Title: Recently Obtained Non-Perturbative 1/N Expansion of Tensor Models

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Abstract: I will present the recently obtained non perturbative 1/N expansion of tensor models. The correlation functions are shown to be analytic in the coupling constant in some domain of the complex plane and to support appropriate scaling bounds at large N. Surprisingly, the non perturbative setting turns out to be a powerful computational tool allowing the explicit evaluation order by order (with bounded rest terms) of the correlations.

The non perturbative $1/N$ expansion of Tensor Models

Răzvan Gurău

Loops '13, PI

Tensor invariants as Edge Colored Graphs

Building blocks: tensors with no symmetry transforming as

$$T'_{b^1\dots b^D} = \sum U^{(1)}_{b^1 a^1} \dots U^{(D)}_{b^D a^D} T_{a^1\dots a^D} \ , \quad \bar{T}'_{p^1\dots p^D} = \sum \bar{U}^{(1)}_{p^1 q^1} \dots \bar{U}^{(D)}_{p^D q^D} \bar{T}_{q^1\dots q^D}$$

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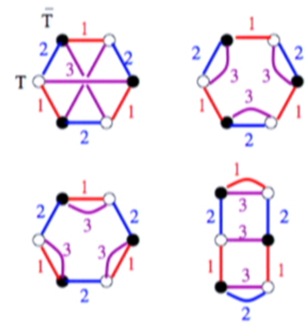
Invariants: colored graphs



$$\mathrm{Tr}_{\mathcal{B}}(T,\bar T) = \sum \prod_{v} T_{a^1_v\dots a^D_v} \prod_{\bar v} \bar T_{q^1_{\bar v}\dots q^D_{\bar v}} \prod_{c=1}^{D} \prod_{l^c=(w,\bar w)} \delta_{a^c_w q^c_{\bar w}}$$

- White (black) vertices for T ($\bar T$).
- Edges for $\delta_{a^c q^c}$ colored by c, the position of the index.

Invariant Actions for Tensor Models

The most general single trace tensor model

$$S(T,\bar{T}) = \sum T_{a^1\dots a^D}\bar{T}_{q^1\dots q^D}\prod_{c=1}^{D}\delta_{a^c q^c} + \sum_{\mathcal{B}} t_{\mathcal{B}}\mathrm{Tr}_{\mathcal{B}}(\bar{T},T)$$

$$Z(t_{\mathcal{B}}) = \int [d\bar{T}dT]\, e^{-N^{D-1}S(T,\bar{T})}$$

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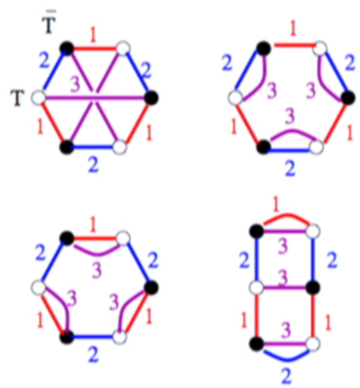
Feynman graphs: "vertices" $\mathcal{B}$.



$$\int_{\bar{T},T} e^{-N^{D-1}\left(\sum T_{a^1\dots a^D}\bar{T}_{q^1\dots q^D}\prod_{c=1}^{D}\delta_{a^c q^c}\right)} \mathrm{Tr}_{\mathcal{B}_1}(\bar{T},T)\mathrm{Tr}_{\mathcal{B}_2}(\bar{T},T)\dots$$

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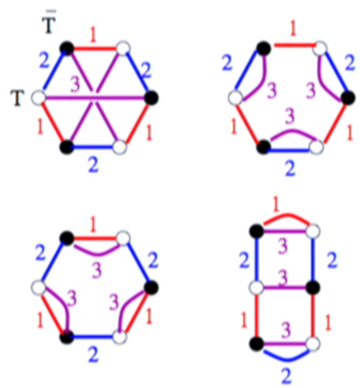

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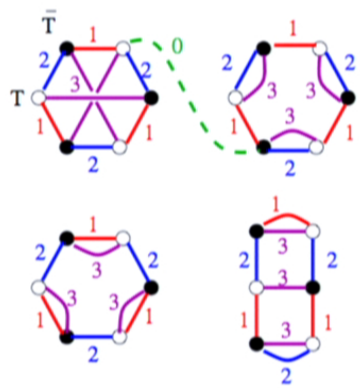
Feynman graphs: "vertices" $\mathcal{B}$. Gaussian integral: Wick contractions of T and $\bar{T}$ ("propagators") $\to$ dashed edges to which we assign the fictitious color 0.

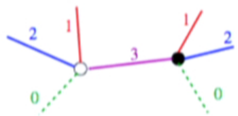


$$\int_{\bar{T},T} \qquad e^{-N^{D-1}\left(\sum T_{a^1\dots a^D} \bar{T}_{q^1\dots q^D} \prod_{c=1}^{D} \delta_{a^c q^c}\right)}$$

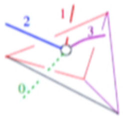
$$\sum\left(\prod \delta_{\dots}\right) \quad \underbrace{T_{a^1a^2a^3}\bar{T}_{p^1p^2p^3}}_{\sim \frac{1}{N^{D-1}} \delta_{a^1p^1}\delta_{a^2p^2}\delta_{a^3p^3}} \quad \dots$$

Colored Graphs as gluings of colored simplices

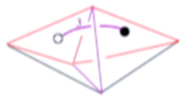
White and black $D+1$ valent vertices connected by edges with colors $0, 1 \dots D$.



Vertex $\leftrightarrow$ colored D simplex .



Edges $\leftrightarrow$ gluings along $D-1$ simplices respecting all the colorings



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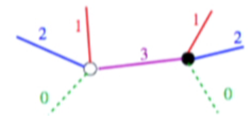

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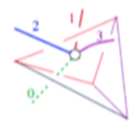

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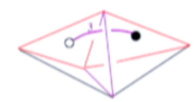

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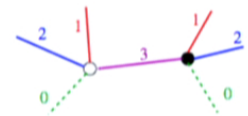

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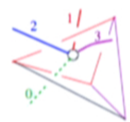

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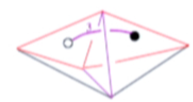

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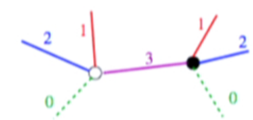

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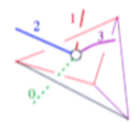

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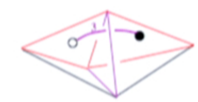

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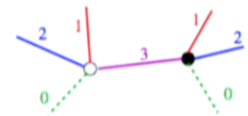

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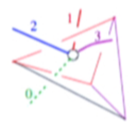

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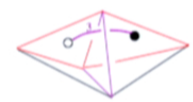

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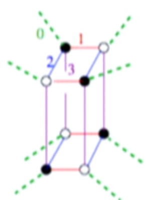

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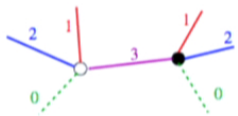

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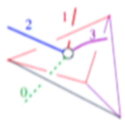

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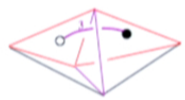

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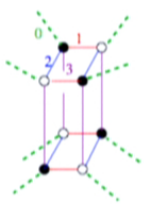

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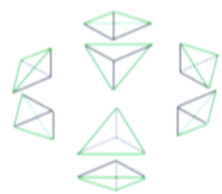

Gluing along all $D-1$ simplices except 0: "chunk" in D dimensions

The general framework

Observables = invariants $\mathrm{Tr}_{\mathcal{B}}$ encoding boundary triangulations.
Expectations =

$$\Big\langle \mathrm{Tr}_{\mathcal{B}_1} \mathrm{Tr}_{\mathcal{B}_2} \dots \mathrm{Tr}_{\mathcal{B}_q} \Big\rangle = \frac{1}{Z(t_{\mathcal{B}})} \int [d\bar{T} dT] \; \mathrm{Tr}_{\mathcal{B}_1} \mathrm{Tr}_{\mathcal{B}_2} \dots \mathrm{Tr}_{\mathcal{B}_q} \; e^{-N^{D-1} S(T,\bar{T})}$$

correlations between boundary states given by sums over all bulk triangulations compatible with the boundary states

- $\Big\langle \mathrm{Tr}_{\mathcal{B}} \Big\rangle$: $\mathcal{B}$ to vacuum amplitude
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Remarks:

- The path integral yields a canonical sum over "histories".
- Weight of a triangulation: discretized EH, $B \wedge F$, etc.
- Need to take some kind of limit in order to go from triangulations to continuum geometries.

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Suppose we are interested in "Interesting Quantity"

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(Mathematical counterpart of: Interesting Quantity $=$ Measured $+$ Error Bar)

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This is not the case if |Rest| is not < Good bound

Computing Explicit in the absence of |Rest| < Good Bound is at best naive and at worst nonsensical.

The quartic tensor model

Our aim is to compute correlations

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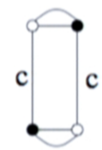
The simplest quartic invariants correspond to "melonic" graphs with four vertices $\mathcal{B}^{(4),c}$

$$\sum\Big(T_{a^1\dots a^D}\bar T_{q^1\dots q^D}\prod_{c'\neq c}\delta_{a^{c'}q^{c'}}\Big)\delta_{a^c p^c}\delta_{b^c q^c}\Big(T_{b^1\dots b^D}\bar T_{p^1\dots p^D}\prod_{c'\neq c}\delta_{b^{c'}p^{c'}}\Big)$$



The simplest interacting theory: coupling constants $t_{\mathcal{B}} = \begin{cases} \frac{\lambda}{2}\,, & \mathcal{B} = \mathcal{B}^{(4),c} \\ 0\,, & \text{otherwise} \end{cases}$

Amplitudes and Dynamical Triangulations

Expand in λ (Feynman graphs):

$$\left\langle \frac{1}{N} \mathrm{Tr}_{\mathcal{B}^{(2)}} \right\rangle = \sum_{D+1 \text{ colored graphs } G} A^G(N)$$

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Each graph is dual to a triangulation

$$A^G(N) = e^{\kappa_{D-2}(\lambda,N) Q_{D-2} - \kappa_D(\lambda,N) Q_D}$$

Discretized Einstein Hilbert action on the dual triangulation with Q_D equilateral D-simplices and Q_{D-2} $(D-2)$-simplices.

The $1/N$ expansion

Two parameters: λ and N.

1) Feynman expansion: $K_2 = 1 - D\lambda - \frac{1}{N^{D-2}} D\lambda + \sum_G A^G(N) \quad A^G(N) \sim \lambda^2$

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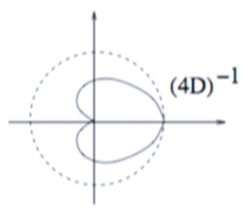
3) non perturbative: $K_2 = \frac{(1+4D\lambda)^{\frac{1}{2}} - 1}{2D\lambda} + \dots + \mathcal{R}_N^{(p)}(\lambda)$

$\mathcal{R}_N^{(p)}(\lambda)$ analytic in $\lambda = |\lambda| e^{\imath\varphi}$ in the domain



$$|\mathcal{R}_N^{(p)}(\lambda)| \leq \frac{1}{N^{p(D-2)}} \frac{|\lambda|^p}{\left(\cos\frac{\varphi}{2}\right)^{2p+2}} \, p! \, A \, B^p$$

The $N \to \infty$ limit

Good Bound ensures $\lim_{N\to\infty} |\mathcal{R}_N^{(1)}(\lambda)| = 0$, hence

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- is the sum of an infinite family of graphs of spherical topology ("melons")
- becomes critical for $\lambda \to -(4D)^{-1}$
- in the critical regime infinite graphs (representing infinitely refined geometries) dominate

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The Double Scaling Limit

The graphs can be reorganized as

$$K_2 = \sqrt{(4D)^{-1}+\lambda}\sum_{p\geq 0}\frac{c_p}{\left(N^{D-2}[(4D)^{-1}+\lambda]\right)^p} + Rest$$

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- canonical path integral formulation.
- built in scales (tensors of size N^D).
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- with weights the discretized (Einstein Hilbert, $B \wedge F$, etc.) action.
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Question: What precise model in this framework describes our universe?

- we don't know hence we concentrate on universal predictions.

Conclusions

More results: GFT/Tensor Models session, Thursday 25th July, 14h30

The tensor track is largely open and begs to be explored!

A personal list of open questions:

- non perturbative results
  - extend the non perturbative treatment to other models.
  - extend the analyticity domain of the rest and study the discontinuity of the rest on the negative real axis (non perturbative cut effects are crucial for unitarity and the role of time)
- study the geometry of the space emerging under multiple scalings.
  - algebra of constraints, Hausdorff and spectral dimensions, geodesics.
- Effective field theory description of the confined phase.
- Phenomenological implications.
- ....

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