Title: Phenomenology - 3

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Abstract:

Loops 2013

Quantum information processing in spacetime

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Relativistic quantum information and metrology

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Marcus Huber (Bristol/Barcelona)
Göran Johansson (Chalmers)
Jorma Louko (Nottingham)
Robert Mann (Waterloo)
Enrique Solano (Bilbao)
Tim Ralph (Queensland)

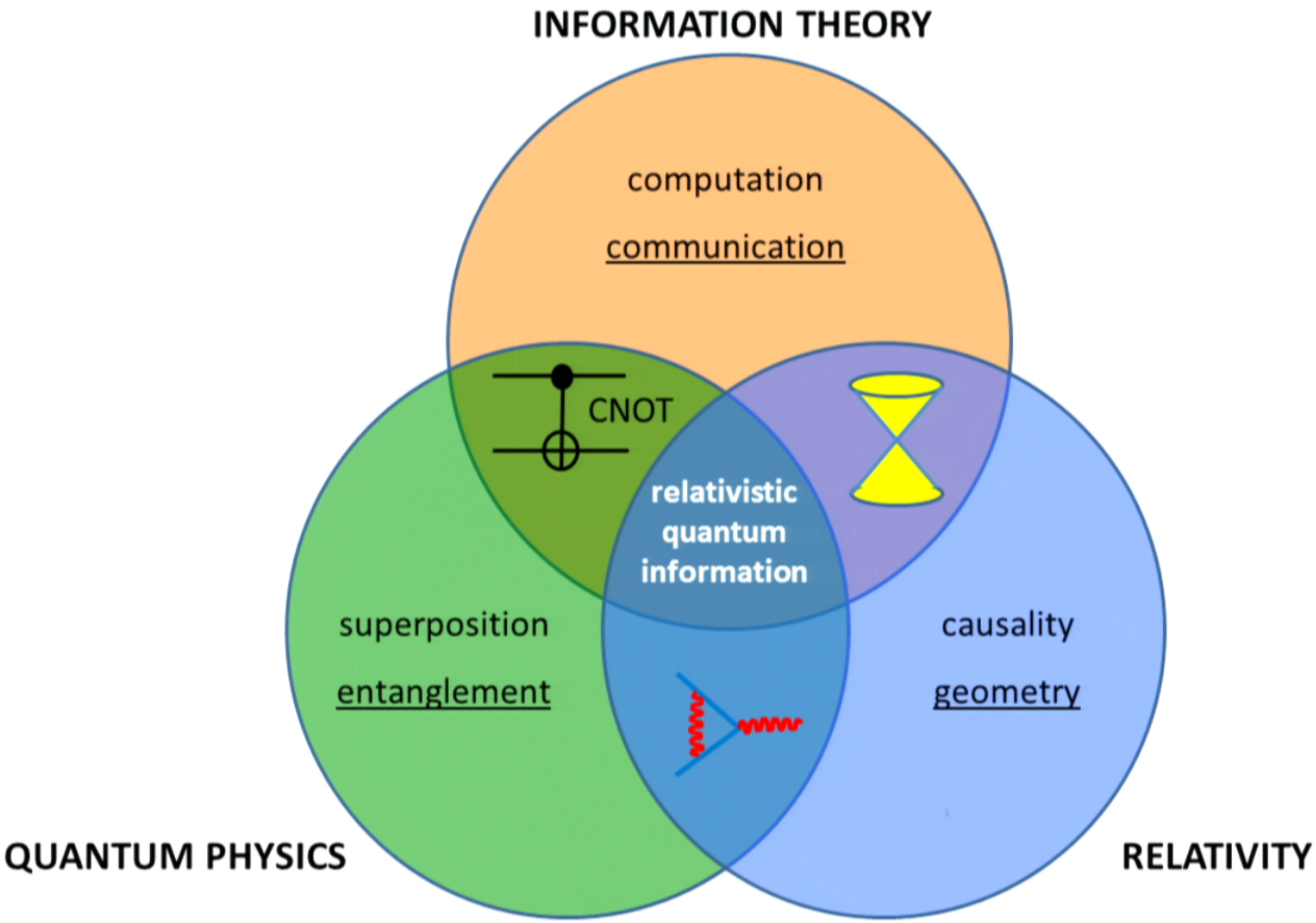
FUNDING: EPSRC (THANKS!!!!)

http://rqinottingham.weebly.com/

OUTLINE

- Motivation: Why relativistic quantum information
- QFT: Particle creation in a moving cavity
- Effects of gravity and motion on entanglement
- Earth and Space –based experiments
- Exploiting relativistic effects in quantum technologies

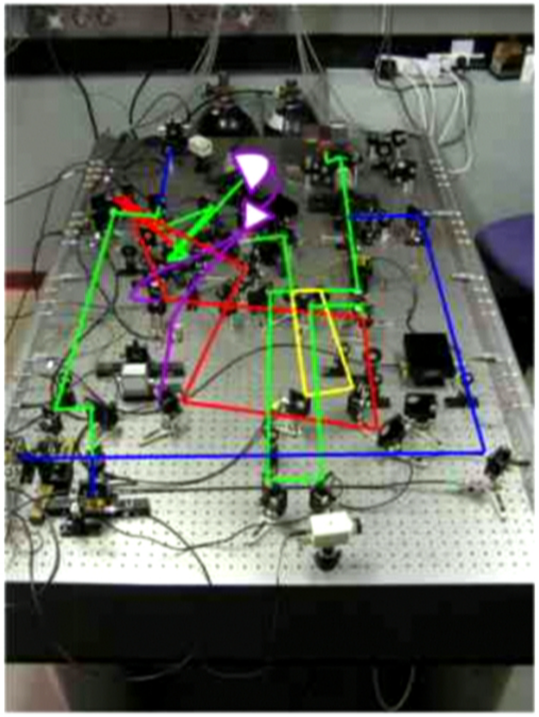
Relativistic quantum information



Why relativistic quantum information?

Real world experiments

Table-top

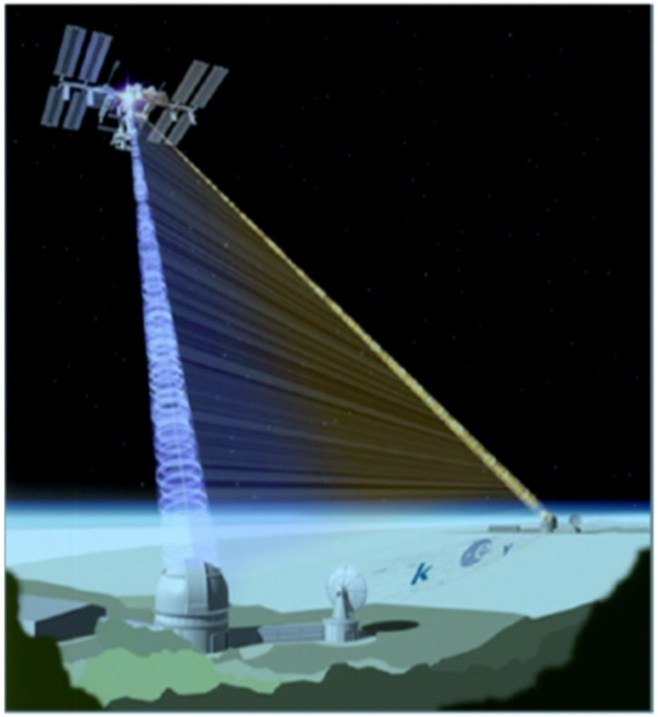


PHOTONS HAVE <u>NO</u> NON-RELATIVISTIC APPROXIMATION

Real world experiments

143 km

Future experiments



Space-QUEST project: distribute entanglement from the International Space Station.

Space Optical Clock project

QUANTUS: quantum gases in microgravity

STE-QUEST: Space-Time Explorer and Quantum Equivalence Principle Space Test

Relativistic regimes

GPS:

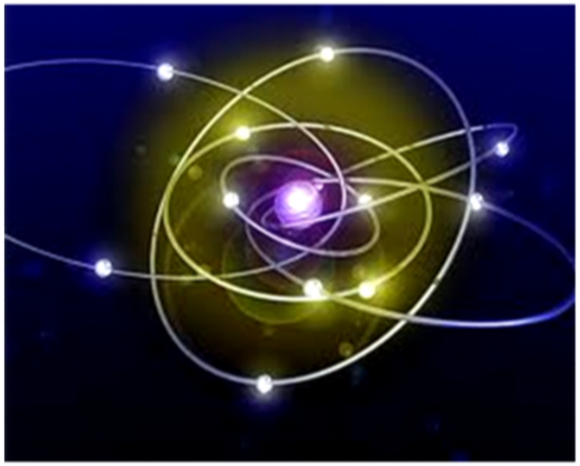
At these regimes relativity kicks in!

Quantum information: classical technologies are reaching quantum regimes

Relativistic QI: quantum technologies are reaching relativistic regimes!

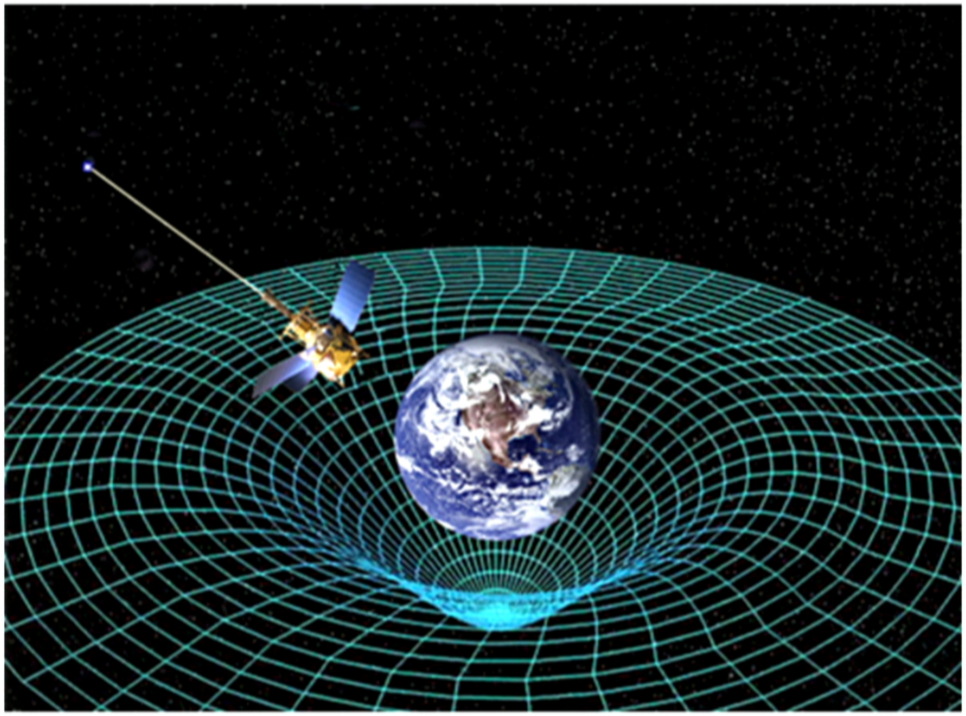
Our understanding of nature

QUANTUM PHYSICS



RELATIVITY

Future quantum technologies in space



Can relativistic effects help?

Quantum Communications
Gravitometers, sensors, clocks

Quantum field theory in curved spacetime

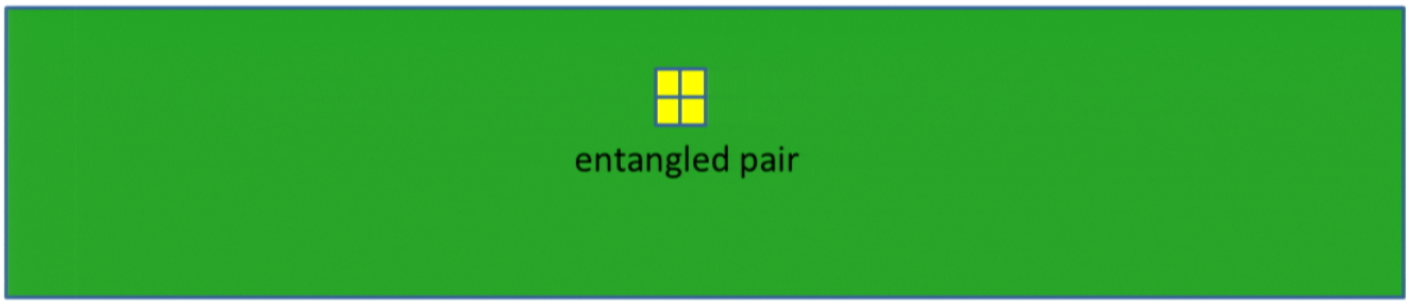
- Classical spacetime + quantum fields
- Incorporates Lorentz invariance
- Combines quantum mechanics with relativity at scales reachable by near-future experiments

First experimental demonstrations!

- Hawking radiation (Unruh, Faccio, Koenig, Steinhauer)
- Unruh effect
- Dynamical Casimir effect (Delsing)
- Expanding Universe (Westbrook)

Entanglement

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



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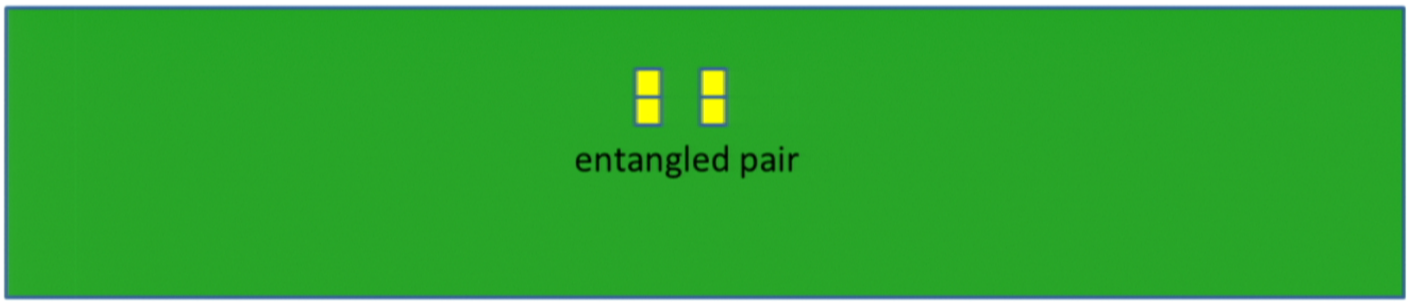

Entanglement

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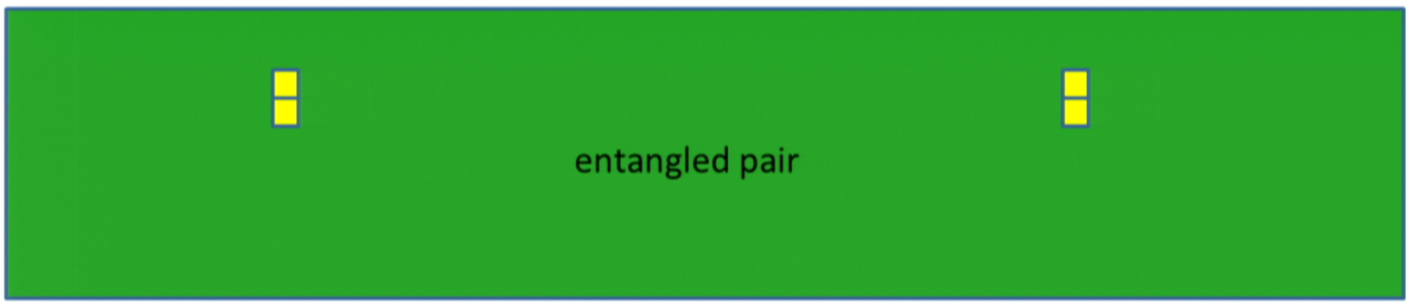


Inertial cavity

Minkowski coordinates (t, x)

$$\Box\phi(t,x) = 0$$ field equation

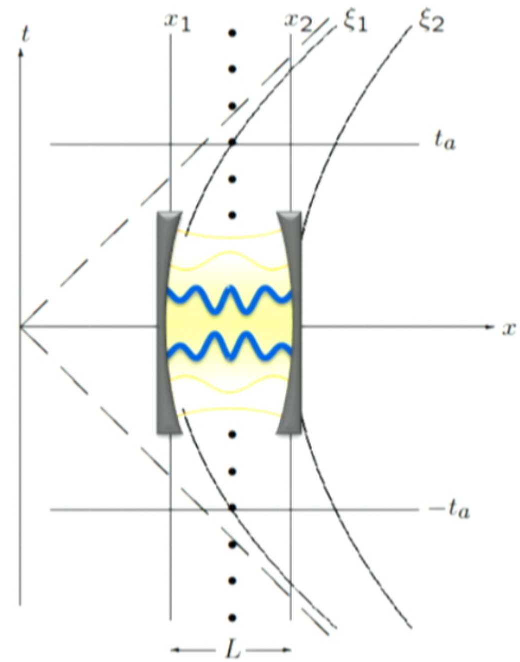
solutions: plane waves+ boundary

$$u_k(x,t) = \frac{1}{\sqrt{k\pi}} \sin\left[\frac{k\pi}{L}(x - x_A)\right] e^{-i\omega_k t},$$

$$\omega_k = \frac{1}{L}\sqrt{(k\pi)^2 + m^2},$$

creation and annihilation operators

$$\hat{\phi}(x,t) = \sum_n (u_n(t,x)\hat{a}_n + u_n^*(t,x)\hat{a}_n^\dagger)$$



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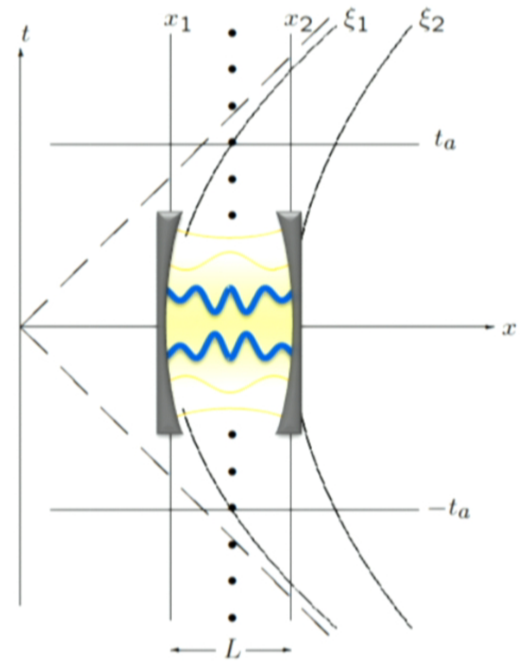
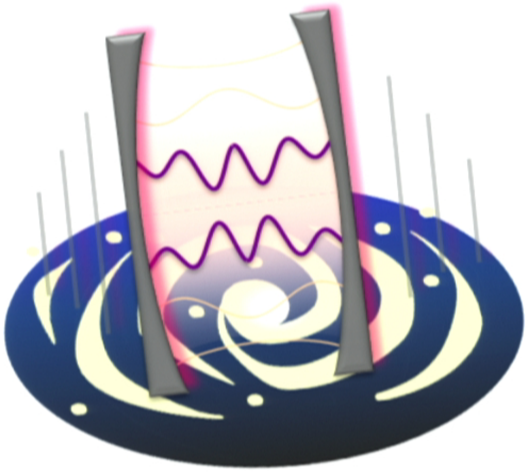


entanglement generated

Friis, Bruschi, Louko & Fuentes PRD 2012
Friis and Fuentes invited at JMO 2012
Bruschi, Louko, Faccio & Fuentes 2012



results

non-uniform motion creates entanglement

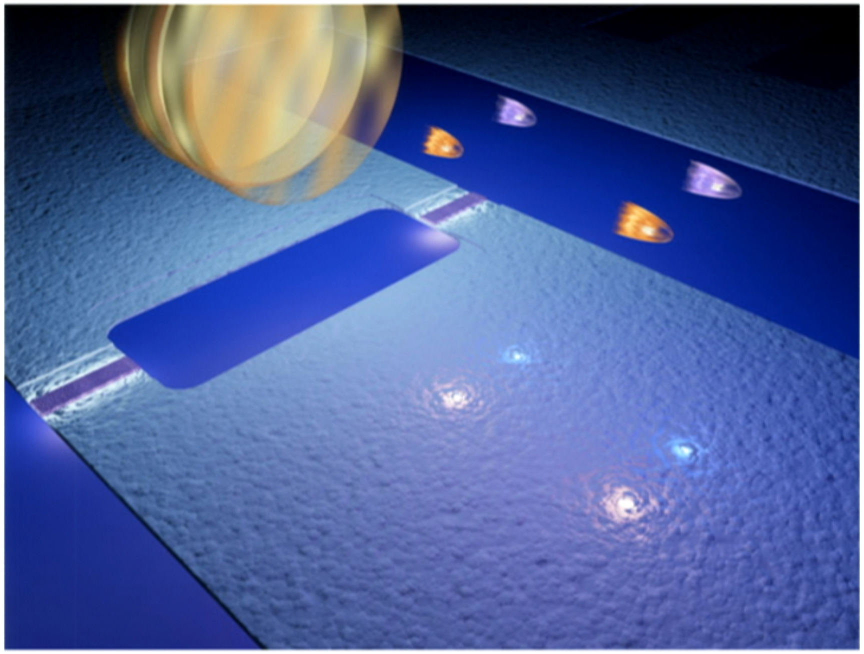
gravity creates entanglement

Negativity for initial separable squeezed state

$$\mathcal{N} = \left(\Re(G_k^* \beta_{kk'}^{(1)})^2 + (\,\Im(G_k^* \beta_{kk'}^{(1)})\cosh(s) - \Im(G_k^* \alpha_{kk'}^{(1)})\sinh(s))^2\right)^{1/2}$$

Dynamical Casimir effect

Delsing's group at Chalmers University

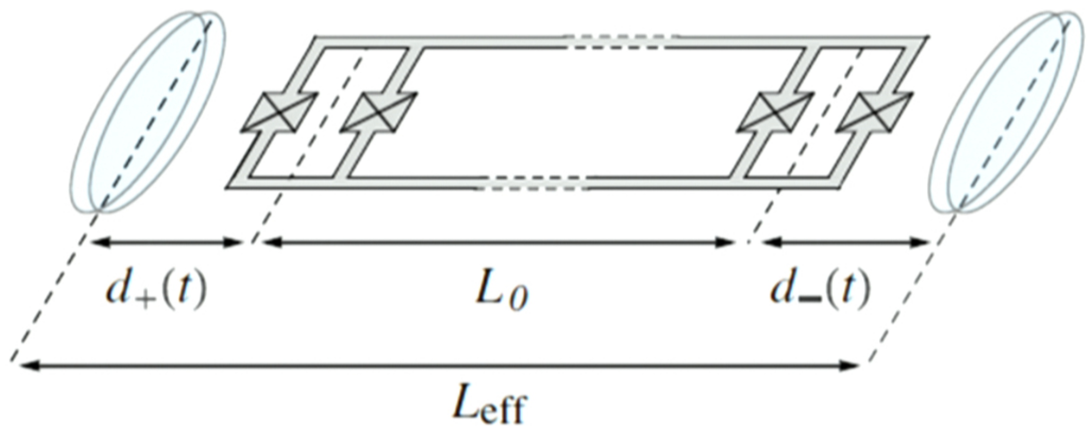


Art by
Philip Krantz
(Chalmers)

Testing QFT: particle creation by a moving boundary

Earth-based experiments

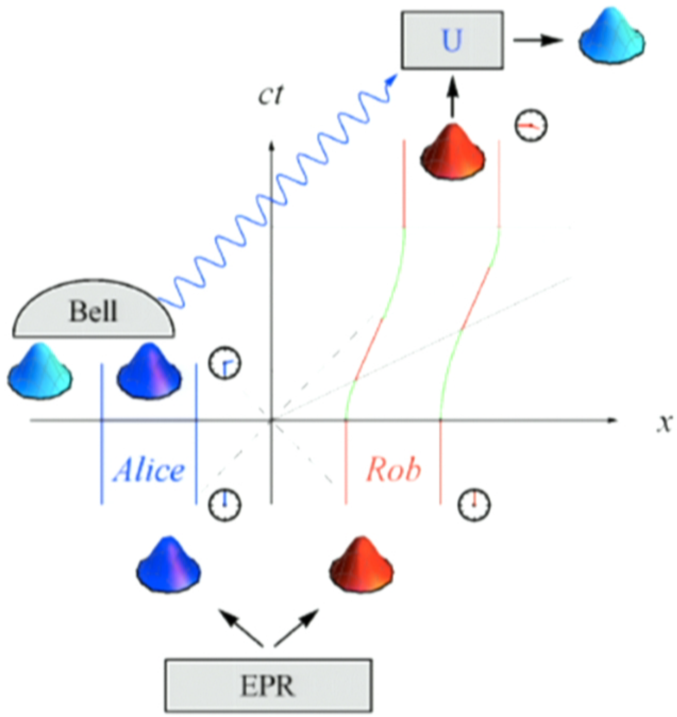
Friis, Lee, Truong, Sabin, Solano, Johansson & Fuentes PRL 2012



simulate field inside a cavity which travels in a spaceship using superconducting circuits

teleporation goes relativistic

Friis, Lee, Truong, Sabin, Solano, Johansson & Fuentes PRL 2013



the fidelity of teleportation is effected by motion
it is possible to correct by local rotations and trip planning

BEC in motion

mean field

$$\hat{\Psi} = \Psi\left(1 + \hat{\Pi}\right)$$

quantum fluctuations

$$\Box \Pi = 0$$

$$\Box = 1/\sqrt{-\mathfrak{g}}\,\partial_a(\sqrt{-\mathfrak{g}}\mathfrak{g}^{ab}\partial_b)$$

effective metric

$$\mathfrak{g}_{ab} = \rho\frac{c}{c_s}\left[g_{ab} + \left(1 - \frac{c_s^2}{c^2}v_a v_b\right)\right]$$

Space-based experiments

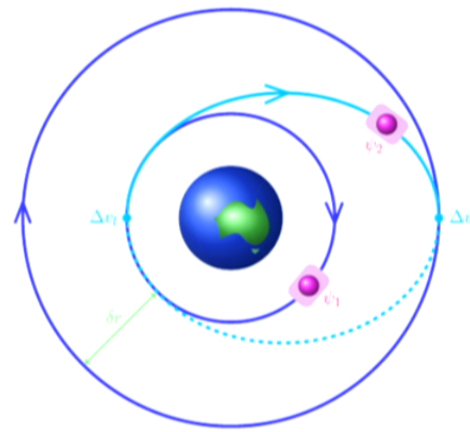
Bruschi, Sabin, White, Baccetti, Oi, Fuentes

arXiv:1306.1933

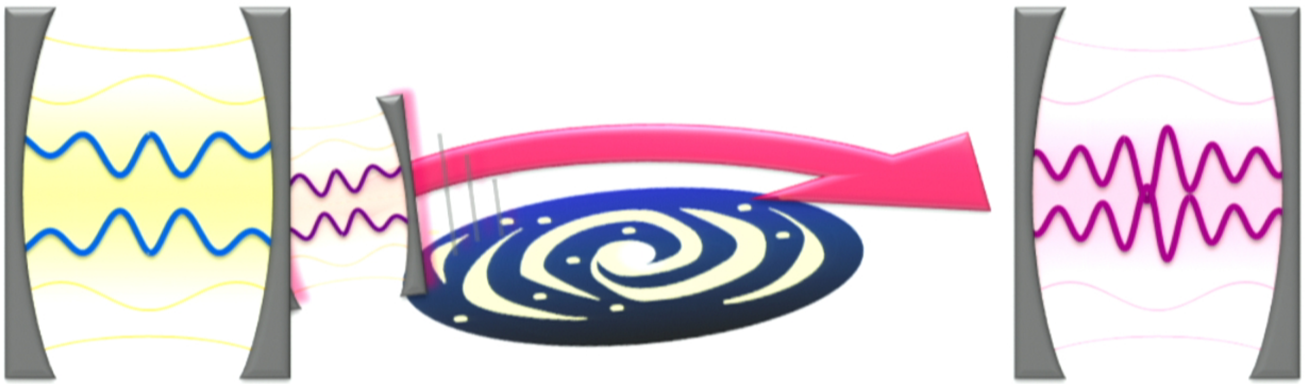
"Quantum experiments in space are a completely new game – one, perhaps, with unknown rules"

experiments, which will use a continuous beam, but it is something to watch out for (*Physical Review Letters*, vol 110, p 060501).

For a dedicated test, Ivette Fuentes, a quantum theorist based at the University of Nottingham in the UK, and her colleagues propose analysing the entanglement between Bose-Einstein condensates. These large collections of atoms, chilled to near absolute zero, behave as one quantum system. The idea is to hold entangled condensates in two separate satellites, and then kick one into a different orbit. Calculations indicate that the acceleration needed for a change in orbital radius of just 400 metres would



Exploiting relativity in quantum technologies

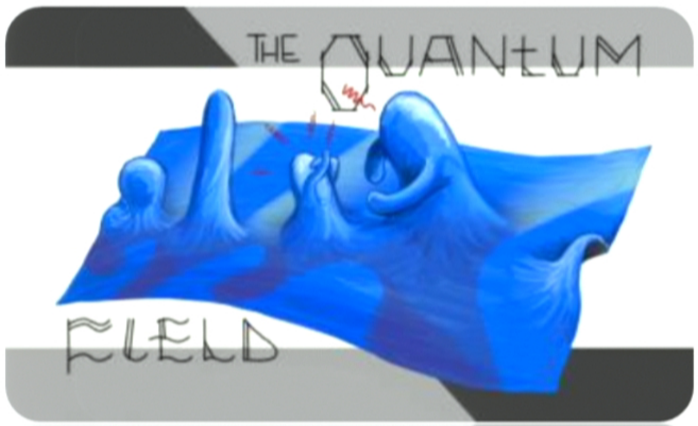


- The relativistic motion can be used to implement quantum gates
- Relativistic effects can be used to improve the precision of measurement devices: accelerometers, sensors, etc.

Relativistic Quantum Metrology

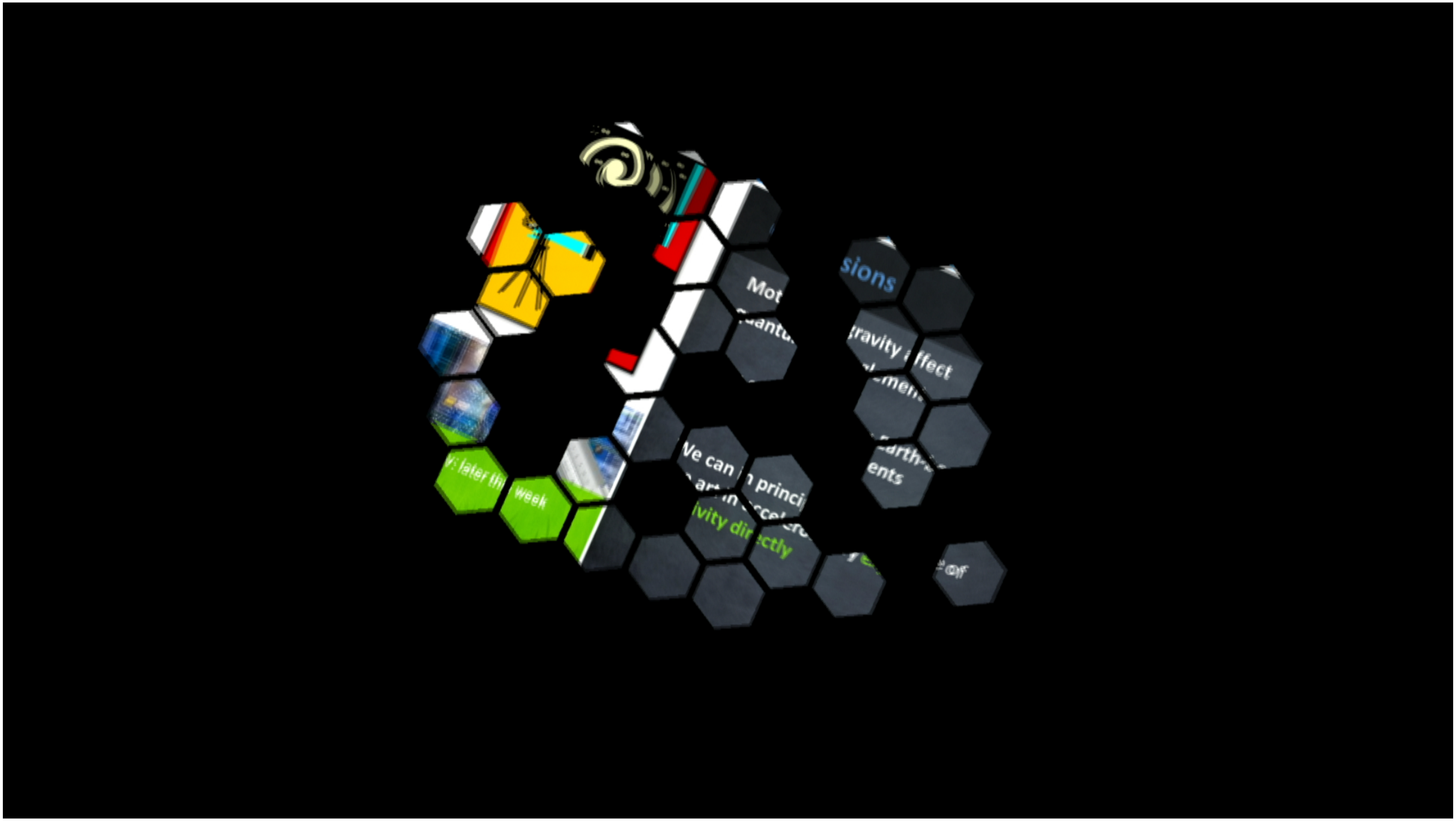
Quantum Metrology

- Exploits quantum properties for ultrasensitive devices for measuring fields, frequencies, time.



RELATIVITIC Quantum Metrology: General framework

- Measuring parameters in QFT: accelerations, proper time, gravitational field strenght: accelerometers, gravitometers, sensors, etc.



Conclusions

Motion and gravity affect quantum entanglement

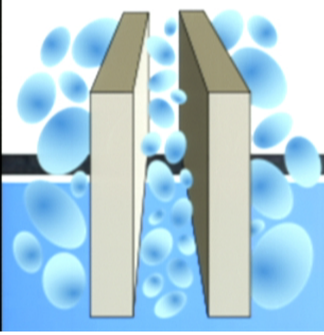
This can be tested in Earth-based and space-based experiments

We can in principle beat the state of the art in accelerometry exploiting relativity directly

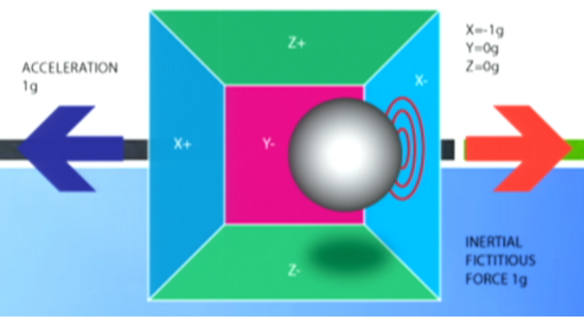
Exploiting genuinely quantum field theory effects such as boundary modulation resonances with or without particle creation, *in principle …*

we can beat the

STATE OF THE ART



Commercial accelerometers: $\frac{\Delta a}{g} \sim 10^{-4}$



THANK YOU

Antonino Marcianò

Fudan University

Dartmouth University

Gravitational origin of weak interaction's chirality

In collaboration with L. Smolin and S. Alexander

arXiv:1212.5246

Loops'13 July 22nd 2013

Motivation

- Why is the WI maximally parity violating ?
- What is the standard model chiral?
- Why is there a similarity bewteen WI and gravity?

... an example of gauge theory

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \imath\, g_1 A^a_{L\,\mu}\mathcal{J}_L^{a\,\mu} + \imath\, g_2 A^a_{R\,\mu}\mathcal{J}_R^{a\,\mu}$$

$g_1 \neq g_2$ parity violating theory

$g_1 = g_2$ parity symmetric theory

$g_1 = 0, \quad g_2 \neq 0;$
$g_1 \neq 0, \quad g_2 = 0;$ maximal parity violation

Mother theories for gravi-weak

Nesti, Percacci (J.Phys. A 2008), **Alexander** (arXiv:0706.4481),
Smolin (Phys. Rev. D 2009), **Alexandrov, Krasnov** (Class. Quant. Grav 2009),
Speziale (Phys. Rev. D 2010), **Alexander, A.M., Tacchi** (Phys. Lett. B. 2012)

$$SO(3,1)_{\mathbb{C}} = SL(2,\mathbb{C})_L \times SL(2,\mathbb{C})_R$$

$$\downarrow$$

$$\boxed{G_{\mathrm{GW}} = GL(2,\mathbb{C})_L \times GL(2,\mathbb{C})_R}$$

Spinorial variables

$$A^{ab} \to (A^{AB} + a\,\epsilon^{AB})\,\epsilon^{A'B'} + (A^{A'B'} + a'\,\epsilon^{A'B'})\,\epsilon^{AB}$$

$$B^{ab} \to (B^{AB} + b\,\epsilon^{AB})\,\epsilon^{A'B'} + (B^{A'B'} + b\,\epsilon^{A'B'})\,\epsilon^{AB}$$

$$\Psi^{abcd} \to \Psi^{ABCD}$$

Theory and EOM I

- The mother theory is a parity symmetric bimetric theory
- The bimetric theory is unstable and has a scalar ghost
- Stable parity-violating sector, and perturbative expansion

$$G = SL(2,\mathbb{C})_L \times SL(2,\mathbb{C})_R$$

$$S = \int \frac{\imath}{4\pi G} \Big\{ B^{AB} \wedge F_{AB} - B^{A'B'} \wedge F_{A'B'} + \frac{\lambda}{6G}(B_{AB} \wedge B^{AB} - B_{A'B'} \wedge B^{A'B'})$$
$$-\frac{1}{2}\Psi_{ABCD} B^{(AB} \wedge B^{CD)} + \frac{1}{2}\Psi_{A'B'C'D'} B^{(A'B'} \wedge B^{C'D')} - \Psi_{A'B'AB} B^{A'B'} \wedge B^{AB} \Big\}$$
$$+\frac{\imath g^2}{2}(\Psi^2_{ABCD} + \Psi^2_{A'B'C'D'} + \Psi^2_{ABA'B'})(B_{AB} \wedge B^{AB} - B_{A'B'} \wedge B^{A'B'})$$

Theory and EOM II

$$\frac{\delta}{\delta B}\quad \begin{aligned} F_{AB} &= \Psi_{ABCD}\, B^{CD} + \Psi_{ABA'B'}\, B^{A'B'} - \left(\frac{\lambda}{3G} + 4\pi G g^2 \Psi^2\right) B_{AB} \\ F_{A'B'} &= \Psi_{A'B'C'D'}\, B^{C'D'} - \Psi_{A'B'AB}\, B^{AB} + \left(-\frac{\lambda}{3G} + 4\pi G g^2 \Psi^2\right)\, B_{A'B'} \end{aligned}$$

$$\frac{\delta}{\delta \Psi}\quad \Psi_{ABCD} = \frac{1}{8\pi G g^2 W}\, B_{(AB} \wedge B_{CD)} \qquad \Psi_{A'B'C'D'} = -\frac{1}{8\pi G g^2 W}\, B_{(A'B'} \wedge B_{C'D')}$$

$$\Psi_{ABA'B'} = \frac{1}{4\pi G g^2 W}\, B_{AB} \wedge B_{A'B'}$$

$$\frac{\delta}{\delta A}\quad \mathcal{D} \wedge B_{AB} = \mathcal{D}' \wedge B_{A'B'} = 0$$

$$W = B_{AB} \wedge B^{AB} - B_{A'B'} \wedge B^{A'B'}$$

Theory and EOM II

$$\frac{\delta}{\delta B}\qquad \begin{aligned} F_{AB} &= \Psi_{ABCD}\, B^{CD} + \Psi_{ABA'B'}\, B^{A'B'} - \left(\frac{\lambda}{3G} + 4\pi G g^2 \Psi^2\right) B_{AB} \\ F_{A'B'} &= \Psi_{A'B'C'D'}\, B^{C'D'} - \Psi_{A'B'AB}\, B^{AB} + \left(-\frac{\lambda}{3G} + 4\pi G g^2 \Psi^2\right) B_{A'B'} \end{aligned}$$

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$$\Psi_{ABA'B'} = \frac{1}{4\pi G g^2 W}\, B_{AB} \wedge B_{A'B'}$$

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$$W = B_{AB} \wedge B^{AB} - B_{A'B'} \wedge B^{A'B'}$$

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Theory and EOM III

New reality conditions via Urbantke metrics

$$\tilde{g}^{L}_{\mu\nu} = \varepsilon^{\gamma\delta\rho\sigma} B^{B}_{\mu\gamma A} B^{A}_{\nu\delta C} B^{C}_{\rho\sigma B}$$
$$\tilde{g}^{R}_{\mu\nu} = \varepsilon^{\gamma\delta\rho\sigma} B^{B'}_{\mu\gamma A'} B^{A'}_{\nu\delta C'} B^{C'}_{\rho\sigma B'}$$

$$\downarrow$$

$$S^{\mathrm{wrc}} = S + \int \lambda^{ab}_{R}\Big(\tilde{g}^{R}_{ab} - (\tilde{g}^{R}_{ab})^{*}\Big) + \lambda^{ab}_{L}\Big(\tilde{g}^{L}_{ab} - (\tilde{g}^{L}_{ab})^{*}\Big)$$

Symmetric vs asymmetric phase I

Symmetric phase

$$8 \ \ \mathrm{DOF} \ \ (2+5+1)$$

$$B_{AB} = e_A^{\ A'} \wedge e_{BA'} + g^2 b_{AB}$$

$$B_{A'B'} = f_{A'A} \wedge f^A_{\ B'} + g^2 b_{A'B'} \qquad f^{AA'} = \chi \ e^{AA'}$$

Asymmetric phase

$$\lambda \gg 1 \qquad g << 1 \qquad \lambda g^2 = \xi$$

$$B_{A'B'} = -\pi G g^2 \left(\delta_\xi \mathbb{1} + \gamma_\xi \star\right) F_{A'B'} + g^6 b_{A'B'}$$

Symmetric vs asymmetric phase II

Leading order

$$S^{(0)} = \int \frac{\imath}{4\pi G}\,\Sigma^{AB}\wedge F_{AB} + \frac{\lambda}{12\pi G^2}\,e - \frac{e}{4g_{YM}^2}F^{A'B'}_{\mu\nu}F_{A'B'\rho\sigma}g^{\mu\rho}g^{\nu\sigma} - \imath\,\Theta\,F^{A'B'}\wedge F_{A'B'} + \frac{9G^2}{(16\pi)^2\lambda^2 e}\left(F_{(A'B'}\wedge F_{C'D')}\right)^2$$

where

$$(4g_{YM}^2)^{-1} = -g^2[\delta_\xi\,\gamma_\xi(\xi\pi^2/3 - 1/64 - 74) + \gamma_\xi]$$

$$\Theta = g^2[(\delta_\xi^2 + \gamma_\xi^2)(\xi\pi^2/6 - 1/128 - 37) + \delta_\xi]$$

and

$$\xi \sim 10^{-1} \longrightarrow g^2\,g_{YM}^2 \sim 10^{-4}$$

Dark hypercharge and new interactions

$$A^{ab} \longrightarrow A^{AB}\,\epsilon^{A'B'} + A^{A'B'}\,\epsilon^{AB}$$

$$\boxed{G = SL(2,\mathbb{C})_L \times SL(2,\mathbb{C})_R}$$

$$A^{AB} + a\,\epsilon^{AB} \qquad A^{A'B'} + a'\,\epsilon^{A'B'}$$

$$\boxed{G_{\rm GW} = GL(2,\mathbb{C})_L \times GL(2,\mathbb{C})_R}$$

$$\boxed{S = S^{SL(2,\mathbb{C})_\mathbb{C}} + S^{U(1)_\mathbb{C}}}$$

- New intercations involving dark hypergharge
- Universal four-points coupling $(F \wedge F)^2$
- Quite small coupling constant $\lambda_4 = \dfrac{9\,g^2\,G^2}{256\,\xi^2} \sim \dfrac{9\,g^2}{256\,M_p^4 \xi^2}$

Concluding remarks

New theoretical framework

- A new model for the gravitational origin of the weak interactions chirality
- New interaction vertices

New predictions

- Left handed spinor that transform like a doublet scalar (Higgs?)
- Right handed spinor that transforms like a weak singlet spin 1/2 particle (sterile neutrino?)
- The two would transform into each other under parity

Longitudinal and Transverse Relative Locality

Leonardo Barcaroli
with
G. Amelino-Camelia and N. Loret

Universitá La Sapienza di Roma

LOOPS 13

July 25, 2013

Amelino-Camelia, Matassa, Mercati, Rosati *et al.*, Phys. Rev. Lett. 106, (2011).

Amelino-Camelia, LB, Loret *et al.*, Int J Theor Phys 51, 3359-3375 (2012).

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Relative Locality

- A relativistic theory $\rightarrow$ $\mathcal{C}(p) = \mathcal{C}(p')$
- Two invariant scales (DSR) $\rightarrow$ (c, ℓ)
- Phenomenology oriented $\rightarrow$ $O(\ell)$

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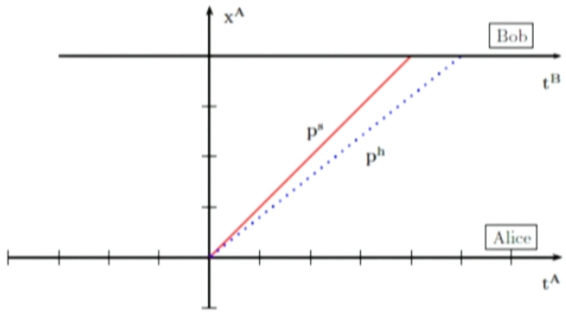
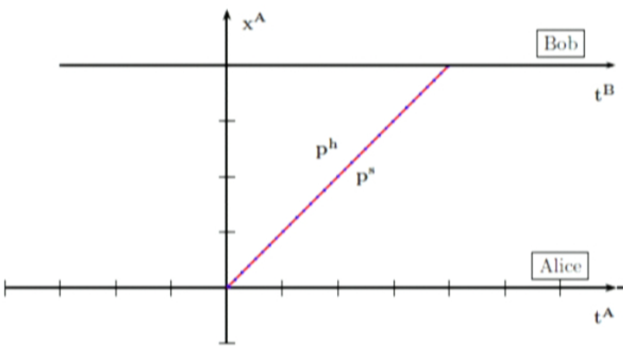
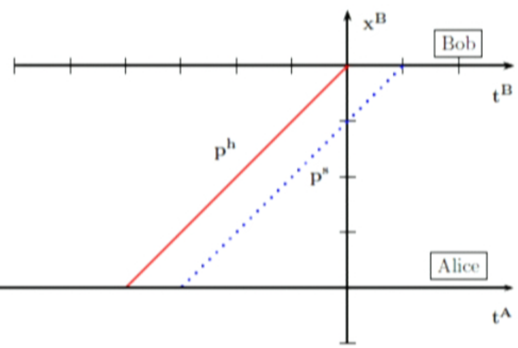
Background

Previous studies focused mainly on time delays and related inferences







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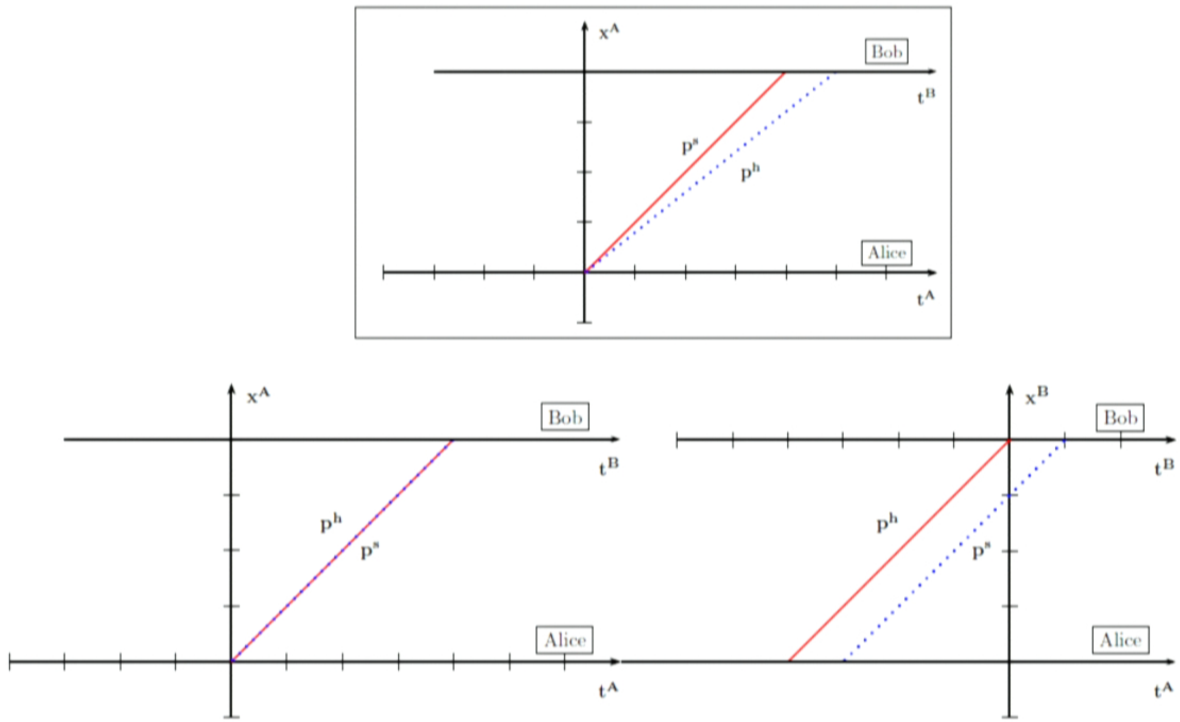

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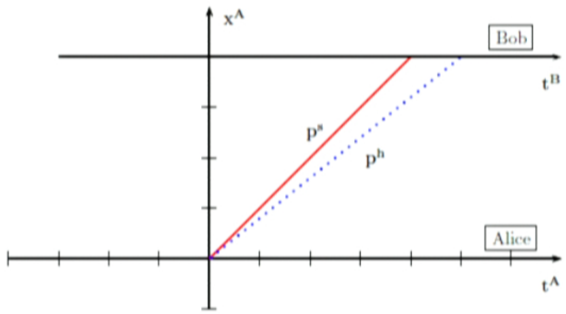

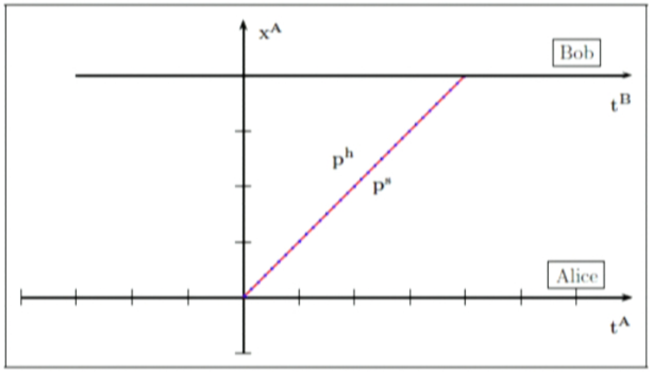

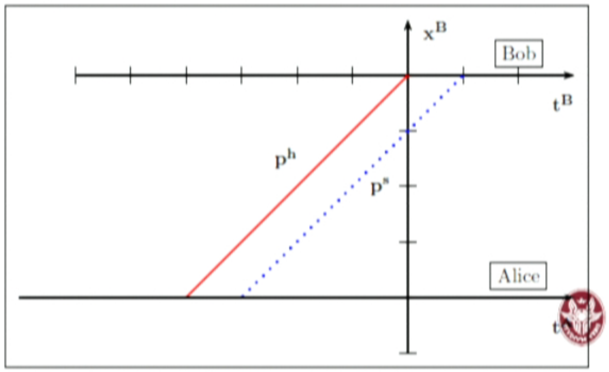

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Background 2

Only *longitudinal* features.

- Longitudinal $\Rightarrow$ parallel to the direction connecting two distant observers
- Transverse $\Rightarrow$ orthogonal to the direction connecting two distant observers

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Background 2

Only *longitudinal* features.

- Longitudinal $\Rightarrow$ parallel to the direction connecting two distant observers
- Transverse $\Rightarrow$ orthogonal to the direction connecting two distant observers

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The model

We used the model given in a previous work[1]
Deformation of the 2+1D Poincaré

$$
\begin{aligned}
&\{\mathcal{R},\mathcal{N}_i\} = \epsilon_{ij}\mathcal{N}_j,\\
&\{\mathcal{N}_i,\mathcal{N}_j\} = (-1+3(\alpha-\beta-\gamma-\frac{1}{2})\ell p_0)\epsilon_{ij}\mathcal{R},\\
&\{\mathcal{R},p_0\} = 0,\\
&\{\mathcal{R},p_i\} = \epsilon_{ij}p_j,\\
&\{\mathcal{N}_i,p_0\} = (1-\ell\,\alpha\,p_0)p_i,\\
&\{\mathcal{N}_i,p_j\} = \delta_{ij}p_0+\\
&+\ell[\delta_{ij}(\beta|\vec{p}|^2+(1-\alpha+\gamma)p_0^2)-(\beta+\gamma-\frac{1}{2})p_ip_j]
\end{aligned}
$$

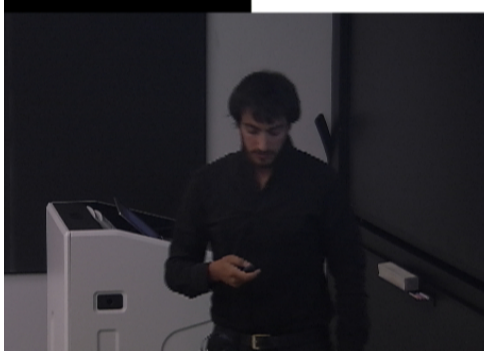
[1] Amelino-Camelia, LB, Loret, Phys. Rev. Lett. 106, (2011).

The model

We used the model given in a previous work[1]

Deformed casimir

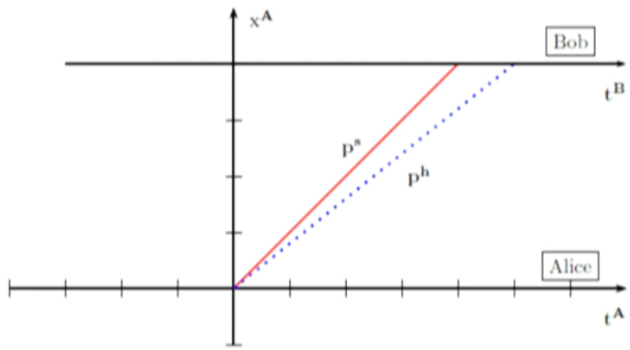
$$\mathcal{C}_\ell(p) = p_0^2 - |\vec{p}|^2 + \ell(2\gamma p_0^3 + (1-2\gamma)p_0|\vec{p}|^2)$$



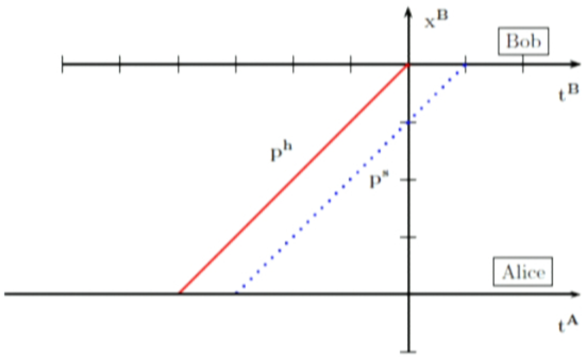
[1] ...elino-Camelia, LB, Loret, Phys. Rev. Lett. 106, (2011).

Aside on the symplectic structure

$$\{x^\mu, p_\nu\} = \delta^\mu_\nu$$



$$\{x^\mu, p_\nu\} = \delta^\mu_\nu + O(\ell)$$



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The recipe

- $\mathcal{C}$ as Hamiltonian of evolution in affine parameter τ.

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- $\frac{dA}{d\tau} = \{\mathcal{C}, A\} \quad \Rightarrow \quad x^i - \bar{x}^i = \mathsf{v}^i(p;\ell)(t - \bar{t})$
- $A \underset{a}{\rightarrow} \mathcal{A}_a \quad \mathcal{A}_a \triangleright f(x,p) = \sum_{n=0}^{\infty} \frac{1}{n!}\{a \cdot A, f\}$

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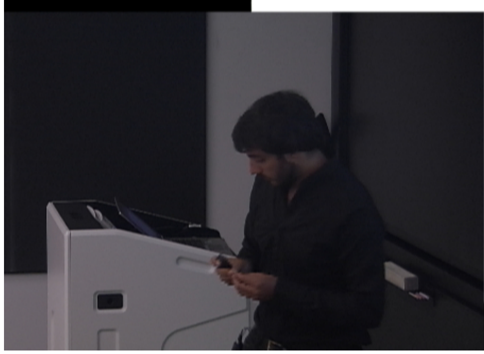
Non removable Relative Locality

We want to relate Alice (the emitter) to Bob (the distant boosted observer).



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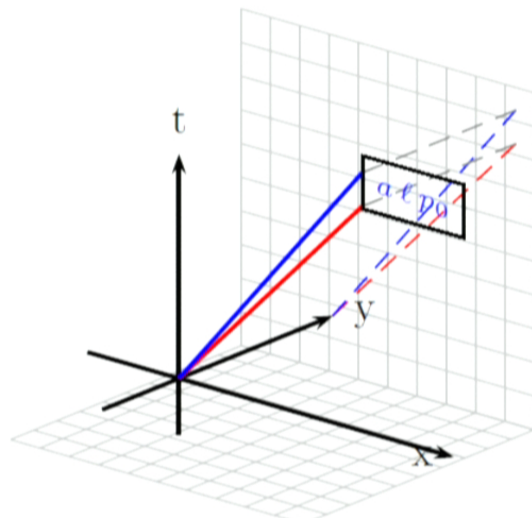

Figure : The emission in Alice's frame

Non removable Relative Locality

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Longitudinal Relative Locality

$$x^{\mu}_{B} = \mathcal{B}_{\xi} \triangleright \mathcal{T}_{a} \triangleright x^{\mu}_{A} \quad (2)$$

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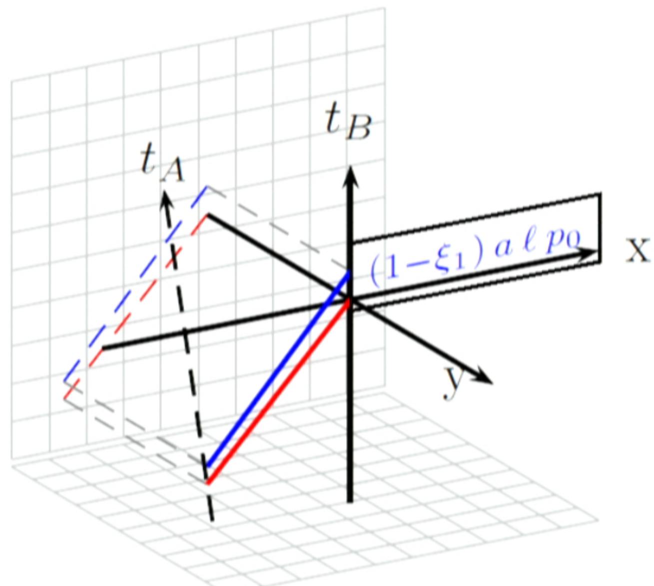

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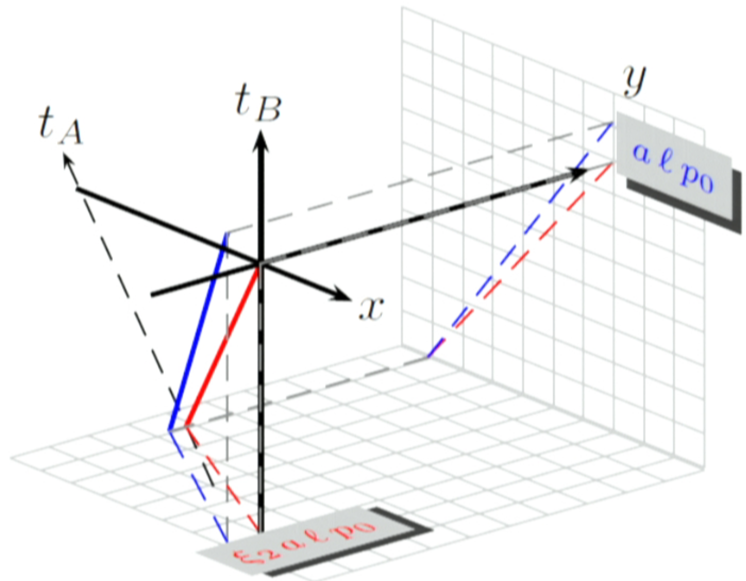
Transverse Relative Locality

$$x_B^{\mu} = \mathcal{B}_{\xi} \triangleright \mathcal{T}_a \triangleright x_A^{\mu} \quad (6)$$

$$\xi_T = (0, \xi_2) \quad (7)$$



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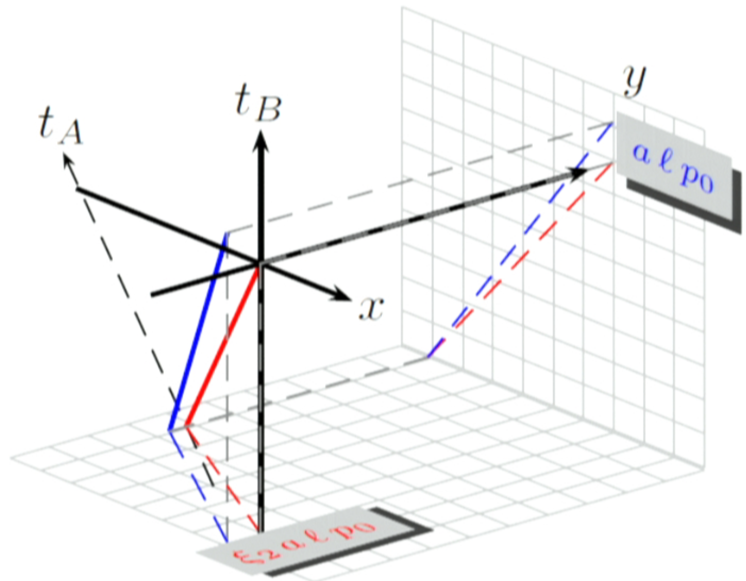

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Results:

$$\text{Time delay} \quad \Rightarrow \quad \Delta t^{@B} = a\ell p_0 \tag{8}$$

$$\text{Rigid shift} \quad \Rightarrow \quad \Delta y = \xi_2 a\ell p_0 \tag{9}$$

$$\text{Dual Gravity Lensing}^2 \quad \Rightarrow \quad \Delta\theta = \xi_2 \left(\alpha - \beta - \gamma - \frac{1}{2}\right) \ell p_0 \tag{10}$$

[2] Freidel and Smolin, arXiv:1103.5626, (2011).

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Transverse Relative Locality

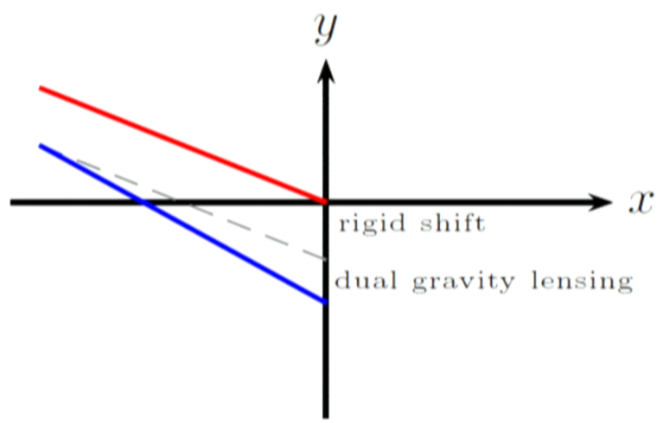
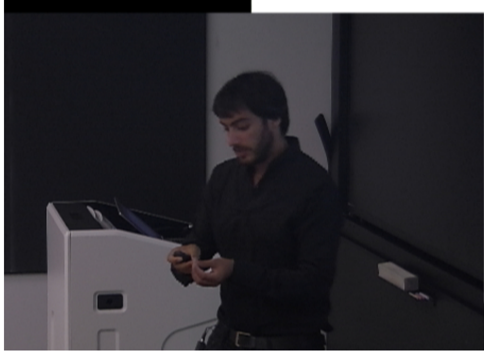


Figure : $\Delta y = \xi_2 a \ell p_0, \quad \Delta\theta = \xi_2 \left(\alpha - \beta - \gamma - \frac{1}{2}\right) \ell p_0$



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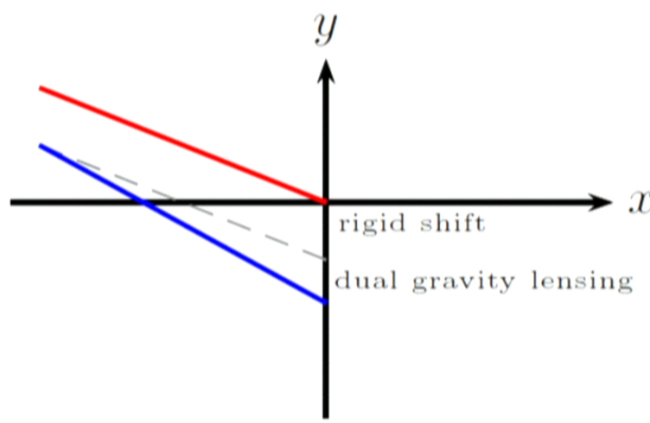

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Transverse Relative Locality

Results:

Time delay $\Rightarrow$ $\Delta t^{@B} = a\ell p_0$ (6)

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Algebra: Lorentz sector

$\{\mathcal{N}_i, \mathcal{N}_j\} = -(1 + 3\left(\alpha - \beta - \gamma - \frac{1}{2}\right)\ell p_0)\epsilon_{ij}\mathcal{R}$

[2] Freidel and Smolin, arXiv:1103.5626, (2011).

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Conclusions

- We distinguished two kinds of RL features
- We gave two explicit examples of non removable features of both types

Thank you!

Probing the quantum nature of spacetime by diffusion

Astrid Eichhorn

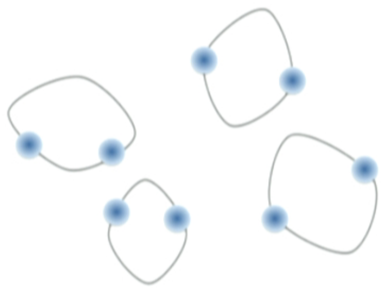
Perimeter Institute, Waterloo

Loops 13, 25th July 2013

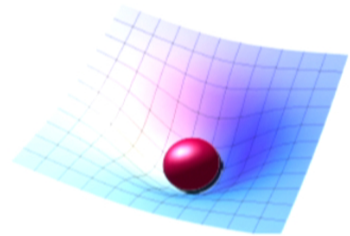
PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Quantum fields and gravitational fields

quantum fields:



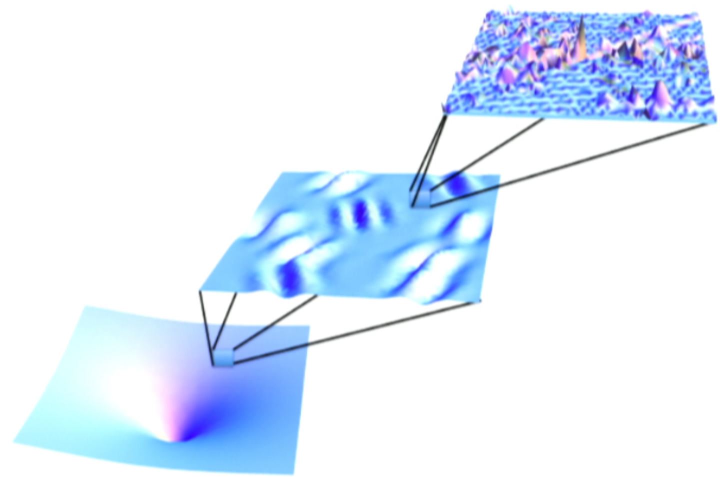
gravity:



→ quantum gravity:

spacetime fluctuations
at the Planck scale

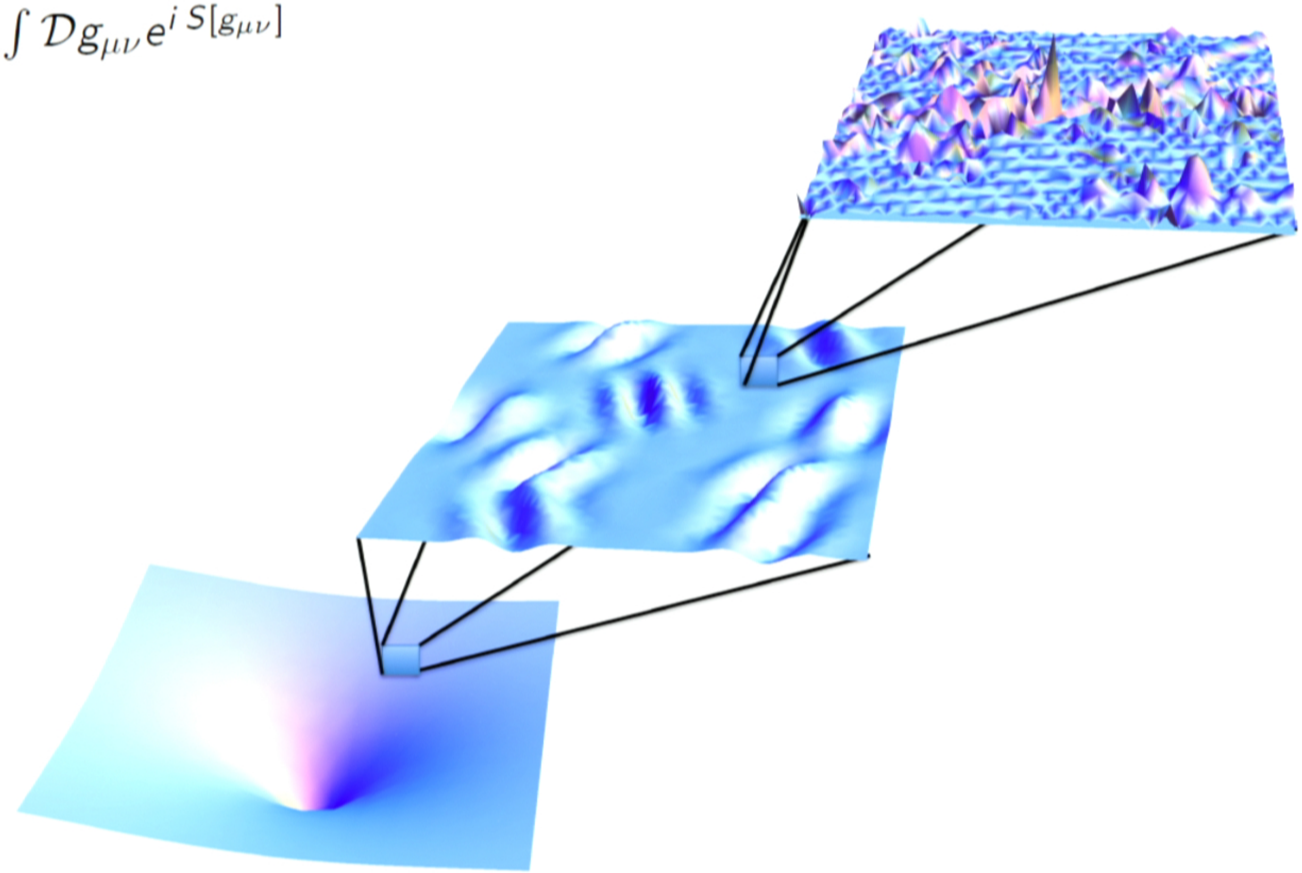
$$M_{\text{Planck}} = \sqrt{\frac{\hbar c}{G}} \sim 10^{19}\text{GeV}/\text{c}^2$$



→ Asymptotically Safe gravity, Loop Quantum Gravity/ Spin foams, Horava-Lifshitz, causal dynamical triangulations, group field theory,....

What is the corresponding spacetime like?

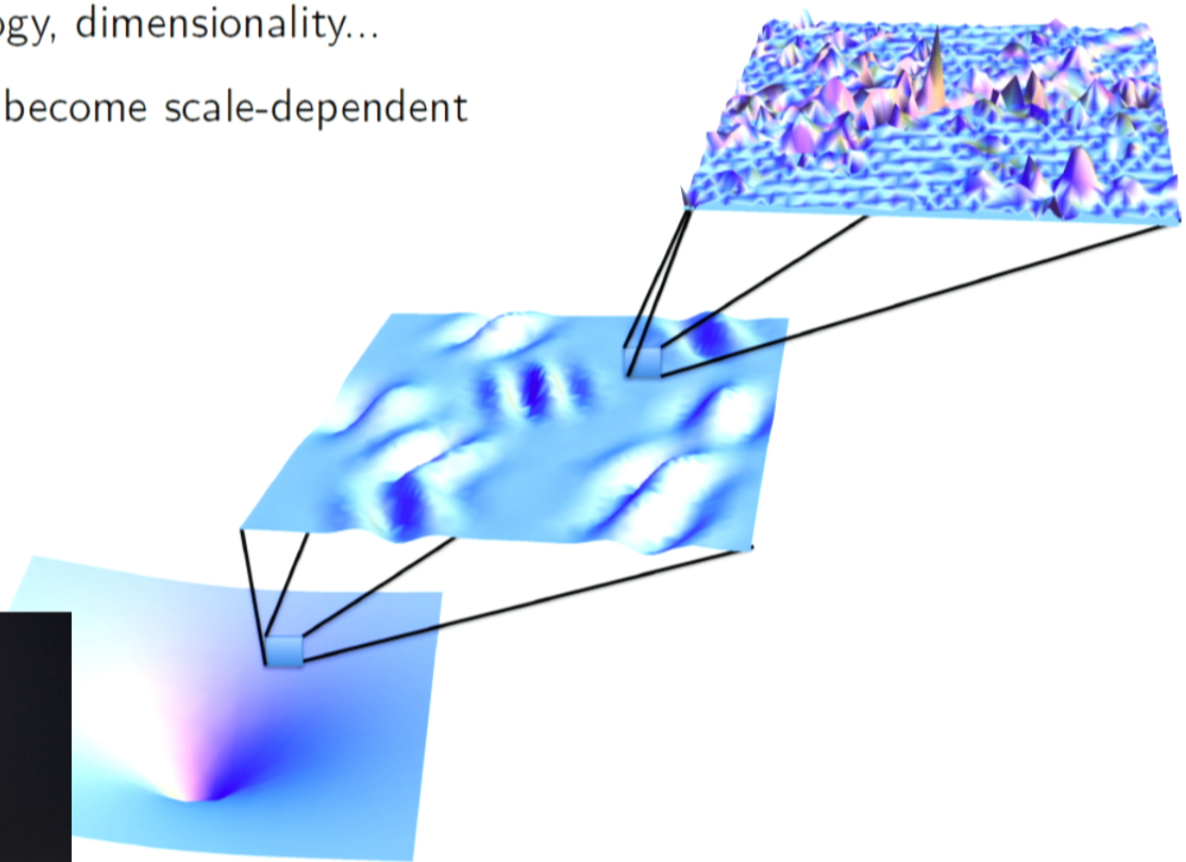
$\int \mathcal{D}g_{\mu\nu} e^{i\,S[g_{\mu\nu}]}$



What is the corresponding spacetime like?

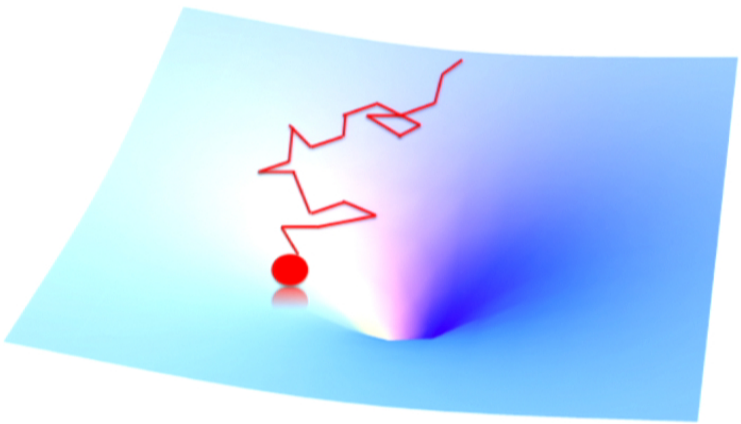
topology, dimensionality...

could become scale-dependent



Testing by diffusion

Probe the properties by a (fictitious) diffusing particle:



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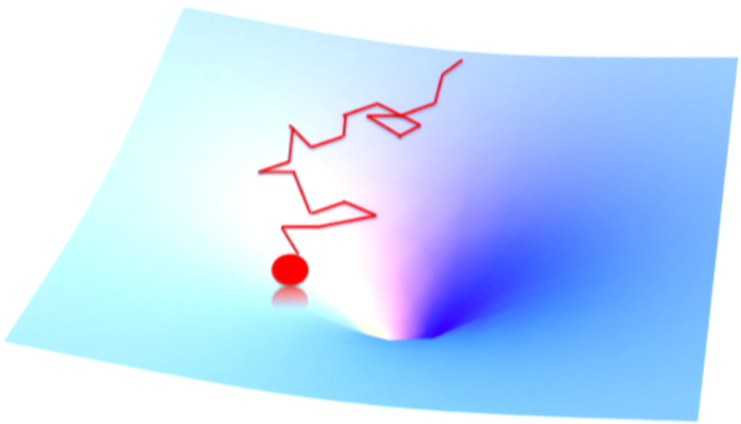

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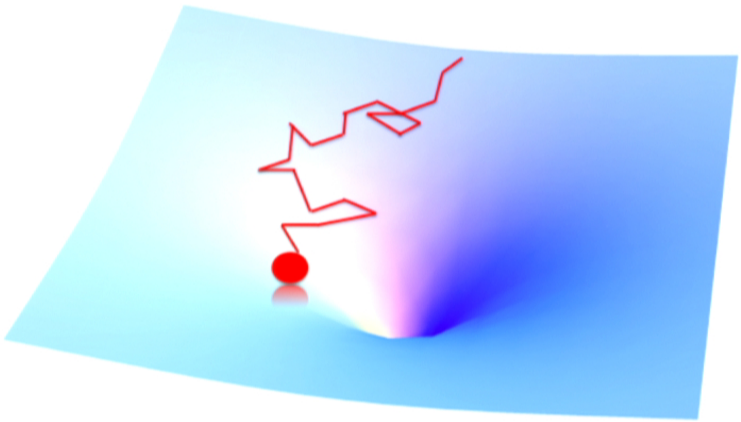


standard Brownian motion

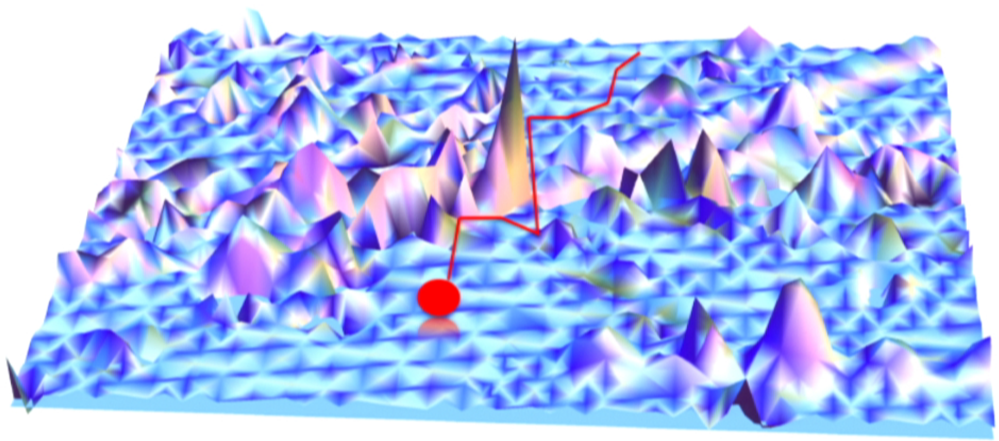
$\left(\partial_\sigma - \nabla_x^2\right) P(x, x', \sigma) = 0$ & initial condition $P(x, x', 0) = \delta^4(x - x')$

probability density $P(x, x', \sigma) = \frac{1}{(4\pi\sigma)^2} e^{-\frac{|x-x'|^2}{4\sigma}}$

$\rightarrow$ spectral dimension $d_s = -2 \frac{\partial \ln P(x,x,\sigma)}{\partial \ln \sigma}$

Testing by diffusion

Probe the quantum regime by a (fictitious) diffusing particle:



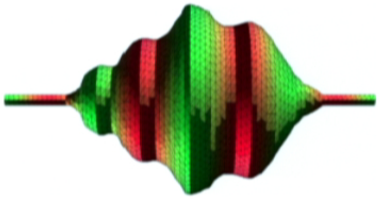
What is the diffusion equation?
What is the spectral dimension?

Testing by diffusion

setting: $\int \mathcal{D}g_{\mu\nu} e^{-S[g_{\mu\nu}]}$

- let random walker explore each single configuration $g_{\mu\nu}$ (mod. gauge), then average over configurations

  Causal/ Euclidean Dynamical Triangulations



[Ambjorn, Jurkiewicz, Loll (2012)]

- let random walker explore 'averaged' configuration $\langle g_{\mu\nu} \rangle$

  Asymptotic Safety, Loop Quantum Gravity, Horava-Lifshitz gravity

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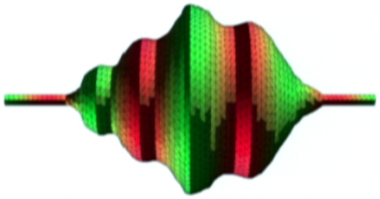

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Diffusion equation for asymptotic safety

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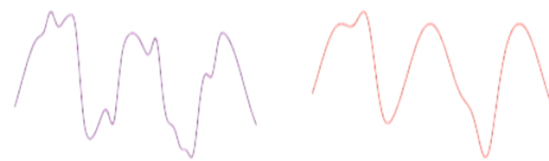

momentum scale k:
family of effective metrics $\langle g_{\mu\nu} \rangle_k$

scale-invariance in fixed point regime: $\langle g^{\mu\nu} \rangle_k \sim \frac{k^2}{k_0^2} \langle g^{\mu\nu} \rangle_{k_0}$ [Lauscher, Reuter, 2005]

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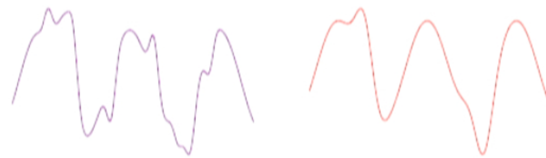

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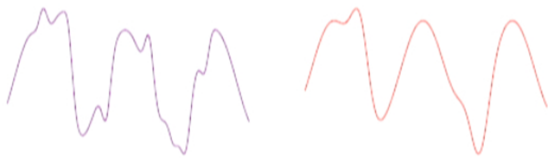

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RG-scale identification

physical scales of the system: particle momentum p, diffusion time σ

k: effective fields at k cannot 'resolve' smaller distances $\Rightarrow k \sim p$?

$\to (\partial_\sigma + \nabla^4) P(x, x', \sigma) = 0$ [Lauscher, Reuter, 2005]

Diffusion equations for quantum gravity (?)

$\rightarrow (\partial_\sigma + \nabla^4) P(x, x', \sigma) = 0$ [Lauscher, Reuter, 2005]

similar equation:

Loop Quantum Gravity [Modesto, 2005]

area operator: $\langle A \rangle \sim \sqrt{l^2 (l^2 + l^2_{\mathrm{Planck}})}$

$\Rightarrow \langle g^{\mu\nu} \rangle \sim l^{-2} \Rightarrow$ same diffusion equation as asymptotic safety

Horava-gravity [Horava, 2009; Sotiriou, Visser, Weinfurtner, 2011]

higher spatial derivatives $\rightarrow$ perturbative renormalizability

$\nabla^2 \rightarrow \partial_t^2 + (\nabla_x^2)^3 \Rightarrow$ similar higher-order equation

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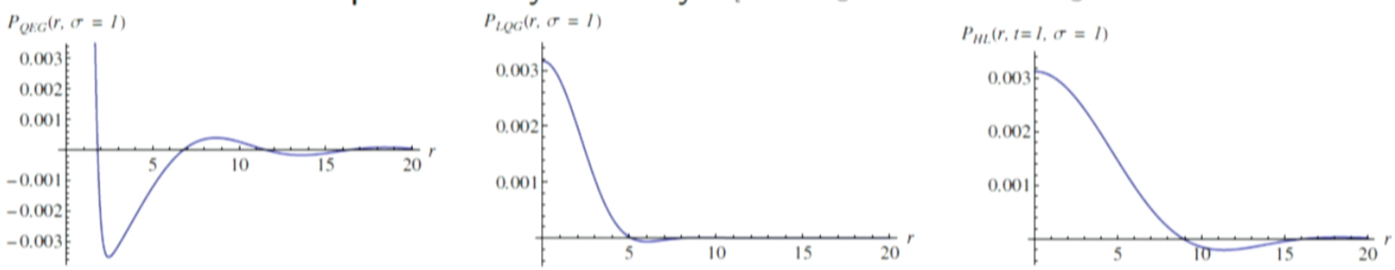
$\rightarrow$ solution not a probability density! [G. Calcagni, AE, F. Saueressig, 1304.7247, PRD 87, 124028]



Quantum-gravity-'improved' diffusion equation I

$k \sim f(\sigma)$: at small diffusion times, particle probes small-scale spacetime
$(\partial_\sigma - k^2\nabla^2)\, P(x, x', \sigma) = 0 \rightarrow$ dimensional analysis: $k \sim \sigma^{-\frac{1}{4}}$

Diffusion equation I for asymptotic safety & LQG

$$\left(\frac{\partial}{\partial \sigma^{\frac{1}{2}}} - \nabla_x^2\right) P(x, x', \sigma) = 0$$

solution: $P(|x - x'| = r, \sigma) = \frac{1}{\left(4\pi\sigma^{1/2}\right)^2} e^{-\frac{r^2}{4\sigma^{1/2}}}$ [G. Calcagni, AE, F. Saueressig, 1304.7247, PRD 87, 124028 (2013)]

positive definite $\rightarrow$ diffusion probability!

Quantum-gravity-'improved' diffusion equation II

fractional (Caputo) derivative: $\partial^\beta f(\sigma) = \frac{1}{\Gamma[1-\beta]} \int_0^\sigma \frac{\mathrm{d}\sigma'}{(\sigma-\sigma')^\beta} \partial_{\sigma'} f(\sigma')$

Diffusion equation II for asymptotic safety & LQG

$$\left(\partial_\sigma^{1/2} - \nabla_x^2\right) P(x, x', \sigma) = 0$$

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[G. Calcagni, AE, F. Saueressig, 1304.7247, PRD 87, 124028 (2013)]

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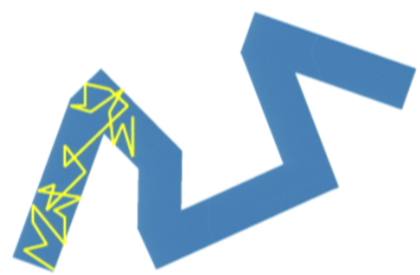
[G. Calcagni, AE, F. Saueressig, 1304.7247, PRD 87, 124028 (2013)]

positive definite!

same equation describes diffusion in a crack!

[Burdzy, Khoshnevisan, 1998]

(iterated Brownian motion:
Brownian motion in Brownian time)

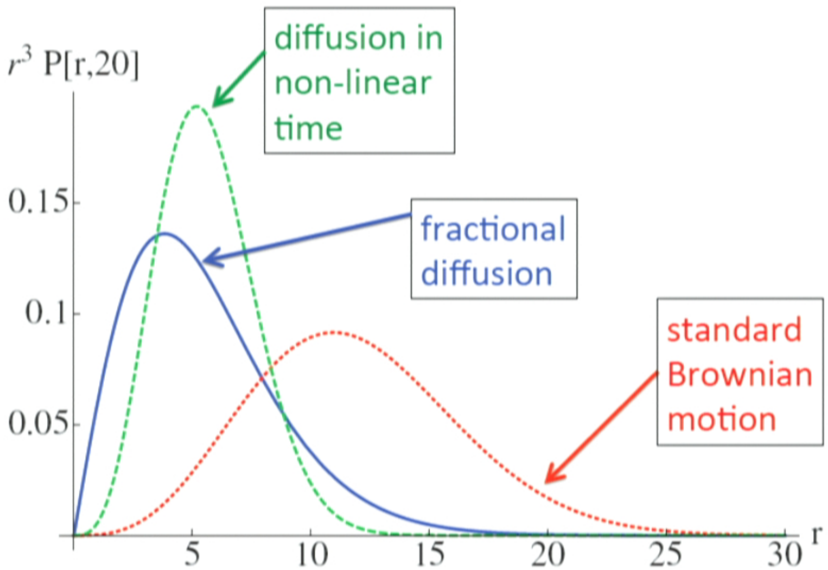


particle 'hindered' by quantum fluctuations $\rightarrow$ diffuses as if in a crack

Quantum fluctuations slow down the diffusion

mean-square displacement:

$\langle r^2 \rangle \sim \sqrt{\sigma}$ standard Brownian motion $\langle r^2 \rangle \sim \sigma$



Subdiffusion

qm fluctuations of spacetime 'drag' the particle

quantum gravity effects lead to subdiffusion (never superdiffusion 'kicking particle')

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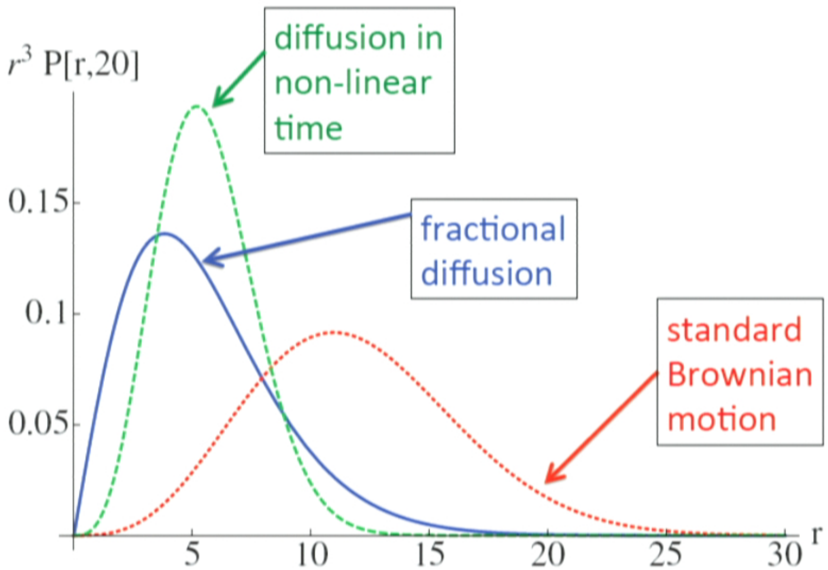

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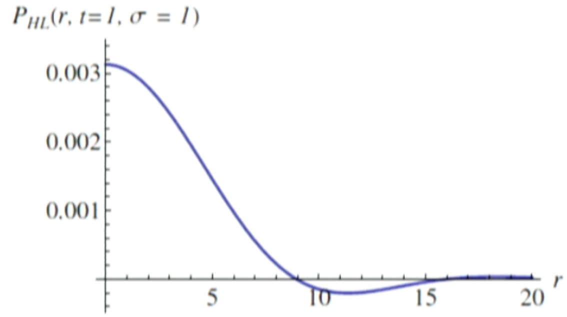
higher spatial derivatives: $\nabla^2 \to \partial_t^2 + (\nabla_x^2)^3$

diffusion equation $\left(\partial_\sigma - \partial_t^2 - (\nabla_x^2)^3\right) P(x, x', \sigma) = 0$ [Horava, 2009]

$\to$ solution not positive semi-definite



Horava-Lifshitz gravity

$(\partial_\sigma - g^{\mu\nu}\nabla_\mu\nabla_\nu)\, P(x, x', \sigma) = 0$

higher spatial derivatives: $\nabla^2 \to \partial_t^2 + (\nabla_x^2)^3$

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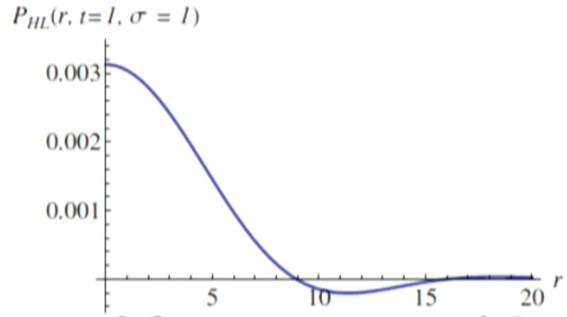

anisotropy between t and $x \Rightarrow$ cannot use QG improvement of ∂_σ

Observation: source-terms can restore positivity!
$\left(\partial_\sigma - \partial_t^2 - (\nabla_x^2)^3\right) P(x, x', \sigma) = \mathcal{S}$ [G. Calcagni, AE, F. Saueressig, 1304.7247, PRD 87, 124028]

Spectral dimension: flowing dimensionality

for asymptotic safety: $d_s = d$ in the infrared, $d_s = \frac{d}{2}$ in the ultraviolet

[Lauscher, Reuter, 2005; Reuter, Saueressig, 2011; Calcagni, AE, Saueressig, 2013]

for Horava-Lifshitz gravity: $d_s = 1 + (d-1)/z$ ($z = 3$ for $d = 4$) in the ultraviolet

[Horava, 2009; Sotiriou, Visser, Weinfurtner, 2011; Calcagni, AE, Saueressig, 2013]

for Loop-Quantum gravity: $d_s = 4$ in the infrared, $d_s = 2$ in the ultraviolet

[Modesto, 2005]

Is there one underlying physical mechanism? Do these approaches 'see' the same quantum spacetime?

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[Modesto, 2005]

Is there one underlying physical mechanism? Do these approaches 'see' the same quantum spacetime?

degeneracy problem

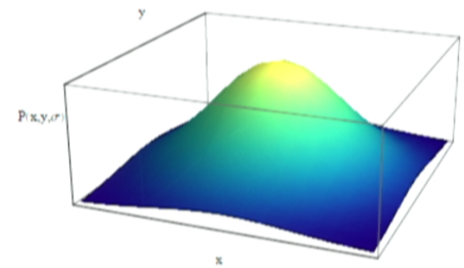
different stochastic processes result in same d_s

How can we distinguish accidental degeneracy?

Probability density as a probe of quantum geometry

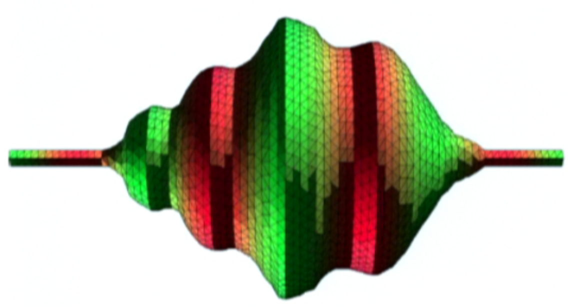
d_s: 'rough' probe of quantum spacetime

agrees in many quantum gravity approaches



study probability density!
$\Rightarrow$ no agreement between Horava-Lifshitz gravity and Asymptotic Safety

Do Causal/Euclidean Dynamical Triangulations agree with one of the two?



$d_s = 2$ in 4d CDT, HL and AS

$P_{AS}(x, x', \sigma) \quad \neq \quad P_{HL}(x, x', \sigma)$
$\rightarrow P_{CDT}(x, x', \sigma)$?

[Ambjorn, Jurkiewicz, Loll (2012)]

Probability density as a probe of quantum geometry

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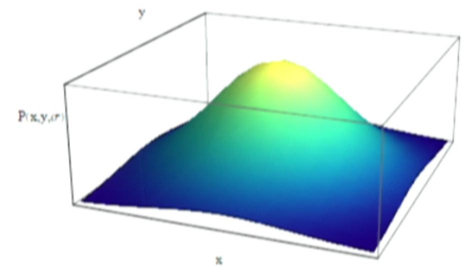

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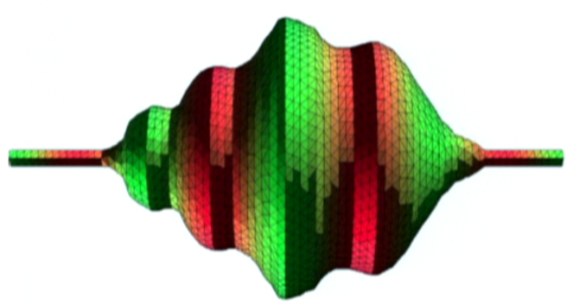

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$P_{AS}(x, x', \sigma) \quad \neq \quad P_{HL}(x, x', \sigma)$
$\rightarrow P_{CDT}(x, x', \sigma)$?

[Ambjorn, Jurkiewicz, Loll (2012)]

Summary: What are the properties of quantum spacetime?

Probe by random walker

$$(\partial_\sigma - \langle g^{\mu\nu} \rangle_k \nabla_\mu \nabla_\nu)\, P(x, x', \sigma) = 0$$

problem: 'naive' QG improvement yields 'negative probabilities'

asymptotic safety (& LQG)

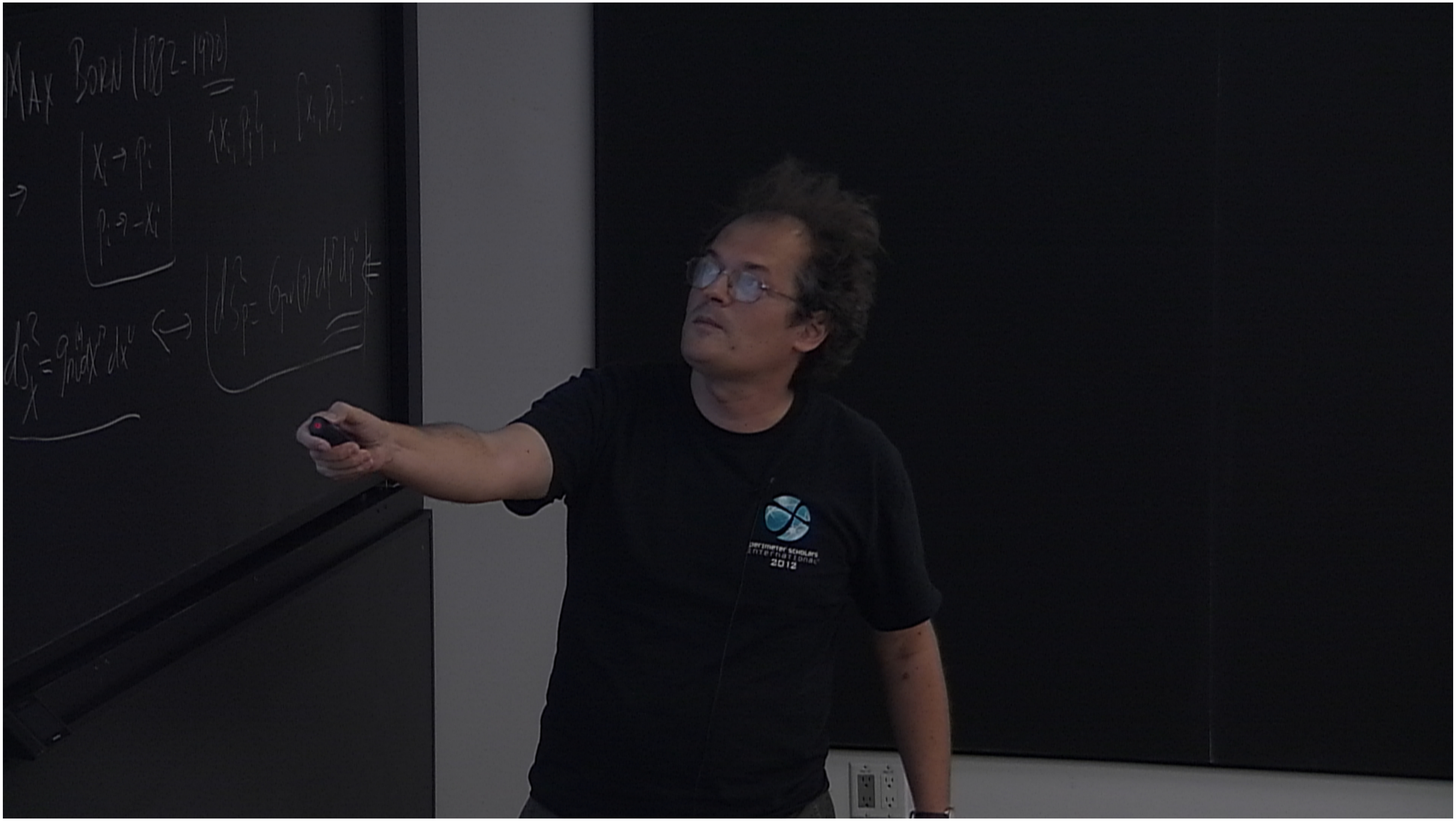
scale-identification $k \sim \sigma$ (non-linear time or fractional time operator)
$\Rightarrow$ positive definite solution
quantum effects lead to subdiffusion $\quad d_s = d/2$

Horava-Lifshitz

additional source terms $\rightarrow$ positive probability density and $d_s = 2$

Outlook: Use probability density as more refined probe of quantum geometry

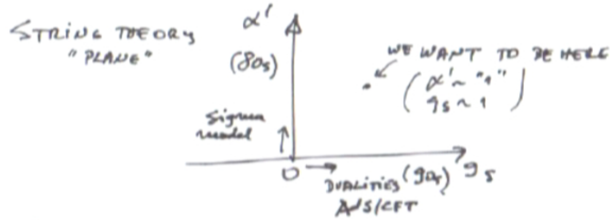
Thank you for your attention!



MAX BORN (1882-1970)

GENERALIZED T-DUALITY, STRING THEORY & THE REAL WORLD

[L. FREIDEL (PERIMETER), R. G. LEIGH (URBANA) & D. M. (VIRGINIA TECH)]



$\alpha' \sim \ell_s^2$ (ℓ_s - SPACE-TIME SIZE OF THE STRING)

DUALITIES : $g_s \to \frac{1}{g_s}$

(g_s large → M-theory ...) also AdS/CFT

T-DUALITY: $R \to \frac{\alpha'}{R}$ (MINIMAL LENGTH?)

- WANT TO UNDERSTAND T-DUALITY!
- WANT TO RELATE $\alpha' \Rightarrow \frac{L^4}{\alpha'}$ BY GENERALIZING T-DUALITY

⇒ STRING THEORY IN PHASE SPACE! (IN GENERAL, DYNAMICAL PHASE SPACE)

RELATE TO THE REAL WORLD (VACUUM SELECTION, OBSERVED VACUUM ENERGY ETC. (LAST PART OF THE TALK))

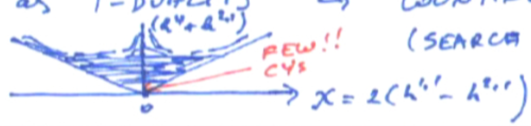
WHY IS T-DUALITY CENTRAL TO STRING THEORY?

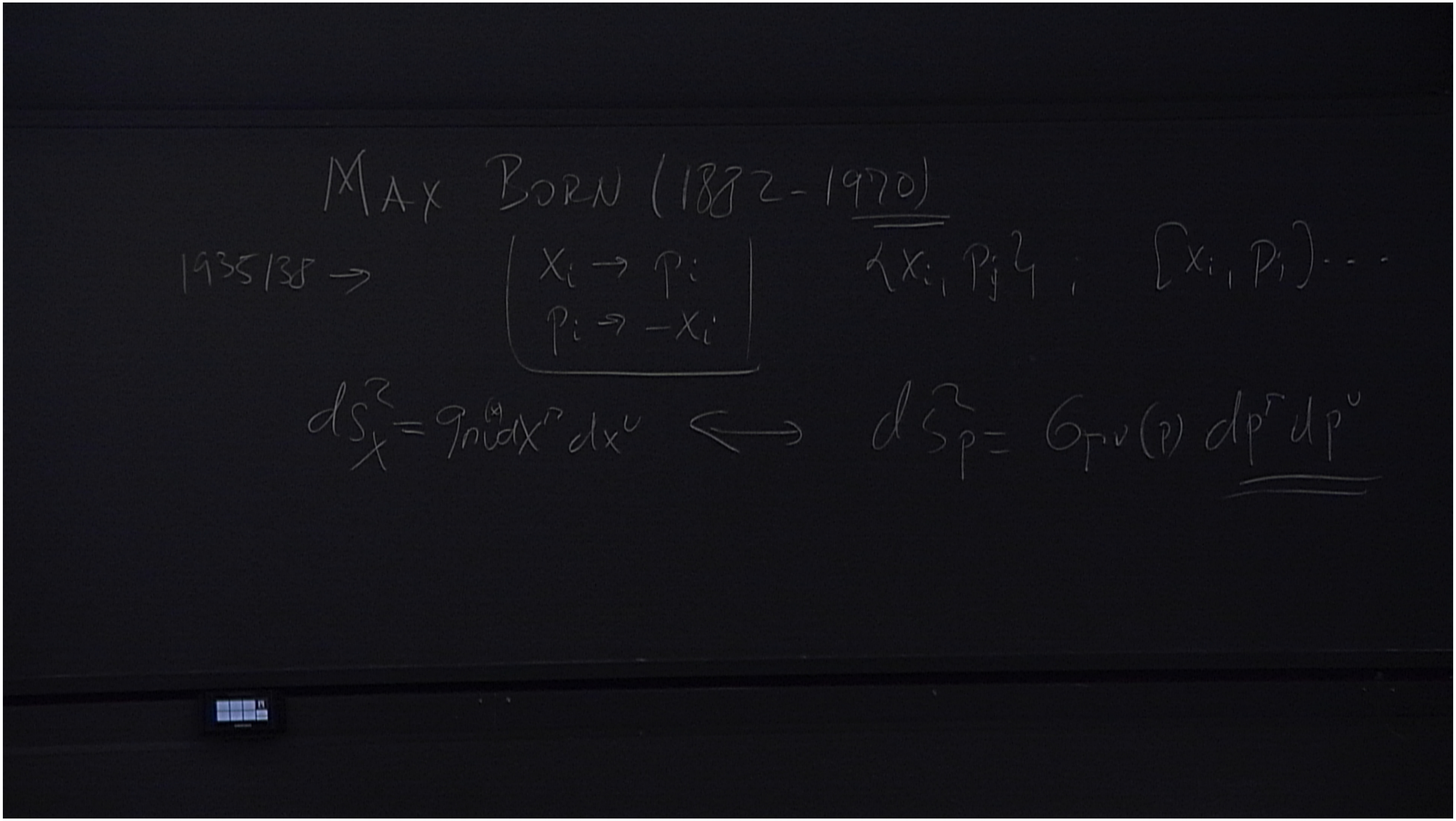
I) high energy behavior (GROSS-MENDE "$\Delta x \sim \alpha' \Delta p$" AT HIGH ENERGY)
$\underline{S=0}$!! TOPOLOGICAL PHASE AT SHORT DISTANCE

II) high temperature behavior (HAGEDORN TEMPERATURE: $T_H \sim \frac{1}{L_s}$)
ABOVE T_H, $F(T) \sim \frac{T^2}{T_H^2} \Rightarrow$ $\underline{2d}$ FIELD THEORY!! (CFT)
↑ free energy
$\lfloor P+\bar{P}=0 \rfloor$

III) T-DUALITY OF OPEN STRINGS → $\underline{\text{D-BRANES}}$!

IV) Mirror symmetry as T-DUALITY → COUNTING OF STRING VACUA (SEARCH FOR THE STANDARD MODEL)
(OF CYs)




MAX BORN (1882-1970)
1935/38 →
$x_i \to p_i$
$p_i \to -x_i$
$\{x_i, p_j\}$; $[x_i, p_i]$ . . .
$ds_x^2 = g_{\mu\nu}^{(x)} dx^\mu dx^\nu \longleftrightarrow d\tilde{s}_p^2 = G_{\mu\nu}(p)\, dp^\mu dp^\nu$

TECHNICAL DETAILS: POLYAKOV (FLAT METRIC) $*$ - 2d Hodge duality (3)

$$S_P(X) = \frac{1}{2\alpha'}\int \eta_{\mu\nu} * d\bar{X}^\mu \wedge d\bar{X}^\nu$$ (ASSUME NON-COMPACTNESS) ; $(\sigma, \tau) \in [0,2\pi] \times [0,1]$ cylinder

$d\bar{X}(\sigma,\tau)$ is PERIODIC W.R.T. σ ; PERIOD 2π.

$\Rightarrow \bar{X}^\mu$ is QUASI-PERIODIC $\bar{X}^\mu(\sigma + 2\pi, \tau) = \bar{X}^\mu(\sigma,\tau) + \bar{p}^\mu$

$\bar{p}^\mu$ is THE QUASI-PERIOD OF $\bar{X}^\mu$. IF $\bar{p}^\mu$ NON-ZERO NO A-PRIORI SPACE-TIME INTERP.

INSTEAD, STRING PROPAGATES INSIDE A PORTION OF PHASE SPACE. (NON-GEOMETRIC BACKGROUND)

T-DUALITY AS A FOURIER TRANSFORM:

$$|\Psi\rangle : \quad \Psi(x^\mu(\sigma)) = \int_{\bar{X}|\partial\Sigma = x} \mathcal{D}\bar{X} \int_{Met(\Sigma)} \mathcal{D}\gamma \, e^{\frac{i}{2\pi\alpha'}\frac{1}{2}\int \eta_{\mu\nu}(* d\bar{X}^\mu \wedge d\bar{X}^\nu)}$$ Here $\bar{X}$-periodic

FOURIER TRANSFORM: (F.T.)

$$\tilde{\Psi}(Y_\mu(\sigma)) = \int_{Y|_{\partial\Sigma = y}} \mathcal{D}Y \int_{Met(\Sigma)} \mathcal{D}\gamma \, e^{\frac{i}{2\pi\alpha'}\frac{1}{2}\int \eta_{\mu\nu}(* dY_\mu \wedge dY_\nu)}$$ Here Y- quasiperiodic

FIRST ORDER ACTION: $\hat{S} = \int_\Sigma (X^\mu dP_\mu + \frac{\alpha'}{2}\eta^{\mu\nu}(* P_\mu \wedge P_\nu))$: E.O.M. $* P_\mu = \frac{1}{\alpha'}\eta_{\mu\nu} dX^\nu$

(Note $P_\mu = dY_\mu$ locally. by integrating $\bar{X}$ to get $dP = 0$) $\Leftrightarrow \hat{S} = -\frac{1}{\alpha'}S_P + \int_{\partial\Sigma} X^\mu P_\mu$ Kernel of the F.T.

We can do this on genus g Σ etc.

Note: Vertex operators $V_{p_i}(x) \to e^{i p_i X(z)}$ & dual $\tilde{V}_{p_i}(Y) \to e^{i \sum p_i \int_{e_i} * dY_i}$ non-local operator (z - are endpoints of a graph Γ with edges e_i)

Electro-magnetic duality on the world sheet $e_1 m_2 + e_2 m_1 \in \mathbb{Z}$ (electric charges, magnetic charges) $\Rightarrow O(D,D)$

START FROM $S = \int P_\mu \wedge dX^\mu + \frac{\alpha'}{2}(*P_\mu \wedge P^\mu)$; "HAMILTONIAN" ANALYSIS

$P_\mu = \Pi_\mu d\sigma + \chi_\mu d\tau \Rightarrow S$, INTEGRATE OUT $\chi \Rightarrow S = \int \Pi \dot{X} - (\frac{\alpha'}{2}\Pi\cdot\Pi + \frac{1}{2\alpha'}X'\cdot X')$

$\dot{X} \equiv \partial_\tau X$; $X' \equiv \partial_\sigma X$ (e.o.m. $\alpha'\Pi = \dot{X}$; $\alpha'\chi = X'$)

INTRODUCE $Y' = \Pi$ (Y - DUAL COORDINATE) $\Rightarrow S = \int Y'\dot{X} - \frac{1}{2}(\alpha' Y'\cdot Y' + \frac{1}{\alpha'}X'\cdot X')$

USE $O(D,D)$: $\mathbb{X}^A = \begin{pmatrix} X^A \\ Y^A \end{pmatrix}$, $\eta_{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $H_{AB} \equiv \begin{pmatrix} \eta_{\mu\nu}/\alpha' & 0 \\ 0 & \alpha'\eta^{\mu\nu} \end{pmatrix}$

$\Rightarrow S = \int \frac{1}{2}\dot{\mathbb{X}}^A \mathbb{X}'^B \eta_{AB} - \frac{1}{2}\mathbb{X}'^A\mathbb{X}'^B H_{AB}$ ("CHIRAL BOSON"-LIKE)

NOTE: DUALITY EQUATIONS $*dX = \alpha' dY \Rightarrow$ e.o.m. for X & Y: $\Delta\mathbb{X}^A = 0$. (STRING THEORY ≡ SELF-DUALITY)

HAMILTONIAN ($\mathcal{H}$) & DIFFEOMORPHISM (D) CONSTRAINTS:

$\mathcal{H} \equiv S H \mathbb{X}' + \mathbb{X}' H \mathbb{X}'$; $S^A \equiv \dot{\mathbb{X}}^A - (\eta^{-1}H\mathbb{X}')^A$

$D \equiv -G + \mathbb{X}'\eta\mathbb{X}'$; $G \equiv \frac{1}{2}(\mathbb{X}'^A\mathbb{X}'^B + \dot{\mathbb{X}}^A\dot{\mathbb{X}}^B)\eta_{AB} - \dot{\mathbb{X}}^A\mathbb{X}'^B H_{AB}$

(Flat case: $1 = (\eta^{-1}H)^2 \Rightarrow$ e.o.m. $S = 0 \Rightarrow \mathcal{H} = (X'^2 + Y'^2)$; $D = X'\cdot Y'$ ← usual form!!)

Propagator in phase space:

$$\langle \mathbb{X}^A(\sigma,\tau)\,\mathbb{X}^B(0,0)\rangle = \frac{1}{4\pi}\left[(\eta^{AB} + H^{AB})\ln|\tau+\sigma| - (\eta^{AB} - H^{AB})\ln|\tau-\sigma|\right]$$

ORTHOGONAL PROJECTORS: $P_{AB} \equiv H_{AB} + \eta_{AB}$; $\bar{P}_{AB} \equiv H_{AB} - \eta_{AB}$ ($P + \bar{P} = 2H$, $P\eta^{-1}P = P$, $\bar{P}\eta^{-1}\bar{P} = \bar{P}$ etc)

COVARIANT ACTION $S_c = \frac{1}{2}\int \det e\, (P_{AB}\,\partial\mathbb{X}^A\bar{\partial}\mathbb{X}^B + P_{AB}\,\bar{\partial}\mathbb{X}^A\bar{\partial}\mathbb{X}^B)$

$ds^2 = e_a \bar{e}_b\, dx^a dx^b$; $(\partial, \bar{\partial})$ - inverse frame fields $\partial \equiv e^a\partial_a$ etc.

Full action $S = S_c + \bar{S}_c$; S_c - chiral action

(5)

CURVED PHASE SPACE: $\eta_{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; $H_{AB} \equiv \begin{pmatrix} G^{-1}_{\mu\nu}/\alpha' & (BG^{-1})^{\mu}{}_{\nu} \\ -(G^{-1}B)^{\nu}{}_{\mu} & \alpha'[G + BG^{-1}B]^{\mu\nu} \end{pmatrix}$

[In "curved space-time" $S_P = \frac{1}{2\alpha'} \int \left(G_{\mu\nu}(x) * dx^\mu \wedge dx^\nu + B_{\mu\nu} dx^\mu \wedge dx^\nu \right)$]

PHASE SPACE ACTION: $\hat{S} = \int \frac{1}{2} \left(\mathbb{X}^{\cdot A} \mathbb{X}'^{B} \eta_{AB} - \mathbb{X}'^{A} \mathbb{X}'^{B} H_{AB} \right)$

DERIVATION OF THE STRINGY UNCERTAINTY PRINCIPLE: (" $\Delta x \Delta p \sim 1 + \alpha' \Delta p^2$ ")

LOOK AT PHASE SPACE OF ZERO MODES: (p_a, x^a) & $(\tilde{p}^a, y_a) \rightarrow$ DUAL PHASE SPACE

$[p_a, x^b] = \delta^b_a$, $[\tilde{p}^a, y_b] = \delta^a_b$

HAMILTONIAN & DIFFEO CONSTRAINTS: $\mathcal{H} = 0$, $D = 0$

$\mathcal{H} = \alpha' \frac{p^2}{2} + \frac{1}{2\alpha'} \tilde{p}^2 - N$; $D = p \cdot \tilde{p} - M$ (N, M integers in the quantum string context)

($\mathcal{H} = \frac{1}{2} \mathbb{P}_A H^{AB} \mathbb{P}_B - N$; $D = \frac{1}{2} \mathbb{P}_A \eta^{AB} \mathbb{P}_B - M$)

$\mathbb{X}^A = \begin{pmatrix} x^a/\ell_s \\ y_a \ell_s \end{pmatrix}$; $\mathbb{P}_A = \begin{pmatrix} p_a \ell_s \\ \tilde{p}_a/\ell_s \end{pmatrix}$ ($\alpha' \sim \ell_s^2$) Note $[\mathcal{H}, \mathbb{X}^A] = -iH^{AB}\mathbb{P}_B$

$[D, \mathbb{X}^A] = -i\eta^{AB}\mathbb{P}_B$

Relativistically invariant positions

$\hat{x}^a = x^a - \alpha' p^a \tau - \tilde{p}^a \sigma$; $\hat{Y}_a = y_a - (\alpha')^{-1} \tilde{p}^a \tau - p_a \sigma$

(Particle: $h = p^2 - m^2 = 0$; $\hat{x}^a_{\text{particle}} \equiv x^a - \frac{p^a}{p^2}(x \cdot p)$; $[\hat{x}^a_p, \hat{x}^b_p] = L^{ab}_p$ Lorentz generator)

FOR THE STRING WE FIND

$[\hat{x}^a, \hat{y}^b] = \frac{N K^{ab} - M L^{ab}}{2(N^2 - M^2)}$

$[\hat{x}^a, \hat{x}^b] = \alpha' \frac{N L^{ab} - M K^{ab}}{2(N^2 - M^2)}$

$[\hat{Y}^a, \hat{Y}^b] = (\alpha')^{-1} \frac{N L^{ab} - M K^{ab}}{2(N^2 - M^2)}$

$L^{ab} \equiv (x \wedge p)^{ab} + (y \wedge \tilde{p})^{ab}$

$K^{ab} \equiv (\alpha')^{-1}(x \wedge \tilde{p})^{ab} + \alpha'(y \wedge p)^{ab}$

$[L^{ab}, L^{cd}] = \eta^{bc} L^{ad} + \dots$

$[L^{ab}, K^{cd}] = \eta^{bc} K^{ad} + \dots$

$[K^{ab}, K^{cd}] = \eta^{bc} L^{ad} + \dots$

THE COSMOLOGICAL CONSTANT SEE-SAW MECHANISM

(LAY NAM CHANG, D.M, TATSU TAKEUCHI)
1004.4220

WHY DOES THE VACUUM WEIGH SO LITTLE ($(10^{-3}eV)^4$) $\overset{\shortparallel}{\epsilon^4}$

(WHAT IS THE VACUUM? HOW DO WE WEIGH?)

THE VACUUM ENERGY RELATES UV & IR

- IR → $S \sim \Lambda \int \sqrt{-g}$
  - Λ ⇝ SIZE OF THE UNIVERSE R
  - $\Lambda \sim \frac{1}{R^2}$
- UV → # OF D.O.F $\sum_p \frac{1}{2}\hbar\omega_p$

UV/IR NON-DECOUPLING ⇐

(EFFECTIVE FIELD THEORY (EFT) FAILS! YET EFT WORKS IN THE IR)

A TRUE QUANTUM GRAVITY (QG) PROBLEM

WE HAVE IN MIND STRING THEORY AS A QG (NOT AN EFT BUT CONSISTENT WITH EFT IN THE IR)

(2)

NUMEROLOGY $\epsilon \sim \frac{M_{sm}^2}{M_P}$; $M_{sm} \sim 1\,TeV$, $M_P \sim 10^{19}\,GeV$

THIS SUGGESTS A SEE-SAW MECHANISM ($M_P \to \infty \Rightarrow \epsilon \to 0 \Rightarrow$ NATURALNESS)

(UNLIKE NEUTRINO MASS SEE-SAW, WHICH IS BASED ON EFT, COSMOLOGICAL CONSTANT SEE-SAW SHOULD BE A QG EFFECT)

LOOK AT $\int^{\epsilon} d^3p\, \frac{1}{2} \hbar \omega_p \sim \epsilon^4$ ($\omega_p \sim p$ FOR MASSLESS FIELDS)

⇓ WANT

SIMILAR TO PLANCK'S SOLUTION OF THE BLACK-BODY PROBLEM $\Leftarrow \int^{\epsilon} d^3p\, \rho(p^2)\, \frac{1}{2} \hbar \omega_p$

a) CHANGE THE WEIGHT $\rho(p^2)$ (PHASE-SPACE)

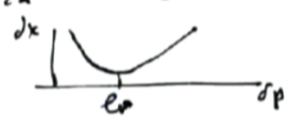
b) CHANGE ϵ (MAKE IT $\epsilon \sim \frac{M_{sm}^2}{M_P}$)

$\left(\sum \frac{1}{2} kT \sim T^4 \right) \Rightarrow \rho\left(\frac{\omega}{T}\right) \sim \frac{1}{e^{\frac{\hbar\omega}{kT}} - 1}$ (PHASE-SPACE QUANTIZED) COUNTING CHANGED

ONE WAY TO GET $\rho(p^2)$: EFFECTIVE $\hbar(p^2) = \hbar A(p^2)$

$\Rightarrow \frac{1}{i\hbar} [x_a, p_a] = A(p^2)\, \delta_{ij} + \cdots$

STRING PERTURBATION THEORY (STRING - STRING SCATTERING)



$\delta x\, \delta p \sim \frac{\hbar}{2} \left(1 + \beta\, \delta p^2 \right)$

③

IF $\frac{1}{i\hbar}[x_a, p_b] = A(p^2) + B(p^2)\, p_i p_j$ AND $[p_a, p_b] = 0$

$A \sim 1+\beta p^2$, $B \sim \beta'$

JACOBI $\Rightarrow$ $\frac{1}{i\hbar}[x_a, x_b] = -\left[2\,(A + Bp^2)\frac{dA}{dp^2} - AB\right] L_{ij}$; $L_{ij} = \frac{x_i p_j - x_j p_i}{A}$; $[x_i, x_j] \neq 0$!!

CLASSICAL LIMIT : $\frac{1}{i\hbar}[\,,\,] \Rightarrow \{\,,\,\}$ (POISSON) THEN CLASSICAL PHASE-SPACE VOLUME:

$$\frac{d^D x\, d^D p}{A^{D-1}(A + Bp^2)}$$

SO (B=0) $\int d^3p\; p \Rightarrow \int \frac{d^3p}{A^3(p^2)}\, p = 4\pi \int^{\epsilon} dp\, \frac{p^3}{A^3(p^2)}$

$\rho(p^2) = A^{-3}(p^2)$

EVEN IF $A^{-3}(p^2) \sim \frac{p^4}{M_p^4} \Rightarrow \hbar(p^2) = \hbar A(p^2)$ BLOWS UP IN THE IR !

NEED SOMETHING ELSE! $[p_i, p_j] = 0 \Rightarrow [p_i, p_j] \neq 0$!!

(STILL WANT $\delta x\, \delta p \sim \frac{\hbar}{2}(1 + \beta\, \delta p^2)$ BECAUSE OF UV/IR $\delta x \sim \frac{\hbar}{2}\beta\, \delta p$!! (DEEP UV))

WANT TO UNDERSTAND VACUUM SELECTION & VACUUM ENERGY } GENERALIZED T-DUALITY
WANT TO UNDERSTAND FUNDAMENTALS OF STRING THEORY

a) PHENOMENOLOGY OF THE MINIMAL LENGTH ℓ_s !
$\Delta x \Delta p \sim 1 + \alpha' \Delta p^2$ ⇐ DERIVE !

b) NON-COMPACT T-DUALITY (WILSON LOOP / SCATTERING AMPLITUDE DUALITY)
↑ UNDERSTAND (ALDAY - MALDACENA, BERKOVITS - MALDACENA ETC. ETC.)

c) LESSONS FROM CANONICAL QUANTUM GRAVITY
↳ CURVED MOMENTUM SPACE

(FREIDEL LIVINE : 2+1 gravity ⊕ matter ⟶ integrate out gravity (2+1 grav topological) ⇐ VIOLATES EFFECTIVE FIELD THEORY !! (LOCALITY)
↳ $[P_i, P_j] \neq 0$ (!!)
("RELATIVE LOCALITY") ⟷

FREIDEL, LEIGH, D.M.: GENERALIZED T-DUALITY (NON-COMPACT IN GENERAL)
NO SPACE-TIME INTERPRETATION IN GENERAL ; CURVED PHASE SPACE
BORN RECIPROCITY $X \Leftrightarrow P$

MAIN CLAIM : STRING THEORY SHOULD BE FORMULATED IN CURVED (AND DYNAMICAL) PHASE SPACE
(TECHNICAL DETAILS BELOW)
WITH $\alpha' \to 0$ TO $\alpha' \to \infty$ DUALITY
(LONG DISTANCE "SPACE-TIME PHASE") (SHORT DISTANCE "NON-SPACE-TIME PHASE")

$\alpha' \to 0 \Rightarrow$ GRAVITY
$\alpha' \to \infty \Rightarrow$ HIGH-SPIN THEORY (FRADKIN, VASILIEV)

IMPORTANT PIONEERING WORK: A. TSEYTLIN (1989 !!) "COVARIANT" T-DUALITY
Schwarz.. Siegel... double field theory Hull, Zwiebach

CURVED PHASE SPACE : E. FRADKIN & CO. → I. BARS

③

IF $\frac{1}{i\hbar}[x_a, p_b] = A(p^2) + B(p^2)\, p_i p_j$ AND $[p_a, p_b] = 0$

$A \sim 1+\beta p^2$, $B \sim \beta'$

JACOBI $\Rightarrow$ $\frac{1}{i\hbar}[x_a, x_b] = -\left[2(A+Bp^2)\frac{dA}{dp^2} - AB\right] L_{ij}$; $L_{ij} = \frac{x_i p_j - x_j p_i}{A}$; $[x_i, x_j] \neq 0$!!

CLASSICAL LIMIT : $\frac{1}{i\hbar}[\,,\,] \Rightarrow \{\,,\,\}$ (Poisson) THEN CLASSICAL PHASE-SPACE VOLUME:

$$\frac{d^D x\, d^D p}{A^{D-1}(A+Bp^2)}$$

SO $(B=0)$ $\int d^3p\, \rho \Rightarrow \int \frac{d^3p}{A^3(p^2)}\, \rho = 4\pi \int^{\epsilon} dp \frac{p^3}{A^3(p^2)}$

$\rho(p^2) = A^{-3}(p^2)$

EVEN IR $A^{-3}(p^2) \sim \frac{p^4}{M_p^4} \Rightarrow \hbar(p^2) = \hbar A(p^2)$ BLOWS UP IN THE IR!

NEED SOMETHING ELSE! $[p_i, p_j] = 0 \Rightarrow [p_i, p_j] \neq 0$!!

(STILL WANT $\delta x\, \delta p \sim \frac{\hbar}{2}(1+\beta\, \delta p^2)$ BECAUSE OF UV/IR $\delta x \sim \frac{\hbar}{2}\beta\, \delta p$!! (DEEP UV))

4

HEURISTICALLY, $ds^2 = g_{ab}\, dx^a dx^b \Longrightarrow \delta s_x^2 = g_{ab}\, \delta x^a \delta x^b$

DYNAMICAL SPACE-TIME (GR)

($\delta x \sim$ QUANTUM FLUCTUATION!)

NOW, IF $\delta x \sim \hbar\, \partial p \Rightarrow \delta s_p^2 = G_{ab}\, \delta p^a \delta p^b \rightsquigarrow$ DYNAMICAL ENERGY-MOMENTUM SPACE! (QG)

($\delta p \sim$ quantum fluctuation)

FLUCTUATING ENERGY-MOMENTUM SPACE (i.e. "GRAVITY" IN ENERGY-MOMENTUM SPACE)

(FOUNDATIONAL issues REGARDING quantum theory ...)

USUALLY WE EXPECT "FOAMY" SPACE-TIME IN QG.

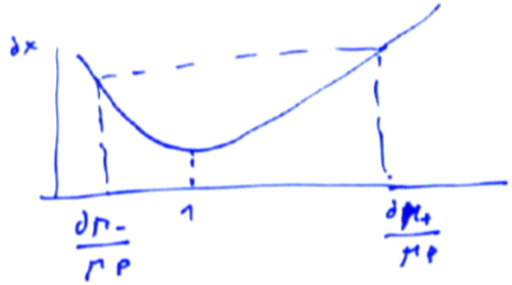
NOW, NOT ONLY SPACE-TIME "FOAM" BUT ENERGY-MOMENTUM "FOAM" AS WELL

GENERICALLY IN QG EXPECT HOLOGRAPHY: $\left(\frac{\ell}{\ell_p}\right)^2$ BH SCALING $\left(S \sim \frac{A}{4G_N}\right)$

OF SPACE-TIME FOAM CELLS (in D=4!) $\left(\frac{\ell^4}{c}\right) \Big/ \left(\frac{\ell^2}{\ell_p}\right) = \frac{\ell^2 \ell_p^2}{c} \sim \delta\ell^4$

$\Rightarrow \delta\ell \sim \ell^{1/2} \ell_p^{1/2}$ (BROWNIAN SCALING ; NG-VAN DAM)

USE THIS REASONING IN ENERGY-MOMENTUM SPACE: $\delta\mu \sim \mu^{1/2} \mu_p^{1/2}$



$\delta\mu_+$ - trans-Planckian (not EFT!)
$\delta\mu_-$ - sub-Planckian (EFT!)
$\delta\mu_\pm \sim \mu_\pm^{1/2} \mu_P^{1/2}$ (energy-momentum form)
$(\delta\mu_\pm^2 \sim \mu_\pm \mu_P) \Rightarrow \underbrace{\mu_- \delta\mu_+^2 = \mu_+ \delta\mu_-^2 \sim \mu_P^3}_{\text{see-saw}}$

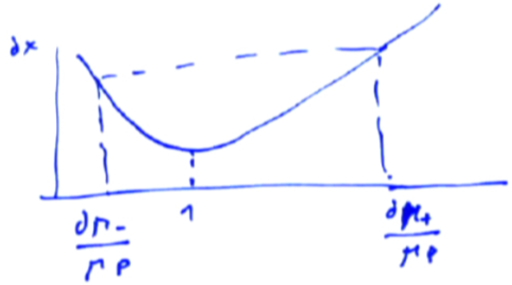
Thus EFT sees: $\mu_- = \frac{\delta\mu_-^2}{\mu_P} \Rightarrow \epsilon^4 \sim \mu_-^4 = \frac{\delta\mu_-^8}{\mu_P^4}$

If $\delta\mu_- \sim M_{SM}$ (RELATION TO THE HIERARCHY PROBLEM?) $\Rightarrow$ ✓✓ $(\epsilon^4 \sim (10^{-3} eV)^4)$

PHYSICS: JAMMING in NON-EQUILIBRIUM STAT PHYSICS
(OR "FREEZING BY HEATING" $\rightsquigarrow$ EXAMPLE TRAFFIC JAM)

$\Rightarrow$ INTEGRATE IN ("HEAT UP") $\rightsquigarrow$ FREEZE VACUUM ENERGY (low ϵ !!)

NON-DECOUPLING OF UV & IR (FERMILAB HOLOMETER?)
(VACUUM ENERGY REVEALS TRANS-PLANCKIAN D.O.F JUST AS NEUTRINO SEE-SAW REVEALS GUT D.O.F)
APPLICATION TO COLLIDER PHYSICS (WORK IN PROGRESS)



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