

Title: 13/14 PSI- Complex Analysis - Lecture 2

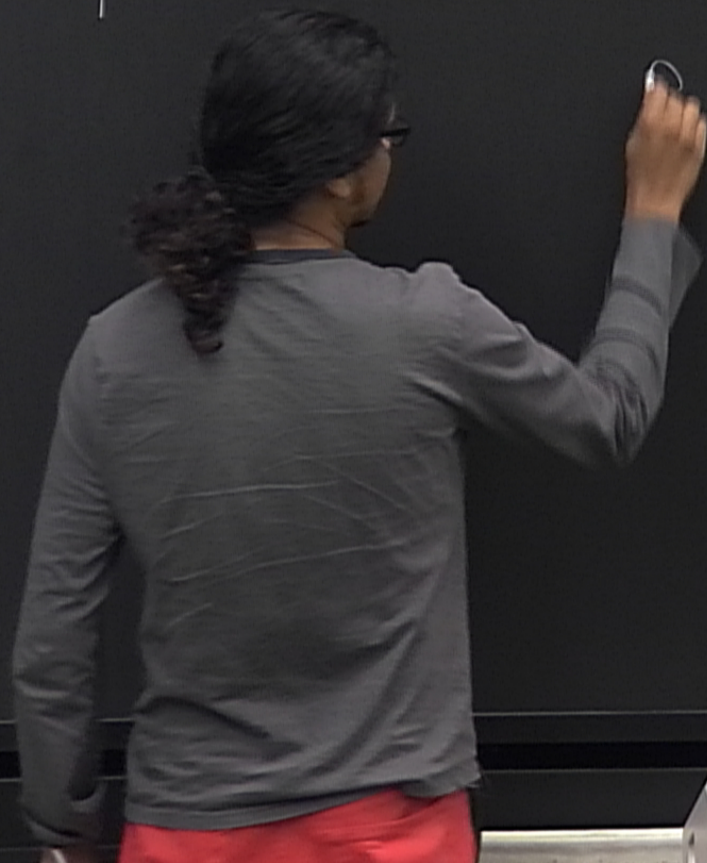
Date: Aug 15, 2013 09:00 AM

URL: <http://pirsa.org/13080007>

Abstract:



$$f: U \subset \mathbb{C} \rightarrow \mathbb{C}$$



$$f: U \subset \mathbb{C} \rightarrow \mathbb{C}$$

$$f'(z) = \frac{1}{z^2}$$

$$f: U \subset \mathbb{C} \rightarrow \mathbb{C}$$

$$f(z) = \frac{1}{\bar{z}^2}$$

→ $\mathbb{C}$

$$f(z) = \frac{1}{z^2}$$

$$2. f(z) = e^z = 1 + z + \frac{z^2}{2!} + \dots$$

$$f(z) = u(x, y) + i v(x, y)$$

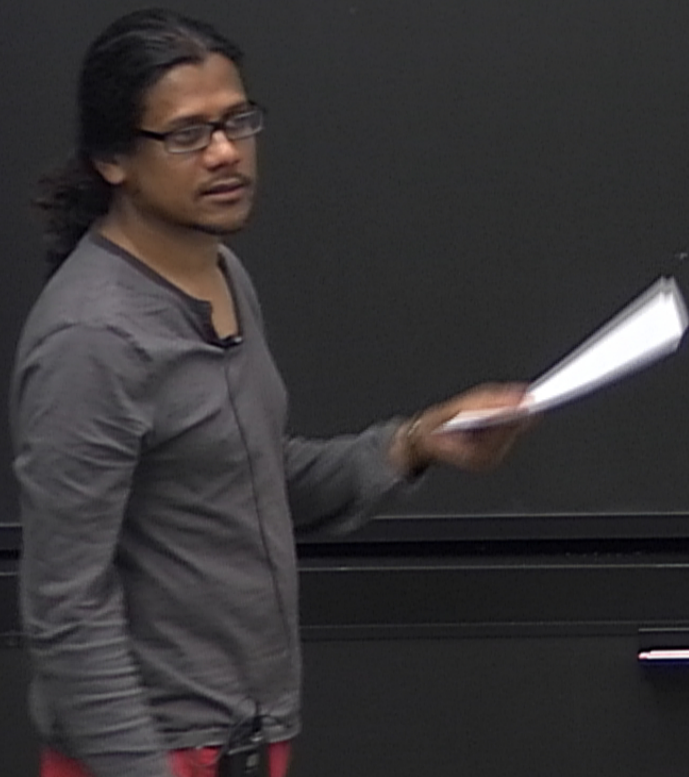
→ $\mathbb{C}$

$$2. f(z) = e^z = 1 + z + \frac{z^2}{2!} + \dots$$

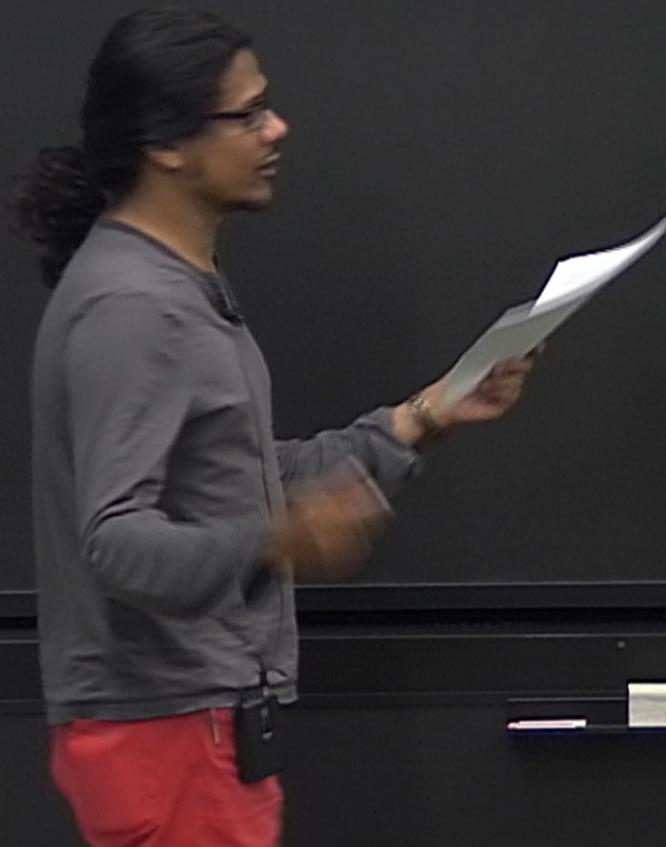
$$f(z) = \frac{1}{z^2}$$

$$f(z) = u(x, y) + i v(x, y)$$

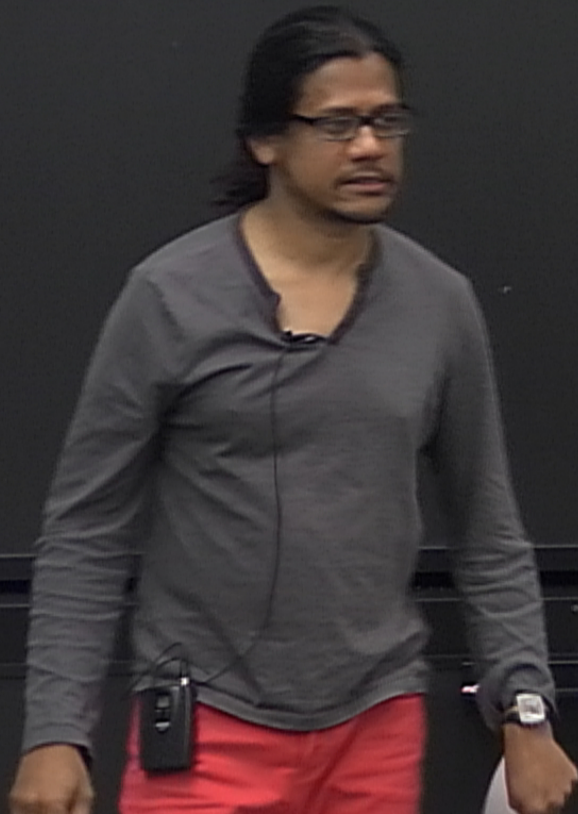
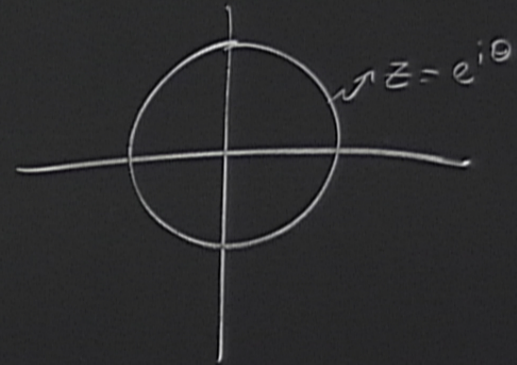
$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$



$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$



$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$



1. z^2

$$\sqrt{z}$$

$$\log z = \log |z| +$$

2. e^z

$$\sqrt{z}$$

$$\log z = \log |z| + i \arg(z)$$

$$\sqrt{z}$$

$$\log z = \log |z| + i \arg(z)$$

$$= \log |z| + i\theta_p + i2\pi k$$

$$\sqrt{z}$$

$$\log z = \log |z| + i \arg(z)$$

$$= \log |z| + i\theta_p + i2\pi k$$

$$\sinh^{-1} z = \log [w + \sqrt{w^2 + 1}]$$

$$q = e^z$$

$$\sinh z = w$$

$$\sinh^{-1} z = \log [w + \sqrt{w^2 + 1}]$$

$$\sinh z = \frac{e^z - e^{-z}}{2}$$

$$q = e^z$$

$$\sinh z = w$$

$$f = e^z$$

$$\sinh z = w$$

$$w = 0,$$



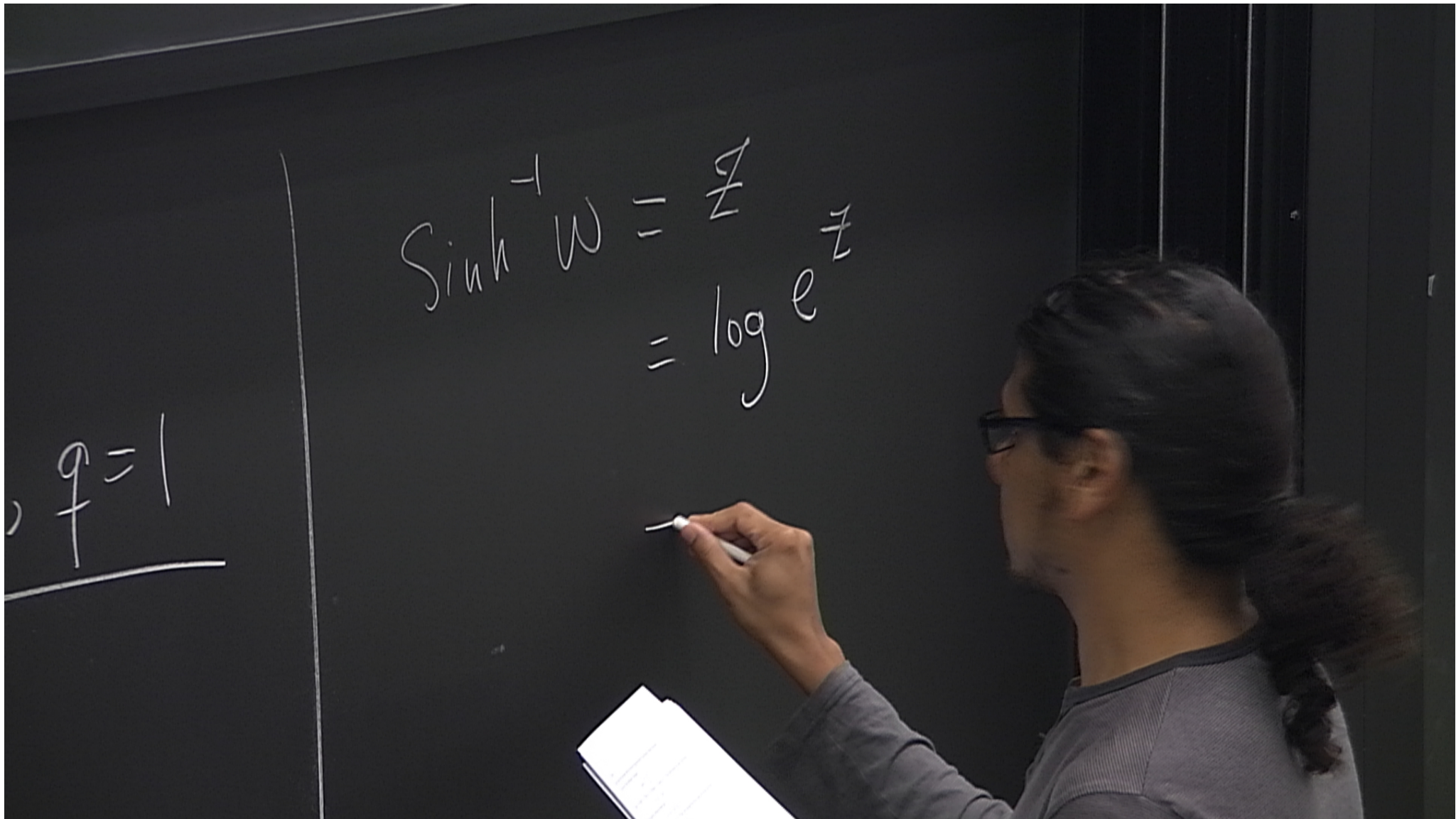
$$q = e^z$$

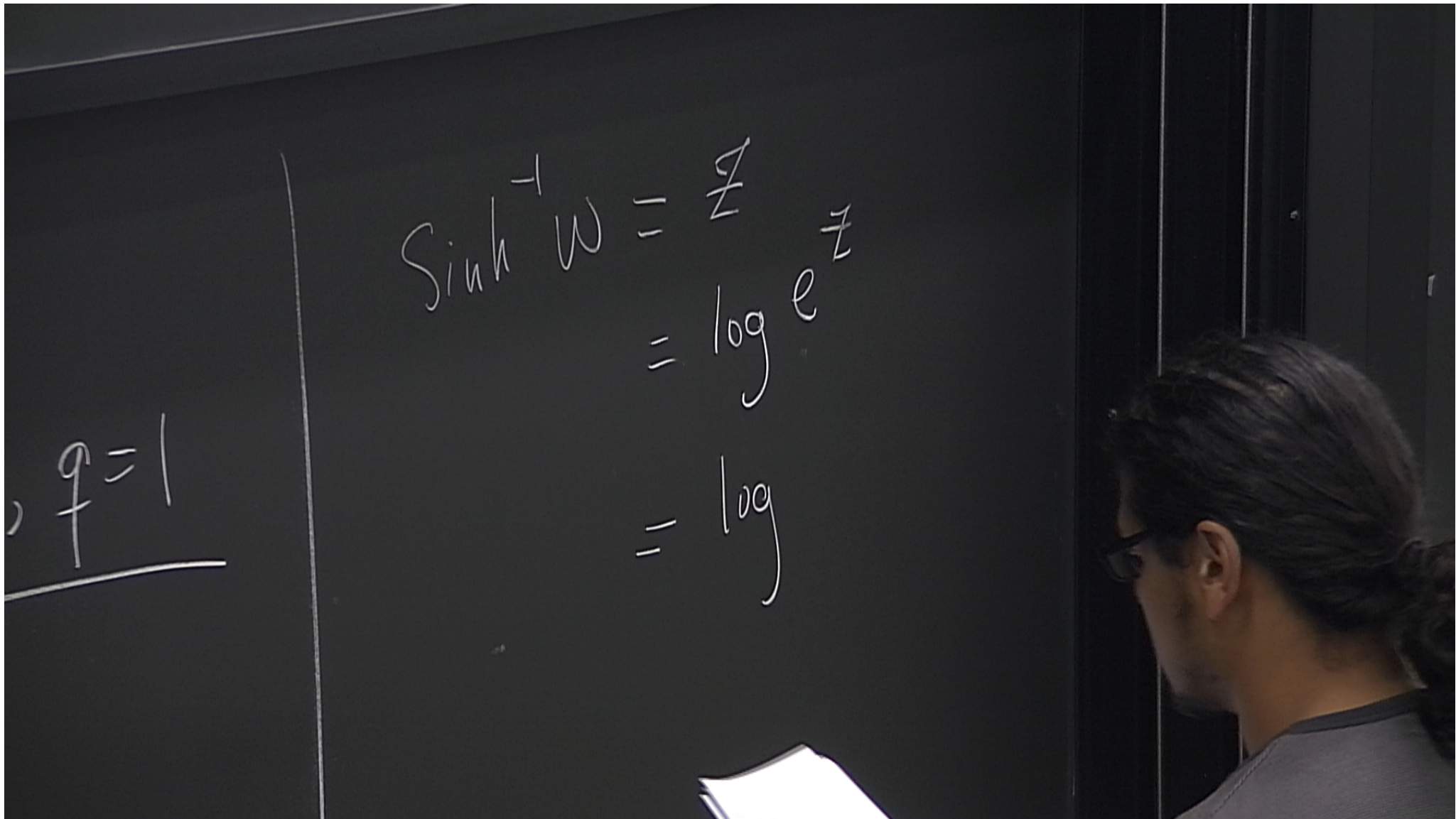
$$\sinh z = w$$

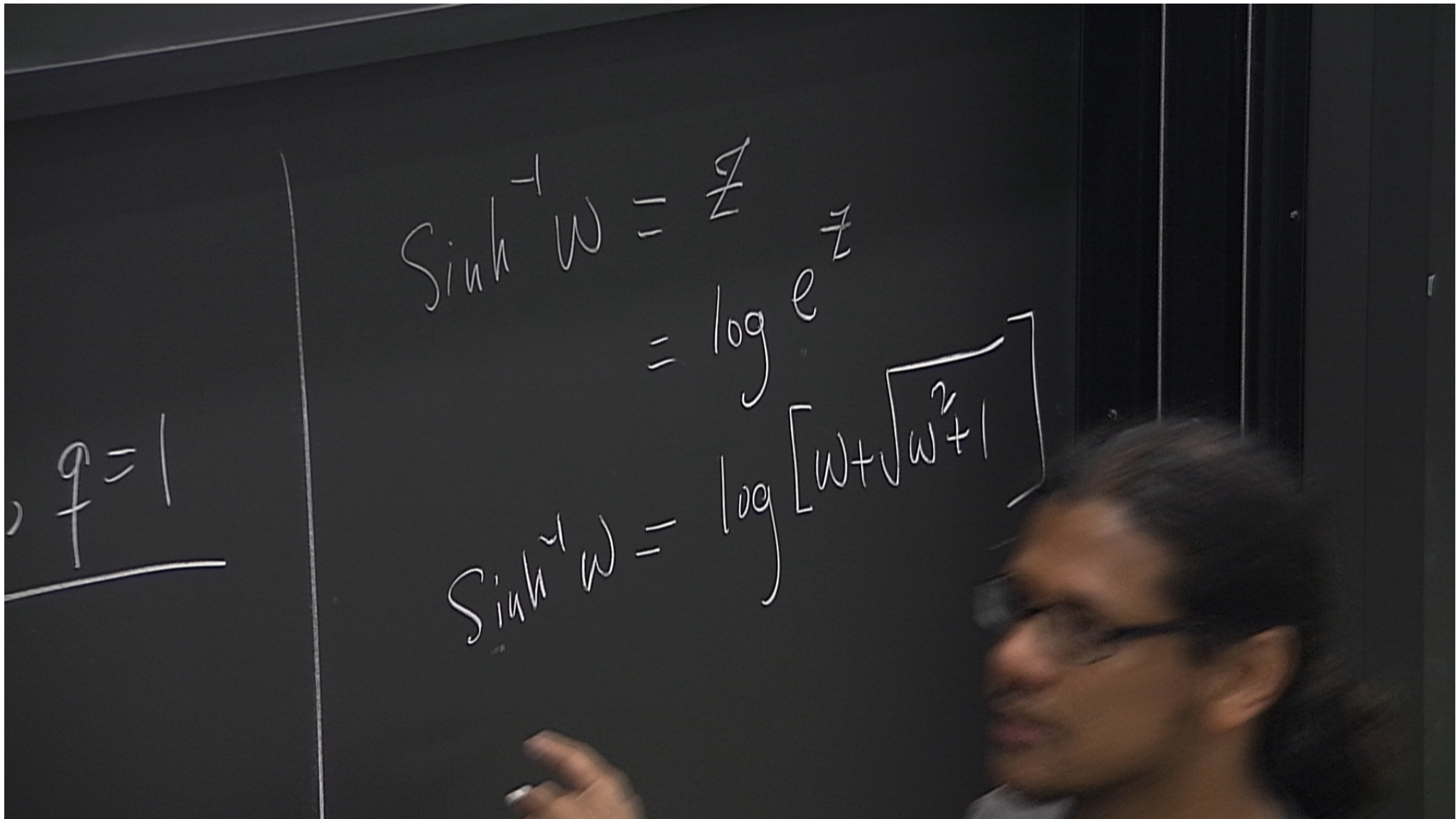
$$w=0, \quad z=0, \quad q=1$$

$$q = e^z$$

$$\text{Sinh } z = w$$
$$\underline{w=0, \quad z=0, \quad q=1}$$







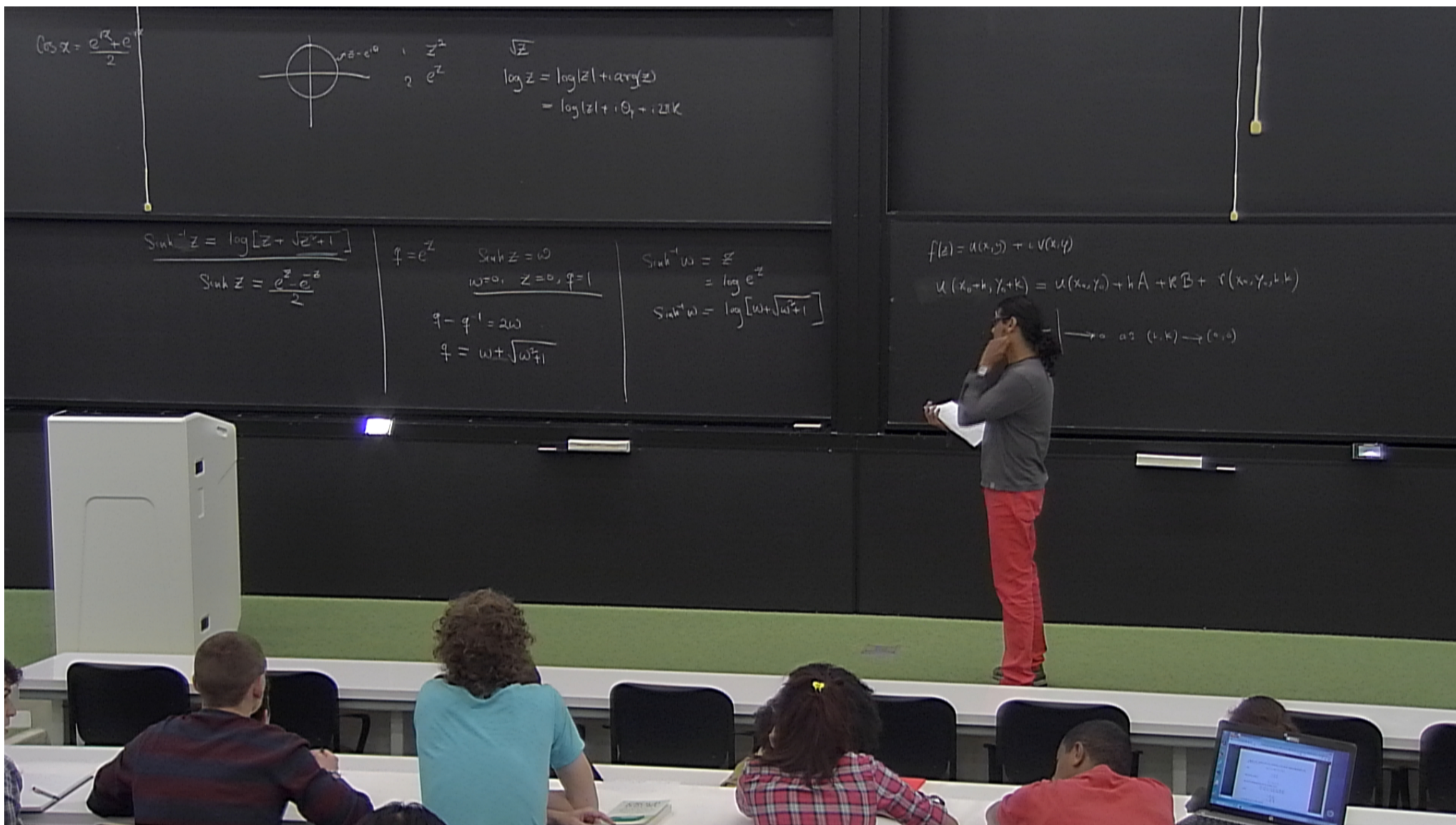
$$f(z) = u(x, y) + i v(x, y)$$

$$u(x_0 + h, y_0) = u(x_0, y_0) + h A + k B + r(x_0, y_0)$$

$$f(z) = u(x, y) + i v(x, y)$$

$$u(x_0 + h, y_0 + k) = u(x_0, y_0) + hA + kB + r(x_0, y_0, h, k)$$

$$\left| \frac{r}{\sqrt{h^2 + k^2}} \right| \rightarrow 0 \text{ as } (h, k) \rightarrow (0, 0)$$



$$= \bar{z}$$

$$= \log e^{\bar{z}}$$

$$w = \log[w+z]$$

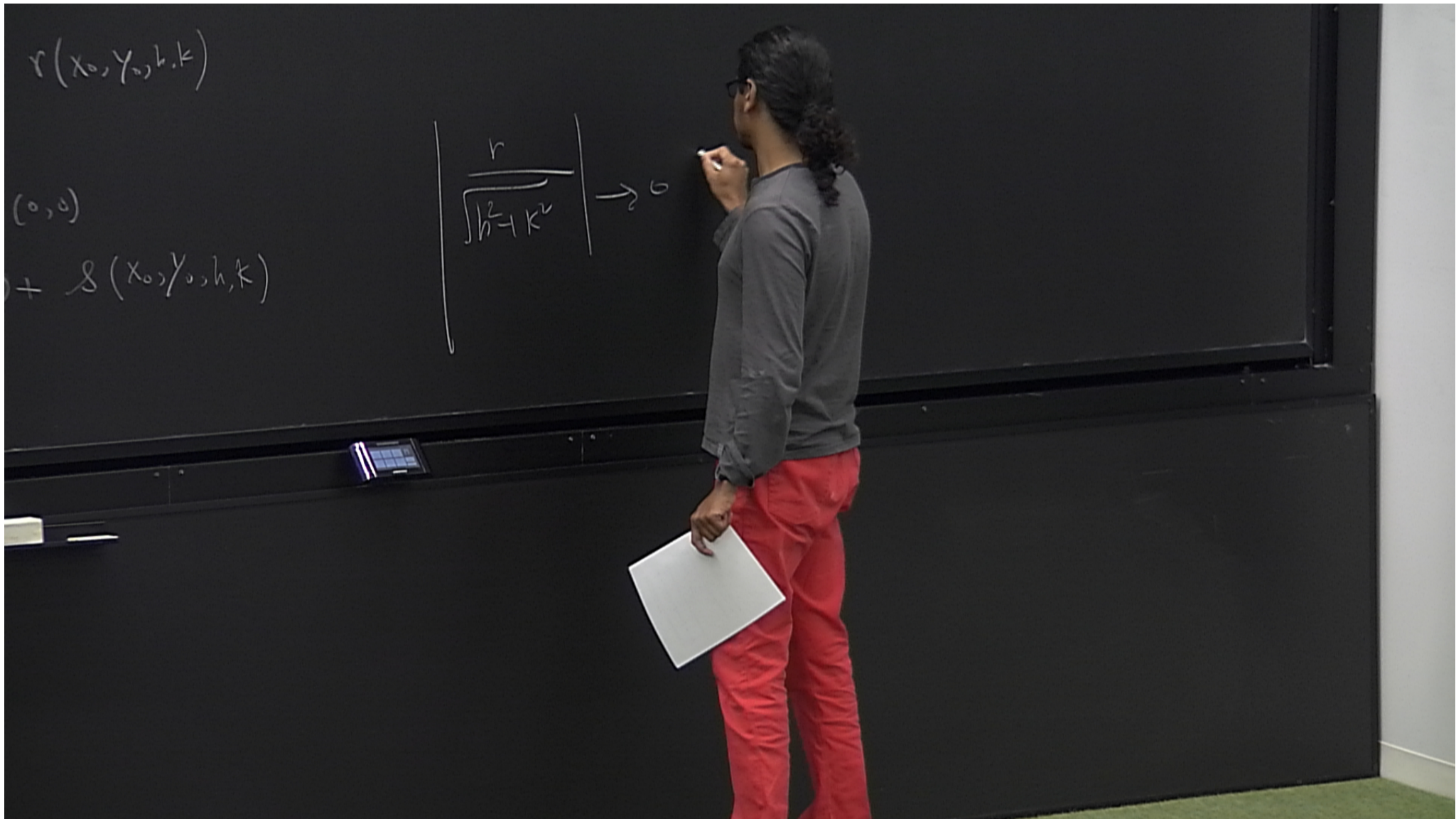
$$f(z) = u(x, y) + i v(x, y)$$

$$u(x_0+h, y_0+k) = u(x_0, y_0) + hA + kB + r(x_0, y_0, h, k)$$

$$\left| \frac{r}{\sqrt{h^2+k^2}} \right|$$

$$\rightarrow 0 \text{ as } (h, k) \rightarrow (0, 0)$$

$$v(x_0+h, y_0+k) = v(x_0, y_0) + hC + kD + s(x_0, y_0, h, k)$$



$$A = \frac{\partial u(x_0, y_0)}{\partial x}$$

$$B = \frac{\partial u(x_0, y_0)}{\partial y}$$

$$C = \frac{\partial v(x_0, y_0)}{\partial x}$$

$$D = \frac{\partial v(x_0, y_0)}{\partial y}$$

$$A = \frac{\partial u(x_0, y_0)}{\partial x}$$

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$$D = \frac{\partial v(x_0, y_0)}{\partial y}$$

$$f = u + iv$$

$$f(z_0 + w) = f(z_0) +$$

$$A = \frac{\partial u(x_0, y_0)}{\partial x}$$

$$B = \frac{\partial u(x_0, y_0)}{\partial y}$$

$$C = \frac{\partial v(x_0, y_0)}{\partial x}$$

$$D = \frac{\partial v(x_0, y_0)}{\partial y}$$

$$f = u + iv$$

$$f(z_0 + w) = f(z_0) + wE + \sigma(z,$$

$$= \bar{z}$$

$$= \log e^z$$

$$w = \log[w]$$

$$A = \frac{\partial u(x_0, y_0)}{\partial x}$$

$$B = \frac{\partial u(x_0, y_0)}{\partial y}$$

$$C = \partial$$

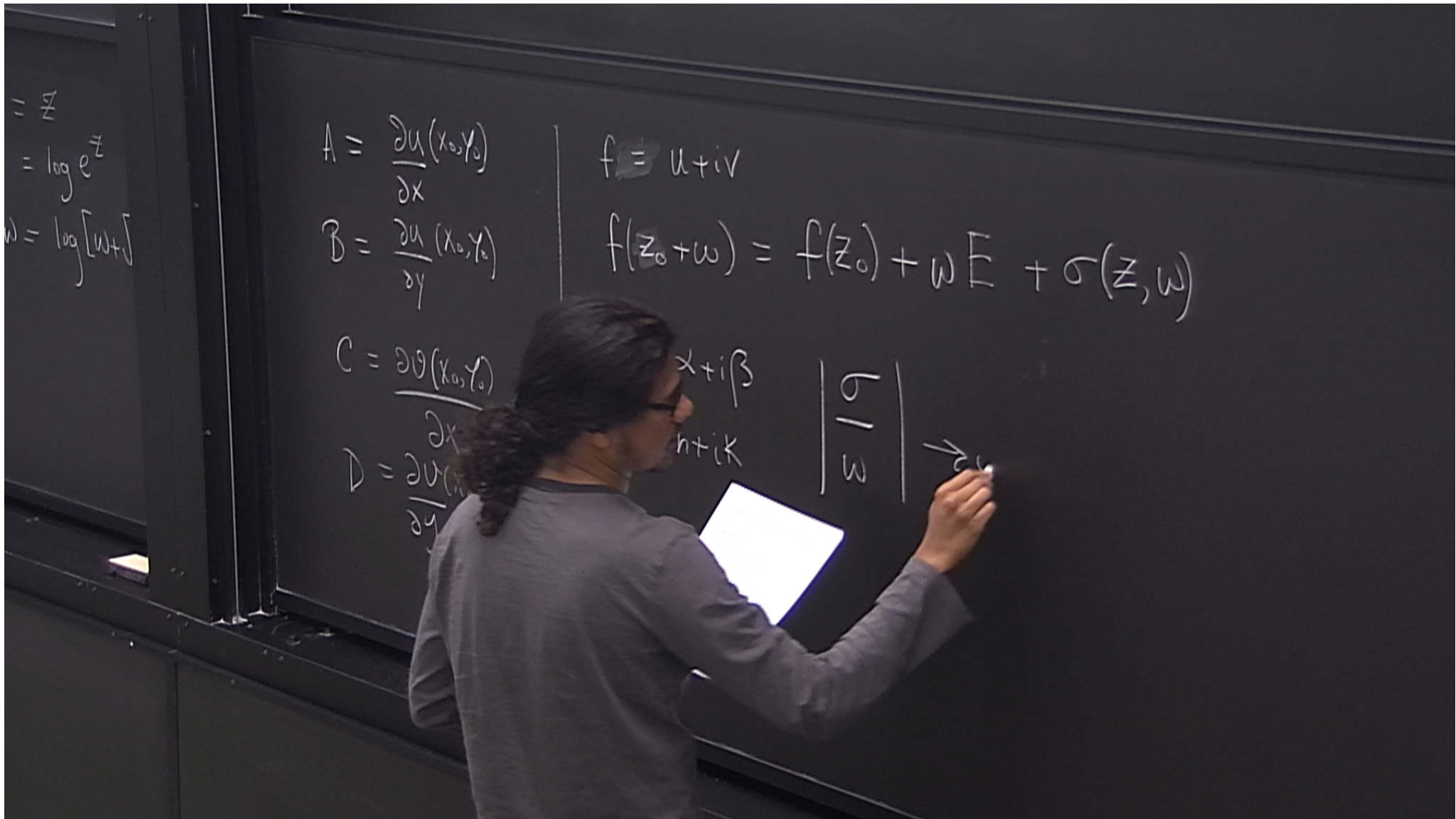
$$D =$$

$$f = u + iv$$

$$f(z_0 + w) = f(z_0) + wE + \sigma(z, w)$$

$$E = \alpha + i\beta$$

$$w = h + ik$$



$$= \bar{z}$$

$$= \log e^{\bar{z}}$$

$$w = \log[w]$$

$$A = \frac{\partial u(x_0, y_0)}{\partial x}$$

$$B = \frac{\partial u(x_0, y_0)}{\partial y}$$

$$C = \frac{\partial v(x_0, y_0)}{\partial x}$$

$$D = \frac{\partial v(x_0, y_0)}{\partial y}$$

$$f = u + iv$$

$$f(z_0 + w) = f(z_0) + wE + \sigma(z, w)$$

$$E = \alpha + i\beta$$

$$w = h + ik$$

$$\left| \frac{\sigma}{w} \right| \rightarrow 0 \quad w \rightarrow 0$$

$$A = \frac{\partial u(x_0, y_0)}{\partial x}$$

$$B = \frac{\partial u(x_0, y_0)}{\partial y}$$

$$C = \frac{\partial v(x_0, y_0)}{\partial x}$$

$$D = \frac{\partial v(x_0, y_0)}{\partial y}$$

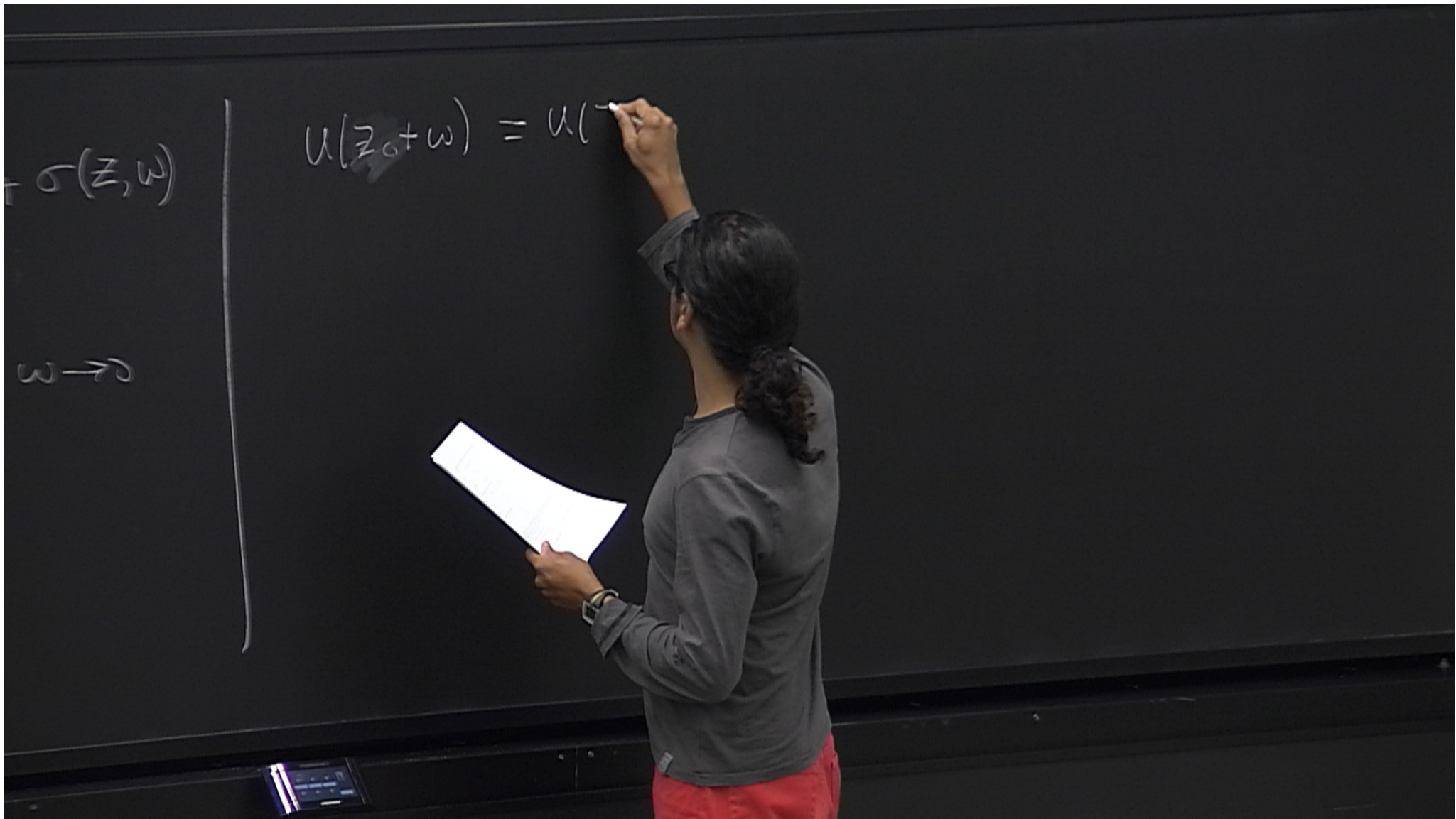
$$f = u + iv$$

$$f(z_0 + w) = f(z_0) + wE + \sigma(z, w)$$

$$E = \alpha + i\beta$$

$$w = h + ik$$

$$\left| \frac{\sigma}{w} \right| \rightarrow 0 \quad w \rightarrow 0$$



$$u(z_0 + w) = u(z_0) + h\alpha - k\beta + \operatorname{Re} \sigma$$

$$v(z_0 + w) = v(z_0) + h\beta + k\alpha + \operatorname{Im} \sigma$$

$$\alpha = \frac{\partial u}{\partial x}$$

$\sigma(z, w)$

$$u(z_0 + w) = u(z_0) + h\alpha - k\beta + \operatorname{Re} \sigma$$

$$v(z_0 + w) = v(z_0) + h\beta + k\alpha + \operatorname{Im} \sigma$$

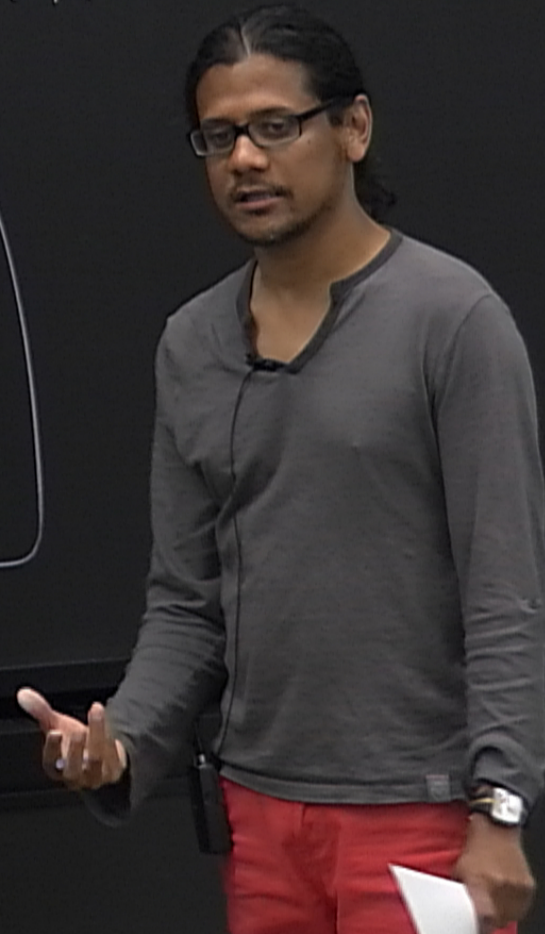
$$\alpha = \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \beta = -\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} \quad \left| \begin{matrix} (x_0, y_0) \end{matrix} \right.$$

(z, w)

$$u(z_0 + w) = u(z_0) + h\alpha - k\beta + \operatorname{Re} \sigma$$

$$v(z_0 + w) = v(z_0) + h\beta + k\alpha + \operatorname{Im} \sigma$$

$$\alpha = \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$
$$\beta = -\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} \quad \left(\begin{matrix} x_0 \\ y_0 \end{matrix} \right)$$



Analytic functions satisfy Cauchy-Riemann Condⁿs

Ho

$$\begin{aligned}\operatorname{Sinh}^{-1} w &= \mathbb{Z} \\ &= \log e^{\mathbb{Z}} \\ \operatorname{Sinh}^{-1} w &= \log[w + \sqrt{w^2 + 1}]\end{aligned}$$

Analytic functions satisfy Cauchy-Riemann Condⁿs

Holomorphic

$$f_1 = (x+iy)^{\vee}(x-iy)$$

$$f_2 = \operatorname{Re} \mathbb{Z}$$

$$\begin{aligned}\operatorname{Sinh}^{-1} w &= \mathbb{Z} \\ &= \log e^{\mathbb{Z}} \\ \operatorname{Sinh}^{-1} w &= \log[w + \sqrt{w^2 + 1}]\end{aligned}$$

Analytic functions satisfy Cauchy-Riemann Condⁿs

Holomorphic

$$f_1 = (x+iy)^{\vee}(x-iy)$$

$$f_2 = \operatorname{Re} \mathbb{Z}$$

$$f_3 = \operatorname{Im} \sqrt{\mathbb{Z}}$$

Analytic functions satisfy Cauchy-Riemann Cond^{ns}

Holomorphic

$$f_1 = (x+iy)^n (x-iy)$$

$$f_2 = \operatorname{Re} z$$

$$f_3 = \operatorname{Im} \sqrt{z}$$

$$f_4 = z^3$$

fy Cauchy-Riemann Cond^{ns}

$$(x+iy)^{\vee}(x-iy)$$

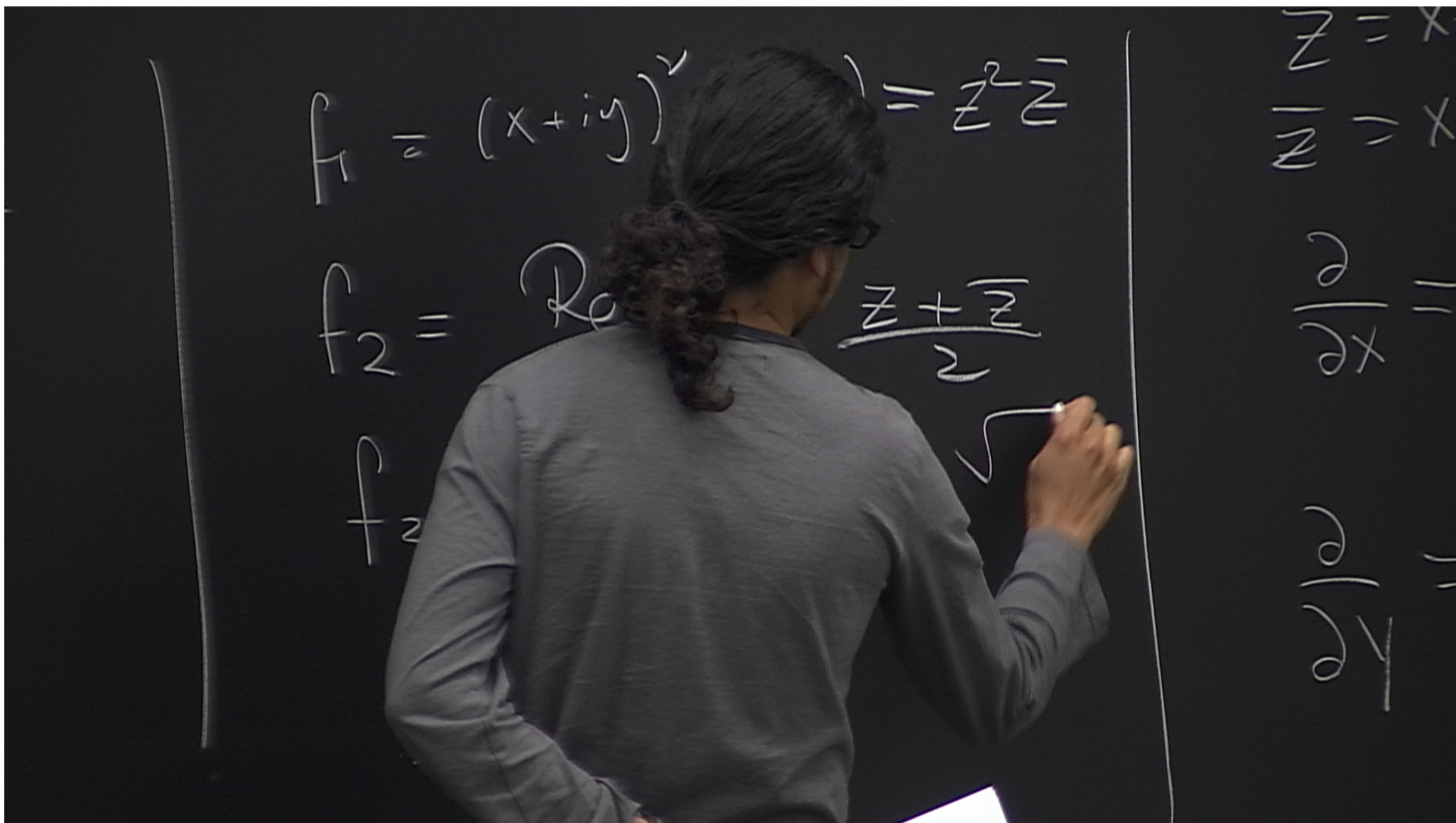
$$\operatorname{Re} z$$

$$\operatorname{Im} \sqrt{z}$$

$$z^2$$

$$z = x + iy$$

$$\bar{z} = x - iy$$



$$f_1 = (x+iy)^2 (x-iy) = z^2 \bar{z}$$

$$f_2 = \operatorname{Re} z = \frac{z + \bar{z}}{2}$$

$$f_3 = \operatorname{Im} \sqrt{z} = \frac{\sqrt{z} - \sqrt{\bar{z}}}{2i}$$

$$f_4 = z^3$$

$$z = x + iy$$

$$\bar{z} = x - iy$$

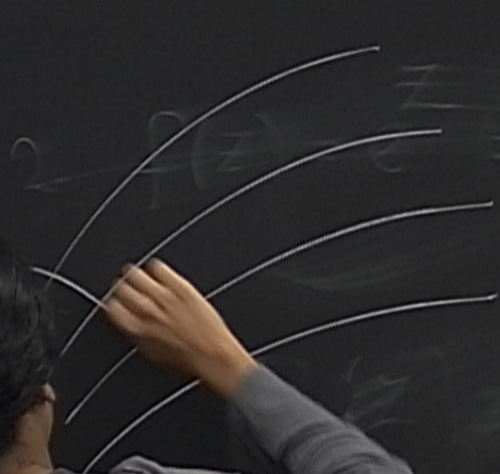
$$\frac{\partial}{\partial x} =$$

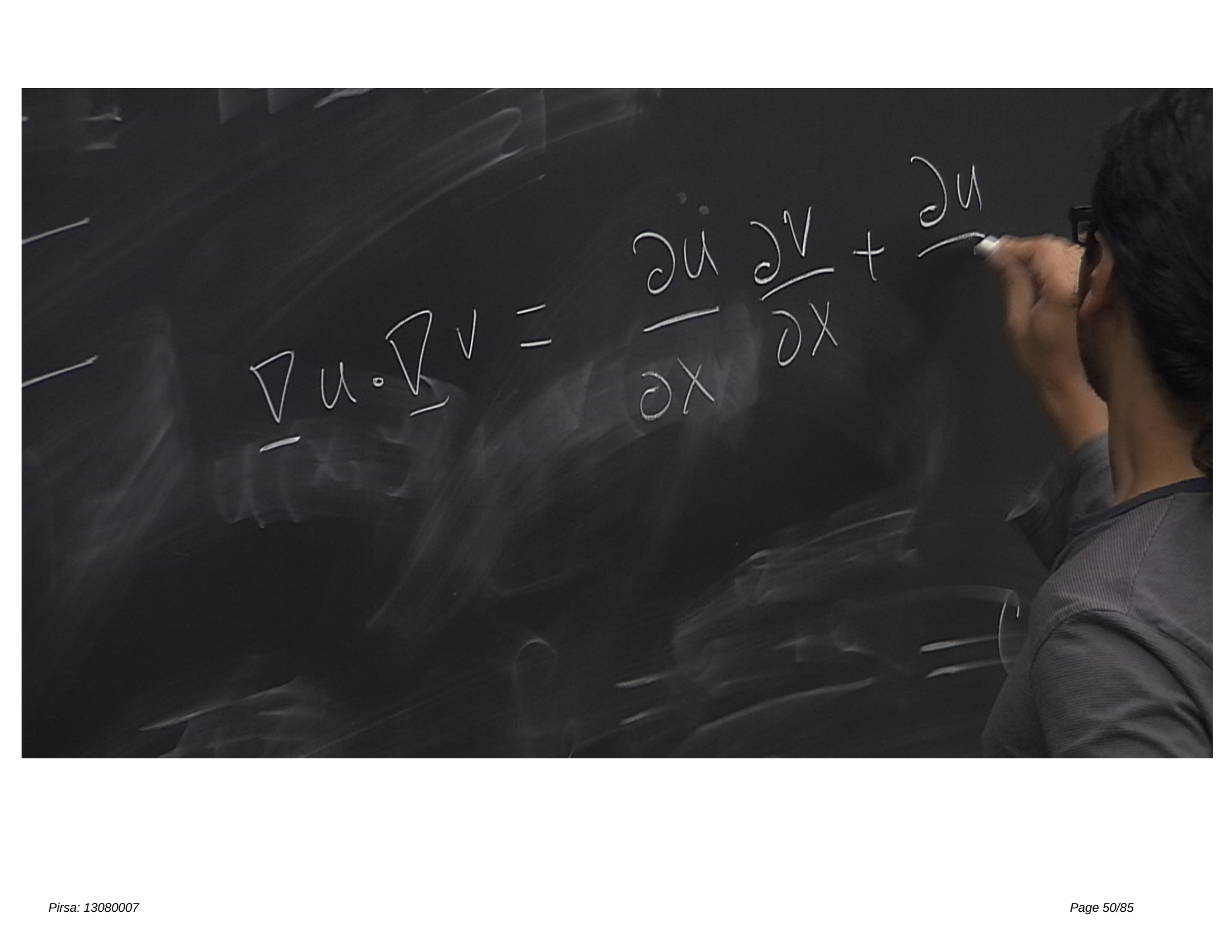
$$\frac{\partial}{\partial y} =$$

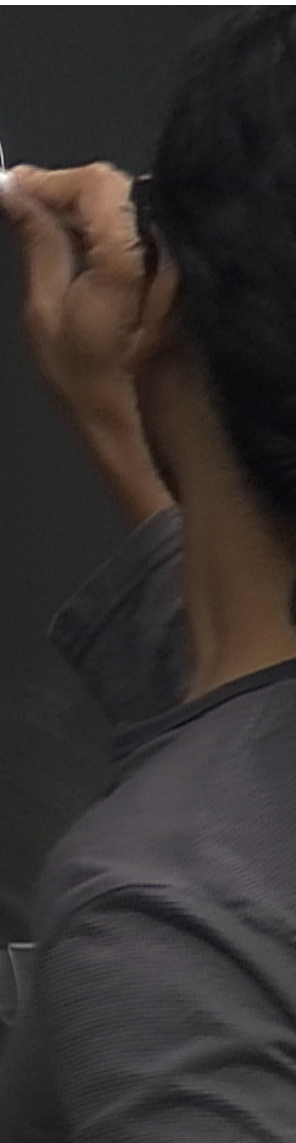
$$I = \begin{bmatrix} -u \\ v \end{bmatrix}$$

$$\nabla \cdot \underline{I} = 0$$

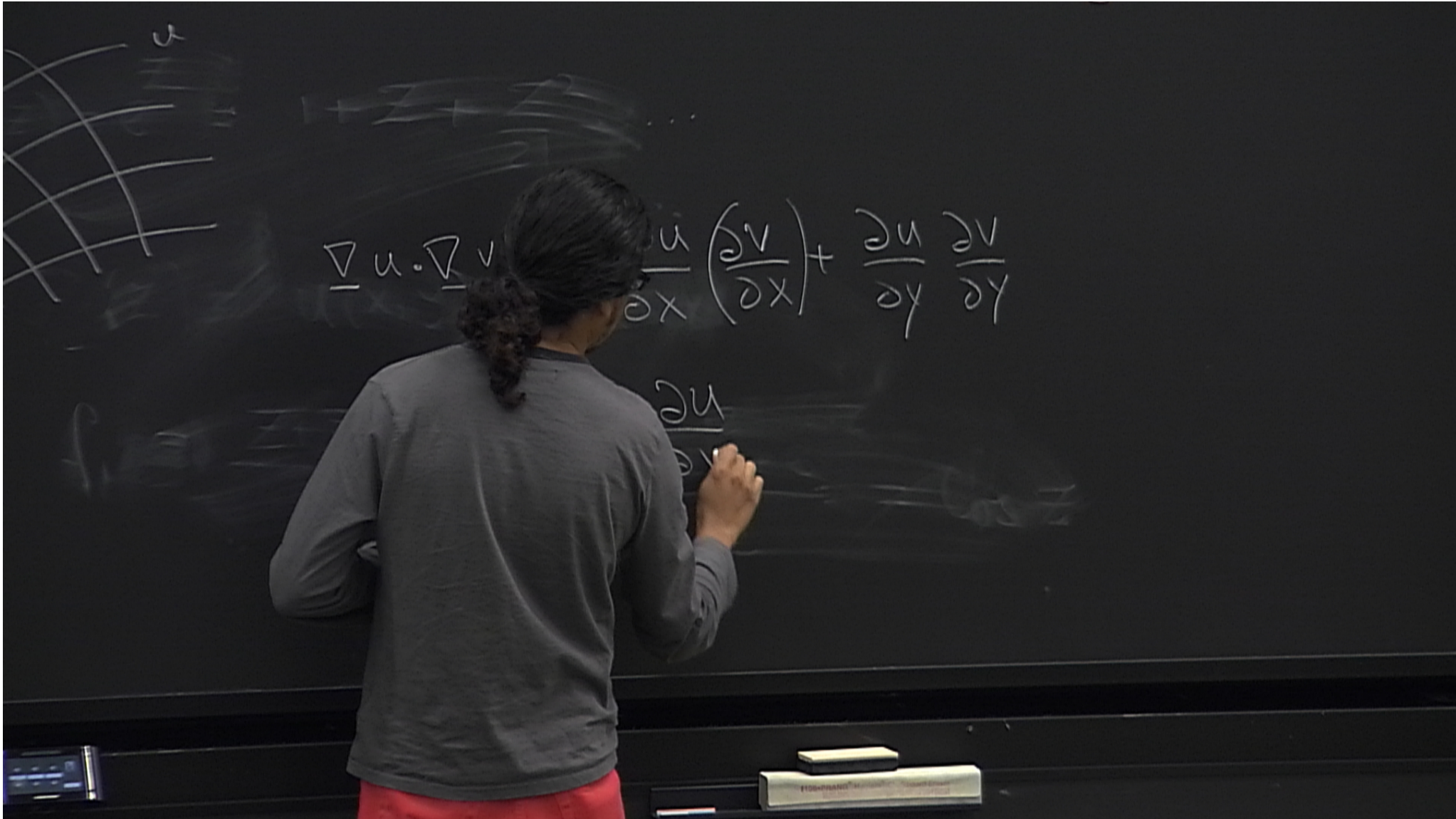
$$\nabla \times \underline{I} = 0$$

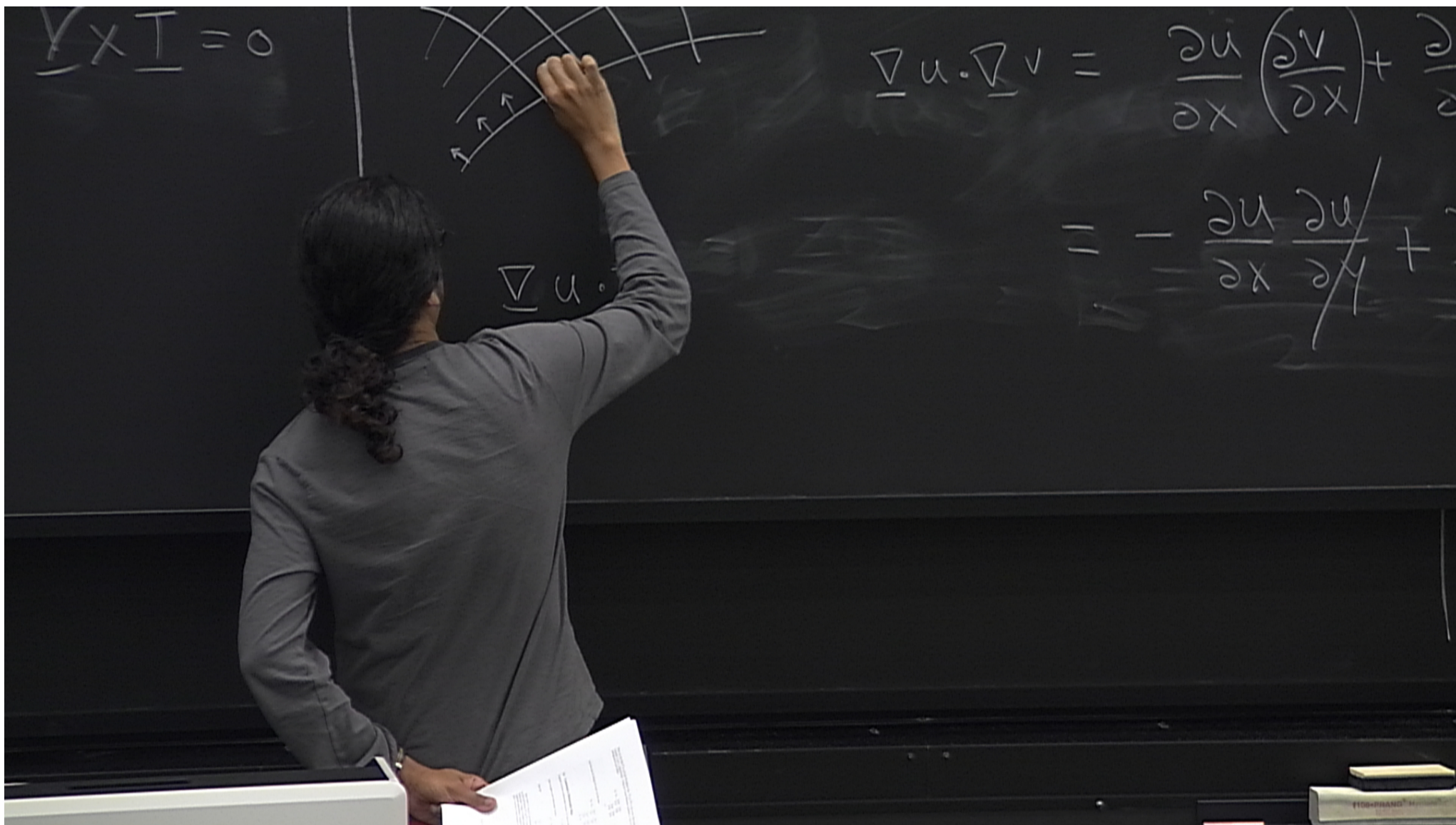


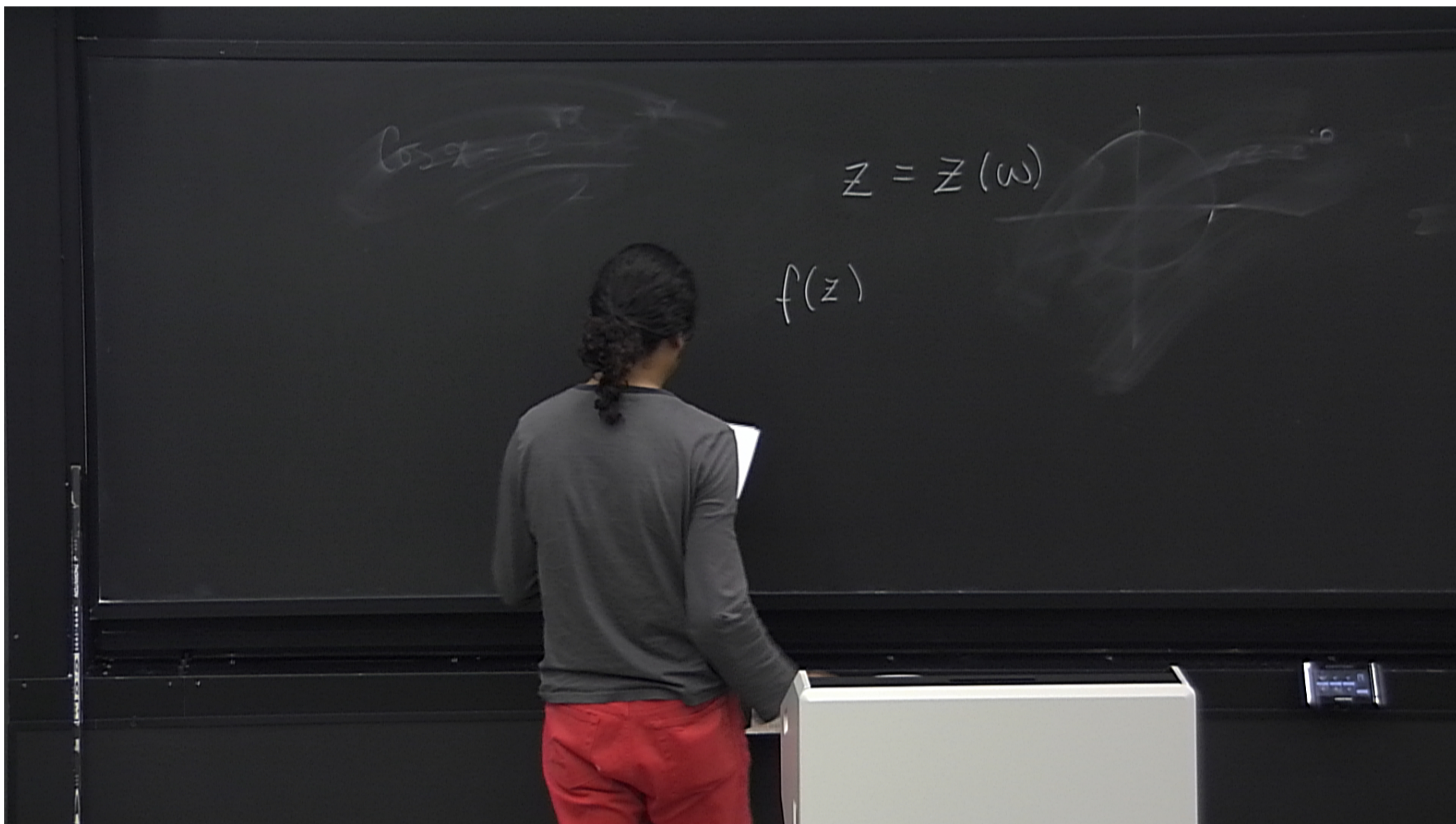
A person with dark hair and glasses, wearing a grey shirt, is seen from the side, writing on a dark chalkboard. They are holding a piece of white chalk in their right hand, which is positioned near the end of the equation. The chalkboard has some faint, illegible markings from previous writing.
$$\nabla u \cdot \nabla v = \frac{\partial u}{\partial x} \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

A person is writing a mathematical formula on a chalkboard. The formula is $\nabla u \cdot \nabla v = \frac{\partial u}{\partial x} \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \frac{\partial v}{\partial y}$. The person's hand is visible on the right side of the image, holding a piece of chalk and writing the final part of the formula.
$$\nabla u \cdot \nabla v = \frac{\partial u}{\partial x} \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \frac{\partial v}{\partial y}$$

$$\nabla u \cdot \nabla v = \frac{\partial u}{\partial x} \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \frac{\partial v}{\partial y}$$





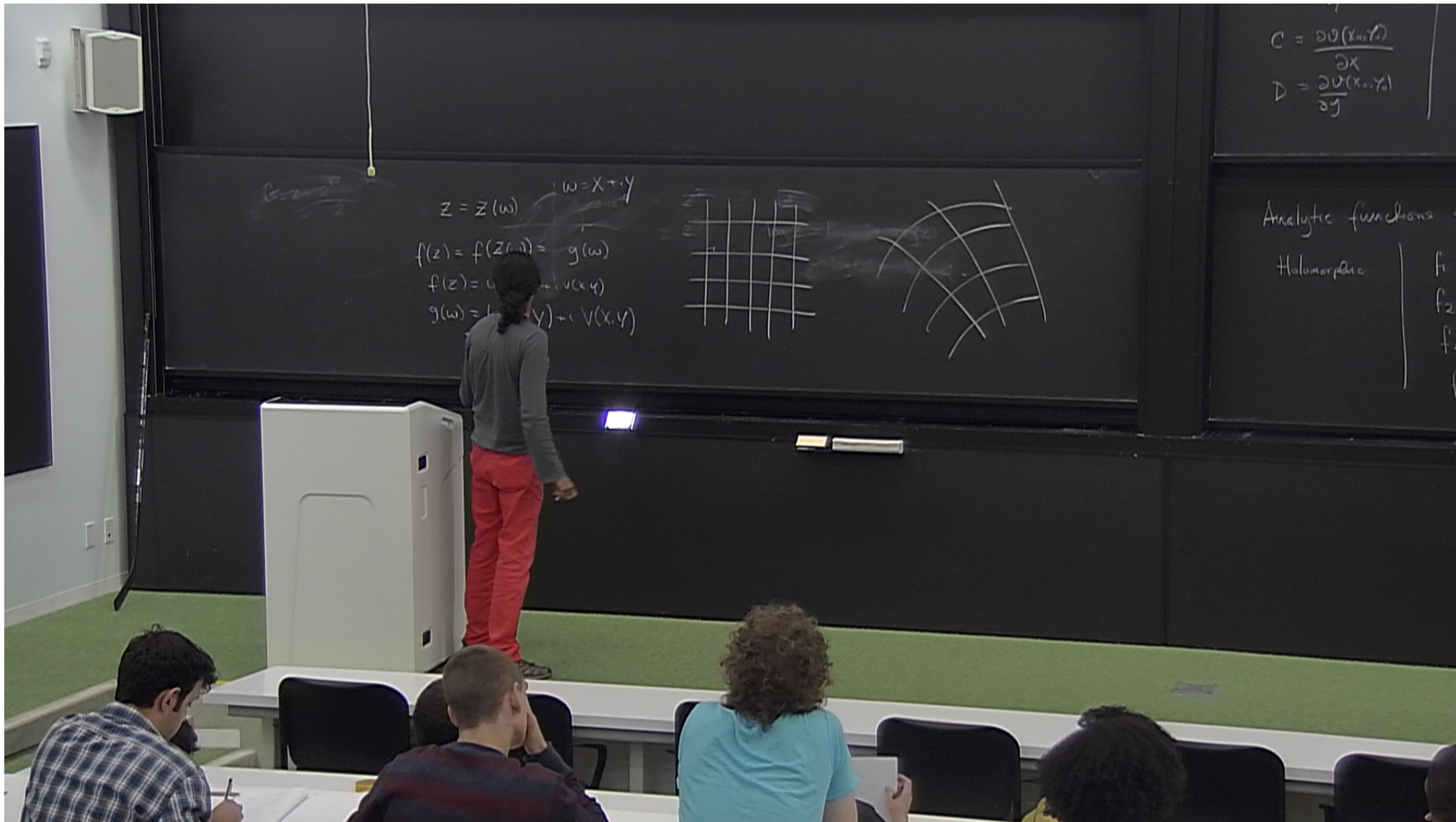


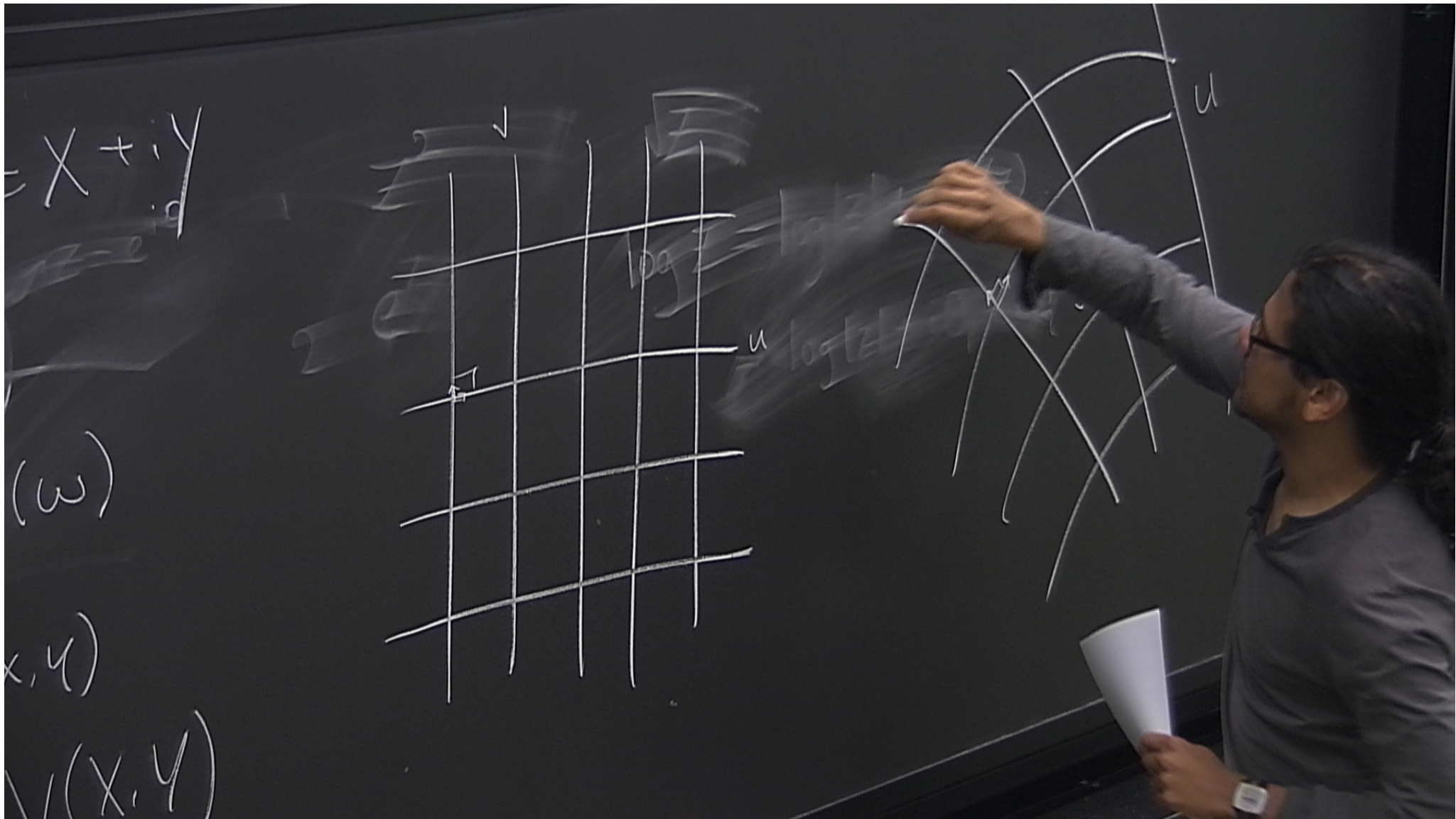
$$z = z(w)$$

$$f(z) = f(z(w)) = g(w)$$

$$f(z) = u(x, y) + i v(x, y)$$

$$g(w)$$





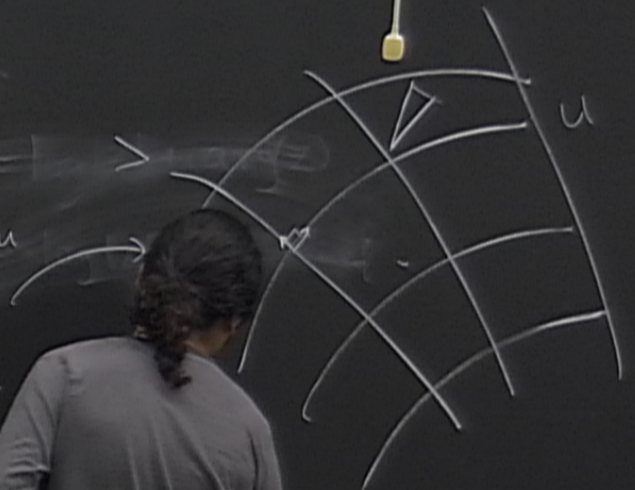
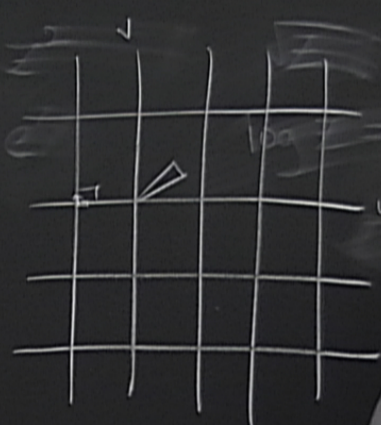
$$z = z(w)$$

$$w = x + iy$$

$$f(z) = f(z(w)) = g(w)$$

$$f(z) = u(x, y) + i v(x, y)$$

$$g(w) = U(x, y) + i V(x, y)$$

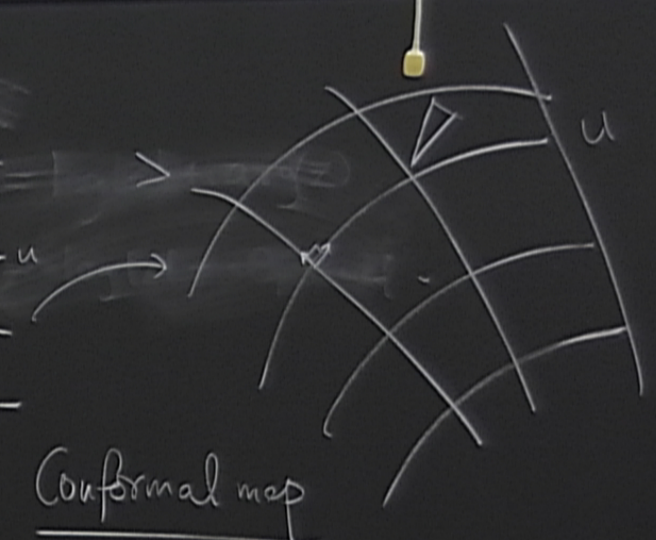
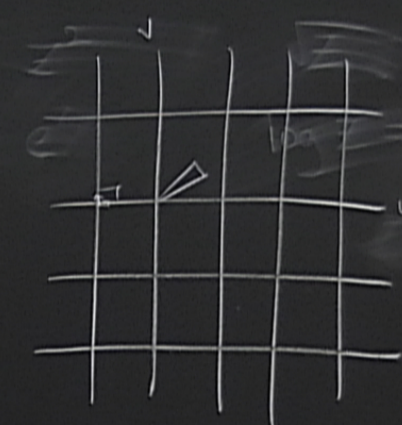


$$z = z(w) \quad w = x + iy$$

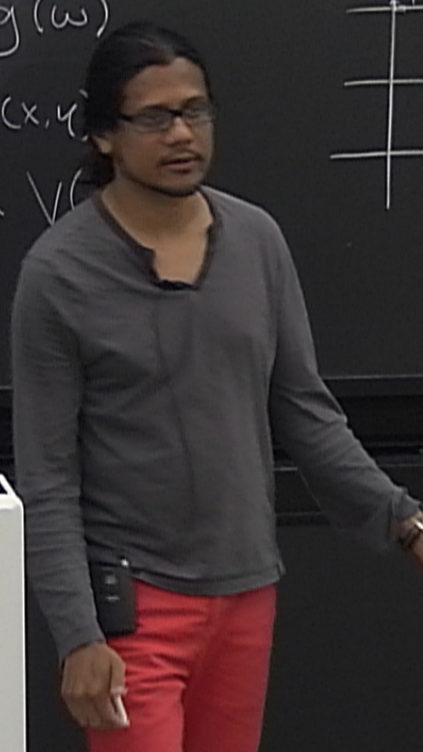
$$f(z) = f(z(w)) = g(w)$$

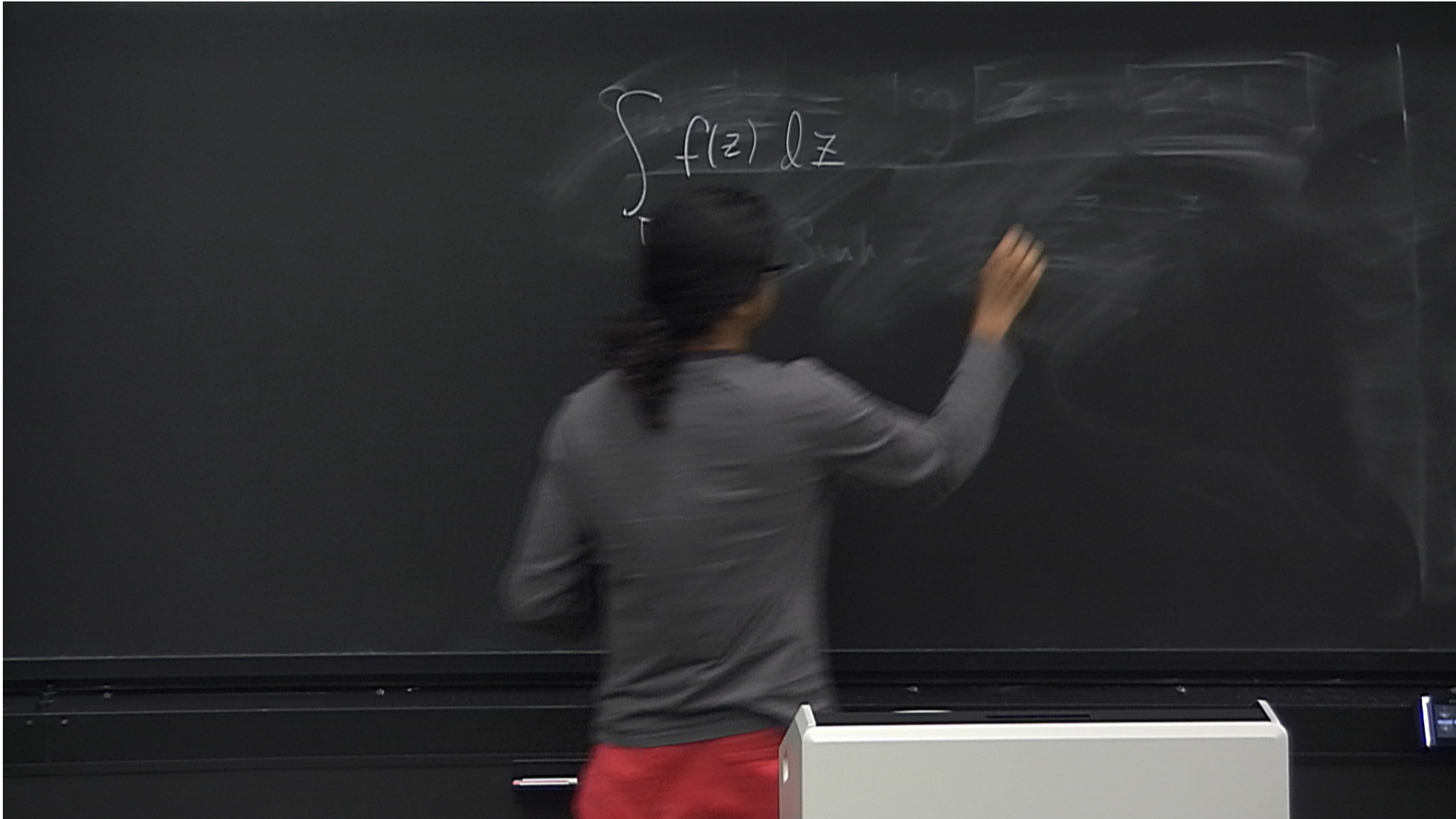
$$f(z) = u(x, y) + i v(x, y)$$

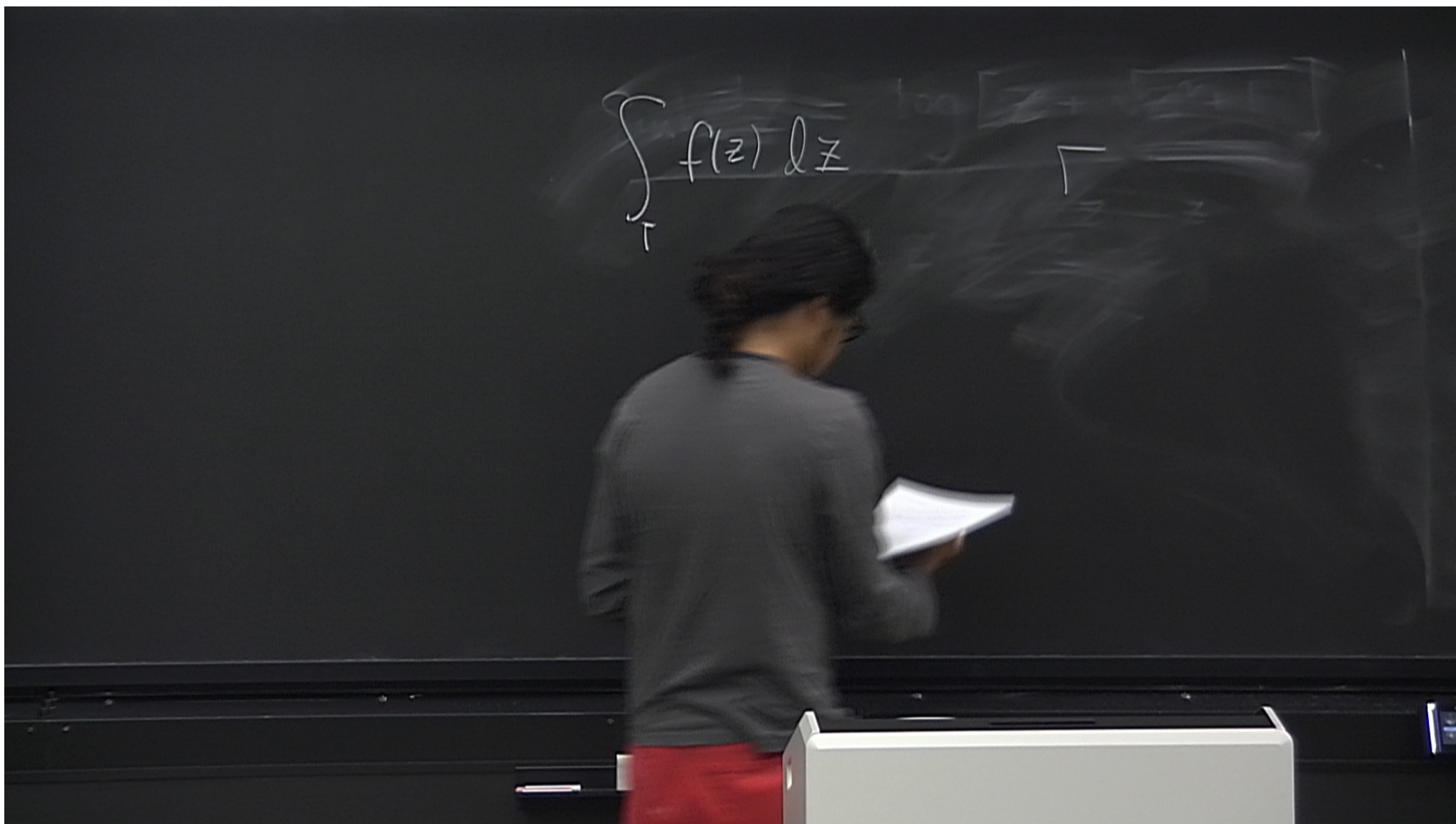
$$g(w) = U(x, y) + i V(x, y)$$

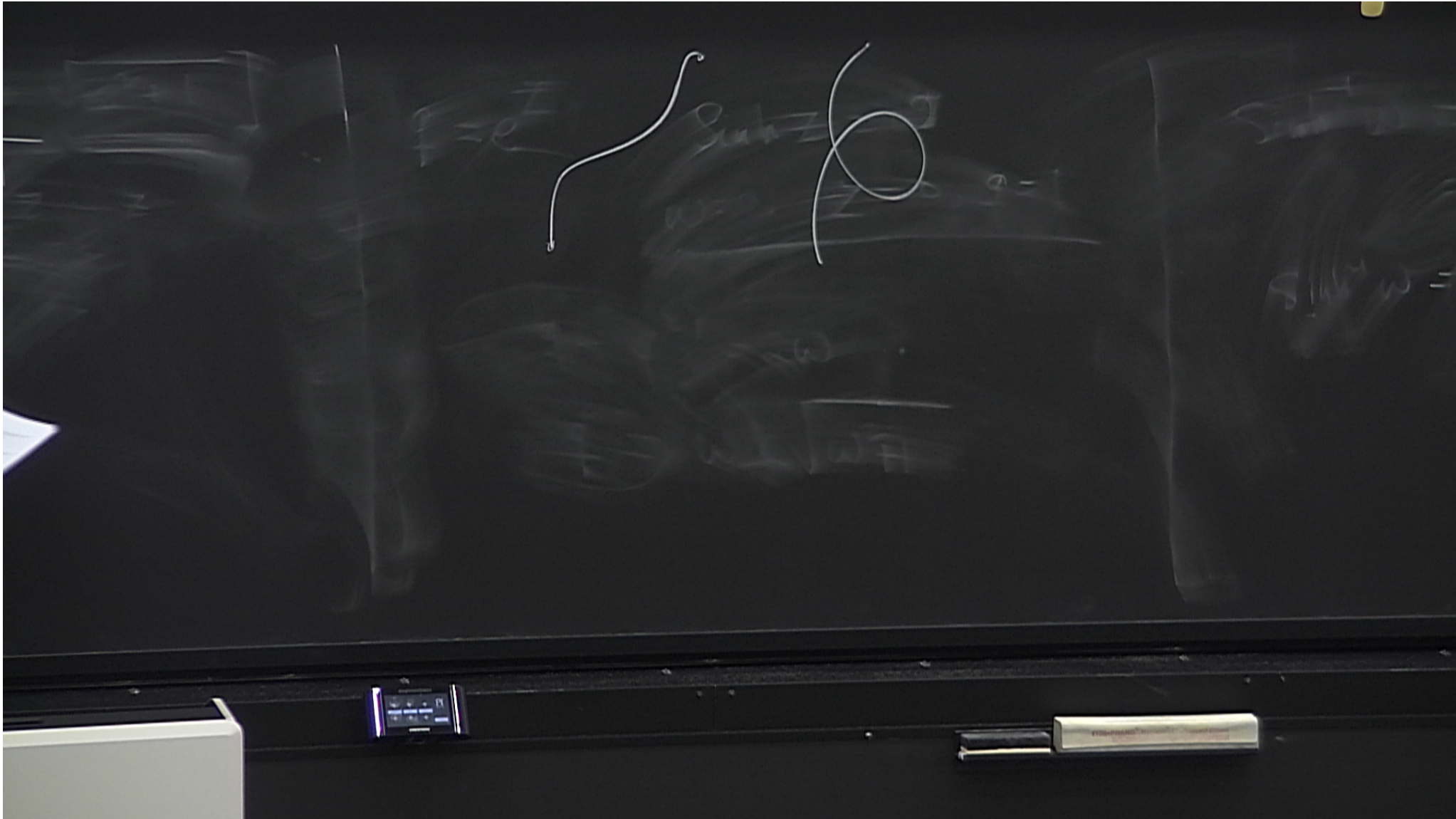


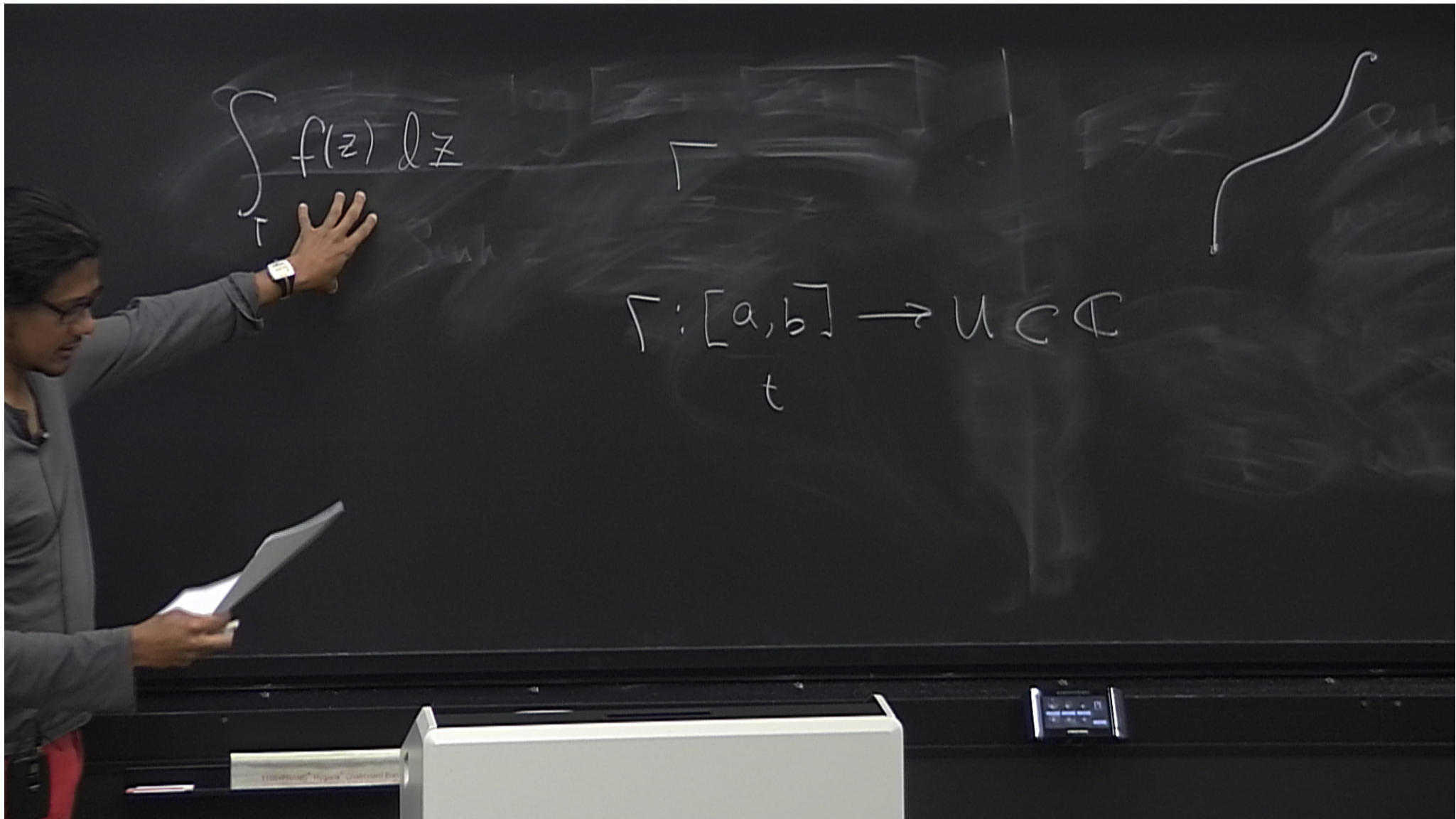
Conformal map





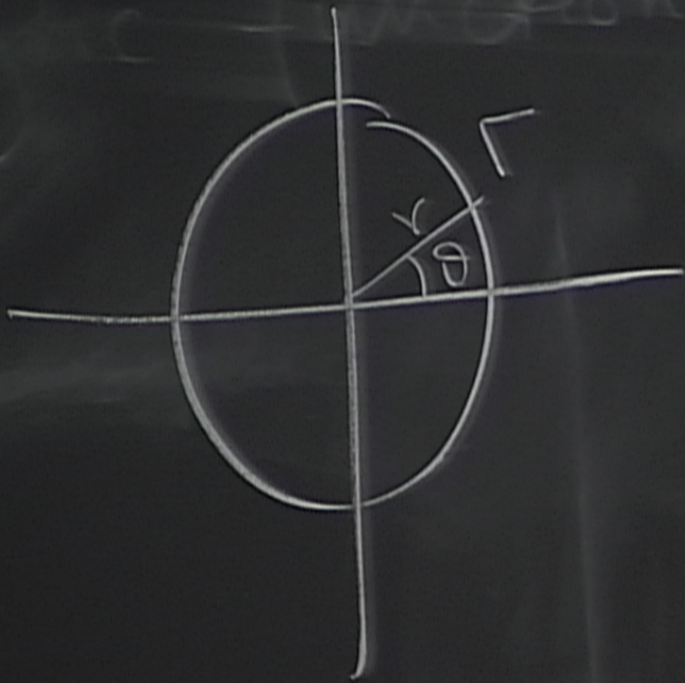






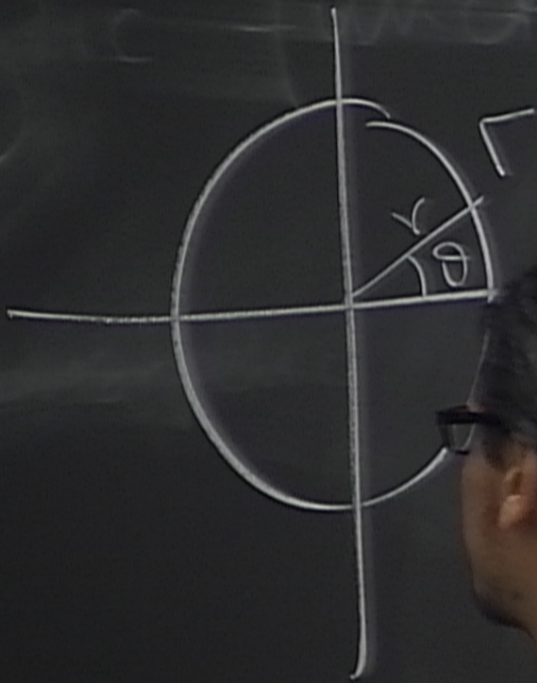
$U \subset \mathbb{C}$

$$\int_{\gamma} f(z) dz := \int_a^b f(\gamma(t)) \gamma'(t) dt$$



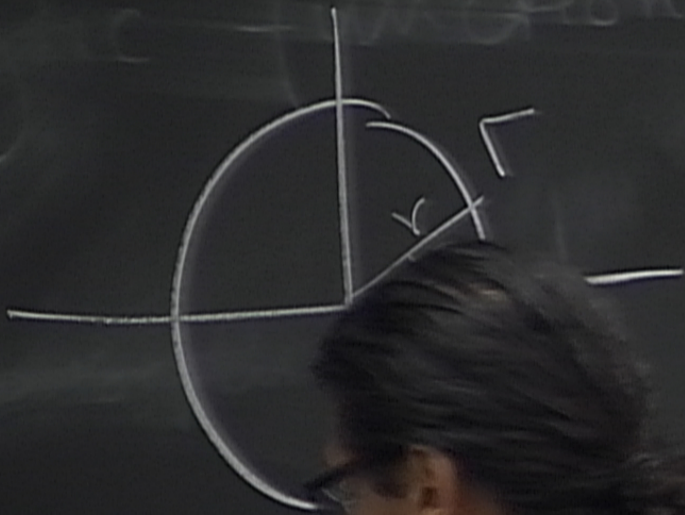
$$f(z) = z^n$$

$$\oint_{\Gamma}$$



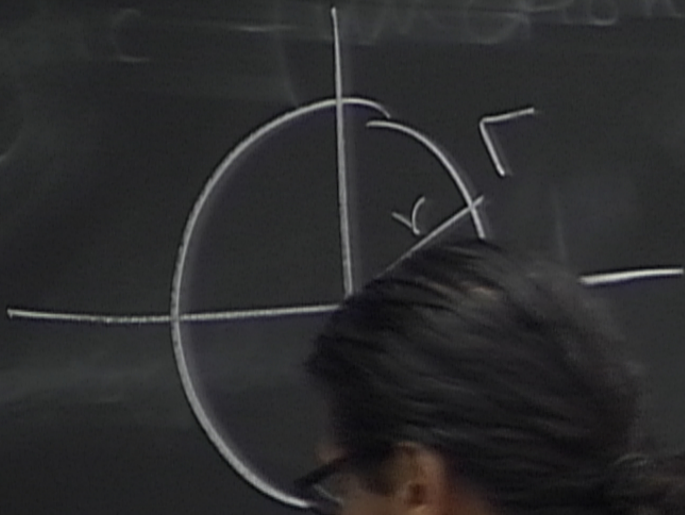
$$f(z) = z^n$$

$$\oint_{\Gamma} f(z) dz$$



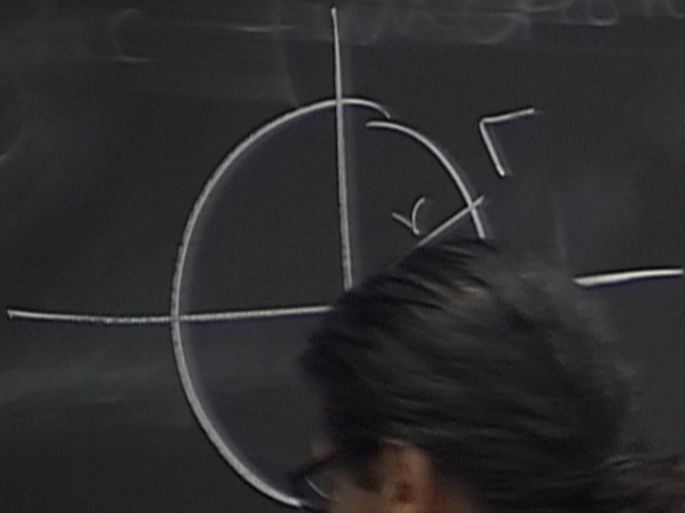
$$f(z) = z^n$$

$$\oint_{\Gamma} f(z) dz$$



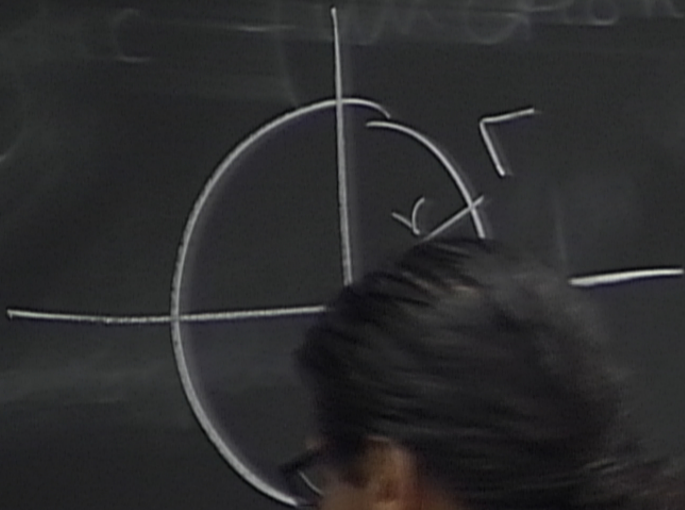
$$f(z) = z^n$$

$$\oint_{\Gamma} f(z) dz$$



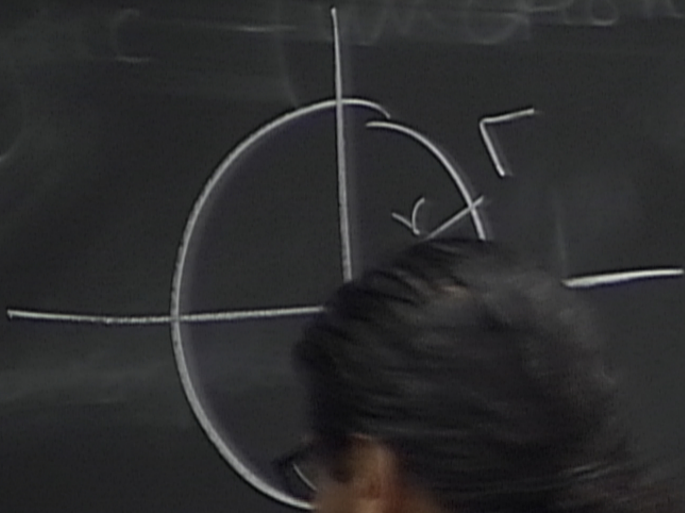
$$f(z) = z^n$$

$$\oint_{\Gamma} f(z) dz$$



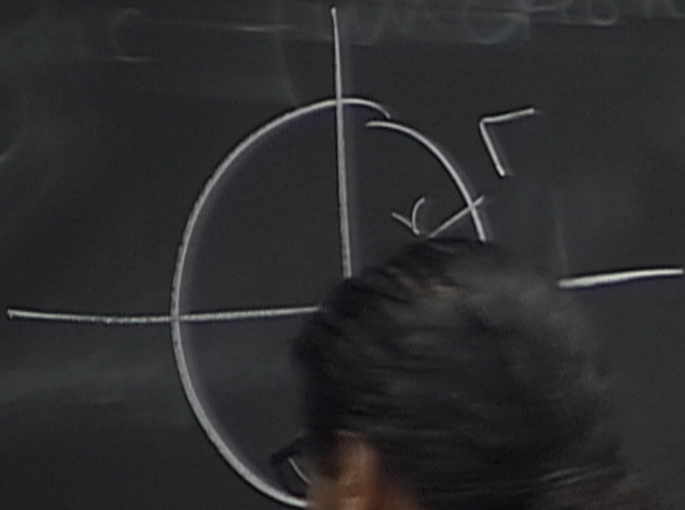
$$f(z) = z^n$$

$$\oint_{\Gamma} f(z) dz$$



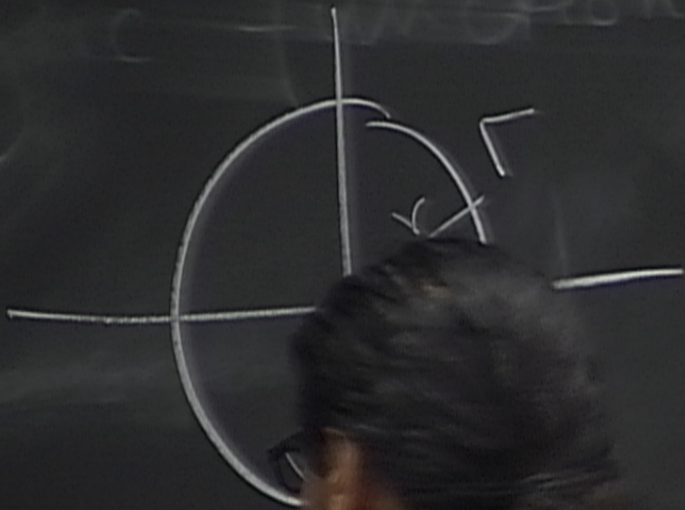
$$f(z) = z^n$$

$$\oint_{\Gamma} f(z) dz$$



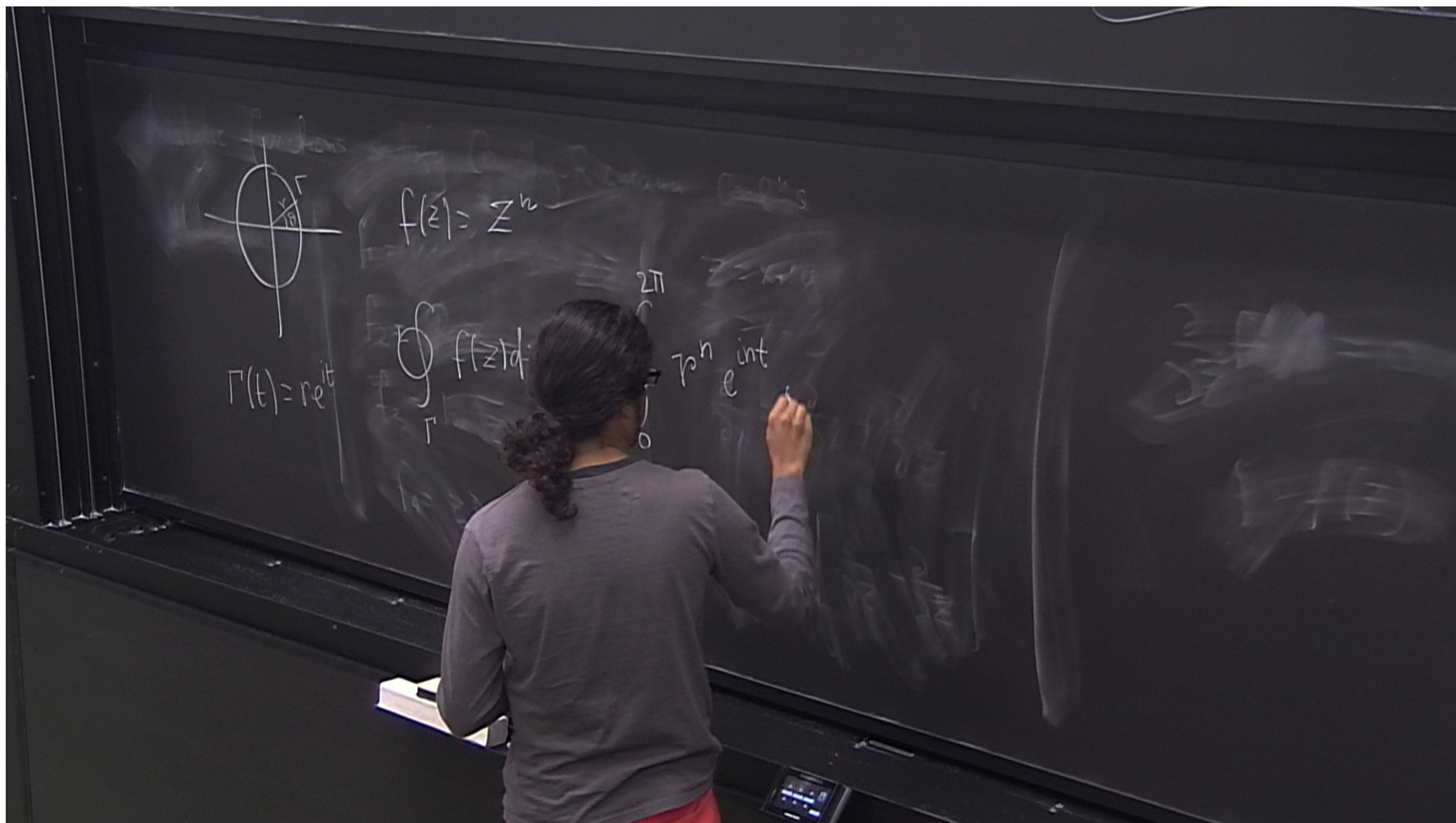
$$f(z) = z^n$$

$$\oint_{\Gamma} f(z) dz$$

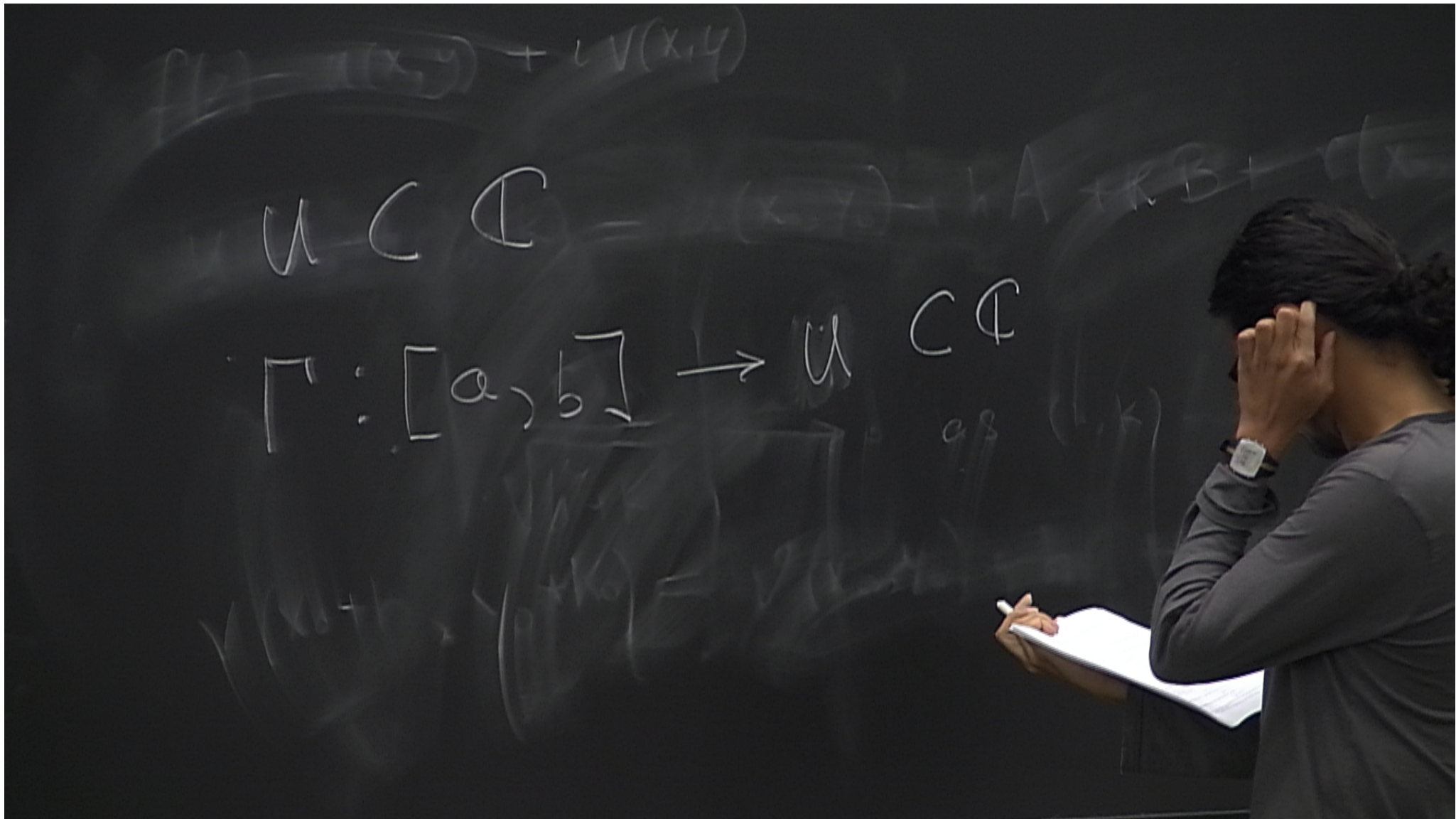


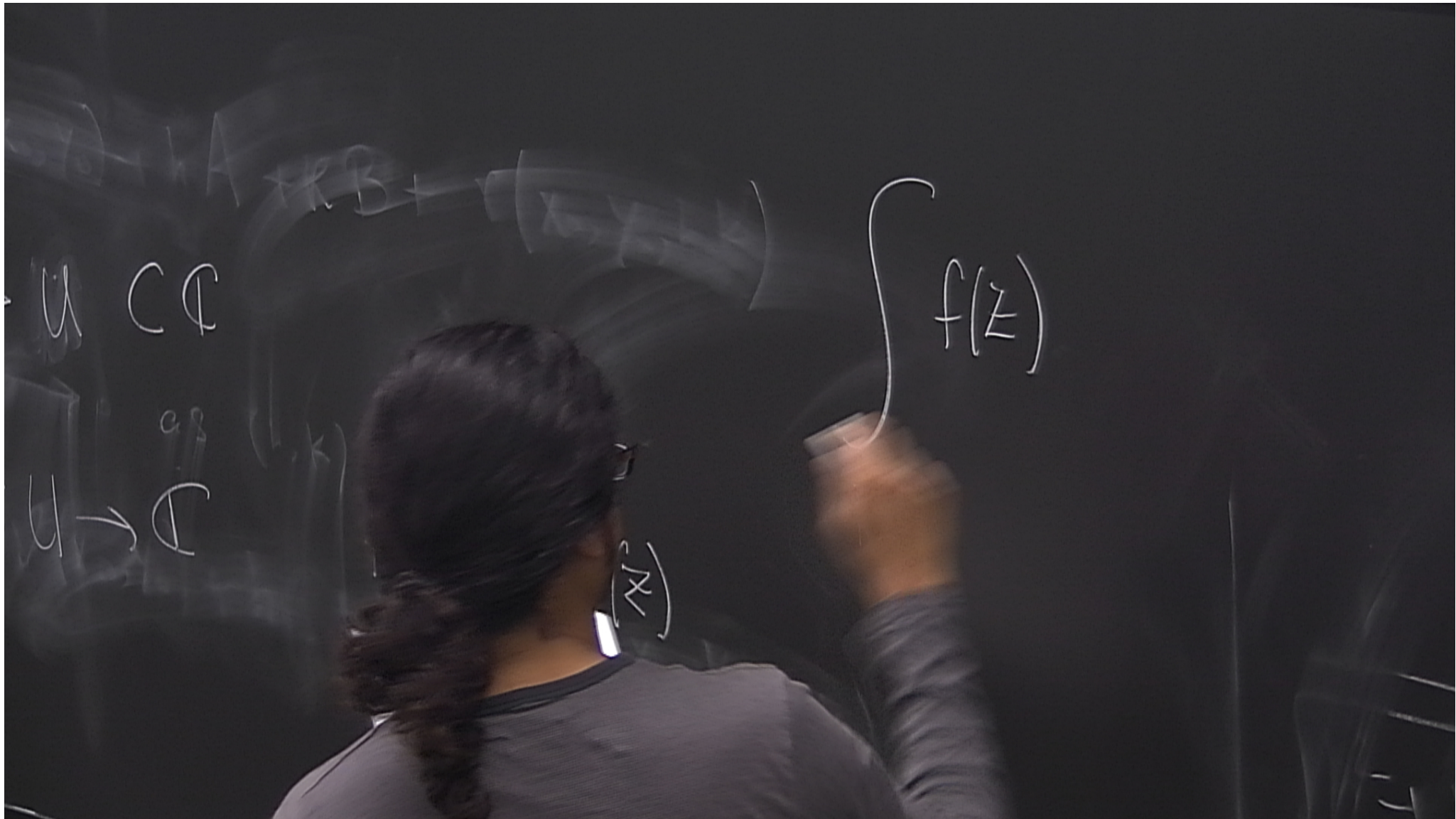
$$f(z) = z^n$$

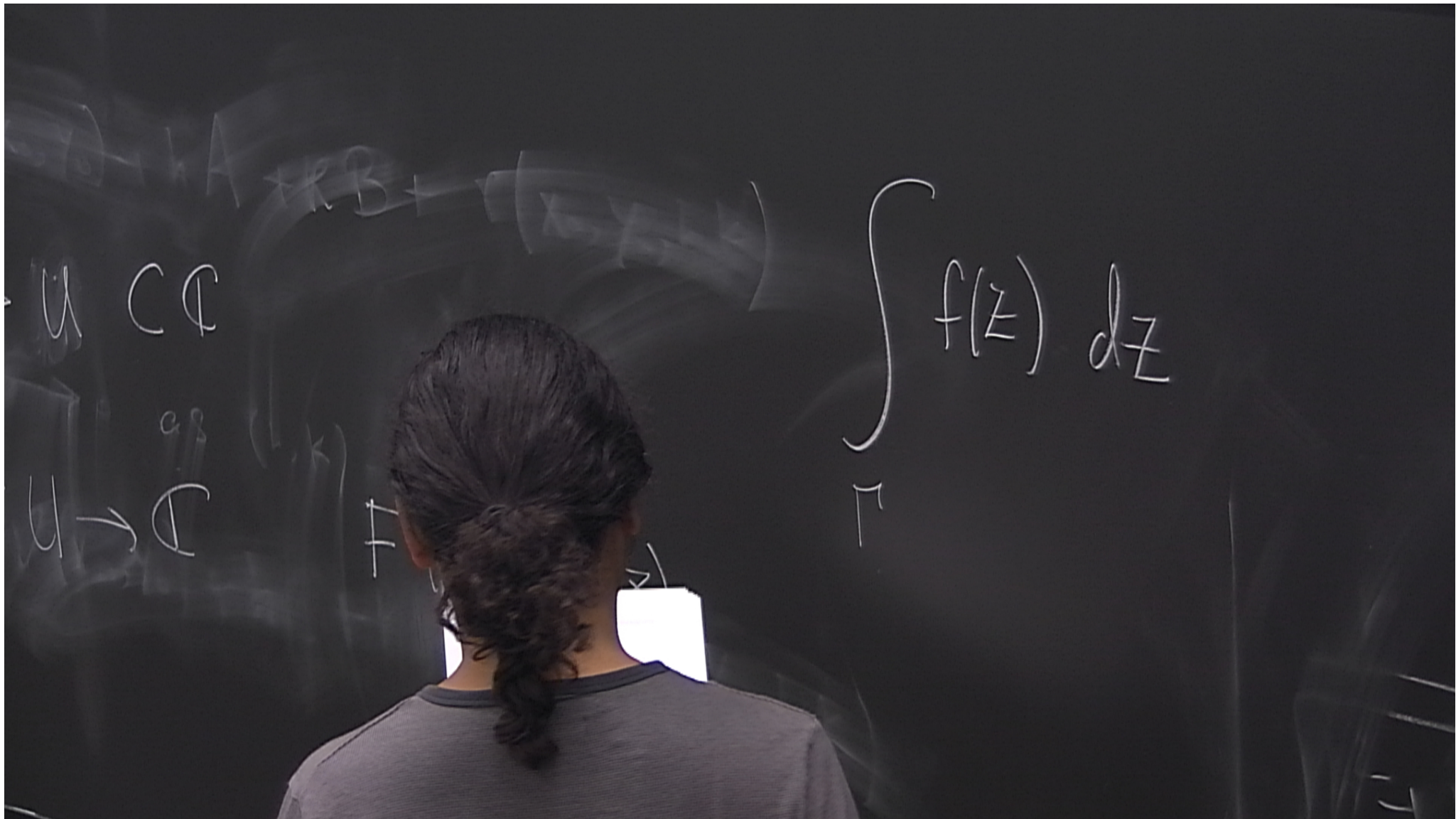
$$\oint_{\Gamma} f(z) dz$$



$$\oint \frac{dz}{z} = 2\pi i$$

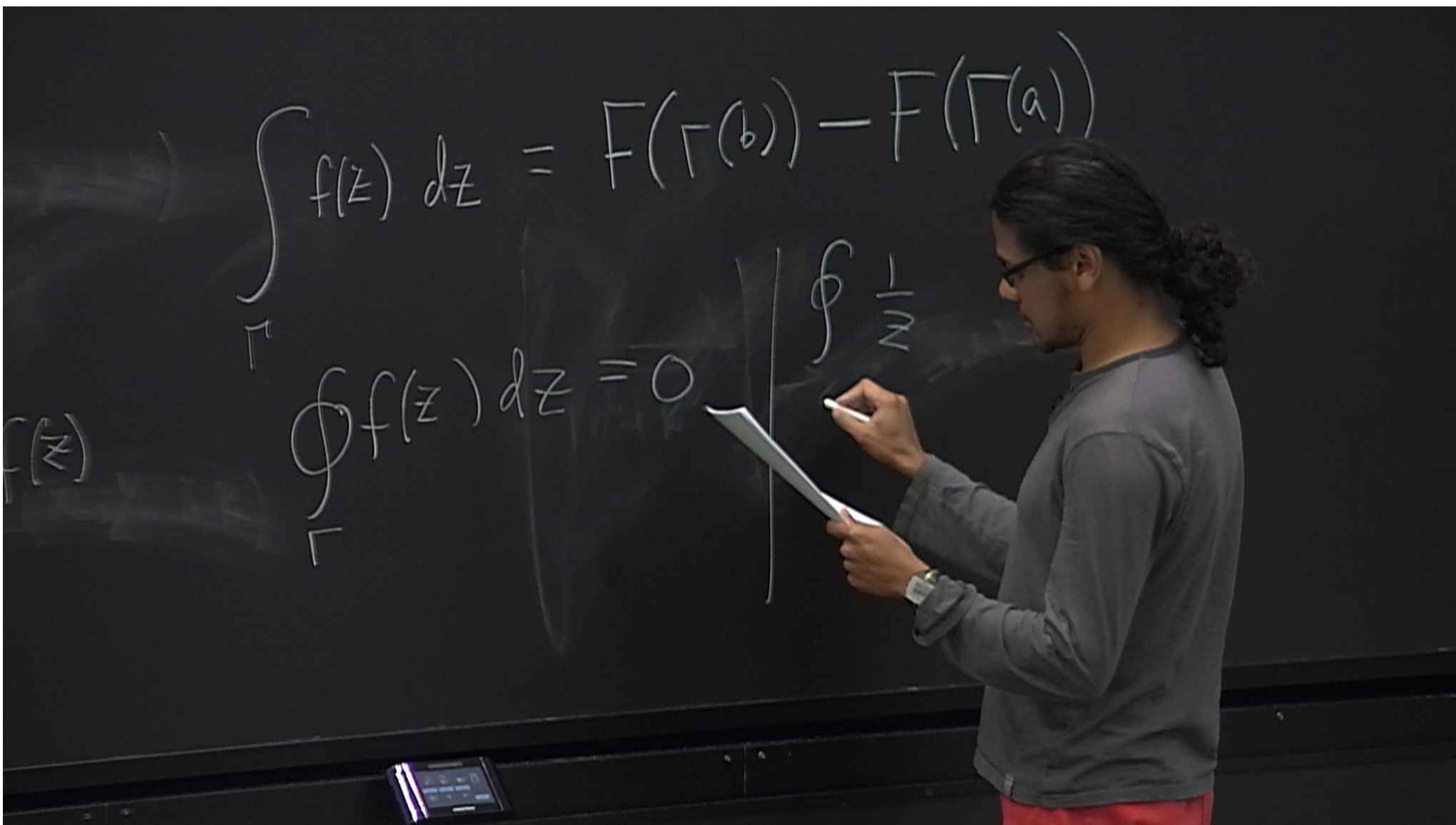






$$\int_{\Gamma} f(z) dz = F(\Gamma(b)) - F(\Gamma(a))$$

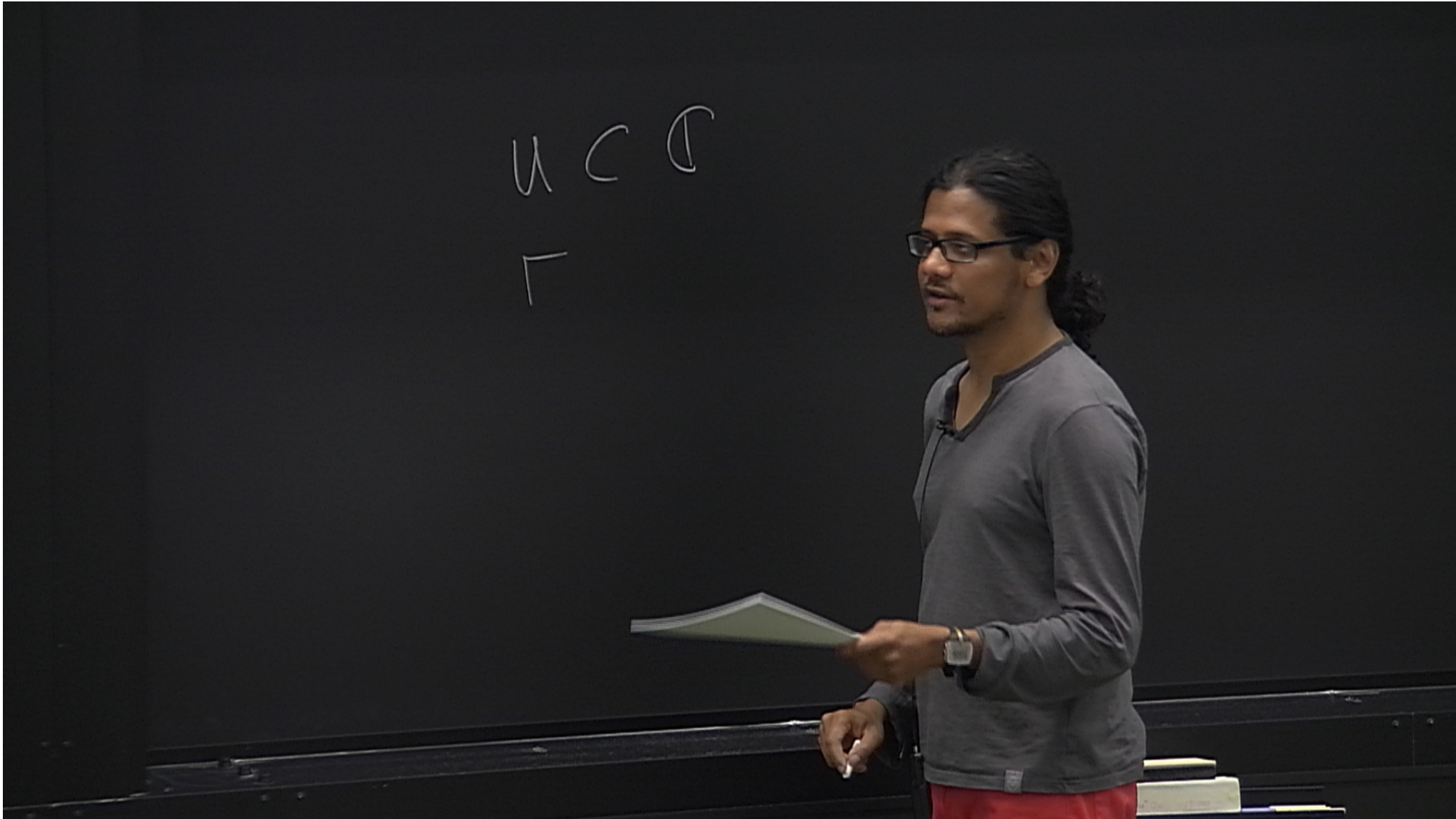
$$\oint_{\Gamma} f(z) dz = 0$$

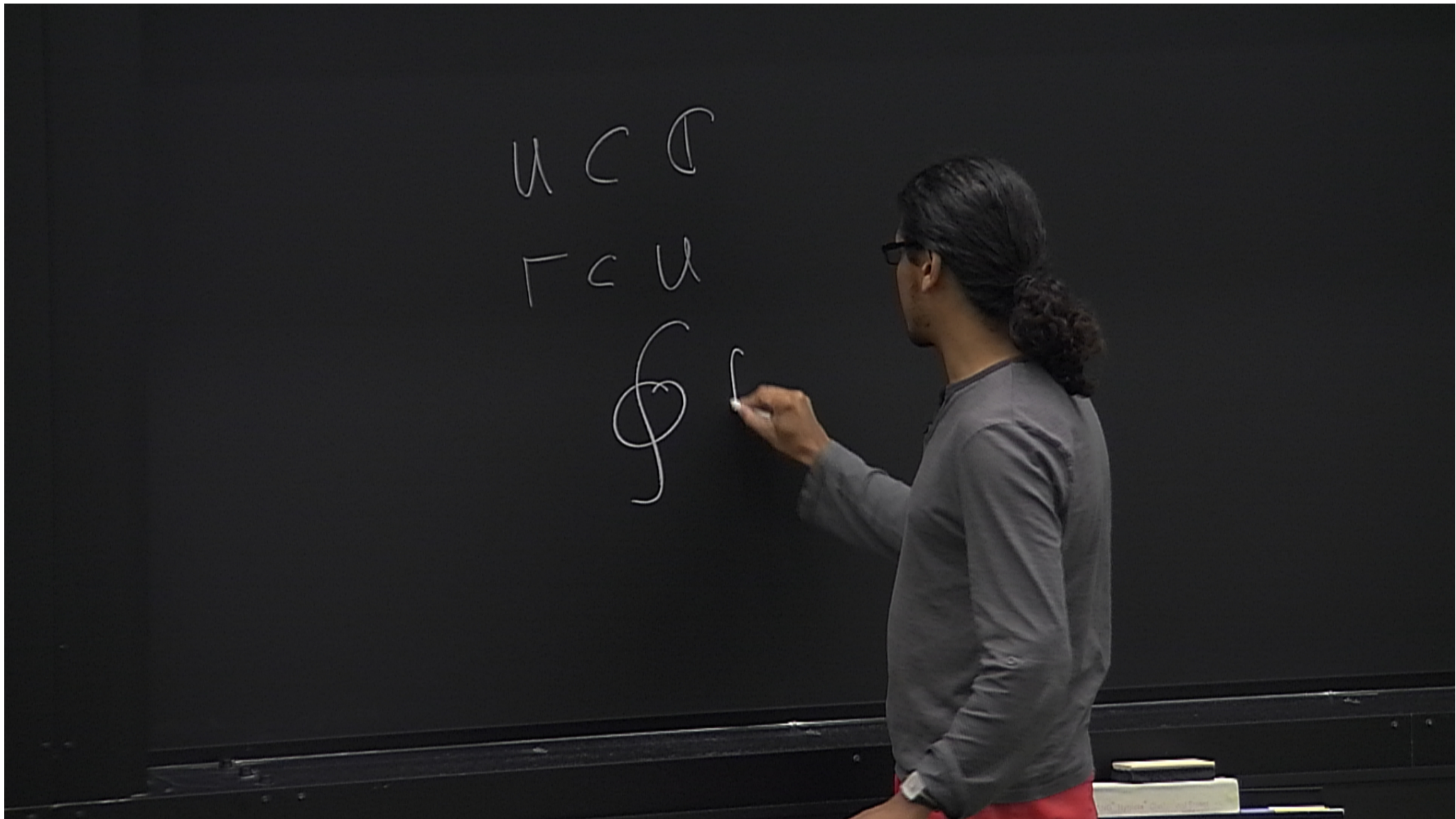


$$\int_{\Gamma} f(z) dz = F(\Gamma(b)) - F(\Gamma(a))$$

$$\oint_{\Gamma} f(z) dz = 0$$

$$\oint_{\Gamma} \frac{1}{z} dz = 2\pi i$$





$$U \subset \mathbb{D}$$

$$\Gamma \subset U$$

$$\oint_{\Gamma} f(z) dz = 0$$