Title: Group Field Theory and Tensor Models - 2

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Abstract:

Beta Functions of $U(1)^d$ Gauge Invariant Just Renormalizable Tensor Models

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Abelian TGFT with gauge invariance

This part addresses a summary of the results obtained in ours previous work [arXiv:1211.2618]. We mainly present the model and its renormalization. TGFTs over a group G are defined by a complex field φ over d copies of group G, i.e.

$$\begin{array}{rccc} \varphi : & G^d & \longrightarrow & \mathbb{C} \\ & (g_1, \cdots, g_d) & \longmapsto & \varphi(g_1, \cdots, g_d) . \end{array} \tag{1}$$

The gauge invariance condition is achieved by imposing that the fields obey the relation

$$\varphi(hg_1, \ldots, hg_d) = \varphi(g_1, \ldots, g_d), \quad \forall h \in G . \tag{2}$$

For Abelian TGFTs, one fixes the group $G = U(1)$. In the momentum representation, the field writes

$$\varphi(g_1, \cdots, g_d) = \sum_p \varphi_{[p]} e^{ip_1\theta_1} e^{ip_2\theta_2} \cdots e^{ip_d\theta_d}, \quad \theta_k \in [0, 2\pi),$$

where we denote $\varphi_{[p]} = \varphi_{12\cdots d} := \varphi(p_1, p_2, \cdots, p_d)$, with $p_k \in \mathbb{Z}$ and $g_k = e^{i\theta_k} \in U(1)$.

Locality principle: Interactions

The generalized locality principle of the TGFTs requires to define the interactions as the sum of tensor invariants. From now, we will focus on $d = 6, 5$, and define two models described by

$$\mathfrak{S}_4[\bar\varphi, \varphi] = \sum_{p_1, \cdots, p_6} \bar\varphi_{654321}\, \delta(\sum_i^6 p_i)(p^2 + m^2)\, \varphi_{123456} + \tfrac{1}{2}\lambda^{(4)}_{4,1}\, V^6_{4,1}, \quad (4)$$

$$\begin{aligned}\mathfrak{S}_6[\bar\varphi, \varphi] &= \sum_{p_1, \cdots, p_5} \bar\varphi_{54321}\, \delta(\sum_i^5 p_i)(p^2 + m^2)\, \varphi_{12345} \\ &+ \tfrac{1}{2}\lambda^{(6)}_{4,1}\, V^5_{4,1} + \tfrac{1}{2}\lambda_{4,2} V_{4,2} + \tfrac{1}{3}\lambda_{6,1} V_{6,1} + \lambda_{6,2} V_{6,2}, \quad (5)\end{aligned}$$

where $\delta(\sum_i^d p_i)$ should be understood as a Kronecker symbol $\delta_{\sum_i^d p_i, 0}$ and $p^2 = \sum_i^d p_i^2$, $d = 6, 5$.

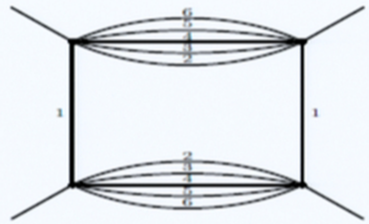

Figure: Vertex representation of φ^4_6-model

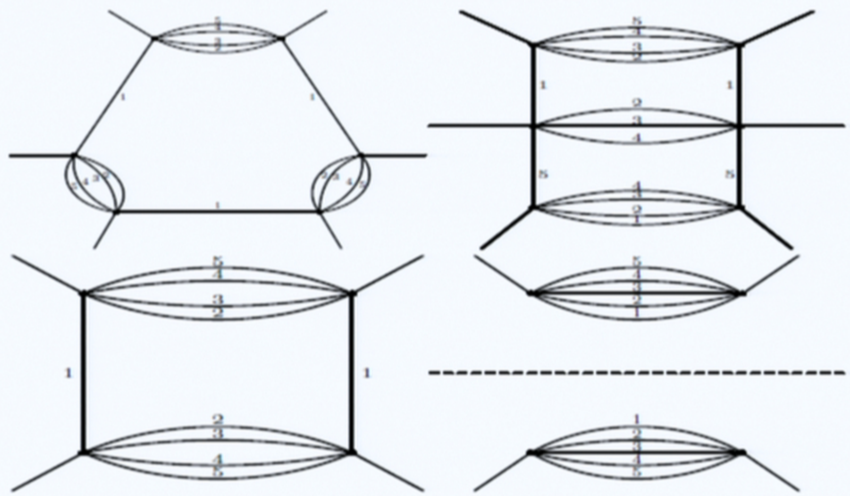

Figure: Vertex representation of φ^6_5-model

Multiscale analysis

Let $\mathcal{L}$ and $\mathcal{F}$ be the sets of internal lines and faces of the graph $\mathcal{G}$. The divergence degree of the amplitude of a graph associated with both models can be written

$$\omega_d(\mathcal{G}) = 2L - F + R \tag{11}$$

where $L = |\mathcal{L}|$, $F = |\mathcal{F}|$ and R is the rank of matrix $\big(\epsilon_{lf},\ l \in \mathcal{L},\ f \in \mathcal{F}\big)$, defined by

$$\epsilon_{lf}(\mathcal{G}) = \begin{cases} 1 & \text{if } l \in f \text{ and their orientation match,} \\ -1 & \text{if } l \in f \text{ and their orientation do not match,} \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

Let $\rho(\mathcal{G})$ be defined as $\rho(\mathcal{G}) = F(\mathcal{G}) - R(\mathcal{G}) - (d-2)\big(L(\mathcal{G}) - V(\mathcal{G}) + 1\big)$.

$$\begin{aligned} \omega_d &= -\frac{2}{(d-1)!}\big(\widetilde{\omega}(\mathcal{G}) - \omega(\partial\mathcal{G})\big) - (C_{\partial\mathcal{G}} - 1) - \frac{d-3}{2}N + (d-1) \\ &\quad + \frac{d-3}{2}n \cdot V - (d-1)V - R \qquad (13) \\ &= \frac{1}{2}\Big[-(d-4)N + (d-4)n \cdot V - 2(d-2)V + 2(d-2) + \rho(\mathcal{G}) \Big] \end{aligned}$$

Renormalization: Divergent graphs

Theorem: Vignes-Tourneret and Samary

The models φ_6^4 defined by $\mathfrak{S}_4$ and φ_5^6 defined by $\mathfrak{S}_6$ are perturbatively renormalizable at all orders.
The proof of this statement rests on a power counting theorem which can be summarized by the following table giving the list of primitively divergent graphs

	N	$\omega(\mathcal{G})$	$\omega(\partial\mathcal{G})$	$C_{\partial\mathcal{G}} - 1$	$\omega_d(\mathcal{G})$
φ_6^4	4	0	0	0	0
	2	0	0	0	2
φ_5^6	6	0	0	0	0
	4	0	0	0	1
	4	0	0	1	0
	2	0	0	0	2
	2	0	0	0	1

(14)

Table: Divergent graphs of both models

Beta functions of φ^4_6-model

Theorem

The wave function renormalization is

$$Z = 1 - \frac{12\pi^2}{5\sqrt{5}}\lambda_4\,\mathcal{I} + O(\lambda_4^2). \tag{15}$$

The sum of all amputated 1PI four-point functions computated at one-loop and at low external momenta is

$$\Gamma_4(0) \quad = \quad -\lambda_4 + \frac{\pi^2}{\sqrt{5}}\lambda_4^2\mathcal{I} + O(\lambda_4^2). \tag{16}$$

At one-loop, the renormalized coupling constant associated with λ_4 is given by

$$\lambda_4^{\rm ren} = \lambda_4 + \frac{19\pi^2}{5\sqrt{5}}\lambda_4^2\mathcal{I} + O(\lambda_4^2), \quad \text{with} \quad \mathcal{I} = \int_0^\infty \frac{e^{-\alpha m^2}}{\alpha}$$

such that the β-function of the model with single wave-function renormalization and single coupling constant is given by $\beta = -\frac{19}{5}$ model is then assyptotically free.

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Effective (quantum?)
cosmology/
Friedmann equation

A possible derivation of EFT for Cosmology

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Field equations
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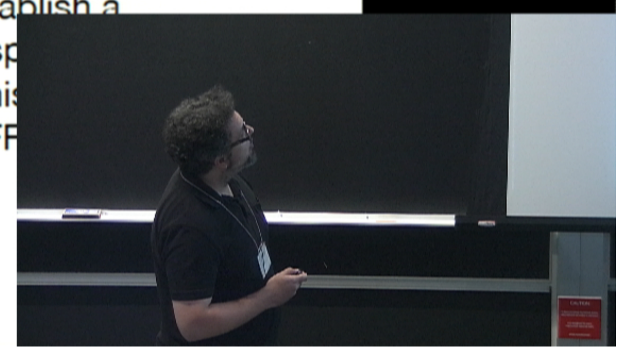


2

GFT in 2nd quantization

SU(2),SL(2,C),SO(4),...

$$\vec{g}_i = (g_{(i,1)}, \ldots, g_{(i,4)})$$

$$\left[\hat{\varphi}(g_1, \ldots g_4), \hat{\varphi}^\dagger(h_1, \ldots h_4)\right] = \prod_{i=1}^{4} \delta(g_i h_i^{-1})$$

$$\Big(\mathcal{K}(\vec{g};\vec{h})\hat{\varphi}(\vec{h}) + \mathcal{V}(\vec{g},\vec{g}_2,\vec{g}_3,\vec{g}_4,\vec{g}_5)\hat{\varphi}(\vec{g}_2)\hat{\varphi}(\vec{g}_3)\hat{\varphi}(\vec{g}_4)\hat{\varphi}(\vec{g}_5) +$$

$$\mathcal{U}(\vec{g},\vec{g}_2,\vec{g}_3,\vec{g}_4,\vec{g}_5)\hat{\varphi}^\dagger(\vec{g}_2)\hat{\varphi}^\dagger(\vec{g}_3)\hat{\varphi}^\dagger(\vec{g}_4)\hat{\varphi}^\dagger(\vec{g}_5) + \ldots\Big)\,|\psi\rangle = \mathbf{0}$$

Perturbative expansion of the partition function

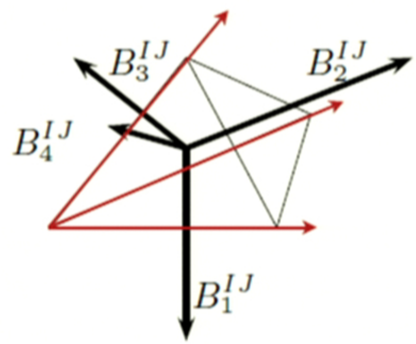
$\downarrow$

Perturbative expansion of the state (Wick theorem applied to ladder operators)

Geometric content/II

$$g_{ab} = g_{ij} v_a^i v_b^j$$

Metric tensor: need the triad



Embedding procedure: take the tetrahedron, embed it into a 3D group manifold (that will be determined by selfconsistency) such that the edges are aligned with a set of left invariant vector fields.

Continuum limit ~ take a lot of them

Furthermore, take them in such a way that the metric (in the left inv. frame) is constant.

- Obtain a state associated to homogeneous (anisotropic) cosmologies!
- It has a hydrodynamic interpretation

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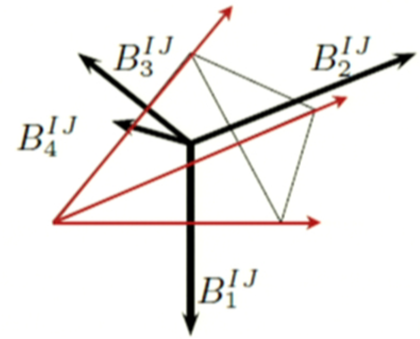

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Towards EFT

- Impose the EOM of GFT: ask that the state are physical
- The equations for the mean field: Wheeler-deWitt equation?

$$\langle\sigma|\hat{\mathcal{O}}[\varphi,\overline{\varphi}]\hat{\mathcal{C}}|\sigma\rangle = 0 \quad \xrightarrow{\text{among other things}} \quad \mathcal{H}_{(\hat{\mathcal{C}})} \triangleright \sigma = 0$$

Depends on the GFT model

Nonlinear!!

- To get an effective cosmological dynamics, we need to translate the equations for the mean field into equations relating these expectation values (~ Ehrenfest theorem)

$$0 = \mathcal{F}(\langle a\rangle_\psi, \langle H\rangle_\psi, \cdots) \sim \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} + \text{corrections}$$

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7


CAUTION

A special case

$$\int (dh)^4 \mathcal{K}(g_I, h_J)\xi(h_J k_J^{-1}) = 0$$

- Consider a Riemannian model: SO(4) gets reduced to SU(2) once the simplicity constraints have been imposed.

$$\xi : \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{SU}(2) \to \mathbb{C}$$

$$K(g_I, h_I) = \delta(g_I h_I^{-1}) \left(\triangle_{\mathrm{SU}(2)^4} + \mu\right) = \delta(g_I h_I^{-1}) \left(\sum_{I=1}^{4} \triangle_{\mathrm{SU}(2)} + \mu\right)$$

- WKB-like approximation:

$$\xi(g_1, g_2, g_3, g_4) = A(g_1, g_2, g_3, g_4) \exp\left(\frac{i}{\kappa} S(g_1, g_2, g_3, g_4)\right)$$

- Restrict to the leading order in the WKB expansion

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- Parametrization for SU(2) $\quad g = \sqrt{1-\pi^2}\mathbb{I}_2 - i\pi^i\sigma_i, \qquad ||\vec{\pi}|| \leq 1$

$$\triangle_{\mathrm{SU}(2)} = \left(\delta^{ij} - \pi^i\pi^j\right) \frac{\partial}{\partial \pi^i}\frac{\partial}{\partial \pi^j}$$

- Remember the geometrical content of the model

$$\frac{\partial S}{\partial \pi_I^i} = B_I^i \sim a^2 \qquad\qquad B_I = a_I^2 T_I, \qquad \pi_I = p_I V_I$$

- Final equation:

$$\sum_{I=1}^{4} \left(B_I \cdot B_I - (\pi_I \cdot B_I)^2\right) \overset{\kappa\to 0}{=} 0$$

- Reduce to the isotropic sector: $\quad a_I = \gamma_I a, \qquad p_I = \beta_I p$

$$\boxed{a^4(p^2 - c^2) \overset{\kappa\to 0}{=} 0} \longrightarrow$$

Corresponds to k=1 FRW, G=SU(2) (Selfconsistency)

a=0 is spurious (add matter)

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GFT & correlation functions

- First: we need for consistency of the interpretation "small tetrahedra": the local curvature radius is much larger than the size of the tetrahedron

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Approximate states
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Comments

- General procedure: design your spinfoam/GFT model and apply the routine
- Shows that we might need LQG data in GFT to give physical meaning
- Allows comparison with other approaches (LQC, spinfoams, WDW)
- Keeps alive part of the sum over geometries (bulk and boundary)
- No background lattice (only the embedding procedure)
- Cosmology as a simple form of GFT hydrodynamics (see geometrogenesis/emergent gravity)
- Can keep under control approximations and decide how good/bad is our result.

Gielen, Oriti, LS 1308.----


CAUTION


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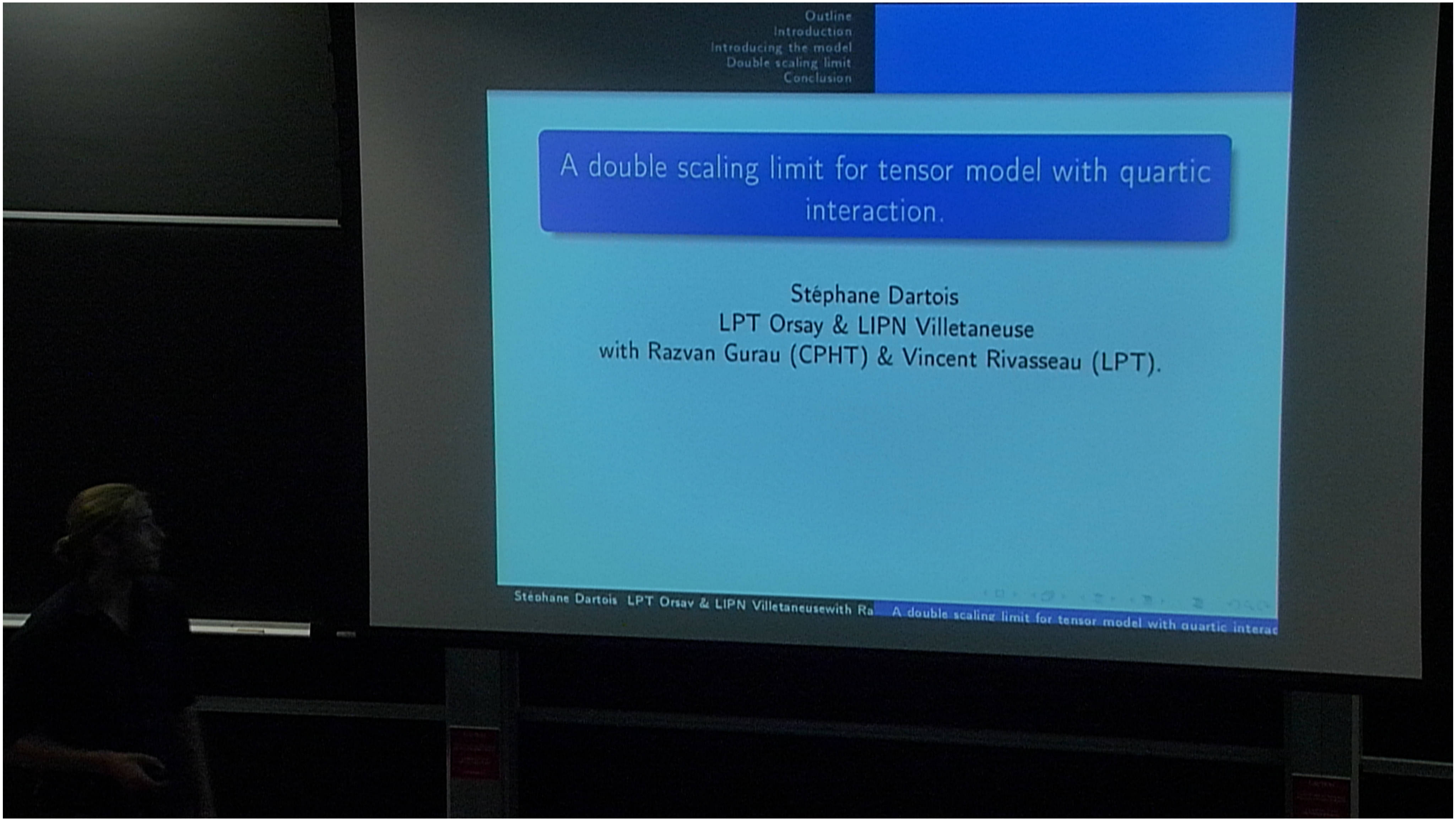
Outline
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Introducing the model
Double scaling limit
Conclusion
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with Razvan Gurau (CPHT) & Vincent Rivasseau (LPT).
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- T^4 model.
- Loop Vertex Expansion graphs.
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- Pruning, Reduction.
- Leading graphs of this scaling.
- Resumming cherry trees.
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Recall known results

Large N limit of matrix model, also called here *single scaling*:

- $G_{2,\text{planar}}(\lambda) = \frac{-1-36\lambda+(1+24\lambda)^{3/2}}{216\lambda^2}$

All spheres but only spheres survive this limit.

For string theory one looks at double scaling limit i.e. $N \to \infty$ and $\lambda \to \lambda_c$, $\kappa^{-1} = N^{5/4}(\lambda - \lambda_c)$.

- $G_{2,\text{double scaling}} = \sum_h a_h \kappa^{2h}$.

Unfortunately *not (Borel) summable !*

For colored tensor models there also exists a single scaling:

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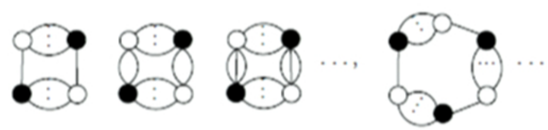
Tensor models and tensor invariant.

Actions of tensor models are polynomial of tensor $U(N)$ invariants.

- **Matrix**: one invariant: trace operator. Action = trace of polynomials of the matrix. Invariants are represented by cycles.

- **Tensor**: plenty of invariants $\Leftrightarrow$ plenty of different contraction patterns. Invariants represented as colored graphs.



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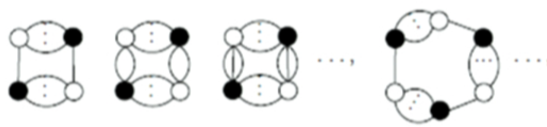


T^4 tensor model.

The double scaling is studied for a melonic $T\bar{T}T\bar{T}$ interaction, i.e. this tensor invariant :

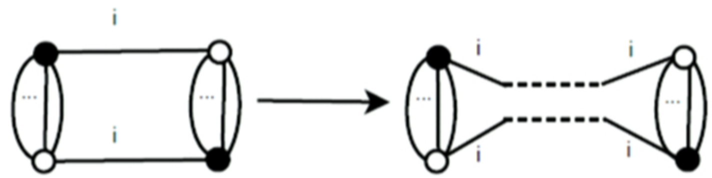


Figure : The interaction term and its intermediate field representation.

One writes the interaction term as:

$$\exp(-\frac{\lambda}{4}\sum_{j,\{k_p\},\{m_q\}} T_{k_1\cdots k_j\cdots k_n}\bar{T}_{m_1\cdots k_j\cdots m_n}T_{m_1\cdots m_j\cdots m_n}\bar{T}_{k_1\cdots m_j\cdots k_n})$$

$$= \int d\mu(\sigma)e^{-\frac{1}{2}\sum_j Tr\left((\sigma^{(j)})^2\right)-\sqrt{\lambda/2}\sum_{j,\{k_p\},\{m_j\}} T_{k_1\cdots k_j\cdots k_n}\bar{T}_{k_1\cdots m_j\cdots k_n}\sigma^{(j)}_{k_j m_j}}.$$

Integrating out the T, $\bar{T}$ fields leads to intermediate field theory.

The Loop Vertex Expansion for tensor models.

The model can be constructed by looking at intermediate field representation and corresponding Feynman graphs.

Graphs of LVE:

- edges are made of sigma field.
- vertices are made of propagator of the original theory. Can be of any degree.

Constructive because this is the Borel sum of the pertubation series for any observable O:

$$O = \sum_{\mathcal{T}} \sum_{\mathcal{G}|\mathcal{G}\supset\mathcal{T}} w(\mathcal{G},\mathcal{T})\mathcal{A}(\mathcal{G}).$$

Convergent ! Very convenient: *melons become trees* in intermediate field representation. Can track $1/N$ factors by tracking the number of loops in the LVE graphs !

What is double scaling ?

- A scaling selecting graphs with optimal combinatorial ratio between powers of $1/N$ and powers of λ.

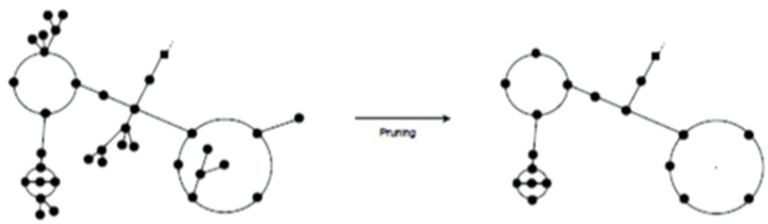
Looking at the 2-point function:

$$G_2 = G_{2,\text{melon}} + \sum_{\bar{G}} \frac{1}{N^{h(\bar{G})}} \frac{1}{(\lambda - \lambda_c)^{e(\bar{G})}}$$
$$= G_{2,\text{melon}} + \sum_{e \geq 1} \sum_{\bar{G}, e(\bar{G})=e} \left(\frac{1}{N^{h(\bar{G})/e(\bar{G})}(\lambda - \lambda_c)} \right)^e .$$

Pruning, Reduction : Computing LVE graphs.

To understand the new graphs, we introduce two procedures:

- Pruning:



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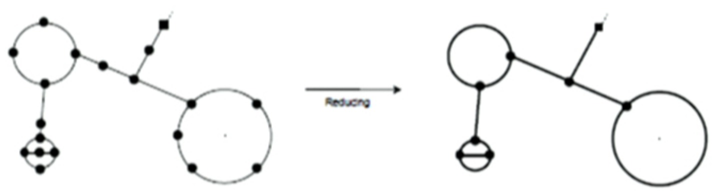

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Pruning, Reduction : Computing LVE graphs.

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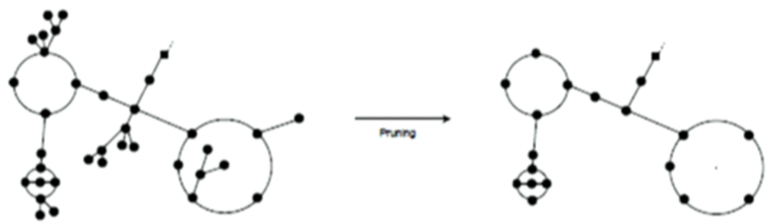

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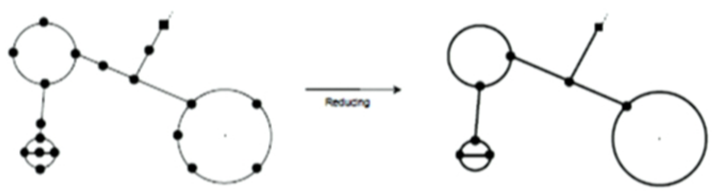

$\rightarrow$ Reduced graphs amplitude are the sum of the amplitudes of all the LVE graphs reducing to it.

Identifying leading graphs.

Using reduced graphs we can identify the family of graphs having the minimal $h(\bar{G})/e(\bar{G})$ ratio:

$$(h(\bar{G})/e(\bar{G}))_{\min} = 1.$$

Set $x = 2N(\lambda - \lambda_c)$ finite, $N \to \infty$ enhances the contribution of the identified family. In dimension D<6 this family maximizes, at fixed number of loops in the reduced graph, the number of 1 PR bars with monocolored loops. We call them *cherry trees*.
Moreover: We can bound the contribution of the amplitude of the non-cherry graphs:

$$|\mathcal{G}^{L,x}_{2,rest}(N)| \leq N^{1/2-D/2} K_L x^{-\frac{3}{2}L+\frac{1}{2}}.$$

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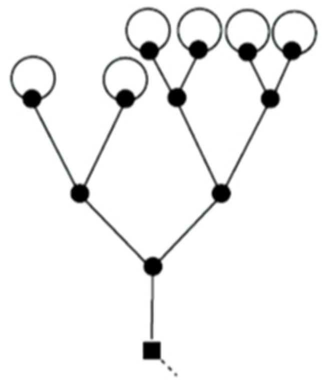
A graphical example of a cherry tree:



We can resum graphs of this form and obtain for the 2-point function:

$$\mathcal{G}^{x}_{2,cherry}(N) = 2 - 4N^{1-D/2}\sqrt{D(x-x_c)}.$$

With a critical point at $x_c = \frac{1}{4(D-1)}$. Melonic critical point: $\lambda_c = \frac{1}{8D}$.

Conclusion & Outlook.

- Double scaling limit allows to compute sum over more triangulations. Sum still runs over spheres.
- As already noticed by Gurau in the analysis of toy model for tensor double scaling, the physical interpretation of the new scaling variable is not clear with respect to GR.

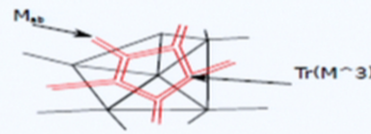
- Critical exponent of $\mathcal{G}_{2,\text{cherry}}$ is still $\gamma = 1/2$ of branched polymer. Investigate the Hausdorff and spectral dimensions of these Cherry trees to confirm (?) they are branched polymer.
- Multiple scaling limit ? A way to sum over more and more graphs (topology ?).
- Details in arXiv:1307.5281. One could also be interested in arXiv:1307.5279 by Razvan Gurau and Gilles Schaeffer.

Tensorial Group Field Theories

Matrix Models 80's [Rev. Di Francesco, 9506153]

• Matrix models : a statistical description for gravity in 2D realized using random triangulations of a manifold;

$$Z_{matrix} = \int dM\, e^{-\frac{1}{2}\mathrm{Tr}M^2 + \frac{g}{\sqrt{N}}\mathrm{Tr}M^3} = e^{Z_{\mathrm{QG}}} \qquad (1)$$



Important tool: 1/N expansion ['t Hooft, Nucl. Phys. B. **72** (74)] $\rightsquigarrow$ Selection of *genus* $= 0$ sector (planar graphs) of the model.

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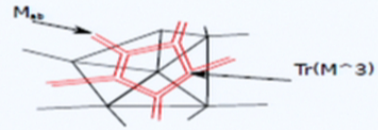

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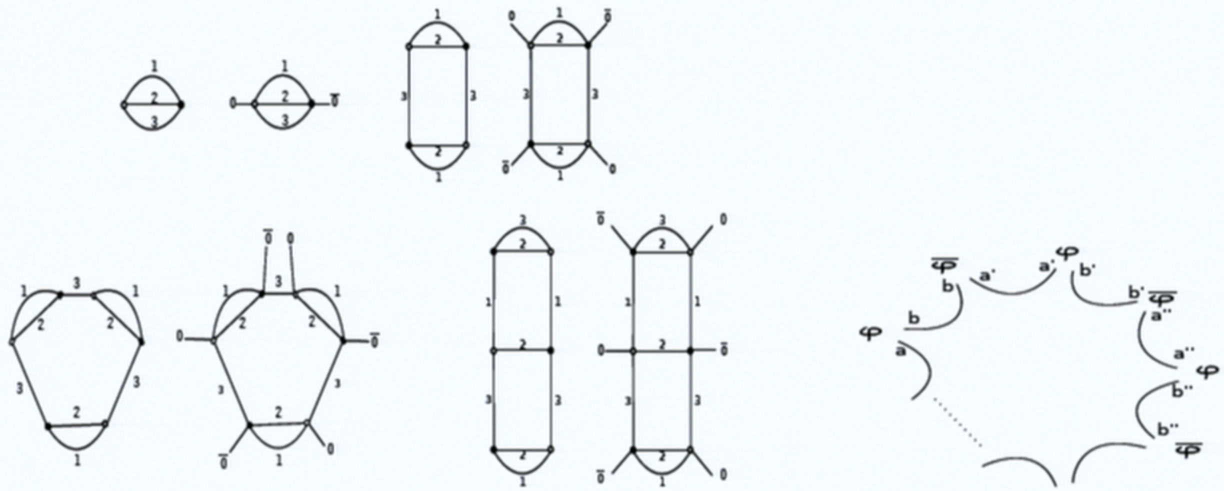
Figure: 3D simplex: A triangle $\sim$a field; the interaction $\sim$ tetrahedron.


CAUTION

Gurau's 1/N limit and the colored world

- Colored tensor models improves a lot the topology associated with simplicial complexes: $T^{a}_{n_1 \ldots n_d}$ (Comment by J.Gaumis (PI) "AhAh!? You don't have enough indices?")
- Allows to understand a 1/N limit: Most dominant amplitudes (called Melons) are associated with the sphere topology ($\forall d$).
- Trace invariants of the melonic kind and Trace invariants in matrix theory



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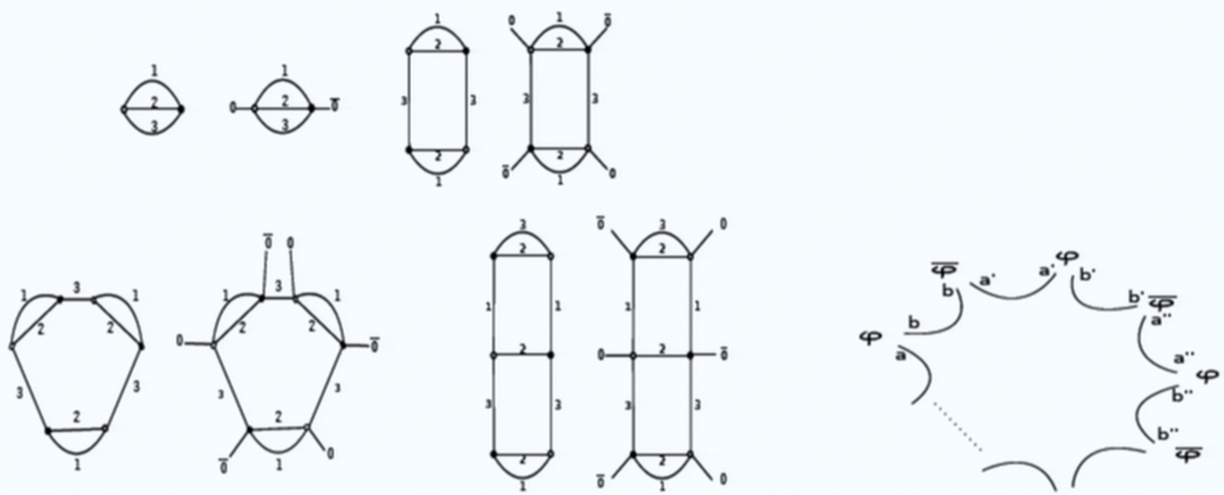

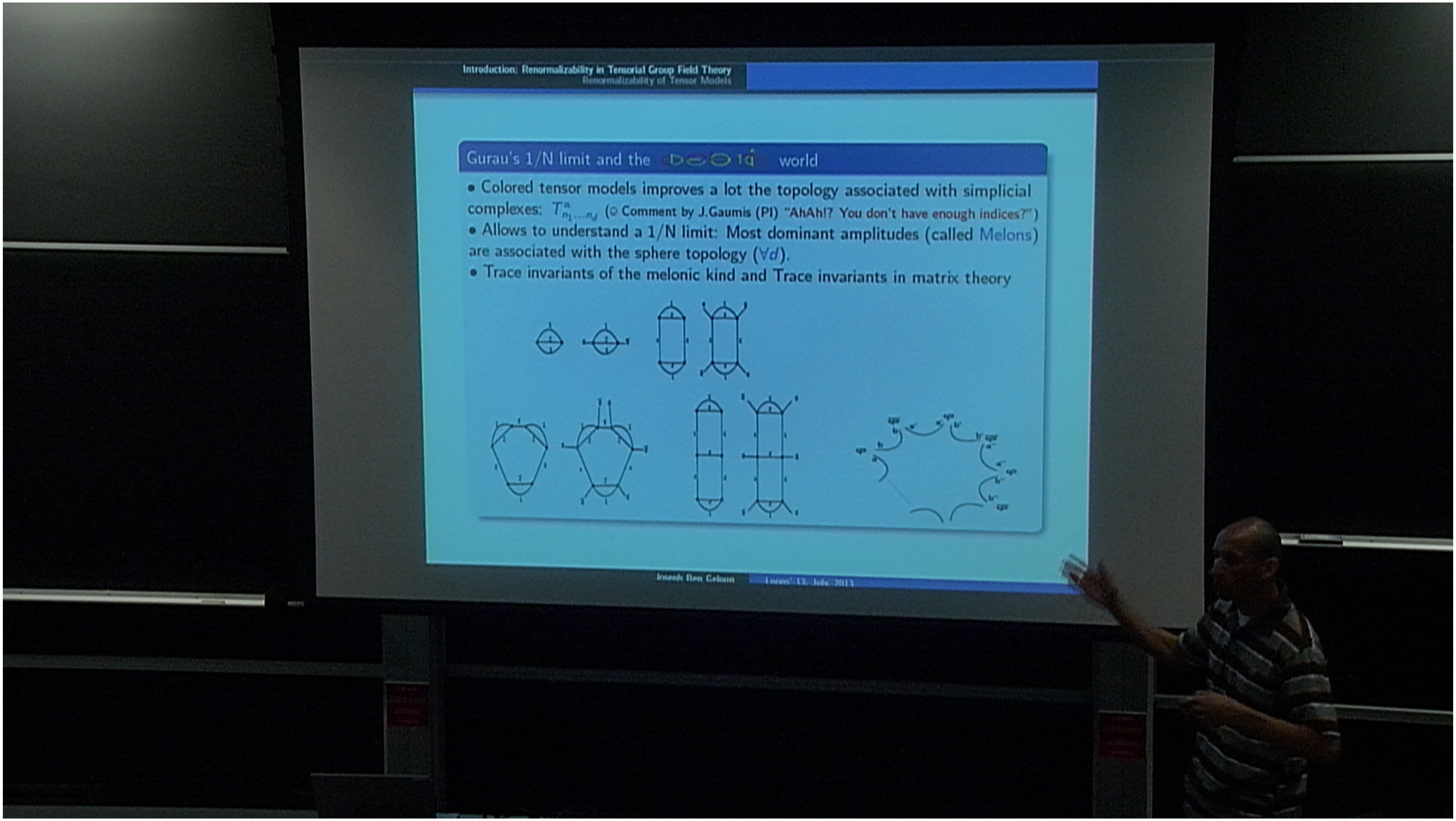
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Joseph Ben Geloun
Loops' 13, July 2013


CAUTION

Space of models

3 initial constraints

(i) Field are defined on a background which is a compact group manifold $G = U(1)^D$ or $SU(2)^D$.

(ii) The propagator = a "stranded" sum momenta of the form p^{2a} with $0 < a \leq 1$; Might be essential in order to achieve Osterwalder-Schraeder positivity axiom [Rivasseau, 1209.5284]? At $a = 1$, Laplacian dynamics. (We ll see that a is however severely constrained by Renorm.)

(iii) The interactions involved are unitary tensor invariants.

Rank $d \geq 2$ complex tensor field: $\varphi : G^d \to \mathbb{C}$. Fourier mode decomp.:

$$\varphi(h_1, h_2, \dots, h_d) = \sum_{P_{l_l}} \tilde{\varphi}_{P_{l_1}, P_{l_2}, \dots, P_{l_d}} D^{P_{l_1}}(h_1) D^{P_{l_2}}(h_2) \dots D^{P_{l_d}}(h_d) \,, \qquad (2)$$

$$S^{\text{kin}} = \sum_{P_{[l]}} \bar{\varphi}_{P_{[l]}} \Big(\sum_{s=1}^{d} |P_{l_s}|^a + \mu^2 \Big) \varphi_{P_{[l]}} \,, \qquad (3)$$

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Updated list of models

TGFT (type)	G_D	$\Phi^{k_{\max}}$	d	a	Renormalizability	UV behavior
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	$U(1)^2$	Φ^4	4	1	Just-	AF
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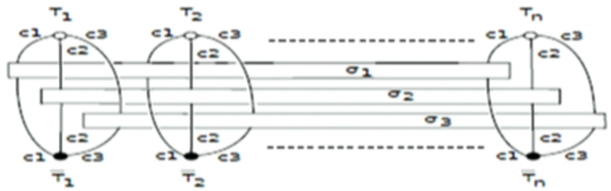
Future Prospects: Counting (and classifying ?) tensor invariants

(In collaboration with Sanjaye Rangoolam: 1307.6490)



- Determination of possible graph amounts to count triples

$$(\sigma_1, \sigma_2, \sigma_3) \in (S_n \times S_n \times S_n) \qquad (\sigma_1, \sigma_2, \sigma_3) \sim (\gamma_1\sigma_1\gamma_2, \gamma_1\sigma_2\gamma_2, \gamma_1\sigma_3\gamma_2) \tag{5}$$

- Counting points in the double coset

$$\mathcal{S}_3(n) = \mathrm{Diag}(S_n)\backslash(S_n \times S_n \times S_n)/\mathrm{Diag}(S_n)\,. \tag{6}$$

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- Relation with different counting: S_n-TFT !!!

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$$1, 4, 11, 43, 161, 901, 5579, 43206, 378360, 3742738, \ldots \tag{8}$$

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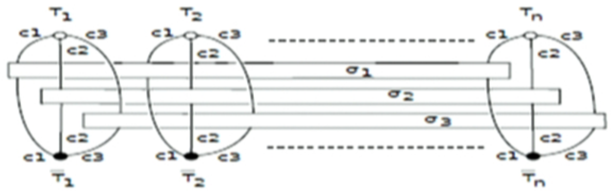

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