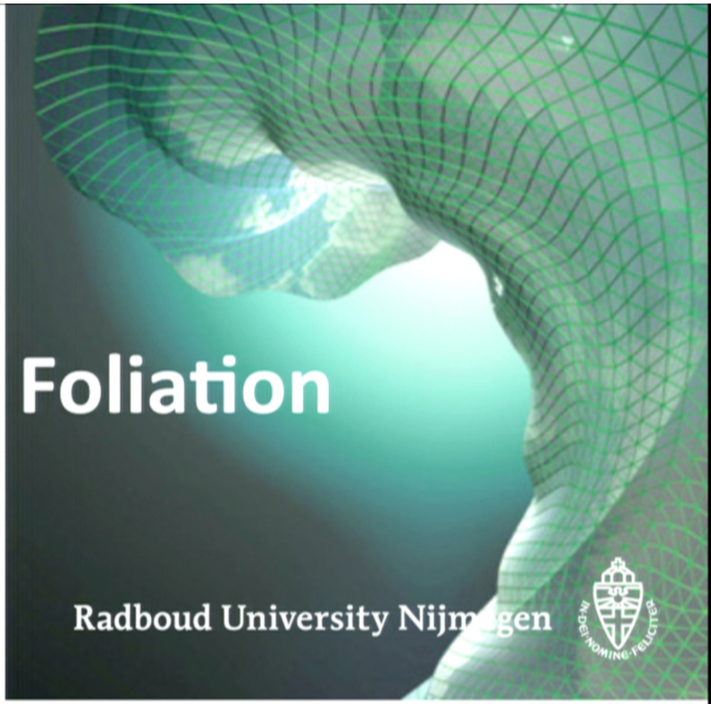
Title: Causal Dynamical Triangulations without Preferred Foliation

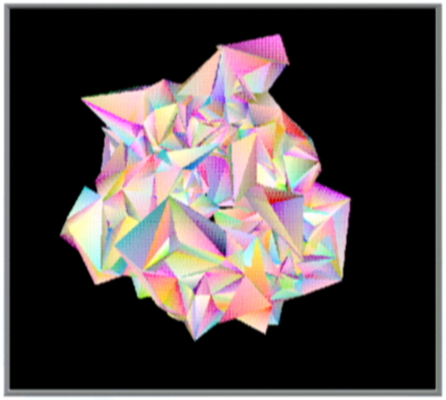
Date: Jul 25, 2013 11:45 AM

URL: http://pirsa.org/13070071

Abstract: We introduce a generalized version of the Causal Dynamical Triangulations (CDT) formulation of quantum gravity, in which the regularized, triangulated path integral histories maintain their causal properties, but do not have a preferred proper-time foliation. An extensive numerical study of the associated nonperturbative path integral in 2+1 dimensions shows that it can nevertheless reproduce the emergence of an extended de Sitter universe on large scales, a key feature of CDT quantum gravity. This suggests that the preferred foliation normally used in CDT is not a crucial (although convenient) part of its background structure.

Causal Dynamical Triangulations without Preferred Foliation





triangulated model of quantum space

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Perimeter Institute, 25 July 2013

Questions for quantum gravity:

- What are the quantum laws underlying General Relativity?
- What are the quantum origins of space and time?
- Can we explain gravitational attraction from first principles?
- What is the quantum microstructure of spacetime?
- Which observables capture its properties?

Questions for theories of quantum gravity:

How much of the classical structure of GR is

- present, not subject to quantum fluctuations (fixed background)? - *e.g. spacetime dimension in perturbative quantum gravity*
- present, and subject to quantum fluctuations? - *e.g. metric, topology*
- present (initially), but changed by quantum fluctuations/instabilities? - *e.g. spacetime dimension in nonperturbative quantum gravity*
- not present, but regained dynamically ("emergent")? - *e.g. causality and time in nonperturbative Euclidean quantum gravity*

Apart from their choices of elementary *degrees of freedom* and a *dynamical principle*, different approaches to quantum gravity can be distinguished by how much ***background structure*** they use, e.g. whether metric, differentiable and manifold structure, topology, dimension etc. are fixed a priori or part of dynamics, and which *extra structures and assumptions* they use, e.g. additional symmetries, a choice of preferred "variables", extra dimensions, ...

From what "works" and what doesn't, we try to learn about quantum gravity proper.

In my talk today, I will examine a piece of "background structure" in CDT quantum gravity.

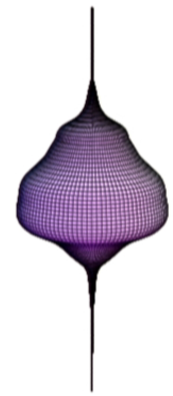
***based on*:**
S. Jordan & R. Loll, Phys. Lett. B 724 (2013) 155 [arXiv:1305.4582], as well as this week's arXiv: 1307.5469

Quantum Gravity from Causal Dynamical Triangulations (CDT)*

... is a *nonperturbative* implementation of the gravitational path integral, much in the spirit of lattice quantum field theory, but based on *dynamical* lattices, reflecting the dynamical nature of spacetime geometry.

CDT is currently the only candidate quantum theory of gravity which can generate *dynamically* a spacetime with semiclassical properties from pure quantum excitations, without using a background metric.

(C)DT has also given us crucial *new* insights into nonperturbative dynamics and pitfalls.



(PRL 93 (2004) 131301, PRD 72 (2005) 064014, PLB 607 (2005) 205)

* main collaborators: J. Ambjørn, T. Budd, A. Görlich, S. Jordan, J. Jurkiewicz

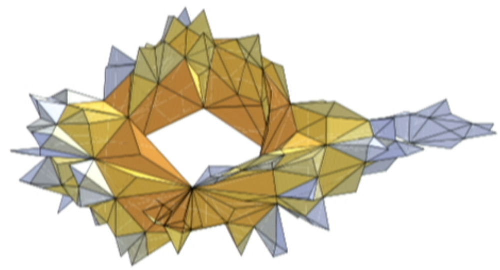
Two more CDT talks in parallel sessions:

Joshua Cooperman (UC Davis) - Tue, 5.40pm (→ pirsa.org)

Lisa Glaser (NBI, Copenhagen) - Thu, 3.50pm (space)

Key points of the CDT approach:

- Few ingredients/priors:
  - quantum superposition principle
  - locality and causal structure (*not* Euclidean quantum gravity)
  - notion of (proper) time
  - Wick rotation
  - standard tools of quantum field theory

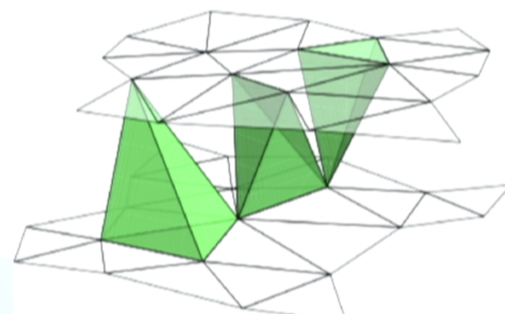


triangulated torus

- Few free parameters (Λ, G_N, Δ)
- Robustness of construction; universality
- At intermediate stage, approximate curved spacetimes by triangulations
- Crucial: nonperturb. computational tools to extract quantitative results

Key results:

- dynamical "emergence" of spacetime
- scale-dependent dimensionality (2 $\rightarrow$ 4)
- nontrivial phase structure
- second-order phase transition!



piece of causal triangulation

Nonperturbative gravitational path integral via CDT

PI becomes a "democratic", regularized sum over piecewise flat spacetimes:

$$Z(G_N,\Lambda) := \lim_{\substack{a\to 0\\ N\to\infty}} \sum_{\substack{\text{inequiv.}\\ \text{triangul.s}\\ T\in\mathcal{G}_{a,N}}} \frac{1}{C(T)}\, \mathrm{e}^{iS^{\mathrm{Regge}}_{\mathrm{G_N},\Lambda}[T]}$$

Regge action

Newton's constant

cosmol. constant

|Aut(T)|

edge length a = diff-invariant UV regulator

Each triangulation T represents a different curved spacetime, consisting of N simplices, which can be "Wick-rotated" to a Riemannian space.

IMPORTANT: the causal structure of CDT is essential! This does not work in Euclidean signature (DT) - no sensible classical limit (~mid-90s).

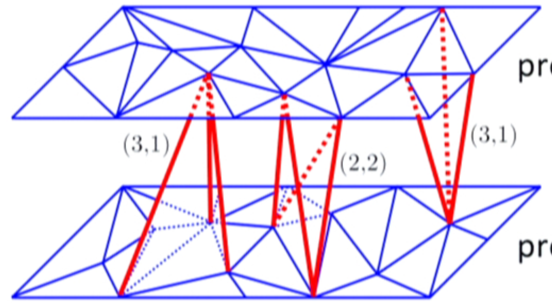
In CDT, ***"causality"*** is enforced through a preferred slicing by simplicial spatial hypermanifolds, with an associated preferred ***"proper time"***.

What is the role of the distinguished foliation?

Does the preferred time/ foliation affect the results?

Not a gauge choice (there are no coordinates).

Continuum interpretation of "*t*" on large scales only.



Previous work on relaxing the strict proper time foliation:
F. Markopoulou, L. Smolin, NPB 739 (2006) 120 [hep-th/0409057];
T. Konopka, PRD 73 (2006) 024023 [hep-th/0505004]

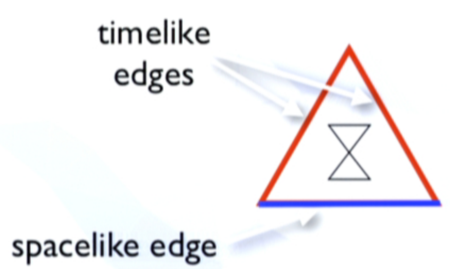
We will introduce a generalization of CDT quantum gravity, where the causal structure and the preferred time are dissociated (in fact, there will not be a preferred time).

NEW: CDT quantum gravity without *distinguished* foliation

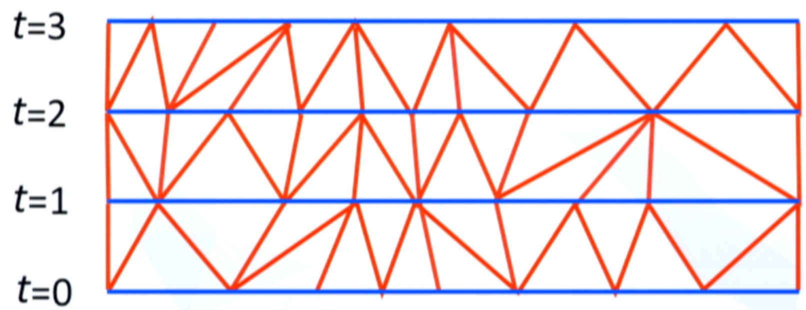
- standard path integral formulation needs a time t; propagator $G(g_{in},g_{out};t)$ satisfies

$$G(g_{in}, g_{out}; t) = \sum_g G(g_{in}, g; t_1) G(g, g_{out}; t_2), \quad t = t_1 + t_2$$

- *proper time* is a natural geometric choice; in standard CDT, slices of constant proper time t=0, 1, 2, 3, ... coincide with simplicial submanifolds, consisting of purely spatial (d-1)-simplices



building block of standard 1+1 CDT (with light cone)



building causal spacetimes from proper-time strips in standard CDT quantum gravity

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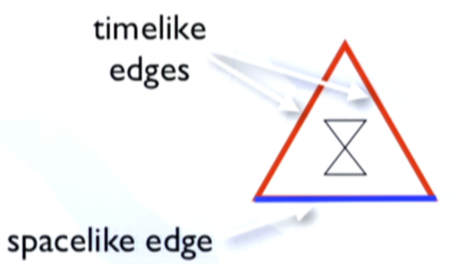

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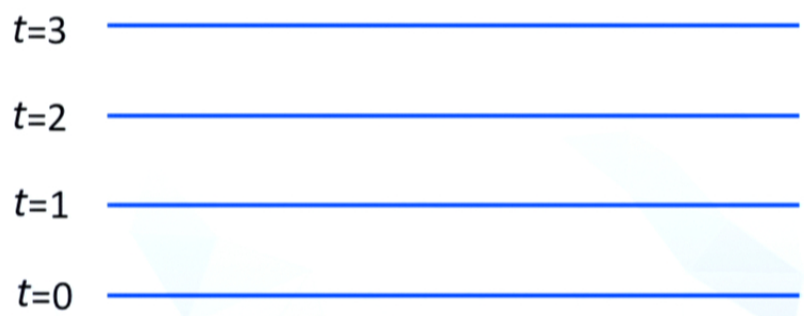
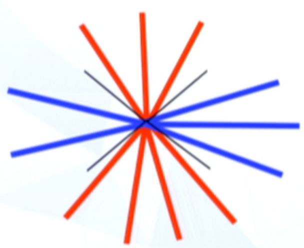


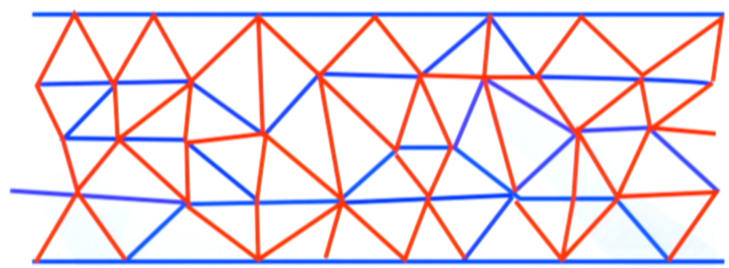
removing timelike links exhibits the strictly foliated structure of CDT quantum gravity

Relax the strict slicing while retaining causality

- introduce additional elementary building blocks, e.g. in 1+1 dimensions

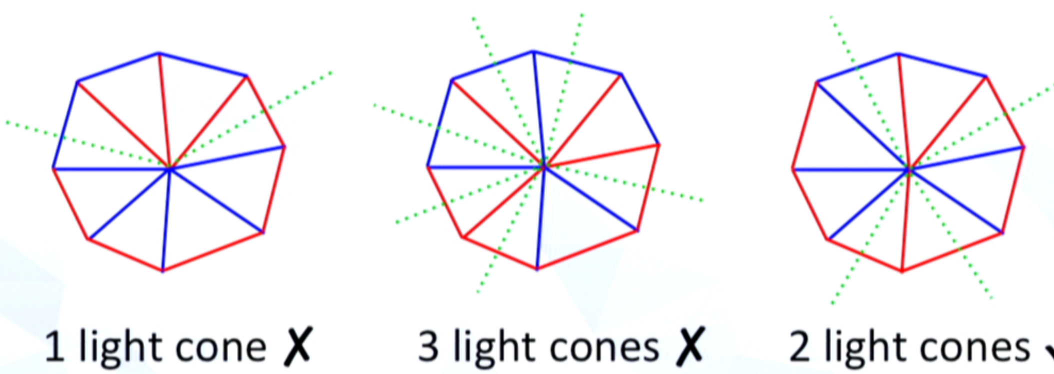
- impose local causality conditions at vertices:





building causal spacetimes in generalized CDT from these two building blocks

Examples of noncausal vertices in 2D



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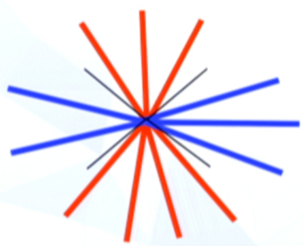

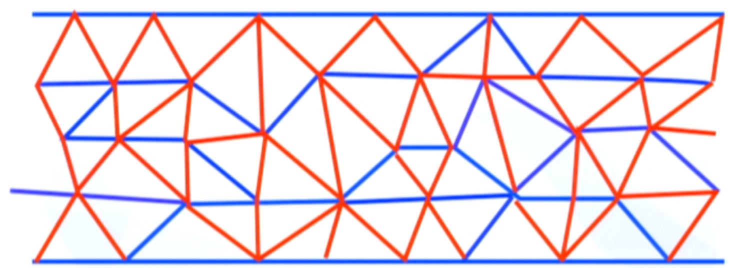

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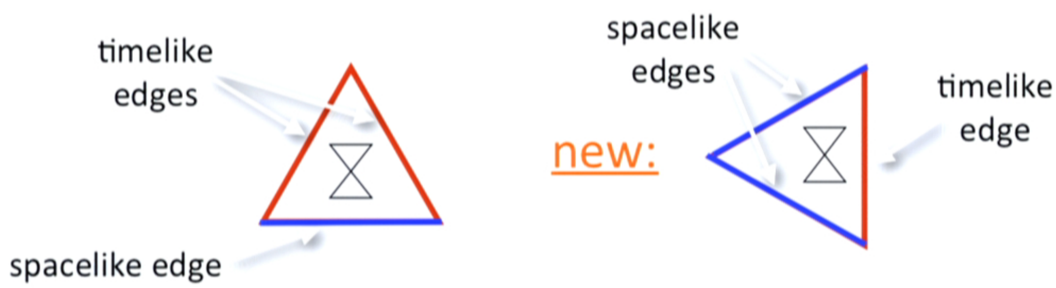

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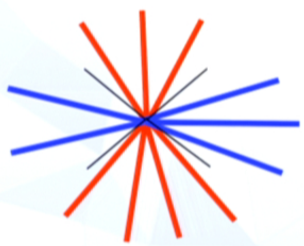
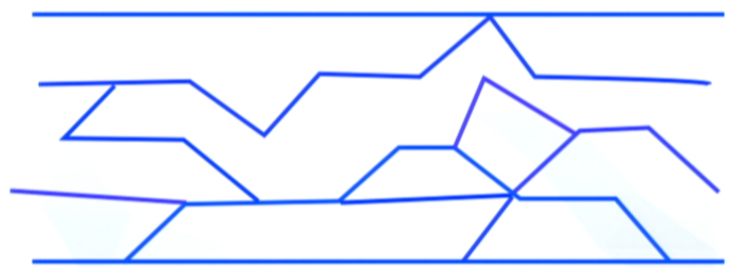




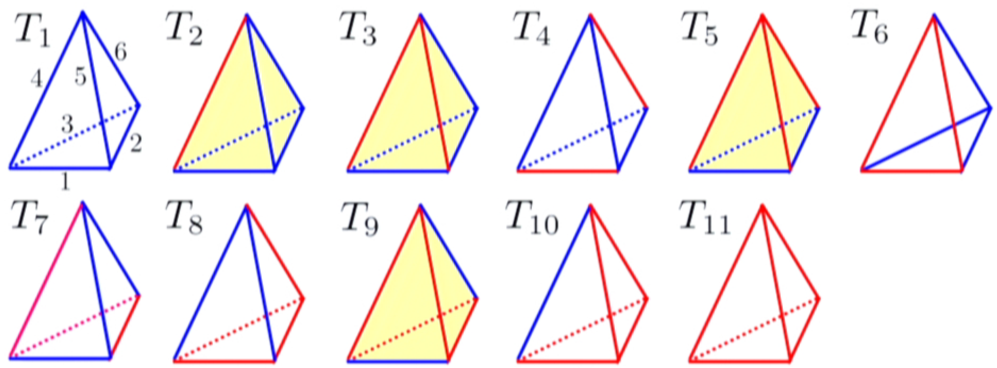
removing timelike links results in a structure with branches and bubbles

"Nonfoliated" CDT quantum gravity in 3D

- computational complexity already quite formidable in three dimensions:
  - implementation of Monte Carlo moves
  - long simulation times
- similar large-scale properties as in 4D of the dynamically generated quantum spacetime:
  - emergence of Euclidean de Sitter universe J. Ambjørn, J. Jurkiewicz, R.L., PRD 64 (2001) 044011 [hep-th/0011276]; D. Benedetti, J. Henson, PRD 80 (2009) 124036 [arXiv: 0911.0401]
- plenty of recent interesting work in 3D CDT C. Anderson, S. Carlip, J. Cooperman, P. Hořava, R. Kommu, P. Zulkowski, PRD 85 (2012) 044027 [arXiv: 1111.6634]; J. Cooperman, J. Miller, arXiv: 1305.2932; T. Budd, R.L., PRD 88 (2013) 024015 [arXiv: 1305.4702], ...

Selecting Minkowskian building blocks in 3D

all flat tetrahedra that can be built from just two edge lengths



spacelike links — length assignment $\ell_{space}^2 = a^2$

timelike links — length assignment $\ell_{time}^2 = -\alpha a^2$, $\alpha > 0$

tetrahedra marked in yellow are Minkowskian for all values of $\alpha > 0$

Kinematics of nonfoliated CDT in 3D

re-introducing time:

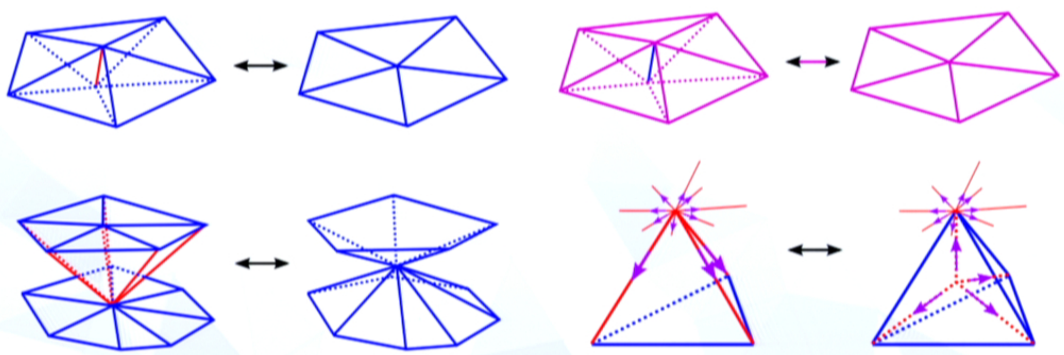
spacetime topology is $[0,1] \times S^2$, with the spatial slices contracted to points at 0 and 1, yielding effectively an S^3-topology

time coordinate of a vertex $v := d_{\uparrow}(v) - d_{\downarrow}(v)$

$d_{\uparrow}(v) \sim$ average length of future-oriented paths from v to “north pole”

$d_{\downarrow}(v) \sim$ average length of past-oriented paths from v to “south pole”

examples of new Monte Carlo moves:

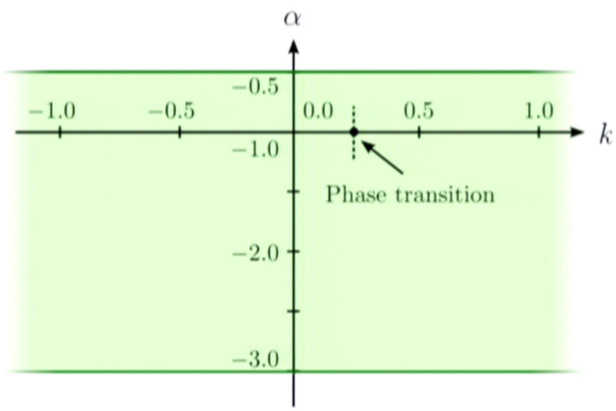


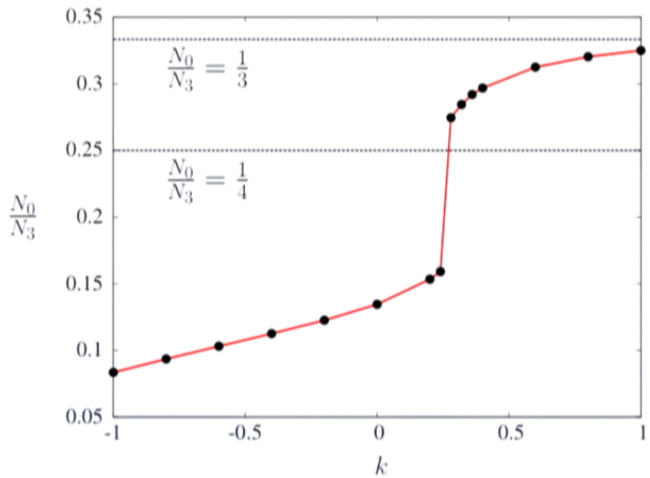
“bubble moves”

pinching move

“moving a pole”

Phase diagram of nonfoliated CDT





the existence of a Wick rotation requires that $-3 \leq \alpha \leq -0.5$

$k \sim$ inverse bare Newton constant

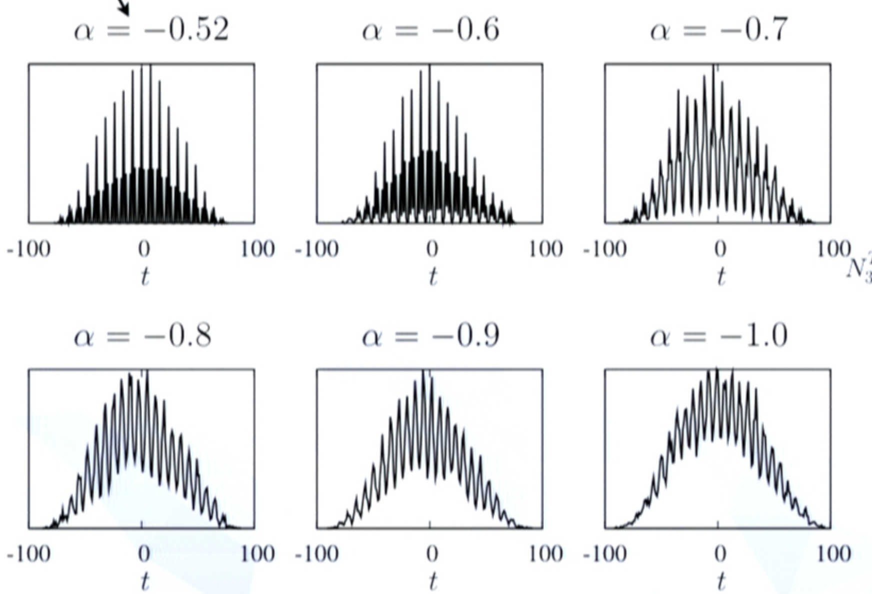
phase transition at $k^{crit} \approx 0.25$, measured along the "line of isotropy" $\alpha = -1$

Looking at the behaviour of the order parameter N_0/N_3, we find two phases, of low ($k < k^{crit}$) and high vertex density ($k > k^{crit}$). Interesting physics is found in the former.

What happens to the foliation (part 1)?

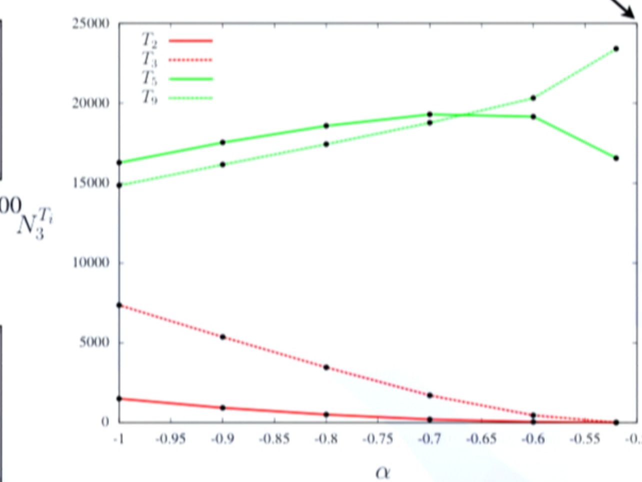
strict CDT foliation ⇔ #(T_2-tetrahedra) = #(T_3-tetrahedra) = 0

emerges near kinematical phase boundary α=-0.5



tetrahedron distributions as function of their time label (*k*=0)

kinematical phase boundary



multiplicity of tetrahedra of the various types, as function of α (*k*=0)

What happens to the foliation (part 2)?

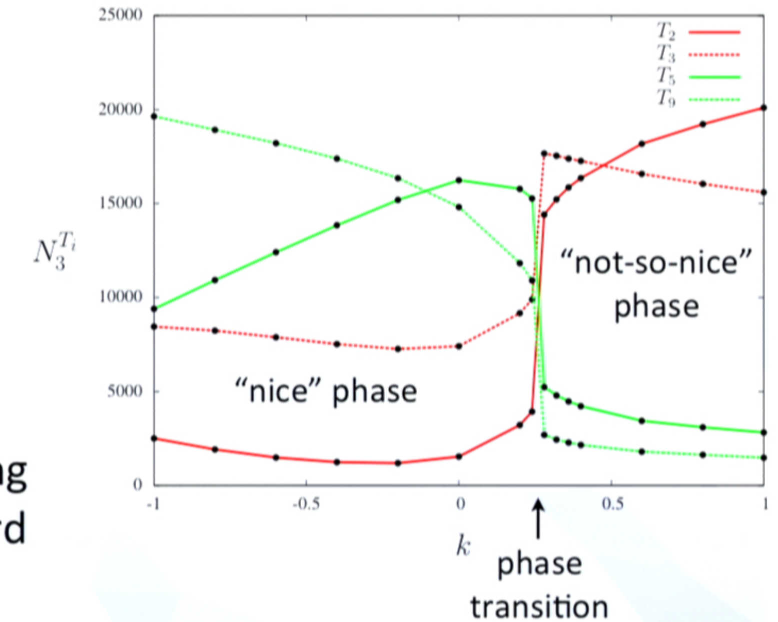
Depending on the location in the phase of low vertex density, the foliation of the simplicial geometries gets more and more diluted.

CDT tetrahedra

new tetrahedra

On the line α=-1, the alternating pattern weakens further toward lower k, but remains visible.

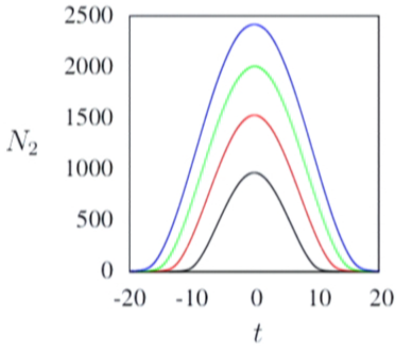
multiplicity of tetrahedra of the various types, as function of *k* (α=-1):

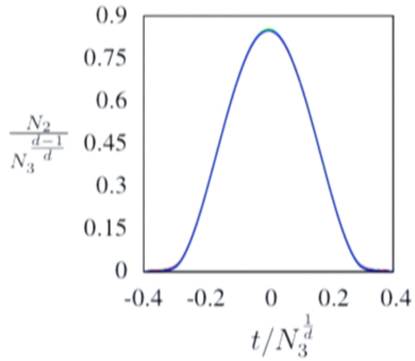


⇒ **in the investigated region k < k^{crit} the degree of foliatedness changes smoothly from strictly foliated to barely visible**

Phases of geometry in nonfoliated CDT

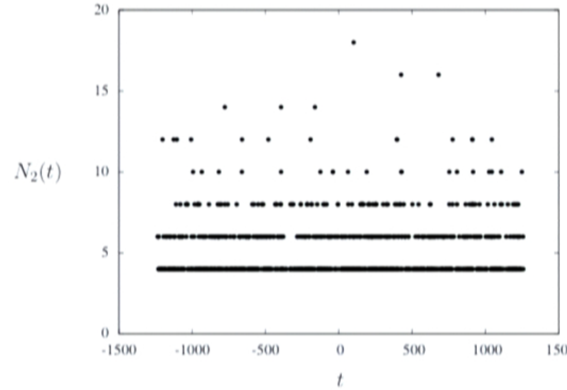
below the phase transition ($k < k^{crit}$):





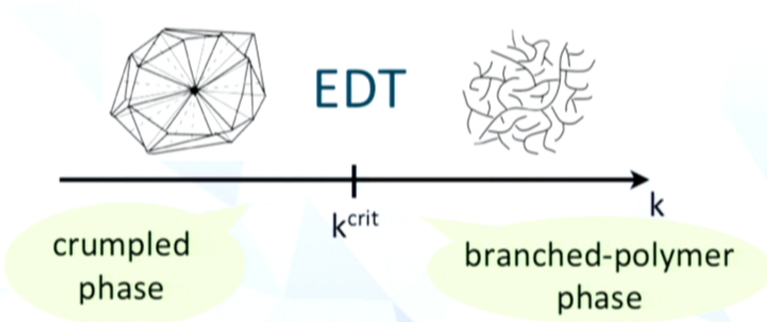
extended volume profiles $\langle N_2(t) \rangle$

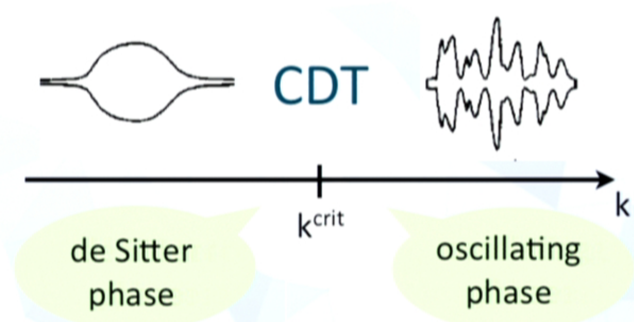
above the phase transition ($k > k^{crit}$):



one-dimensional tube

To be compared to:

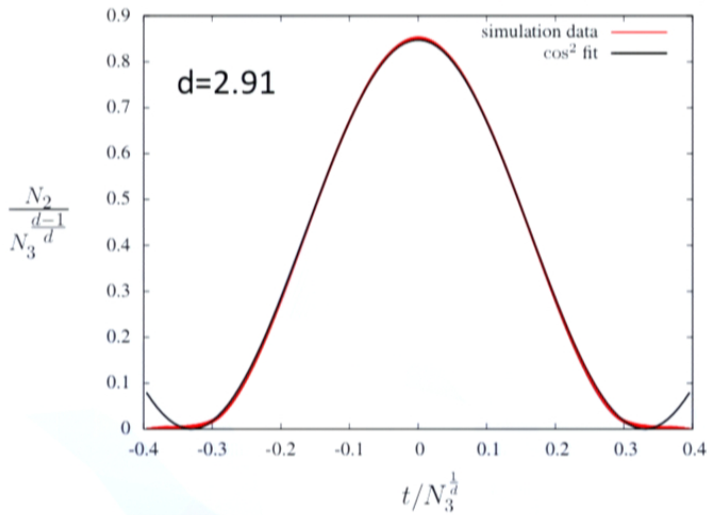




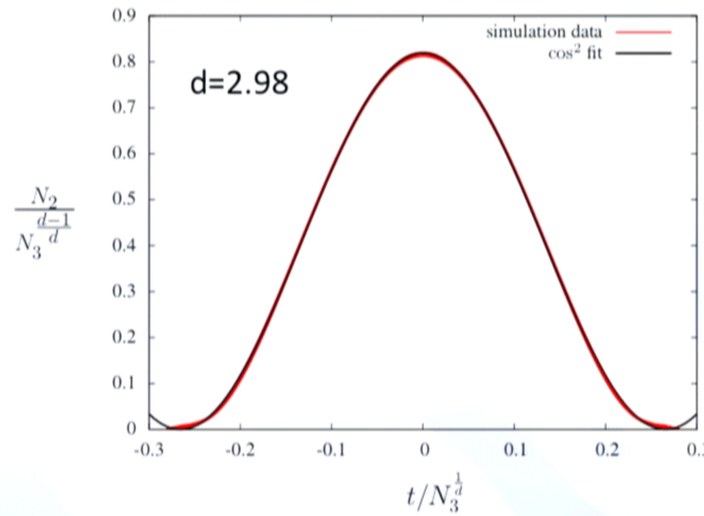
Recovering de Sitter space

Expectation value $\langle N_2(t)\rangle$ of the measured volume profiles, rescaled with best value for dimension *d* from finite-size scaling:

close to phase transition: $(k,\alpha)=(0.0,-1.0)$



further away from transition: $(k,\alpha)=(-0.8,-1.0)$



Perfect match to the volume profile of a three-dimensional Euclidean *de Sitter space*, as function of Euclidean proper time *t=iτ*, with scale factor $a(t)^2$ given by

$$ds^2 = dt^2 + a(t)^2 d\Omega^2_{(2)} = dt^2 + c^2 \cos^2\left(t/c\right) d\Omega^2_{(2)}$$

← volume el. S^2

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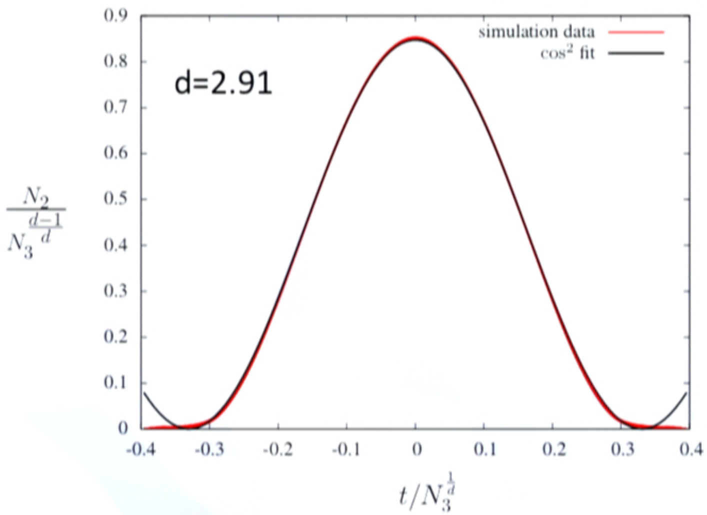

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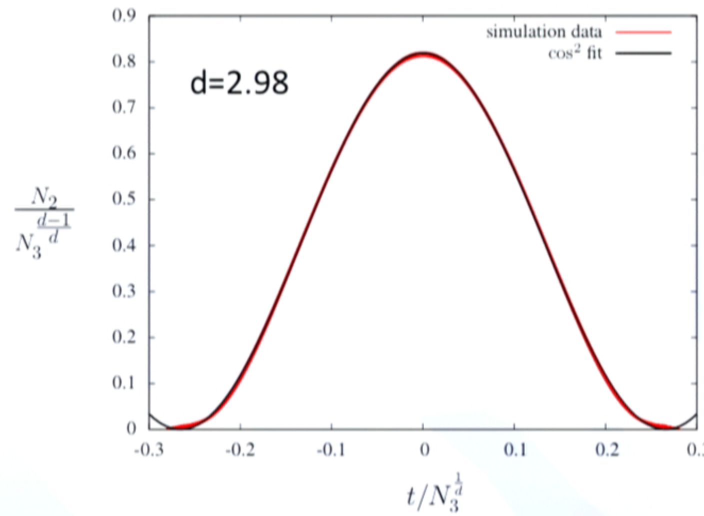

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Conclusion

- we got rid of the preferred time and associated foliation, while maintaining a well-behaved local causal structure
- the nonperturbative path integral has two phases (as usual in 3D)
- in the "nice", *physical* phase we found:
  - a range of degrees of "foliatedness" of simplicial geometry (strong to weak)
  - an excellent matching of volume profiles to a Euclidean de Sitter universe throughout
- this strongly suggests:
  - distinguished foliation is *not* an essential part of CDT's "background structure" (unlike in Hořava-Lifshitz gravity), and the two models are in the same universality class
  - because of the computational complexities, we may want to stick with the usual formulation of CDT quantum gravity ...

To learn more

about these results:

S. Jordan and R. Loll,
Phys. Lett. B 724 (2013) 155 [arXiv:1305.4582], and arXiv: 1307.5469

about CDT in general:

J. Ambjørn, A. Görlich, J. Jurkiewicz and R. Loll,
Physics Report 519 (2012) 127-212 [arXiv: 1203.3591]