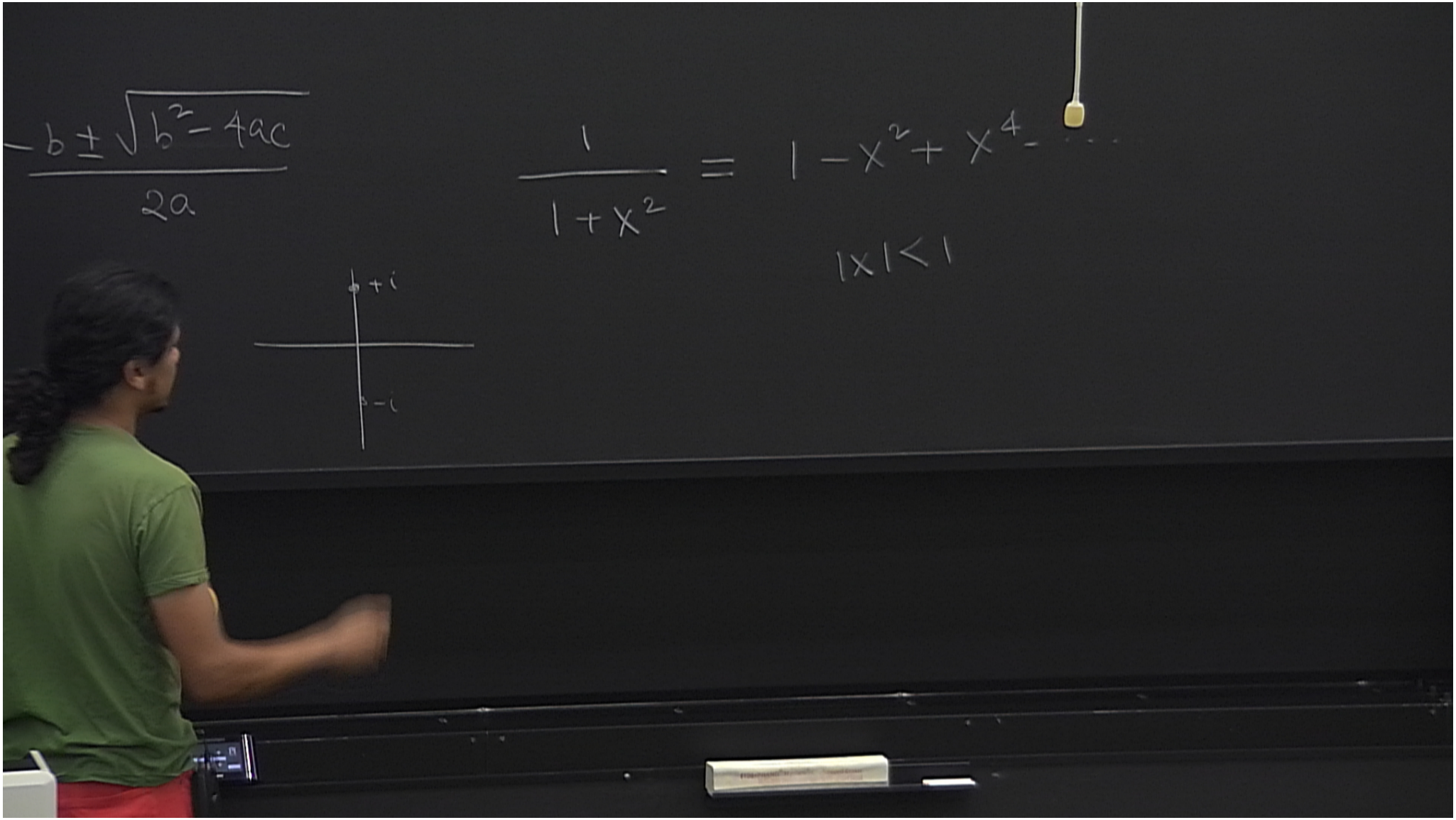
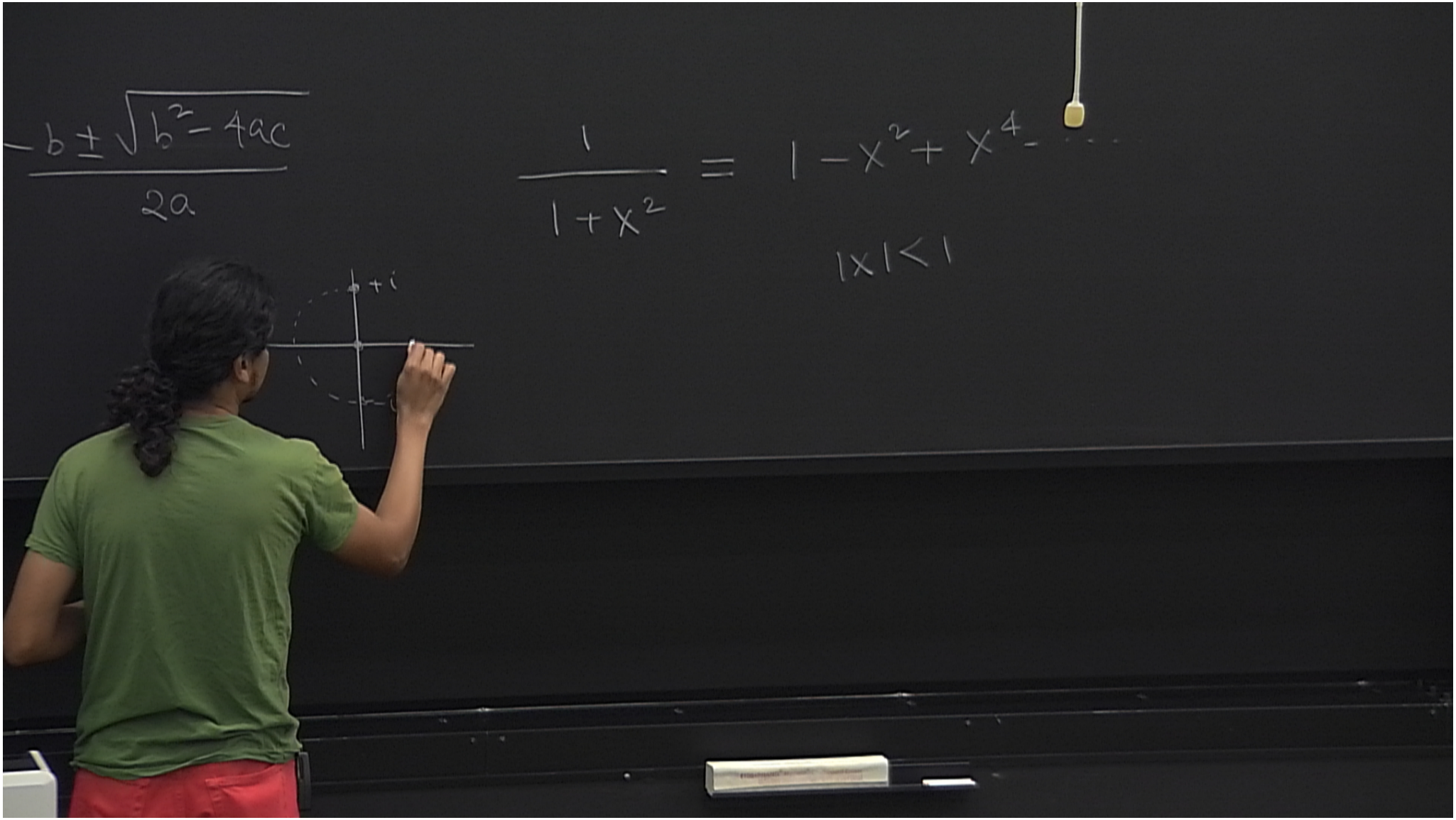
Title: 13/14 PSI- Complex Analysis - Lecture 1

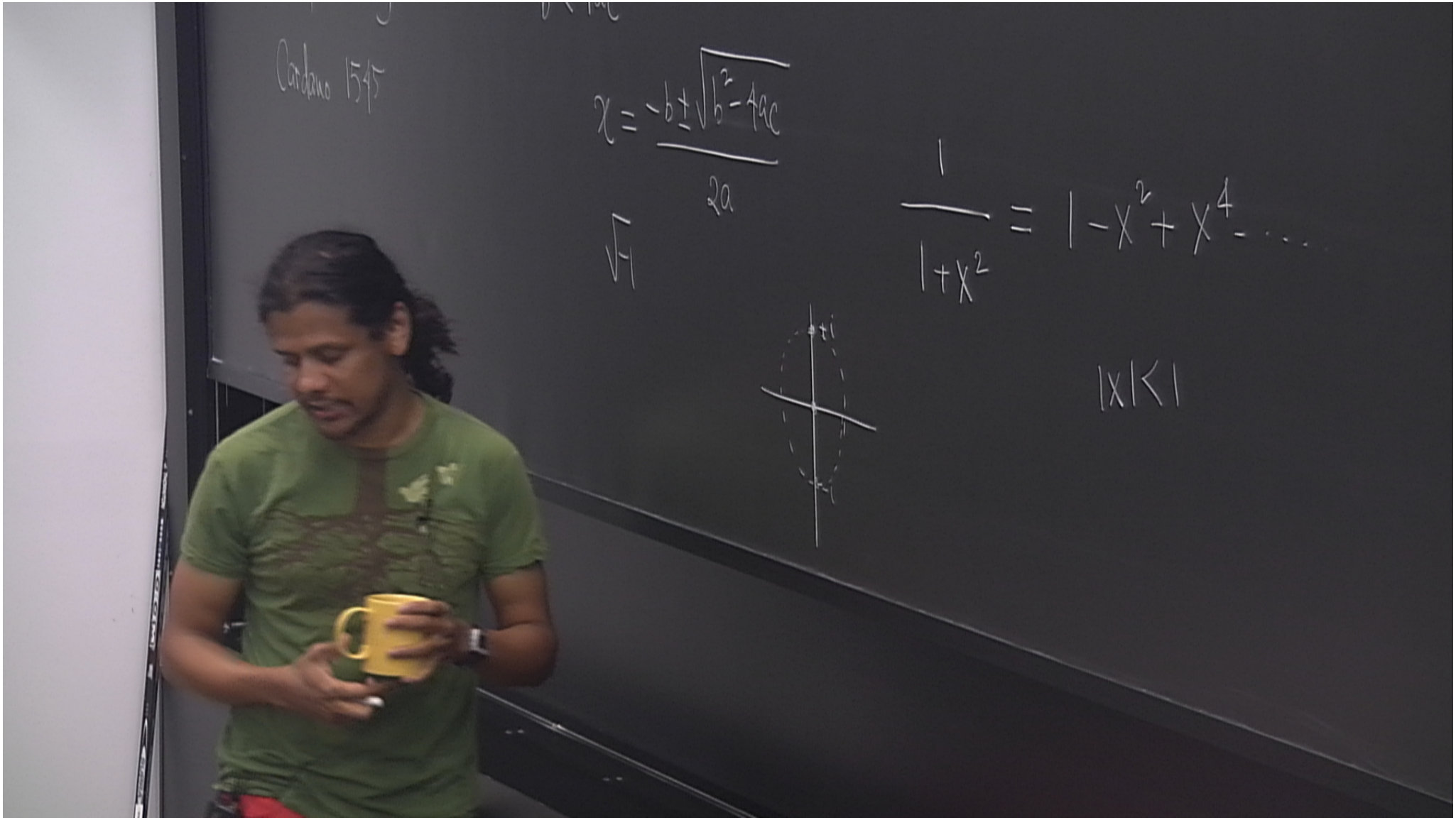
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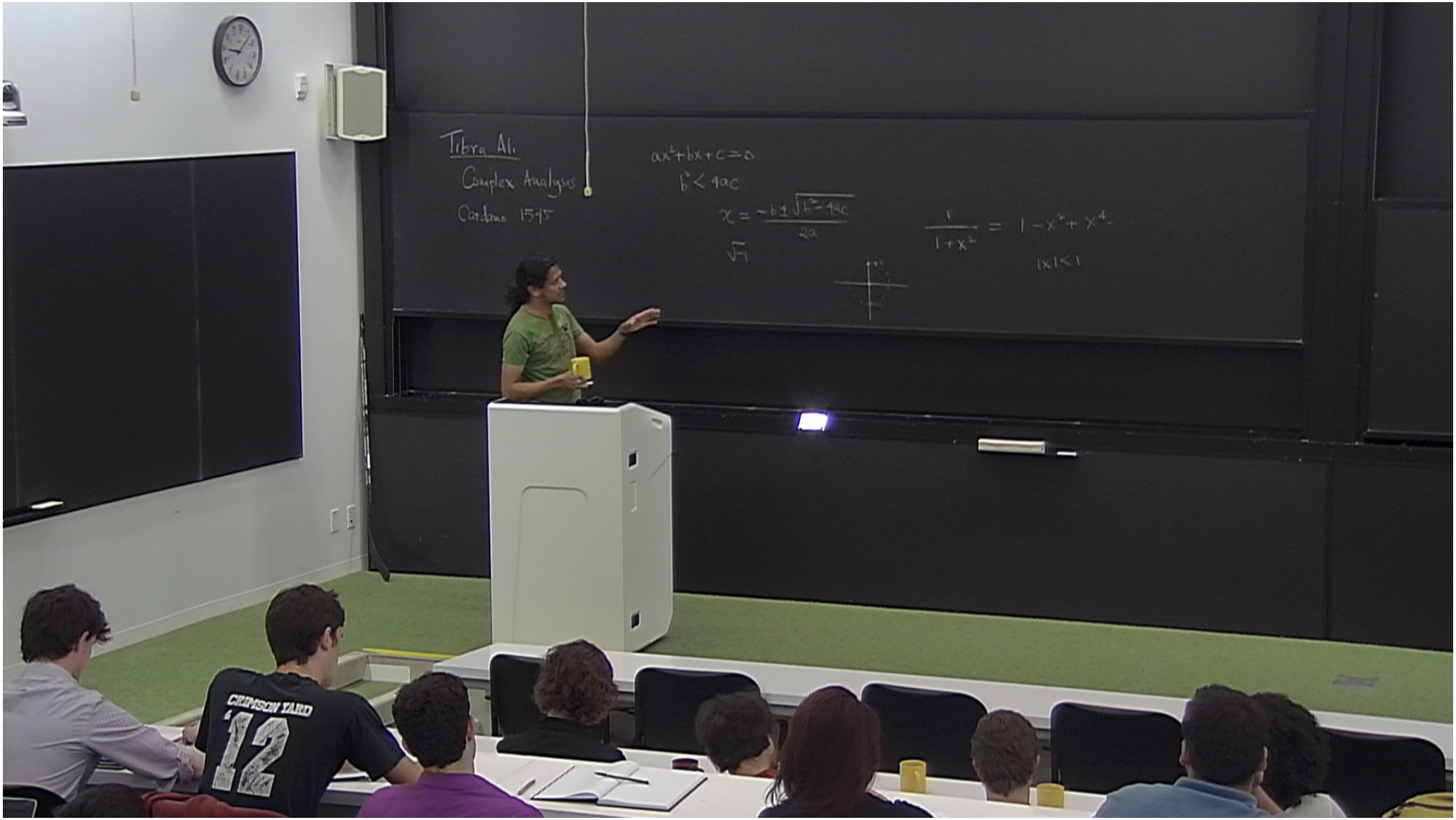
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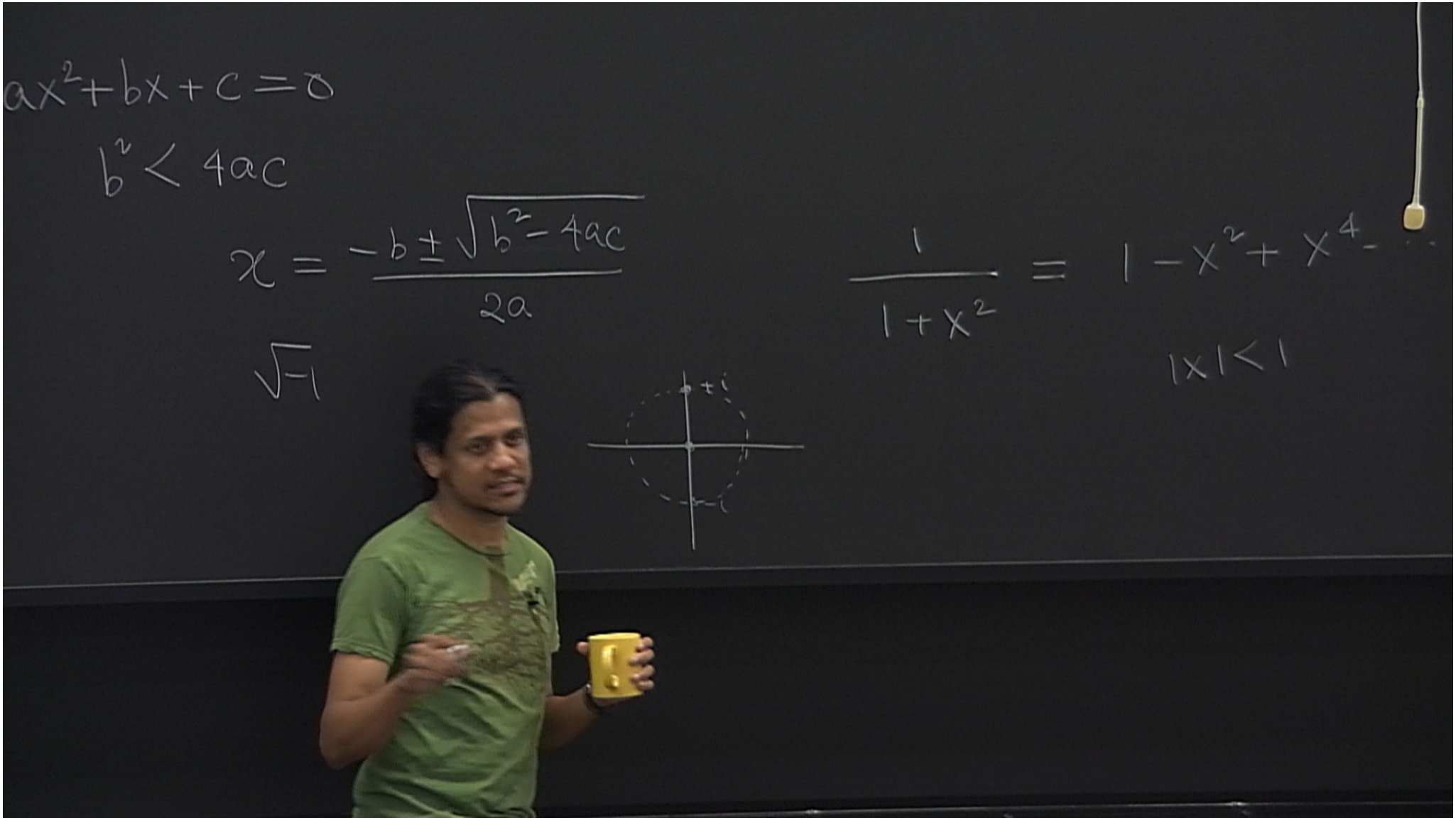
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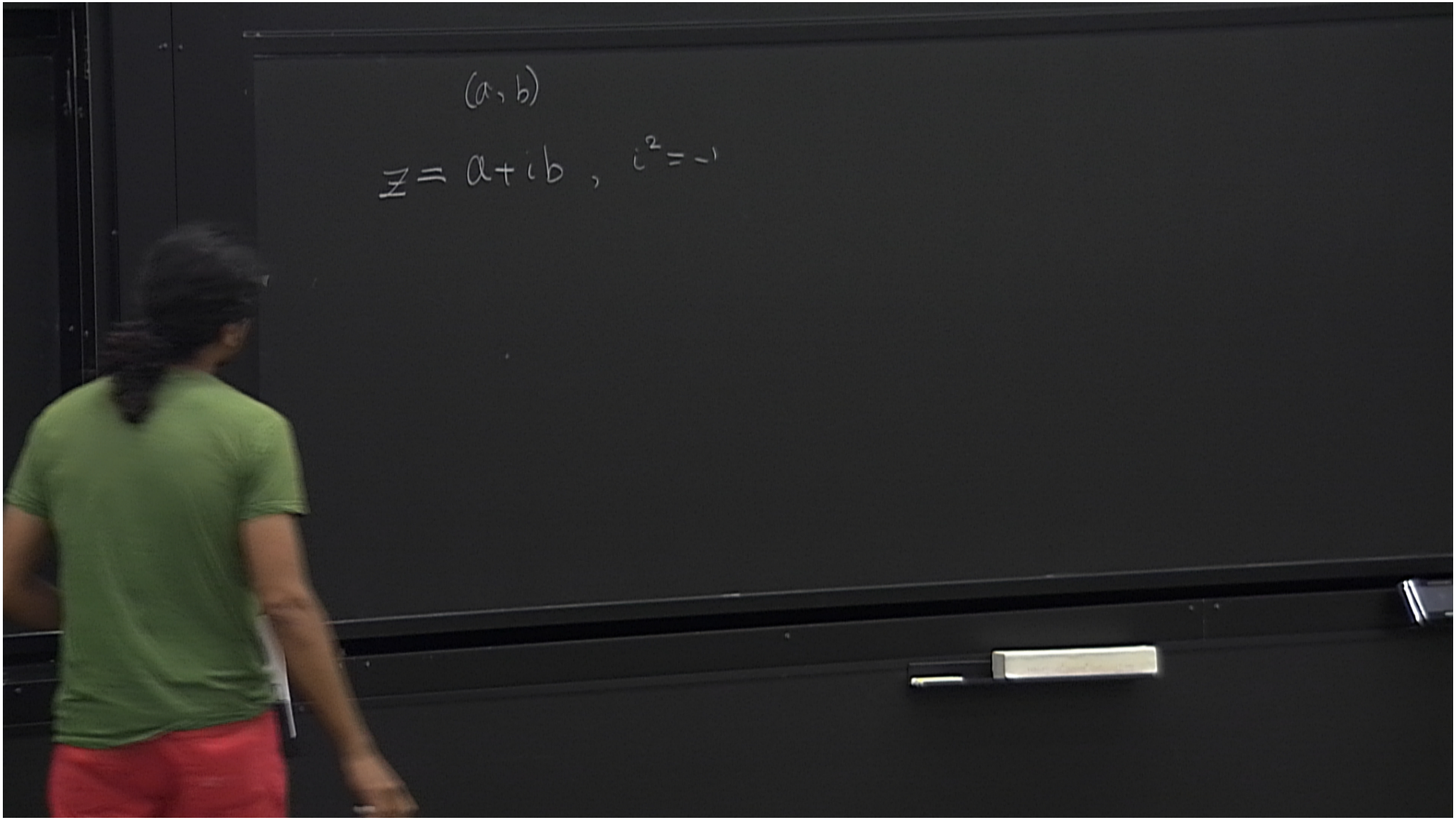

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$\frac{1}{1+x^2} = 1 - x^2 + x^4 - \cdots$
$|x| < 1$
$+i$
$-i$

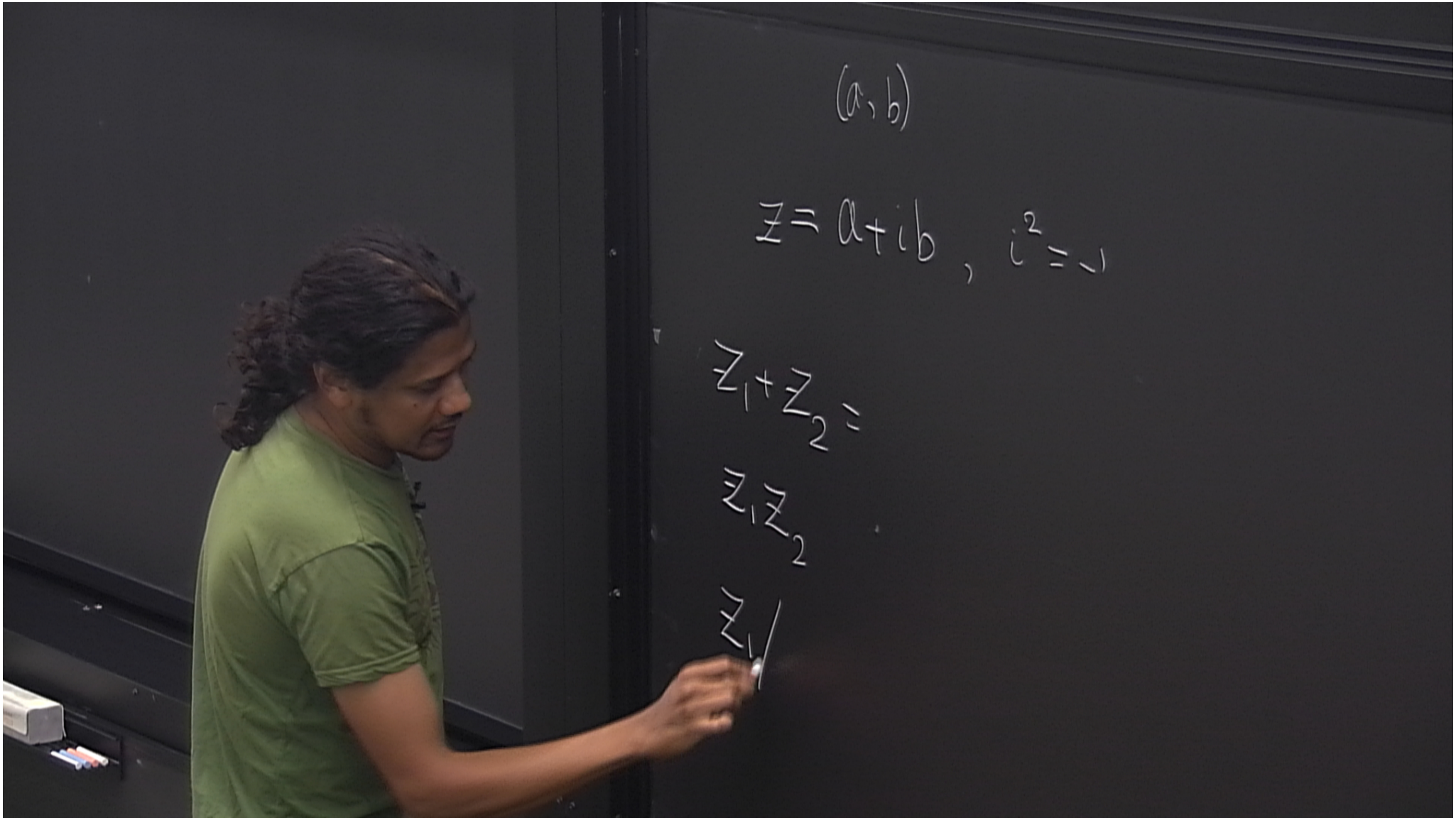

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$\frac{1}{1+x^2} = 1 - x^2 + x^4 - \cdots$
$|x| < 1$
$+i$

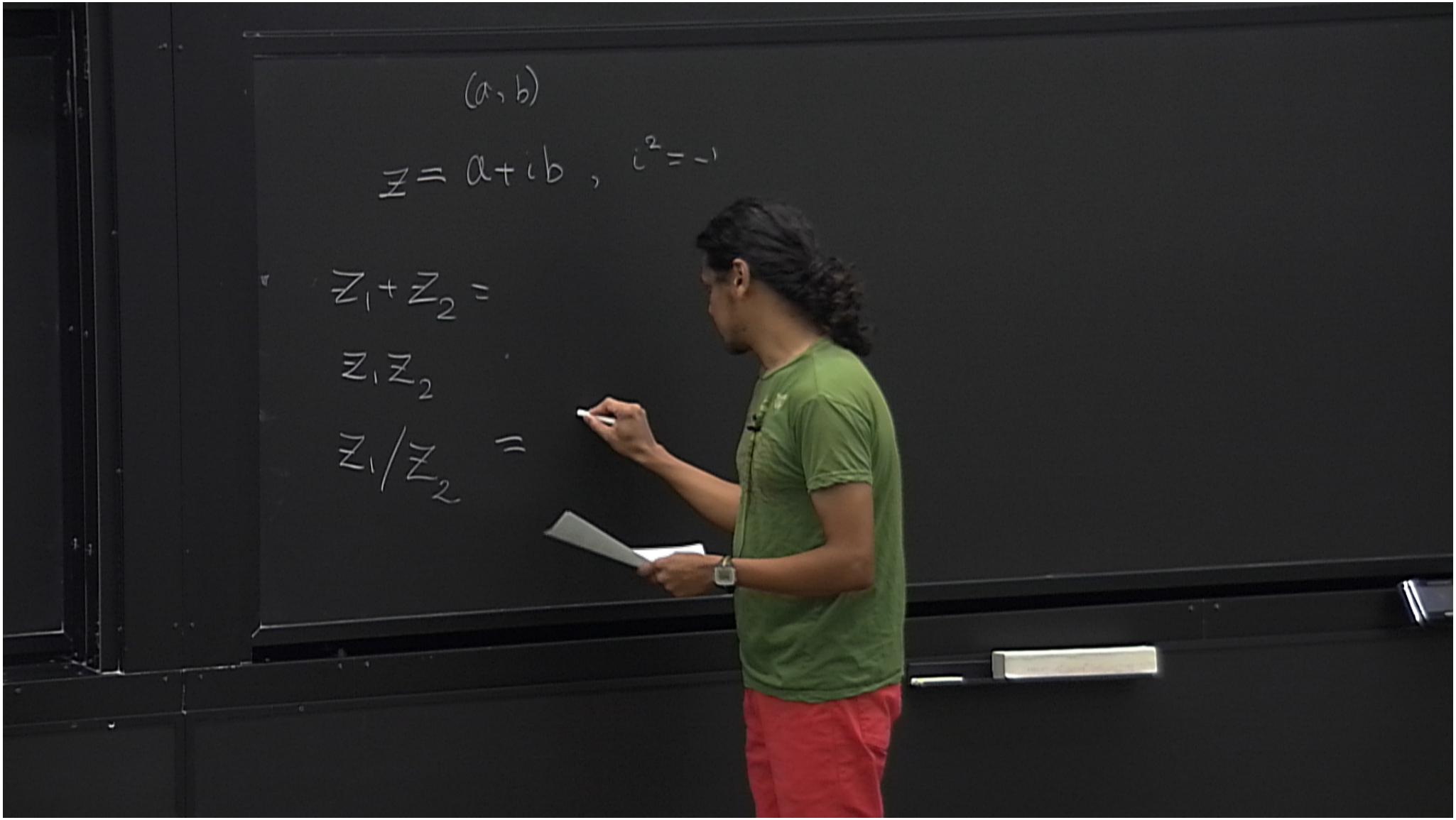

Cardano 1545
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$\sqrt{-1}$
$\frac{1}{1+x^2} = 1 - x^2 + x^4 - \dots$
$+i$
$-i$
$|x| < 1$

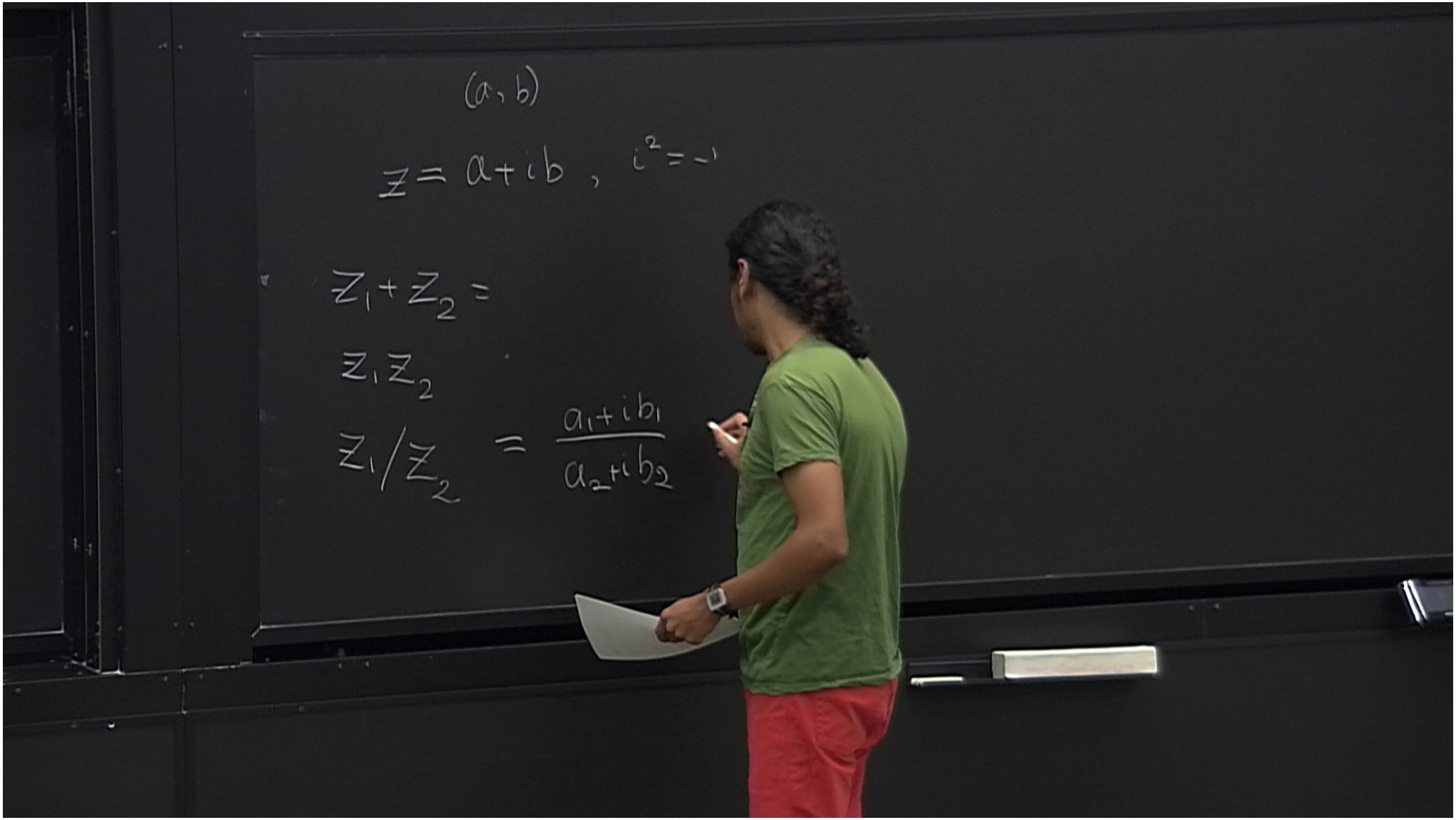

Tibra Ali
Complex Analysis
Cardano 1545
$ax^2+bx+c=0$
$b^2<4ac$
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$\sqrt{-1}$
$\frac{1}{1+x^2}=1-x^2+x^4-$
$|x|<1$
CRIMSON YARD
12

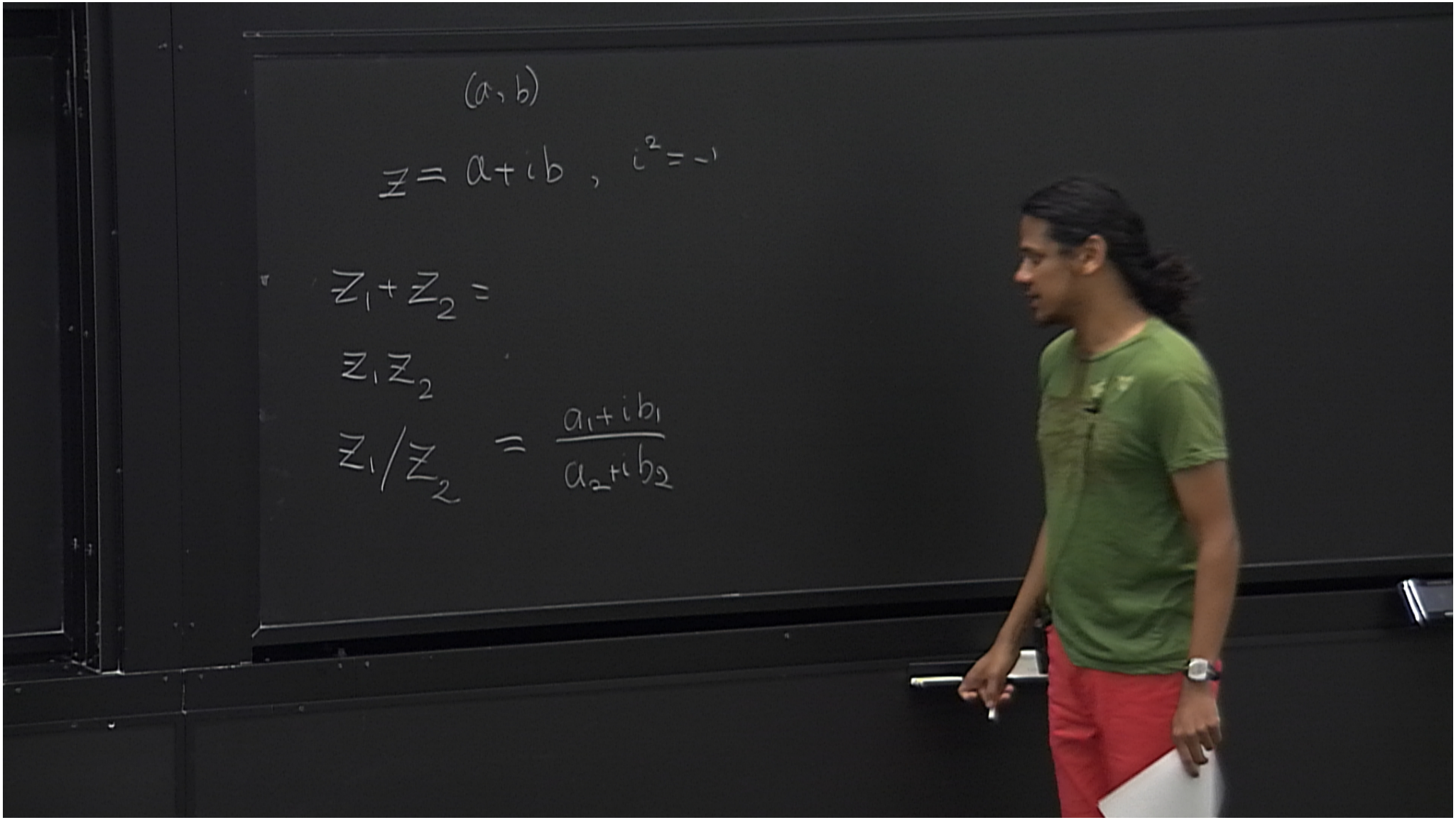

$ax^2+bx+c=0$
$b^2<4ac$
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$\sqrt{-1}$
$\pm i$
$\frac{1}{1+x^2}=1-x^2+x^4-\cdots$
$|x|<1$

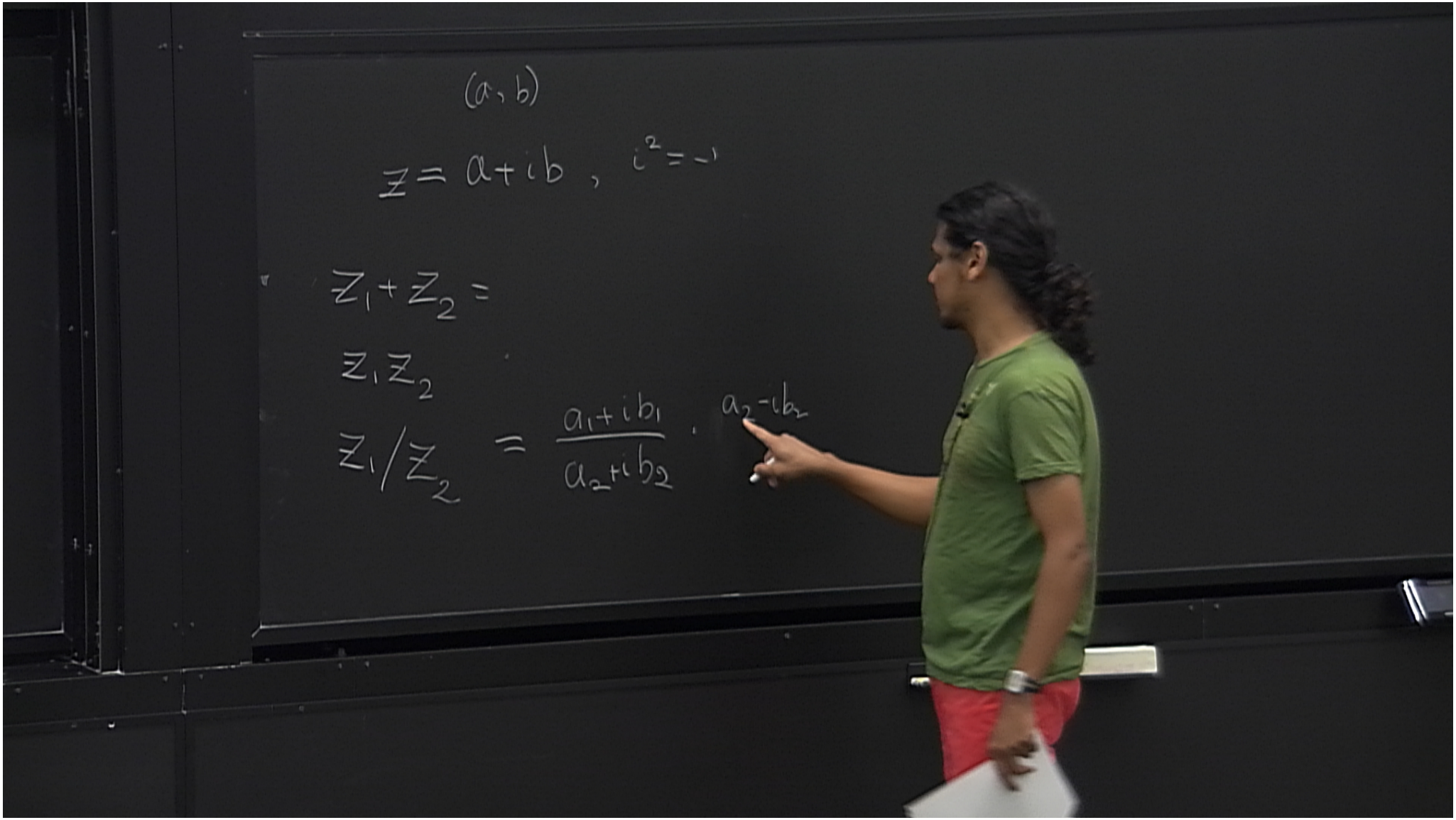

(a, b)
z = a + ib , i² = -1

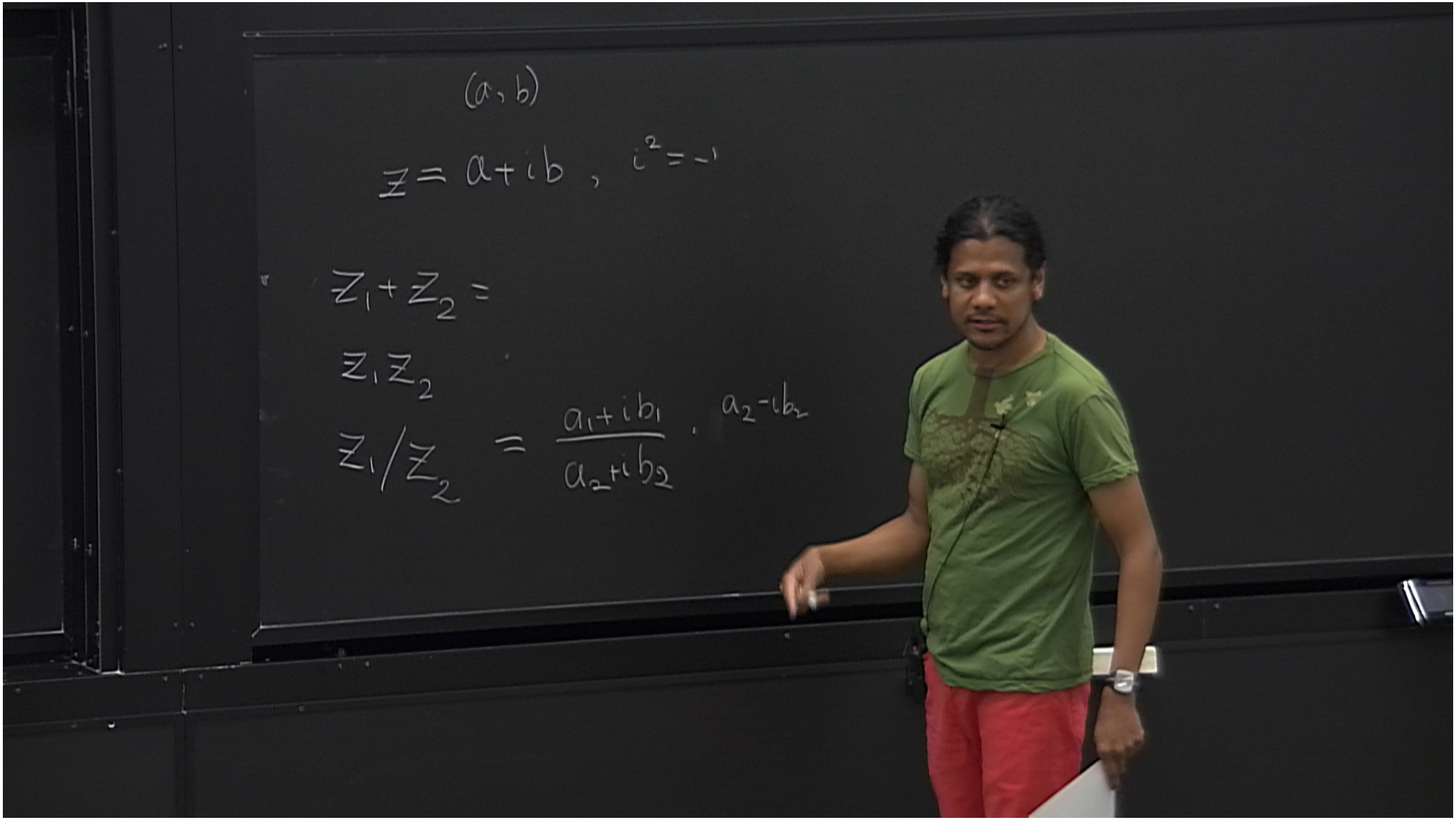

(a, b)
z = a + ib , i² = -1
z₁ + z₂ =
z₁ z₂
z⁻¹

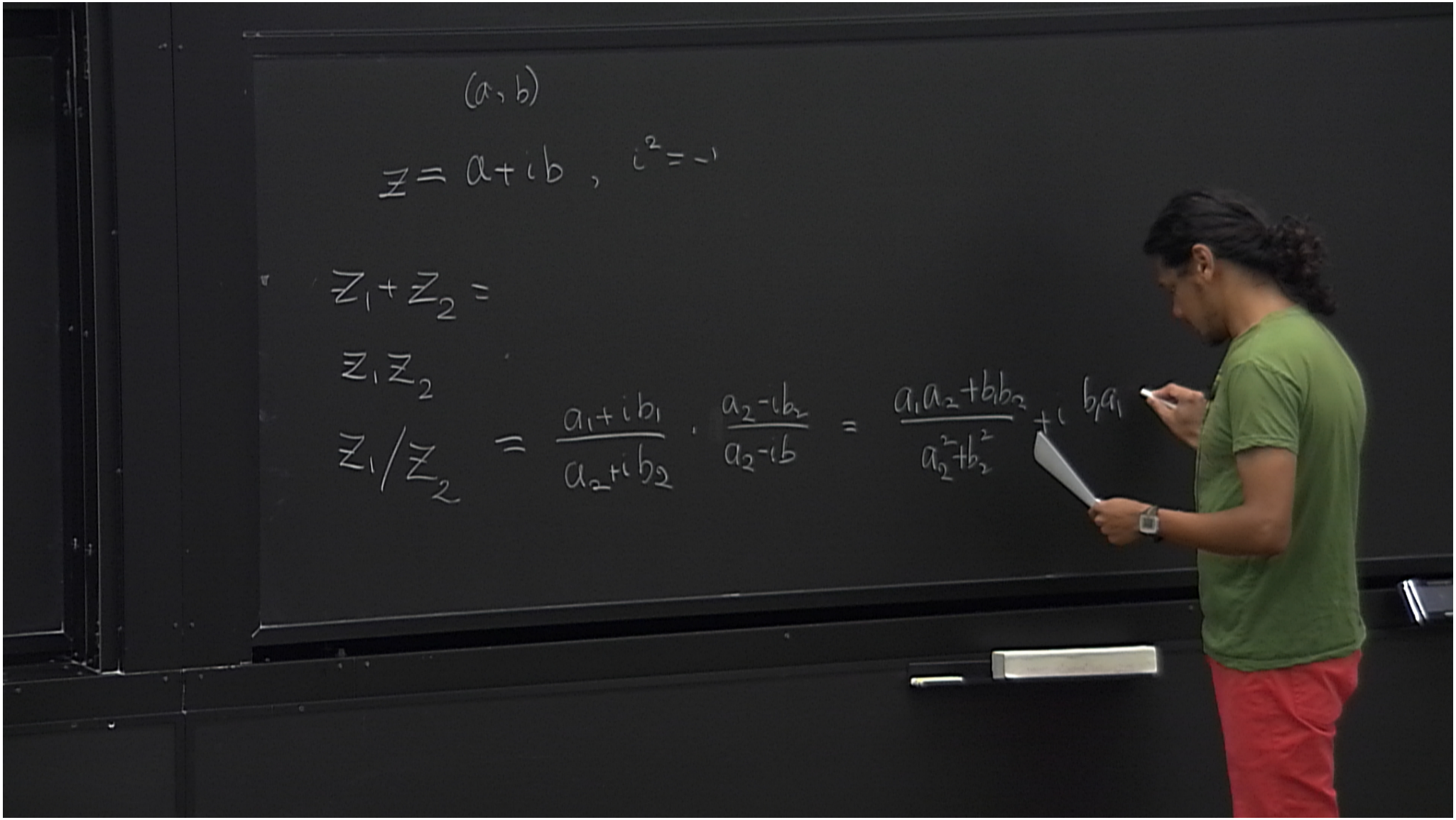

(a, b)
z = a + ib , i² = -1
z₁ + z₂ =
z₁ z₂
z₁ / z₂ =

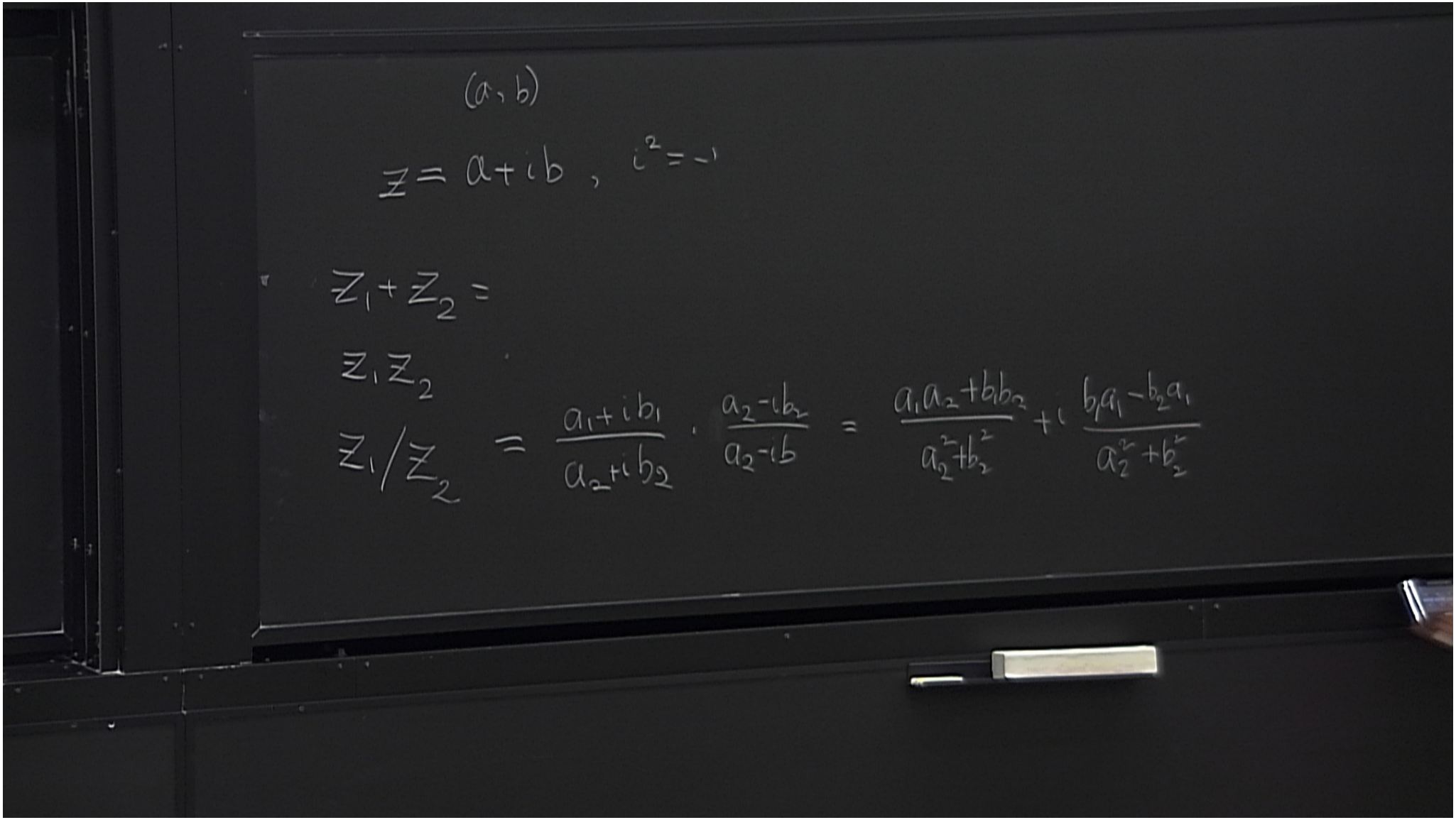

(a, b)
$z = a + ib$, $i^2 = -1$
$z_1 + z_2 =$
$z_1 z_2$
$z_1 / z_2 = \frac{a_1 + ib_1}{a_2 + ib_2}$

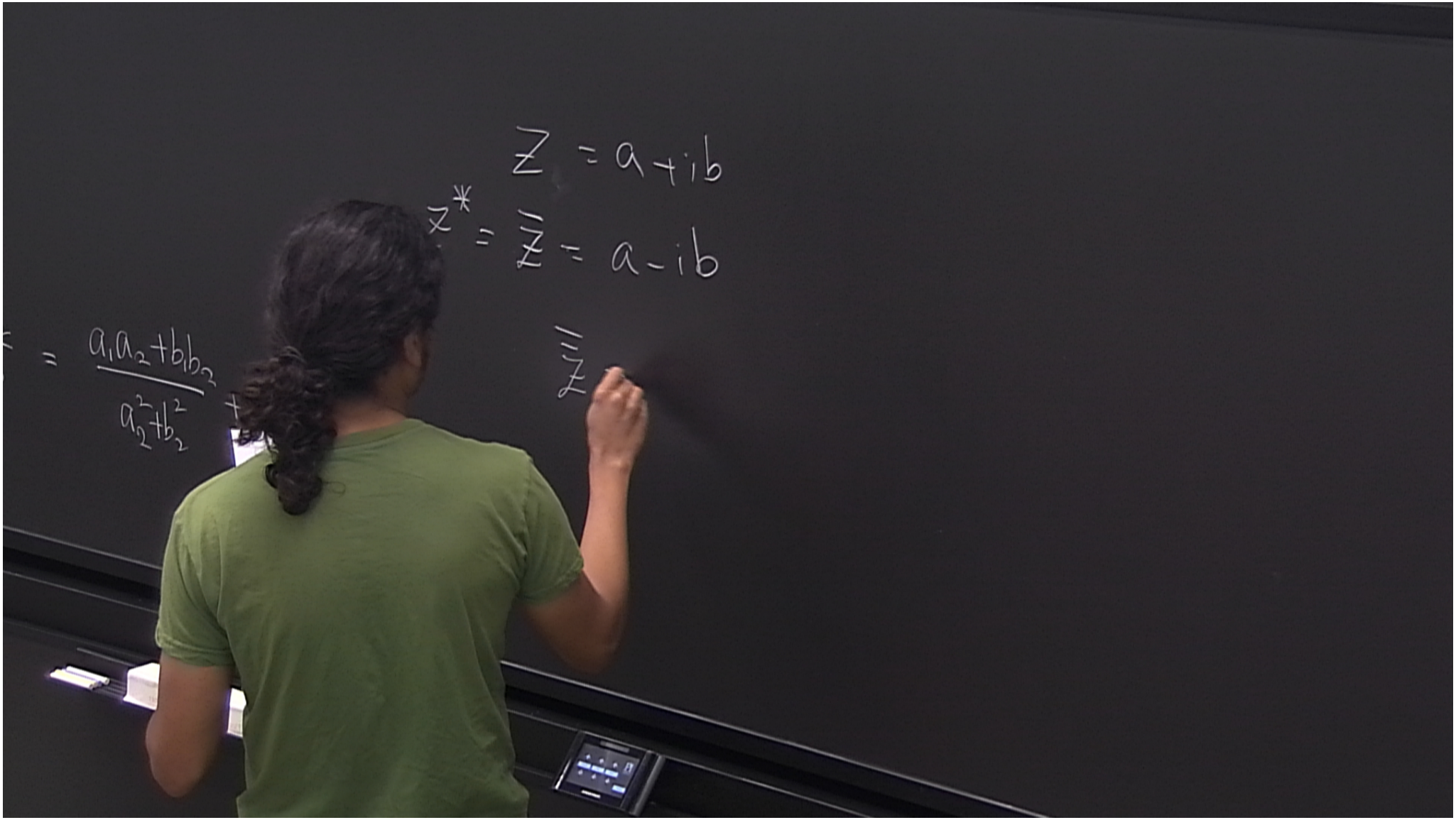

(a, b)
z = a + ib , i² = -1
z₁ + z₂ =
z₁ z₂
z₁ / z₂ = (a₁ + ib₁) / (a₂ + ib₂)

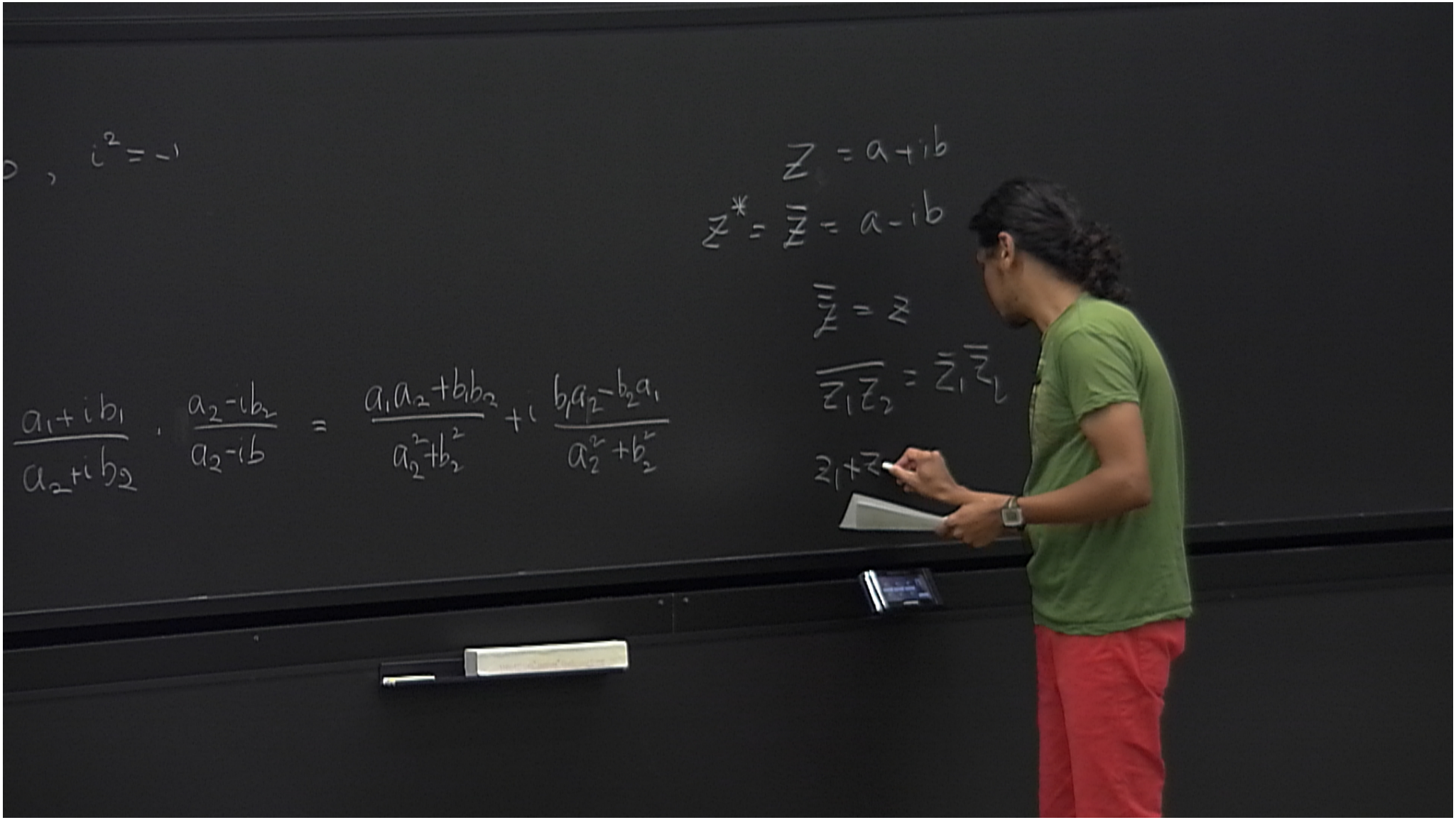

(a, b)
$z = a + ib$, $i^2 = -1$
$z_1 + z_2 =$
$z_1 z_2$
$z_1 / z_2 = \frac{a_1 + ib_1}{a_2 + ib_2} \cdot a_2 - ib_2$

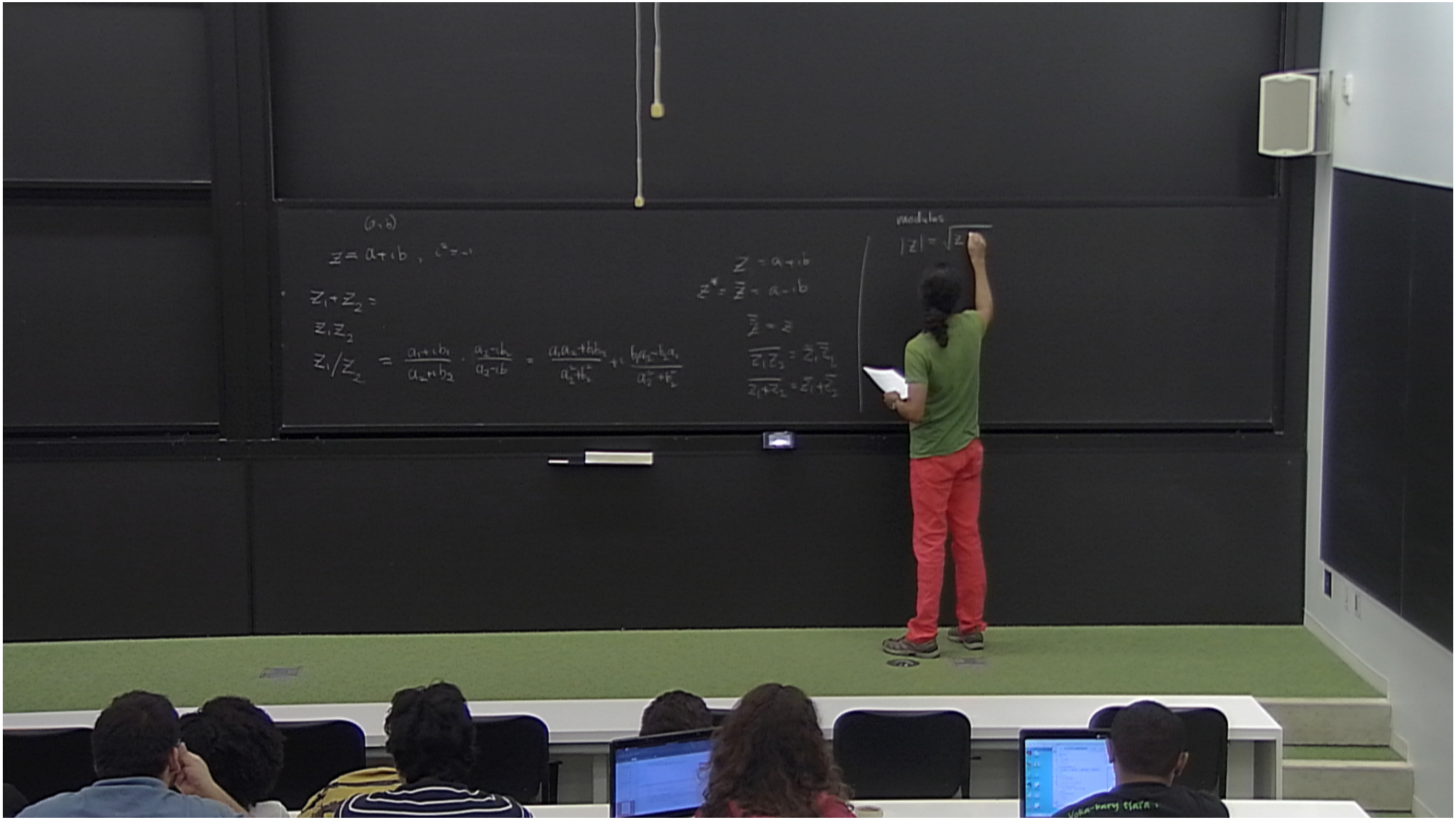

(a, b)
$z = a + ib$, $i^2 = -1$
$z_1 + z_2 =$
$z_1 z_2$
$z_1 / z_2 = \frac{a_1 + ib_1}{a_2 + ib_2} \cdot a_2 - ib_2$

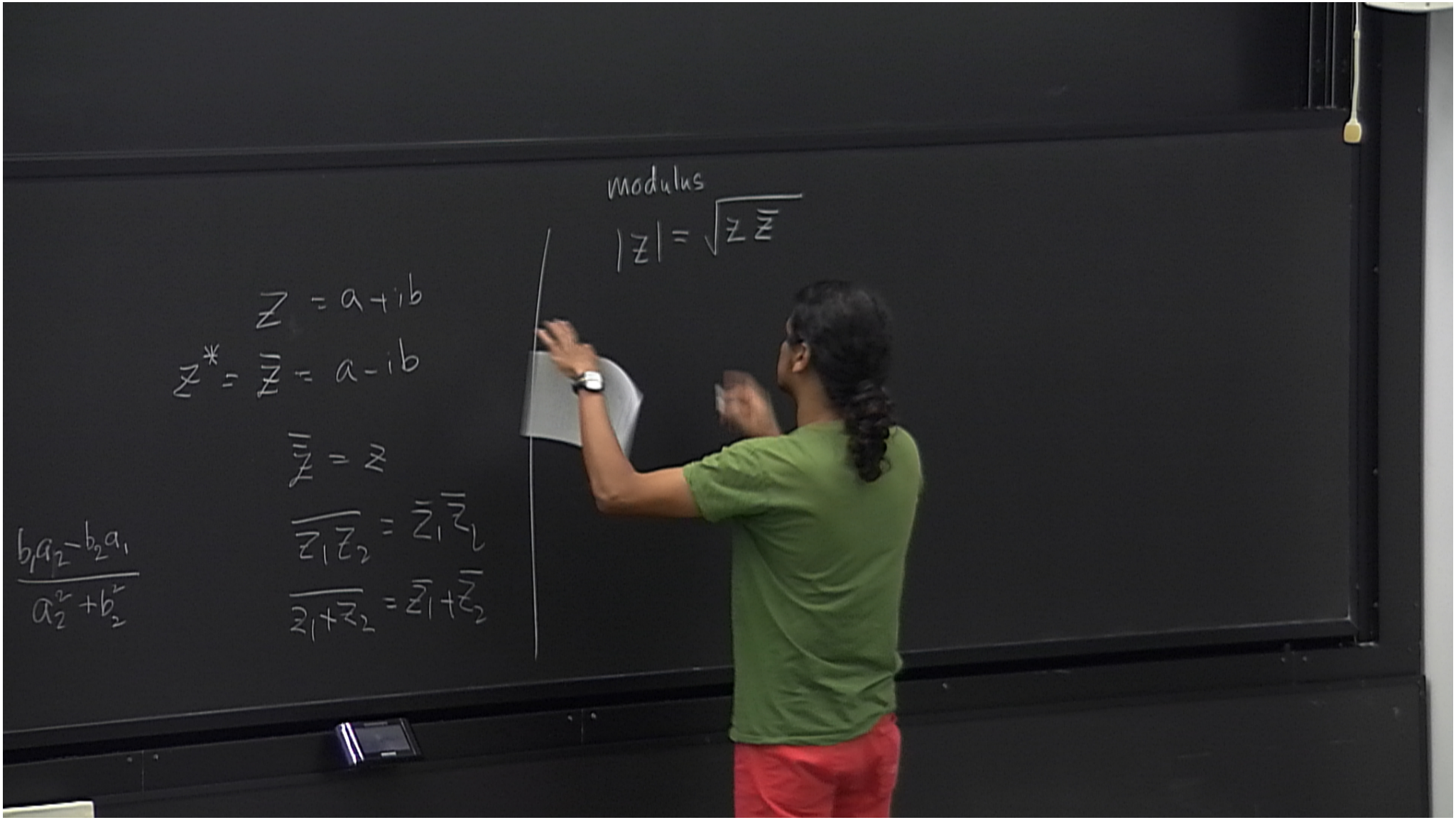

(a, b)
$z = a + ib$, $i^2 = -1$
$z_1 + z_2 =$
$z_1 z_2$
$z_1 / z_2 = \frac{a_1 + ib_1}{a_2 + ib_2} \cdot \frac{a_2 - ib_2}{a_2 - ib} = \frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2} + i\, b_1 a_1$

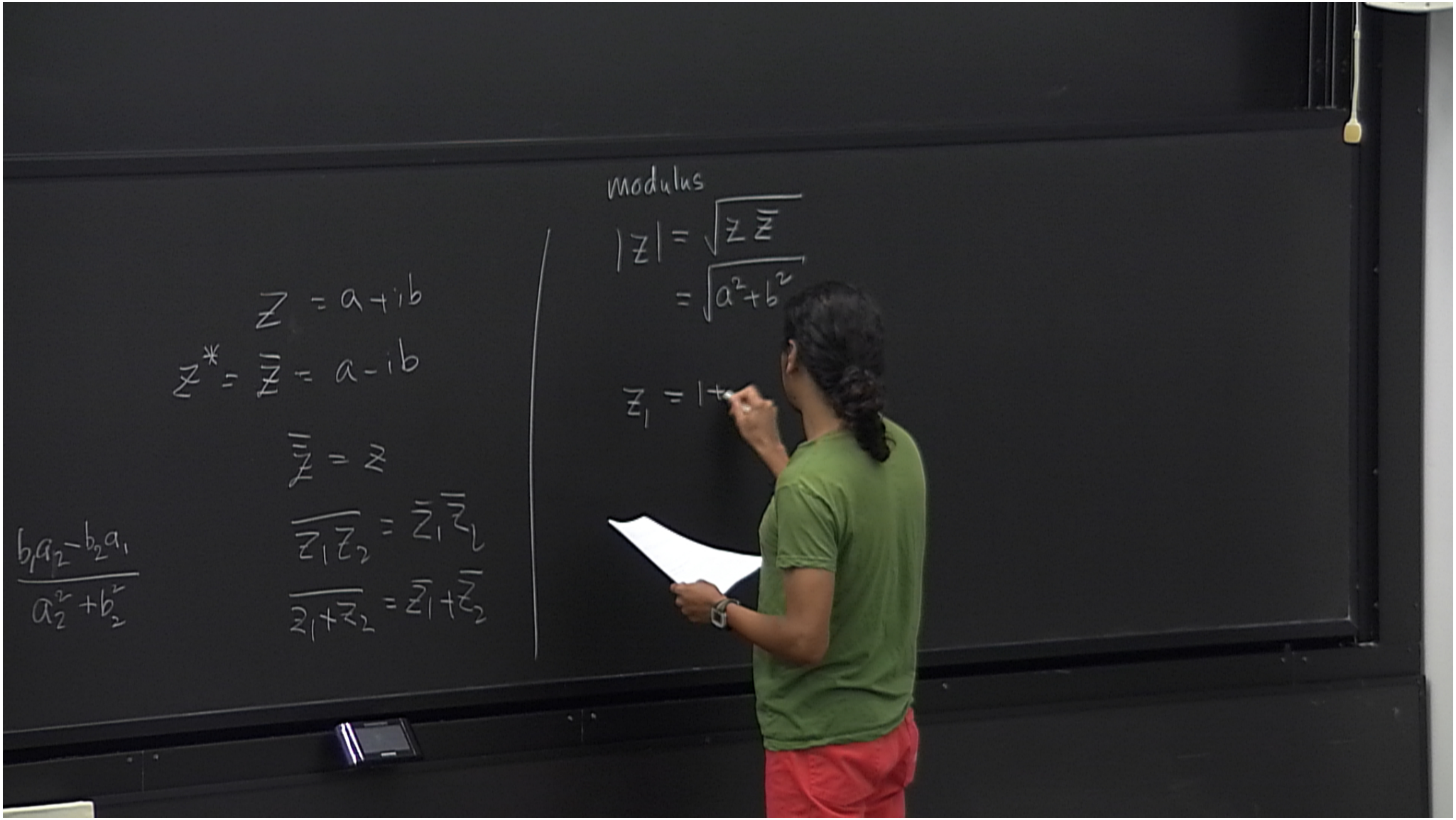

(a, b)
$z = a + ib$, $i^2 = -1$
$z_1 + z_2 =$
$z_1 z_2$
$z_1 / z_2 = \frac{a_1 + ib_1}{a_2 + ib_2} \cdot \frac{a_2 - ib_2}{a_2 - ib_2} = \frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2} + i \frac{b_1 a_1 - b_2 a_1}{a_2^2 + b_2^2}$

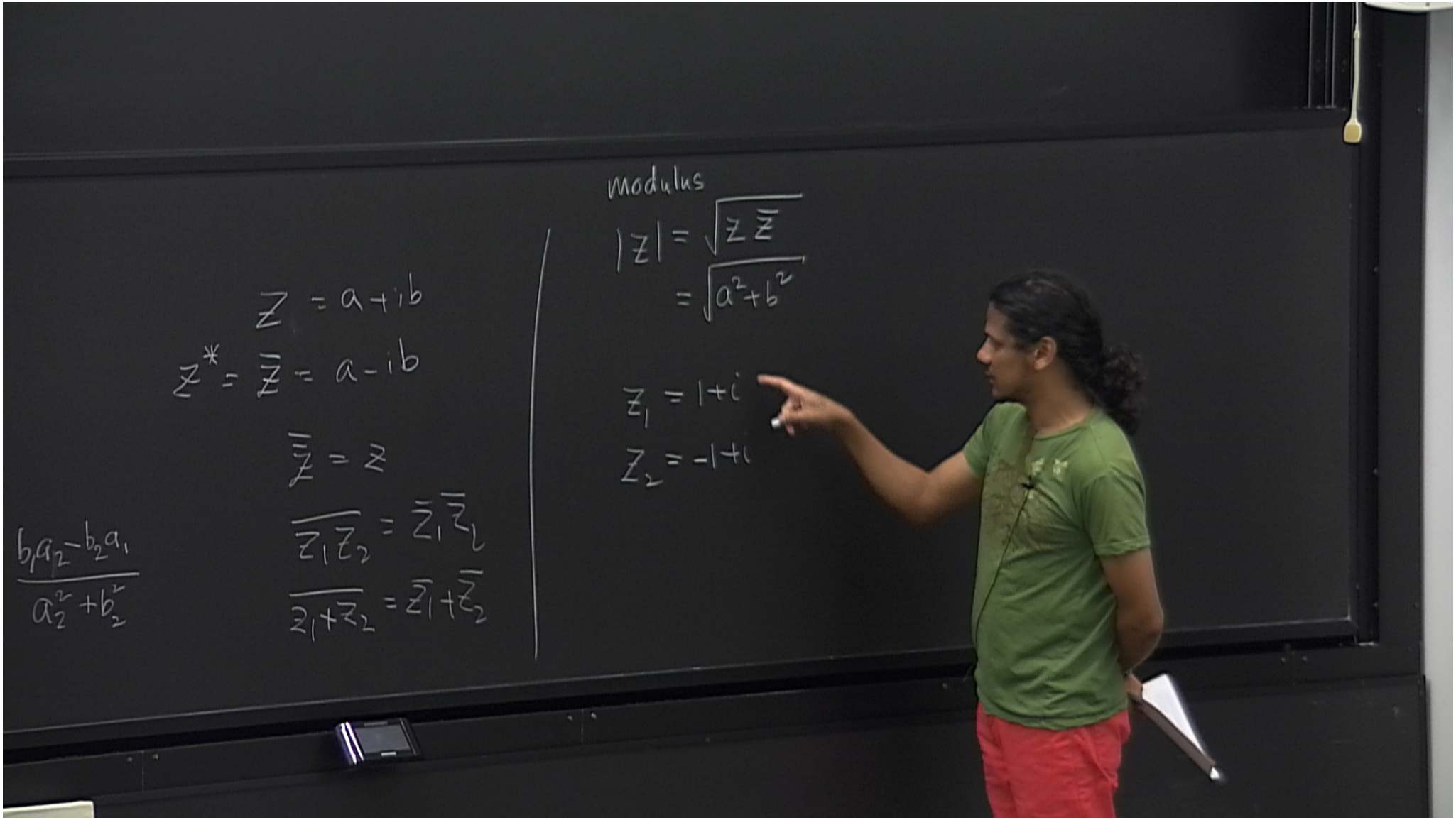

z = a + ib
z* = z̄ = a - ib
= a1a2 + b1b2 / a2^2 + b2^2

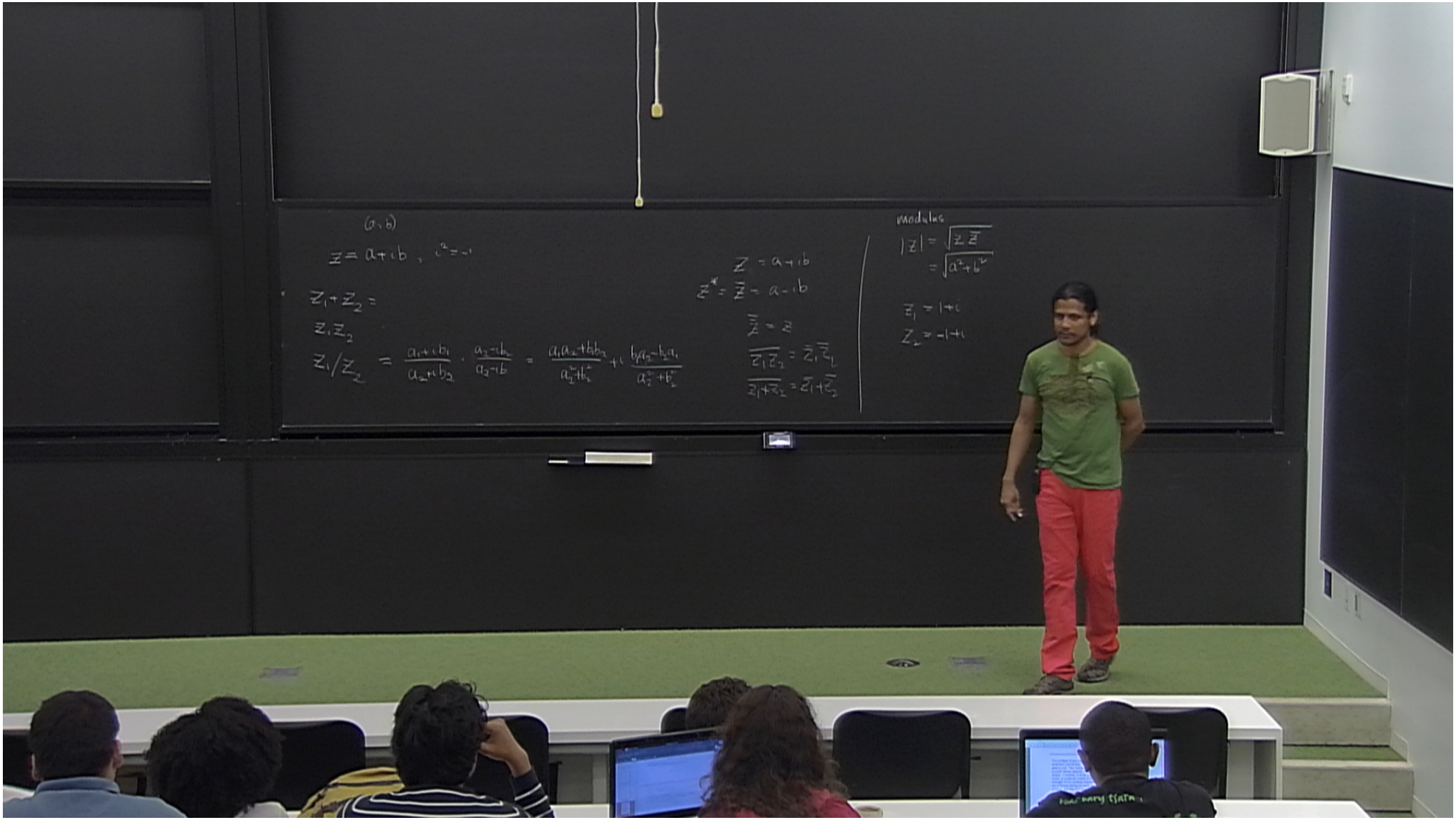

$i^2 = -1$
$z = a + ib$
$z^* = \bar{z} = a - ib$
$\bar{\bar{z}} = z$
$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$
$z_1 + z$
$\frac{a_1 + ib_1}{a_2 + ib_2} \cdot \frac{a_2 - ib_2}{a_2 - ib_2} = \frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2} + i \frac{b_1 a_2 - b_2 a_1}{a_2^2 + b_2^2}$

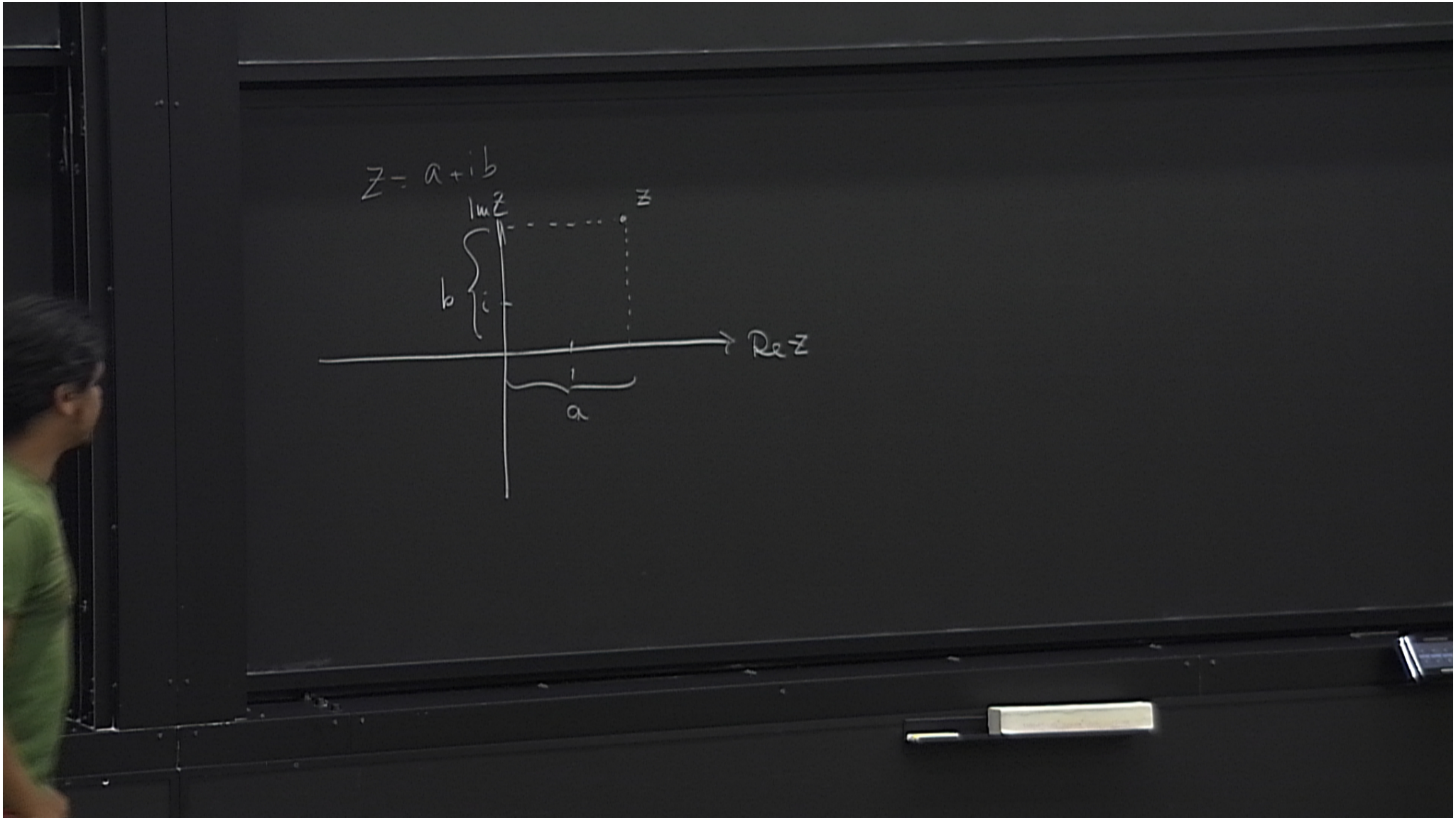

(a,b)
z = a+ib , i² = -1
z₁+z₂ =
z₁z₂
z₁/z₂ = (a₁+ib₁)/(a₂+ib₂) · (a₂-ib₂)/(a₂-ib₂) = (a₁a₂+b₁b₂)/(a₂²+b₂²) + i (b₁a₂-b₂a₁)/(a₂²+b₂²)
z = a+ib
z* = z̄ = a-ib
z̿ = z
z₁z₂ = z̄₁z̄₂
z₁+z₂ = z̄₁+z̄₂
modulus
|z| = √z

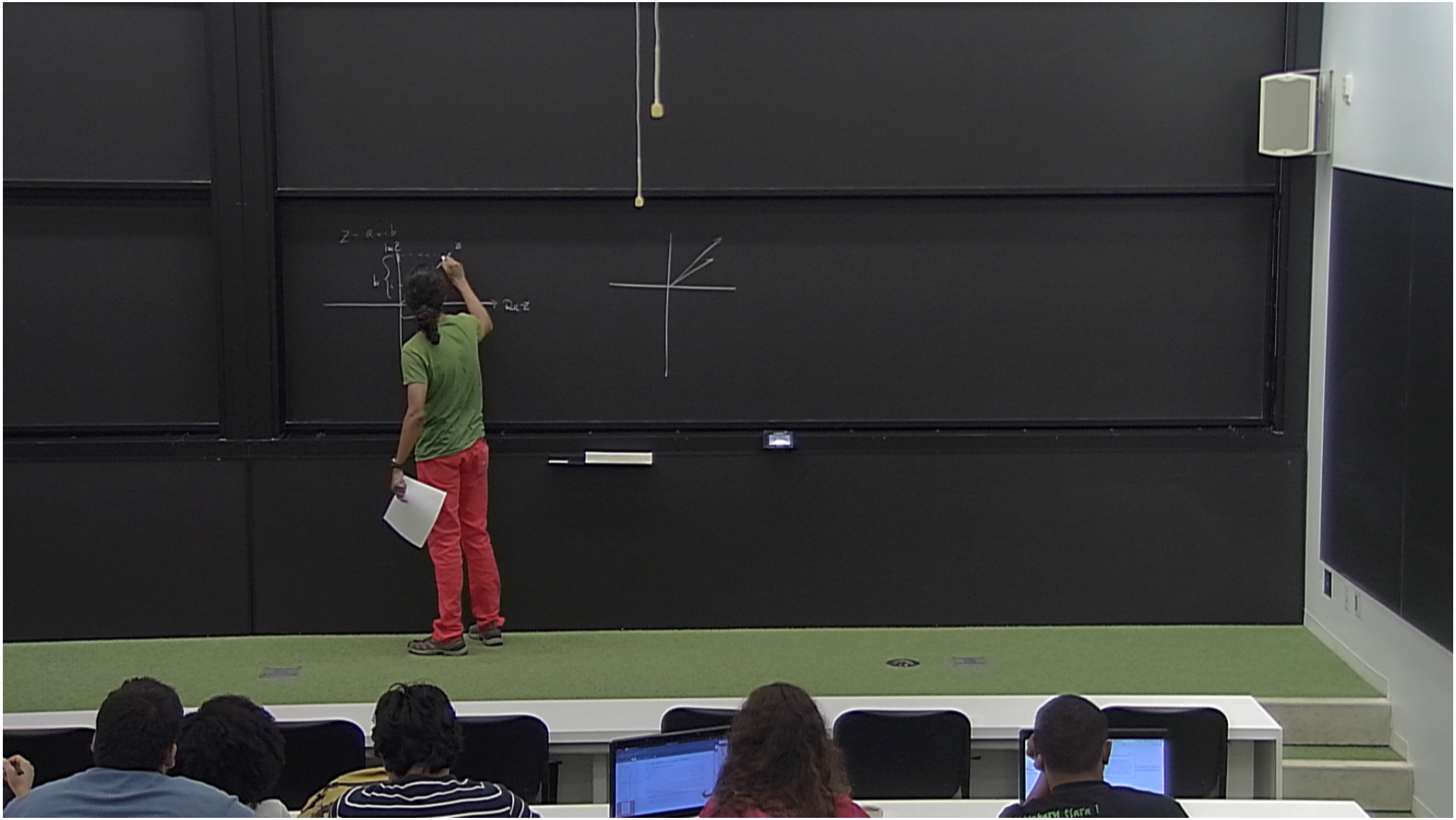

modulus
$|z| = \sqrt{z\bar{z}}$
$z = a + ib$
$z^* = \bar{z} = a - ib$
$\bar{\bar{z}} = z$
$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$
$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$
$\frac{b_1 a_2 - b_2 a_1}{a_2^2 + b_2^2}$

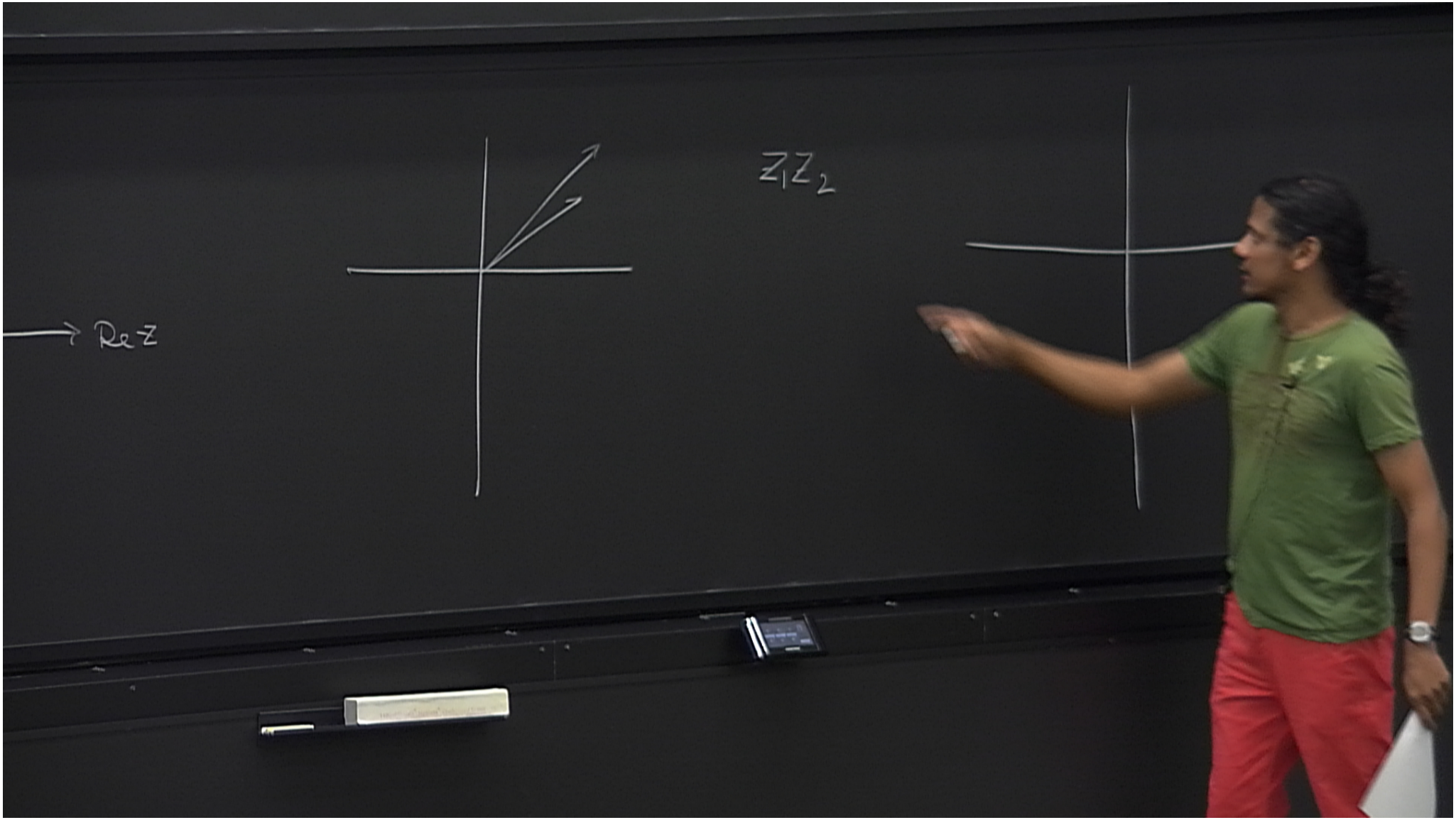

modulus
$z = a + ib$
$z^* = \bar{z} = a - ib$
$\bar{\bar{z}} = z$
$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$
$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$
$|z| = \sqrt{z\bar{z}}$
$= \sqrt{a^2 + b^2}$
$z_1 = 1 +$
$\frac{b_1 a_2 - b_2 a_1}{a_2^2 + b_2^2}$

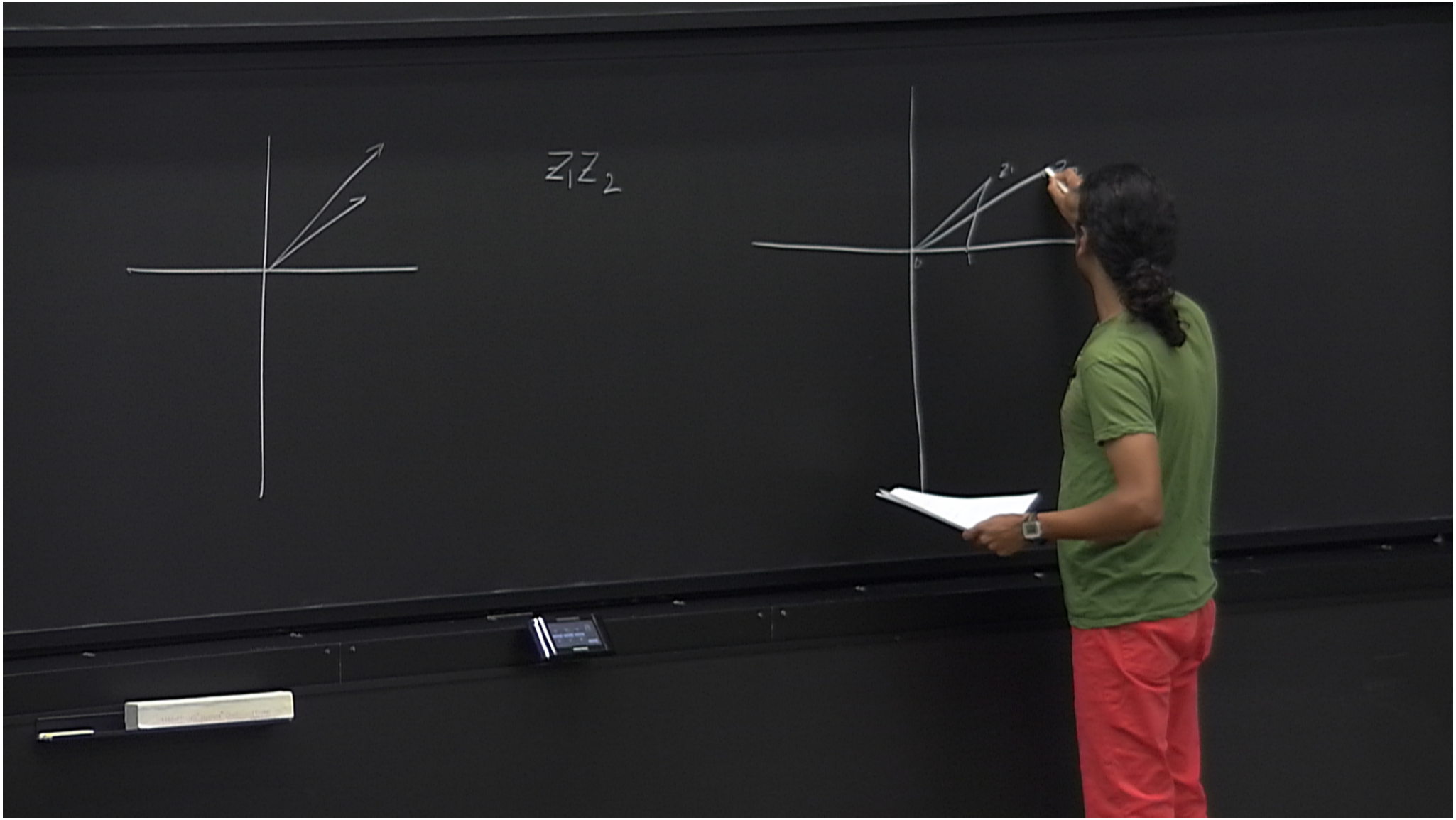

modulus
$|z| = \sqrt{z\bar{z}}$
$= \sqrt{a^2+b^2}$
$z = a+ib$
$z^* = \bar{z} = a-ib$
$\bar{\bar{z}} = z$
$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$
$\overline{z_1+z_2} = \bar{z}_1 + \bar{z}_2$
$z_1 = 1+i$
$z_2 = -1+i$
$\frac{b_1 a_2 - b_2 a_1}{a_2^2+b_2^2}$

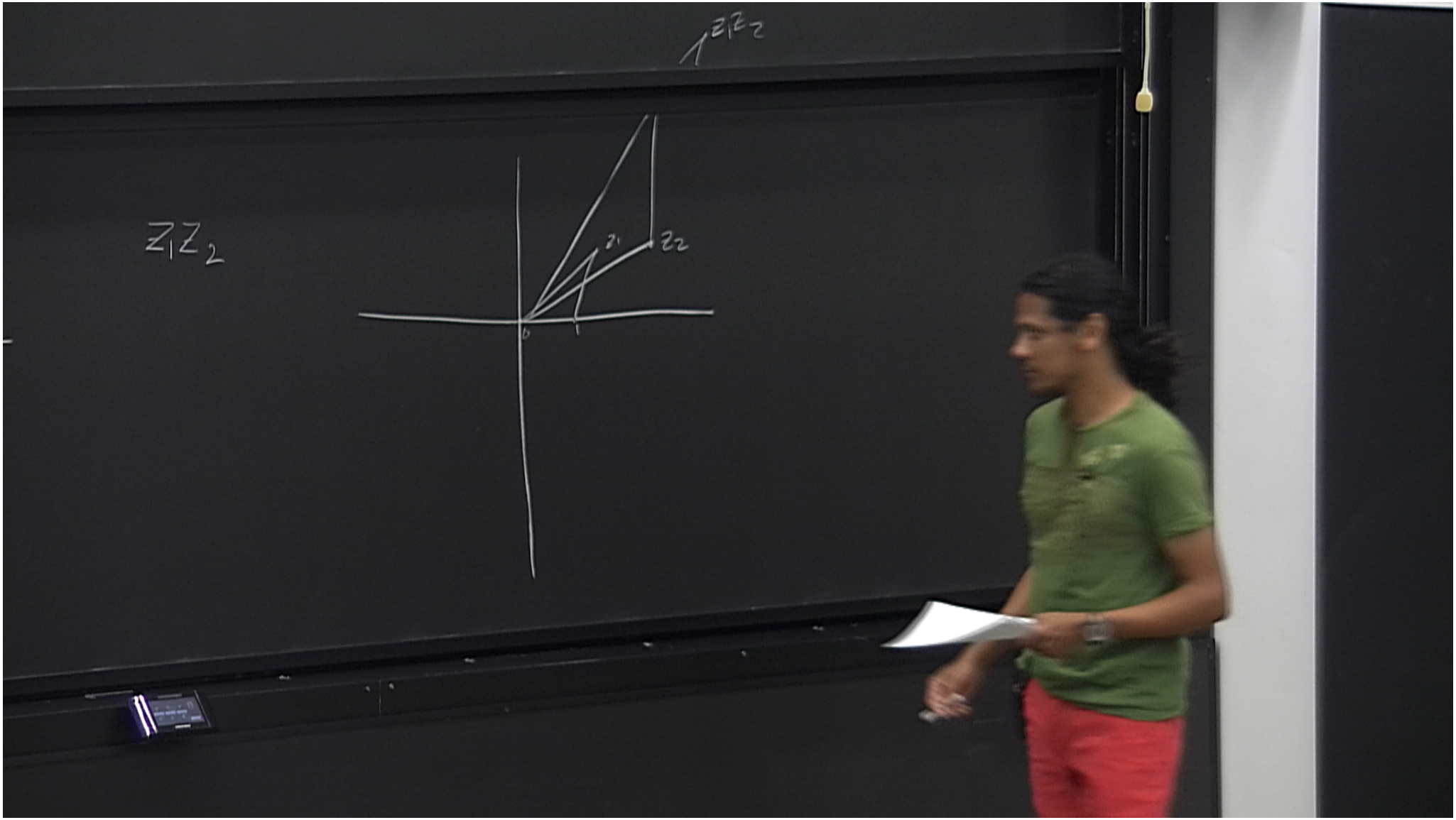

(a,b)
z = a+ib , i² = -1
z₁+z₂ =
z₁z₂
z₁/z₂ =
z = a+ib
z* = z̄ = a-ib
z̄̄ = z
z₁z₂ = z̄₁z̄₂
z₁+z₂ = z̄₁+z̄₂
modulus
|z| = √(z z̄)
= √(a²+b²)
z₁ = 1+i
z₂ = -1+i

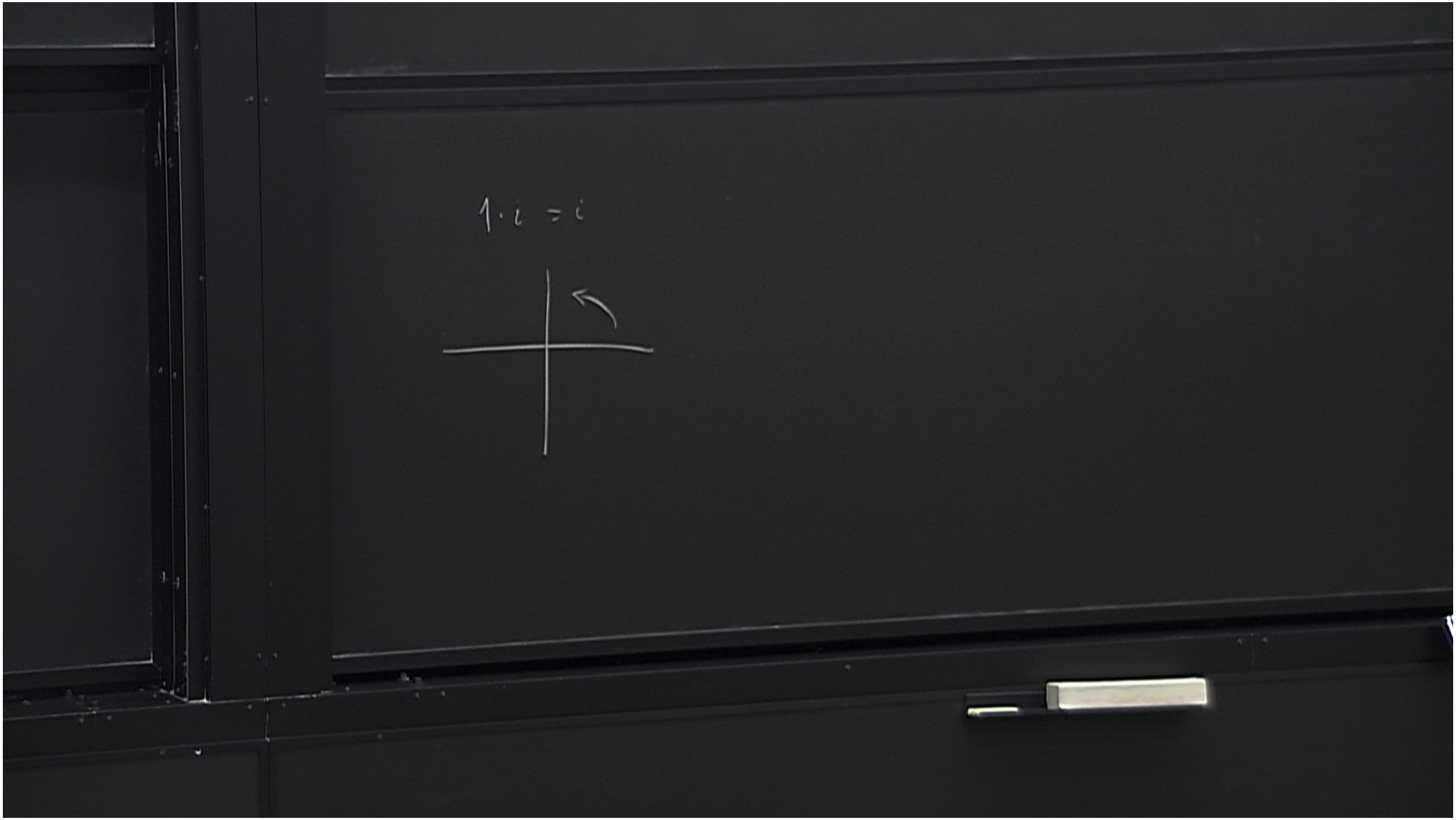

z = a + ib
Im z
z
b
i
Re z
1
a

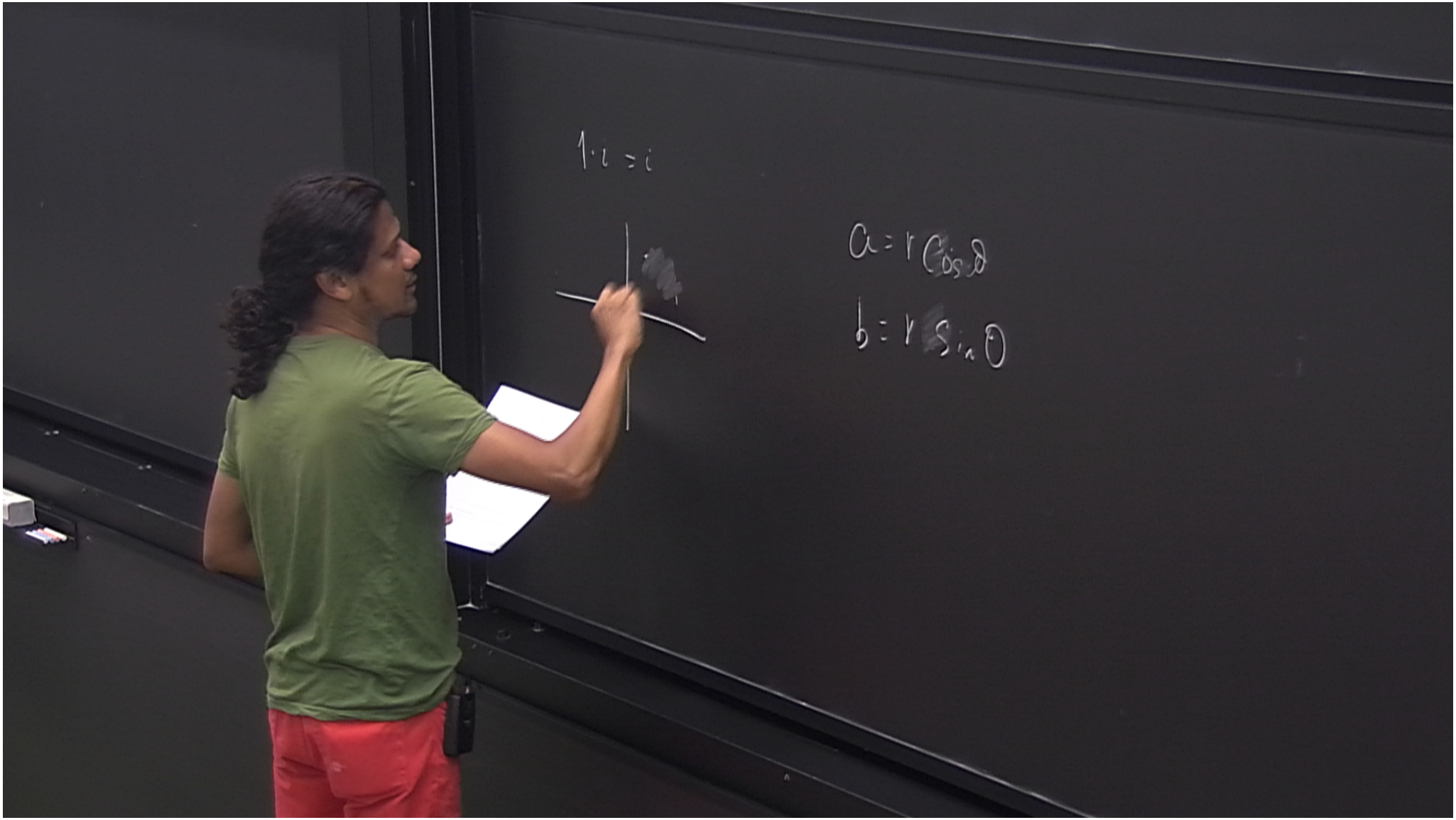


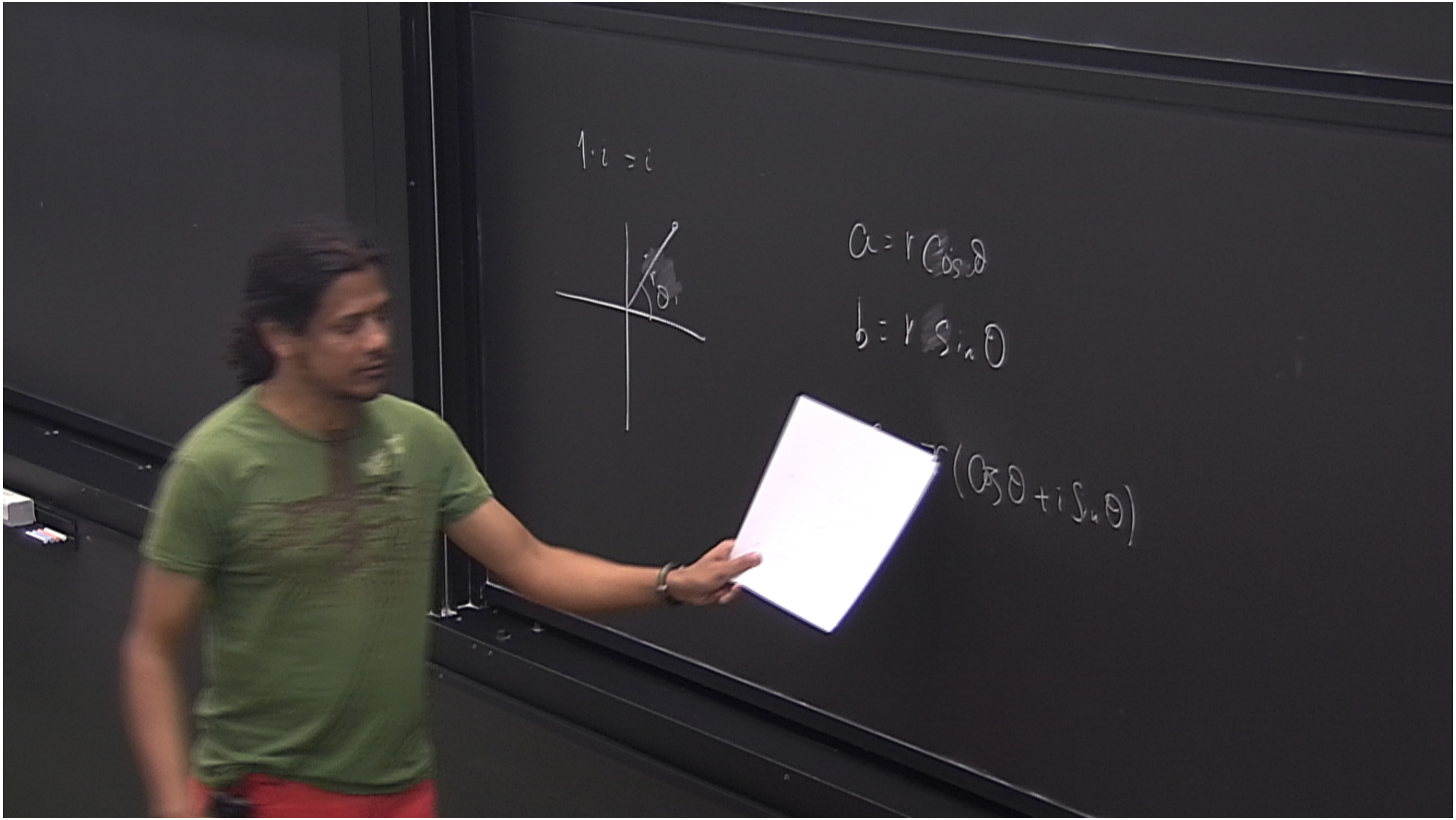

Re z
$z_1 z_2$

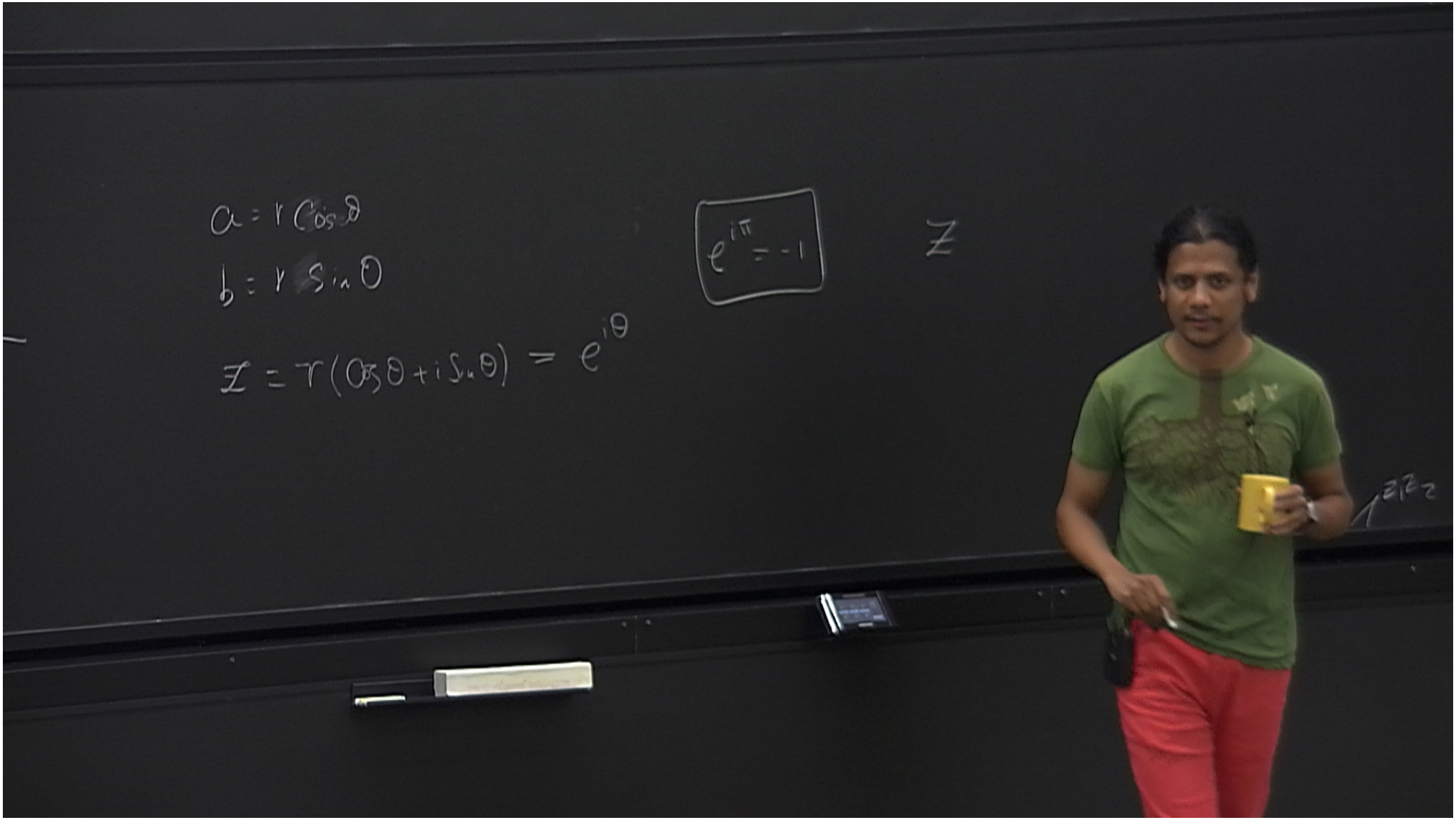

$z_1 z_2$

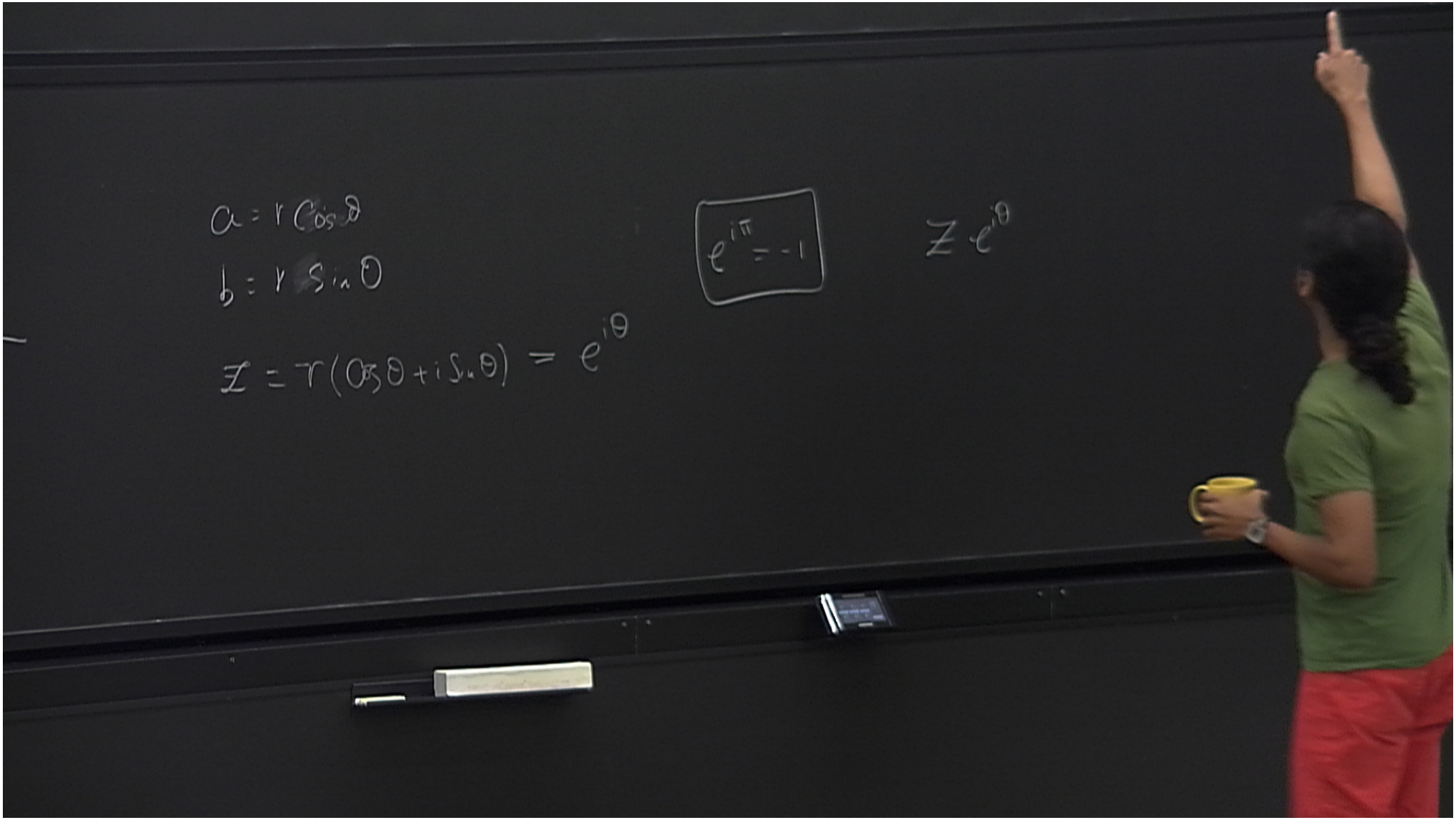

$z_1 z_2$
z_1
z_2

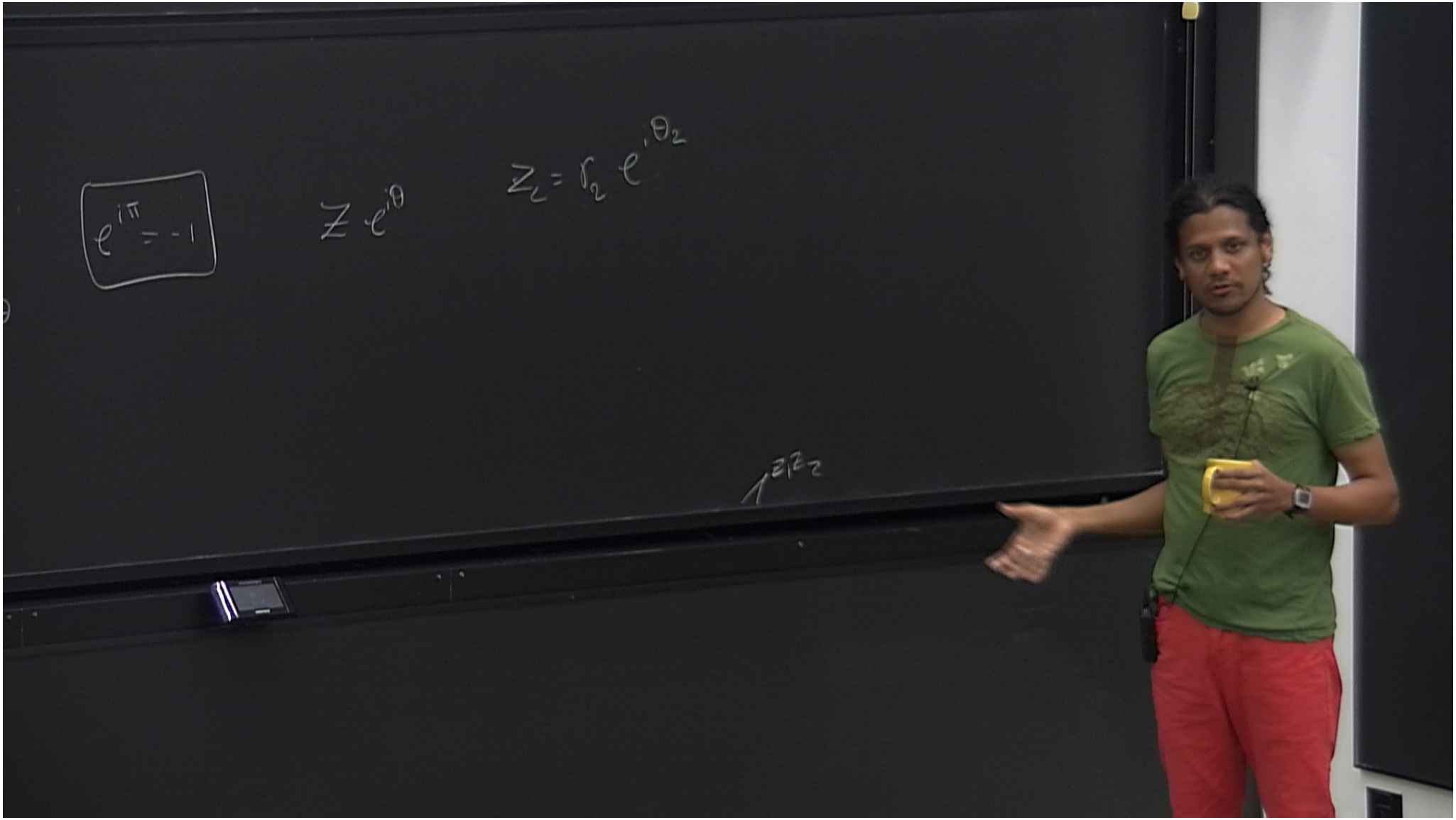

1 · i = i

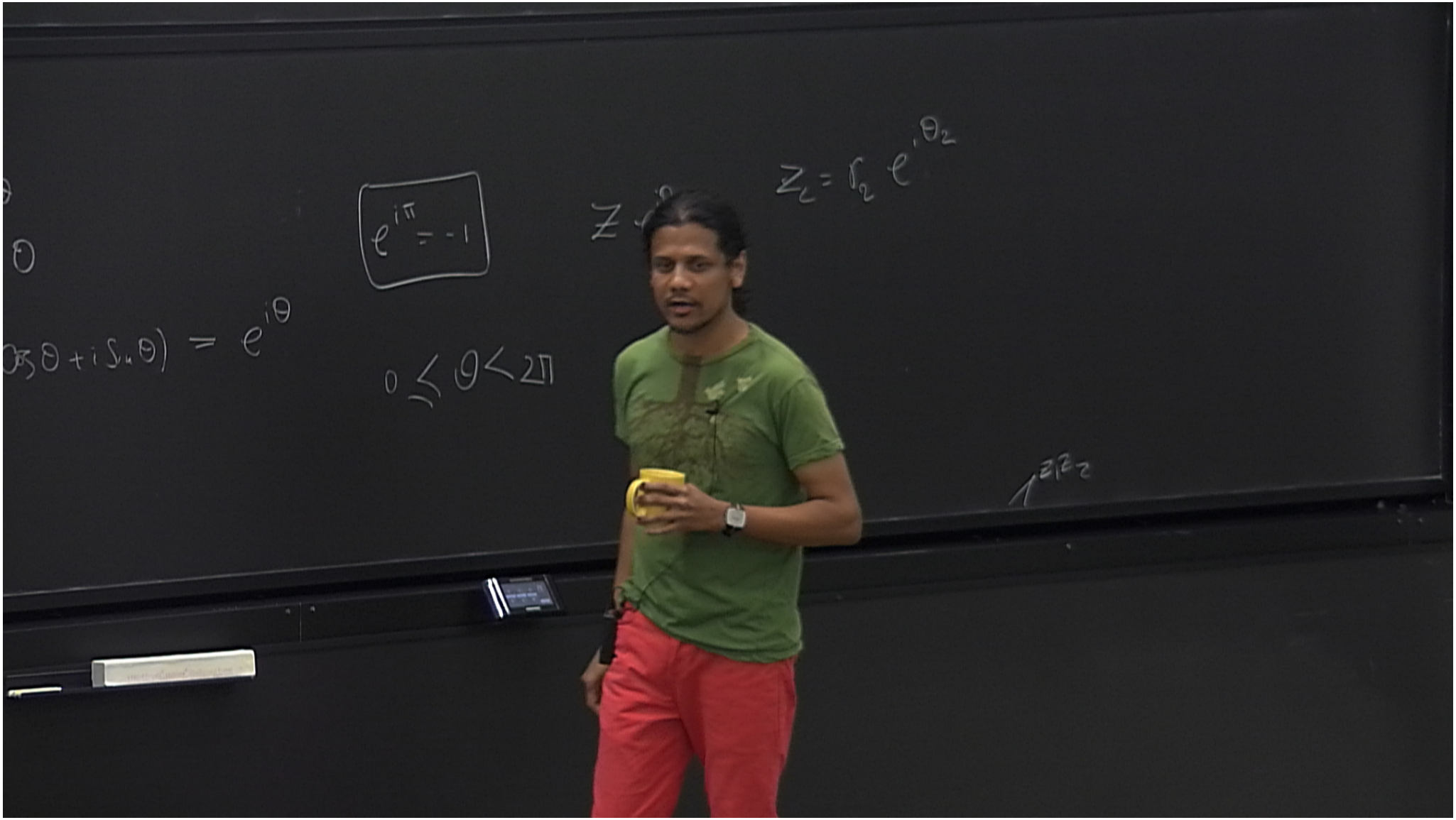

1·i = i
a = r cos θ
b = r sin θ

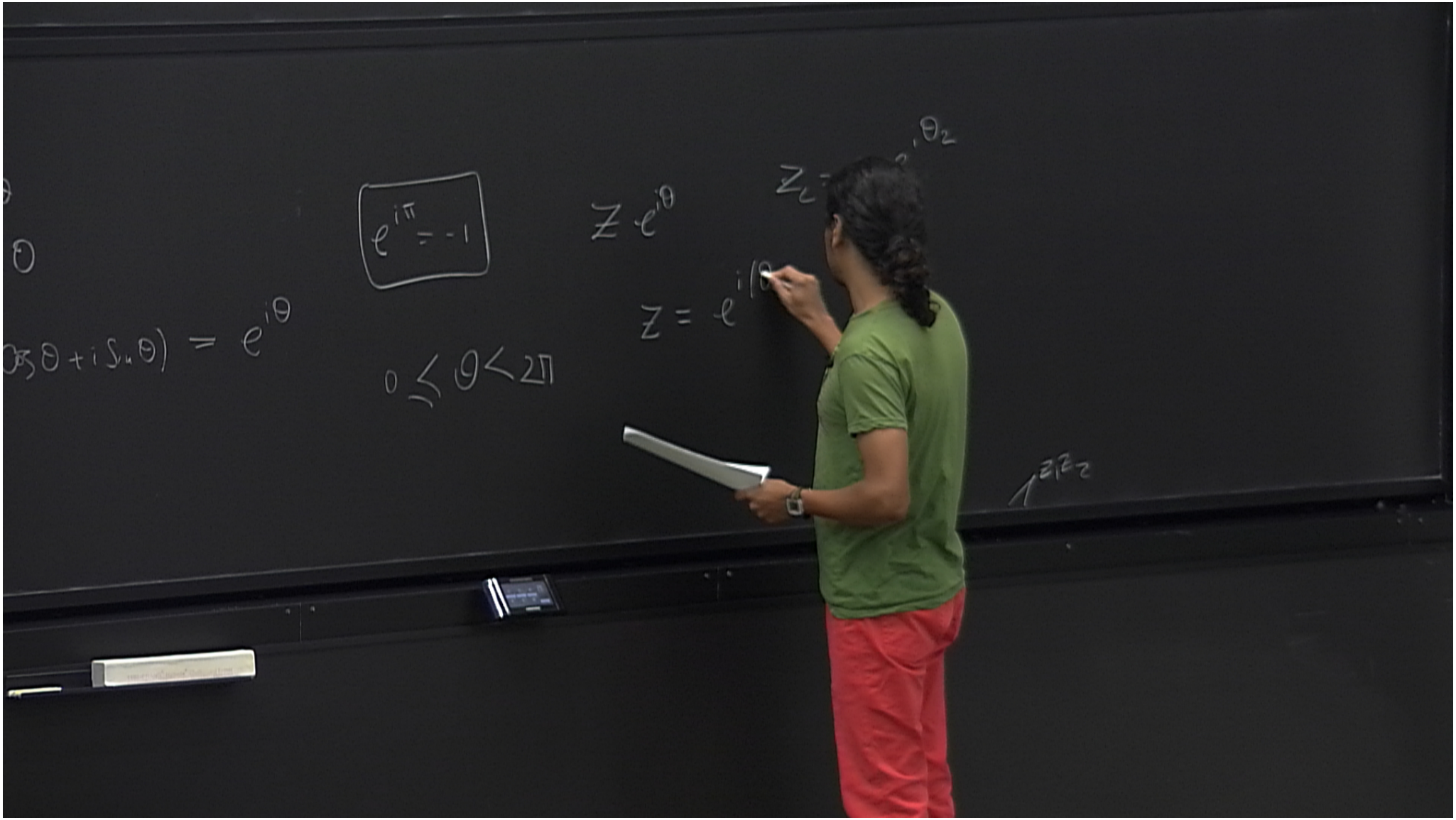

1·i = i
θ
a = r cos θ
b = r sin θ
(cos θ + i sin θ)

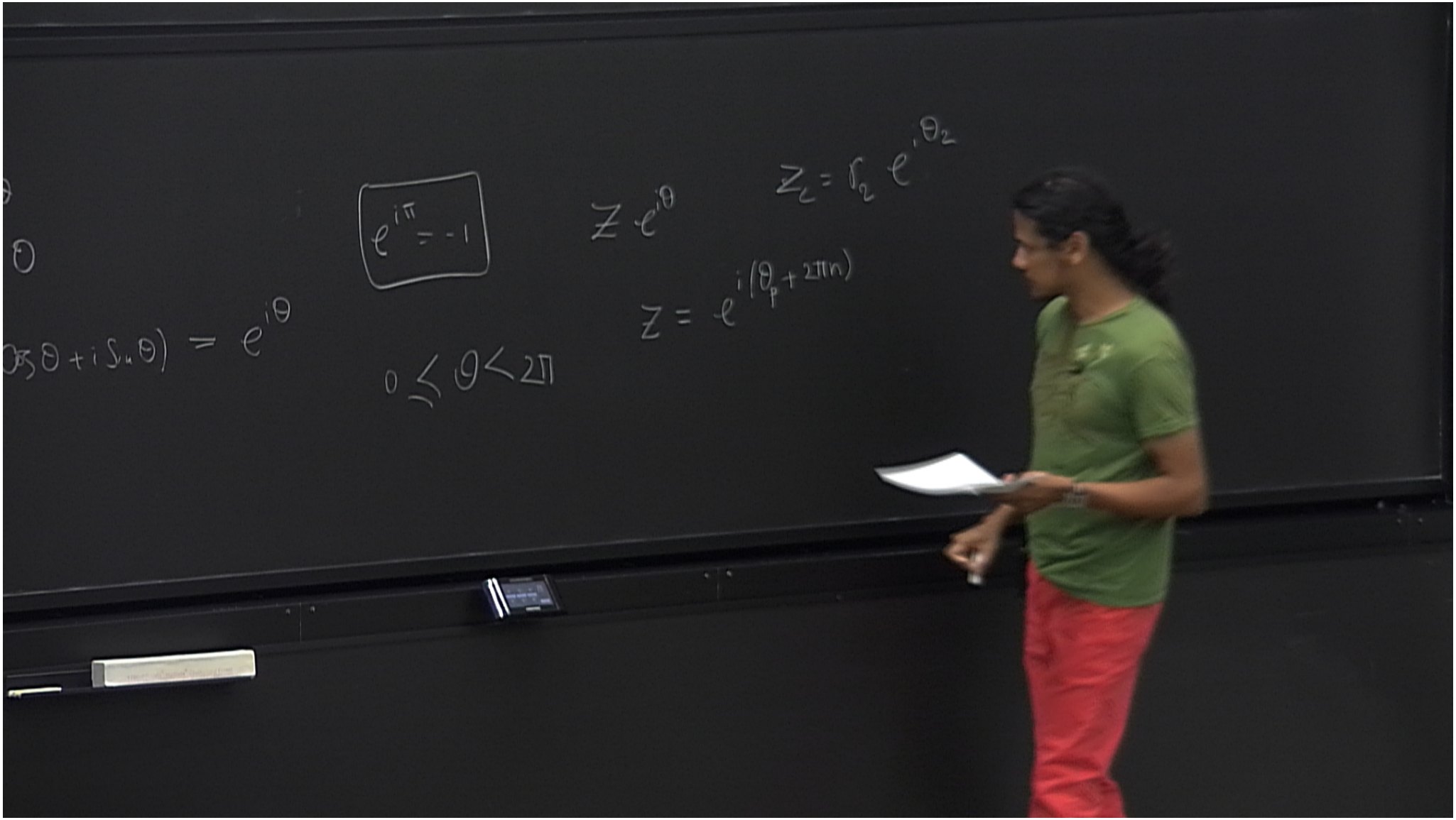

a = r Cos θ
b = r Sin θ
z = r(Cos θ + i Sin θ) = e^{iθ}
e^{iπ} = -1
z

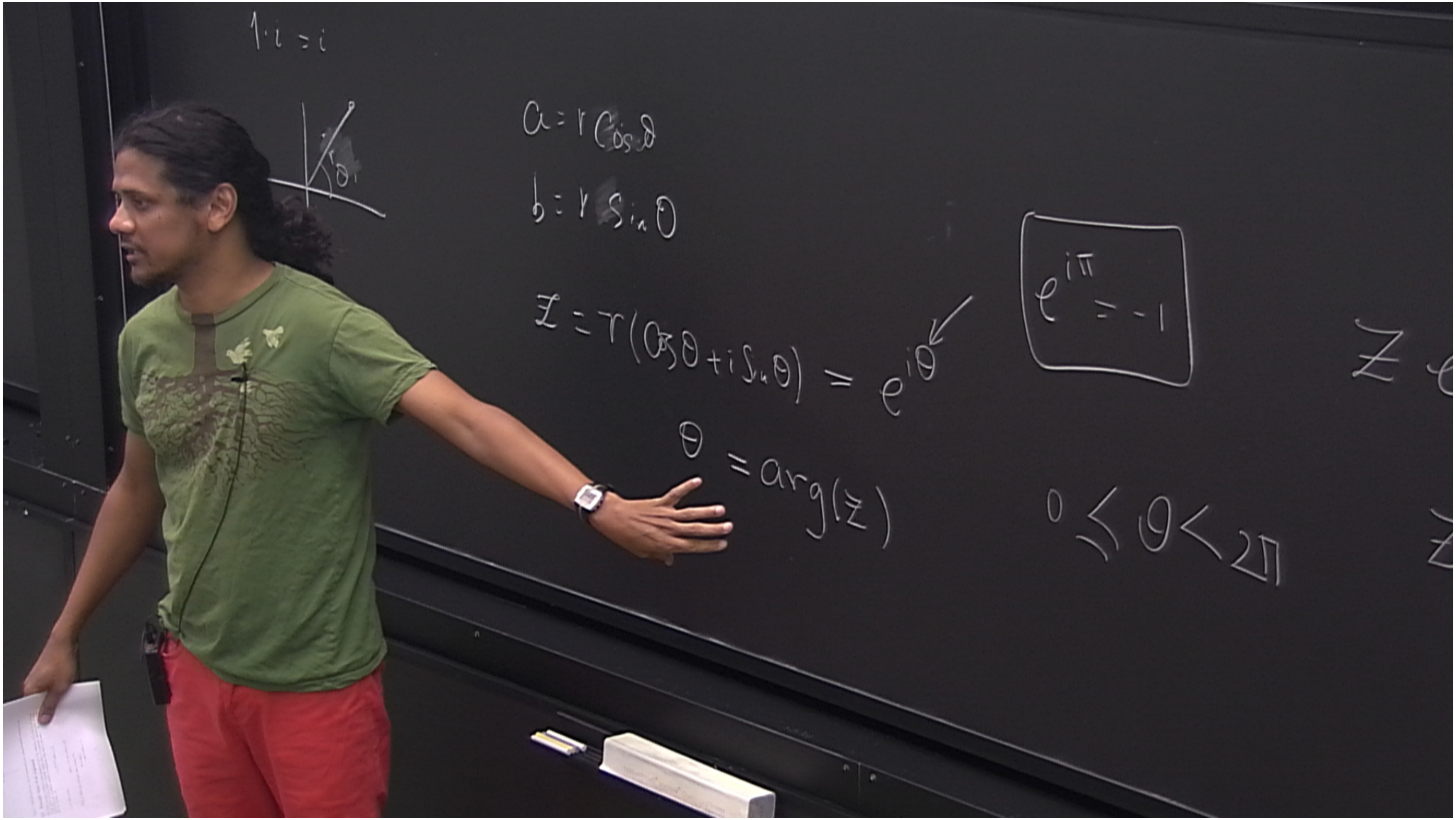

a = r Cos θ
b = r Sin θ
z = r(Cos θ + i Sin θ) = e^{iθ}
e^{iπ} = -1
z e^{iθ}

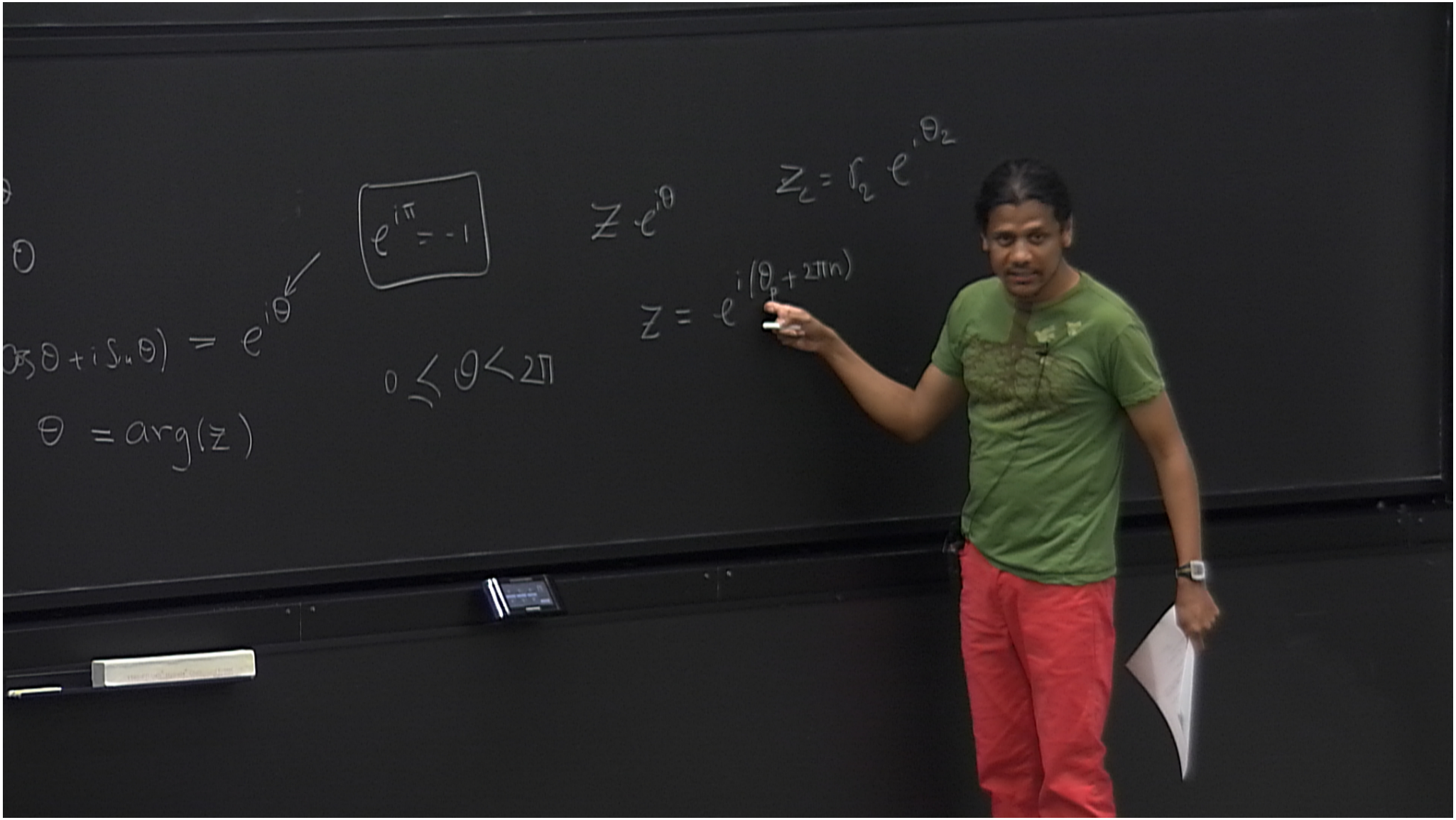

$e^{i\pi} = -1$
$z = e^{i\theta}$
$z_2 = r_2 e^{i\theta_2}$
$1^{z_1 z_2}$

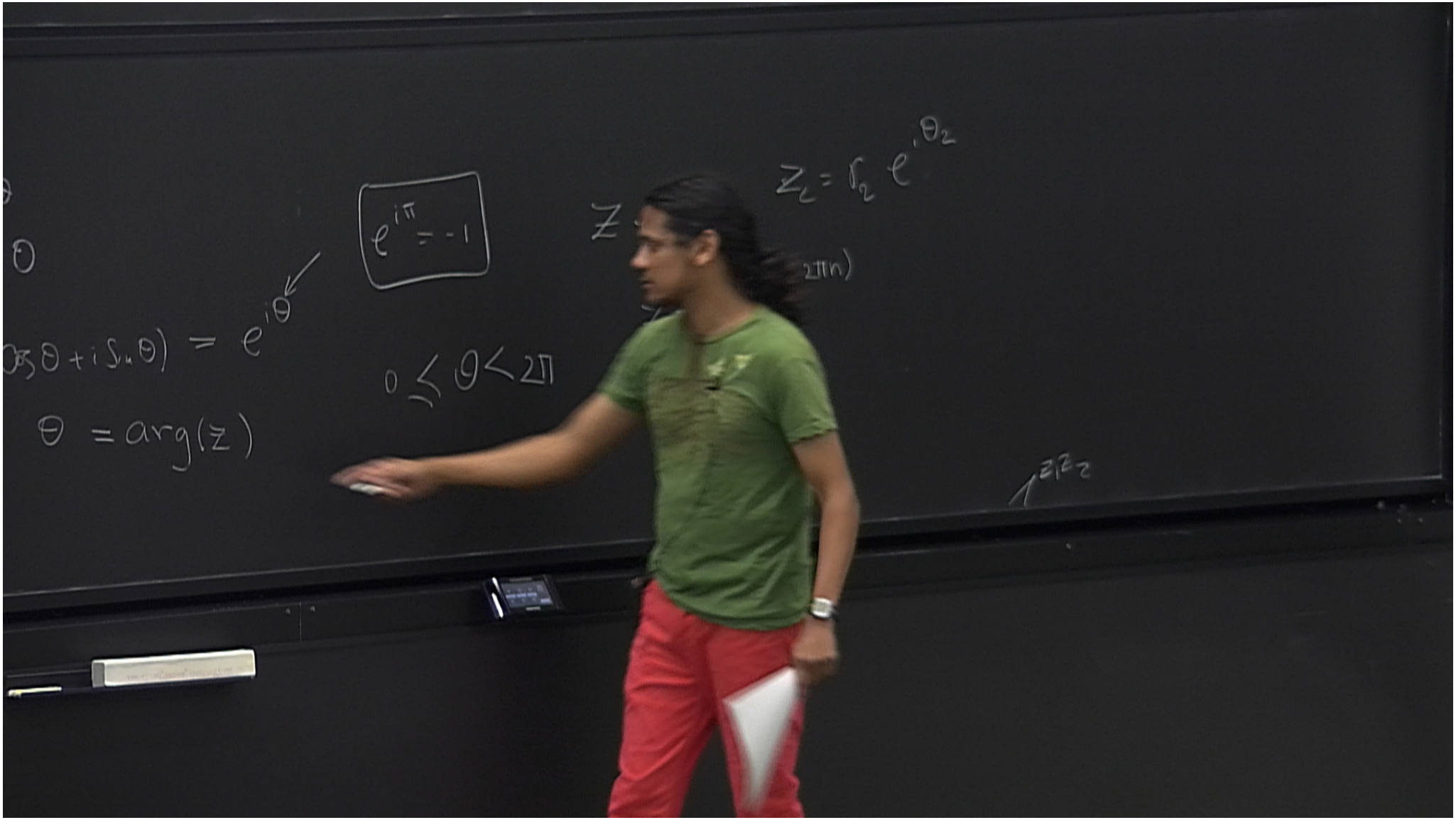

$e^{i\pi} = -1$
$z_2 = r_2 e^{i\theta_2}$
$0 \leq \theta < 2\pi$

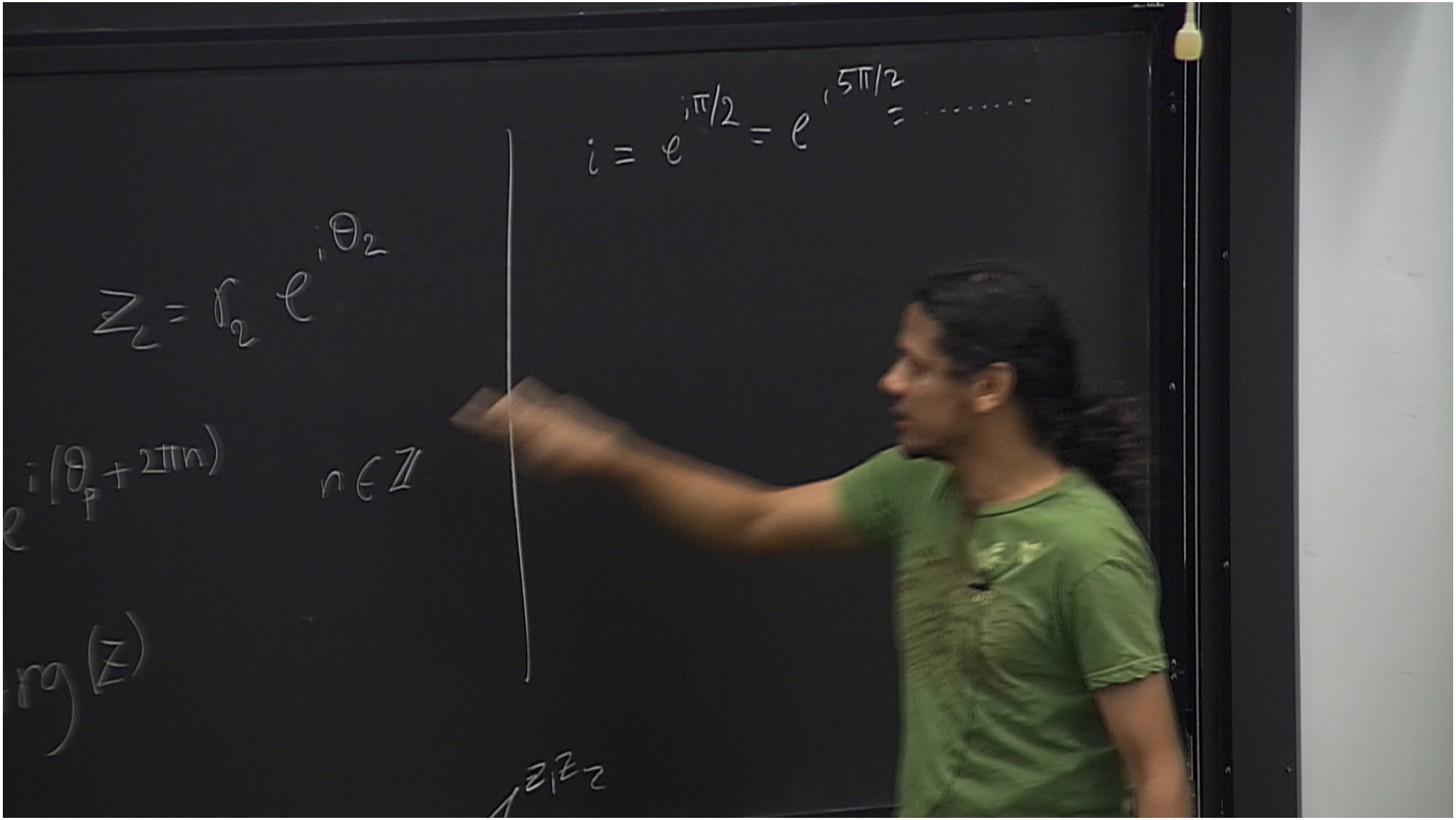

$e^{i\pi} = -1$
$z = e^{i\theta}$
$0 \leq \theta < 2\pi$

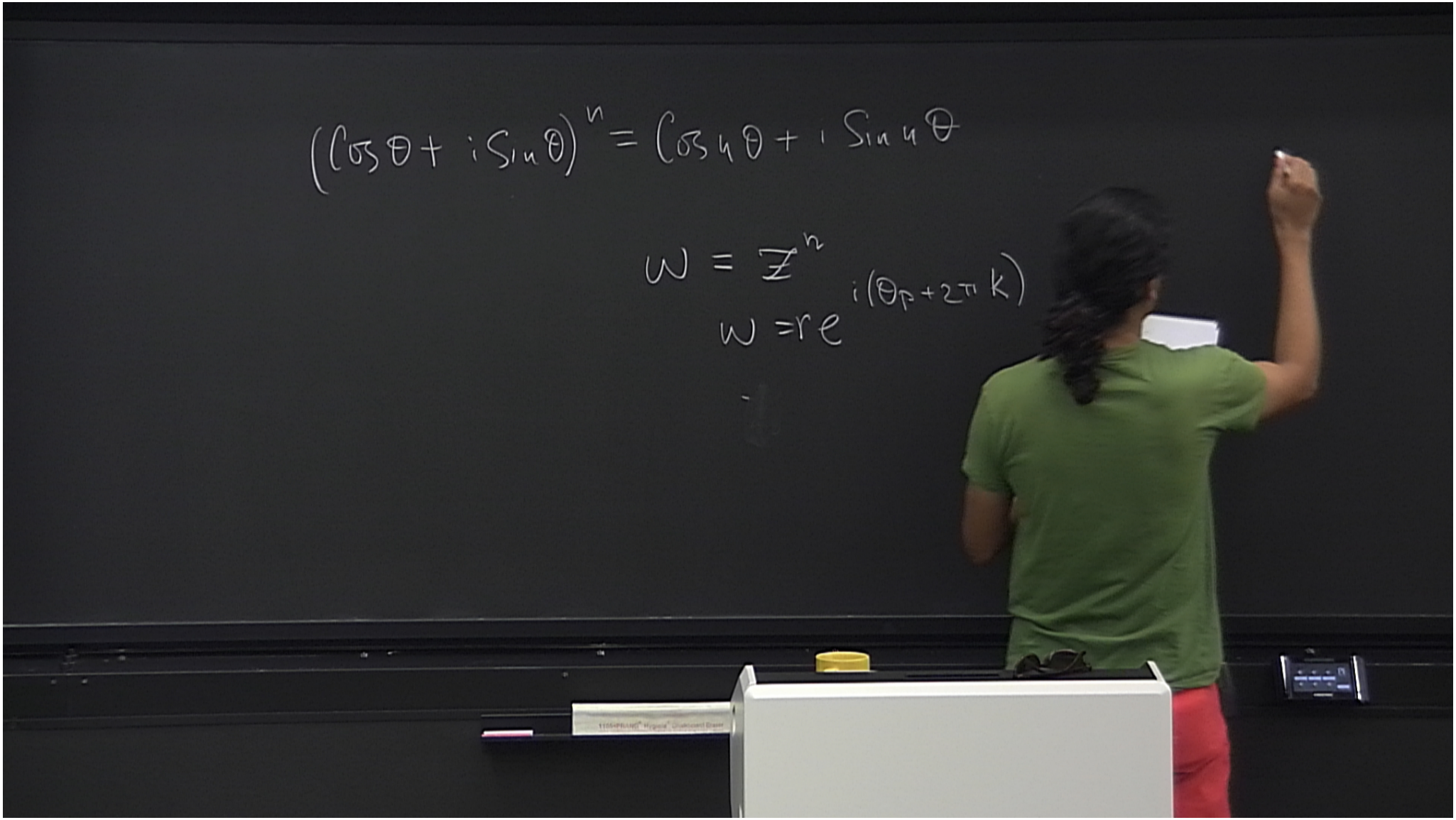

$e^{i\pi} = -1$
$z = e^{i\theta}$
$z_2 = r_2 e^{i\theta_2}$
$z = e^{i(\theta_p + 2\pi n)}$
$0 \leq \theta < 2\pi$

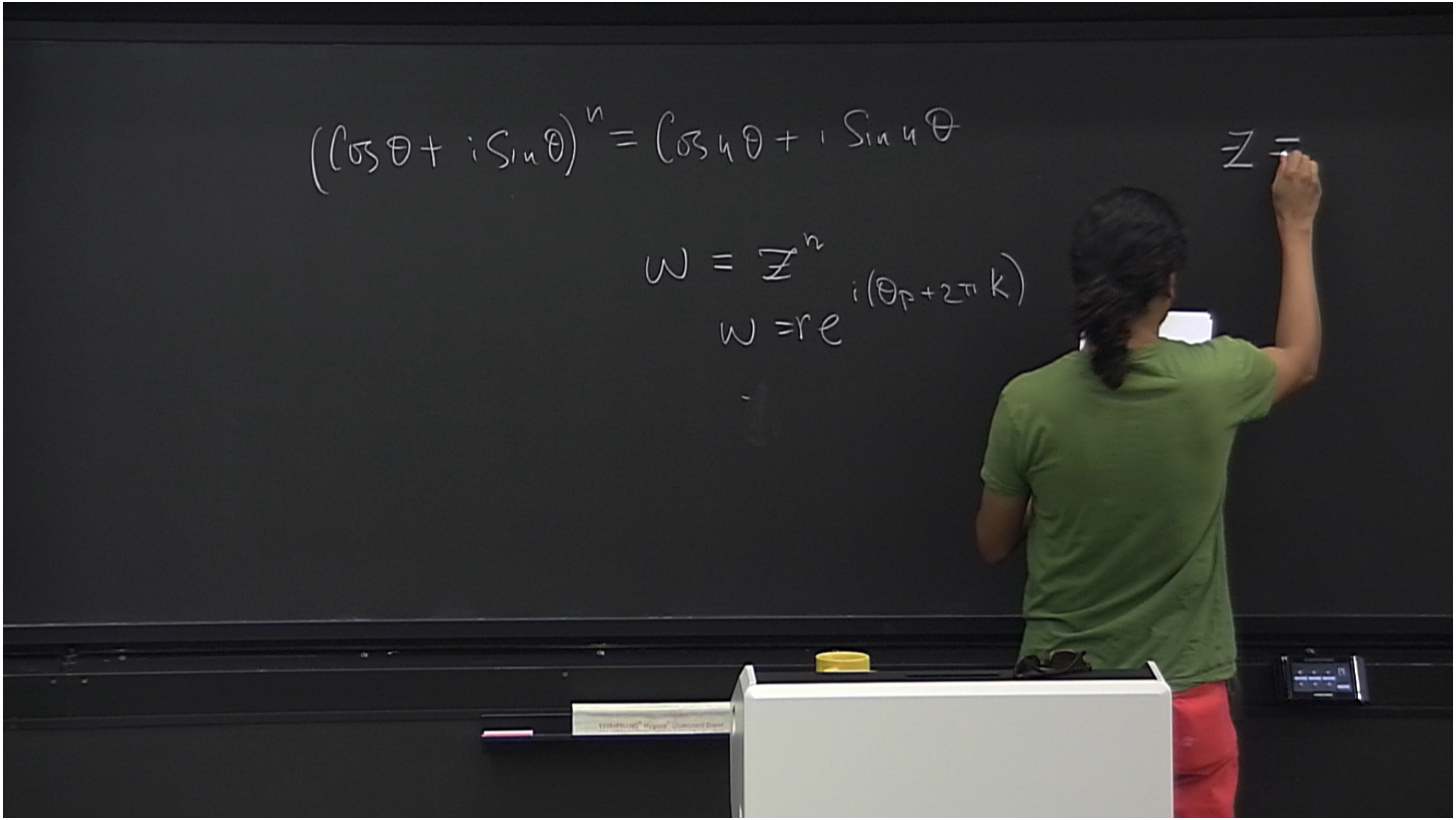

1·i = i
a = r cos θ
b = r sin θ
z = r(cos θ + i sin θ) = e^{iθ}
e^{iπ} = -1
θ = arg(z)
0 ≤ θ < 2π

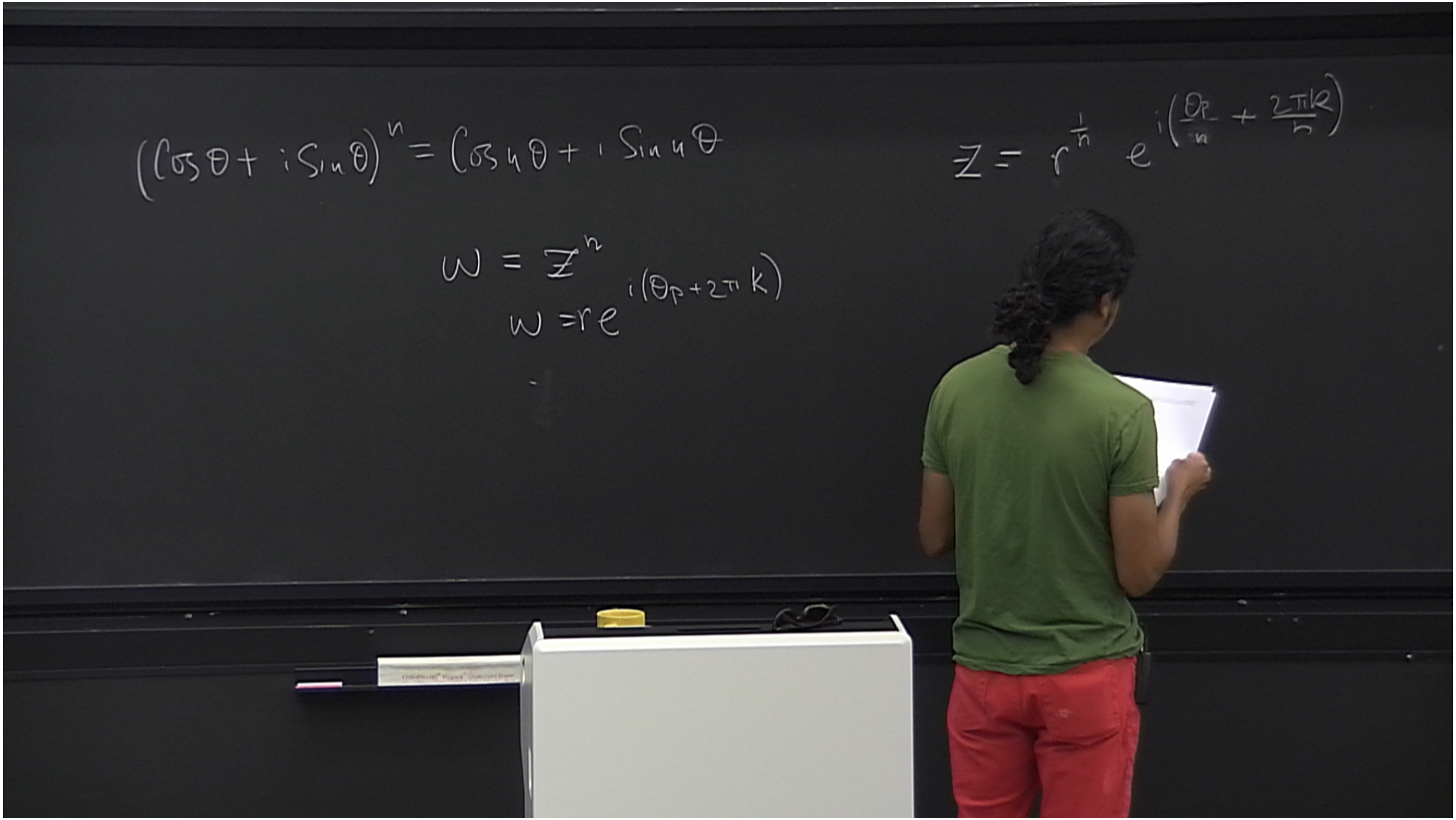

$e^{i\pi} = -1$
$z = r e^{i\theta}$
$z_2 = r_2 e^{i\theta_2}$
$z = e^{i(\theta_0 + 2\pi n)}$
$0 \leq \theta < 2\pi$
$\theta = \arg(z)$

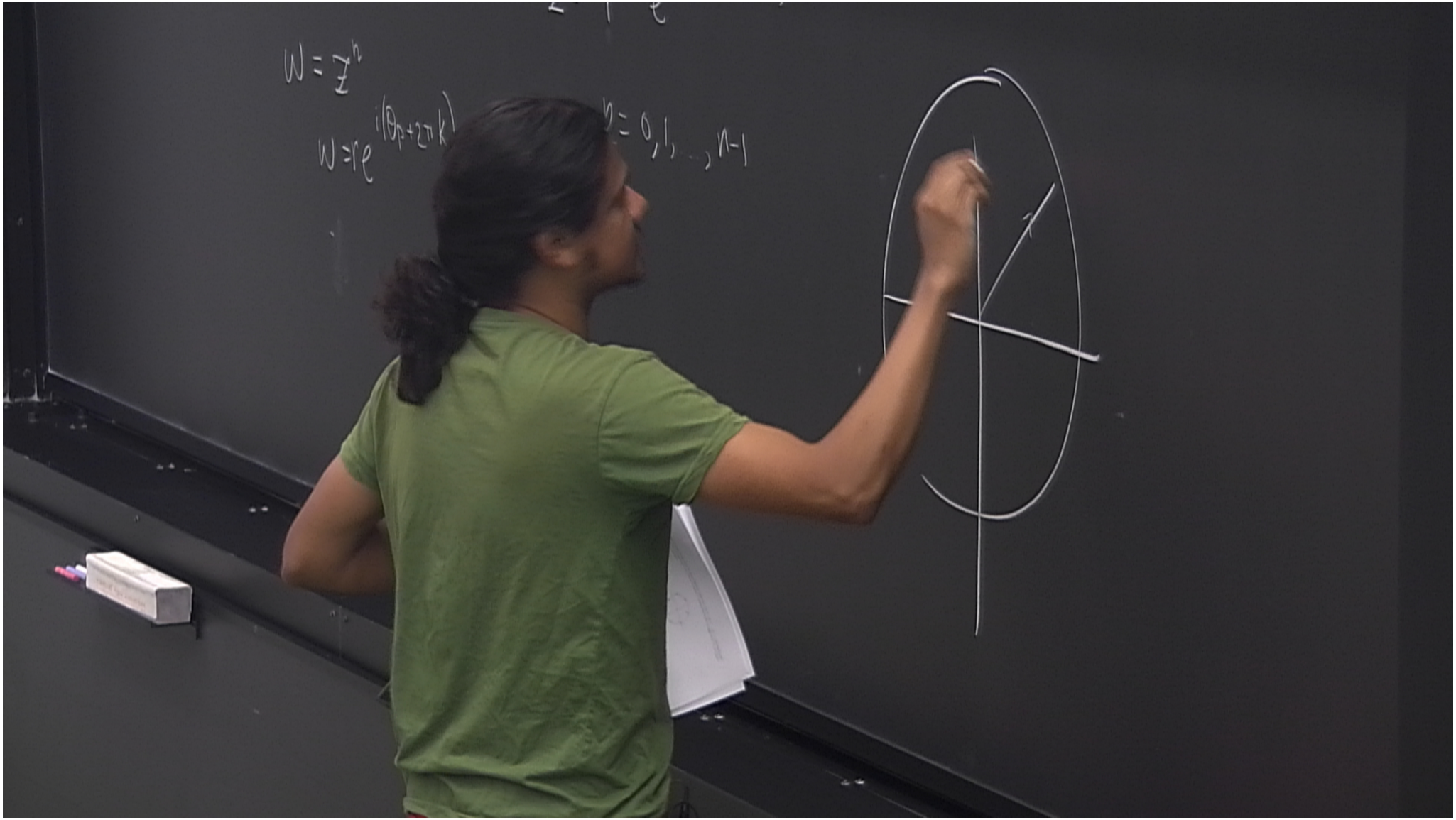

$e^{i\pi} = -1$
$z_2 = r_2 e^{i\theta_2}$
$0 \leq \theta < 2\pi$
$\theta = \arg(z)$
$z_1 z_2$

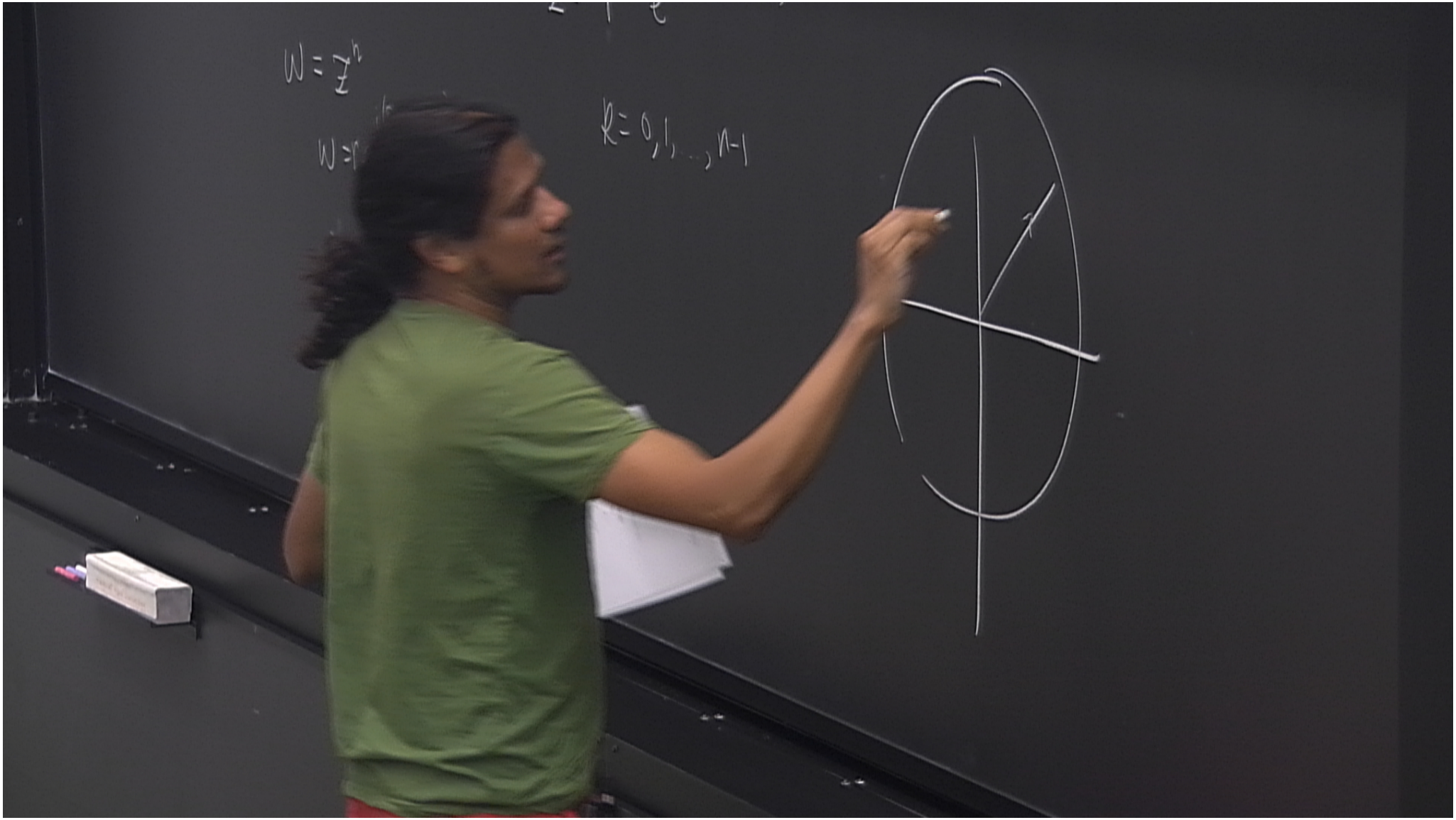

$i = e^{i\pi/2} = e^{i5\pi/2} = \cdots\cdots$
$z_2 = r_2 e^{i\theta_2}$
$n \in \mathbb{Z}$

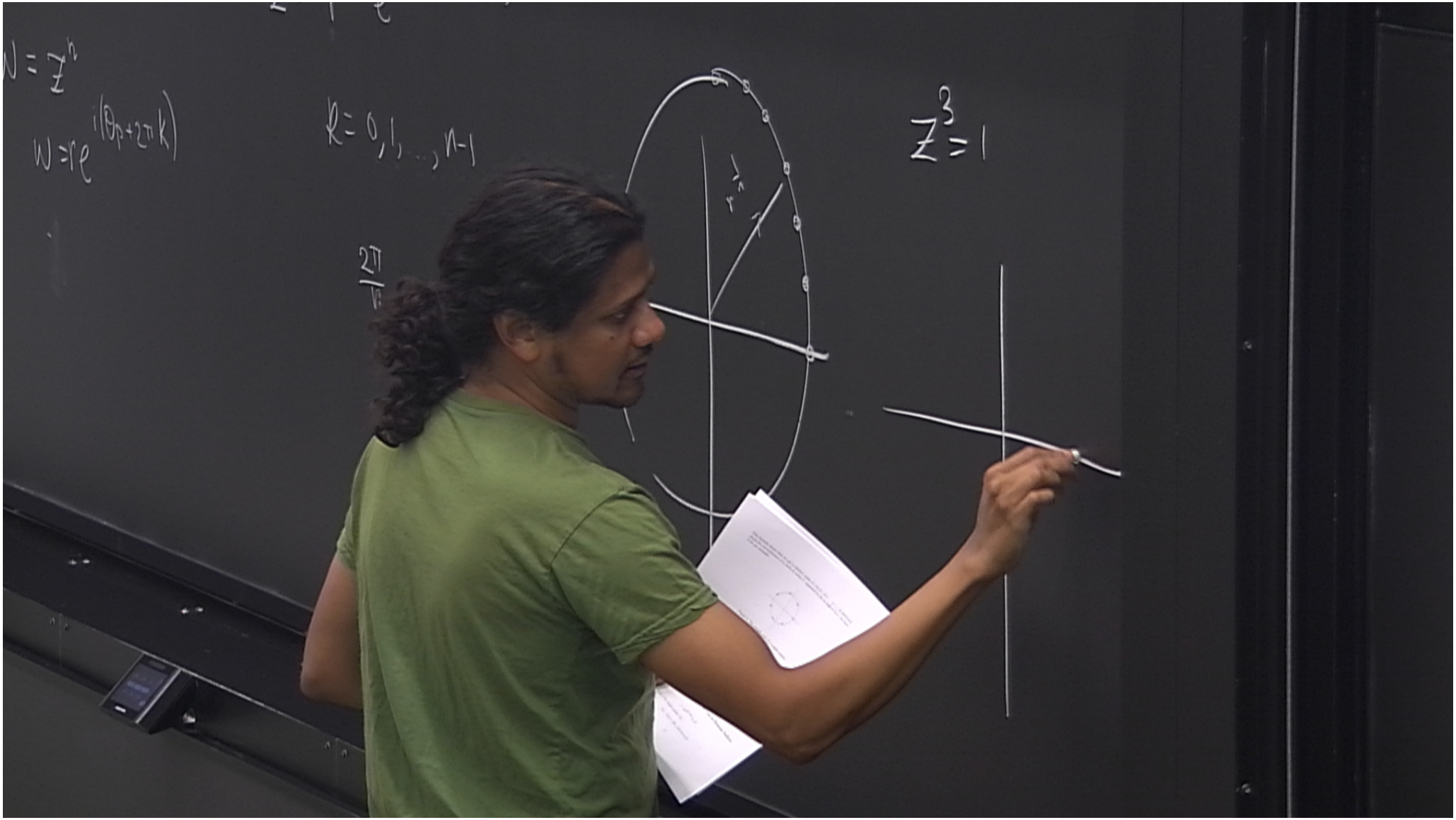

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$
$w = z^n$
$w = re^{i(\theta_p + 2\pi k)}$

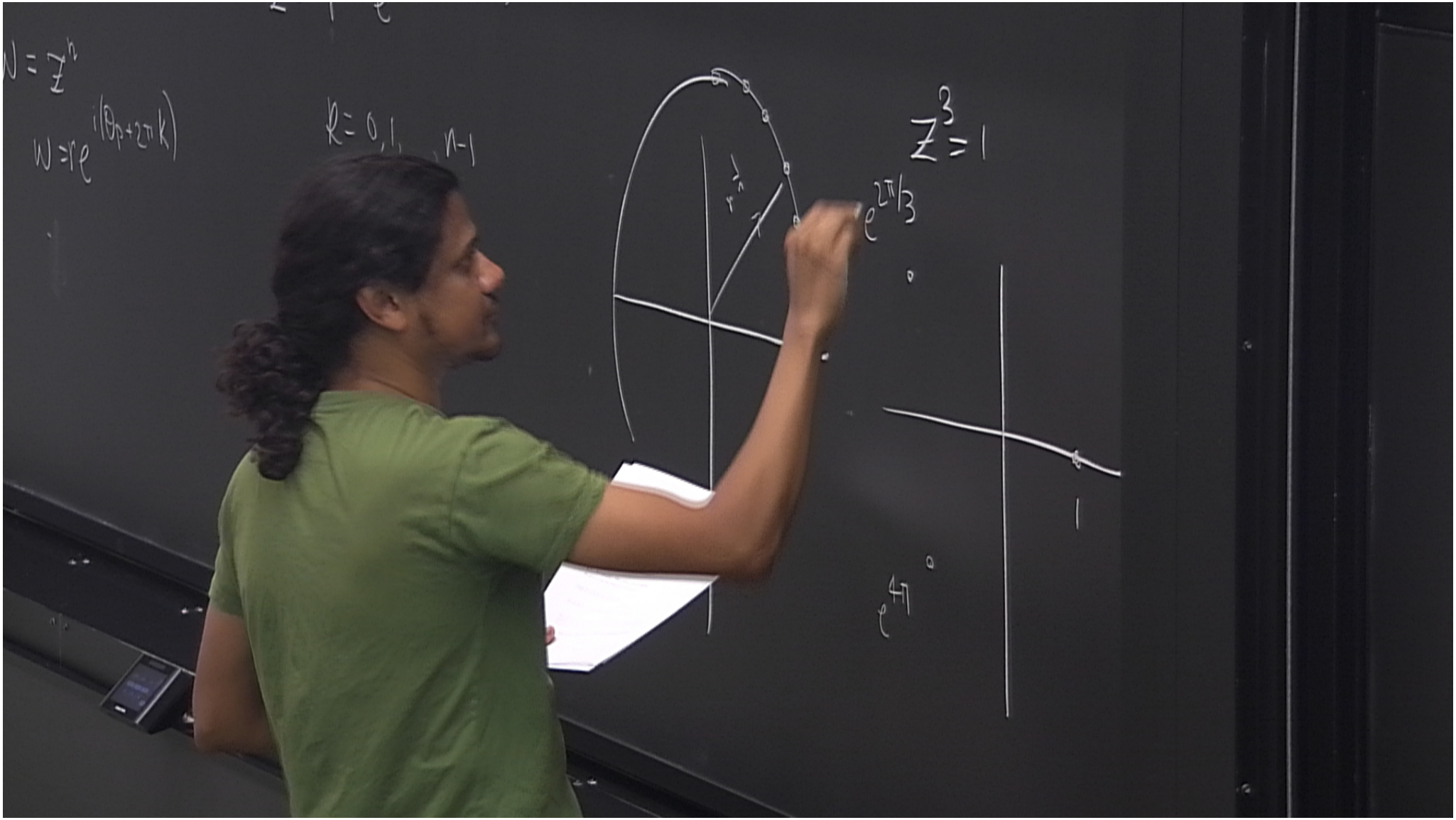

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$
$z =$
$w = z^n$
$w = re^{i(\theta_p + 2\pi k)}$

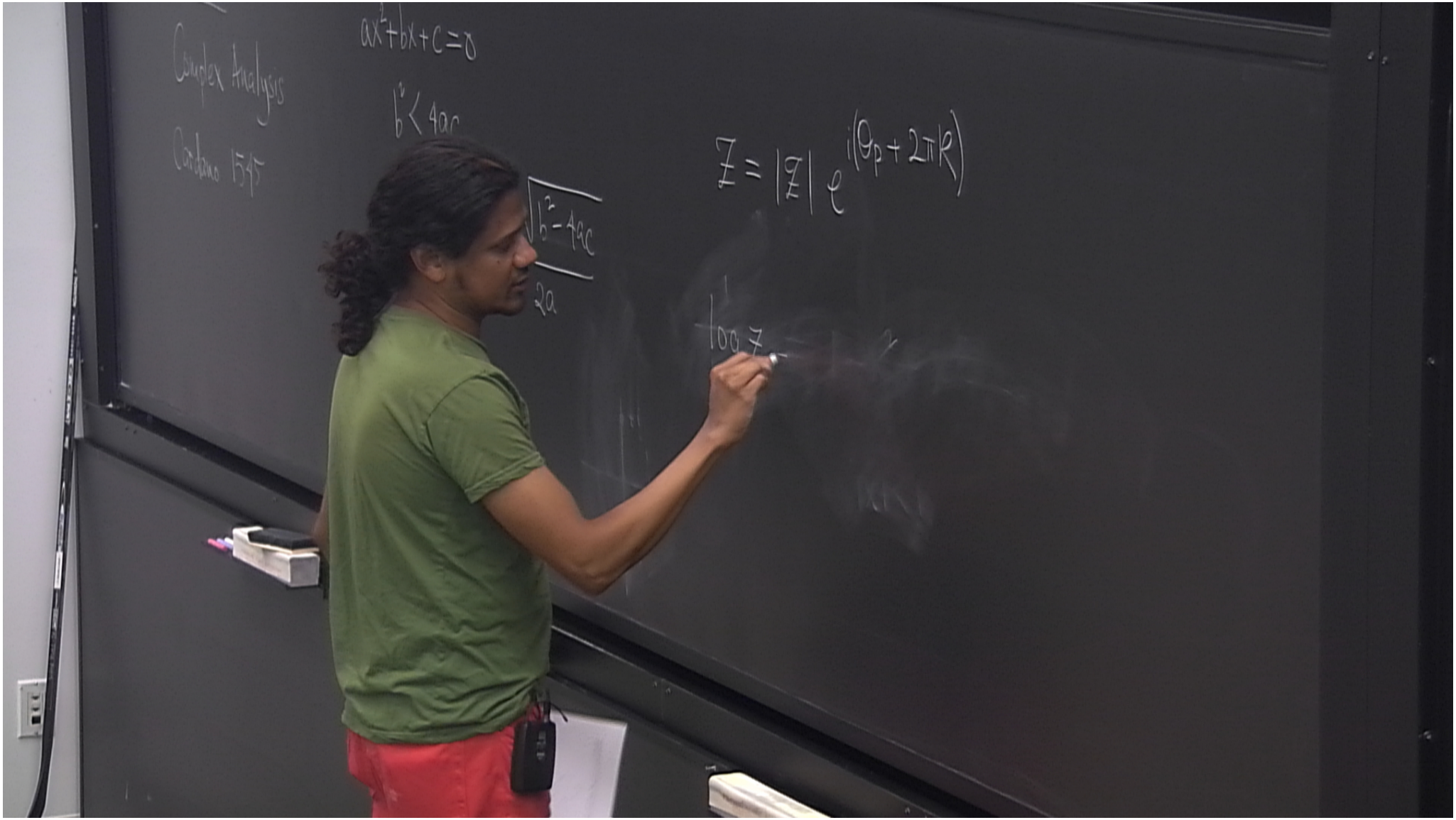

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$
$z = r^{\frac{1}{n}} e^{i\left(\frac{\theta_p}{n} + \frac{2\pi k}{n}\right)}$
$w = z^n$
$w = re^{i(\theta_p + 2\pi k)}$

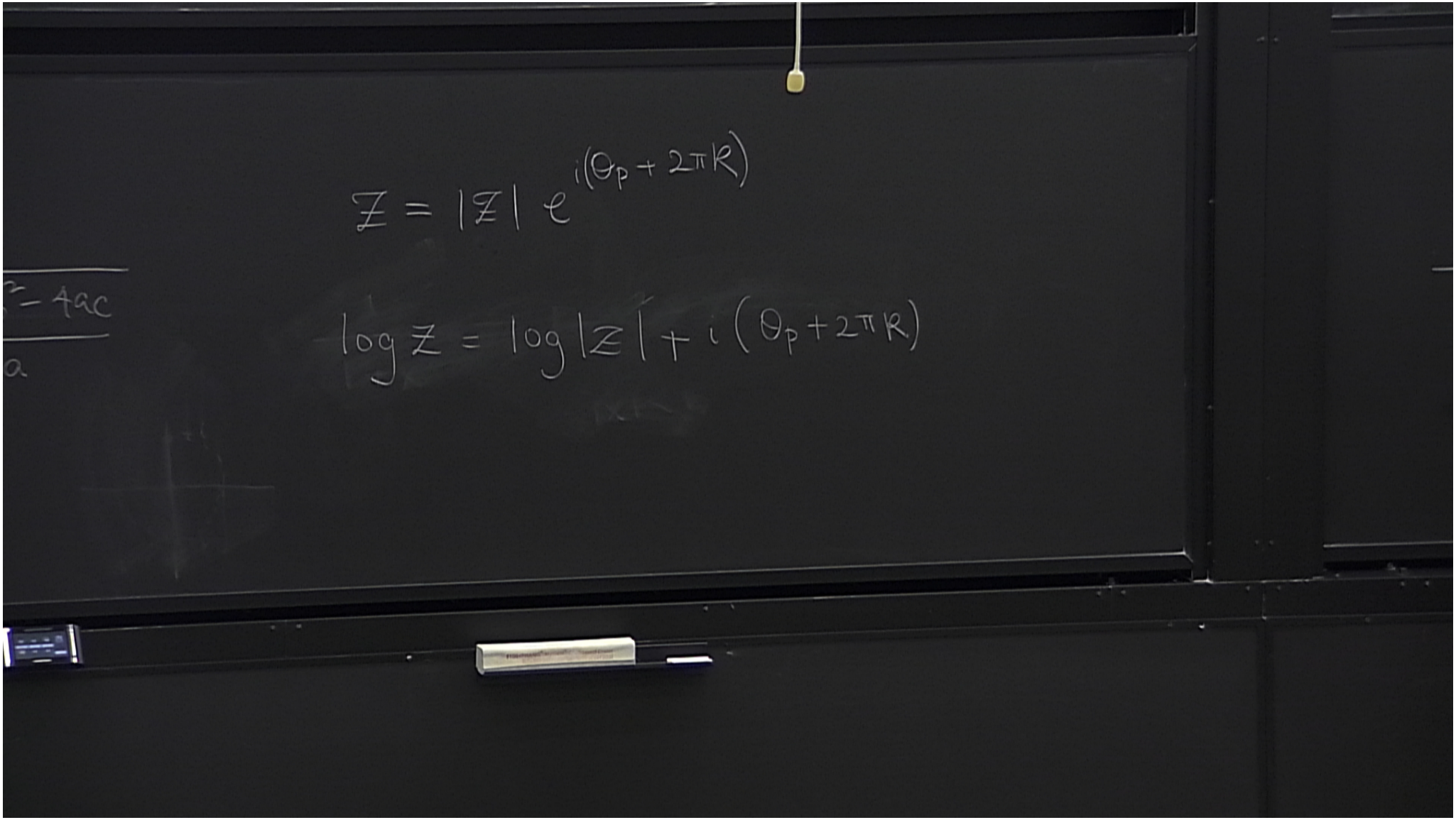

$w = z^n$
$w = re^{i(\theta + 2\pi k)}$
$n = 0, 1, \ldots, n-1$

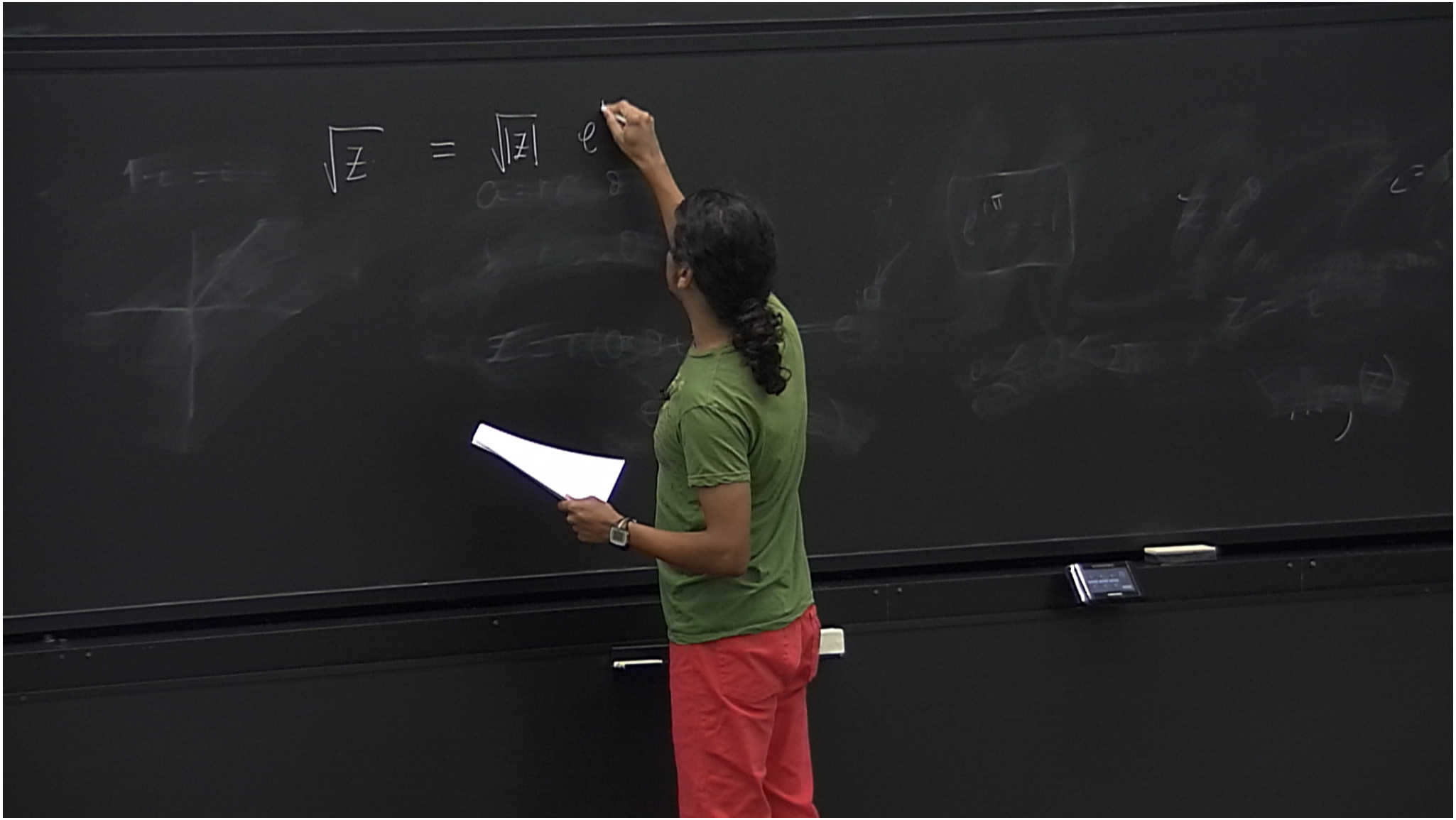

$w = z^n$
$k = 0, 1, \ldots, n-1$

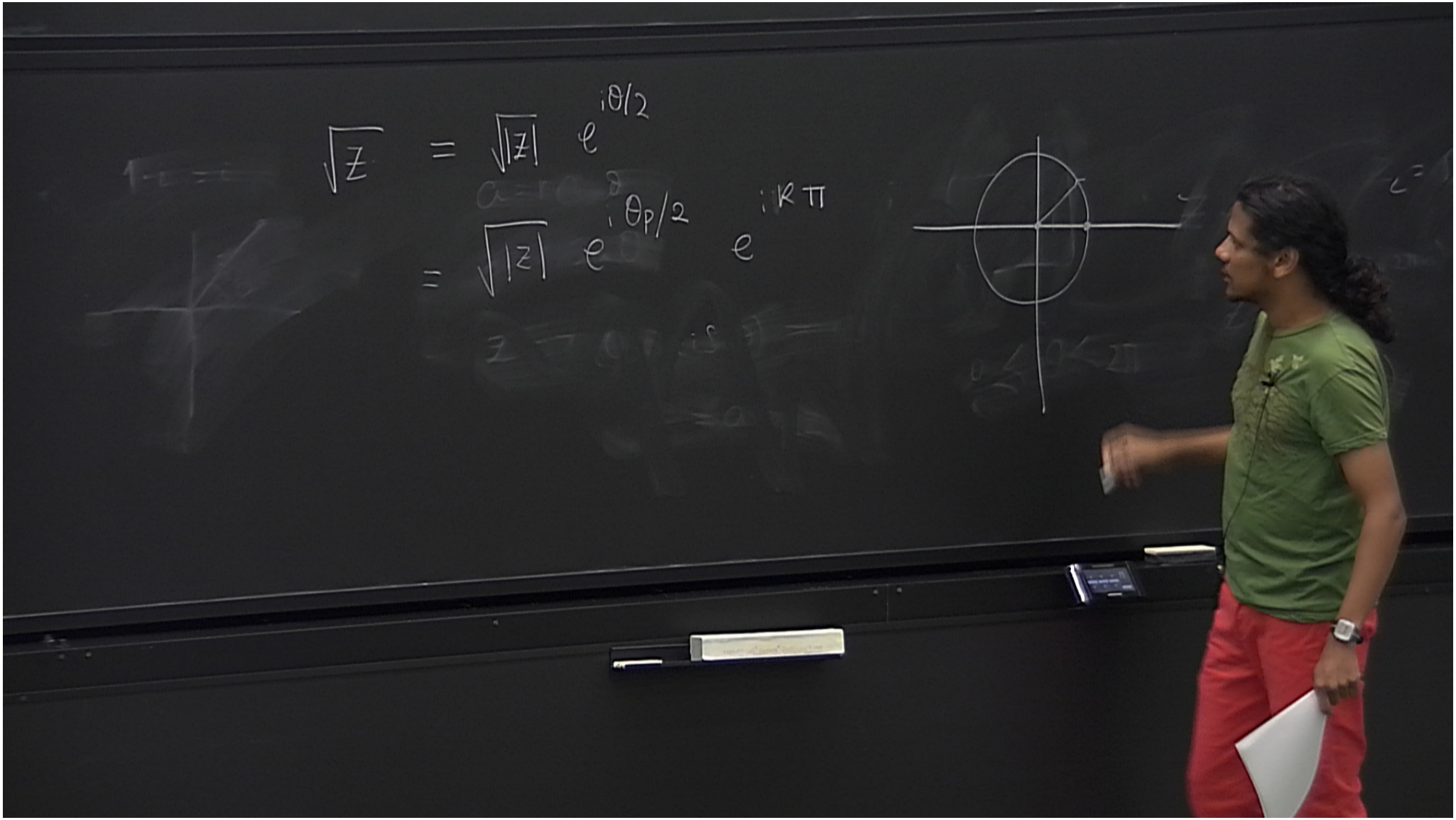

$w = z^n$
$w = re^{i(\theta + 2\pi k)}$
$k = 0, 1, \ldots, n-1$
$\frac{2\pi}{n}$
$z^3 = 1$

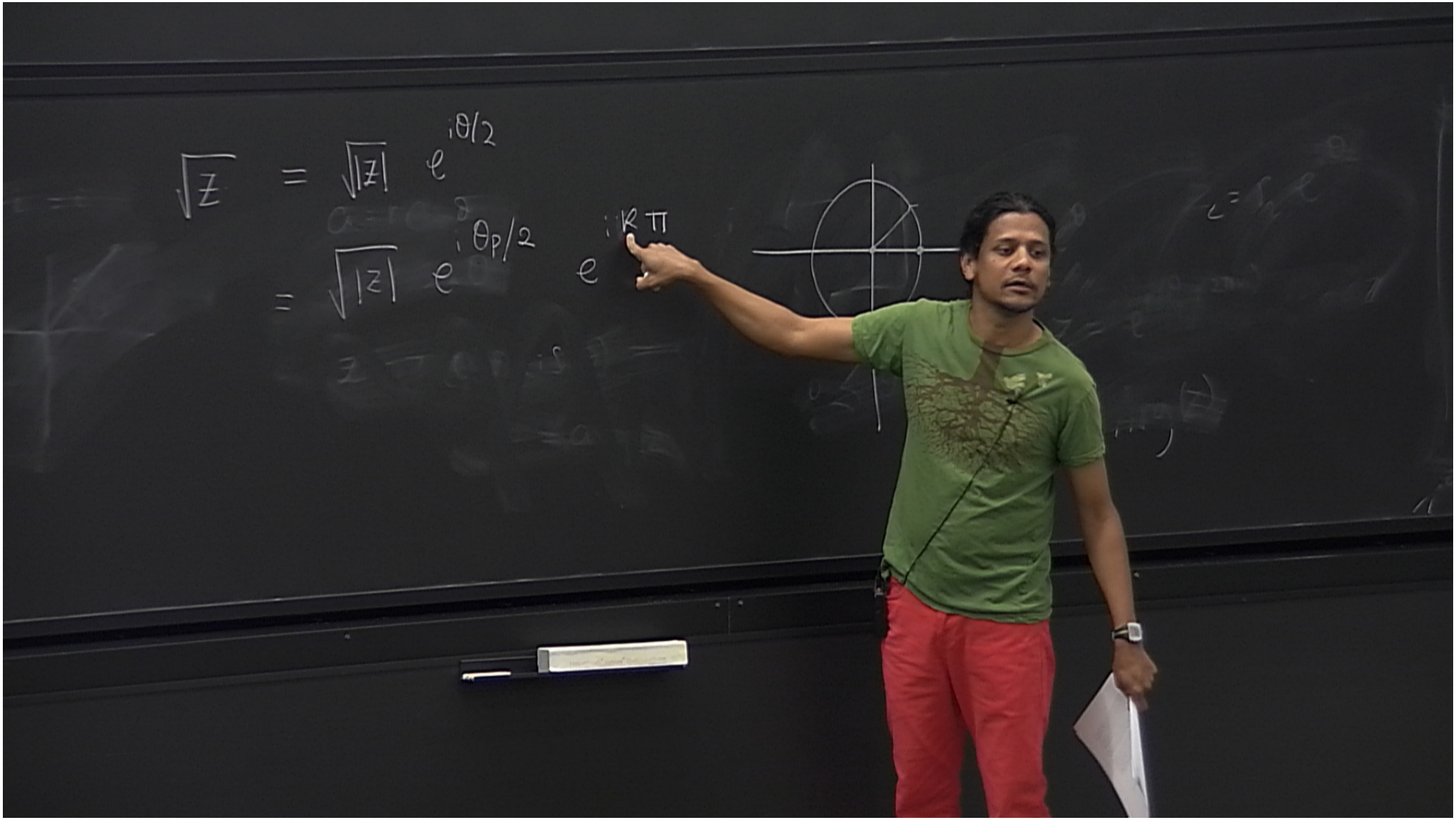

$w = z^n$
$w = re^{i(\theta + 2\pi k)}$
$k = 0, 1, \ldots n-1$
$z^3 = 1$
$e^{2\pi/3}$
1
$e^{4\pi}$

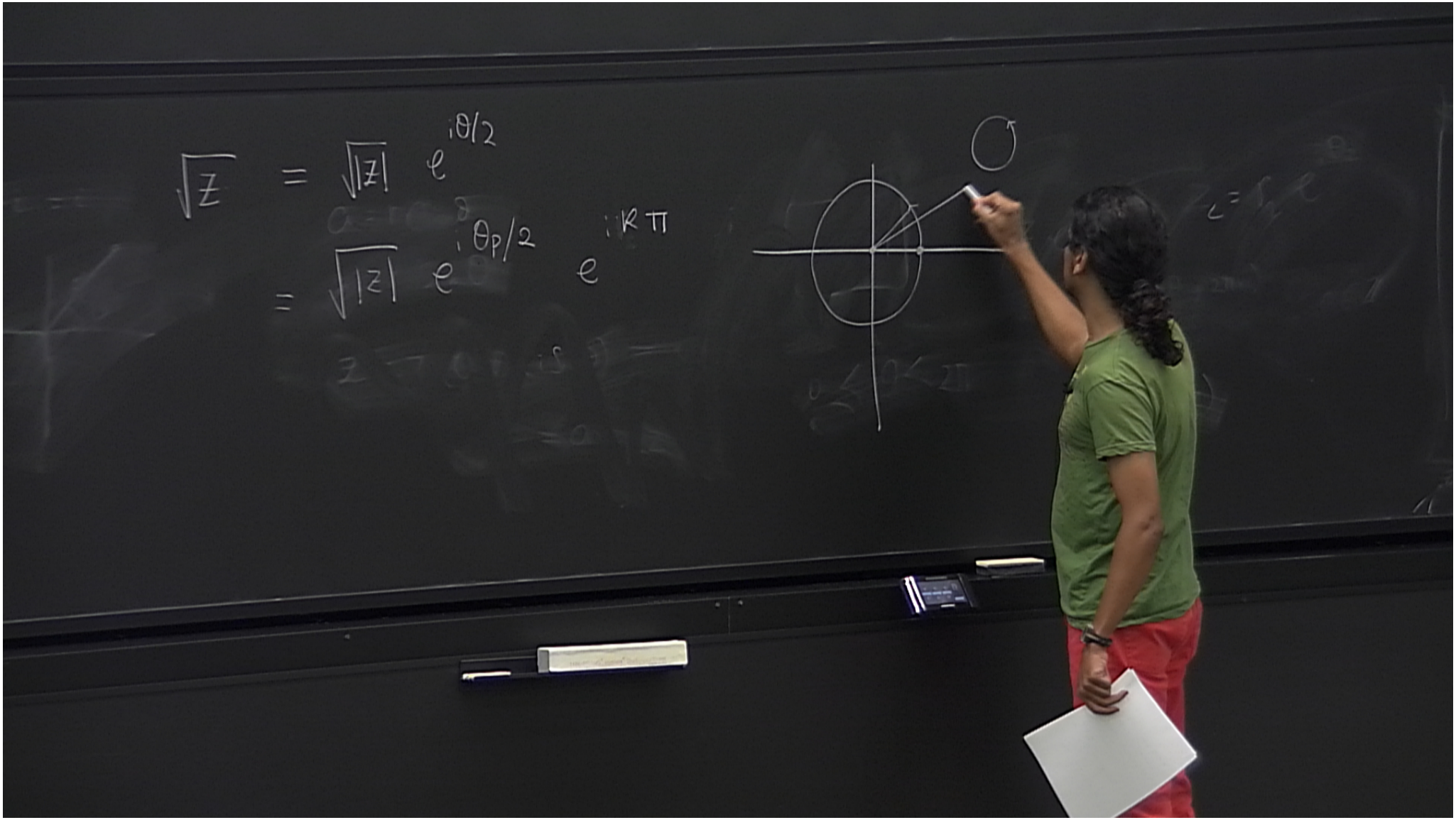

Complex Analysis
Cardano 1545
$ax^2+bx+c=0$
$b^2<4ac$
$\sqrt{b^2-4ac}$
$2a$
$z=|z|e^{i(\theta_p+2\pi R)}$
$\log z$

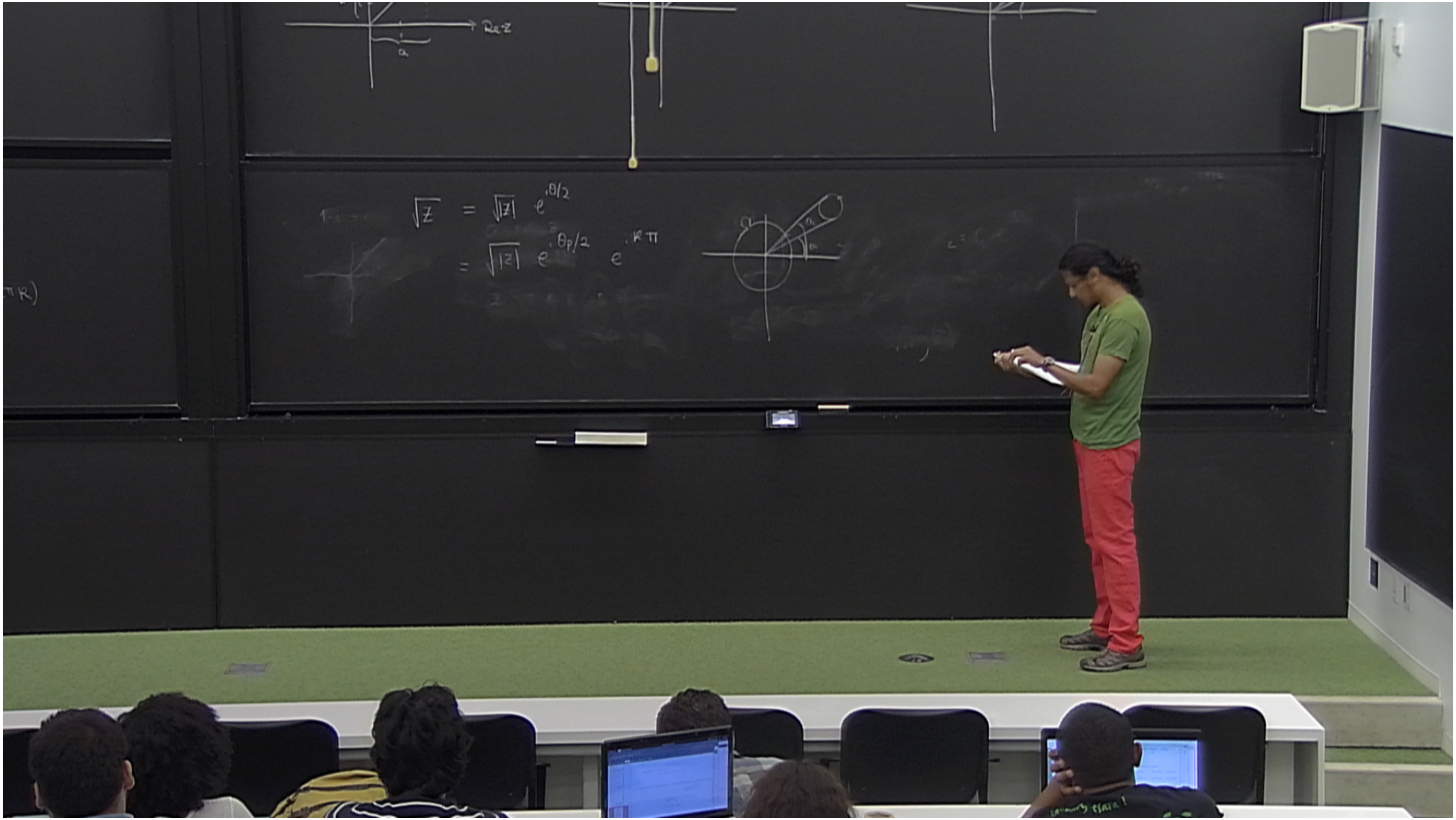

$Z = |Z| e^{i(\theta_p + 2\pi R)}$
$\log Z = \log|Z| + i(\theta_p + 2\pi R)$

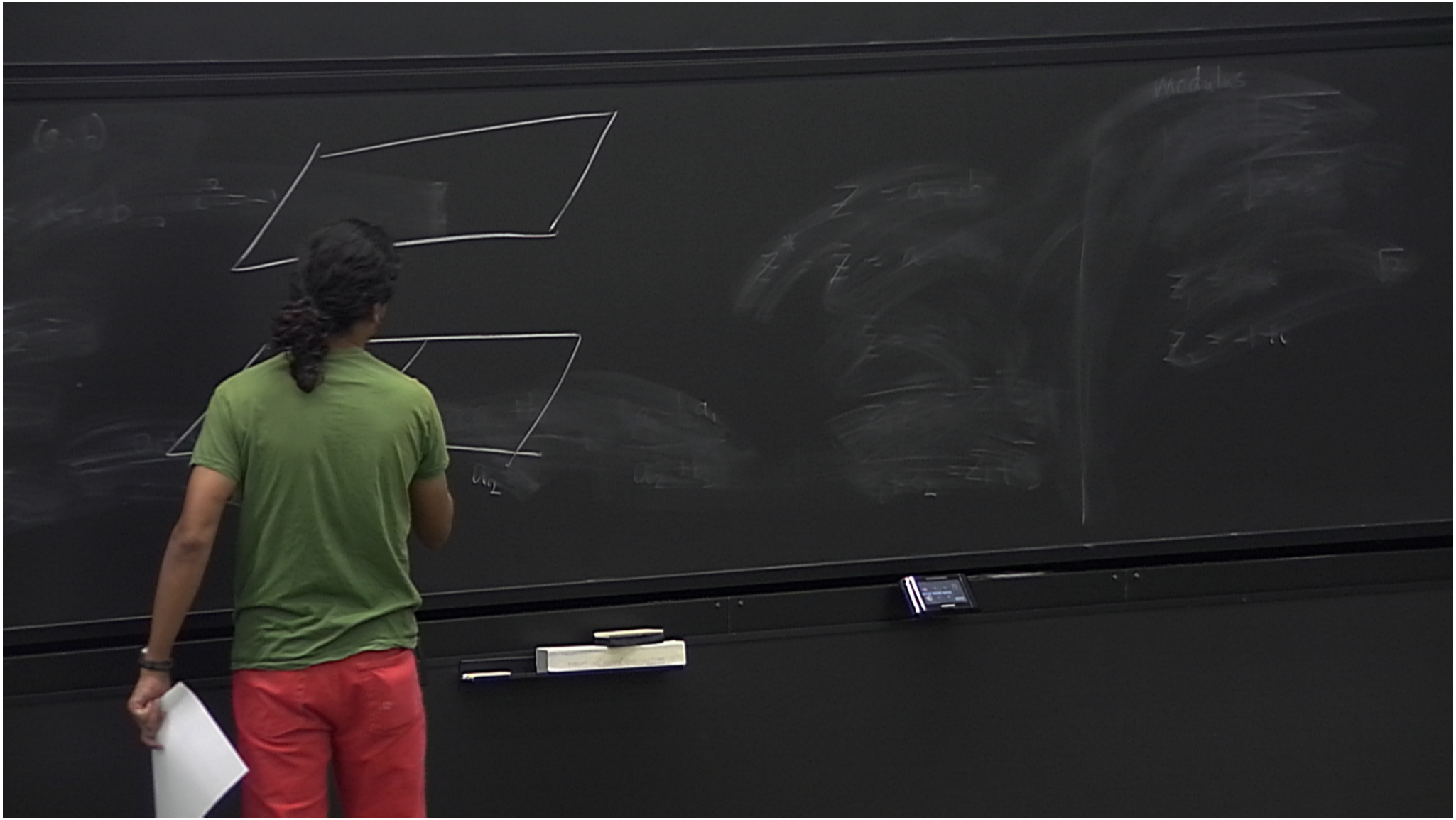

$\sqrt{z} = \sqrt{|z|}\, e$

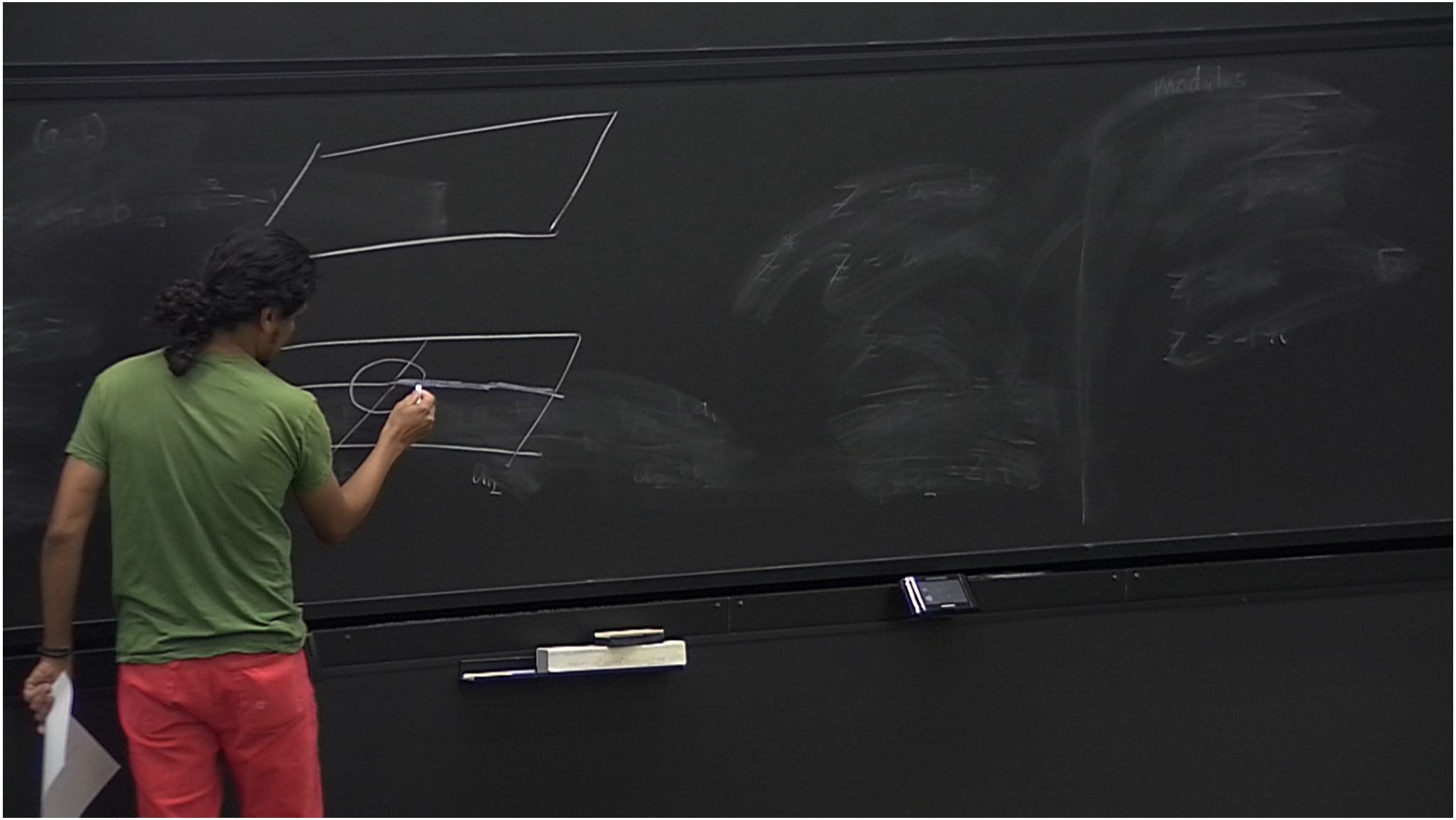

$\sqrt{z} = \sqrt{|z|}\, e^{i\theta/2}$
$= \sqrt{|z|}\, e^{i\theta_p/2}\, e^{ik\pi}$

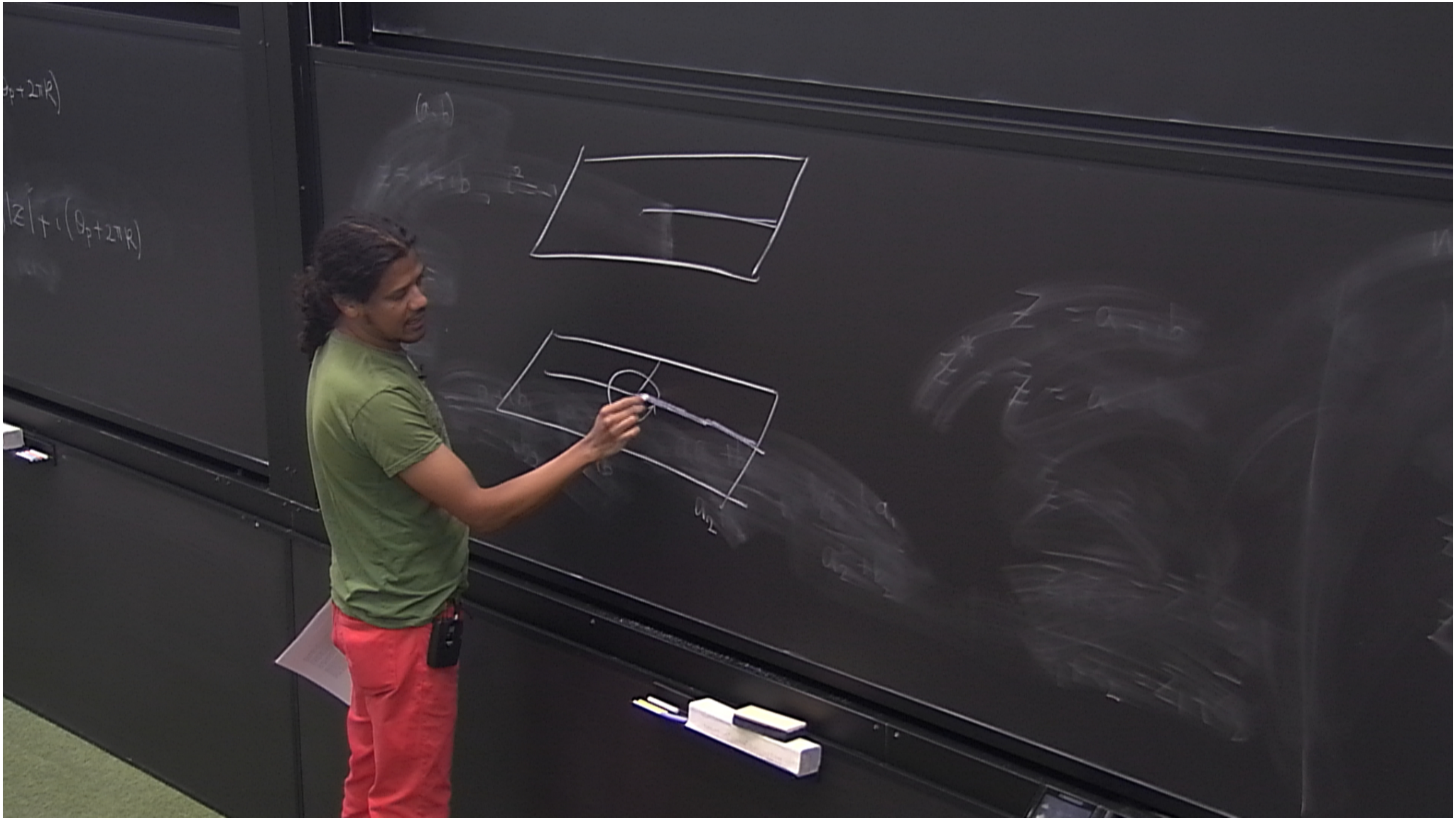

$\sqrt{z} = \sqrt{|z|}\, e^{i\theta/2}$
$= \sqrt{|z|}\, e^{i\theta_p/2}\, e^{i k\pi}$

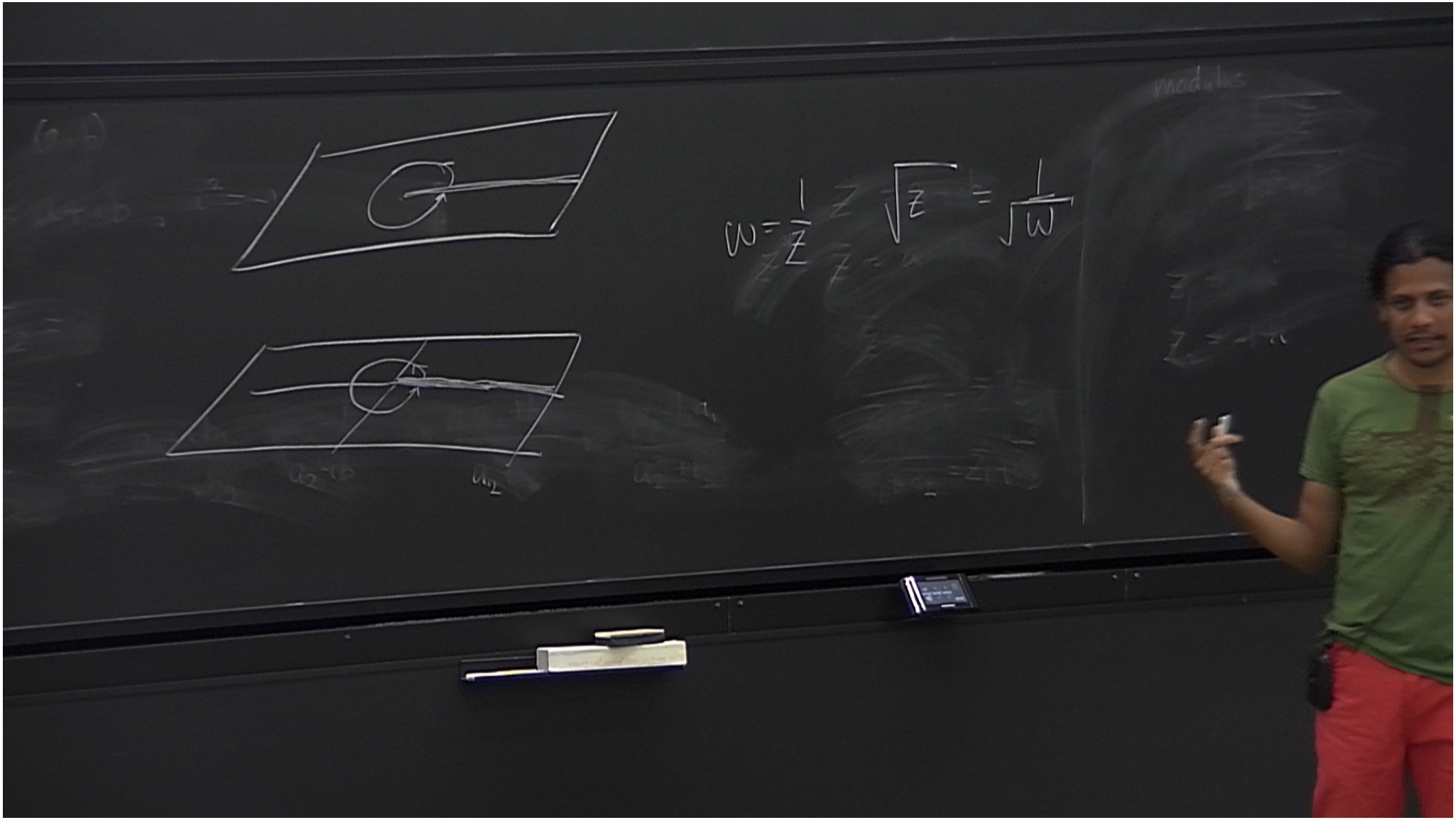

$\sqrt{z} = \sqrt{|z|}\, e^{i\theta/2}$
$= \sqrt{|z|}\, e^{i\theta_p/2}\, e^{ikn}$

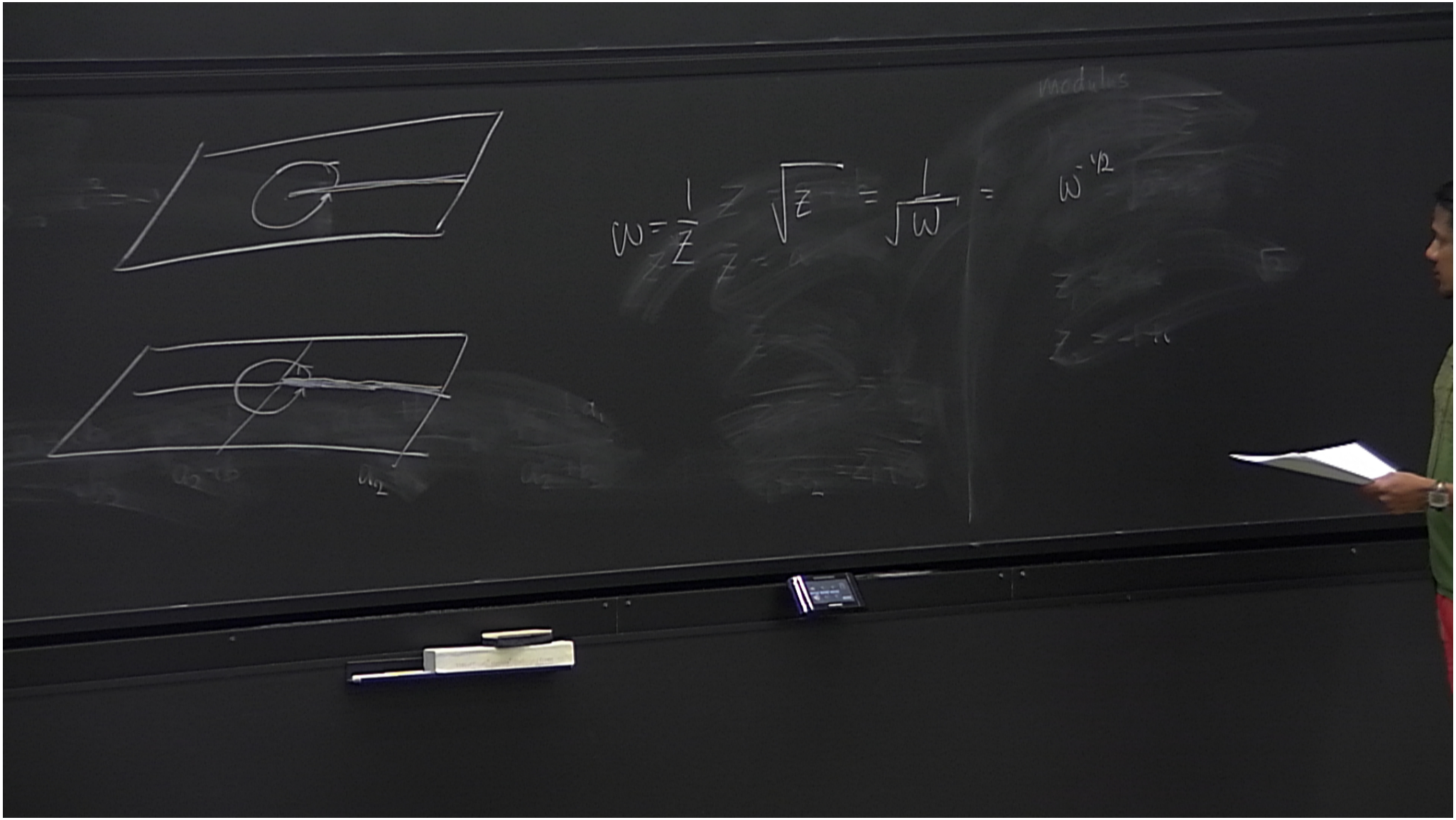

Re z
$\sqrt{z} = \sqrt{|z|}\, e^{i\theta/2}$
$= \sqrt{|z|}\, e^{i\theta_p/2}\, e^{ik\pi}$


modules

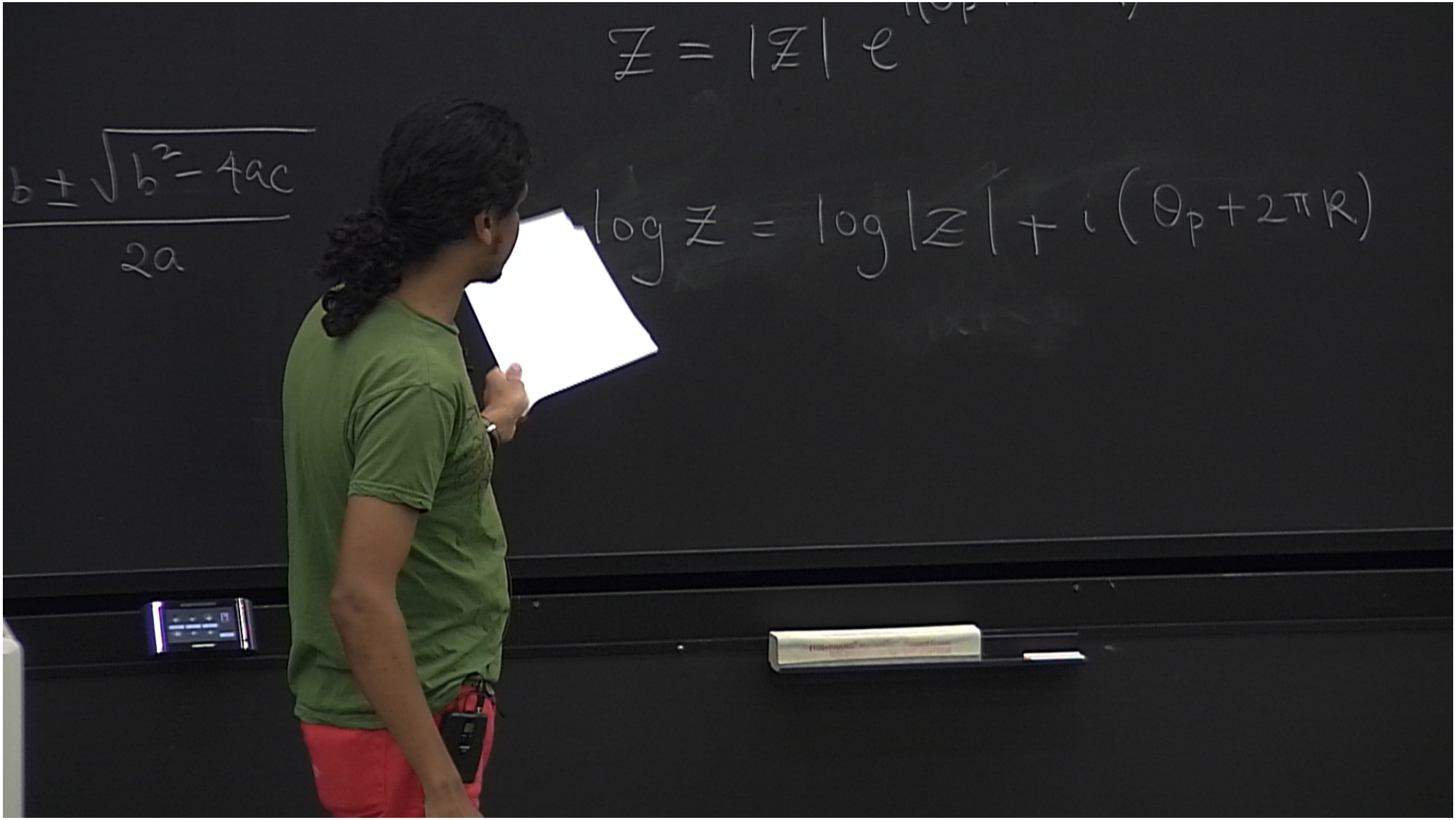

modules

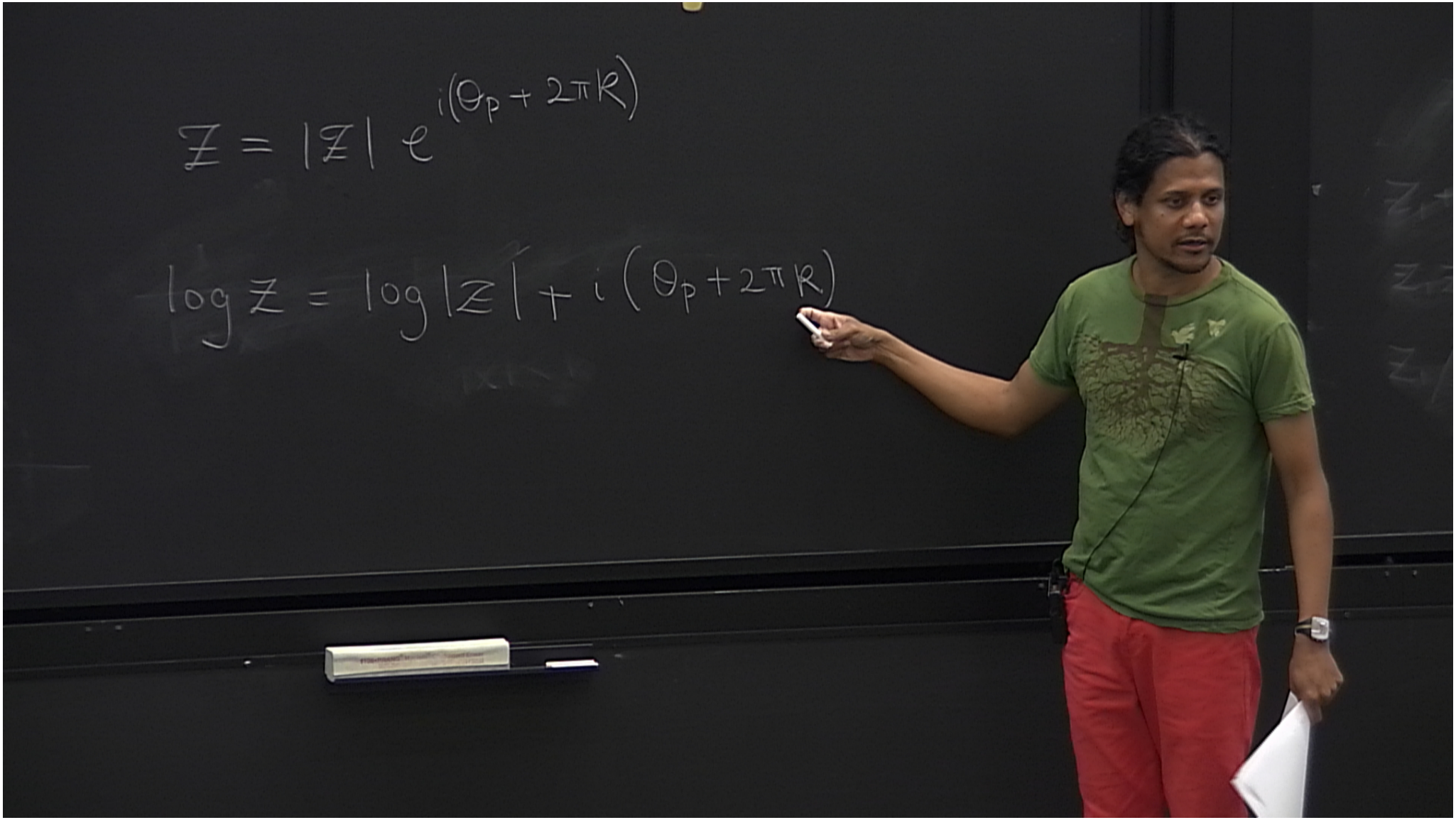


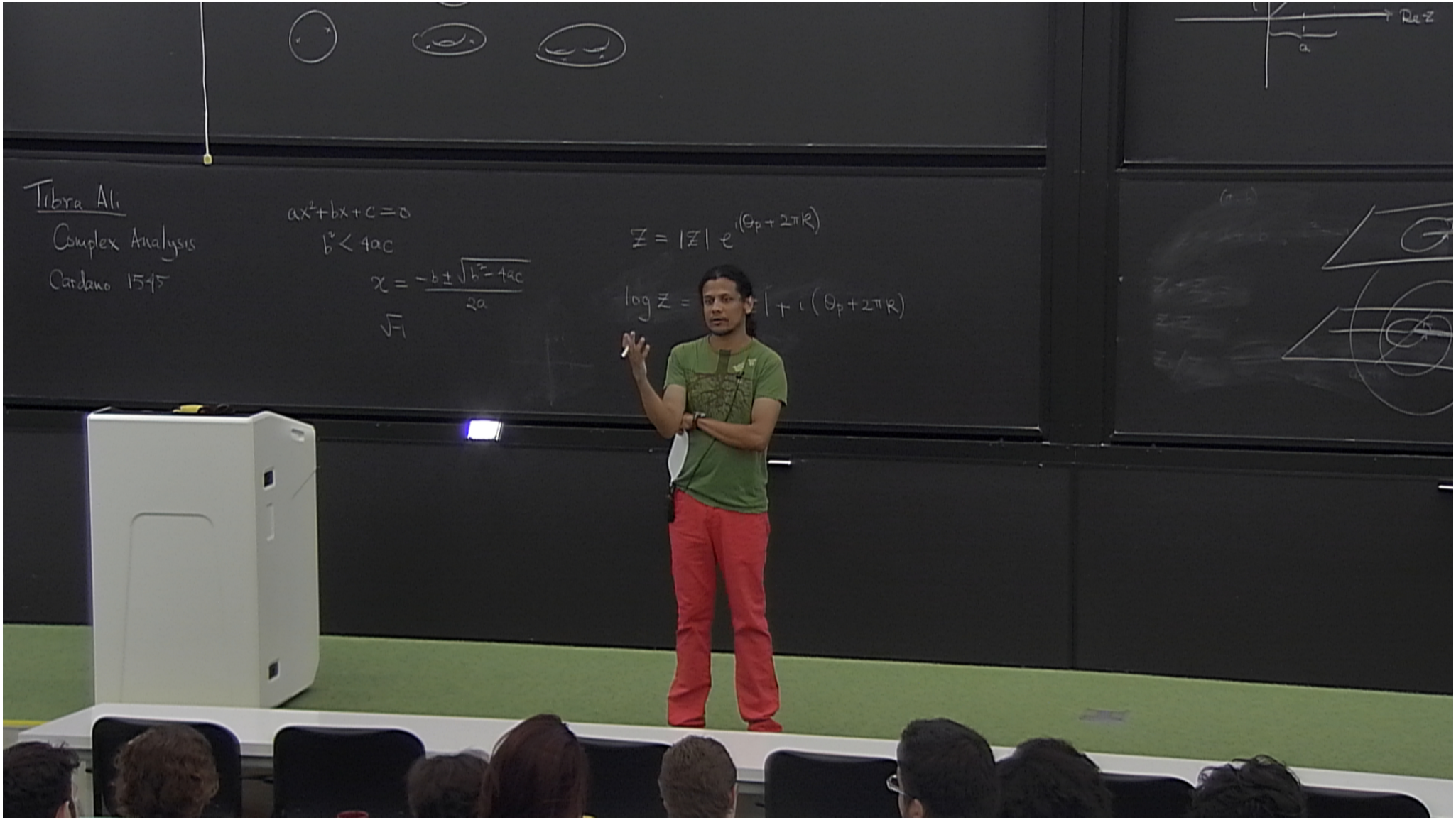

modulus

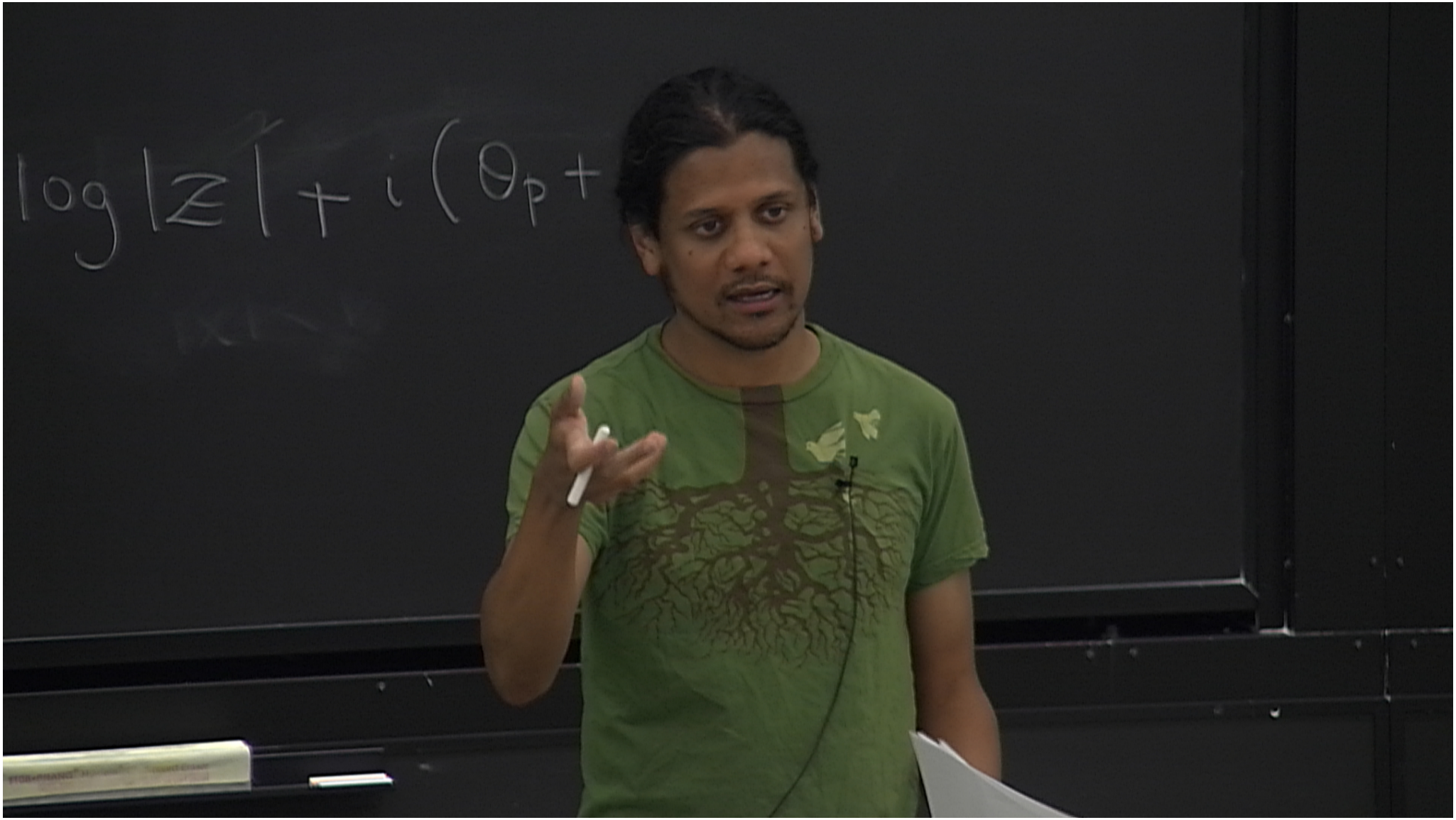

modulus
$w = \frac{1}{z}$
$\sqrt{z} = \frac{1}{\sqrt{w}} = w^{-1/2}$

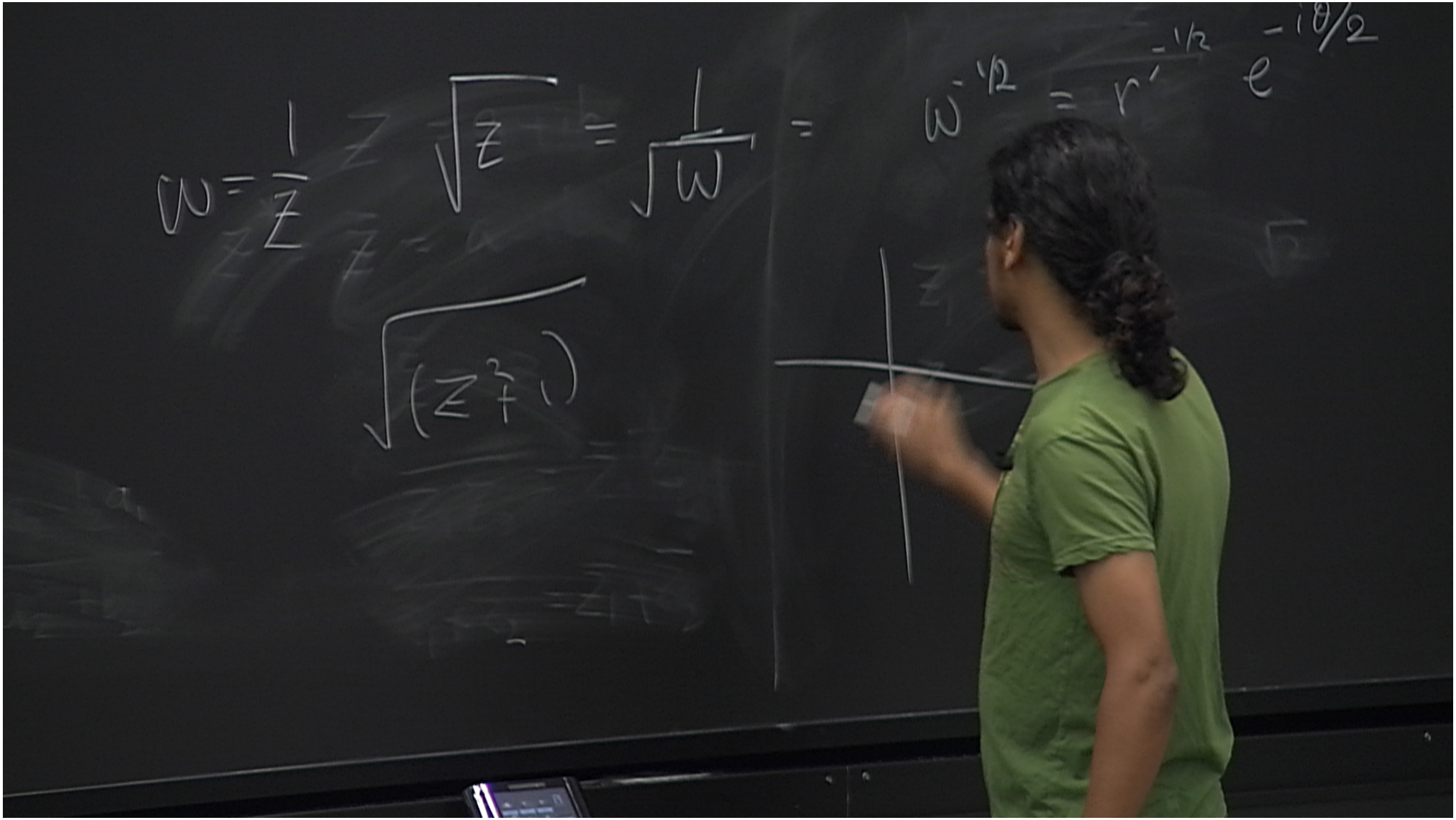

$z = |z| e^{(\theta_z + 2\pi k)}$
$\log z = \log|z| + i(\theta_z + 2\pi k)$
$w = \frac{1}{z}$
$\sqrt{z} = \frac{1}{\sqrt{w}} = w^{-1/2}$
modulus

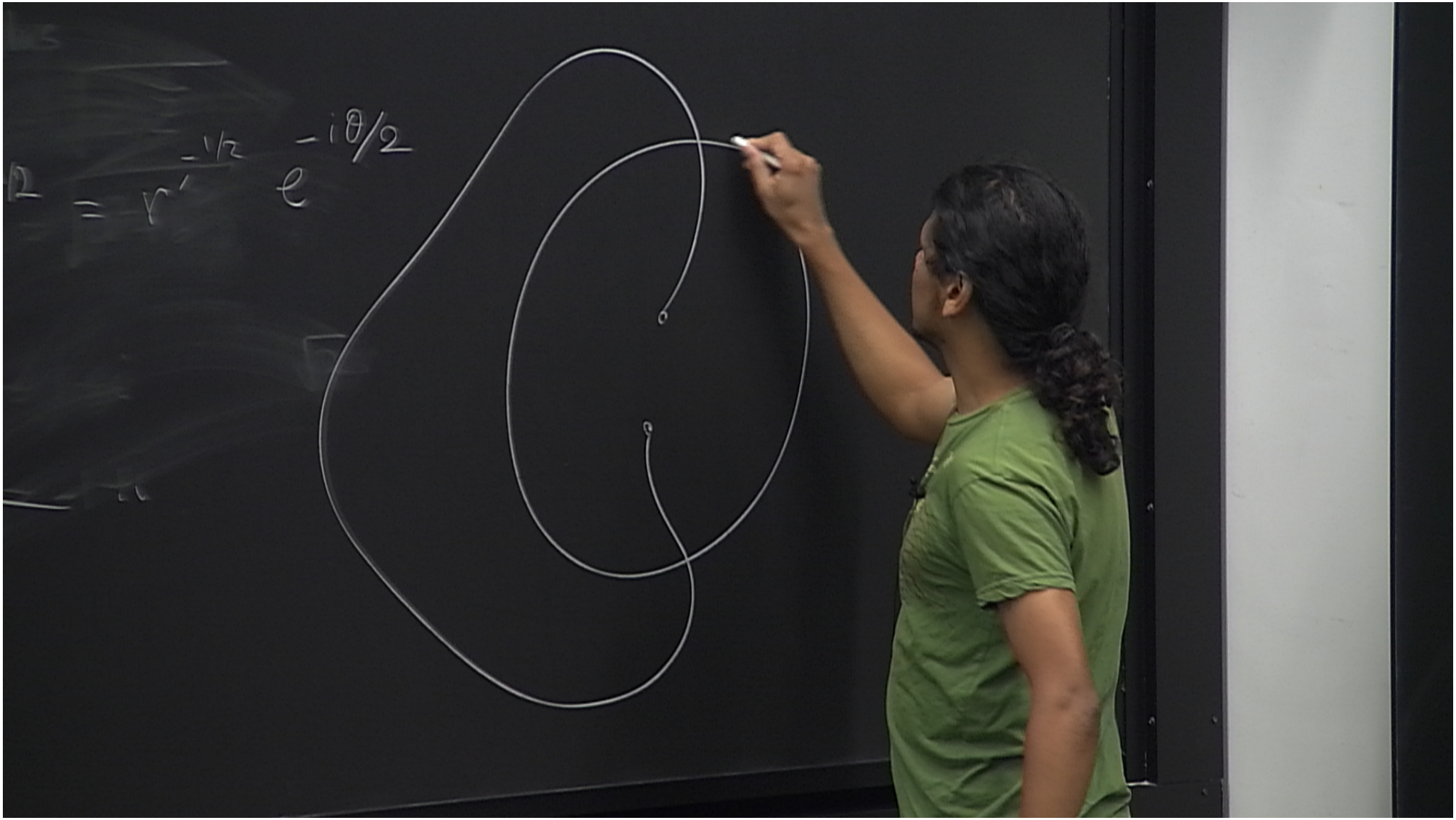

$z = |z| e^{i(\theta_p + 2\pi R)}$
$\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$
$\log z = \log|z| + i(\theta_p + 2\pi R)$

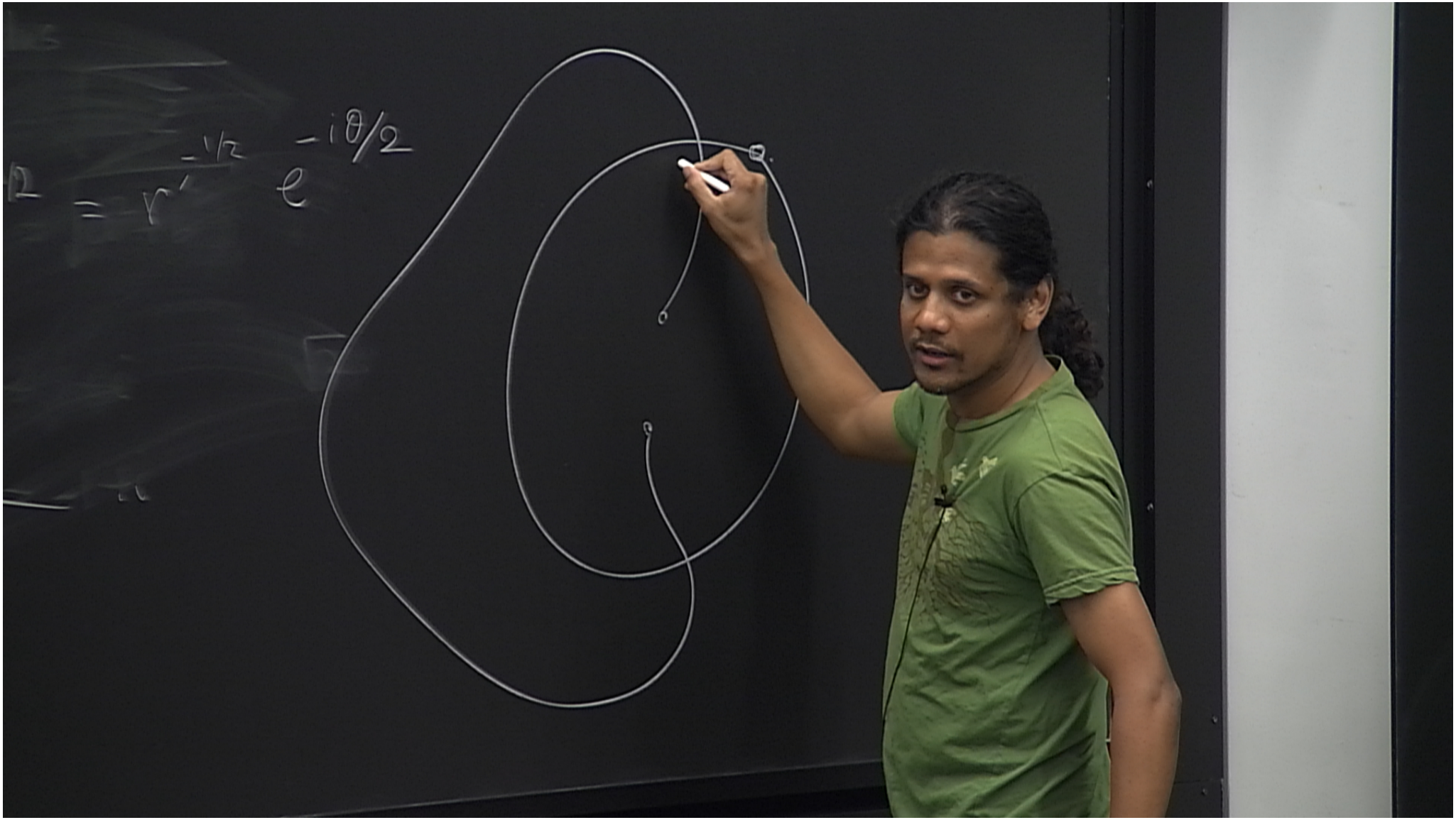

$Z = |Z| e^{i(\theta_p + 2\pi R)}$
$\log Z = \log|Z| + i(\theta_p + 2\pi R)$

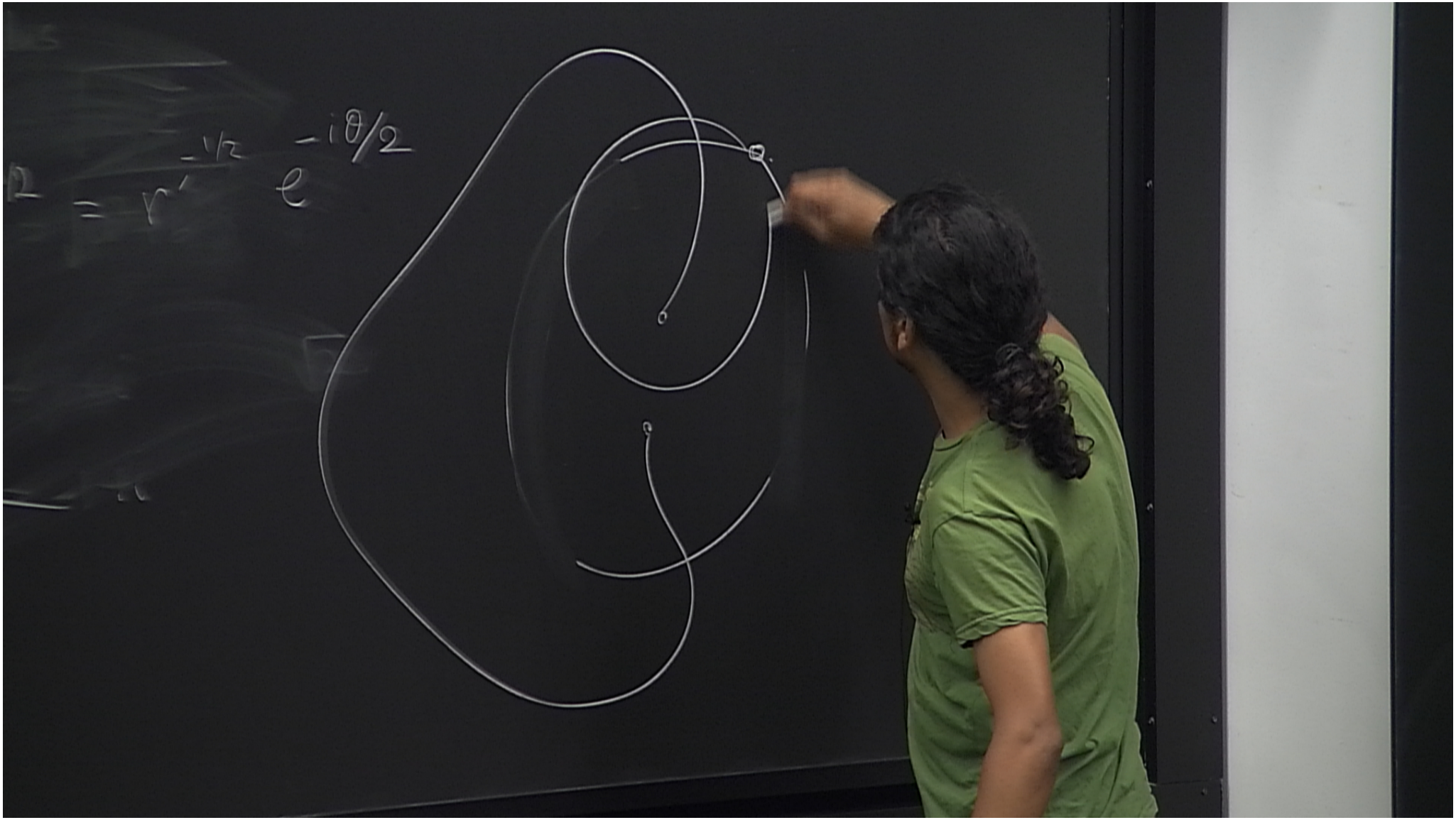

Tibra Ali
Complex Analysis
Cardano 1545
$ax^2+bx+c=0$
$b^2<4ac$
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$\sqrt{-1}$
$z=|z|e^{i(\theta_p+2\pi R)}$
$\log z=$
$+i(\theta_p+2\pi R)$
Re z


log|z| + i(θp +


w = 1/z
√z = 1/√w
w^{-1/2} = r^{-1/2} e^{-iθ/2}
√(z²+1)


$= r^{-1/2} e^{-i\theta/2}$


$= r^{-1/2} e^{-i\theta/2}$


$= r^{-1/2} e^{-i\theta/2}$