Title: Spinfoam Formulation of Loop Quantum Gravity

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Abstract: Recently there are a lot of progresses in developing the spinfoam formulation of loop quantum gravity. In this talk I give an overview of the subject. I introduce the formalism and the motivation of the theory, and I discuss the application of spinfoam formulation in black hole and cosmology. I also discuss the inclusion of the quantum matter fields and cosmological constant in the formalism. The inclusion of cosmological constant motivates a Chern-Simons formulation of LQG. Finally I discuss the semiclassical low-energy approximation of the spinfoam formulation, where Einstein gravity appears as the leading contribution.

Spin Foam Formulation of LQG :

Questions & Answers

Muxin Han

Centre de Physique Théorique, Marseille

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Covariant Path Integral Formulation of QG:

$$Z[h_{ab}^{in}, h_{cd}^{out}]$$

$$= \int_{h_{ab}^{in}}^{h_{ab}^{out}} \mathcal{D}g_{\alpha\beta} \; e^{\frac{i}{\ell_P^2}\int_M d^Dx\sqrt{-g}\, R + \cdots}$$

high curvature corrections, boundary terms

[Misner '57, Hartle, Hawking '83]

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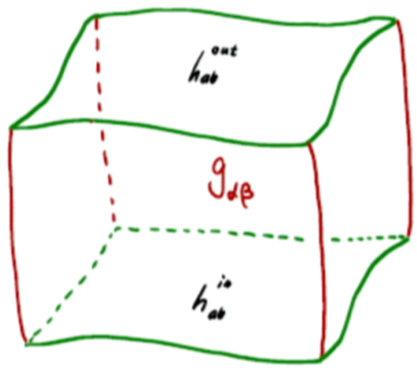
Summing over histories of 3-geometries :



Σ^{out} : (D-1)-dim

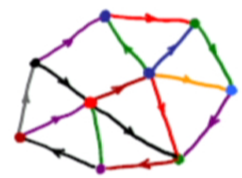
M : D-dim

Σ^{in} : (D-1)-dim

[Misner '57, Hartle, Hawking '83]

Adapt into LQG Framework:

Quantum 3-geometry = Spin-network state (Γ, j_l, i_n)



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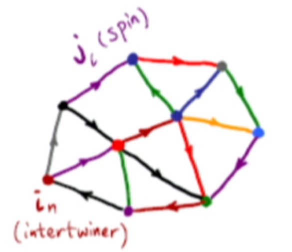

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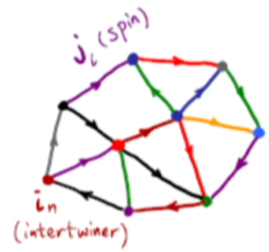

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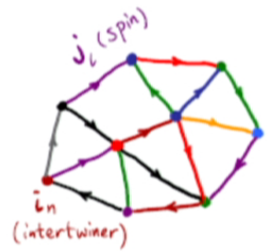

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LQG Hilbert space: $\mathcal{H} = \overline{\bigcup_\Gamma \mathcal{H}_\Gamma / \sim}$

$$\mathcal{H}_\Gamma = L^2\left(SU(2)^{\#\,\mathrm{link}} / SU(2)^{\#\,\mathrm{node}}\right)$$

[Rovelli, Smolin, Ashtekar, Lewandowski, Thiemann, Sahlmann, Okolow, Fleischhack, Marolf, Mourão, ⋯]

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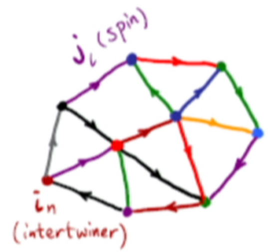

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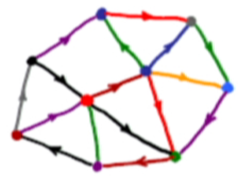
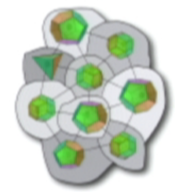
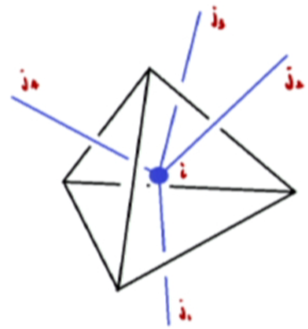
Quantum 3-geometry



$\sim$





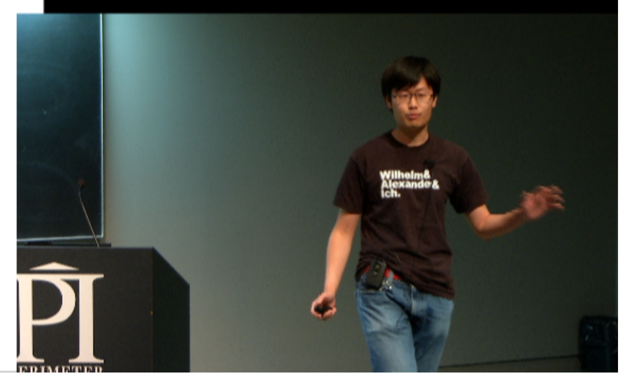
Nodes : quanta of space volume (quantum number i_n)

Links : quanta of area (quantum number j_l)

Spectra of area and volume are discrete

e.g. $\quad \text{Area}(S) = 8\pi\gamma\hbar G\sqrt{j(j+1)}$

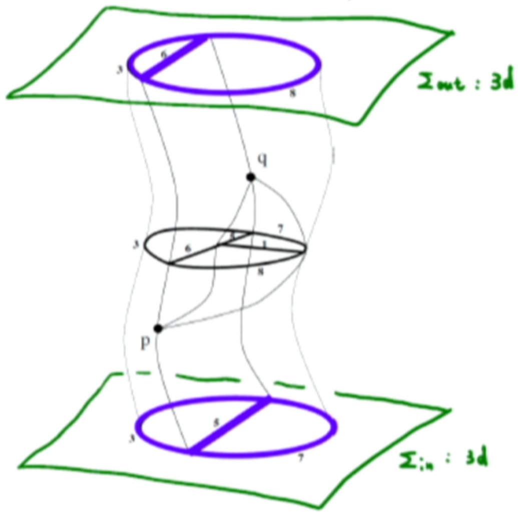
Barbero-Imirzi parameter



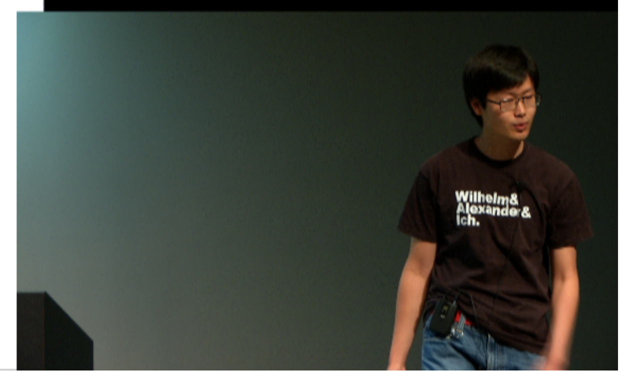
Summing over histories of 3-geometries :

histories of spin-networks = spinfoam

$(\Gamma_{out}, j_{out}, i_{out}) = S_{out}$



$(\Gamma_{in}, j_{in}, i_{in}) = S_{in}$



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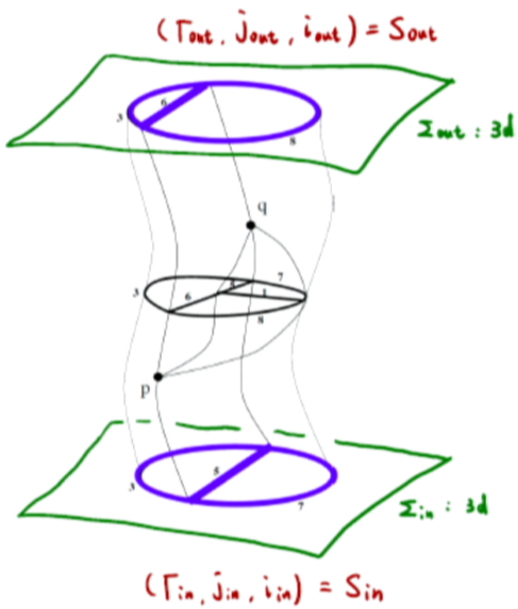

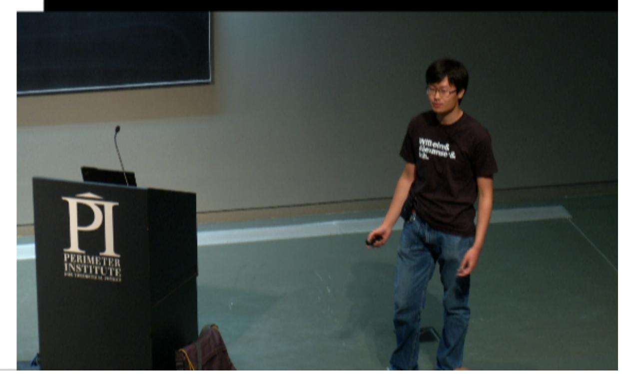

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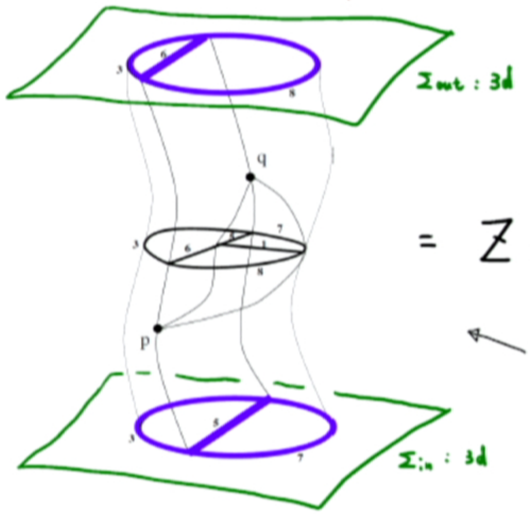


boundary 3-geometry

$$= Z(K ; S_{in}, S_{out})$$

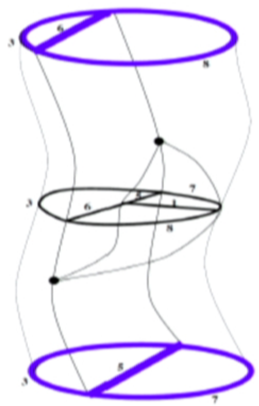
2-complex

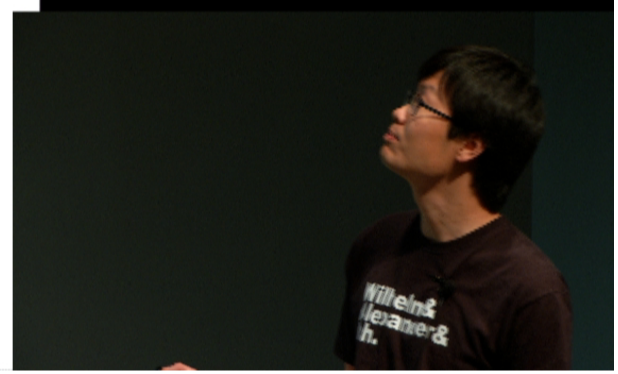
$(\Gamma_{in}, j_{in}, i_{in}) = S_{in}$

[Barrett, Crane, Freidel, Krasnov, Engle, Pereira, Rovelli, Livine, Speziale, Perez, Baez, Lewandowski, Kaminski, Kisielowski, Thiemann, Baratin, Flori, Oriti, Girelli, Dupuis, Hellmann, Dittrich, Bahr, Bianchi, Krajewski, Bonzom, Wieland, Alesci, Perini, Ding, Magliaro, Zhang, Fairbairn, Meusburger, Noui, Conrady, MH, ...]

Formal Definition: [Lewandowski, Kaminski, Kisielowski, 09]

A spin foam $= (\mathcal{K}, j_f, i_e)$





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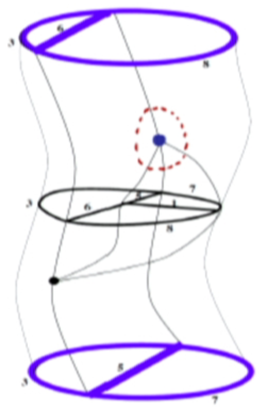
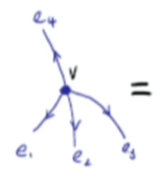
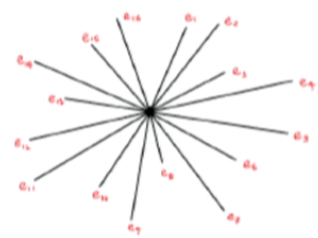




$= A_v(j_f,\ i_e)$

Vertex amplitude in general :



$$A_v(j_f,\ i_e) := \mathrm{Tr}\Big(\bigotimes_{\text{outgoing } e} I_e \ \bigotimes_{\text{incoming } e'} I_{e'}^{*} \Big)$$

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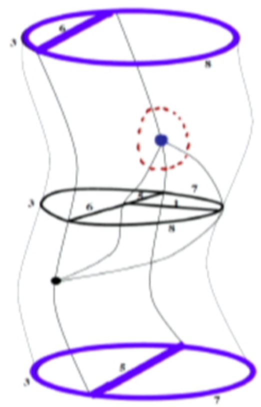
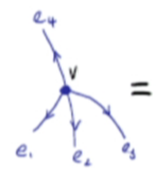




$= A_v(j_f, i_e)$

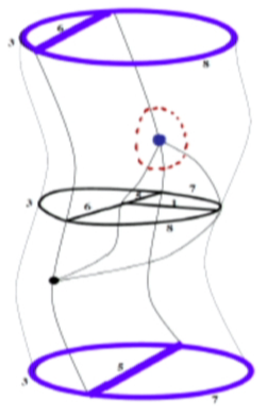
vertex amplitude

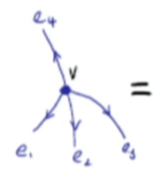
$$Z = \sum_{j_f, i_e} \prod_f w(j_f) \prod_v A_v(j_f, i_e)$$

Spinfoam amplitude

Formal Definition: [Lewandowski, Kaminski, Kisielowski, 09]

A spin foam $= (K, j_f, i_e)$





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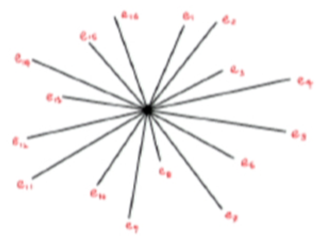

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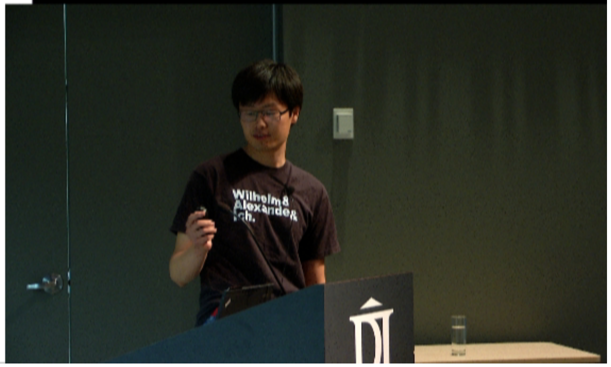
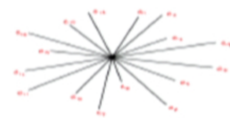


Vertex amplitude:



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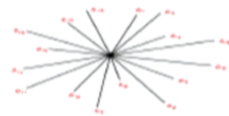

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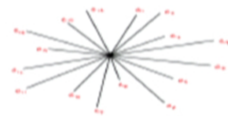
$SL(2, \mathbb{C})$ intertwiner (of spacetime)
evolved from i_e (of space)

$$Y: \begin{bmatrix} SU(2) \text{ unitary irrep} \\ j \end{bmatrix} \longrightarrow \begin{bmatrix} SL(2,\mathbb{C}) \text{ unitary irrep} \\ (p,m) = (\gamma j, j) \end{bmatrix}$$

$$|j, m\rangle \longmapsto |\gamma j, j; j, m\rangle$$

Vertex amplitude :



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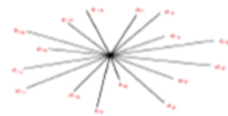

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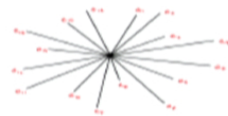
$$I_e = P^{\text{inv}}_{SL(2,\mathbb{C})} \circ Y(i_e)$$

SL(2,C) unitary irrep matrix $\downarrow$ $\swarrow$ SU(2) intertwiner

$$= \int_{SL(2,\mathbb{C})} \mathrm{d}g \bigotimes_{i=1}^{4} D^{2j_i, 2j_i\gamma}_{(j_i', m_i')(j_i, m_i)}(g)\ i^{m_1 \cdots m_4}$$

Vertex amplitude :



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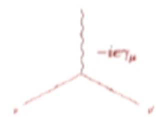
$A_v(j_f, i_e)$: EPRL/FK vertex amplitude.

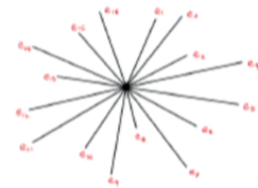
[Engle, Pereira, Rovelli, Livine, Freidel, Krasnov, Speziale]

QFT

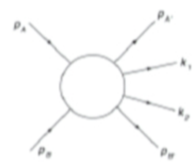
Spinfoam

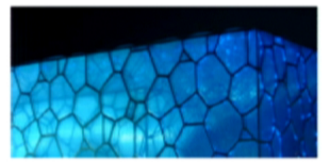
vertex:





amplitude:





Other Equivalent formulations of Z : (different factorizations)

Face amplitude & characters: [Bianchi, Rovelli, '10]

$$Z = \sum_{j_f} \int_{SL(2,\mathbb{C})} dg_{ev} \int_{SU(2)} dh_{ef} \underbrace{\prod_f \dim(j_f)\, \chi^{\gamma j_f, j_f}\Big(\prod_{e\in f} g^{*}_{ef}\Big) \prod_{e\in f} \chi^{j_f}(h_{ef})}_{\text{face amplitude}}$$

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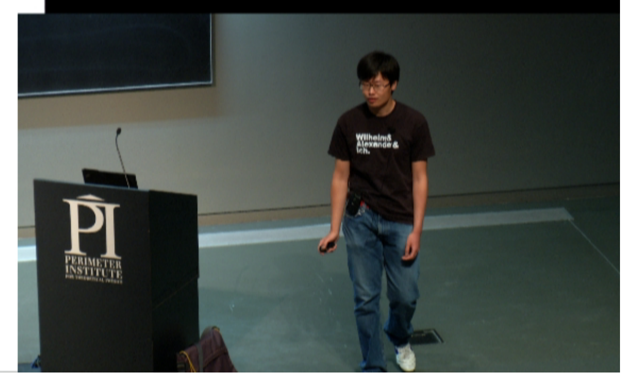

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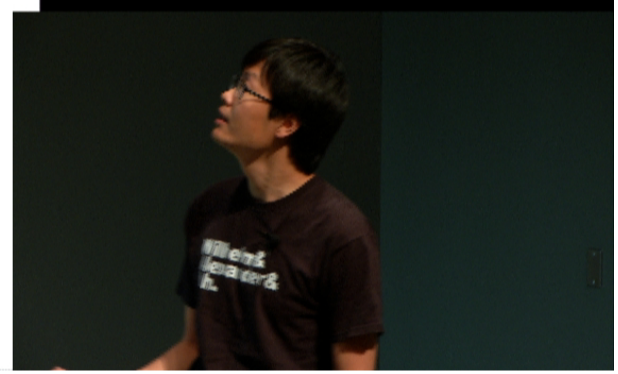

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- **Edge Projectors:** [Lewandowski, Kaminski, Kisielowski, Bahr, Hellmann '10]

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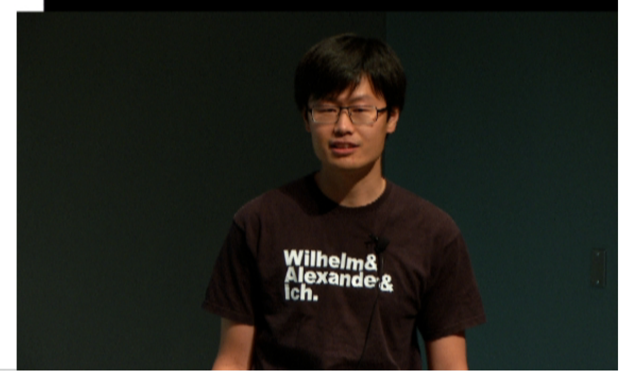

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- **Coherent State Path Integral** [Freidel, Conrady, Barrett, Hellmann, Fairbairn, Pereira, Gomes, Dowdall, Krajewski, MH, etc.]

$$Z = \sum_{j_f} \prod_f \dim(j_f) \int_{SL(2,\mathbb{C})} dg_{ev} \int_{\mathbb{C}P^1} dz_{vf}\ e^{S[j_f, g_{ve}, z_{vf}]}$$

(useful in semiclassical analysis)

Holonomy Spin Foam Model

[Bahr, Dittrich, Hellmann, Kaminski, Perini]

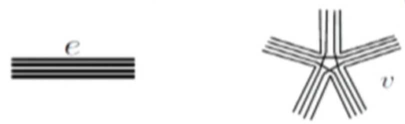
$$Z = \int_{Spin(4)} dg_{ev} \int_{Spin(4)} dg_{ef} \prod_{f} \omega(g_f) \prod_{e \subset f} E(g_{ef})$$

Distributions

(useful in semiclassical analysis & continuum limit, see Hellmann talk)

Group Field Theory

[Freidel, Rivasseau, Gurau, Oriti, Krajewski, Ryan, many others ...]



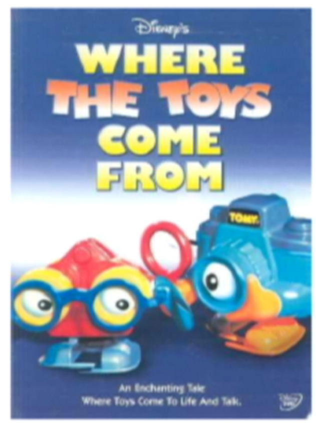
(see talks by Rivasseau, Gurau, Oriti, Krajewski, Ryan, etc)

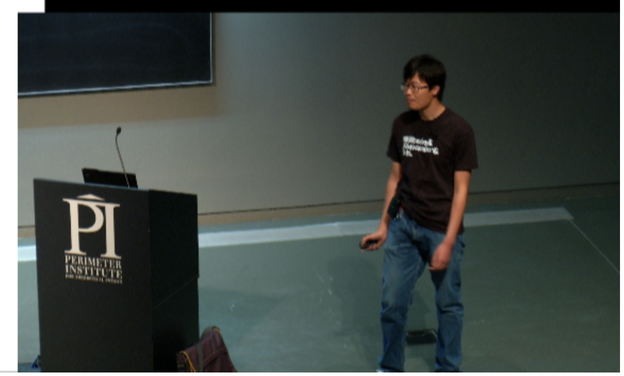
Spinor-Twistor Formulation

[Freidel, Speziale, Livine, Wieland, Dupuis, Tamborino]

(powerful in geometrical interpretations, See talks by Livine, Speziale, Wieland)

Q: Where does it come from ?

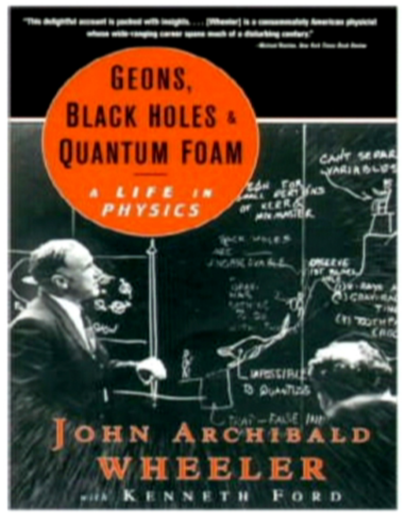




Wheeler & Hawking's Quantum Foam

J.A. Wheeler, *Geometrodynamics and the issue of the final state*, in Relativity, groups and topology, C. DeWitt and B.S. DeWitt eds., Gordon and Breach, New York U.S.A. (1964).

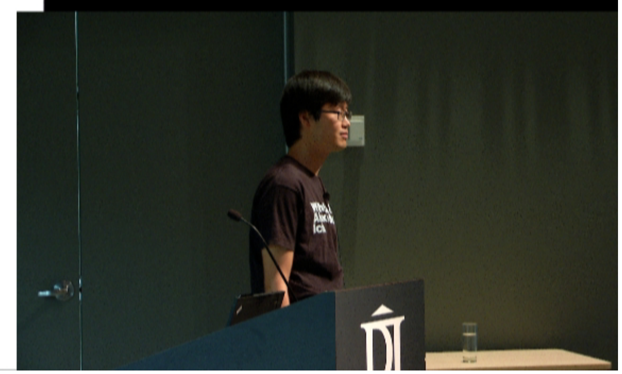
S.W. Hawking, *Space-time foam*, Nucl. Phys. B 144 (1978) 349.



Heisenberg Uncertainty Principle

$\Downarrow$

A Large amount of energy created in short distance $\sim \ell_p$



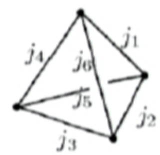
3d Quantum Gravity:

3d gravity

$$Z = \int \mathcal{D}e\,\mathcal{D}A\; e^{\,i\int_M \mathrm{Tr}\, e\wedge F}$$

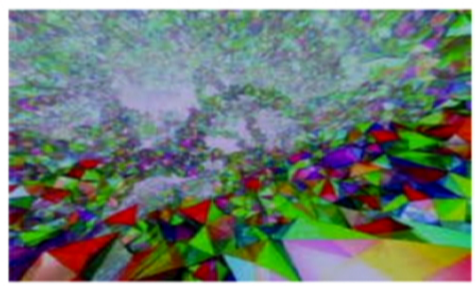
Discretization

$$Z_{\mathrm{PR}} = \sum_{j} \prod_{f} \dim(j_f) \prod_{v}$$



[Ponzano, Regge '68]

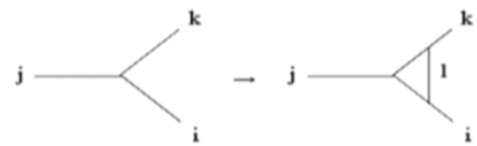
3d Quantum Spacetime made by tetrahedra



Canonical LQG

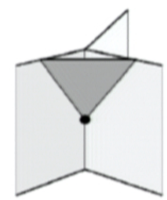
Thiemann 's Hamiltonian Constraint Operator : [Thiemann '97]

$\hat{H}(N)$:



Covariant picture : history of spin-network

$$\langle s, \int DN\, e^{i\hat{H}(N)}\, s' \rangle \quad \sim$$



Physical inner product

[Reisenberger, Rovelli, '97]

$$\boxed{\text{Spinfoam Amplitude} = \text{LQG Physical Inner Product}}$$

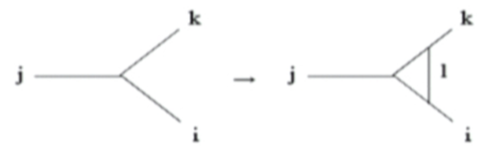
Recent new progresses :

[Alesci, Thiemann, Zipfel, Liegener, '11 - 13]

Canonical LQG

Thiemann's Hamiltonian Constraint Operator: [Thiemann '97]

$\hat{H}(N)$:



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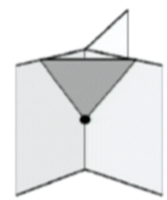


↑ Physical inner product

[Reisenberger, Rovelli, '97]

$$\boxed{\text{Spinfoam Amplitude} = \text{LQG Physical Inner Product}}$$

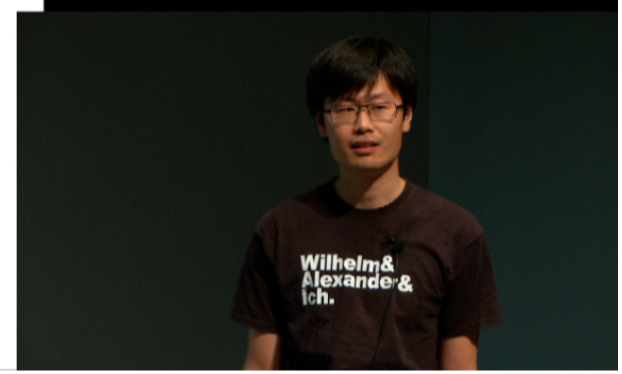
Recent new progresses:

[Alesci, Thiemann, Zipfel, Liegener, '11 -13]

Covariant Path Integral

Plebanski Gravity 4d

$$Z = \int \mathcal{D}B\,\mathcal{D}A\,\mathcal{D}\varphi \quad e^{i\int B\wedge F + \varphi\cdot B\wedge B}$$



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$$Z = \int \mathcal{D}B\, \mathcal{D}A\, \mathcal{D}\varphi \quad e^{i\int B\wedge F + \varphi \cdot B\wedge B}$$

$$= \int \mathcal{D}B\, \mathcal{D}A \quad e^{i\int B\wedge F} \quad \delta(\text{simplicity})$$

$$\epsilon_{IJKL} B^{IJ}_{\mu\nu} B^{KL}_{\rho\sigma} = e\ \epsilon_{\mu\nu\rho\sigma}$$

Sector I$\pm$: $\quad B^{IJ} = \pm e^I \wedge e^J$ $\longrightarrow$ first order Einstein gravity

Sector II$\pm$: $\quad B^{IJ} = \pm \frac{1}{2}\epsilon^{IJ}{}_{KL} e^K \wedge e^L$

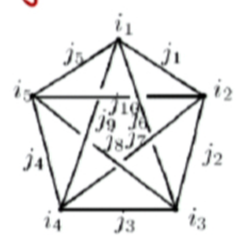
Covariant Path Integral

Plebanski Gravity 4d

$$Z = \int \mathcal{D}B\,\mathcal{D}A\,\mathcal{D}\varphi \quad e^{i\int B\wedge F + \varphi\cdot B\wedge B}$$

$$= \underbrace{\int \mathcal{D}B\,\mathcal{D}A \quad e^{i\int B\wedge F}}_{\text{Topological BF theory}} \; \delta(\cancel{\text{simplicity}})$$

$$= \sum_{j_f, i_v} \prod_f \dim(j_f) \prod_v$$



[Ooguri '92]

{15j} - symbole
vertex amplitude

Covariant Path Integral

Plebanski Gravity 4d

$$Z = \int \mathcal{D}B\,\mathcal{D}A\,\mathcal{D}\varphi \quad e^{i\int B\wedge F + \varphi\cdot B\wedge B}$$

$$= \int \mathcal{D}B\,\mathcal{D}A \quad e^{i\int B\wedge F} \; \delta(\text{simplicity})$$

impose simplicity to BF

EPRL/FK Spinfoam Amplitude

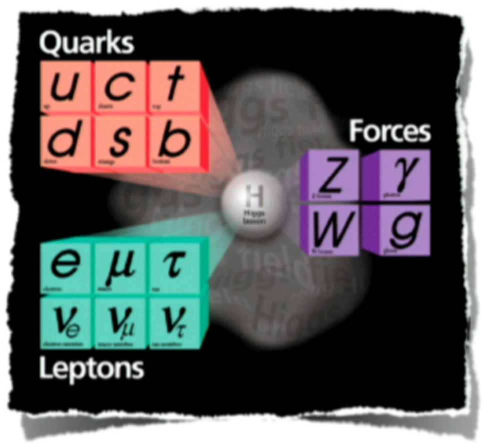
$$Z = \sum_{j_f, i_e} \prod_f \dim(j_f) \prod_v A_v(j_f\ i_e)$$

Many different ways to understand the simplicity constraint in the formalism :

Linear simplicity , Master constraint, coherent state, Gupta-Bleuler spinor/twistor ......

[Barrett, Crane, Freidel, Krasnov, Engle, Pereira, Rovelli, Livine, Speziale, Conrady, Ding, Dupuis, Bonzom, Wieland, Baratin, Oriti, MH, ...]

Q: Where is elementory matter ?



Spinfoam Amplitude for Gravity + Fermion + Yang-Mills :

[Bianchi, Perini, Magliaro, Rovelli, Wieland, MH, '10]

$$Z = \sum_{\{c\}} \sum_{j_f} \int_{SL(2,\mathbb{C})} dg_{ve} \int_{SU(2)} dh_{ef} \int_G dU_{ve}$$
$$\prod_f d_{j_f} \, \chi^{\gamma j_f, j_f}\Big(\prod_{e \in \partial f} (g_{es_e} h_{ef} g^{-1}_{et_e})^{\epsilon_{ef}} \Big) \prod_{e \in \partial f} \chi^{j_f}(h_{ef})$$
$$\prod_c (-1)^{|c|} \, \chi^{\frac{1}{2}}\Big(\prod_{e \in c_n} (g_{es_e} U_{es_e} U^{\dagger}_{et_e} g^{\dagger}_{et_e})^{\epsilon_{ec}} \Big).$$

Yang-Mills
Wilson-line

fermion cycles

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$$\prod_c (-1)^{|c|} \, \chi^{\frac{1}{2}}\Big(\prod_{e\in c_n} (g_{es_e} U_{es_e} U^{\dagger}_{et_e} g^{\dagger}_{et_e})^{\epsilon_{ec}}\Big).$$

fermion cycles

Yang-Mills Wilson-line

∃ another Yang-Mills coupling in Barrett-Crane model

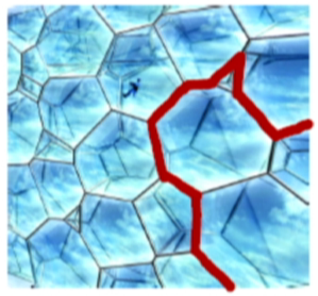
[Oriti, Pfeiffer '02]

$\boxed{\text{Our knowledge of matter coupling in spinfoam is limited, especially in the semiclassical limit ...}}$

Some properties of spinfoam fermion:

[Rovelli, MH, '11]

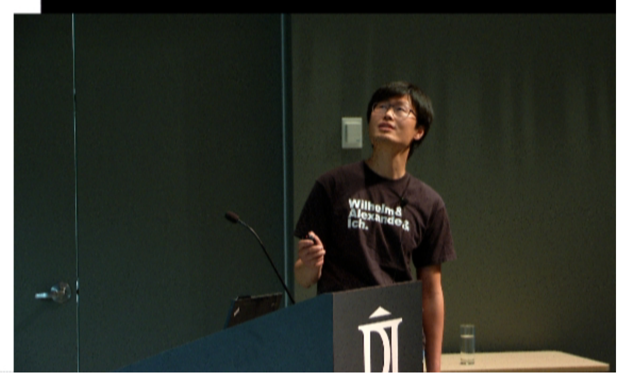
Fermion correlators : World-lines on spinfoam spacetime



$$= \sum_{\text{path}} P e^{-S_F}$$

Discretization of fermion world-line action

Integrated in spinfoam amplitude



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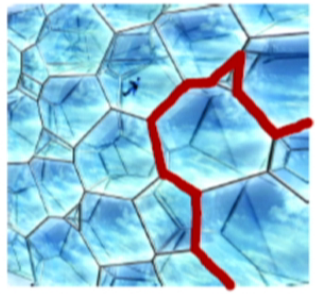


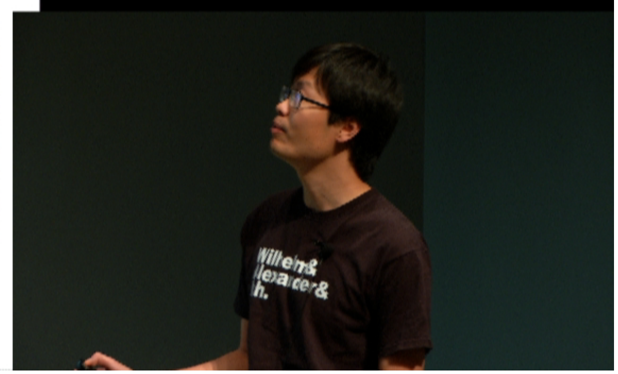
$$= \sum_{\text{path}} P e^{-S_F}$$

Spinfoam PCT symmetry :

PT

$\exists$ spinfoam analog of $\overline{M}$ with $-O=(-T,\, \varepsilon_{\alpha\beta\gamma\delta})$

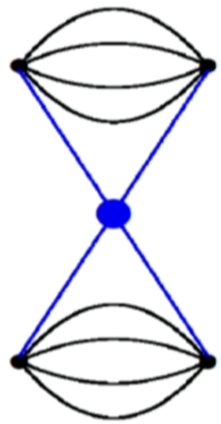
$$\boxed{\begin{aligned} &\langle \bar\psi^c_{v_1} \cdots \bar\psi^c_{v_n}\, \psi^c_{v_{n+1}} \cdots \psi^c_{v_{n+m}} \rangle_{\text{spinfoam}(\overline{M})} \\ &\quad = \langle \bar\psi_{v_1} \cdots \bar\psi_{v_n}\, \psi_{v_{n+1}} \cdots \psi_{v_{n+m}} \rangle^*_{\text{spinfoam}(M)} \end{aligned}}$$



Spinfoam cosmology:

[Bianchi, Krajewski, Rovelli, Vidotto]

Approximation by simple spinfoam



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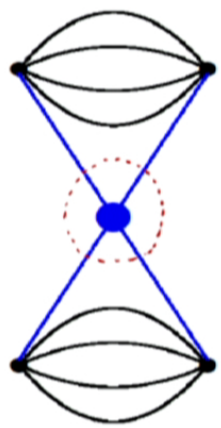


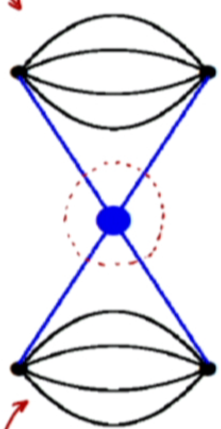
A single spinfoam vertex

Spinfoam cosmology:

[Bianchi, Krajewski, Rovelli, Vidotto]

Approximation by simple spinfoam

spatial dipole spin-network S_{out}



A single spinfoam vertex

spatial dipole spin-network S_{in}

Dipole approximation:

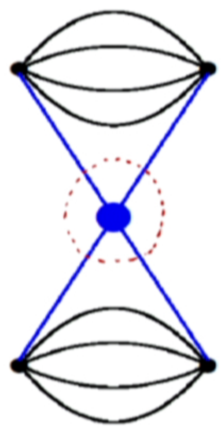
curvature radius >> scale of dipole

Spinfoam cosmology: [Bianchi, Krajewski, Rovelli, Vidotto]

Approximation by simple spinfoam

(homogeneous, isotropic)

Final semiclassical state $(a, \dot{a})_{out}$



Initial semiclassical state $(a, \dot{a})_{in}$

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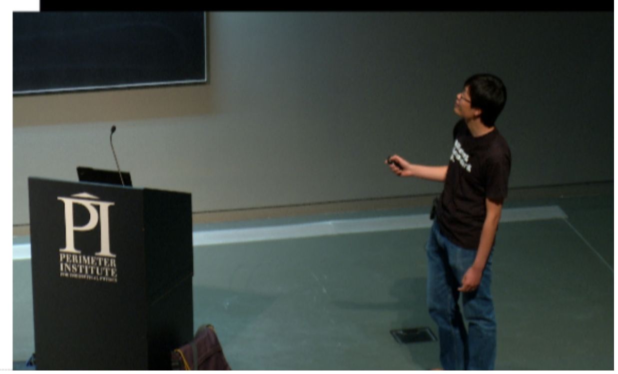
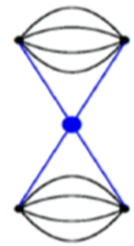
Dipole approximation:

curvature radius >> scale of dipole



Spinfoam cosmology:



Q: How to go beyond dipole ?

- analyzing more transitions
  [Hellmann, Kisielowski, Lewandowski, Puchta]
- more links, U(N) framework, spinor/twistor
  [Livine, Borja, Garay, Vidotto, Diaz-Polo, Freidel, Martin-Benito]
- . . .

Q: Relation with LQC ?

- Hamitonian constraint $\hat{C}$  $= 0$ [Livine, Martin-Benito '11]
- Path integral from LQC [Ashtekar, Campiglia, Henderson]
- . . .

Q: Singularity ?

- seems to be resolved by the minimal area [Rovelli, Vidotto, '13]
- . . .

Q: Beyond $k=0$? Beyond homogeneous – isotropic ?
Matters ? Inflation ?

. . . . . .

Implement Λ in full theory:

Cosmological constant & Quantum group in LQG (4-dim)

Implement Λ in full theory:

Cosmological constant & Quantum group in LQG (4-dim)

- Seth Major, Lee Smolin, *Quantum deformation of quantum gravity*. Nucl.Phys. B473 (1996) 267-290
- R. Borissov, S. Major, L. Smolin, *The Geometry of quantum spin networks*. Class.Quant.Grav. 13 (1996) 3183-3196
- L. Smolin, *The Bekenstein bound, topological quantum field theory and pluralistic quantum cosmology*. [gr-qc/9508064]
- L. Smolin, *Linking topological quantum field theory and nonperturbative quantum gravity*. J.Math.Phys. 36 (1995) 6417-6455
- L. Smolin, *Quantum gravity with a positive cosmological constant*. [arXiv:hep-

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$$\boxed{\text{Cosmological constant} \sim q\text{-deformation} \sim \text{Boundary Chern-Simons} \sim \text{IR-finiteness}}$$

Quantum group Spinfoam formulation: (4-dim)

Replace the usual structure group by Quantum Group

[Noui, Roche, Fairbairn, Meusburger, MH]

- $SL(2,\mathbb{C}) \rightsquigarrow SL(2,\mathbb{C})_q \quad q \in \mathbb{R}$
- $Spin(4) \rightsquigarrow SU(2)_q \times SU(2)_q \quad q \in \mathbb{C},\ q^n = 1$

Quantum group Spinfoam amplitude:

$$Z = \sum_{j_f, i_e}^{j_{max}} \prod_f \dim(j_f)_q \prod_v A_v(j_f\ i_e)_q$$

$$j_{max} \sim \frac{1}{\ln q} \sim \frac{1}{\Lambda}$$

Quantum Group ~ Chern-Simons Theory

[Witten, Reshetikhin, Turaev, Buffenoir, Noui, Roche ...]

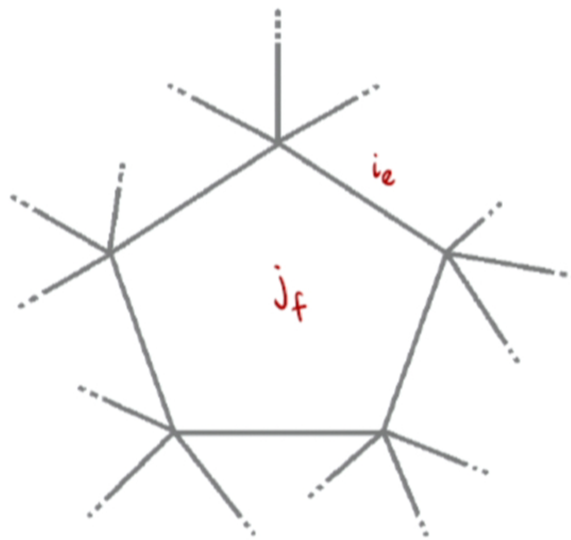
$$S_{CS}[A] = \frac{h}{4\pi}\int \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A\wedge A \wedge A\right) + c.c.$$

$h = k + is$ may be complex for complex G

$k \in \mathbb{Z}$

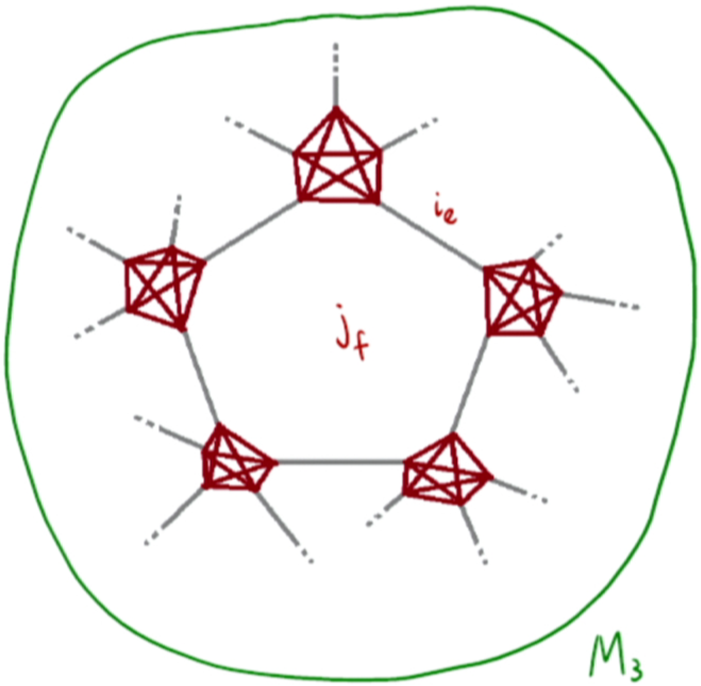
Chern-Simons Formulation of q-deformed Spinfoam

Given a usual spinfoam amplitude :



Chern-Simons Formulation of q-deformed Spinfoam

blow up each spinfoam vertex



Chern-Simons Formulation of q-deformed Spinfoam

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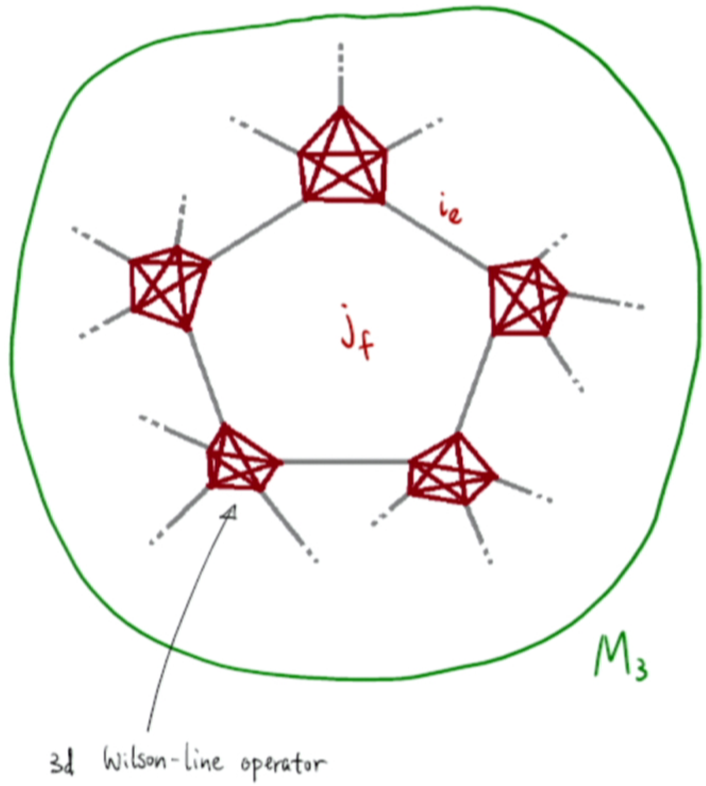
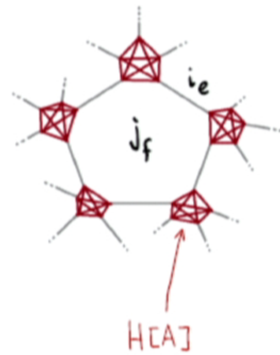


Chern-Simons Formulation of q-deformed Spinfoam

q-deformed spinfoam amplitude :

3-manifold

$$Z = \sum_{j,i} \int$$



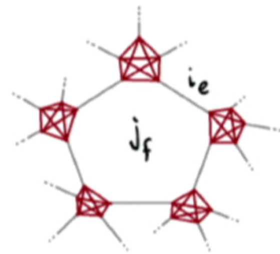
$$e^{iS_{cs}[A,\, M_3]}\, \mathcal{D}A$$

3d G-connection

Chern-Simons Formulation of q-deformed Spinfoam

q-deformed spinfoam amplitude :

$$Z = \sum_{j,i} \int$$



$$e^{iS_{CS}[A,\, M_3]}\, \mathcal{D}A$$

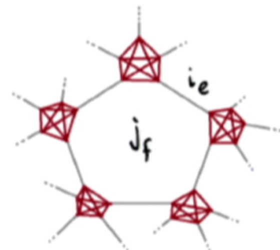
$$\boxed{Z_{LQG_q}(M_4) = Z_{CS\text{-Line Defects}}(M_3)}$$

? # of D.O.F $\propto$ Area (rather than Volume) ?

Chern-Simons Formulation of q-deformed Spinfoam

q-deformed spinfoam amplitude :

$$Z = \sum_{j,i} \int$$

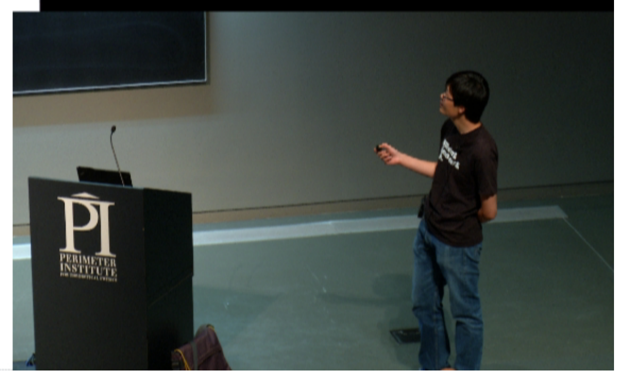


$$e^{iS_{CS}[A,\, M_3]}\, \mathcal{D}A$$

$$\boxed{Z_{LQG_\Lambda}(M_4) = Z_{CS\text{-Line Defects}}(M_3)}$$

Convenient for semiclassical analysis

$$Z \sim \text{"}\, e^{iS_{GR,\Lambda}} \,\text{"} \qquad \Lambda \sim [CS \text{ coupling}]^{-1}$$



Q: Nontrivial dynamical effects from knotting ?

Q: Enlarge $\mathcal{H}$ and q-deform $\mathcal{H}$?

- Major, Smolin, Wan ⋯ 1995 - 2008
- Lewandowski, Okolow, 2005 - 2008
- Girelli, Dupuis, 2013 [Dupuis' talk]

How do we do semiclassical analysis : [Hellmann's talk]

[Ashtekar, Lewandowski, Rovelli, Smolin, Thiemann, Sahlmann, Winkler, Giesel, Marolf,
Barrett, Oriti, Freidel, Krasnov, Engle, Pereira, Livine, Speziale, Perez, Ma, Mourão
Hellmann, Kaminski, Conrady, Krajewski, Bonzom, Ding, Dittrich, Bahr, Bianchi,
Wieland, Alesci, Perini, Magliaro, Zhang, Fairbairn, Baratin, MH, ...]

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Regime to find Einstein : [MH '13]

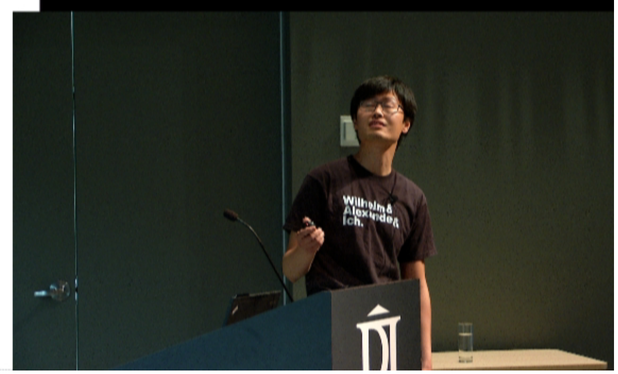
area of each plaquette

4d curvature radius

$$\ell_P^2 \ll a \ll L^2$$

Analog of semiclassicality in canonical LQG

[Sahlmann, Thiemann, Winkler, '01]



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Regime to find Einstein: [MH '13]

area of each plaquette

4d curvature radius

$$\ell_P^2 \ll a \ll L^2$$

semiclassical   low-energy

scaling: $j \sim \frac{a}{\ell_P^2} \gg 1$ $\Theta \sim \frac{a}{L^2} \ll 1$

(Large spin)   (small deficit angle)

Analog of semiclassicality in canonical LQG
[Sahlmann, Thiemann, Winkler, '01]

Large-j Approximation: (more in Hellmann' talk)

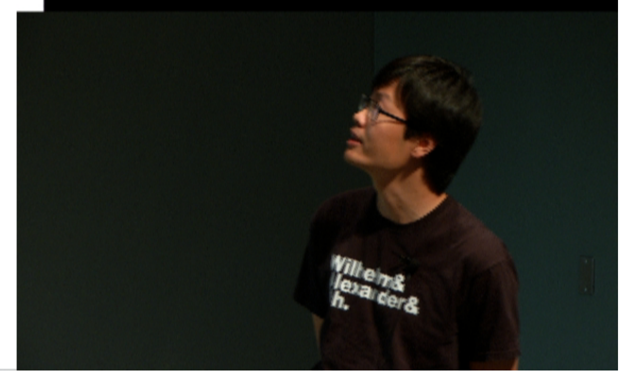
[Barrett, Freidel, Hellmann, Fairbairn, Dowdall, Gomes, Pereira, Conrady, Zhang
Krajewski, MH ···]

Path Integral + Stationary Phase : $j \gg 1$

$$Z = \sum_{j_f} \prod_f \dim(j_f) \int_{SL(2,\mathbb{C})} dg_{ev} \int_{\mathbb{CP}^1} dz_{vf} \; e^{S[j_f, g_{ve}, z_f]}$$

Krajewski's spinfoam action

Spinfoam critical data → Simplicial geomety
+ spacetime / time orientation



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Vector	$i\Phi_f$	0

Lorentzian geometry, globally Spacetime oriented, Time oriented

$$[\cdots] \equiv Z_j \sim e^{\frac{i}{\ell_P^2} S_{Regge} + O\left(\frac{1}{j^m}\right)}$$

$\Theta \ll 1$ Expansion :

[MH '13]

$\exists$ Einstein sector in large-j regime of the state-sum :

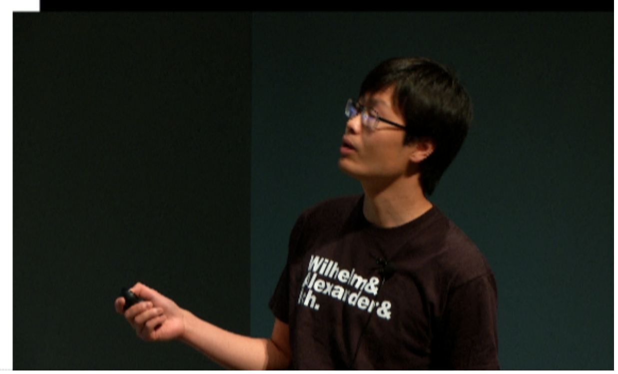
$$Z \sim \sum_{l} \mu(l)\, e^{\frac{i}{l_p} S_{Regge}[l] + o\left(\frac{1}{j^m}\right) + o\left(\gamma^n \Theta^n\right)}$$

$\Theta \sim aR$

2 - parameter expansion :

$\frac{1}{j}$: quantum corrections

a : high energy (curvature) corrections



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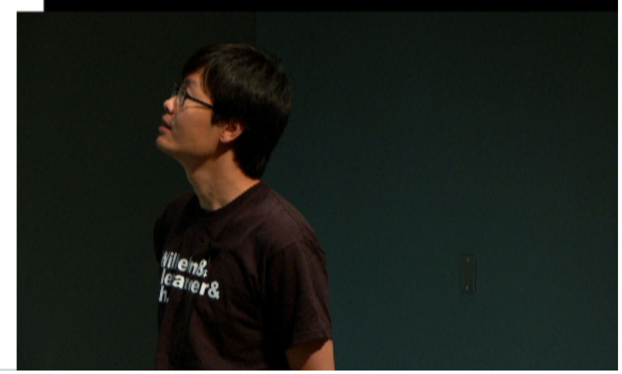
- expansion seems to be formal, BUT ...

assume they are small $\longrightarrow$ restriction of Θ as $j \to \infty$



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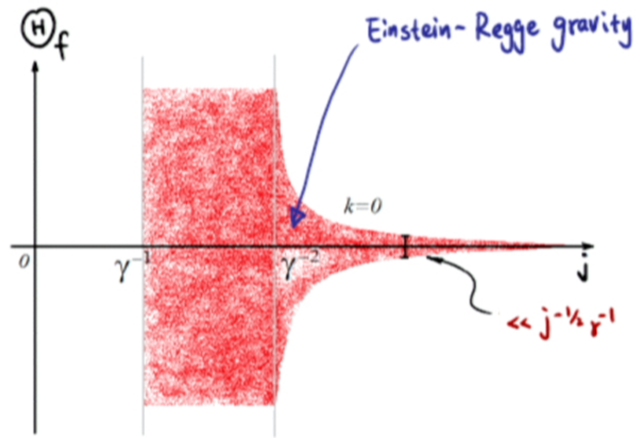

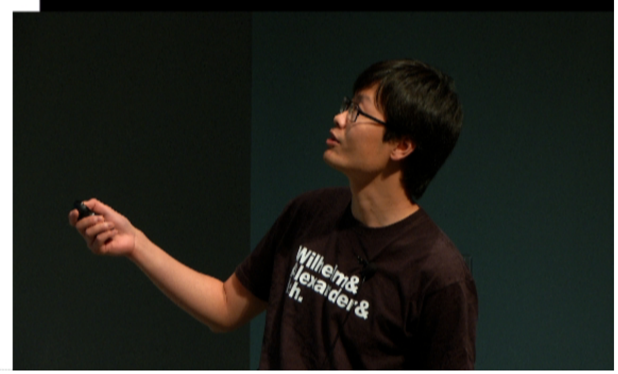

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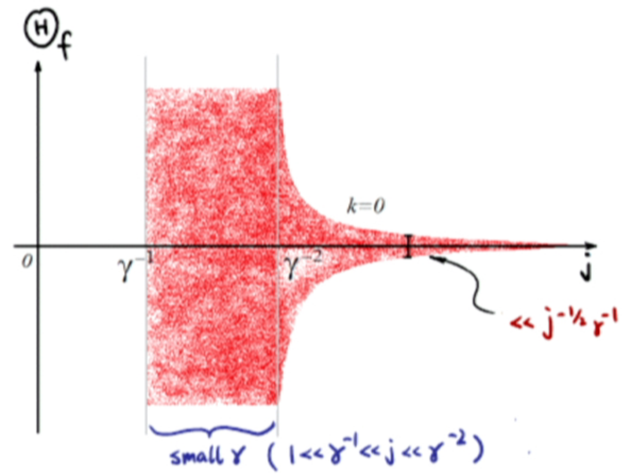
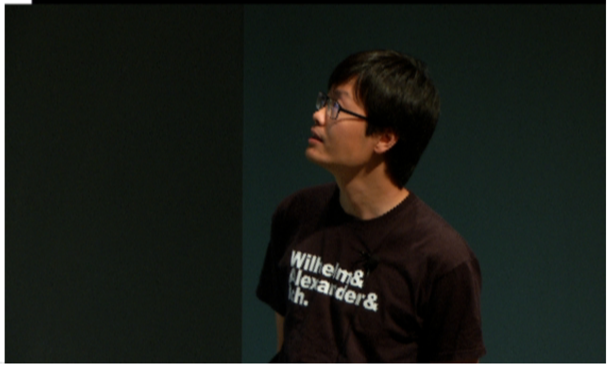


Q: Small γ ?

– graviton n-pt function [Rovelli, Eugenio, Alesci, Speziale, Perini, Magliaro, Ding, Zhang, Modesto …]



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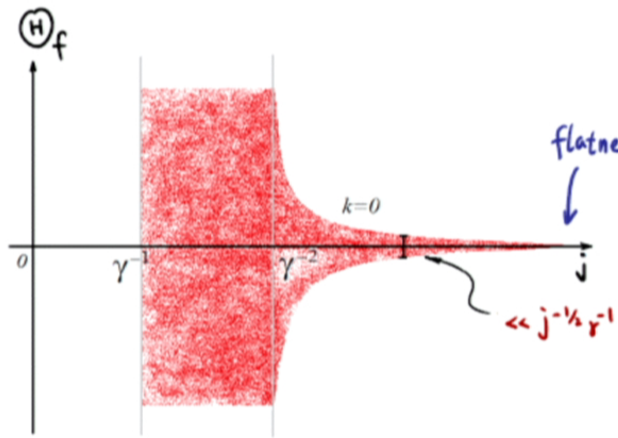

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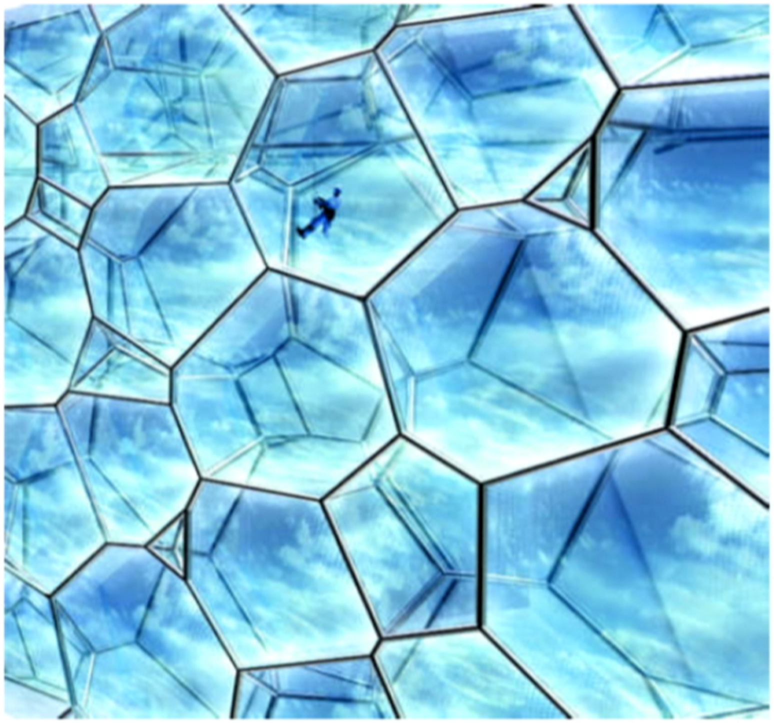
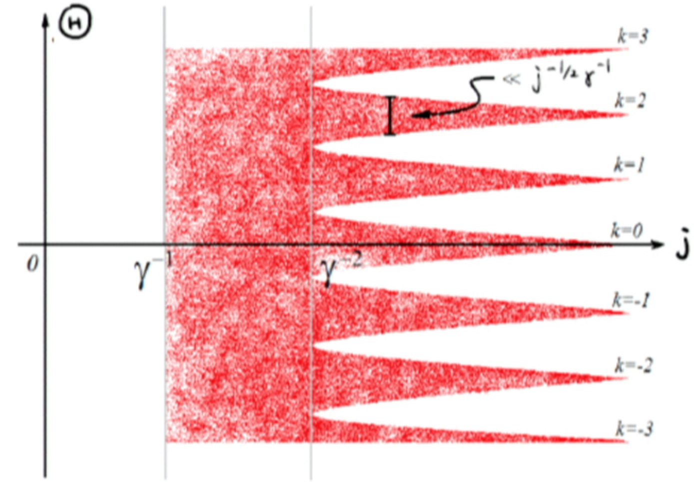
Q: Flatness ? [talks by Hellmann, Kaminski]

[Bonzom, Hellmann, Kaminski, Perini]



Beyond Einstein sector:



Q: Even more beyond?

- different orientations?
- topological sectors?

to be done ...

What do we have ?

A nonperturbative formulation . finiteness

$$\boxed{Z_{LQG_4}(M_4) = Z_{CS\text{-Line Defects}}(M_3)}$$

A perturbative formulation . 2-parameter expansion.

$$\boxed{Z \sim \sum_{l} \mu(l)\, e^{\frac{i}{\ell_p} S_{Regge}[l] + o\left(\frac{1}{j^m}\right) + o\left(\gamma^n \Theta^n\right)}}$$

- UV completion ?
- Continuum limit ?
- ...

Q: Semiclassical low-energy effective theory ?

Q: More understandings on Chern-Simons formulation ?

Q: Observable consequences ? cosmology ? particles physics ?

Q: Top-down approach to black hole physics ?

. . .

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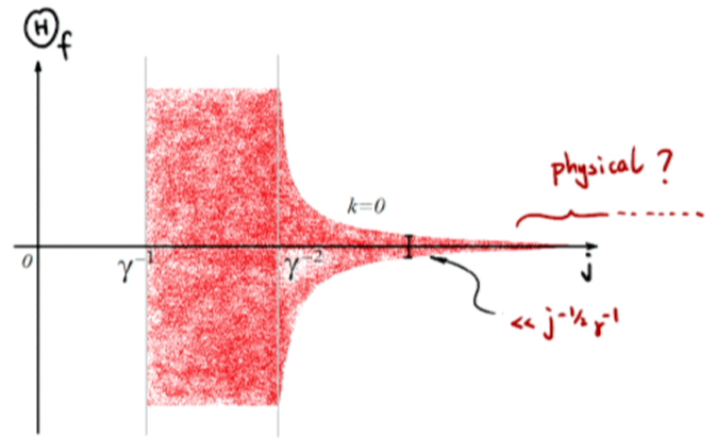

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Q: Flatness ? [Hellmann's talk]

[Bonzom, Hellmann, Kaminski, Perini]

Q: Is it still physical/semiclassical if j is too large ?

Spinfoam Amplitude for Gravity + Fermion + Yang-Mills :

[Bianchi, Perini, Magliaro, Rovelli, Wieland, MH, '10]

$$Z = \sum_{\{c\}} \sum_{j_f} \int_{SL(2,\mathbb{C})} dg_{ve} \int_{SU(2)} dh_{ef} \int_G dU_{ve}$$

$$\prod_f d_{j_f} \, \chi^{\gamma j_f, j_f}\Big(\prod_{e\in\partial f} (g_{es_e} h_{ef} g^{-1}_{et_e})^{\epsilon_{ef}}\Big) \prod_{e\in\partial f} \chi^{j_f}(h_{ef})$$

$$\prod_c (-1)^{|c|} \, \chi^{\frac{1}{2}}\Big(\prod_{e\in c_n} (g_{es_e} U_{es_e} U^\dagger_{et_e} g^\dagger_{et_e})^{\epsilon_{ec}}\Big).$$

fermion cycles

Yang-Mills
wilson-line

∃ another Yang-Mills coupling in Barrett-Crane model

[Oriti, Pfeiffer '02]

Our knowledge of matter coupling in spinfoam is limited, especially in the semiclassical limit ...

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