Title: Spontaneous Dimensional Reduction?

Date: Jul 26, 2013 09:00 AM

URL: http://pirsa.org/13070070

Abstract: Several lines of evidence hint that quantum gravity at distances a bit larger than the Planck scale may become effectively two-dimensional. I will summarize the evidence for this "spontaneous dimensional reduction," and suggest a further argument based on the effect of vacuum fluctuations on light cones. If this description proves to be correct, it suggests an interesting relationship between small-scale quantum spacetime and the behavior of cosmologies near a spacelike singularity.

Spontaneous Dimensional Reduction?

Steve Carlip

U.C. Davis

Loops '13
Perimeter Institute, Waterloo, Canada
July 2013

Accumulating bits of evidence for "spontaneous dimensional reduction"

- Lattice approaches to path integral ("causal dynamical triangulations")
- Exact renormalization group analysis
- Strong coupling approximation to the Wheeler-DeWitt equation
- High temperature string theory
- A number of others . . .

Are these hints telling us something important?

Accumulating bits of evidence for "spontaneous dimensional reduction"

- Lattice approaches to path integral ("causal dynamical triangulations")
- Exact renormalization group analysis
- Strong coupling approximation to the Wheeler-DeWitt equation
- High temperature string theory
- A number of others . . .

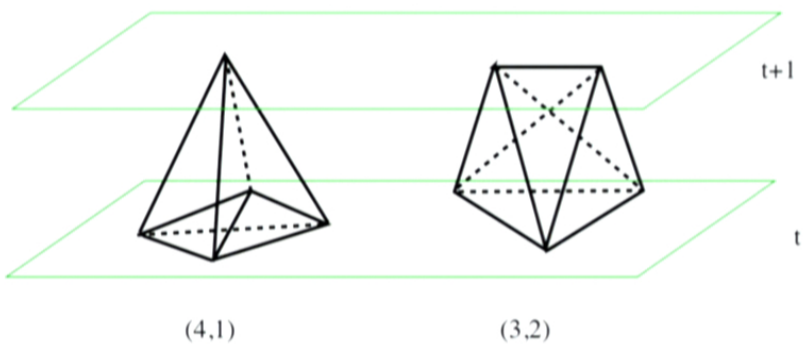
Are these hints telling us something important?

Causal dynamical triangulations

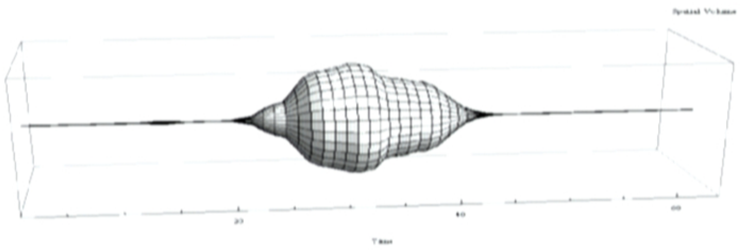
Approximate path integral by sum over discrete triangulated manifolds

$$\int [dg] e^{iI_{EH}[g]} \Rightarrow \sum e^{iI_{Regge}[\Delta]}$$

Fix causal structure ($\Rightarrow$ no topology change)



Nice "de Sitter" phase



- Volume profile fits (Euclidean) de Sitter
- Volume fluctuations fit quantum minisuperspace

But what about small scale structure?

How do you measure the "dimension" of a space that is not a nice manifold?

Spectral dimension d_S: dimension of spacetime seen by random walker

Basc idea: more dimensions $\Rightarrow$ slower diffusion

Heat kernel $K(x, x'; s)$: $\left(\frac{\partial}{\partial s} - \Delta_x\right) K(x, x'; s) = 0$

$$K(x, x'; s) \sim (4\pi s)^{-d_S/2} e^{-\sigma(x,x')/2s} (1 + \dots)$$

Ambjørn, Jurkiewicz, and Loll; Benedetti and Henson; Kommu:

- $d_S(\sigma \to \infty) = 4$,
- $d_S(\sigma \to 0) \approx 2$

Propagator $G(x, x') \sim \int_0^\infty ds\, K(x, x'; s) \sim \begin{cases} \sigma^{-1}(x, x') & \sigma \text{ large} \\ \log|\sigma(x, x')| & \sigma \text{ small} \end{cases}$

Short distances: characteristic behavior of a propagator in two dimensions

(Cooperman: physical scale for reduction $\sim 15\ell_p$)

Spectral dimension d_S: dimension of spacetime seen by random walker

Basc idea: more dimensions $\Rightarrow$ slower diffusion

Heat kernel $K(x, x'; s)$: $\left(\frac{\partial}{\partial s} - \Delta_x\right) K(x, x'; s) = 0$

$$K(x, x'; s) \sim (4\pi s)^{-d_S/2} e^{-\sigma(x,x')/2s} (1 + \dots)$$

Ambjørn, Jurkiewicz, and Loll; Benedetti and Henson; Kommu:

- $d_S(\sigma \to \infty) = 4$,
- $d_S(\sigma \to 0) \approx 2$

Propagator $G(x, x') \sim \int_0^\infty ds\, K(x, x'; s) \sim \begin{cases} \sigma^{-1}(x, x') & \sigma \text{ large} \\ \log|\sigma(x, x')| & \sigma \text{ small} \end{cases}$

Short distances: characteristic behavior of a propagator in two dimensions

(Cooperman: physical scale for reduction $\sim 15\ell_p$)

Spectral dimension d_S: dimension of spacetime seen by random walker

Basc idea: more dimensions $\Rightarrow$ slower diffusion

Heat kernel $K(x, x'; s)$: $\left(\frac{\partial}{\partial s} - \Delta_x\right) K(x, x'; s) = 0$

$$K(x, x'; s) \sim (4\pi s)^{-d_S/2} e^{-\sigma(x,x')/2s} (1 + \dots)$$

Ambjørn, Jurkiewicz, and Loll; Benedetti and Henson; Kommu:

- $d_S(\sigma \to \infty) = 4$,
- $d_S(\sigma \to 0) \approx 2$

Propagator $G(x, x') \sim \int_0^\infty ds\, K(x, x'; s) \sim \begin{cases} \sigma^{-1}(x, x') & \sigma \text{ large} \\ \log|\sigma(x, x')| & \sigma \text{ small} \end{cases}$

Short distances: characteristic behavior of a propagator in two dimensions

(Cooperman: physical scale for reduction $\sim 15\ell_p$)

Spectral dimension d_S: dimension of spacetime seen by random walker

Basc idea: more dimensions $\Rightarrow$ slower diffusion

Heat kernel $K(x, x'; s)$: $\left(\frac{\partial}{\partial s} - \Delta_x\right) K(x, x'; s) = 0$

$$K(x, x'; s) \sim (4\pi s)^{-d_S/2} e^{-\sigma(x,x')/2s} (1 + \dots)$$

Ambjørn, Jurkiewicz, and Loll; Benedetti and Henson; Kommu:

- $d_S(\sigma \to \infty) = 4,$
- $d_S(\sigma \to 0) \approx 2$

Propagator $G(x, x') \sim \int_0^\infty ds\, K(x, x'; s) \sim \begin{cases} \sigma^{-1}(x, x') & \sigma \text{ large} \\ \log|\sigma(x, x')| & \sigma \text{ small} \end{cases}$

Short distances: characteristic behavior of a propagator in two dimensions

(Cooperman: physical scale for reduction $\sim 15\ell_p$)

Renormalization group

Lauscher, Reuter, Niedermaier, etc.:

Look at renormalization group flow for Einstein gravity plus higher derivative terms

- Truncate effective action
- Use exact renormalization group methods
- Find evidence for non-Gaussian fixed point, asymptotic safety (cf Saueressig's talk)

At fixed point:

- anomalous dimensions ⇔ two-dimensional field theory
- propagators $\sim \log|x - x'|$
- spectral dimension $d_S \sim 2$

General argument (Percacci and Perini):

If gravity has non-Gaussian UV fixed point,
propagator must behave as $\ln|x - x'|$

High temperature string theory (Atick&Witten)

At high temperatures, free energy of a gas of strings is

$$F/VT \sim T \sim \textit{free energy of a 2D QFT}$$

"... a lattice theory with a (1+1)-dimensional field theory on each lattice site" (1988)

Loop quantum gravity (Modesto)

Area spectrum $A \sim \ell_j^2$ for large areas, but $A \sim \ell_p \ell_j$ for small areas

Causal sets

Myrheim-Meyer dimension for a random causal set is ~ 2.38

Other hints

- Gas of Planck-scale virtual black holes (Crane, Smolin)?
- Multifractal geometry (Calgani)?
- Noncommutative geometry (Connes)?
- Anisotropic scaling (Hořava)?

High temperature string theory (Atick&Witten)

At high temperatures, free energy of a gas of strings is

$$F/VT \sim T \sim \textit{free energy of a 2D QFT}$$

"...a lattice theory with a (1+1)-dimensional field theory on each lattice site" (1988)

Loop quantum gravity (Modesto)

Area spectrum $A \sim \ell_j^2$ for large areas, but $A \sim \ell_p \ell_j$ for small areas

Causal sets

Myrheim-Meyer dimension for a random causal set is ~ 2.38

Other hints

- Gas of Planck-scale virtual black holes (Crane, Smolin)?
- Multifractal geometry (Calgani)?
- Noncommutative geometry (Connes)?
- Anisotropic scaling (Hořava)?

Short distance approximation

Wheeler-DeWitt equation:

$$\left\{16\pi\ell_p^2 G_{ijkl}\frac{\delta}{\delta g_{ij}}\frac{\delta}{\delta g_{kl}} - \frac{1}{16\pi\ell_p^2}\sqrt{g}\,{}^{(3)}R\right\}\Psi[g] = 0$$

"strong coupling" ($G \to \infty$) $\Leftrightarrow$ "small distance" ($\ell_p \to \infty$)

$\Leftrightarrow$ "ultralocal" (no spatial derivatives)

Classical solution:

- Kasner at each point if $\ell_p \to \infty$
- normally BKL/Mixmaster if ℓ_p large but finite

  (Kasner eras with bounces in which axes change)

Any signs of "dimensional reduction"?

Which dimensions are picked out?

Short distance approximation

Wheeler-DeWitt equation:

$$\left\{16\pi\ell_p^2 G_{ijkl}\frac{\delta}{\delta g_{ij}}\frac{\delta}{\delta g_{kl}} - \frac{1}{16\pi\ell_p^2}\sqrt{g}\,{}^{(3)}\!R\right\}\Psi[g] = 0$$

"strong coupling" ($G \to \infty$) ⇔ "small distance" ($\ell_p \to \infty$)

⇔ "ultralocal" (no spatial derivatives)

Classical solution:

- Kasner at each point if $\ell_p \to \infty$
- normally BKL/Mixmaster if ℓ_p large but finite

  (Kasner eras with bounces in which axes change)

Any signs of "dimensional reduction"?

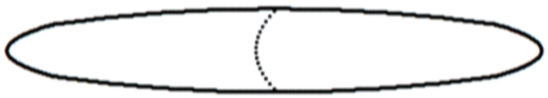
Which dimensions are picked out?

Kasner Space is effectively (1+1)-dimensional

$$ds^2 = dt^2 - t^{2p_1}dx^2 - t^{2p_2}dy^2 - t^{2p_3}dz^2$$

Start timelike geodesic at $t = t_0$, $\mathbf{x} = 0$ with random initial velocity

Look at proper distance along each axis:



Particle horizon shrinks to line as $t \to 0$

Geodesics explore a nearly one-dimensional space!

Various approximations of heat kernel (Futamase, Berkin):

$$K(x, x; s) \sim \frac{1}{(4\pi s)^2}(1 + Qs) \quad \text{with } Q \sim \frac{1}{t^2}$$

Small t: Q term dominates, $d_S \sim 2$

[Hu and O'Connor (1986): "effective infrared dimension"]

For BKL behavior, "preferred" dimension changes chaotically in space and time; known probability distributions

Asymptotic silence?

Cosmology near generic spacelike singularity:

- Asymptotic silence: light cones shrink to timelike lines
- Asymptotic locality: inhomogeneities fall outside shrinking horizons faster than they grow

⇒ "anti-Newtonian" limit (as if $c \to 0$)

⇒ spatial points decouple; BKL behavior

Underlying physics: extreme focusing near initial singularity

Is this also true at very short distances?

Mielczarek: asymptotic silence near critical density in loop quantum cosmology
(Barrau's talk Monday: $c_s \to 0$)

Pierce: shape of light cones in causal dynamical triangulations (in progress)

Page 10 of 15

Vacuum fluctuations and the Raychaudhuri equation

Expansion of a bundle of null geodesics: $\theta = \frac{1}{A}\frac{dA}{d\lambda}$

Raychaudhuri equation:

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma_a{}^b\sigma_b{}^a + \omega_{ab}\omega^{ab} - 16\pi G T_{ab}k^a k^b$$

Semiclassically:

- Expansion and shear focus geodesics
- Vorticity remains zero if it starts zero
- What about stress-energy tensor?

Fewster, Ford, and Roman:

Vacuum fluctuations of $T_{ab}k^a k^b$ are usually negative (defocusing)

But lower bound, long positive tail (focusing)

Vacuum fluctuations and the Raychaudhuri equation

Expansion of a bundle of null geodesics: $\theta = \frac{1}{A}\frac{dA}{d\lambda}$

Raychaudhuri equation:

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma_a{}^b\sigma_b{}^a + \omega_{ab}\omega^{ab} - 16\pi G T_{ab}k^a k^b$$

Semiclassically:

- Expansion and shear focus geodesics
- Vorticity remains zero if it starts zero
- What about stress-energy tensor?

Fewster, Ford, and Roman:

Vacuum fluctuations of $T_{ab}k^a k^b$ are usually negative (defocusing)

But lower bound, long positive tail (focusing)

– Frequent negative fluctuations will defocus geodesics, but their effect is limited
– Rare large positive fluctuations will strongly focus geodesics
– Once the focusing is strong enough, nonlinearities take over

"Gambler's ruin":

Whatever the odds, if you bet long enough against a House with unlimited resources, you always lose in the end.

Back-of-the envelope estimate:

Let $\min(T_{ab}k^a k^b) = -\mathcal{T}$
Let "smearing time" be Δt
Let ρ be the probability of a positive vacuum fluctuation with a value $> 2\mathcal{T}$

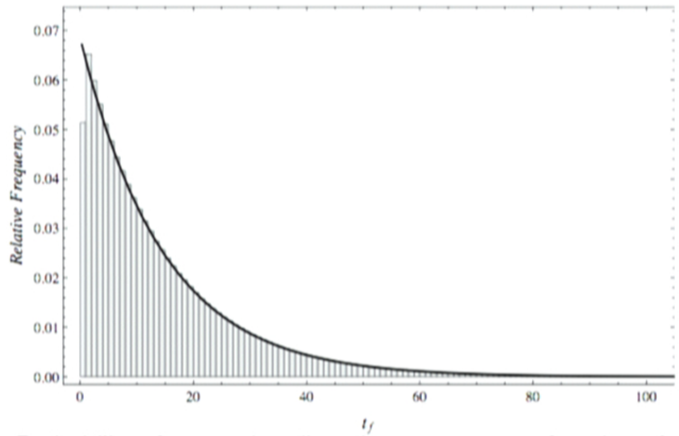
Then the time for θ to be driven to $-\infty$
is approximately described by an exponential distribution

$$\frac{\rho}{\Delta t} e^{-\rho t/\Delta t}$$

with a mean value $\sim 15.4\Delta t$

Simulation for dilaton gravity (Mosna, Pitelli, S.C.):

- Dimensionally reduce to two dimensions
- For matter: massless scalar field (central charge $c = 1$)
- Take $\Delta t = t_p$
- Assume fluctuations are independent (not quite right. . .)
- Run simulation 10 million times, measure time to $\theta \to -\infty$

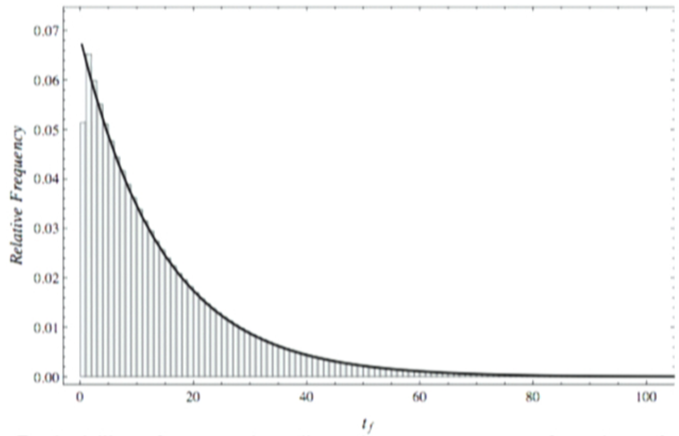


Probability of expansion diverging to $-\infty$ as a function of Planck time steps. Solid line is exponential distribution.

Full (3+1)-dimensional version in progress . . .

Simulation for dilaton gravity (Mosna, Pitelli, S.C.):

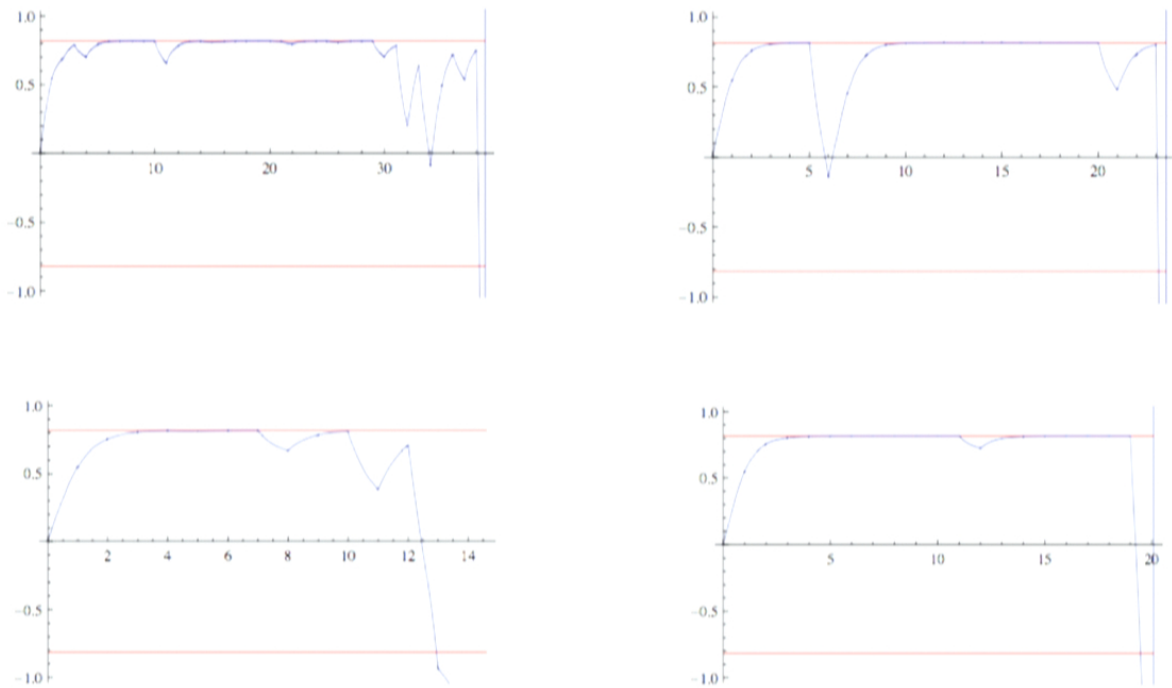
- Dimensionally reduce to two dimensions
- For matter: massless scalar field (central charge $c = 1$)
- Take $\Delta t = t_p$
- Assume fluctuations are independent (not quite right. . .)
- Run simulation 10 million times, measure time to $\theta \to -\infty$



Probability of expansion diverging to $-\infty$ as a function of Planck time steps. Solid line is exponential distribution.

Full (3+1)-dimensional version in progress . . .

Some typical runs



Short-distance picture (at perhaps $\sim 15\ell_P$):

- short distance asymptotic silence
- "random" direction at each point in space
  - not changing too rapidly in space: regions of size $\gg \ell_p$ fairly independent
  - evolving in time; "bouncing," axes rotating, etc.
- effective two-dimensional behavior:

  dynamics concentrated along preferred direction
- Lorentz violation near Planck scale, but "nonsystematic"

Can we use this?

- 't Hooft, Verlinde and Verlinde, Kabat and Ortiz: eikonal approximation

$$ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta + h_{ij}dy^i dy^j$$

with different natural scales for the two metrics

- Haba: lower dimensional gravity provides natural cutoff for field theory