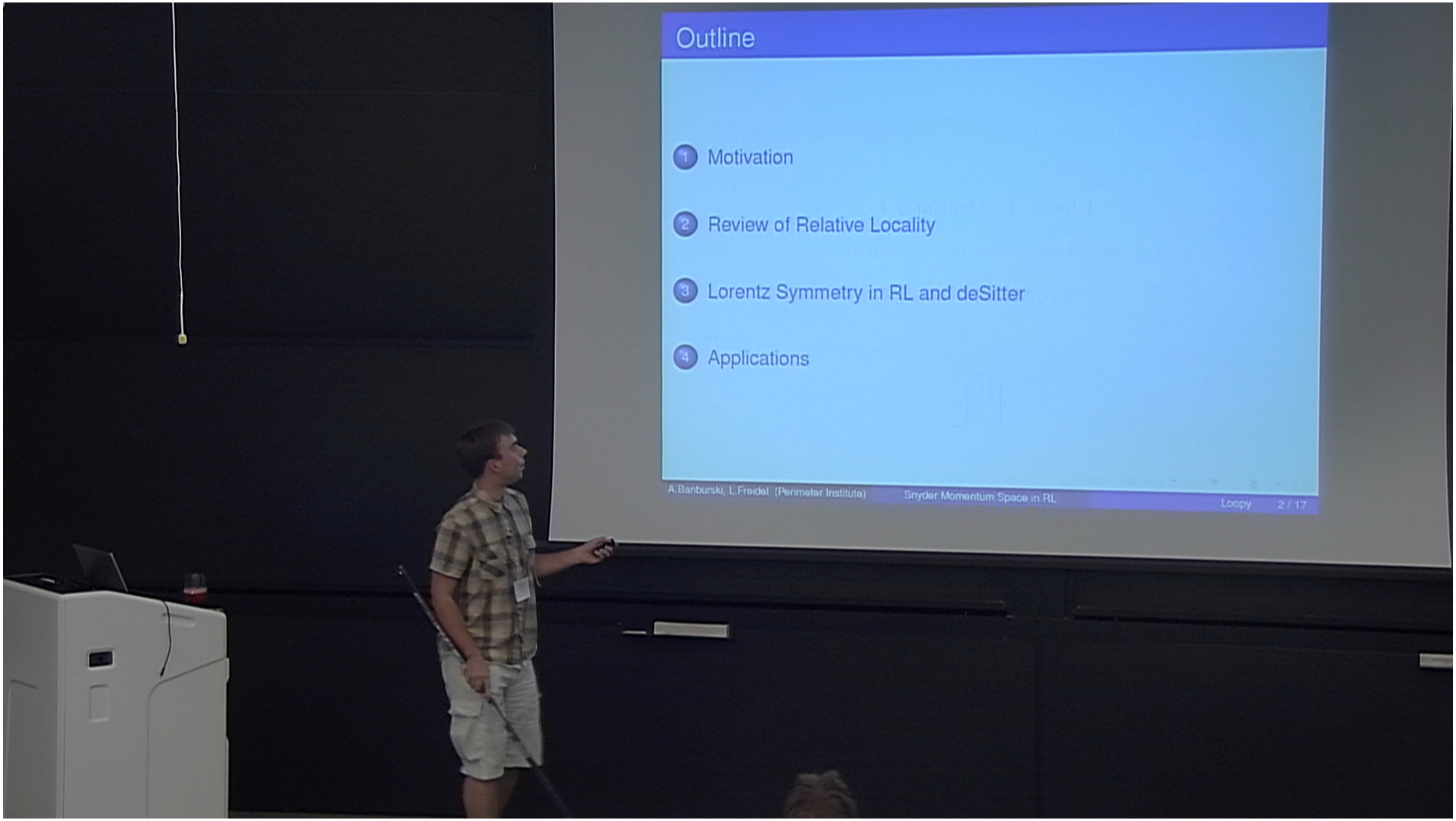
Title: Phenomenology - 2

Date: Jul 23, 2013 04:40 PM

URL: http://pirsa.org/13070056

Abstract:


Outline
1 Motivation
2 Review of Relative Locality
3 Lorentz Symmetry in RL and deSitter
4 Applications
A.Banburski, L.Freidel (Perimeter Institute)
Snyder Momentum Space in RL
Loopy
2 / 17

Outline

Motivation

- The usual approaches of phenomenology of QG, including noncommutative field theories or DSR have always until now explicitly broken Lorentz invariance:
  - in dispersion relation; or
  - in addition rule for momenta
- Is it possible to have in 3+1 dimensions a (non-local) deformation preserving full Lorentz Invariance, as in 2+1 QG?
- The answer is: YES!

Review of Relative Locality

- Relative Locality is postulated as a limit of QG in which $\hbar \to 0$ and $G_N \to 0$, but their ratio $\sqrt{\frac{\hbar}{G_N}} = m_P$ is fixed.
- Fundamental measurements are those of momenta and energies of particles, so it is natural to describe physics in momentum space, which does not have to be a priori flat.
- Take momentum space as a manifold. Have to define a notion of composition of momenta

Action of RL

The action for a point particle in Relative Locality is given by

$$S = \sum_J \int ds \left(x_J^\mu \dot{p}_\mu^J + \mathcal{N}_J \left(D^2(p) - m^2 \right) \right) + \sum_i z_i^\mu \mathcal{K}^i(p(0))_\mu$$

Equations of motion are

$$\dot{p}_\mu^J = 0 \quad \dot{x}_J^\mu = \mathcal{N}_J \frac{\delta \left(D^2(p) - m^2 \right)}{\delta p_\mu}$$

$$D^2(p) = m^2 \quad x_J^\mu(0) = \pm z^\nu \frac{\delta \mathcal{K}_\nu}{\delta p_\mu^J}$$


A.Banburski, L.Freidel (Perimeter Institute)

Lorentz Symmetry in RL and deSitter

We want to construct a momentum manifold $\mathcal{M}$ equipped with

- metric g
- addition rule $\oplus : \mathcal{M} \times \mathcal{M} \to \mathcal{M}$
- action of Lorentz group $p \to \Lambda(p)$, $\Lambda \in \mathrm{SO}(1,3)$ preserving g and $\oplus$
- $g_{\Lambda(p)}(\mathrm{d}_p\Lambda(X), \mathrm{d}_p\Lambda(Y)) = g_p(X, Y)$, $\qquad \Lambda(p \oplus q) = \Lambda(p) \oplus \Lambda(q)$

The second condition excludes κ-Poincaré, in which we have

$$\Lambda(p \oplus q) = \Lambda(p) \oplus \Lambda'_p(q)$$

with Λ'_p depending on p.

Lorentz Symmetry in RL and deSitter

We want to construct a momentum manifold $\mathcal{M}$ equipped with

- metric g
- addition rule $\oplus : \mathcal{M} \times \mathcal{M} \to \mathcal{M}$
- action of Lorentz group $p \to \Lambda(p)$, $\Lambda \in \mathrm{SO}(1,3)$ preserving g and $\oplus$
- $g_{\Lambda(p)}(\mathrm{d}_p\Lambda(X), \mathrm{d}_p\Lambda(Y)) = g_p(X, Y), \qquad \Lambda(p \oplus q) = \Lambda(p) \oplus \Lambda(q)$

The second condition excludes κ-Poincaré, in which we have

$$\Lambda(p \oplus q) = \Lambda(p) \oplus \Lambda'_p(q)$$

with Λ'_p depending on p.

Lorentz Symmetry in RL and deSitter

We want to construct a momentum manifold $\mathcal{M}$ equipped with

- metric g
- addition rule $\oplus : \mathcal{M} \times \mathcal{M} \to \mathcal{M}$
- action of Lorentz group $p \to \Lambda(p)$, $\Lambda \in \mathrm{SO}(1,3)$ preserving g and $\oplus$
- $g_{\Lambda(p)}(\mathrm{d}_p\Lambda(X), \mathrm{d}_p\Lambda(Y)) = g_p(X, Y), \qquad \Lambda(p \oplus q) = \Lambda(p) \oplus \Lambda(q)$

The second condition excludes κ-Poincaré, in which we have

$$\Lambda(p \oplus q) = \Lambda(p) \oplus \Lambda'_p(q)$$

with Λ'_p depending on p.

Finding the manifold $\mathcal{M}$

- The compatibility condition for addition can be written as

$$\Lambda L_p \Lambda^{-1} = L_{\Lambda(p)}$$

- Let I be the identity of the addition $\oplus$, then $\Lambda(I) = I$.
- Consider the group of all left multiplications and their inverses $\mathcal{L} \equiv \{L_{p_1}^{\pm 1} \cdots L_{p_n}^{\pm 1} \,|\, p_i \in \mathcal{M}\}$.
- Also consider the subgroup of $\mathcal{L}$ which leaves the identity invariant $\mathcal{G} \equiv \{L \in \mathcal{L} | \, L(1) = 1\}$. This group is left invariant by the adjoint action of the Lorentz group $\Lambda \mathcal{G} \Lambda^{-1} = \mathcal{G}$.

Finding the manifold $\mathcal{M}$

- The compatibility condition for addition can be written as

$$\Lambda L_p \Lambda^{-1} = L_{\Lambda(p)}$$

- Let I be the identity of the addition $\oplus$, then $\Lambda(I) = I$.
- Consider the group of all left multiplications and their inverses $\mathcal{L} \equiv \{L_{p_1}^{\pm 1} \cdots L_{p_n}^{\pm 1} \,|\, p_i \in \mathcal{M}\}$.
- Also consider the subgroup of $\mathcal{L}$ which leaves the identity invariant $\mathcal{G} \equiv \{L \in \mathcal{L} |\, L(1) = 1\}$. This group is left invariant by the adjoint action of the Lorentz group $\Lambda \mathcal{G} \Lambda^{-1} = \mathcal{G}$.
- The manifold $\mathcal{M} = \mathcal{L}/\mathcal{G}$

The simplest solution is homogenous with $\mathcal{G} = \mathrm{SO}(1,3)$, $\mathcal{L} = \mathrm{SO}(1,4)$. hence $\mathcal{M} = \mathrm{SO}(1,4)/\mathrm{SO}(1,3)$, which is the de Sitter space.

Constructing the addition rule

Let us work in embedding coordinates $-P_0^2 + P_1^2 + P_2^2 + P_3^2 + P_4^2 = 1$.

Constructing the addition rule

Let us work in embedding coordinates $-P_0^2 + P_1^2 + P_2^2 + P_3^2 + P_4^2 = 1$.

- We know that $L_P \in \mathrm{SO}(1,4)$. Requiring Lorentz invariance tells us that L_P has to be a tensor that depends only on P_A and $I_A = \delta_A^4$

$$(L_P)_A^B = \delta_A^B + aP_AP^B + bI_AI^B + cP_AI^B + dI_AP^B,$$

where a, b, c and d must be functions of the invariants $P \cdot P$, $I \cdot I$ and $P \cdot I$.

Constructing the addition rule

Let us work in embedding coordinates $-P_0^2 + P_1^2 + P_2^2 + P_3^2 + P_4^2 = 1$.

- We know that $L_P \in \mathrm{SO}(1,4)$. Requiring Lorentz invariance tells us that L_P has to be a tensor that depends only on P_A and $I_A = \delta_A^4$

$$(L_P)_A^B = \delta_A^B + aP_AP^B + bI_AI^B + cP_AI^B + dI_AP^B,$$

  where a, b, c and d must be functions of the invariants $P \cdot P$, $I \cdot I$ and $P \cdot I$.
- We can solve this to get

$$(P \oplus Q)_4 = 2P_4Q_4 - P \cdot Q,$$

$$(P \oplus Q)_\mu = Q_\mu + P_\mu \frac{Q_4 + 2P_4Q_4 - P \cdot Q}{1 + P_4}$$

Properties of the addition

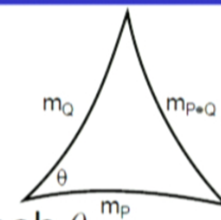
- Can show that $P_4 = \cosh m$ and $P_\mu P^\mu = -\sinh^2 m$.

$$(P \oplus Q)_4 = \cosh m_P \cosh m_Q + \sinh m_P \sinh m_Q \cosh \theta.$$

θ - rapidity of boost needed for changing rest frames from P to Q.

Properties of the addition

- Can show that $P_4 = \cosh m$ and $P_\mu P^\mu = -\sinh^2 m$.

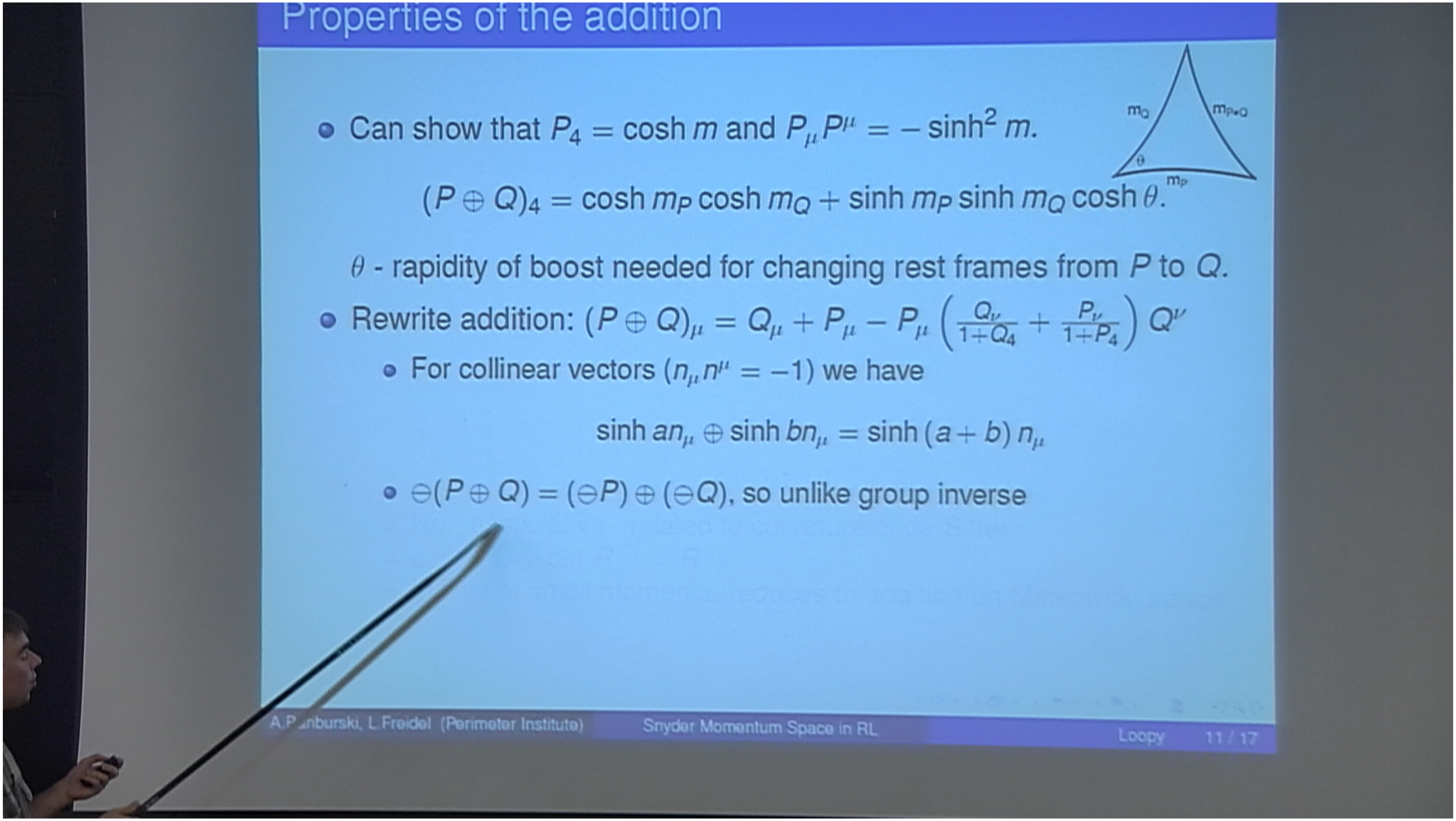


$$(P \oplus Q)_4 = \cosh m_P \cosh m_Q + \sinh m_P \sinh m_Q \cosh \theta.$$

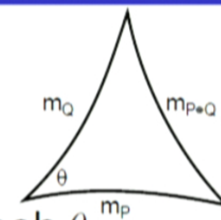
  θ - rapidity of boost needed for changing rest frames from P to Q.

- Rewrite addition: $(P \oplus Q)_\mu = Q_\mu + P_\mu - P_\mu \left(\frac{Q_\nu}{1+Q_4} + \frac{P_\nu}{1+P_4} \right) Q^\nu$
  - For collinear vectors ($n_\mu n^\mu = -1$) we have

$$\sinh a n_\mu \oplus \sinh b n_\mu = \sinh (a+b)\, n_\mu$$


Properties of the addition
Can show that $P_4 = \cosh m$ and $P_\mu P^\mu = -\sinh^2 m$.
$(P \oplus Q)_4 = \cosh m_P \cosh m_Q + \sinh m_P \sinh m_Q \cosh\theta$.
m_Q
$m_{P\oplus Q}$
θ
m_P
θ - rapidity of boost needed for changing rest frames from P to Q.
Rewrite addition: $(P \oplus Q)_\mu = Q_\mu + P_\mu - P_\mu \left(\frac{Q_\nu}{1+Q_4} + \frac{P_\nu}{1+P_4} \right) Q^\nu$
For collinear vectors ($n_\mu n^\mu = -1$) we have
$\sinh a n_\mu \oplus \sinh b n_\mu = \sinh (a+b)\, n_\mu$
$\ominus(P \oplus Q) = (\ominus P) \oplus (\ominus Q)$, so unlike group inverse
A.Pachburski, L.Freidel (Perimeter Institute)
Snyder Momentum Space in RL
Loopy
11 / 17

Properties of the addition



- Can show that $P_4 = \cosh m$ and $P_\mu P^\mu = -\sinh^2 m$.

$$(P \oplus Q)_4 = \cosh m_P \cosh m_Q + \sinh m_P \sinh m_Q \cosh \theta.$$

  θ - rapidity of boost needed for changing rest frames from P to Q.

- Rewrite addition: $(P \oplus Q)_\mu = Q_\mu + P_\mu - P_\mu \left(\frac{Q_\nu}{1+Q_4} + \frac{P_\nu}{1+P_4} \right) Q^\nu$
  - For collinear vectors ($n_\mu n^\mu = -1$) we have

$$\sinh a n_\mu \oplus \sinh b n_\mu = \sinh (a+b)\, n_\mu$$

  - $\ominus(P \oplus Q) = (\ominus P) \oplus (\ominus Q)$, so unlike group inverse
  - Non-associative - related to curvature of de Sitter
  - In limit of small momenta, reduces to addition on Minkowski space

Properties of the addition

- Can show that $P_4 = \cosh m$ and $P_\mu P^\mu = -\sinh^2 m$.

$$(P \oplus Q)_4 = \cosh m_P \cosh m_Q + \sinh m_P \sinh m_Q \cosh \theta.$$

  θ - rapidity of boost needed for changing rest frames from P to Q.

- Rewrite addition: $(P \oplus Q)_\mu = Q_\mu + P_\mu - P_\mu \left(\frac{Q_\nu}{1+Q_4} + \frac{P_\nu}{1+P_4} \right) Q^\nu$
  - For collinear vectors ($n_\mu n^\mu = -1$) we have

$$\sinh a n_\mu \oplus \sinh b n_\mu = \sinh (a+b)\, n_\mu$$

  - $\ominus(P \oplus Q) = (\ominus P) \oplus (\ominus Q)$, so unlike group inverse
  - Non-associative - related to curvature of de Sitter
  - $L_P^{-1} = L_{\ominus P}$ but $R_P^{-1} \neq R_{\ominus P}$
  - In limit of small momenta, reduces to addition on Minkowski space

Properties of the addition

- Can show that $P_4 = \cosh m$ and $P_\mu P^\mu = -\sinh^2 m$.

$$(P \oplus Q)_4 = \cosh m_P \cosh m_Q + \sinh m_P \sinh m_Q \cosh \theta.$$

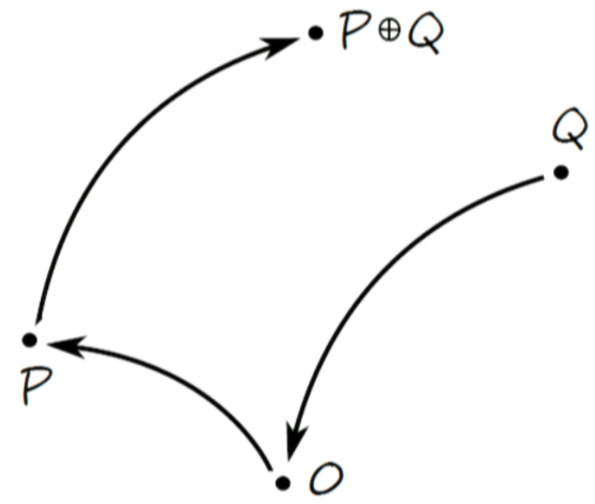
  θ - rapidity of boost needed for changing rest frames from P to Q.
- Rewrite addition: $(P \oplus Q)_\mu = Q_\mu + P_\mu - P_\mu \left(\frac{Q_\nu}{1+Q_4} + \frac{P_\nu}{1+P_4} \right) Q^\nu$
  - For collinear vectors ($n_\mu n^\mu = -1$) we have

$$\sinh a n_\mu \oplus \sinh b n_\mu = \sinh (a+b)\, n_\mu$$

  - $\ominus(P \oplus Q) = (\ominus P) \oplus (\ominus Q)$, so unlike group inverse
  - Non-associative - related to curvature of de Sitter
  - $L_P^{-1} = L_{\ominus P}$ but $R_P^{-1} \neq R_{\ominus P}$
  - In limit of small momenta, reduces to addition on Minkowski space

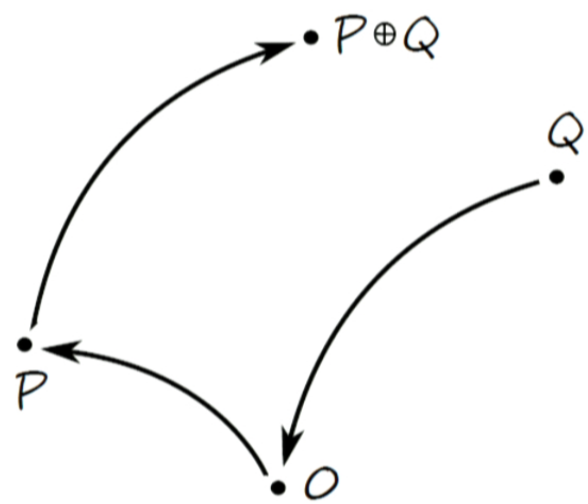
Geometric understanding of the addition

One can also show that $P \oplus Q \equiv \exp_P \circ U_P^I \circ \exp_I^{-1} Q$.



Geometric understanding of the addition

One can also show that $P \oplus Q \equiv \exp_P \circ U_P^I \circ \exp_I^{-1} Q$.



Emergence of Snyder spacetime

- Can change to 4d coordinates by using frame fields $e^{\mu}_{A}(p) \equiv \partial^{\mu} P_A$
- Convenient to work in $P_\mu = p_\mu$ with metric $g^{\mu\nu} = \eta^{\mu\nu} + p^\mu p^\nu / P_4^2$
- Using $\{p_\mu, x^\nu\} = \delta^\nu_\mu$ and $X^A = e^A_\mu x^\mu$ we get

Emergence of Snyder spacetime

- Can change to 4d coordinates by using frame fields $e^{\mu}_{A}(p) \equiv \partial^{\mu} P_A$
- Convenient to work in $P_\mu = p_\mu$ with metric $g^{\mu\nu} = \eta^{\mu\nu} + p^\mu p^\nu / P_4^2$
- Using $\{p_\mu, x^\nu\} = \delta^\nu_\mu$ and $X^A = e^A_\mu x^\mu$ we get

$$\left\{P_A, X^B\right\} = \delta^B_A - P_A P^B, \quad \left\{X^A, X^B\right\} = J^{AB}$$

classical version of CRs of Snyder quantum spacetime: to include minimal length scale, while preserving Lorentz invariance, promote x^μ to a hermitian operator $x^\mu = i\ell\left(P_4 \frac{\partial}{\partial P_\mu} - P^\mu \frac{\partial}{\partial P_4}\right)$

- In RL framework can get same Poisson brackets for interaction coordinates z if we consider tree processes of the form

Emergence of Snyder spacetime

- Can change to 4d coordinates by using frame fields $e^{\mu}_{A}(p) \equiv \partial^{\mu} P_A$
- Convenient to work in $P_{\mu} = p_{\mu}$ with metric $g^{\mu\nu} = \eta^{\mu\nu} + p^{\mu} p^{\nu} / P_4^2$
- Using $\{p_{\mu}, x^{\nu}\} = \delta^{\nu}_{\mu}$ and $X^A = e^{A}_{\mu} x^{\mu}$ we get

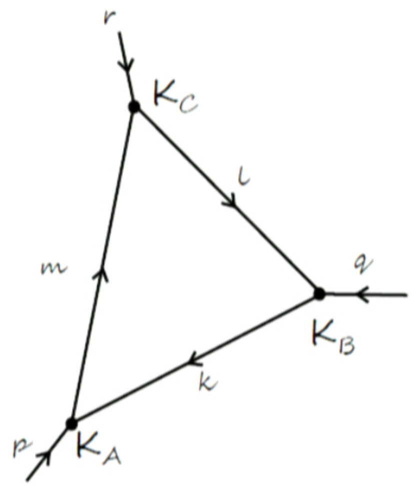
$$\left\{P_A, X^B\right\} = \delta^B_A - P_A P^B, \quad \left\{X^A, X^B\right\} = J^{AB}$$

  classical version of CRs of Snyder quantum spacetime: to include minimal length scale, while preserving Lorentz invariance, promote x^{μ} to a hermitian operator $x^{\mu} = i\ell \left(P_4 \frac{\partial}{\partial P_{\mu}} - P^{\mu} \frac{\partial}{\partial P_4} \right)$
- In RL framework can get same Poisson brackets for interaction coordinates z if we consider tree processes of the form $\mathcal{K} = (\sum Q_i) \oplus P$.

Loop processes

When we consider loop processes in RL with this framework, it seems impossible to avoid "x-dependence". This is due to the curvature of the momentum space.

$$\tau_l \hat{l}^{\mu} + \tau_k \hat{k}^{\nu} \left[U^l_k\right]^{\mu}_{\nu} + \tau_m \hat{m}^{\nu} \left[U^l_m\right]^{\mu}_{\nu} = x^{\nu}_{k,\,A} \left[U^l_k\right]^{\rho}_{\nu} (1\!\!1^{\rho}_{\mu} - \prod_{loop} U^{\rho}_{\mu}) \sim \mathit{curvature}$$

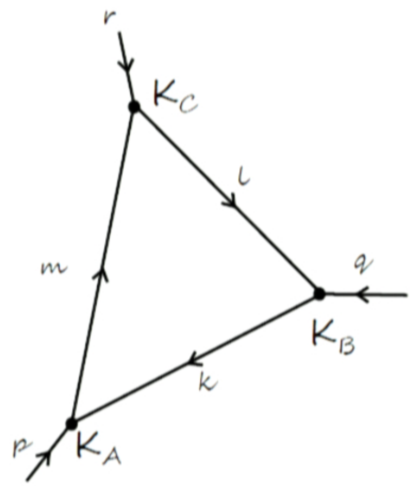


Because every loop depends on specific x, we cannot universally synchronize more than 2 clocks in a single process.

Loop processes

When we consider loop processes in RL with this framework, it seems impossible to avoid "x-dependence". This is due to the curvature of the momentum space.

$$\tau_l \hat{l}^\mu + \tau_k \hat{k}^\nu \left[U_k^l\right]_\nu^\mu + \tau_m \hat{m}^\nu \left[U_m^l\right]_\nu^\mu = x_{k,\ A}^\nu \left[U_k^l\right]_\nu^\rho (1\!\!1_\mu^\rho - \prod_{loop} U_\mu^\rho) \sim curvature$$

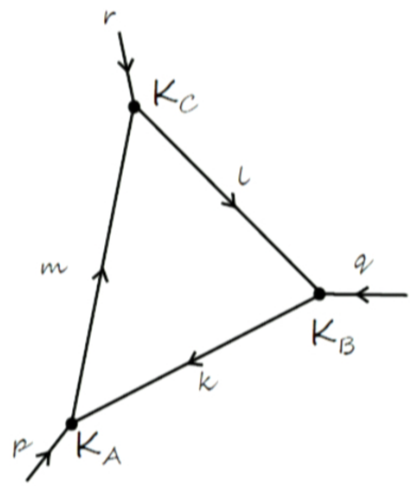


Because every loop depends on specific x, we cannot universally synchronize more than 2 clocks in a single process.

Loop processes

When we consider loop processes in RL with this framework, it seems impossible to avoid "x-dependence". This is due to the curvature of the momentum space.

$$\tau_l \hat{l}^{\mu} + \tau_k \hat{k}^{\nu} \left[U^l_k\right]^{\mu}_{\nu} + \tau_m \hat{m}^{\nu} \left[U^l_m\right]^{\mu}_{\nu} = x^{\nu}_{k,\,A} \left[U^l_k\right]^{\rho}_{\nu} \left(\mathbb{1}^{\rho}_{\mu} - \prod_{loop} U^{\rho}_{\mu}\right) \sim curvature$$



Because every loop depends on specific x, we cannot universally synchronize more than 2 clocks in a single process.

Summary

- We have constructed the first example of 3+1 defomration of Relativity preserving full Lorentz invariance.
- It turnes out to be related to the first ever studied quantum spacetime - Snyder.
- This provides a new (hopefully fruitful) model for QG phenomenology.

Summary

- We have constructed the first example of 3+1 defomration of Relativity preserving full Lorentz invariance.
- It turnes out to be related to the first ever studied quantum spacetime - Snyder.
- This provides a new (hopefully fruitful) model for QG phenomenology.

- Outlook
  - Understand the x-dependence and find more phenomenological predictions
  - Is this a unique construction?

Summary

- We have constructed the first example of 3+1 defomration of Relativity preserving full Lorentz invariance.
- It turnes out to be related to the first ever studied quantum spacetime - Snyder.
- This provides a new (hopefully fruitful) model for QG phenomenology.

- Outlook
  - Understand the x-dependence and find more phenomenological predictions
  - Is this a unique construction?

Scalar Field Theory in a Curved Momentum Space

Trevor Rempel

Perimeter Institute for Theoretical Physics

July 23, 2013

Scalar Field Theory in a Curved Momentum Space

Trevor Rempel

Perimeter Institute for Theoretical Physics

July 23, 2013

Introduction

Motivation

- Relative locality was originally formulated in the "classical non–gravitational" limit:

$$\hbar, G_{\mathrm{N}} \to 0 \text{ keeping } m_p = \sqrt{\hbar/G_{\mathrm{N}}} \text{ constant}$$

- First step towards "turning $\hbar$ back on"

Introduction

Motivation

- Relative locality was originally formulated in the "classical non–gravitational" limit:

$$\hbar, G_N \to 0 \text{ keeping } m_p = \sqrt{\hbar/G_N} \text{ constant}$$

- First step towards "turning $\hbar$ back on"

Set–up

- Momentum space is a non-linear manifold $\mathcal{M}$
- Spacetime emerges as the cotangent planes $T_p^*\mathcal{M}$ to points in momentum space – Trivial geometry
- Phase space is the cotangent bundle $T^*\mathcal{M}$
- Spacetime is no longer absolute, each observer constructs their own spacetime as momentum dependent slices of phase space

Combination of Momenta

To describe interactions we need a method for combining momenta. Define a rule, $\oplus$, given by

$$\begin{aligned} \oplus : \mathcal{M} \times \mathcal{M} &\to \mathcal{M} \\ (p, q) &\mapsto p \oplus q \end{aligned} \tag{1}$$

Significance of Combination Rule

We can use the combination rule to define a connection on momentum space

$$\Gamma^{\mu\nu}_{\rho}(0) = \frac{\partial}{\partial p_\mu}\frac{\partial}{\partial q_\nu}(p \oplus q)_\rho\Big|_{p,q=0} \tag{3}$$

- Covariant derivatives are defined in terms of this connection
- In general this connection differs from the standard metric compatible one

Significance of Combination Rule

We can use the combination rule to define a connection on momentum space

$$\Gamma^{\mu\nu}_{\rho}(0) = \frac{\partial}{\partial p_\mu}\frac{\partial}{\partial q_\nu}(p \oplus q)_\rho\Big|_{p,q=0} \tag{3}$$

- Covariant derivatives are defined in terms of this connection
- In general this connection differs from the standard metric compatible one

It turns out that the torsion measures the failure of the combination rule to commute

$$T^{\mu\nu}_{\rho}(0) = \Gamma^{[\mu\nu]}_{\rho}(0) = \frac{\partial}{\partial p_\mu}\frac{\partial}{\partial q_\nu}(p \oplus q - q \oplus p)_\rho\Big|_{p,q=0} \tag{4}$$

Defining Mass

It is assumed that the metric on momentum, $g_{\mu\nu}(p)$, is known. Given a path $\gamma(\tau)$ connecting points p_0, p_1 in momentum space, the distance between these points is

$$D_\gamma(p_0, p_1) = \int_a^b \sqrt{-g^{\mu\nu}\left(\gamma(\tau)\right) \frac{d\gamma_\mu}{d\tau} \frac{d\gamma_\nu}{d\tau}} d\tau \quad (6)$$

Defining Mass

It is assumed that the metric on momentum, $g_{\mu\nu}(p)$, is known. Given a path $\gamma(\tau)$ connecting points p_0, p_1 in momentum space, the distance between these points is

$$D_\gamma(p_0, p_1) = \int_a^b \sqrt{-g^{\mu\nu}(\gamma(\tau)) \frac{d\gamma_\mu}{d\tau} \frac{d\gamma_\nu}{d\tau}} d\tau \tag{6}$$

- A geodesic is a curve which extremizes this distance
- If γ is a geodesic we write $D_\gamma(p_0, p_1) = D(p_0, p_1)$

Given a particle with momentum p, we define its mass to be the geodesic distance from the origin:

$$D^2(p, 0) = D^2(p) = -m^2 \tag{7}$$

Modified Feynman Rules

Rule 4) – Integrate over all momenta

- Introduce a measure on momentum space – $d\mu(p)$.
- Define a delta function, $\delta(p,q)$, which is compatible with $d\mu(p)$:

$$\int d\mu(p)\delta(p,q)f(p) = f(q) \tag{8}$$

Rule 5) – Symmetry Factor

- Requires no modification

Rule 1) – Factor associated with the propagator

- Propagator has a single simple pole when a particle goes on shell which suggests

$$p^2 + m^2 \to D^2(p) + m^2 \tag{9}$$

Modified Feynman Rules

Rule 2) – Factor associated with external point

- Requires no modification

Rule 3) – Factor associated with vertex

- Combination rule $\oplus$ is not associative or commutative so $p \oplus q \oplus k$ is ambiguous
- Factor we write down should reflect statistics of the particles
- Assume scalar fields still obey Bose statistics – Vertex factor should be symmetric on interchange of momentum labels
- We denote the vertex factor by, $-g\mathcal{F}(p, q, k)$, where

$$\mathcal{F}(p, q, k) = \frac{1}{6}\big[\delta(p, \ominus(q \oplus k)) + \delta(p, \ominus(k \oplus q)) + \delta(q, \ominus(p \oplus k)) + \delta(q, \ominus(k \oplus p)) + \delta(k, \ominus(p \oplus q)) + \delta(k, \ominus(q \oplus p))\big]$$

Momentum Space Action

The generating functional for this theory can be written as

$$Z(J) \propto \exp\left(-\frac{g}{3!}\int d\mu(p)\int d\mu(q)\int d\mu(k)\mathcal{F}(p,q,k)\frac{\delta}{\delta J(p)}\frac{\delta}{\delta J(q)}\frac{\delta}{\delta J(k)}\right)$$
$$\times \exp\left(\frac{i}{2}\int d\mu(p)J(p)\left(D^2(p)+m^2\right)^{-1}J(\ominus p)\right)$$

Inserting a path integral over the field $\varphi(p)$ allows us to extract the corresponding action, which is given by

$$S = -\frac{1}{2}\int d\mu(p)\left(D^2(p)+m^2\right)\varphi(p)\varphi(\ominus p)$$
$$+\frac{g}{3!}\int d\mu(p)d\mu(q)d\mu(k)\mathcal{F}(p,q,k)\varphi(\ominus p)\varphi(\ominus q)\varphi(\ominus k)$$

The fields commute so the factor $\mathcal{F}(p,q,k)$ collapses to the single term $\delta(p, \ominus(q\oplus k))$

Momentum Space Action

Integrating out the delta function and imposing the reality condition $\varphi(\ominus p) = \varphi^*(p)$ we find

$$S = -\frac{1}{2}\int d\mu(p)\left(D^2(p) + m^2\right)\varphi(p)\varphi^*(p) + \frac{g}{3!}\int d\mu(p)d\mu(q)\varphi(p\oplus q)\varphi^*(p)\varphi^*(q) \tag{10}$$

Momentum Space Action

Integrating out the delta function and imposing the reality condition $\varphi(\ominus p) = \varphi^*(p)$ we find

$$S = -\frac{1}{2}\int d\mu(p)\left(D^2(p) + m^2\right)\varphi(p)\varphi^*(p) + \frac{g}{3!}\int d\mu(p)d\mu(q)\varphi(p \oplus q)\varphi^*(p)\varphi^*(q) \tag{10}$$

- Want to explore the spacetime properties of this action, particularly locality
- Need to Fourier transform into spacetime

World Function

Introduce Synge's[1] world function

$$\sigma(p, p') = \frac{1}{2}\int_0^1 d\tau g^{\mu\nu}(\gamma(\tau))\dot{\gamma_\mu}\dot{\gamma_\nu}, \tag{11}$$

where $\gamma_\mu(\tau)$ is a geodesic connecting p and p'.

[1] J.L.Synge, "Relativity: The General Theory"

World Function

Introduce Synge's[1] world function

$$\sigma(p,p') = \frac{1}{2}\int_0^1 d\tau g^{\mu\nu}(\gamma(\tau))\dot{\gamma_\mu}\dot{\gamma_\nu}, \tag{11}$$

where $\gamma_\mu(\tau)$ is a geodesic connecting p and p'.

- $\sigma(p,p')$ is a bi–scalar
- Integrand in (11) is constant along a geodesic so

$$\sigma(p,p') = \frac{1}{2}D^2(p,p') \tag{12}$$

Using the notation

$$\nabla_{p_a}\sigma(p,p') = \sigma^a(p,p') \qquad \text{and} \qquad \nabla_{p'_a}\sigma(p,p') = \sigma^{a'}(p,p'),$$

we find that the world–function satisfies the differential equation

$$2\sigma(p,p') = \sigma_a(p,p')\sigma^a(p,p') \tag{13}$$

[1] J.L.Synge, "Relativity: The General Theory"

Fourier Kernel

Fix a point $p' \in \mathcal{M}$ and let $x^{\mu'} \in T^*_{p'}\mathcal{M}$, then the kernel

$$\exp\left(ix^{\mu'}\sigma_{\mu'}(p,p')\right)$$

is covariant for all $p, p' \in \mathcal{M}$.

- Dependence on p' persists even when momentum space is flat

Define $R_p(q) = q \oplus p$ introduce the translated world function

$$\sigma^R(p,p') = \sigma(R_{p'}(p), p')$$

which does have the correct flat momentum space limit.

- Take the Fourier kernel to be

$$\exp\left(ix^{\mu'}\sigma^R_{\mu'}(p,p')\right)$$

Plane Waves and Transport Operator

Define a "covariant plane wave" based at the point $p' \in \mathcal{M}$ as

$$e_{p'}(p,x) = \mathcal{V}^{1/2}(R_{p'}(p),p')\exp\left(-ix^{\mu'}\sigma^{R}_{\mu'}(p,p')\right),$$

where $x \in T^*_{p'}\mathcal{M}$.

- Plane waves are eigenfunctions of the Laplacian on $T^*_{p'}\mathcal{M}$

$$\Box_x e_{p'}(p,x) = -D^2(R_{p'}(p),p')e_{p'}(p,x)$$

Action in Spacetime

Recall our momentum space action

$$S = -\frac{1}{2}\int d\mu(p)\left(D^2(p) + m^2\right)\varphi(p)\varphi^*(p)$$
$$+ \frac{g}{3!}\int d\mu(q)\int d\mu(k)\varphi(q \oplus k)\varphi^*(q)\varphi^*(k).$$

Kinetic Term

We would like to use that

$$D^2(p)e_0(p,x) = -\Box_x e_0(p,x)$$

Kinetic Term

We would like to use that

$$D^2(p)e_0(p,x) = -\Box_x e_0(p,x)$$

- Need to translate the $e_{p'}$ appearing the Fourier transform of $\varphi(p)$ to e_0, use the above relation and then translate the result back to p'.

Performing this computation we obtain

$$(\Box\hat{\varphi})_{p'}(x) = \int d\mu(y) T_{p',0}(x,y) \Box_y \hat{\varphi}_0(y).$$

Kinetic term can be written as

$$\int d\mu(p) D^2(p)\varphi(p)\varphi^*(p) = -\int d\mu(x) \left(\hat{\varphi}_{p'} \circ (\Box\hat{\varphi})_{p'}\right)(x).$$

UNIVERSITA' DEGLI STUDI DI ROMA
"LA SAPIENZA"

INTRODUCING LATESHIFT

NICCOLÒ LORET

[arXiv:1305.5062]

WITH: GIOVANNI AMELINO-CAMELIA, LEONARDO BARCAROLI AND GIULIA GUBITOSI

LOOPS 13 CONFERENCE

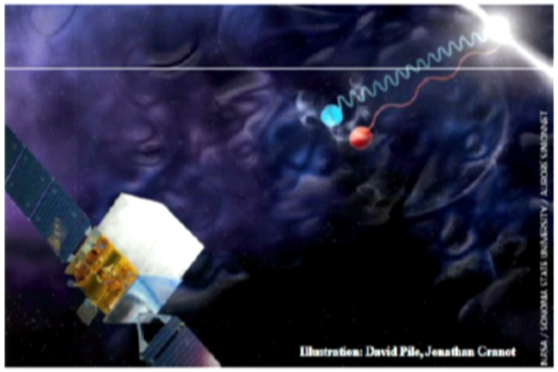
PERIMETER INSTITUTE JULY 22-26, 2013

Introducing Lateshift

• We call Lateshift a Relative-Locality effect such that

*an observer (**Bob**) measures different time-of-arrival for two photons with different energies emitted simultaneously by the emitter (**Alice**).*

$$c \sim 1 + \frac{\eta}{M_P} p$$



Deformed spacetime symmetries

$$E^2 = p^2 + m^2 + \eta p^2 \frac{E^\alpha}{E_P^\alpha} + \mathcal{O}\left(\frac{E^{\alpha+3}}{E_P^{\alpha+1}}\right)$$

→ **Violation**

→ **Deformation**

Deformed spacetime symmetries

$$E^2 = p^2 + m^2 + \eta p^2 \frac{E^\alpha}{E_P^\alpha} + \mathcal{O}\left(\frac{E^{\alpha+3}}{E_P^{\alpha+1}}\right)$$

→ Violation

→ Deformation

If we want to preserve Lorentz invariance we should think to deform Poincaré algebra, for example:

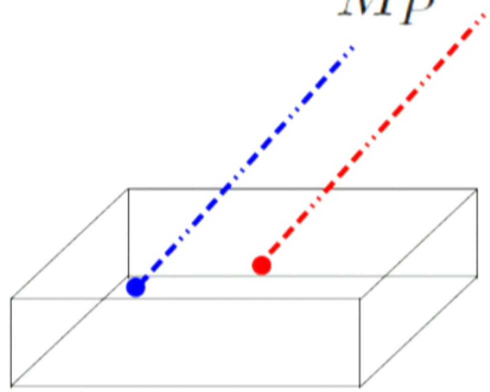
$$\mathcal{C} = P_0^2 - P^2 + \ell P_0 P^2$$

Where

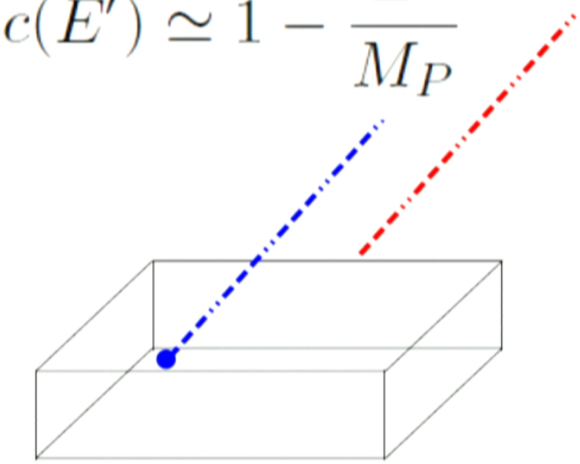
$$\ell \sim 1/M_P$$

THE NONLOCALITY PROBLEM

$$c(E) \simeq 1 - \frac{E}{M_P}$$

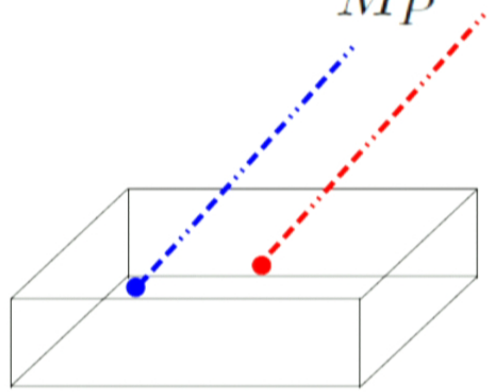


$$c(E') \simeq 1 - \frac{E'}{M_P}$$

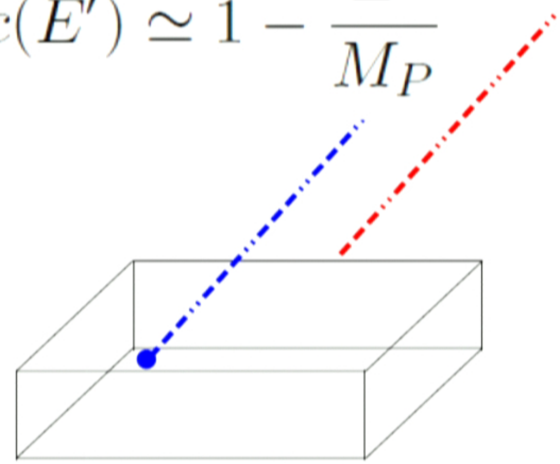


THE NONLOCALITY PROBLEM

$$c(E) \simeq 1 - \frac{E}{M_P}$$

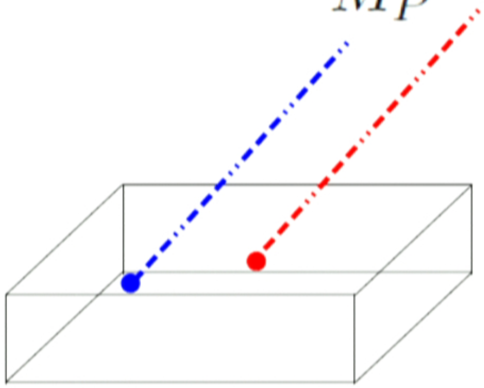


$$c(E') \simeq 1 - \frac{E'}{M_P}$$

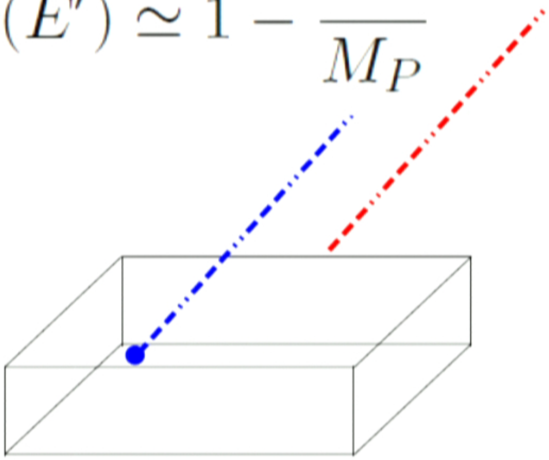


THE NONLOCALITY PROBLEM

$$c(E) \simeq 1 - \frac{E}{M_P}$$



$$c(E') \simeq 1 - \frac{E'}{M_P}$$

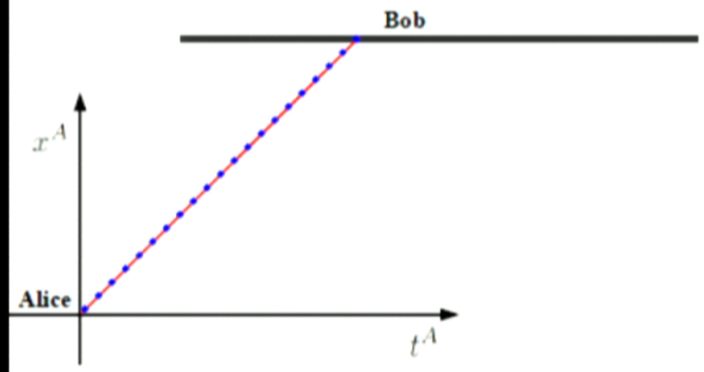


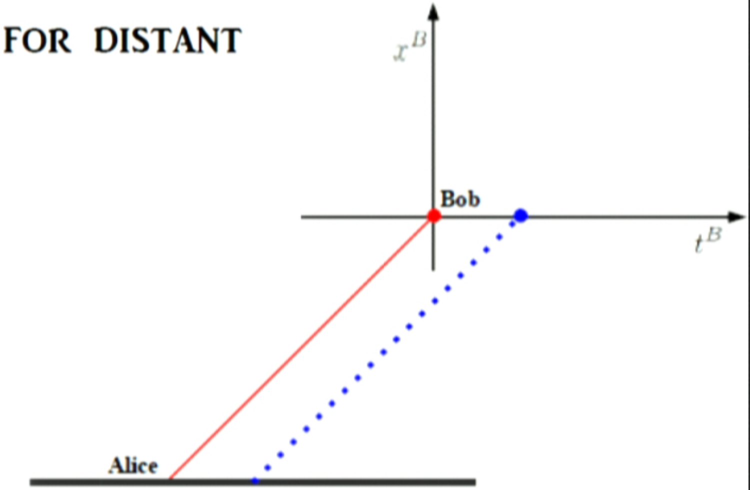
UNDEFORMED RULES OF BOOST TRANSFORMATION FOR THE COORDINATES OF THE EMISSION POINTS OF PARTICLES, BUT DEFORMED BOOST TRANSFORMATIONS FOR THEIR VELOCITIES.

SUCH CRITERIA OF "SELECTIVE APPLICABILITY" OF DEFORMED BOOSTS CANNOT PRODUCE A CONSISTENTLY RELATIVISTIC PICTURE.

About Relative Locality

Nonlocalities still exist but only for distant observers.





We formalize that as a curvature of momentum–space

$$\mathcal{D}(p,0) = \int_0^1 \sqrt{\zeta^{\mu\nu}\dot{p}_\mu \dot{p}_\nu}$$

•This interpretation describes nonlocalities as a dual redshift effect on momentum space.

About Redshift

$$E_B = E_A e^{-Ha^0}$$

De Sitter spacetime

$$ds^2 = (dx^0)^2 - e^{2Hx^0}(dx^1)^2$$

About Redshift

$$E_B = E_A e^{-Ha^0}$$

De Sitter spacetime

$$ds^2 = (dx^0)^2 - e^{2Hx^0}(dx^1)^2$$

DEFORMED TRANSLATION OPERATORS

- GENERALIZED TRANSLATION OPERATORS CHARGES:

$$p_0 \longrightarrow \Pi_0 = p_0 - H x^1 p_1$$
$$p_1 \longrightarrow \Pi_1 = p_1$$

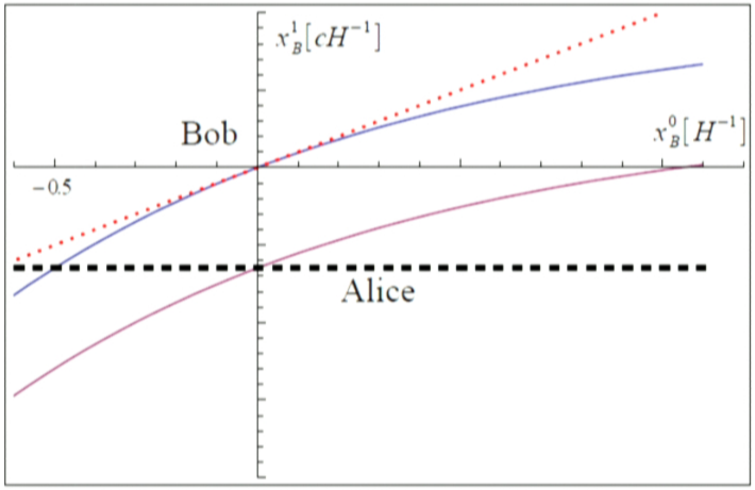
Deformed translation operators

- Generalized translation operators charges:

$$p_0 \longrightarrow \Pi_0 = p_0 - H x^1 p_1$$
$$p_1 \longrightarrow \Pi_1 = p_1$$

- Wordlines

$$x_A^1(x^0) = \left(\frac{1 - e^{-Hx^0}}{H} \right)$$



- Coordinate transformations

$$p_1^B = p_1^A e^{-a^0 H}$$
$$x_B^1 = e^{a^0 H} \left(x_A^1 - \frac{a^1}{a^0} \frac{1 - e^{-a^0 H}}{H} \right)$$

DeSitter momentum-space

We describe the time delay effect as a property of spacetime translations in theories with De Sitter-like curved momentum space

With algebra $\{p_1, p_0\} = 0$

$$\{\mathcal{N}, p_0\} = p_1, \;\; \{\mathcal{N}, p_1\} = \frac{1 - e^{-2\ell p_0}}{2\ell} - \frac{\ell}{2}(p_1)^2$$

DeSitter momentum–space

We describe the time delay effect as a property of spacetime translations in theories with De Sitter–like curved momentum space

With algebra $\{p_1, p_0\} = 0$

$$\{\mathcal{N}, p_0\} = p_1, \quad \{\mathcal{N}, p_1\} = \frac{1 - e^{-2\ell p_0}}{2\ell} - \frac{\ell}{2}(p_1)^2$$

And Casimir operator

$$\mathcal{C}_\ell = \left(\frac{2}{\ell}\sinh\left(\frac{\ell p_0}{2}\right)\right)^2 - e^{\ell p_0} p_1^2$$

$(\mathcal{C}_\ell = 0)$

$$p_1(p_0) = \frac{1 - e^{-\ell p_0}}{\ell}$$

Momentum dependant velocity?

We use this condition of onshelness as Hamiltonian → $\frac{d\chi^\mu}{d\tau} \equiv \dot{\chi}^\mu = \{\mathcal{C}_\ell, \chi^\mu\}$

Non-commutative coordinates

We take inspiration from κ-Minkowski noncommutative relation between coordinates

$$\{\chi^0, \chi^1\} = \ell \chi^1$$

$$\chi^0 = x^0 - \ell x^1 p_1 \qquad \chi^1 = x^1$$

NON-COMMUTATIVE COORDINATES

WE TAKE INSPIRATION FROM
K-MINKOWSKI NONCOMMUTATIVE
RELATION BETWEEN COORDINATES

$$\{\chi^0, \chi^1\} = \ell \chi^1$$

$$\chi^0 = x^0 - \ell x^1 p_1 \qquad \chi^1 = x^1$$

K-MINKOWSKI DEFORMED SYMPLECTIC SECTOR

$$\{p_1, \chi^1\} = -1\,, \quad \{p_1, \chi^0\} = \ell p_1,$$
$$\{p_0, \chi^1\} = 0\,, \quad \{p_0, \chi^0\} = -1.$$

MEANWILE IN DE SITTER SPACETIME...

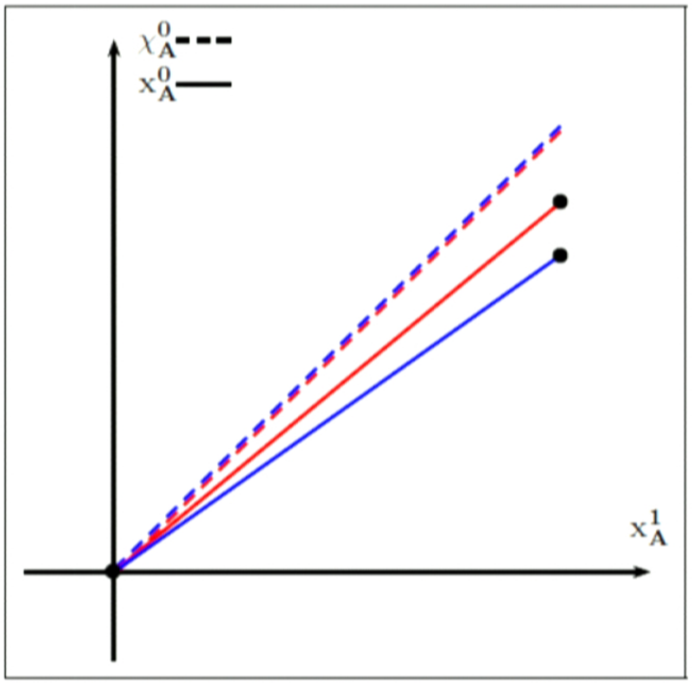
$$\{\Pi_0, x^0\} = 1\,, \quad \{\Pi_0, x^1\} = -H x^1$$
$$\{\Pi_1, x^0\} = 0\,, \quad \{\Pi_1, x^1\} = 1\,,$$

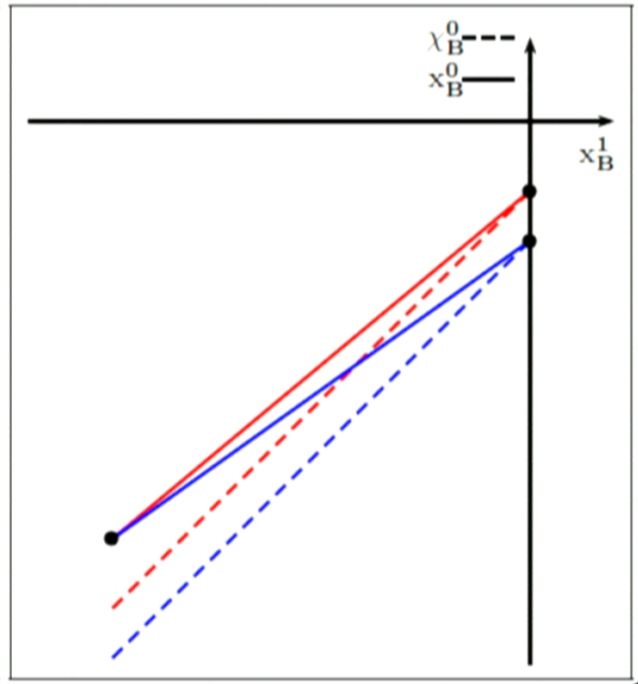
$$\{\Pi_0, \Pi_1\} = H \Pi_1$$

TAMING NONLOCALITIES

ACCORDING TO ALICE

ACCORDING TO BOB



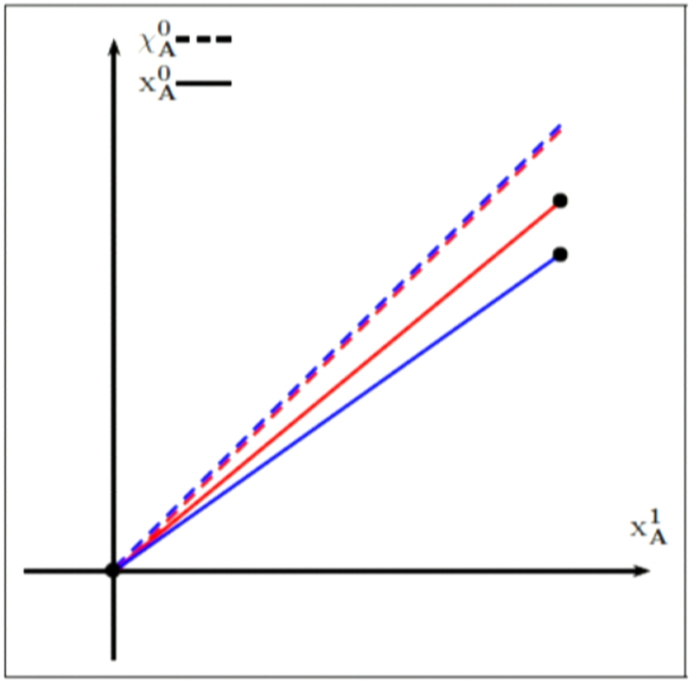


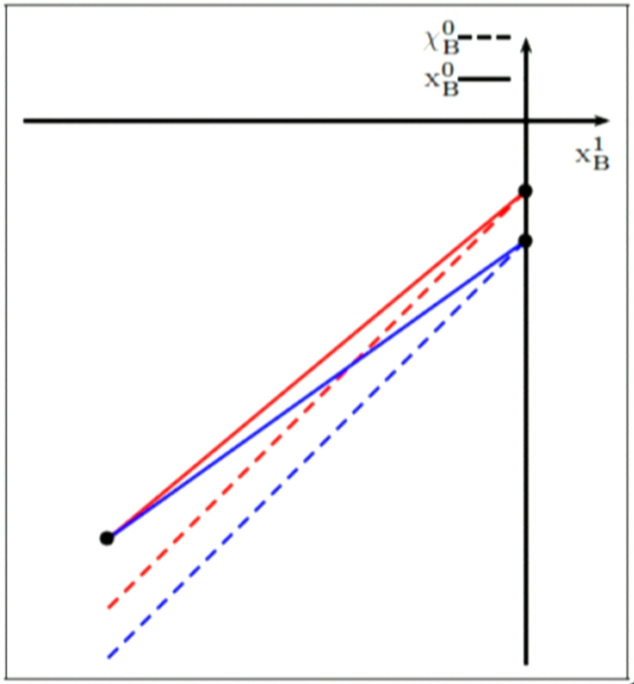
BOB OBSERVES THE SAME PHYSICAL EFFECT WITH BOTH COORDINATES

TAMING NONLOCALITIES

ACCORDING TO ALICE

ACCORDING TO BOB

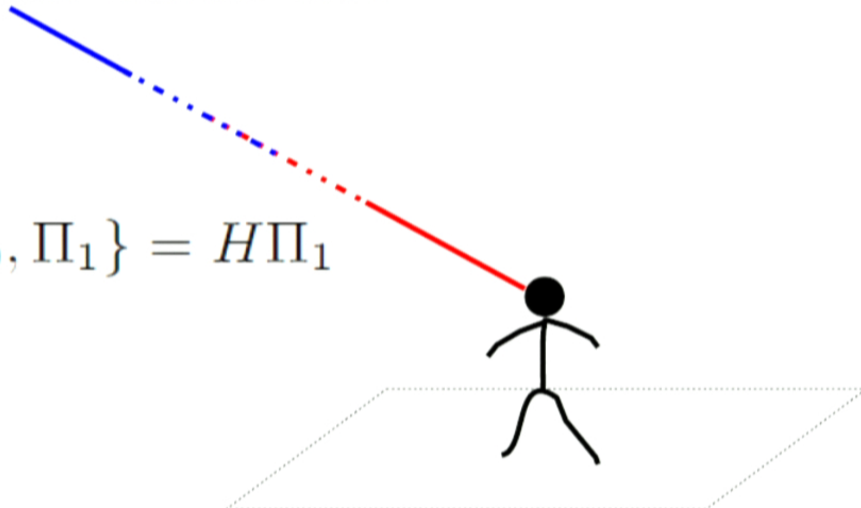




BOB OBSERVES THE SAME PHYSICAL EFFECT WITH BOTH COORDINATES

Comparing curvatures

- Same energy
- Different emission times

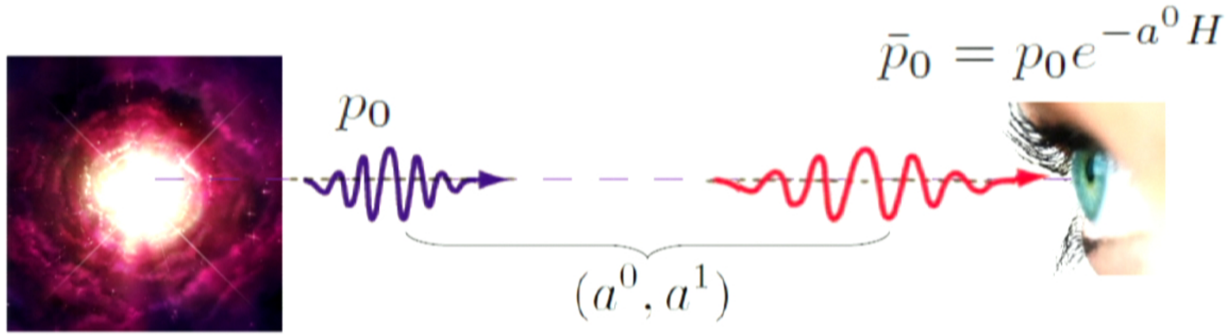


$$\{\Pi_0, \Pi_1\} = H\Pi_1$$

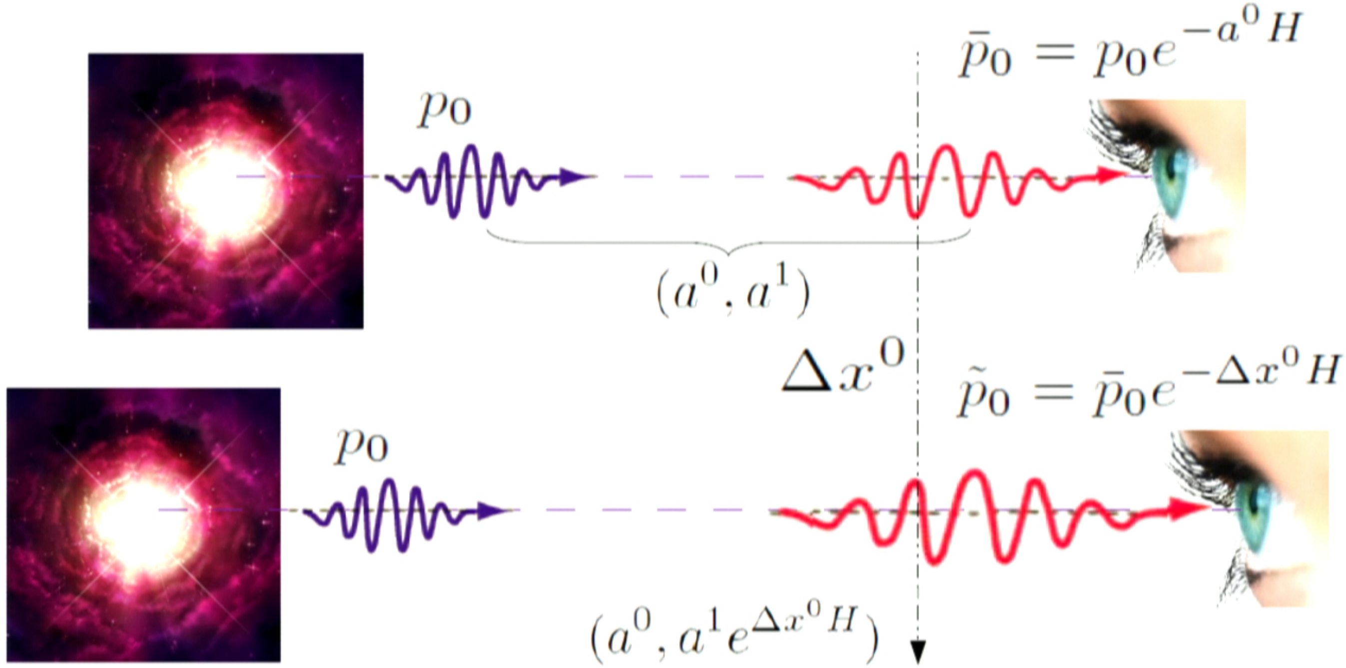
$$\{x^0, x^1\} = 0, \quad \{p_0, p_1\} = 0$$

Spacetime curvature

Summarizing redshift



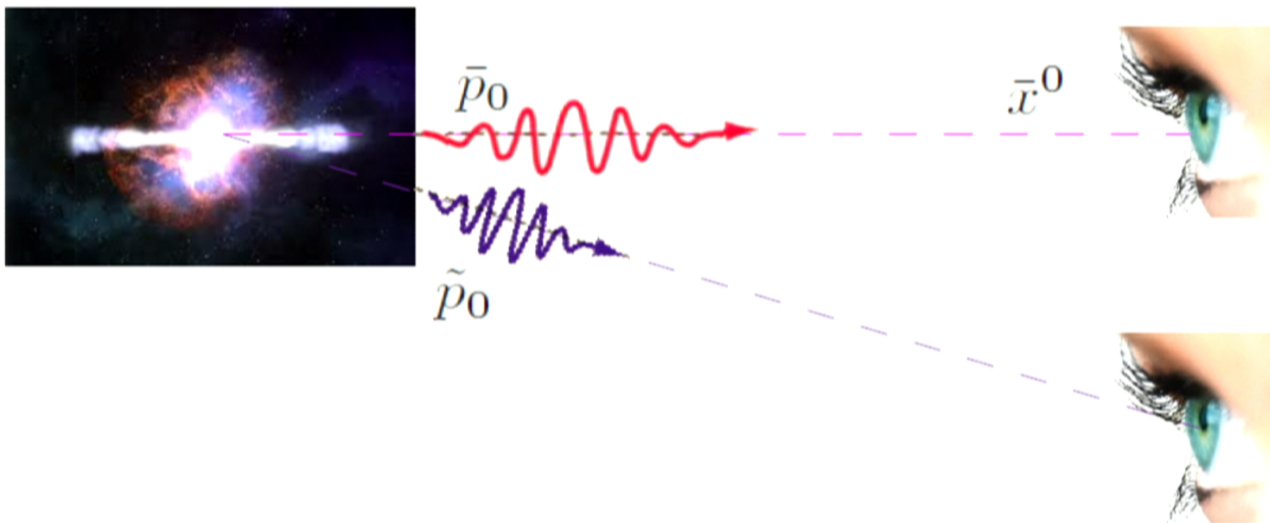
SUMMARIZING REDSHIFT



$$\frac{\Delta p_0}{\bar{p}_0} = 1 - e^{-\Delta x^0 H}$$

SUMMARIZING LATESHIFT

$$x^1 = x^0 e^{\ell p_0}$$



Introducing Lateshift

What's the role of the Planck-scale-curved geometry of momentum space in the correlations between emission and detection times, the travel times between a given emitter (Alice) and a given detector (Bob)?

↓

We have shown that these Planck-scale corrections to travel times can be exactly described, under a Relative locality perspective, as a dual Redshift effect or Lateshift.

Introducing Lateshift

What's the role of the Planck-scale-curved geometry of momentum space in the correlations between emission and detection times, the travel times between a given emitter (Alice) and a given detector (Bob)?

↓

We have shown that these Planck-scale corrections to travel times can be exactly described, under a Relative locality perspective, as a dual Redshift effect or Lateshift.

↓

They are manifestations of momentum-space curvature of exactly the same type (up to every detail of the technical derivation) already known for ordinary redshift produced by spacetime curvature.

↓

We can identify the novel notion of relative momentum-space locality as a known but under-appreciated feature associated to ordinary redshift produced by spacetime curvature, and this can be described in complete analogy with the relative spacetime locality.

Semidualisation in 3d gravity I

Bernd Schroers, Heriot-Watt University, Edinburgh

Loops 13 @ PI, July 2013

based on Prince Osei and Bernd Schroers, *On the semiduals of local isometry groups in 3d gravity*, J. Math. Phys. 53 (2012)
and (mainly)
Classical r-matrices via semidualisation, 2013, to appear

Motivation

Rotation(Boost)-Momentum-Position algebra
(JPX)

$$[J_a, J_b] = \epsilon_{abc} J^c, \quad [J_a, P_b] = \epsilon_{abc} P^c, \quad [J_a, X_b] = \epsilon_{abc} X^c \quad [P_a, X_b] = \delta_{ab}.$$

Spacetime isometry algebra
(JP)

Momentum space isometry algebra
(JX)

$$[J_a, J_b] = \epsilon_{abc} J^c, \quad [J_a, P_b] = \epsilon_{abc} P^c, \quad [P_a, P_b] = \ldots.$$

$$[J_a, J_b] = \epsilon_{abc} J^c, \quad [J_a, X_b] = \epsilon_{abc} X^c \quad [X_a, X_b] = \ldots$$

Semiduality or Born reciprocity

Motivation

Rotation(Boost)-Momentum-Position algebra
(JPX)

$$[J_a, J_b] = \epsilon_{abc} J^c, \quad [J_a, P_b] = \epsilon_{abc} P^c, \quad [J_a, X_b] = \epsilon_{abc} X^c \quad [P_a, X_b] = \delta_{ab}.$$

Spacetime isometry algebra
(JP)

Momentum space isometry algebra
(JX)

$$[J_a, J_b] = \epsilon_{abc} J^c, \quad [J_a, P_b] = \epsilon_{abc} P^c, \quad [P_a, P_b] = \ldots.$$

$$[J_a, J_b] = \epsilon_{abc} J^c, \quad [J_a, X_b] = \epsilon_{abc} X^c \quad [X_a, X_b] = \ldots$$

Semiduality or Born reciprocity

Motivation

Rotation(Boost)-Momentum-Position algebra
(JPX)

$$[J_a, J_b] = \epsilon_{abc} J^c, \quad [J_a, P_b] = \epsilon_{abc} P^c, \quad [J_a, X_b] = \epsilon_{abc} X^c \quad [P_a, X_b] = \delta_{ab}.$$

Spacetime isometry algebra
(JP)

Momentum space isometry algebra
(JX)

$$[J_a, J_b] = \epsilon_{abc} J^c, \quad [J_a, P_b] = \epsilon_{abc} P^c, \quad [P_a, P_b] = \ldots.$$

$$[J_a, J_b] = \epsilon_{abc} J^c, \quad [J_a, X_b] = \epsilon_{abc} X^c \quad [X_a, X_b] = \ldots$$

$\longleftrightarrow$

Semiduality or Born reciprocity

Double cross sum decomposition

$\mathfrak{g}$ is real Lie algebra $[J_a, J_b] = f_{ab}{}^c J_c.$

complexify with $\theta^2 = -\lambda$ and $Q_a = \theta J_a,$

$$[J_a, J_b] = f_{ab}{}^c J_c, \quad [Q_a, J_b] = f_{ab}{}^c Q_c, \quad [Q_a, Q_b] = \lambda f_{ab}{}^c J_c.$$

look for $Q'_a = Q_a + F^b{}_a J_b,$ so that

$$[J_a, J_b] = f_{ab}{}^c J_c, \quad [Q'_a, J_b] = f_{ab}{}^c Q'_c + L_{ab}{}^c J_c, \quad [Q'_a, Q'_b] = g_{ab}{}^c Q'_c.$$

double cross sum structure: $\mathfrak{g} \bowtie \mathfrak{m}$

Condition on F:

$$[F(X), F(Y)] - F([X, F(Y)] + [F(X), Y]) = -\lambda[X, Y] \quad \forall\, X, Y \in \mathfrak{g}.$$

r-matrices from semiduality

Define dual generators $P^a(Q'_b) = \delta^a_b$

Semidual Lie brackets $[J_a, J_b] = f_{ab}{}^c J_c, \quad [P^a, J_b] = f_{bc}{}^a P^c, \quad [P^a, P^b] = 0$

... and co-commutators
$$\delta(P^a) = g_{cb}{}^a P^c \otimes P^b$$
$$\delta(J_a) = L_{ba}{}^c \left(J_c \otimes P^b - P^b \otimes J_c\right)$$

Theorem:

1. Semidual Lie bialgebra is co-boundary with $r = F^b{}_a P^a \wedge J_b$
2. Modified classical Yang-Baxter equation is equivalent to factorisation condition for the map F

r-matrices from semiduality

Define dual generators $P^a(Q'_b) = \delta^a_b$

Semidual Lie brackets $[J_a, J_b] = f_{ab}{}^c J_c, \quad [P^a, J_b] = f_{bc}{}^a P^c, \quad [P^a, P^b] = 0$

... and co-commutators

$$\delta(P^a) = g_{cb}{}^a P^c \otimes P^b$$
$$\delta(J_a) = L_{ba}{}^c \left(J_c \otimes P^b - P^b \otimes J_c\right)$$

Theorem:

1. Semidual Lie bialgebra is co-boundary with $r = F^b{}_a P^a \wedge J_b$
2. Modified classical Yang-Baxter equation is equivalent to factorisation condition for the map F

Solutions II

$$F = \beta V \langle V, \cdot \rangle + \alpha \, \mathrm{ad}_V, \qquad \beta \in \mathbb{R}, \quad \alpha \in \{0, 1\}, \quad \alpha \langle V, V \rangle = -\lambda.$$

gives $\mathfrak{m} = \mathbb{R} \ltimes \mathbb{R}^2$ with action depending on α, β, λ:

$$[X_1, X_2] = 0$$

$$[T, X_1] = aX_1 + bX_2$$
$$[T, X_2] = cX_1 + dX_2$$

Solutions II

$$F = \beta V \langle V, \cdot \rangle + \alpha \, \mathrm{ad}_V, \qquad \beta \in \mathbb{R}, \quad \alpha \in \{0, 1\}, \quad \alpha \langle V, V \rangle = -\lambda.$$

gives $\mathfrak{m} = \mathbb{R} \ltimes \mathbb{R}^2$ with action depending on α, β, λ:

$$[X_1, X_2] = 0$$

$$[T, X_1] = aX_1 + bX_2$$
$$[T, X_2] = cX_1 + dX_2$$

Solutions II

$$F = \beta V \langle V, \cdot \rangle + \alpha \, \mathrm{ad}_V, \qquad \beta \in \mathbb{R}, \quad \alpha \in \{0, 1\}, \quad \alpha \langle V, V \rangle = -\lambda.$$

gives $\mathfrak{m} = \mathbb{R} \ltimes \mathbb{R}^2$ with action depending on α, β, λ:

$$[X_1, X_2] = 0$$

$$[T, X_1] = aX_1 + bX_2$$
$$[T, X_2] = cX_1 + dX_2$$

Solutions II

$$F = \beta V \langle V, \cdot \rangle + \alpha \, \mathrm{ad}_V , \qquad \beta \in \mathbb{R}, \quad \alpha \in \{0, 1\}, \quad \alpha \langle V, V \rangle = -\lambda.$$

gives $\mathfrak{m} = \mathbb{R} \ltimes \mathbb{R}^2$ with action depending on α, β, λ:

$$[X_1, X_2] = 0$$

$$[T, X_1] = aX_1 + bX_2$$
$$[T, X_2] = cX_1 + dX_2$$

Solutions II

$$F = \beta V \langle V, \cdot \rangle + \alpha \, \mathrm{ad}_V, \qquad \beta \in \mathbb{R}, \quad \alpha \in \{0, 1\}, \quad \alpha \langle V, V \rangle = -\lambda.$$

gives $\mathfrak{m} = \mathbb{R} \ltimes \mathbb{R}^2$ with action depending on α, β, λ:

$$[X_1, X_2] = 0$$

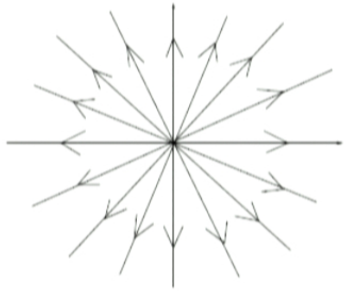
$$[T, X_1] = aX_1 + bX_2$$
$$[T, X_2] = cX_1 + dX_2$$

Solutions II

$$F = \beta V \langle V, \cdot \rangle + \alpha\, \mathrm{ad}_V, \qquad \beta \in \mathbb{R}, \quad \alpha \in \{0, 1\}, \quad \alpha \langle V, V \rangle = -\lambda.$$

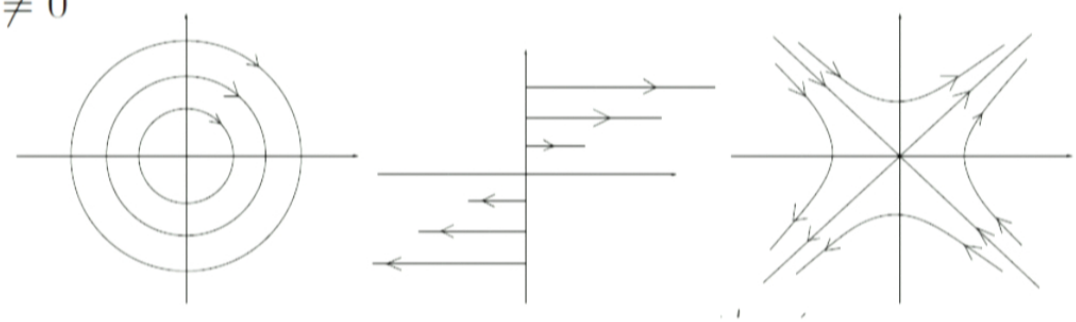
action depending on α, β, λ:

$\alpha = 1, \beta = 0$



Kappa-Poincare!

$\alpha = 0, \beta \neq 0$



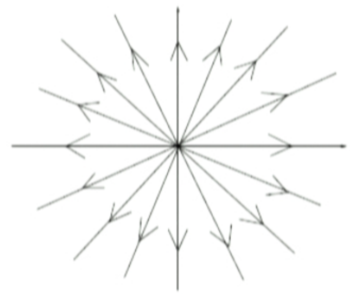
timelike spacelike lightlike

Solutions II

$$F = \beta V \langle V, \cdot \rangle + \alpha\, \mathrm{ad}_V, \qquad \beta \in \mathbb{R}, \quad \alpha \in \{0,1\}, \quad \alpha \langle V, V \rangle = -\lambda.$$

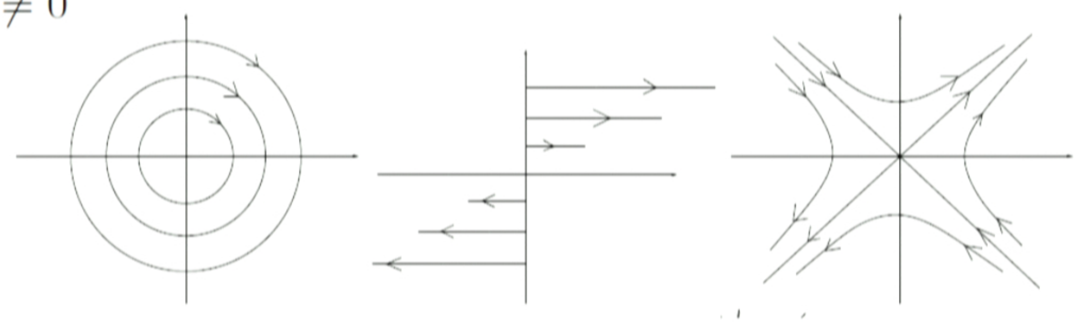
action depending on α, β, λ:

$\alpha = 1, \beta = 0$



Kappa-Poincare!

$\alpha = 0, \beta \neq 0$



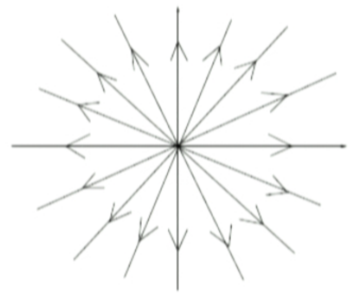
timelike spacelike lightlike

Solutions II

$$F = \beta V\langle V,\cdot\rangle + \alpha\,\mathrm{ad}_V, \qquad \beta\in\mathbb{R},\quad \alpha\in\{0,1\},\quad \alpha\langle V,V\rangle = -\lambda.$$

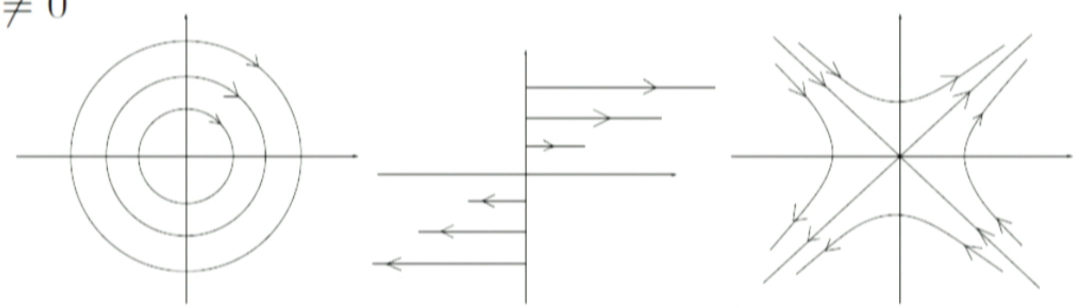
action depending on α, β, λ:

$\alpha = 1, \beta = 0$



Kappa-Poincare!

$\alpha = 0, \beta \neq 0$

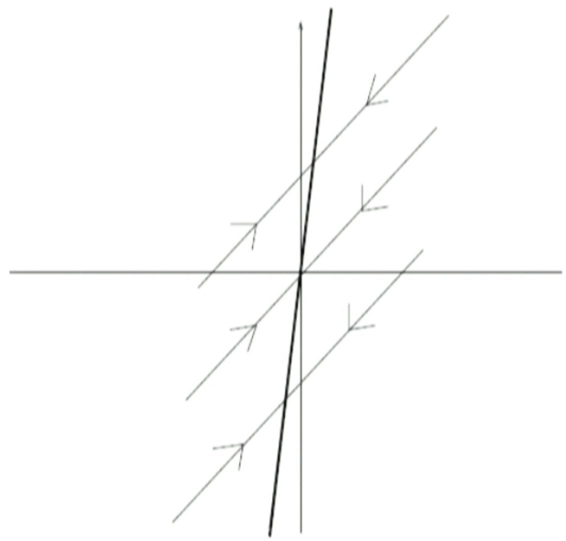


timelike spacelike lightlike

Solutions III

Two special solutions: S not diagonalisable $\quad \mathfrak{m} = \mathbb{R} \oplus L(2)$

Degenerate case of $\mathfrak{m} = \mathbb{R} \ltimes \mathbb{R}^2$ with action



$r_{\rm LJ} = \beta P_N \wedge J_1.$ or $r_{\rm SJ} = \beta P_N \wedge N + \sqrt{\lambda}(Q_1 \wedge J_1 + \epsilon^b{}_{a1} P^a \wedge J_b),$

Conclusion

- Semiduality switches X non-commutativity for classical r-matrix and P non-co-commutativity (momentum space curvature)

- In 3d get a complete list of `non-trivial' r-matrices and a correspondence between r-matrices and the Bianchi classification of 3d Lie algebras

- Theoretical framework for studying `JPX algebra' in any dimension in a unified language

Solutions II

$$F = \beta V \langle V, \cdot \rangle + \alpha \, \mathrm{ad}_V, \qquad \beta \in \mathbb{R}, \quad \alpha \in \{0, 1\}, \quad \alpha \langle V, V \rangle = -\lambda.$$

gives $\mathfrak{m} = \mathbb{R} \ltimes \mathbb{R}^2$ with action depending on α, β, λ:

$$[X_1, X_2] = 0$$

$$[T, X_1] = aX_1 + bX_2$$
$$[T, X_2] = cX_1 + dX_2$$

Semidualisation in 3d gravity II

Prince K. Osei

AIMS-Ghana, Biriwa & University of Ghana, Legon

LOOPS 13, PI

July, 2013

based on work with B. J Schroers

Semidualisation in 3d gravity II

Prince K. Osei

AIMS-Ghana, Biriwa & University of Ghana, Legon

LOOPS 13, PI

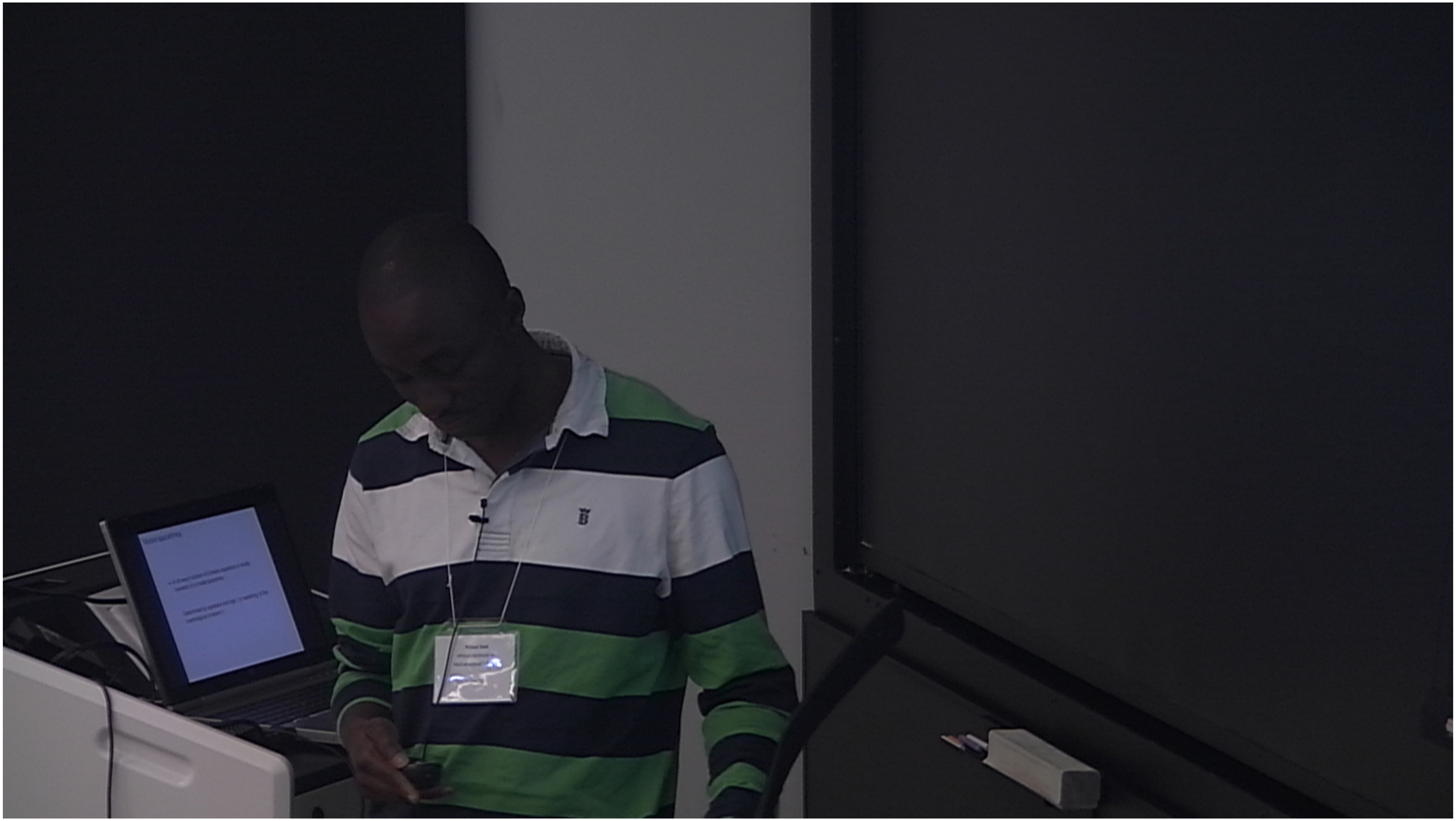
July, 2013

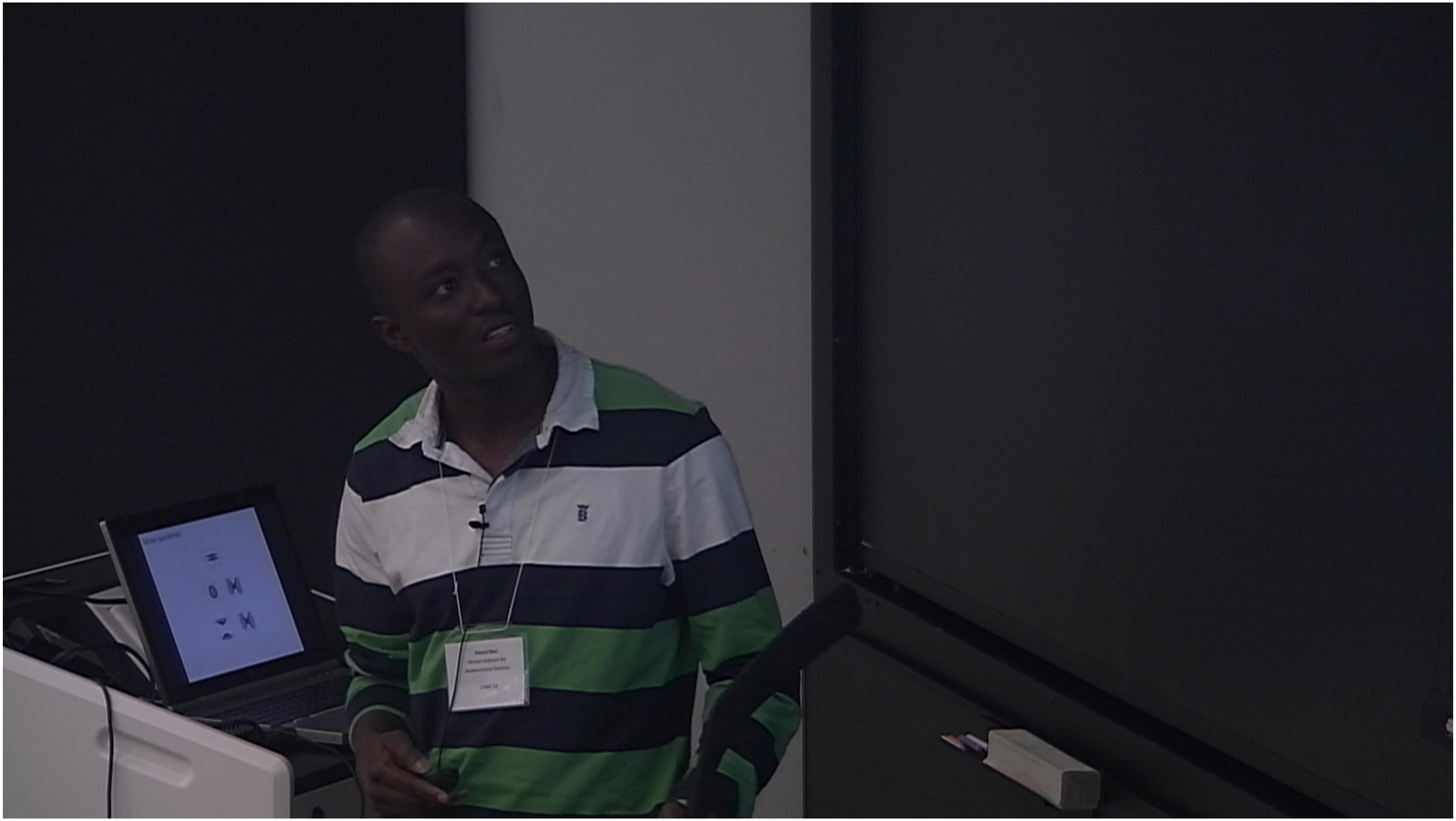
based on work with B. J Schroers

Model spacetimes

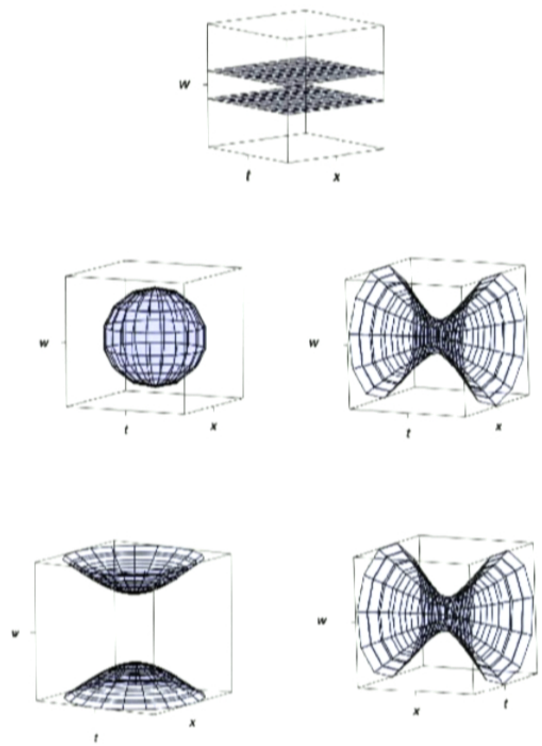
- In 3d every solution of Einstein equations is locally isometric to a model spacetime

  Determined by signature and sign (or vanishing) of the cosmological constant Λ





Model spacetimes



Isometry groups of 3d gravity

Isometry groups of the local model spacetimes play a fundamental role in 3d gravity:

- Construction of globally non-trivial solutions of the Einstein equations on a general 3-manifold;
- In the Chern-Simons formulation of 3d gravity, they play the role of gauge groups.

Isometry groups of 3d gravity

Λ	Euclidean sig.($c^2 < 0$)	Lorentzian sig.($c^2 > 0$)
$\Lambda = 0$	$ISO(3) = SU(2) \ltimes \mathbb{R}^3$	$ISO(2,1) = SU(1,1) \ltimes \mathbb{R}^3$
$\Lambda > 0$	$SO(4) \cong \frac{(SU(2) \times SU(2))}{\mathbb{Z}_2}$	$SO(3,1) \cong SL(2,\mathbb{C})/\mathbb{Z}_2$
$\Lambda < 0$	$SO(3,1) \cong \frac{SL(2,\mathbb{C})}{\mathbb{Z}_2}$	$SO(2,2) \cong \frac{(SL(2,\mathbb{R}) \times SL(2,\mathbb{R}))}{\mathbb{Z}_2}$

Lie algebras local isometry groups

The Lie algebras, denoted by $\mathfrak{g}_\lambda$, are the six-dimensional Lie algebra with generators J_a and P_a, $a = 0, 1, 2$ with Lie brackets

$$[J_a, J_a] = \varepsilon_{abc} J^c, \quad [J_a, P_b] = \varepsilon_{abc} P^c \qquad [P_a, P_b] = \lambda \varepsilon_{abc} J^c.$$

where

$$\lambda = -c^2 \Lambda.$$

Quantum picture

- Is based on the application of the combinatorial quantisation program (CQP) to the Chern-Simons formulation of 3d gravity
- provides a systematic way of studying the role of quantum groups and non-commutative geometry in $3d$ gravity.

Quantum picture

- A QIG is found via a classical r-matrix which is required to be compatible with the Cherns-Simons action in a certain sense
- The CQP does not uniquely define a QIG, but defines an equivalent class of quantum groups

Semiduals of local isometry groups

- Consider some factorisations of the local isometry groups arising in 3D gravity
- use them to construct associated bicrossproduct quantum groups via semidualisation.

Factorisation of local isometry groups

	Euclidean signature	Lorentzian signature
$\lambda > 0$	$\widetilde{SO}(4) = SU(2) \bowtie SU(2)$	$\widetilde{SO}(2,2) = \begin{cases} SL(2,\mathbb{R}) \bowtie SL(2,\mathbb{R}) \\ SL(2,\mathbb{R}) \bowtie_s AN(2) \end{cases}$
$\lambda = 0$	$\tilde{E}_3 = SU(2) \bowtie \mathbb{R}^3$	$\tilde{P}_3 = \begin{cases} SL(2,\mathbb{R}) \bowtie \mathbb{R}^3 \\ SL(2,\mathbb{R}) \bowtie_l AN(2) \end{cases}$
$\lambda < 0$	$SL(2,\mathbb{C}) = SU(2) \bowtie AN(2)$	$SL(2,\mathbb{C}) = SL(2,\mathbb{R}) \bowtie_t AN(2)$

Factorisation of local isometry groups

	Euclidean signature	Lorentzian signature
$\lambda > 0$	$\widetilde{SO}(4) = SU(2) \ltimes\!\!\!\rtimes SU(2)$	$\widetilde{SO}(2,2) = \begin{cases} SL(2,\mathbb{R}) \ltimes\!\!\!\rtimes SL(2,\mathbb{R}) \\ SL(2,\mathbb{R}) \bowtie_s AN(2) \end{cases}$
$\lambda = 0$	$\tilde{E}_3 = SU(2) \ltimes\!\!\!\rtimes \mathbb{R}^3$	$\tilde{P}_3 = \begin{cases} SL(2,\mathbb{R}) \ltimes\!\!\!\rtimes \mathbb{R}^3 \\ SL(2,\mathbb{R}) \bowtie_l AN(2) \end{cases}$
$\lambda < 0$	$SL(2,\mathbb{C}) = SU(2) \bowtie AN(2)$	$SL(2,\mathbb{C}) = SL(2,\mathbb{R}) \bowtie_t AN(2)$

Semiduals of local Isometry groups

	Euclidean signature	Lorentzian signature
$\lambda > 0$	$D(U(\mathfrak{su}(2)))$	$D(U(\mathfrak{sl}(2,\mathbb{R})))$ $\mathbb{C}(AN(2))\blacktriangleright\!\!\triangleleft_s U(\mathfrak{sl}(2,\mathbb{R}))$
$\lambda = 0$	$(\mathbb{R}^*)^3 \rtimes\!\!\!\triangleleft U(\mathfrak{su}(2,\mathbb{R}))$	$(\mathbb{R}^*)^3 \rtimes\!\!\!\triangleleft U(\mathfrak{sl}(2,\mathbb{R}))$ $\mathbb{C}(AN(2))\blacktriangleright\!\!\triangleleft_l U(\mathfrak{sl}(2,\mathbb{R}))$
$\lambda < 0$	$\mathbb{C}(AN(2))\blacktriangleright\!\!\triangleleft U(\mathfrak{su}(2))$	$\mathbb{C}(AN(2))\blacktriangleright\!\!\triangleleft_t U(\mathfrak{sl}(2,\mathbb{R}))$

Conclusion

Interpretation of semiduality

- The interpretation of semiduality proposed by (B. J Schroers , S. Majid) as the exchange of the cosmological length scale and the Planck mass in the context of 3D quantum gravity is confirmed and elaborated.

	Original regime	Semidual regime
Cosmological time scale	$\frac{1}{\sqrt{\lambda}}$	∞
Planck mass	∞	$\frac{1}{\sqrt{\lambda}}$

THANK YOU!!!