Title: Quantum Cosmology - 3

Date: Jul 23, 2013 04:40 PM

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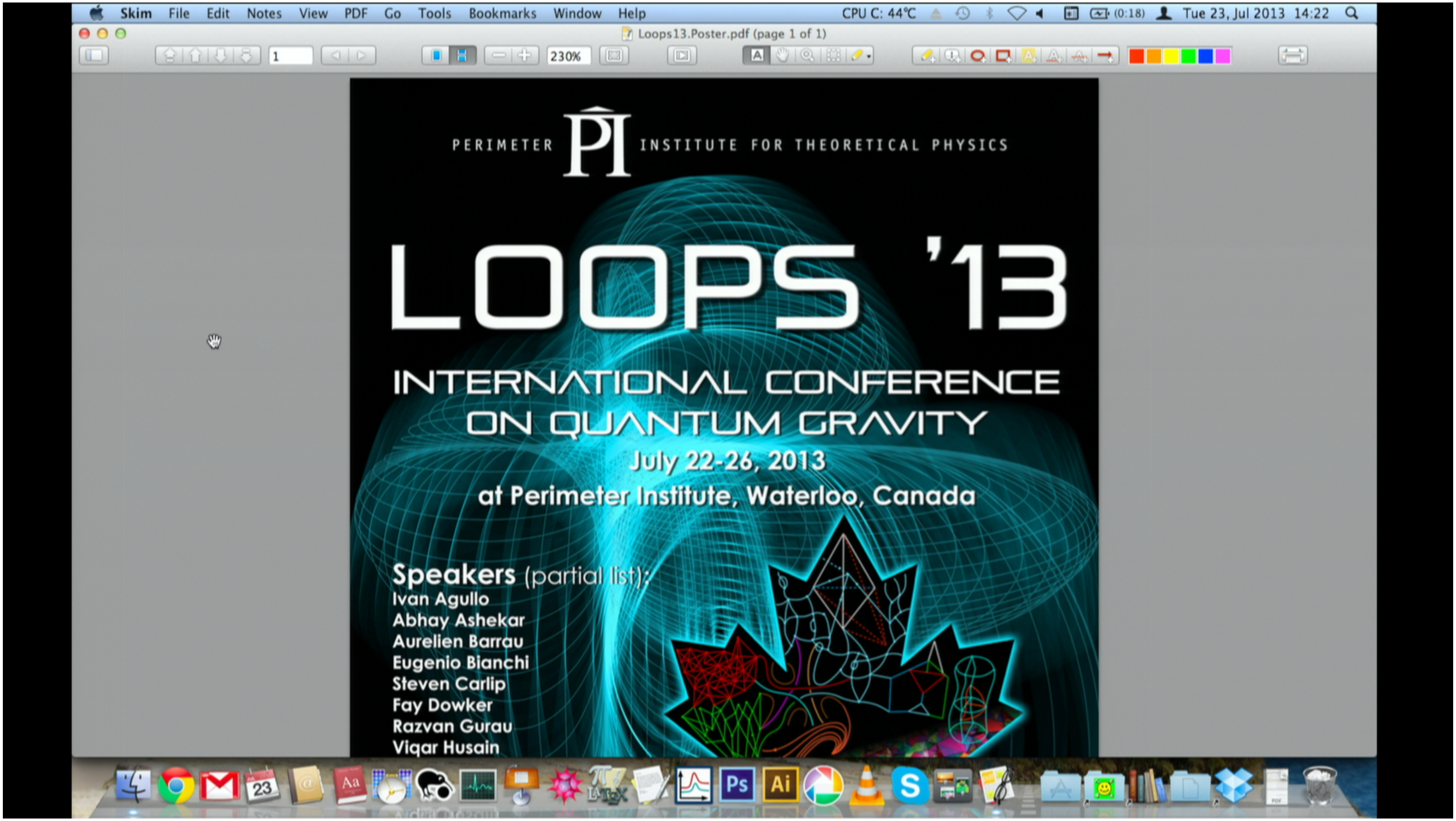
Abstract: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PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS
LOOPS '13
INTERNATIONAL CONFERENCE ON QUANTUM GRAVITY
July 22-26, 2013
at Perimeter Institute, Waterloo, Canada
Speakers (partial list):
Ivan Agullo
Abhay Ashekar
Aurelien Barrau
Eugenio Bianchi
Steven Carlip
Fay Dowker
Razvan Gurau
Viqar Husain

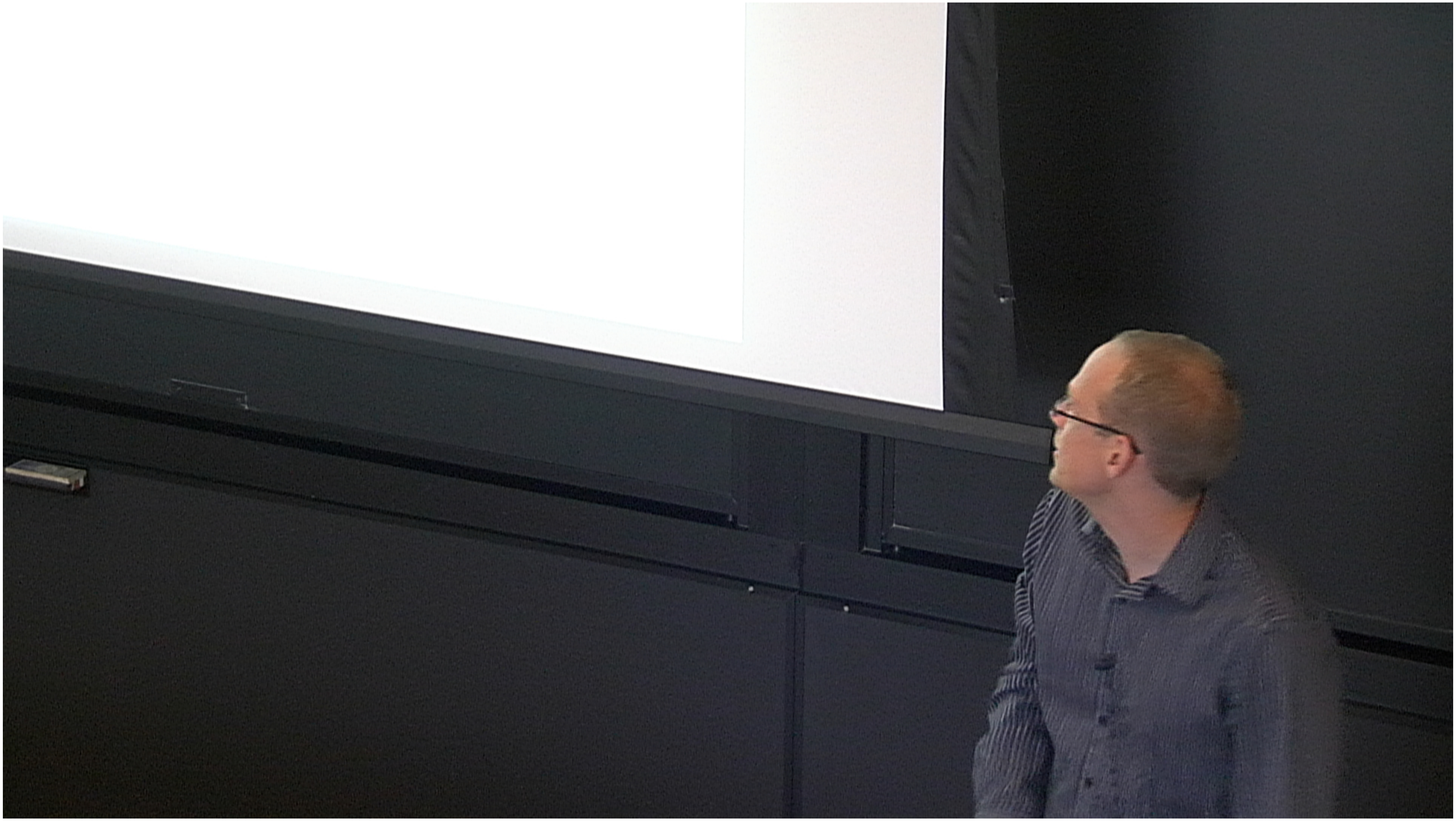
Searching for Other Universes

Matt Johnson
Perimeter Institute/York University

Really?

- An infinite number of individually infinite universes in an infinite expanding background?

Surely I can't be serious!



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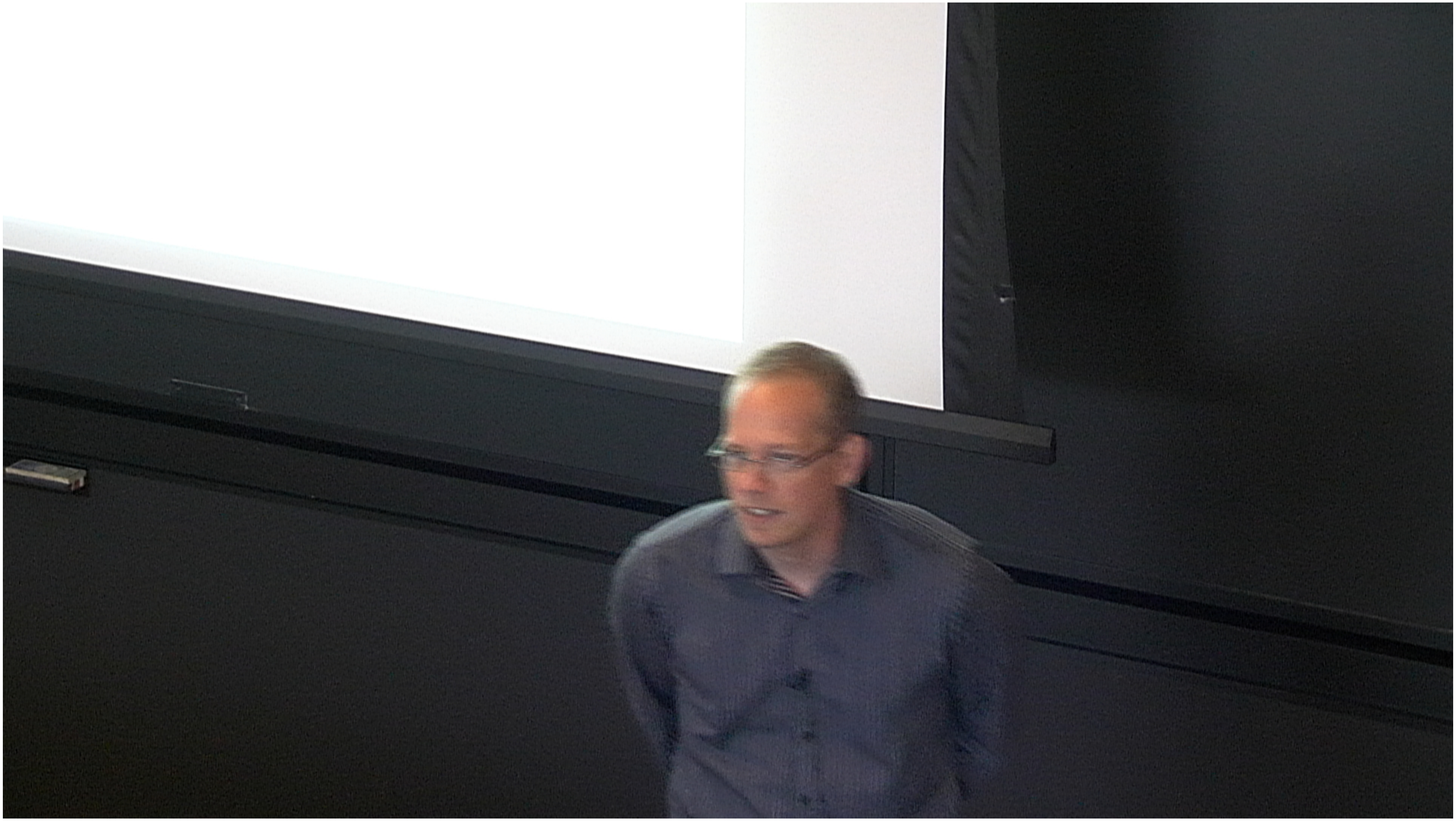
- Eternal inflation is a direct consequence of:

non-unique vacuum
state

(possible in standard model)

(common in BSM physics)

(inevitable in string theory)



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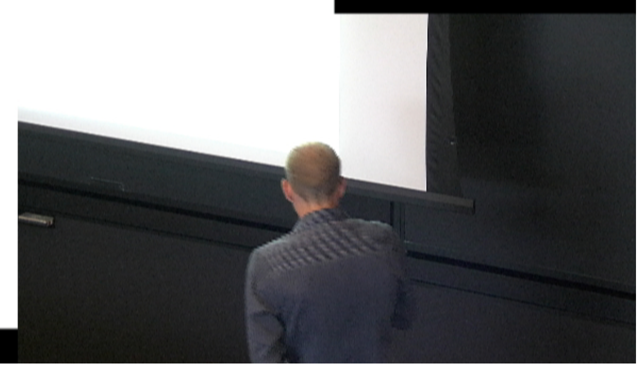

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- Eternal inflation is a direct consequence of:

non-unique vacuum state	Quantum field theory
(possible in standard model)	(works fantastically)
(common in BSM physics)	
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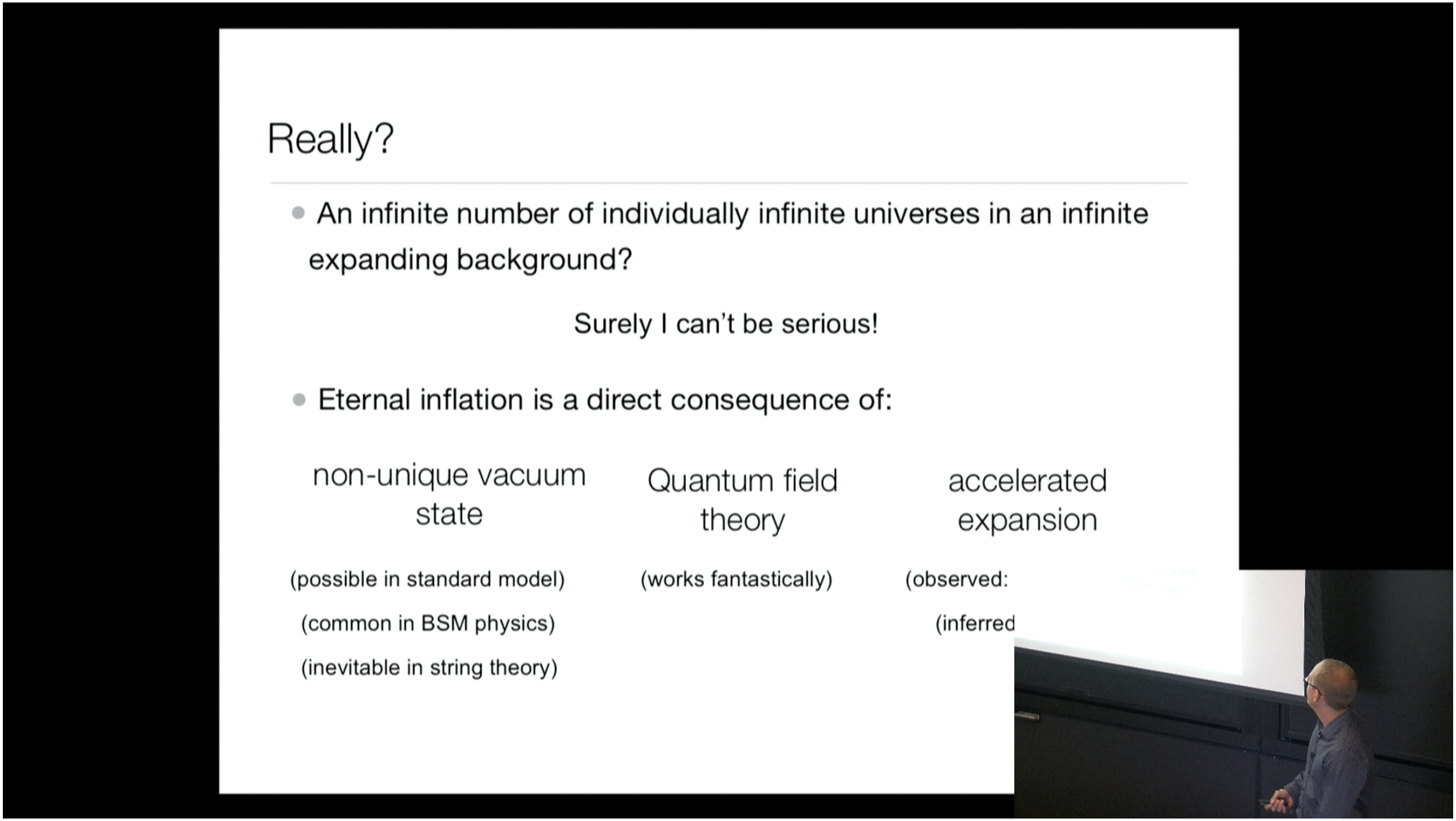
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Eternal inflation is a direct consequence of:
non-unique vacuum state
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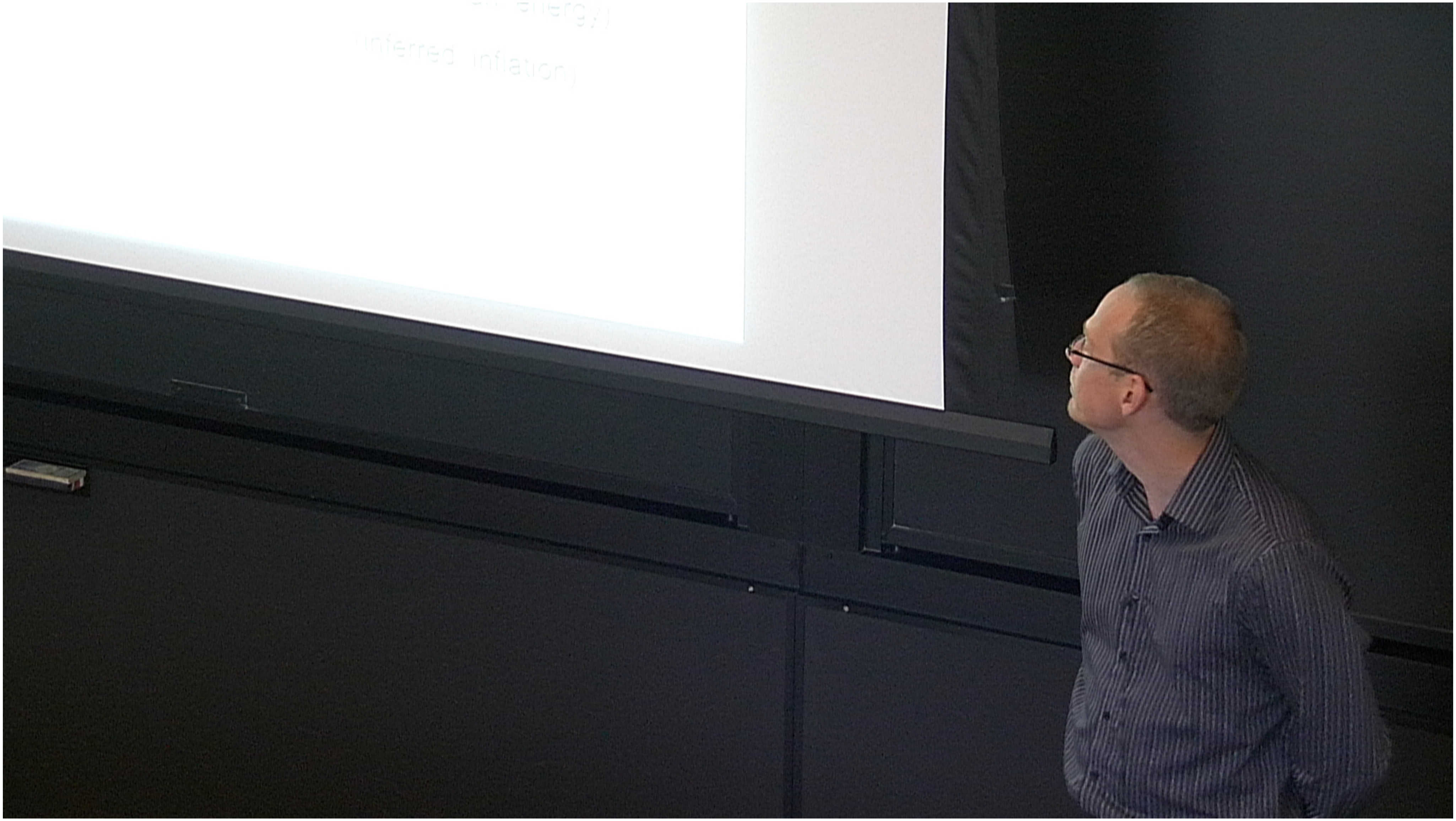
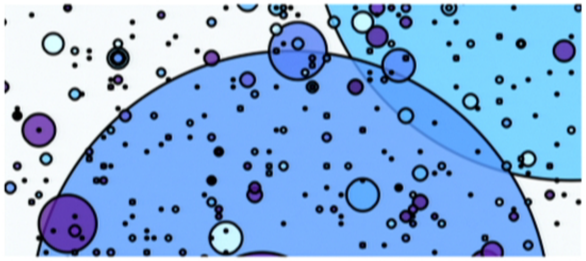
(observed:
(inferred



Observational Tests of Eternal Inflation

- Strong theoretical motivation, but is eternal inflation experimentally verifiable?

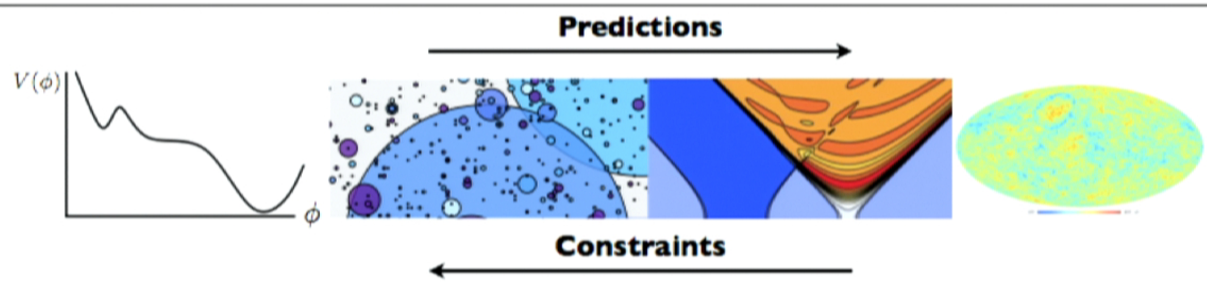
Our bubble does not evolve in isolation....



The collision of our bubble with others provides an observational test of eternal inflation.

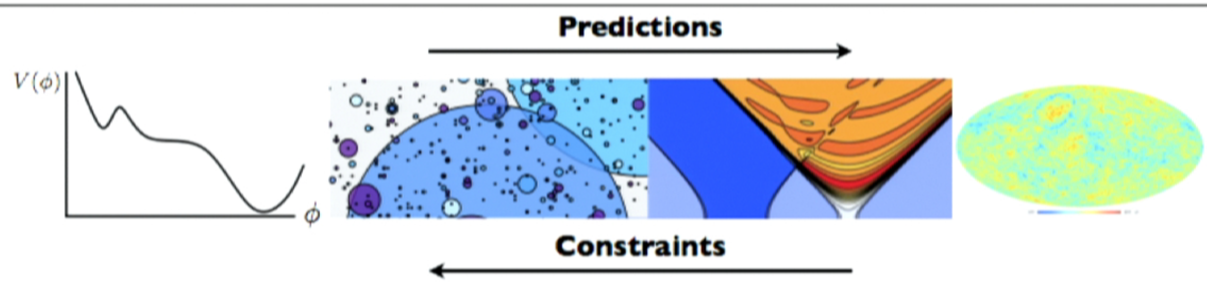
Aguirre, MCJ, Shomer

Making predictions and testing models



Scalar field Lagrangian	Eternal inflation	Bubble collisions	Observational signatures
Possible models? Motivation from theory? Constraints from observation?	Probabilities and inferences? Relation to other versions of the multiverse? How are model parameters manifest?	Predictions from specific models? Phenomenology? Perturbations inside the bubble?	Detailed signature for temperature and polarization? Best data analysis strategies?

Making predictions and testing models



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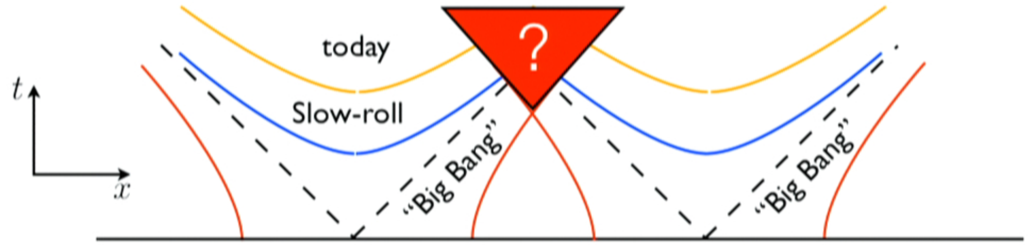
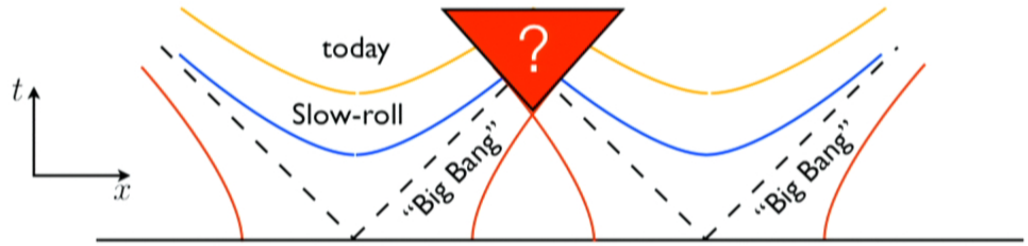
Bubble collisions



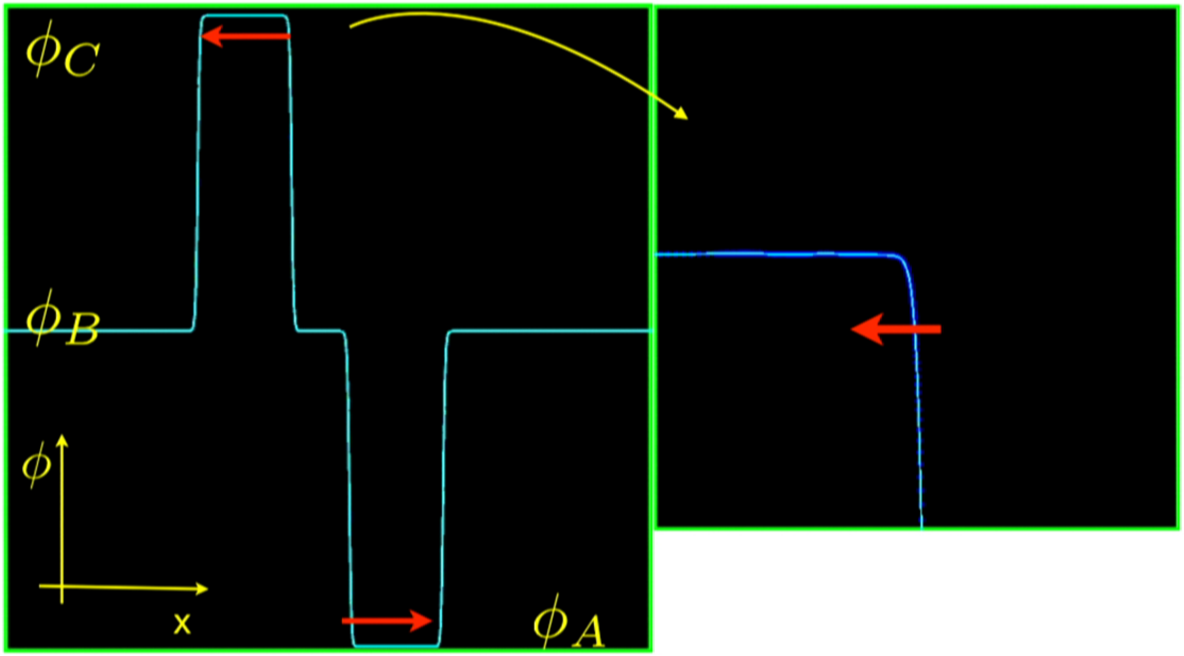
- Collisions are always in our past.
- The outcome is fixed by the potential and kinematics.

Bubble collisions



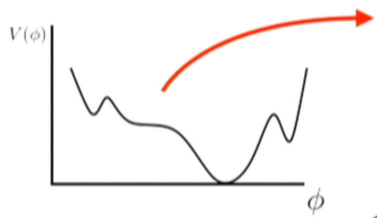
- Collisions are always in our past.
- The outcome is fixed by the potential and kinematics.
- To study what happens, need full GR.
  - We want to find the post-collision cosmology: GR.
  - Huge center of mass energy in the collision.
  - Non-linear potential, non-linear field equations.

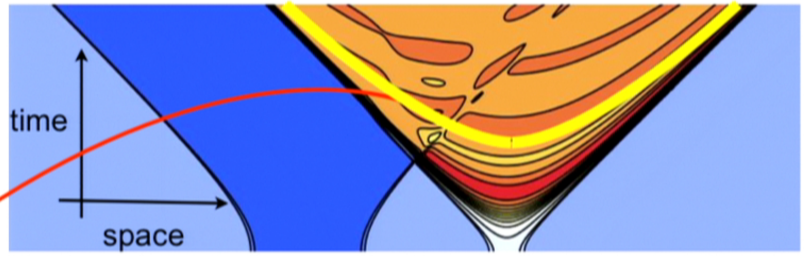
Numerical solutions

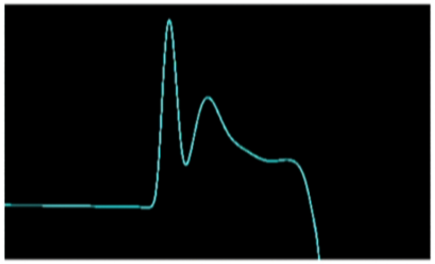


Observational Signatures

- Bubble collisions perturb the epoch of inflation inside our bubble.







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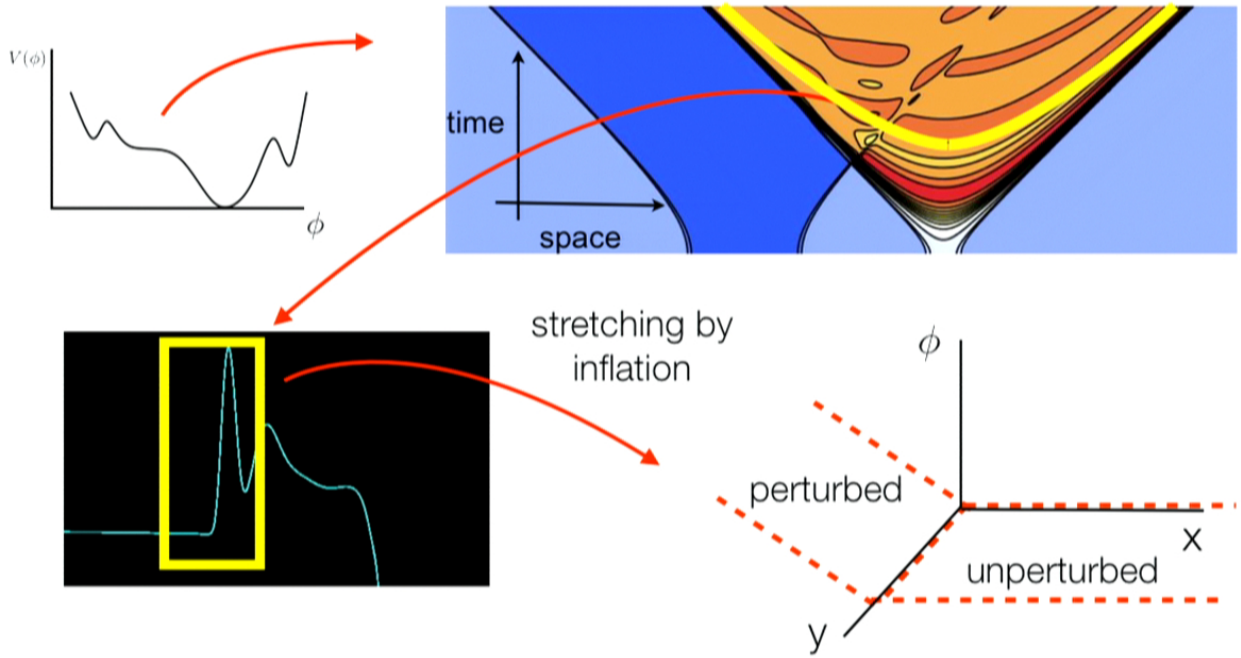
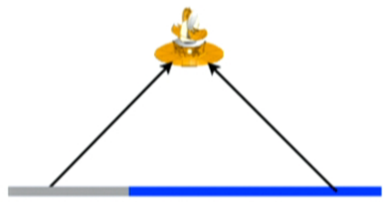


Observational Signatures



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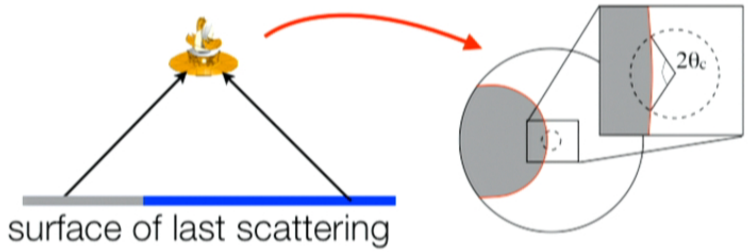

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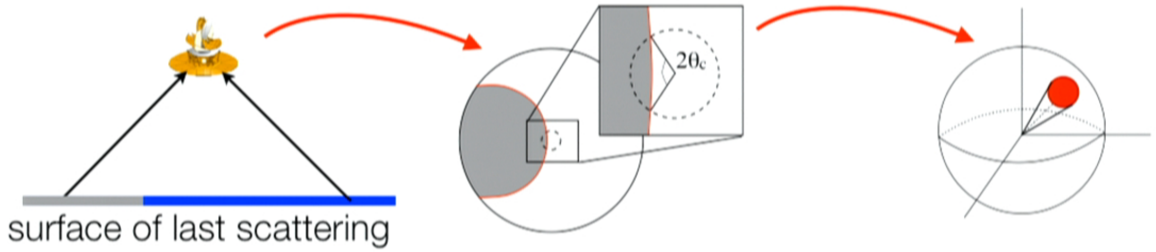
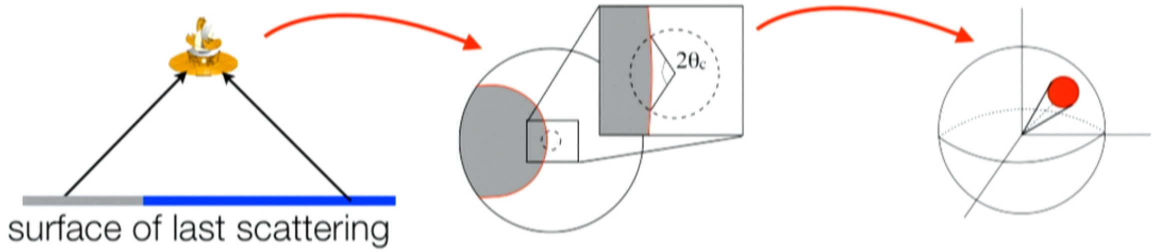


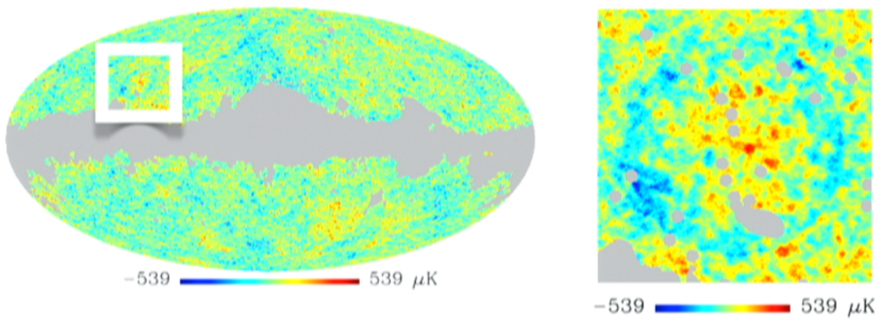
Symmetry+causality: effects confined to a disc.

Observational Signatures



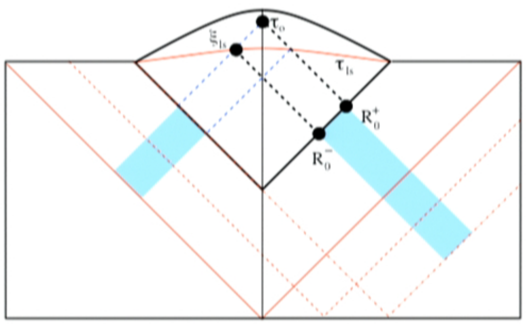
Symmetry+causality: effects confined to a disc.

- Generic signature (thanks inflation!):



Counting collisions

- Counting only collisions whose disc of influence is smaller than the whole sky:



$$N \simeq \frac{16\pi\lambda}{3H_F^4}\left(\frac{H_F^2}{H_I^2}\right)\sqrt{\Omega_c}$$

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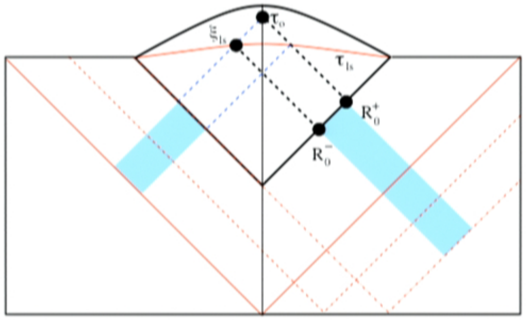

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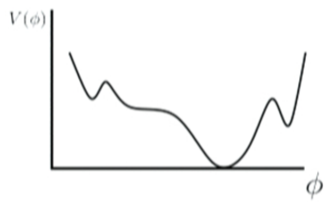
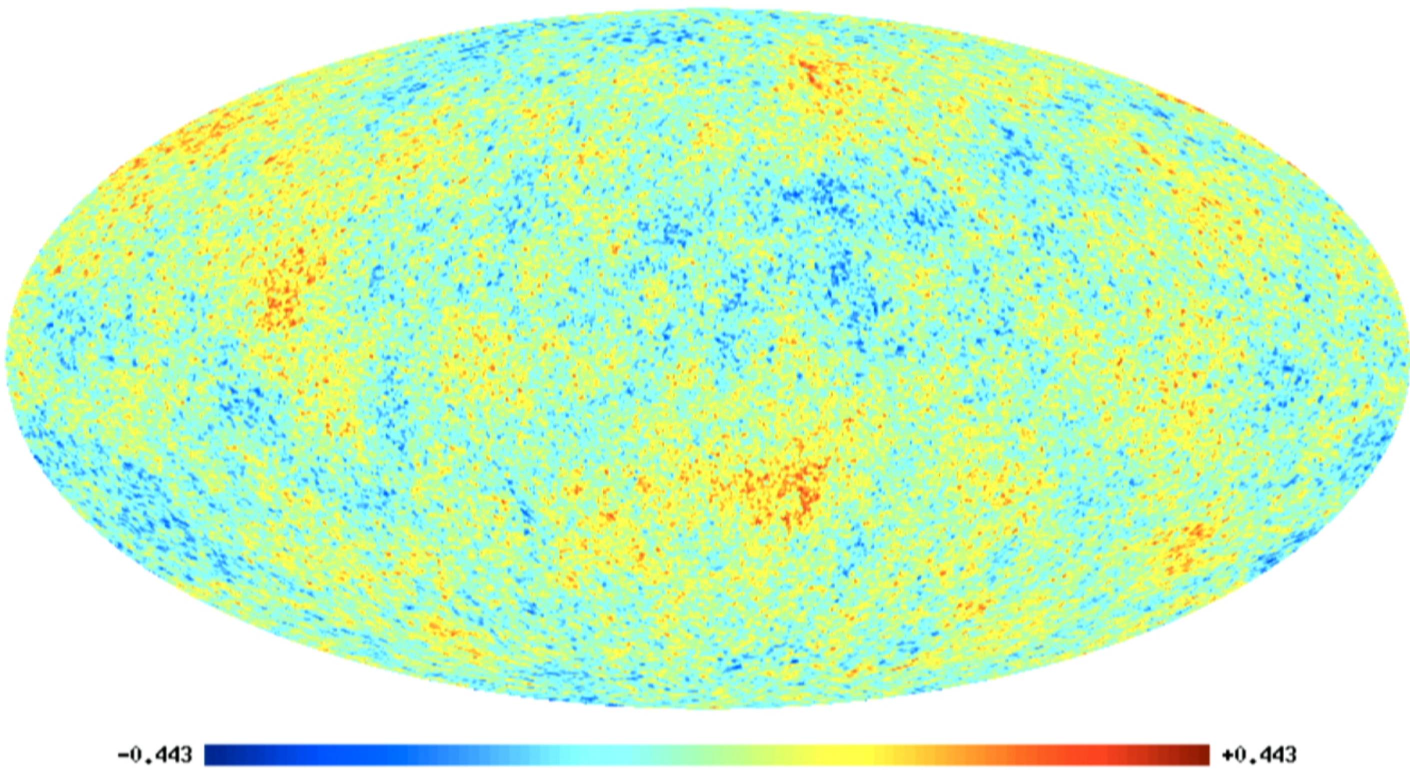
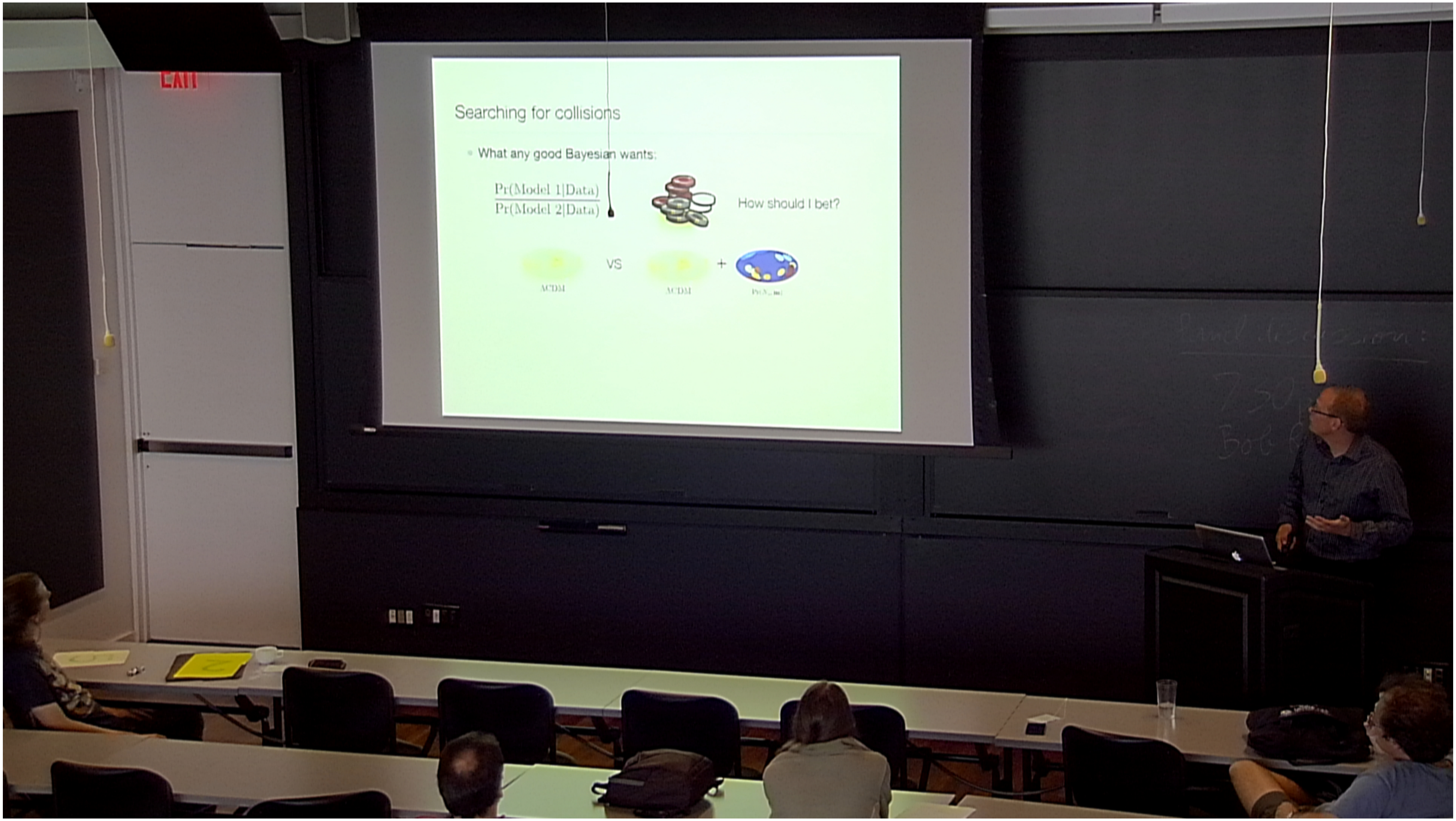
Bubble collisions model

- The model:



Collisions (realistic) + CMB + instrumental noise



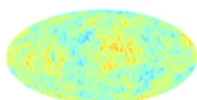

Searching for collisions
What any good Bayesian wants:
Pr(Model 1|Data)
Pr(Model 2|Data)
How should I bet?
VS
+
ΛCDM
ΛCDM

Searching for collisions

- What any good Bayesian wants:

$$\frac{\Pr(\text{Model 1}|\text{Data})}{\Pr(\text{Model 2}|\text{Data})}$$

How should I bet?



ΛCDM

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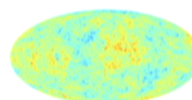
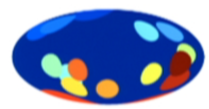


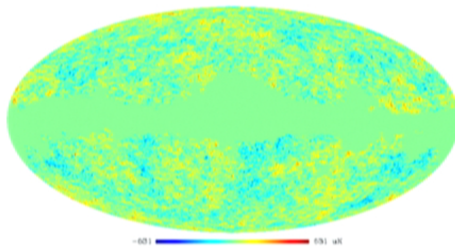
ΛCDM

+

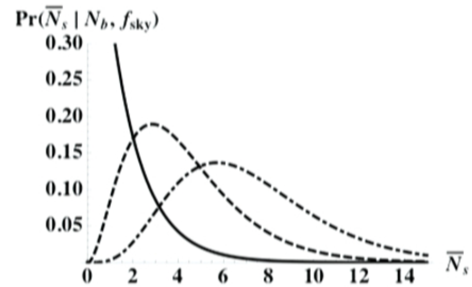


$\Pr(N_s, \mathbf{m})$

Searching for collisions



$$Pr(\bar{N}_s|\mathbf{d})$$



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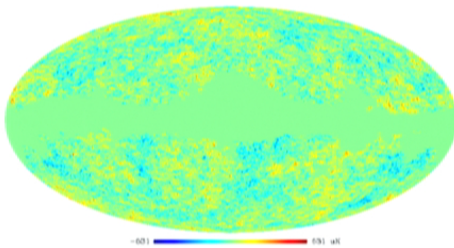

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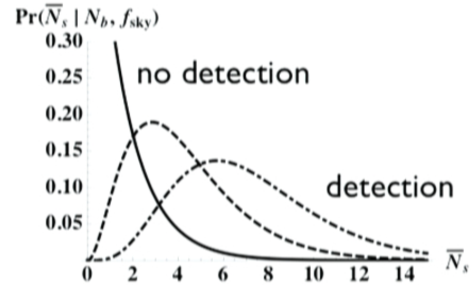
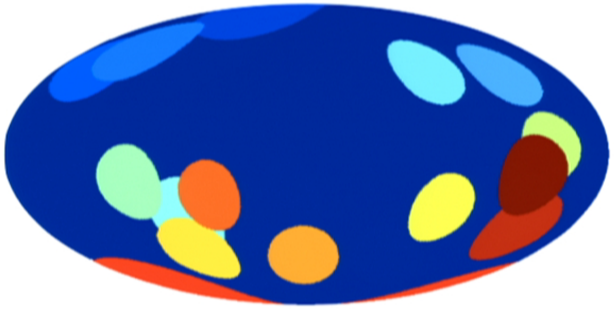


- To calculate this, need to test for:
  - Arbitrary number of templates
  - Arbitrary position on the sky
  - Arbitrary amplitude, shape, and size (lying within prior $\mathrm{Pr}(N_s, \mathbf{m})$)

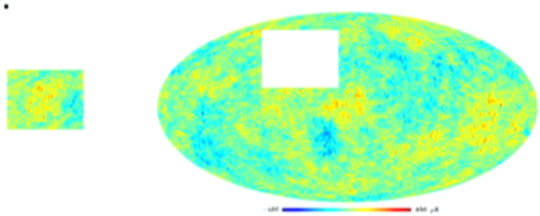
Searching for collisions

- Solution:
  - Locate candidate features with a blind analysis.



Searching for collisions

- For one candidate:



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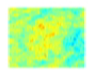
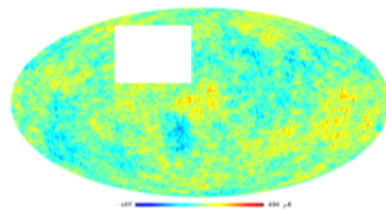




$$\rho_b = \frac{\int d\mathbf{m} \Pr(\mathbf{m}) L_b(\mathbf{d}|\mathbf{m})}{L_b(\mathbf{d}|\mathbf{0})}$$

Evidence ratio in the blob: how much better does one describe the data by adding a template?

- Pixel-based likelihood $L_b(\mathbf{d}|\mathbf{m})$ contains: CMB cosmic variance, beam, and spatially varying noise.
- Flat prior on amplitude and shape, prior on size and position from theory.

Searching for collisions

- The general expression for N_b candidates:

Theory prior

Cosmic variance

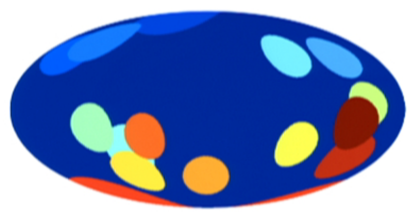
All combos of templates and blobs

$$\Pr(\bar{N}_{\rm s}|\mathbf{d}, f_{\rm sky}) \propto \Pr(\bar{N}_{\rm s})\, e^{-f_{\rm sky}\bar{N}_{\rm s}} \sum_{N_{\rm s}=0}^{N_{\rm b}} \frac{(f_{\rm sky}\bar{N}_{\rm s})^{N_{\rm s}}}{N_{\rm s}!} \sum_{b_1, b_2, \ldots, b_{N_{\rm s}}=1}^{N_{\rm b}} \left[\prod_{s=1}^{N_{\rm s}} \rho_{b_s} \prod_{i,j=1}^{N_{\rm s}} (1-\delta_{s_i,s_j}) \right]$$

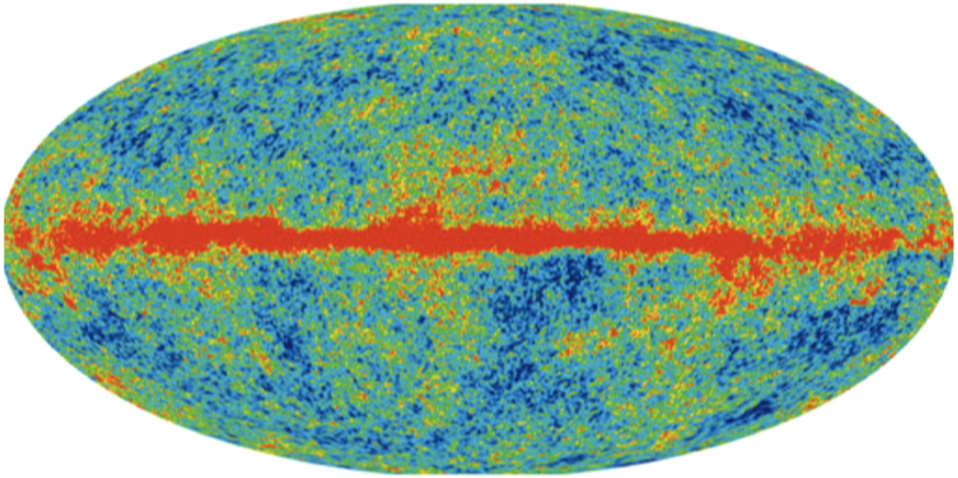
Expected number of features

Poisson process

Evidence ratio in each blob

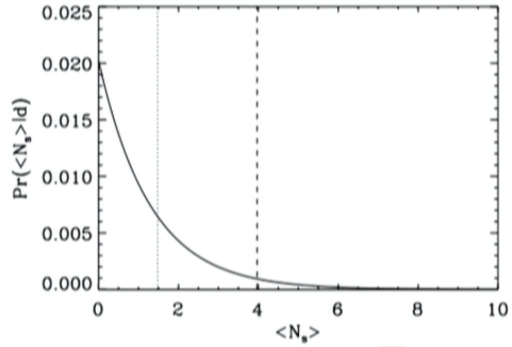


WMAP7 W-Band (94 GHz)



The WMAP7 W-Band data.......

WMAP7 W-Band (94 GHz) : Posterior



- The posterior is peaked around $\bar{N}_s = 0$

The data does not support the bubble collision hypothesis.

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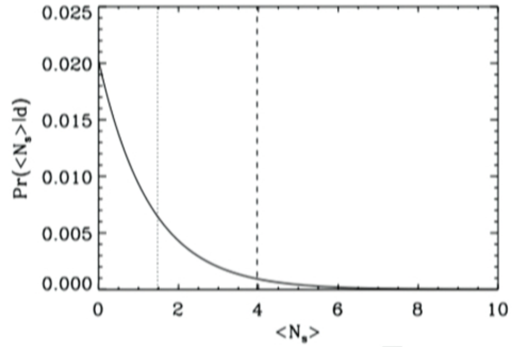

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- From the shape of the posterior, we can rule out

$$\bar{N}_s < 1.6 \text{ at } 68\% \text{ CL}$$

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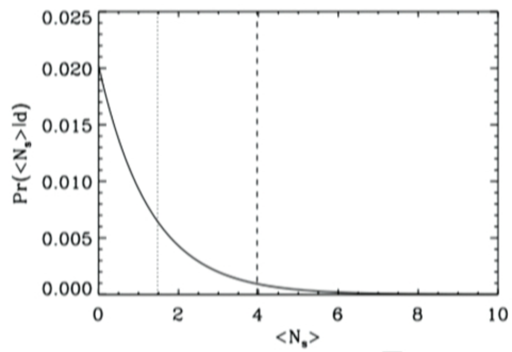

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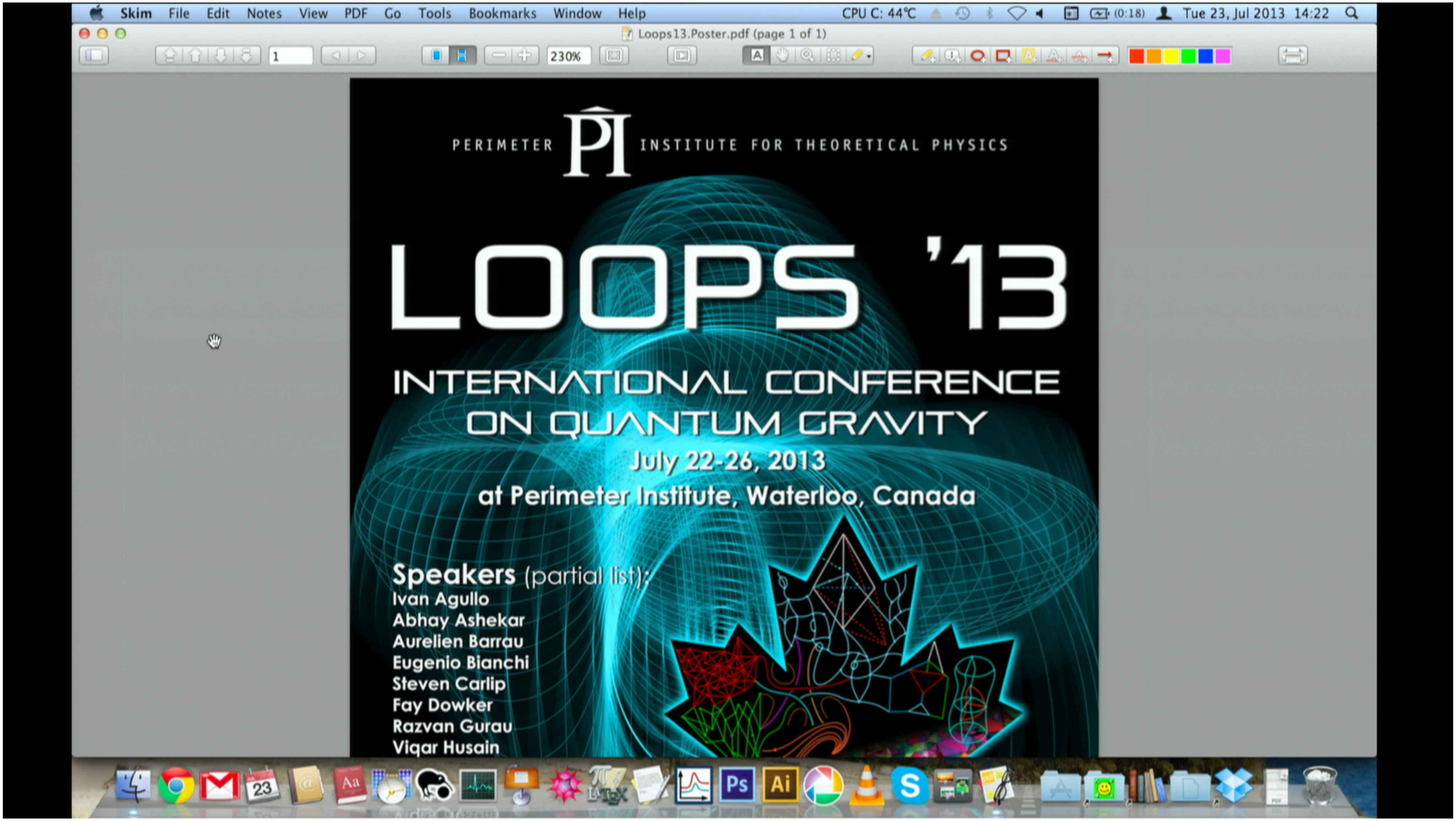
Bob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Viqar Husain

LOOPS '13, Perimeter Institute
July 23, 2013

Potential observational effects from Wheeler–DeWitt quantum cosmology

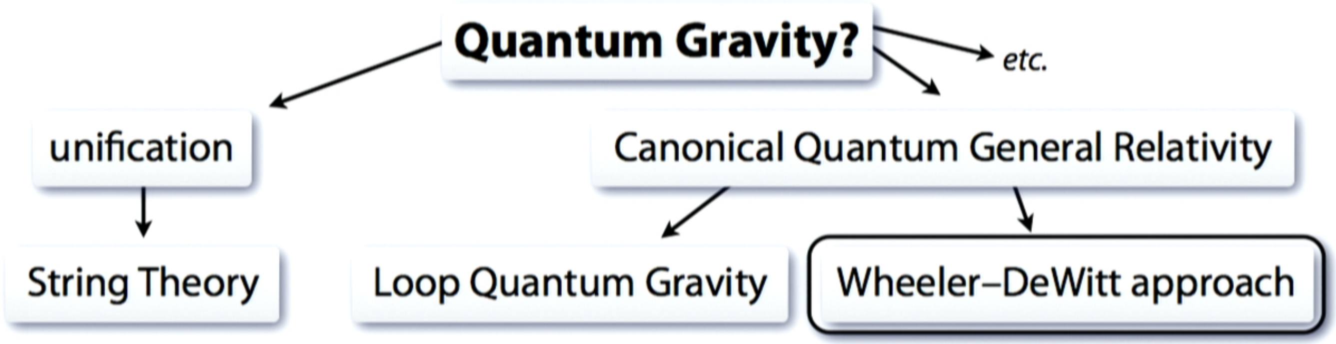
joint work with: **C. Kiefer, D. Bini, G. Esposito, and F. Pessina**

- Phys. Rev. Lett. **108**, 021301 (2012), arXiv:1103.4967.
- Int. J. Mod. Phys. D **21**, 1241001 (2012), arXiv:1205.5161.
- Phys. Rev. D **87**, 104008 (2013), arXiv:1303.0531.

Universität zu Köln

Manuel Krämer
Institute for Theoretical Physics
University of Cologne

The problem with Quantum Gravity



- observational guidance needed to distinguish the candidate theories
- *problem:* quantum-gravitational effects might only become dominant in the Planck regime

$$m_{\mathrm{P}} = \sqrt{\frac{\hbar c}{G}} \simeq 1.22 \times 10^{19}\,\mathrm{GeV}/c^2$$

- effects are expected for: ➔ black holes *(Hawking radiation)*
  ➔ very early universe *(Cosmic Microwave Background)*

The problem with Quantum Gravity

2

Quantum Gravity? → unification, Canonical Quantum General Relativity, *etc.*

unification → String Theory

Canonical Quantum General Relativity → Loop Quantum Gravity, **Wheeler–DeWitt approach**

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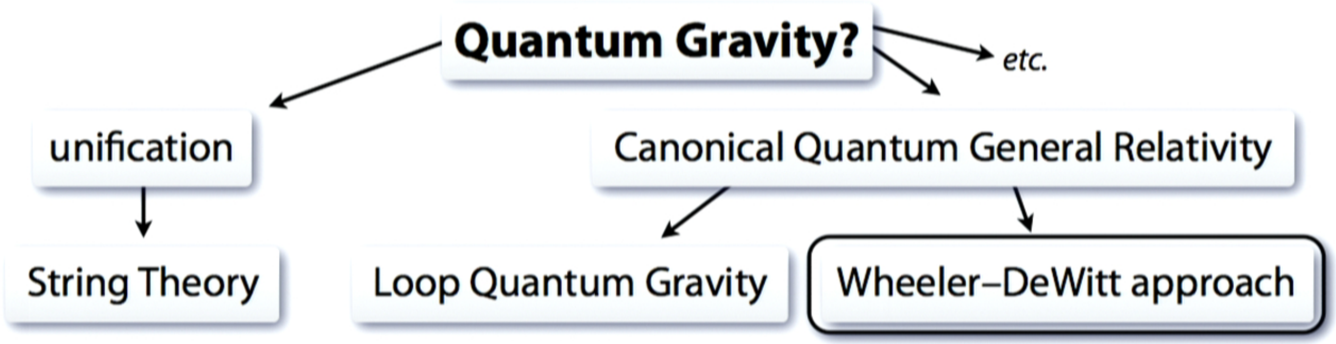

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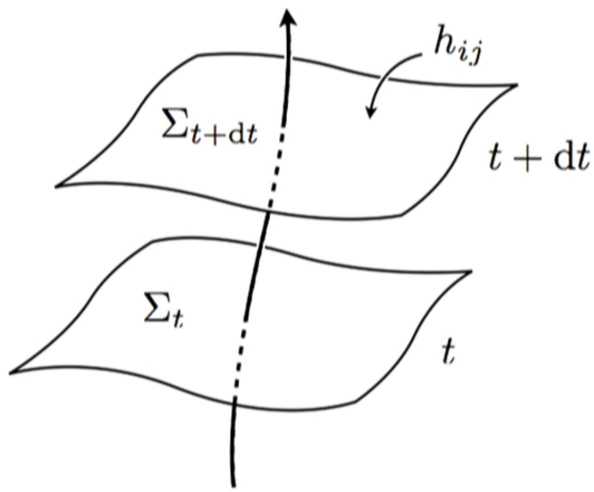
Wheeler–DeWitt approach *(Quantum Geometrodynamics)*

- canonical quantization of Hamiltonian formulation of General Relativity
- 3+1 decomposition by foliating spacetime (ADM formalism)
- *Canonical variables:*
  induced spatial metric h_{ij}
  and its conjugate momentum



➡ **Wheeler–DeWitt equation**

- functional differential equation for a wave functional Ψ defined on the *superspace* of all 3-geometries

$$\mathcal{H}\,\Psi[h_{ij}(\mathbf{x}),\phi(\mathbf{x})] = 0$$

$$\left[-16\pi G\,\hbar^2\,G_{ijkl}\,\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} - \frac{\sqrt{h}}{16\pi G}\left({}^{(3)}R - 2\,\Lambda\right) + \mathcal{H}_{\mathrm{mat}}[h_{ij},\phi]\right]\Psi[h_{ij},\phi] = 0$$

Wheeler–DeWitt equation

$$\left[-16\pi G\,\hbar^2\,G_{ijkl}\,\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} - \frac{\sqrt{h}}{16\pi G}\left({}^{(3)}R - 2\,\Lambda\right) + \mathcal{H}_{\rm mat}[h_{ij},\phi]\right]\Psi[h_{ij},\phi] = 0$$

DeWitt metric; $\det(h_{ij})$; 3-dim. Ricci scalar; cosmol. constant; matter field

- timeless (GR: *dynamical time* **vs.** QM: *absolute time* ➔ QG: *no time*)
- might not hold at the most fundamental level, but as an effective eq.
- Born–Oppenheimer approximation with respect to $m_{\rm P}^2 \propto G^{-1}$

$\mathcal{O}(m_{\rm P}^2)$: Hamilton–Jacobi equation of GR ➔ recovery of Einstein eq.

$\mathcal{O}(m_{\rm P}^0)$: functional Schrödinger equation for matter field; WKB time
➔ recovery of QFT in curved spacetime

$\mathcal{O}(m_{\rm P}^{-2})$: quantum-gravitational correction terms to Schrödinger eq.

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➡ *dominant QG contribution for the power spectrum of cosmol. perturbations?*

Wheeler–DeWitt equation

$$\left[-16\pi G\,\hbar^2\,G_{ijkl}\,\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} - \frac{\sqrt{h}}{16\pi G}\left({}^{(3)}R - 2\,\Lambda\right) + \mathcal{H}_{\rm mat}[h_{ij},\phi]\right]\Psi[h_{ij},\phi] = 0$$

DeWitt metric $\det(h_{ij})$ 3-dim. Ricci scalar cosmol. constant matter field

- timeless (GR: *dynamical time* **vs.** QM: *absolute time* ➔ QG: *no time*)
- might not hold at the most fundamental level, but as an effective eq.
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$\mathcal{O}(m_{\rm P}^2)$: Hamilton–Jacobi equation of GR ➔ recovery of Einstein eq.

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Semiclassical approximation

- Born–Oppenheimer approximation, WKB ansatz: $\Psi_k(\alpha, f_k) = \mathrm{e}^{\mathrm{i}\,S(\alpha,f_k)}$
  - expansion of $S(\alpha, f_k)$: $S = m_\mathrm{P}^2\, S_0 + m_\mathrm{P}^0\, S_1 + m_\mathrm{P}^{-2}\, S_2 + \ldots$
  - insert WKB ansatz into WDW eq. and equate terms of equal power of m_P
    - $\mathcal{O}(m_\mathrm{P}^2)$: Hamilton–Jacobi equation
    - $\mathcal{O}(m_\mathrm{P}^0)$: define $\psi_k^{(0)}(\alpha, f_k) \equiv \gamma(\alpha)\,\mathrm{e}^{\mathrm{i}\,S_1(\alpha,f_k)}$

→ introduce WKB time

$$\frac{\partial}{\partial t} \equiv -\,\mathrm{e}^{-3\alpha}\,\frac{\partial S_0}{\partial \alpha}\,\frac{\partial}{\partial \alpha}$$

→ **Schrödinger equation**

$$\mathrm{i}\,\frac{\partial}{\partial t}\,\psi_k^{(0)} = \mathcal{H}_k \psi_k^{(0)}$$

- $\mathcal{O}(m_\mathrm{P}^{-2})$: **quantum-gravitationally corrected Schrödinger equation**

$$\mathrm{i}\,\frac{\partial}{\partial t}\,\psi_k^{(1)} = \mathcal{H}_k \psi_k^{(1)} - \frac{\mathrm{e}^{3\alpha}}{2\,m_\mathrm{P}^2\,\psi_k^{(0)}}\left[\frac{(\mathcal{H}_k)^2}{V}\,\psi_k^{(0)} + \mathrm{i}\,\frac{\partial}{\partial t}\left(\frac{\mathcal{H}_k}{V}\right)\psi_k^{(0)}\right]\psi_k^{(1)}$$

$\mathrm{e}^{6\alpha} H^2$

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V → $\mathrm{e}^{6\alpha} H^2$

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V ← $\mathrm{e}^{6\alpha} H^2$ → V

Derivation of the power spectrum

- solve Schrödinger eq. with ansatz: $\psi_k^{(0)}(t, f_k) = \mathcal{N}_k^{(0)}(t)\, \mathrm{e}^{-\frac{1}{2}\,\Omega_k^{(0)}(t)\, f_k^2}$
- density contrast δ_k at the time when mode k reenters Hubble radius

$$\delta_k(t_{\text{enter}}) \propto \left| \frac{\mathrm{d}}{\mathrm{d}t} \mathfrak{Re}\left[\Omega_k^{(0)}(t)\right]^{-1/2} \right|_{t_{\text{exit}}} \propto k^{-3/2}$$

➡ **power spectrum:** $\mathcal{P}^{(0)}(k) \propto k^3 \left|\delta_k(t_{\text{enter}})\right|^2 \propto H^4 \left|\dot{\phi}(t)\right|_{t_{\text{exit}}}^{-2}$

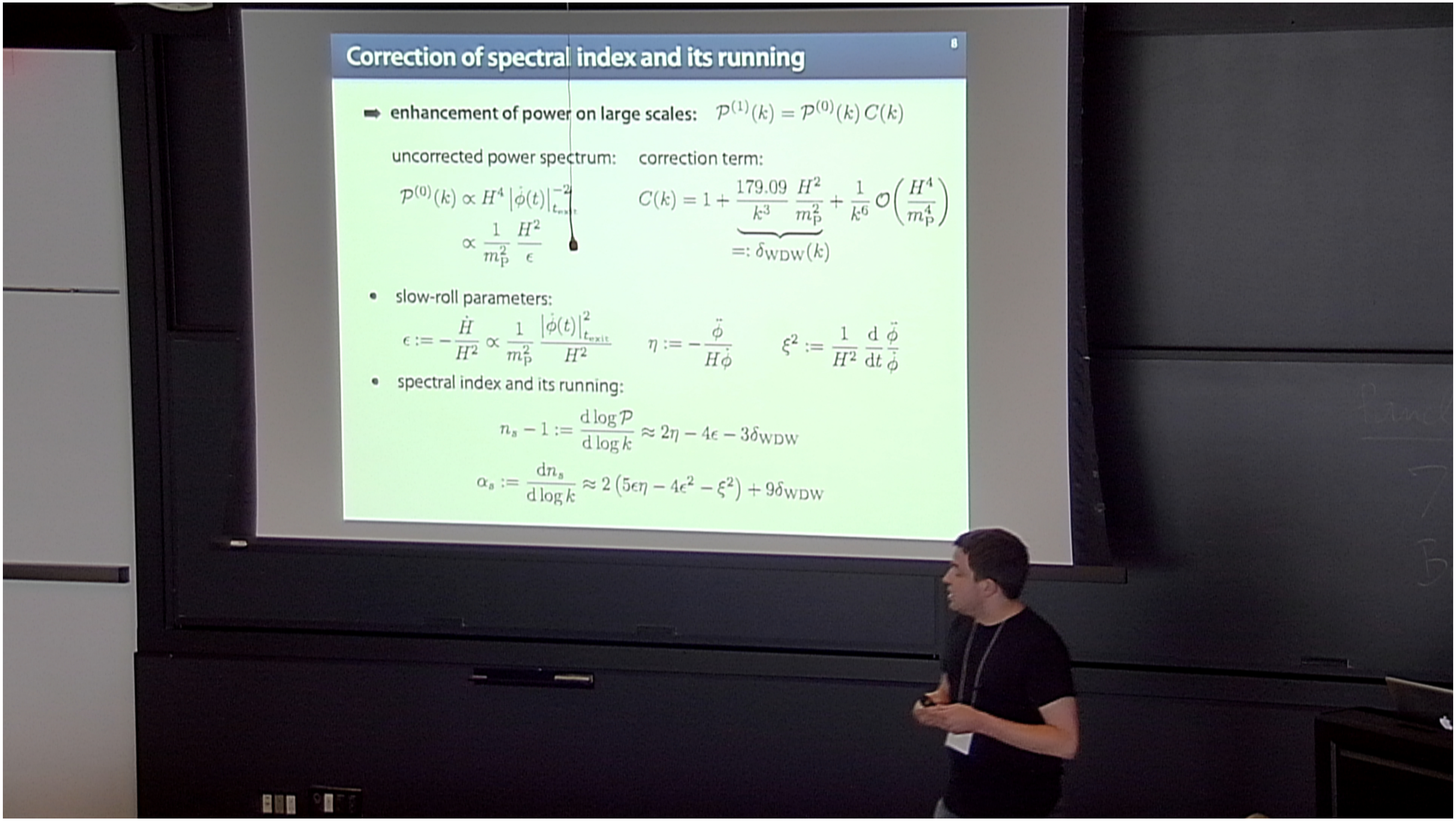
- approximate solution for *QG corrected* Schrödinger eq. with ansatz:

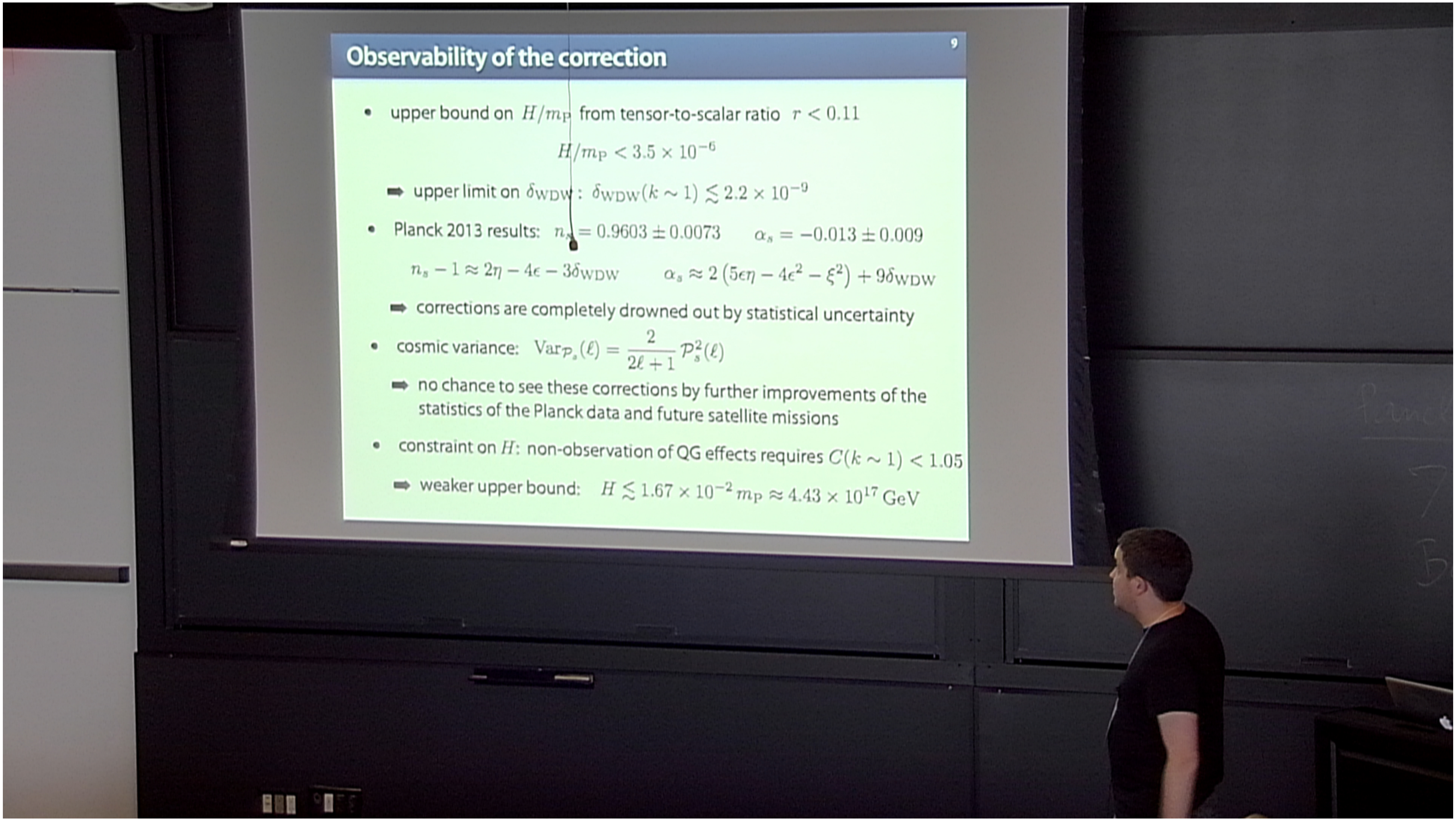
$$\psi_k^{(1)}(t, f_k) = \left(\mathcal{N}_k^{(0)}(t) + \frac{1}{m_{\mathrm{P}}^2}\,\mathcal{N}_k^{(1)}(t)\right) \exp\left[-\frac{1}{2}\left(\Omega_k^{(0)}(t) + \frac{1}{m_{\mathrm{P}}^2}\,\Omega_k^{(1)}(t)\right) f_k^2\right]$$

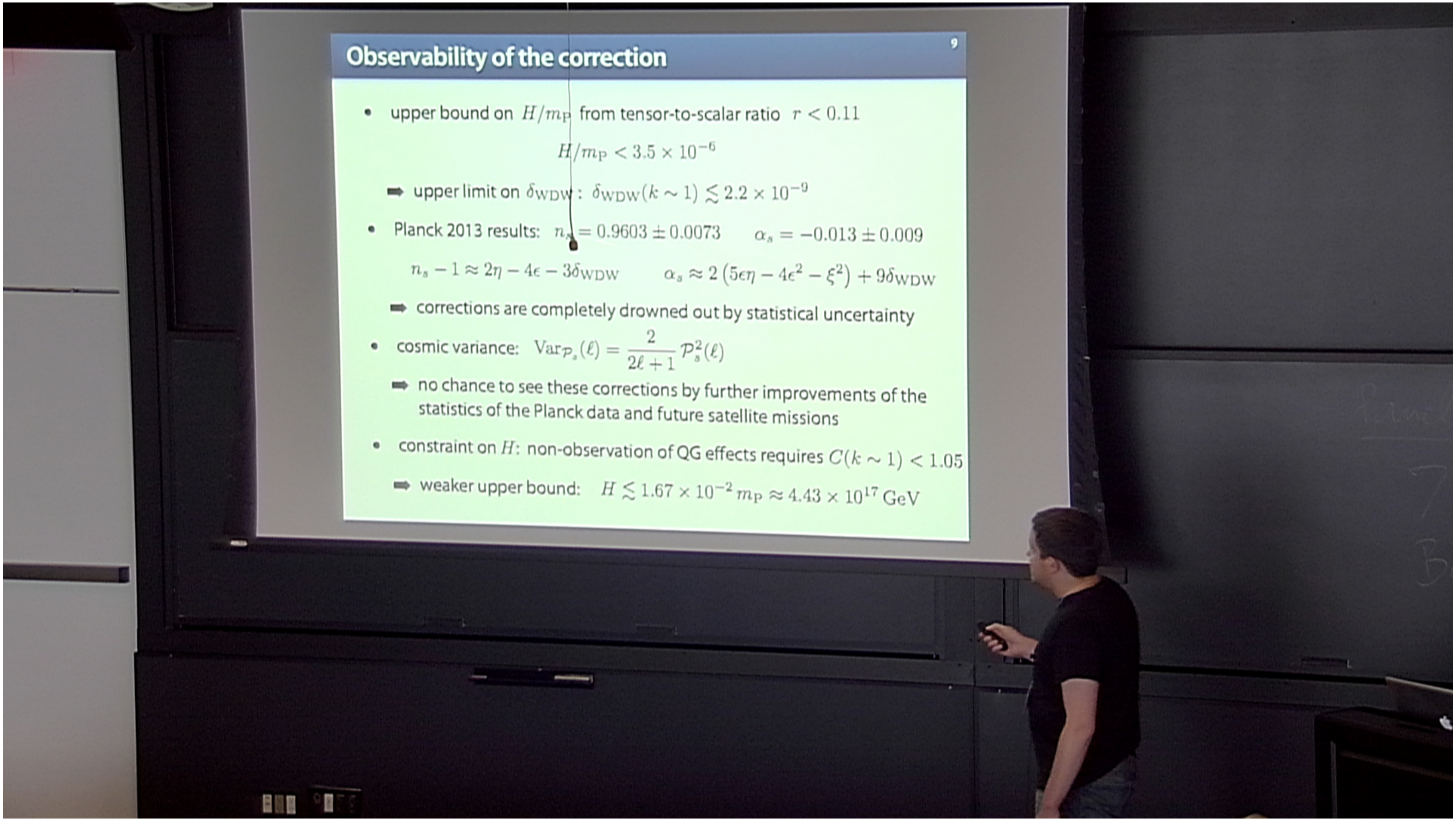
- boundary condition: $\Omega_k^{(1)}(t) \to 0$ as $t \to \infty$

➡ ***QG corrected* power spectrum:** $\mathcal{P}^{(1)}(k) = \mathcal{P}^{(0)}(k)\, C(k)$ with

$$C(k) = 1 + \frac{179.09}{k^3}\,\frac{H^2}{m_{\mathrm{P}}^2} + \frac{1}{k^6}\,\mathcal{O}\left(\frac{H^4}{m_{\mathrm{P}}^4}\right)$$


Correction of spectral index and its running
8
enhancement of power on large scales: $\mathcal{P}^{(1)}(k) = \mathcal{P}^{(0)}(k)\,C(k)$
uncorrected power spectrum:
$\mathcal{P}^{(0)}(k) \propto H^4 \left|\dot\phi(t)\right|^{-2}_{t_{\rm exit}} \propto \frac{1}{m_{\rm P}^2}\frac{H^2}{\epsilon}$
correction term:
$C(k) = 1 + \underbrace{\frac{179.09}{k^3}\frac{H^2}{m_{\rm P}^2}}_{=:\,\delta_{\rm WDW}(k)} + \frac{1}{k^6}\,\mathcal{O}\!\left(\frac{H^4}{m_{\rm P}^4}\right)$
slow-roll parameters:
$\epsilon := -\frac{\dot H}{H^2} \propto \frac{1}{m_{\rm P}^2}\frac{\left|\dot\phi(t)\right|^2_{t_{\rm exit}}}{H^2} \qquad \eta := -\frac{\ddot\phi}{H\dot\phi} \qquad \xi^2 := \frac{1}{H^2}\frac{\rm d}{{\rm d}t}\frac{\ddot\phi}{\dot\phi}$
spectral index and its running:
$n_s - 1 := \frac{{\rm d}\log\mathcal{P}}{{\rm d}\log k} \approx 2\eta - 4\epsilon - 3\delta_{\rm WDW}$
$\alpha_s := \frac{{\rm d}n_s}{{\rm d}\log k} \approx 2\left(5\epsilon\eta - 4\epsilon^2 - \xi^2\right) + 9\delta_{\rm WDW}$


Observability of the correction
9
upper bound on H/m_P from tensor-to-scalar ratio $r < 0.11$
$H/m_P < 3.5 \times 10^{-6}$
upper limit on δ_{WDW}: $\delta_{WDW}(k \sim 1) \lesssim 2.2 \times 10^{-9}$
Planck 2013 results: $n_s = 0.9603 \pm 0.0073$ $\alpha_s = -0.013 \pm 0.009$
$n_s - 1 \approx 2\eta - 4\epsilon - 3\delta_{WDW}$ $\alpha_s \approx 2\left(5\epsilon\eta - 4\epsilon^2 - \xi^2\right) + 9\delta_{WDW}$
corrections are completely drowned out by statistical uncertainty
cosmic variance: $\mathrm{Var}_{\mathcal{P}_s}(\ell) = \frac{2}{2\ell+1}\mathcal{P}_s^2(\ell)$
no chance to see these corrections by further improvements of the statistics of the Planck data and future satellite missions
constraint on H: non-observation of QG effects requires $C(k \sim 1) < 1.05$
weaker upper bound: $H \lesssim 1.67 \times 10^{-2}\, m_P \approx 4.43 \times 10^{17}\,\mathrm{GeV}$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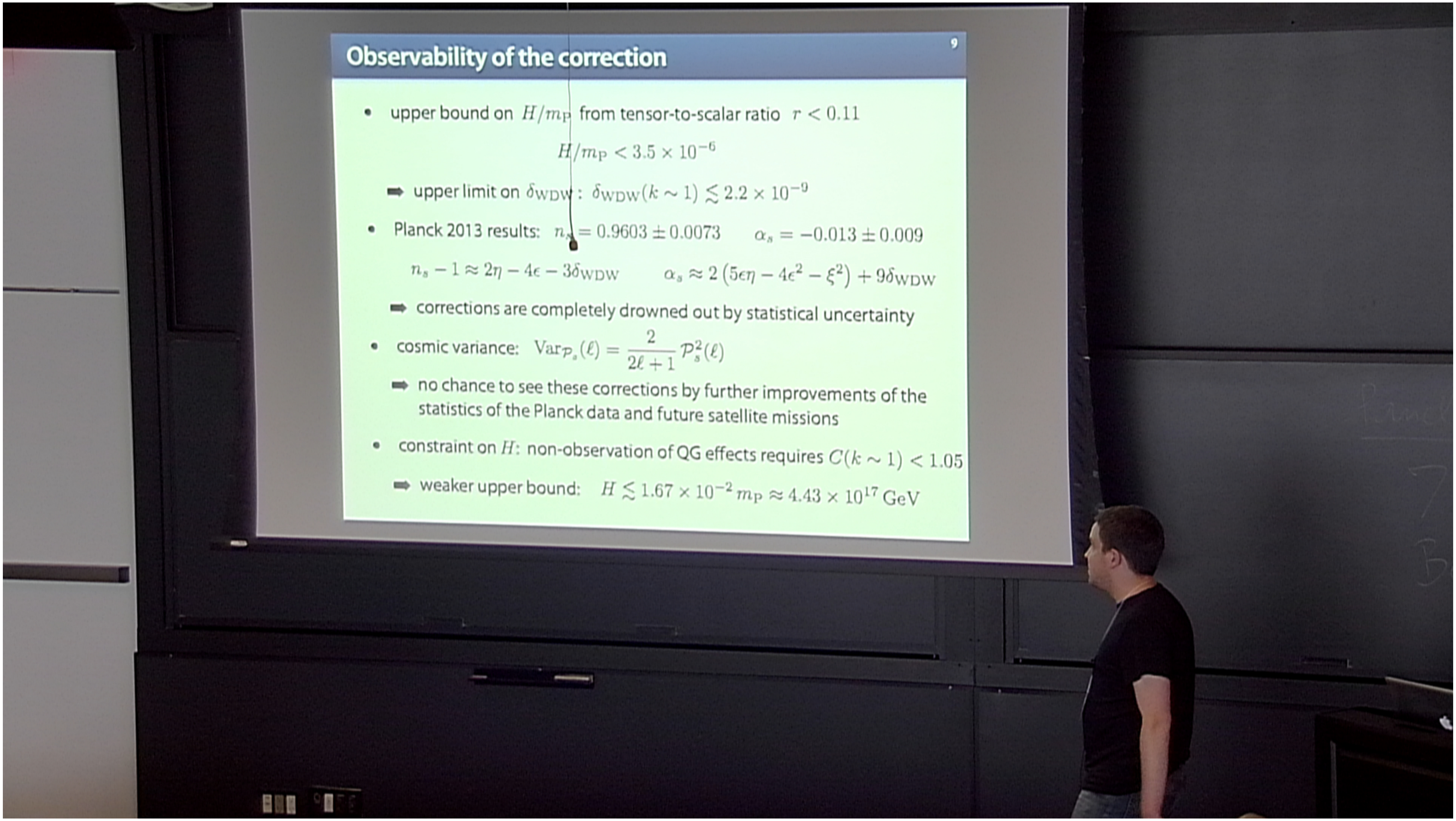
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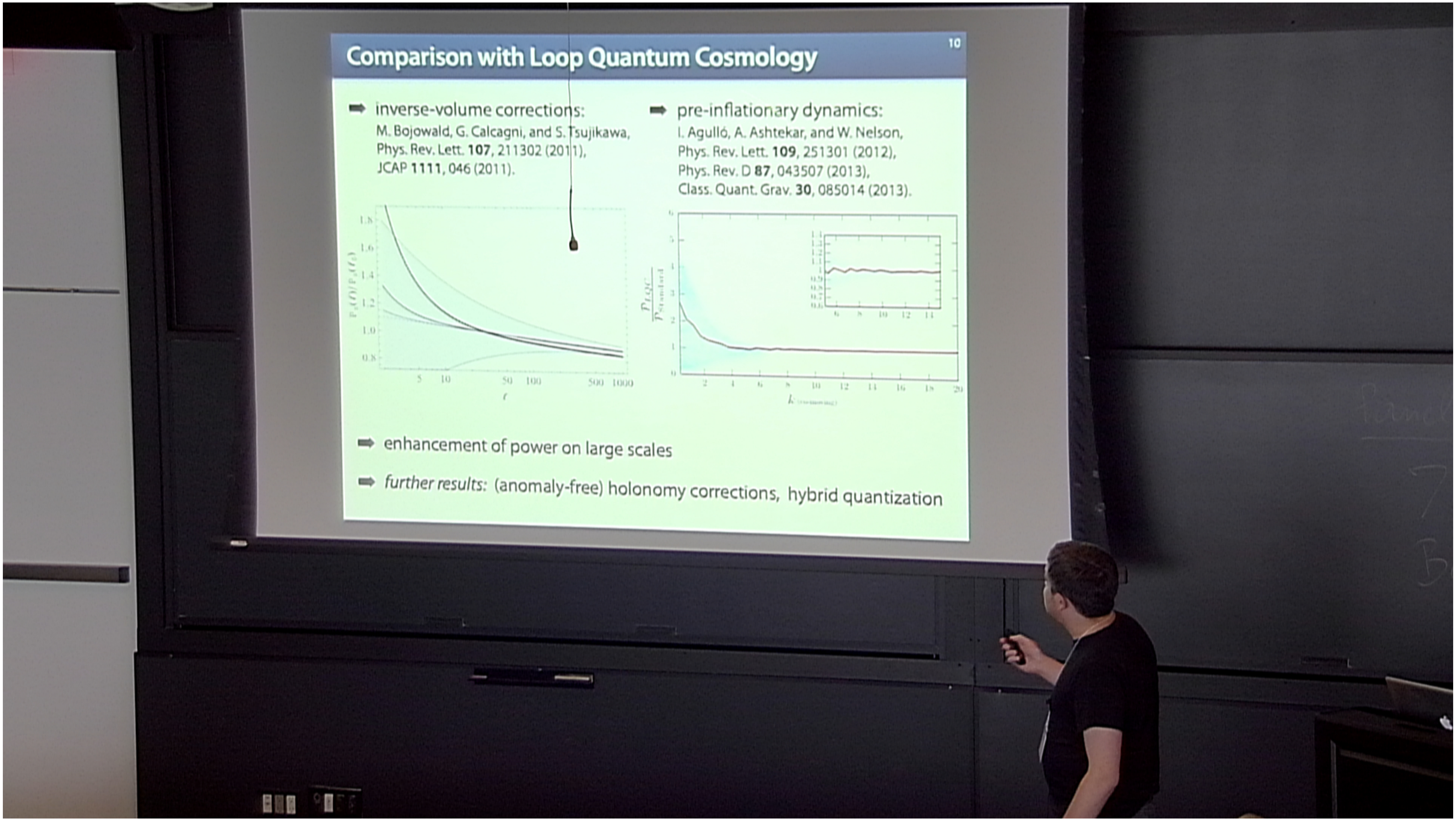
Comparison with Loop Quantum Cosmology
10
inverse-volume corrections:
M. Bojowald, G. Calcagni, and S. Tsujikawa,
Phys. Rev. Lett. 107, 211302 (2011),
JCAP 1111, 046 (2011).
pre-inflationary dynamics:
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further results: (anomaly-free) holonomy corrections, hybrid quantization

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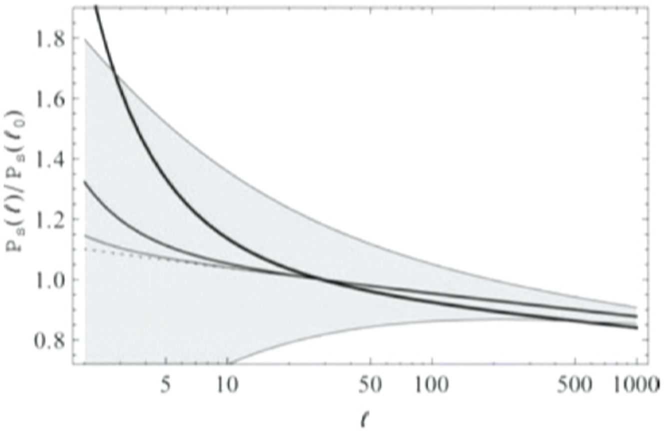

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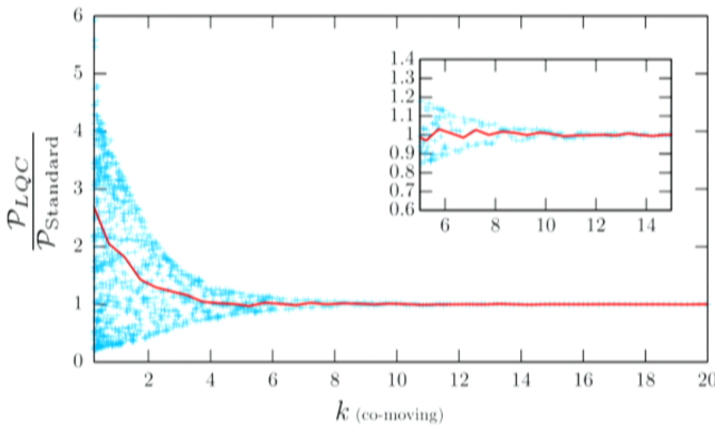

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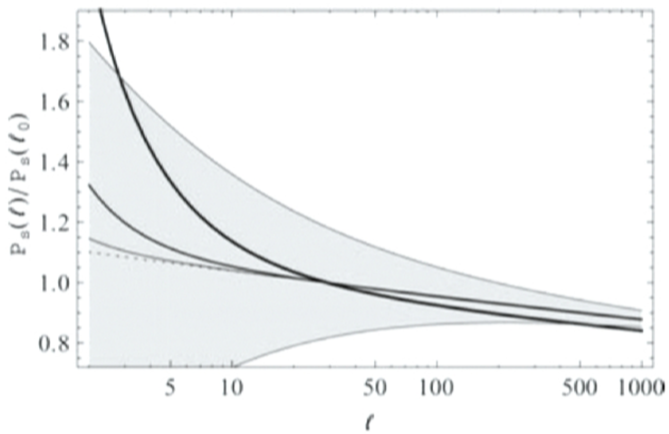
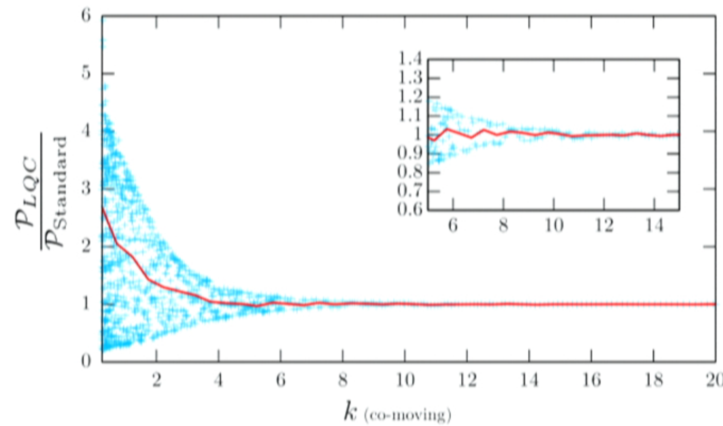




➡ enhancement of power on large scales

➡ *further results:* (anomaly-free) holonomy corrections, hybrid quantization

Summary

- quantization of an inflationary universe with perturbations of a scalar field
- semiclassical approximation to the Wheeler–DeWitt equation
  - derivation of the power spectrum of these perturbations and quantum-gravitational corrections to it
    - quantum-gravitational correction term induces scale dependence, modification of power on largest scales
    - too small to be observable (cosmic variance)
    - weak constraint on energy scale during inflation

➡ comparison with other approaches to Quantum Gravity

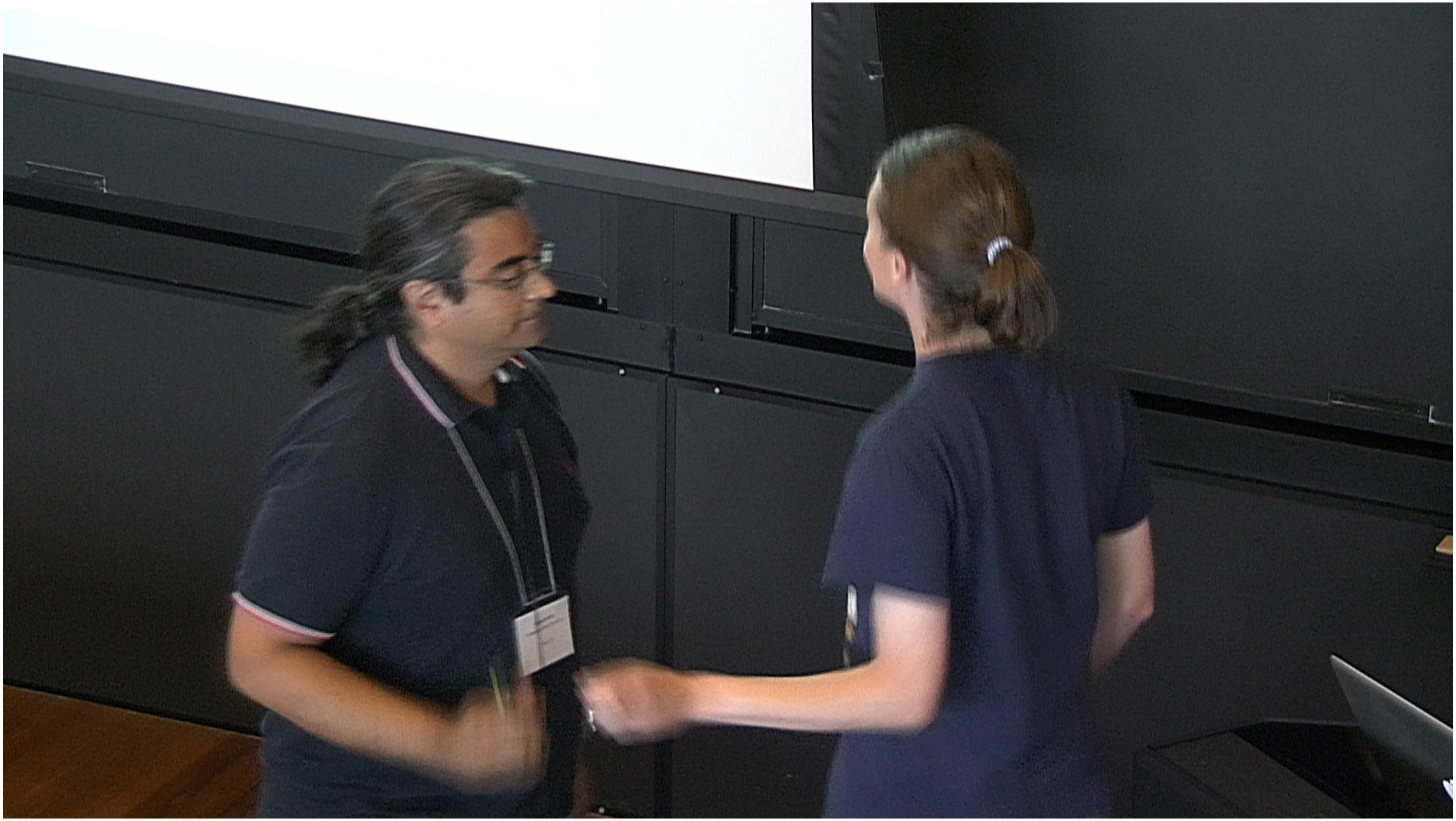
  - LQC: inverse-volume corrections, pre-inflationary dynamics
    - also lead to enhancement of power on large scales

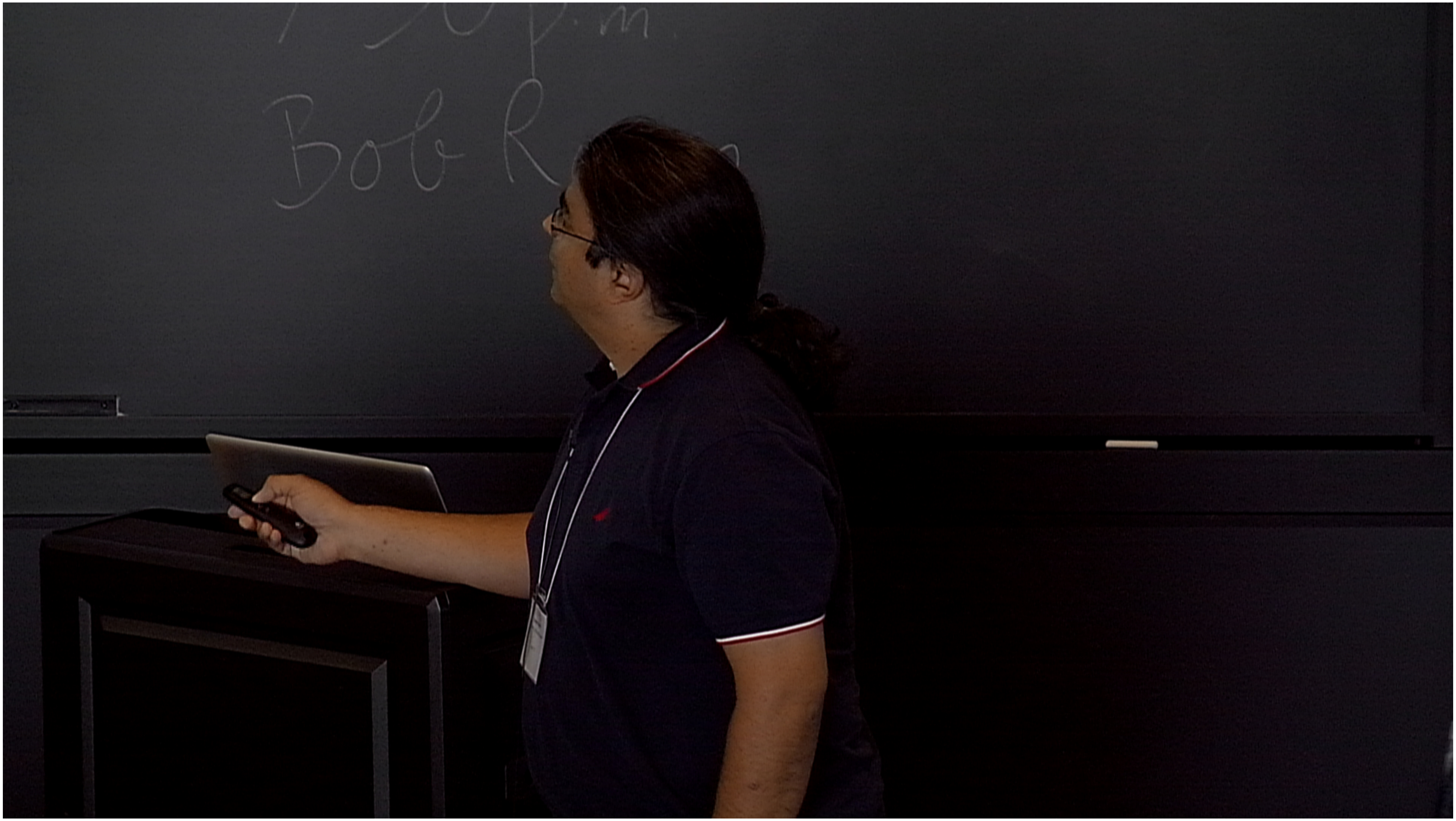
11

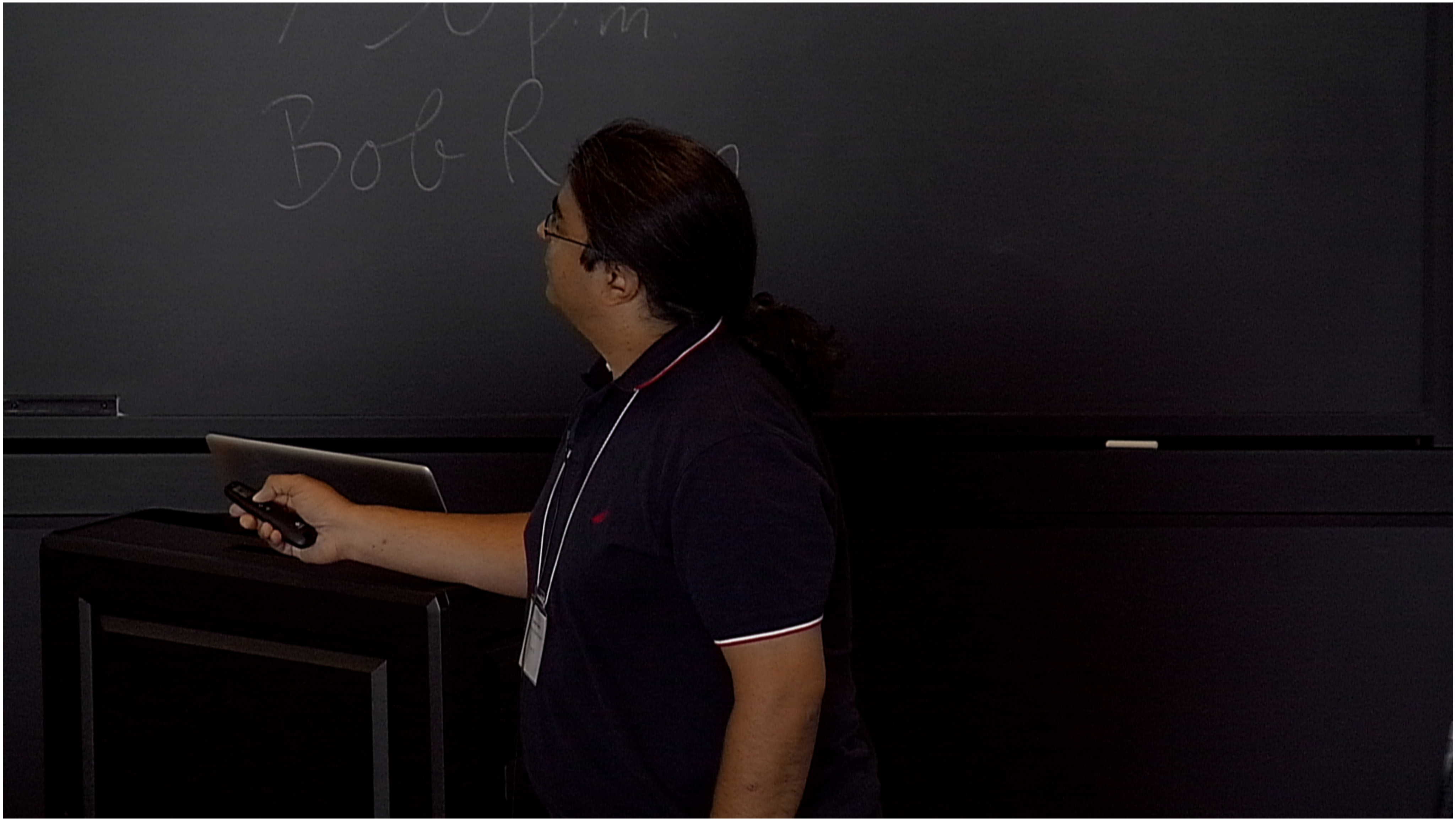
Primordial polymer perturbations

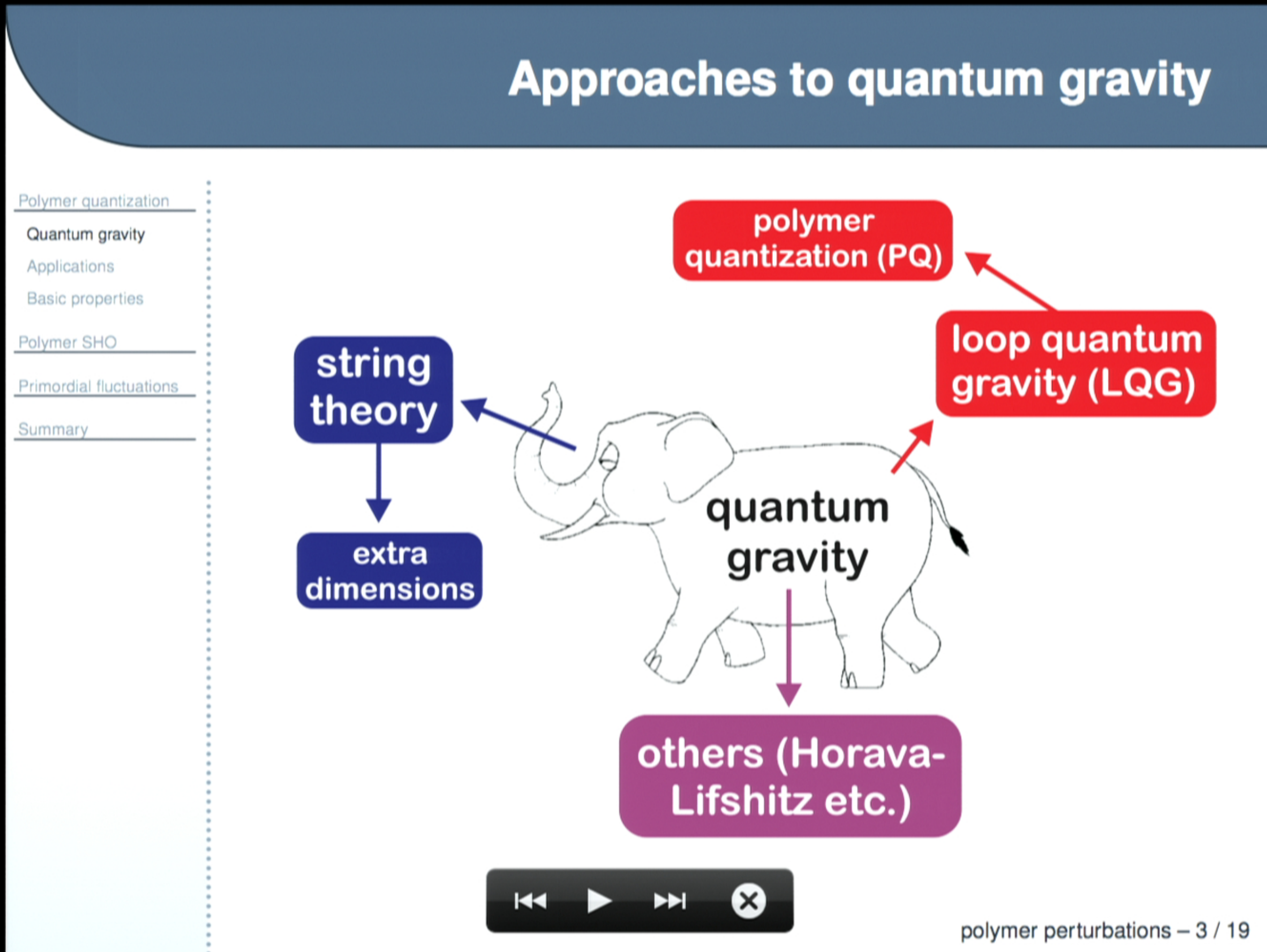
Sanjeev Seahra

July 23, 2013

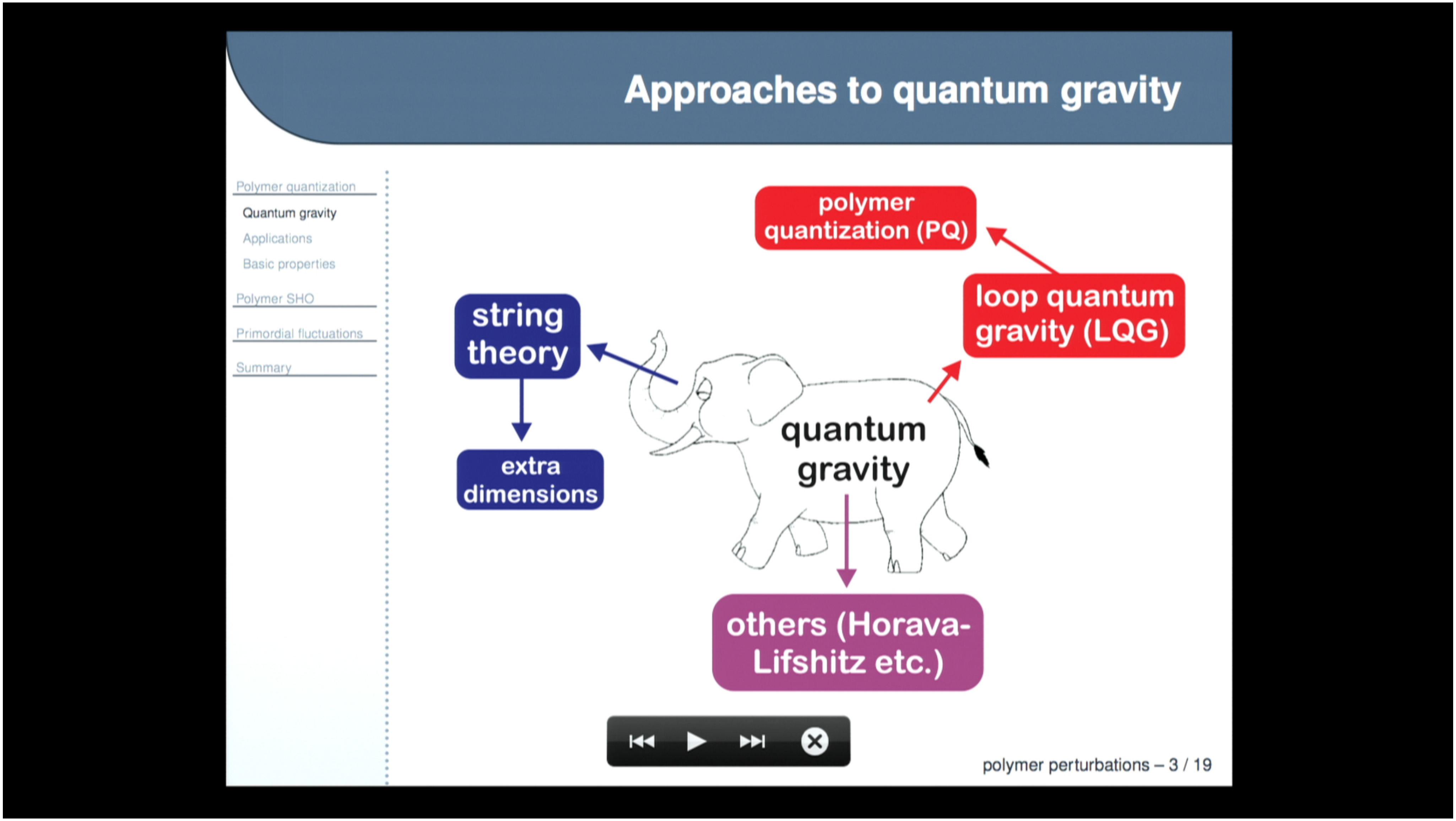



Bob





Approaches to quantum gravity
Polymer quantization
Quantum gravity
Applications
Basic properties
Polymer SHO
Primordial fluctuations
Summary
polymer quantization (PQ)
loop quantum gravity (LQG)
string theory
extra dimensions
quantum gravity
others (Horava-Lifshitz etc.)
polymer perturbations – 3 / 19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polymer perturbations – 3 / 19

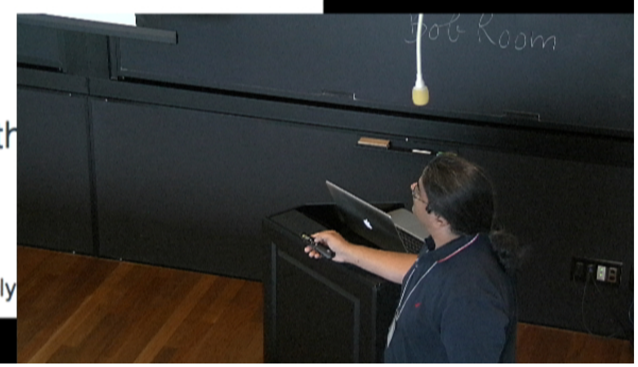
Applications of PQ to scalar fields

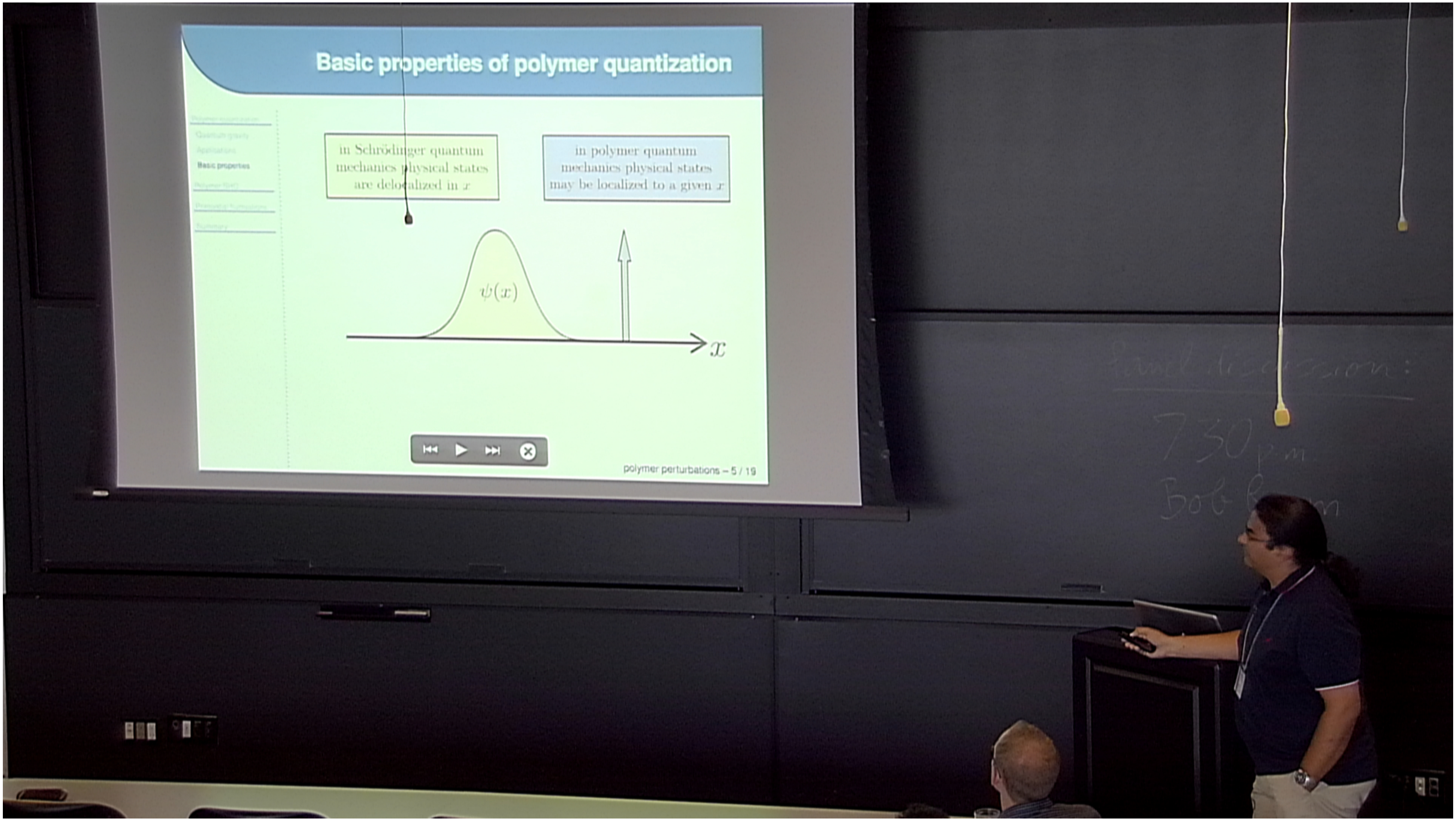
- polymer quantization of a scalar field corrects QFT at high energies, hence it affects the:
  - ☐ small scale behaviour of dispersion relations
  - ☐ quantum cosmology of a free scalar coupled to FRW geometry at high density
  - ☐ ultraviolet behaviour of the free particle propagator
  - ☐ generation of primordial perturbations during inflation
  - ☐ high temperature behaviour of blackbody radiation

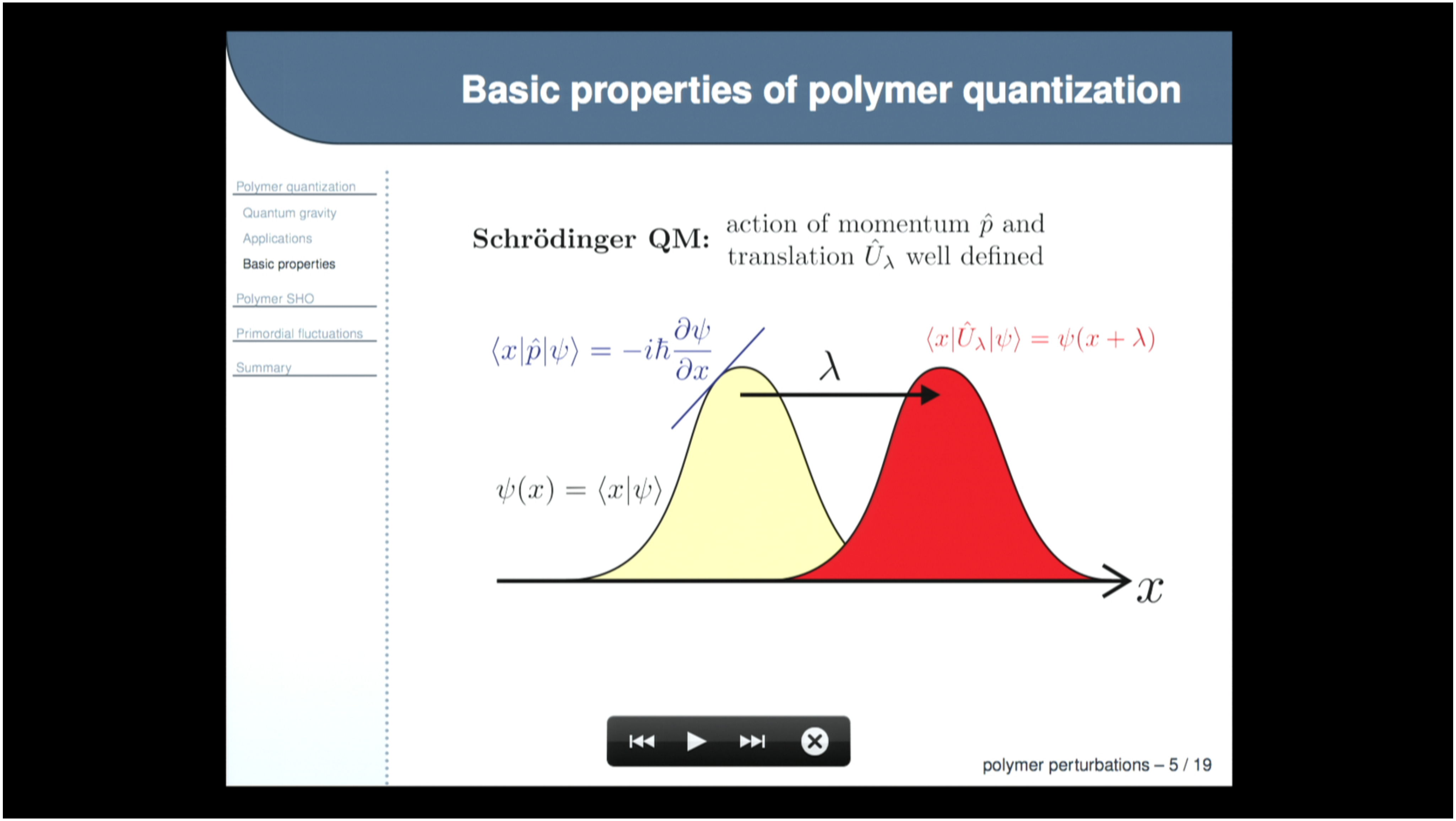
this talk

(see Viqar Husain's talk on Friday for oth

poly




Basic properties of polymer quantization
in Schrödinger quantum mechanics physical states are delocalized in x
in polymer quantum mechanics physical states may be localized to a given x
$\psi(x)$
x
polymer perturbations – 5 / 19



Basic properties of polymer quantization
Polymer quantization
Quantum gravity
Applications
Basic properties
Polymer SHO
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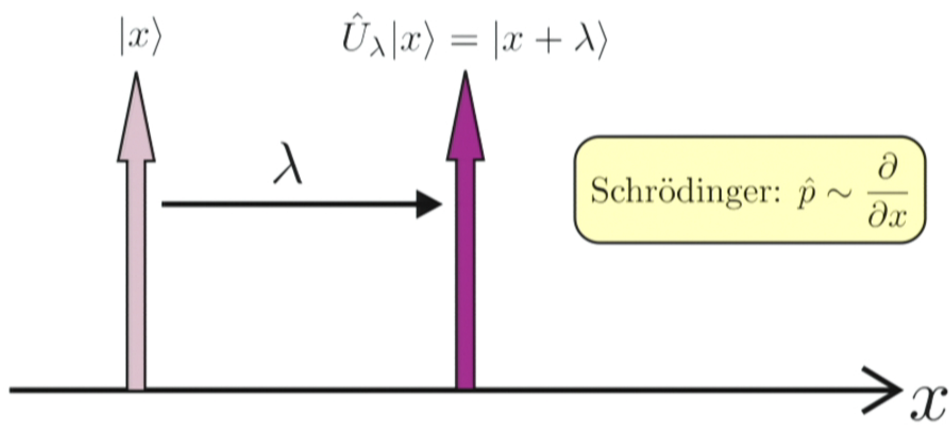
Summary
Schrödinger QM: action of momentum $\hat{p}$ and translation $\hat{U}_\lambda$ well defined
$\langle x|\hat{p}|\psi\rangle = -i\hbar\frac{\partial\psi}{\partial x}$
$\langle x|\hat{U}_\lambda|\psi\rangle = \psi(x+\lambda)$
λ
$\psi(x) = \langle x|\psi\rangle$
x
polymer perturbations – 5 / 19

Basic properties of polymer quantization

polymer QM: translation $\hat{U}_\lambda$ well-defined



Schrödinger: $\hat{p} \sim \dfrac{\partial}{\partial x}$

"$\partial/\partial x$" of $|x\rangle$ not well-defined $\Longrightarrow$ Schrödinger-like momentum not defined

Basic properties of polymer quantization

momentum in polymer QM given by:

$$\hat{p} \mapsto \hat{p}_{\lambda_\star} \equiv i\hbar \left(\frac{\hat{U}_{\lambda_\star} - \hat{U}^\dagger_{\lambda_\star}}{2\lambda_\star} \right)$$

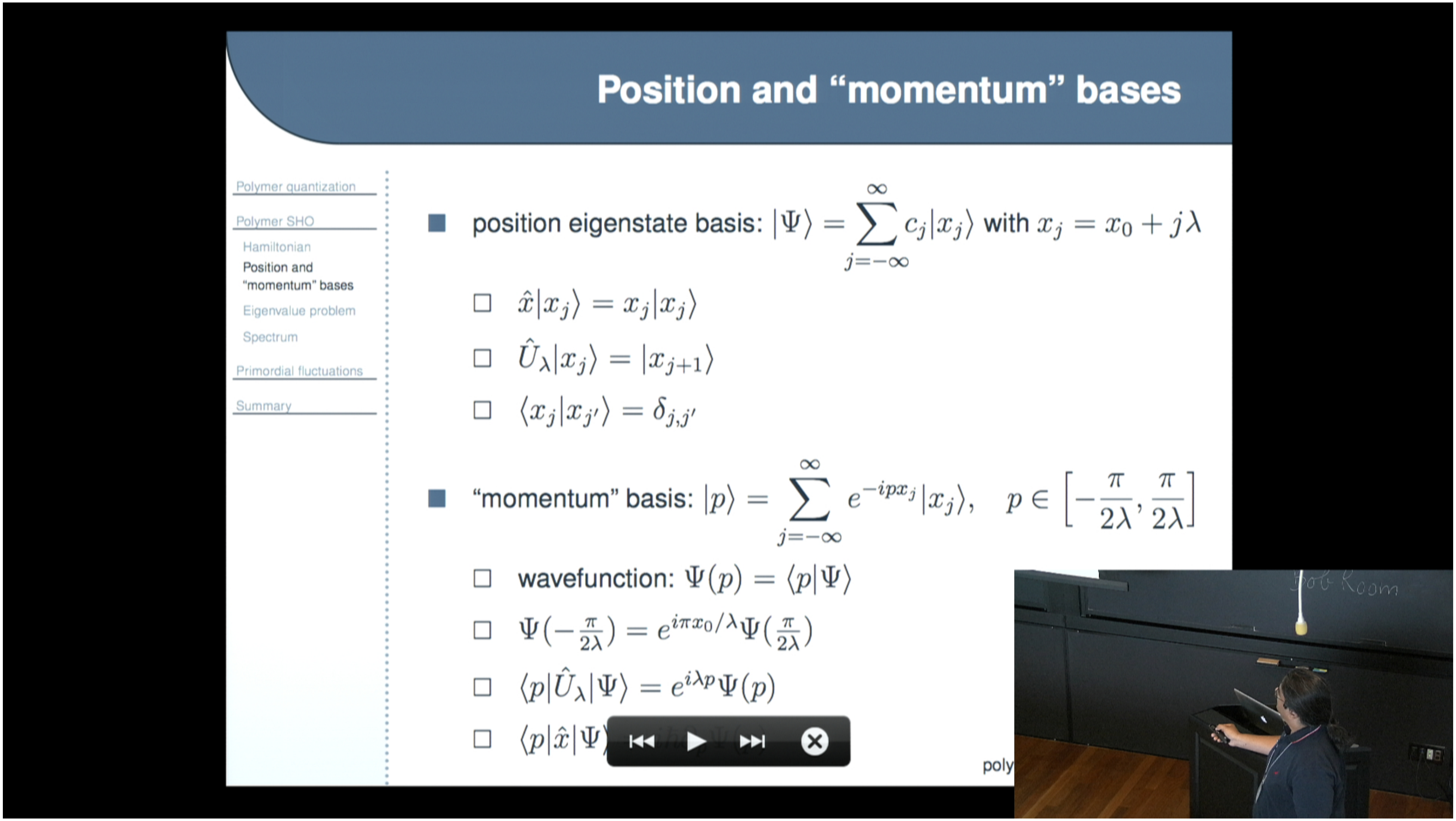
finite difference stencil of $-i\hbar\,\partial_x$ with width $\lambda_\star$

parameter of the quantization defines an energy scale $M_\star$

energy $\ll M_\star \quad \Rightarrow \quad$ recover Schrödinger QM

energy $\gg M_\star \quad \Rightarrow \quad$ deviations from Schrödinger QM

Worked example: the polymer simple harmonic oscillator


Position and "momentum" bases
Polymer quantization
Polymer SHO
Hamiltonian
Position and "momentum" bases
Eigenvalue problem
Spectrum
Primordial fluctuations
Summary
position eigenstate basis: $|\Psi\rangle = \sum_{j=-\infty}^{\infty} c_j |x_j\rangle$ with $x_j = x_0 + j\lambda$
$\hat{x}|x_j\rangle = x_j|x_j\rangle$
$\hat{U}_\lambda|x_j\rangle = |x_{j+1}\rangle$
$\langle x_j|x_{j'}\rangle = \delta_{j,j'}$
"momentum" basis: $|p\rangle = \sum_{j=-\infty}^{\infty} e^{-ipx_j}|x_j\rangle, \quad p \in \left[-\frac{\pi}{2\lambda}, \frac{\pi}{2\lambda}\right]$
wavefunction: $\Psi(p) = \langle p|\Psi\rangle$
$\Psi(-\frac{\pi}{2\lambda}) = e^{i\pi x_0/\lambda}\Psi(\frac{\pi}{2\lambda})$
$\langle p|\hat{U}_\lambda|\Psi\rangle = e^{i\lambda p}\Psi(p)$
$\langle p|\hat{x}|\Psi\rangle$
poly

Eigenvalue problem

- projecting eigenvalue equation $\hat{H}|\Psi\rangle = E|\Psi\rangle$ onto $|p\rangle$:

$$E\Psi = \frac{\omega}{2}\left[-\frac{\partial^2}{\partial y^2} + \frac{\sin^2(\sqrt{g}y)}{g}\right]\Psi, \quad y = \frac{p}{\sqrt{m\omega}}, \quad g = \frac{m\omega}{M_\star^2}$$

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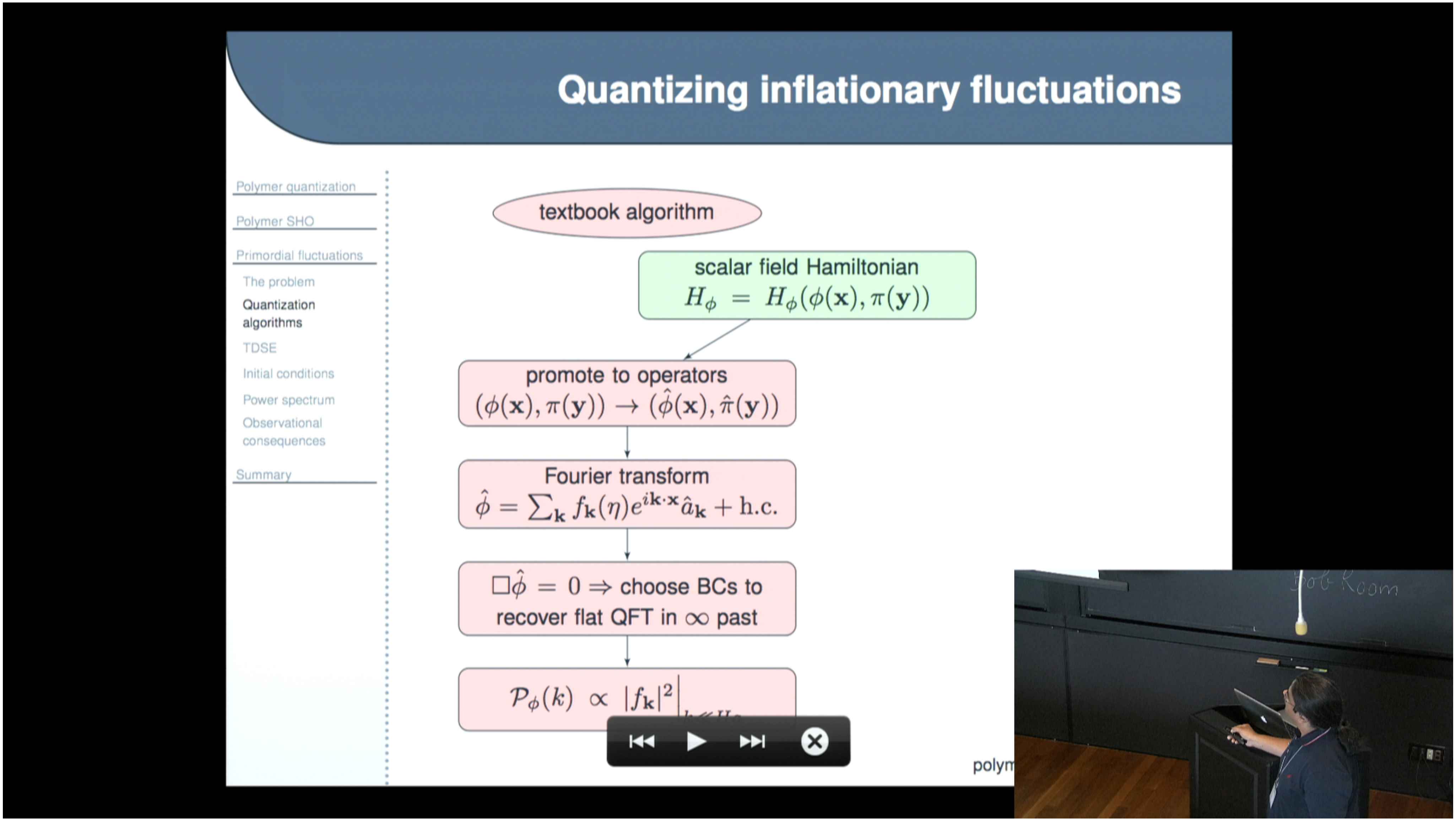
- "polymer coupling" g tells us close to Schrödinger we are
  - i.e. as $g \to 0$ we recover $E\Psi = \frac{1}{2}\omega\left[-\partial_y^2 + y^2\right]\Psi$
  - impose Dirichlet BCs to recover the ordinary SHO

The problem

- here we consider an inhomogeneous massless scalar in a de Sitter background

$$ds^2 = \begin{cases} -dt^2 + a^2 d\mathbf{x}^2 & a = \exp(Ht) \\ a^2(-d\eta^2 + d\mathbf{x}^2) & a = -(H\eta)^{-1} \end{cases}$$

$$H_\phi = \int d^3x\, a^3 \left[\frac{1}{2a^6}\pi^2 + \frac{1}{2a^2}(\nabla\phi)^2 \right]$$


Quantizing inflationary fluctuations
Polymer quantization
Polymer SHO
Primordial fluctuations
The problem
Quantization algorithms
TDSE
Initial conditions
Power spectrum
Observational consequences
Summary
textbook algorithm
scalar field Hamiltonian
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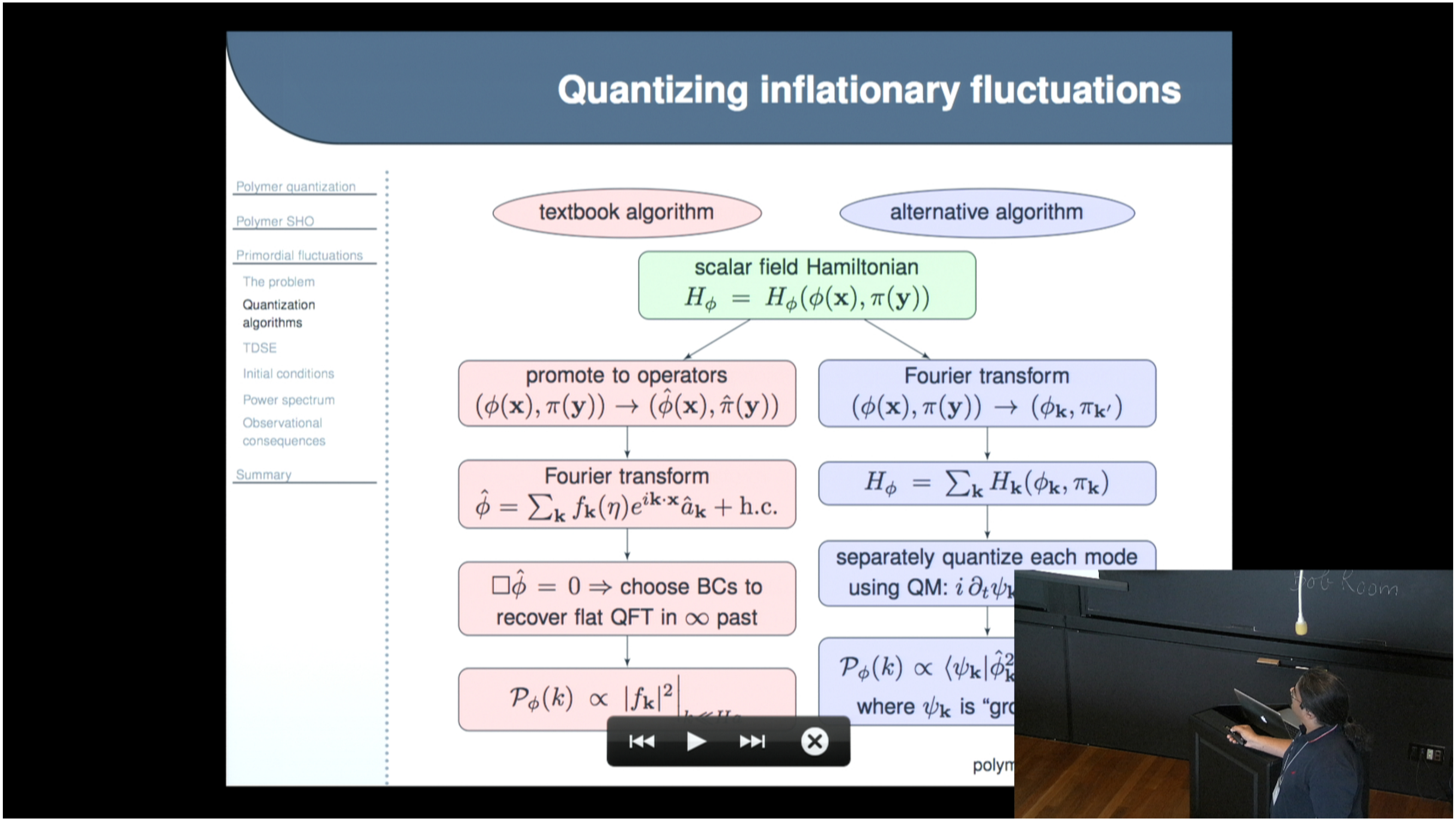
promote to operators
$(\phi(\mathbf{x}), \pi(\mathbf{y})) \to (\hat\phi(\mathbf{x}), \hat\pi(\mathbf{y}))$
Fourier transform
$\hat\phi = \sum_{\mathbf{k}} f_{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} \hat a_{\mathbf{k}} + \text{h.c.}$
$\Box\hat\phi = 0 \Rightarrow$ choose BCs to recover flat QFT in ∞ past
$\mathcal{P}_\phi(k) \propto |f_{\mathbf{k}}|^2$
polym
Bob Room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separately quantize each mode
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Bob Room

Time dependent Schrödinger equation

Polymer quantization

Polymer SHO

Primordial fluctuations

- The problem
- Quantization algorithms
- TDSE
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- Power spectrum
- Observational consequences

Summary

- after (lots of) manipulations, polymer evolution of a given mode's wavefunction governed by

$$i\frac{\partial\psi_{\mathbf{k}}}{\partial\eta} = \frac{k}{2}\left[-\frac{\partial^2}{\partial y^2} + \frac{\sin^2(\sqrt{g}y)}{g}\right]\psi_{\mathbf{k}}$$

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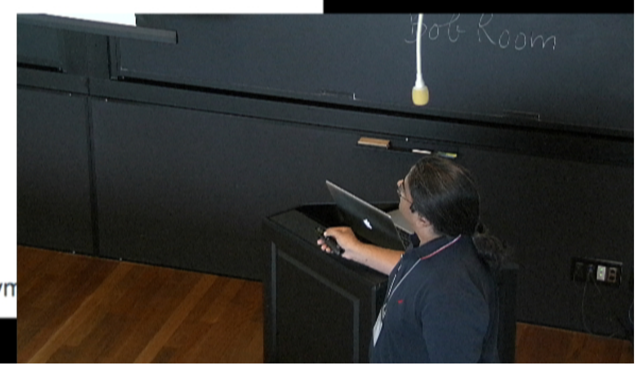

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- same equation as for the polymer SHO, **but** the polymer coupling is time-dependent

$$g = \frac{k}{M_\star a} = \frac{\text{physical wavenumber}}{\text{polymer energy scale}}$$

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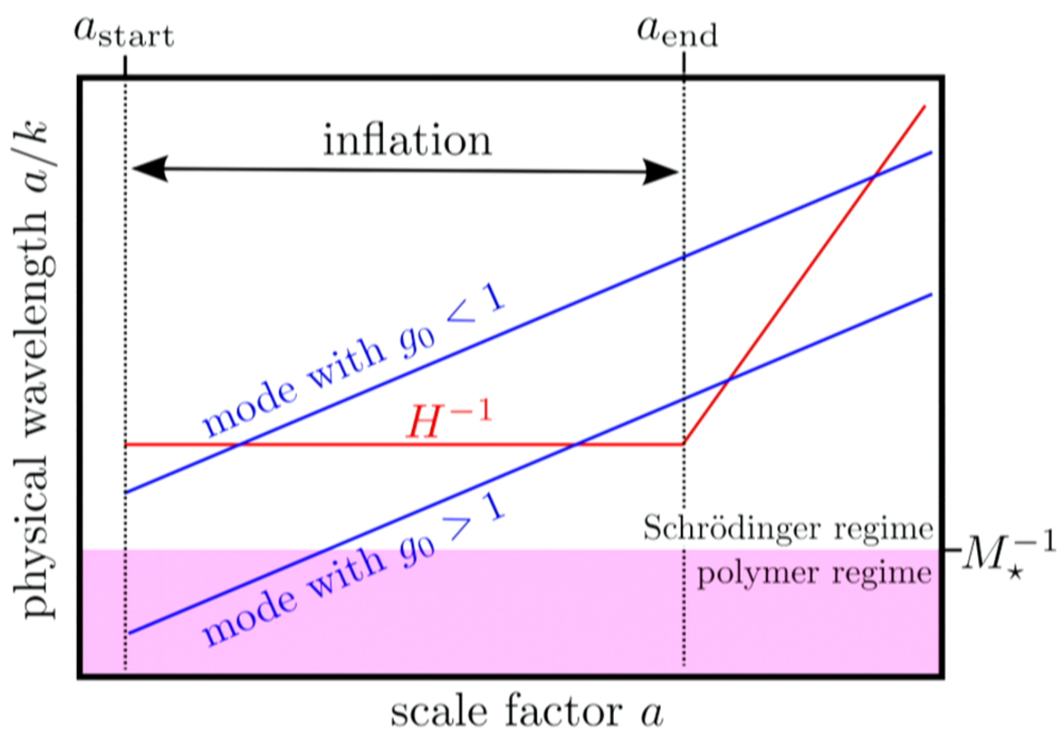

Initial conditions

- assume each mode is in instantaneous ground state of

$$\mathcal{A} = \frac{k}{2}\left[-\frac{\partial^2}{\partial y^2} + \frac{\sin^2(\sqrt{g}y)}{g}\right]$$

  at the start of inflation

  - reduces to the usual Bunch-Davies ICs for $g \to 0$

- quantum state at end of inflation determined by polymer coupling at start of inflation $g_0 = k/k_\star$
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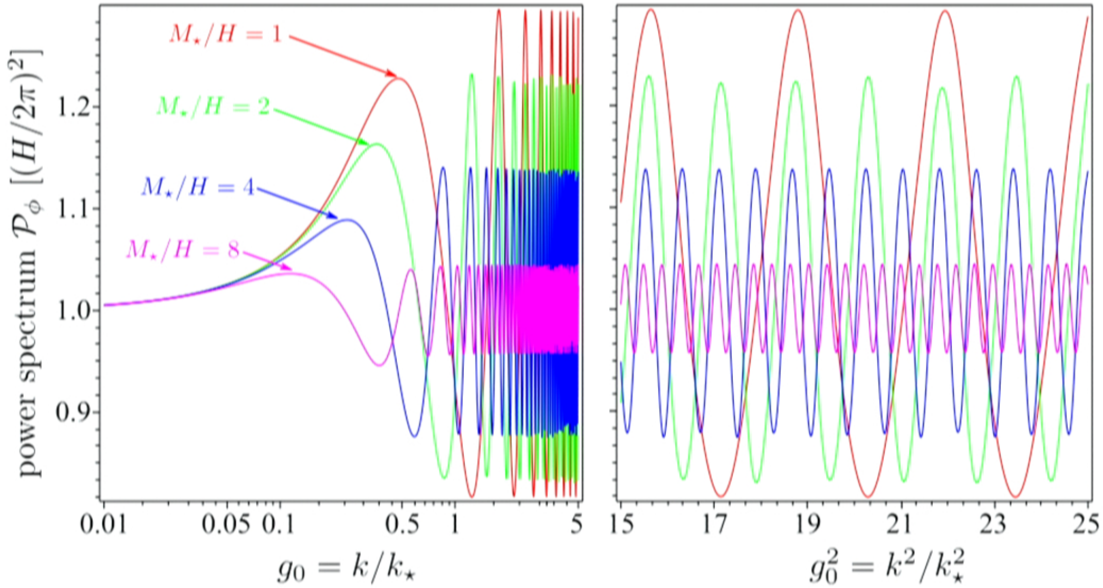
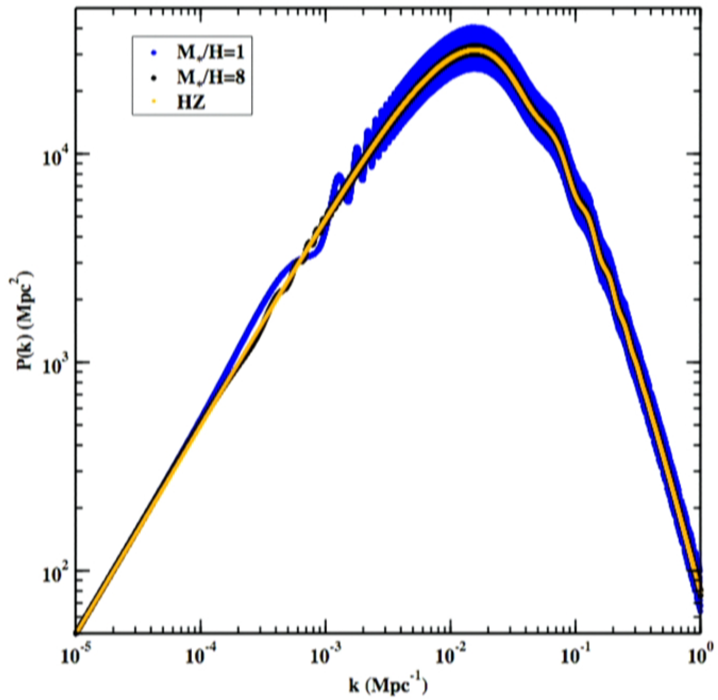
Results for the power spectrum



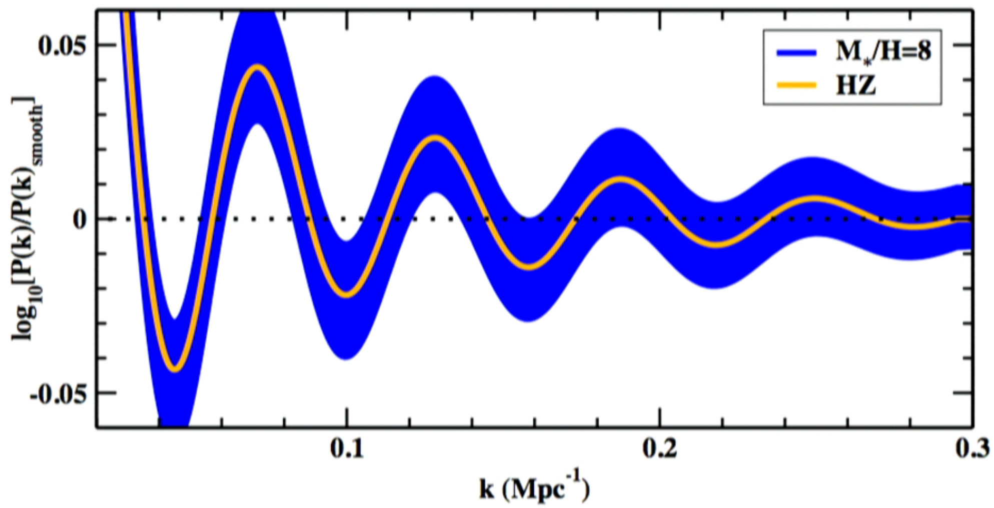
- recover standard result $\mathcal{P}_\phi = \mathcal{P}_0 = (H/2\pi)^2$ for $g_0 \ll 1$
- polymer effects vanish for $M_\star/H \to \infty$

Observational consequences



Present day matter power spectrum with $k_\star = 5 \times 10^{-4}\,\mathrm{Mpc}^{-1}$

Observational consequences



- baryon acoustic oscillations with $k_\star = 5 \times 10^{-4}\,\mathrm{Mpc}^{-1}$
- $M_\star/H \sim 1$ already ruled out by current observations
- future surveys (e.g. Euclid) will be able to rule out $M_\star/H \lesssim 10$

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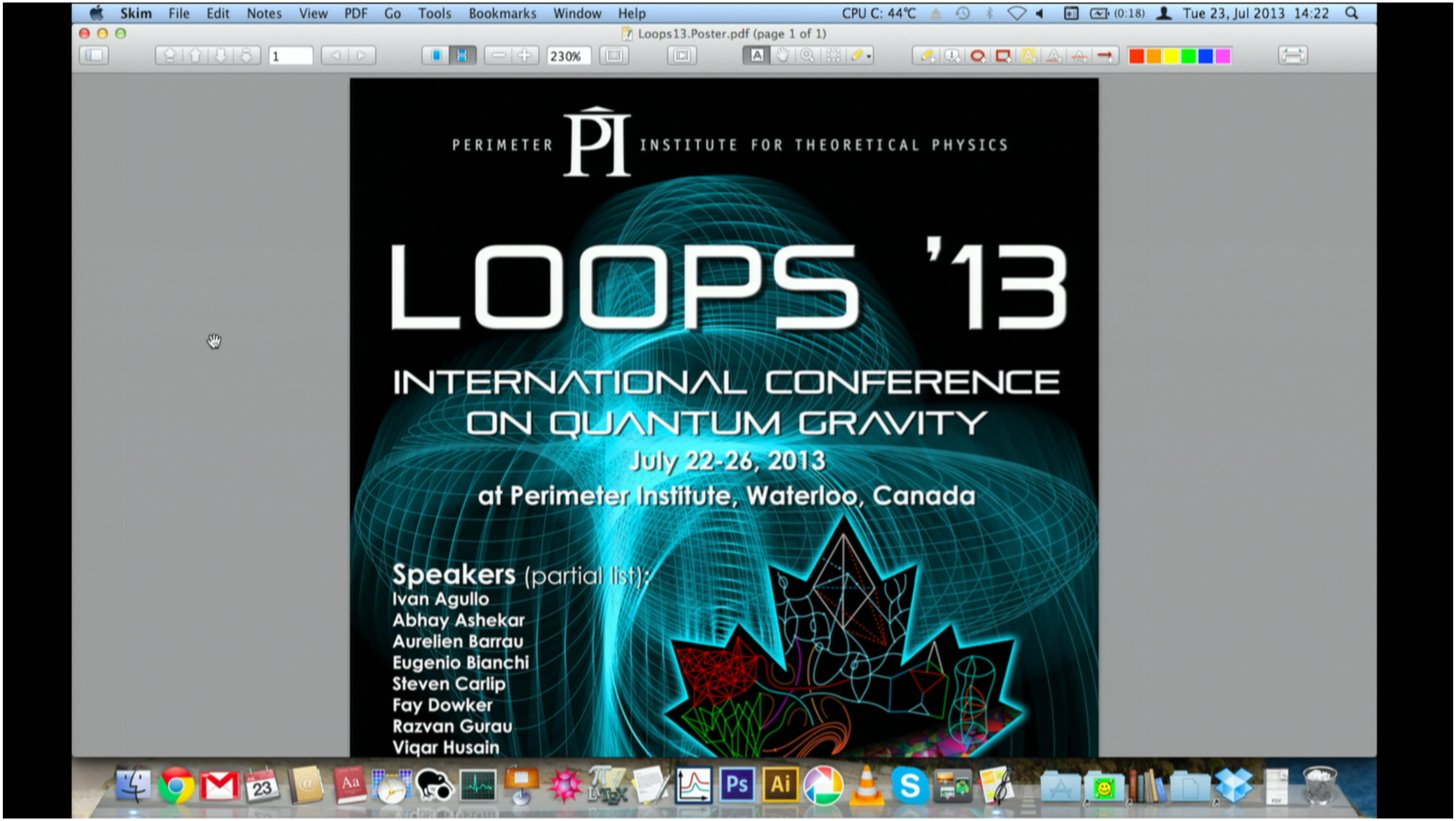
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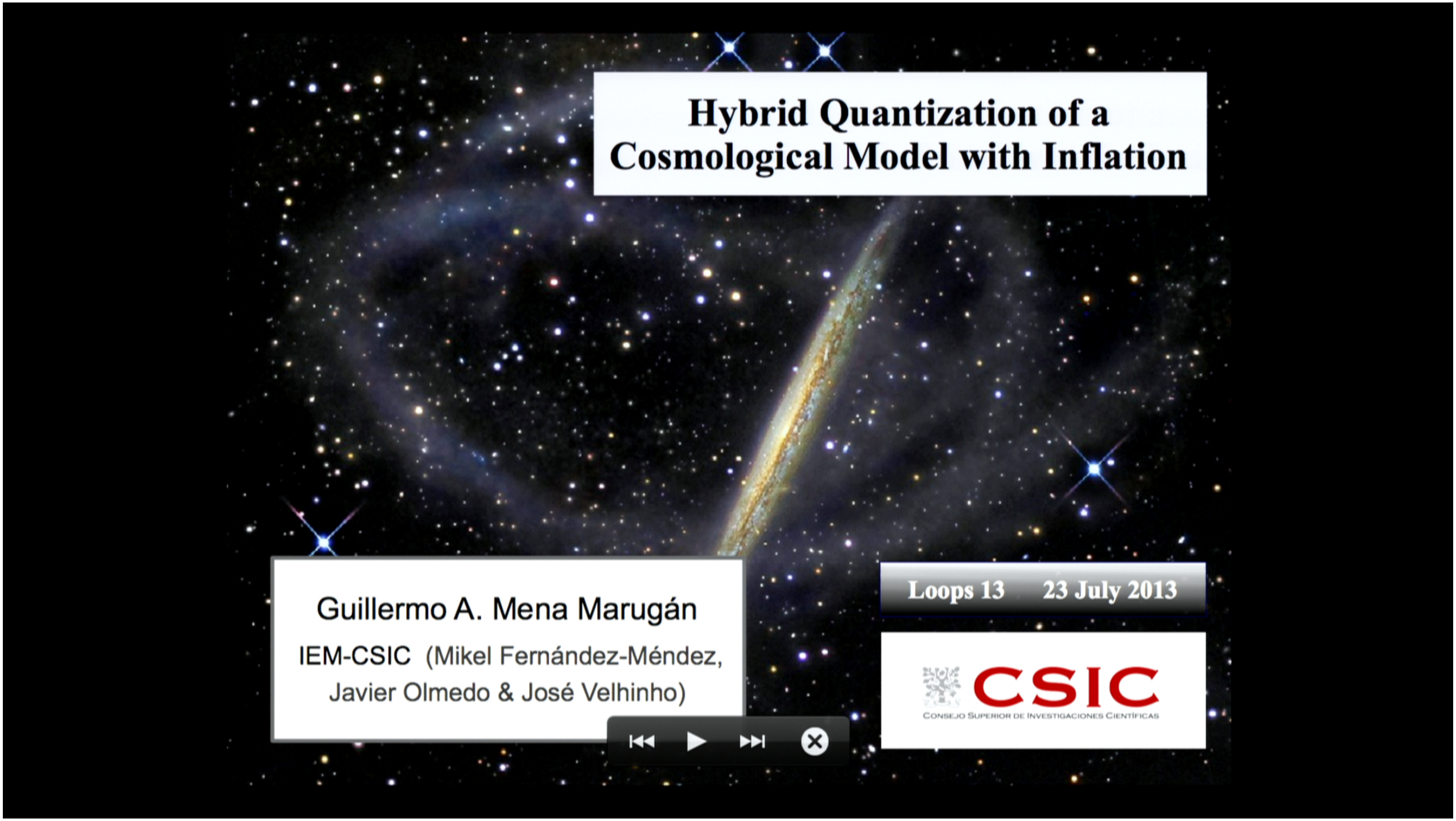
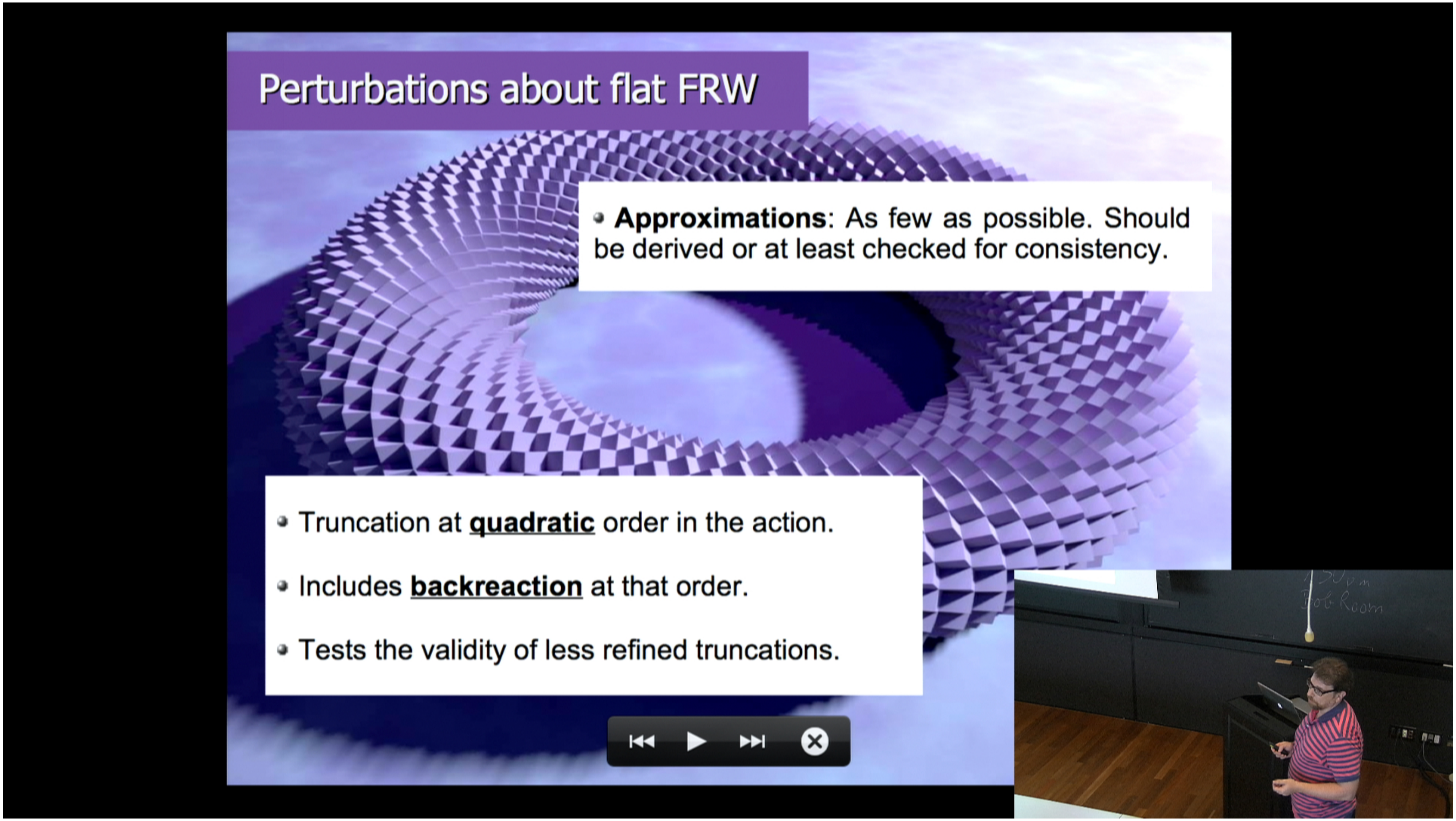
PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS
LOOPS '13
INTERNATIONAL CONFERENCE ON QUANTUM GRAVITY
July 22-26, 2013
at Perimeter Institute, Waterloo, Canada
Speakers (partial list):
Ivan Agullo
Abhay Ashekar
Aurelien Barrau
Eugenio Bianchi
Steven Carlip
Fay Dowker
Razvan Gurau
Viqar Husain

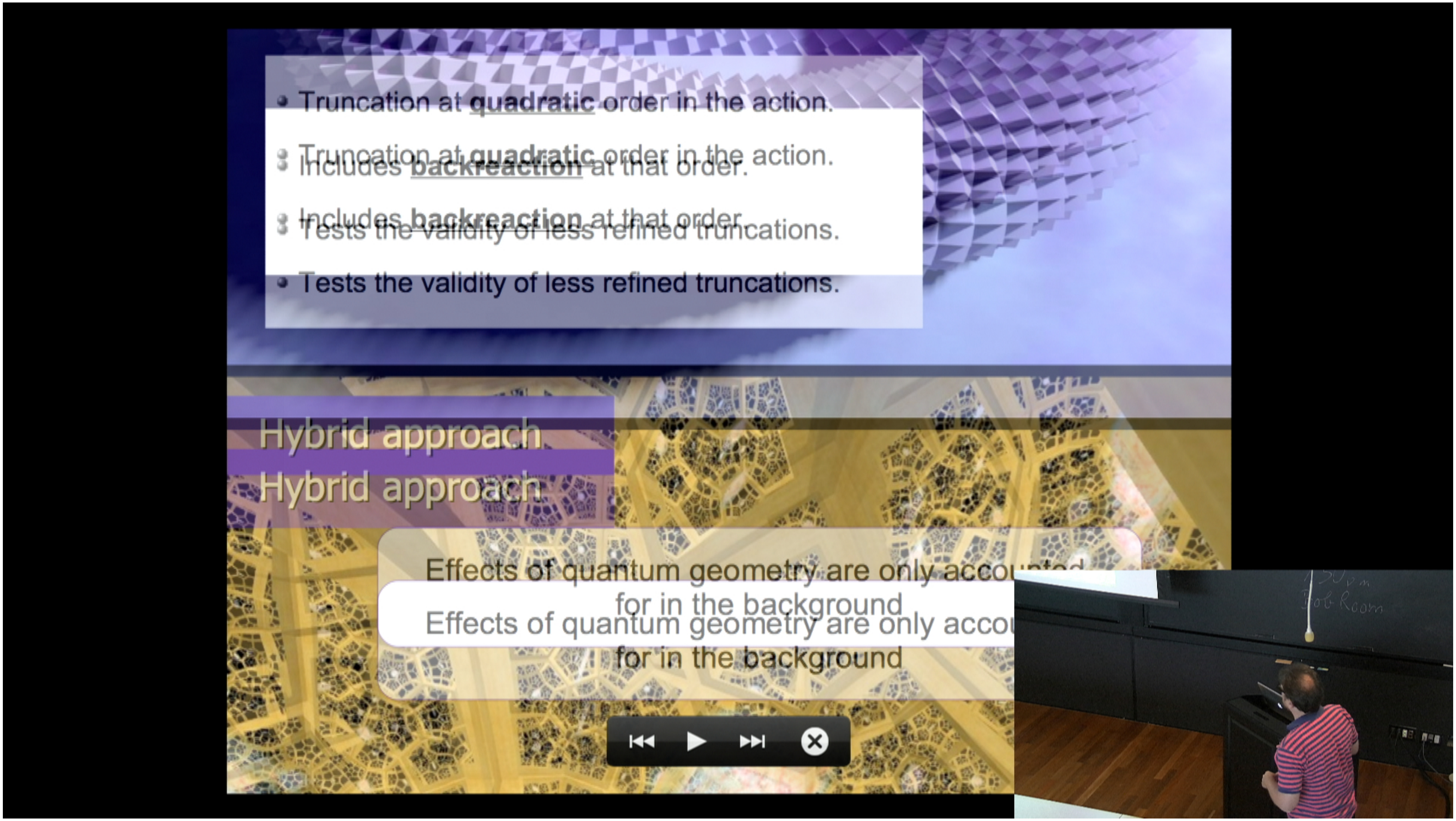

Hybrid Quantization of a
Cosmological Model with Inflation
Guillermo A. Mena Marugán
IEM-CSIC (Mikel Fernández-Méndez,
Javier Olmedo & José Velhinho)
Loops 13 23 July 2013
CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

The model

- We consider **perturbed** FRW universes filled with a **massive** scalar field.
- The model can generate inflation.

- The most interesting case is flat spatial topology. It is also the simplest.
- The effects of **spatial curvature** can be studied by considering, e.g., spherical topology.
- We assume **compact** spatial sections.


Perturbations about flat FRW
Approximations: As few as possible. Should be derived or at least checked for consistency.
Truncation at quadratic order in the action.
Includes backreaction at that order.
Tests the validity of less refined truncations.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Effects of quantum geometry are only accounted for in the background

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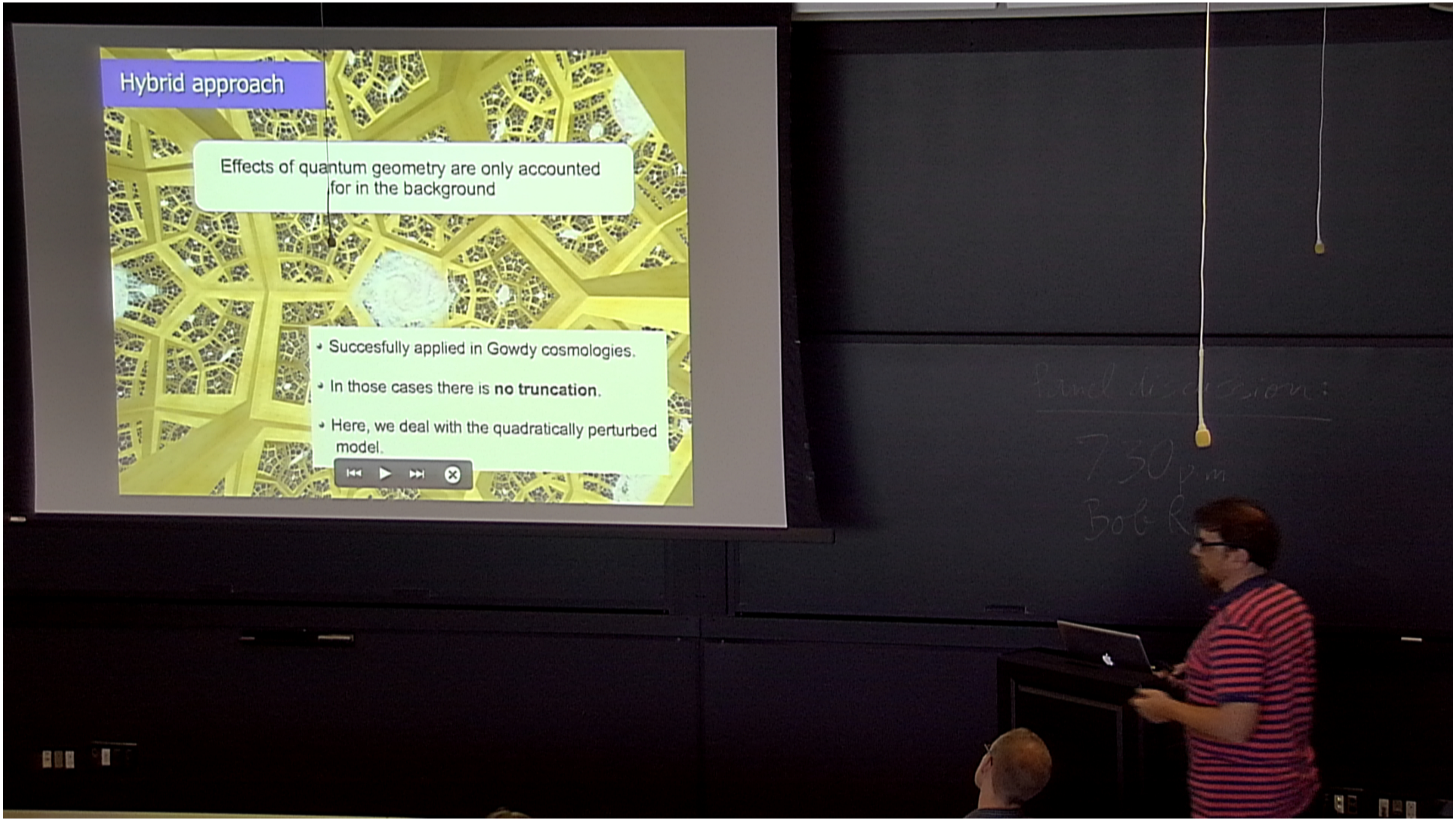
- Succesfully applied in Gowdy cosmologies.
- In those cases there is **no truncation**.
- Here, we deal with the quadratically perturbed model.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Uniqueness of the Fock description

- The **ambiguity** in selecting a Fock representation in QFT can be restricted by:

  - appealing to *background* **symmetries**.

  - demanding the **UNITARITY** of the quantum evolution.

- There is additional ambiguity in the **separation of the background** and the field. This introduces time-dependent canonical field transformations.

- Remarkably, our proposal selects a **UNIQUE canonical pair** and a **UNIQUE Fock representation** for their CCR's.

- One can even consider non-local transformations respecting mode decoupling.

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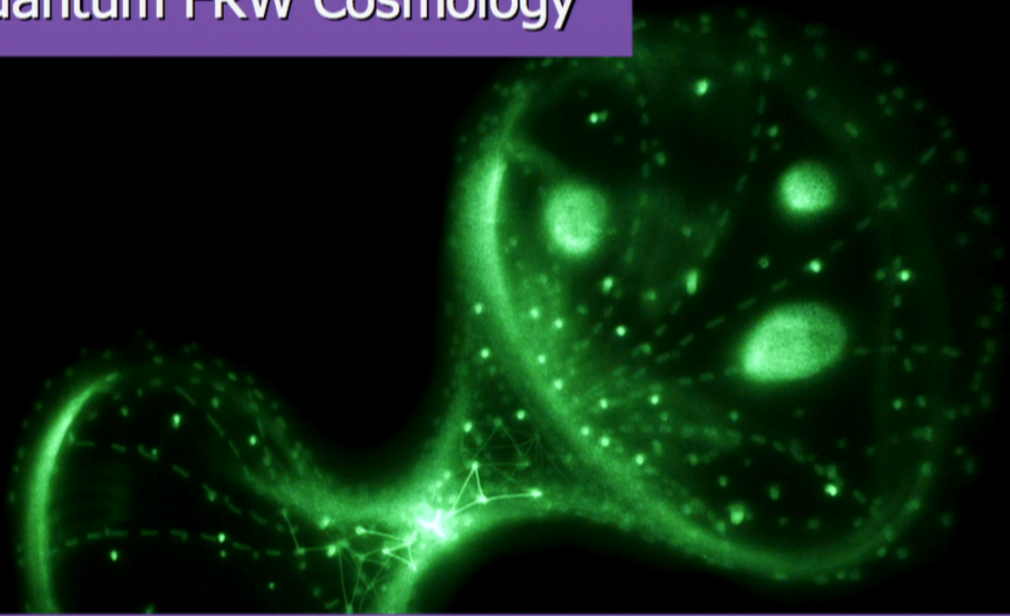
Loop Quantum FRW Cosmology



- Specific **proposal** such that:
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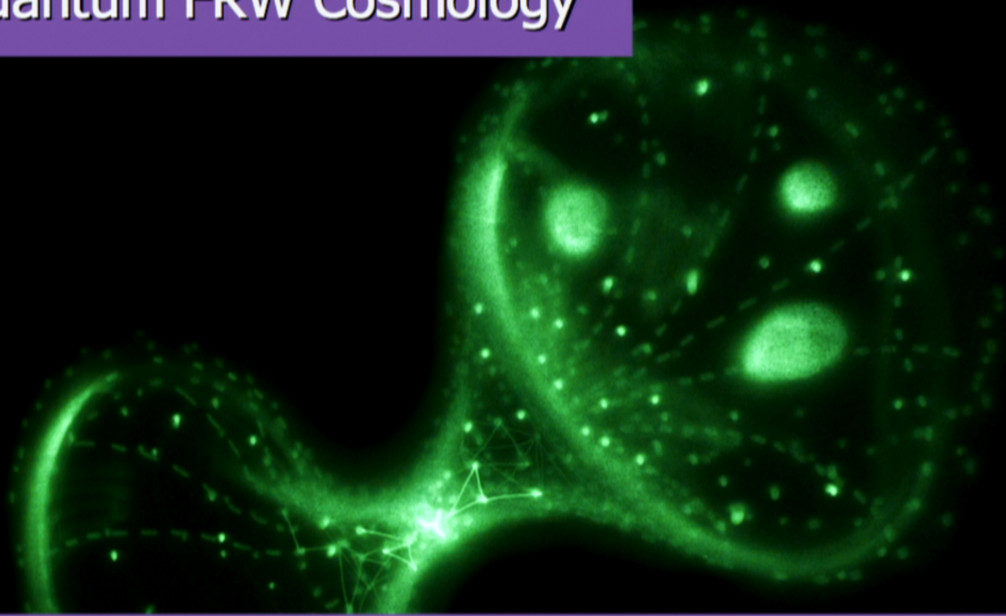

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Classical system: FRW

- **Massive scalar field minimally coupled to a compact, flat FRW universe.**

Geometry: $A_a^i = c\,{}^0e_a^i(2\pi)^{-1}$; $\quad E_i^a = p\sqrt{{}^0e}\,{}^0e_i^a(2\pi)^{-2}$. $\qquad \{c, p\} = 8\pi G\gamma/3$.

$a^2 = e^{2\alpha} = |p|(2\pi\sigma)^{-2}$; $\quad \pi_\alpha = -pc(\gamma 8\pi^3\sigma^2)^{-1}$. $\qquad \sigma^2 = G(6\pi^2)^{-1}$.

Matter: $\varphi = (2\pi)^{3/2}\sigma\phi$; $\quad \pi_\varphi = (2\pi)^{-3/2}\sigma^{-1}\pi_\phi$.

$V = |p|^{3/2}$.

Hamiltonian constraint:

$$C_0 = -\frac{6}{\gamma^2}\sqrt{|p|}\,c^2 + \frac{8\pi G}{V}(\pi_\phi^2 + m^2V^2\phi^2).$$

Classical system: Inhomogeneities

- We expand inhomogeneities in a (real) **Fourier basis**:

$$Q_{\vec{n},+}=(2\pi^{3/2})^{-1}\cos\vec{n}\cdot\vec{\theta}, \qquad Q_{\vec{n},-}=(2\pi^{3/2})^{-1}\sin\vec{n}\cdot\vec{\theta}. \qquad \omega_n^2=\vec{n}\cdot\vec{n}.$$

- We only consider **scalar perturbations, excluding zero modes**.

$$h_{ij}=(\sigma e^{\alpha})^2\left[{}^0h_{ij}+2\epsilon(2\pi)^{3/2}\sum\left\{a_{\vec{n},\pm}(t)Q_{\vec{n},\pm}\,{}^0h_{ij}+b_{\vec{n},\pm}(t)\left(\frac{3}{\omega_n^2}(Q_{\vec{n};\pm})_{,ij}+Q_{\vec{n},\pm}\,{}^0h_{ij}\right)\right\}\right],$$

$$N=\sigma N_0(t)\left[1+\epsilon(2\pi)^{3/2}\sum g_{\vec{n},\pm}(t)Q_{\vec{n},\pm}\right], \qquad N_i=\epsilon(2\pi)^{3/2}\sigma^2 e^{\alpha}\sum\frac{k_{\vec{n},\pm}(t)}{\omega_n}(Q_{\vec{n},\pm})_{;i},$$

$$\Phi=\frac{1}{\sigma}\left[\frac{\varphi(t)}{(2\pi)^{3/2}}+\epsilon\sum f_{\vec{n},\pm}(t)Q_{\vec{n},\pm}\right].$$

- At **quadratic** order in perturbations, one obtains:

$$H=\frac{N_0\sigma}{16\pi G}C_0+\epsilon^2\sum\left(N_0H_2^{\vec{n},\pm}+N_0 g_{\vec{n},\pm}H_1^{\vec{n},\pm}+k_{\vec{n},\pm}\tilde{H}_1^{\vec{n},\pm}\right).$$

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Longitudinal gauge: Hamiltonian

- We adopt a **longitudinal gauge**.
- After **REDUCTION**, the background variables are corrected with quadratic perturbations to form a canonical set.
- The remaining **Hamiltonian constraint** reads:

$$H = \frac{N_0 \sigma}{16 \pi G} C_0 + \epsilon^2 N_0 \sum H_2^{\vec{n},\pm}, \qquad H_2^{\vec{n},\pm} 2 e^{\bar{\alpha}} = \bar{E}_{\bar{f}\bar{f}} \bar{f}^2_{\vec{n},\pm} + \bar{E}_{\bar{f}\pi} \bar{f}_{\vec{n},\pm} \pi_{\bar{f}_{\vec{n},\pm}} + \bar{E}_{\pi\pi} \pi^2_{\bar{f}_{\vec{n},\pm}},$$

$$\bar{E}^n_{\bar{f}\bar{f}} = \omega_n^2 + e^{2\bar{\alpha}} m^2 \sigma^2 - \frac{e^{-4\bar{\alpha}}}{2} \left(\pi^2_{\bar{\alpha}} + 15 \pi^2_{\bar{\varphi}} + 3 e^{6\bar{\alpha}} m^2 \sigma^2 \bar{\varphi}^2 \right) - \frac{3}{\omega_n^2} e^{-8\bar{\alpha}} \left(e^{6\bar{\alpha}} m^2 \sigma^2 \bar{\varphi} - 2 \pi_{\bar{\alpha}} \pi_{\bar{\varphi}} \right)^2.$$

$$\bar{E}^n_{\bar{f}\pi} = -\frac{3}{\omega_n^2} e^{-6\bar{\alpha}} \pi_{\bar{\varphi}} \left(e^{6\bar{\alpha}} m^2 \sigma^2 \bar{\varphi} - 2 \pi_{\bar{\alpha}} \pi_{\bar{\varphi}} \right), \qquad \bar{E}^n_{\pi\pi} = 1 - \frac{3}{\omega_n^2} e^{-4\bar{\alpha}} \pi^2_{\bar{\varphi}}.$$

Robustness under gauge fixing

- A unitary transformation relates the reduced variables for the inhomogeneities with the **Mukhanov-Sasaki** variables.
- Similar results are obtained in the gauge of flat spatial sections.
- Moreover, the same **symplectic** structure for **gauge invariants** is obtained.

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Solutions to the constraint

- If the matter field serves as a **clock**, we can:

☆ Consider positive (negative) frequency states with respect to to that time.
☆ Use a Born-Oppenheimer-like **approximation** $\Psi = \chi_0(v,\phi)\psi(\phi, N[\vec{f}_{\vec{n},\pm}])$.
☆ **Neglect** the field momentum of the inhomogeneities versus that of the homogeneous part.

- This leads to a sort of ***effective*** QFT for the inhomogeneities.

$$-i\hbar\partial_\phi\psi = \frac{\epsilon^2}{2}\frac{\langle {}^{(0)}\hat{\Theta}_2 + {}^{(1)}\hat{\Theta}_2\hat{H}_0\rangle_{\chi_0}}{\langle \hat{H}_0\rangle_{\chi_0}}\psi .$$

$$\hat{H}_0^2 = {}^{(0)}\hat{\pi}_\phi^2 - \frac{\hat{C}_0}{8\pi G},$$

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Physical states

- An alternate **perturbative** scheme:

$$(\Psi| = (\Psi|^{(0)} + \epsilon^2 (\Psi|^{(2)} \ldots$$

- **FRW solution:** $(\Psi|^{(0)} \hat{C}_0 = 0,$

$$\hat{C}_0 = -\frac{6}{\gamma^2}\hat{\Omega}_0^2 + 8\pi G\left(\hat{\pi}_\phi^2 + m^2\hat{\phi}^2\hat{V}^2\right).$$

- ***Evolution* of the perturbations:**

$$(\Psi|^{(2)}\hat{C}_0 = -(\Psi|^{(0)}\left(\sum \hat{C}_2^{\vec{n},\pm}\right)^\dagger.$$

- Solutions are characterized by their initial data at **minimum volume.**
- From these data we arrive, e.g., at the **physical Hilbert space** $H_{kin}^{matt} \otimes \mathscr{F}$.

Conclusions

- We have considered a perturbed FRW universe with a **massive** scalar field.

  ✯ The action has been truncated to second order in the perturbations.
  ✯ A hybrid quantization scheme has been adopted.

- The system is endowed with a **symplectic structure** and a **Hamiltonian** constraint. **Backreaction** is included.

- **No internal time** is needed.

- Under some **controlled approximations**, if a matter clock is available, one may reach an effective QFT with first-order evolution equations.

- In our analysis, the dynamics are **UNITARY** in the QFT regime.

- One can characterize quantum states from data at **minimum volume**.