Title: Towards a Consistent Quantum Dynamics for Euclidean LQG: A Weak Coupling Limit

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Abstract: Spacetime covariance in canonical quantum gravity is tied to the existence of an anomaly free representation of its constraint algebra. I will argue that establishing the existence of such a representation in the LQG context requires the consideration of   higher than unit density weight Hamiltonian constraints. Smolin's weak coupling limit of Euclidean gravity, while simpler than full blown gravity , still exhibits an open constraint algebra isomorphic to that of gravity and offers an ideal testing ground for the investigation of the quantum constraint algebra of such higher density constraints. I will report on recent progress on this issue in the context of an LQG type quantization of this system. Certain features of the constructions such as the encoding of the action of the quantum constraint in terms of operator valued diffeomorphisms may play a key role in the definition of a consistent quantum dynamics for LQG.

Towards a Consistent Quantum Dynamics for Euclidean LQG: a weak coupling limit

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Introductory Remarks:

- In Hamiltonian formulation of generally cov theory, evolution is along foliation of background/emergent spacetime. Evolution along different foliations are consistent in sense that they yield same spacetime.
  Consistency coded in structure of classical constraint algebra (H-K-T). Expect sptime cov in quantum theory only if quantum constraint yield anomaly free repn of this algebra.
- In this talk, focus on most nontrivial part of the constraint algebra: commutator between a pair of Hamiltonian constraints.
- Status of Ham constraint in LQG:
  In the mid 1990's Thiemann constructed a Ham constraint for LQG in his seminal QSD papers.
  Very hard to do this.

Difficulty: C_{ham} made out of local fields like curvature. Basic oprtrs nonlocal holonomies. Classically can extract curvature from holonomy by shrinking loop down to point.
But limit not defined in quantum theory (background independence).So:

- Fix triangulation T_δ. Construct finite T_δ approximants to relevant local fields from holonomy- flux variables.
- From these, construct C_{ham,T_δ} at finite triangulation.
- Replace holonomy- flux by corresponding operators, obtain $\hat{C}_{ham,T_\delta}$ and construct its continuum limit.
- Limit exists on diffeo inv states. Commutator vanishes. (TT). Internally consistent. All well?

- My view: The calculation is not an effective probe for the existence (or not) of an anomaly.

- Consider simplest implementation of TT Ham constraint:
  Let $[\hat{C}_{ham}(N), \hat{C}_{ham}(M)] =$ LHS.
  Let $\{C_{ham}\widehat{(N), C}_{ham}(M)\} =$ RHS.

- LHS=0: Second constraint doesnt act on deformations generated by first.

- RHS=0: "Too many factors of δ":
  $d^3x \sim \delta^3 \quad \hat{E}^a_i \sim \frac{\widehat{\text{flux}}}{\delta^2} \quad \widehat{\sqrt{q}} \sim \frac{\widehat{\text{Volume}}}{\delta^3} \quad \hat{F}^i_{ab} \sim \frac{\widehat{\text{holonomy}}}{\delta^2}.$
  RHS$= \delta \times (\widehat{\text{Diffeo}} - \mathbf{1})$

- What if states NOT diffeo inv?: No anomaly. (L-M)
  What if states not diff inv AND if more factors of δ^{-1}?: could be an anomaly (L-M-G-P)

Higher Density Wt Constraints

- Recall that $\widehat{\sqrt{q}} \sim \frac{\widehat{\text{Volume}}}{\delta^3}$
- Higher density weight $\rightarrow$ more powers of $\widehat{\sqrt{q}}$ $\rightarrow$ more powers of δ^{-1} $\rightarrow$ RHS $\neq 0$.
- Question: Is LHS also non- zero for higher density wt constraints? If so, is LHS= RHS?
- Moral of this story: To meaningfully probe the question of existence of anomaly, one must employ higher than unit density wt Ham constr.
- TT: density wt 1 Ham constr $= \widehat{\text{Finite}}$.
  So higher density= $\frac{\widehat{\text{Finite}}}{\delta^m}$: singular!

Lessons from Toy Models:

- Sanity Check: Toy Models:
  PFT, H-K models. (Alok Laddha, MV)

- Non -trivial anomaly free constr algebra requires singular constr oprtrs.

- Constraint action in terms of spatial diffeos:
  $\hat{C}_\delta(N) = \frac{1}{\delta}(\widehat{\mathrm{smalldiffeo}} - \mathbf{1})$

- $\delta \to 0$ limit well defined on 'off shell' states $\{\Psi_f\}$. Each $\Psi_f =$ non- normalizable sum of spin nets labelled by a smooth function f on spatial manifold.

- $\hat{C}_\delta(N)\Psi_f = \Psi_g$, $g = \frac{f(x_\delta)-f(x)}{\delta} \to \mathcal{L}_N f$.
  This yields nontrivial repn of constr algebra.

- In PFT, H-K constr algebras are Lie algebras whereas gravity has structure fns + other complications.
- Laddha replaced $SU(2)$ triad rotation group of Euclidean gravity by $U(1)^3$. Constr algebra isomorphic to gravity. Same as Smolin's $G_{\mathrm{Newton}} \rightarrow 0$ limit of Euclidean gravity(Campiglia).
- This talk: examine constraint algebra in LQG type quantztn of this $U(1)^3$ model.
- Henderson- Laddha- Tomlin :2+1.
  Here 3+1 (Tomlin-MV). If time: diffeo cov (MV).

Strategy:

- Use high density $\hat{C}$ so as to get $\hat{C}_\delta \sim \frac{\widehat{\mathtt{Finite}-\mathbf{1}}}{\delta}$.
- Code action in terms of diffeos.
- Define $\delta \to 0$ by using suitable $\{\Psi_f\}$.
- Whenever pble use structures generelizable to $SU(2)$.
- KEY POINT:Diffeos move vertices of spinnets. In LHS 2nd constr acts on moved vertices generated by first. Movement manifest thru derivative of f from δ^{-1}. In RHS δ^{-1} factors cancel "too many δ". LHS, RHS both nontrivial.


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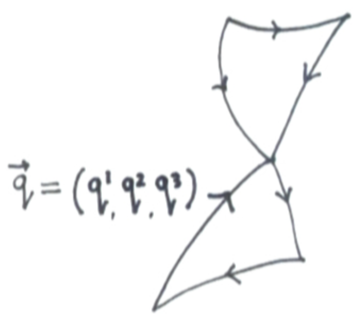
The $U(1)^3$ Theory:

- **Phase Space:** Triplet of $U(1)$ elec fields E^a_i, conj. $U(1)$ conncts A^i_a, $i = 1, 2, 3$.
- **Constraints:** $\partial_a E^a_i = 0 \quad E^a_i F^i_{ab} = 0 \quad \frac{1}{\sqrt{q}}\epsilon^{ijk} E^a_i E^b_j F_{abk} = 0$,
  $F^i_{ab} := \partial_a A^i_b - \partial_b A^i_b$
- **Quantum kinematics:**
  $|\alpha, \{\vec{q}\}\rangle \equiv \psi_{\alpha,\{\vec{q}\}}(A) =$ product of edge holonomies.



The Hamiltonian Constraint:

- Density Weight for '$\frac{1}{\delta}$':
  Wt $4/3$ constraint, lapse wt $-1/3$.
  $C(N) = \int \epsilon^{ijk} (\frac{NE_i^a}{q^{\frac{1}{3}}}) F_{abk} E_j^b$
- Action thru diffeos: $N^a F_{ab}^i = \mathcal{L}_{\vec{N}} A_b^i - \partial_b (N^c A_c^i)$.
  'Electric Shift' $N_i^a := \frac{NE_i^a}{q^{\frac{1}{3}}}$.

$$C(N) = \int \mathcal{L}_{\vec{N}_i} A_{bk} \, \epsilon^{ijk} \, E_j^a + \text{Gauss Law.}$$

$$\hat{C}(N)\psi(A) = -i \int \mathcal{L}_{\vec{N}_i} A_{bk} \, \epsilon^{ijk} \, \frac{\delta\psi(A)}{\delta A_j^a}$$

$$\hat{C}_\delta(N)\psi(A) \sim \frac{1}{\delta} \left(\psi(A_a^j + \delta \mathcal{L}_{\vec{N}_i} A_{bk} \, \epsilon^{ijk}) - \psi(A_a^j) \right)$$

Note that this is not just a diffeo since 'internal' indices are being shuffled.

Let ψ be a chargenet state. Then it turns out that $\hat{C}_\delta(N)$ deforms state by combination of diffeos + charge reshuffling.
"Ham constraint= Diffeos + Charge Flips"

The Quantum Shift:

- The Quantum Shift: $\hat{N}_i^a(x) = N\hat{E}_i^a \widehat{\frac{1}{q^{1/3}}}$

  $\widehat{\frac{1}{q^{1/3}}}$ non zero only at vertices. At vertex v:

  $N\hat{E}_i^a \widehat{\frac{1}{q^{1/3}}} |\alpha, \{\vec{q}\}\rangle \sim \quad N(v)\big(\sum_I \; q_I^i \hat{e}_I^a\big) \; \lambda_v \, |\alpha, \{\vec{q}\}\rangle$

- Coordinate Dependence: Need coordinate patch to regulate, define quantum shift. $N(v)\hat{e}_{I_v}^a$ **coordinate dependent**.

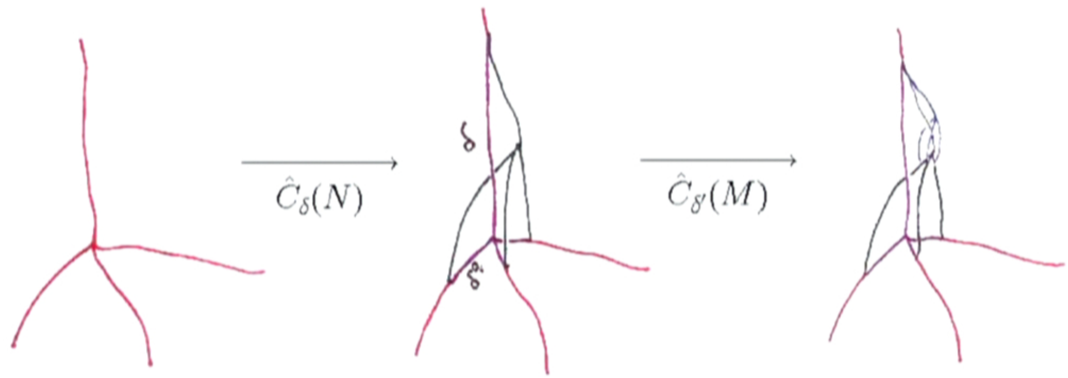
- Deformations generated by Quantum Shift:

  Fix i. M- valent vertex.

  Quantum Shift$=$ Sum of M vectors, Ith vector along Ith edge.

  Yields sum of M deformed chargenets, Ith one deformed along Ith edge.

State Deformations:



- (LHS) is a commutator between 2 C's. At finite triangulation: $(LHS)_{\delta,\delta'} := \hat{C}_{\delta'}(M)\hat{C}_\delta(N) - M \leftrightarrow N$

RHS:A Remarkable Identity.

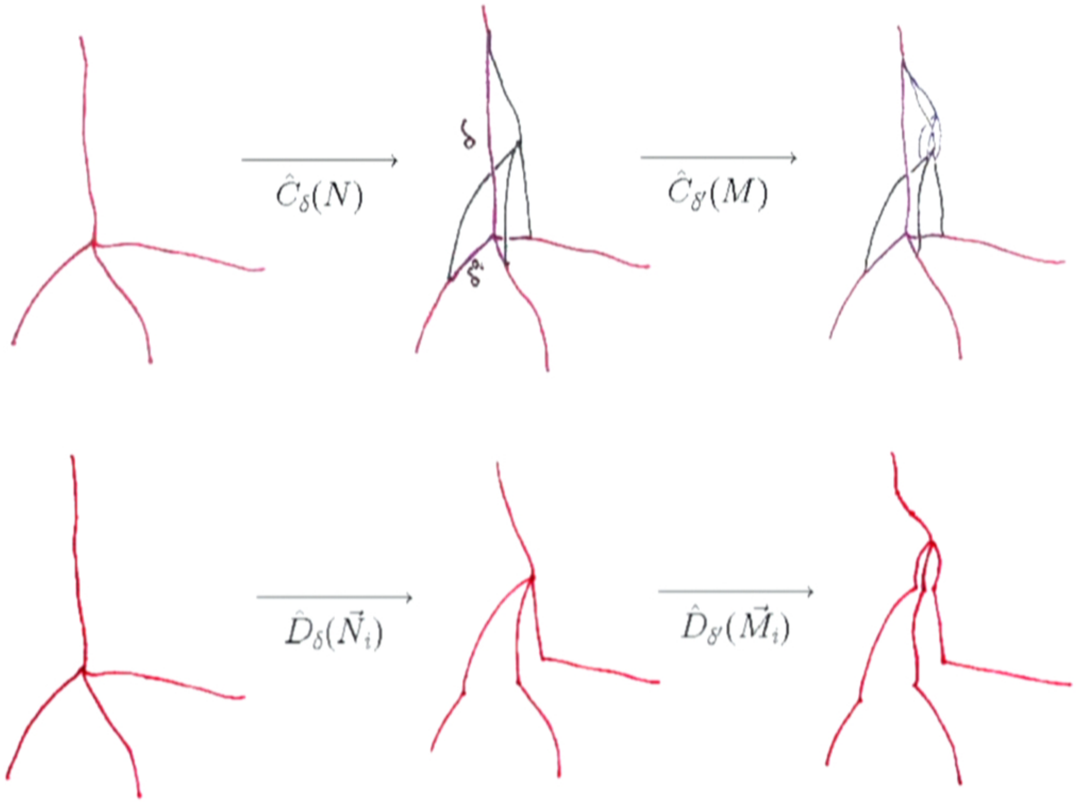
- $(RHS) = \{\widehat{C(M), C}(N)\} \sim$ Diffeo Constraint.
  Could try to write $(RHS)_\delta = \frac{\widehat{\text{Diffeo}}-\mathbf{1}}{\delta}$.
- But, if we could write (RHS) also as commutator, then easier to compare $(RHS)_{\delta,\delta'}, (LHS)_{\delta,\delta'}$. Can we?
- Consider Ham constr of density wt $2(1-\alpha)$.
  Define $M_i^a := ME_i^a q^{-\alpha}$.
  Let $D(\vec{M}_i)$ be diffeo constraint smeared with M_i^a.
- Beautiful Identity:
  $(2\alpha - 1)\ \{C(M), C(N)\} \ = \ \sum_{i=1}^{3}\ \{D(\vec{M}_i), D(\vec{N}_i)\}$.

- Valid also for $SU(2)$.
- Identity **trivialises** if $\alpha = 1/2$ i.e. density 1!

- Our case: $\alpha = 1/3$: $\quad (RHS)_{\delta,\delta'} = -3\big(\widehat{D_{\delta'}(\vec{M}_i)}\widehat{D_\delta(\vec{N}_i)} - M \leftrightarrow N\big)$

Action of $\widehat{D(\vec{N}_i)}$:

- $\widehat{D(\vec{N}_i)}$ exactly like $\hat{C}(N)$ without charge flips i.e. only 'diffeo'.



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The Continuum Limit

- Define Off Shell States Ψ_f, $f : \Sigma^n \rightarrow \mathbf{C}$.
  $(\Psi_f| := \sum \langle \bar{s}| f(\text{vertices of } \bar{s})$.
  Sum over set of bras B.
- Cont. Lim of LHS: Compute: $\lim_{\delta\rightarrow 0} \lim_{\delta'\rightarrow 0} (\Psi_f|(LHS)_{\delta,\delta'}|s\rangle$
  for all off shell states, all chargenets $|s\rangle$.
  Similarly for RHS. Want to show equality.
- If B contains double Ham deformations of state ($LHS \neq 0$), must also contain double ('electric') diffeo deformations of state ($RHS \neq 0$); and *vice versa*.
- To check (and ensure) this:
  Given double Ham deformed state, need to infer the undeformed state and then double diffeo deform it; and *vice versa* infer undeformed state from double diffeo deformation then double Ham deform it.

Ancestry:

- Given double Ham deformation, undeformed 'parent' state is embedded in deformed one. Indeed, unique "ancestry" can be traced for an 'nth generation child' obtained by n Ham deformations.
- For 'double diffeo child', no parental embedding so no unique lineage. Nevertheless, sufficient 'genetic' traces exist so as to find all possible parents using certain notion of 'future completion of 1- past'.
- In summary, careful study of causal structure enables the construction of set of parents whose double deformations yield B so that in the continuum limit $LHS = RHS$.

  However: $LHS = RHS$ in non- covariant setting....

Diffeomorphism Covariance:

- Want: $\hat{U}(\phi)\hat{C}(N)\hat{U}^{\dagger}(\phi) = \hat{C}(\phi_* N)$

- Need to choose coordinate patches covariantly so that deformations of diffeo related chrgenets are diffeomorphic. (Proof uses 'Grot- Rovelli' nondegeneracy + properties of cyclic subbgroups of $GL(3, R)$.)

- Want to preserve $LHS = RHS$:

- Covariance acts as magnifying glass. Calculation becomes much more sensitive to details of deformations. Need to specify deformations so that they are 'conical'.
- Need to specify 'Short distance' behaviour of f in Ψ_f.

Final Result/Concluding Remarks:

- **Final Result (Single vertex v):**

$$\{N(v)\partial_a M(v) - N \leftrightarrow M\} \sum_{I_v} 2\nu_v^{-2/3} \nu_{vI_v}^{-2/3} \hat{e}^a_{I_v} \hat{e}^b_{I_v} \cos^2 \frac{\theta_{I_v}}{2} \sum_i (q^i_{I_v})^2$$

$$\left(\frac{\sum_{J_v \neq I_v} \sum_{K_v \neq I_v} g_{cd}(\hat{e}^c_{K_v} - \hat{e}^c_{J_v})(\hat{e}^d_{K_v} - \hat{e}^d_{J_v})}{4(M-1) g_{ef}(\hat{e}^e_{I_v} \hat{e}^f_{I_v})} \right)^{\frac{1}{3}} f_2(v, .., v) \partial_b g(v)$$

Compare Classical: $\int \mathrm{d}^3x \; (M\partial_a N - N\partial_a M) \quad q^{-\frac{2}{3}} E^a_i E^b_i \quad F^j_{bc} E^c_j$

- **Concluding Remarks:**

- Continuum Limit defined in terms of oprtr topology induced by off shell states. Would be good to improve this to genuine representation space. Seems that f may have to acquire additional dependence on tangent bundle etc.
- Physically correct? Need Dirac Observables (FB-EV)
- $SU(2)$ case of Euclidean Gravity. Good reasons for optimism.
- Can this feed into other more sptime- covariant ways of tackling dynamics?

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