Title: Renormalization - 3

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Abstract:

On the running of the coupling constants in gravity

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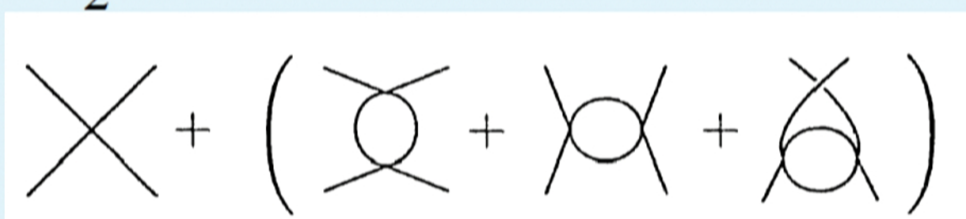
LOOPS 2013

M.A., J. Donoghue, M. El-Houssiney; Arxiv: 1101.3229
M.A., J. Donoghue; Arxiv: 1111.2875

RG in field theory, review

- The perturbation expansion parameter is dimensionless parameter
- Perturbatively renormalized theory: only a finite number of counter terms for all loop orders
- E.g. $\ell = \frac{1}{2}\left(\partial_{\mu}\varphi\right)^{2} + \lambda\varphi^{4}$



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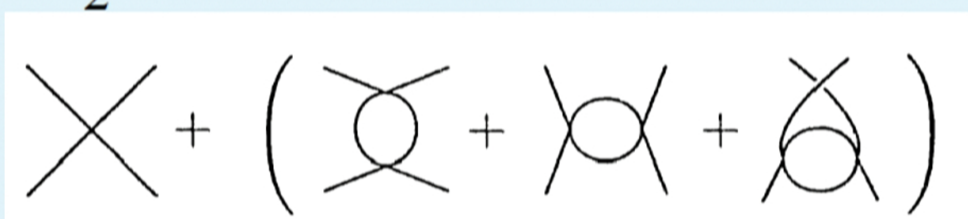

RG in field theory, review

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- Perturbatively renormalized theory: $\lambda\varphi^4$ theory

$$-iM = -i\lambda + \frac{3i\lambda^2}{32\pi^2}\left[\frac{2}{\varepsilon} + \log 4\pi - \gamma\right] - \frac{i\lambda^2}{2(4\pi)^2}\left[\log\left(\frac{-s}{\mu^2}\right) + \log\left(\frac{-t}{\mu^2}\right) + \log\left(\frac{-u}{\mu^2}\right)\right]$$

renormalize at $s = t = u = -4M^2/3$

$$-i\lambda(M) = -i\lambda + \frac{3i\lambda^2}{32\pi^2}\left[\frac{2}{\varepsilon} + \log 4\pi - \gamma\right] - \frac{3i\lambda^2}{2(4\pi)^2}\log\left(\frac{4M^2}{3\mu^2}\right)$$

$$\beta(\lambda) = \frac{3\lambda^2}{16\pi^2}$$ Universal!

$$-iM = -i\lambda(E) - \frac{i\lambda^2(E)}{2(4\pi)^2}\left[\log\left(\frac{s}{2E^2}\right) + \log\left(\frac{-t}{2E^2}\right) + \log\left(\frac{-u}{2E^2}\right) + i\pi\right]$$

RG in field theory, review

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- The same result if we perform on-shell renormalization

$$s = 2E^2, t = u = -E^2, \quad \log(-s) = \log s + i\pi$$

- Beta-function is universal, it sums the logs and it is robust against symmetry crossing problem
- Non-analytic pieces: $\log(-q\hat{}2)$ are long range quantum effects

RG in field theory, review

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Effective field theory

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- We do not have a dimensionless expansion parameter, instead we expand in powers of $\frac{E}{\Lambda}$
- Effective field theories are not perturbatively renormalized: you need an infinite number of counter terms

Gravity as an effective field theory

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- Einstein gravity is an effective field theory:

$$\ell = \Lambda + \frac{2}{\kappa^2} R \quad , \kappa^2 = 32\pi G$$

- We take $\frac{E}{M_p}$ as our expansion parameter
- Gravity is not perturbatively renormalized: loop corrections demand an infinite number of counter terms

$$\ell = \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots$$

- Pure gravity is finite to one-loop: since $R_{\mu\nu} = 0$

Standard results in quantum gravity

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- Quantum corrections to lower order operators are absorbed in higher order operators

Graviton propagator

scalar

't Hooft and Veltman
dim reg

$$c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu}$$

- Non-local remnants will have effect on the low energy physics:

$$\log(-q^2), \sqrt{-q\hat{}2}$$

Quantum piece

Classical piece

M. Anber, Loops 2013 7/26/2013

Running couplings in gravity

- Does the inclusion of gravity improves the UV behavior of the coupling constant of non-asymptotically free theory, like QED or Yukawa? Asymptotic safety people, Robinson and Wilczek 2006

- Since this work, many other works appeared : run or not to run? Improve or not?

- Does the running make sense in the presence of gravity?

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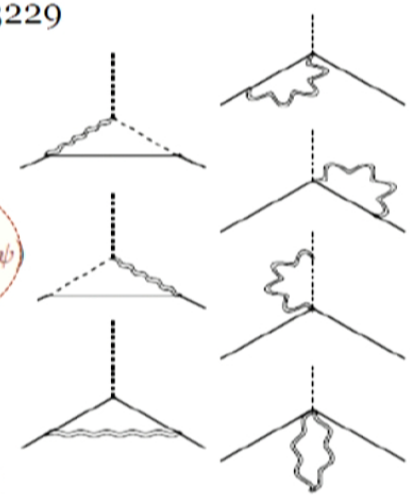
Running couplings in gravity

- Yukawa + gravity
  $g\varphi\bar{\psi}\psi$

M.A., J. Donoghue, M. El-Houssiney; Arxiv: 1101.3229

$$\begin{aligned}\mathcal{A} &= -ig + iM^2\left(2\xi_1 g_2 - \xi_2 g_3\right) + g_1 p_{1\mu} p_{2\nu}\sigma^{\mu\nu} \\ &- \frac{g\kappa^2}{8(4\pi)^2}\left[-\frac{\eta\xi_1}{\xi_1-\eta\xi_2}\mathcal{S}_1 + \frac{\xi_2}{\xi_1-\eta\xi_2}\mathcal{S}_2\right] p_{1\mu}p_{2\mu}\sigma^{\mu} \\ &+ \frac{5ig^3}{2(4\pi)^2}\mathcal{S}_1 - \frac{ig\kappa^2}{4(4\pi)^2}\left(\xi_1+\eta\xi_2\right)M^2\mathcal{S}_2 \\ &+ \frac{ig\kappa^2}{4(4\pi)^2}\left(\xi_1+2\eta\xi_2\right)M^2\mathcal{S}_1\,, \qquad (43)\end{aligned}$$

$$\begin{aligned}\mathcal{O}_1 &= \phi\partial_\mu\bar{\psi}\sigma^{\mu\nu}\partial_\nu\psi\,, \\ \mathcal{O}_2 &= \phi\left(\bar{\psi}\partial^2\psi + \partial^2\bar{\psi}\psi\right) \\ \mathcal{O}_3 &= \phi\partial_\mu\bar{\psi}\partial^\mu\psi\,.\end{aligned}$$



$\eta = 1$, time-like

$\eta = -1$, space-like

$$\mathcal{S}_1 = \left[\frac{2}{\epsilon} - \gamma + \log 4\pi - \log\left(\frac{\xi_1 M^2}{\mu^2}\right)\right],$$

$$\mathcal{S}_2 = \left[\frac{2}{\epsilon} - \gamma + \log 4\pi - \log\left(\frac{2(\xi_1+\eta\xi_2)M^2}{\mu^2}\right)\right]$$

$p_1^2 = p_2^2 = -\xi_1 M^2$, and $p_1 \cdot p_2 = -\xi_2 M^2$

Running couplings in gravity

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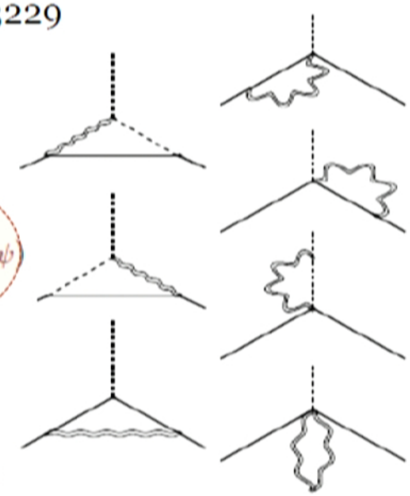

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$p_1^2 = p_2^2 = -\xi_1 M^2$, and $p_1 \cdot p_2 = -\xi_2 M^2$

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Running couplings in gravity

- Trying to define a running coupling:

$$\mathcal{A} = -ig(M) + iM^2\left(2\xi_1 g_2(M) - \xi_2 g_3(M)\right) + g_1(M) p_{1\mu} p_{2\nu} \sigma^{\mu\nu} .$$

$$g_1(M) = g_1 - \frac{\eta g \kappa^2}{8\,(4\pi)^2} \log\left(\frac{M^2}{\mu^2}\right) .$$

$$\frac{\partial \mathcal{A}}{\partial(\xi_1 M^2)} = 2ig_2(M)$$

$$\frac{\partial \mathcal{A}}{\partial(\xi_2 M^2)} = -ig_3(M) .$$

$$\beta(g) = \frac{5g^3}{(4\pi)^2} - \frac{\eta \xi_2 g \kappa^2}{2\,(4\pi)^2} M^2 ,$$

$$\beta(g_1) = -\frac{\eta g \kappa^2}{4\,(4\pi)^2} ,$$

$$\beta(g_2) = \frac{5g^3}{(4\pi)^2 \xi_1 M^2} ,$$

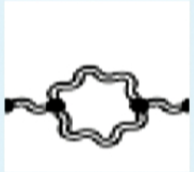
$$\beta(g_3) = \frac{\eta g \kappa^2}{2\,(4\pi)^2} .$$

Non-universal
Symmetry crossing problem!

- The running of the coupling does not make sense!

Running of the gravitational coupling

- Let us see if we can define a running of the gravitational coupling κ M.A., J. Donoghue; Arxiv: 1111.2875
- 1) vacuum polarization



$$G(q^2) = G\left[1 + \frac{1}{60\pi}Gq^2 \ln\left(\frac{-q^2}{\mu_2^2}\right) + \frac{7}{10\pi}Gq^2 \ln\left(\frac{-q^2}{\mu_1^2}\right)\right].$$

For space like: increase in G
For time like: decrease in G
In Euclidean: increase in G

Running of the gravitational coupling

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$$\left(\frac{d\sigma}{d\Omega}\right)_{tree} + \left(\frac{d\sigma}{d\Omega}\right)_{rad.} + \left(\frac{d\sigma}{d\Omega}\right)_{nonrad.} =$$

Donoghue and Torma

$$= \frac{\kappa^4 s^5}{2048\pi^2 t^2 u^2}\left\{1 + \frac{\kappa^2 s}{16\pi^2}\left[\ln\frac{-t}{s}\ln\frac{-u}{s} + \frac{tu}{2s^2} f\left(\frac{-t}{s}, \frac{-u}{s}\right)\right.\right.$$

$$\left.\left. - \left(\frac{t}{s}\ln\frac{-t}{s} + \frac{u}{s}\ln\frac{-u}{s}\right)\left(3\ln(2\pi^2) + \gamma + \ln\frac{s}{\mu^2} + \frac{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(1)}(\gamma_{ij})}{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(0)}(\gamma_{ij})}\right)\right]\right\}$$

$$s = 2E^2,\ t = u = -E^2$$

$$G^2(E) = G^2\left[1 + \frac{\kappa^2 E^2\left(\ln^2 2 + \frac{1}{8}\left(\frac{2297}{180} + \frac{63\pi^2}{64}\right)\right)}{8\pi^2}\right]$$

G increases with increasing E

$$\mathcal{A}(+,-;+,-)$$

$$1 + \frac{\kappa^2 E^2\left(\frac{29}{10}\ln 2 - \frac{67}{45}\right)}{16\pi^2}$$

G increases with increasing E
But different coefficient

Non-universality!

Running of the gravitational coupling

$$s = 2E^2,\ t = u = -E^2.$$

$$\mathcal{M}_{total} = \mathcal{M}_{tree} + \mathcal{M}_h = i\frac{9\kappa^2 E^2}{8}\left[1 - \frac{\kappa^2 E^2}{360\,(4\pi)^2}\left(609\ln\frac{E^2}{\mu^2} + (340\pi^2 + (123 - 340\ln 2)\ln 2)\right)\right]$$

$$G(E) = G\left[1 - \frac{\kappa^2 E^2}{360\,(4\pi)^2}\left(609\ln\frac{E^2}{\mu^2} + (340\pi^2 + (123 - 340\ln 2)\ln 2)\right)\right]$$

G decreases with increasing E!
Opposite to the pure gravity case!

Running of the gravitational coupling

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- Different scalars (only t-channel)

$$A + B \to A + B$$

$$\mathcal{M}_{total} = \frac{i\kappa^2 E^2}{2}\left[1 - \frac{\kappa^2 E^2}{10(4\pi)^2}\left((19 + 10\ln 2)\ln\left(\frac{E^2}{\mu^2}\right) + 5\left(\pi^2 - (\ln 2 - 1)\ln 2\right)\right)\right]$$

G decreases with increasing E!

$$A + A \to B + B$$

Symmetry crossing

$$\mathcal{M}_{total} = \frac{i\kappa^2 E^2}{8}\left[1 + \frac{\kappa^2 E^2}{10(4\pi)^2}\left(9\ln\left(\frac{E^2}{\mu^2}\right) - 5\pi^2 + (19 + 5\ln 2)\ln 2\right)\right]$$

G increases with increasing E!

Crossing symmetry problem!

Conclusion

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- No universal and useful definition of running constants in gravity, at least in the perturbative region!
- Raises questions about nonperturbative attempts in gravity: asymptotic safety program

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Dilaton Quantum Gravity

A Functional Renormalization Group Approach

Tobias Henz

with Jan Martin Pawlowski, Andreas Rodigast & Christof Wetterich

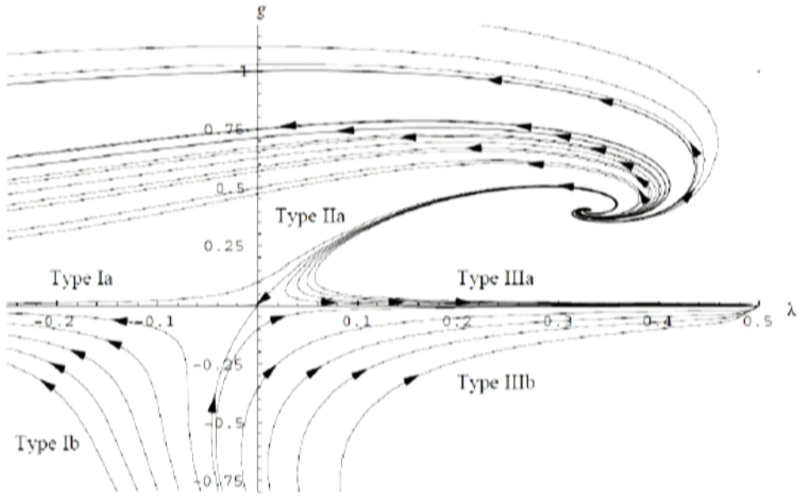
Institute for Theoretical Physics, Heidelberg University

based on arXiv:1304.7743

Loops'13

Perimeter Institute, July 2013

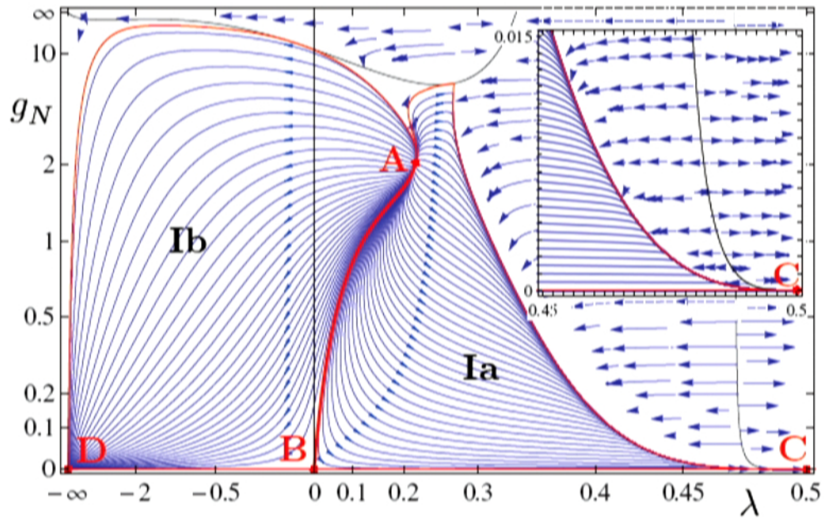
The Flow Diagram of Quantum Einstein Gravity



Reuter, 1998

Coupling Constants approach a nontrivial UV Fixed Point
$\Longrightarrow$ Prospect of Gravity being Asymptotically Safe

The Flow Diagram of Quantum Einstein Gravity



Christiansen, Litim, Pawlowski, Rodigast, 2012

Stable Infrared Scenarios

$\Longrightarrow$ Prospect of UV and IR consistent theory

Asymptotically Safe Quantum Gravity

FRG Technicalities	Coupling to SM
• Regulator Dependence • Background Dependence • Truncation Stability	• Yang Mills Theory • Background Dependence • Asymptotic Freedom

Asymptotically Safe Quantum Gravity

FRG Technicalities

- Regulator Dependence
- Background Dependence
- Truncation Stability

Coupling to SM

- Yang Mills Theory
- Background Dependence
- Asymptotic Freedom

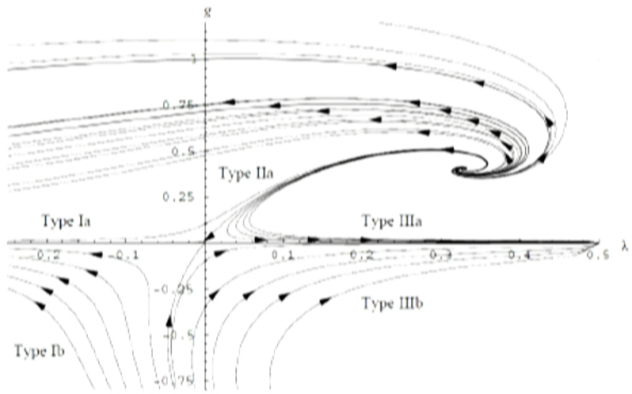
Infrared Limit

- Trajectory UV $\rightarrow$ IR
- GR as limiting case?

Hierarchy Problem

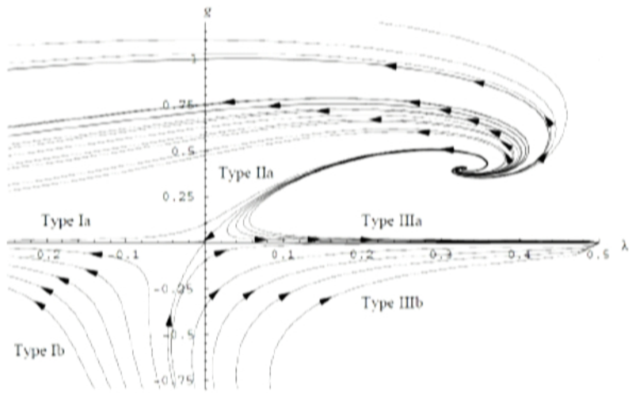
- $M_{SM} \approx 10^2$ GeV
- $M_{Planck} \approx 10^{19}$ GeV

What defines a fixed point?



⇒ realistic gravity on pure fixed point trajectory
physical interpretation for RG scale k

What defines a fixed point?



Naturally, $\partial_t \tilde{g}_i = 0$.

$\Rightarrow$ at a fixed point, the scale becomes irrelevant

Strategy: encode scale in a real scalar field χ $[\chi] = k$

on the fixed point	$\leftrightarrow$	away from fixed point
dilatation symmetry intact		dilatation symmetry broken
scale invariance		nonzero Planck mass

$\Rightarrow$ realistic gravity on pure fixed point trajectory
physical interpretation for RG scale k

Setup & Action

$$\Gamma_k\left[g_{\mu\nu}\right] = \frac{1}{16\pi G_{N,k}} \int \mathrm{d}^d x \sqrt{g}\left(2\Lambda_k - R[g_{\mu\nu}]\right)$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$\Gamma_k\left[g_{\mu\nu},\chi\right] = \int \mathrm{d}^d x \sqrt{g}\left(V_k[\chi] - \frac{1}{2}F_k[\chi]R[g_{\mu\nu}] + \frac{1}{2}g_{\mu\nu}\partial^\mu\chi\partial^\nu\chi\right)$$

First investigated by Narain & Percacci, 2010
Extensions:

- investigation of IR limit
- trajectory UV $\leftrightarrow$ IR
- improved UV analysis

Setup & Action

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Dilatation Symmetry

- Dilatations $\leftrightarrow$ Conformal Transformations
  $g_{\mu\nu}(x) \mapsto \Omega(x) g_{\mu\nu}(x)$
  with $\Omega = const.$
- Dilatation $\leftrightarrow$ **global** resetting of the physical scale
- Dilatation Symmetry $\leftrightarrow$ Physical Scale is introduced only by expectation value of the scalar field
- Arising Goldstone Boson: Dilaton

Dilatation Symmetric Actions

Γ is invariant under dilatations

$\Updownarrow$

all couplings have scaling dimension 0.

$$d = 4 : \Gamma = \int \mathrm{d}^d x \sqrt{g} \left(v\chi^4 - \frac{1}{2} f \chi^2 R + \frac{1}{2} g_{\mu\nu} \partial^\mu \chi \partial^\nu \chi \right)$$

Physical Motivation

Infrared Limit	Hierarchy Problem
• Trajectory UV $\to$ IR	• $M_{SM} \approx 10^2$ GeV
• GR as limiting case?	• $M_{Planck} \approx 10^{19}$ GeV

Characterisation of Fixed Point via Dilatation Symmetry

a scale invariant IR limit for Einstein-Hilbert Quatum Gravity

b generation of Planck mass via breaking of dilatation symmetry

c quintessence cosmology scenarios with vanishing cosmological constant

Fixed Point Action

Let $y = \frac{\chi^2}{k^2}$, $\quad V_k(\chi^2) = k^4 y^2\, v_k(y), \quad F_k(\chi^2) = k^2 y\, f_k(y)$

$\Rightarrow$ Dilatation Symmetric parts factored out

proposed fixed point for large y

$$\lim_{y\to\infty} f(y) = \xi \qquad \lim_{y\to\infty} v(y) = 0.$$

$$\Rightarrow \quad \Gamma = \int d^4x \sqrt{g} \left(\tfrac{1}{2} g^{\mu\nu} \partial_\mu \chi\, \partial_\nu \chi - \tfrac{1}{2} \xi \chi^2\, R \right) \qquad \text{(Jordan)}$$

$$\Rightarrow \quad \Gamma = \int d^4x \sqrt{g} \left(\tfrac{1}{2} g^{\mu\nu} \partial_\mu \phi\, \partial_\nu \phi - \tfrac{1}{2} M^2\, R \right) \qquad \text{(Einstein)}$$

Properties of the Fixed Point

- Let $f_0 = \lim_{y\to\infty} f_k(y)$

$\Rightarrow$ Gravitational Interaction scales like $f_0^{-1}\chi^{-2}$

$\Rightarrow$ Gravity induced flow from $f_0^{-1}y^{-1}$ with $\lim_{y\to\infty} f_0^{-1}y^{-1} = 0$

$\Rightarrow$ For $\lim_{y\to\infty} v_k(y) = 0$ free scalar theory

- Flow of free scalar theory can at most produce a constant term $\propto v_{-2}$ in V and
- For $g_N = 0$ no flow of gravitational interaction

$\Rightarrow$ Asymptotically

$$\lim_{y\to\infty} v(y) = v_{-2}\, y^{-2} + \dots$$
$$\lim_{y\to\infty} f(y) = \xi + f_{-1}\, y^{-1} + \dots$$

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Fixed Point Solutions

1 for $y = \chi^2/k^2 \to 0$ or ∞: Taylor Expansions in y or y^{-1}

Infrared: $y \to \infty$

- $\chi \to \infty$ or $k \to 0$
- finite limits for flow generators ζ_V, ζ_F
- closed set of flow equations

$\Rightarrow$ $\lim_{y\to\infty} v = 0$, free parameter $\xi = \lim_{y\to\infty} f$

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$\Rightarrow$ $\lim_{y\to\infty} v = 0$, free parameter $\xi = \lim_{y\to\infty} f$

Ultraviolet: $y \to 0$

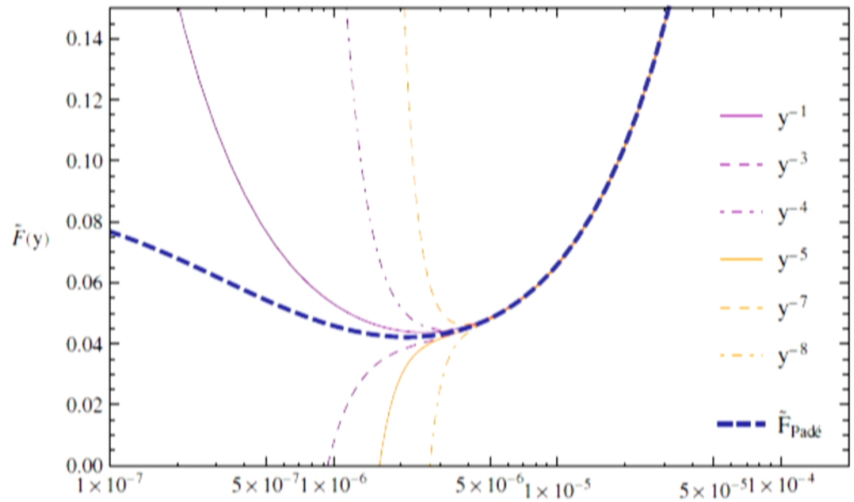
- $\chi \to 0$ or $k \to \infty$
- set of flow equations not closed
- treat V_0 and F_0 as "free" parameters

Closing the Gap: Possible Global Solutions

a Einstein-Hilbert Gravity + Scalar Field

$\Leftrightarrow \tilde{V} = y^2 v(y) = 0.008620$ and $\tilde{F} = y f(y) = 0.04751$

b Continuations of Taylor Expansions around $y = \infty$



Closing the Gap: Possible Global Solutions

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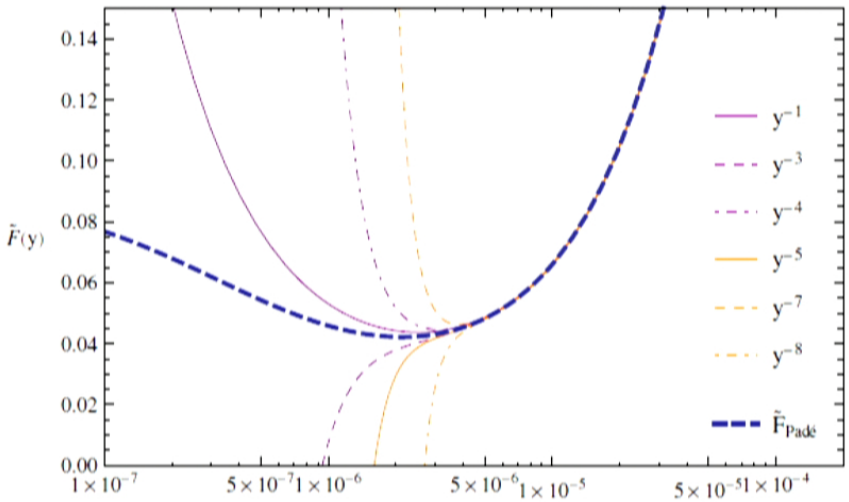


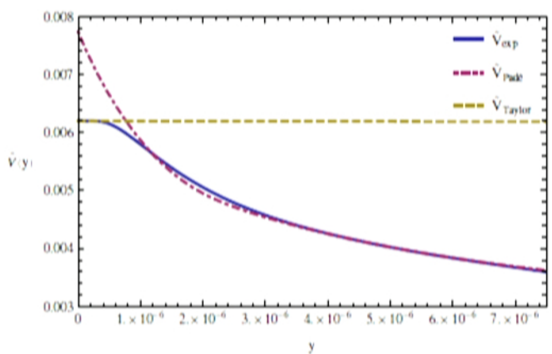
Padé ressumation

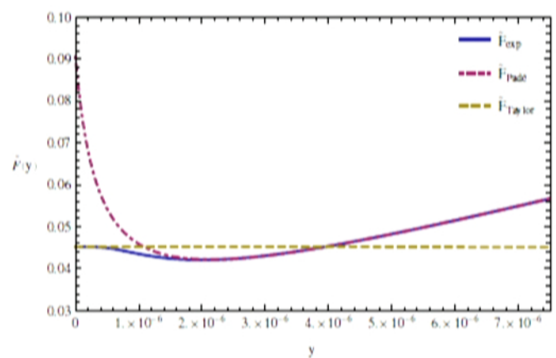
Strategy:

- stick with Taylor Solution for as long as possible
- smoothen solution for $y \to 0$
- connect to known solution for small y

Padé resummation:

- Expand both numerator and denominator in powers of y^{-1}
- dependency on parameter ξ remains





Physical Implications, Summary & Outlook

- Dilatation symmetric infrared limit due to
$$\lim_{y\to\infty} V = 0, \quad \lim_{y\to\infty} F = \xi\chi^2$$
- Planck mass is generated by spontaneous breaking of Dilatation Symmetry via $\langle\chi\rangle \neq 0$
- Realistic gravity possible on a pure fixed point trajectory
- quintessence cosmology for small deviations from fixed point and $y \gg 1$
- cosmological constant vanishes on and away from FP in the IR

- different classes of solutions / enlarged truncation
- wave function renormalisation and flat backgrounds
- adding matter

Thank you!

Fixed points of f(R) quantum gravity

Loops 13, Perimeter Institute.

Kevin Falls, University of Sussex.

Search for quantum gravity

- We want to combine the theories of general relativity and quantum mechanics.
- Theory should:
  1) Be free from unphysical divergences
  2) Be predictive
  3) Recover classical general relativity at low energies
- Two lengths scales associated to GR and QM

$$r = 2GM \qquad \lambda = \frac{\hbar}{M}.$$

- We expect quantum gravity to become important when $\frac{\lambda}{r} \propto \frac{\hbar G}{r^2} \approx 1$

$$M \approx M_P = \sqrt{\frac{\hbar}{G}}, \qquad \lambda \approx \ell_P = \sqrt{\hbar G}.$$

Beyond this scale fluctuations of spacetime become important.

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Gravity as a QFT: Absorbing divergences

- Does gravity exist as a fundamental quantum field theory in four dimensions?
- Gravity based on the Einstein-Hilbert action is power counting non-renormalisable.

$$[G] = -2$$

- Gomis and Weinberg '96: Gravity is renomalisable in the modern sense i.e upon the inclusions of an infinite number of terms in the bare action.
- This ensures that gravity exists as an effective field theory at low energies.

$$S = \int d^d x \sqrt{g} \left\{ \frac{\Lambda}{8\pi G_N} - \frac{1}{16\pi G_N} R + g_2 R^2 + g_{2b} R_{\mu\nu} R^{\mu\nu} + ... \right\}$$

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Asymptotic Safety: Restoring predictivity

- In order to restore predictivity at high energies Weinberg '79 proposed that gravity maybe asymptotically safe. This scenario requires the existence of a UV fixed point at high energies i.e.

$$\beta_i(\lambda_j^*) = 0 \qquad \text{for all (essential) } \mathcal{O}_i$$

with a finite number of relevant directions.

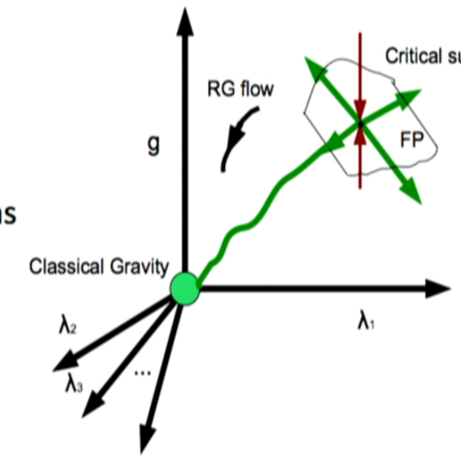
- Linearising around the fixed point

$$\beta_i(\lambda_j) = \sum_j M_{ij}(\lambda_j - \lambda_j^*)$$

- The general solution can be expressed as

$$\lambda_i - \lambda_i^* = \sum_A C_A V_i^A e^{t\vartheta_A}$$

$$t = \log k/k_0$$



- Number of eigenvalues $\mathrm{Re}(\vartheta_A) < 0$= Number of free parameters

Relevant operators

- Classically the relevance of an operator is determined by its canonical mass dimension

$$\beta_i = -d_i \lambda_i + \text{quantum corrections}$$

- In four dimensional gravity there are two local relevant operators $\sqrt{\gamma}$ and $\sqrt{\gamma}R.$
- Classical scaling is recovered at the IR fixed point $k \ll M_P$

$$g \to 0$$

- At a non-perturbative fixed point $g^* \neq 0$ quantum corrections will modify the beta functions.
- **Provided these corrections are bounded only a finite number can change sign.**
- Nonetheless there is no a-priori way to know which operators will be relevant. We must carry out a non-perturbative calculations.

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A bootstrap approach

- **Step 1)** Taking this to be true as a working hypothesis we can take the action to be approximated by a finite set of operators up to some maximum canonical mass dimension D.
- **Step 2)** The impact of the quantum fluctuations on the scaling exponents is quantified using methods e.g. RG, lattice, holography.
- **Step 3)** Increase the maximum D and repeat the analysis.
- Studying the convergence step by step the original assumption will be confirmed or refuted.

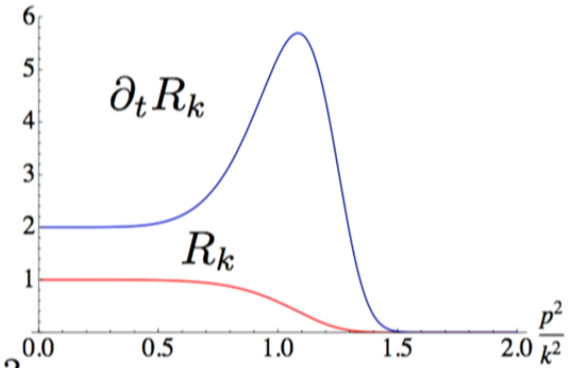
F(R) approximation

- Action

$$\Gamma_k = \int d^4x \sqrt{\gamma}\, F(R)\,.$$

- RG equation

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\frac{1}{\Gamma_k^{(2)} + R_k}\partial_t R_k\,,$$



- IR Regulator suppress modes $p^2 \ll k^2$
- For practical purposes use "optimized" regulator

$$R_k = (k^2 - p^2)\theta(k^2 - p^2)$$

- Effective average action

$$\begin{aligned} \Gamma_k[\phi] &\to S[\phi] \quad \text{for} \quad k^2 \to \infty\,, \\ \Gamma_k[\phi] &\to \Gamma[\phi] \quad \text{for} \quad k^2 \to 0 \end{aligned}$$

F(R) approximation

- Dimensionless quantities

$$\frac{F_k(R)}{k^4} = \frac{1}{16\pi} f(\rho)\,, \quad \rho = \frac{R}{k^2}$$

- Flow equation (Machado, Saueressig; Codello, Percacci, Rahmede)

$$\partial_t f + 4f - 2\rho\, f' = I[f]\,.$$

- LHS gives the canonical scaling, RHS encodes the fluctuations

$$I[f] = I_0[f] + \partial_t f'\, I_1[f] + \partial_t f''\, I_2[f]\,.$$

- Polynomial approximation

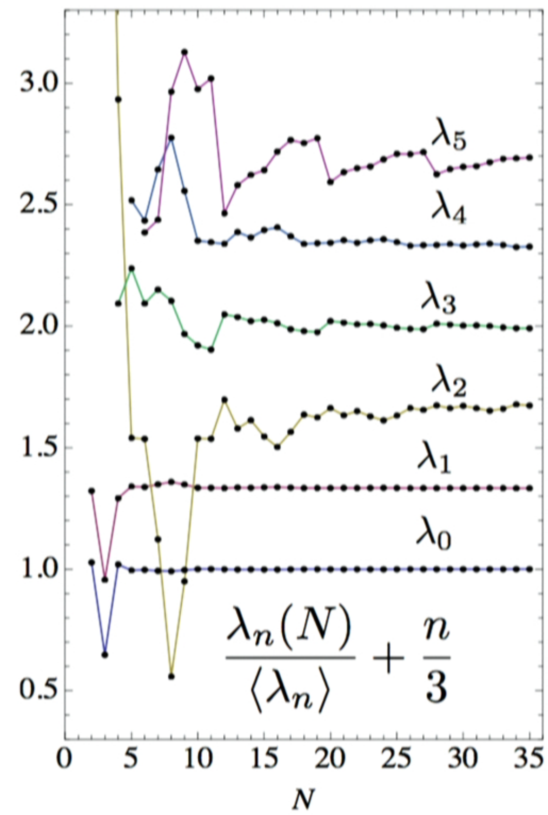
$$f(\rho) = \sum_{n=0}^{N-1} \lambda_n \rho^n \qquad g = -\frac{1}{\lambda_1} \qquad g_{\text{eff}} \equiv -\frac{1}{f'(\rho)}$$

UV fixed point in f(R) quantum gravity

- For a non-Gaussian fixed point we look for solutions to

$$4f_* - 2\rho\, f'_* = I_0[f_*]\,,$$

- Consistent fixed point found up to N=35. ArXiv: 1301.4191. KF, D. Litim, K. Nikolakopoulos and C. Rahmede.
- Previous results up to N=11 (Bonanno, Contillo, Percacci).
- Convergence of fixed point values.
- Three relevant directions!
- Convergence of critical exponents.



Critical exponents.

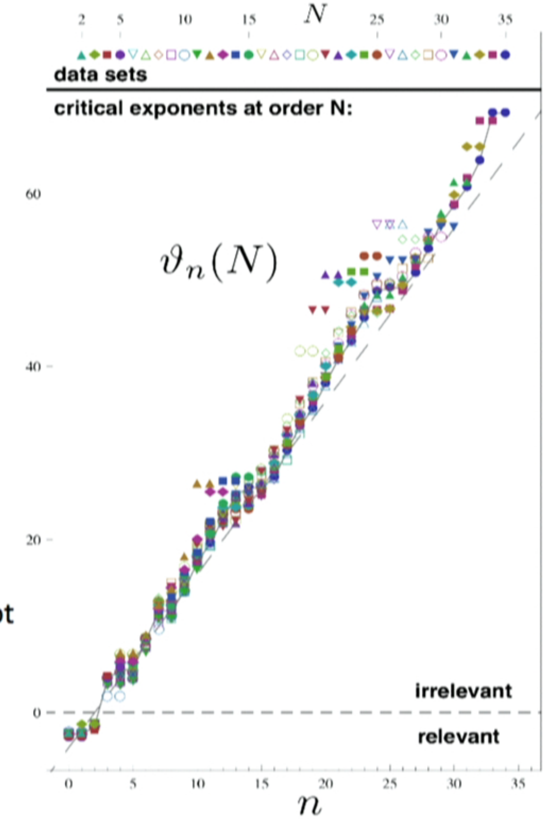
- Three relevant directions for N=3 to N=34.
- Complex conjugate pairs of critical exponents
- Increasing N leads to a new more irrelevant eigenvalue in agreement with our initial assumption.
- Near Gaussianity: Make a linear fit,

$$\vartheta_n \approx a \cdot n - b$$

- In the IR $a_{\mathrm{G}} = 2,\ b_{\mathrm{G}} = 4\,.$
- At the UV fixed point we obtain

$$a_{\mathrm{UV}} = 2.17 \pm 5\%$$
$$b_{\mathrm{UV}} = 4.06 \pm 10\%$$

- Extrapolating to higher orders we have good reason to believe that higher order terms will not introduce new relevant directions.



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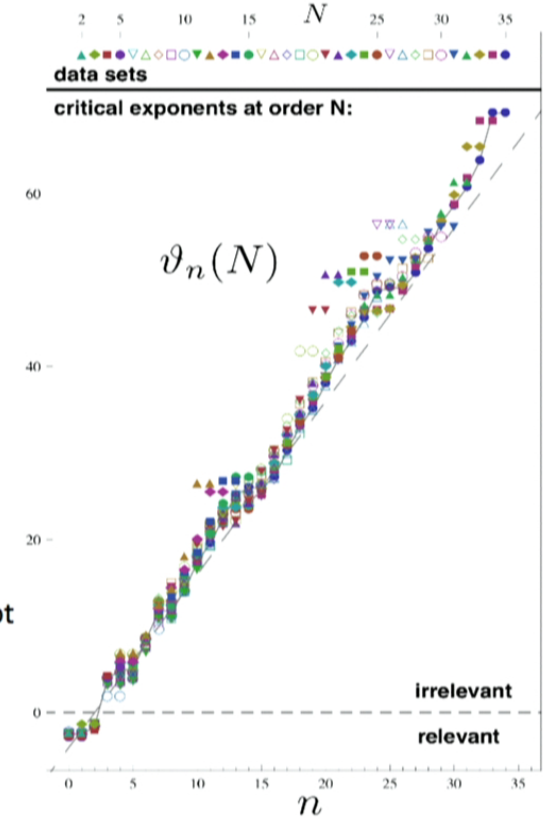


Flow into the IR: Classical limit

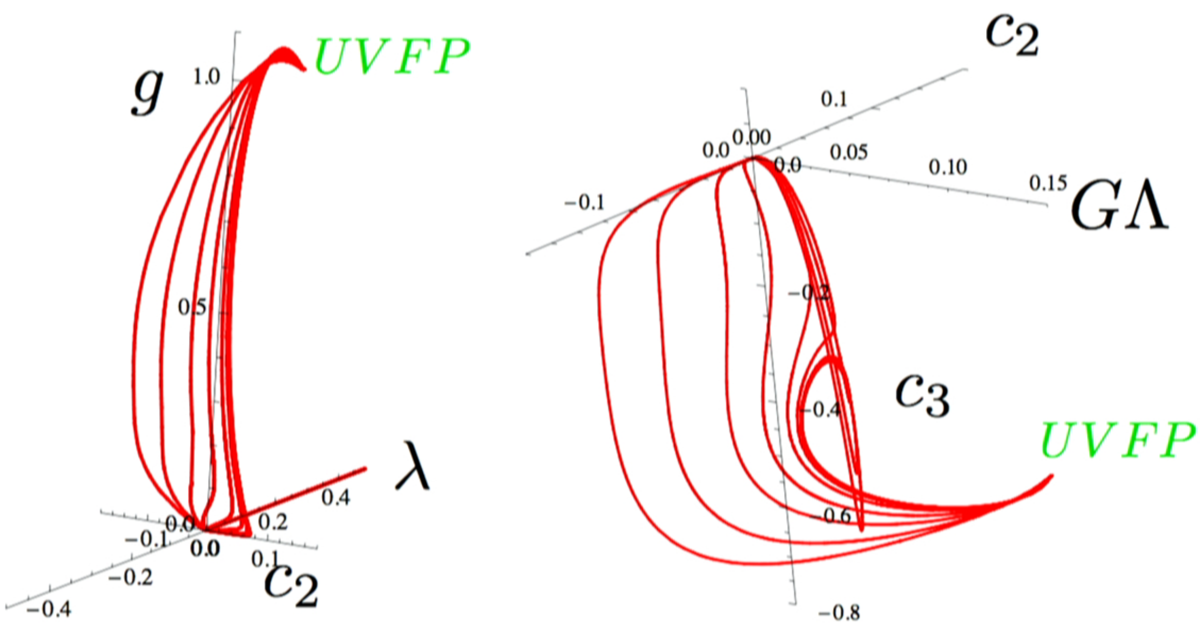
- Integrated the RG flow into the IR with R^3 approximation:

$$f(\rho) = \frac{1}{g}(2\lambda - \rho + c_2\rho^2 + c_3\rho^3)$$

- Gaussian FP where all couplings vanish
- Cosmological constant is an IR repulsive direction
- Dimensionless product $G\Lambda$ can be tuned to be arbitrarily small.
- The UV fixed point has one irrelevant direction.

Flow into the IR

- One parameter family of trajectories which go from UV fixed point to classical general relativity in IR.



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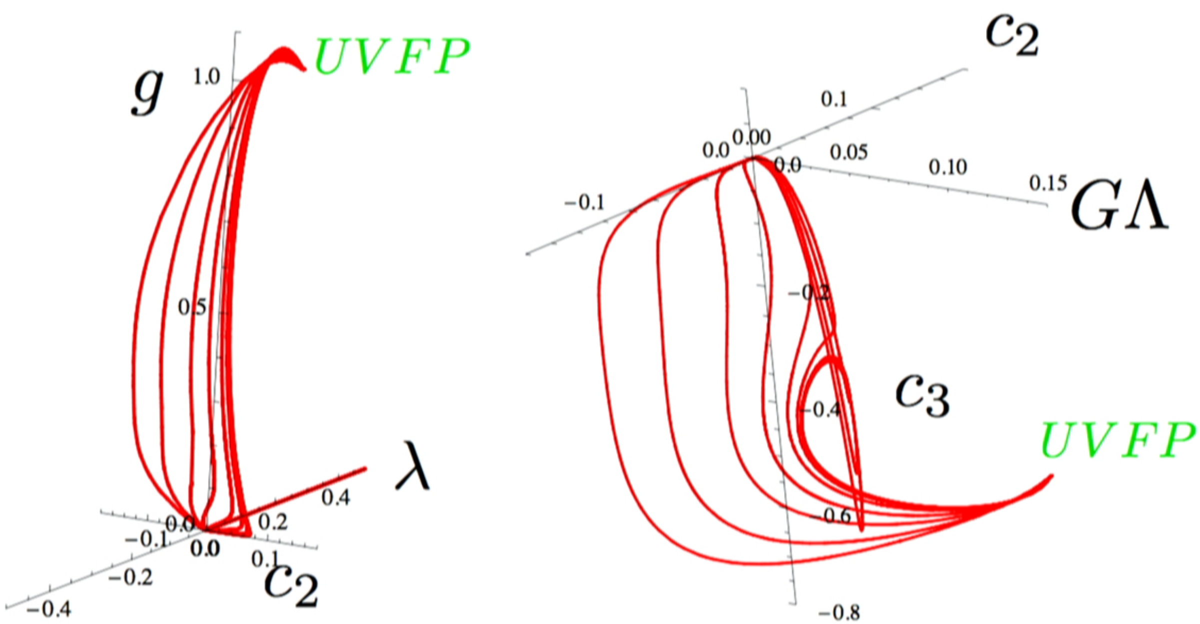

Conclusions and open questions

- Within the F(R) approximation a UV fixed point with near Gaussian exponents has been found.
- Three relevant directions
- Agreement with Benedetti arXiv:1301.4422 :

existence of a fixed point $\rightarrow$ finite number of relevant directions

- Classical limit is found R^3 approximation.
- Does this pattern carry over to the full theory?
- What about the inclusion of matter?