

Title: 13/14 PSI - Integral Transformations & Green's Function - Lecture 4

Date: Aug 30, 2013 09:00 AM

URL: <http://pirsa.org/13080031>

Abstract:

IV) ADVANCED APPLICATIONS



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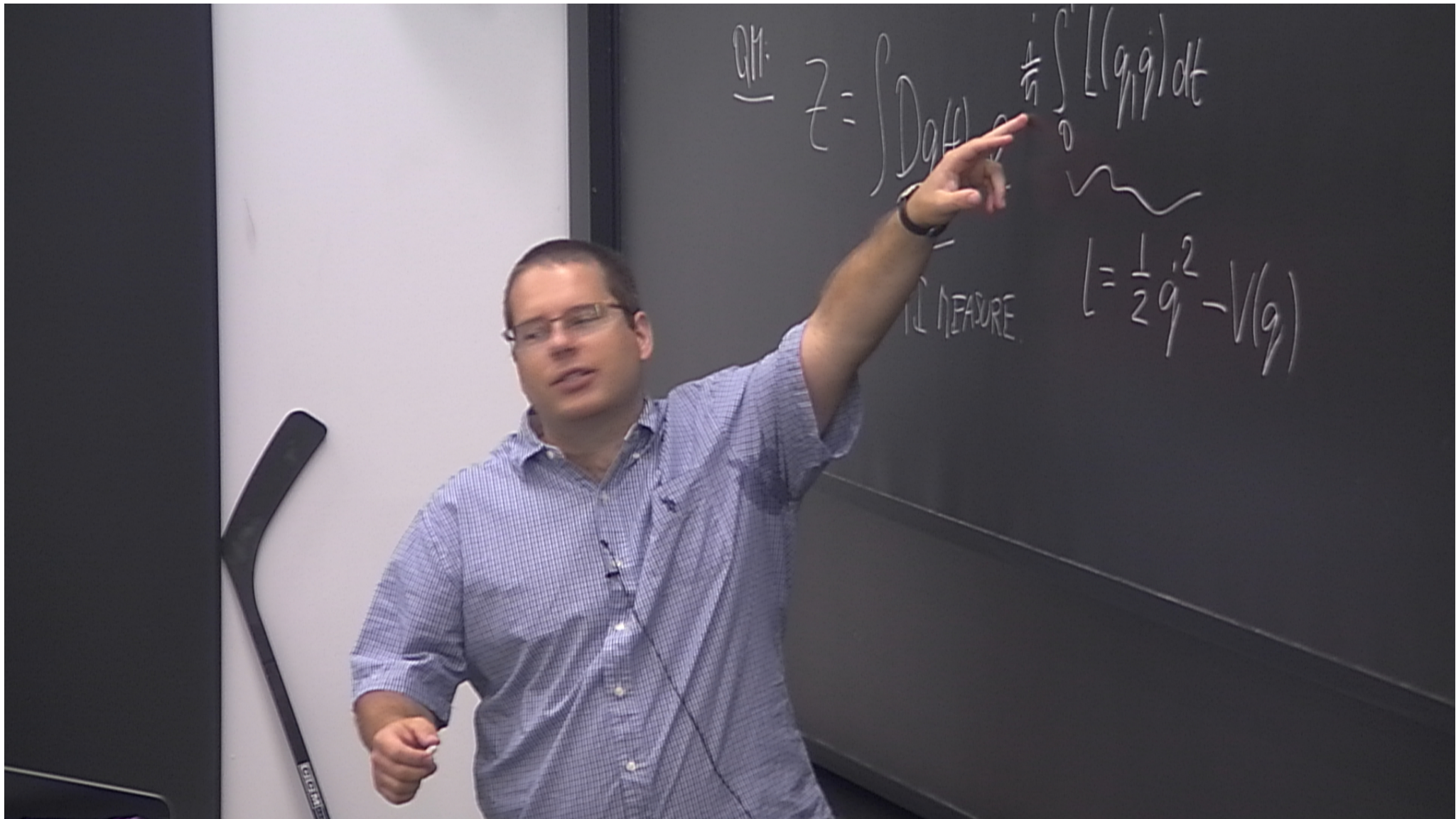


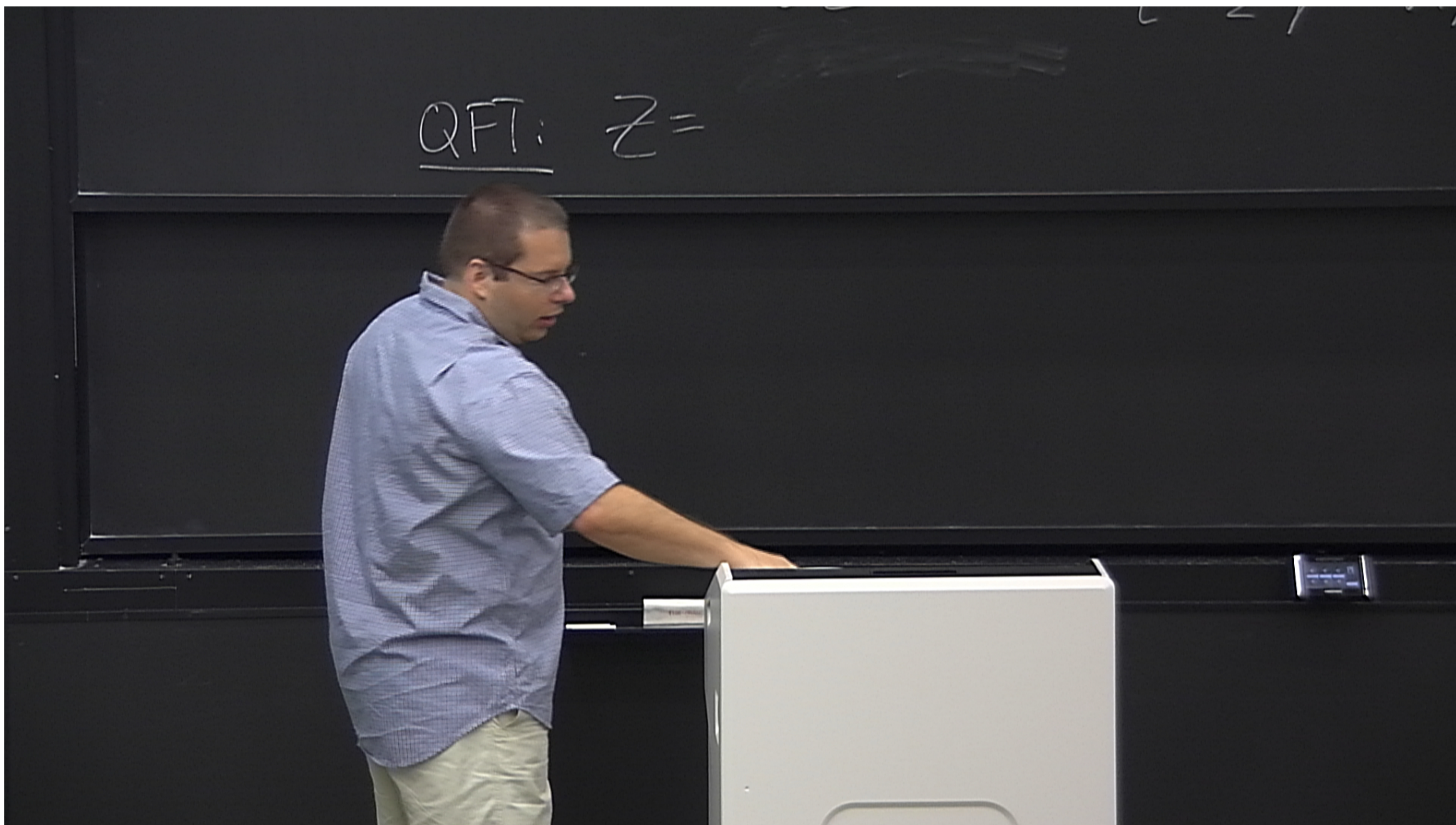
IV) ADVANCED APPLICATIONS

II PROPAGATOR

$$\frac{1}{\hbar} \int_0^T L(q, \dot{q}) dt$$

$$L = \frac{1}{2} \dot{q}^2 - V(q)$$

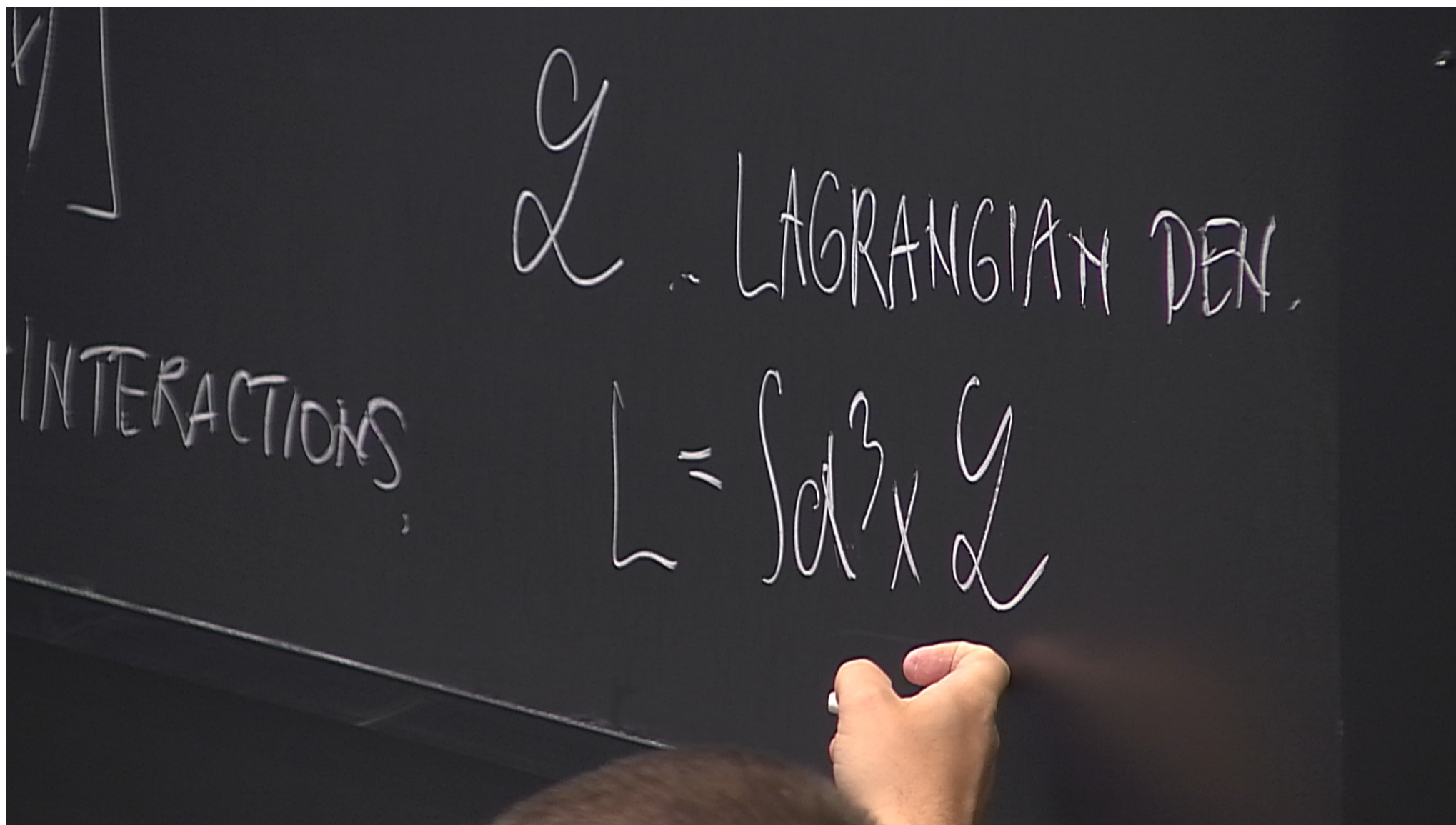


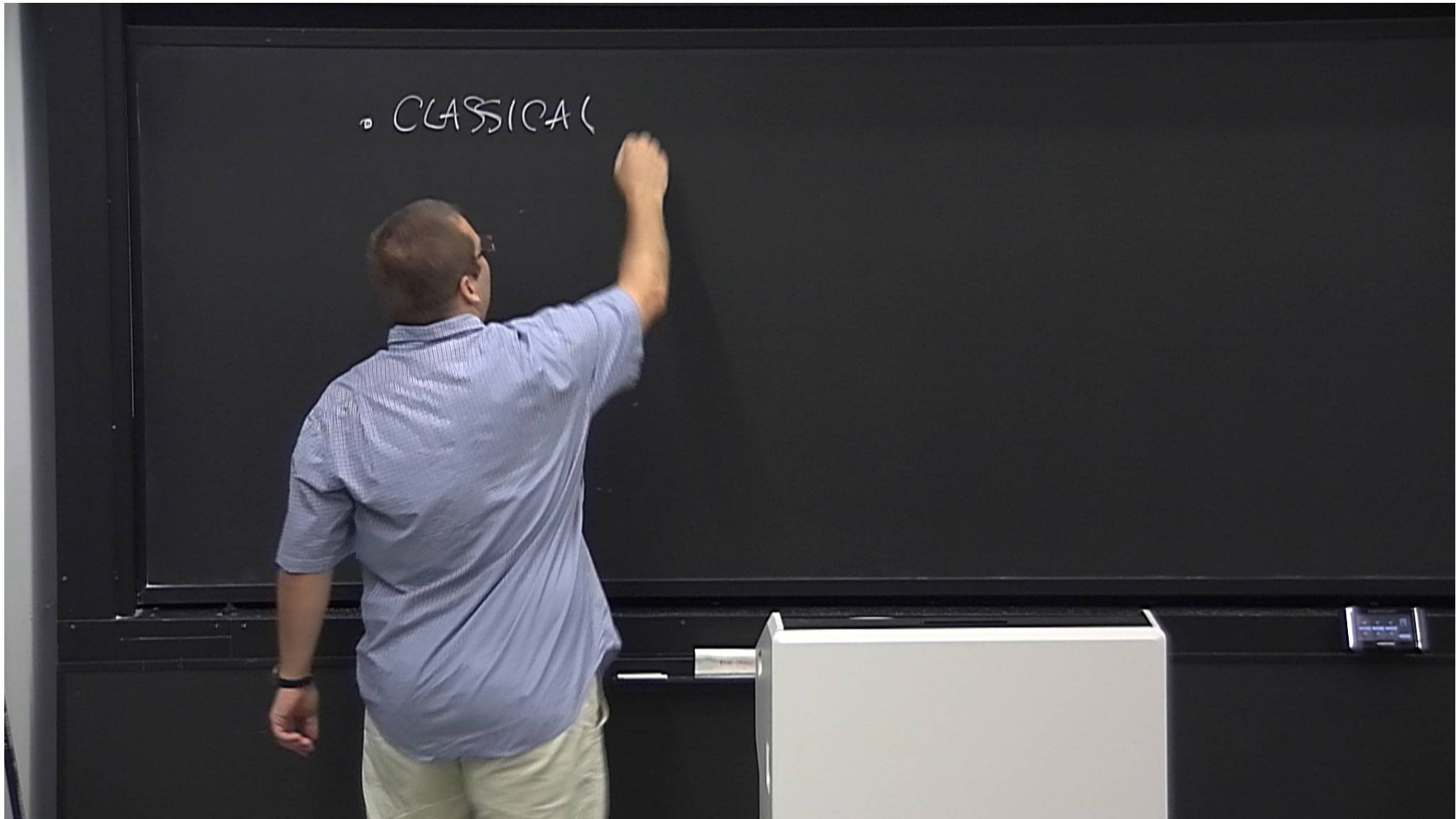


QFT: $Z = \int D$

$$\text{QFT: } Z = \int D\varphi e^{i \int_{t_1}^{t_2} L dt}$$

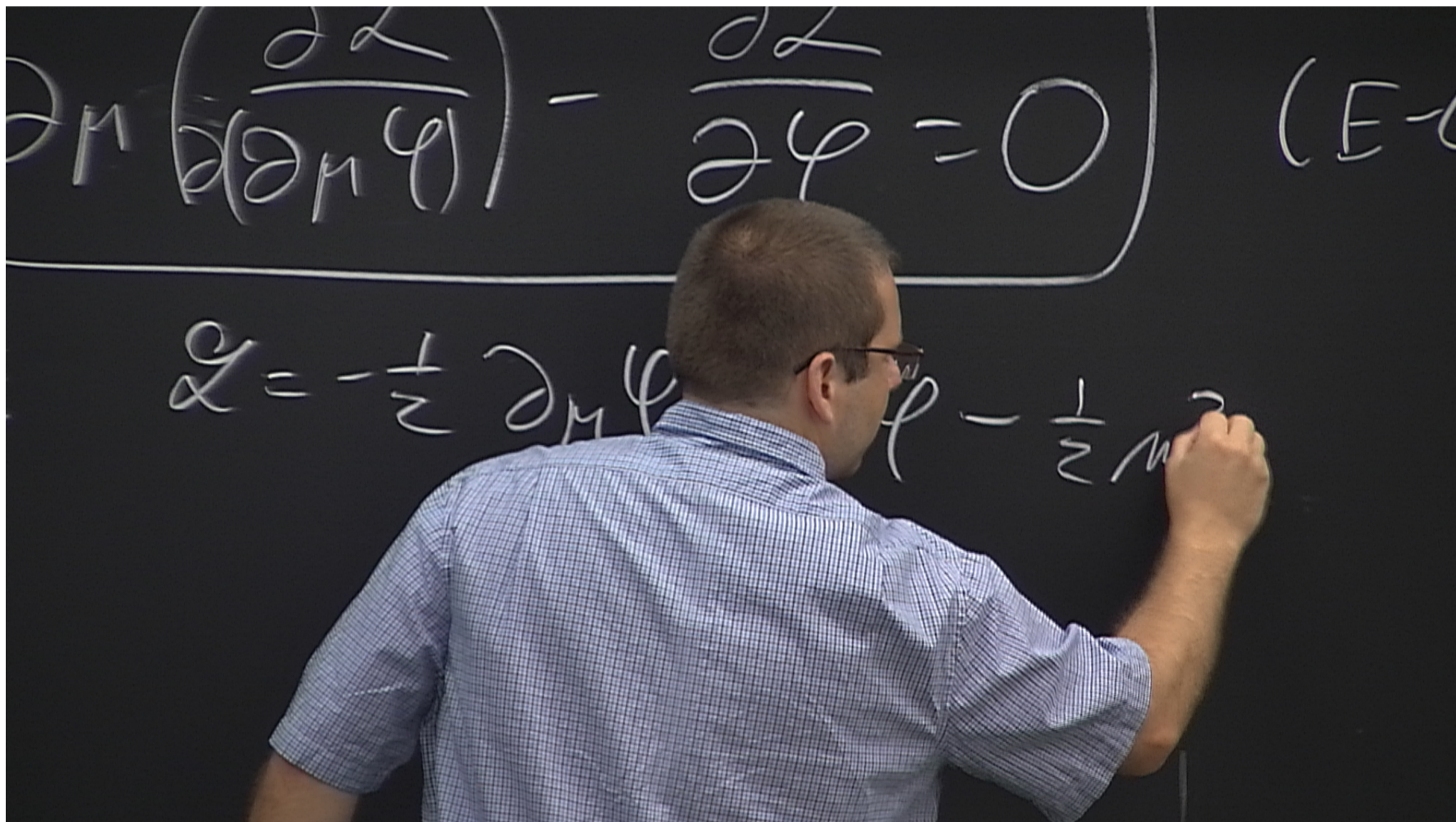
$$\partial = (\partial_t, \nabla)$$





• CLASSICAL EOM:





QFT: $\mathcal{L} = \int d^4x$

FREE FIELD

$\partial = (\partial_+, \nabla)$

• CLASSICAL EOM: $\left[\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi)} \right) - \frac{\partial \mathcal{L}}{\partial \varphi} = 0 \right] \quad (E-L)$

FREE FIE $\mathcal{L} = -\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} m^2 \varphi^2$

$$\square = \partial^\mu \partial_\mu = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

FREE FIELD

$$\partial = (\partial_t, \nabla)$$

INTERACTIONS

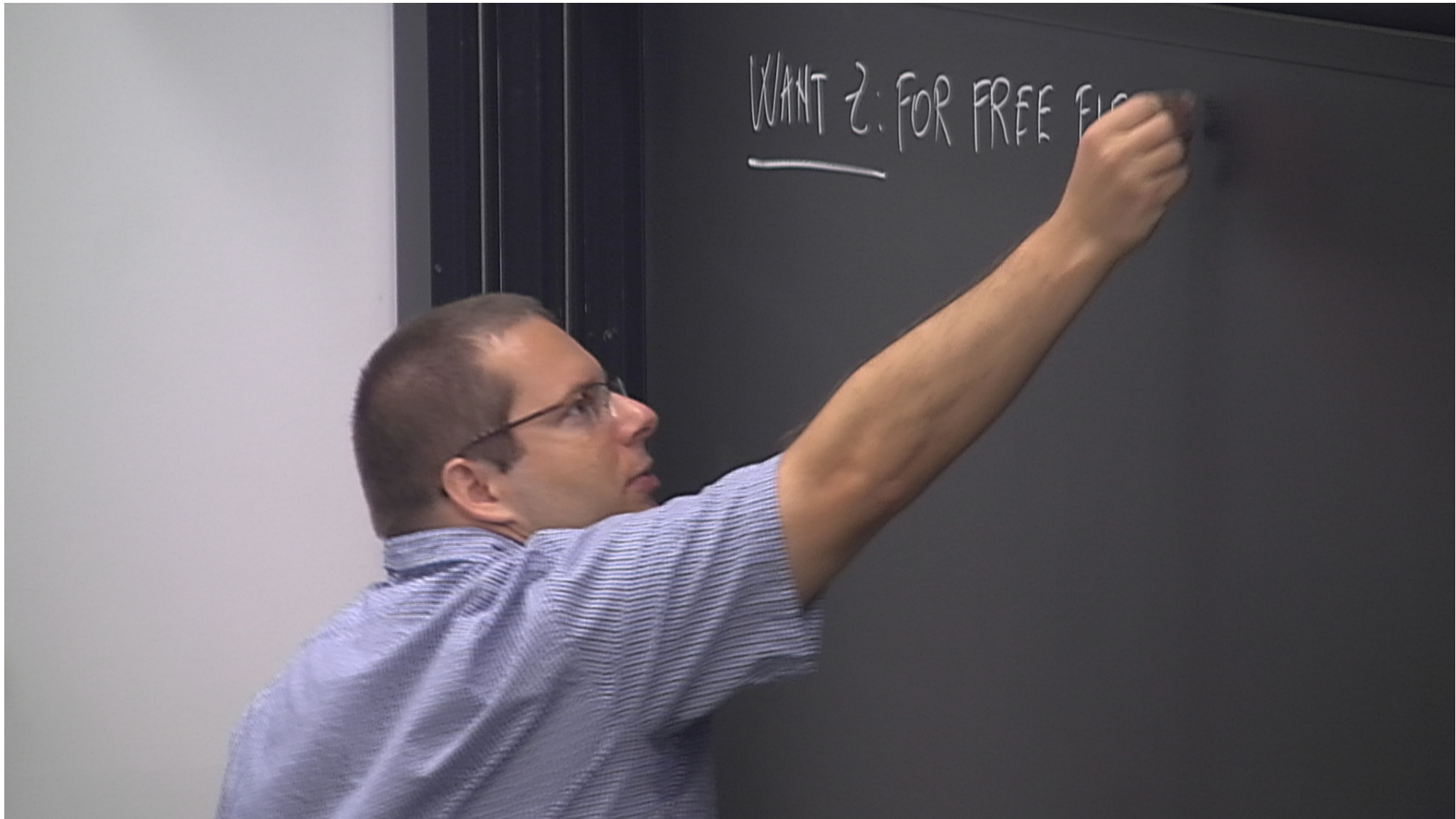
$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi)} \right) - \frac{\partial \mathcal{L}}{\partial \varphi} = 0 \quad (E-L)$$

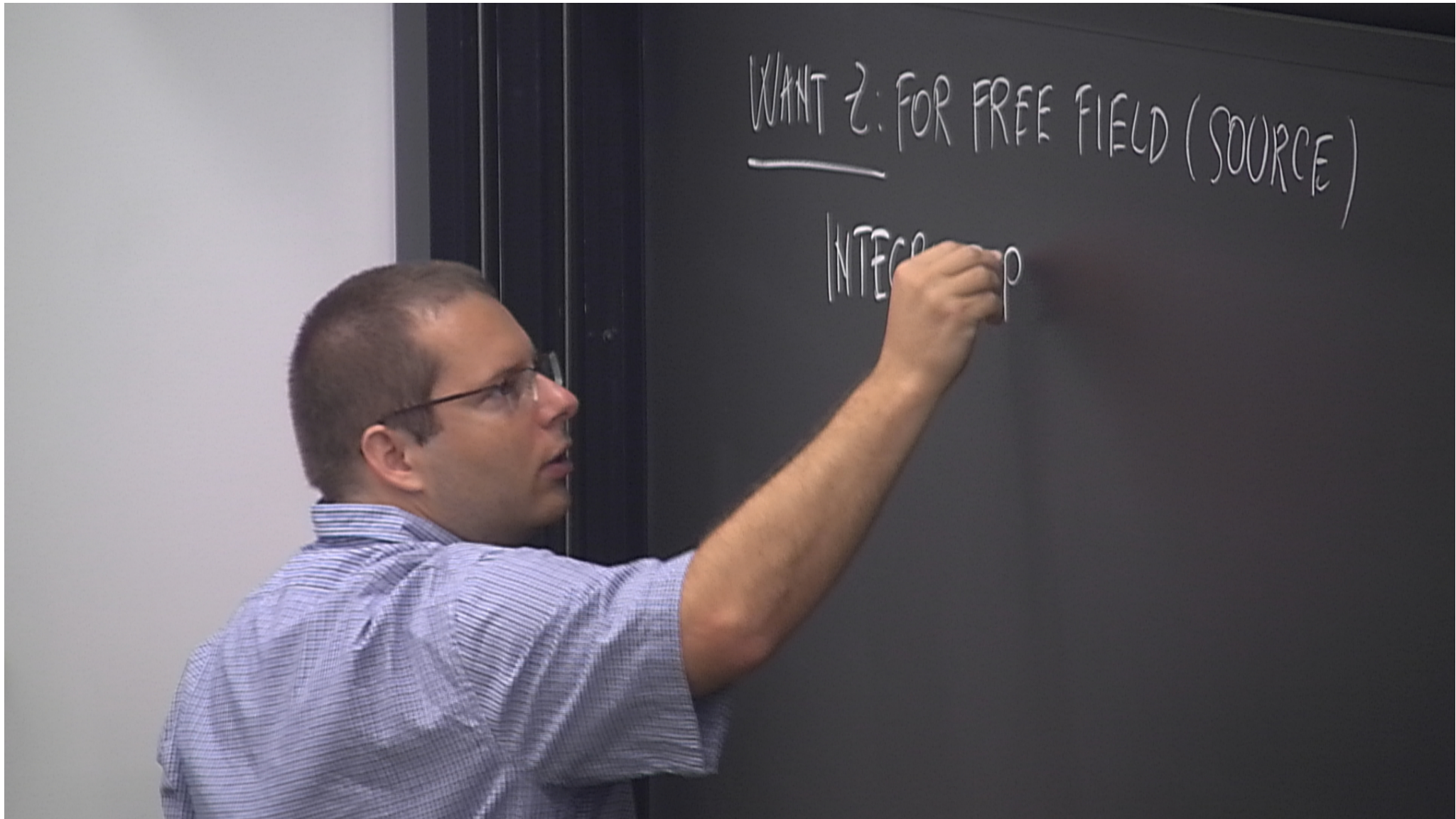
$$\square = \partial^\mu \partial_\mu = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} m^2 \varphi^2$$

$$\partial_\mu (-\partial^\mu \varphi) + m^2 \varphi = 0$$

$$(-\square + m^2) \varphi$$





WANT z : FOR FREE FIELD (SOURCE)

INTEGRATE BY PARTS

$$z = \int i \exp$$

WANT Z: FOR FREE FIELD (SOURCE)

INTEGRATE BY PARTS

$$= \int D\varphi \exp\left(i \int d^4x\right)$$

WANT Z : FOR FREE FIELD (SOURCE)

INTEGRATE BY PARTS

$$Z = \int D\varphi \exp \left(i \int d^4x \frac{1}{2} \varphi (\partial^2 - m^2) \varphi + J\varphi \right)$$

(I)

REIND:

$$\int_{-\infty}^{\infty} dq_1 \dots dq_N \quad l^{\frac{1}{2}} q_0 A$$

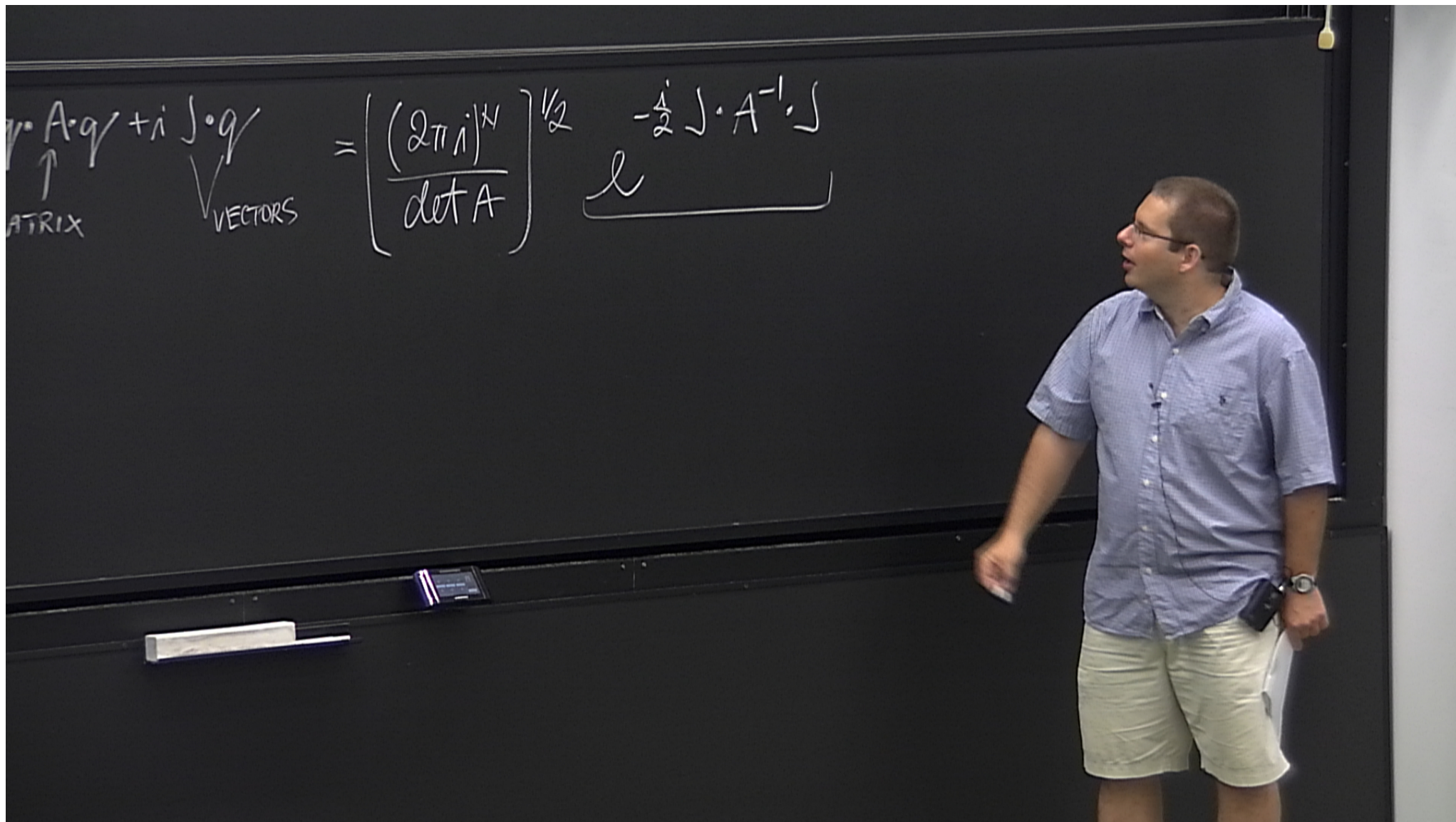
$$\int_{-\infty}^{\infty} dq_1 \dots dq_n \, e^{i \frac{\Delta}{2} q_0 A \cdot q}$$

REIND

$$\int_{-\infty}^{\infty} dq_1 \dots dq_N \, e^{i \frac{1}{2} q \cdot A \cdot q + i \int q \cdot V}$$

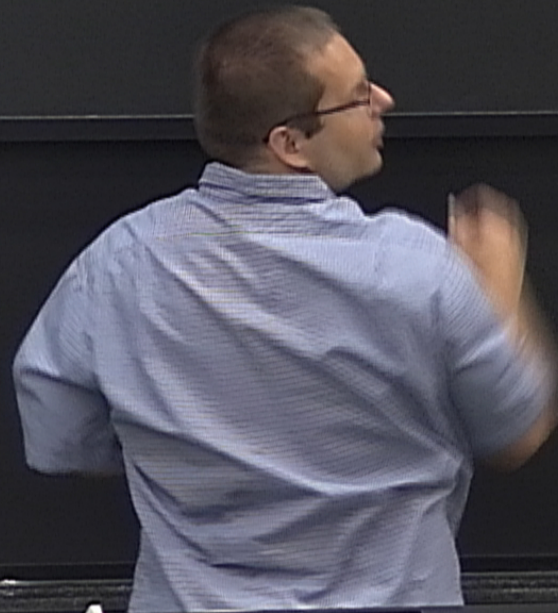
↑
MATR

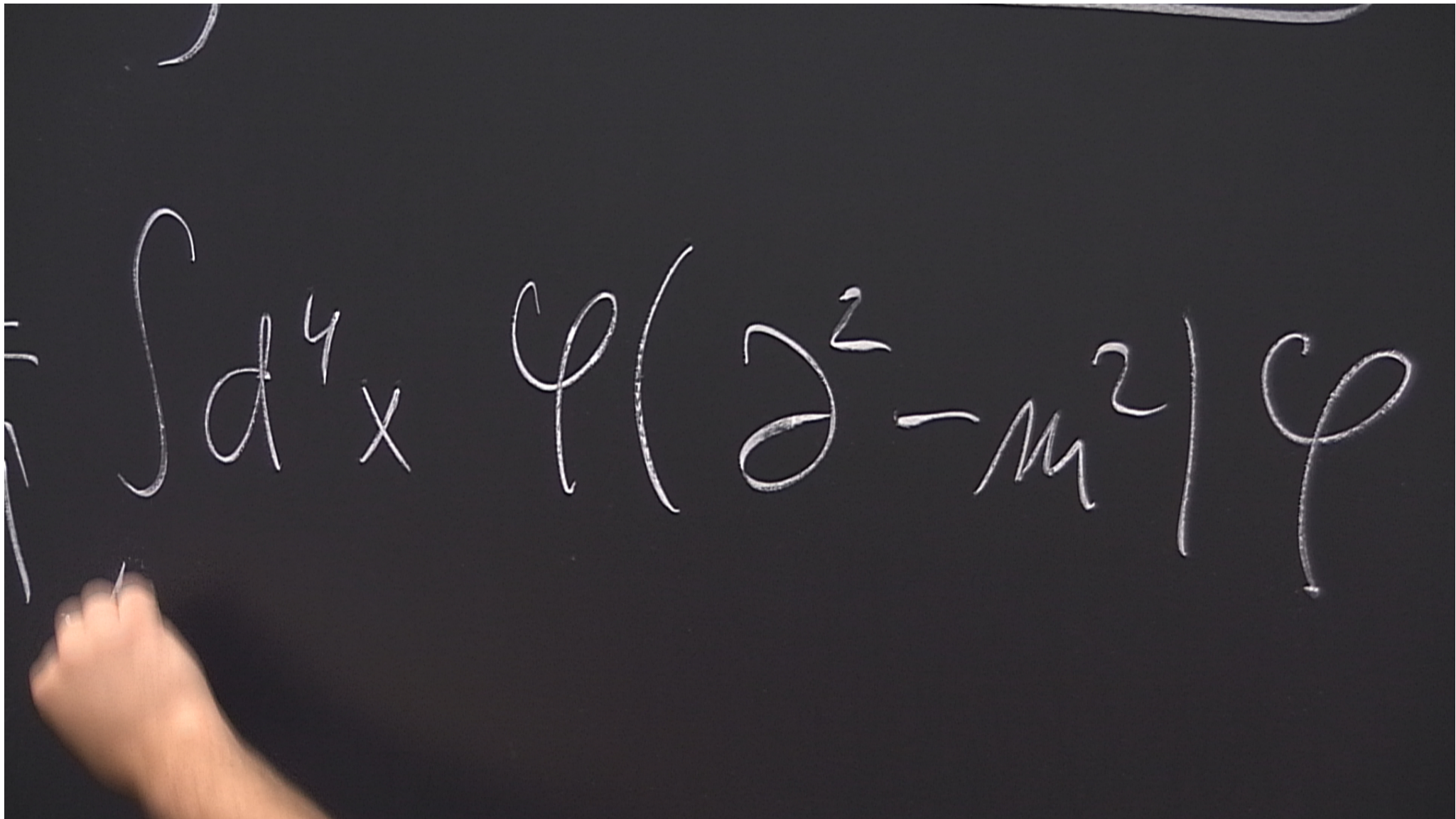
↓
VECTORS



$$Z = \int D\varphi \exp \left(i \int d^4x \underbrace{\frac{1}{2} \varphi (\partial^2 - m^2) \varphi}_A + \underbrace{J\varphi}_{\infty\text{-DIM VECTOR (EACH } x)} \right)$$

(I)





A photograph of a chalkboard with a handwritten mathematical expression in white chalk. The expression is $\int d^4x \, \varphi (\partial^2 - m^2) \varphi$. A person's hand is visible on the left side, holding a piece of chalk and having just finished writing the expression. The chalkboard is dark, and the chalk is white, creating a high contrast. The handwriting is cursive and fluid.

$$\int d^4x \, \varphi (\partial^2 - m^2) \varphi$$

$$\int d^4x \varphi(\partial^2 - m^2) \varphi$$

WANT

$$= \int d^4x d^4z \varphi(x)$$

NEED INVERSE TO A:

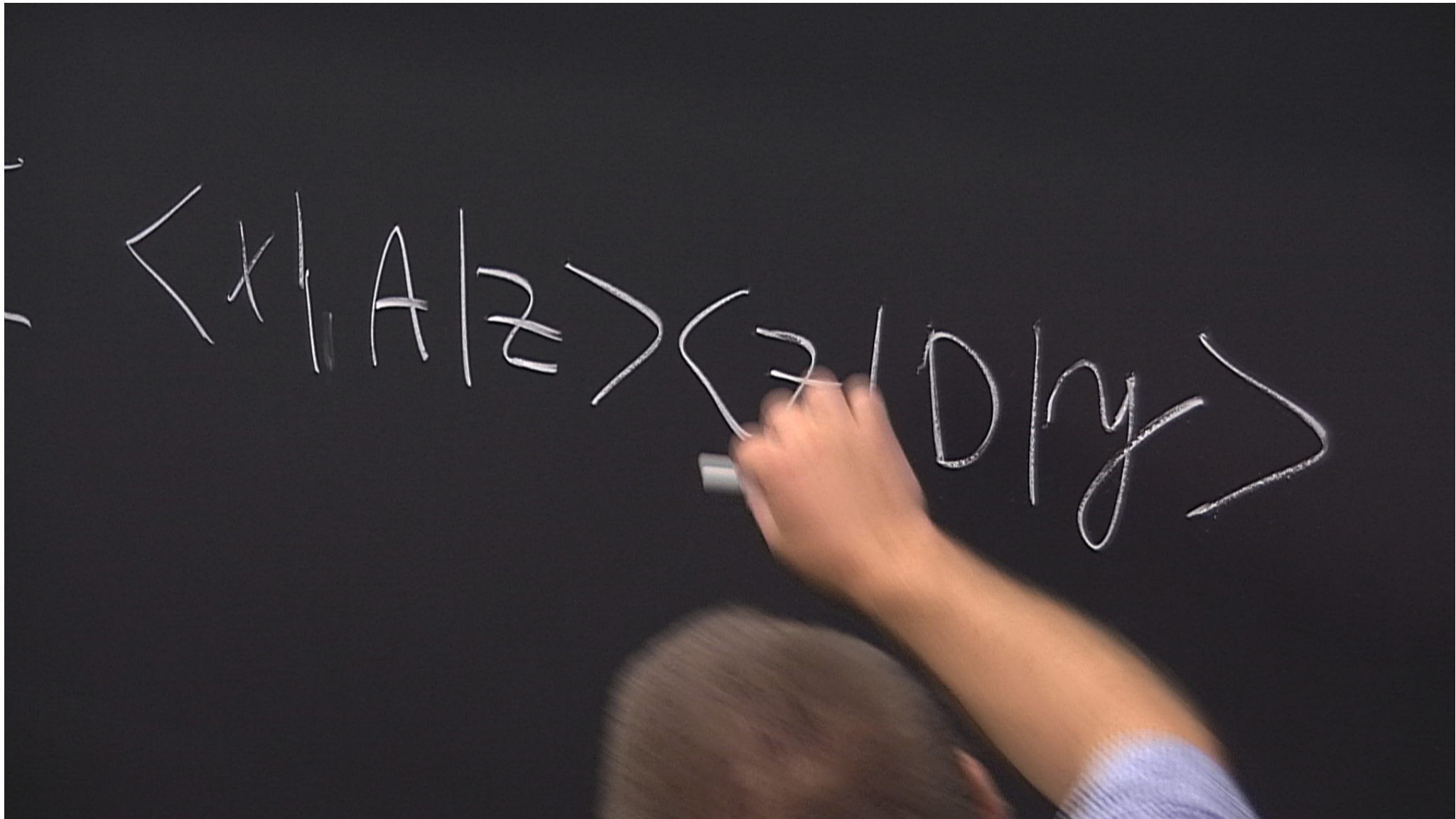
$$D = A^{-1}$$

$E | AD$

NEED INVERSE TO A:

$$D = A^{-1}$$

$$\langle x | AD | y \rangle$$

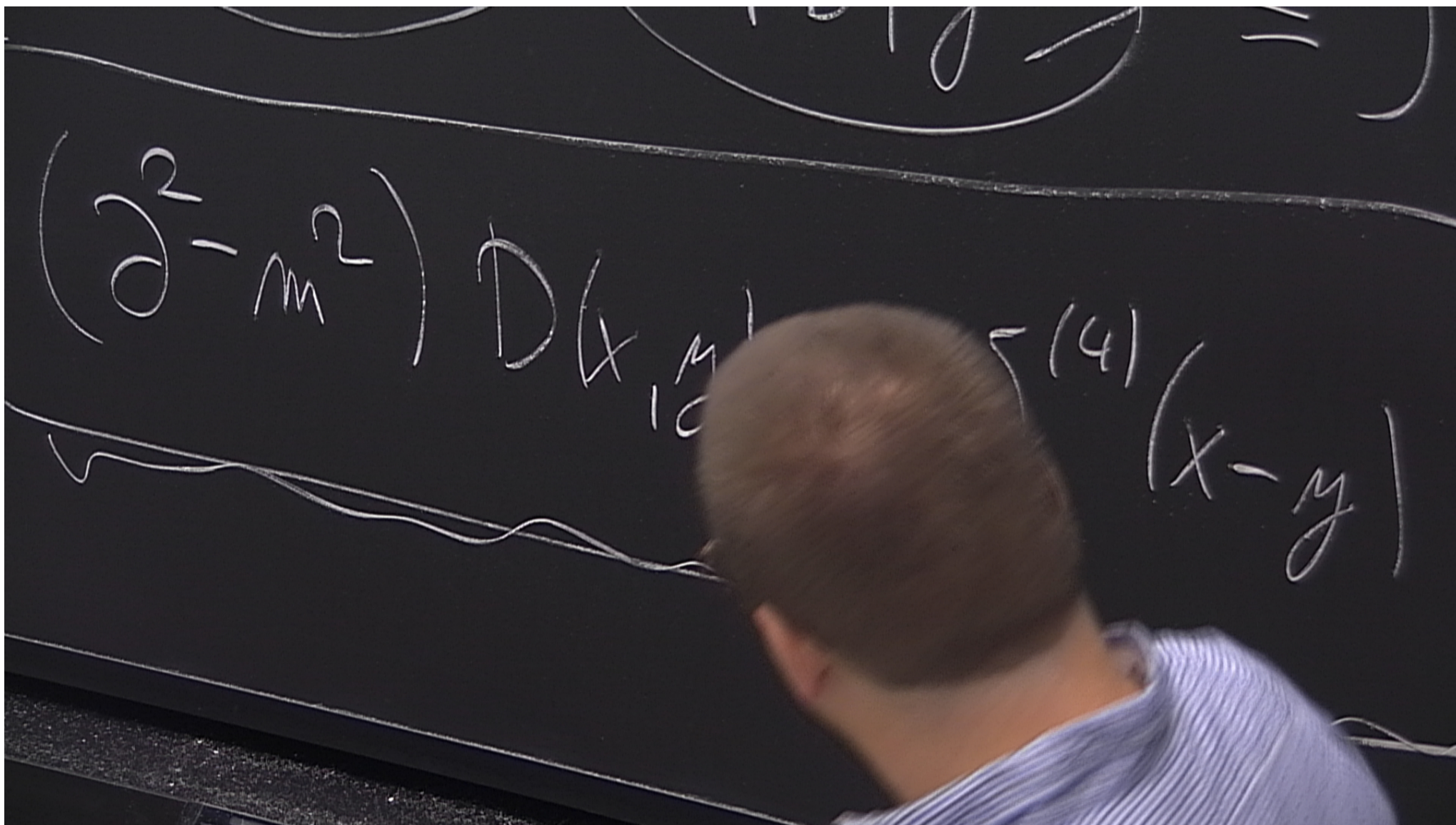


$$= \sum_z \langle x | A | z \rangle \langle z | D | y \rangle$$

$D(z, y)$

$$D(z, y)$$

$$\langle z | D | y \rangle = \int d^4 z$$

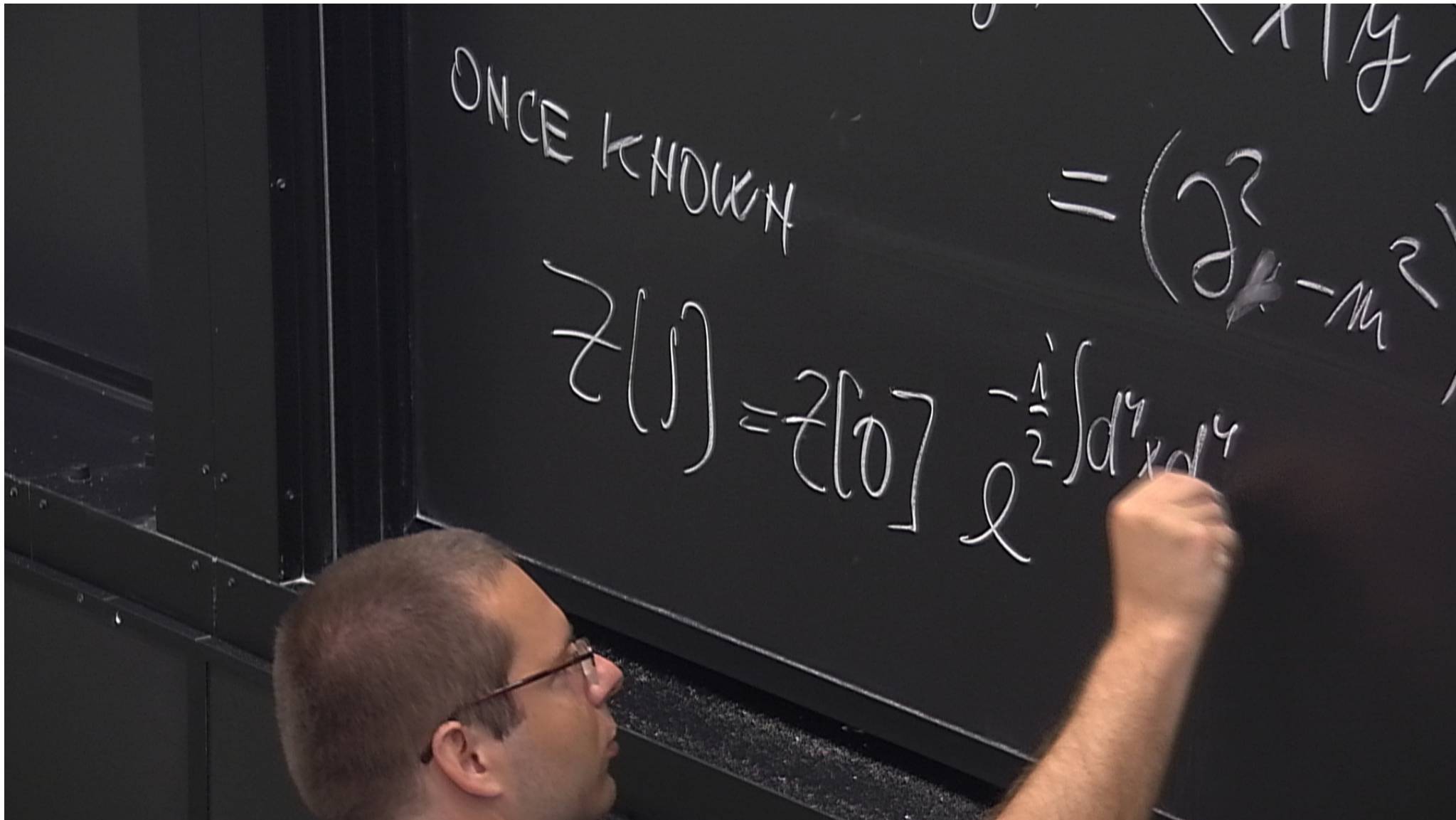


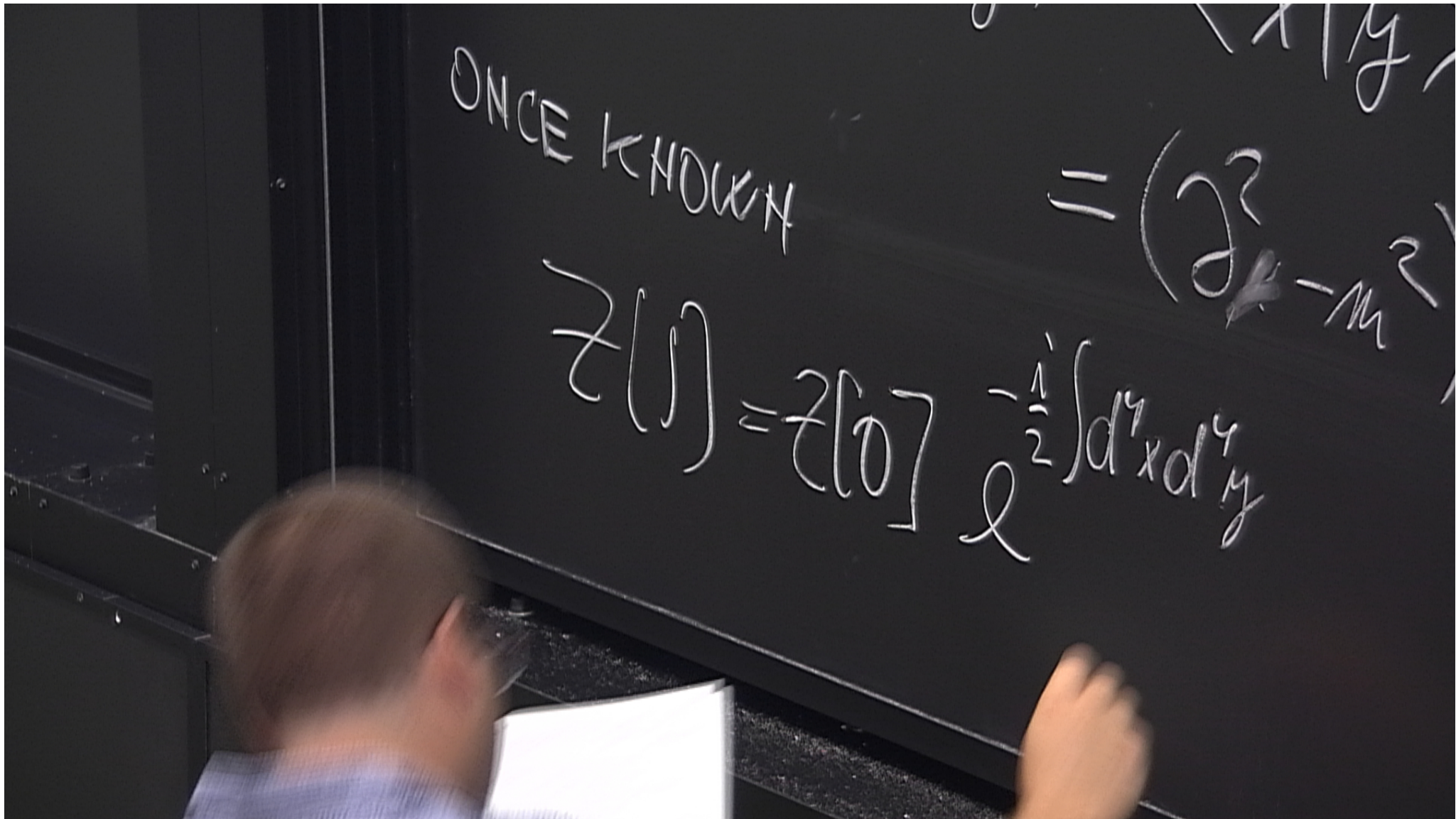
GREEN'S FUNCTION

$(\partial_x - m)$ $D(z)$

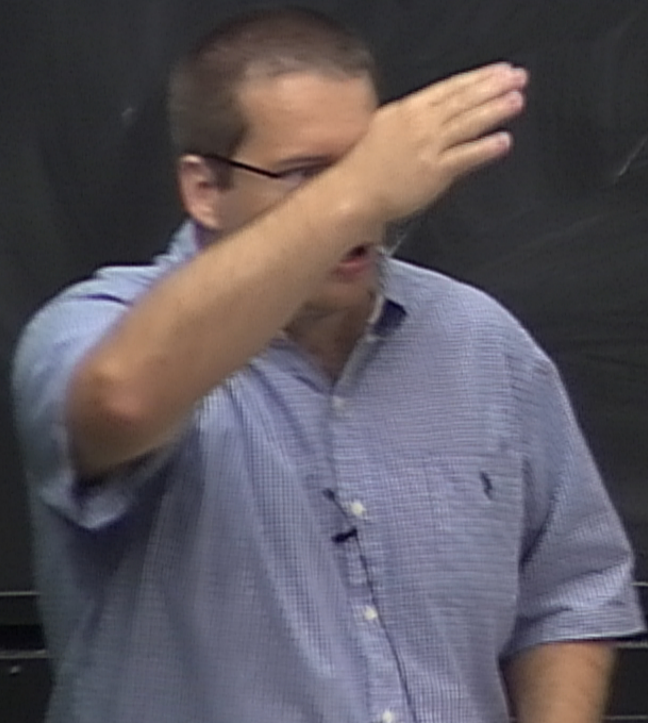
GREEN'S FUNCTION FOR

~ PROPAGATOR





• FOR (I) TO CONVERGE



SET

$$m^2 \rightarrow m^2 - i\varepsilon$$

$$i(-m^2) \rightarrow -im^2 - \varepsilon$$

FEYNMAN'S PRESCRIPTION

- SOLVE FOR (D) USING FT.

$$\frac{1}{2}$$

$$1(-m^2) \rightarrow -im^2 - \varepsilon$$

$$\frac{1}{k^2 + i\varepsilon} e^{ik \cdot x} = \frac{1}{(2\pi)^4} \int d^4 k e^{ik \cdot x}$$

$$\Rightarrow \boxed{Dk = -k^2 - m^2 + 1\varepsilon}$$

EXPLICITLY $k = (k^0, \vec{k})$ $x = (t, \vec{x})$



$$\Rightarrow \boxed{Dk = -k^2 - m^2 + 1\varepsilon}$$

E EXPLICITLY $k = (k^0, \vec{k})$ $x = (t, \vec{x})$

$$^2 = -k^0^2 -$$

GOING BACK TO X-SPACE

MORE EXPLICITLY

$$k = (k^0, \vec{k})$$

$$x = (t, \vec{x})$$

$$k^2 = -k^0^2 + \vec{k}^2$$

X-SPACE

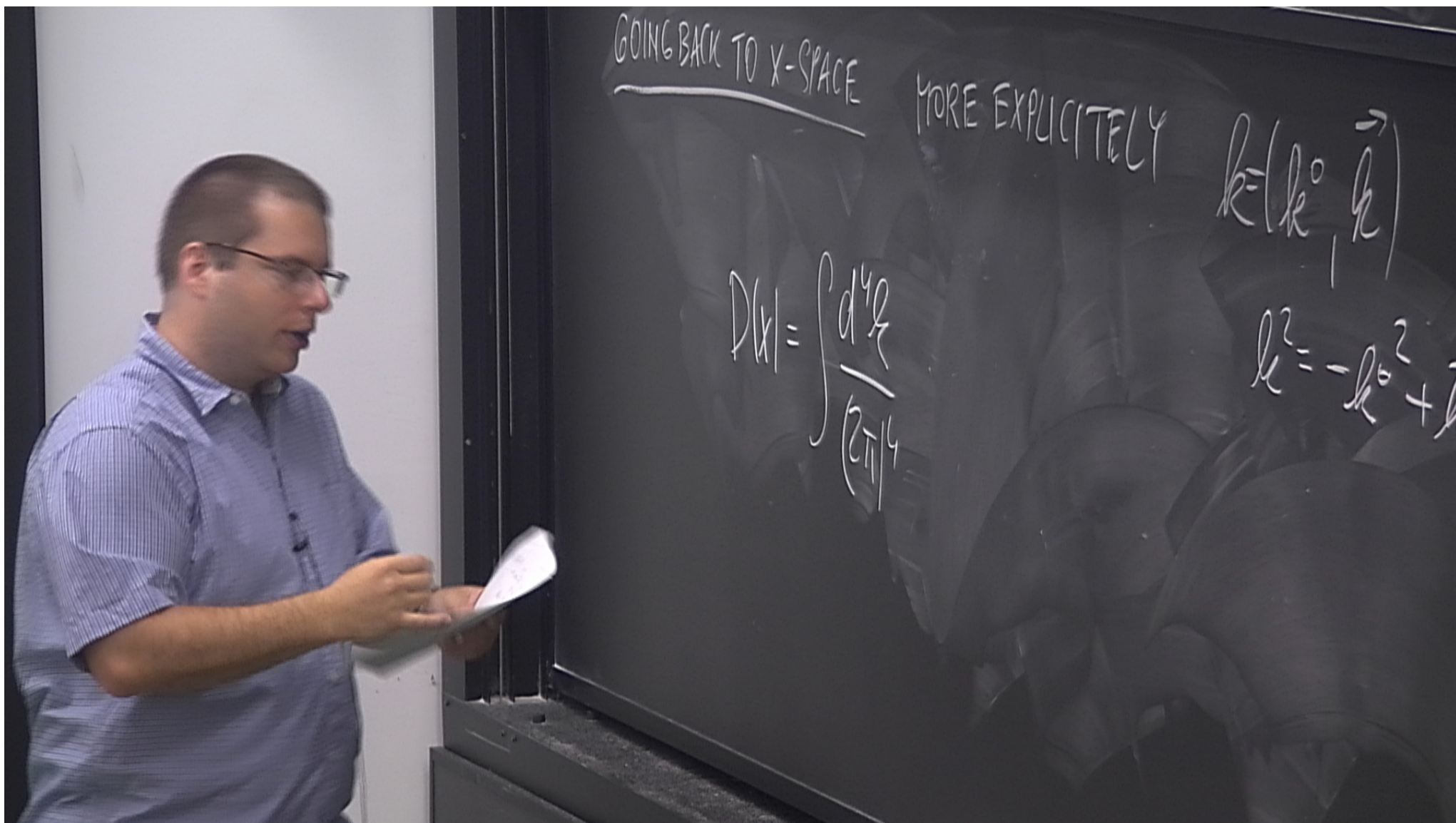
MORE EXPLICITLY

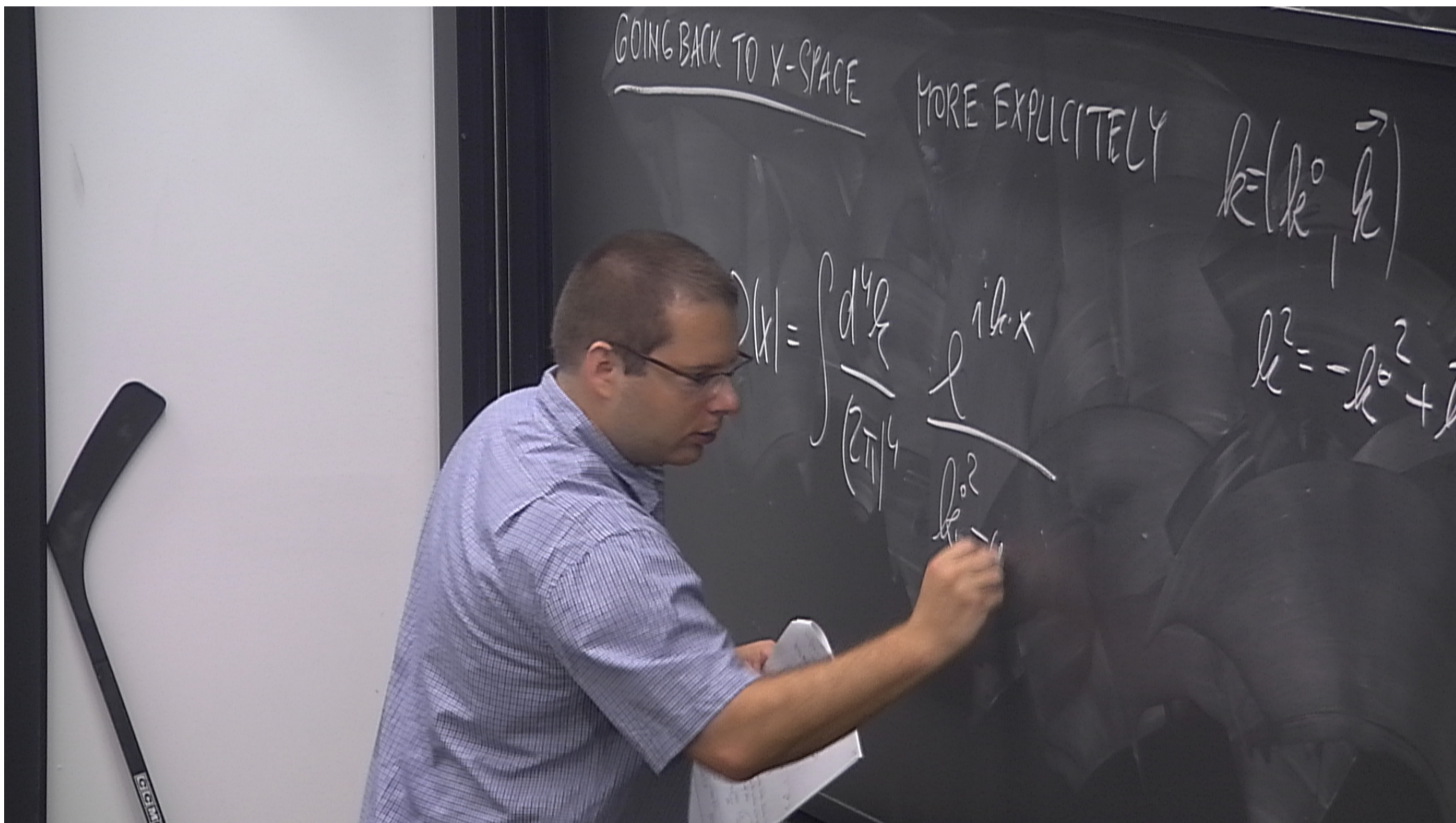
$$k = (k^0, \vec{k})$$

$$X = (t, \vec{x})$$

$$k^2 = -k^0{}^2 + \vec{k}^2$$

$$| \quad \omega_k = \sqrt{\vec{k}^2 + m^2}$$





$$\begin{aligned}
 & -i(\omega_k t - \vec{k} \cdot \vec{x}) \\
 & i(\omega_k t + \vec{k} \cdot \vec{x}) \\
 & \Theta(t) + \ell
 \end{aligned}$$

$$i(\omega t - \vec{p} \cdot \vec{x})$$

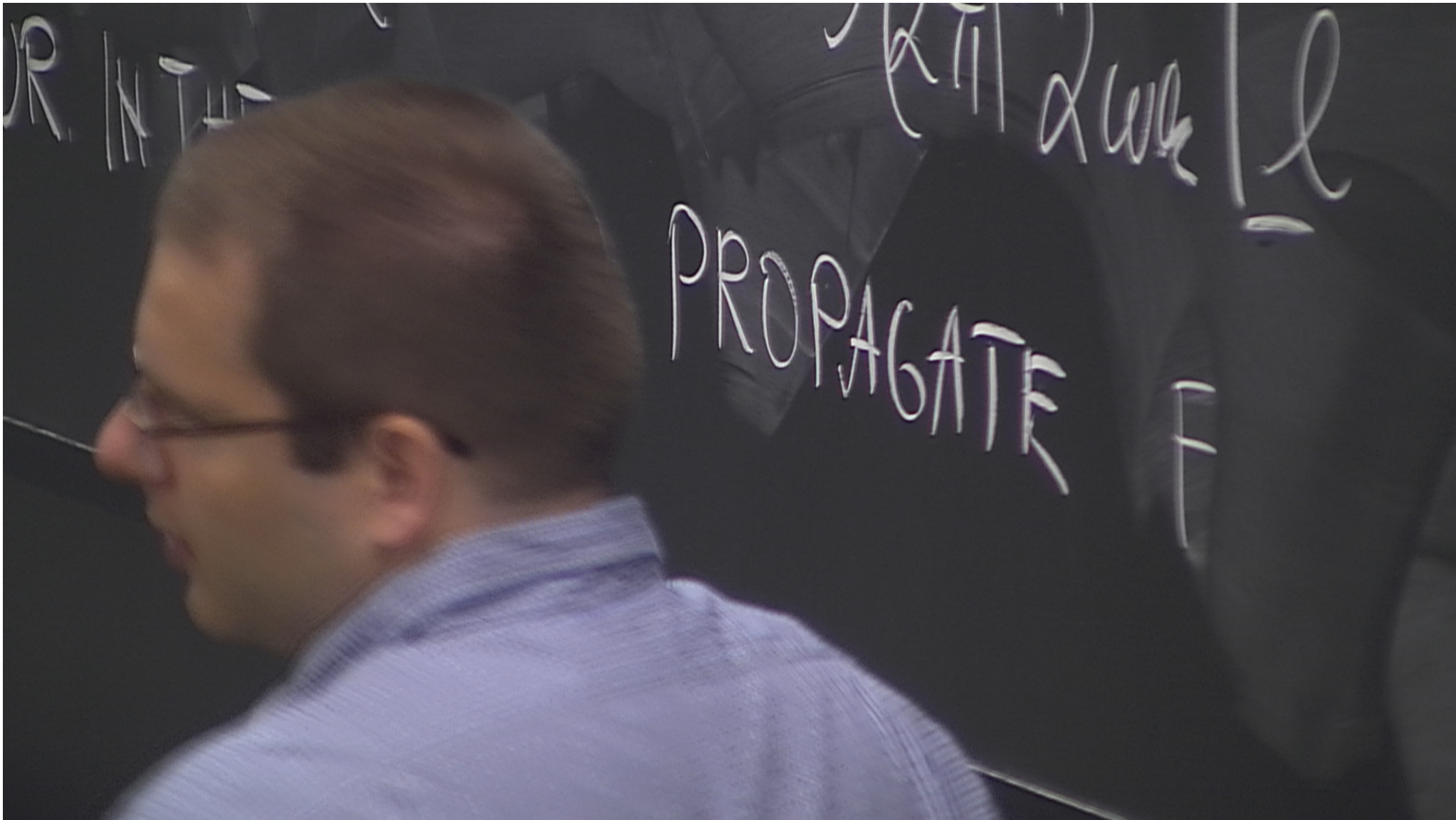
$$i(\omega t + \vec{p} \cdot \vec{x})$$

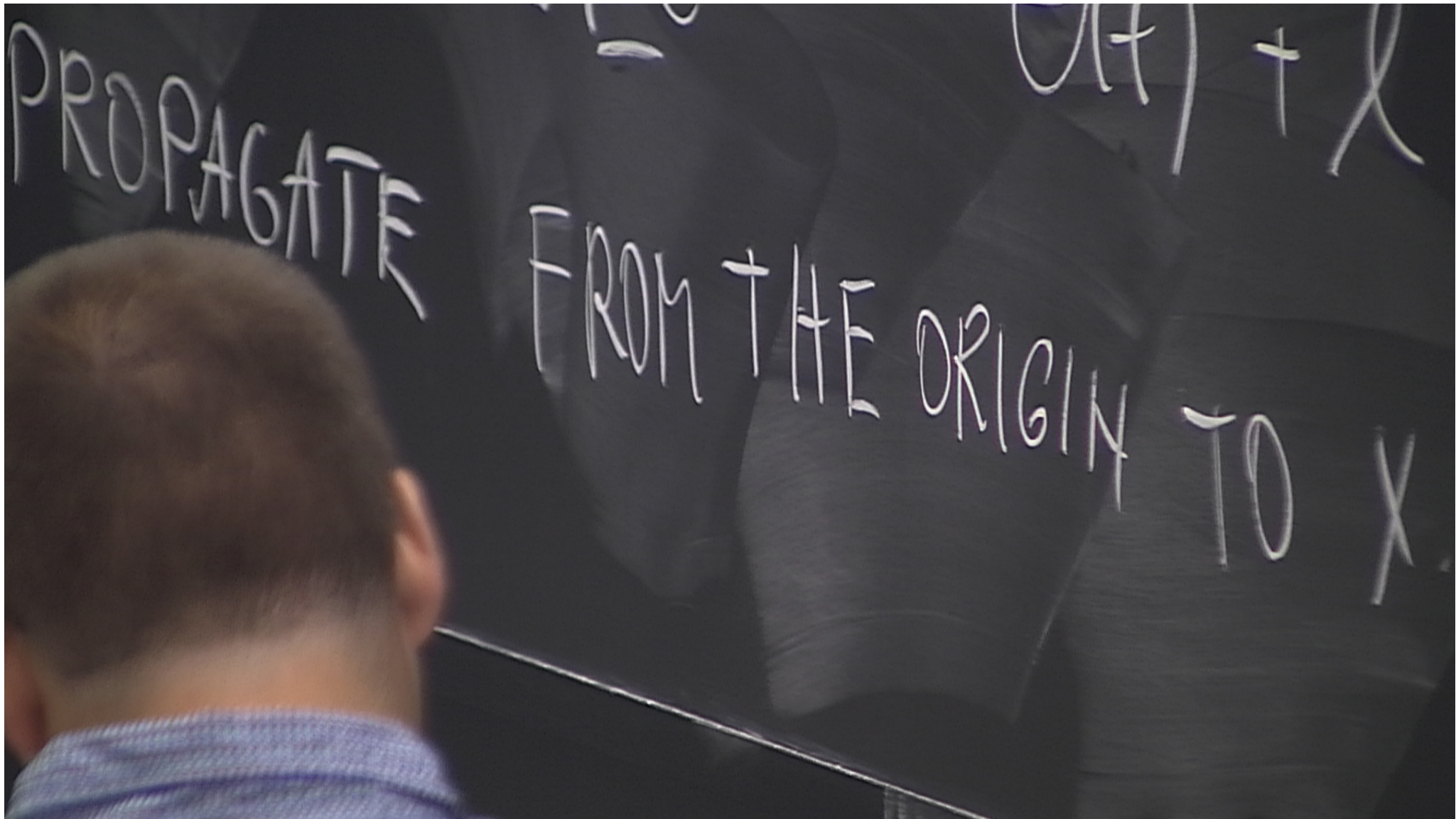
$$\theta(t) + \lambda$$

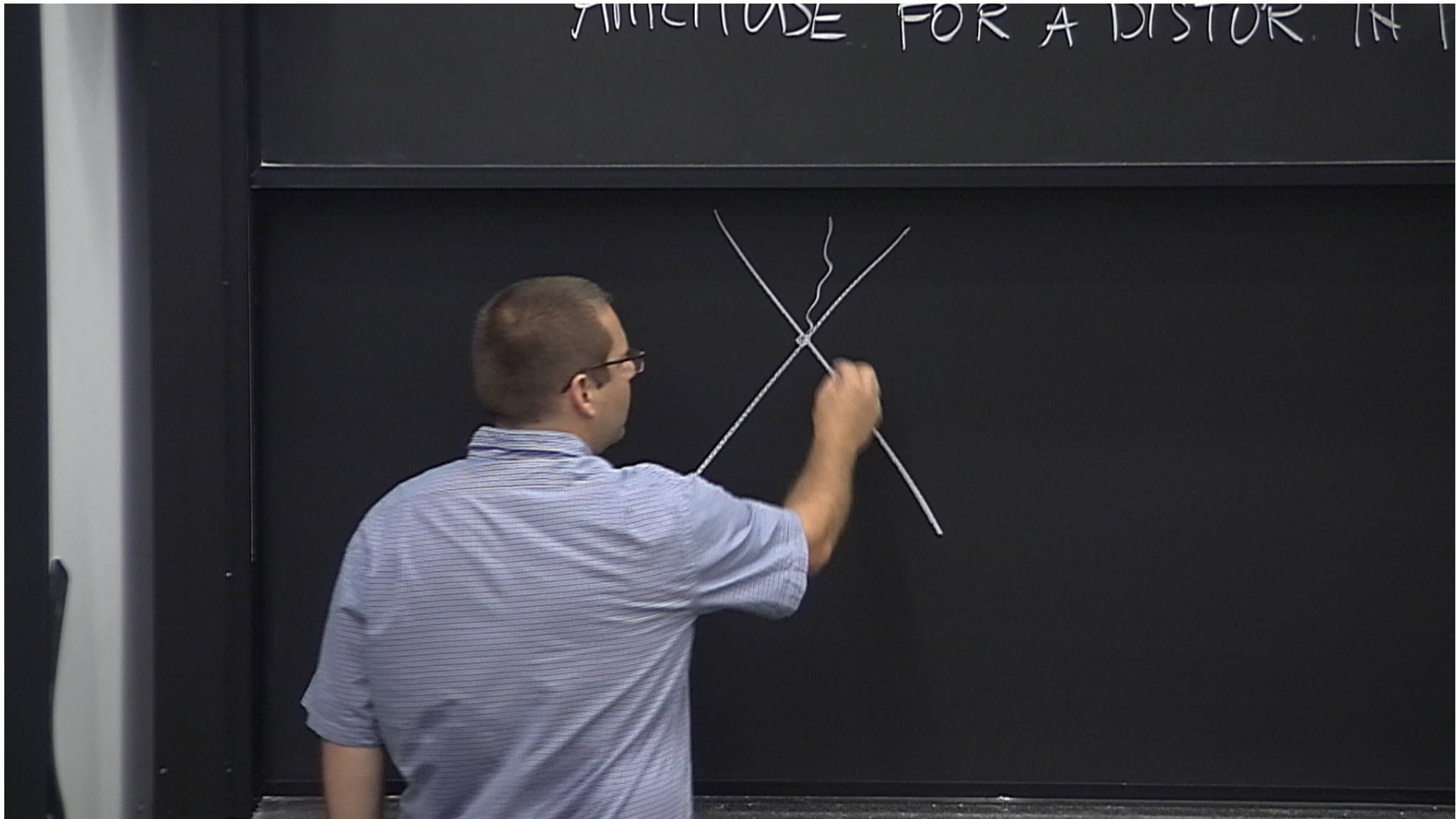
$$\theta(-t)$$

$$D(x) = \int \frac{d^4 q}{(2\pi)^4} \frac{e^{i q \cdot x}}{q^2 - m^2 + i\epsilon}$$

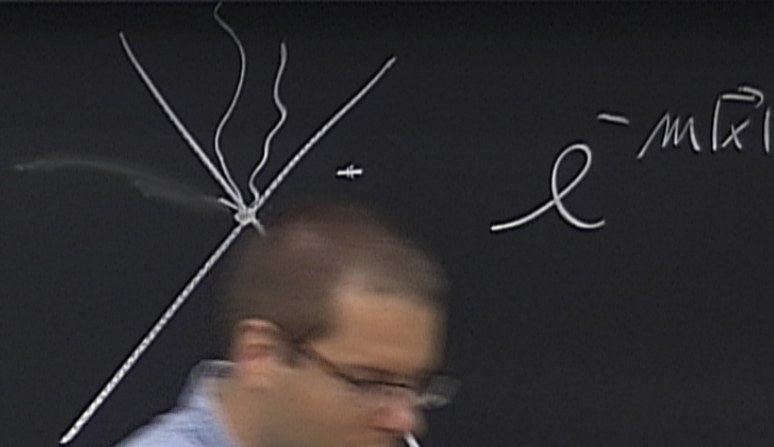
ITUDE



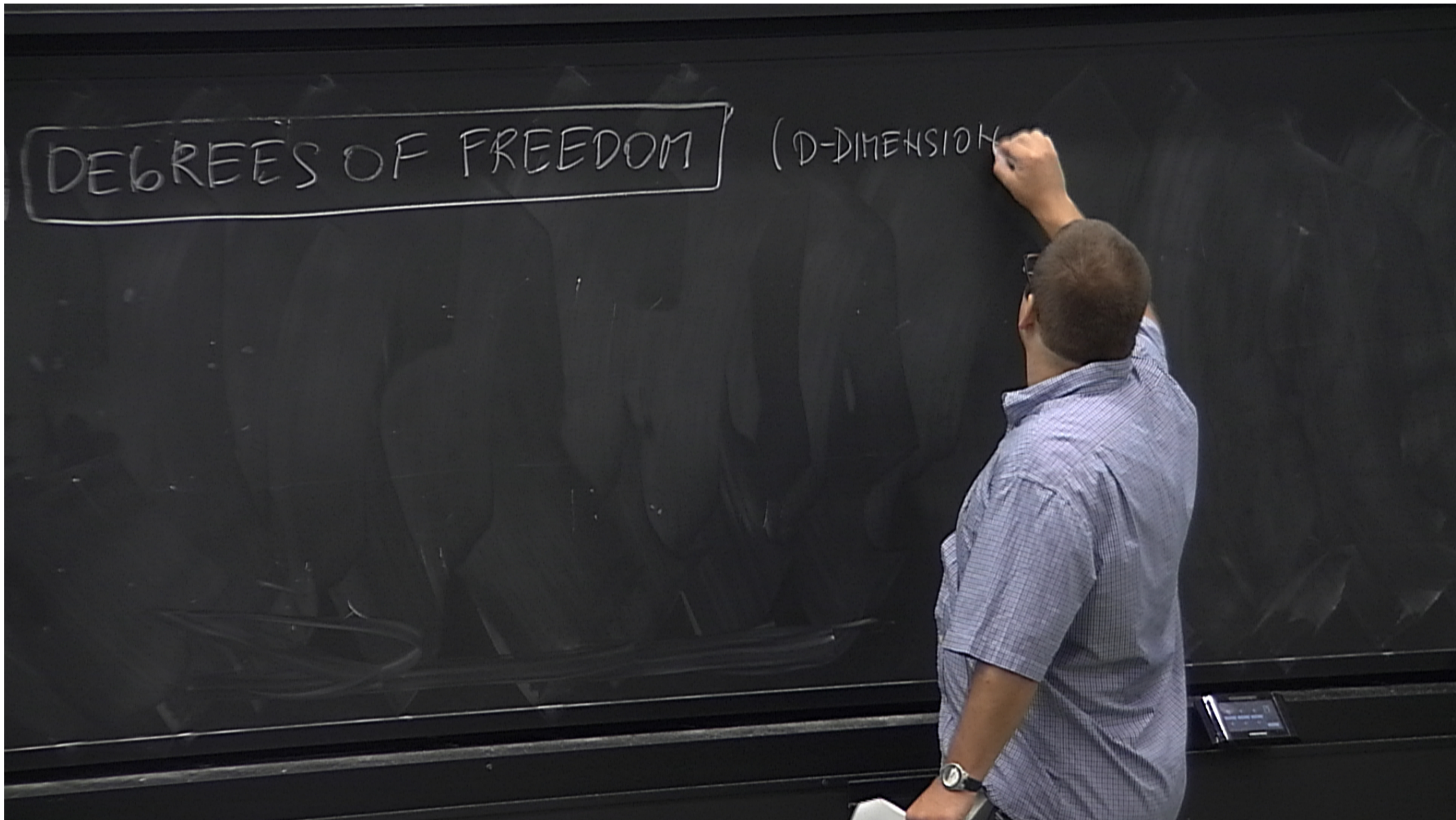




AMPLITUDE FOR A DISTOR. IN 1



2) DEGREES OF FREEDOM



a) SCALAR FIELD

$$r_1 = \int \frac{d^D p}{(2\pi)^D} \phi(p) e^{i \cdot}$$

DEGREES OF FREEDOM (D-DIMENSIONS)

a) SCALAR FIELD

$$(\partial^2 - m^2) \varphi = 0$$

$$\Rightarrow (p^2 + m^2)$$

$$\text{FT: } \phi(x) = \int \frac{d^D p}{(2\pi)^D} \phi(p) e^{i p \cdot x}$$

$$\langle \varphi | A | \varphi \rangle = \int d^4x \int d^4z \underbrace{\langle \varphi | x \rangle}_{\varphi(x)} \underbrace{A(x, z)}_{\delta(x-z)(\partial_x^2 - m^2)} \underbrace{\langle z | \varphi \rangle}_{\varphi(z)} = \int d^4x \varphi(x) (\partial_x^2 - m^2) \varphi(x)$$

↑
WANT

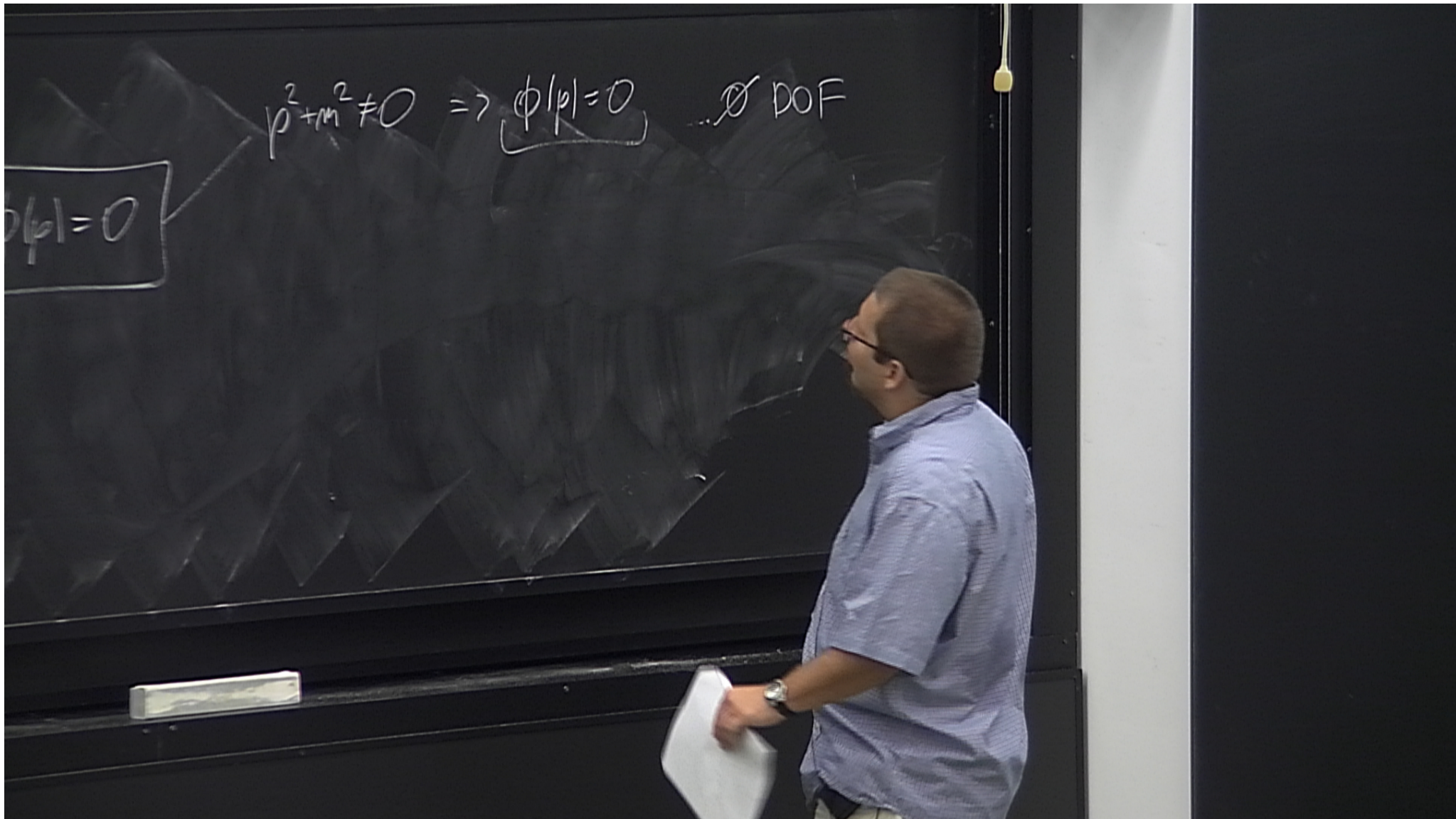
$$= \int d^4x d^4z \varphi(x) \delta(x-z) (\partial_x^2 - m^2) \varphi(z)$$

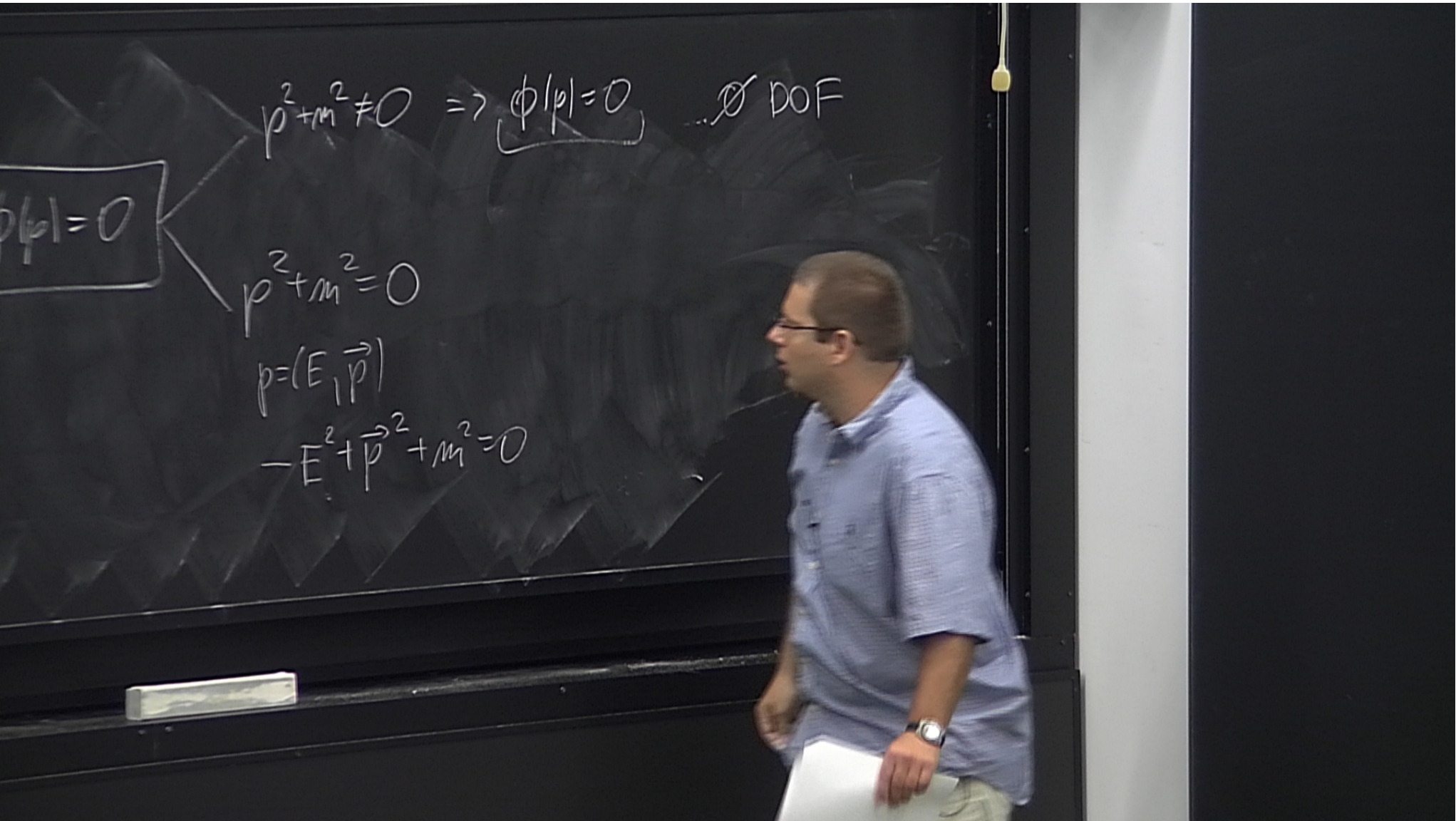
$$A(x, z) = \delta(x-z) (\partial_x^2 - m^2)$$

2) DEGREES OF FREEDOM (D-DIMENSIONS)

a) SCALAR FIELD $(\partial^2 - m^2) \varphi = 0 \Rightarrow (p^2 + m^2) \phi(p) = 0$

FT: $\phi(x) = \int \frac{d^D p}{(2\pi)^D} \phi(p) e^{i p \cdot x}$





$$p^2 + m^2 \neq 0 \Rightarrow \underbrace{\phi|p|=0}_{\text{no DOF}}$$

$$p^2 + m^2 = 0$$

$$p = (E, \vec{p})$$

$$-E^2 + \vec{p}^2 + m^2 = 0$$

MASS SHELL CONDITION

$\Rightarrow \underbrace{\phi|p \text{ EXISTS.}}$



$$p^2 + m^2 \neq 0 \Rightarrow \phi|p| = 0 \quad \emptyset \text{ DOF}$$

$$p^2 + m^2 = 0$$

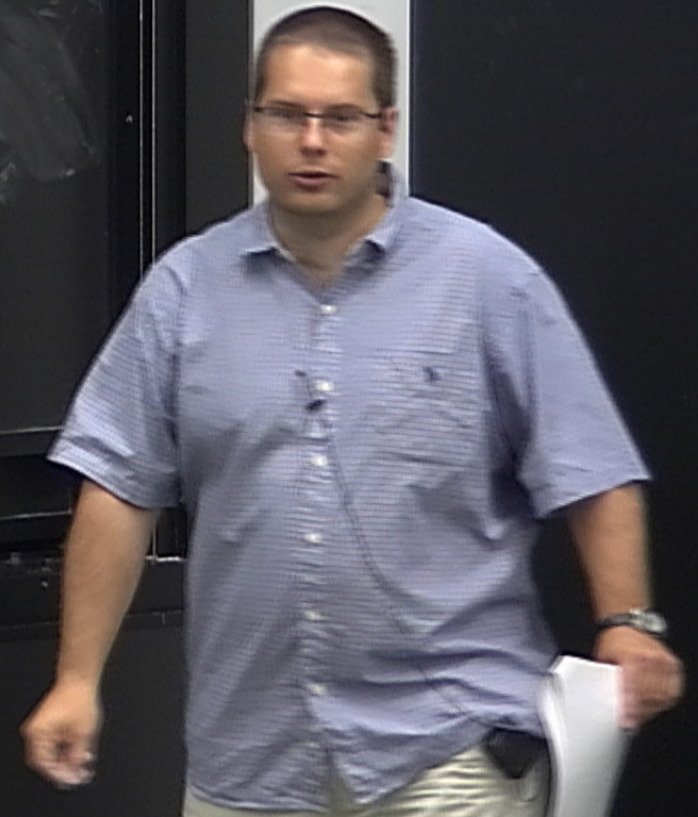
$$p = (E, \vec{p})$$

$$-E^2 + \vec{p}^2 + m^2 = 0$$

MASS SHELL CONDITION

$\Rightarrow \phi|p|$ EXISTS.

1 DOF



$$k = (k^0, \vec{k}) \quad x = (t, \vec{x})$$

$$k^2 = -k^0^2 + \vec{k}^2, \quad \omega_k = \sqrt{\vec{k}^2 + m^2}$$

$$= -i \int \frac{d^3k}{(2\pi)^3 2\omega_k} \left[\frac{-i(\omega_k t - \vec{k} \cdot \vec{x})}{\Theta(t)} + \frac{i(\omega_k t + \vec{k} \cdot \vec{x})}{\Theta(-t)} \right]$$

PROPAGATE FROM THE ORIGIN TO X

PARTICLE CANNOT GET OUTSIDE THE LIGHT CONE

"LEAK" OUT OVER A DISTANCE $\propto$

b) GAUGE FIELD

$$FT: \phi(x) = \int \frac{d^D p}{(2\pi)^D} \phi(p) e^{i p \cdot x}$$

$$p = (E, \vec{p})$$

$$-E^2 + \vec{p}^2 + m^2 = 0$$

b) GAUGE FIELD

RULE: IN ANY GAUGE THEORY

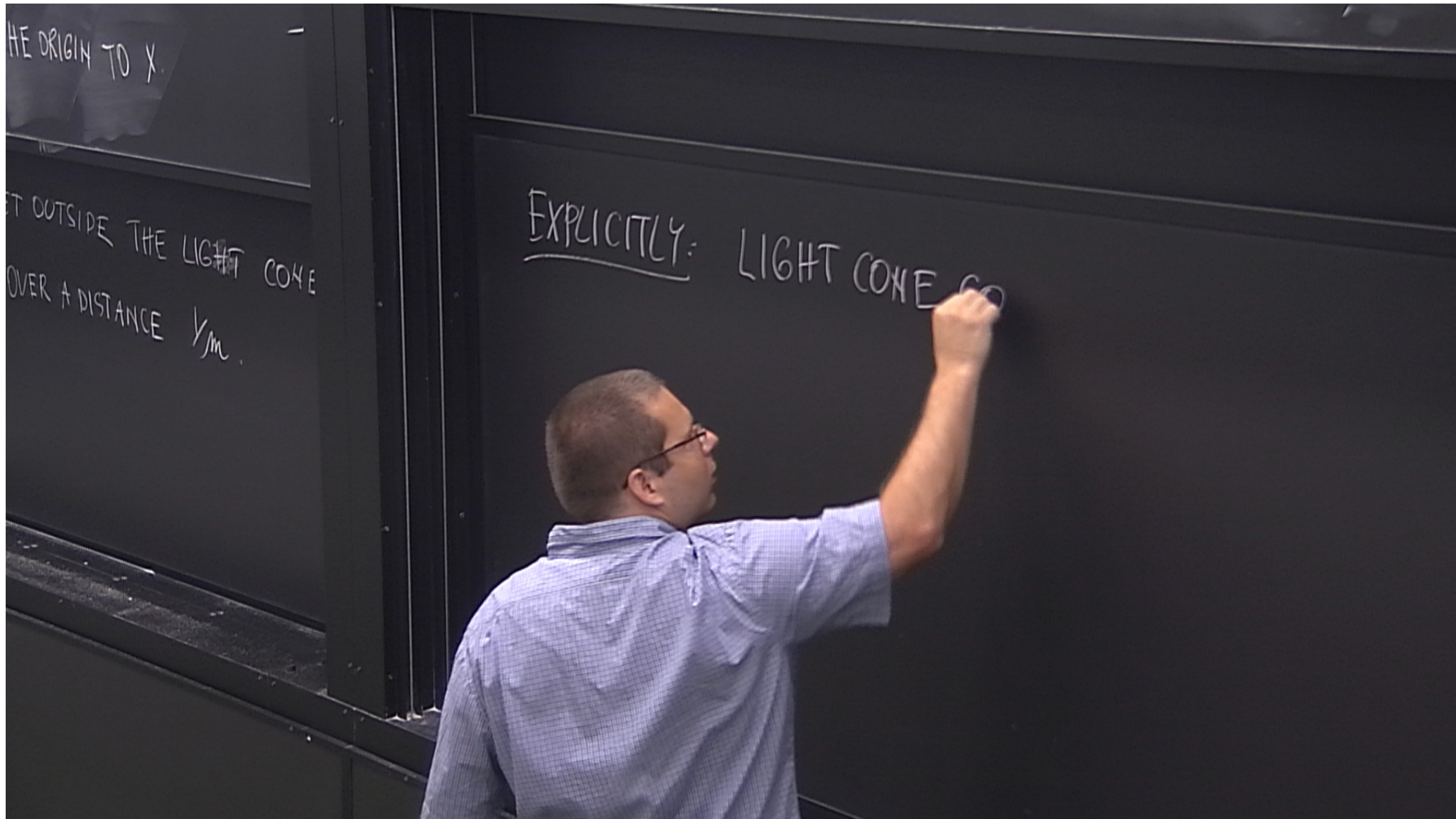
$$\# \text{ DOF} = \# \text{ APPARENT DOF} - 2 \times \text{DOF OF THE GAUGE FUNCTION}$$

FM:

$$\# \text{ DOF} = \# \text{ APPAREN}$$

EM: $AM = (\underbrace{\phi, \vec{A}}_{3})$

IONS OF THE PHOTON



EXPLICITLY: LIGHT CONE COORDINATES

$$A = (A^+, A^-, A)$$

EXPLICITLY: LIGHT CONE COORDINATES

$$A = (A^+, A^-, \underbrace{A^2, A^3, A^4, \dots}_{A^I})$$

DINATES

$$A^0 = \frac{1}{\sqrt{2}}$$

(

I 2 D-1

DINATES

$$A^0 = \frac{1}{\sqrt{2}} (A^+ + A^-)$$

$$A^1 = \frac{1}{\sqrt{2}} ($$

I 2 D-1

THE ORIGIN TO X.

ET OUTSIDE THE LIGHT CONE
OVER A DISTANCE $\frac{1}{\sqrt{m}}$.

EXPLICITLY: LIGHT CONE COORDINATES

$$A = (A^+, \bar{A}^-, \underbrace{A^2, A^3, A^4, \dots}_{A^I})$$

$$A^0 = \frac{1}{\sqrt{2}}$$

$$A^1 = \frac{1}{\sqrt{2}}$$

$$A^I$$

$$I \dots 2 \dots D-1$$

$$A \cdot B = -A$$

$$A^0 = \frac{1}{\sqrt{2}} (A^+ + A^-)$$

$$A^1 = \frac{1}{\sqrt{2}} (A^+ - A^-)$$

I 2 D-1

$$-A^- B^+ - A^+ B^- + A^{\pm} P$$

Δ GAUGE FUNCTION. 1

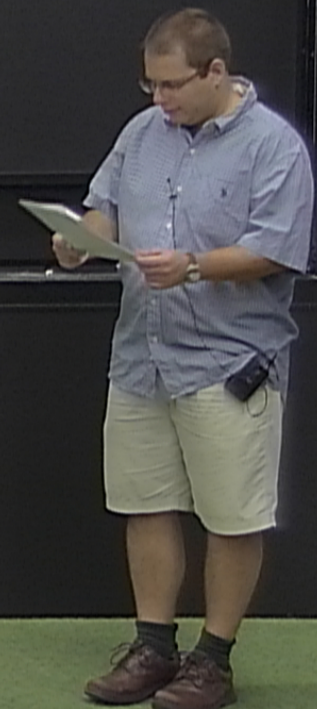
EXPLICITLY: LIGHT CONE COORDINATES

$$A = (A^+, A^-, \underbrace{A^2, A^3, A^4, \dots}_{A^I}) \quad \begin{aligned} A^0 &= \frac{1}{\sqrt{2}} (A^+ + A^-) \\ A^1 &= \frac{1}{\sqrt{2}} (A^+ - A^-) \end{aligned}$$

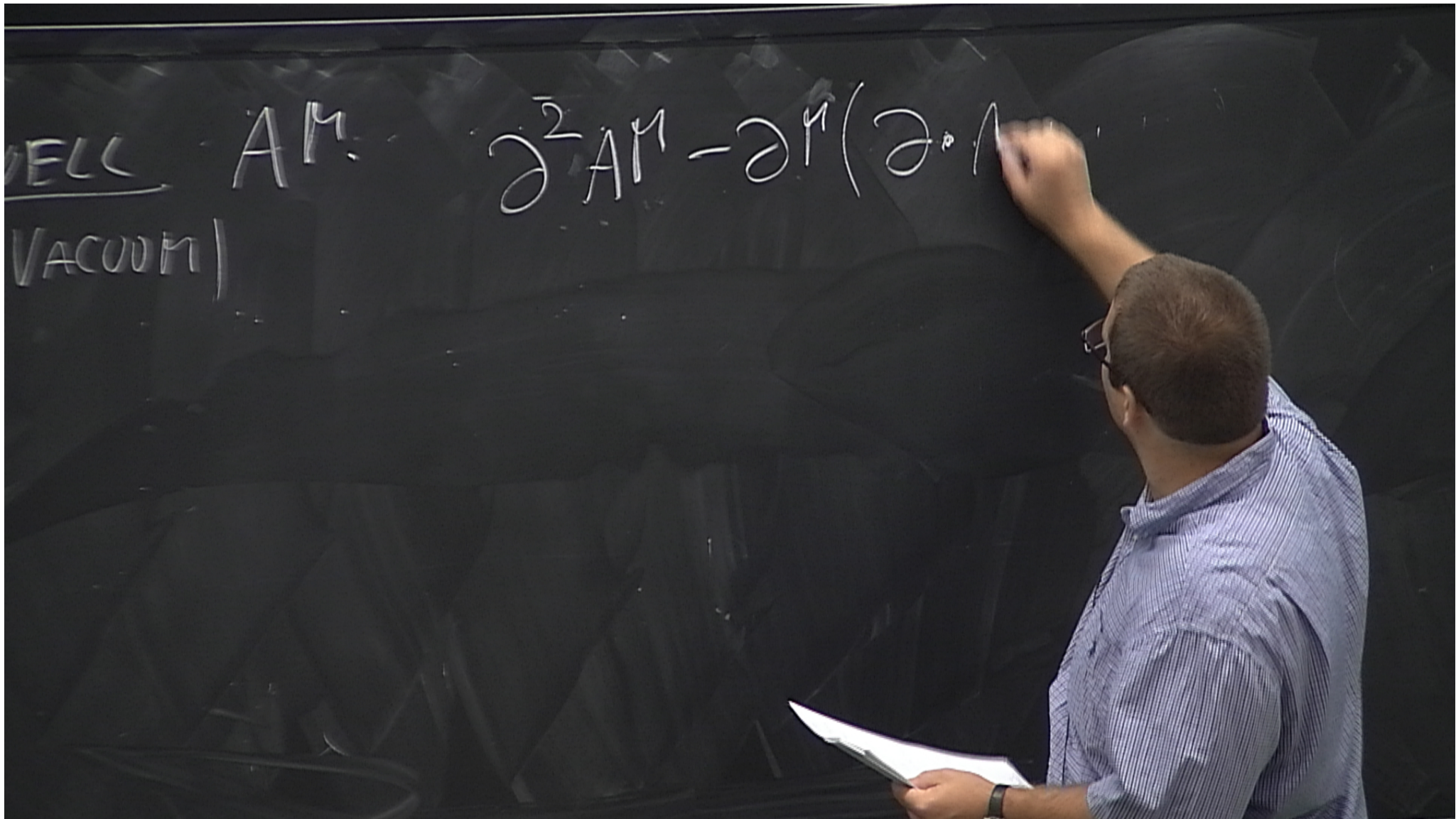
$A^I \quad I = 2, 3, \dots, D-1$

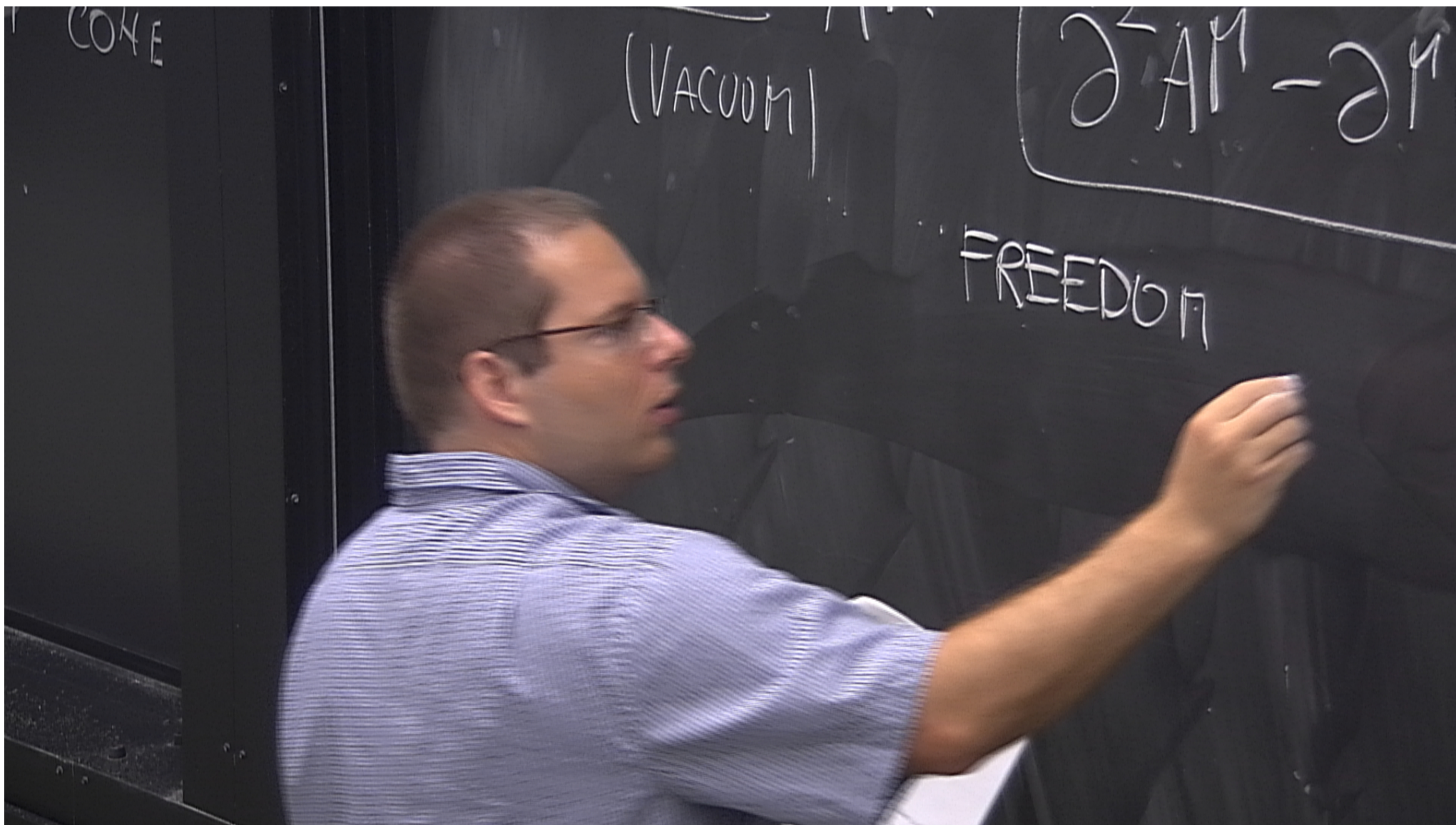
$$\Leftrightarrow \eta = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & \mathbb{I} \end{pmatrix}$$

$$A \cdot B = -A^0 B^0 + A^1 B^1 + A^I B^I = -A^- B^+ - A^+ B^- + A^I B^I$$



C) MAXWELL A.M.





$$-p^2 A^\mu(p) + p^\mu (p \cdot A) = 0$$

$$\delta A_\mu = i p_\mu \Delta$$

WLOG: $p^+ \neq 0$

$$\delta A^+ = \lambda' p^+ \lambda$$

$$1=0$$

WLOG: $p^+ \neq 0$

$$\delta A^+ = \lambda p^+ \underline{\Lambda}$$

CHOOSE $\underline{\Lambda}$

$$1=0$$

$$+ p^I A^I$$

$\Rightarrow$

$$A^- = \frac{p^I A^I}{p^+}$$

$$p A^1 + p^4 A^L$$

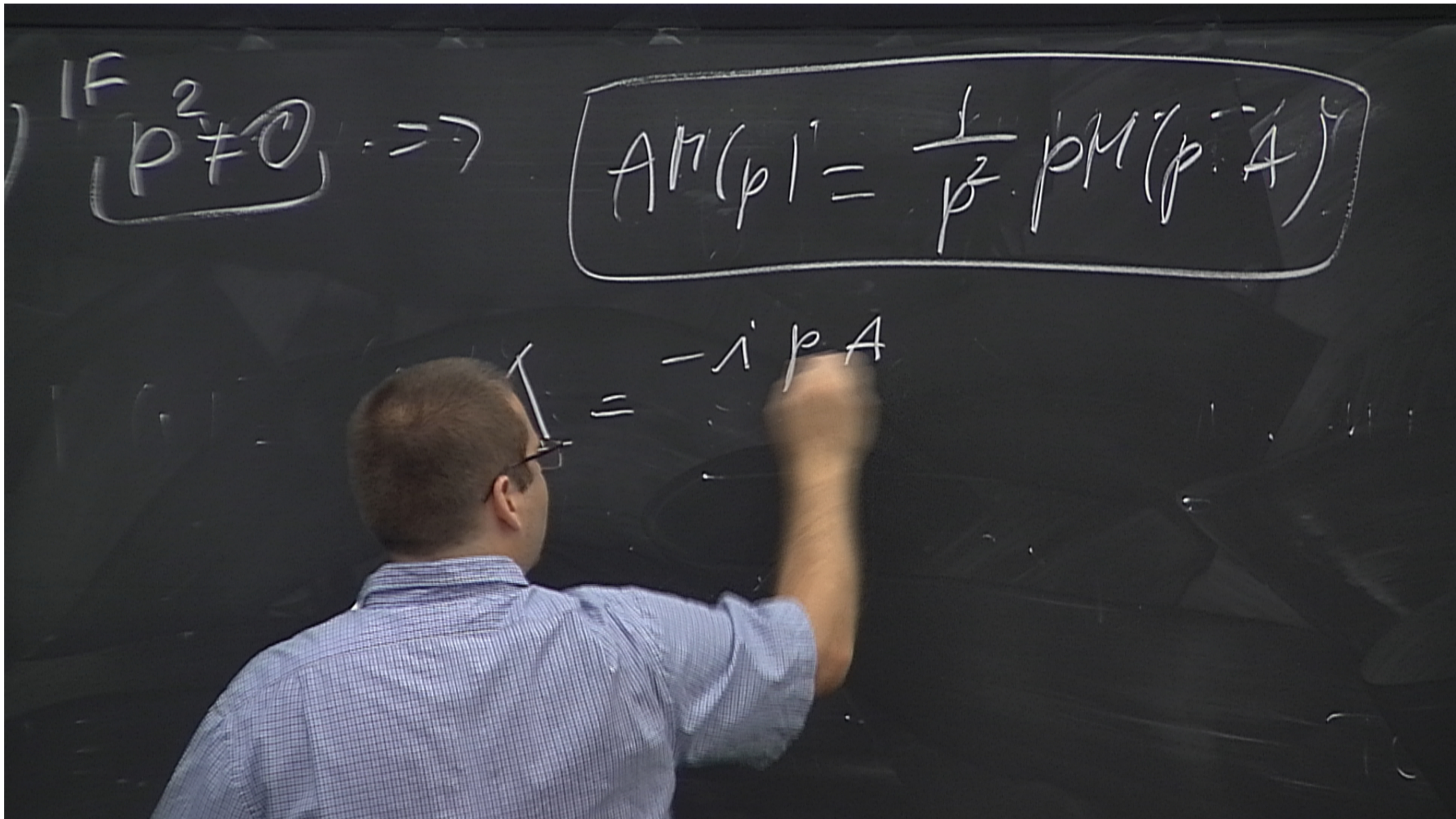
$\Rightarrow$

I AM LEFT WITH

$$p^2 A^I = 0$$

$\neq 0 \Rightarrow$

$$AM(p) = \frac{1}{p^2} pM(p, A)$$



$$\underline{\Delta} = \frac{-i p A}{p^2 \dots}$$

2) 1 PHOTON STATE

$$\underline{\Delta} = \frac{-i p A}{p^2 \dots}$$

$\nearrow$ ELI

2) 1 PHOTON STATE

$\sum_{I=1}^{D-1}$

$\{ \sum_I a_{p^+, p^I}^I +$

$|\Omega\rangle$