Title: Neutron Stars and the Dense Matter Equation of State

Date: Aug 08, 2013 01:00 PM

URL: http://pirsa.org/13080000

Abstract: Implications of
recently well-measured neutron star masses, particularly near and above 2 solar masses, for the equation of state (EOS) of neutron star matter will be highlighted. Model independent upper
limits to thermodynamic properties in neutron stars, which only depend on the neutron star maximum mass, established from causality considerations will be presented. The need for non-perturbative treatments of quark matter in neutron stars is revealed through studies of self-bound quark matter stars, and of nucleon-quark hybrid stars. The extent to which several well-measured masses and radii of individual neutron stars can establish a model-independent EOS through an inversion of the stellar structure equations will be briefly discussed.

Observable Physical Properties governed by M & R

Examples

- Binding energy of a neutron star:
  $B.E \simeq (0.6 \pm 0.05)\,\frac{GM^2}{Rc^2}\,\left(1 - \frac{GM}{Rc^2}\right)^{-1}$;
  Nearly 99% this B.E. is carried by neutrinos emitted during the birth of a neutron star.
- (Limiting) Frequency of rotation:
  $\Omega_K = 7.7 \times 10^3\,\left(\frac{M_{max}}{\mathrm{M}_\odot}\right)^{1/2}\left(\frac{R_{max}}{10\ \mathrm{km}}\right)^{3/2}\ s^{-1}$
  Not yet known if this "allowed" limiting frequency has been attained.
- Moments of Inertia:
  $I_{max} = 0.6 \times 10^{45}\,\frac{(M_{max}/M_\odot)(R_{max}/10\ \mathrm{km})^2}{1-0.295(M_{max}/M_\odot)/(R_{max}/10\ \mathrm{km})}\ \mathrm{g\ cm}^2$
  Accurate pulse timing techniques being developed to measure I in a double neutron star binary.
- ...
- See Phys. Rep. 442 (2007) 109 for more examples.

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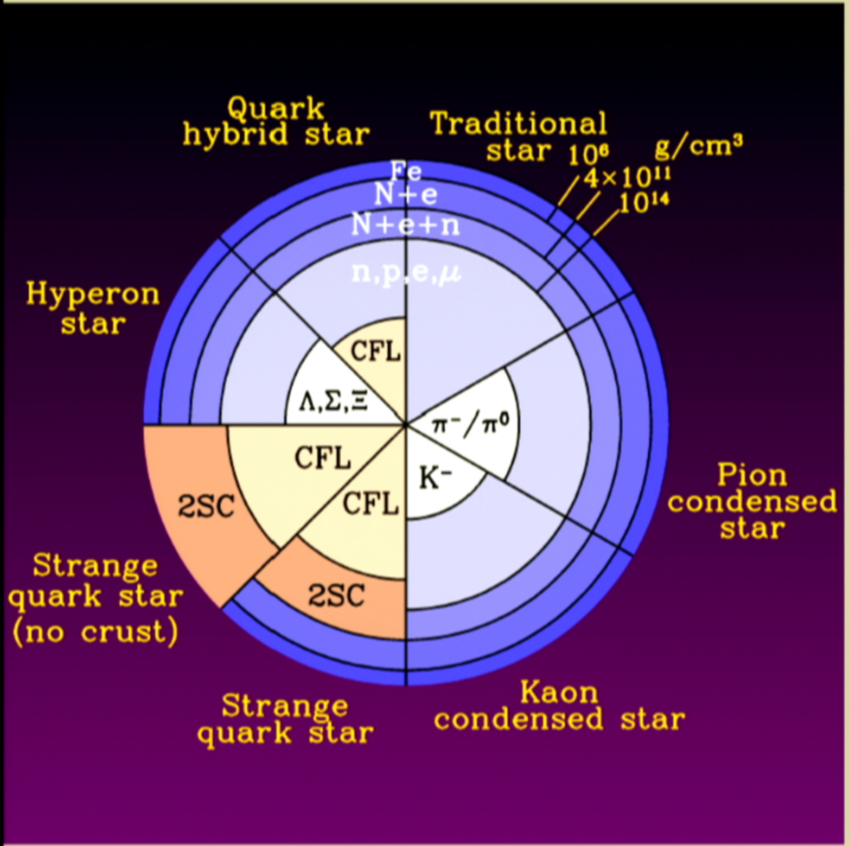
The American Pie



Courtesy A. W. Steiner & Prakash. Many exotic scenarios for interior composition, but none confirmed!

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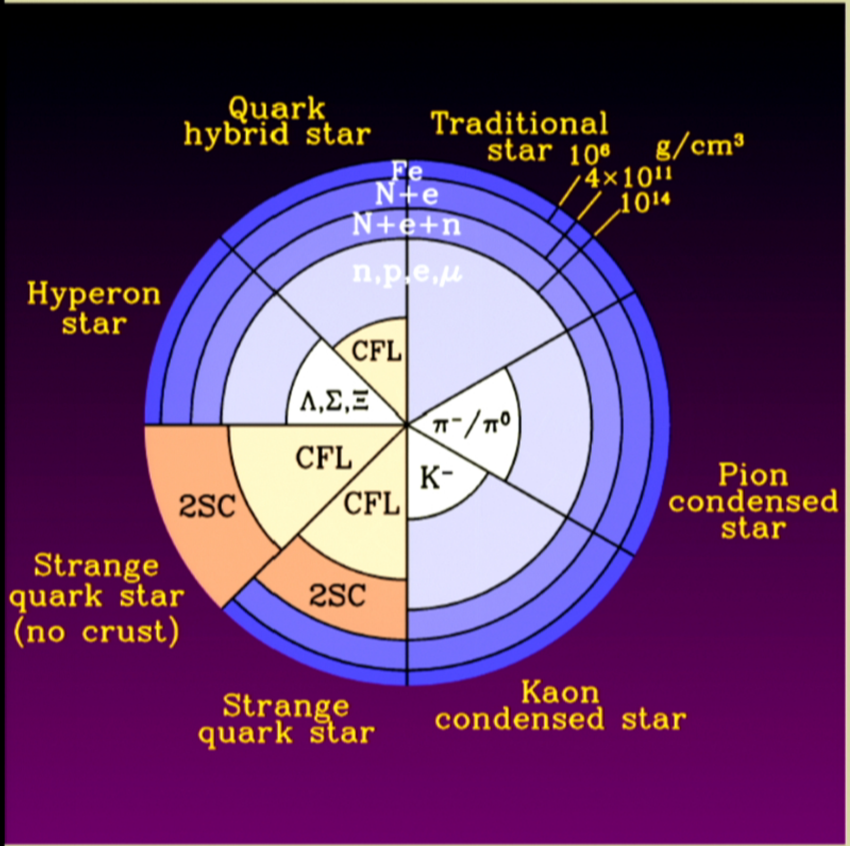

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Composition of Dense Stellar Matter

- Crustal Surface :
  electrons, nuclei, dripped neutrons, $\cdots$ set in a lattice
  new phases with lasagna, sphagetti, $\cdots$ like structures
- Liquid (Solid?) Core :
  $n, p, \Delta, \cdots$ leptons: $e^{\pm}, \mu^{\pm}, \nu_e's, \nu_\mu's$
  $\Lambda, \Sigma, \Xi, \cdots$
  $K^-, \pi^-, \cdots$ condensates
  $u, d, s, \cdots$ quarks
- Constraints :
  1. $n_b = n_n + n_p + n_\Lambda + \cdots$: baryon # conservation
  2. $n_p + n_{\Sigma^+} + \cdots = n_e + n_\mu$: charge neutrality
  3. $\mu_i = b_i \mu_n - q_i \mu_\ell$: energy conservation

$$\Rightarrow$$

$$\mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_n \quad \mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_e \quad \mu_p = \mu_{\Sigma^+} = \mu_n - \mu_e$$

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$$\mu_{K^-} = \mu_e = \mu_\mu = \mu_n - \mu_p$$

$$\Rightarrow$$

$$\mu_d = \mu_u + \mu_e = \mu_s = (\mu_n + \mu_e)/3$$
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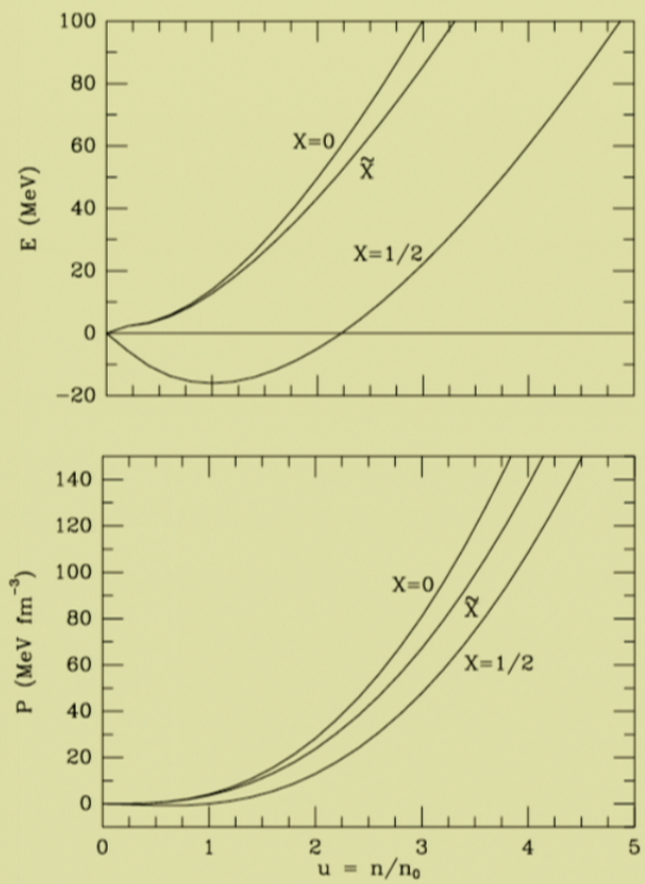
Nucleonic Equation of State



- Energy (E) & Pressure (P) vs. scaled density ($u = n/n_0$).
- Nuclear matter equilibrium density $n_0 = 0.16 \text{ fm}^{-3}$.
- Proton fraction $x = n_p/(n_p + n_n)$.
- Nuclear matter : $x = 1/2$.
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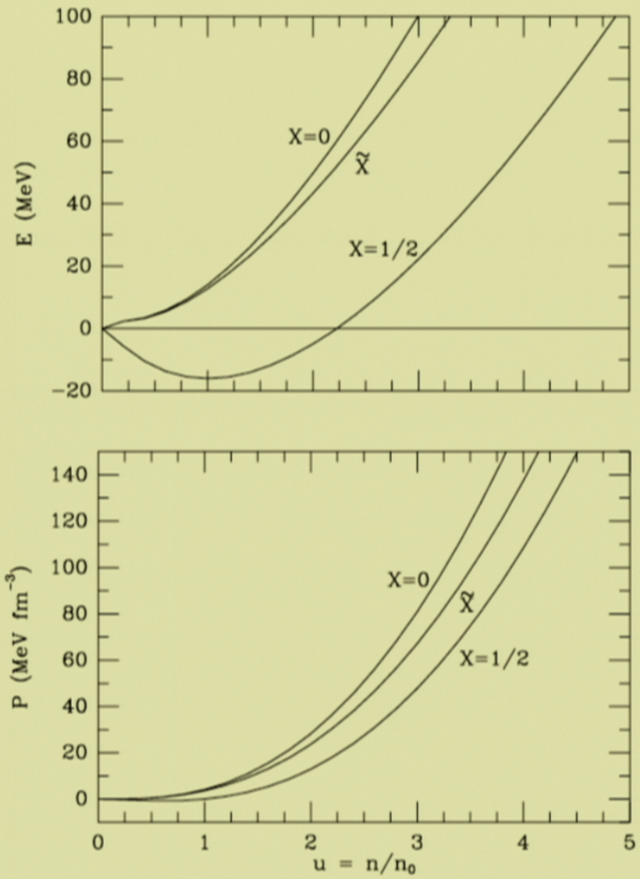

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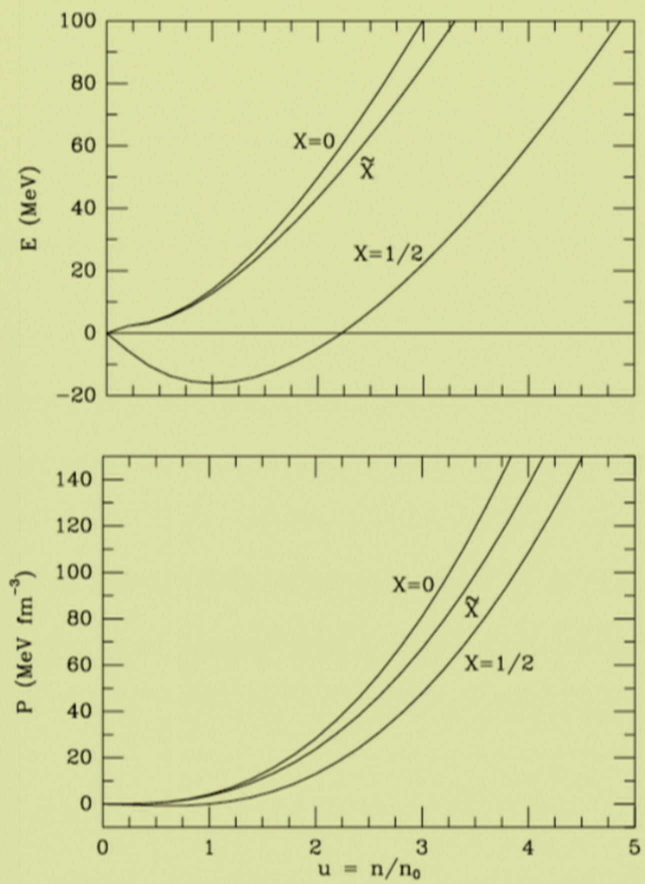

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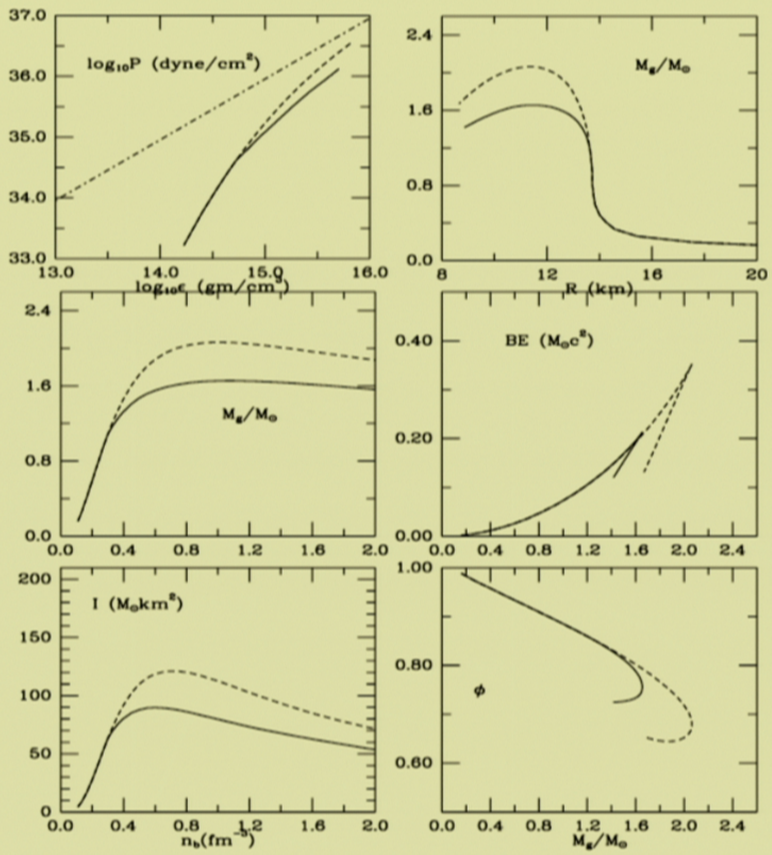
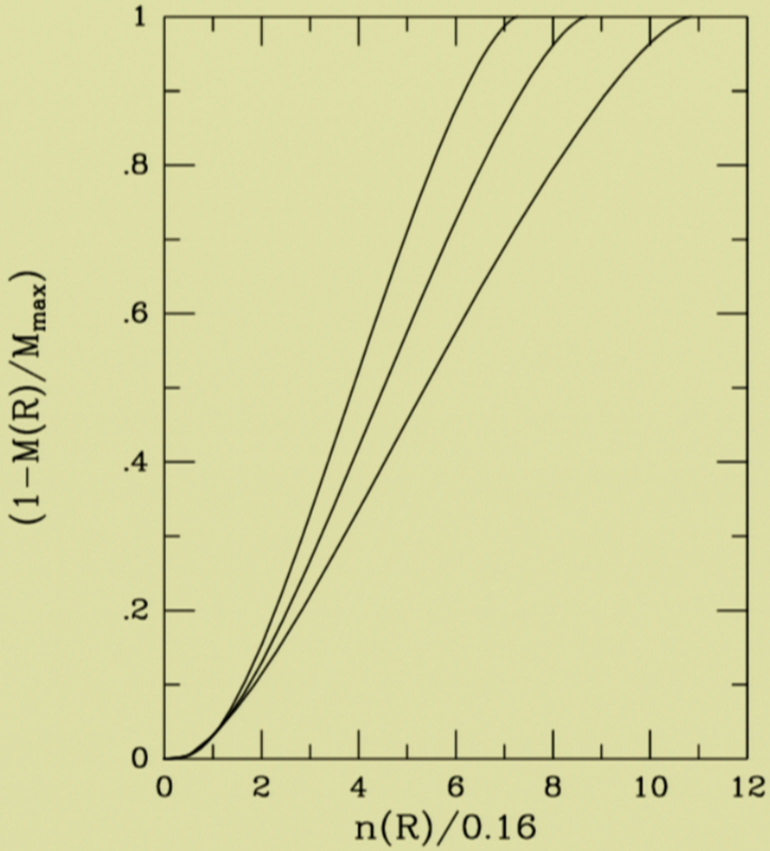
Results of Star Structure



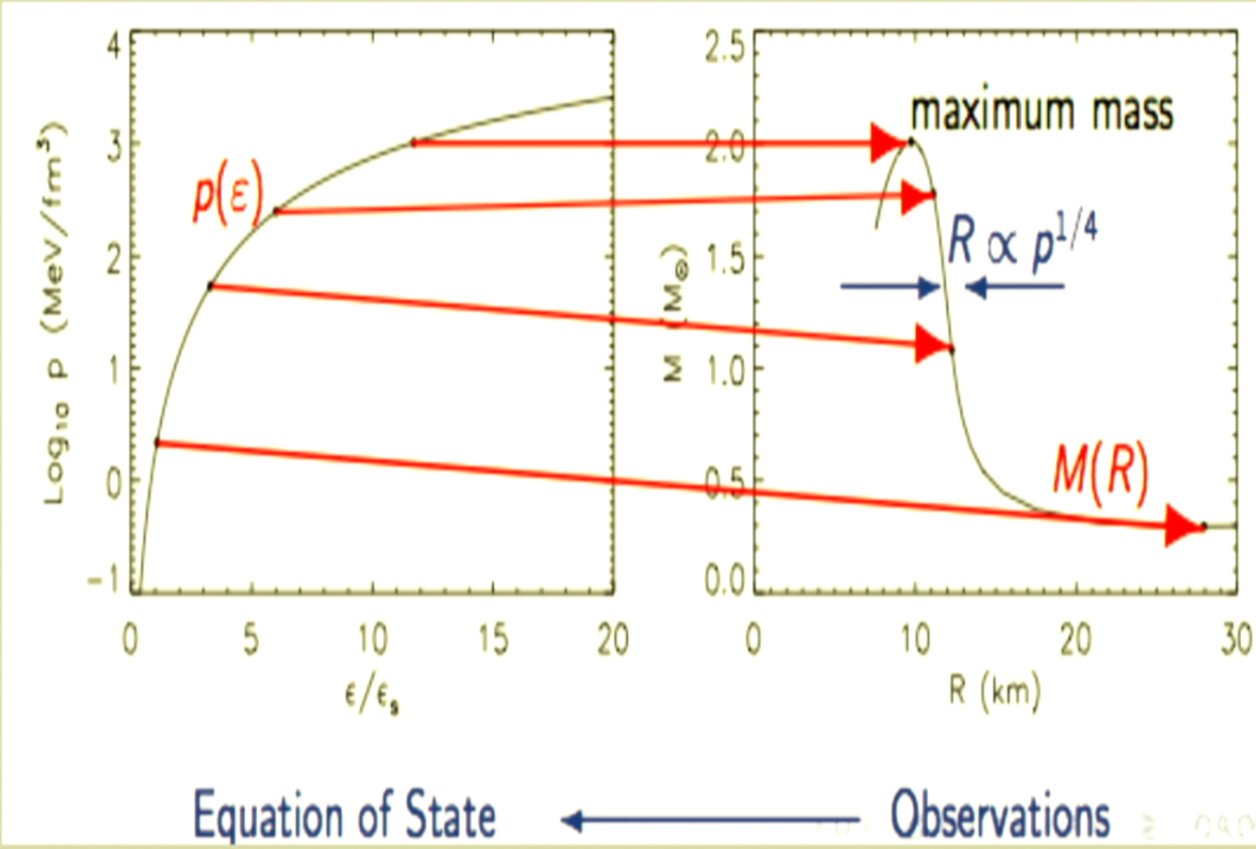
- Stellar properties for soft & stiff (by comparison) EOS's.
- Causal limit : $P = \epsilon$.
- M_g : Gravitational mass
- R : Radius
- BE : Binding energy
- n_b : Central density
- I : Moment of inertia
- ϕ : Surface red shift , $e^{\phi/c^2} = (1 - 2GM/c^2R)^{-1/2}$.

Growth of Mass with Density



- $\frac{M_{max}}{M_\odot}$ & $\frac{n_c}{n_0}$
- Left: 1.94 & 7.27
- Middle: 1.72 & 8.68
- Right: 1.46 & 10.84
- PAL: PRL 61 (1988) 716

Growth of Mass with Radius



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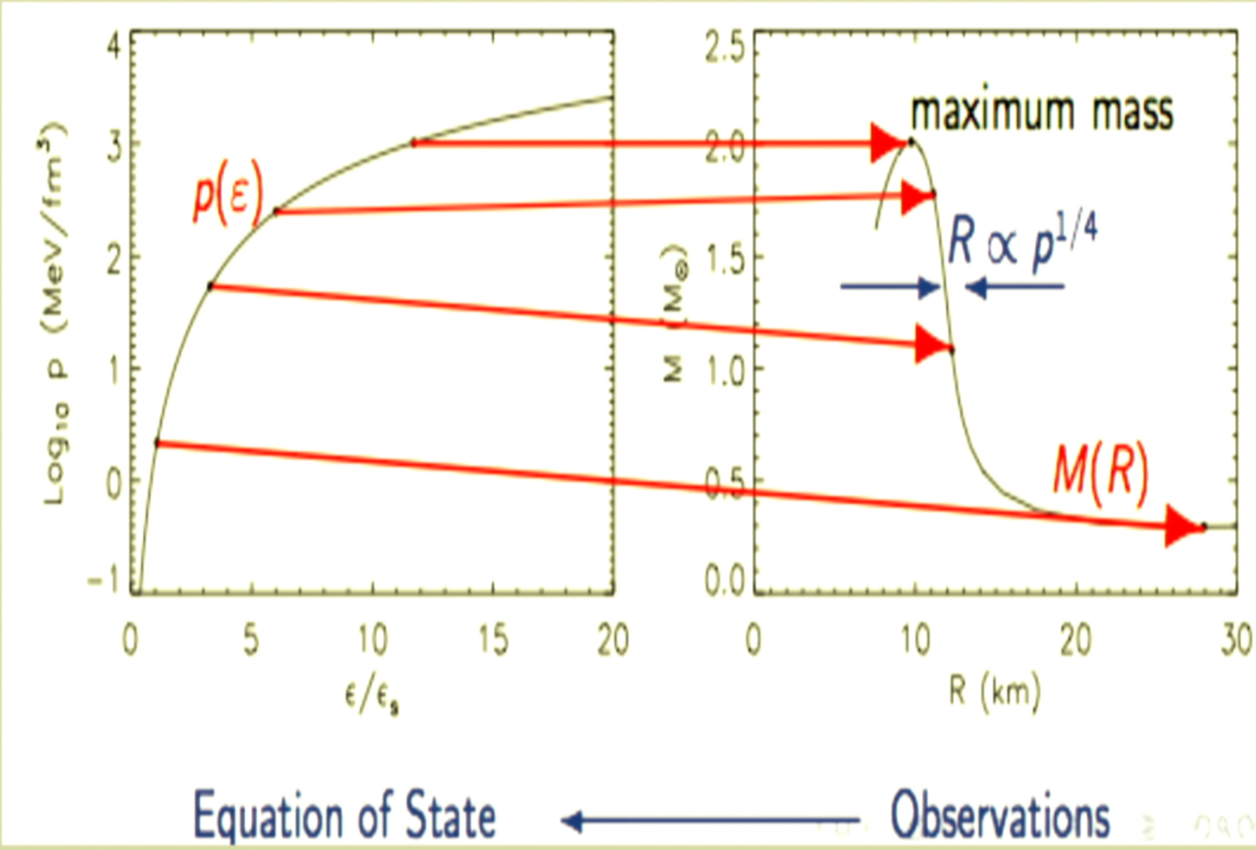
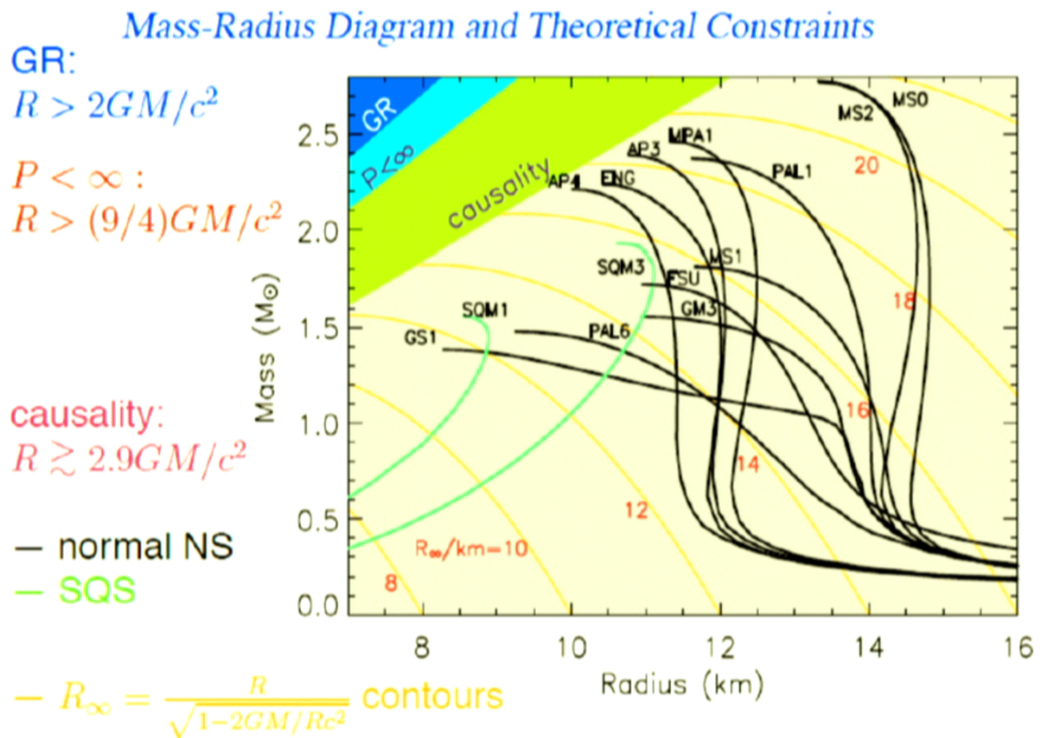


M. Prakash The Equation of State of Neutron Star Matter

Model Predictions



Lattimer & Prakash, Phys. Rep. **442**, 109 (2007).

Model Predictions

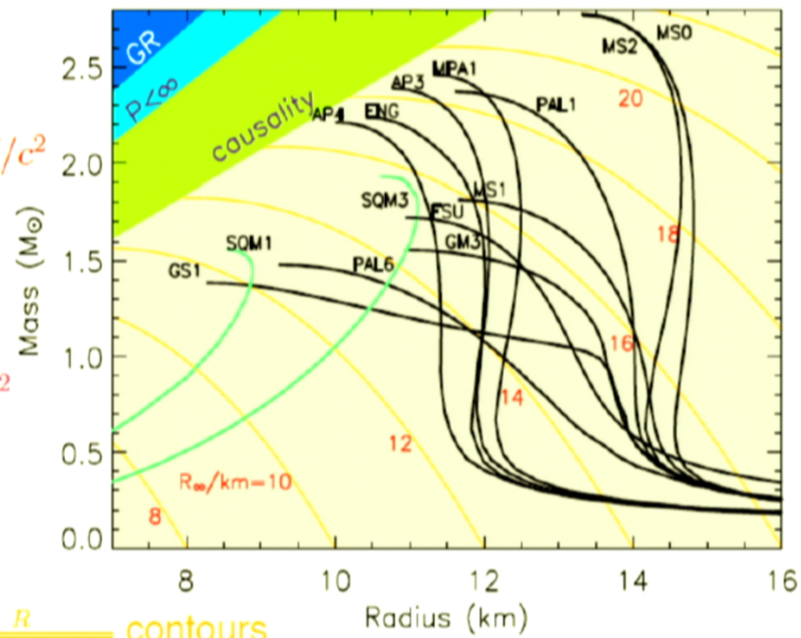
Mass-Radius Diagram and Theoretical Constraints

GR:
$R > 2GM/c^2$

$P < \infty$:
$R > (9/4)GM/c^2$

causality:
$R \gtrsim 2.9GM/c^2$

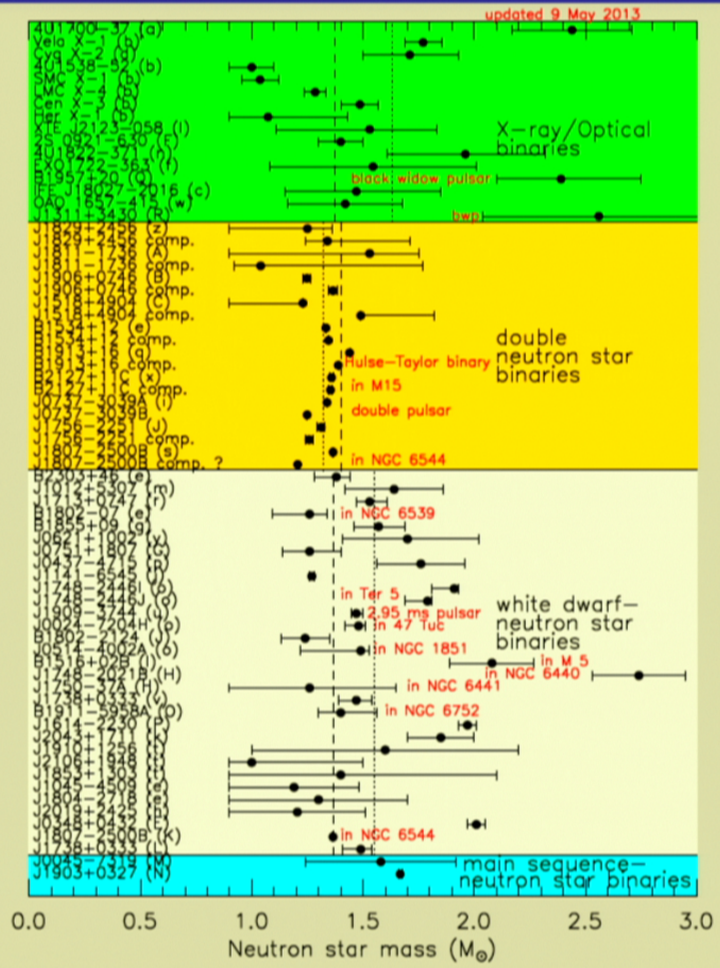
— normal NS
— SQS



— $R_\infty = \frac{R}{\sqrt{1-2GM/Rc^2}}$ contours

Lattimer & Prakash, Phys. Rep. **442**, 109 (2007).

Measured Neutron Star Masses



- Mean & error weighted mean in $M_\odot$
- X-ray binaries: 1.609 & 1.393
- Double NS binaries: 1.325 & 1.403
- WD & NS binaries: 1.599 & 1.362
- Lattimer & Prakash, PRL, **94** (2005) 111101; Updated regularly by Lattimer in http://www.stellarcollapse.org

Measurement of Neutron Star Masses - I

- Accurate measurements are for pulsars in bound binary systems
- 5 Keplerian orbital parameters can be precisely measured by pulse-timing techniques:
  1. the binary period, P,
  2. the projection of the pulsars's semimajor axis on the line of sight $a_p \sin i$ (i: the binary's inclination angle),
  3. the orbit's eccentricity e,
  4. the time of periastron T_0, and
  5. the longitude of periastron, ω.

The Mass Function f_p (combines two observational parameters) :

$$f_p = \left(\frac{2\pi}{P}\right)^2 \left(\frac{a_p \sin i}{c}\right)^3 \frac{\mathrm{M}_\odot}{\mathrm{T}_\odot} = \frac{(M_c \sin i)^3}{M^2}\mathrm{M}_\odot$$

$M = M_p + M_c$; M_p & M_c are pulsar & companion masses in $\mathrm{M}_\odot$
$\mathrm{T}_\odot = \mathrm{GM}_\odot/\mathrm{c}^3 = 4.9255\ \mu$s.

Measurement of Neutron Star Masses - II

- General Relativistic Effects :

1. Advance of the Periastron of the Orbit :

$$\dot{\omega} = 3\left(\frac{2\pi}{P}\right)^{5/3}(M\mathrm{T}_\odot)^{2/3}(1-e^2)^{-1}$$

2. Combined effect of variations in the transverse Doppler shift & gravitational redshift :

$$\gamma = e\left(\frac{P}{2\pi}\right)^{1/3}\frac{M_c(M+M_c)}{M^{4/3}}\,\mathrm{T}_\odot^{2/3}$$

3. Orbital period decay due to the emission of gravitational radiation :

$$\dot{P} = -\frac{192\pi}{5}\left(\frac{2\pi\mathrm{T}_\odot}{P}\right)^{5/3}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)(1-e^2)^{-7/2}\frac{M_pM_c}{M^{1/3}}$$

Measurement of Neutron Star Masses - III

- Shapiro Delays : General relativistic delay in pulse arrival time

$$\delta_S(\phi) = 2M_c\mathrm{T}_\odot \ln\left[\frac{1 + e\cos\phi}{1 - \sin(\omega + \phi)\sin i}\right]$$

ϕ : True anomaly, the angular parameter defining the position of the pulsar in its orbit relative to the periastron.

- Amplitude of Shapiro Delays :

$$\Delta_S \simeq 2M_C\mathrm{T}_\odot \left|\ln\left[\left(\frac{1 + e\sin\omega}{1 - e\sin\omega}\right)\left(\frac{1 + \sin i}{1 - \sin i}\right)\right]\right|$$

- For nearly circular orbits & almost edge on configurations

$$\Delta_S \simeq 4M_C\mathrm{T}_\odot \ln(2/\cos i)$$

Lattimer & Prakash (2010)

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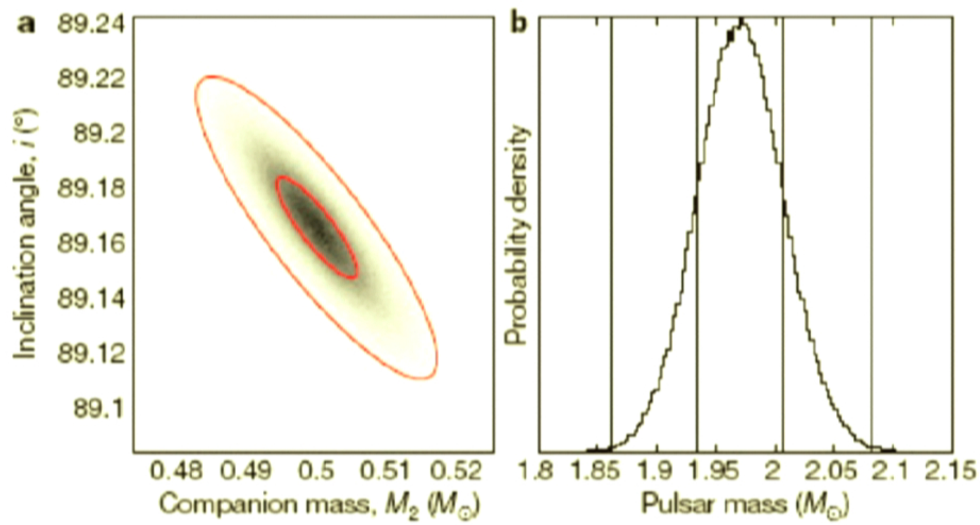
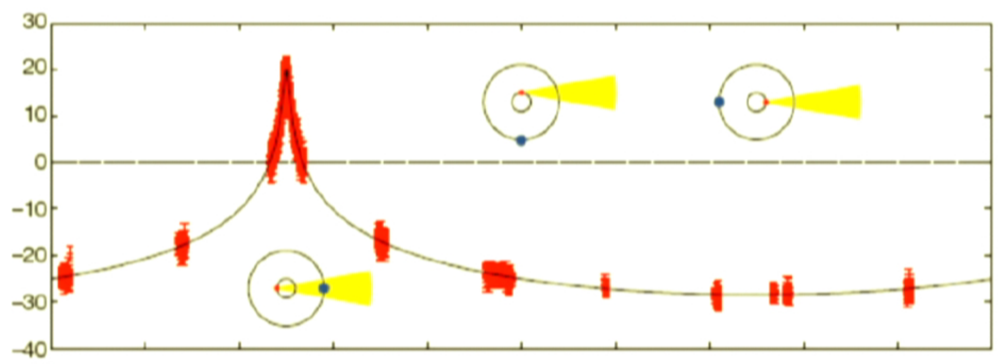
A 2-solar-mass neutron star



A 3.15 ms pulsar (PSR J1614-2230) in an 8.69 d orbit with a 0.5 $\mathrm{M}_\odot$ white dwarf companion. Shapiro delay yields edge-on inclination: $\sin i = 0.99984$. Distance > 1 kpc & $B \simeq 10^8$ G.
Demorest et al. , Nature, **467**, 1081 (2010).

Shapiro Delay Signal in PSR J1614-2230



Functional form of the Shapiro delay when all other model parameters are fixed at their best-fit values.

y-axis : Timing Dealay (μ s) ; x-axis : Orbital Phase [0-1(0.1)]

Demorest et al., Nature **461**, 1081 (2010).

What Controls Shapiro Delays

Pulsar	M_c	i	Full	Theory
J0437-4715	0.254	42.42	4.08	4.1
B1855+09	0.258	86.7	17.94	18.03
J1713+0747	0.28	72.0	10.11	10.19
J1640+2224	0.15	84	8.67	8.71
J0737-3039A	1.2489	88.7	109.64	110.21
J1903+0327	1.029	77.47	44.56	44.79
J1909-3744	0.2038	86.58	14.03	14.1
J1614-2230	0.500	89.17	48.29	48.54
J1802-2124	0.780	80	37.25	37.44

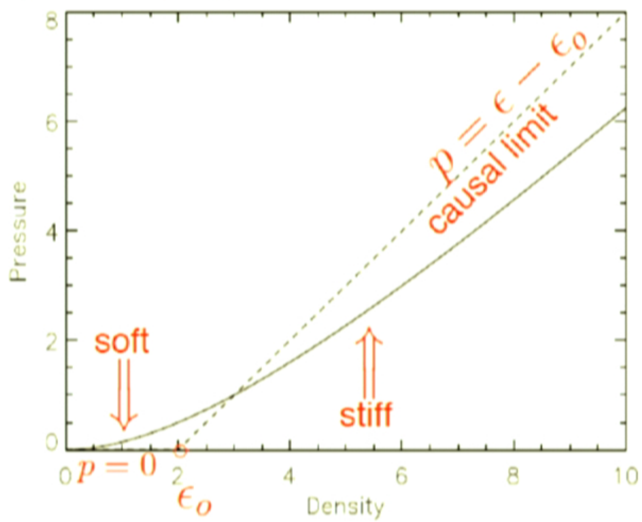
M_c : Companion mass in $\mathrm{M}_\odot$.
i : Approximate inclination of the source in degrees
Full : Peak-of-cusp to bottom-of-delay Shapiro signal amplitue in μs
Theory (LP10) : $\Delta_s \simeq 4 M_c T_\odot \ln(2/\cos(i))$ μs ; $T_\odot = 4.9255$ μs

Table courtesy of Scott Ransom & Paul Demorest (NRAO)

Maximally Compact Neutron Stars -I

Extreme Properties of Neutron Stars

- The most compact and massive configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff".

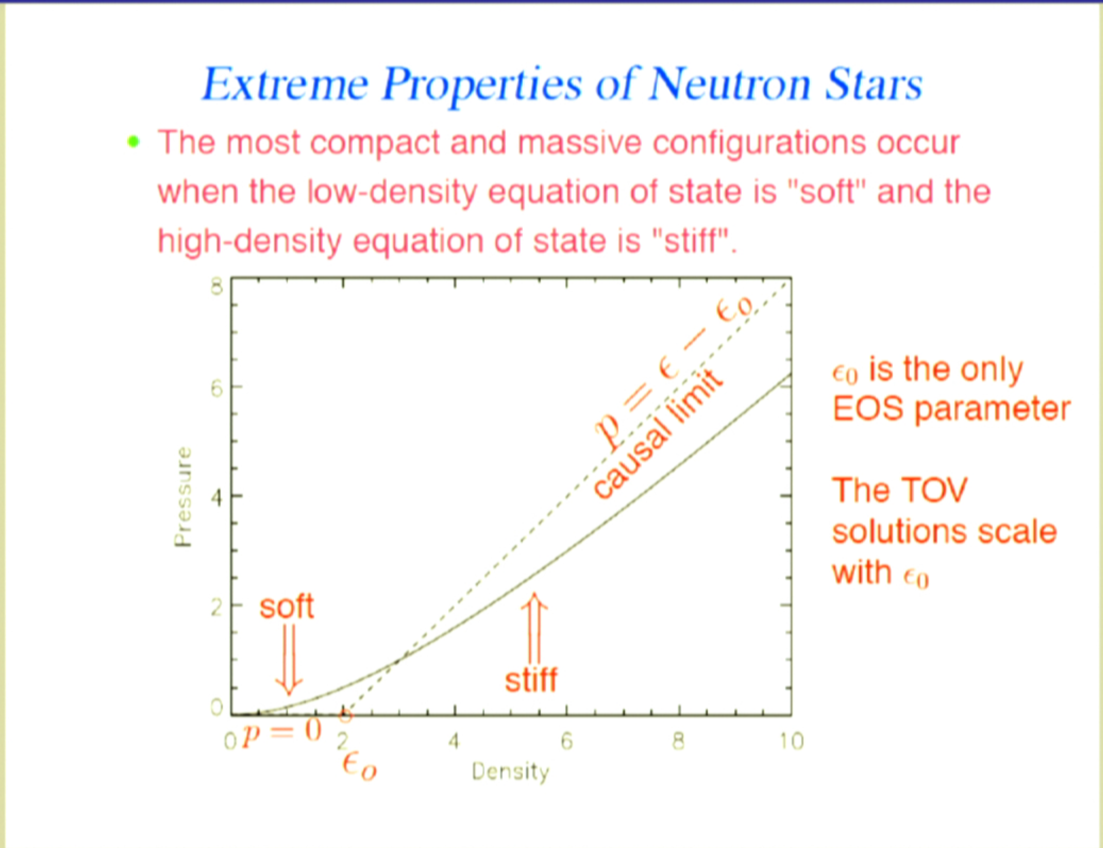


ϵ_0 is the only EOS parameter

The TOV solutions scale with ϵ_0

See, e.g., Lattimer, Prakash, Masak & Yahil, ApJ. **355**, 241 (1990); Lattimer & Prakash, GEB's Festschrift (2011)

Maximally Compact Neutron Stars -I



See, e.g., Lattimer, Prakash, Masak & Yahil, ApJ. **355**, 241 (1990); Lattimer & Prakash, GEB's Festschrift (2011)

Maximally Compact Neutron Stars -II

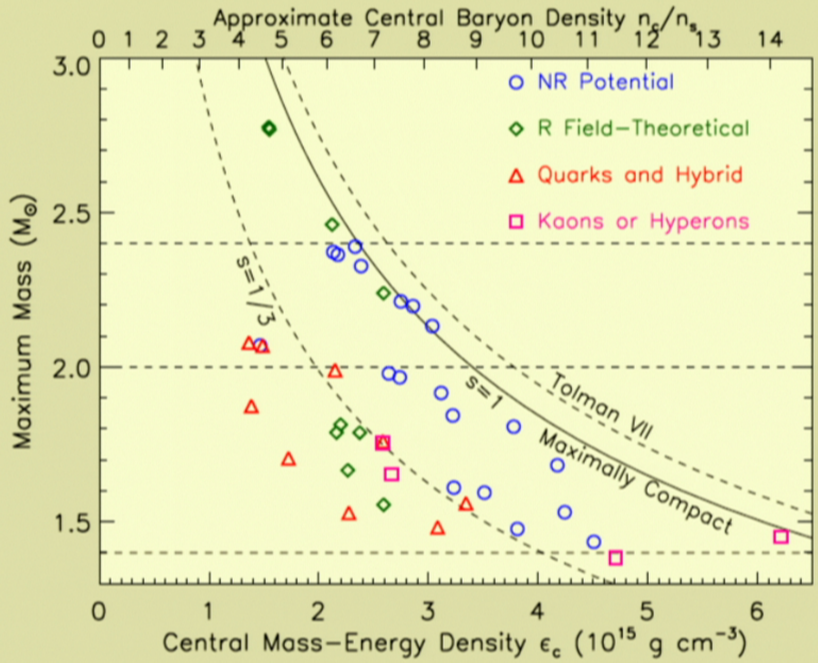
Extreme Properties of Neutron Stars

- $M_{max} = 4.1\ (\varepsilon_s/\varepsilon_0)^{1/2}\ \mathrm{M}_\odot$
- $M_{B,max} = 5.41\ (m_B c^2/\mu_o)(\varepsilon_s/\varepsilon_0)^{1/2}\ \mathrm{M}_\odot \geq 2.63\ \mathrm{M}_\odot$
- $R_{min} = 2.82\ GM/c^2 = 4.3\ (M/\mathrm{M}_\odot)\ \mathrm{km} \geq 8.5$ km
- $\mu_{B,max} = 2.09$ GeV
- $\varepsilon_{c,max} = 50\ (\mathrm{M}_\odot/M_{largest})^2\ \varepsilon_s \leq 13.1\ \varepsilon_s$
- $p_{c,max} = 33\ (\mathrm{M}_\odot/M_{largest})^2\ \varepsilon_s \leq 8.6\ \varepsilon_s$
- $n_{B,max} = 38\ (\mathrm{M}_\odot/M_{largest})^2\ n_s \leq 9.8\ n_s$
- $\mathrm{BE}_{max} = 0.34\ M$
- $P_{min} = 0.74\ (\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}\ \mathrm{ms} \geq 0.43$ ms

- ϵ_0 & μ_0: energy density & chem. potential where pressure $p = 0$
- $\epsilon_s \simeq 150\ \frac{\mathrm{MeV}}{\mathrm{fm}^{-3}}$ energy density at 0.16 fm^{-3}

See, e.g., Lattimer, Prakash, Masak & Yahil, ApJ. **355**, 241 (1990); Lattimer & Prakash, GEB's Festschrift (2011)

Ultimate Energy Density of Cold Matter



- Tolman VII: $\epsilon = \epsilon_c(1-(r/R)^2)$
- $\epsilon_c \propto (M_\odot/M)^2$
- **Crucial to establish an upper limit to** M_{max} .

Lattimer & Prakash, PRL, **94** (2005) 111101;
GEB's Festschrift (2011)

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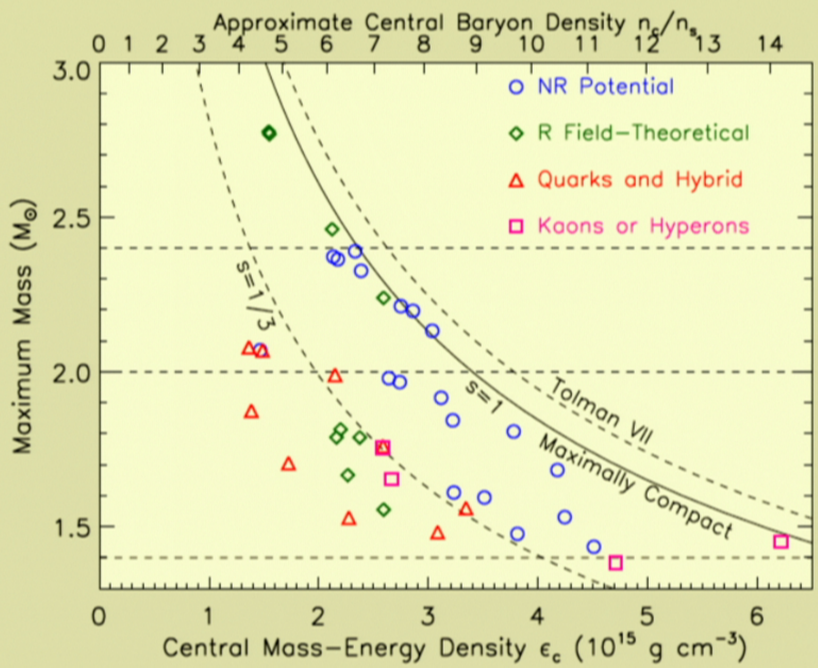

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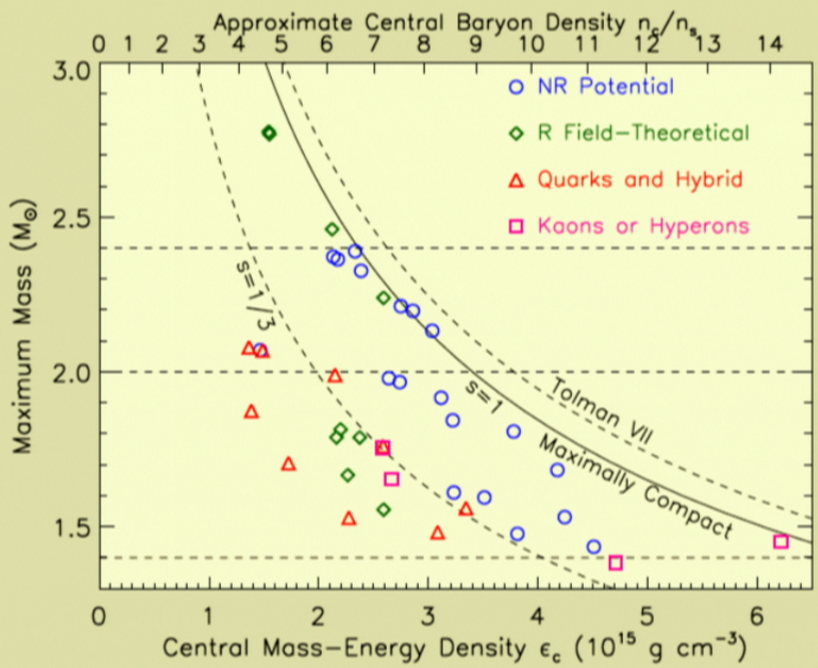
- $\epsilon_c \propto (M_\odot/M)^2$
- **Crucial to establish an upper limit to** M_{max} .

Lattimer & Prakash, PRL, **94** (2005) 111101;
GEB's Festschrift (2011)

Ultimate Energy Density of Cold Matter



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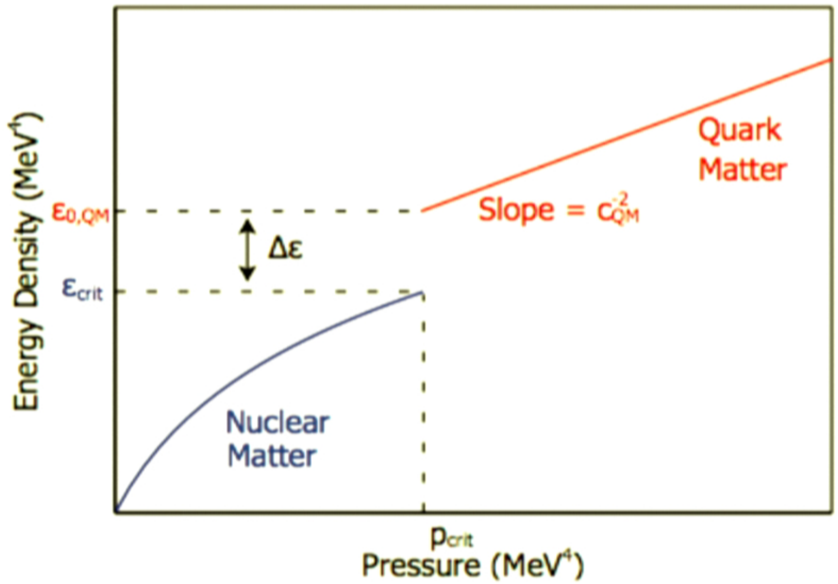
Matter with Quarks

Generic quark matter EoS

Generic ansatz: $\varepsilon(p) = \varepsilon_{\rm crit} + \Delta\varepsilon + c_{\rm QM}^{-2}(p - p_{\rm crit})$



Alford, Han, Prakash, unpublished

M. Prakash | The Equation of State of Neutron Star Matter

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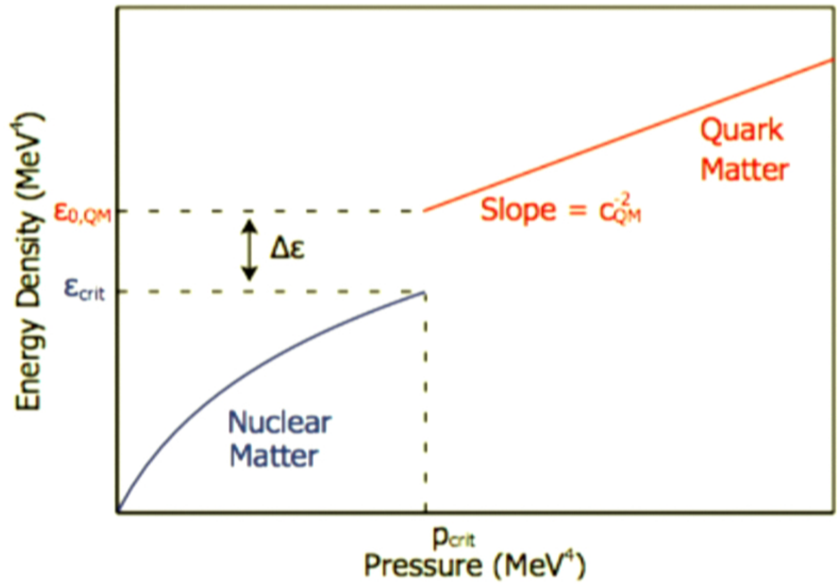

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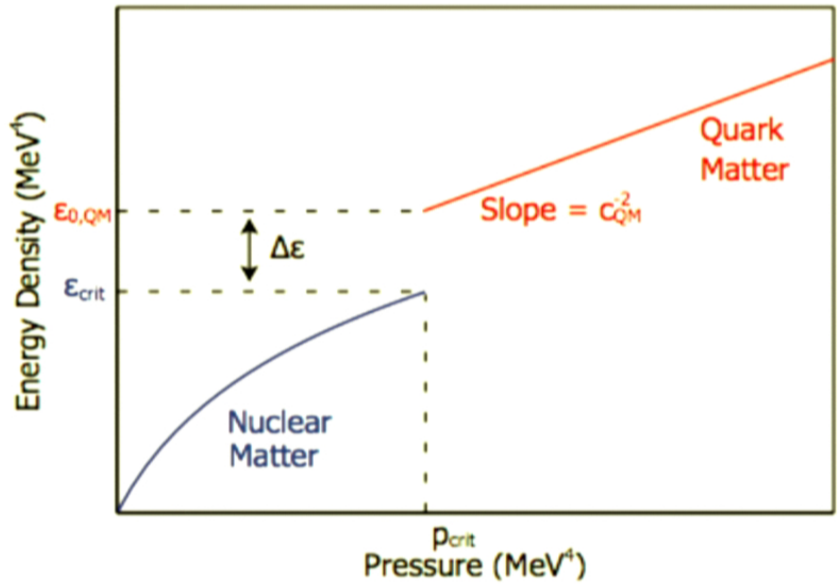


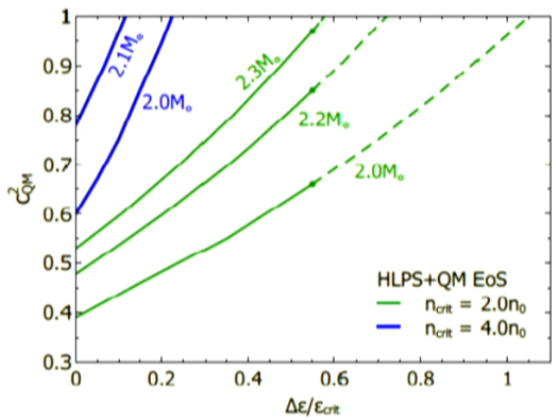
Alford, Han, Prakash, unpublished

Matter with Quarks

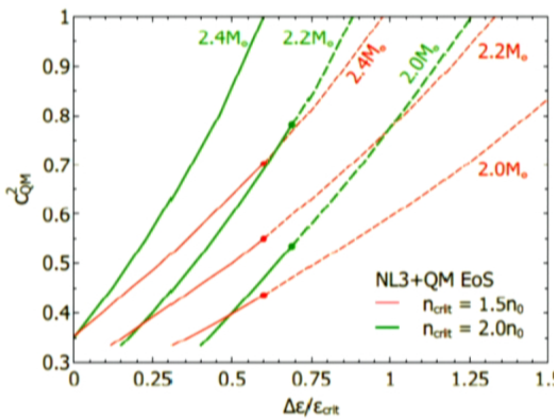
Constraints on the quark matter EoS (2)

Generic ansatz: $\varepsilon(p) = \varepsilon_{\rm crit} + \Delta\varepsilon + c_{\rm QM}^{-2}(p - p_{\rm crit})$

QM + Soft Nuclear Matter



QM + Hard Nuclear Matter



Alford, Han, Prakash, unpublished

1. Observations can constrain QM EoS but not rule out generic QM
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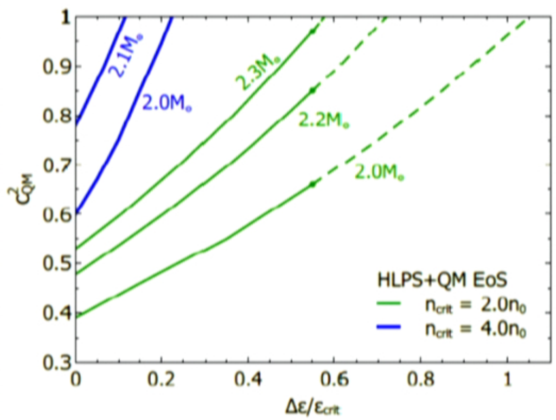

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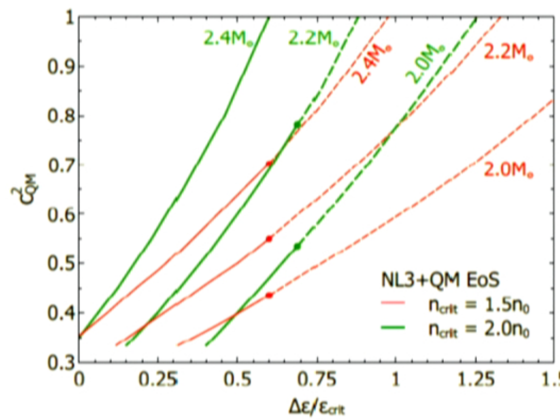

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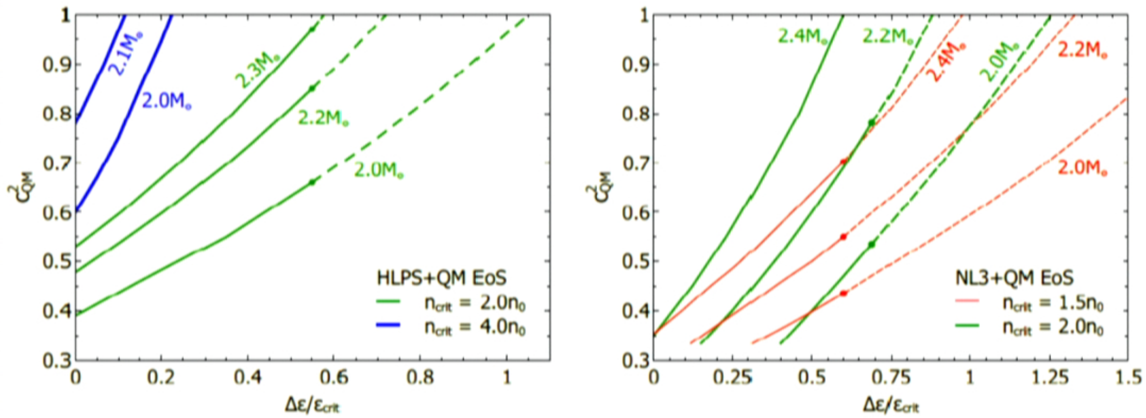
QM + Soft Nuclear Matter QM + Hard Nuclear Matter



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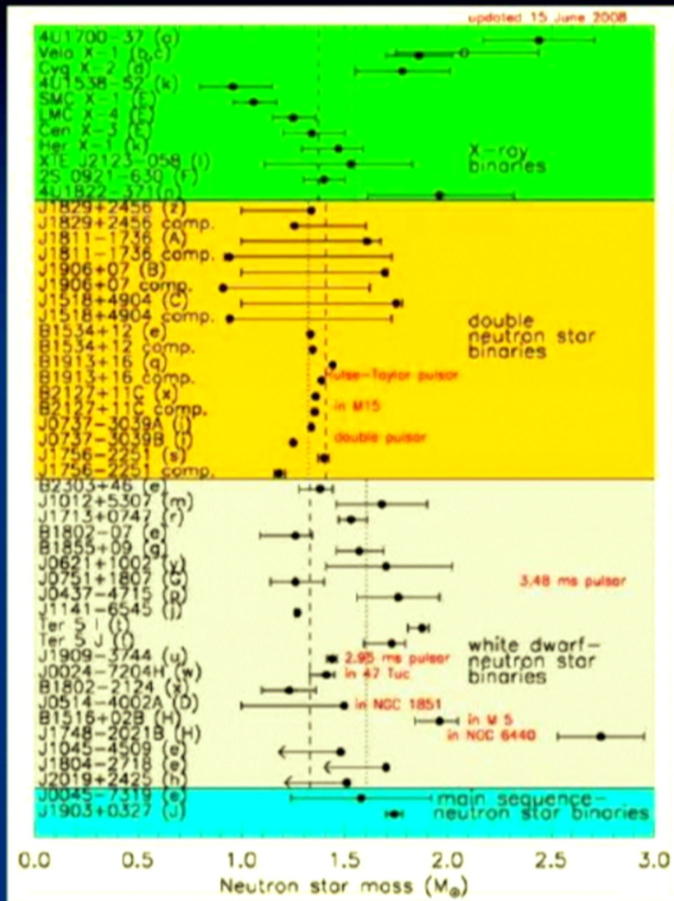
Estimates of Radiation Radii



Masses are well measured in binary systems but radii are not precisely known.

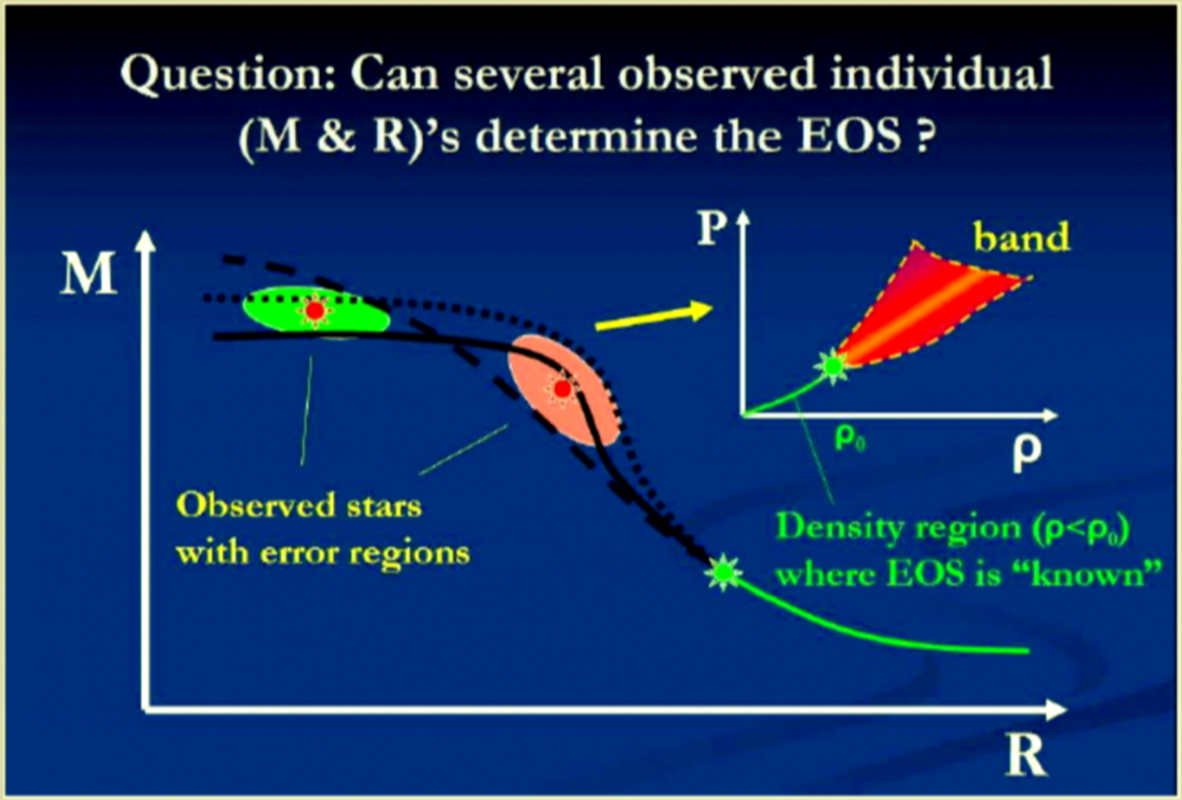
The other systems, in which the radiation radius R_∞ is measured, do not have reliable masses.

Object	R_∞ (km)	D (kpc)	$kT_{eff,\infty}$ (eV)	Ref.
Omega Cen (Chandra)	13.5 ± 2.1	$5.36 \pm 6\%$	66^{+4}_{-5}	Rutledge et al. ('02)
Omega Cen (XMM)	13.6 ± 0.3	$5.36 \pm 6\%$	67 ± 2	Gendre et al. ('02)
M13 (XMM)	12.6 ± 0.4	$7.80 \pm 2\%$	76 ± 3	Gendre et al. ('02)
47 Tuc X7 (Chandra)	$14.5^{+1.6}_{-1.4}$ ($1.4\ M_\odot$)	$5.13 \pm 4\%$		Rybicki et al. ('05)
M28 (Chandra)	$14.5^{+6.9}_{-3.8}$	$5.5 \pm 10\%$	90^{+30}_{-10}	Becker et al. ('03)
EXO 0748-676 (Chandra)	13.8 ± 1.8 ($2.10 \pm 0.28\ M_\odot$)	9.2 ± 1.0		Ozel ('06)

$$R_\infty = R/\sqrt{1-(2GM/c^2R)}\,; \qquad F = 4\pi T_{eff}^4 (R_\infty/D)^2$$

Atmospheric (sometimes magnetic) modeling required.

Deconstructing a neutron star - I



Postnikov, Steiner, Prakash & Lattimer

Ongoing work

Deconstructing a neutron star - III

Inversion scheme: (M & R)'s to EOS

Recast TOV using new variable h

$$dh = \frac{dp}{p + \rho\,(p)}$$

Advantages:
- finite at surafce & center
- m & r are now dependent variables

$$\frac{dr^2}{dh} = -2\,r^2\,\frac{r - 2\,m}{m + 4\,\pi\,r^3\,p}\,,$$

$$\frac{dm}{dh} = -4\,\pi\,r^3\,\rho\,\frac{r - 2\,m}{m + 4\,\pi\,r^3\,p}$$

With EOS

$p\,(h)$ and $\rho\,(h)$

At star's center

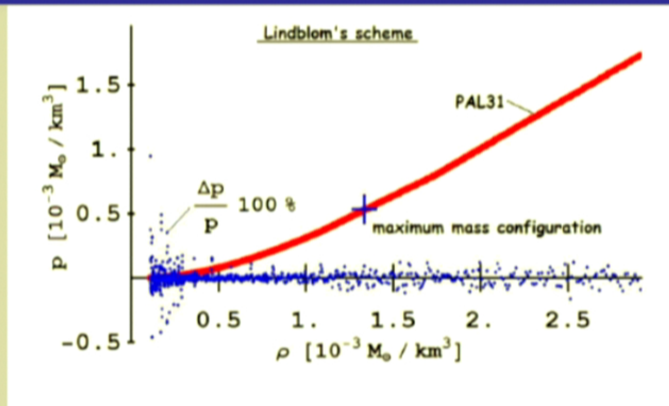
$$r^2\,(h) = \frac{3\,(h_c - h)}{2\,\pi\,(3\,p_c + \rho_c)}\left(1 + \frac{3\,(d\rho\,/\,dh)_c + 15\,p_c - 5\,\rho_c}{10\,(3\,p_c + \rho_c)}\,(h_c - h) + \ldots\right),$$

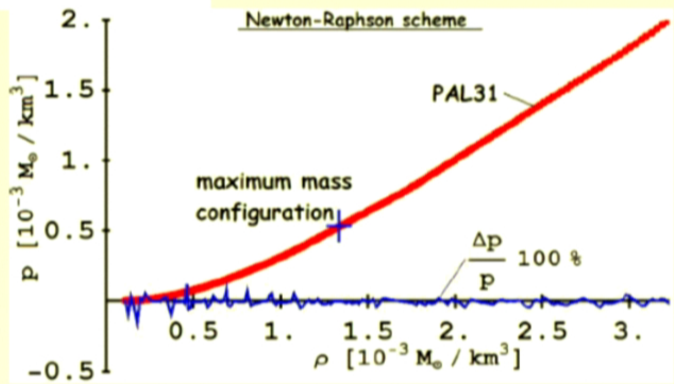
$$m\,(h) = \frac{4\,\pi}{3}\,r^3\,(h)\,\rho_c\left(1 - \frac{3\,(d\rho\,/\,dh)_c}{5\,\rho_c}\,(h_c - h) + \ldots\right)$$

Postnikov, Steiner, Prakash & Lattimer

Ongoing work

Tests with a model EOS





PAL31: $K_0 = 240$ MeV & $F(u) = u$ with n/n_0.

Postnikov, Steiner, Prakash & Lattimer

Ongoing work

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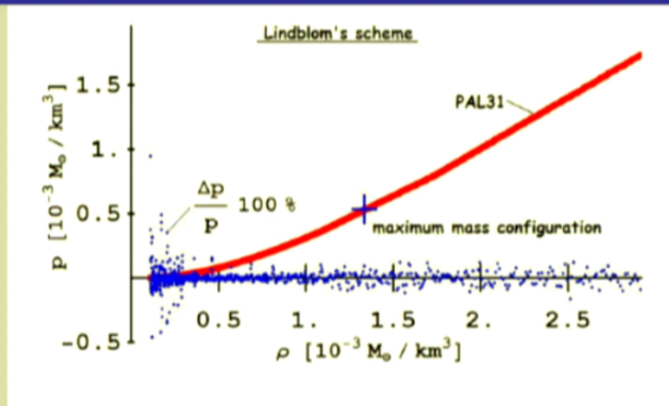

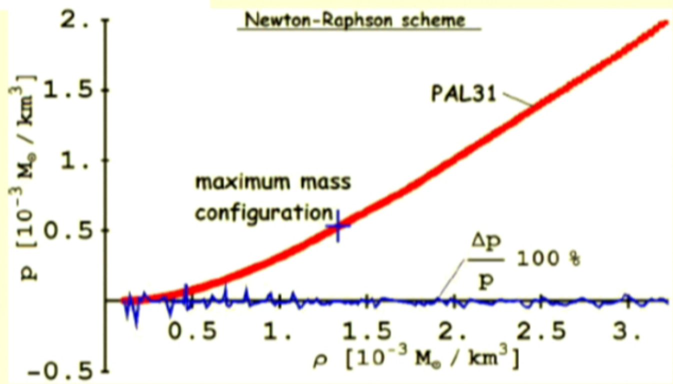

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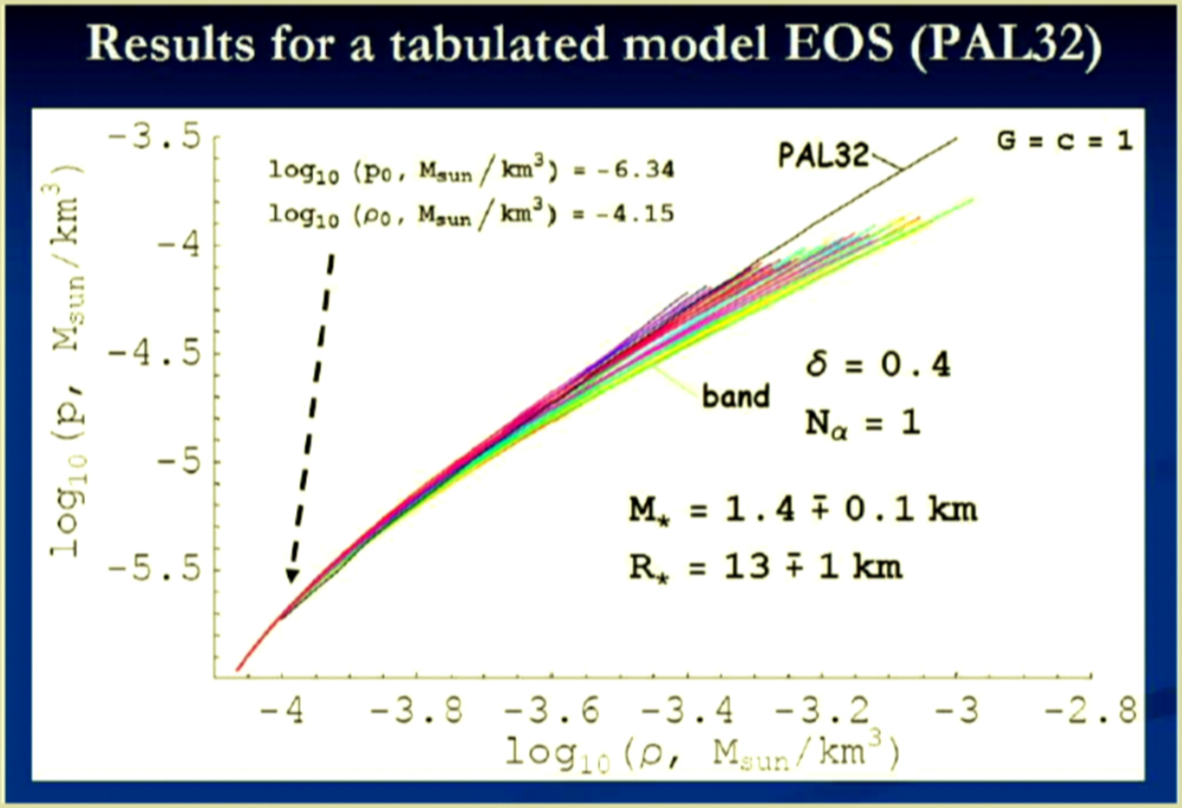
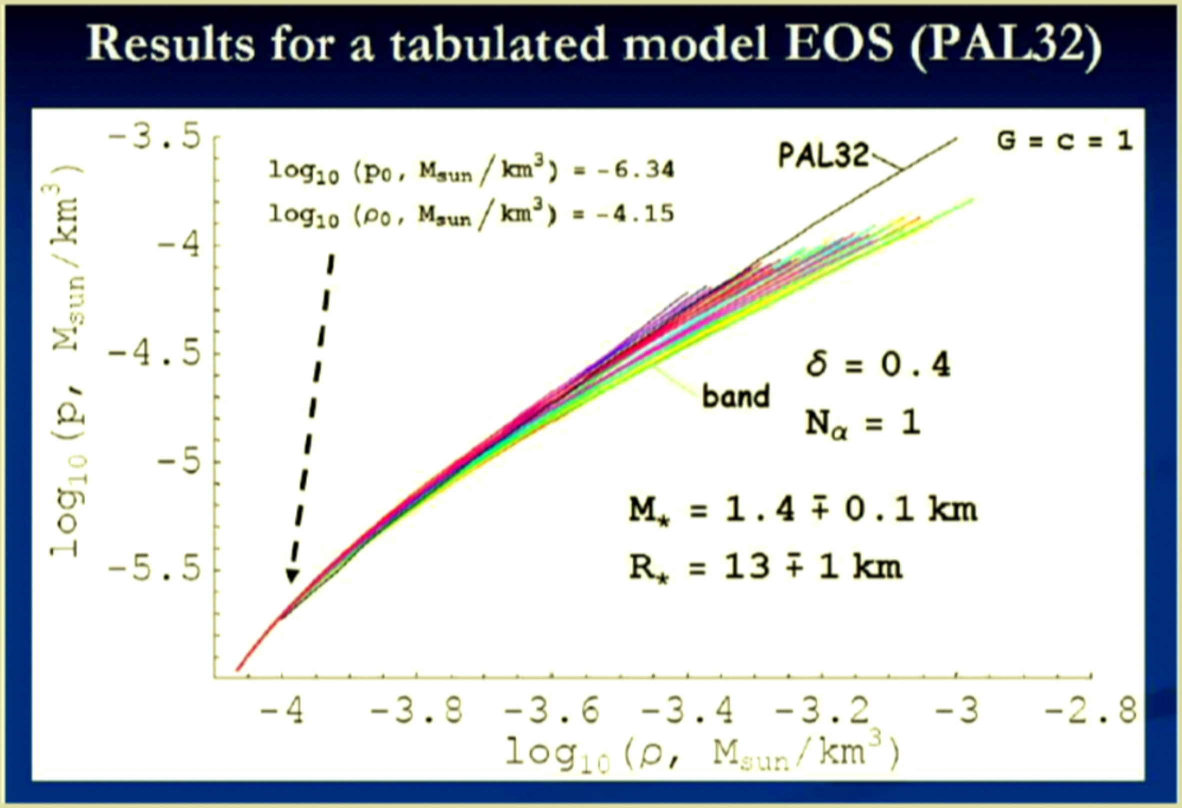
Postnikov, Steiner, Prakash & Lattimer
Ongoing work

Deconstruction at work - I



Postnikov, Steiner, Prakash & Lattimer
Ongoing work

Deconstruction at work - I

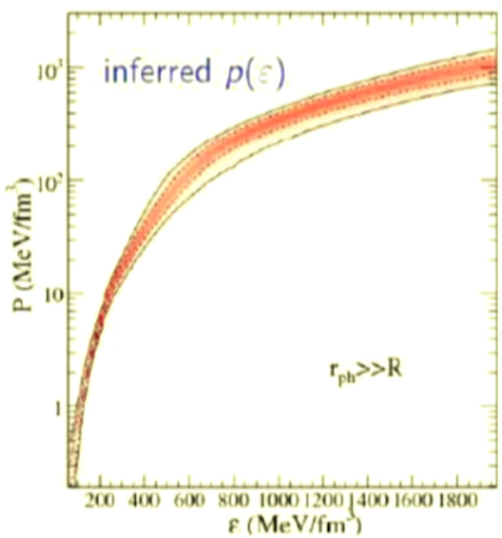


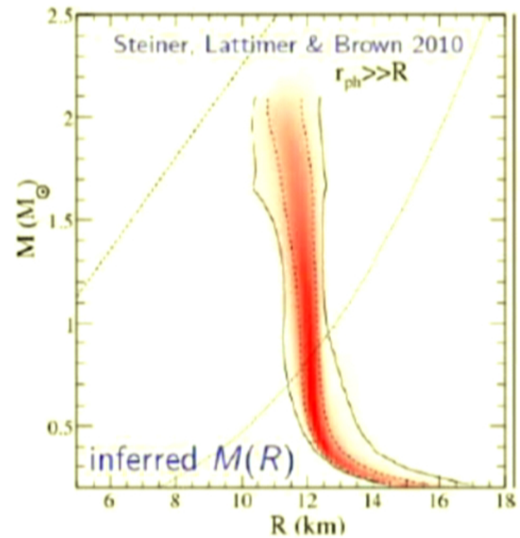
Postnikov, Steiner, Prakash & Lattimer
Ongoing work

Deconstruction at work - II

Bayesian TOV Inversion

- $\varepsilon < 0.5\varepsilon_0$: Known crustal EOS
- $0.5\varepsilon_0 < \varepsilon < \varepsilon_1$: EOS parametrized by K, K', S_v, γ
- $\varepsilon_1 < \varepsilon < \varepsilon_2$: n_1; $\varepsilon > \varepsilon_2$: Polytropic EOS with n_2
- EOS parameters $(K, K', S_v, \gamma, \varepsilon_1, n_1, \varepsilon_2, n_2)$ uniformly distributed
- M and R probability distributions for 8 neutron stars treated equally.





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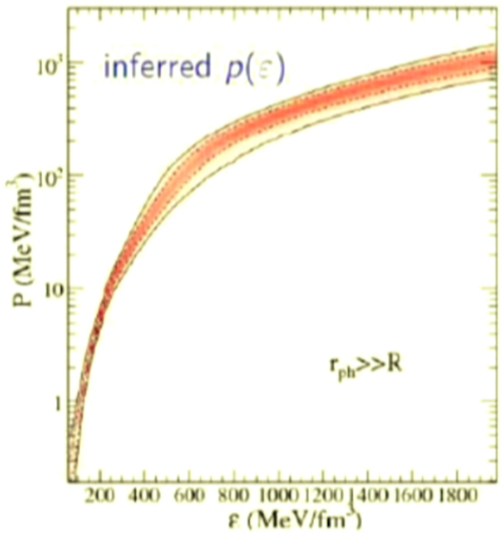
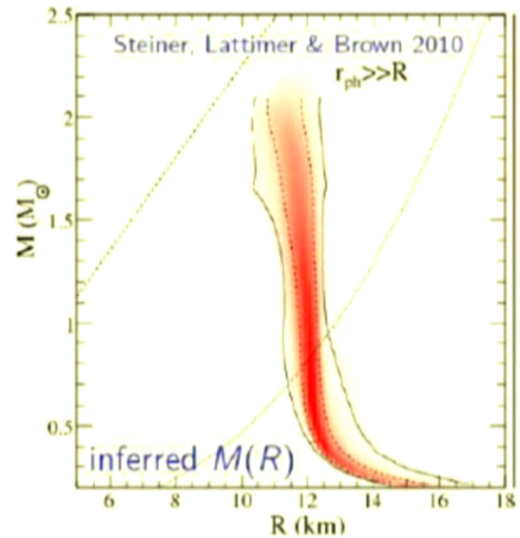




J.M. Lattimer | What a 2 Solar Mass Neutron Star Means

M. Prakash | The Equation of State of Neutron Star Matter

Simultaneous mass and radius measurements - I

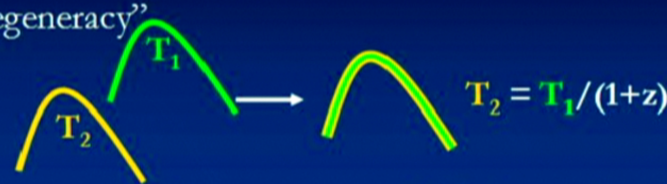
- Thermal X-ray and optical fluxes from isolated and quiescent neutron stars.
- Type I X-ray bursts on neutron star surfaces.
- Quasi-periodic oscillations from accreting neutron stars.
- Moments of inertia through spin-orbit coupling, observable through pulsar timing in extremely compact binaries.
- Pulsar glitches, which constrain properties of neutron star crusts.

Possible next steps?

Individual M & R measurement from continuous spectra

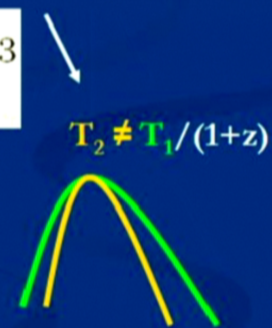
"You cannot measure a redshift (z) from blackbody emission due to photon energy (E_γ) - temperature (kT) degeneracy"

$$I(E_\gamma) \propto (E_\gamma/kT)^3 \frac{1}{e^{E_\gamma/kT} - 1}$$



"But the free-free opacity breaks this degeneracy. This spectrum, redshifted, permits determination of the redshift."

$$I(E_\gamma) \propto (E_\gamma/kT)^3 \frac{1}{e^{E_\gamma/kT} - 1} \kappa_{ff,o}(E_o/E_\gamma)^3$$



$R_\infty \rightarrow$ Normalization

$T_\infty \rightarrow$ Peak of the spectrum

$z = \frac{GM_{NS}}{c^2 R_{NS}} \rightarrow$ Second Derivative at the Peak of the Spectrum

$$(R_\infty = \frac{R_{NS}}{\sqrt{1 - \frac{2GM_{NS}}{c^2 R_{ns}}}}, z = \frac{GM_{NS}}{c^2 R_{NS}}) \rightarrow (R_{NS}, M_{NS})$$

Bob Rutledge
McGill University

Summary & Outlook

- The two-solar-mass neutron star sets upper bounds on the maximum energy density, pressure and chemical potential at the star's center. However, the composition remains uncertain from M and R measurements only.
- Accurate data on masses and radii of several (say 5 to 7) individual neutron stars can establish a model-independent dense matter equation of state. Theory in place, waiting for data!!
- The hunt for sub-millisecond pulsars must continue!
- Precise laboratory experiments, particularly those involving neutron-rich nuclei, are sorely needed to pin down the near-nuclear aspects of the symmetry energy (masses, neutron skin thicknesses, collective excitations, etc.)
- More data, the merrier! Theory needs aggravation to become better.

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