Title: Asymptotic Dynamics: Spin Foam Partition Functions in an Asymptotic Regime

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Abstract: Spin foam models are models for space time built from discrete chunks of quantized geometry. In the asymptotic regime the classical geometry is regained.
In the last year we have seen rapid developments in our understanding of this geometry at the level of the entire partition function. In particular it was found that the geometries that contribute to the partition function in the asymptotic regime satisfy accidental curvature constraints.
I will discuss the classic results and role of asymptotics, the recent results and their impact on the interpretation of these models.

LOOPS 13

Spin foams

Asymptotic geometry of the partition function

July 25th 2013
Frank Hellmann

Spin foam models

are
- discrete (2-complex: vertices, edges and faces)
- with quantized geometry

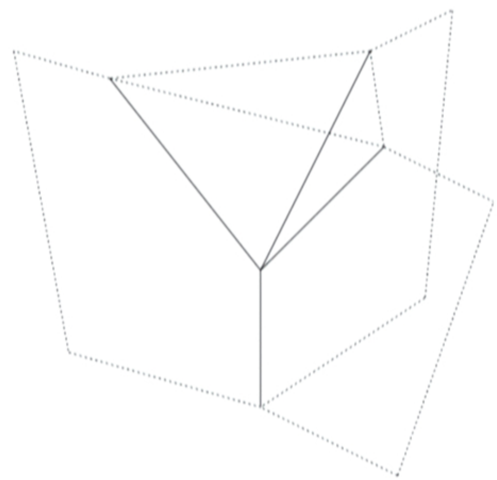
to provide covariant dynamics for network states.

Many contributors over the years:

Alesci, Alexandrov, Baez, Baratin, Barbieri, Barrett, Bahr, Bianchi, Bonzom Carrozza, Cianfrani, Conrady, Crane, DePietri, Dittrich, Dupuis, Ding, Engle, Fairbairn, Freidel, Gambini, Girelli Gomes, Hal, Han, Hnybida, Immirzi, Kaminski, Kisielowski, Krasnov, Lewandowski, Livine, Louapre Magliaro, Martin-Benito, Meusburger, Mikovic, Modesto, Noui, Oeckl, Oriti, Perez, Pereira, Perini, Pfeiffer, Pullin, Raasakka, Reisenberger, Riello, Roche, Rovelli, Ryan, Smerlak, Speziale, Steinhaus, Thiemann, Vidotto, Vojinovic, Wieland, Wilson-Ewing, Zapata, Zhang, Zipfel ...

Spin foam models

Space time picture
for operators on
graph Hilbert spaces
(Loop quantum gravity)



Feynman diagrams for
a type of non local
Interaction (Group field theory)

Lattice gauge gravity

Fiducial structure for
the construction of
topological quantum field theories

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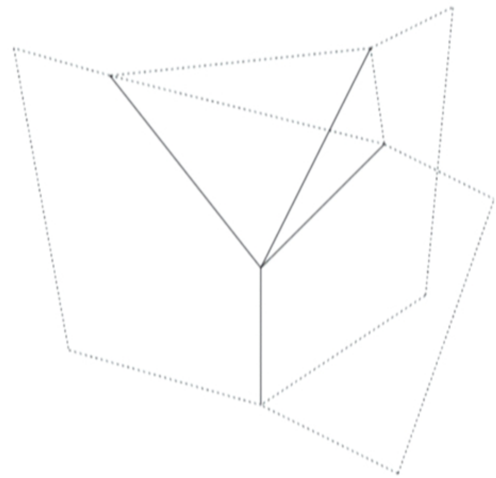

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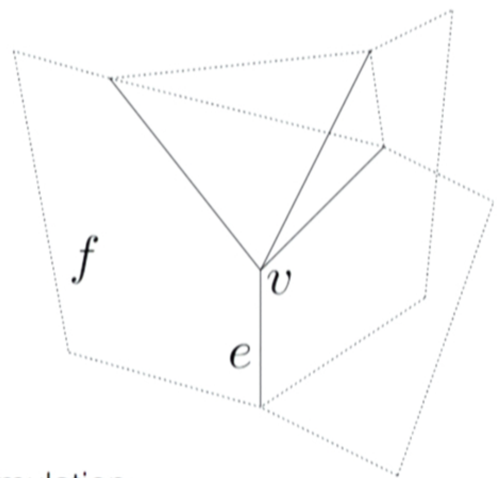
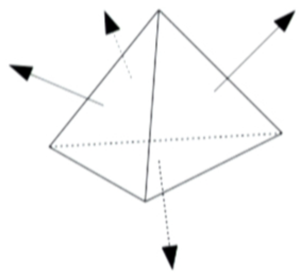


State sum formulation:

$$\mathcal{Z} = \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v(j_f, i_e)$$

Vertex asymptotics:

$$\mathcal{Z} = \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v(j_f, i_e)$$



Phase space of 3d geometry

Quantization ⇒

⇐ Asymptotics

$$i_e \in \mathrm{Inv}(j_f \otimes j_{f'} \otimes j_{f''} \otimes j_{f'''})$$

SU(2) representation theoretic data

E.g. coherent tetrahedral states ala Livine Speziale '07 $\psi_e(j_f, n_f)$

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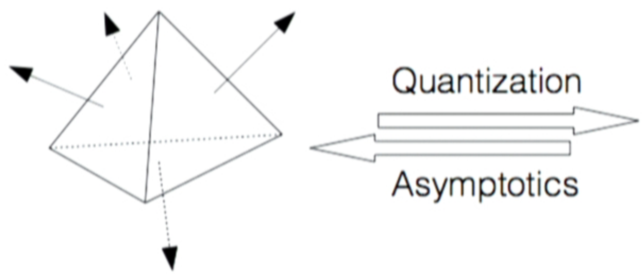

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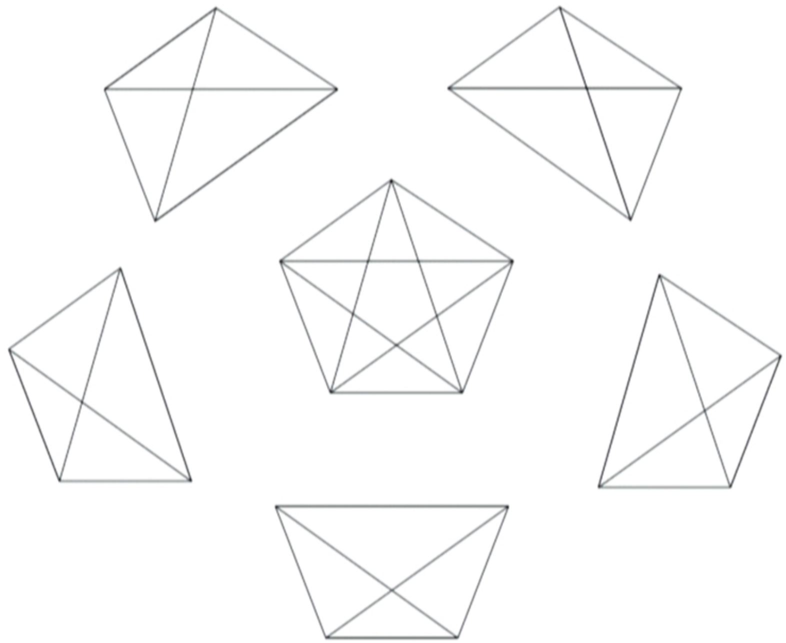

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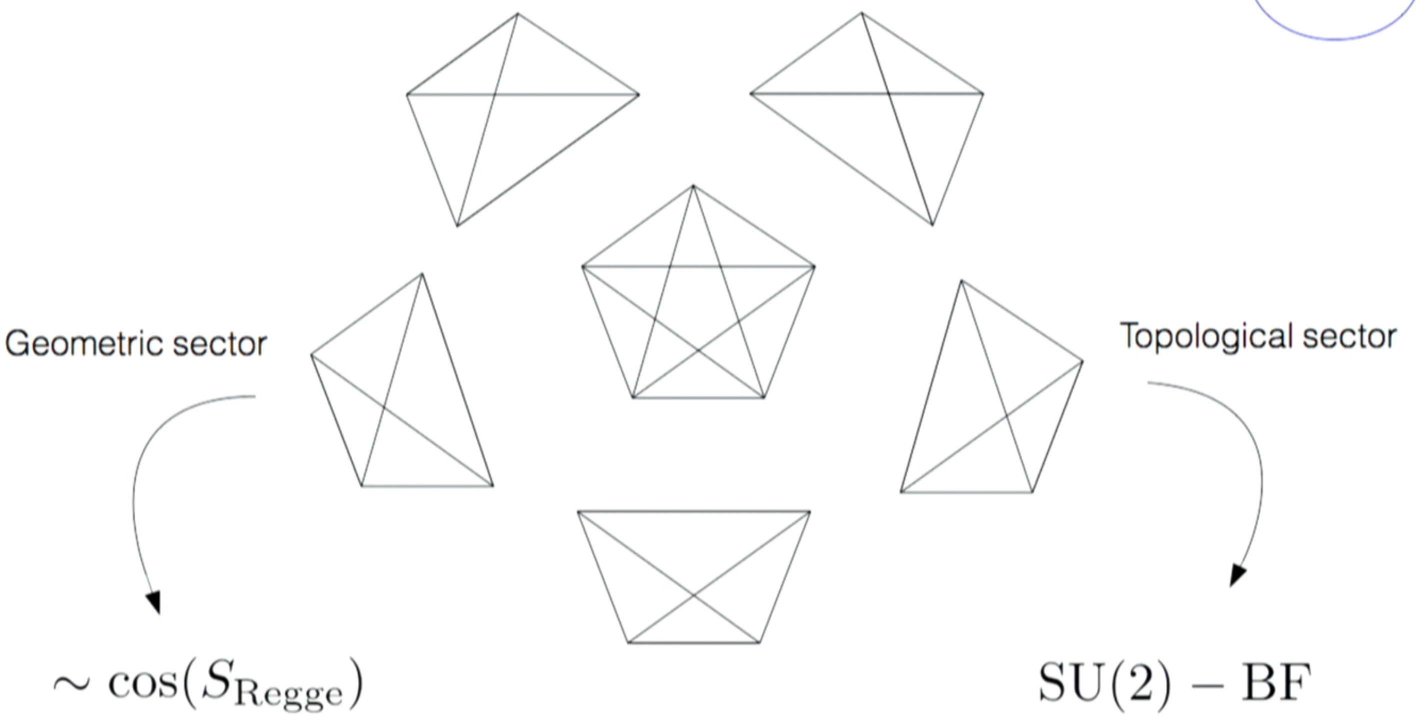

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Vertex	Asymptotics
Barrett and Crane	Barrett and Williams '99,Baez, Christiansen, Egan '02, Barrett and Steele '03, Freidel and Louapre '03, Kaminski, Steinhaus.
Engle, Pereira, Rovelli and Livine, Freidel and Krasnov	Conrady and Freidel '09, Barrett et.al. '09-'11, Han and Zhang '11

Work on the BF sector and the cosine:

Bahr, Dittrich, Engle, Livine, Oriti, Ryan, Speziale, and many others... Since '99 and ongoing...

More recent proposals for new vertex amplitudes: Dupuis and Livine, and Baratin and Oriti.

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Based on these were calculated observables on the single simplex
(and similar simple triangulations):

Propagator: Alesci, Bianchi, Ding, Magliaro, Modesto, Perini, Rovelli, Speziale

Dipole boundaries: Rovelli, Vidotto, Bianchi, FH, Kisielowski, Lewandowski, Puchta

Effective action: Mikovic, Vojinovic

Radiative corrections: Riello

This tests: The geometricity. The boundary action. The operators.

This does not test: The dynamics. Continuum physics.

It is unclear how to treat the sum over representation.

It is unclear how to study refinement.

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Ignore this for now...

Cause for concern: Bonzom '09

Flatness!?

Amenable to study through asymptotics

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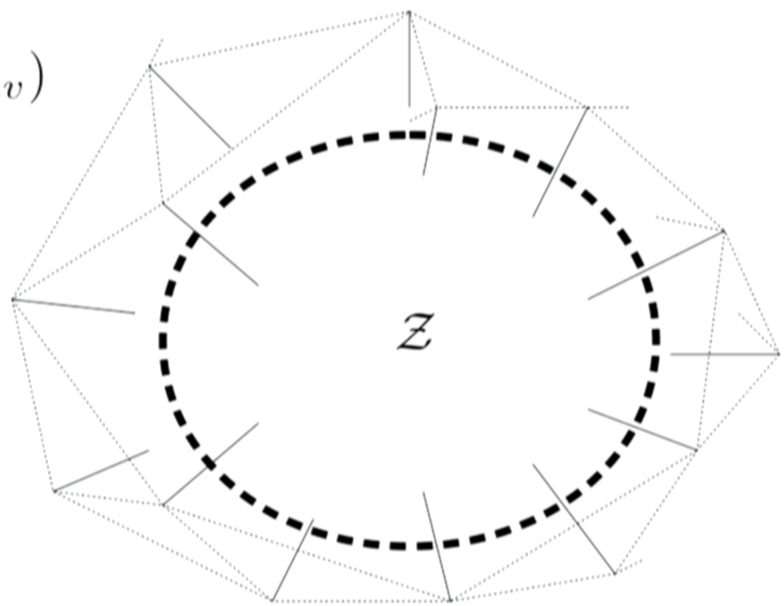
$\langle \psi(\lambda j_e, n_{ev}) | \mathcal{Z} \rangle$

$\psi(\lambda j_e, n_{ev})$



Asymptotic boundary state, no assumption on the interior.

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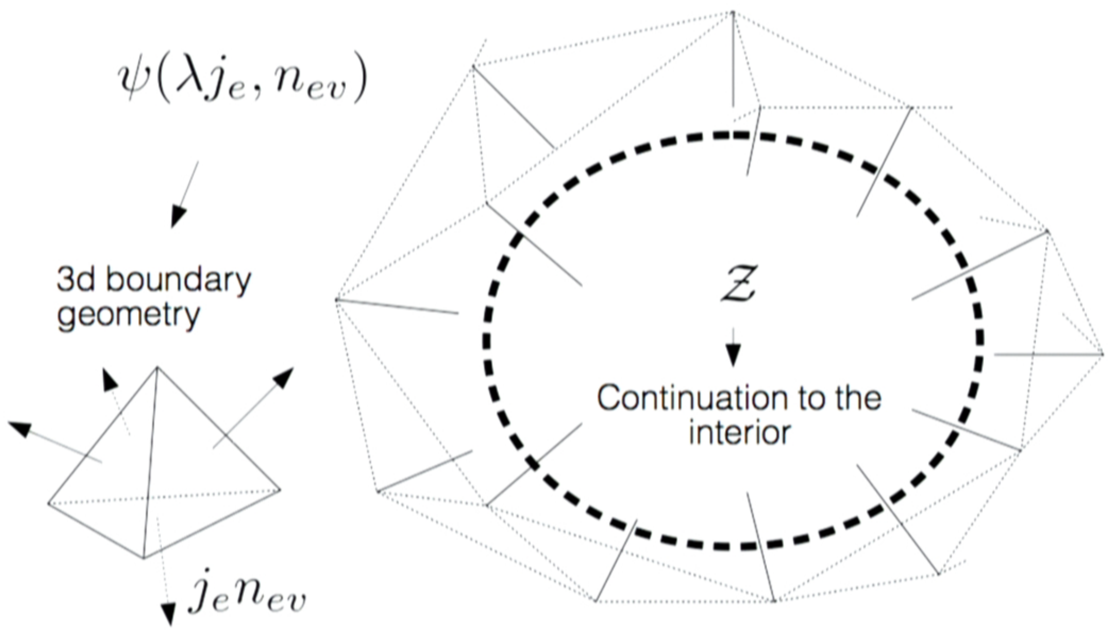

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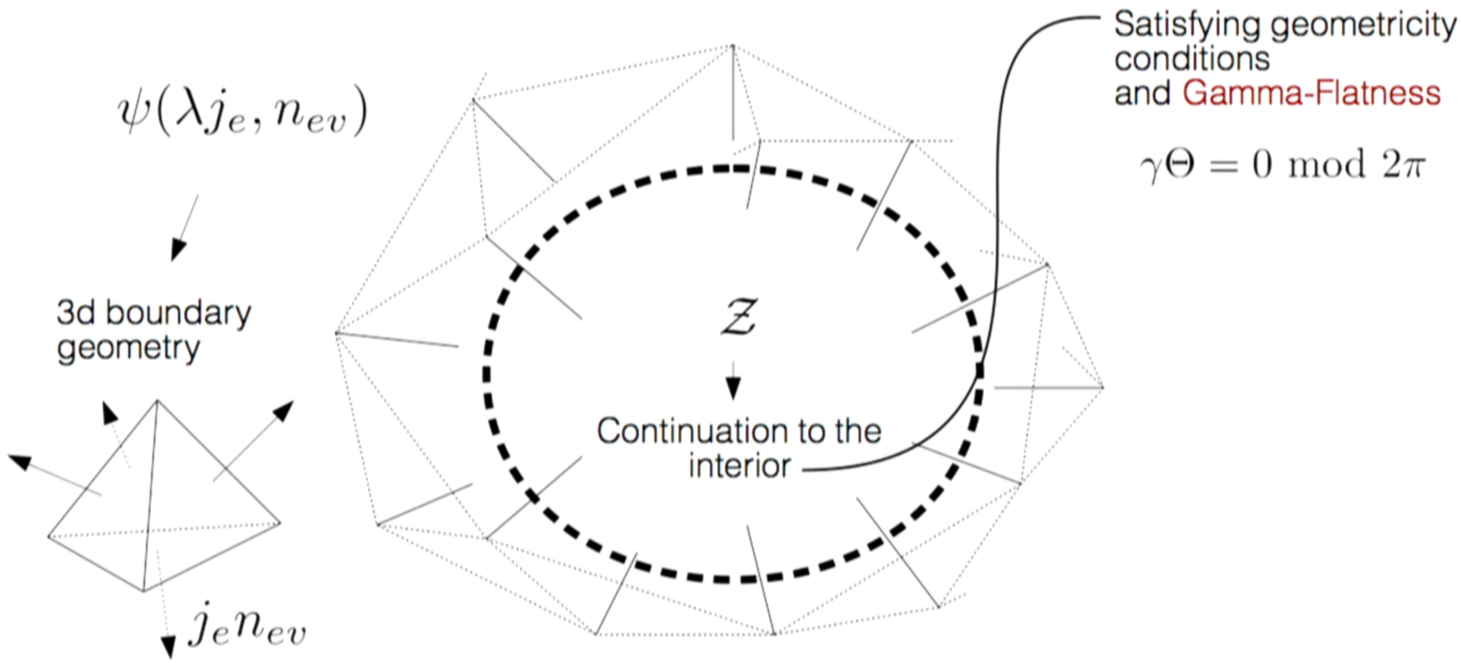

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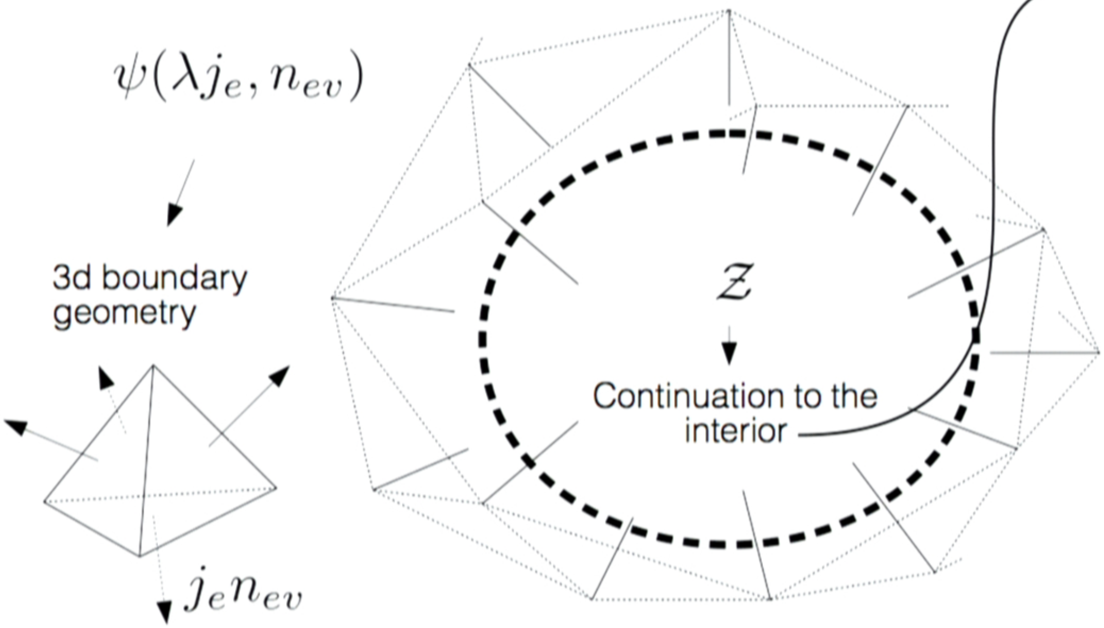

Satisfying geometricity conditions and Gamma-Flatness

$$\gamma\Theta = 0 \bmod 2\pi$$

Asymptotic boundary state, no assumption on the interior.

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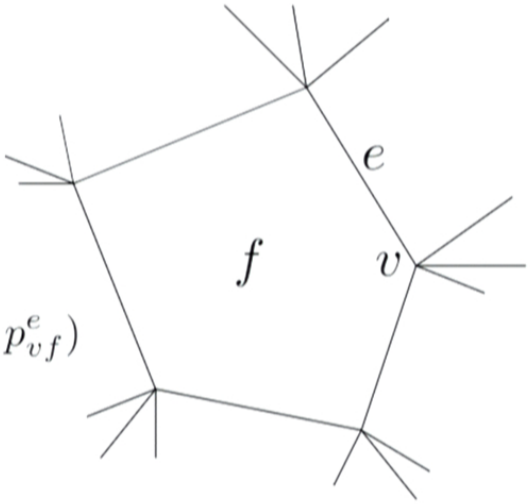
Theorem (FH,Kaminski):

Partition function exists if the interior continuation is bounded whenever it exists, is asymptotically suppressed if the boundary data allows no continuation.

Interior continuation is given by

$\{p^v_{ee'}, p^e_{fv}, g_{ev}, g_{ef}\}$

satisfying geometricity conditions:



$$p^v_{ee'} = -p^v_{e'e}$$
$$p^e_{fv} = -p^e_{fv'}$$
$$p^e_{fv} = g_{ev} p^v_{ee'}$$

$$N^0 \cdot (1 + \gamma\star) p^e_{fv} = 0$$
$$g_{ef} = \exp(\xi_{ef} \star p^e_{vf})$$
$$\sum_f p^e_{fv} = 0$$

$$g_{ev} \cdots g_{v''e} = g_{ef}$$

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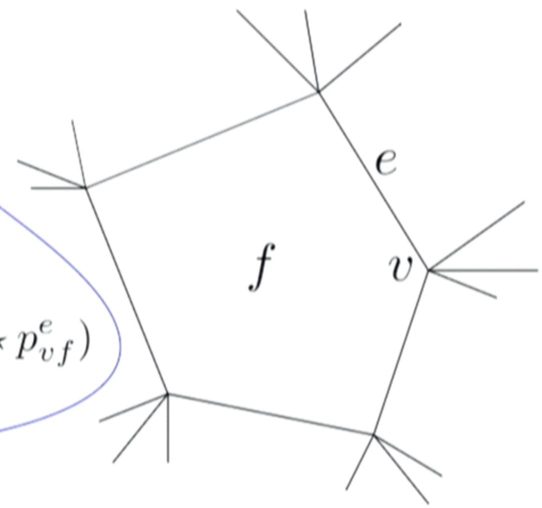

3d geometry

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Metric comptaible connection

4-simplex reconstruction

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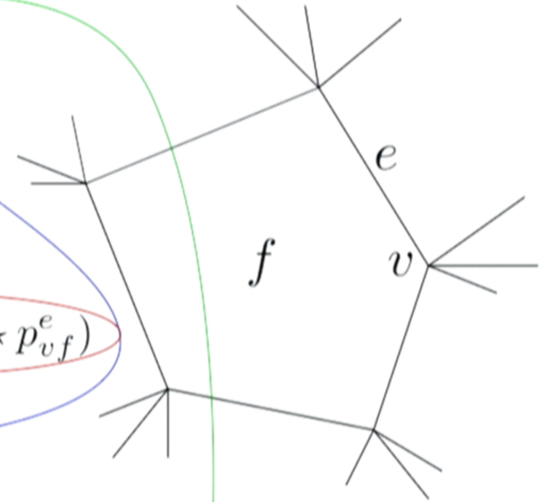

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$\gamma\Theta = 0 \mod 2\pi$

Metric compatible connection

4-simplex reconstruction

Iff? Extra equations? (Regge?)

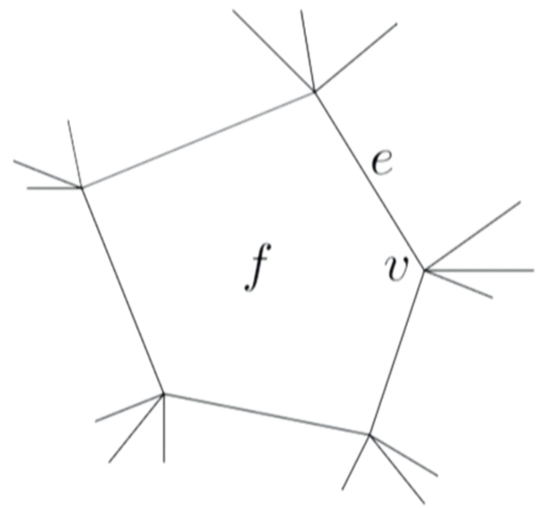
Holonomy spin foams. The dual formulation.

$\langle \psi(\lambda j_e, n_{ev}) | \mathcal{Z} \rangle$

Idea: Perform the spin sum explicitly, obtain formulation in terms of group elements.

$$\mathcal{Z} = \int \mathrm{d}g_{ef} \mathrm{d}g_{ev} \prod_{ef} E(g_{ef}) \prod_{f} \omega(\tilde{g}_f)$$

$$\tilde{g}_f = g_{ve} g_{ef} g_{ev'} \cdots g_{e'''v}$$



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In the spirit of Pfeiffer and Oeckls work.

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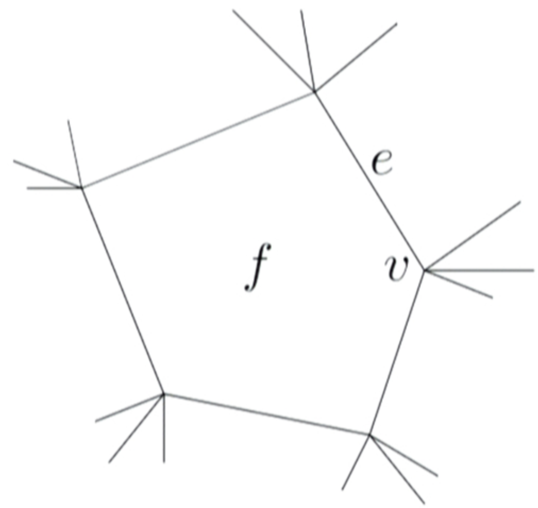
The important part lives here.

Measure factors live here.



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Method: Wave front sets

How does having functions on the group help with the large j limit? Take U(1):

$$f(e^{i\phi}) = \sum_n f^{(n)} e^{i\phi/n}$$

large representations ~ short wavelength on the group ~ local structure ~ smoothness

Wave front sets are a refinement of the notion of the singular support of a distribution to the cotangent bundle (phase space) of the space (configuration space, the group manifold) on which the distribution is defined. They probe the same regime as the large representation limit does.

This is why the holonomy formulation allows us to study the large j limit:

The large j limit is captured by the failure of distributions to be smooth.
The failure of smoothness propagates in a natural way under convolution.

(Really: large covector limit for a very general class of coherent states.)

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$E(g_{ef})$

Simplicity

$$N^0 \cdot (1 + \gamma\star) p^e_{fv} = 0$$

$$g_{ef} = \exp(\xi_{ef} \star p^e_{vf})$$

Lagrange multipliers

$$\int \mathrm{d}g A(g) B(g) C(g)$$

$$\sum_f p^e_{fv} = 0$$

Closure
(3d boundary geometry)

$$\omega(\ldots g_{ve} g_{ef} g_{ev'} \ldots)$$

$$p^v_{ee'} = -p^v_{e'e}$$

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Parallel transport
Orientation
(4d bulk geometry)

$\omega(\tilde{g}_f)$

$$\tilde{g}_f = 1$$

Fixing the Lagrange multipliers

Asymptotic dynamics:

Flatness:

First reported by Bonzom '11 (Path integral, no gamma)
FH, Kaminski '12, '13 (Wave front sets)
Perini '12 (Naive spin variation)
Han '13 (State sum, double expansion)
And Wieland '13 (Classical spinor networks)

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Interpreting spin foam models:

Spin foam partition function → Asymptotic geometricity

Integral over Regge manifolds → Continuum limit

Continuum general relativity

This does not work anymore. Most Regge manifolds do not appear in the state sum (e.g. 3-3 move), unclear how the Regge equations of motion should arise without curvature variations.

→ Modify the model. Different limit (gamma to zero).

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Re-Interpreting spin foam models:

Spin foam partition function

Asymptotic geometricity
+ add simplices

locally flat geometry
~ continuum limit

Need to study how flatness occurs to understand what macroscopic conditions (Einstein equations!) arise.

Barrett, Dittrich, Rovelli, Han

(Does not allow us to simply ignore the problematic stuff.)

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Re-Interpreting spin foam models:

Spin foam partition function → (Asymptotic geometricity + add simplices) → locally flat geometry ~ continuum limit

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What's next:

We have:

Sufficient condition for existence of the partition function.
Necessary condition for non-suppressed-ness of boundary data.

Can we have:

Necessary condition for existence? Degree of non-existence?

Recent works by Riello, Dittrich, Bonzom, as well as previous work on measure factors by Perez and Rovelli suggest we are far from optimal.

Relevance for GFT? Ala Riello? (Ben Geloun, Gurau, Rivasseau, et.al.)

Sufficient condition for non-suppressed-ness?

What equations of motion (if any) do our discrete geometries satisfy?

Relevance for coarse graining? (Dittrich, Martin-Benito, Steinhaus, et.al.)

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