Title: Discrete Approaches - 3

Date: Jul 25, 2013 04:40 PM

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Abstract:

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Asymptotic behaviour of lorentzian polyhedra propagator

Based on *arXiv:1307.4747*

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Waterloo, 25th of July 2013
Loops 13 Conference

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Outline

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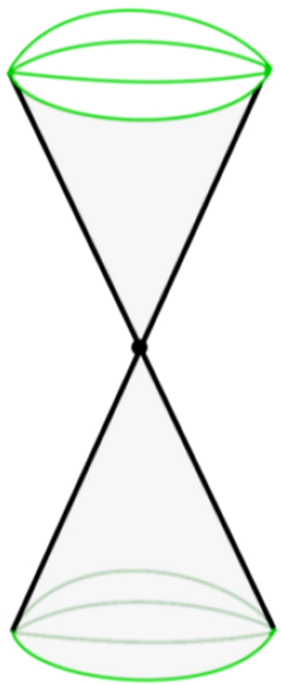
Motivation: Dipole cosmology



[Bianchi, Rovelli, Vidotto, Borja, Garay,. . . , 2010-2013]

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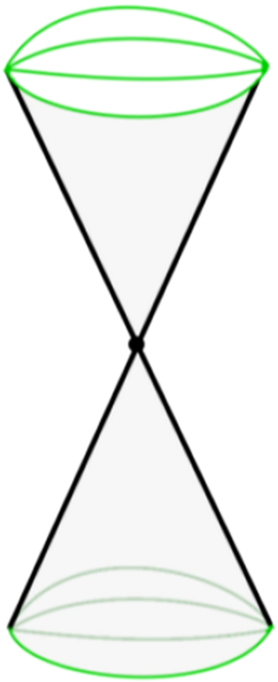

[Bianchi, Rovelli, Vidotto, Borja, Garay,. . . , 2010-2013]

$$W(z) = \sum_{\{j_\ell\}} \prod_{\ell=1}^{4} (2j_\ell + 1)\, e^{-2t\hbar j_\ell(j_\ell+1) - i\lambda v_0 j_\ell^{\frac{3}{2}} - izj_\ell} \times$$

$$\times \int_{SL(2,\mathbb{C})} \mathrm{d}g \prod_{\ell=1}^{4} \langle j_\ell |\, u^\dagger_{\vec{n}_\ell} Y^\dagger g\, Y\, u_{\vec{n}'_\ell}\, | j_\ell \rangle_{j_\ell}$$

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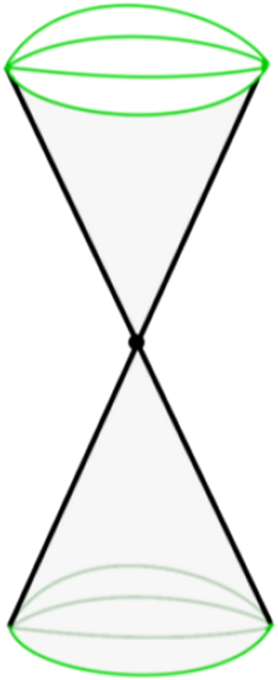

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\begin{aligned}
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&\quad \times \int_{SL(2,\mathbb{C})} \mathrm{d}g \prod_{\ell=1}^{4} \langle j_\ell|\, u^\dagger_{\vec{n}_\ell} Y^\dagger g\, Y\, u_{\vec{n}'_\ell}\, |j_\ell\rangle_{j_\ell} \\
&= \sum_{\{j_\ell\}} \prod_{\ell=1}^{4} (2j_\ell + 1)\, e^{-2t\hbar j_\ell(j_\ell+1) - i\lambda v_0 j_\ell^{\frac{3}{2}} - izj_\ell} \, \langle \iota|\, \mathbb{T}\, |\iota'\rangle
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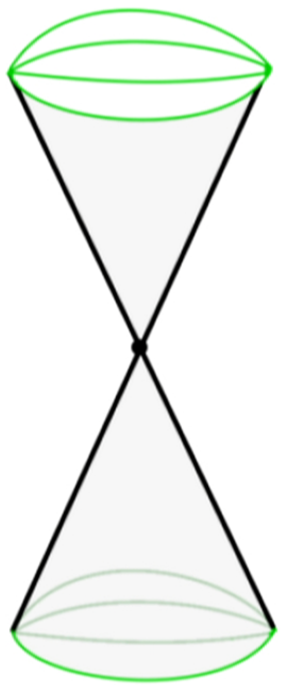

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$$= \sum_{\{j_\ell\}} \prod_{\ell=1}^{4} (2j_\ell + 1)\, e^{-2t\hbar j_\ell(j_\ell+1) - i\lambda v_0 j_\ell^{\frac{3}{2}} - izj_\ell} \langle \iota |\, \mathbb{T}\, | \iota' \rangle$$

$$\text{with} \quad \iota^{(\prime)} := \int_{SU(2)} \mathrm{d}u \prod_{\ell=1}^{4} u \cdot u_{n^{(\prime)}_{\vec{\ell}}}\, | j_\ell \rangle$$

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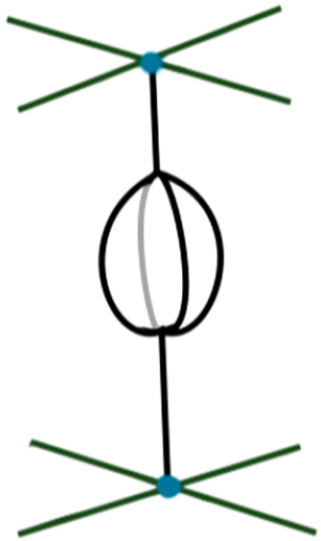
Motivation: "Melonic" radiative correction



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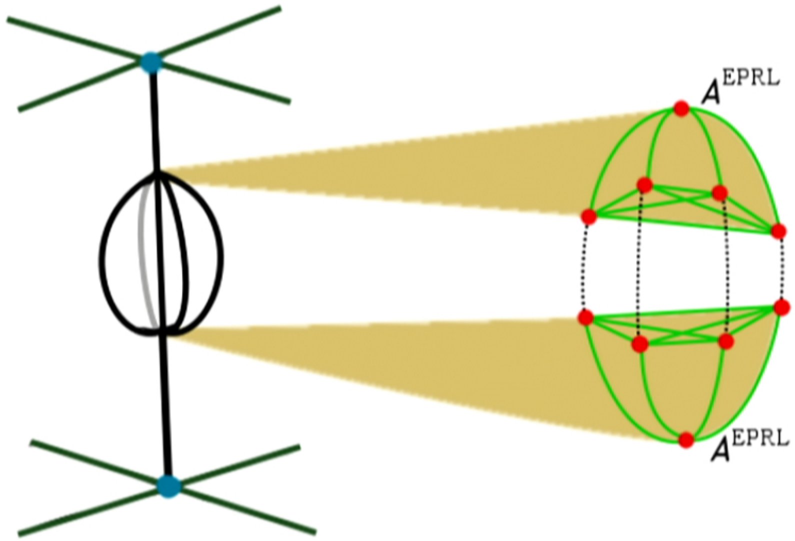

[Riello, 2013]

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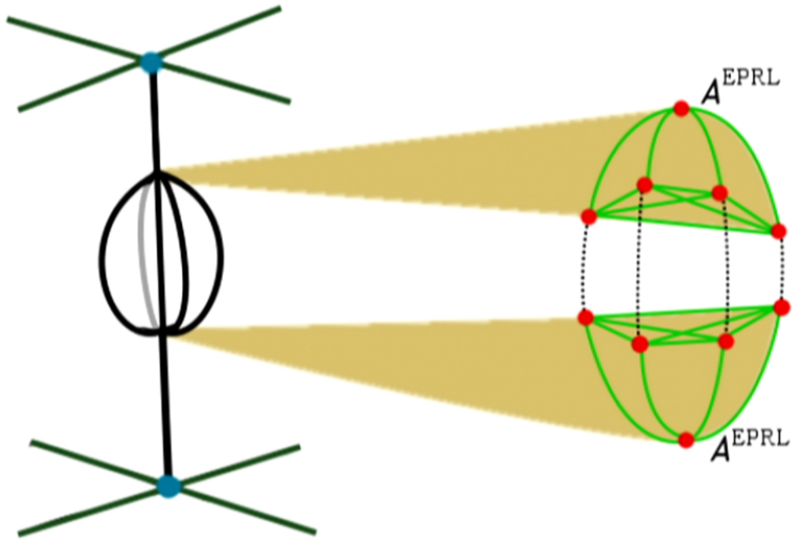

[Riello, 2013]

Let Λ be the maximum spin of the internal faces of the bubble.
Then the self-energy correction to the spin-foam edge is:

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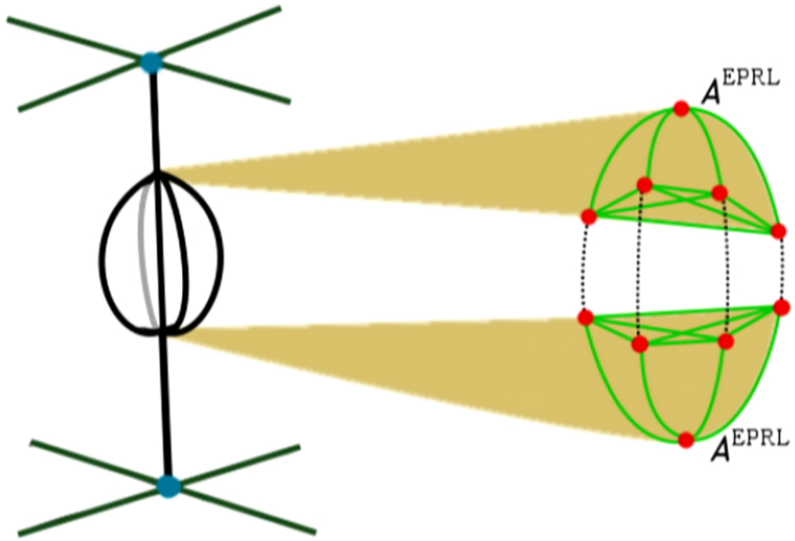

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Let Λ be the maximum spin of the internal faces of the bubble.
Then the self-energy correction to the spin-foam edge is:

$$W^{\Lambda} \sim \Lambda^{6(\mu-1)} \int_{SL(2,\mathbb{C})^2} \mathrm{d}g_1 \mathrm{d}g_2 \sum_{\{n_i\}} \prod_{i=1}^{4} \langle m_i | Y^{\dagger} g_1 Y | n_i \rangle \langle n_i | Y^{\dagger} g_2 Y | \tilde{m}_i \rangle$$

$$= \Lambda^{6(\mu-1)} \cdot \mathbb{T}^2 \underset{\text{for}\,\mu=1}{=} \ln \Lambda \cdot \mathbb{T}^2$$

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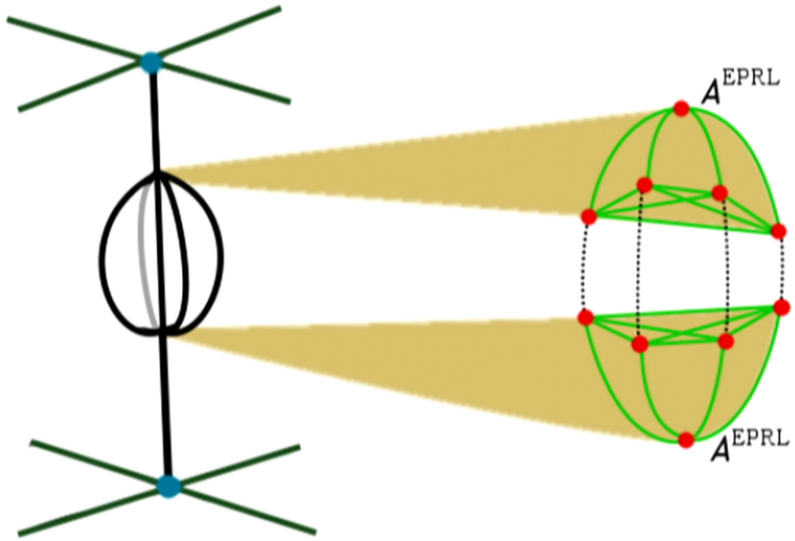

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Definition

Def: Lorenzian polyheadra propagator

Given a set of spins $j_1, \ldots, j_N$ we define an opertor

$$\mathbb{T} := \int_{SL(2,\mathbb{C})} \mathrm{d}g \, \left[Y^\dagger g Y \right]^{(j_1 \otimes \cdots \otimes j_N)}$$

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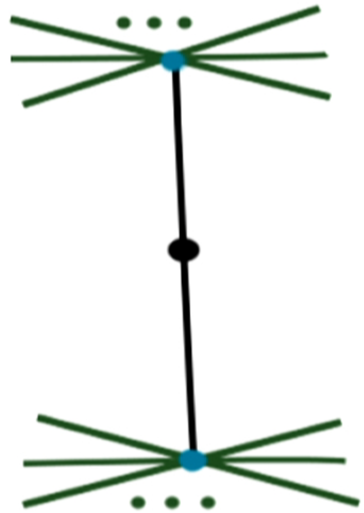

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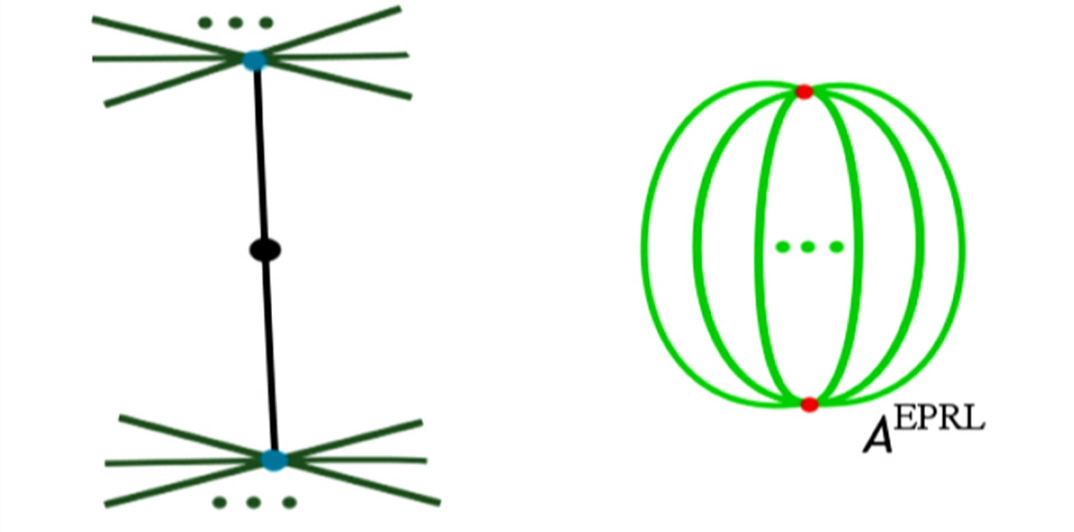

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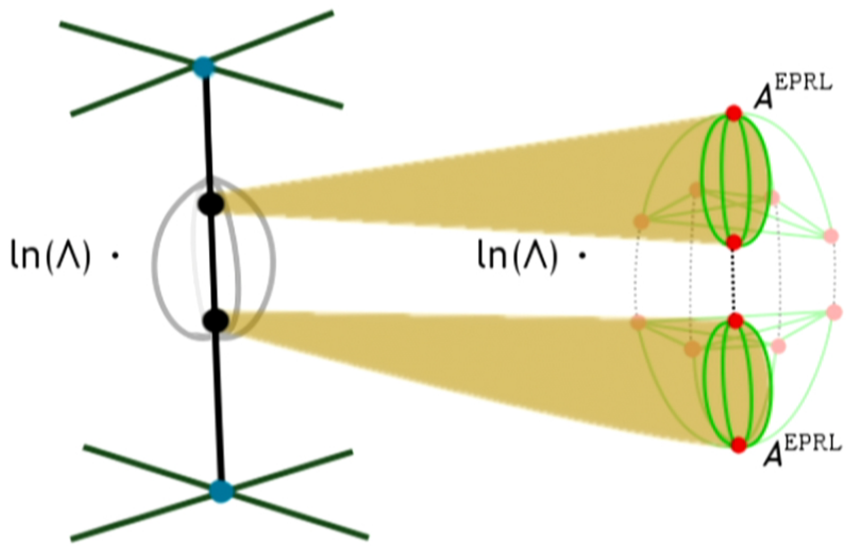

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A little technical introduction

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In the intertweiner basis, the matrix elements of $\mathbb{T}$ are

$$\mathbb{T}^{\iota}_{\iota'} := \int_{SL(2,\mathbb{C})} \mathrm{d}g \, \langle \iota | Y^\dagger g Y | \iota' \rangle$$
$$=: \int_{SL(2,\mathbb{C})} \mathrm{d}g \, \Phi^{\iota}_{\iota'}(g) = \int_{SL(2,\mathbb{C})} \mathrm{d}g \sum_{\vec{m},\vec{m}'} \overline{\iota_{\vec{m}}} \iota'_{\vec{m}'} \Phi^{\vec{m}'}_{\vec{m}}(g)$$

with $\iota^{(\prime)}_{\vec{m}} = \langle \vec{m} | \, \big| \iota^{(\prime)} \big\rangle$ and $\Phi^{\vec{m}}_{\vec{m}'}(g) := \prod_{i=0}^{N} D^{(\gamma j_i, j_i)}(g)^{j_i, m_i'}_{j_i, m_i}$

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Assuming, that $\Phi^{\iota}_{\iota'}(g)$ satisfy the assumptions of the SPA theorem, and anticipating, that the maximum of the integrand is at the unity, the $\mathbb{T}$ operator will be given by the formula

$$\mathbb{T} = J^{-d/2} \mu(\mathbf{1}) \Phi^{\iota}_{\iota'}(\mathbf{1}) \frac{1}{\sqrt{|\partial^2 \phi(\mathbf{1})|}}$$

with $J = \max_{i=1,\dots,N}\{j_i\}$, d - the dimention of manyfold we integrate on, $\phi = \lim_{J\to\infty} \frac{1}{J} \ln\left[\Phi(g,J)\right]$, $\left|\partial^2 f\right|$- the determinant of the Hessian matrix of function f, and $\mu(g)$ - the integral measure.

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Symetries: I

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It's straightforward to see, that $J_i \Phi^{\iota}_{\iota'}(g) \equiv 0$
Indeed: J_i are $SU(2)$ generators, thus they commute with the Y map, and $J_i |\iota\rangle = 0$, so

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So the integral over $SU(2) \in SL(2,\mathbb{C})$ is trivial (gives a constant factor). We need to integrate over the boosts $g = e^{\vec{\eta}\cdot\vec{K}}$.

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Symetries: II

Consider now a boost in arbitrary direction $\vec{n}$.
Since

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Thus the behaviour of $\Phi^{\iota}_{\iota'}(e^{\eta K_3})$ is crucial in further calculation.

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Let us now deffine the function $f^{(j)}_m(\eta)$

$$\langle m|\, Y^\dagger g Y \,|m'\rangle_j =: \delta_{m,m'} f^{(j)}_m(\eta)$$

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Obviously

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Assymptotics

To use the SPA method in integrating $\Phi^{\iota}_{\iota'}(g)$, we have be sure, that our integrand decay sufficiently fast for g far from the critical point.

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One can proove, that

$$\left| f^{(j)}_{m}(\eta) \right| \leq \left(e^{1-2\eta-e^{-2\eta}} \right)^{\frac{(j+1)^2-m^2}{4(2j+3)}}$$

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$$\begin{aligned}\Phi^{\iota}_{\iota'}(e^{\eta\vec{n}\cdot\vec{K}}) &= \langle\iota|\,Y^\dagger u^{-1}e^{\eta K_3}uY\,|\iota'\rangle = \langle\iota|\,u^{-1}Y^\dagger e^{\eta K_3}Yu\,|\iota'\rangle \\ &= \langle\iota|\,Y^\dagger e^{\eta K_3}Y\,|\iota'\rangle = \Phi^{\iota}_{\iota'}(e^{\eta K_3})\end{aligned}$$

Thus the behaviour of $\Phi^{\iota}_{\iota'}(e^{\eta K_3})$ is crucial in further calculation.
Let us now deffine the function $f^{(j)}_m(\eta)$

$$\langle m|\,Y^\dagger gY\,|m'\rangle_j =: \delta_{m,m'}f^{(j)}_m(\eta)$$

Obviously

$$\Phi^{\iota}_{\iota'}\left(e^{\eta K_3}\right) = \sum_{\vec{m}} \overline{\iota_{\vec{m}}}\,\iota'_{\vec{m}} \prod_{i=1}^{N} f^{(j)}_m(\eta)$$

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$$\left|f^{(j)}_{m}(\eta)\right| \leq \left(e^{1-2\eta-e^{-2\eta}}\right)^{\frac{(j+1)^2-m^2}{4(2j+3)}}$$

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and thus

$$|\Phi^{\iota}_{\iota'}(\eta)| \leq \left(e^{1-2\eta-e^{-2\eta}}\right)^{\frac{J}{12}\sum_{i=1}^{N} x_i + \frac{1}{J}} \ll 1 \quad \text{for J} \gg 1 \text{ and } \eta > 0$$

where $x_i := \frac{j_i}{J}$.

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$$= \left(\frac{j_i(j_i+1)}{3}\right)^k \langle\iota|\,|\iota'\rangle$$

and since it works for polynomials, it works for any series:

$$\sum_{\vec{m}} \overline{\iota_{\vec{m}}} \iota'_{\vec{m}} f(m_i^2) = f\left(\frac{j_i(j_i+1)}{3}\right) \langle\iota|\,|\iota'\rangle$$

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Lorentzian Polyheadra Propagator

The leading order of the LPP operator is

$$\mathbb{T} = \left(\frac{1}{4\pi}\right)^2 \left[\frac{6\pi}{J\left(1+\gamma^2\right)\sum_{i=1}^{N} x_i}\right]^{\frac{3}{2}} \mathbf{1}_{\vec{j}} + O\left(J^{-5/2}\right)$$

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Moreover note, that the LPP operator is diagonal in the $|\vec{m}\rangle$ basis, i.e.

$$\mathbb{T}^{\vec{m}}_{\vec{m}'} = \left(\frac{1}{4\pi}\right)^2 \left(\left[\frac{6\pi}{J\left(1+\gamma^2\right)\sum_{i=1}^{N} x_i}\right]^{\frac{3}{2}} + \left(\frac{1}{J}\right)^{\frac{5}{2}} T^{(1)}_{\vec{m}}\right) \delta^{\vec{m}}_{\vec{m}'}$$

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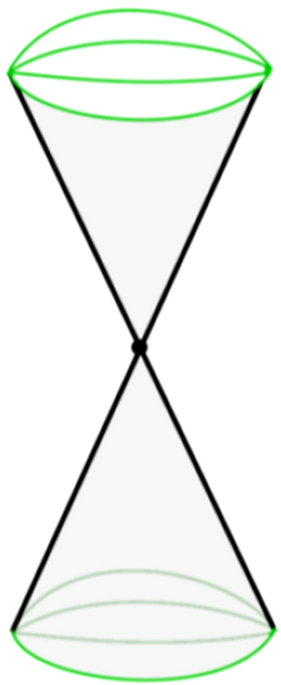
Application in DC



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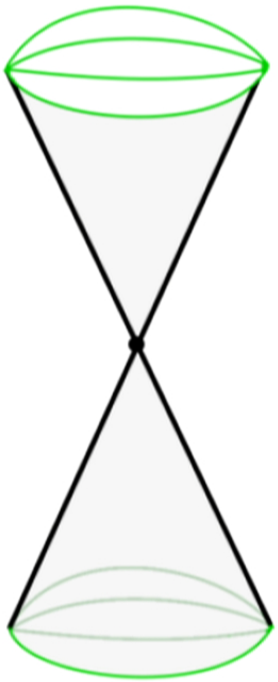

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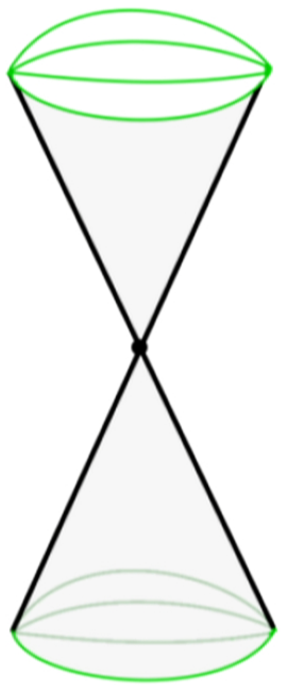

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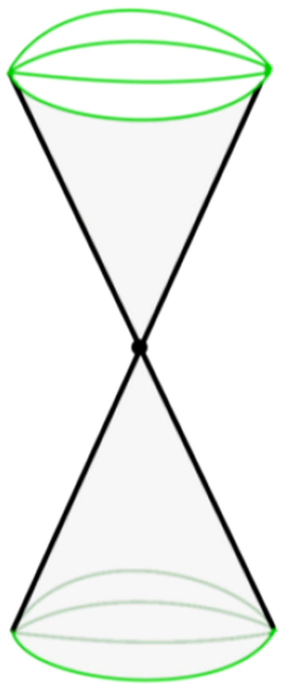
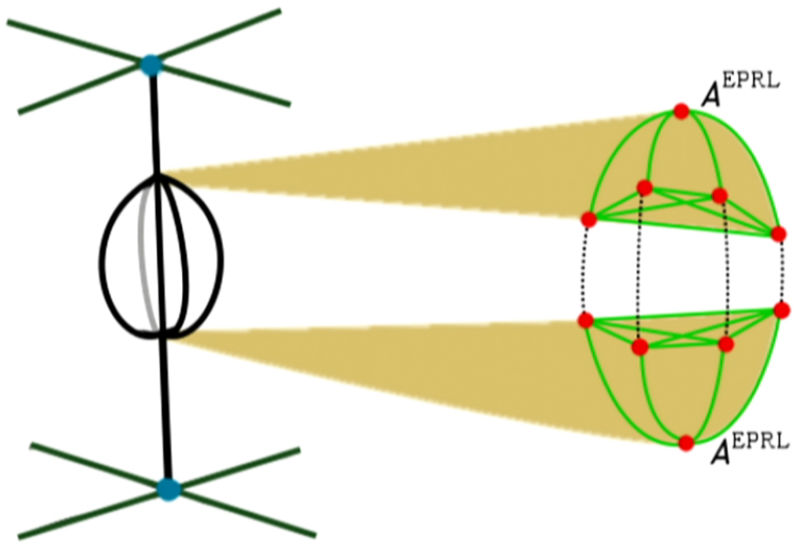


In Dipole Cosmology model the main result (i.e. recovery of the classical trajectory) does not change. Only the factor in front of the transition amplitude becomes shape-sensitive:

$$W(z) = \sum_{\{j_\ell\}} \frac{N(x_i)}{j_0^3} \prod_{\ell=1}^{4} (2j_\ell + 1)\, e^{-2t\hbar j_\ell(j_\ell+1) - i\lambda v_0 j_\ell^{\frac{3}{2}} - izj_\ell}$$

Application in bubble divergences



Let's now go back to the transition amplitude of the "melonic" bubble:

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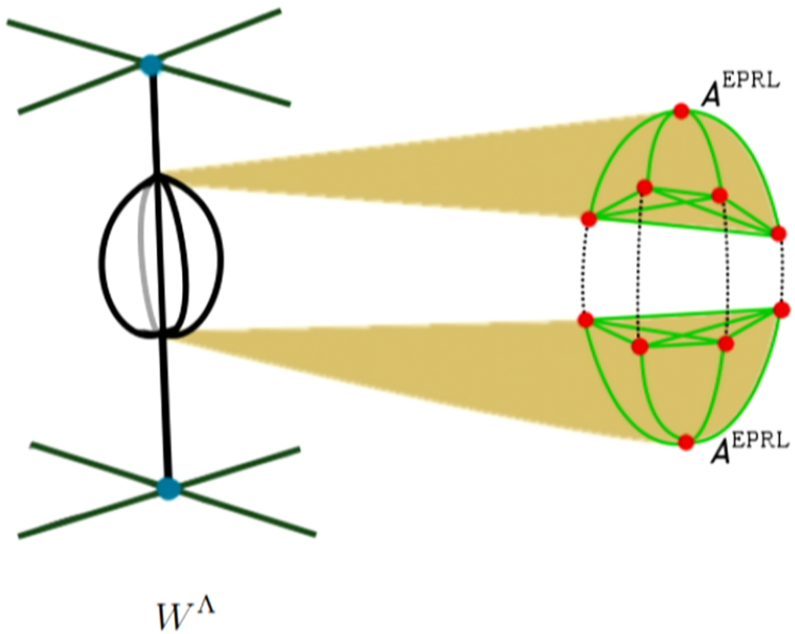
Application in bubble divergences



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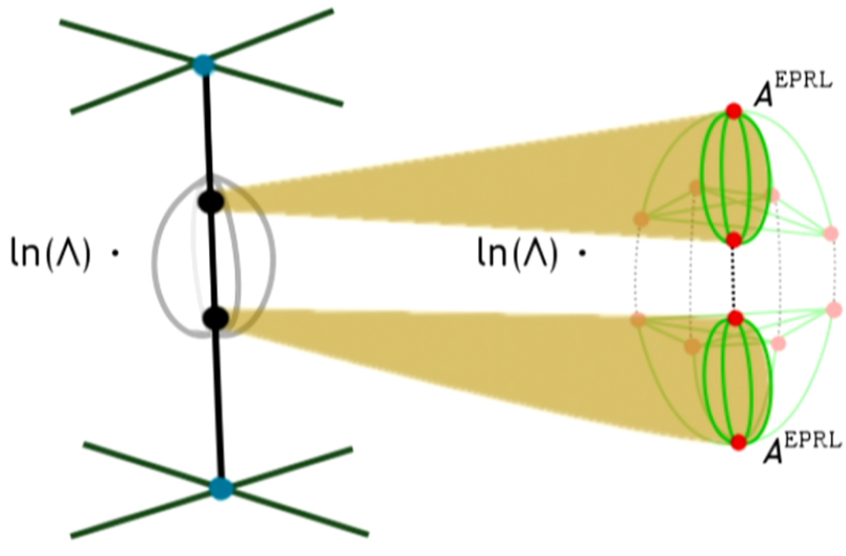

$$W^\Lambda \sim \ln\Lambda \;\cdot\; \mathbb{T}^2$$

Let's now go back to the transition amplitude of the "melonic" bubble:

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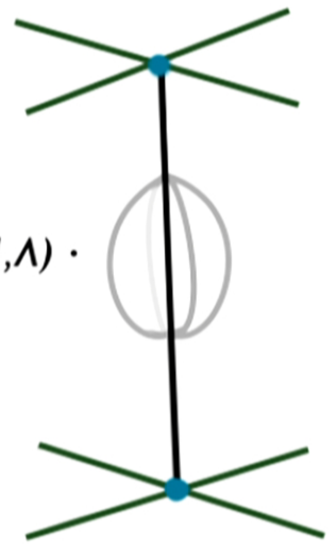
Application in bubble divergences

$a(J,\Lambda)\,\cdot$



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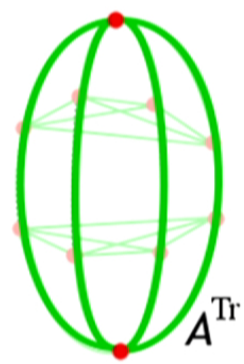
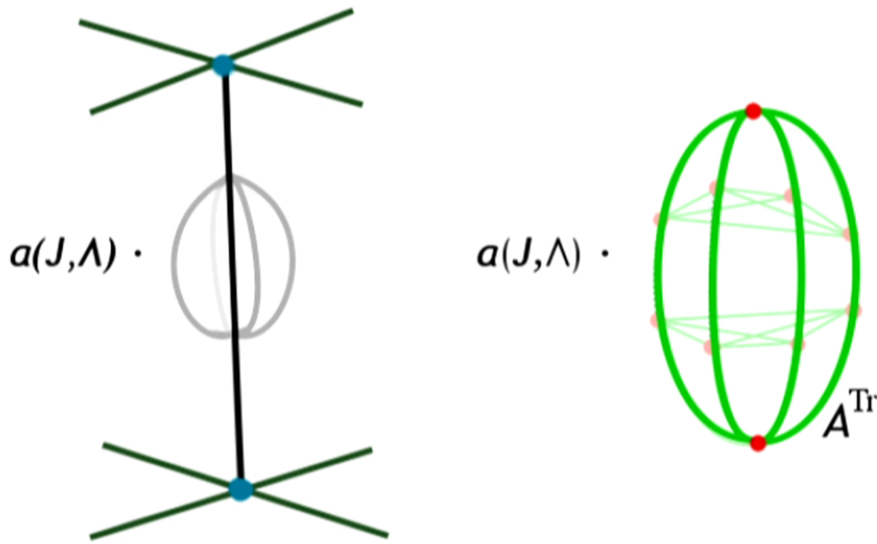


Let's now go back to the transition amplitude of the "melonic" bubble:

$$W^{\Lambda} \sim \ln\Lambda \;\cdot\; \mathbb{T}^2 = a(J,\Lambda)\cdot\mathbf{1}$$

Application in bubble divergences



Let's now go back to the transition amplitude of the "melonic" bubble:

$$W^{\Lambda} \sim \ln\Lambda \;\cdot\; \mathbb{T}^2 = a(J,\Lambda)\cdot\mathbf{1} = \frac{27}{32\pi(1+\gamma^2)^3\left(\sum x_i\right)^3}\,\frac{\ln\Lambda}{J^3}\,\mathbf{1}$$

Application in bubble divergences

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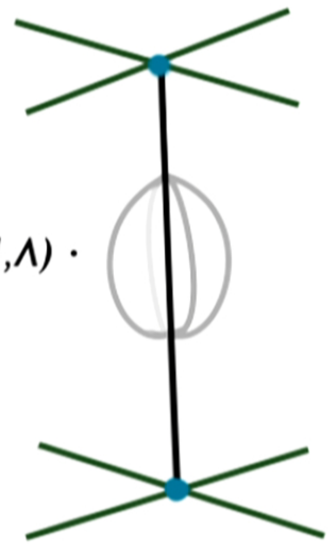

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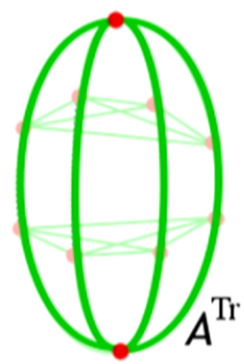

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For $\Lambda = 10^{120}$ we have $a(J,\Lambda) \leq \frac{74.2}{\left[(1+\gamma^2)\sum x_i\right]^3} < 9.28$

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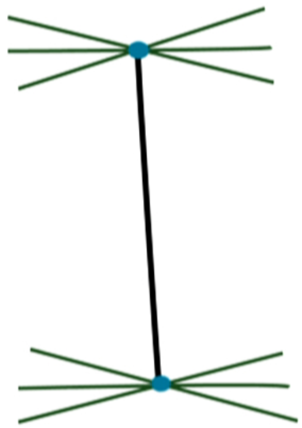
(New) Application in renormalisation?

Recall, that $\mathbb{T} = \frac{\alpha\cdot\alpha(\vec{x})}{J^{3/2}}\mathbf{1}_{\vec{j}} + \cdots$.
Note, that the α is always smaller than 1. One can thus consider a series:



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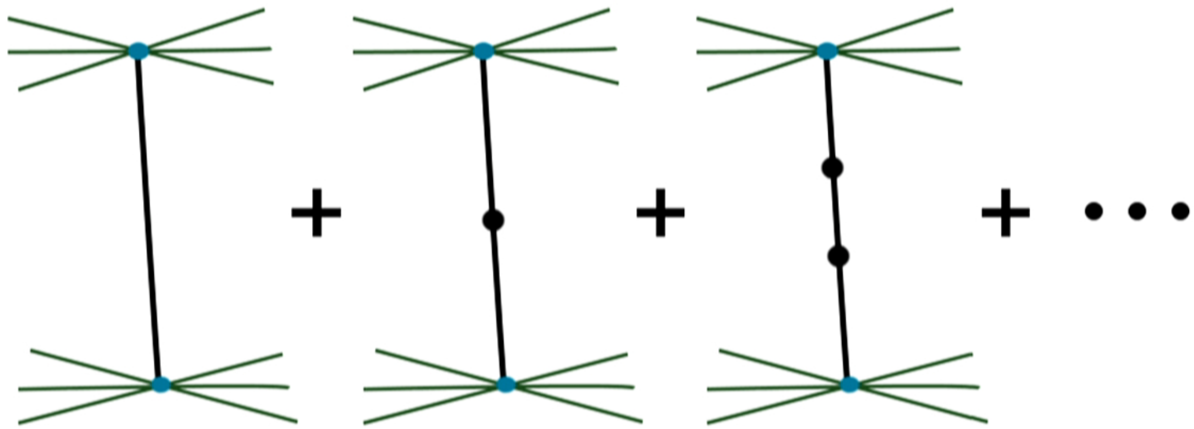

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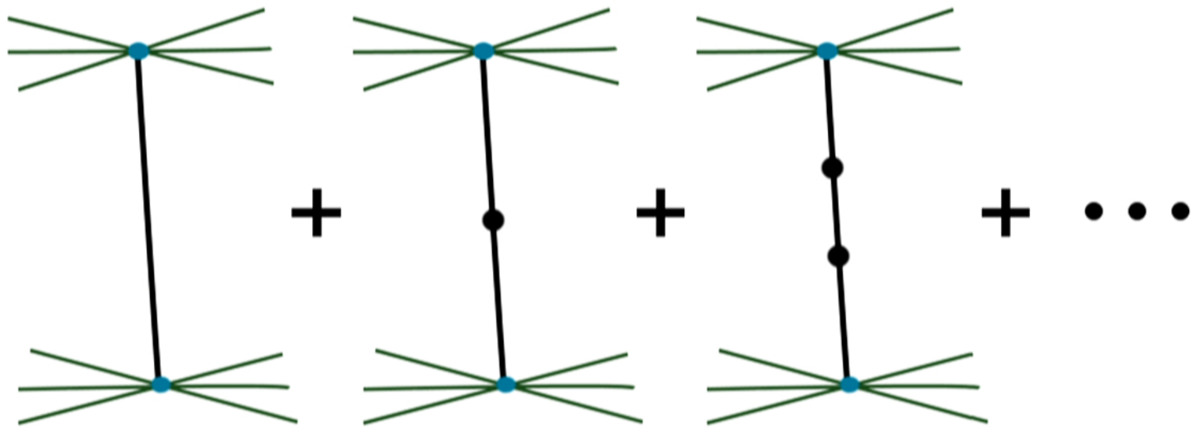

It sum to:

$$\mathbb{T}^R = \frac{\mathbf{1}_{\vec{j}}}{\mathbf{1}_{\vec{j}} - \mathbb{T}} = \frac{1}{1 - \frac{\alpha}{A^{3/2}}} \mathbf{1}_{\vec{j}} = \frac{A^{3/2}}{A^{3/2} - \alpha} \mathbf{1}_{\vec{j}} \quad \text{for } \alpha < 0.181$$

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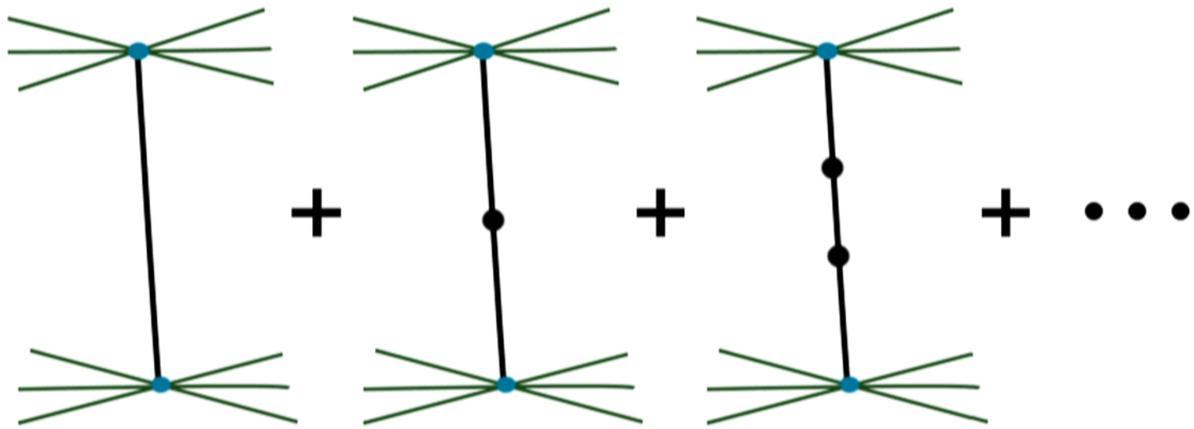

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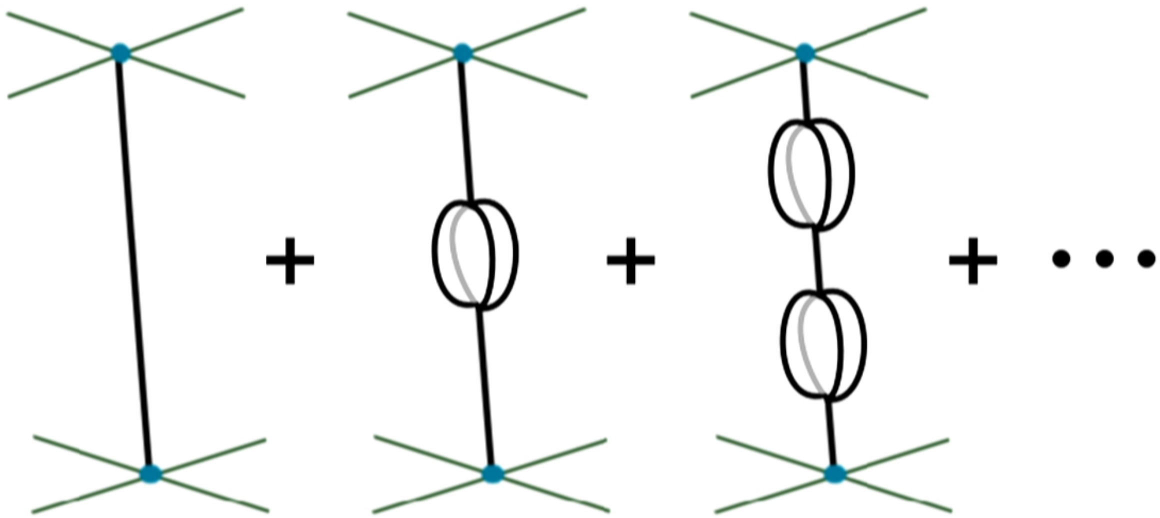
and the sum allways converges.

(New) Application in renormalisation?

The same can be done for a series of "mellonic" bubbles on an edge:



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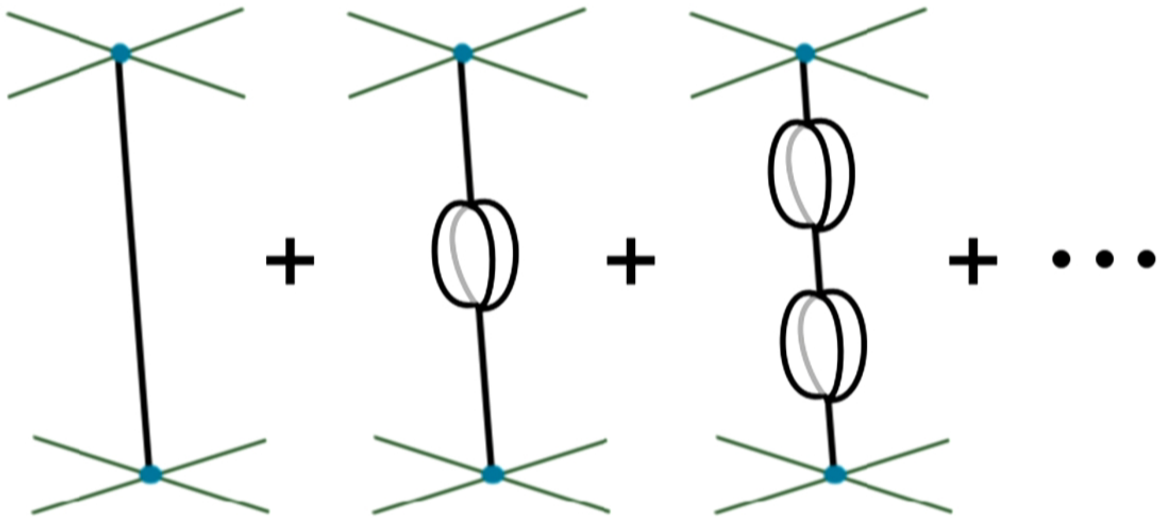

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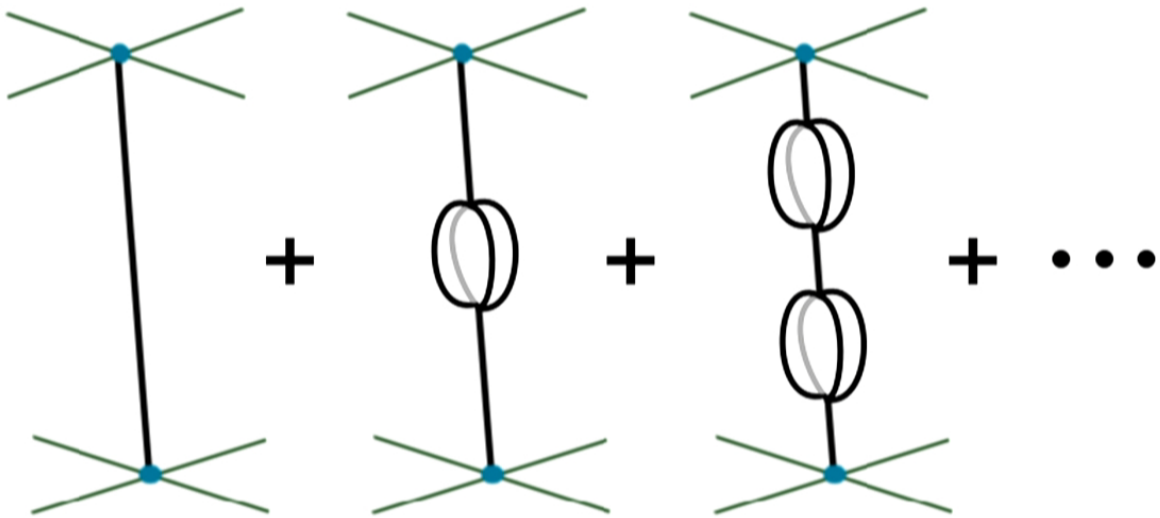

$$\mathbb{T}^{R,bubble} = \frac{\mathbf{1}_{\vec{j}}}{\mathbf{1}_{\vec{j}} - \mathbb{T}^2 \cdot \ln \Lambda}$$

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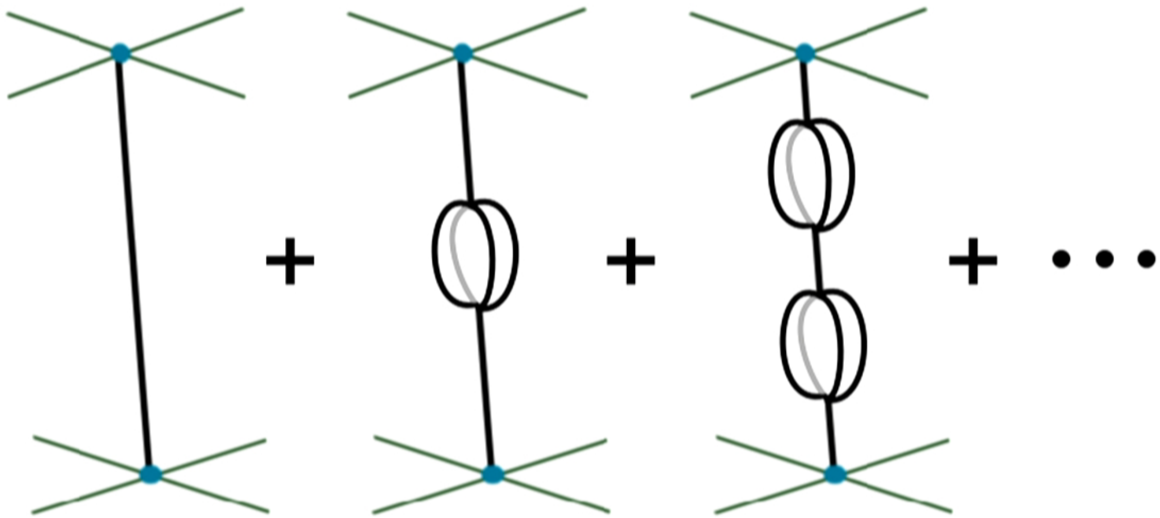

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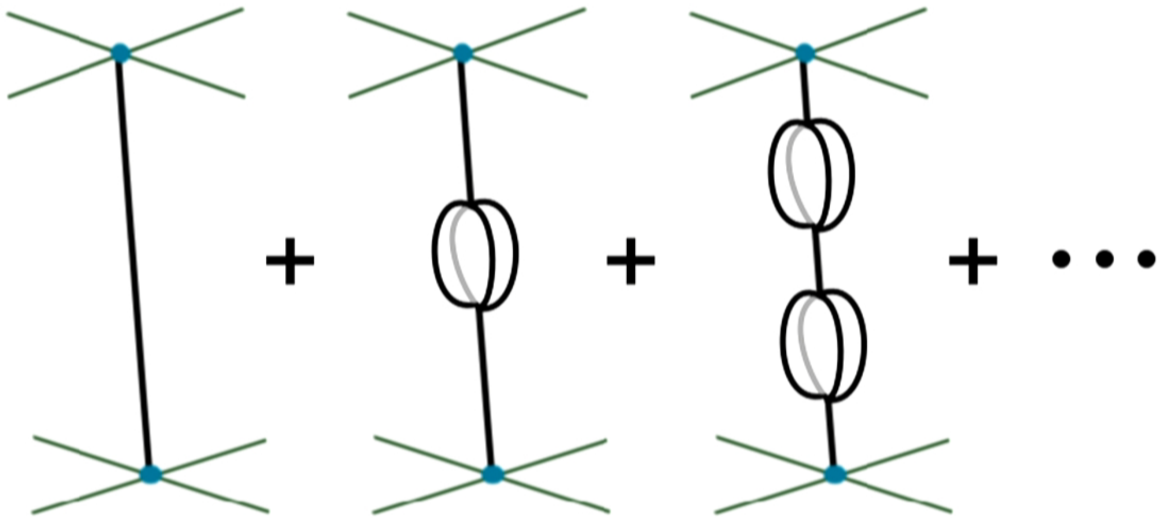

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Assume now, that Λ - the maximum spin - is the inverse cosmological constant,

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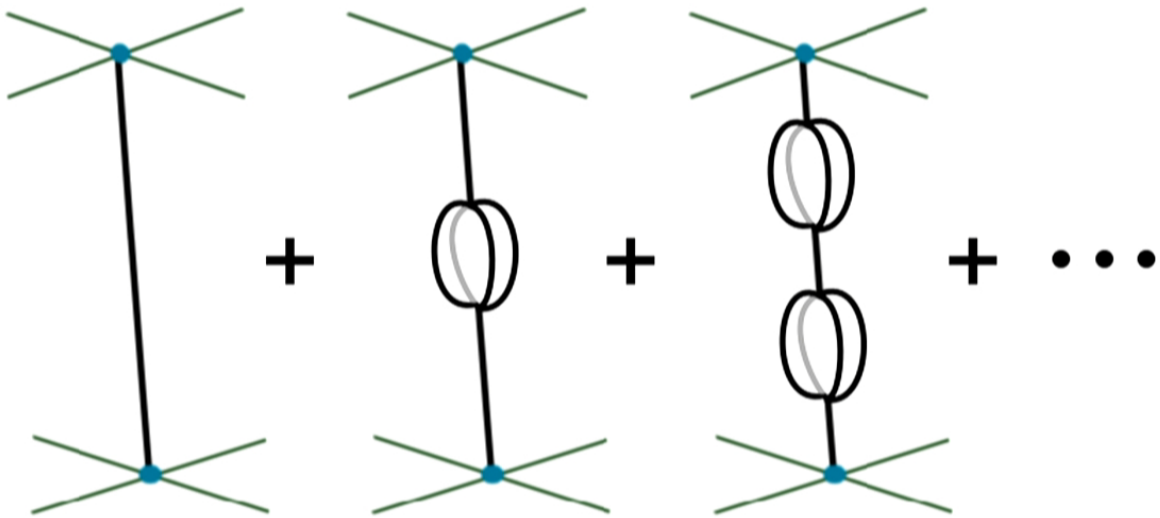

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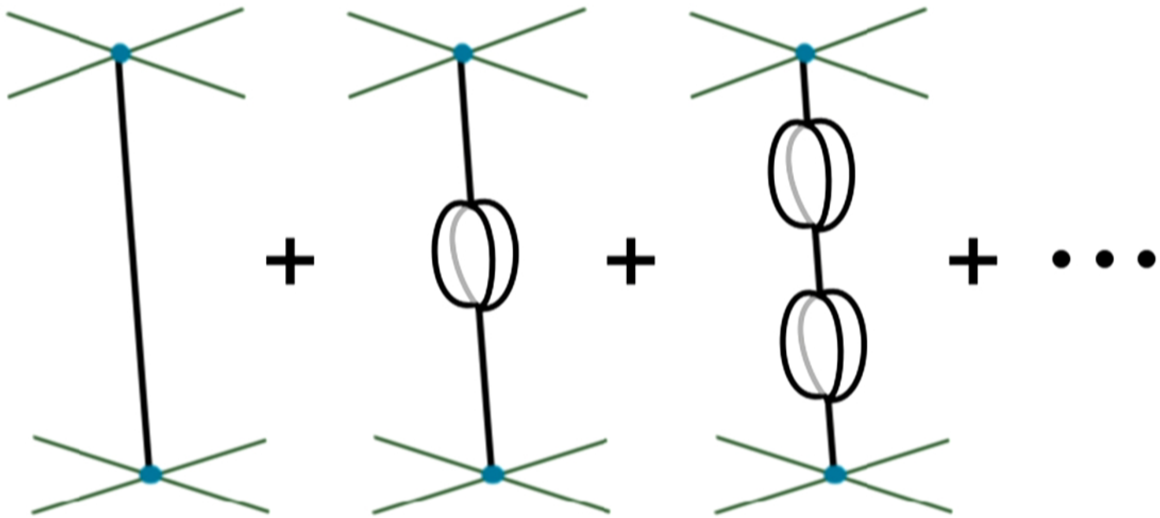

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Assume now, that Λ - the maximum spin - is the inverse cosmological constant, thus in Planck units $\Lambda = 10^{120}$. Then the factor $a(J,\Lambda) < \frac{9.276}{J^3}$, so $a(J,\Lambda) \underset{\text{for } J>2}{<} 1$, and the sum converges!

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Summary

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Summary

- The operator $\mathbb{T}^{\iota}_{\iota'} := \int_{SL(2,\mathbb{C})} \mathrm{d}g \, \langle \iota | \, Y^{\dagger} g \, Y \, | \iota' \rangle$ has been studied

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Further directions

- Subleading order
- Further study of applications in renormalisation

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- Subleading order
- Further study of applications in renormalisation
- Understanding of the factor $\frac{\alpha}{A^{3/2}}$

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Many thanks to PI for inviting me here.

And thank you all for your attention!

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How to solve your theory: cylindrically consistent dynamics

Bianca Dittrich
(Perimeter Institute)

[BD, 1205. 6127, New J. Phys. 12]
[Bahr, BD et al 09-11]

LOOPS '13

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Somebody gives you a theory of quantum gravity.

Typical: Comes as a description of amplitudes for ‘fundamental building blocks’.

What to do with this?

How to describe a background independent theory ?

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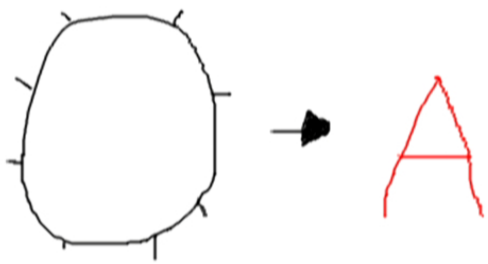
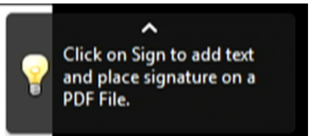
Define dynamic of theory via: [Oeckl: generalized boundary formalism
Perez, Rovelli: transition amplitudes as observables]

Amplitude map:

boundary with (geom) data/
boundary wave function → complex number

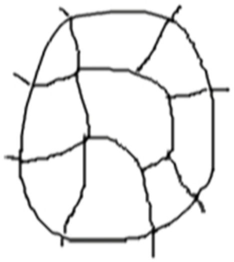


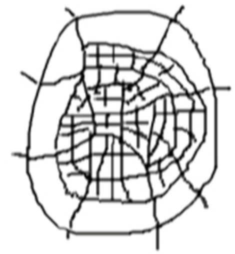
- test states describing boundary states
carry finite amount of information: might have discrete features
(projective / inductive limit construction [LQG: Isham, Ashtekar, Lewandowski, ...])



How do we get this amplitude map?

(a)





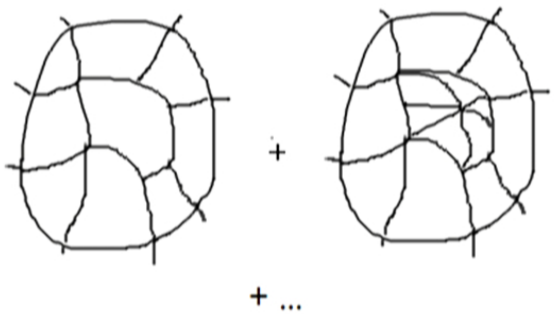
regularized path integral:
glue (fundamental) regions

refinement limit to loose dependence on auxiliary discretization
(hope that details of limit do not matter)

question: refinement of boundary?

How do we get this amplitude map?

(b)



sum over bulk discretizations
(group field theory)

[Rovelli, Smerlak: should be the same as (a)]

(c)

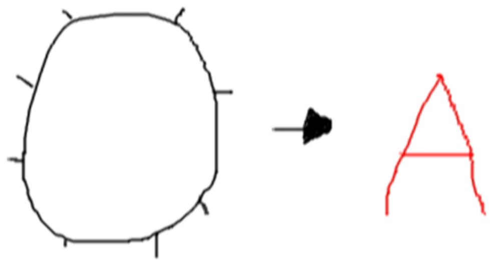
canonical:
coefficient of expansion of physical solution over graphs

[Halliwell, Hartle, Rovelli (a) should give (c)]

(d) ...

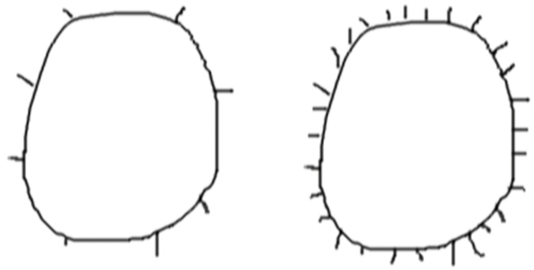
Hope:

well defined amplitude map



Is that sufficient?

Cylindrical consistency



two states describing equivalent boundary data

[LQG: Isham, Ashtekar, Lewandowski, ...]

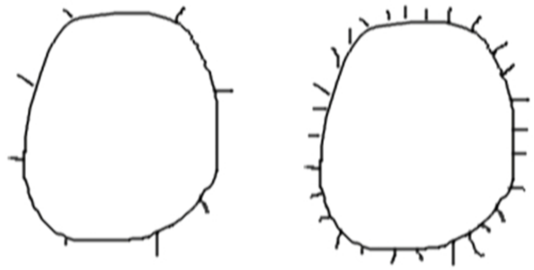
$\iota_{bb'}: \quad \mathcal{H}_b \to \mathcal{H}_{b'}$ embedding of coarser into finer boundary Hilbert space

$A_b : \mathcal{H}_b \mapsto \mathbb{C}$ amplitude map

$(\iota_{bb'})^\star A_{b'}(\psi_b) = A_b((\psi_b))$ demand cylindrical consistency for amplitude map

Amplitude does not depend on which graph/discrete structure we represent boundary data.
It is defined in the continuum (limit).

Cylindrical consistency



two states describing equivalent boundary data

[LQG: Isham, Ashtekar, Lewandowski, ...]

$\iota_{bb'}: \quad \mathcal{H}_b \to \mathcal{H}_{b'}$	embedding of coarser into finer boundary Hilbert space
$A_b : \mathcal{H}_b \mapsto \mathbb{C}$	amplitude map
$(\iota_{bb'})^\star A_{b'}(\psi_b) = A_b((\psi_b))$	demand cylindrical consistency for amplitude map

Amplitude does not depend on which graph/discrete structure we represent boundary data.
It is defined in the continuum (limit).

Hope:

amplitude map is cylindrical consistent.

How might we get such a map?

by using the idea of cylindrical consistency in the construction of the refinement limit.

Back up:

What are convenient (for the dynamics) families of embedding maps?

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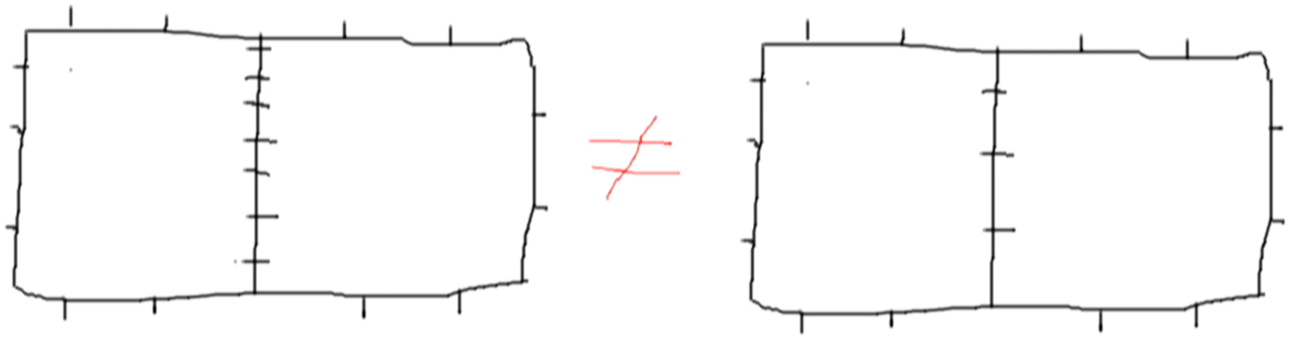

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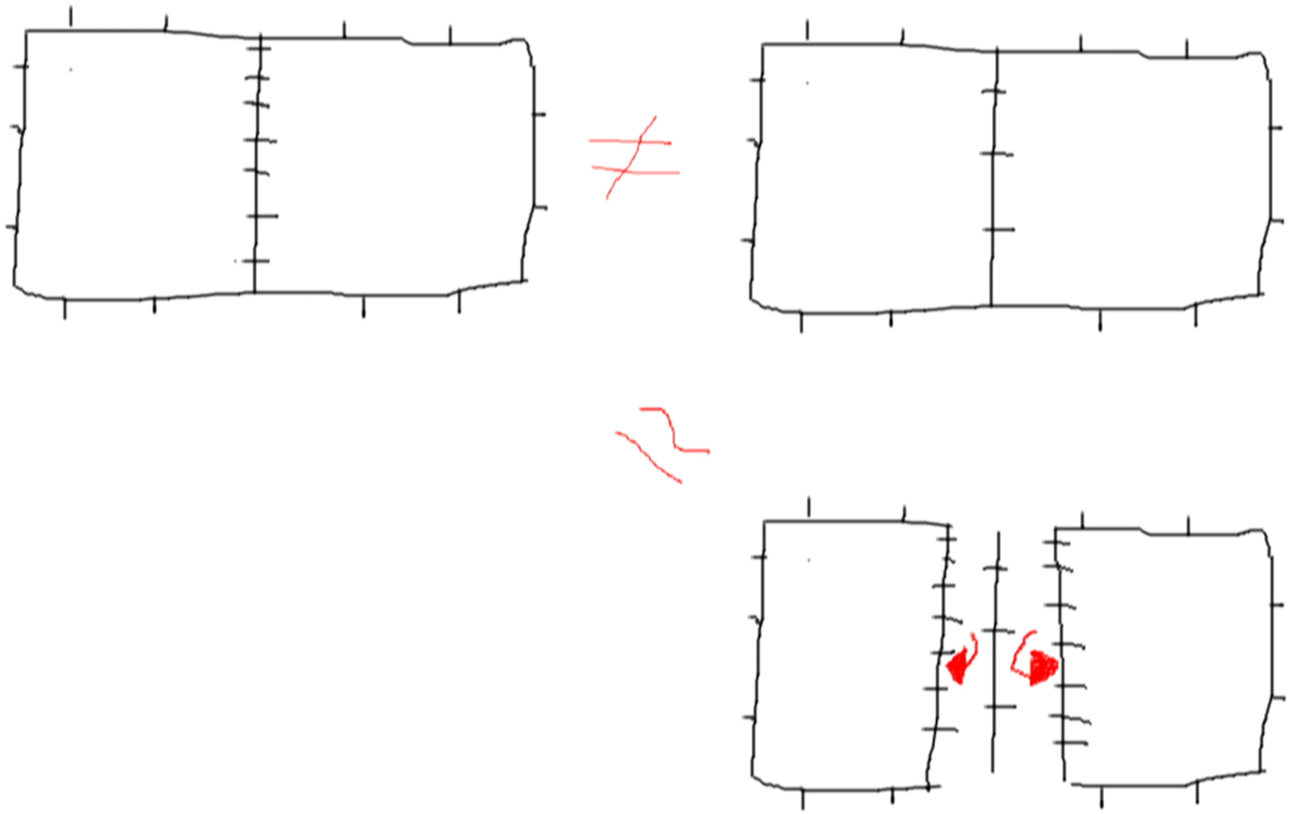

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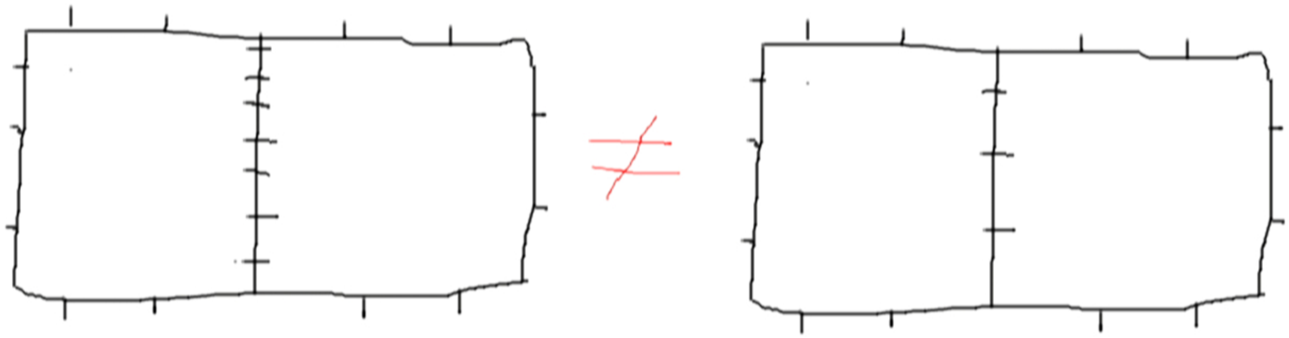
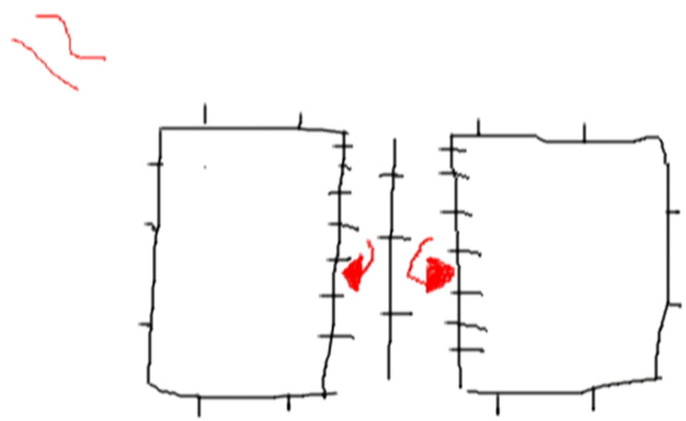




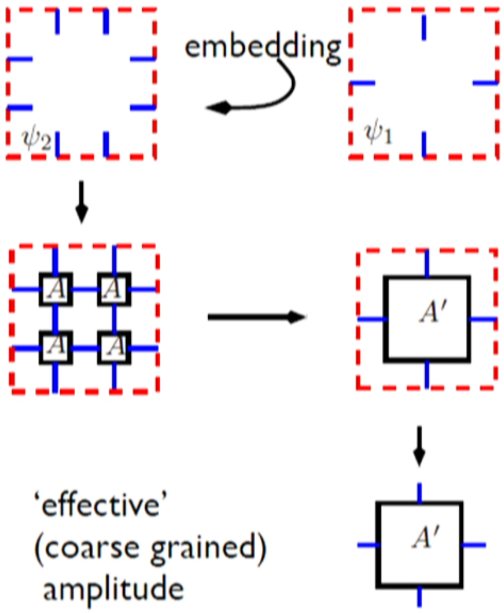
Convenient embedding maps allow us to replace gluing along very fine boundaries by gluing along coarser boundaries: **define good truncations!**

Embedding maps defined by dynamics!

LQG:

embedding map corresponding to kinematical vacuum describing completely degenerate geometry with vanishing volume, areas, ...

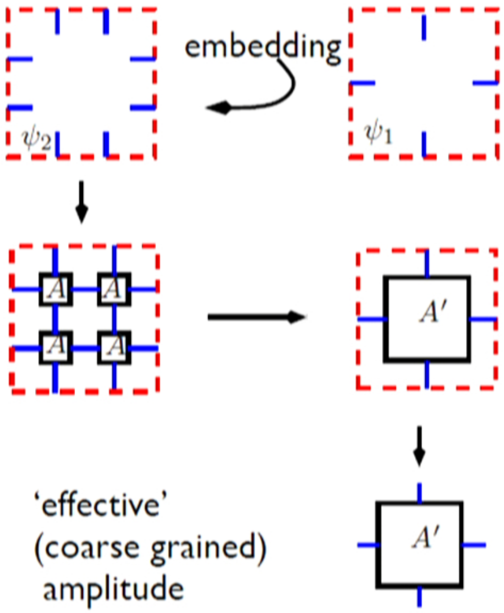
Such embedding maps allow an effective way to find the refinement limit.



Can now iterate.
Fixed point:
refinement/
continuum limit.

This procedure does refinement limit in bulk and boundary.

Such embedding maps allow an effective way to find the refinement limit.



Can now iterate.
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How to get these embedding maps?

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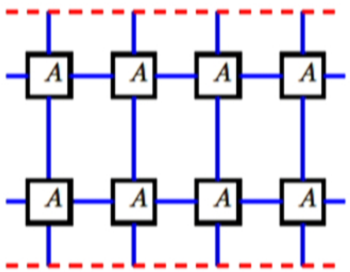
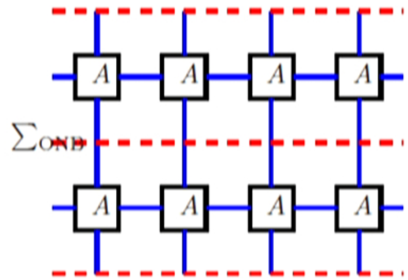
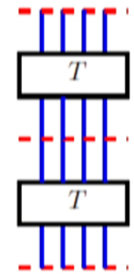
Motivation: transfer operator technique



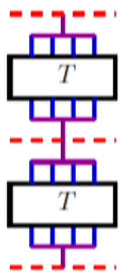
Transition amplitude between two states $\langle\psi_1|\mathcal{A}|\psi_2\rangle$



insert id $= \sum_{\text{ONB}} |\psi\rangle\langle\psi|$



$\mathcal{A} = T^N$

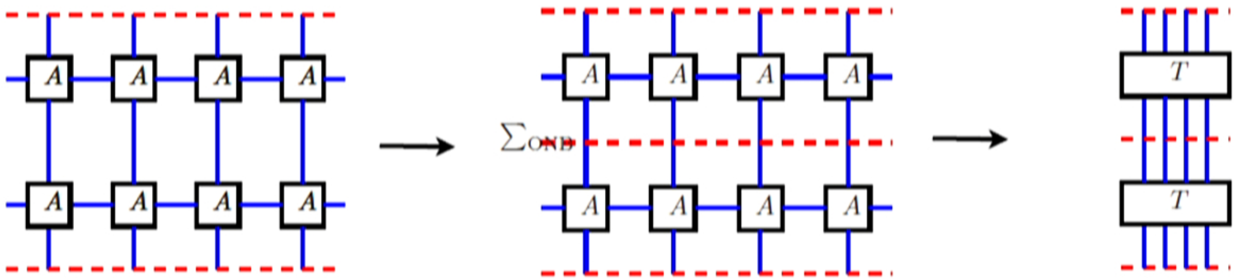


Truncate by restricting $\sum_{\text{ONB}}$ to the eigenvectors of T with the χ largest (in mod) eigenvalues.

Expect good approximation if ψ_1, ψ_2 are in span of these eigenvectors.

But: explicit diagonalization of T difficult.

Motivation: transfer operator technique



Transition amplitude between two states $\langle \psi_1 | \mathcal{A} | \psi_2 \rangle$

insert id $= \sum_{\mathrm{ONB}} |\psi\rangle\langle\psi|$

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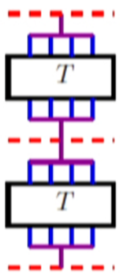

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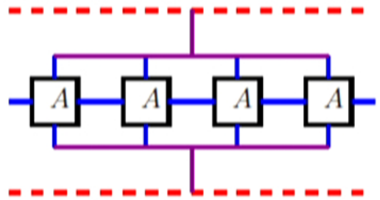
Dynamically determined embedding maps



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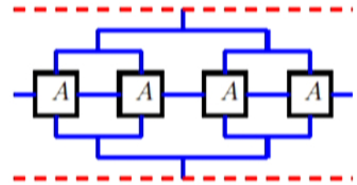
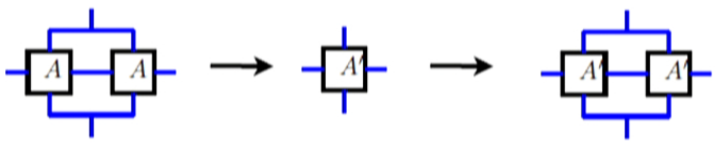
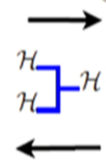


Localize truncations, diagonalize only subparts of transfer operator

iteration procedure



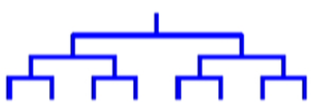
blocking



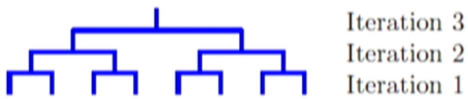
Determined by (generalized) EV-decomposition.

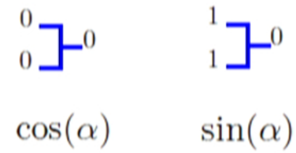
embedding

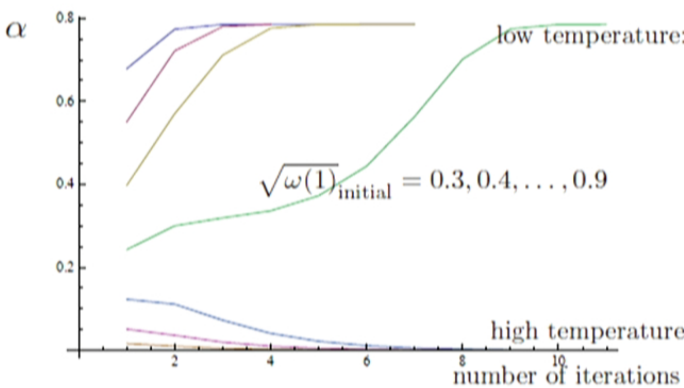
embedding map after 3 iterations

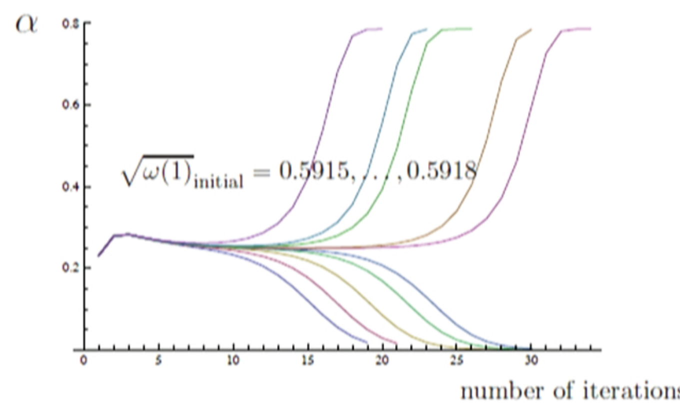


Example: Ising model







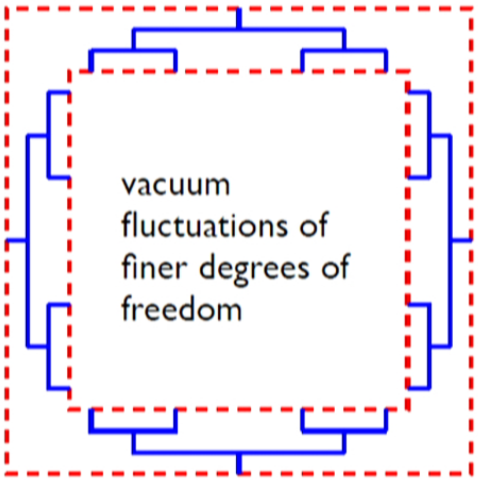


Plateau (scale free dynamics) of almost constant embedding maps around phase transition

Background scale free!

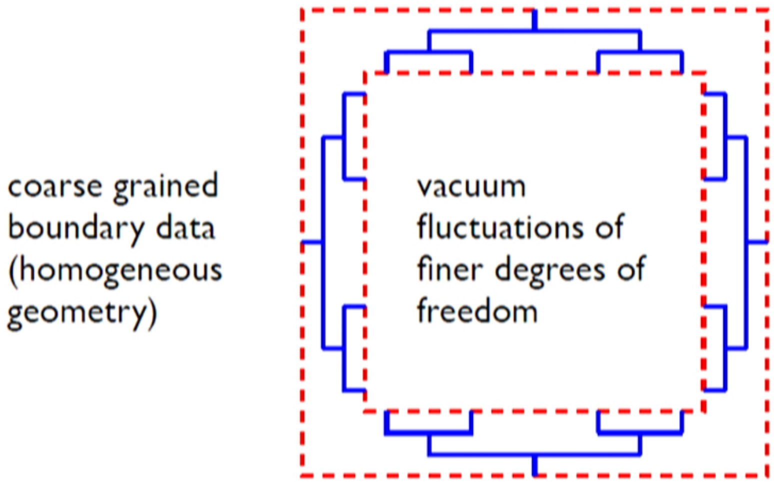
Build up the physical vacuum

coarse grained boundary data (homogeneous geometry)



Embeddings determined by the dynamics of the system. Represent the physical vacuum for finer degrees of freedom.

Build up the physical vacuum



Embeddings determined by the dynamics of the system. Represent the physical vacuum for finer degrees of freedom.

Other examples:

•tensor network renormalization methods: condensed matter
[Vidal, Levin, Nave, Gu, Wen, ...]

•spin nets: analogue spin foams
[BD, Eckert, Martin-Benito '11][BD, Martin-Benito, Schnetter' 13][BD, Martin-Benito, Steinhaus: to appear]

Coarse graining methods provide

- efficient way to 'solve' the theory as a CONTINUUM theory
- way to understand physical vacuum and your theory at different 'scales'
- renormalization:
  effective way to organize / connect dynamics at different scales

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Causal Set Dynamics: Results in 2D quantum gravity

Sumati Surya

Raman Research Institute

GR20, Warsaw
July 2013

Outline

- The Causal Set Hypothesis
- A Continuum Inspired Dynamics.
- Results in 2D Causal Set Quantum Gravity: The Emergence of the Continuum.
  – S. Surya, Class.Quant.Grav. 29 (2012)
- Open Questions
  – with Lisa Glaser.
  – with J. Henson, D. Rideout and R. Sorkin.

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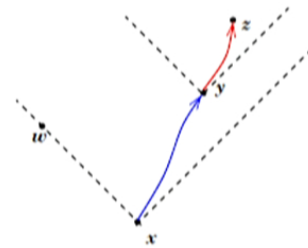
The Causal set Hypothesis

This is based on two fundamental building blocks:

- The Causal Structure Poset $(M, \prec)$



  - M is the *set* of events.
  - $\prec$ is:
    - Acyclic: $x \prec y$ and $y \prec x \Rightarrow x = y$
    - Reflexive: $x \prec x$
    - Transitive: $x \prec y, y \prec z \Rightarrow x \prec z$

The Causal set Hypothesis

This is based on two fundamental building blocks:

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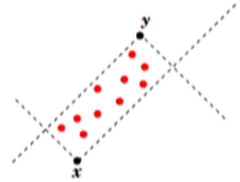
- Fundamental Spacetime Discreteness:

  V has $n \sim V/V_p$ fundamental spacetime atoms.



Be Wise, Discretise! —- Mark Kac

The Causal set Hypothesis

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$\Downarrow$

The underlying structure of spacetime is a locally finite poset $(C, \prec)$ or a *causal set*

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Spacetime from Causal Sets

Causal Structure + Spacetime Volume = Spacetime Geometry

–Malament, Hawking, King, McCarthy, etc.

Causal Structure $\longrightarrow$ Partially Ordered Set

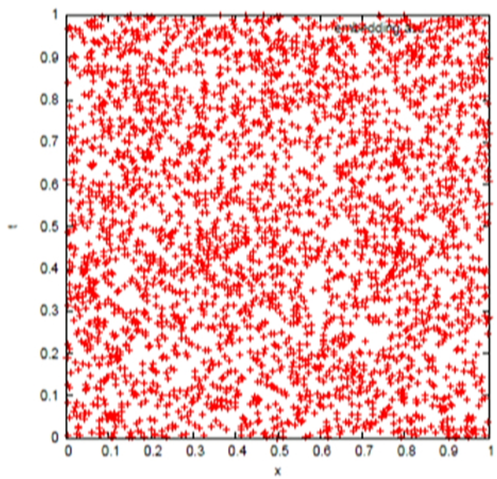
Spacetime Volume $\longrightarrow$ Number

Order + Number $\sim$ Spacetime Geometry

Spacetime from Causal Sets

- Regular lattice does not preserve Number-Volume correspondence
- Random lattice generated via a Poisson process:

$$P_V(n) \equiv \frac{1}{n!} e^{-\rho V} (\rho V)^n, \quad < N >= \rho V$$



A Continuum Inspired Dynamics for Causal sets

- From first principles:

  Quantum Sequential Growth Dynamics using a histories based "**quantum measure**" formulation.

  – F. Dowker, S. Johnston, S. Surya, J.Phys. A43 (2010), R. D. Sorkin, arXiv:1104.0997, J. Henson, Stud.Hist.Philos.Mod.Phys. 36 (2005)

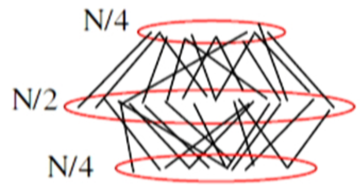
- Continuum Inspired Dynamics:

$$Z = \sum_{c\in\Omega} \exp^{i\frac{S(c)}{\hbar}}$$

  - Sample space Ω is a collection of causal sets. Example, the set of all countable past-finite causal sets.
  - S(C) is a causal set action. An example of this is the Benincasa-Dowker action $S(C)$.

The Sample Space of Causal Sets

- Unimodular gravity: Fix N and then take $N \to \infty$.
- In the large N limit, a generic causal set looks nothing like spacetime:



$$\log P_N = N^2/4 + 3n/2 + O(\log N).$$

– Kleitman and Rothschild, Trans AMS, (1975)

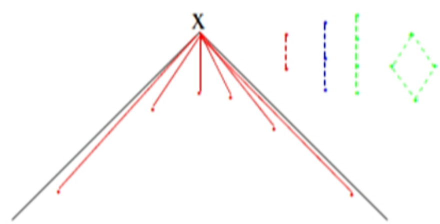
- Microcanonical ensemble: infinite sequence of first order phase transitions
  Important comparisons to lattice gas models with long range interactions.
  – D. Dhar, J.M.P (1978).

The Causal Set Action

- Benincasa-Dowker-Glaser Action for Causal Sets:
  – D. Benincasa and F.Dowker PRL, (2010), F. Dowker and L. Glaser, arXiv:1305.2588.

$$\frac{S^{(d)}(C)}{\hbar} = \zeta_d \left(N + \frac{\beta_d}{\alpha_d} \sum_{i=1}^{n} C_i^{(d)} N_i \right)$$

- N_i: # of i-element order intervals



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Analytic Continuation: Quantum Dynamics → Thermodynamics

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$$Z = \sum_{C \in \Omega} \exp^{i\frac{S(C)}{\hbar}}$$

- Introduce a new parameter β (inverse temperature)

  $i\beta S(C) \rightarrow -\beta S(C)$

- Space of Configurations Ω is unchanged: There is no need for "Euclideanising" Ω.

$$Z = \sum_{C \in \Omega} e^{-\beta \frac{S(C)}{\hbar}}$$

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2D Causal Set Quantum Gravity

Construct a 2D theory of causal sets

$$Z[N] = \sum_{\text{2D orders}} \exp^{-\frac{\beta}{\hbar} S_{2d}}$$

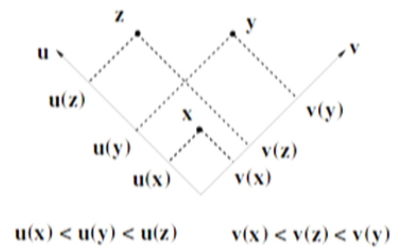
$$\Omega = \{ \text{ 2D orders } \}$$

$S_{2d}(C)$: Benincasa-Dowker Action.

All topologically trivial conformally flat spacetimes $\rightarrow$ 2D orders

A 2D orders

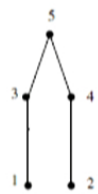
$$U = \{u_1, u_2, \ldots, u_N\} \text{ and } V = \{v_1, v_2, \ldots, v_N\}$$



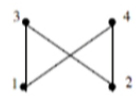
$$x \prec y \quad \Leftrightarrow \quad u(x) < u(y) \quad \text{and} \quad v(x) < v(y)$$

$\Phi(C) = U \cap V$ is a 2D ORDER

The Sample Space of 2D orders

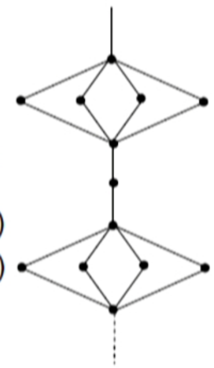


L = (1,3,2,4,5)
M= (2,4,1,3,5)



L = (1,2,3,4)
M= (2,1,4,3)

L = (1,2,3,4)
M= (4,3,2,1)



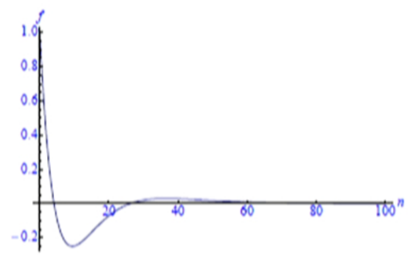
2D random orders ($\sim {}^2\mathbb{M}$) dominate the uniform distribution

– M.H. El-Zahar and N.W. Sauer, Order, (1988), P. Winkler, Order, (1991), G. Brightwell, J. Henson, S. Surya, CQG (2008).

The 2d Benincasa-Dowker Action for a Causal Set

$$S(\epsilon)/\hbar = 4\epsilon\left(N - 2\epsilon\sum_{n=0}^{N-2} N_n f(n,\epsilon)\right)$$

- Mesoscale $l_k >> l_p$: $\boxed{\epsilon = \left(\frac{l_p}{l_k}\right)^2 \in [0,1]}$
- $f(n,\epsilon) = (1-\epsilon)^n - 2\epsilon n(1-\epsilon)^{n-1} + \frac{1}{2}\epsilon^2 n(n-1)(1-\epsilon)^{n-2}$

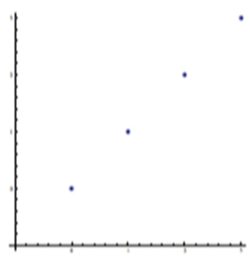


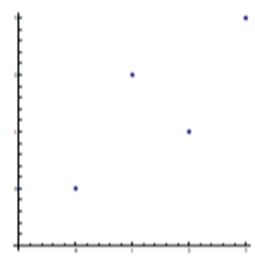
Markov Chain Monte Carlo

The Move:

- $U = (u_1, u_2, \ldots u_i, \ldots u_j, \ldots u_N)$, $V = (v_1, v_2, \ldots v_i, \ldots v_j, \ldots v_N)$
- Pick a pair (u_i, v_i) and (u_j, v_j) at random and exchange: $u_i \leftrightarrow u_j$
- $U' = (u_1, u_2, \ldots u_j, \ldots u_i, \ldots u_N)$, $V' = (v_1, v_2, \ldots v_i, \ldots v_j, \ldots v_N)$
- EXAMPLE:

  $u_2 \leftrightarrow u_3$: $U = (1, 2, 3, 4)$, $V = (1, 2, 3, 4) \longrightarrow U' = (1, 3, 2, 4)$, $V' = (1, 2, 3, 4)$





Covariant Observables

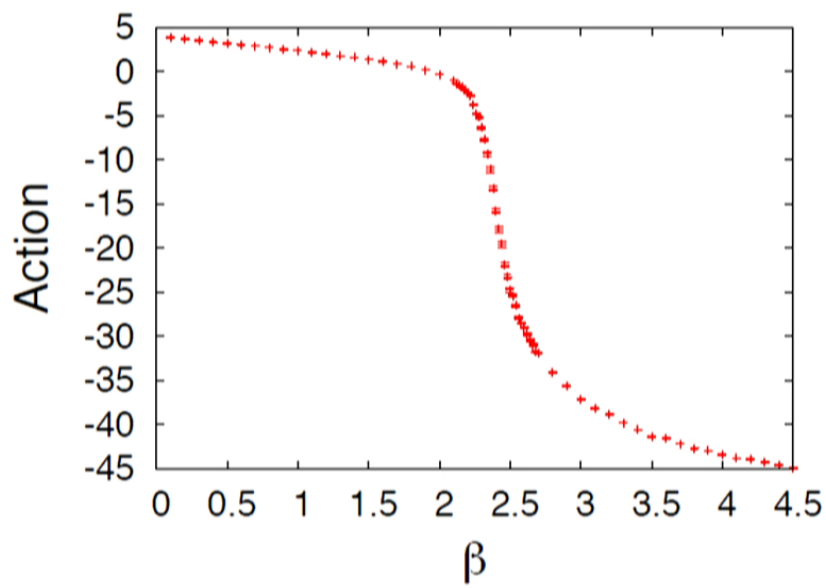
Covariance $\sim$ Label invariance

- Ordering Fraction: $\chi = 2r/N(N-1)$
  r: actual number of relations in the causal set, $N(N-1)/2$: maximum number of possible relations
- Dimension: Spacetime dimension v/s poset dimension
  In 2d Myrheim-Meyer dimension $d_{MM} = \chi^{-1}$
- Action ($\sim$ energy):
  $S(\epsilon)/\hbar = 4N\epsilon \times \left(1 - 2\frac{\epsilon}{N}\sum_{n=0}^{N-2} N_n f(n,\epsilon)\right)$
- N_n: Abundance of n-order intervals
- Height: Length of the longest chain $\sim$ longest time-like distance
- Time asymmetry: Difference in number of minimal and maximal elements

A Cross Over or Phase Transition?

- Fix ϵ
- Plot $\langle \mathcal{O} \rangle (\beta)$

Action vs β



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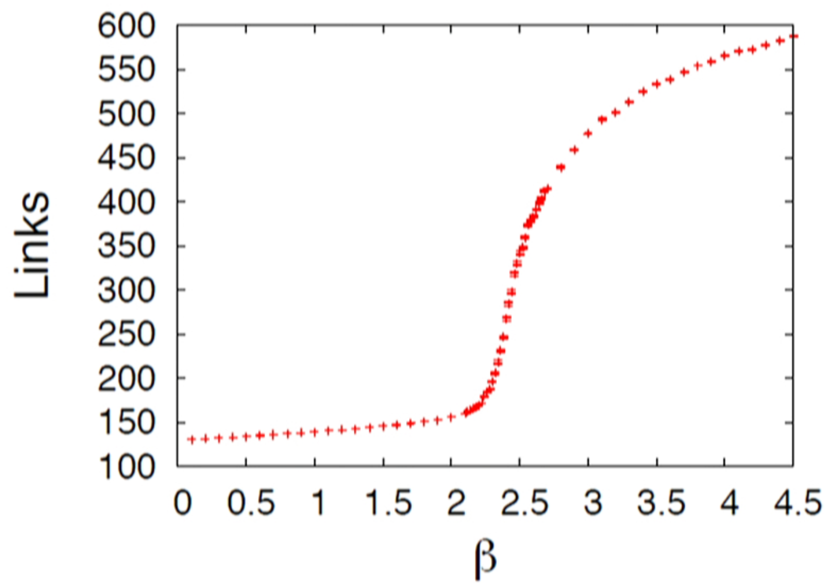
A Cross Over or Phase Transition?

- Fix ϵ
- Plot $\langle \mathcal{O} \rangle(\beta)$

Links vs β



A Cross Over or Phase Transition?

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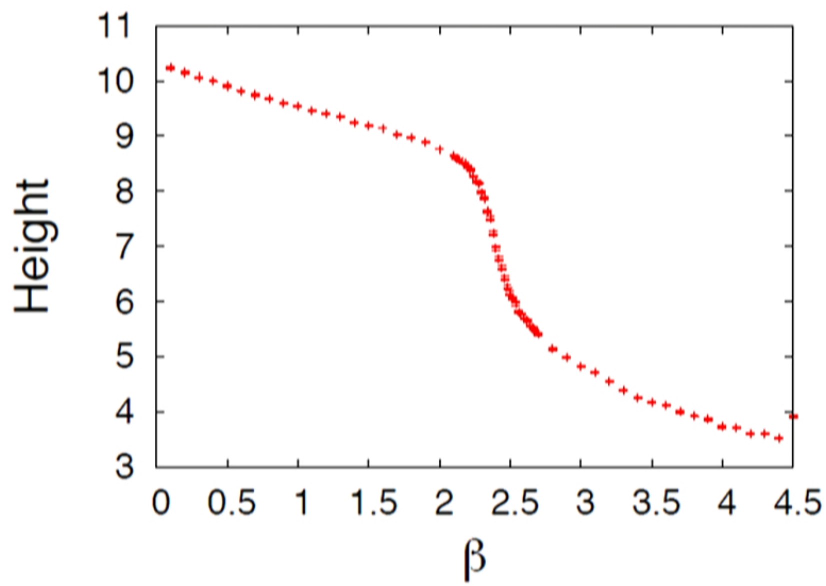
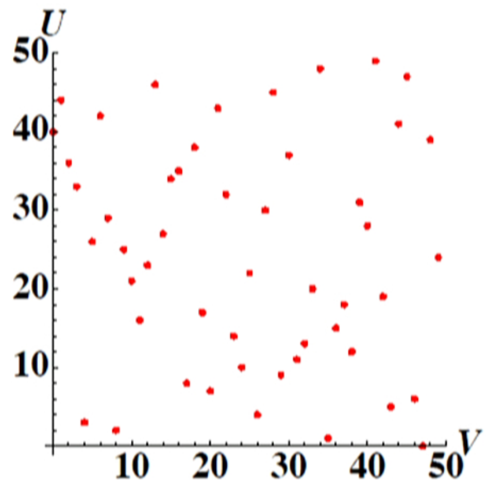
Height v/s β



Continuum Phase



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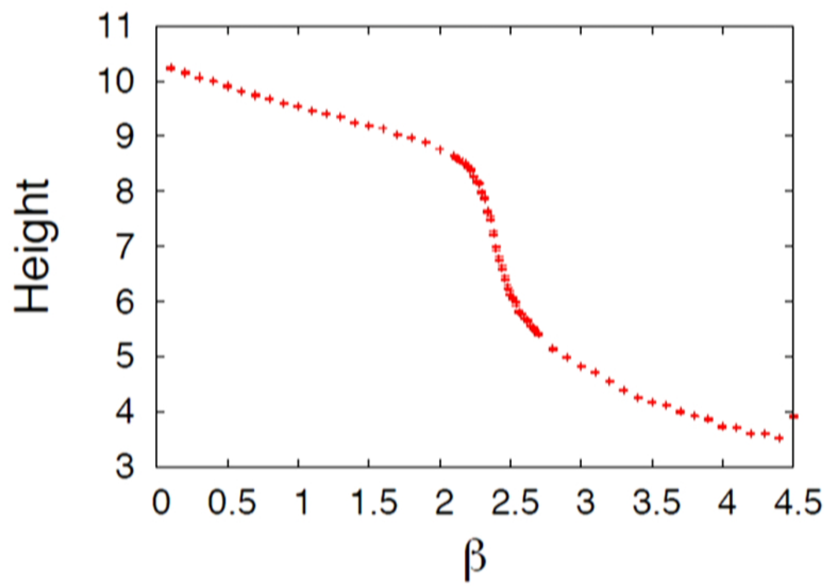
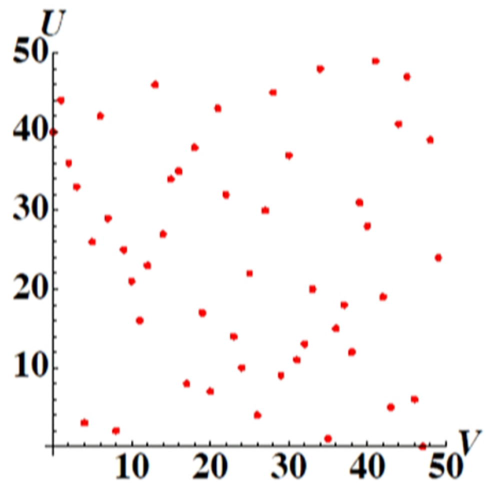


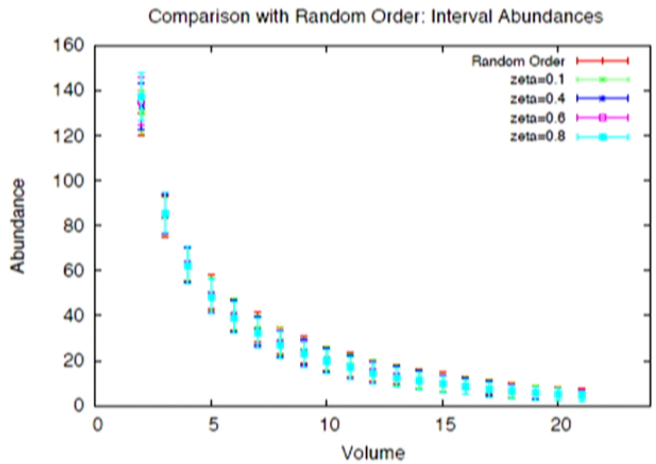
Continuum Phase



Continuum Phase

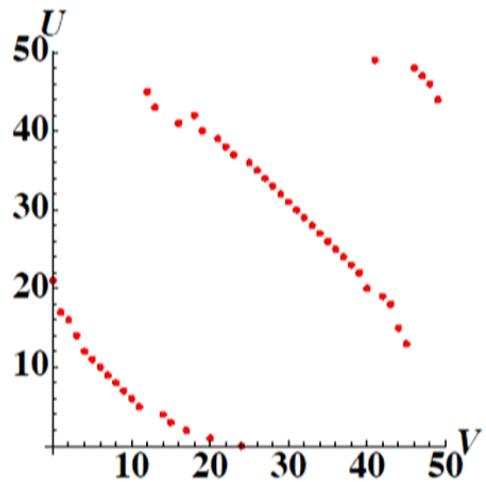
For $\epsilon = 0.12$, $\beta = 0.1$:

- Ordering Fraction: $< \chi >= 0.498 \pm 0.045$. $\Rightarrow$ $< d_{MM} >\sim 2$.
- Height: $< h >= 10.217 \pm 1.401$ (*Height of* $V = 50$ *Minkowski interval is* $\sqrt{100} = 10$)
- Time Asymmetry: $< TA >= -0.007 \pm 2.411$
- Action: $< S > /\hbar = 3.845 \pm 1.256$
- Abundance of Intervals:



Continuum Phase closely resembles the random 2D order aka the Minkowski interval

Crystalline Phase

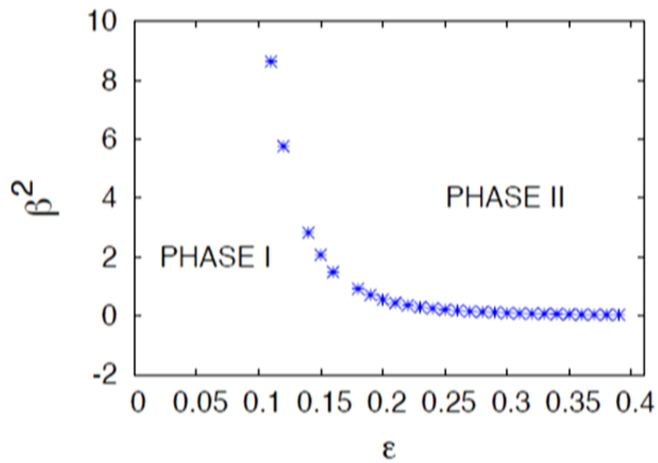


What does this mean?

Caution: Continuum approximation exists without the continuum limit.

$$\beta \rightarrow i\beta$$

Thermodynamics → Quantum Dynamics

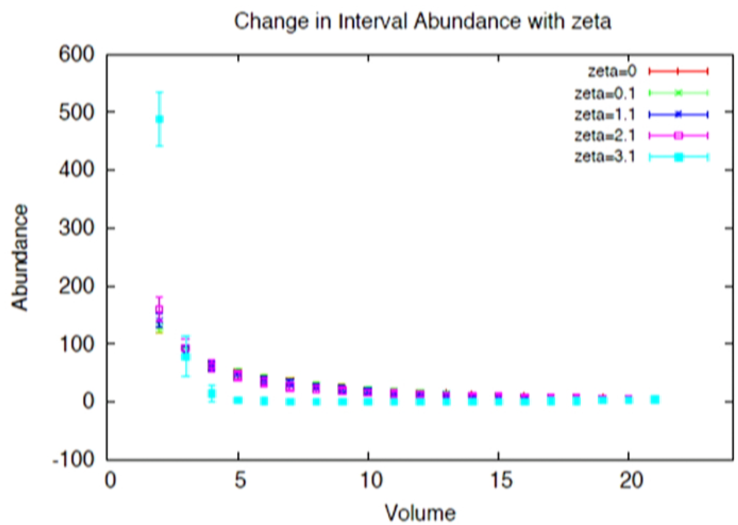


Suggests that the continuum phase may survive the analytic continuation

Crystalline Phase

For $\epsilon = 0.12$, $\beta = 3.1$

- Ordering Fraction: $< \chi >= 0.589 \pm 0.001$. $\Rightarrow$ $< d_{MM} >\sim 1.7$.
- Height: $< h >= 4.631 \pm 0.860$
- Time Asymmetry: $< TA >= -1.327 \pm 5.156$
- Action: $< S > /\hbar = -38.000 \pm 3.197$
- Abundance of Intervals:



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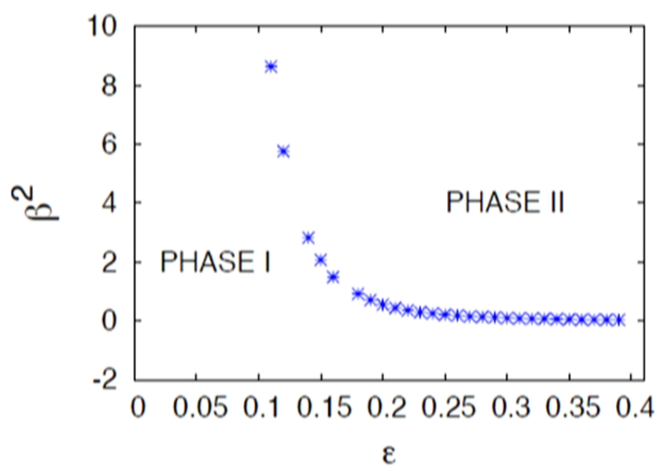
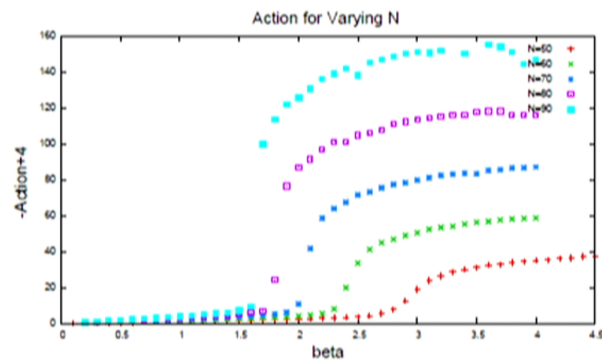


Suggests that the continuum phase may survive the analytic continuation

Finite Size Effects

- How sensitive is the phase diagram to N?



- Monte Carlo RG techniques: Simulations under way with Lisa Glaser.
- System is Non-Extensive with “long range interactions”: is there Finite size scaling?

Conclusions and Open Questions

- MCMC methods can be successfully used to study the quantum dynamics of causal sets using *covariant* observables.
- Appearance of distinct phases separated by a phase transition/cross-over.
- Flat spacetime is emergent in 2D causal set quantum gravity in a precise sense.
- Order of the phase transition: strong hints that it is second order, but questions re. finite size scaling need to be addressed.
- Are there other options for the analytic continuation?
- Does RG help us find a fixed point for the non-locality scale ϵ?
- Does this have implications for MCMC simulations for full 4D causal set quantum gravity with unrestricted sample space Ω? Simulations under way with David Rideout, Joe Henson and Rafael Sorkin.

Thanks to: Rafael Sorkin, David Rideout, Joe Henson, Fay Dowker, Aleksi Kurkela and Lisa Glaser

Continuous Symmetries in Polymer Quantization

Ghanashyam Date

The Institute of Mathematical Sciences, Chennai

July 25, 2013 PITP, Canada

Loops 2013

Introduction

In a quantum theory of gravity, space-time symmetries, in particular the Lorentz symmetry, are thought to be potentially violated. The primary source for this conjecture is supposed to be the natural occurrence of the Planck length/mass which is thought to reflect a fundamental discreteness analogous to that of a lattice structure.

LQG, does reveal discreteness of metrical properties of 'space', quite different from the discreteness of a lattice.

Qn: *Does the metrical discreteness suggest violation of continuous symmetries?*

Introduction (Cont. . . .)

The structure responsible for the metrical discreteness is the specific, non-separable Hilbert space of LQG. Hence the question becomes:

Are certain quantizations in conflict with implementation of continuous symmetries?

This is explored for rotational symmetry in the context of polymer quantum mechanics and polymer quantized scalar field.

(With Nirmalya Kajuri, CQG. 30 (2013) 075010, arXiv:1211.0823)

Plan

Realization of symmetries in Schrodinger Quantum Mechanics

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Plan

Realization of symmetries in Schrodinger Quantum Mechanics

Realization of symmetries in Polymer Quantum Mechanics

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Realization of symmetries in Polymer Quantum Mechanics

The Dual Option

Plan

Realization of symmetries in Schrodinger Quantum Mechanics

Realization of symmetries in Polymer Quantum Mechanics

The Dual Option

Realization of symmetries in Polymer Quantized Scalar Field

Plan

Realization of symmetries in Schrodinger Quantum Mechanics

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Concluding Remarks

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Schrodinger Quantum Mechanics

Basic observables: q^i, p_i ; States: $\langle\vec{k}|\vec{k}'\rangle = \delta^3(\vec{k}-\vec{k}')$

Representation: $p_i|\vec{k}\rangle = \hbar k_i|\vec{k}\rangle$, $q^i|\vec{k}\rangle = i\hbar^{-1}\partial_{k_i}|\vec{k}\rangle$

Rotations matrices: $\Lambda^i{}_m \Lambda^j{}_n \delta^{mn} = \delta^{ij}$

$$q^i_\Lambda := U(\Lambda) q^i \, U(\Lambda)^\dagger = \Lambda^i{}_j q^j \quad , \quad p_i^\Lambda := U(\Lambda) p_i \, U(\Lambda)^\dagger = \Lambda^j{}_i p_j$$

Infinitesimally, $\Lambda^i{}_j := \delta^i{}_j + \epsilon^i{}_j$, $U(\mathbb{1}+\epsilon) := 1 - \frac{i}{\hbar}\epsilon \cdot J$ implies,

$$-\frac{i}{\hbar}[\epsilon \cdot J, q^i] = \epsilon^i{}_j q^j \quad , \quad -\frac{i}{\hbar}[\epsilon \cdot J, p_i] = \epsilon^j{}_i p_j .$$

and $\epsilon^i{}_j := \epsilon_k \mathcal{E}^{ki}{}_j$, $\epsilon \cdot J := \epsilon_k J^k$, leads to,

$$J^k := \mathcal{E}_m{}^{nk} q^m p_n$$

Polymer Quantum Mechanics

Basic observables: $e^{i\vec{k}\cdot\vec{q}}, p_i$; States: $\langle\vec{k}|\vec{k'}\rangle = \delta_{\vec{k},\vec{k'}}$

Representation: $p_i|\vec{k}\rangle = \hbar k_i|\vec{k}\rangle$, $e^{i\vec{k'}\cdot\vec{q}}|\vec{k}\rangle = |\vec{k}+\vec{k'}\rangle$

Action of Rotations ($\Lambda^i{}_m\Lambda^j{}_n\delta^{mn} = \delta^{ij}$) :

$$\left(e^{i\vec{k}\cdot\vec{q}}\right)_\Lambda := U(\Lambda)\left(e^{i\vec{k}\cdot\vec{q}}\right) U(\Lambda)^\dagger = \left(e^{ik_i\Lambda^i{}_j q^j}\right) ,$$

$$p_i^\Lambda := U(\Lambda)p_i\, U(\Lambda)^\dagger = \Lambda^j{}_i p_j$$

$$\therefore U(\Lambda)|\vec{k}\rangle = |\,(\Lambda^{-1})^j{}_i\, k_j\rangle$$

$\therefore$ for every $\sigma > 0$, the subspace spanned by $\{|\vec{k}\rangle, \vec{k}\cdot\vec{k} = \sigma\}$, provides an infinite dimensional, irreducible representation.

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Polymer Quantum Mechanics (Cont. . . .)

A non-trivial invariant Hamiltonian can be constructed from $p \cdot p$ and $e^{-if(p^2)p_i q^i}$ with the action of the latter being,

$$e^{-if(p^2)p\cdot q}|\vec{k}\rangle := |\vec{k}' = \xi\vec{k}\rangle \quad , \quad \int_0^1 d\lambda = \frac{1}{2}\int_{k^2}^{\xi^2 k^2} \frac{dp^2}{p^2 f(p^2)}$$

However, Eigenvalues of such an invariant Hamiltonian are generically infinitely degenerate!

Either (a) rotations cease to be a symmetry (explicit breaking of symmetry) or (b) the symmetry is spontaneously broken.

The Dual Option:

$\mathrm{Cyl} \subset \mathrm{H} \subset \mathrm{Cyl}^*$

Elements of Cyl are suitable countable linear combinations of $|\vec{k}\rangle$ and elements of Cyl* can thus be specified by giving linear functions of $\vec{k}$, $\psi(\vec{k}) := (\Psi|\vec{k}\rangle$.

Every operator A : Cyl → Cyl, defines $\tilde{\mathrm{A}} : \mathrm{Cyl}^* \to \mathrm{Cyl}^*$ by the 'dual action': $(\tilde{\mathrm{A}}\Psi|\mathrm{f}\rangle := (\Psi|\mathrm{Af}\rangle, \forall\ |\mathrm{f}\rangle \in \mathrm{Cyl},\ \forall\ (\Psi| \in \mathrm{Cyl}^*$.

Using the dual $\tilde{U}(\Lambda)$, we can define infinitesimal generators on a subspace of Cyl* as,

$$\frac{i}{\hbar}(\mathrm{J}^{\mathrm{l}}\Psi|\vec{\mathrm{k}}\rangle \quad := \quad \lim_{\epsilon_{\mathrm{l}}\to 0}(\Psi|\frac{\mathrm{U}(\mathbb{1}+\epsilon)-\mathrm{U}(\mathbb{1}-\epsilon)}{2\epsilon_{\mathrm{l}}}|\vec{\mathrm{k}}\rangle$$

$$= \quad (2\epsilon_l)^{-1}\left[(\Psi|\vec{\mathrm{k}}+\overrightarrow{\epsilon\mathrm{k}}\rangle - (\Psi|\vec{k}-\overrightarrow{\epsilon\mathrm{k}}\rangle\right] \quad = \quad \mathcal{E}^{\mathrm{li}}{}_{\mathrm{j}}\mathrm{k}^{\mathrm{j}}\frac{\partial\psi^*}{\partial\mathrm{k}^{\mathrm{i}}}$$

The Dual Option: (Cont. . . .)

Likewise, for each orthonormal triad, $\hat{e}_j,\ j = 1, 2, 3,\ \hat{e}_i \cdot \hat{e}_j = \delta_{ij}$

and a small parameter δ, we have, $U_{\delta \hat{e}_j}(\vec{q}) := e^{i\delta \hat{e}_j \cdot \vec{q}}$ and

$\sin_{\delta \hat{e}_j} := (2i)^{-1}(U_{\delta \hat{e}_j}(\vec{q}) - U_{-\delta \hat{e}_j}(\vec{q}))$ leading to,

$$(\hat{e}_j \cdot \tilde{q}\ \Psi|\vec{k}\rangle := \lim_{\delta \to 0}(\Psi|\frac{\sin_{\delta \hat{e}_j}}{\delta}|\tilde{k}\rangle = \frac{1}{2i\delta}\left[(\Psi|\tilde{k} + \delta\hat{e}_j\rangle - (\Psi|\tilde{k} - \delta\hat{e}_j\rangle\right]$$

$$= \frac{\psi^*(\tilde{k} + \delta\hat{e}_j) - \psi^*(\tilde{k} - \delta\hat{e}_j)}{2i\delta} = -i\hat{e}_j \cdot \tilde{\nabla}_{\tilde{k}}\psi^*$$

It follows,

$$(\ [\hat{e}_m \cdot \tilde{q}\ ,\ \hat{e}_n \cdot \tilde{p}]\ \Psi|\tilde{k}\rangle = (\ \{i\hbar\hat{e}_m \cdot \hat{e}_n\}\ \Psi|\tilde{k}\rangle\ .$$

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Polymer Quantized Scalar Field

Orthonormal states are labelled, for each $n \geq 0$, by *vertex sets*, $V = (\vec{x}_1, \ldots, \vec{x}_n), \vec{x}_i \in \mathbb{R}^3$ and corresponding set of non-zero, real numbers $(\lambda_1, \ldots, \lambda_n)$ and are denoted as

$$\mathcal{N}_{V,\vec{\lambda}}(\phi) := e^{i\sum_j \lambda_j \phi(\vec{x}_j)} \quad \leftrightarrow \quad |V, \vec{\lambda}\rangle.$$

The smeared momenta,

$$P_g := \int d^3x g(\vec{x}) \pi_\phi(x) \; = \; -i\hbar \int d^3x g(\vec{x}) \frac{\delta}{\delta\phi(\vec{x})},$$

satisfy, $[P_f, P_g] = 0$, $P_f^\dagger \; = \; P_f$ and act on the basis states as,

$$P_g \mathcal{N}_{V,\vec{\lambda}} \; = \; \left[\hbar \sum_j \lambda_j g(\vec{x}_j)\right] \mathcal{N}_{V,\vec{\lambda}}$$

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Polymer Quantized Scalar Field (Cont. . . .)

Elements of Cyl* are specified by giving, $(\Psi|V, \vec{\lambda}\rangle =:$

$$\psi^*(\vec{x}_1, \cdots, \vec{x}_n, \lambda_1, \cdots, \lambda_n) \ , \ \vec{x}_i \neq \vec{x}_j, \ \forall \ i \neq j \text{ and } \quad \lambda_i \neq 0, \forall \ i.$$

Following similar steps as before, we can define the infinitesimal generators on a subspace of $\psi^*(V; \vec{\lambda})$ which are

differentiable w.r.t. the $\vec{x}$ arguments.

To define smeared scalar field operators, we need

differentiability w.r.t. the $\vec{\lambda}$ arguments and in addition,

$$\frac{\partial \psi^*}{\partial \lambda_j}(\vec{x}_1, \ldots, \vec{x}_n, \lambda_1, \ldots, \lambda_j, \ldots, \lambda_n)\Big|_{\lambda_j=0} = 0 \ \ , \forall \ j = 1, 2, \ldots, n \ .$$

Polymer Quantized Scalar Field (Cont. . . .)

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$\psi^*(\vec{x}_1, \cdots, \vec{x}_n, \lambda_1, \cdots, \lambda_n)$, $\vec{x}_i \neq \vec{x}_j$, $\forall\ i \neq j$ and $\lambda_i \neq 0, \forall\ i$.

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Polymer Quantized Scalar Field (Cont. . . .)

These lead to the definitions:

$$(J^k \Psi | V, \vec{\lambda}\rangle \;:=\; -\mathrm{i}\hbar\, \mathcal{E}^{\mathrm{ki}}{}_{\mathrm{j}} \sum_{\mathrm{m}=1}^{\mathrm{n}} \mathrm{x}^{\mathrm{j}}_{\mathrm{m}} \frac{\partial \psi^*}{\partial \mathrm{x}^{\mathrm{i}}_{\mathrm{m}}}$$

$$(\widetilde{\phi_f} \Psi | V, \vec{\lambda}\rangle \;:=\; -i \sum_j f(\vec{x}_j) \frac{\partial \psi^*(\vec{x}_1, \ldots, \vec{x}_n, \lambda_1, \ldots, \lambda_n)}{\partial \lambda_j} \;.$$

It is easy to verify,

$$(\widetilde{[\phi_f, P_g]} \Psi | V, \vec{\lambda}\rangle \;=\; (\left\{ +i\hbar \left(\sum_{j=1}^{n} f(\vec{x}_j) g(\vec{x}_j) \right) \right\} \Psi | V, \vec{\lambda}\rangle \;;$$

$$(\widetilde{[J^k, \phi_f]} \Psi | V, \vec{\lambda}\rangle \;=\; i\hbar\, (\widetilde{\phi_{\mathcal{L}_k f}} \Psi | V, \vec{\lambda}\rangle \quad , \quad \mathcal{L}_k f(\vec{x}) := \mathcal{E}^{ki}{}_j x^j \frac{\partial f}{\partial x^i}$$

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Concluding Remarks

Continuous symmetries can be implemented in polymer quantization but with a physically unacceptable price of infinitely degenerate energies.

Therefore these must be broken explicitly or spontaneously.

Alternatively, the polymer quantization may be treated as an intermediate step. By going to the dual Cyl*, it is possible to re-gain infinitesimal symmetries.

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THANK YOU.