Title: Twistor Strings for N=8 Supergravity

Date: Jul 24, 2013 09:45 AM

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Abstract: The perturbative S-matrix of General Relativity has a rich and fascinating geometric structure that is completely obscured by its traditional description in terms of Feynman diagrams. I'll explain a new way of looking at four dimensional supergravity: as a string theory in twistor space. All tree-level amplitudes in the theory can be described by algebraic curves in Penrose's nonlinear graviton

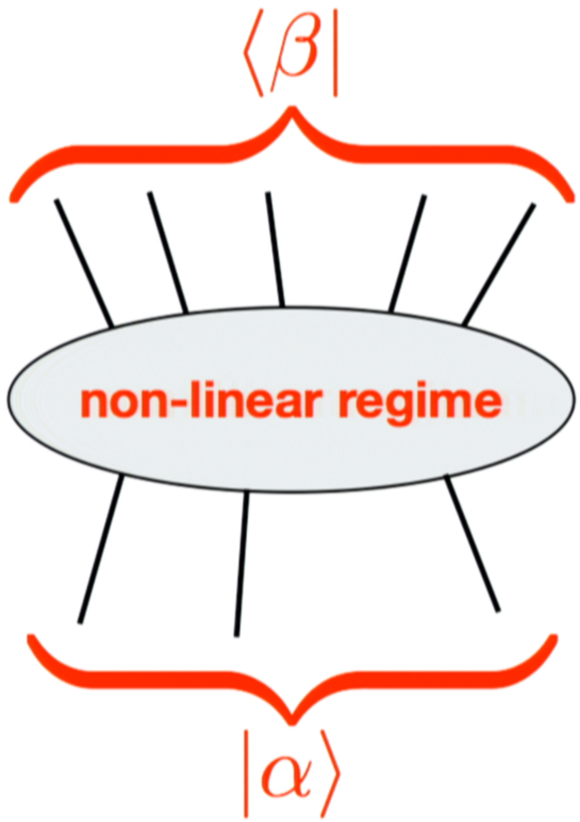
Twistor strings for $\mathcal{N} = 8$ supergravity

David Skinner - IAS & Cambridge

Loops 2013 - Perimeter Institute

http://www.youtube.com/watch?v=W_TO2WESSA4#at=114

Scattering amplitudes are quantum mechanical overlap between states with prescribed asymptotic behaviour



- point of contact between theorists and experimentalists
- encode the dynamics of quantum systems
- live at boundary of asymptotically flat space-time: diffeo invariant observables

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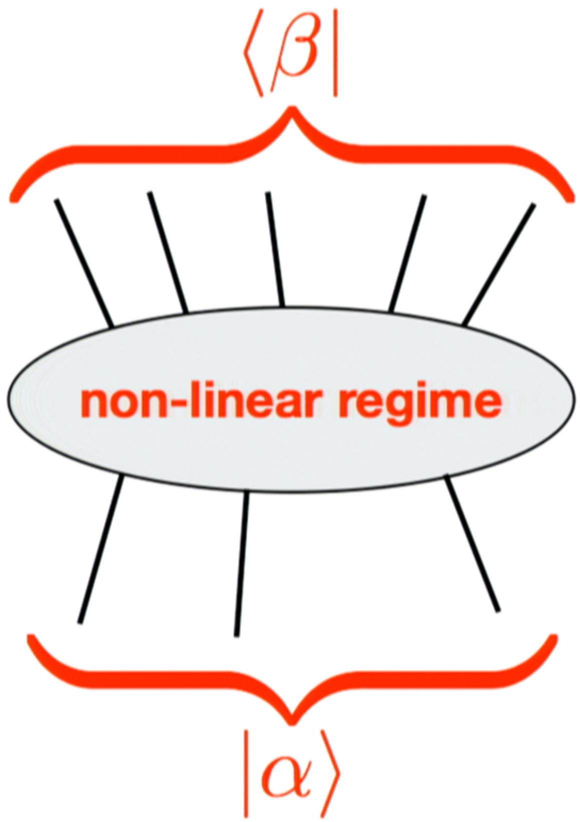

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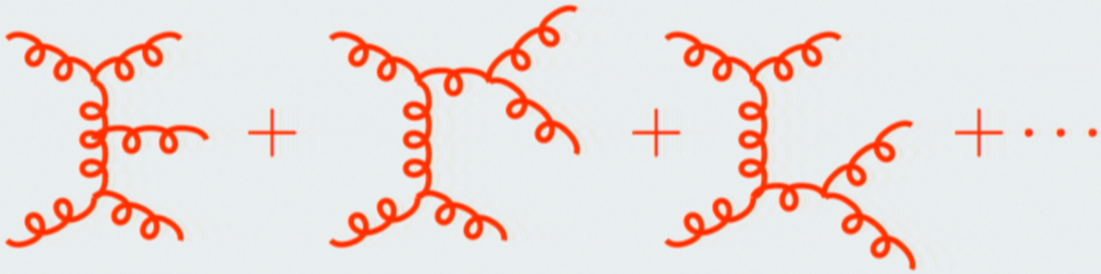
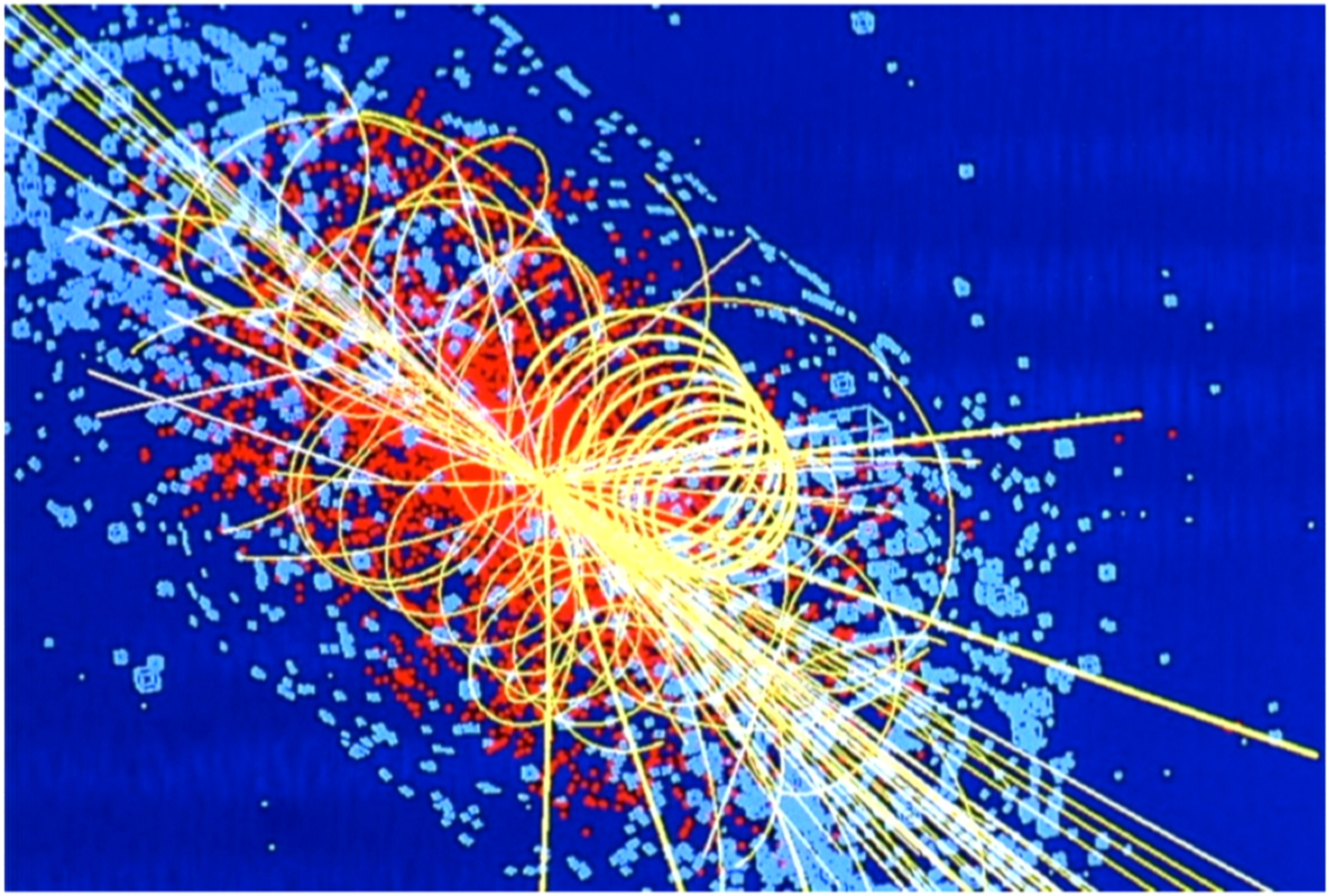
Perturbatively, scattering amplitudes are usually described in terms of Feynman diagrams

In Yang-Mills & gravity - theories we care most about - Feynman diagrams rapidly become very complicated

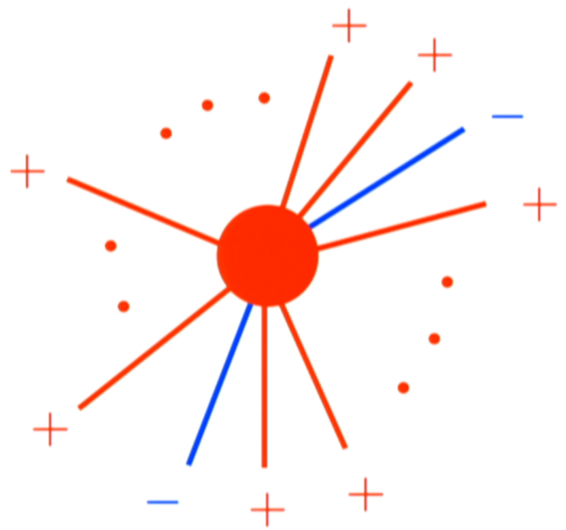


- much effort by many groups in recent years to find better ways to calculate these amplitudes

The original motivation for many of these investigations was very practical:



Remarkably, these investigations showed that multi-particle amplitudes are often dramatically simpler than expected from their Feynman representation



$$\mathcal{M}_n^{\mathrm{MHV}} = \frac{\langle ij \rangle^4 \, \delta^4(\sum p_i)}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}$$

[Parke, Taylor]

$$p^2 = 0 \;\Leftrightarrow\; p_{\alpha\dot\alpha} := p_\mu \sigma^\mu_{\alpha\dot\alpha} = \lambda_\alpha \bar\lambda_{\dot\alpha}$$
$$\langle ij \rangle := \epsilon^{\alpha\beta} \lambda^{(i)}_\alpha \lambda^{(j)}_\beta$$

Standard representations obscure the true nature of scattering amplitudes

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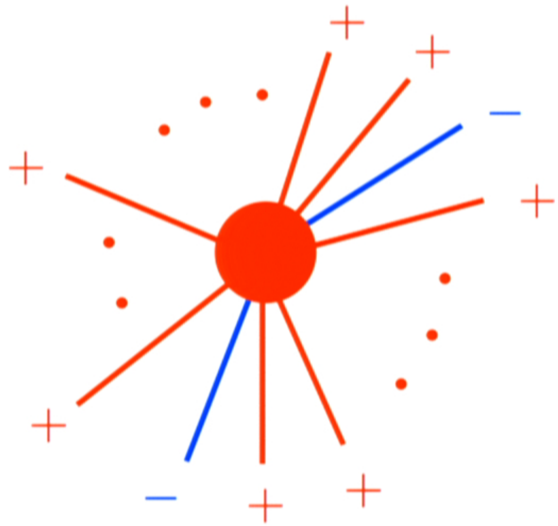

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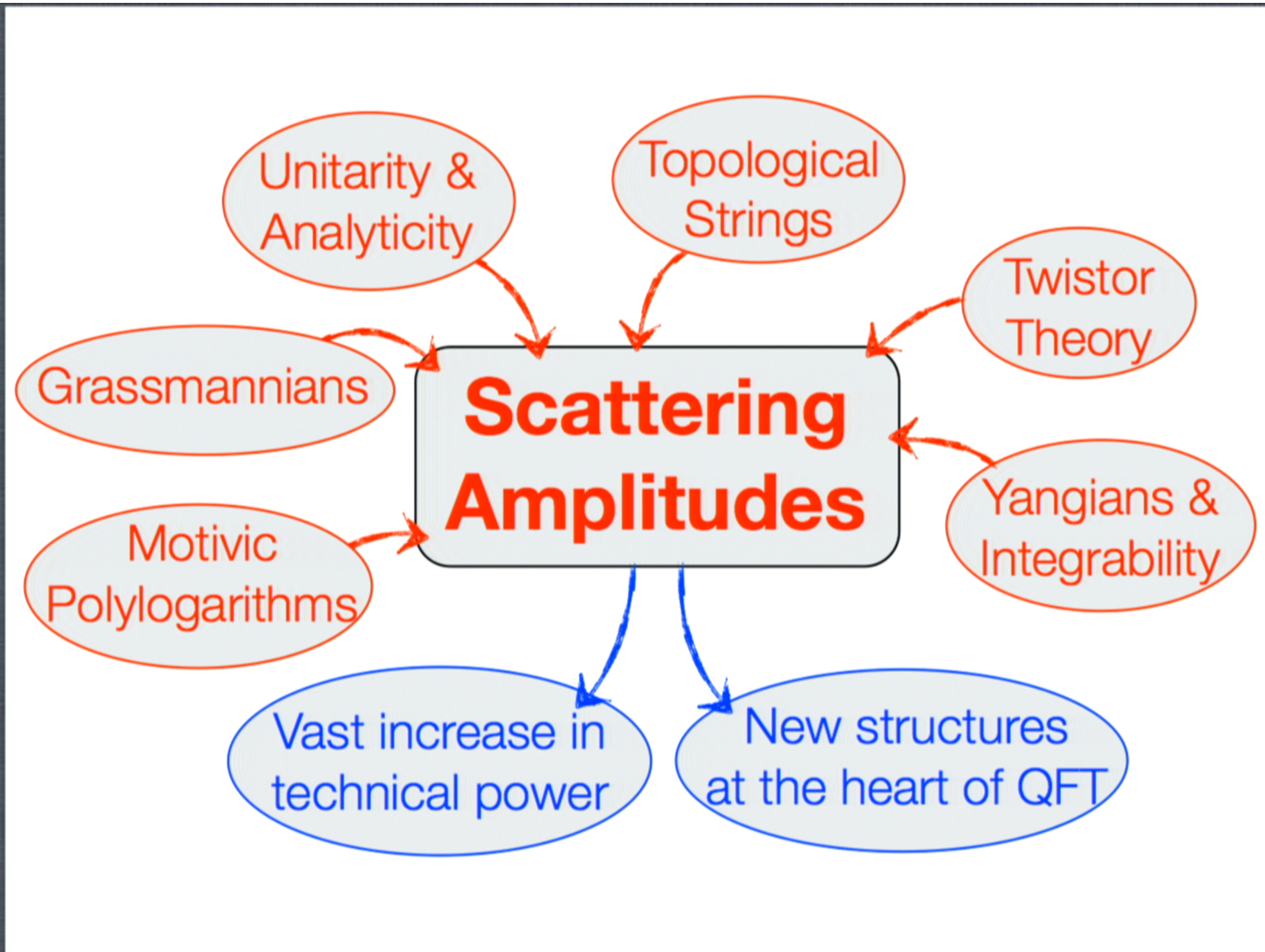
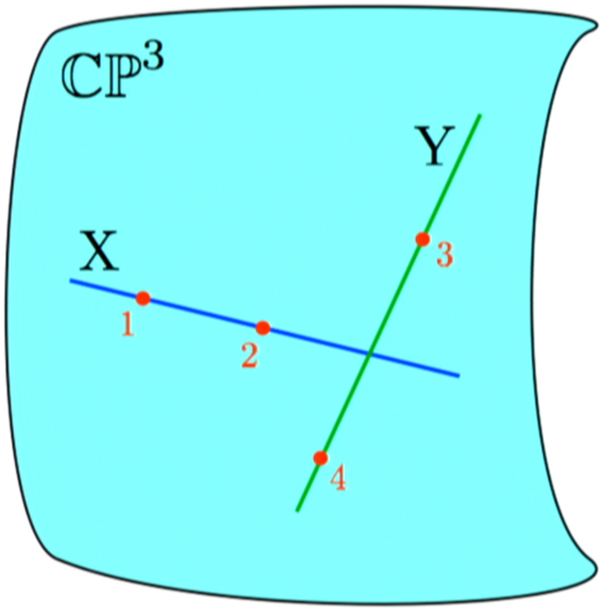
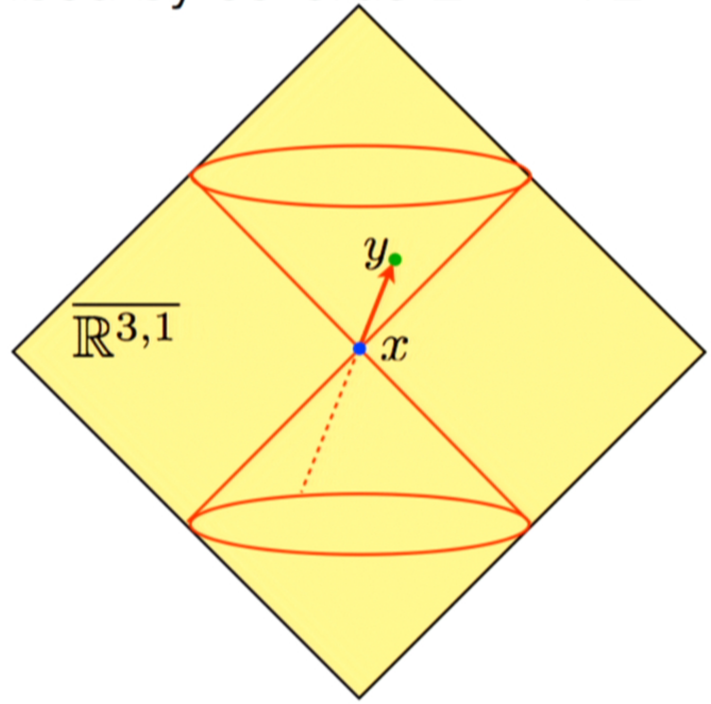

Unitarity &
Analyticity
Topological
Strings
Twistor
Theory
Grassmannians
Scattering
Amplitudes
Yangians &
Integrability
Motivic
Polylogarithms
Vast increase in
technical power
New structures
at the heart of QFT

Twistor space is $\mathbb{CP}^3$, described by co-ords $Z^a \sim rZ^a$





$\mathbb{CP}^1$ in twistor space $\longleftrightarrow$ point in space-time

two lines intersect $\longleftrightarrow$ separation is null

$$\mathrm{X}^{ab} = Z_1^{[a} Z_2^{b]} \qquad \mathrm{Y}^{cd} = Z_3^{[c} Z_4^{d]} \qquad \epsilon(1,2,3,4) \propto (x-y)^2$$

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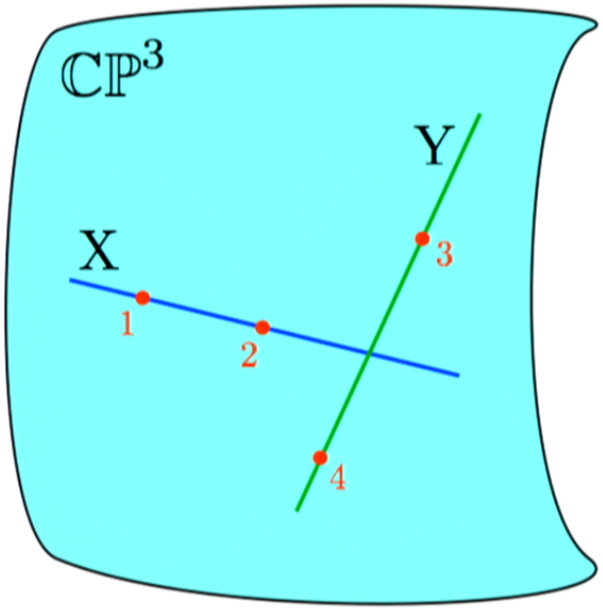

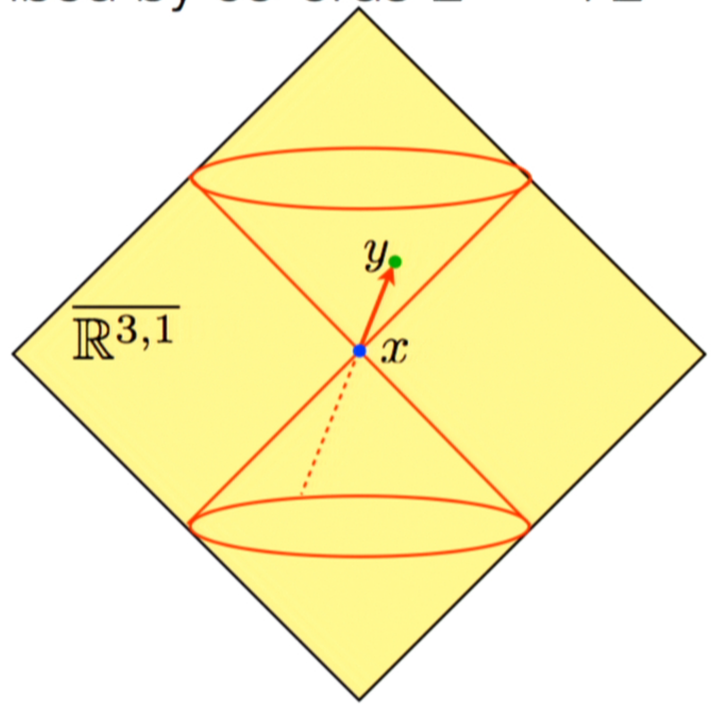

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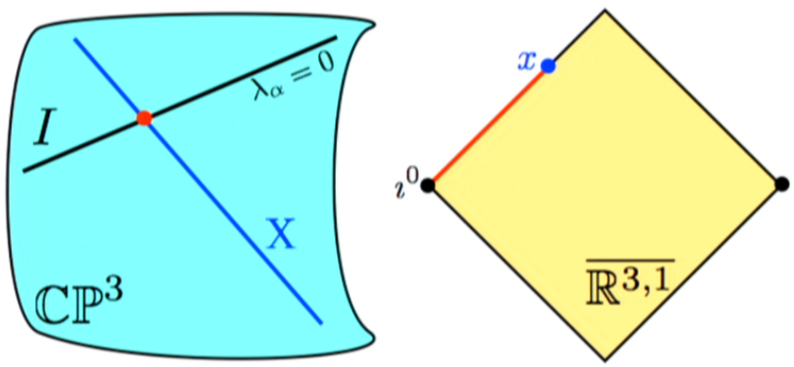
To define a metric, not just a conformal structure, we must also choose an ***infinity twistor*** $I^{ab} = I^{[ab]}$

For flat space-time the infinity twistor represents a line. In



terms of the coords

$$Z^a = (\mu^{\dot\alpha}, \lambda_\alpha),$$

$$I^{ab} = \begin{pmatrix} \epsilon^{\dot\alpha\dot\beta} & 0 \\ 0 & 0 \end{pmatrix}$$

and is the line $\lambda_\alpha = 0$

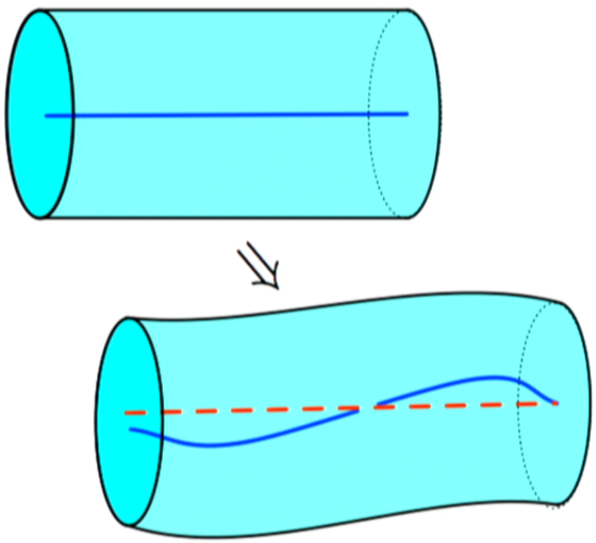
I breaks conformal invariance and sets a ***mass scale***

$$(x-y)^2 = \frac{\epsilon(1,2,3,4)}{\langle 12\rangle \langle 34\rangle} \qquad \langle ij\rangle := \epsilon_{abcd} I^{ab} Z^c_{(i)} Z^d_{(j)}$$

To describe gravity, we deform the complex structure

$$\bar{\partial} \longrightarrow \bar{\partial} + V \qquad V \in H^{0,1}(\mathbb{PT}, T_{\mathbb{PT}})$$

[Penrose; Ward; Atiyah,Hitchin,Singer]



Arbitrary deformations give ***conformal*** gravity. To yield a vacuum Einstein metric, V must be Hamiltonian

$$V = \{h,\ \} = I^{ab}\frac{\partial h}{\partial Z^a}\frac{\partial}{\partial Z^b}$$

w.r.t. the Poisson bracket defined by the infinity twistor

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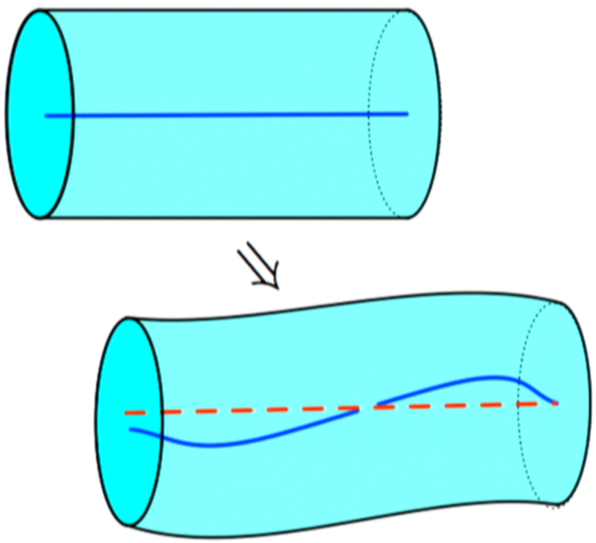

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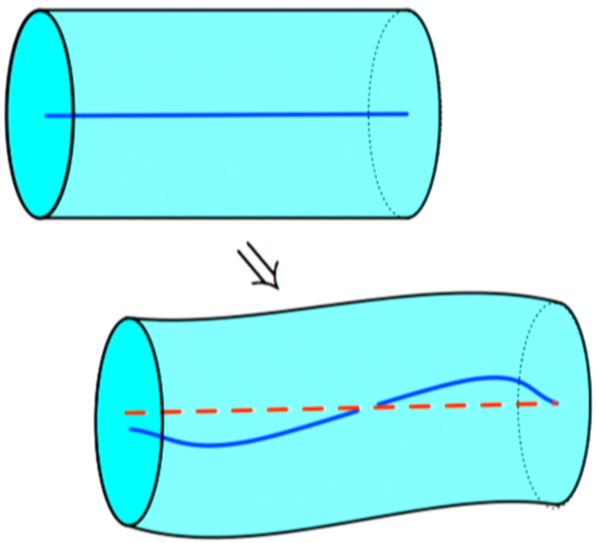

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The infinity twistor is also important in governing the structure of scattering amplitudes

When written on twistor space, the n-particle, g-loop amplitude with $n_\pm$ external gravitons of helicity ± 2 is a monomial with

parity

$n_+ + g - 1$ powers of $I^{ab} \leftrightarrow [\ ,\ ]$

and

$n_- + g - 1$ powers of $I_{ab} \leftrightarrow \langle\ ,\ \rangle$

- g-loop, n-pt Feynman diagram $\propto \kappa^{n+2g-2}$. In twistor space, each κ is accompanied by an infinity twistor
- parity exchanges $[\ ,\ ]$ with $\langle\ ,\ \rangle$
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All ***MHV*** tree amplitudes in $\mathcal{N}=8$ sugra are given by

$$\mathcal{M}_n^{\mathrm{MHV}} = \delta^{4|16}\left(\sum_{i=1}^{n} p_i\right) \frac{\|\mathrm{H}\|^{ijk}_{rst}}{\langle ij\rangle\langle jk\rangle\langle ki\rangle\ \langle rs\rangle\langle st\rangle\langle tr\rangle}$$

on momentum space, where H is the symmetric matrix

$$\mathrm{H}_{ij} = \frac{[ij]}{\langle ij\rangle} \qquad \mathrm{H}_{ii} = -\sum_{j\neq i} \mathrm{H}_{ij} \frac{\langle pj\rangle\langle qj\rangle}{\langle p\,i\rangle\langle q\,i\rangle}$$

[Hodges]

Permutation symmetric ***without explicit sum!***

- determinant suggests correlator of fermion bilinears

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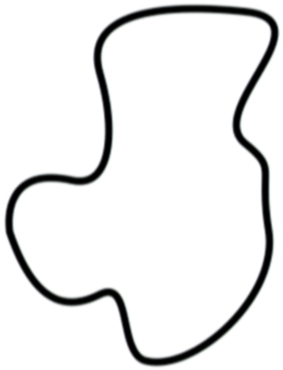
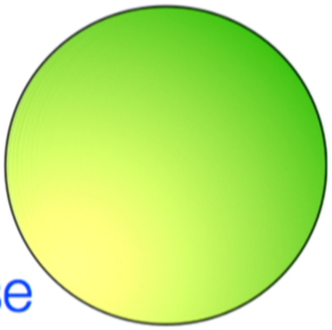
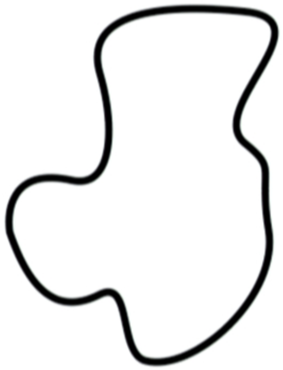
The worldsheet theory



Usually in string theory, worldsheet oscillations lead to an infinite (Regge) tower of extra states, entering at a scale set by the string tension.



The twistor string cannot oscillate, because its tension is infinite. Instead, we see the extended structure because it wraps non-contractible cycles.



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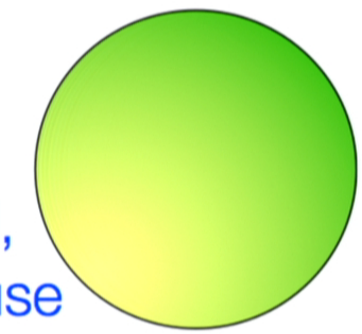
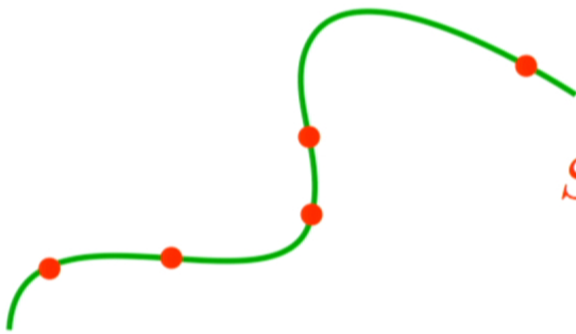


Like the Berkovits - Witten twistor string, the model is based on holomorphic maps to twistor space, here with $\mathcal{N} = 8$ supersymmetry



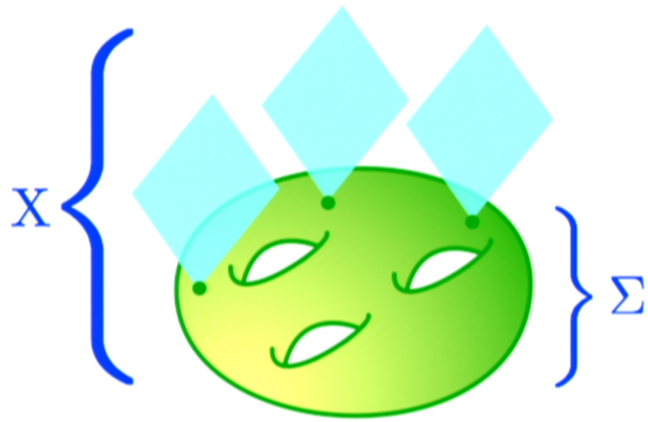
$$S = \int_\Sigma Y_I(\bar{\partial} + \bar{A})Z^I + \cdots$$

$\mathfrak{gl}(1)$ gauge field

Additional fields needed to:

- introduce dependence on infinity twistor
- provide worldsheet version of Hodges' matrix
- cancel anomalies ($\mathbb{CP}^{3|8}$ is not sCY)

Extend Σ to a 1|2-dimensional supermanifold $\mathrm{X} \to \Sigma$, described locally by coords (x, θ^a)



Vectors $\mathcal{V}^a(x,\theta)\dfrac{\partial}{\partial\theta^a}$ in fermionic directions obey $\mathfrak{sl}(1|2)$ algebra

- four bosonic & four fermionic generators
- maximal bosonic subalgebra $\mathfrak{gl}(2)_R \cong \mathfrak{gl}(1) \oplus \mathfrak{sl}(2)$

twist by $\mathfrak{gl}(1)$ scaling of target

θ^a have conformal weight -½ (as in RNS) & charge +1

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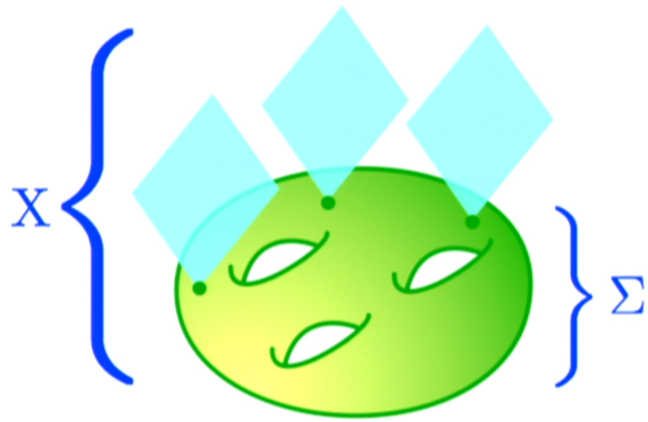

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The matter & ghost fields are

$$\mathcal{Z}^I(x,\theta) = Z^I(x) + \theta^a \rho_a^I(x) + \theta^2 Y^I(x)$$

$$C^a(x,\theta) = \gamma^a(x) + \theta^b N^a_{\ b}(x) + \theta^2 \nu^a(x)$$

$$B_a(x,\theta) = \mu^a(x) + \theta^b M_{ab}(x) + \theta^2 \beta_a(x)$$

In the gauge $\bar{A}_{\mathfrak{sl}(1|2)} = 0$, the worldsheet action is

$$S = \int_X \mathrm{d}^{1|2}x \, \langle \mathcal{Z}, \bar{\partial}\mathcal{Z} \rangle + B_a \bar{\partial} C^a$$

while the (classically) nilpotent BRST operator is

$$Q = \oint \mathrm{d}^{1|2}x \, \langle \mathcal{Z}, C^a \partial_a \mathcal{Z} \rangle - \frac{1}{2} B_a [C, C]^a$$

- BRST operator depends on the infinity twistor $\langle \; , \; \rangle$ breaking conformal invariance

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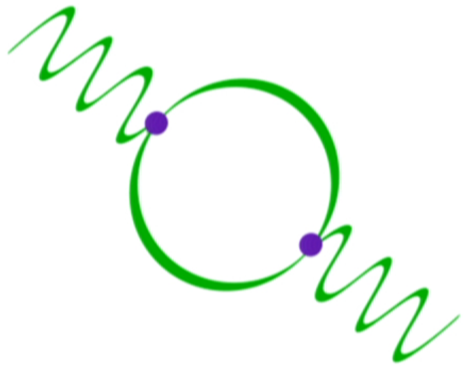
Gauge anomalies cancel iff twistor space has $\mathcal{N} = 8$ supersymmetry



GL(1) anomaly:

$$\sum_i (-1)^{F_i} q_i^2 = (\underset{YZ}{4 - \mathcal{N}}) + \underset{\beta\gamma}{2} + \underset{\mu\nu}{2}$$

SL(2) anomaly:

$$\sum_i \frac{(-1)^{F_i}}{|\mathrm{Aut}\Gamma_i|} \mathrm{tr}_{R_i}(t \cdot t) = \frac{3}{4}(\mathcal{N} - 8)$$

- involves both ghosts and matter; cancellation not solely due to supersymmetry of target space

Positively charged fields have zero modes:

$\mathcal{Z}^I : \; d+1-g$

selection rule relating
MHV level to degree of curve
$n_- = d+1-g$

$\gamma^a : \; d+2-2g$

zero modes of bosonic ghost -
fix residual fermionic symmetry
$\#\gamma_{\mathrm{zm}} = n - \#[\;,\;]$

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Matter vertex operators are similar to RNS string:

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for 'fixed' vertex operators. Integrated operators are

$$V \equiv \int \mathrm{d}^2\theta\, h(\mathcal{Z}) = \int_\Sigma \left[\frac{\partial h}{\partial Z}, Y\right] - \rho^I \frac{\partial}{\partial Z^I}\left[\bar{\rho}, \frac{\partial h}{\partial Z}\right]$$

describing deformations of the worldsheet action

- h is the twistor wavefunction of an $\mathcal{N}=8$ graviton

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$$V \equiv \int \mathrm{d}^2\theta\, h(\mathcal{Z}) = \int_\Sigma \left[\frac{\partial h}{\partial Z}, Y\right] - \rho^I \frac{\partial}{\partial Z^I}\left[\bar{\rho}, \frac{\partial h}{\partial Z}\right]$$

describing deformations of the worldsheet action

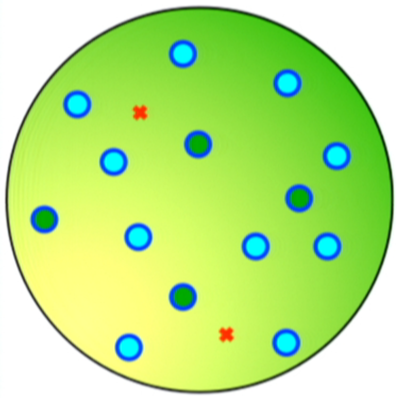
- h is the twistor wavefunction of an $\mathcal{N} = 8$ graviton

Picture changing operators (associated to μ zm) are

$$\Upsilon \equiv \prod_{a=1,2} [Q, \Theta(\mu_a)] = \delta^2(\mu)\langle\rho, Z\rangle\, \bar{\rho}_I Z^I + \cdots$$

All tree-level amplitudes in $\mathcal{N}=8$ supergravity come from the $g=0$ twistor string correlator

[,] dependence lives here



$$\left\langle cU_1 cU_2\, cU_3 \prod_{i=4}^{d+2} \int U_i \prod_{j=d+3}^{n} \int V_j \prod_{k=1}^{d} \Upsilon \right\rangle$$

⟨ , ⟩ dependence from here

- ‘fixed’ vertex op
- integrated vertex op
- PCO

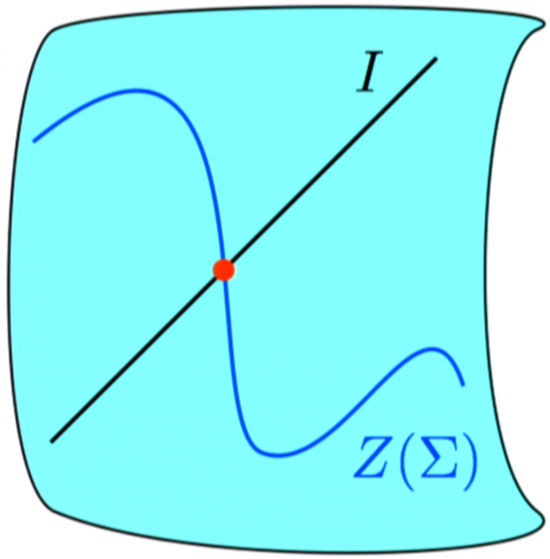
Correlator of PCOs is independent of insertion points

$$\left\langle \prod_{k=1}^{d} \Upsilon(x_k) \right\rangle = \mathrm{R}(\lambda_\alpha)$$

$\delta^2(\mu)\langle \rho Z\rangle \bar{\rho} Z$

the ***resultant*** of the two λ_α components of $Z : \Sigma \to \mathbb{CP}^{3|8}$

[Cachazo]



$\mathrm{R}(\lambda_\alpha) = 0 \quad \longleftrightarrow \quad \lambda_\alpha(x_*) = 0$ for some $x_* \in \Sigma$

$\lambda_\alpha = 0$ is the line I at infinity

The amplitude thus lives on holomorphic curves in $\mathbb{CP}^{3|8} - I$, the 'inside' of space-time

[Casali,DS; Cachazo,He,Yuan]

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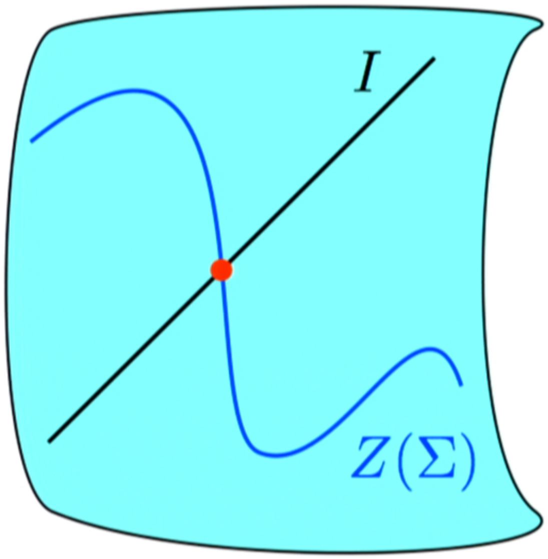

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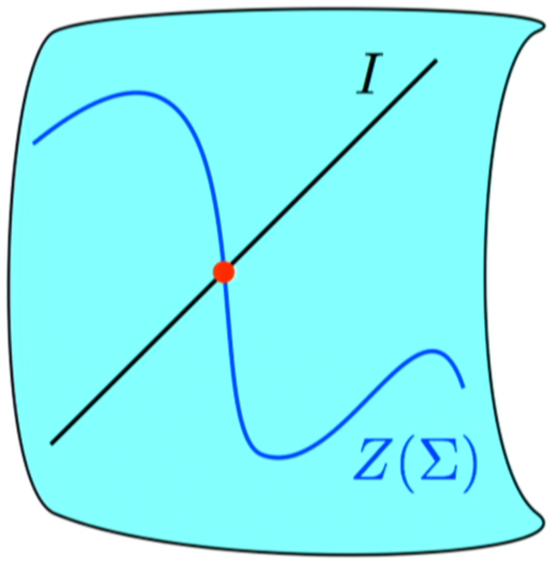

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The remaining correlator of matter vertex operators

$$\left\langle \prod_{k=1}^{d+2} \delta^2(\gamma) h_k \prod_{\ell=d+3}^{n} \left(\left[Y, \frac{\partial h_\ell}{\partial Z} \right] + \rho \frac{\partial}{\partial Z} \left[\bar{\rho}, \frac{\partial h_\ell}{\partial Z} \right] \right) \right\rangle = \frac{\|\Phi\|^{r_1 \cdots r_{d+2}}_{c_1 \cdots c_{d+2}}}{\|\omega_j(x_{r_k})\| \; \|\omega_l(x_{c_m})\|}$$

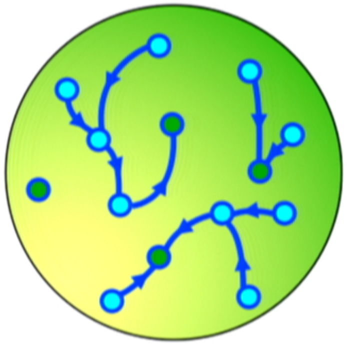
provides a worldsheet generalization of Hodges' matrix, but now valid for ***all N^kMHV amplitudes***

- $$\Phi_{ij} = \frac{1}{x_{ij}} \left[\frac{\partial}{\partial Z_i}, \frac{\partial}{\partial Z_j} \right] \qquad \Phi_{ii} = -\sum_{j \neq i} \Phi_{ij} \prod_{a=0}^{d} \frac{y_a - x_j}{y_a - x_i}$$

  $\bar{\rho}\rho$ contractions $\qquad$ YZ contractions

- $\{\omega_i(x)\}$ is a basis of the space of γ zero modes
- fixed vertex operators correspond to rows & columns absent from $\|\Phi\|^{r_1 \cdots r_{d+2}}_{c_1 \cdots c_{d+2}}$

What do these determinants actually mean?

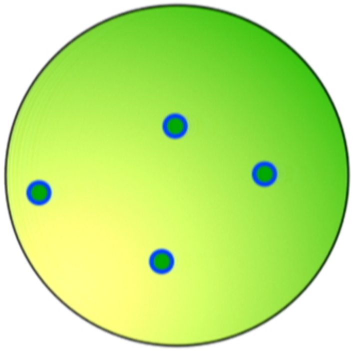


Rather than computing

$$\left\langle \prod_{k=1}^{d+2} \delta^2(\gamma) h_k \prod_{\ell=d+3}^{n} \left(\left[Y, \frac{\partial h_\ell}{\partial Z} \right] + \rho \frac{\partial}{\partial Z} \left[\bar{\rho}, \frac{\partial h_\ell}{\partial Z} \right] \right) \right\rangle$$

using the original free action, we can instead compute

$$\left\langle \prod_{k=1}^{d+2} \delta^2(\gamma) h_k(Z) \right\rangle$$



using the nonlinear action

$$S' = \int_\Sigma Y_I \left(\bar{\partial} Z^I + I^{IJ} \frac{\partial h}{\partial Z^J} \right) \ + \ \text{fermions}$$

obtained by exponentiating an integrated vertex operator

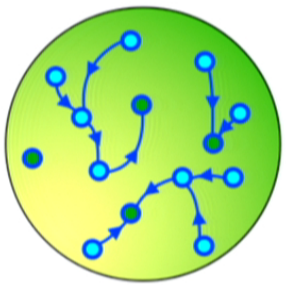
Path integral over Y imposes $\bar{\partial} Z^I + \{h\,, Z^I\} = 0$

- perform field redefinition to $Z'(x)$, defined implicitly by

$$\bar{\partial} Z'^I(x) = \bar{\partial} Z^I(x) + \{h\,, Z^I(x)\}$$

- Jacobian provided by fermion path integral *(c.f. Nicolai map)*

Expanding $h(Z(Z'))$ in fixed vertex op^s "grows a tree"

$\sum$ 

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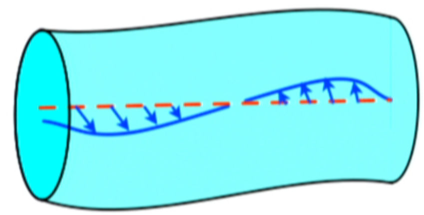

perturbative description of nonlinear graviton background

[Adamo,Mason; Casali,DS]

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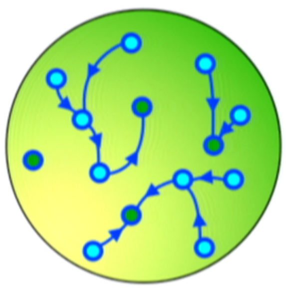
$\sum$ 

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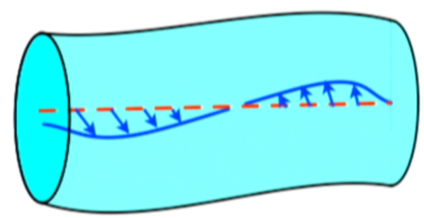

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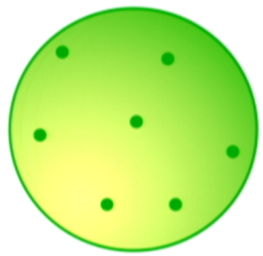
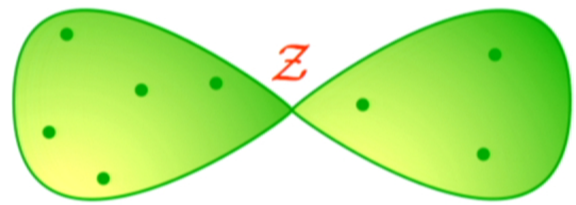
How do we know this formula is correct?

- in twistor space, statement that poles of amplitude occur whenever $(\sum_{\text{subset}} p_i)^2 = 0$ becomes

$\bar{\partial}\Big($  $\Big) = \sum \int \mathrm{D}^{3|8}\mathcal{Z}$ 

corresponding to boundary divisor in $\overline{\mathcal{M}}_{0,n}(\mathbb{PT}, d)$
[Gukov,Motl,Neitzke; Vergu; DS]

- also has correct asymptotics under BCFW shift, so satisfies BCFW recursion relations[Cachazo,Mason,DS]; at a physicist's level of rigour, this is a proof.

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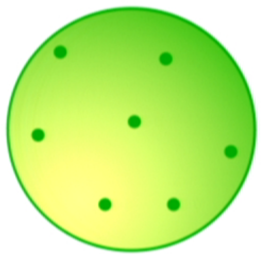
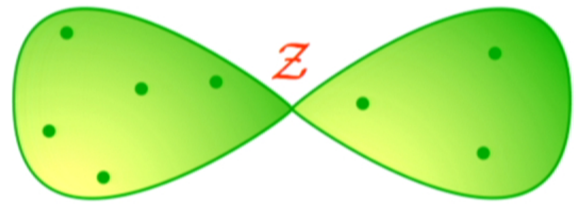
$\bar{\partial}\Big($  $\Big) = \sum \int \mathrm{D}^{3|8}\mathcal{Z}$ 

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I have presented an holomorphic twistor string that computes the classical S-matrix of maximal supergravity

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- integrated vertex operators give maps to nonlinear graviton

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- behaviour at higher genus?
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