Title: Effective Field Theory of Multi-Field Inflation a la Weinberg

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Abstract: We
employ the effective field theory approach for multi-field inflation which is a
generalization of Weinberg's work. In this method the first correction terms in
addition to standard terms in the Lagrangian have been considered. These terms
contain up to the fourth derivative of the fields including the scalar field
and the metric. The results show the possible shapes of the interaction terms
resulting eventually in non-Gaussianity in a general formalism. In addition
generally the speed of sound is different but almost unity. Since in this
method the adiabatic mode is not discriminated initially so we define the
adiabatic as well as entropy modes for a specific two-field model. It has been
shown that the non-Gaussianity of the adiabatic mode and the entropy mode are
correlated in shape and amplitude. It is shown that even for speed close to
unity large non-Gaussianities are possible in multi-field case. The amount of
the non-Gaussianity depends on the curvature of the classical path in the
phase-space in the Hubble unit such that it is large for the large curvature.
In addition it is emphasized that the time derivative of adiabatic and entropy
perturbations do not transform due to the shift symmetry as well as the
original perturbations. Though two specific combinations of them are invariant
under such a symmetry and these combinations should be employed to construct an
effective field theory of multi-field inflation.

Effective Field Theory of Multi-Field Inflation a la Weinberg

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arXiv:1203.2266

map:

- keywords: Effective Field Theory, Inflation

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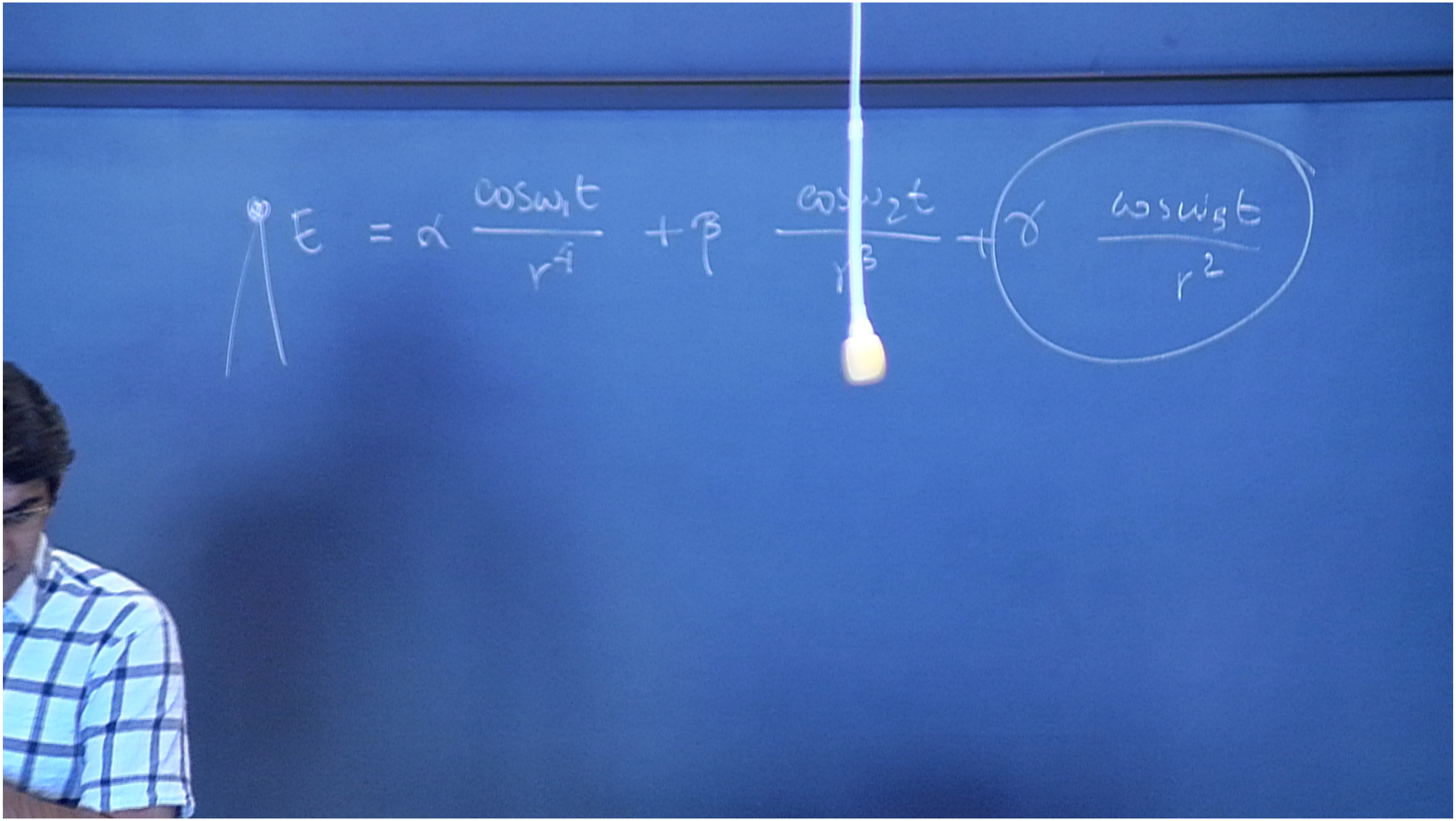
single field	C. Cheung et al. JHEP, arXiv:0709.0293 The EFT of Inflation	S. Weinberg PRD, arXiv:0804.4291 EFT for Inflation
multi-field	L. Senatore and M. Zaldarriaga JHEP, arXiv:1009.2093 The EFT of Multifield Inflation	JCAP, arXiv:1203.2266 EFT of Multi-Field Inflation a la Weinberg


$E = \alpha \frac{\cos \omega_1 t}{r^4} + \beta \frac{\cos \omega_2 t}{r^3} + \gamma \frac{\cos \omega_3 t}{r^2}$

Effective Field Theory

- an *effective* theory:
- is true for a certain domain of energy.

- two cases:
  - as a part of a true theory for whole energy scales
    - using EFT to simplify calculations!
  - in lack of a complete theory for the energy scales of interests
    - using EFT since there is no other choice!

EFT for Multi-Field Inflation

- the most general form of Lagrangian up to the 4th order derivatives:

after simplifications:

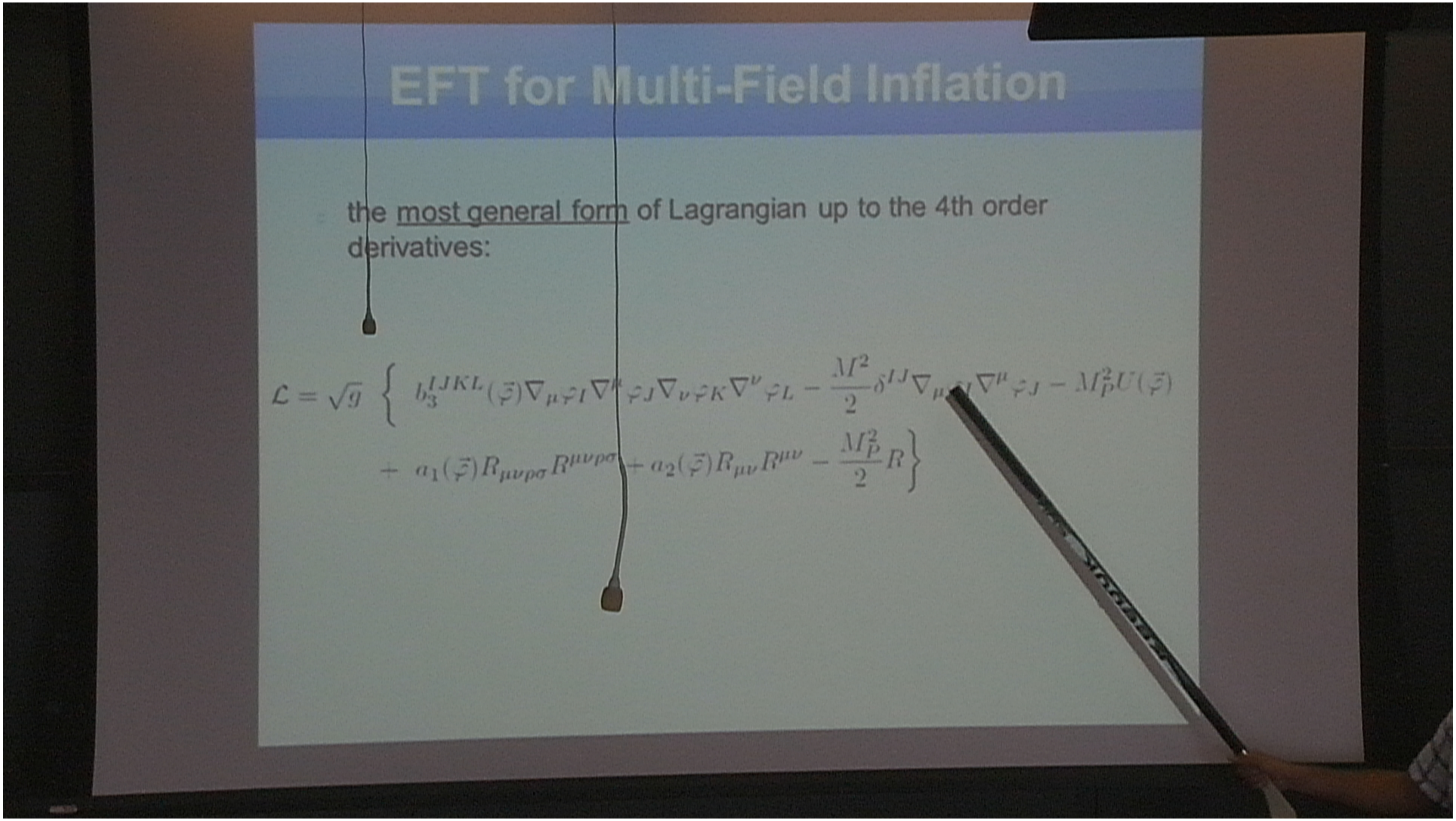
$$\mathcal{L} = \sqrt{g}\ \Big\{\ b_3^{IJKL}(\vec{\varphi})\nabla_\mu\varphi_I\nabla^\mu\varphi_J\nabla_\nu\varphi_K\nabla^\nu\varphi_L - \frac{M^2}{2}\delta^{IJ}\nabla_\mu\varphi_I\nabla^\mu\varphi_J - M_P^2 U(\vec{\varphi})$$
$$+\ a_1(\vec{\varphi})R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + a_2(\vec{\varphi})R_{\mu\nu}R^{\mu\nu} - \frac{M_P^2}{2}R\Big\}$$

EFT for Multi-Field Inflation

- the <u>most general form</u> of Lagrangian up to the 4th order derivatives:

$$
\begin{aligned}
&\Big\{ \; b_1^{IJ}(\vec{\varphi})\Box\varphi_I\Box\varphi_J + b_2^{IJK}(\vec{\varphi})\nabla_\mu\varphi_I\nabla^\mu\varphi_J\Box\varphi_K + b_3^{IJKL}(\vec{\varphi})\nabla_\mu\varphi_I\nabla^\mu\varphi_J\nabla_\nu\varphi_K\nabla^\nu\varphi_L + b_4^{IJ}(\vec{\varphi})\nabla_\mu\varphi_I\nabla^\mu\varphi_J \qquad \text{(A1)}\\
&+ \; b_5(\vec{\varphi}) + b_6^{IJ}(\vec{\varphi})(\nabla^\mu\varphi_I)(\Box\nabla_\mu\varphi_J) + b_7^{I}(\vec{\varphi})(\nabla_\mu\nabla_\nu\varphi_I)^2 + b_8^{IJK}(\vec{\varphi})(\nabla_\mu\varphi_I)(\nabla_\nu\varphi_J)(\nabla^\mu\nabla^\nu\varphi_K) + b_9^{I}(\vec{\varphi})\nabla^\mu\Box\nabla_\mu\varphi_I\\
&+ \; b_{10}^{I}(\vec{\varphi})\Box^2\varphi_I + b_{11}^{I}(\vec{\varphi})(\nabla^\mu)\nabla_\mu\Box\varphi_I + c_1^{IJ}(\vec{\varphi})R\nabla_\mu\varphi_I\nabla^\mu\varphi_J + c_2^{IJ}(\vec{\varphi})R^{\mu\nu}\nabla_\mu\varphi_I\nabla_\nu\varphi_J + c_3^{I}(\vec{\varphi})R\Box\varphi_I\\
&+ \; c_4^{I}(\vec{\varphi})(\nabla^\mu R)(\nabla_\mu\varphi_I) + c_5(\vec{\varphi})\Box R + c_6^{I}(\vec{\varphi})R_{\mu\nu}\nabla^\mu\nabla^\nu\varphi_I + a_1(\vec{\varphi})R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + a_2(\vec{\varphi})R_{\mu\nu}R^{\mu\nu} + a_3(\vec{\varphi})R^2 + a_4(\vec{\varphi})R\Big\}
\end{aligned}
$$

before simplifications!


EFT for Multi-Field Inflation
the most general form of Lagrangian up to the 4th order derivatives:
$\mathcal{L} = \sqrt{g} \Big\{ b_3^{IJKL}(\vec{\varphi}) \nabla_\mu \varphi_I \nabla^\mu \varphi_J \nabla_\nu \varphi_K \nabla^\nu \varphi_L - \frac{M^2}{2} \delta^{IJ} \nabla_\mu \varphi_I \nabla^\mu \varphi_J - M_P^2 U(\vec{\varphi})$
$+ a_1(\vec{\varphi}) R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + a_2(\vec{\varphi}) R_{\mu\nu} R^{\mu\nu} - \frac{M_P^2}{2} R \Big\}$

EFT for Multi-Field Inflation

- the <u>most general form</u> of Lagrangian up to the 4th order derivatives:

$$
\begin{aligned}
\Big\{ \; & b_1^{IJ}(\vec{\varphi})\Box\varphi_I\Box\varphi_J + b_2^{IJK}(\vec{\varphi})\nabla_\mu\varphi_I\nabla^\mu\varphi_J\Box\varphi_K + b_3^{IJKL}(\vec{\varphi})\nabla_\mu\varphi_I\nabla^\mu\varphi_J\nabla_\nu\varphi_K\nabla^\nu\varphi_L + b_4^{IJ}(\vec{\varphi})\nabla_\mu\varphi_I\nabla^\mu\varphi_J \qquad \text{(A1)} \\
+ \; & b_5(\vec{\varphi}) + b_6^{IJ}(\vec{\varphi})(\nabla^\mu\varphi_I)(\Box\nabla_\mu\varphi_J) + b_7^{I}(\vec{\varphi})(\nabla_\mu\nabla_\nu\varphi_I)^2 + b_8^{IJK}(\vec{\varphi})(\nabla_\mu\varphi_I)(\nabla_\nu\varphi_J)(\nabla^\mu\nabla^\nu\varphi_K) + b_9^{I}(\vec{\varphi})\nabla^\mu\Box\nabla_\mu\varphi_I \\
+ \; & b_{10}^{I}(\vec{\varphi})\Box^2\varphi_I + b_{11}^{I}(\vec{\varphi})(\nabla^\mu)\nabla_\mu\Box\varphi_I + c_1^{IJ}(\vec{\varphi})R\nabla_\mu\varphi_I\nabla^\mu\varphi_J + c_2^{IJ}(\vec{\varphi})R^{\mu\nu}\nabla_\mu\varphi_I\nabla_\nu\varphi_J + c_3^{I}(\vec{\varphi})R\Box\varphi_I \\
+ \; & c_4^{I}(\vec{\varphi})(\nabla^\mu R)(\nabla_\mu\varphi_I) + c_5(\vec{\varphi})\Box R + c_6^{I}(\vec{\varphi})R_{\mu\nu}\nabla^\mu\nabla^\nu\varphi_I + a_1(\vec{\varphi})R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + a_2(\vec{\varphi})R_{\mu\nu}R^{\mu\nu} + a_3(\vec{\varphi})R^2 + a_4(\vec{\varphi})R \Big\}
\end{aligned}
$$

before simplifications!

perturbations:

- perturbations in single field model (Weinberg's paper)

$$\mathcal{L} = \sqrt{g}\Big[-\frac{M^2}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - M_P^2 U(\varphi) + f(\varphi)\Big(g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}\Big)^2\Big] \qquad \varphi = \bar{\varphi} + \delta\varphi$$

$$= \bar{\mathcal{L}} - \frac{1}{2}a^3\Big(M^2 + 4f(\bar{\varphi})\dot{\bar{\varphi}}^2\Big) \times \Big(-\dot{\delta\varphi}^2 + a^{-2}(\vec{\nabla}\delta\varphi)^2\Big)$$

$$+ 4a^3 f(\bar{\varphi})\dot{\bar{\varphi}}^2\Big(\dot{\delta\varphi}^2 + \dot{\delta\varphi}^3/\dot{\bar{\varphi}} - a^{-2}\dot{\delta\varphi}(\vec{\nabla}\delta\varphi)^2/\dot{\bar{\varphi}} + \frac{1}{4}\dot{\delta\varphi}^4/\dot{\bar{\varphi}}^2 - \frac{1}{2}a^{-2}\dot{\delta\varphi}^2(\vec{\nabla}\delta\varphi)^2/\dot{\bar{\varphi}}^2 + \frac{1}{4}a^{-4}(\vec{\nabla}\delta\varphi)^4/\dot{\bar{\varphi}}^2\Big)$$

perturbations:

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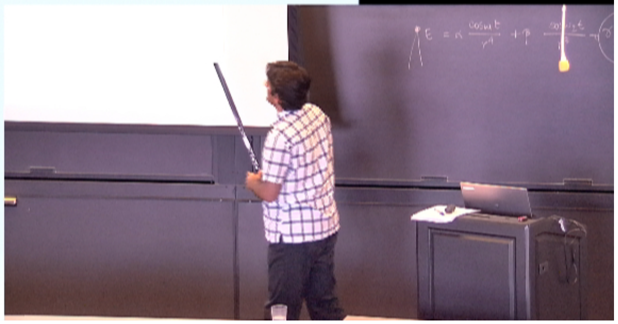
$$\begin{aligned}\mathcal{L} &= \sqrt{g}\Big[-\frac{M^2}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - M_P^2U(\varphi) + f(\varphi)\Big(g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}\Big)^2\Big] \qquad \varphi = \bar{\varphi} + \delta\varphi \\ &= \bar{\mathcal{L}} - \frac{1}{2}a^3\Big(M^2 + 4f(\bar{\varphi})\dot{\bar{\varphi}}^2\Big) \times \Big(-\dot{\delta\varphi}^2 + a^{-2}(\vec{\nabla}\delta\varphi)^2\Big) \\ &+ 4a^3 f(\bar{\varphi})\dot{\bar{\varphi}}^2\Big(\dot{\delta\varphi}^2 + \dot{\delta\varphi}^3/\dot{\bar{\varphi}} - a^{-2}\dot{\delta\varphi}(\vec{\nabla}\delta\varphi)^2/\dot{\bar{\varphi}} + \frac{1}{4}\dot{\delta\varphi}^4/\dot{\bar{\varphi}}^2 - \frac{1}{2}a^{-2}\dot{\delta\varphi}^2(\vec{\nabla}\delta\varphi)^2/\dot{\bar{\varphi}}^2 + \frac{1}{4}a^{-4}(\vec{\nabla}\delta\varphi)^4/\dot{\bar{\varphi}}^2\Big)\end{aligned}$$

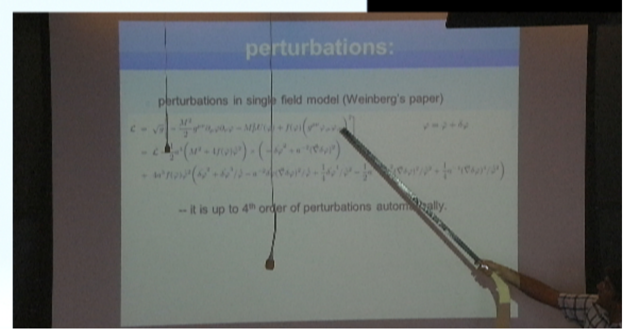


perturbations:

- perturbations in single field model (Weinberg's paper)

$$\begin{aligned}
\mathcal{L} &= \sqrt{g}\Big[-\frac{M^2}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - M_P^2 U(\varphi) + f(\varphi)\Big(g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}\Big)^2\Big] \qquad \varphi = \bar{\varphi} + \delta\varphi \\
&= \bar{\mathcal{L}} - \frac{1}{2}a^3\Big(M^2 + 4f(\bar{\varphi})\dot{\bar{\varphi}}^2\Big) \times \Big(-\dot{\delta\varphi}^2 + a^{-2}(\vec{\nabla}\delta\varphi)^2\Big) \\
&+ 4a^3 f(\bar{\varphi})\dot{\bar{\varphi}}^2\Big(\dot{\delta\varphi}^2 + \dot{\delta\varphi}^3/\dot{\bar{\varphi}} - a^{-2}\dot{\delta\varphi}(\vec{\nabla}\delta\varphi)^2/\dot{\bar{\varphi}} + \frac{1}{4}\dot{\delta\varphi}^4/\dot{\bar{\varphi}}^2 - \frac{1}{2}a^{-2}\dot{\delta\varphi}^2(\vec{\nabla}\delta\varphi)^2/\dot{\bar{\varphi}}^2 + \frac{1}{4}a^{-4}(\vec{\nabla}\delta\varphi)^4/\dot{\bar{\varphi}}^2\Big)
\end{aligned}$$

-- it is up to 4th order of perturbations automatically.



perturbations:

- perturbations in single field model (Weinberg's paper)

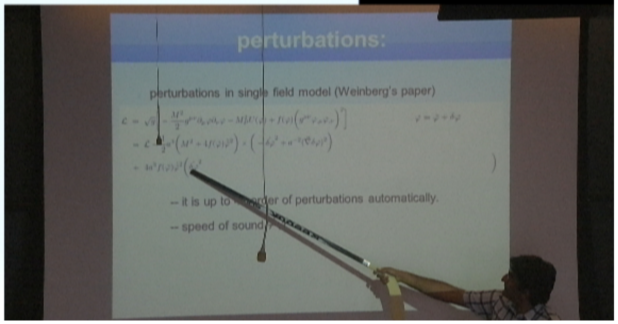
$$\mathcal{L} = \sqrt{g}\Big[-\frac{M^2}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - M_P^2 U(\varphi) + f(\varphi)\Big(g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}\Big)^2\Big] \qquad \varphi = \bar{\varphi} + \delta\varphi$$

$$= \bar{\mathcal{L}} - \frac{1}{2}a^3\Big(M^2 + 4f(\bar{\varphi})\dot{\bar{\varphi}}^2\Big) \times \Big(-\dot{\delta\varphi}^2 + a^{-2}(\vec{\nabla}\delta\varphi)^2\Big)$$

$$+\ 4a^3 f(\bar{\varphi})\dot{\bar{\varphi}}^2\Big(\dot{\delta\varphi}^2 \qquad\qquad \Big)$$

-- it is up to 4th order of perturbations automatically.

-- speed of sound ≠ 1



perturbations:

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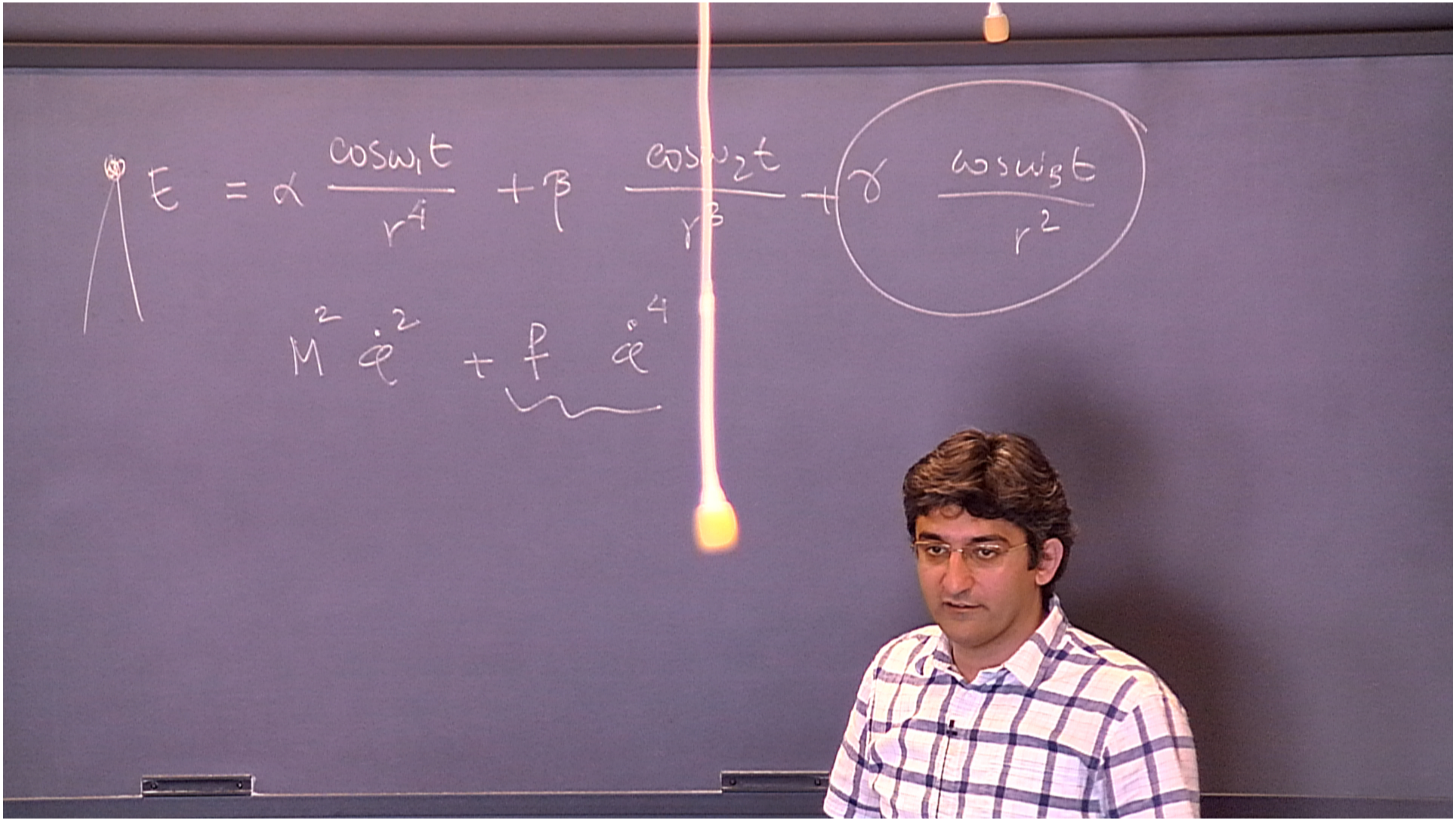
-- it is up to 4th order of perturbations automatically.

-- speed of sound ≠ 1

-- large non-Gaussianity?

-- speed of sound is constrained by validity of EFT!


E = α cosω₁t / r⁴ + β cosω₂t / r³ + γ cosω₃t / r²
M² φ̇² + f φ̇⁴

perturbations:

- two-field case:

  the most general form of the Lagrangian:

$$\mathcal{L} = -a^3\Big\{ -\frac{M_1^2}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{M_2^2}{2}\partial_\mu\chi\partial^\mu\chi - M_P^2 U(\varphi,\chi) + g_1(\varphi,\chi)\big(\partial_\mu\varphi\partial^\mu\varphi\big)^2 + g_2(\varphi,\chi)\big(\partial_\mu\chi\partial^\mu\chi\big)^2$$
$$+ g_3(\varphi,\chi)\big(\partial_\mu\varphi\partial^\mu\varphi\big)\big(\partial_\nu\varphi\partial^\nu\chi\big) + g_4(\varphi,\chi)\big(\partial_\mu\chi\partial^\mu\chi\big)\big(\partial_\nu\chi\partial^\nu\varphi\big)$$
$$+ g_5(\varphi,\chi)\big(\partial_\mu\varphi\partial^\mu\varphi\big)\big(\partial_\nu\chi\partial^\nu\chi\big) + g_6(\varphi,\chi)\big(\partial_\mu\varphi\partial^\mu\chi\big)\big(\partial_\nu\varphi\partial^\nu\chi\big)\Big\}$$

perturbations:

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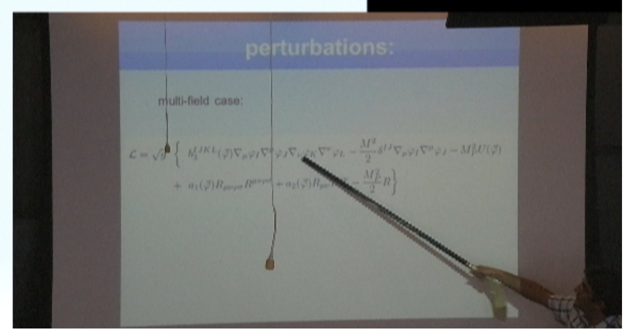
perturbations:

- multi-field case:

$$\mathcal{L} = \sqrt{g}\ \Big\{\ b_3^{IJKL}(\vec{\varphi})\nabla_\mu\varphi_I\nabla^\mu\varphi_J\nabla_\nu\varphi_K\nabla^\nu\varphi_L - \frac{M^2}{2}\delta^{IJ}\nabla_\mu\varphi_I\nabla^\mu\varphi_J - M_P^2 U(\vec{\varphi})$$
$$+\ a_1(\vec{\varphi})R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + a_2(\vec{\varphi})R_{\mu\nu}R^{\mu\nu} - \frac{M_P^2}{2}R\Big\}$$



perturbations:

- two-field case:

  the most general form of the Lagrangian:

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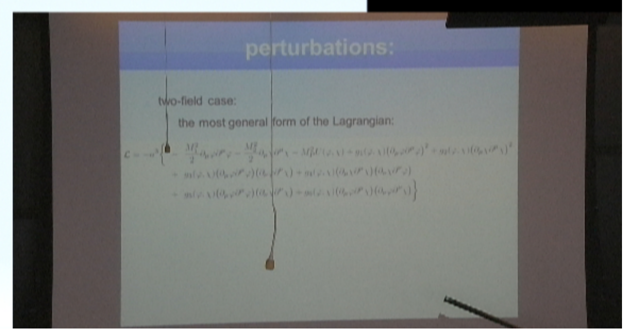
$$+\ g_3(\varphi,\chi)\big(\partial_\mu\varphi\partial^\mu\varphi\big)\big(\partial_\nu\varphi\partial^\nu\chi\big) + g_4(\varphi,\chi)\big(\partial_\mu\chi\partial^\mu\chi\big)\big(\partial_\nu\chi\partial^\nu\varphi\big)$$
$$+\ g_5(\varphi,\chi)\big(\partial_\mu\varphi\partial^\mu\varphi\big)\big(\partial_\nu\chi\partial^\nu\chi\big) + g_6(\varphi,\chi)\big(\partial_\mu\varphi\partial^\mu\chi\big)\big(\partial_\nu\varphi\partial^\nu\chi\big)\Big\}$$



perturbations:

- two-field case:

  the most general form of the Lagrangian:

$$\mathcal{L} = -a^3 \Big\{ -\frac{M_1^2}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{M_2^2}{2}\partial_\mu\chi\partial^\mu\chi - M_P^2 U(\varphi,\chi) + g_1(\varphi,\chi)\big(\partial_\mu\varphi\partial^\mu\varphi\big)^2 + g_2(\varphi,\chi)\big(\partial_\mu\chi\partial^\mu\chi\big)^2$$
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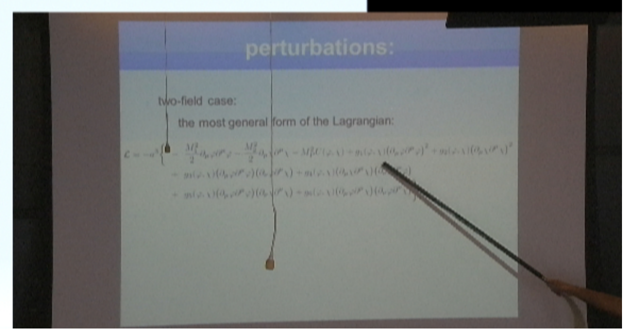
$$+ g_5(\varphi,\chi)\big(\partial_\mu\varphi\partial^\mu\varphi\big)\big(\partial_\nu\chi\partial^\nu\chi\big) + g_6(\varphi,\chi)\big(\partial_\mu\varphi\partial^\mu\chi\big)\big(\partial_\nu\varphi\partial^\nu\chi\big)\Big\}$$

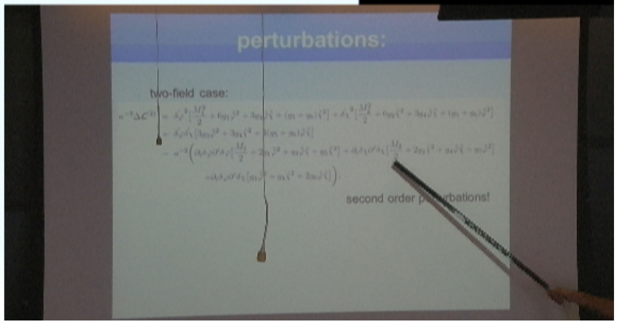


perturbations:

- two-field case:

$$
\begin{aligned}
a^{-3}\Delta\mathcal{L}^{(2)} &= \dot{\delta\varphi}^2\Big[\frac{M_1^2}{2}+6g_1\dot{\bar\varphi}^2+3g_3\dot{\bar\varphi}\dot{\bar\chi}+(g_5+g_6)\dot{\bar\chi}^2\Big]+\dot{\delta\chi}^2\Big[\frac{M_2^2}{2}+6g_2\dot{\bar\chi}^2+3g_4\dot{\bar\varphi}\dot{\bar\chi}+(g_5+g_6)\dot{\bar\varphi}^2\Big]\\
&+\dot{\delta\varphi}\dot{\delta\chi}\Big[3g_3\dot{\bar\varphi}^2+3g_4\dot{\bar\chi}^2+4(g_5+g_6)\dot{\bar\varphi}\dot{\bar\chi}\Big]\\
&-a^{-2}\Big(\partial_i\delta\varphi\partial^i\delta\varphi\Big[\frac{M_1}{2}+2g_1\dot{\bar\varphi}^2+g_3\dot{\bar\varphi}\dot{\bar\chi}+g_5\dot{\bar\chi}^2\Big]+\partial_i\delta\chi\partial^i\delta\chi\Big[\frac{M_2}{2}+2g_2\dot{\bar\chi}^2+g_4\dot{\bar\varphi}\dot{\bar\chi}+g_5\dot{\bar\varphi}^2\Big]\\
&\qquad+\partial_i\delta\varphi\partial^i\delta\chi\Big[g_3\dot{\bar\varphi}^2+g_4\dot{\bar\chi}^2+2g_6\dot{\bar\varphi}\dot{\bar\chi}\Big]\Big),
\end{aligned}
$$

second order perturbations!

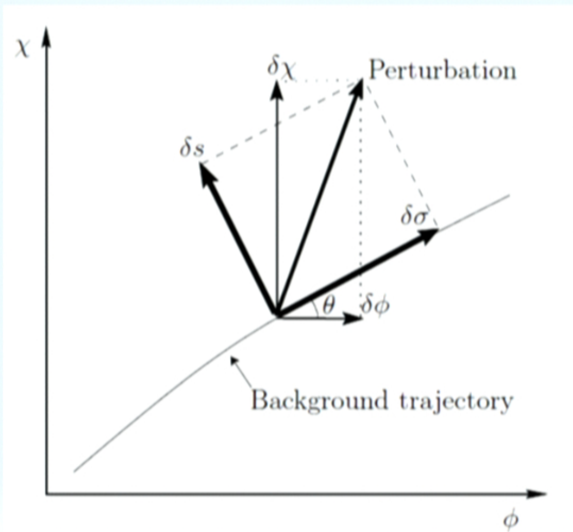


perturbations: adiabatic & entropy modes

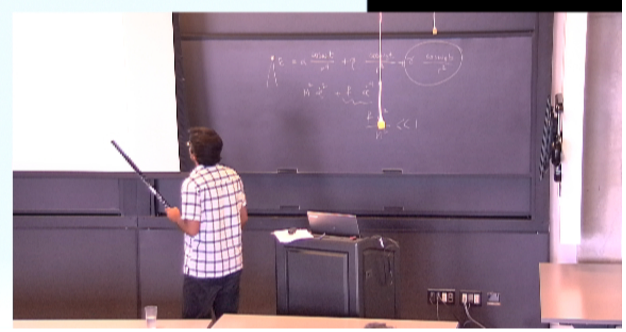
- adiabatic mode: entropy mode:

$$\delta\sigma \equiv \vec{T}.\vec{\delta}, \qquad \delta s \equiv \vec{N}.\vec{\delta}$$

$$\vec{\delta} \equiv (\delta\varphi, \delta\chi)\,, \qquad \vec{T} = (\cos\theta, \sin\theta) \equiv (\dot{\varphi}/\dot{\sigma}, \dot{\chi}/\dot{\sigma})\,, \qquad \vec{N} \equiv (\sin\theta, -\cos\theta)$$



Gordon et al. arXiv:astro-ph/0009131



perturbations: adiabatic & entropy modes

- as an example:

second order perturbation terms (containing time derivatives) due to correction term:

$$
\begin{aligned}
& 6\dot{\sigma}^2(\vec{T}.\dot{\vec{\delta}})^2 \times \left[g_1\cos^4\theta + g_2\sin^4\theta + g_3\cos^3\theta\sin\theta + g_4\cos\theta\sin^3\theta + (g_5+g_6)\cos^2\theta\sin^2\theta\right] \qquad (18)\\
+ & \ \dot{\sigma}^2(\vec{N}.\dot{\vec{\delta}})^2 \times \\
& \left[(g_5+g_6)\Big(\cos^4\theta + \sin^4\theta\Big) + 3(g_4-g_3)\Big(\cos^3\theta\sin\theta - \cos\theta\sin^3\theta\Big) + 2\Big(3(g_1+g_2) + 2(g_5+g_6)\Big)\cos^2\theta\sin^2\theta\right]\\
+ & \ 3\dot{\sigma}^2(\vec{T}.\dot{\vec{\delta}})(\vec{N}.\dot{\vec{\delta}}) \times \\
& \left[-g_3\cos^4\theta + g_4\sin^4\theta + 2\Big(2g_1 - (g_5+g_6)\Big)\cos^3\theta\sin\theta - 2\Big(2g_2 + (g_5+g_6)\Big)\cos\theta\sin^3\theta + 3(g_3-g_4)\cos^2\theta\sin^2\theta\right]
\end{aligned}
$$

$$(\dot{\delta\sigma} - \dot{\theta}\delta s)$$

$$(\dot{\delta s} + \dot{\theta}\delta\sigma)$$

note that just these two combinations appear in this formalism!

shape of non-Gaussianity

- due to previous slide: for example:

$$(\vec{T}.\dot{\vec{\delta}})^3 = \delta\dot{\sigma}^3 - 3\dot{\theta}\delta\dot{\sigma}^2\delta s + 3\dot{\theta}^2\delta\dot{\sigma}\delta s^2 - \dot{\theta}^3\delta s^3$$

equilateral NG in adiabatic mode → local NG in entropy mode

--- in this formalism

the "Cosine" between different kinds of NG is fixed!

amplitude of NG

$$\frac{\mathcal{L}^{(3)}}{\mathcal{L}^{(2)}} = \frac{\{H^3, H^2\dot{\theta}, H\dot{\theta}^2, \dot{\theta}^3\}}{\{H^2, H\dot{\theta}, \dot{\theta}^2\}} \times \left(\frac{f(g_i)}{M^2}\dot{\sigma}^2\right) \times \frac{\delta\sigma}{\dot{\sigma}} = \frac{\{H^3, H^2\dot{\theta}, H\dot{\theta}^2, \dot{\theta}^3\}}{H \times \{H^2, H\dot{\theta}, \dot{\theta}^2\}} \times \left(\frac{f(g_i)}{M^2}\dot{\sigma}^2\right) \times \zeta$$

amplitude of NG

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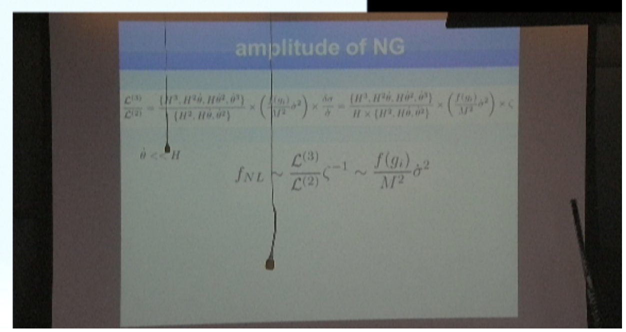
$\dot{\theta} << H$

$$f_{NL} \sim \frac{\mathcal{L}^{(3)}}{\mathcal{L}^{(2)}}\zeta^{-1} \sim \frac{f(g_i)}{M^2}\dot{\sigma}^2$$



amplitude of NG

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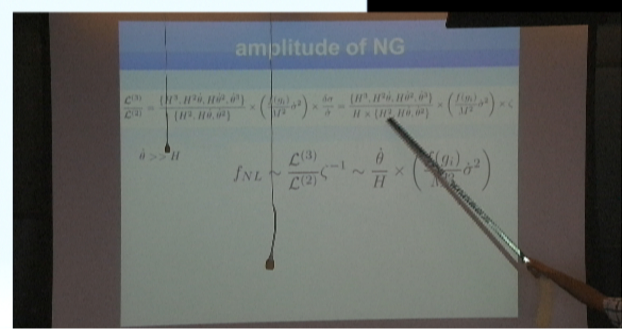

amplitude of NG

$$\frac{\mathcal{L}^{(3)}}{\mathcal{L}^{(2)}} = \frac{\{H^3, H^2\dot{\theta}, H\dot{\theta}^2, \dot{\theta}^3\}}{\{H^2, H\dot{\theta}, \dot{\theta}^2\}} \times \left(\frac{f(g_i)}{M^2}\dot{\sigma}^2\right) \times \frac{\delta\sigma}{\dot{\sigma}} = \frac{\{H^3, H^2\dot{\theta}, H\dot{\theta}^2, \dot{\theta}^3\}}{H \times \{H^2, H\dot{\theta}, \dot{\theta}^2\}} \times \left(\frac{f(g_i)}{M^2}\dot{\sigma}^2\right) \times \zeta$$

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$$f_{NL} \sim \frac{\mathcal{L}^{(3)}}{\mathcal{L}^{(2)}}\zeta^{-1} \sim \frac{\dot{\theta}}{H} \times \left(\frac{f(g_i)}{M^2}\dot{\sigma}^2\right)$$

-- validity condition of EFT i.e. $\frac{f(g_i)}{M^2}\dot{\sigma}^2 < 1$ constrains the amplitude of NG!

-- except if the curvature of classical (background) path be large!

-- or: if by a mechanism (e.g. Vainshtein) one can modify the validity condition of EFT!

compare with Senatore & Zaldarriaga

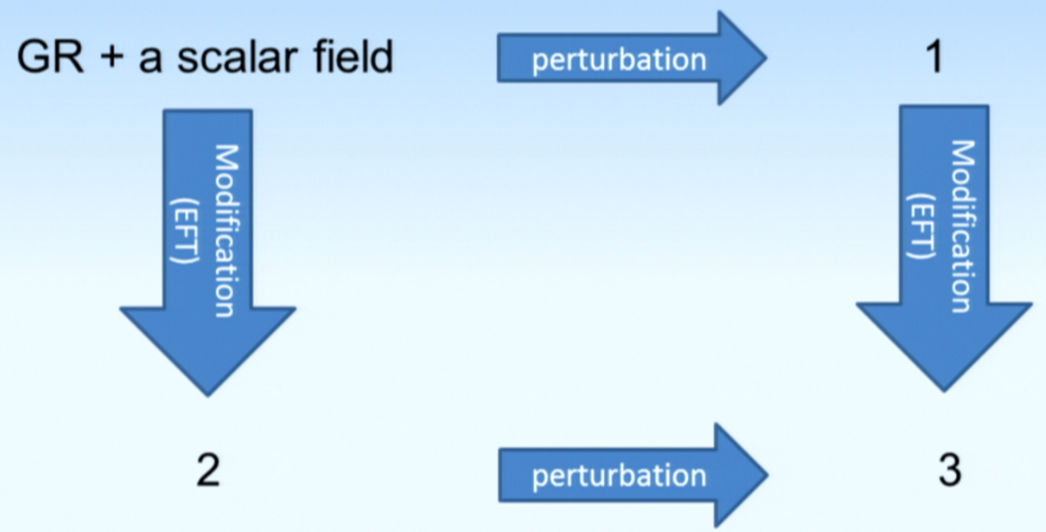
-- Senatore & Zaldarriaga model is based on Cheung et al.'s work!

-- in Cheung's work, EFT is constructed on perturbations' level!

-- since their model is single field, the perturbation is associated to adiabatic mode!

-- so in Senatore & Z., the entropy modes are added into a base with already known adiabatic mode!

compare with Senatore & Zaldarriaga



Cheung et al. and Senatore & Z.: $0 \longrightarrow 1 \longrightarrow 3$

Weinberg and this talk: $0 \longrightarrow 2 \longrightarrow 3$

compare with Senatore & Zaldarriaga

- so in Senatore & Zaldarriaga, the shift symmetry results in a Lagrangian similar to

$$
\begin{aligned}
a^{-3}\Delta\mathcal{L}^{(2)} \;=\;& \dot{\delta\varphi}^2\Big[\frac{M_1^2}{2} + 6g_1\dot{\bar\varphi}^2 + 3g_3\dot{\bar\varphi}\dot{\bar\chi} + (g_5+g_6)\dot{\bar\chi}^2\Big] + \dot{\delta\chi}^2\Big[\frac{M_2^2}{2} + 6g_2\dot{\bar\chi}^2 + 3g_4\dot{\bar\varphi}\dot{\bar\chi} + (g_5+g_6)\dot{\bar\varphi}^2\Big] \\
&+ \dot{\delta\varphi}\dot{\delta\chi}\Big[3g_3\dot{\bar\varphi}^2 + 3g_4\dot{\bar\chi}^2 + 4(g_5+g_6)\dot{\bar\varphi}\dot{\bar\chi}\Big] \\
&- a^{-2}\Big(\partial_i\delta\varphi\partial^i\delta\varphi\Big[\frac{M_1}{2} + 2g_1\dot{\bar\varphi}^2 + g_3\dot{\bar\varphi}\dot{\bar\chi} + g_5\dot{\bar\chi}^2\Big] + \partial_i\delta\chi\partial^i\delta\chi\Big[\frac{M_2}{2} + 2g_2\dot{\bar\chi}^2 + g_4\dot{\bar\varphi}\dot{\bar\chi} + g_5\dot{\bar\varphi}^2\Big] \\
&\qquad + \partial_i\delta\varphi\partial^i\delta\chi\Big[g_3\dot{\bar\varphi}^2 + g_4\dot{\bar\chi}^2 + 2g_6\dot{\bar\varphi}\dot{\bar\chi}\Big]\Big),
\end{aligned}
$$

$$\delta\varphi \to \delta\varphi + c_1 \text{ and } \delta\chi \to \delta\chi + c_2$$

i.e. there are just *derivatives* of adiabatic and entropy perturbations!

compare with Senatore & Zaldarriaga

shift symmetry: $\delta\varphi \to \delta\varphi + c_1$ and $\delta\chi \to \delta\chi + c_2$

due to

$$\delta\sigma \equiv \vec{T}.\vec{\delta}, \qquad \delta s \equiv \vec{N}.\vec{\delta}$$

$$\vec{\delta} \equiv (\delta\varphi, \delta\chi)\,, \qquad \vec{T} = (\cos\theta, \sin\theta) \equiv (\dot{\varphi}/\dot{\sigma}, \dot{\chi}/\dot{\sigma})\,, \qquad \vec{N} \equiv (\sin\theta, -\cos\theta)$$

results in

$$\delta\sigma \to \delta\sigma + (c_1 \cos\theta + c_2 \sin\theta)$$
$$\delta s \to \delta s + (c_1 \sin\theta - c_2 \cos\theta)$$

which causes a new symmetry for adiabatic and entropy modes:

$$\dot{\delta\sigma} - \dot{\theta}\delta s \to \dot{\delta\sigma} - \dot{\theta}\delta s$$
$$\dot{\delta s} + \dot{\theta}\delta\sigma \to \dot{\delta s} + \dot{\theta}\delta\sigma$$

compare with Senatore & Zaldarriaga

shift symmetry: $\delta\varphi \to \delta\varphi + c_1$ and $\delta\chi \to \delta\chi + c_2$

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$$\delta\sigma \equiv \vec{T}.\vec{\delta}, \qquad \delta s \equiv \vec{N}.\vec{\delta}$$

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results in

$$\delta\sigma \to \delta\sigma + (c_1\cos\theta + c_2\sin\theta)$$
$$\delta s \to \delta s + (c_1\sin\theta - c_2\cos\theta)$$

which causes a new symmetry for adiabatic and entropy modes:

$$\dot{\delta\sigma} - \dot{\theta}\delta s \to \dot{\delta\sigma} - \dot{\theta}\delta s \quad \longleftarrow \quad \vec{T}.\dot{\vec{\delta}}$$
$$\dot{\delta s} + \dot{\theta}\delta\sigma \to \dot{\delta s} + \dot{\theta}\delta\sigma \quad \longleftarrow \quad \vec{N}.\dot{\vec{\delta}}$$

conclusions

-- this model does not predict a large non-Gaussianity except:

- -- for a highly curved classical path in phase-space!
- -- or if a shielding mechanism allows large first correction term in EFT.

conclusions

-- this model does not predict a large non-Gaussianity except:

-- for a highly curved classical path in phase-space!

-- or if a shielding mechanism allows large first correction term in EFT.

-- different shapes of non-Gaussianity are correlated!

-- in contrast to Senatore & Zaldarriaga, we suggest EFT for multi-filed inflation should be constructed as

$$\Delta\mathcal{L} \propto \sum c_{n_0,n_1,\ldots,n_N} \left(\vec{T}.\dot{\vec{\delta}}\right)^{n_0} \left(\vec{N}_1.\dot{\vec{\delta}}\right)^{n_1} \left(\vec{N}_2.\dot{\vec{\delta}}\right)^{n_2} \ldots \left(\vec{N}_N.\dot{\vec{\delta}}\right)^{n_N}$$

where small latin indexes run from 1 to 3. It is useful to join together this Lagrangian and the one from single field (5), and to split it into a quadratic and a cubic term. We obtain:

$$S^{(2)} = \int d^4x\sqrt{-g}\left[(2M_2^4 - M_{\rm Pl}^2\dot H)\dot\pi^2 + M_{\rm Pl}^2\dot H\frac{(\partial_i\pi)^2}{a^2} + 2\tilde M_1^{2I}\dot\pi\dot\sigma_I + (1+\tilde e_2^I)\dot\sigma_I\dot\sigma_I + \frac{\partial_i\sigma_I\partial_i\sigma_I}{a^2} + ...\right], \tag{10}$$

and

$$\begin{aligned} S^{(3)} = \int d^4x\sqrt{-g}\Big[&-2M_2^4\,\dot\pi\frac{(\partial_i\pi)^2}{a^2} + \left(2M_2^4 - \frac{4}{3}M_3^4\right)\dot\pi^3 + \\ &-(\tilde M_1^2 + 4\tilde M_2^2)^I\dot\pi^2\dot\sigma_I - \tilde M_1^{2I}\frac{(\partial_i\pi)^2}{a^2}\dot\sigma_I - 2\tilde M_1^{2I}\dot\pi\frac{\partial_i\pi\partial_i\sigma_I}{a^2} \\ &2\,(e_2 - e_3 + e_4)^{IJ}\,\dot\pi\dot\sigma_I\dot\sigma_J - 2e_4^{IJ}\dot\pi\frac{\partial_i\sigma_I\partial_i\sigma_J}{a^2} - 2\tilde e_2^I\frac{\partial_i\pi\partial_i\sigma_I}{a^2}\dot\sigma_I \\ &+\left(\tilde M_4^{-2} - \tilde M_3^{-2}\right)^{IJK}\dot\sigma_I\dot\sigma_J\dot\sigma_K - \tilde M_4^{-2,\,IJK}\dot\sigma_I\frac{\partial_i\sigma_J\partial_i\sigma_K}{a^2} + \dots\Big]. \end{aligned} \tag{11}$$

In both equations, . . . represent higher derivative terms or terms that break the shift symmetry. Let us analyze the quadratic and the cubic Lagrangian separately.

- **Quadratic Lagrangian**

In the π Lagrangian the term in $(\delta g^{00})^2$ induces a speed of sounds different from one for the π Goldstone boson. Because the Lorentz symmetry is spontaneously broken, a speed of sound equal to one is not protected by any symmetry [1]. The same is true for the σ_I fields. In addition to the standard Lorentz invariant kinetic term for the σ_I's the operator proportional to $\tilde e_2$ generates an additional time-kinetic term. This has the effect of changing the speed of