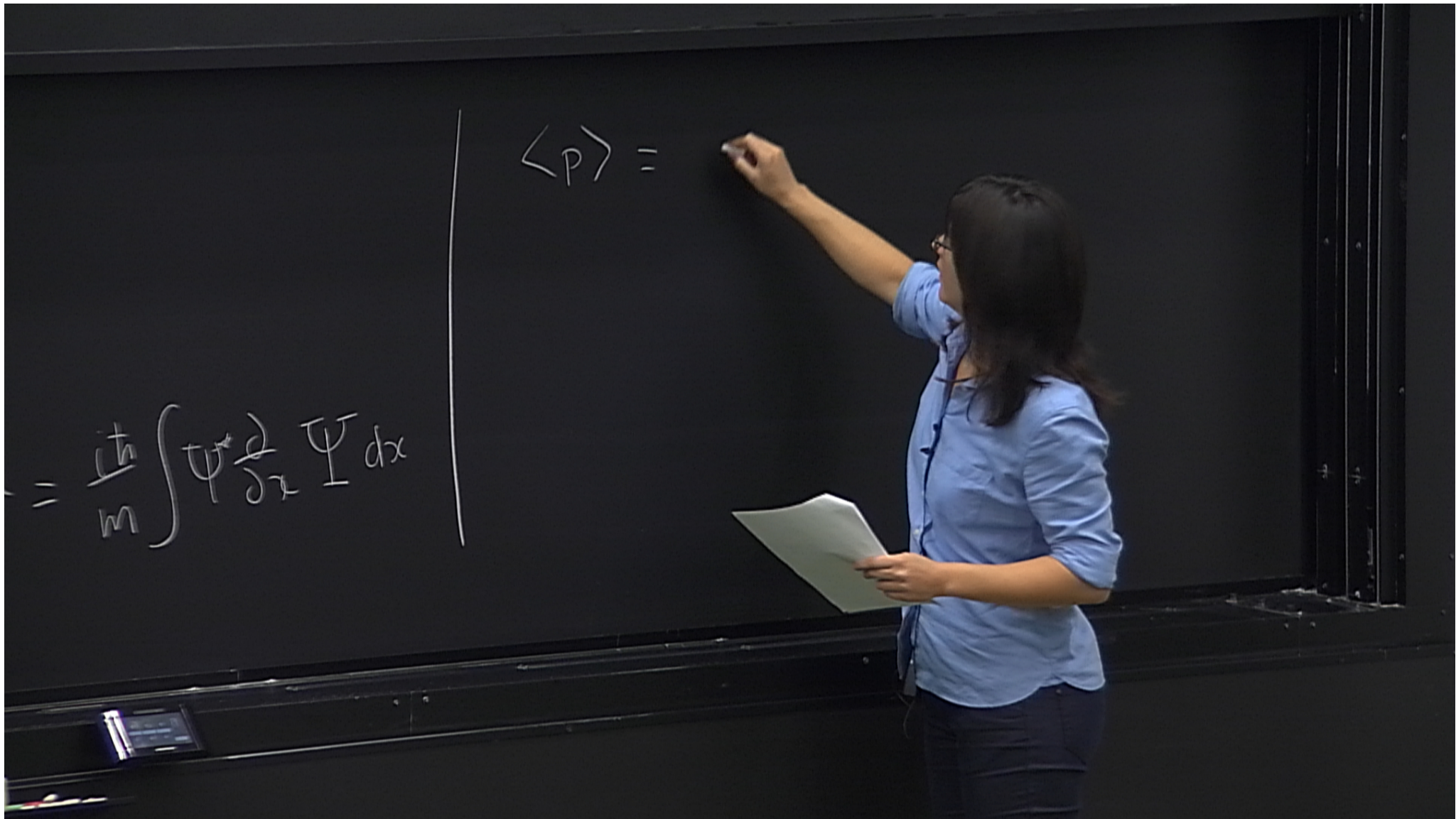
$$\langle x \rangle = \int_{-\infty}^{\infty} x |\Psi(x,t)|^2 dx$$

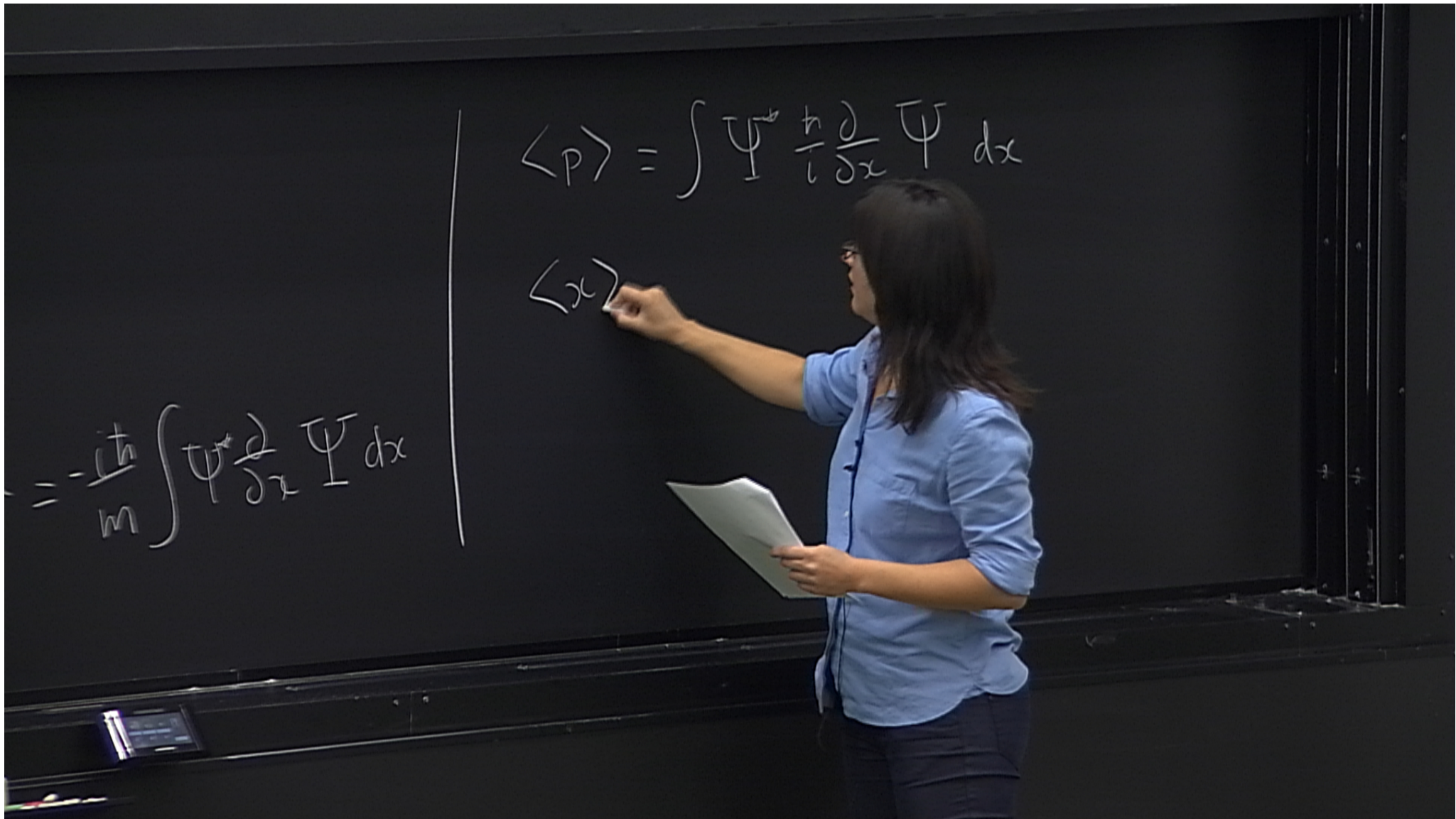
$$\frac{d\langle x \rangle}{dt} = \int x \frac{\partial}{\partial t} |\Psi(x,t)|^2 dx = -\frac{i}{2m} \int (\Psi \nabla^2 \Psi - \nabla^2 \Psi \Psi)$$

s of operators

$\int dx$

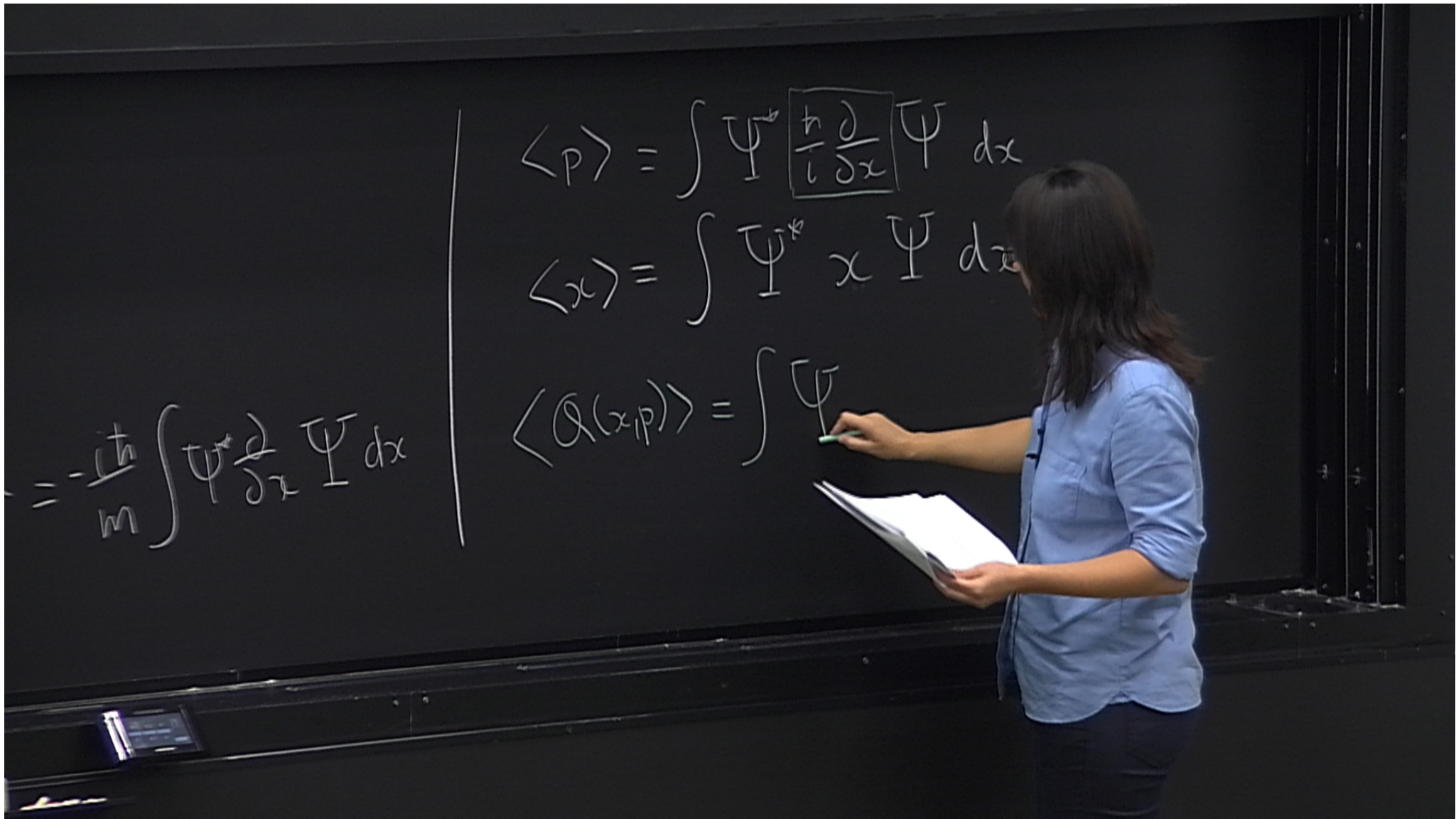
$$\int dx = \frac{i\hbar}{2m} \int \left(\psi^* \frac{\partial \psi}{\partial x} - \frac{\partial \psi^*}{\partial x} \psi \right) dx = \frac{i\hbar}{m} \int \psi^* \frac{\partial \psi}{\partial x}$$





$$\langle p \rangle = \int \Psi^* \left[\frac{\hbar}{i} \frac{\partial}{\partial x} \right] \Psi dx$$

$$\langle x \rangle = \int \Psi^* x \Psi dx$$



values of operators

K.E.

$$\langle T \rangle = -\frac{\hbar^2}{2m} \int \psi^* \frac{\partial^2 \psi}{\partial x^2} dx$$

$$\int |\psi(x,t)|^2 dx$$

$$\int |\psi(x,t)|^2 dx = \frac{i\hbar}{2m} \int \left(\psi^* \frac{\partial \psi}{\partial x} - \frac{\partial \psi^*}{\partial x} \psi \right) dx - \frac{i\hbar}{2m} \int \psi^* \frac{\partial \psi}{\partial x} dx$$

$$\langle p \rangle = \int \psi^* \hat{p} \psi dx$$

$$\langle x \rangle = \int \psi^* \hat{x} \psi dx$$

$$\langle Q(x,p) \rangle = \int \psi^* \hat{Q} \psi dx$$

then $A \cdot \underline{\Psi}$ is a soln.

$$r = \infty; \quad \underline{\Psi} = 0$$

$$[x, p] = xp - px$$

=

$$\begin{aligned}
 [x, p] &= xp - px \\
 &= \frac{\hbar}{i} \left[x \frac{\partial}{\partial x} - \frac{\partial x}{\partial x} \right] \\
 &= \frac{\hbar}{i} [0 - 1]
 \end{aligned}$$

Time dependent S.E.

$$\frac{\partial \Psi}{\partial t} = -$$

$$\frac{\partial \Psi}{\partial t} = -$$

Time dependent S.E.

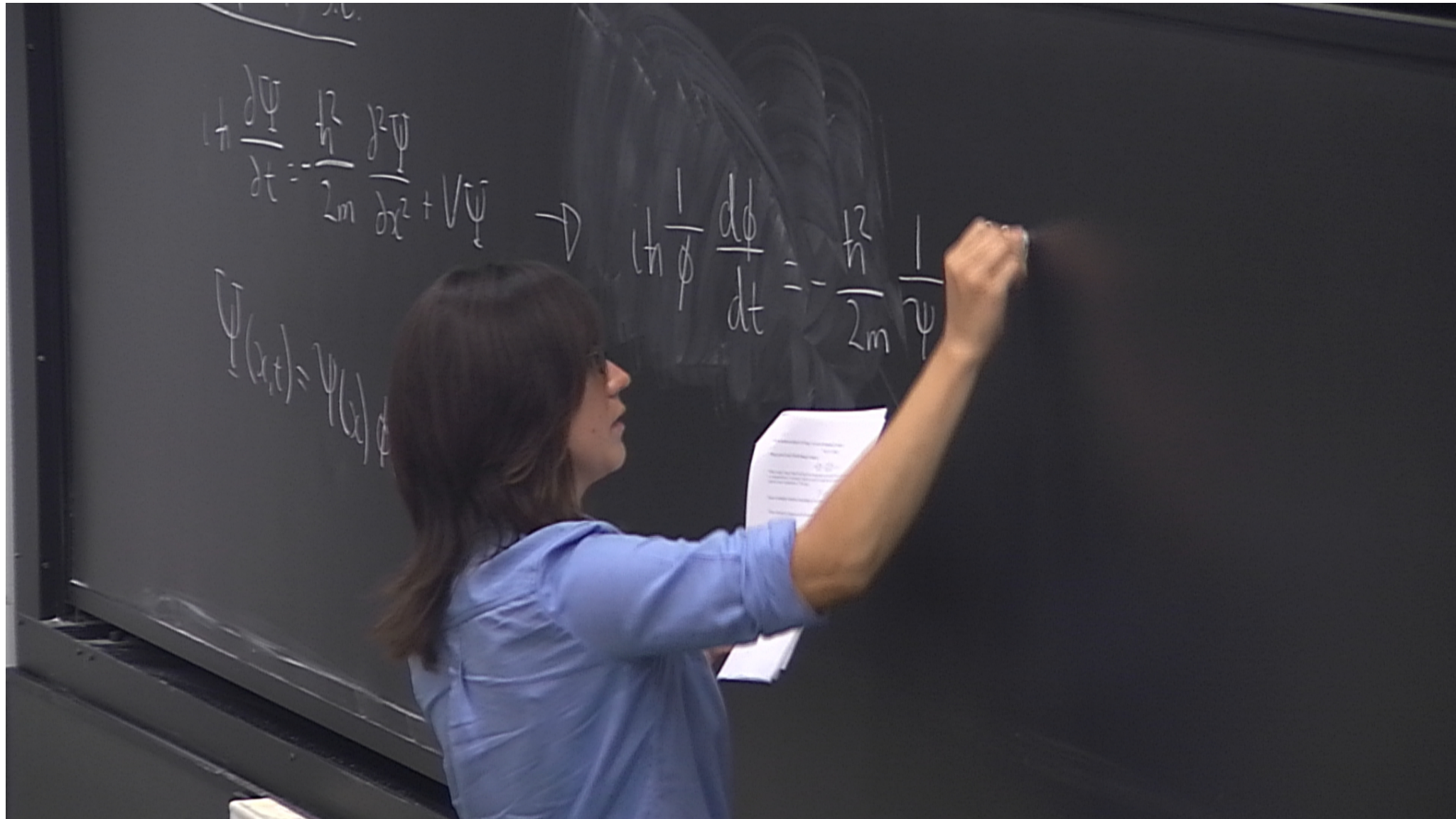
$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

Ψ

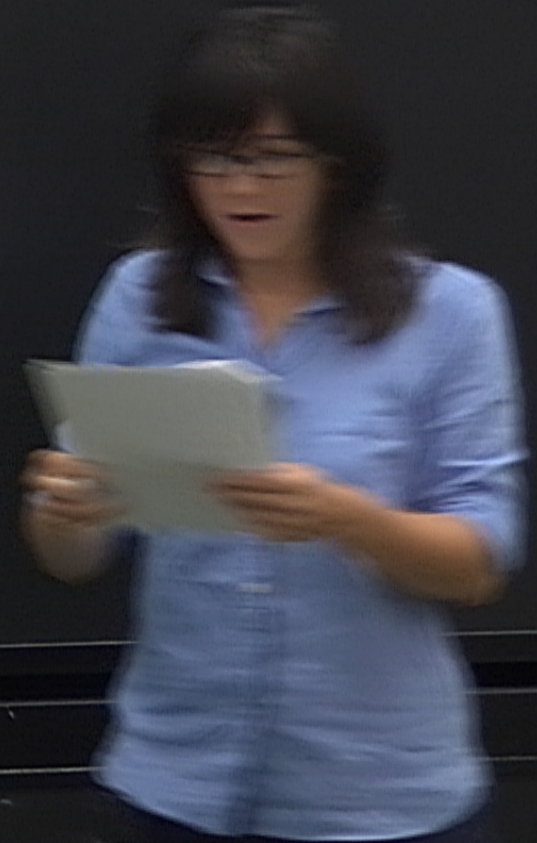
Time dependent S.E.

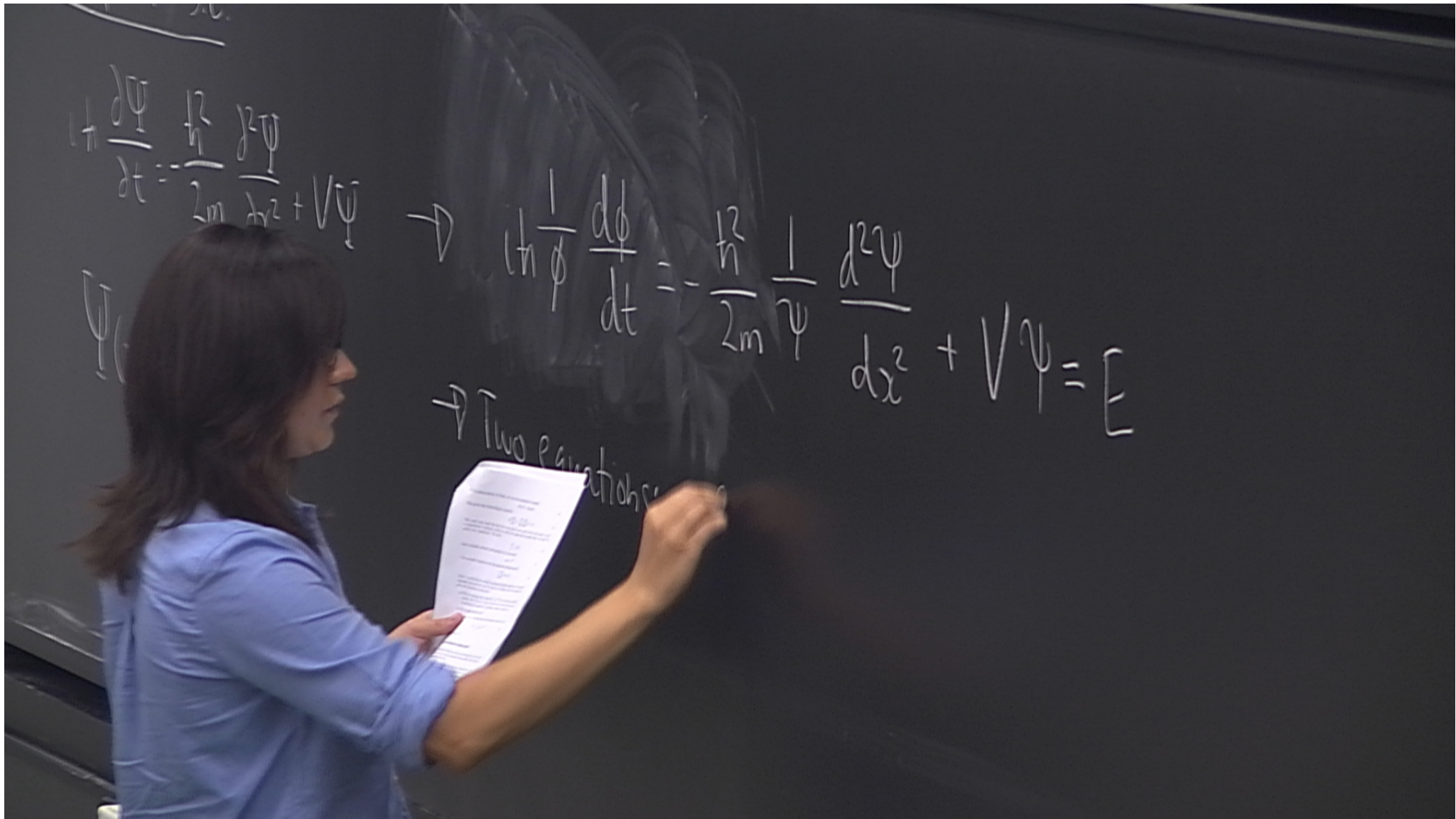
$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

$$\Psi(x,t) = \psi(x)\phi(t)$$



$$\frac{1}{\phi} \frac{d\phi}{dt} = -\frac{\hbar^2}{2m} \frac{1}{\psi} \frac{d^2\psi}{dx^2} + V\psi$$

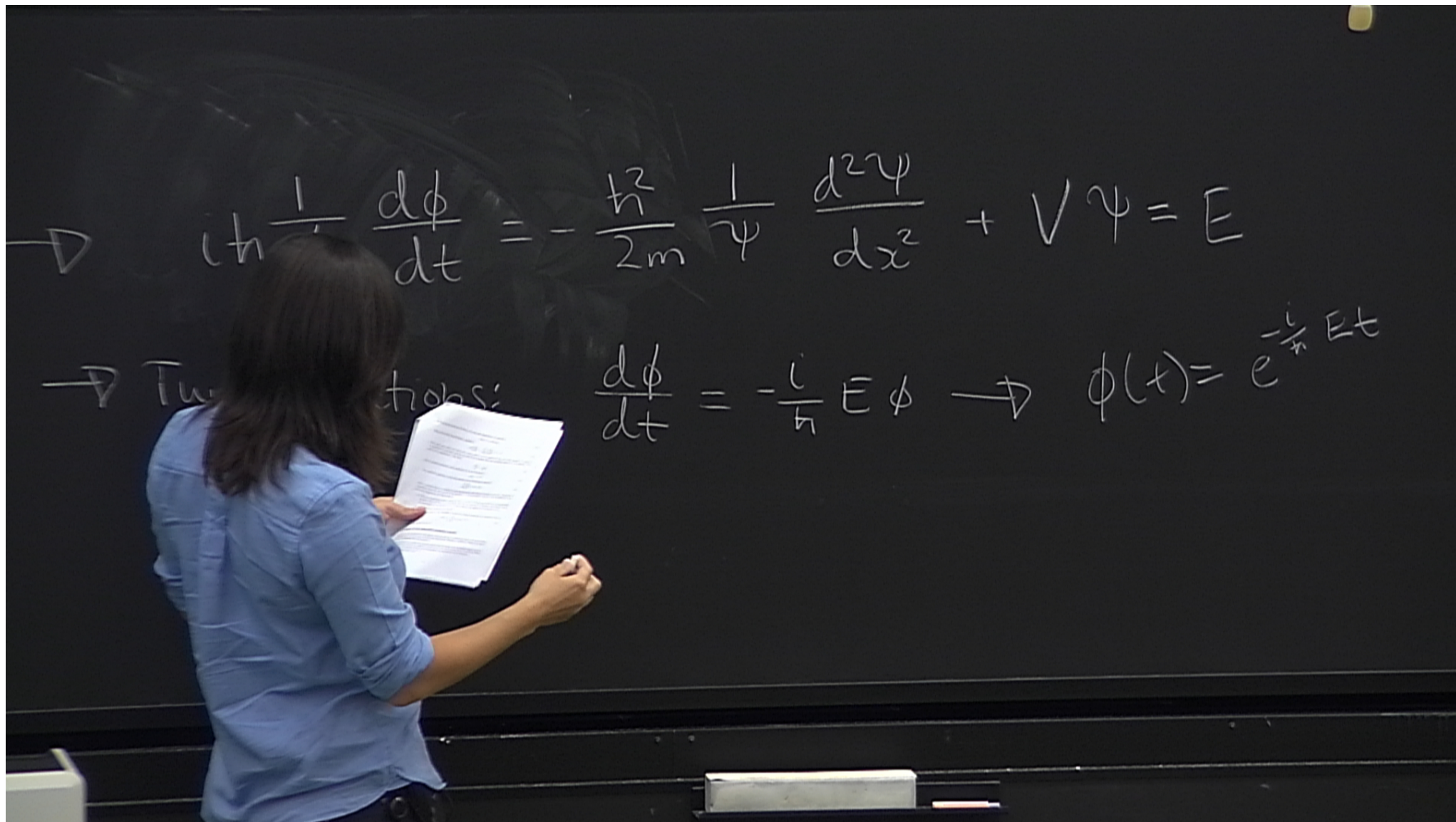


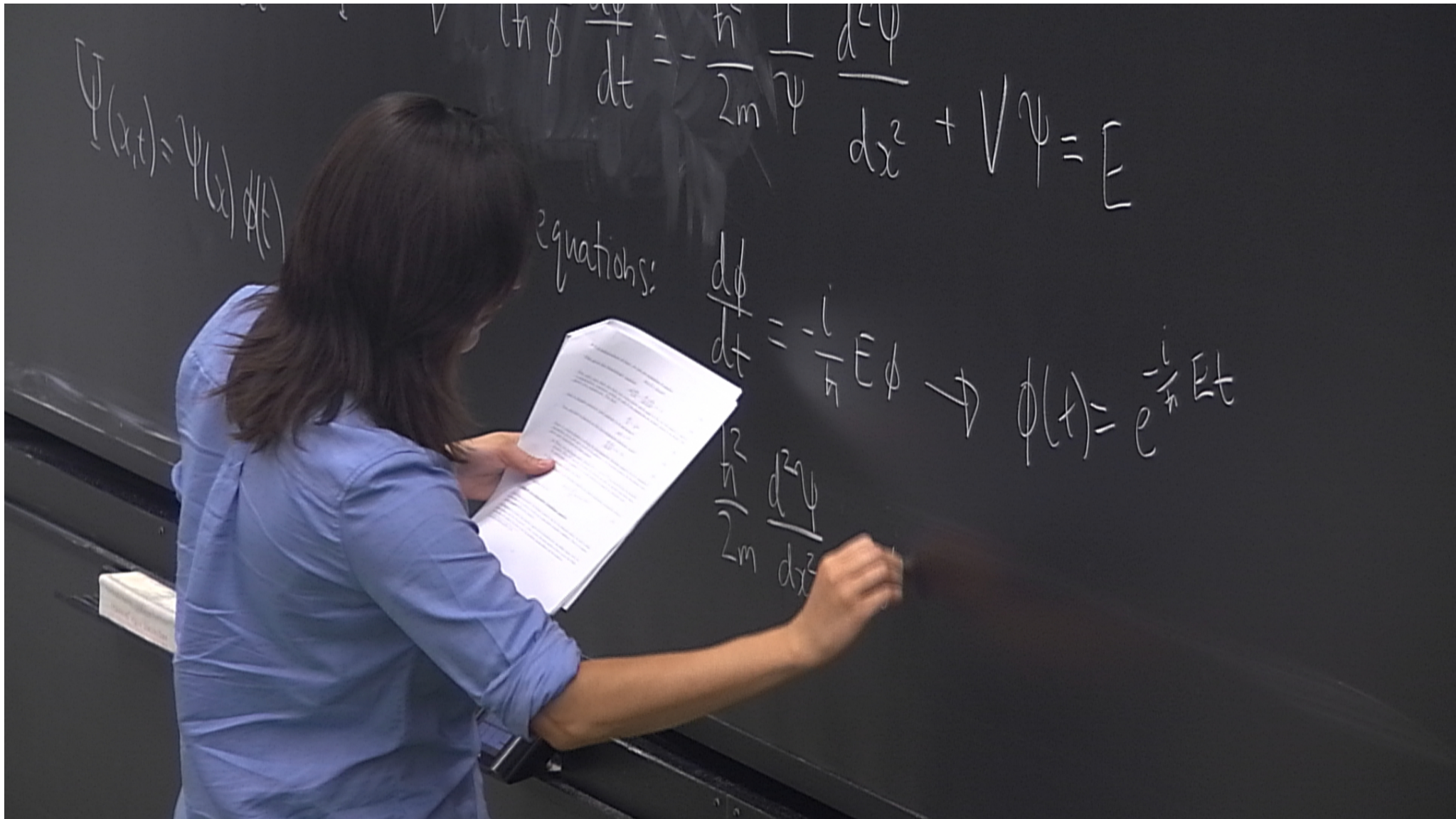


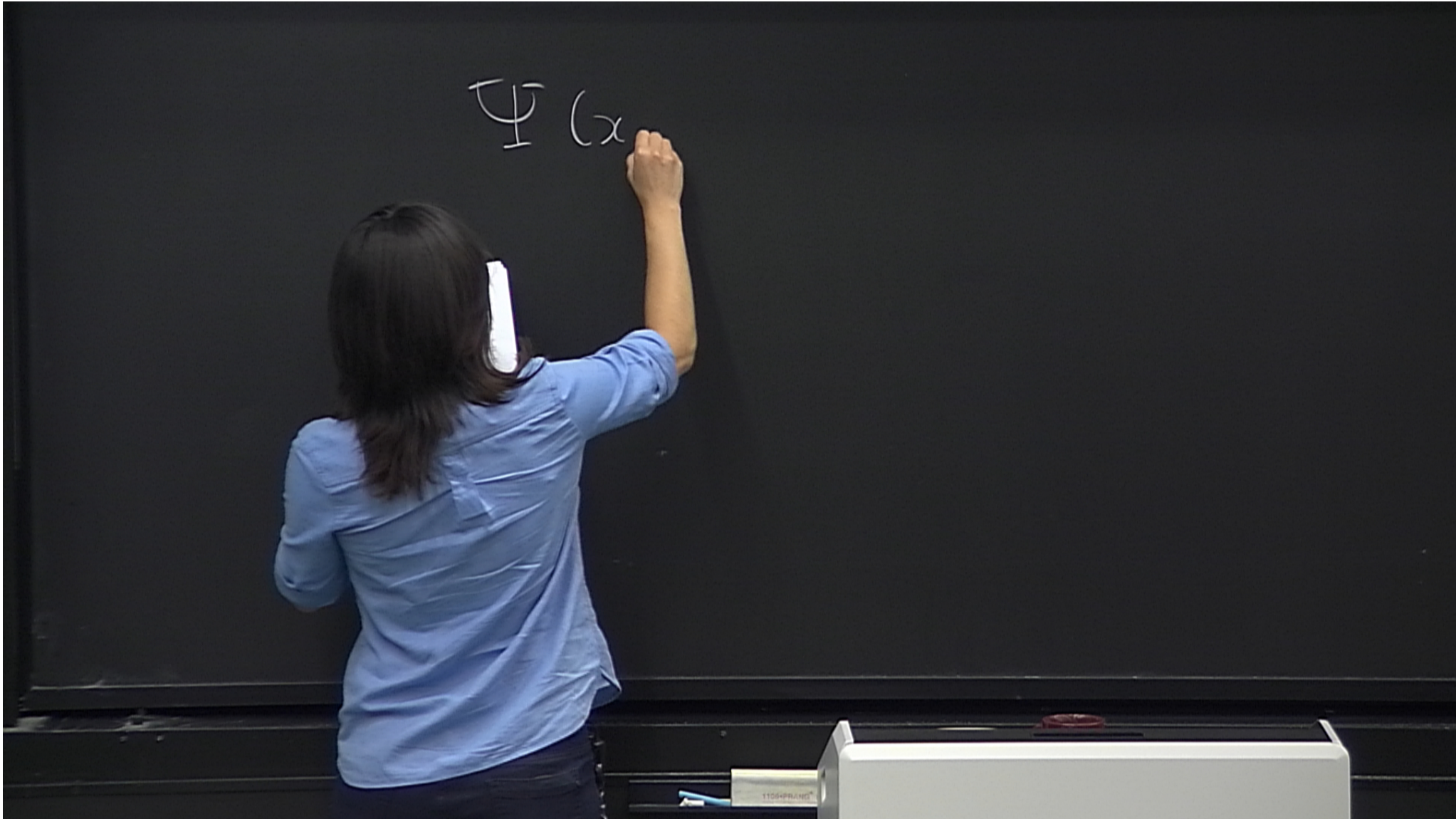
$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

$$\rightarrow i\hbar \frac{1}{\phi} \frac{d\phi}{dt} = -\frac{\hbar^2}{2m} \frac{1}{\psi} \frac{d^2 \psi}{dx^2} + V\psi = E$$

$\rightarrow$ Two parameters

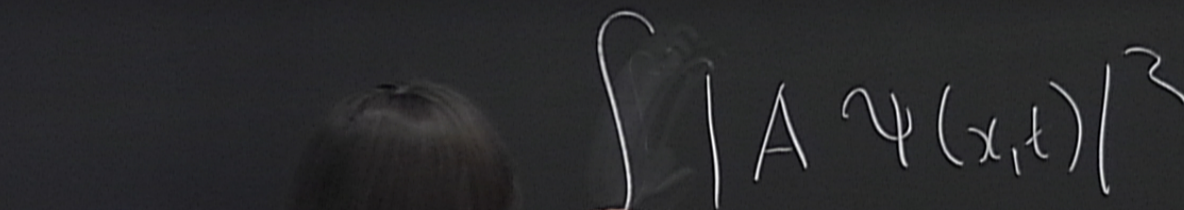






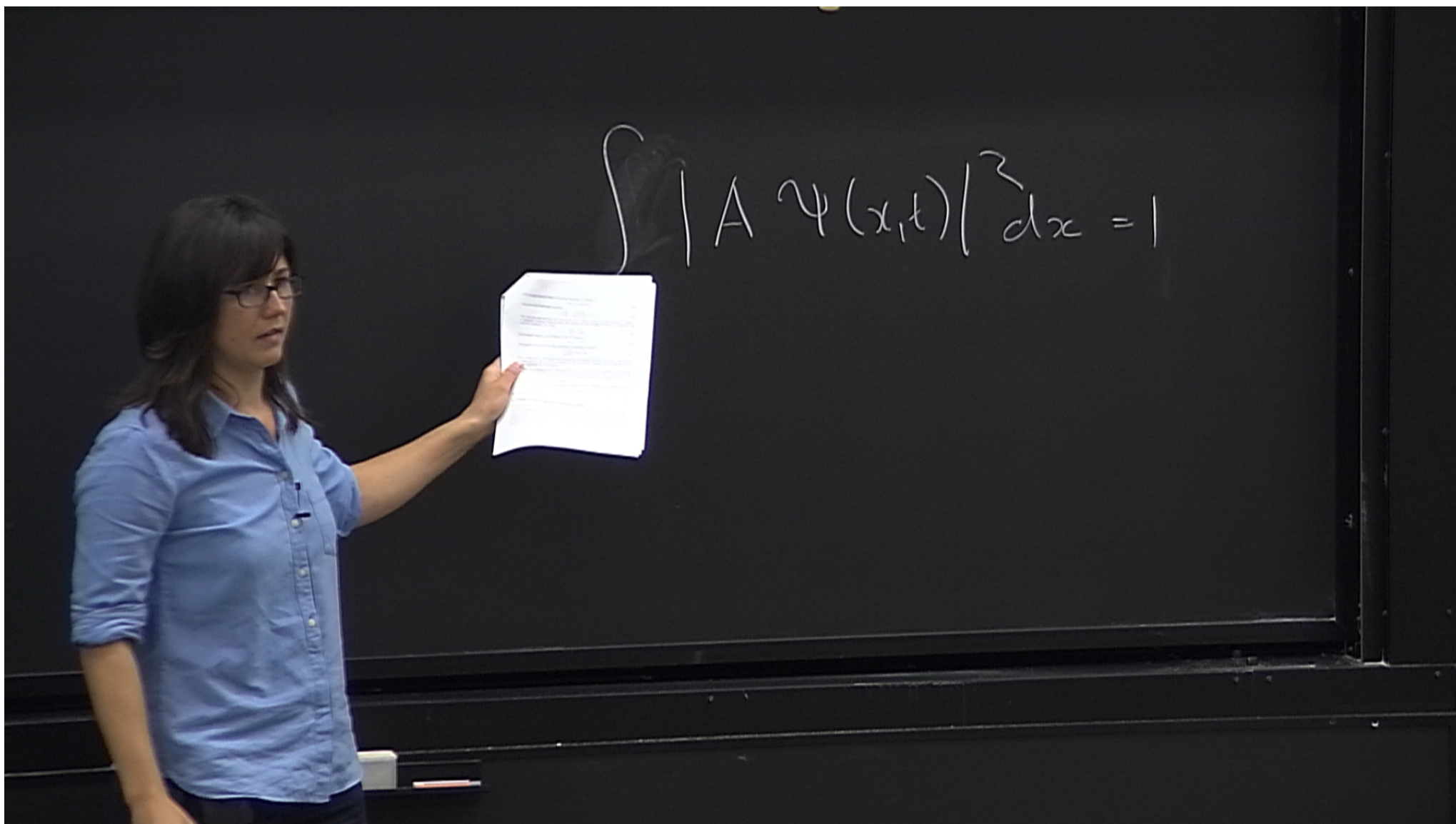
$$\Psi(x,t) = \psi(x) e^{-\frac{i}{\hbar} E t}$$

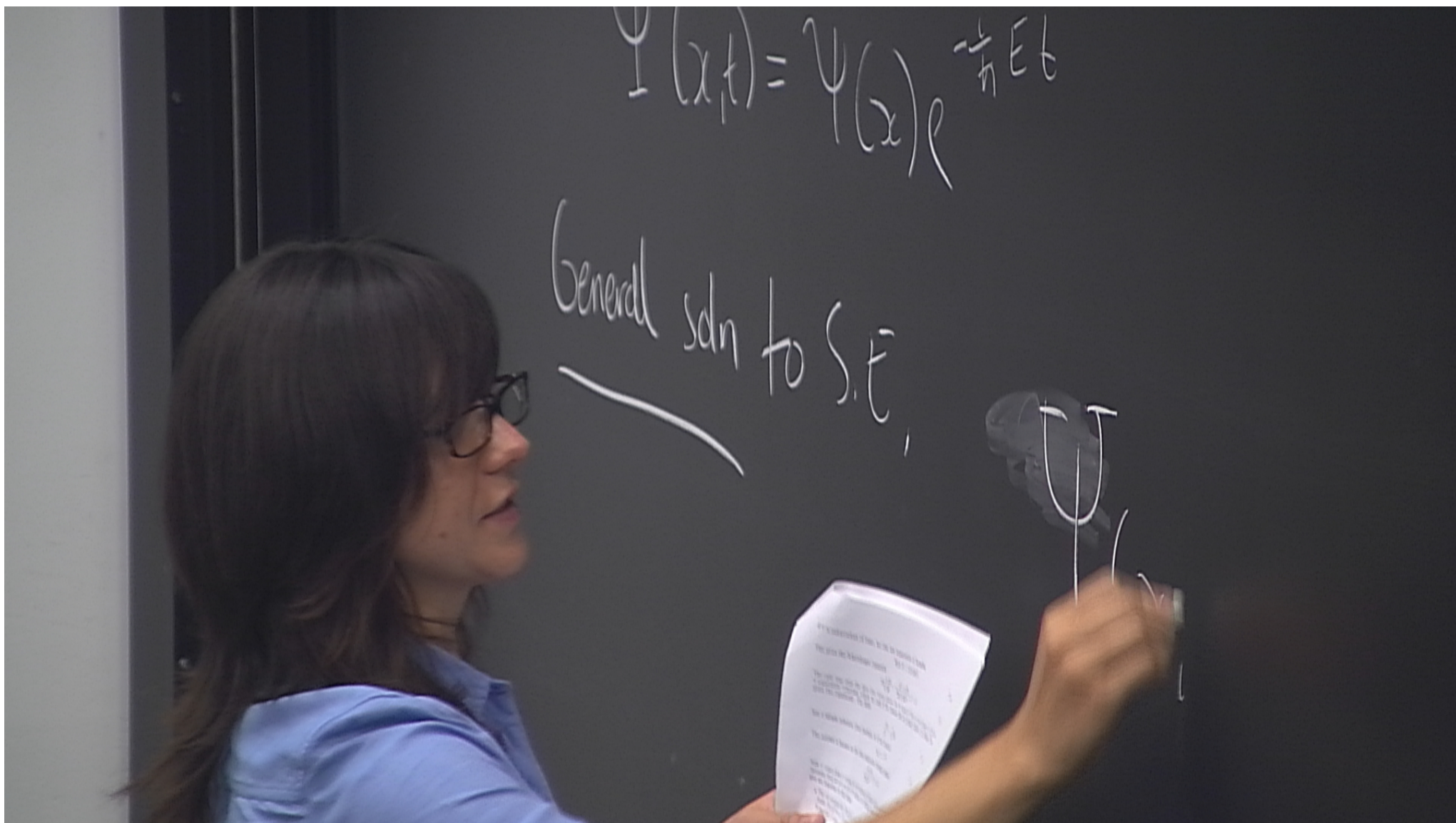
Genera

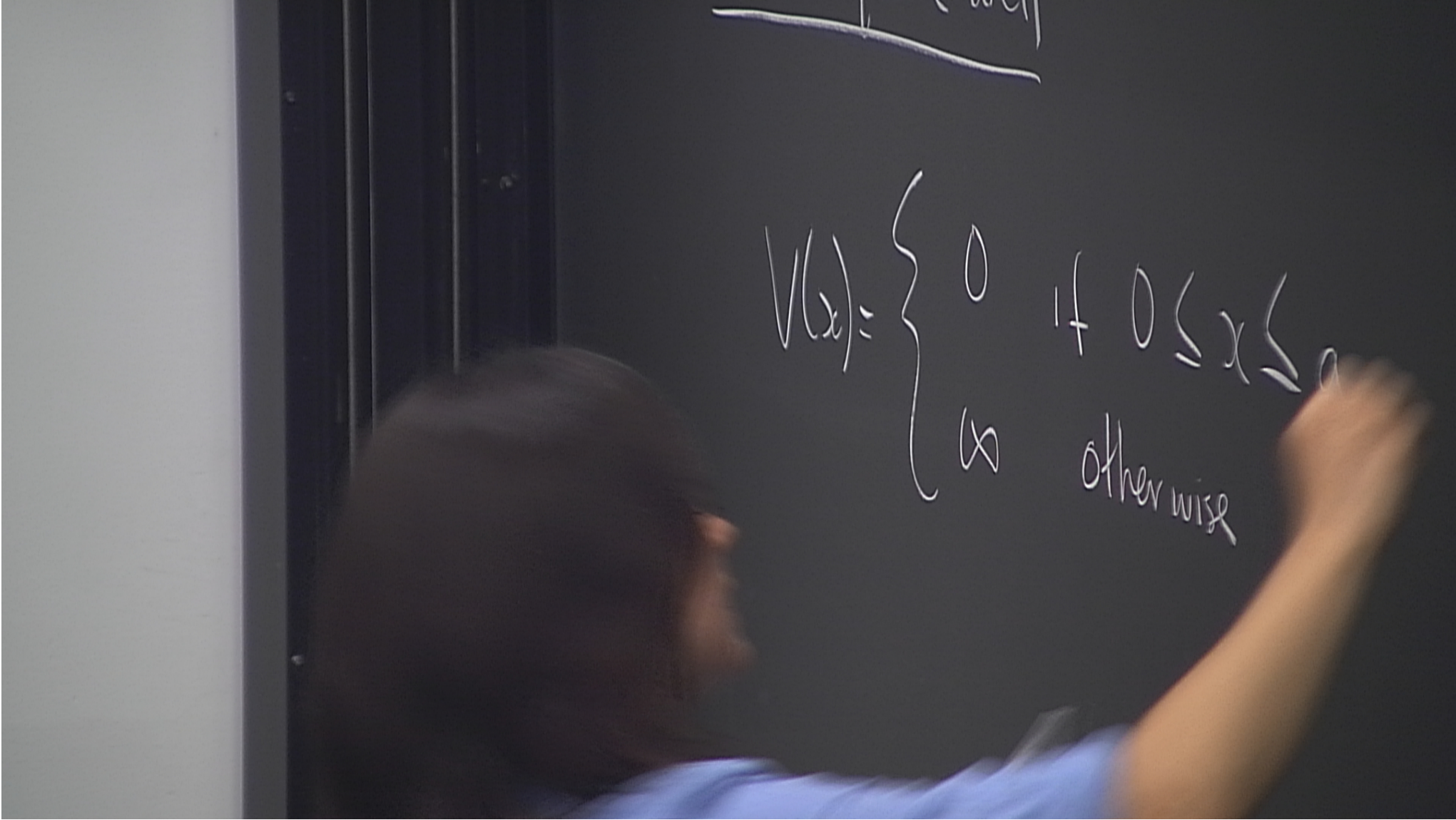


A person with long dark hair, wearing a light blue short-sleeved shirt and dark pants, is standing in front of a blackboard. They are holding a white sheet of paper in their left hand and writing the mathematical expression $\int |\psi(x,t)|^2$ on the blackboard with their right hand. The blackboard is dark and the writing is in white chalk.

$$\int |\psi(x,t)|^2$$



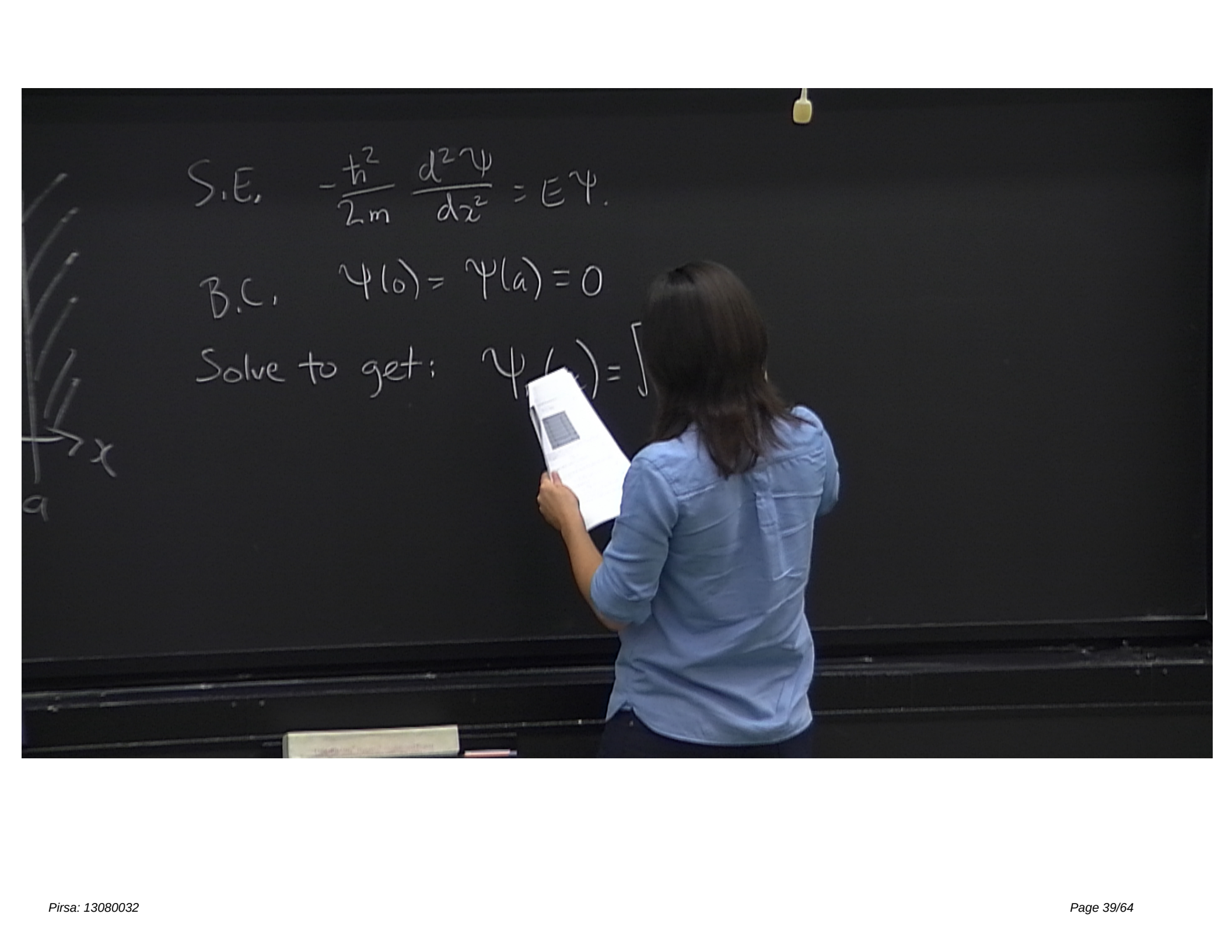


A person with dark hair, seen from the back and side, is writing on a dark chalkboard. They are wearing a light blue shirt. The chalkboard has a piecewise function written in white chalk. Above the main equation, there is a partially visible underlined expression. The function is defined as V(x) = 0 for 0 ≤ x ≤ a, and ∞ otherwise. The person's right hand is visible, holding a piece of chalk and finishing the word 'otherwise'.
$$V(x) = \begin{cases} 0 & \text{if } 0 \leq x \leq a \\ \infty & \text{otherwise} \end{cases}$$

Infinite square well

$$V(x) = \begin{cases} 0 & \text{if } 0 \leq x \leq L \\ \infty & \text{otherwise} \end{cases}$$

$$\text{S.t. } -\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} = E \psi.$$



S.E. $-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} = E\psi.$

B.C. $\psi(0) = \psi(a) = 0$

Solve to get: $\psi(x) = \sin\left(\frac{n\pi x}{a}\right)$

$$\text{S.E.} \quad -\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} = E \psi.$$

$$\text{B.C.} \quad \psi(0) = \psi(a) = 0$$

$$\text{Solve to get: } \psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right) \text{ for } n=1, 2, 3, \dots$$

$$\text{S.E.} \quad -\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} = E \psi.$$

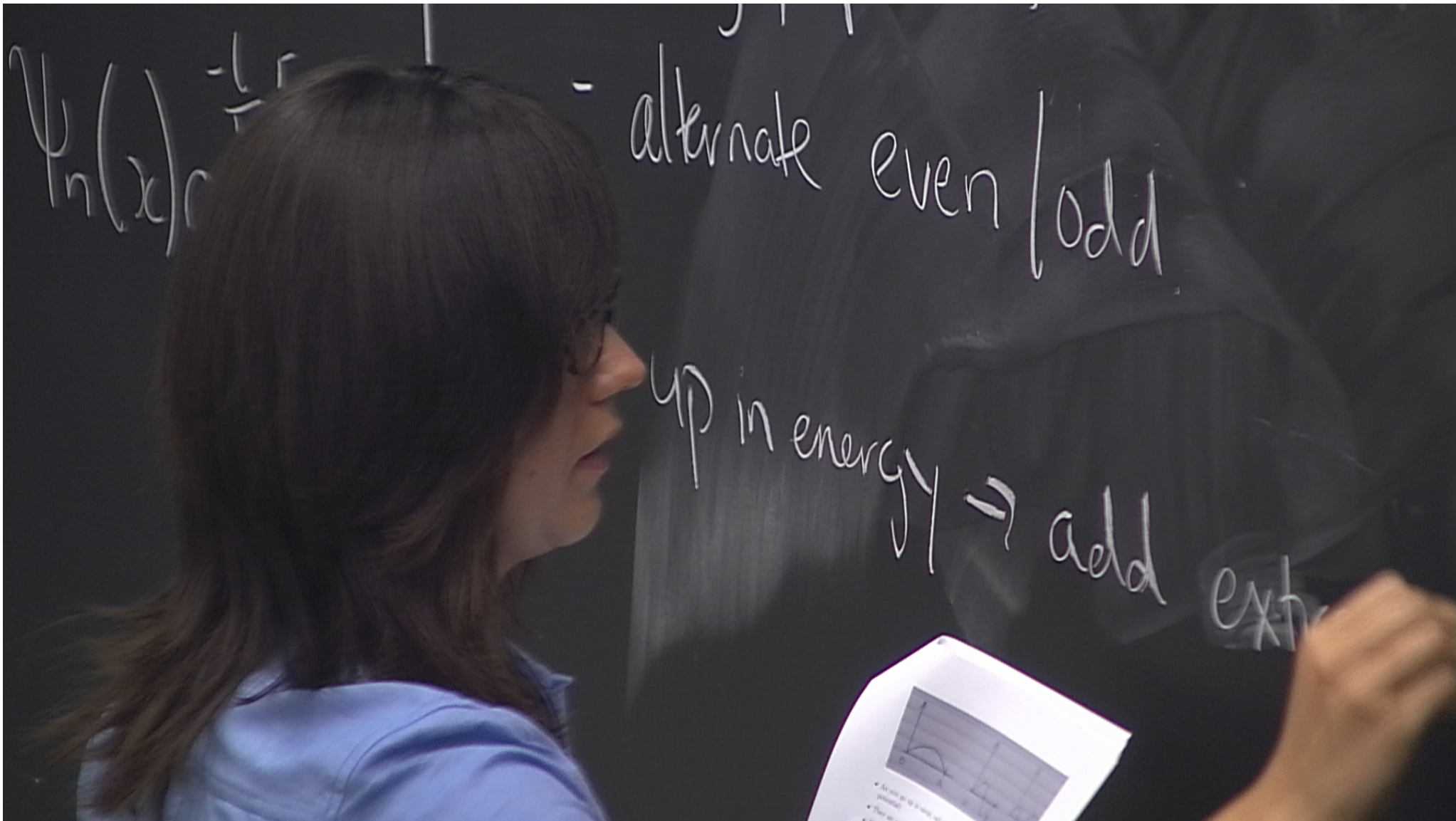
$$\text{B.C.} \quad \psi(0) = \psi(a) = 0$$

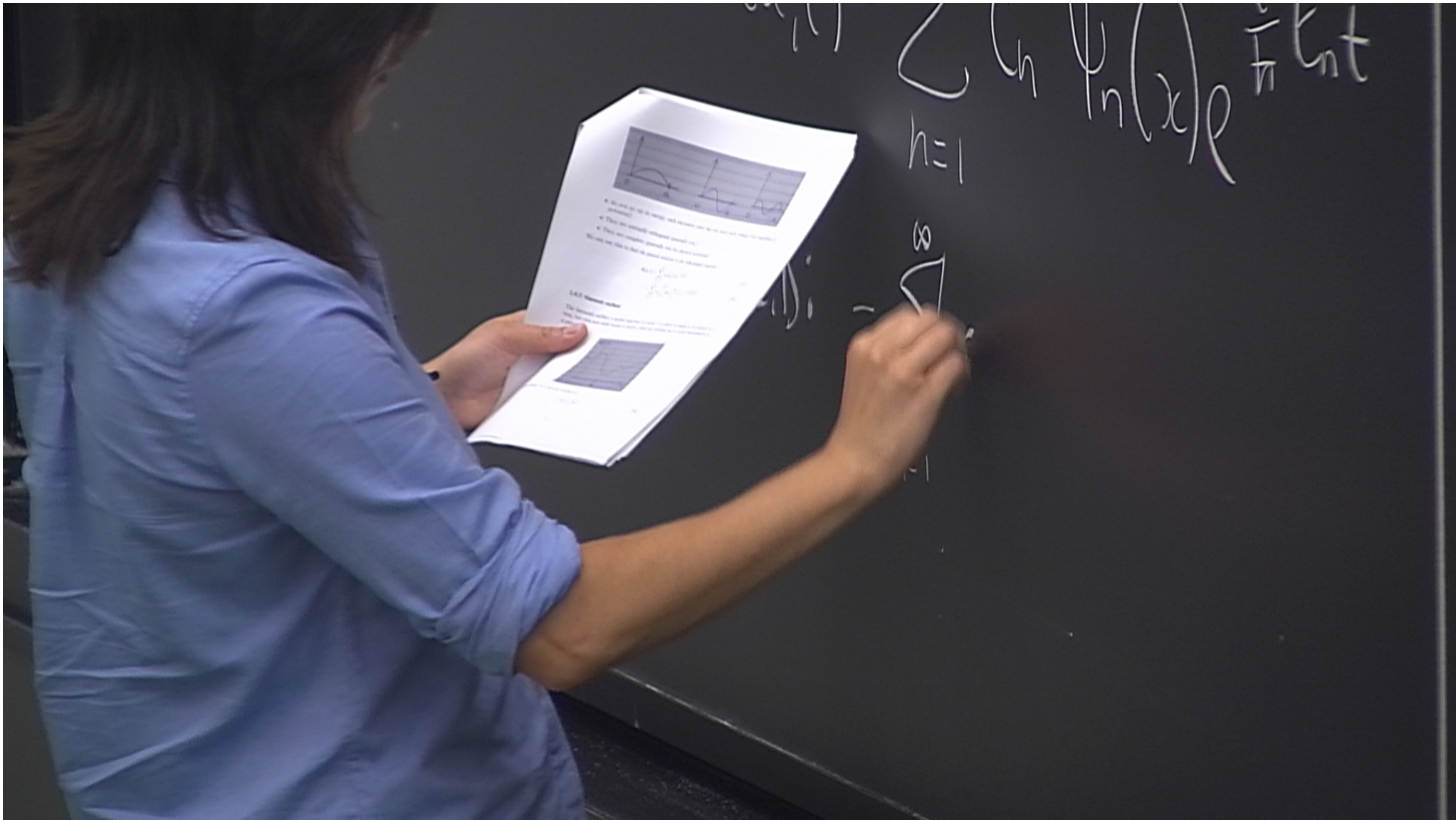
$$\text{Solve to} \quad \psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right) \quad \text{for } n=1, 2, 3, \dots$$

Solns to P.I.B,

interesting

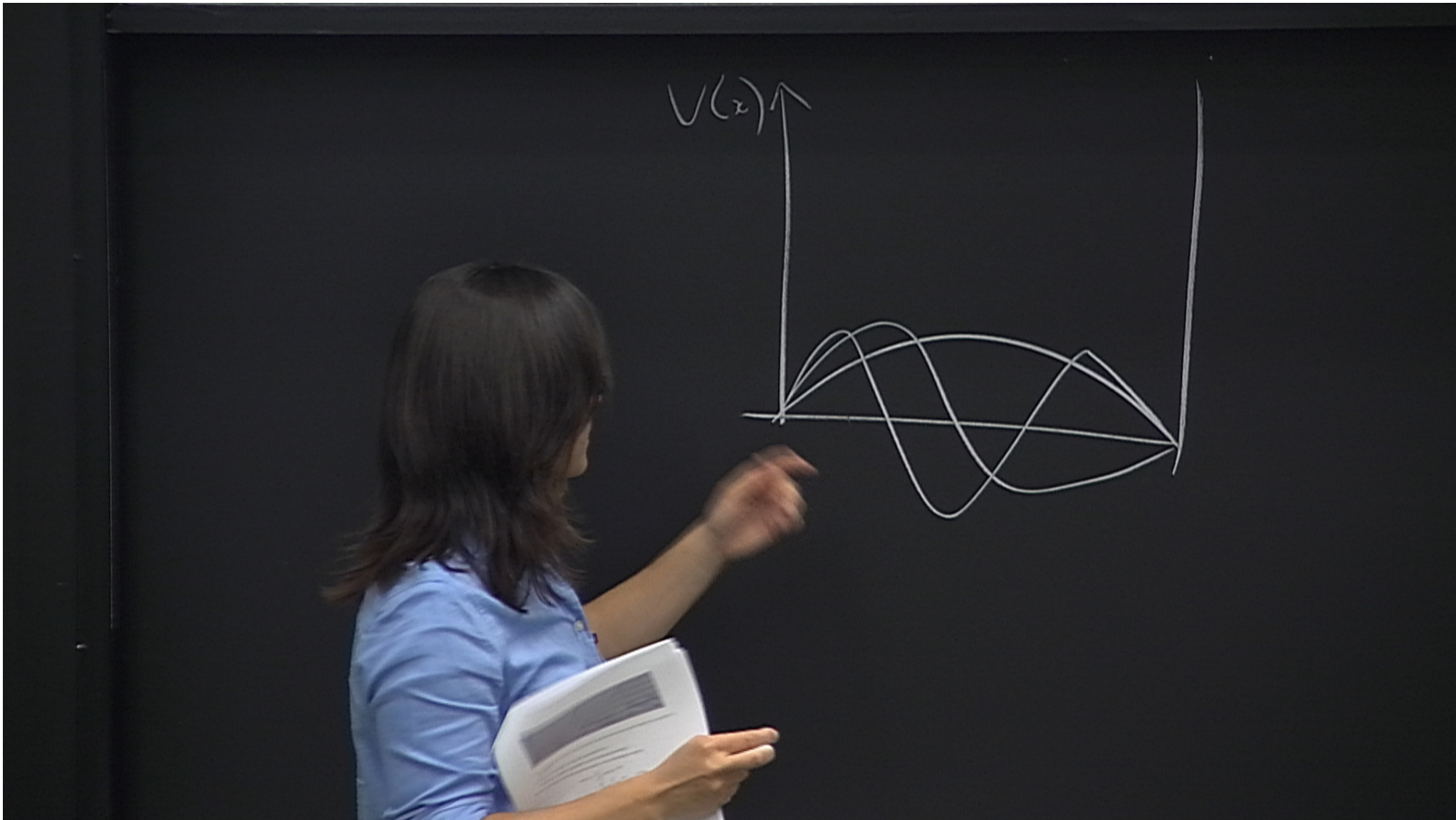
$$(x)_0^{-\frac{1}{n}} E_{nt}$$

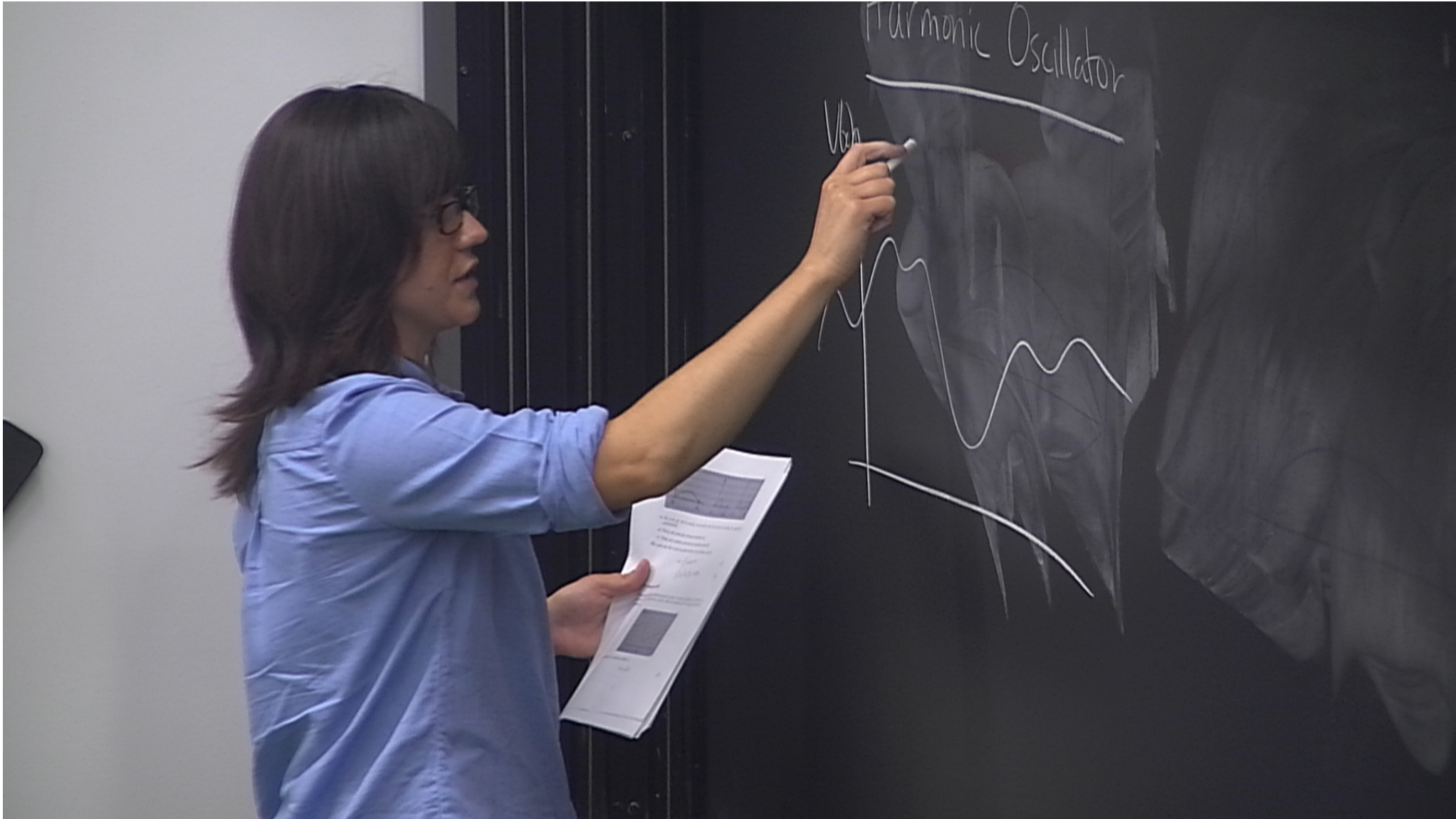




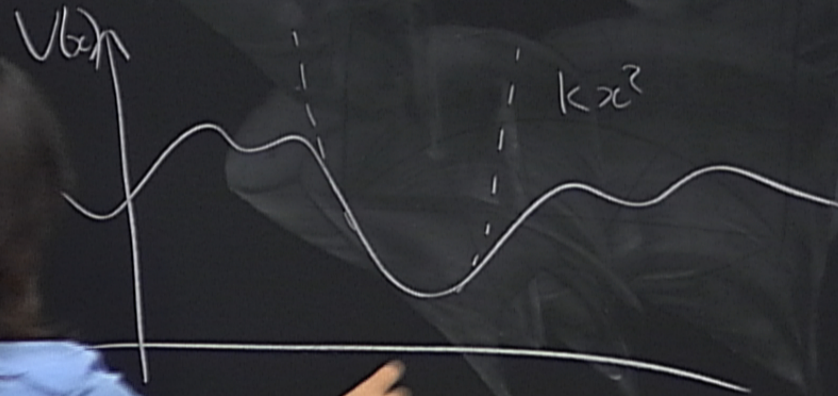
$$= \sum_{n=1}^{\infty} C_n \psi_n(x) e^{-\frac{i}{\hbar} E_n t}$$

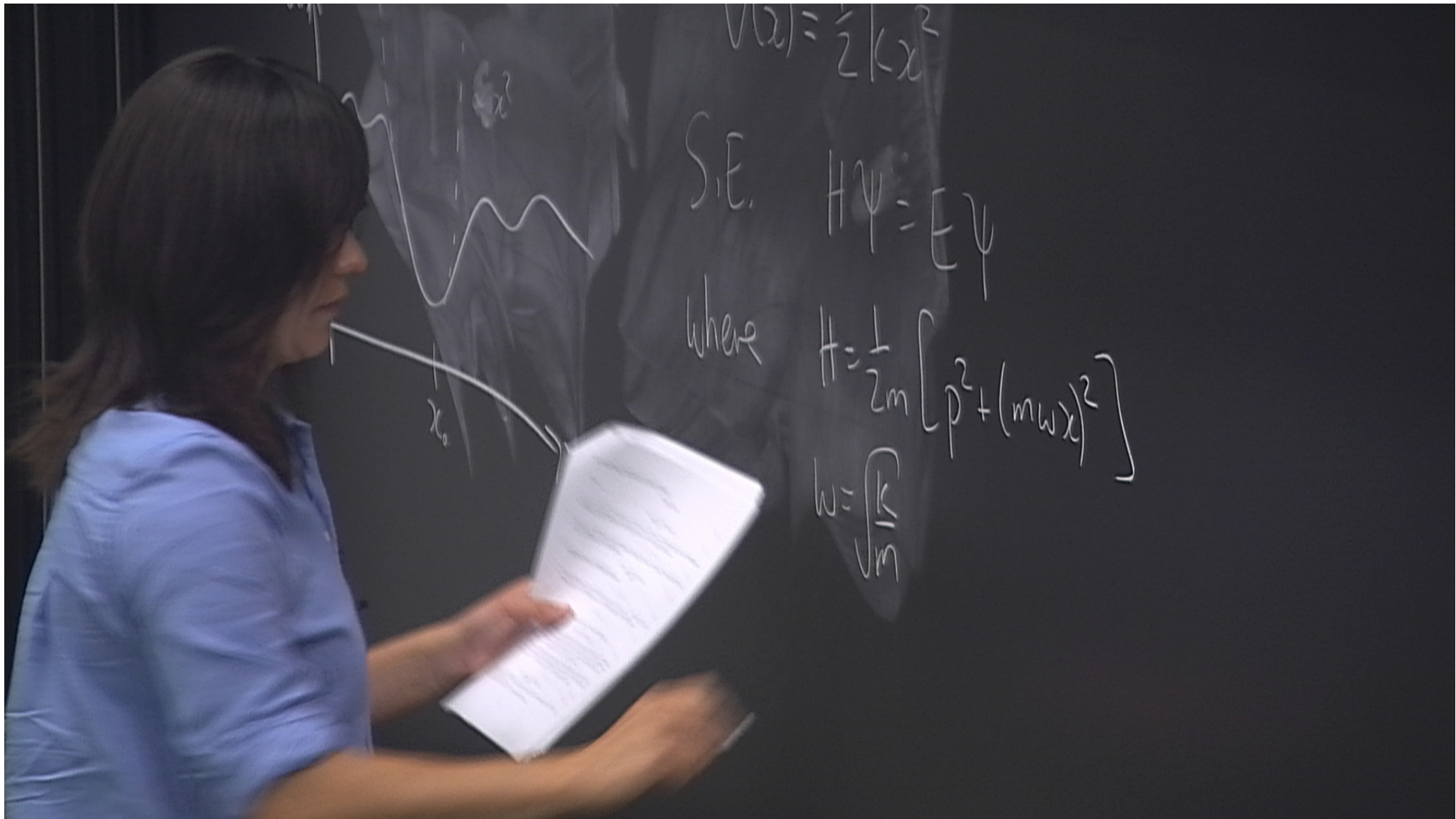
$$= \sum_{n=1}^{\infty} C_n \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right) e^{i \left(\frac{n^2 \pi^2 \hbar}{2ma^2} \right) t}$$





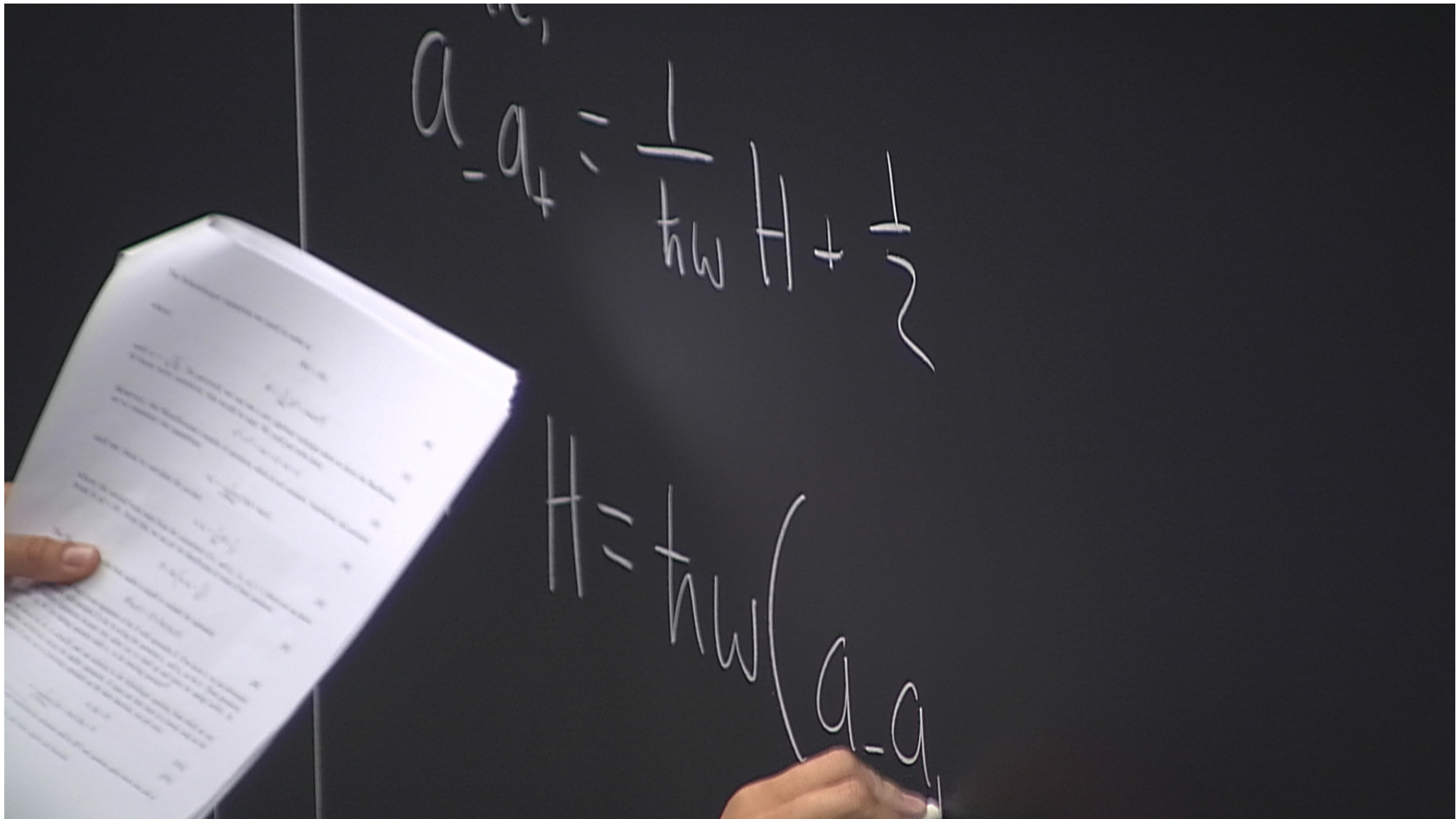
Harmonic Oscillator

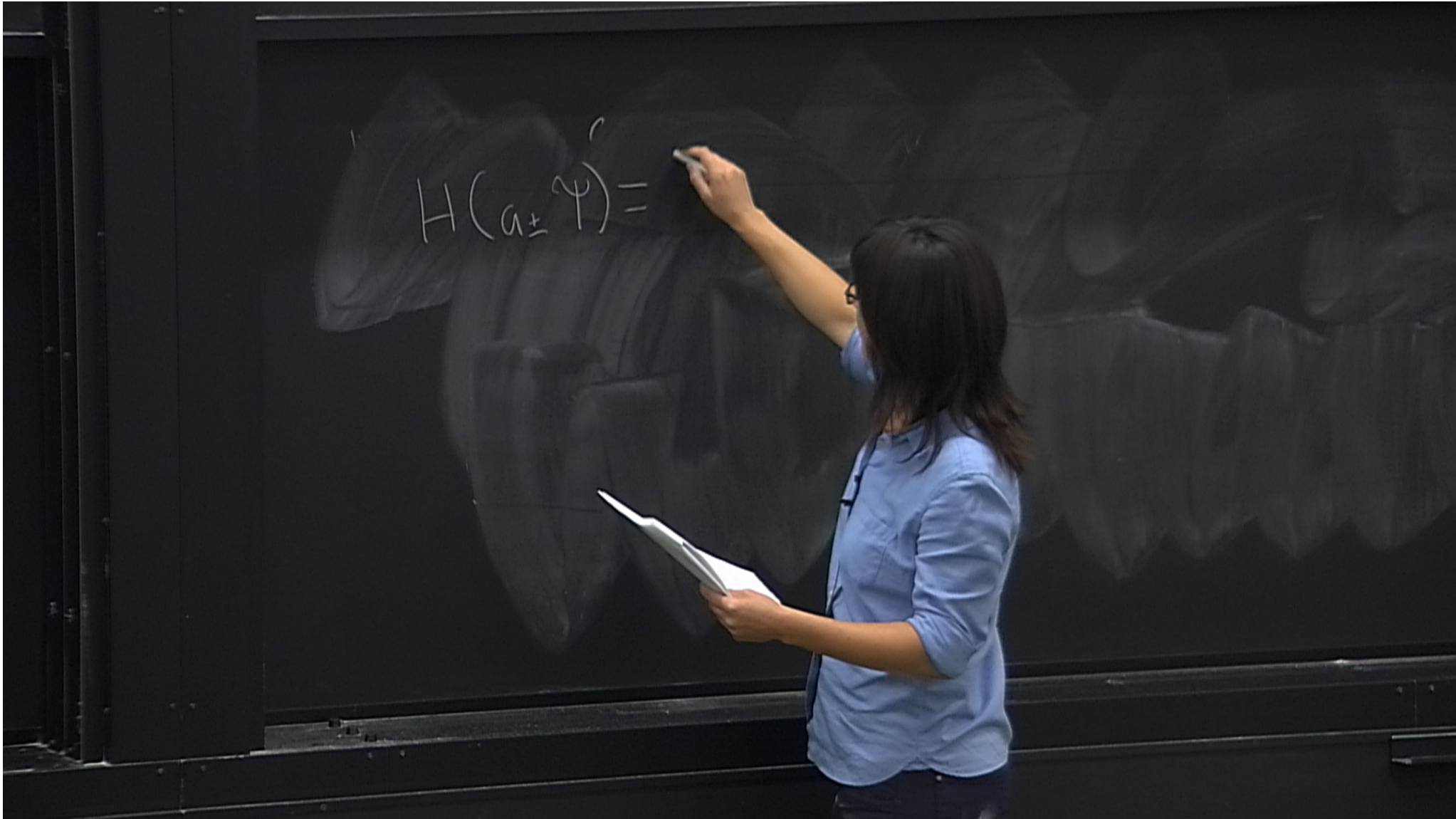


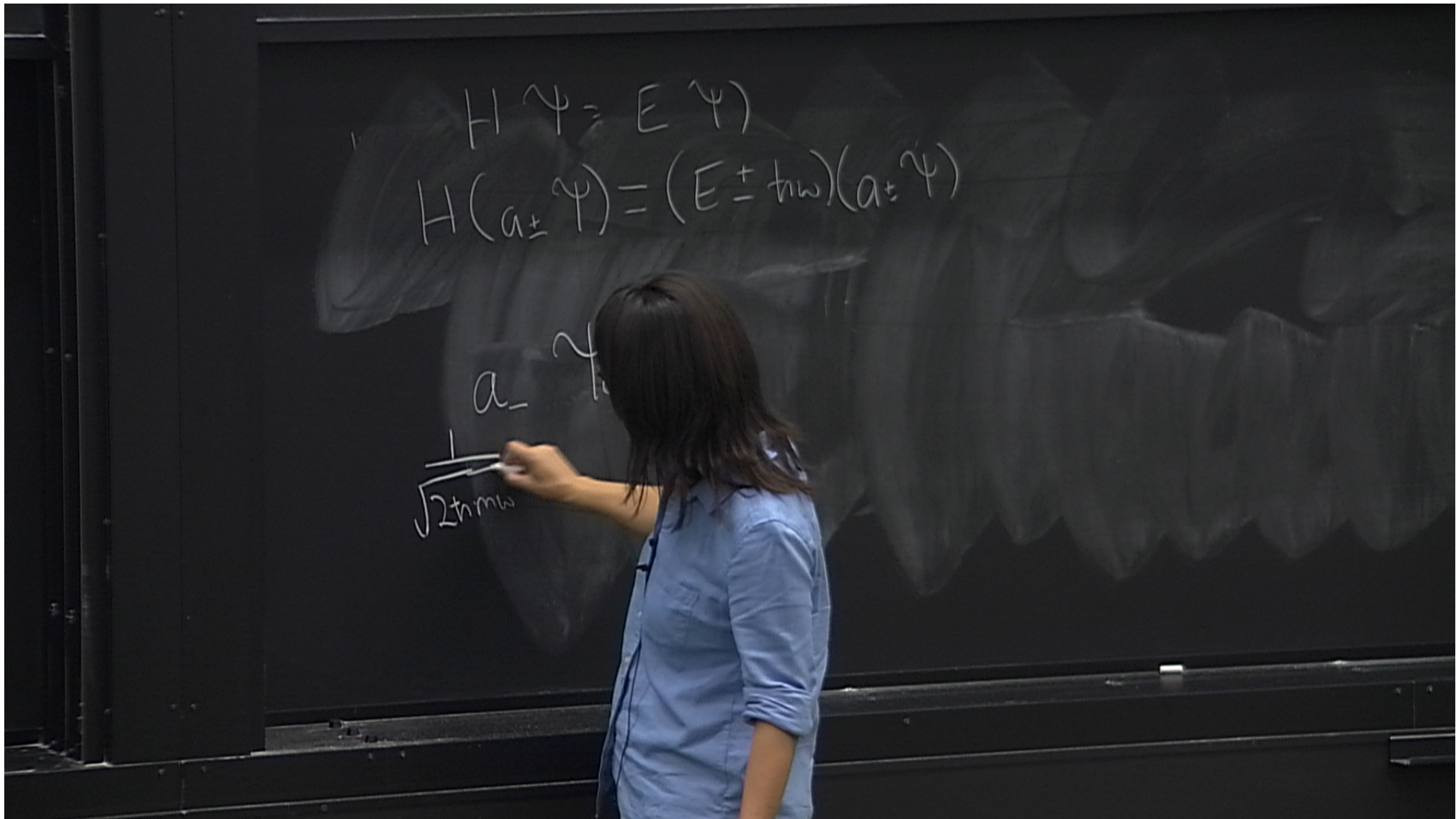


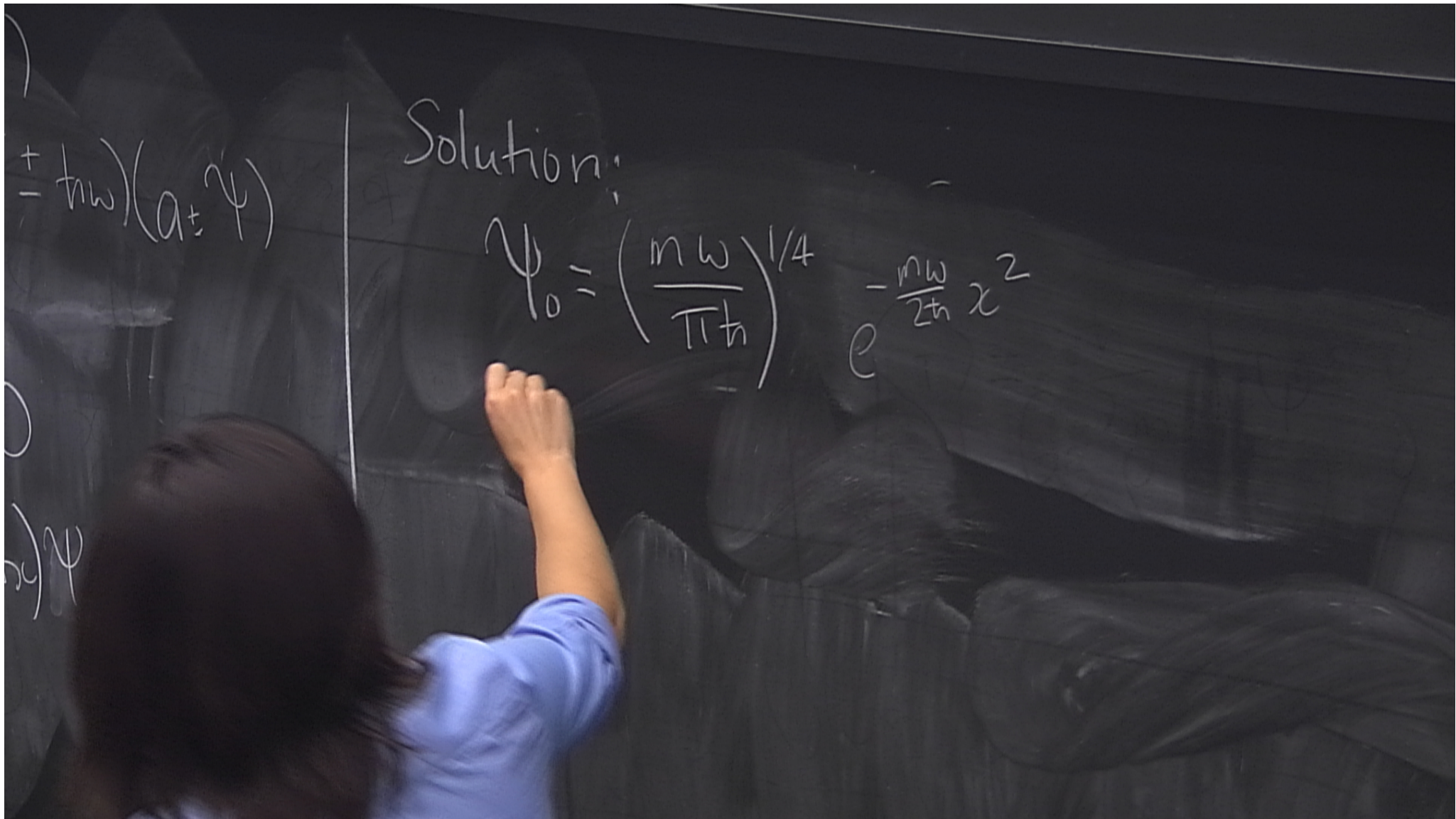
Inspire us to look at operators:

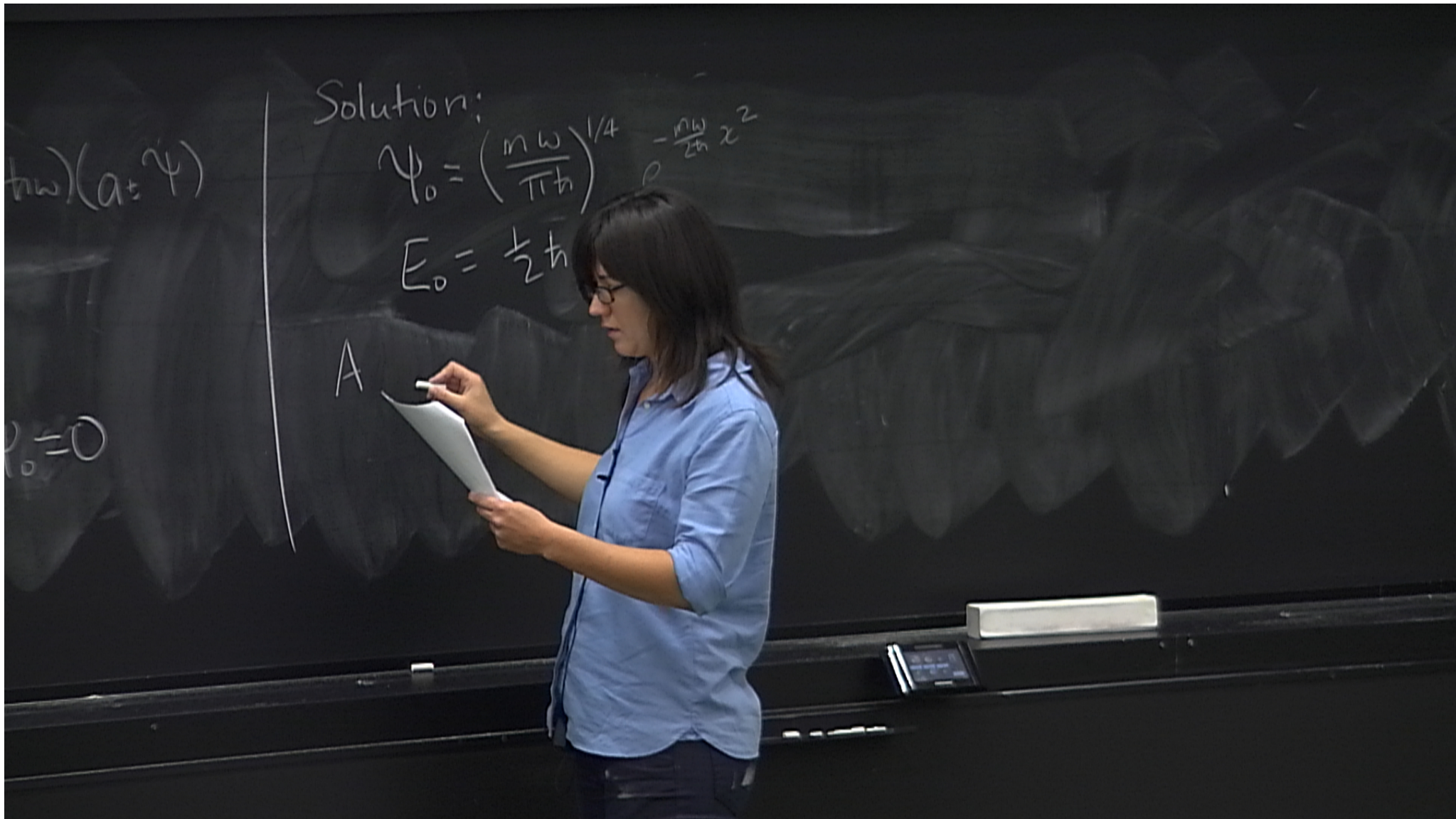
$$a_{\pm} = \left(\begin{array}{c} -i \\ +ip \end{array} \right)$$











Solution:

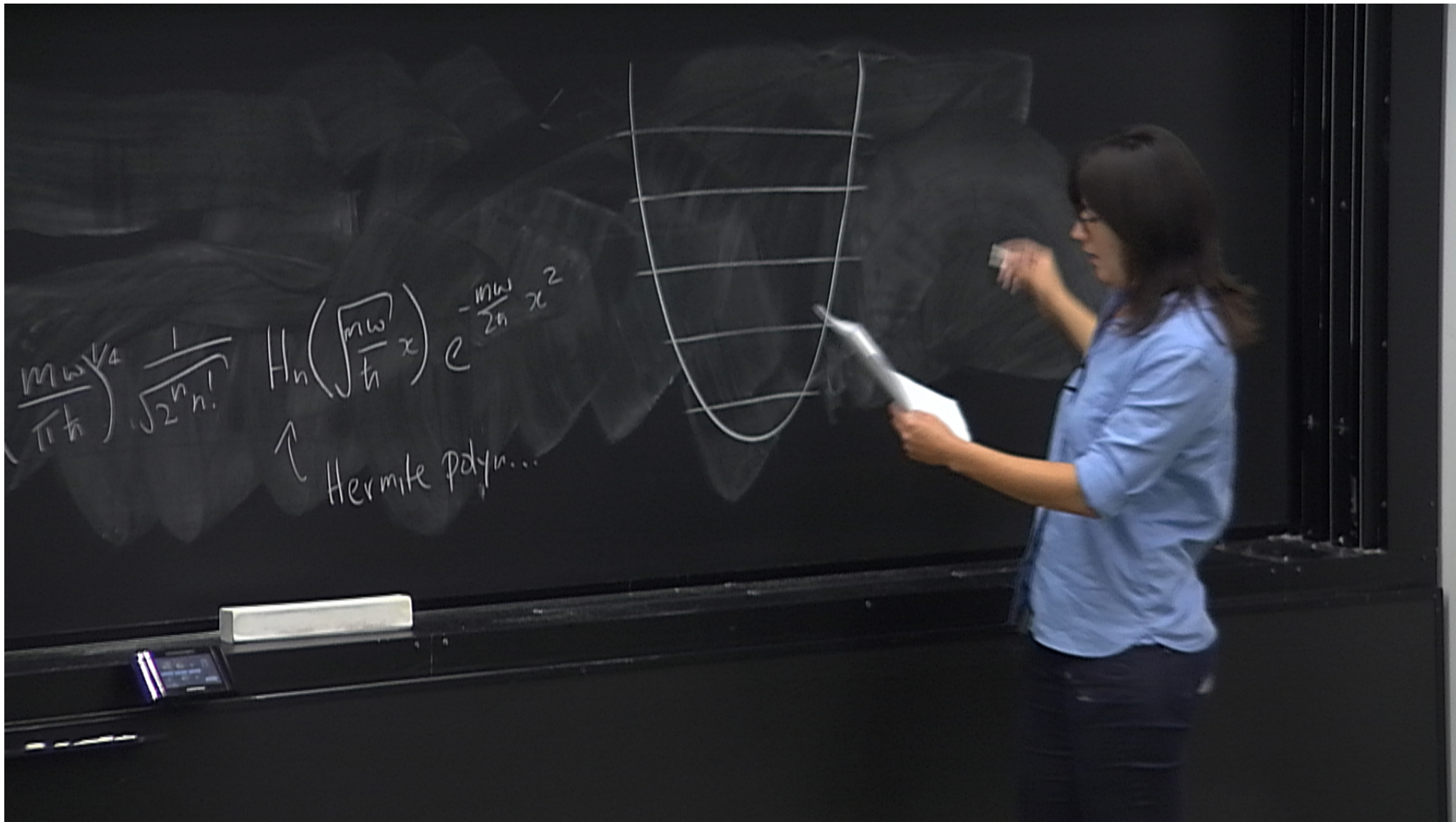
$$\psi_0 = \left(\frac{m\omega}{\pi\hbar} \right)^{1/4} e^{-\frac{m\omega}{2\hbar} x^2}$$

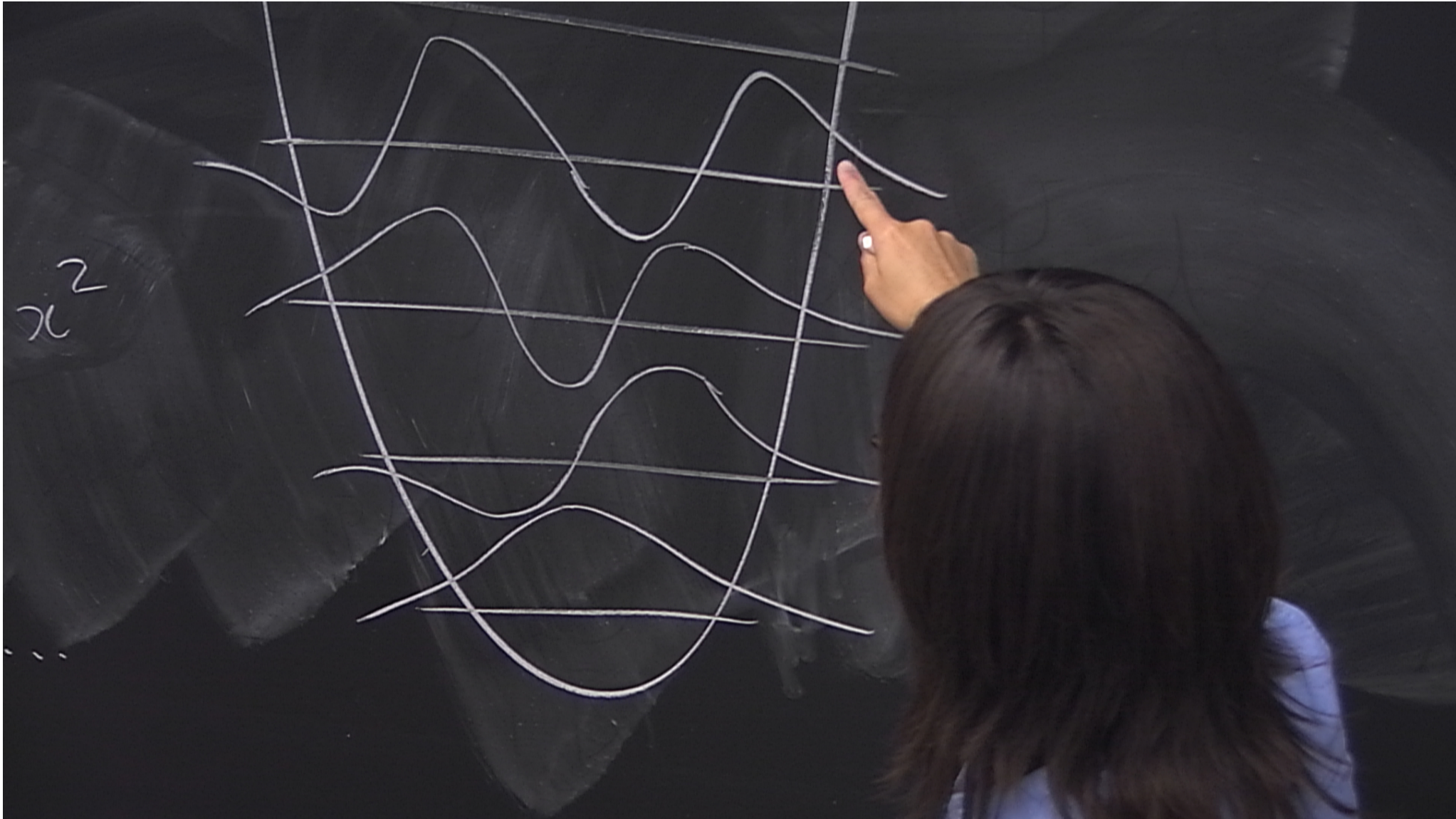
$$E_0 = \frac{1}{2}\hbar\omega$$

All other solns:

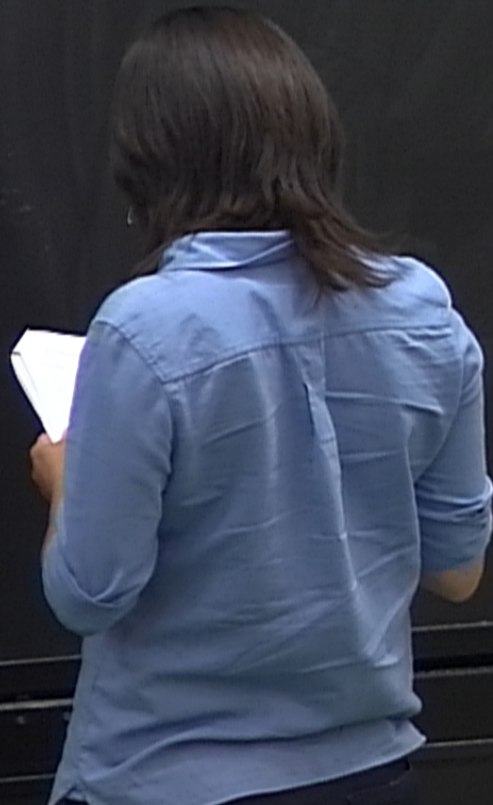
$$\psi_n = \frac{1}{\sqrt{n!}} (a_+)^n \psi_0 = \left(\frac{m\omega}{\pi\hbar} \right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n \left(\sqrt{\frac{m\omega}{\hbar}} x \right) e^{-\frac{m\omega}{2\hbar} x^2}$$

↑





Free particle



Free particle

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} = E \psi$$

Solve: $\Psi(x,t) = Ae^{i(kx - \omega t)}$

Free particle

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} = E \psi$$

Solve: $\psi(x,t) = A e^{i(kx - \frac{\hbar k^2}{2m} t)}$

$$\int_{-\infty}^{+\infty} \psi_k^* \psi_k dx$$

$$\int_{-\infty}^{\infty} \Psi_k^* \Psi_k dx = |A|^2 \int_{-\infty}^{\infty} dx = |A|^2 (\infty)$$

→ physical solns:

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) e^{i(kx - \frac{\hbar k^2}{2m} t)} dk$$

$\phi(k)$

$A e^{i(kx - \frac{\hbar k^2}{2m} t)}$

physical

leads to P.I.B.

interesting properties:

- alternate even/odd
- up in energy $\Rightarrow$ add extra node
- mutually orthogonal
- Complete

inspire us to look at operators

$$a_{\pm} = \frac{1}{\sqrt{2m\hbar\omega}} (\mp p + m\omega x)$$

calculate:

$$a_{-} a_{+} = \frac{1}{\hbar\omega} H + \frac{1}{2}$$

$$\Rightarrow H = \hbar\omega(a_{+} a_{-} + \frac{1}{2})$$

Normalization

$$\int_{-\infty}^{\infty} |\Psi(x,0)|^2 dx = 1$$

Schrodinger equation

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

- if Ψ is a soln, then $A\Psi$ is a soln
- Some solns $\int_{-\infty}^{\infty} |\Psi|^2 dx = \infty$; $\Psi = 0$

$$\begin{aligned} [x,p] &= xp - px \\ &= \hbar \left[x \frac{\partial}{\partial x} - \frac{\partial}{\partial x} x \right] \\ &= \hbar [0 - 1] \\ &= -i\hbar \end{aligned}$$

Free particle

$$-\frac{\hbar^2}{2m} \frac{d^2 \Psi}{dx^2} = E\Psi$$

$$\Psi(x,t) = A e^{i(kx - \frac{\hbar k^2}{2m} t)}$$

$\Rightarrow$ not physical

$$\int_{-\infty}^{\infty} \Psi_k^* \Psi_k dx = |A|^2 \int_{-\infty}^{\infty} dx = |A|^2 (\infty)$$

$\Rightarrow$ physical solns:

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \phi(k) e^{i(kx - \frac{\hbar k^2}{2m} t)} dk$$

$\phi(k)$ is a wave packet

$$\int_{-\infty}^{\infty} \Psi_k^* \Psi_k dx = |A|^2 \int_{-\infty}^{\infty} dx = |A|^2 (\infty)$$

→ physical solns:

$$\Psi(x,t) = \int b(k) e^{i(kx - \frac{\hbar k^2}{2m} t)} dk$$

$\phi(k)$ is a wave