Title: Black Holes - 3

Date: Jul 23, 2013 02:30 PM

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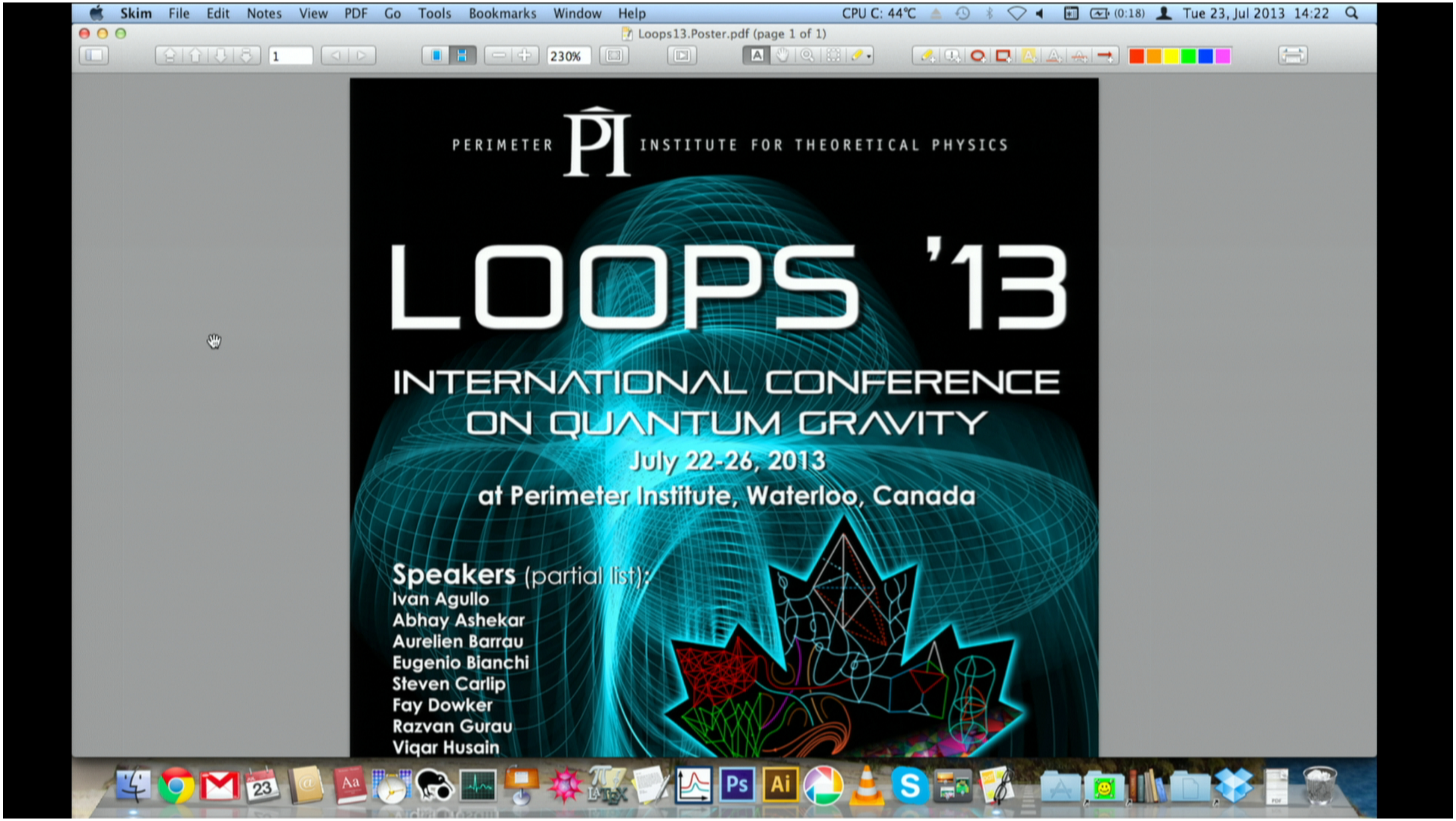
Abstract: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Loops13.Poster.pdf (page 1 of 1)
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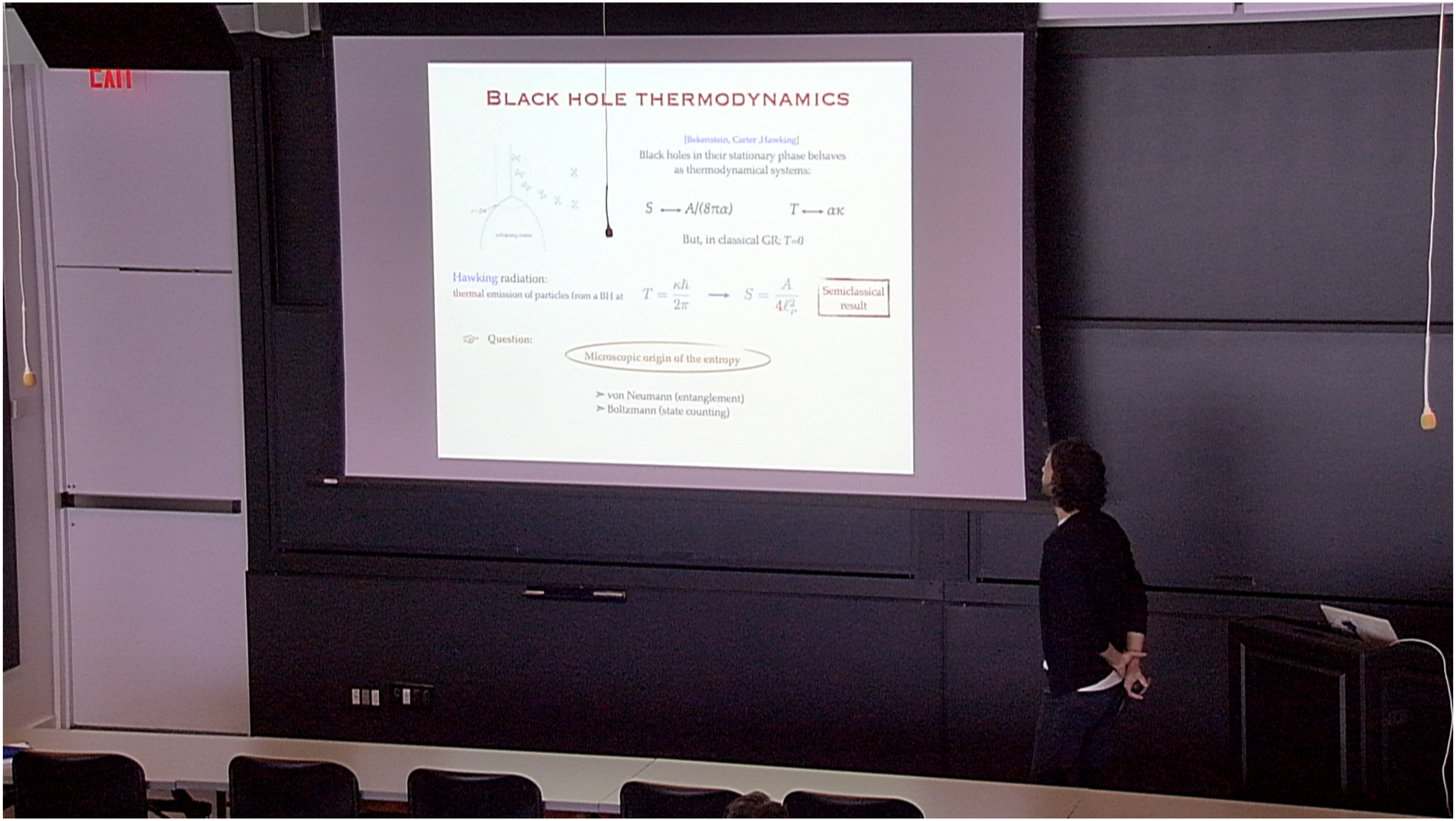
PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS
LOOPS '13
INTERNATIONAL CONFERENCE ON QUANTUM GRAVITY
July 22-26, 2013
at Perimeter Institute, Waterloo, Canada
Speakers (partial list):
Ivan Agullo
Abhay Ashekar
Aurelien Barrau
Eugenio Bianchi
Steven Carlip
Fay Dowker
Razvan Gurau
Viqar Husain

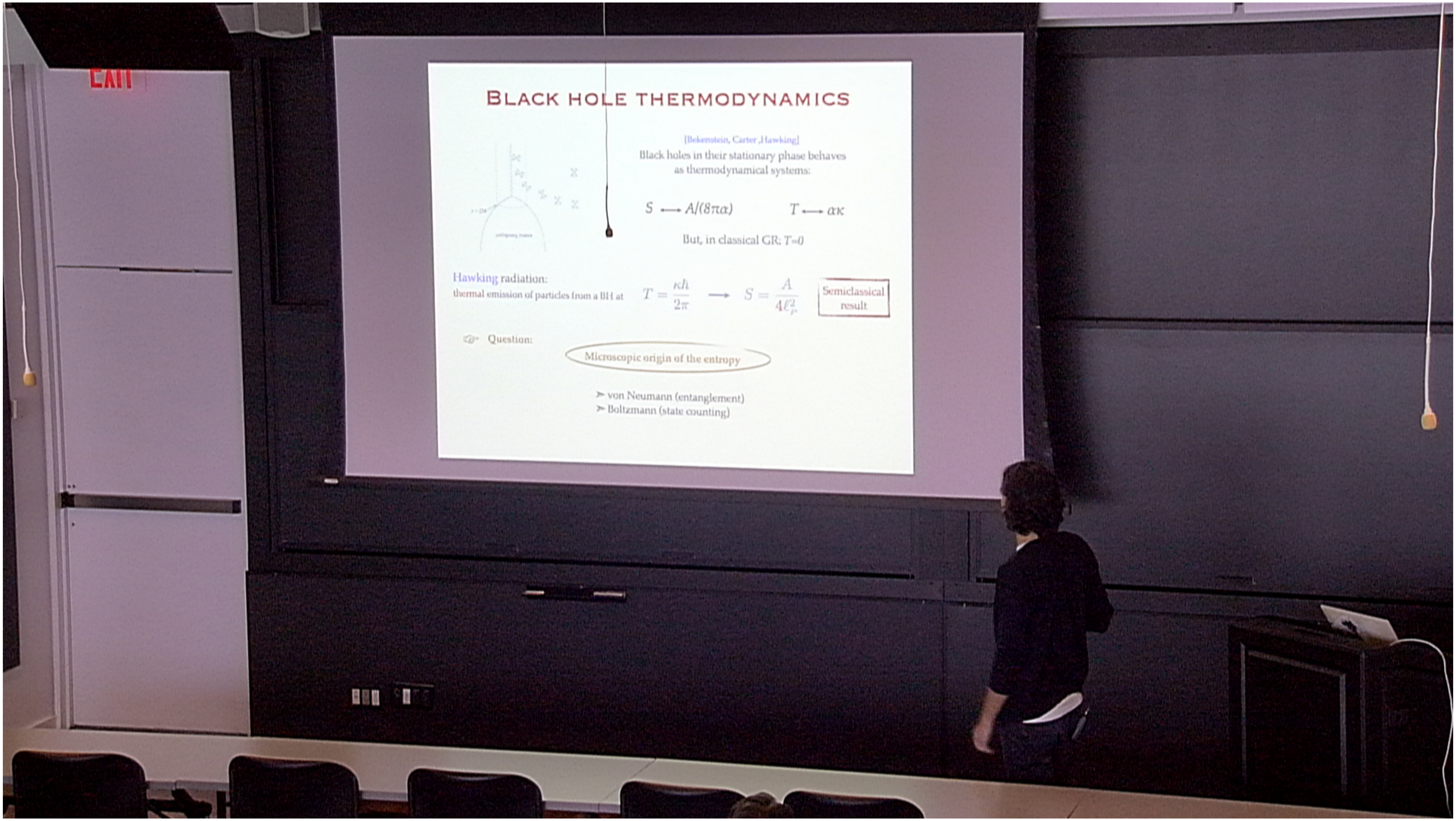
BH ENTROPY FROM KMS-STATES OF QUANTUM ISOLATED HORIZONS

Daniele Pranzetti

arXiv:1305.6714 [gr-qc];

Max-Planck-Institut
für Gravitationsphysik
(Albert-Einstein-Institut)


BLACK HOLE THERMODYNAMICS
[Bekenstein, Carter ,Hawking]
Black holes in their stationary phase behaves
as thermodynamical systems:
$S \longleftrightarrow A/(8\pi\alpha)$
$T \longleftrightarrow \alpha\kappa$
But, in classical GR: $T=0$
Hawking radiation:
thermal emission of particles from a BH at
$T = \frac{\kappa\hbar}{2\pi} \longrightarrow S = \frac{A}{4\ell_p^2}$
Semiclassical result
Question:
Microscopic origin of the entropy
von Neumann (entanglement)
Boltzmann (state counting)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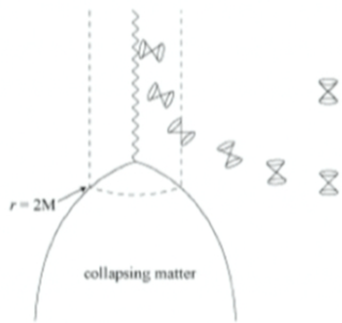
EXIT

BLACK HOLE THERMODYNAMICS



[Bekenstein, Carter ,Hawking]

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☞ **Question:**

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- ➢ Boltzmann (state counting)

A brief history of entropy:

* First ideas [Smolin 95; Rovelli 96; Krasnov 96]
* U(1) C-S theory [Ashtekar, Baez, Corichi, Krasnov 00]
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* State counting: γ = real fixed number [Kaul, Majumdar 98; Domagala, Lewandowski 04; Ghosh, Mitra 04; Agullo, Barbero, Borja, Diaz-Polo, Villasenor 08; Engle, Noui, Perez, DP 11]
* Local observer perspective + Unruh temp. by hand: γ free [Ghosh, Perez 11; Frodden, Ghosh, Perez 11]
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OUTLINE

- KMS-states ⇨ Temperature
- Physical understanding of $\gamma = i$
- von Neumann & Boltzmann entropies


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Horizon temperature

KMS-states = physical extension of Gibbs equilibrium thermal states to infinite dimensional quantum systems

Kubo-Martin-Schwinger

[Haag, Winnink, Hugenholtz]

✦ I ingredient: Lorentz invariance

thermality of a black hole ⇔ thermality of the vacuum in flat space-time restricted to one wedge (Unruh effect)

Euclidean path integral representation

$$\rho_R = \mathrm{tr}_L |0\rangle\langle 0| = \sum_n e^{-2\pi E_n} |n\rangle_{R\,R}\langle n|$$

$|n\rangle$ eigenstates of the boost Hamiltonian

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➢ In LQG:

natural complex structure of *SL(2, C)* in the phase-space

SU(2) kinematical Hilbert space of LQG

➨

$$K^i = \gamma L^i \quad (*)$$

linear simplicity constraints $\Leftrightarrow$ reality conditions

[Wieland 10]

$\Rightarrow$

$|\Gamma, j_l, v_n\rangle$

[Rovelli, Smo

(quantum) geometry

(r,t)-plane $\Leftrightarrow$ *(θ,ϕ)*-plane

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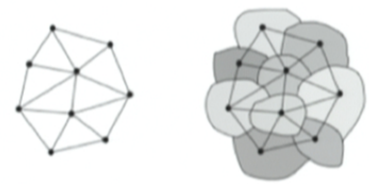

$|\Gamma, j_l, v_n\rangle$

[Rovelli, Smolin 95]

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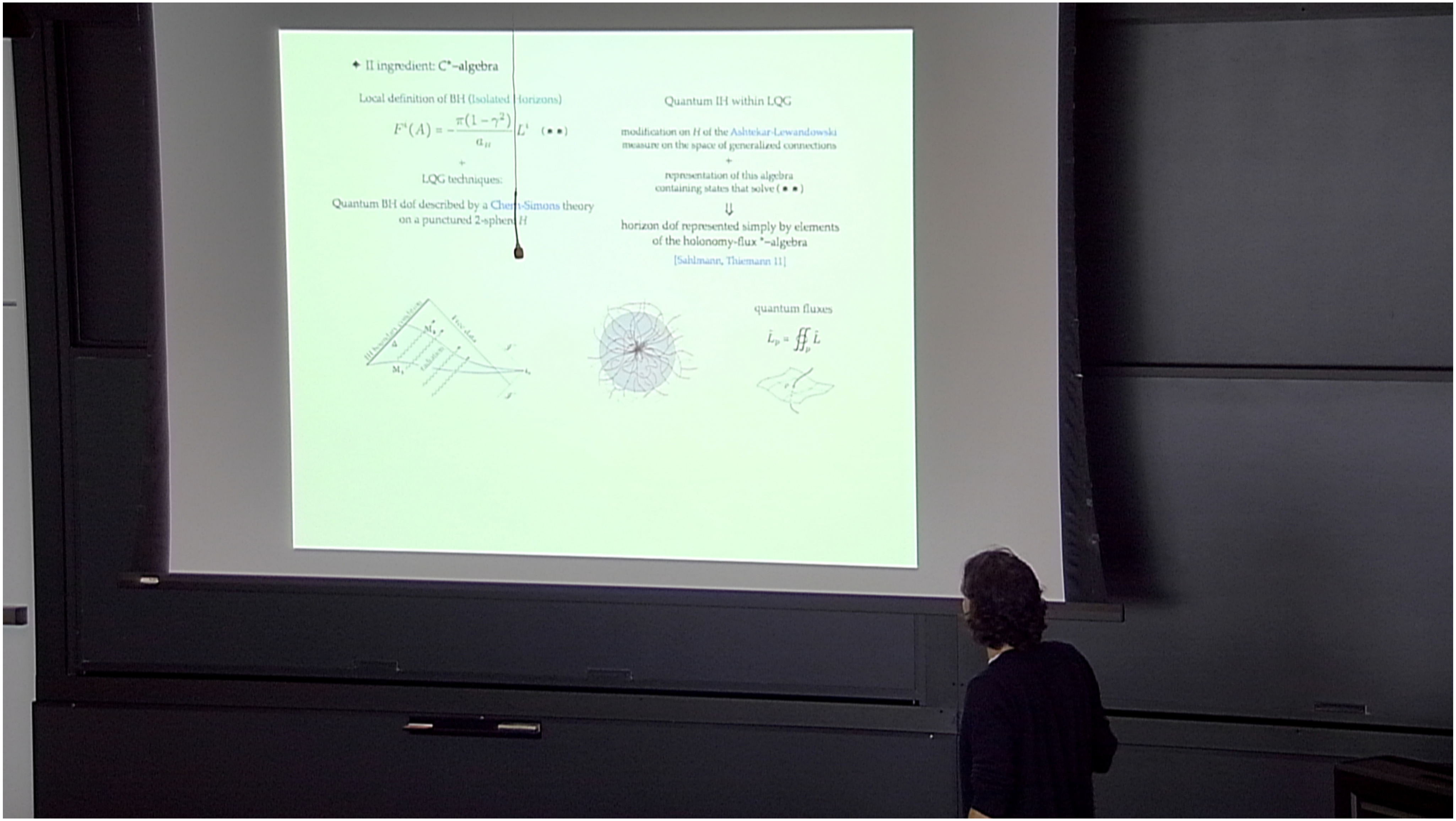
II ingredient: C*–algebra
Local definition of BH (Isolated Horizons)
$F^i(A) = -\frac{\pi(1-\gamma^2)}{a_H} L^i$ (* *)
+
LQG techniques:
Quantum BH dof described by a Chern-Simons theory on a punctured 2-sphere H
Quantum IH within LQG
modification on H of the Ashtekar-Lewandowski measure on the space of generalized connections
+
representation of this algebra containing states that solve (* *)
⇓
horizon dof represented simply by elements of the holonomy-flux *–algebra
[Sahlmann, Thiemann 11]
quantum fluxes
$\hat{L}_p = \iint_p \hat{L}$

✦ II ingredient: $C^{\star}$–algebra

Local definition of BH (Isolated Horizons)

$$F^i(A) = -\frac{\pi(1-\gamma^2)}{a_H} L^i \quad (\ast\ast)$$

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Quantum BH dof described by a Chern-Simons theory on a punctured 2-sphere H

Quantum IH within LQG

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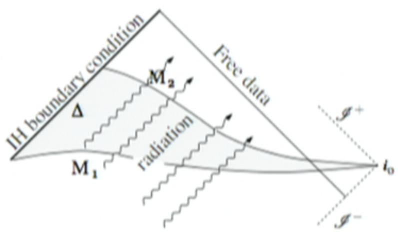
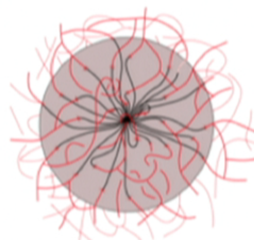
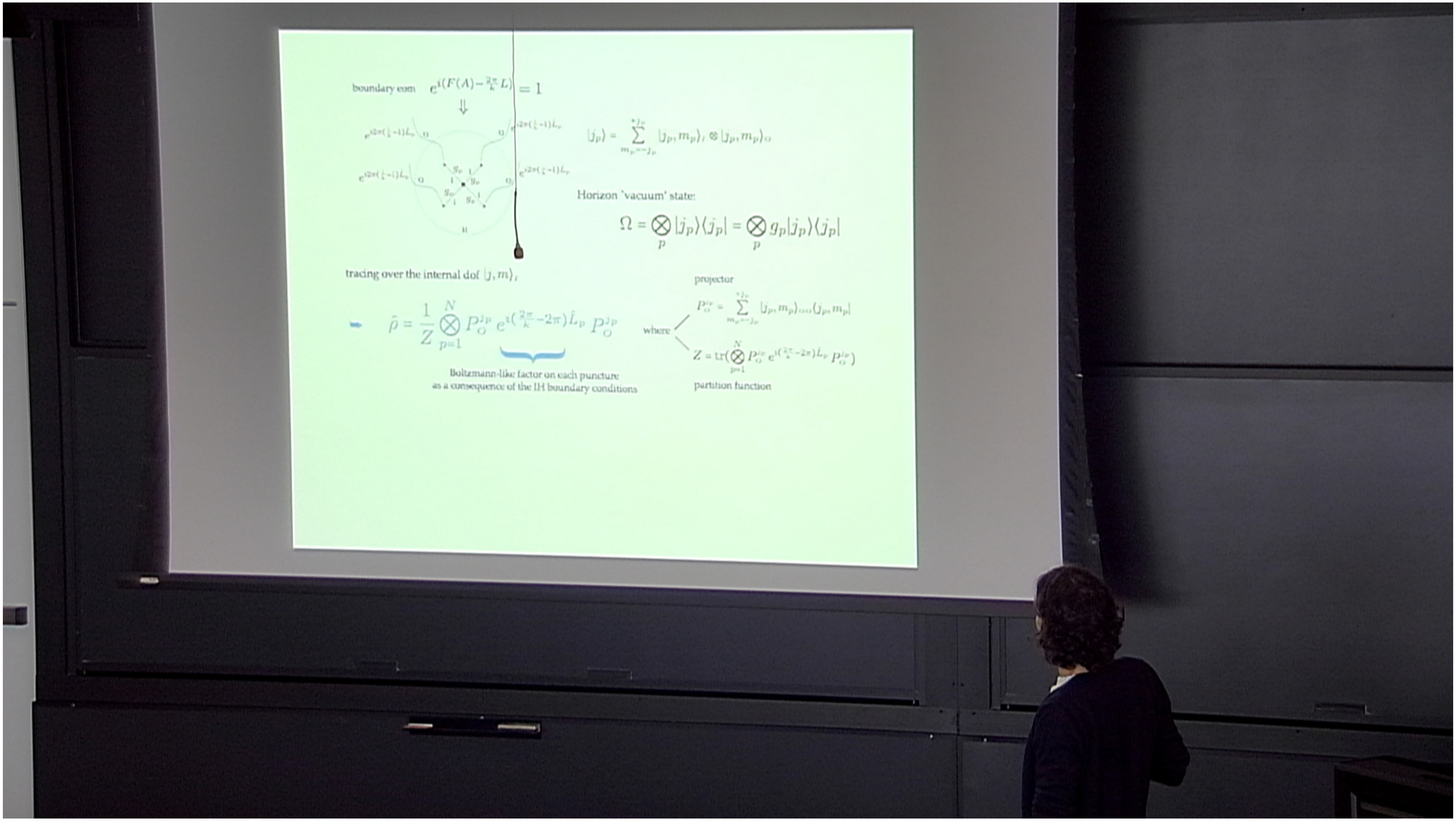




quantum fluxes

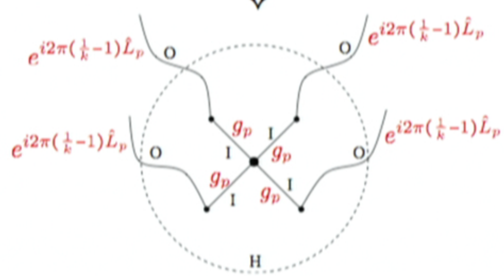
$$\hat{L}_p = \oiint_p \hat{L}$$



boundary con $e^{i(F(A)-\frac{2\pi}{k}L)} = 1$
$|j_p\rangle = \sum_{m_p=-j_p}^{+j_p} |j_p, m_p\rangle_I \otimes |j_p, m_p\rangle_O$
Horizon 'vacuum' state:
$\Omega = \bigotimes_p |j_p\rangle\langle j_p| = \bigotimes_p g_p |j_p\rangle\langle j_p|$
tracing over the internal dof $|j,m\rangle_I$
projector
$\hat{\rho} = \frac{1}{Z} \bigotimes_{p=1}^{N} P_O^{j_p} e^{i(\frac{2\pi}{k}-2\pi)\hat{L}_p} P_O^{j_p}$
where
$P_O^{j_p} = \sum_{m_p=-j_p}^{+j_p} |j_p, m_p\rangle_{OO}\langle j_p, m_p|$
$Z = \mathrm{tr}(\bigotimes_{p=1}^{N} P_O^{j_p} e^{i(\frac{2\pi}{k}-2\pi)\hat{L}_p} P_O^{j_p})$
Boltzmann-like factor on each puncture
as a consequence of the IH boundary conditions
partition function

boundary eom $e^{i(F(A)-\frac{2\pi}{k}L)} = 1$

$\Downarrow$



$$|j_p\rangle = \sum_{m_p=-j_p}^{+j_p} |j_p, m_p\rangle_I \otimes |j_p, m_p\rangle_O$$

Horizon `vacuum' state:

$$\Omega = \bigotimes_p |j_p\rangle\langle j_p| = \bigotimes_p g_p |j_p\rangle\langle j_p|$$

tracing over the internal dof $|j, m\rangle_I$

$$\Rightarrow \quad \hat{\rho} = \frac{1}{Z} \bigotimes_{p=1}^{N} P_O^{j_p} \underbrace{e^{i(\frac{2\pi}{k}-2\pi)\hat{L}_p}}_{} P_O^{j_p}$$

Boltzmann-like factor on each puncture
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projector

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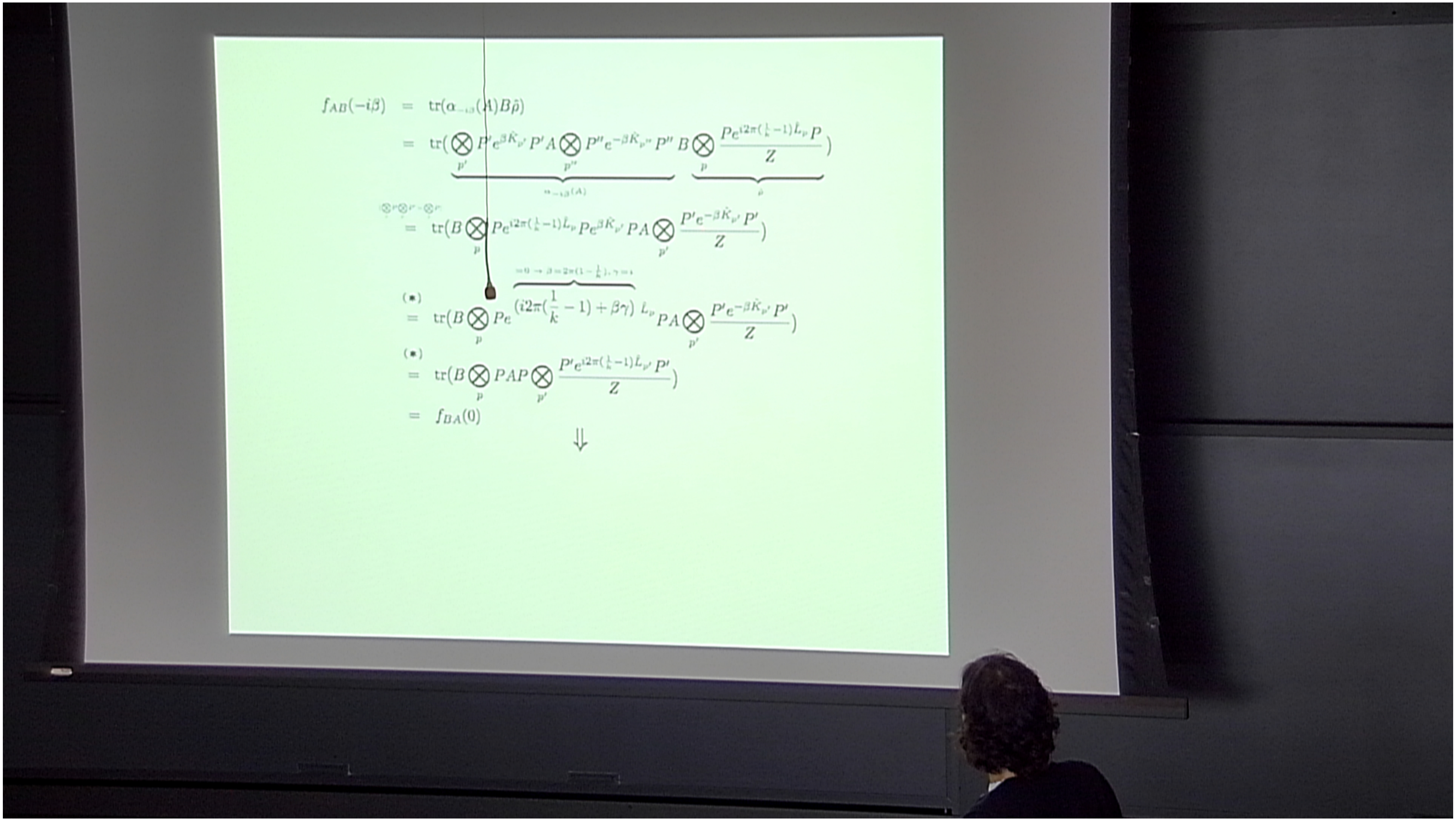
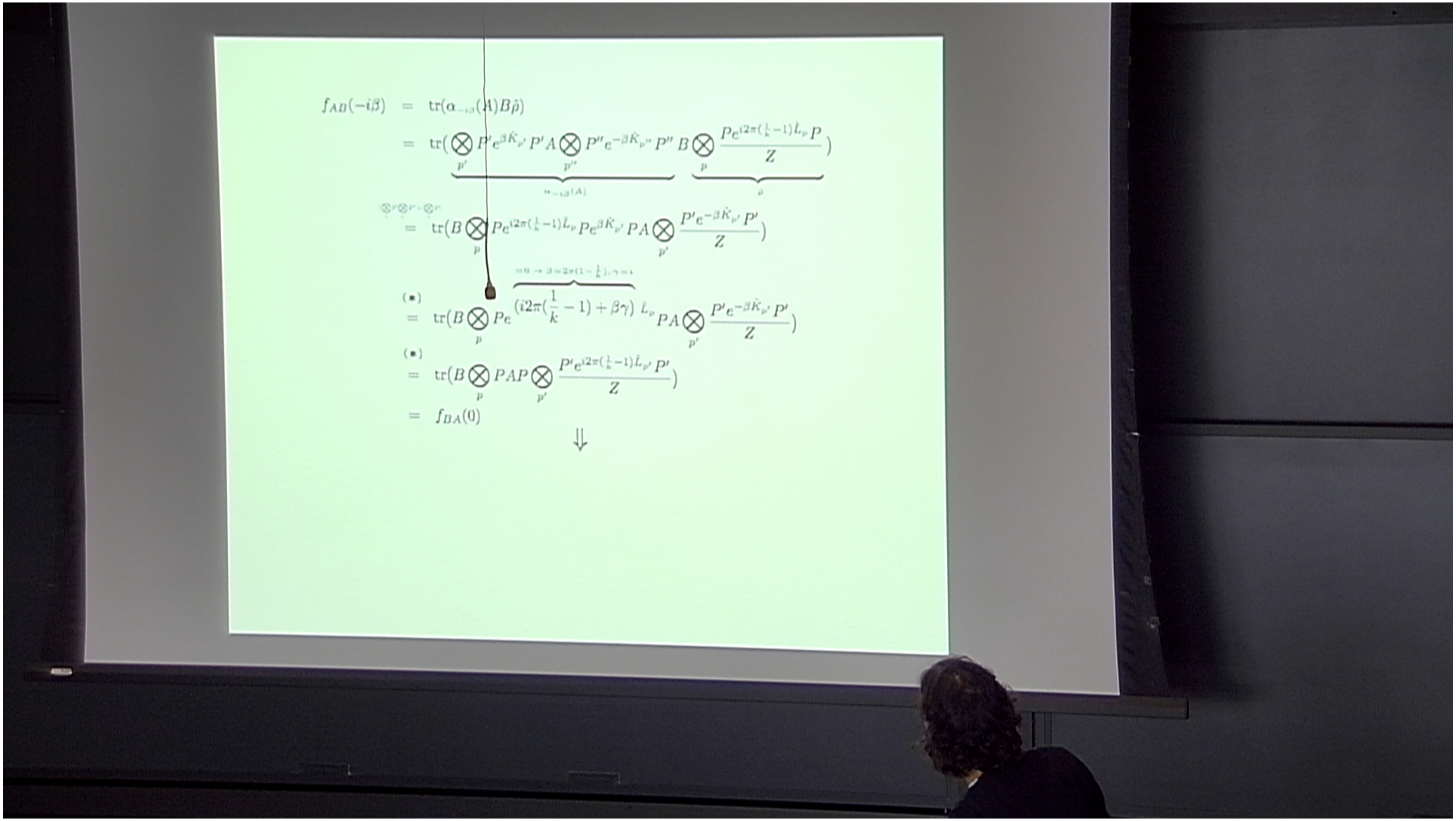
partition function


$f_{AB}(-i\beta) = \mathrm{tr}(\alpha_{-i\beta}(A) B \hat{\rho})$
$= \mathrm{tr}(\underbrace{\bigotimes_{p'} P' e^{\beta \hat{K}_{p'}} P' A \bigotimes_{p''} P'' e^{-\beta \hat{K}_{p''}} P''}\, B \underbrace{\bigotimes_{p} \frac{P e^{i2\pi(\frac{1}{k}-1)\hat{L}_p} P}{Z}})$
$= \mathrm{tr}(B \bigotimes_{p} P e^{i2\pi(\frac{1}{k}-1)\hat{L}_p} P e^{\beta \hat{K}_{p'}} P A \bigotimes_{p'} \frac{P' e^{-\beta \hat{K}_{p'}} P'}{Z})$
$\overset{(*)}{=} \mathrm{tr}(B \bigotimes_{p} P e^{\overbrace{(i2\pi(\frac{1}{k}-1)+\beta\gamma)}\hat{L}_p} P A \bigotimes_{p'} \frac{P' e^{-\beta \hat{K}_{p'}} P'}{Z})$
$\overset{(*)}{=} \mathrm{tr}(B \bigotimes_{p} PAP \bigotimes_{p'} \frac{P' e^{i2\pi(\frac{1}{k}-1)\hat{L}_{p'}} P'}{Z})$
$= f_{BA}(0)$
$\Downarrow$



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$\overset{(*)}{=} \mathrm{tr}(B \bigotimes_{p} P e^{\overbrace{(i2\pi(\frac{1}{k}-1)+\beta\gamma)}^{=0 \to \beta = 2\pi(1-\frac{1}{k}),\ \gamma = i} \hat{L}_p} P A \bigotimes_{p'} \frac{P' e^{-\beta \hat{K}_{p'}} P'}{Z})$
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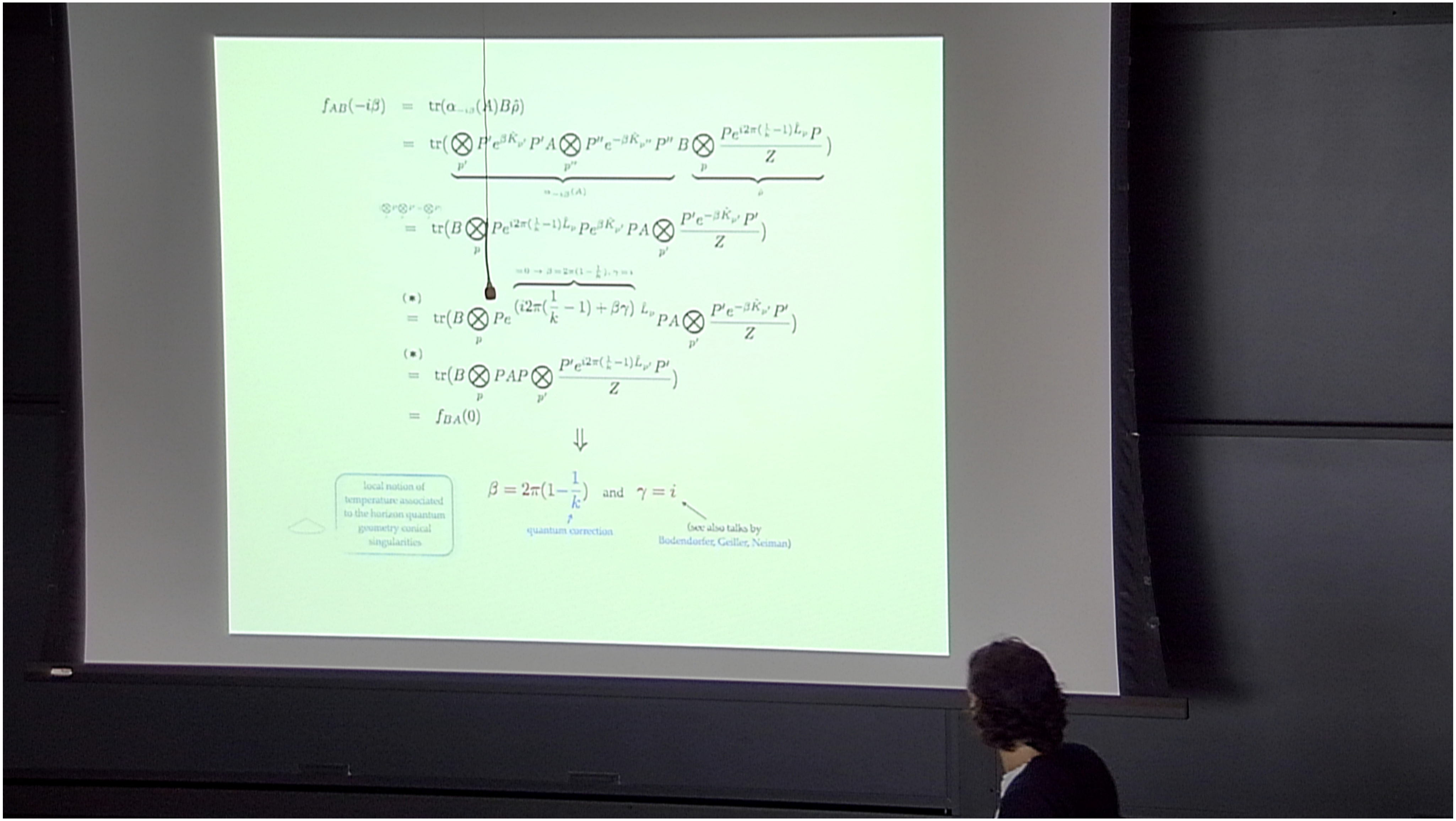

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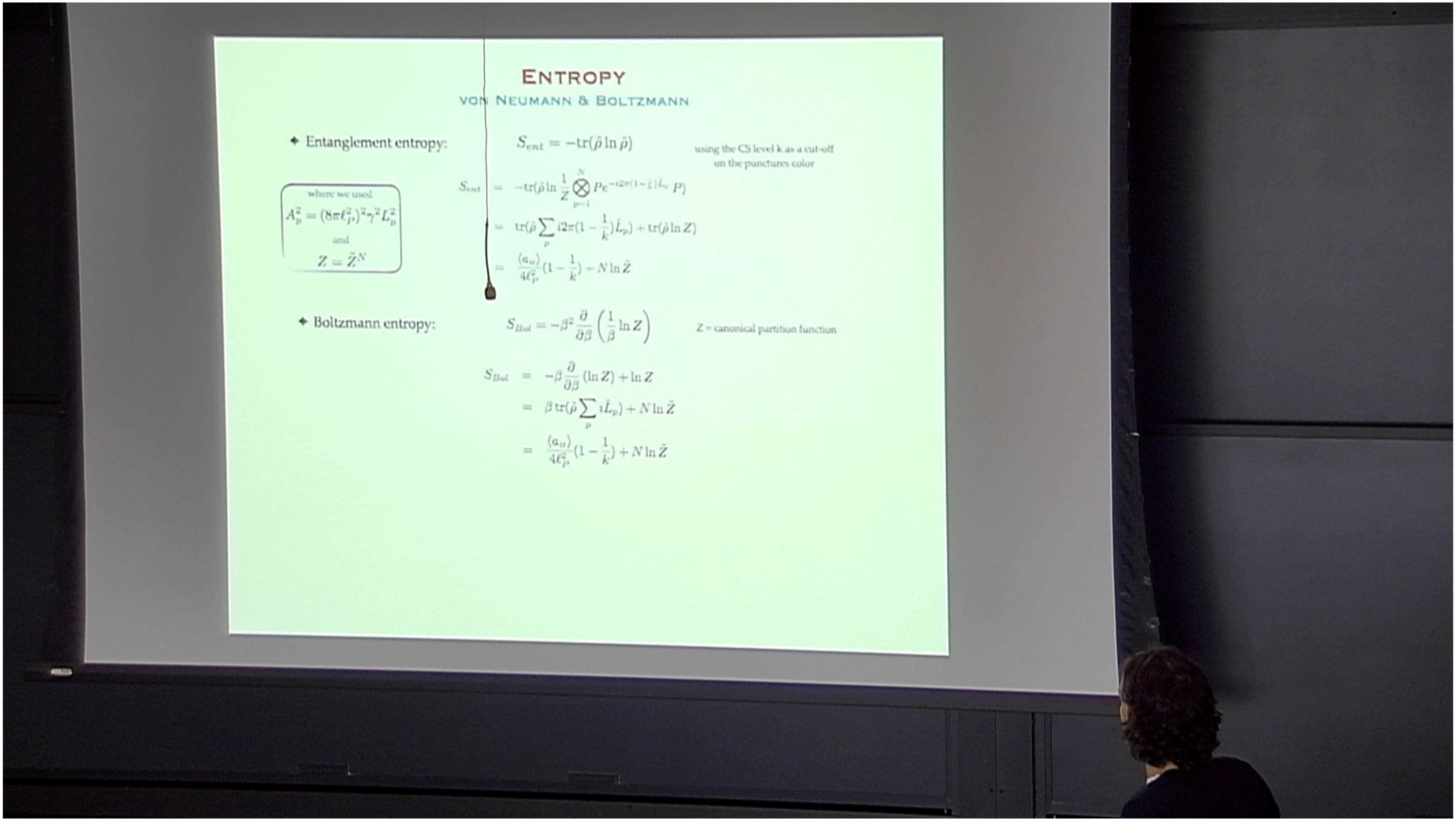
$= f_{BA}(0)$
$\Downarrow$
local notion of temperature associated to the horizon quantum geometry conical singularities
$\beta = 2\pi(1-\frac{1}{k})$ and $\gamma = i$
quantum correction
(see also talks by Bodendorfer, Geiller, Neiman)


ENTROPY
VON NEUMANN & BOLTZMANN
✦ Entanglement entropy:
$S_{ent} = -\mathrm{tr}(\hat{\rho} \ln \hat{\rho})$
using the CS level k as a cut-off
on the punctures color
$S_{ent} = -\mathrm{tr}(\hat{\rho} \ln \frac{1}{Z} \bigotimes_{p=1}^{N} P e^{-i2\pi(1-\frac{1}{k})\hat{L}_p} P)$
$= \mathrm{tr}(\hat{\rho} \sum_p i2\pi(1 - \frac{1}{k})\hat{L}_p) + \mathrm{tr}(\hat{\rho} \ln Z)$
$= \frac{\langle a_H \rangle}{4\ell_P^2}(1 - \frac{1}{k}) + N \ln \tilde{Z}$
where we used
$A_p^2 = (8\pi\ell_P^2)^2\gamma^2 L_p^2$
and
$Z = \tilde{Z}^N$
✦ Boltzmann entropy:
$S_{Bol} = -\beta^2 \frac{\partial}{\partial\beta}\left(\frac{1}{\beta} \ln Z\right)$
Z = canonical partition function
$S_{Bol} = -\beta \frac{\partial}{\partial\beta}(\ln Z) + \ln Z$
$= \beta\,\mathrm{tr}(\hat{\rho} \sum_p i\hat{L}_p) + N \ln \tilde{Z}$
$= \frac{\langle a_H \rangle}{4\ell_P^2}(1 - \frac{1}{k}) + N \ln \tilde{Z}$

Entropy

von Neumann & Boltzmann

✦ Entanglement entropy: $S_{ent} = -\mathrm{tr}(\hat{\rho} \ln \hat{\rho})$

using the CS level k as a cut-off on the punctures color

where we used

$$A_p^2 = (8\pi\ell_P^2)^2\gamma^2 L_p^2$$

and

$$Z = \tilde{Z}^N$$

$$\begin{aligned} S_{ent} &= -\mathrm{tr}(\hat{\rho} \ln \frac{1}{Z} \bigotimes_{p=1}^{N} P e^{-i2\pi(1-\frac{1}{k})\hat{L}_p} P) \\ &= \mathrm{tr}(\hat{\rho} \sum_p i2\pi(1-\frac{1}{k})\hat{L}_p) + \mathrm{tr}(\hat{\rho} \ln Z) \\ &= \frac{\langle a_H \rangle}{4\ell_P^2}(1-\frac{1}{k}) + N \ln \tilde{Z} \end{aligned}$$

✦ Boltzmann entropy: $S_{Bol} = -\beta^2 \frac{\partial}{\partial \beta}\left(\frac{1}{\beta} \ln Z\right)$

Z = canonical partition function

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ENTROPY
VON NEUMANN & BOLTZMANN

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$$\Downarrow$$

$$S_{ent} = S_{Bol}$$

+ consistency with semi-classical analysis

(see [Teitelboim 95] and [Ghosh, Perez 11])

correction associated to the volume of the 2-sphere diffeomerphisms group

modified I law of QIH mechanics: $dE = TdS + \mu dN$

In other words

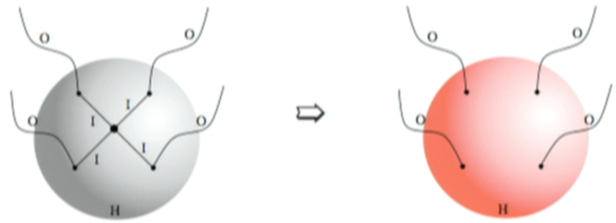
The microscopic derivation of the horizon temperature has an intrinsically geometrical nature

Lorentz symmetry: temperature = period of the rotational symmetry in the (θ,ϕ)-plane

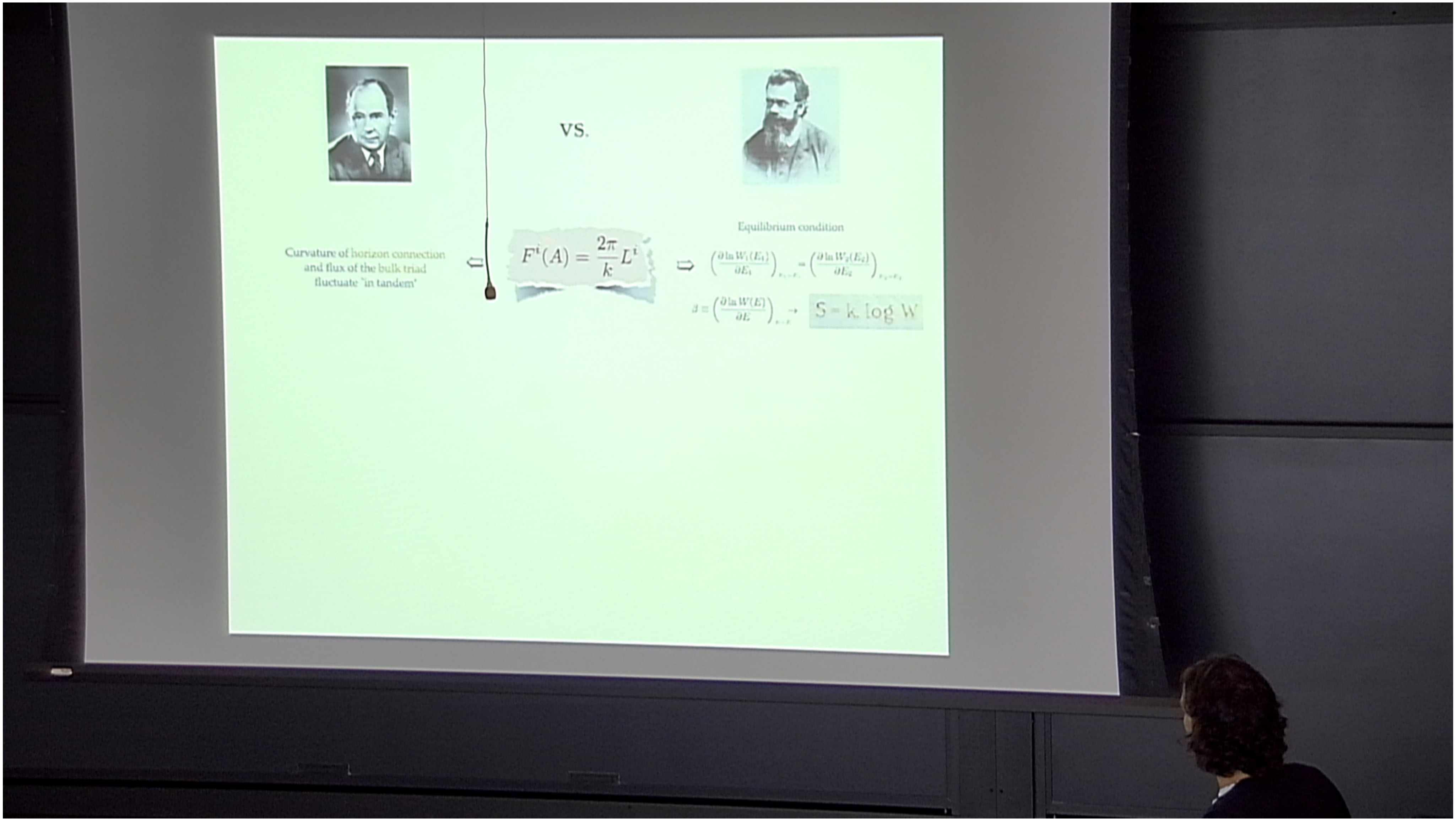
in analogy to the description of [Banados, Carlip, Teitelboim, Zanelli 93]

2 key ingredients:

- imposition of $F^i(A) = \frac{2\pi}{k} L^i$ using the structures of the LQG kinematical Hilbert space
- trace over internal dof



➡ thermality of the horizon originates from the entanglement between physical dof upon passing to the self-dual connection



vs.
Curvature of horizon connection and flux of the bulk triad fluctuate 'in tandem'
$F^i(A) = \frac{2\pi}{k} L^i$
Equilibrium condition
$\left(\frac{\partial \ln W_1(E_1)}{\partial E_1}\right) = \left(\frac{\partial \ln W_2(E_2)}{\partial E_2}\right)$
$\beta \equiv \left(\frac{\partial \ln W(E)}{\partial E}\right) \rightarrow$
S = k. log W

vs.

Curvature of horizon connection
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fluctuate `in tandem’ $\Leftarrow$

$$F^i(A) = \frac{2\pi}{k} L^i$$

$\Rightarrow$ Equilibrium condition

$$\left(\frac{\partial \ln W_1(E_1)}{\partial E_1}\right)_{E_1=\bar{E}_1} = \left(\frac{\partial \ln W_2(E_2)}{\partial E_2}\right)_{E_2=\bar{E}_2}$$

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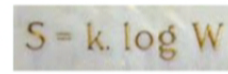

$\Downarrow$

Correlations of the quantum geometry dof across the horizon $\Leftarrow$

Intertwiner structure
encoding

$\Rightarrow$ Ω = number of horizon `quantum shapes'

vs.

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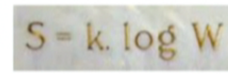
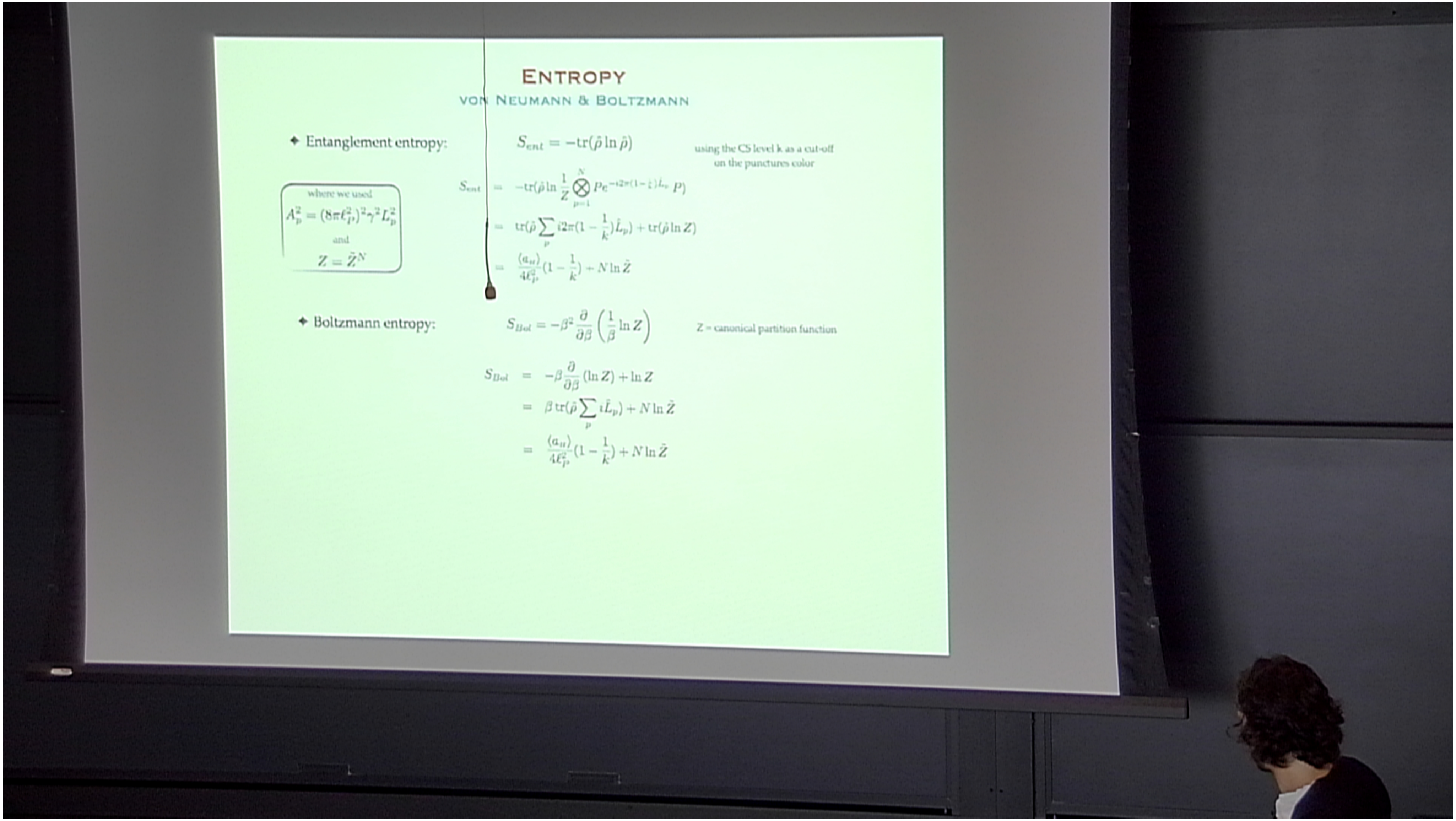


$\Downarrow$

Correlations of the quantum geometry dof across the horizon $\Leftarrow$ **Intertwiner structure** encoding $\Rightarrow$ Ω = number of horizon `quantum shapes'

Near-horizon quantum fields entanglement approach = Original state-counting approach


ENTROPY
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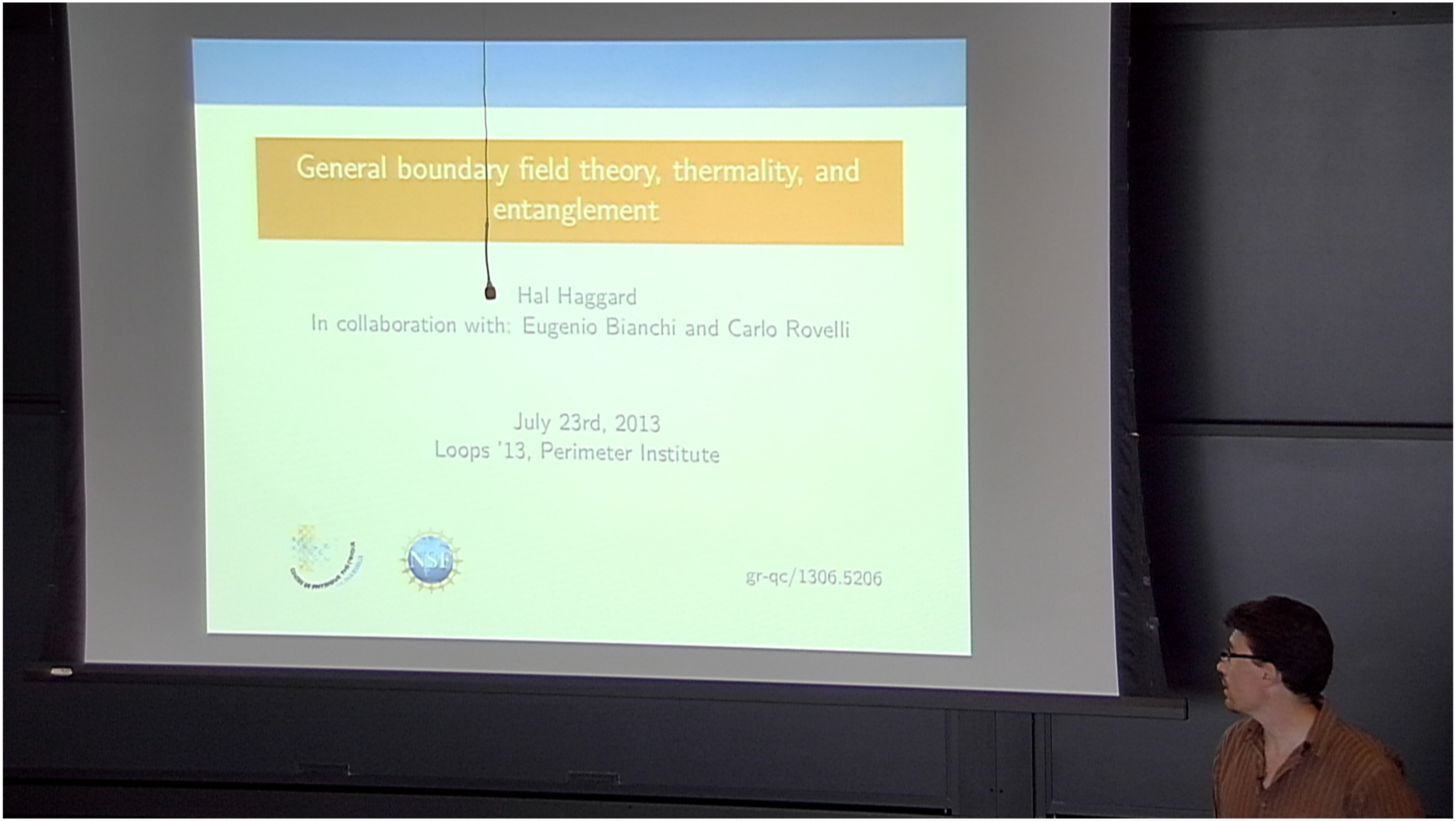
General boundary field theory, thermality, and entanglement
Hal Haggard
In collaboration with: Eugenio Bianchi and Carlo Rovelli
July 23rd, 2013
Loops '13, Perimeter Institute
gr-qc/1306.5206

Questions

What do we learn from the thermality of event horizons?

Is horizon thermality local? Is there a quantum version of the equivalence principle?

How does entanglement speak with gravity and the boundary formalism?

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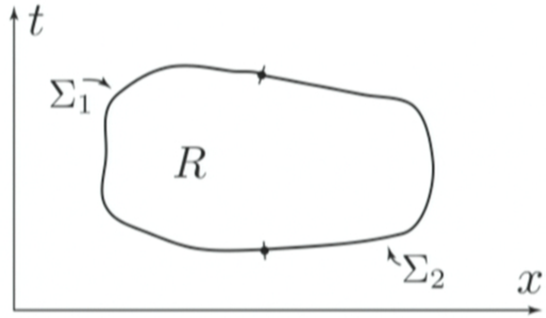
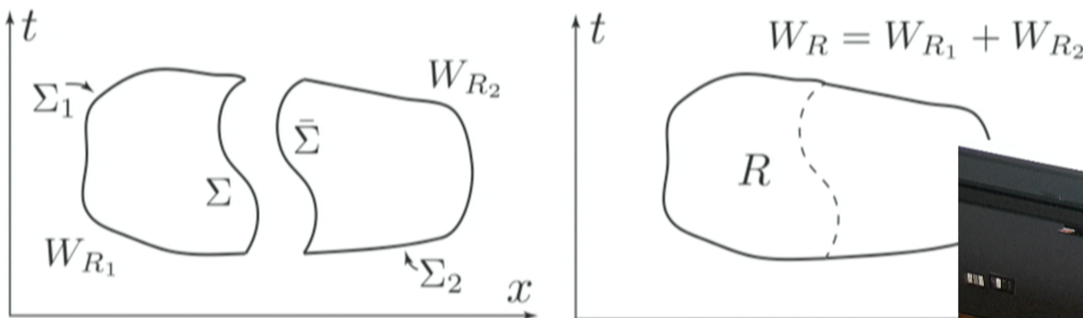
Structure of the general boundary formalism

Fundamental ingredients: 1) Decomposition: Oeckl [1,2,3]



Hilbert space $\mathcal{B}$ of ∂R decomposes: $\mathcal{B} = \mathcal{H}_1 \otimes \mathcal{H}_2$

2) Gluing: glue two manifolds along boundary, amplitudes add



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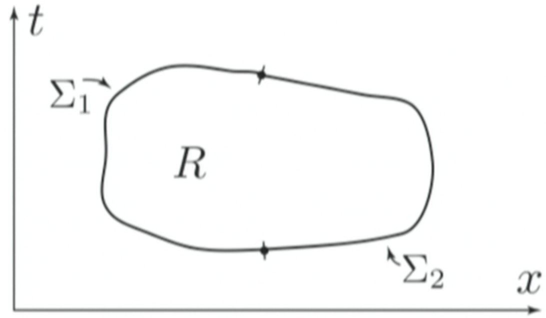

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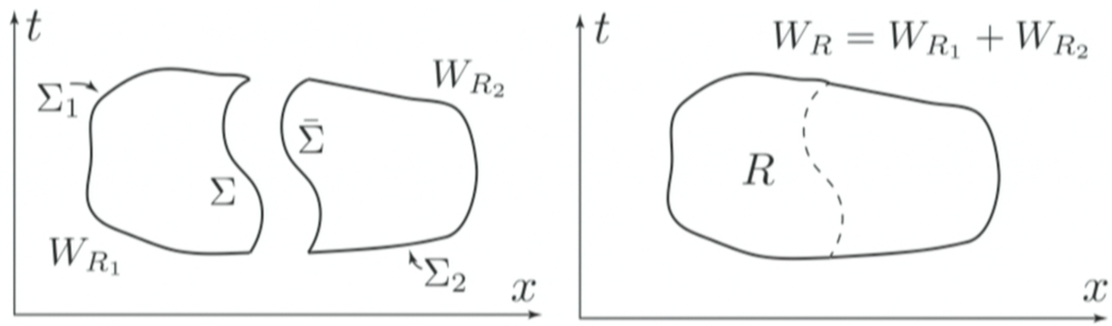


Think of path integrals over regions. Rigorous in context of TQFT.

The Kubo-Martin-Schwinger condition

The KMS condition is standard for identifying thermality.

For a statistical state ρ and observable A

$$\langle A \rangle = \mathrm{Tr}[A\rho],$$

the correlation of A and B is

$$\langle AB \rangle = \mathrm{Tr}[AB\rho],$$

and the time-dependent correlation is

$$\langle A(t)B(0) \rangle = \mathrm{Tr}[e^{iHt}Ae^{-iHt}B\rho].$$

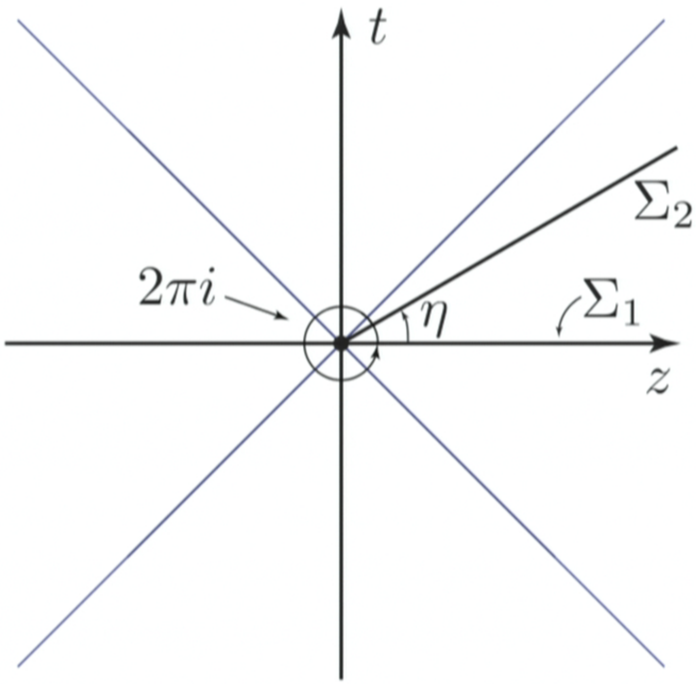
Then the KMS condition is

$$\langle A(t)B(0) \rangle = \langle B(0)A(t+i\beta) \rangle,$$

with β the inverse temperature.

Minkowski vacuum for wedge boundary?

What is the vacuum state in a Rindler wedge? (work w/ E. Bianchi)



The amplitude is:

$W_\Sigma[\varphi] = W_\eta[\varphi_1, \varphi_2]$.

Idea: Vacuum should be the path integral over the exterior of the wedge.

Boost through the light cone by analytically continuing:

- pick up $i\frac{\pi}{2}$ on each crossing of the light cone.

Builds on the ideas of Unruh and Weiss [4].

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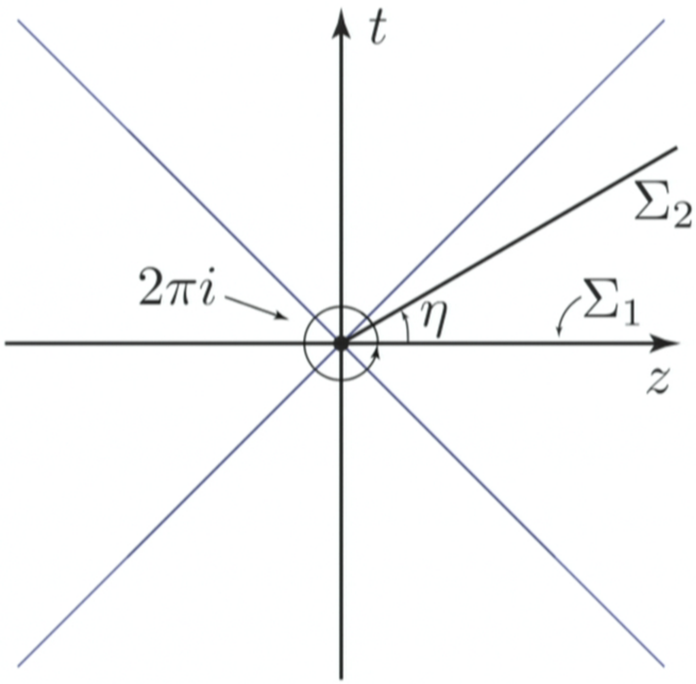

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Wedge vacuum

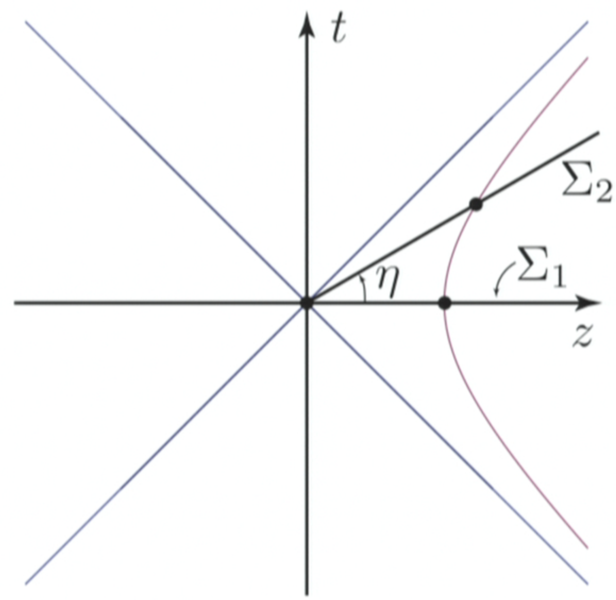
Wedge amplitude $W_\eta[\varphi_1, \varphi_2]$; conjecture $\Psi^0_\eta = W_{-\eta+2\pi i}[\varphi_1, \varphi_2]$.

To check it, do path integral for free scalar field:

$$\langle \varphi_1(x)\varphi_2(x)\rangle_\eta = \frac{\int \mathcal{D}\varphi \Psi^0_\eta \varphi_1\varphi_2 W_\eta}{\int \mathcal{D}\varphi \Psi^0_\eta W_\eta}.$$

For insertions along accelerated trajectory

$$G(\tau) = \langle \varphi_1(x)\varphi_2(x)\rangle_{\eta=a\tau} \sim \frac{1}{\sinh^2\left(\frac{\eta}{2}\right)} = \frac{1}{\sinh^2\left(\frac{a\tau}{2}\right)}$$



The Unruh effect: $G(\tau) = G(\tau + \frac{2\pi}{a} i)$, a thermal correlation.

Rindler horizons

In the near horizon limit all black hole horizons look like the Rindler horizon. (e.g. Jacobson & Parentani [5])

Analogs of black hole results can be derived for Rindler, e.g.:

- $\Delta A_H = \frac{8\pi G}{\kappa} \Delta Q$ Bianch & Satz [6]
- $\Delta S_{\text{gen}} \geq 0$ Wall [7,8]
- $\Delta S_{\text{ent}} = \frac{\Delta A_H}{4G}$ Bianch [9]

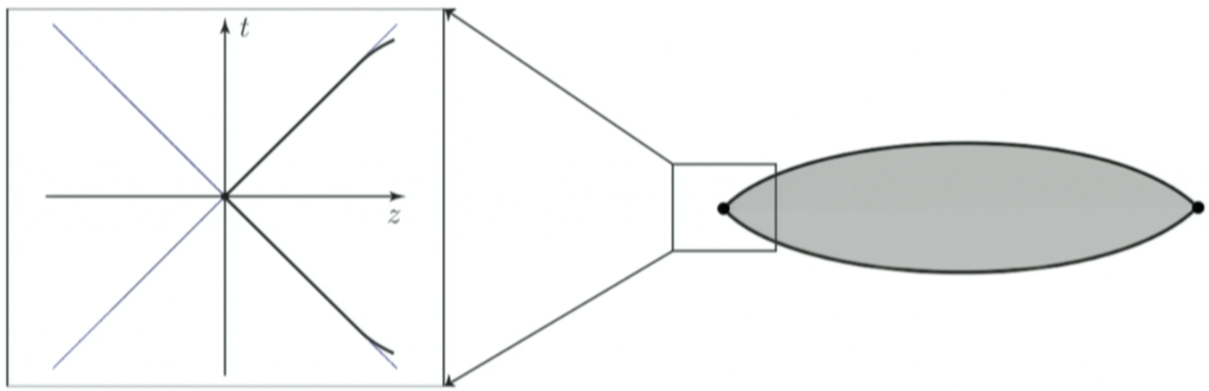
For example, the last work provides new resolutions to:
1) Entanglement entropy of Rindler horizon divergent,
2) tuning of high energy cutoff Λ, 3) species problem.

These works all use the tools of perturbative quantum field theory and provide an accessible treatment of quantum Rindler horizons.

8

Quantum equivalence principle

The above references suggest and support a quantum version of the equivalence principle.



Near a corner a finite spacetime region looks like Rindler

Specific realization: assume that near corners physics is locally Lorentz invariant.

- Below we investigate the consequences of this assumption for non-perturbative, general-boundary gravity.

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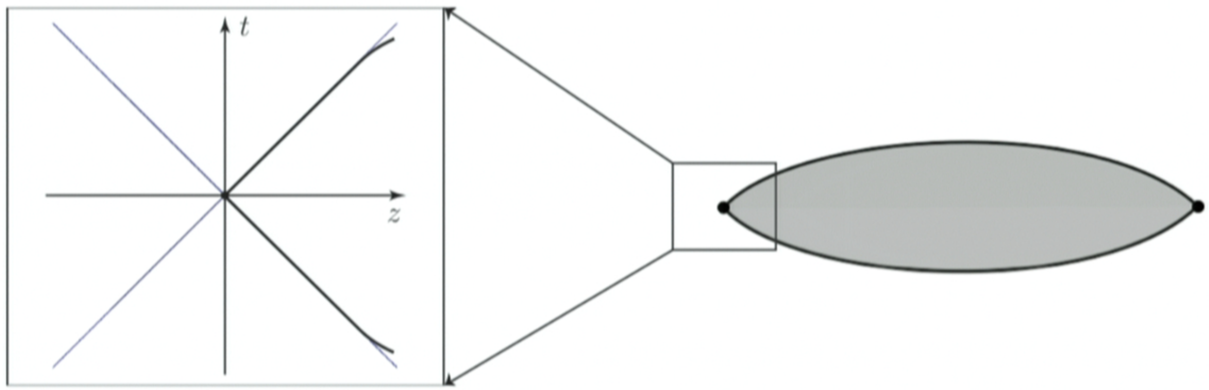

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Non-relativistic formalism

With $\mathcal{H}_t \simeq \mathcal{H}$, define the boundary Hilbert space $\mathcal{B}_t = \mathcal{H}_0 \otimes \mathcal{H}_t^*$.

Mechanics

Two notable structures:

$W_t(\psi \otimes \phi^*) := \langle \phi | e^{-iHt} | \psi \rangle$,

and extend by linearity.

$\sigma : \psi \mapsto \psi \otimes (e^{-iHt}\psi)^*$.

$\Psi = \sigma(\psi)$ satisfy $W_t(\Psi) = 1$, call 'em "physical boundary states".

Stat. Mech.

Statistical state

$\rho = \sum_n c_n |n\rangle \otimes \langle n| \in \mathcal{B}_0$,

s.t. $\sum_n c_n = 1$.

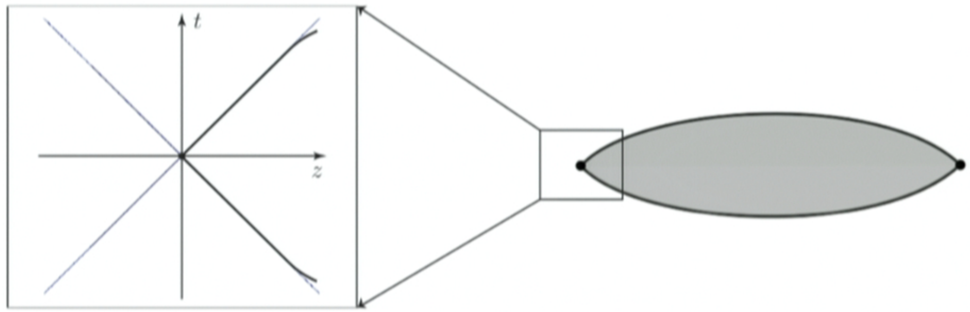
Corresponding element of $\mathcal{B}_t$,

$\rho_t = \sum_n c_n |n\rangle\langle n| e^{iHt}$

and it is physical, $W_t(\rho_t) = 1$.

Quantum Gravity

Finite regions: the quantum equivalence principle & Unruh effect



$\rightsquigarrow$ all local boundary states are mixed. But $\mathcal{B}$ can be made bipartite in many different manners, so better to say *non-separable*.

- Local gravitational states are always entangled states

Remarkably, the complete absence of physical pure sta
that there is no distinction between quantum and stati
fluctuations in quantum gravity. (see also Smolin [10, 11])

Spherical causal domain

Results leading us to reconsider the Cauchy development of spherical regions:

- Foliation leads to repacking of the Rindler trajectories

- Thermal properties subtle: detectors thermalize but region is not in equilibrium

- $m = 0$: Spherical entangling surface $\rightsquigarrow |0\rangle \sim e^{-\pi C_{in}}$, Sorkin-Johnston vacuum [12, 13, 14] in these regions?

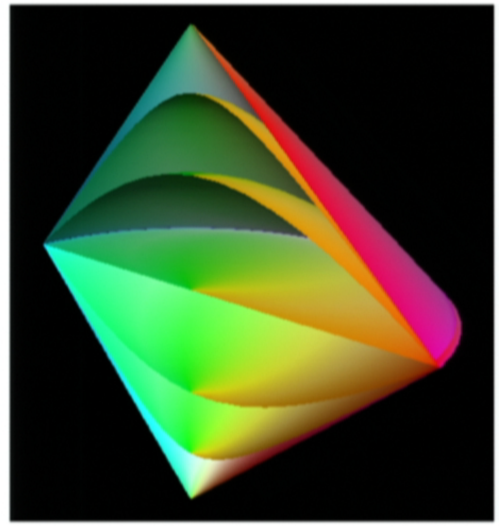
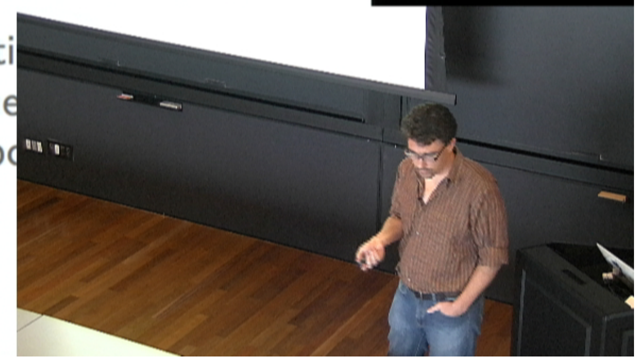


Figure: 2+1 spaceti visualization of sphe domain and hyperbo



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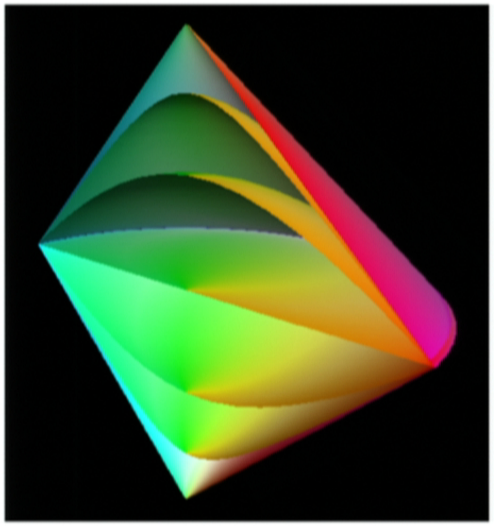


Figure: 2+1 spacetime cutaway visualization of spherical causal domain and hyperbolic foliation

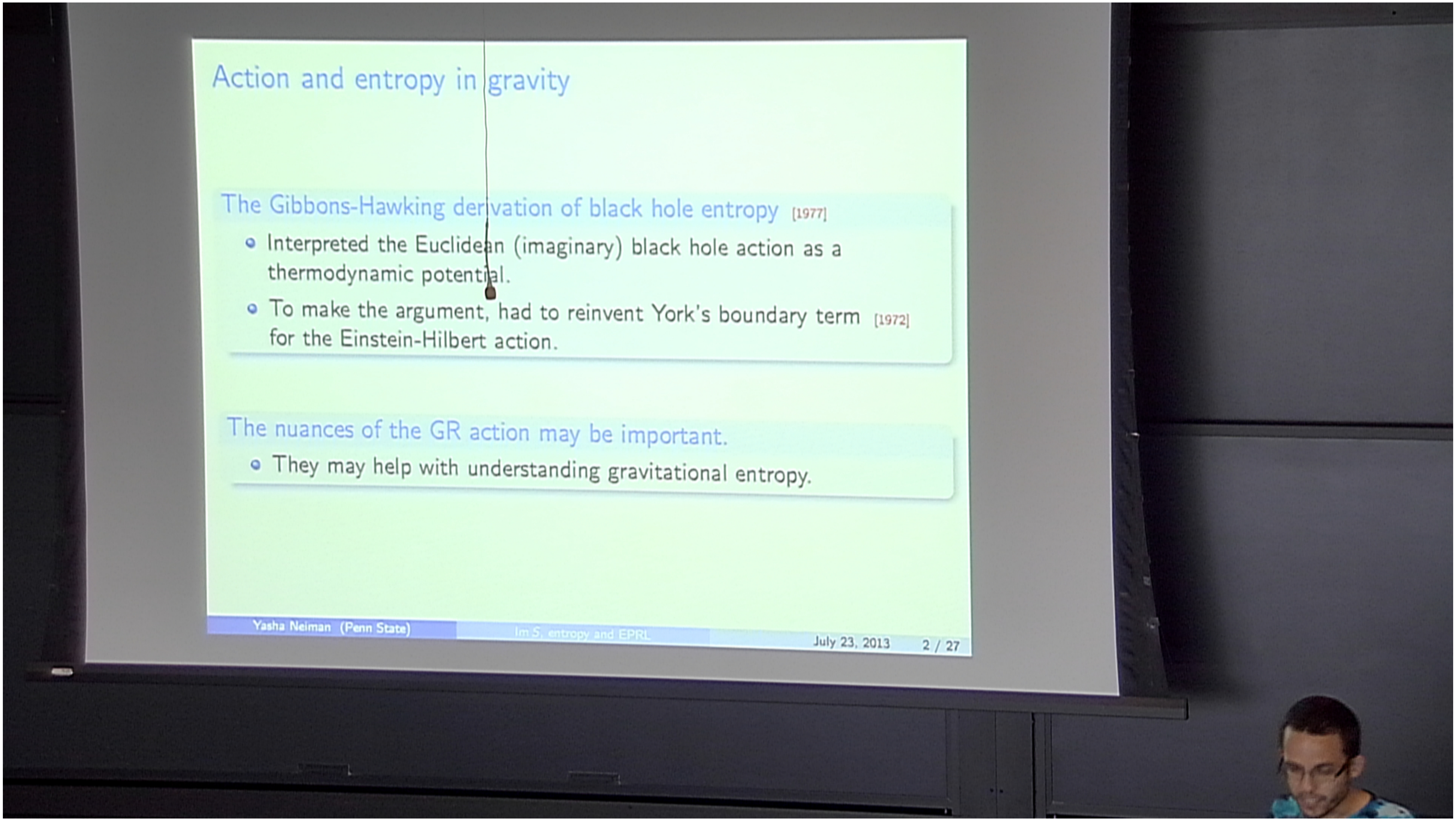
Results

♣ The Minkowski vacuum in the Rindler wedge can be described by $\Psi^0_\eta[\varphi_1, \varphi_2] \neq f(\varphi_1)g(\varphi_2)$.

♦ In quantum gravity finite spacetime regions are the describable physical processes.

These regions are *always* entangled $\rightsquigarrow$ there is no fundamental distinction between statistical and quantum fluctuations.

♠ Revealing new ideas about finite spacetime regions: thermality, vacua, and horizon entanglement


Action and entropy in gravity
The Gibbons-Hawking derivation of black hole entropy [1977]
Interpreted the Euclidean (imaginary) black hole action as a thermodynamic potential.
To make the argument, had to reinvent York's boundary term [1972] for the Einstein-Hilbert action.
The nuances of the GR action may be important.
They may help with understanding gravitational entropy.
Yasha Neiman (Penn State)
Im S, entropy and EPRL
July 23, 2013
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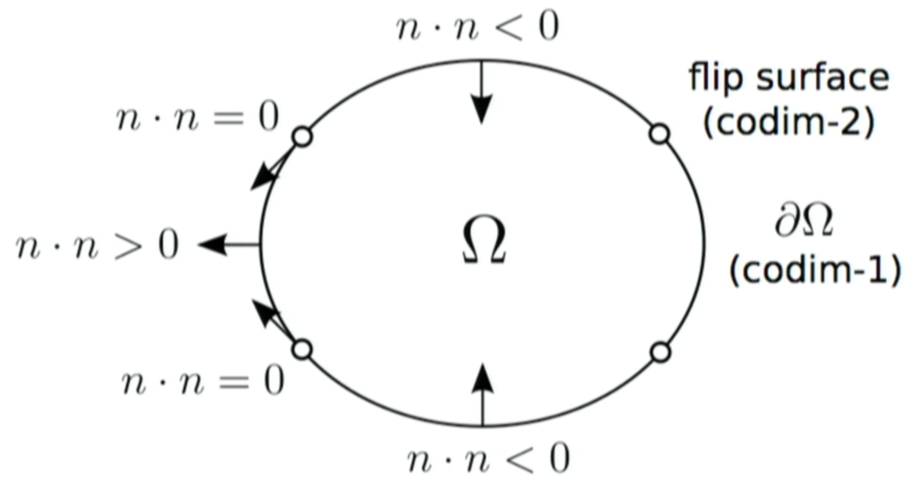
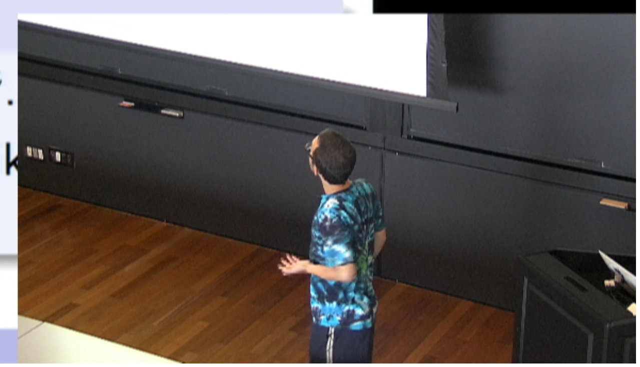
Spacetime regions Ω with smooth boundary $\partial\Omega$



- $\partial\Omega$ doesn't have to be at infinity!

Normal vector n^μ

- Sign of n^μ: chosen so that $n_\mu v^\mu > 0$ for outgoing v^μ.
- Sign of $n \cdot n$: has to flip between spacelike and timelik
  $\Rightarrow$ $n \cdot n$ cannot be constant everywhere!



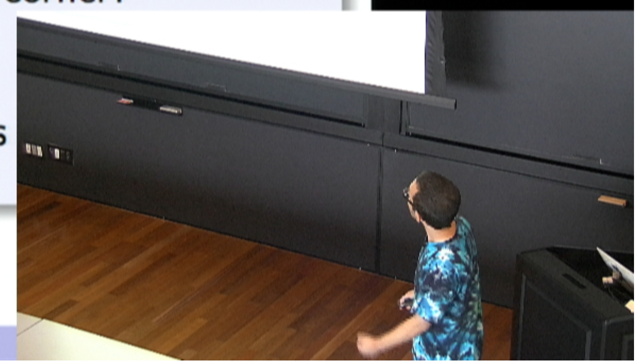
What does it take to know the action?

Know the action formula

- The bulk Lagrangian: $\int_{\Omega} \mathcal{L}\, d^4x$
- The codimension-1 boundary term: $\int_{\partial\Omega} Q\, d^3x$

Know how to evaluate the formula at special codimension-2 surfaces

- Corner surfaces on $\partial\Omega$.
  - Should the boundary integral be carried through the corner?
- Signature-flip surfaces on $\partial\Omega$.
  - When the boundary becomes null, the Q integral has



The γ-independent (Palatini) boundary term [Wieland, 2010]

$$\boxed{S = \frac{1}{2}\int_\Omega \star(e\wedge e)^{IJ}\wedge F_{IJ} + \int_{\partial\Omega}\star(e\wedge e)^{IJ}\wedge\left(\frac{N_I dN_J}{N\cdot N} - \frac{1}{2}A_{IJ}\right)}$$

- $N_I = \frac{1}{3!}\epsilon_{IJKL}\epsilon^{abc}e_a^J e_b^K e_c^L$: densitized normal.

Defining properties of the boundary term

- The A_{IJ} piece cancels the δA_{IJ} variation from the bulk term.
  $\Rightarrow \quad \delta S = 0$ for $\delta e^I\Big|_{\partial\Omega} = 0$.
- The $N_I dN_J$ piece restores gauge invariance.

The $(1/\gamma)$-proportional boundary term

$$\boxed{S_\gamma = \frac{1}{2\gamma}\left(\int_\Omega e^I \wedge e^J \wedge F_{IJ} + \int_{\partial\Omega} e^I \wedge (de_I - e^J \wedge A_{IJ})\right)}$$

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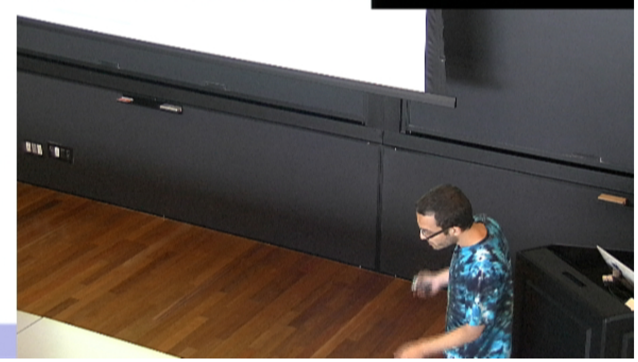

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- Not gauge invariant.

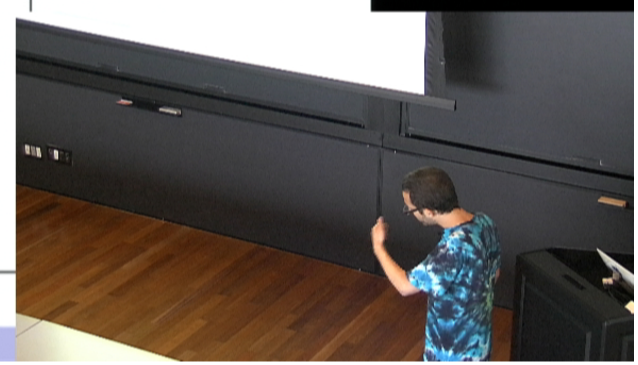
The full Holst action formula

$$\boxed{\begin{aligned} S = \frac{1}{2}\int_\Omega \star(e\wedge e)^{IJ}\wedge F_{IJ} + \int_{\partial\Omega}\star(e\wedge e)^{IJ}\wedge\frac{N_I DN_J}{N\cdot N} \\ + \frac{1}{2\gamma}\left(\int_\Omega e^I\wedge e^J\wedge F_{IJ} + \int_{\partial\Omega} e^I\wedge De_I\right)\end{aligned}}$$

On half-shell (when A_{IJ} and e^I are compatible):

- The Holst and metric actions agree.

$$\boxed{\begin{aligned} \star(e\wedge e)^{IJ}\wedge F_{IJ} &= \sqrt{-g}\,g^{\mu\nu}R_{\mu\nu}\,d^4x \\ \star(e\wedge e)^{IJ}\wedge\frac{N_I DN_J}{N\cdot N} &= h^{ab}\nabla_a N_b\,d^3x \\ e^I\wedge e^J\wedge F_{IJ} &= 0 \\ e^I\wedge De_I &= 0 \end{aligned}}$$



Corner contributions to the action

At a corner surface Σ with area A and corner angle θ:

- The derivative of N_μ has a delta-function singularity.
- The boundary term integrates to:

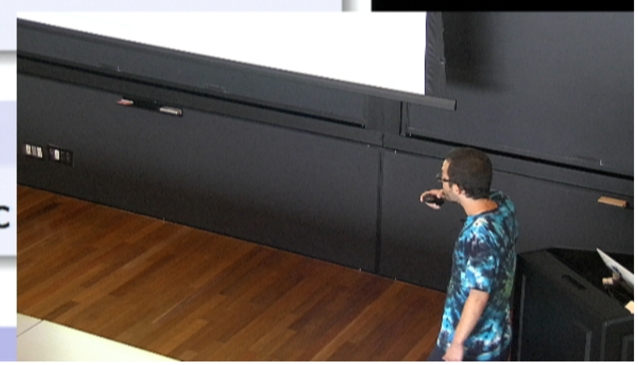
$$\int_{\delta\Sigma} h^{ab}\nabla_a N_b \, d^3x = \boxed{A\theta}$$

Do we include these "corner contributions"? Yes!

- In the continuum, need infinite resolution to tell a corner from a smooth turn.
- In simplicial gravity, this is the whole boundary term.

Required by a stronger variational principle. [Hayward,1993]

- Hold fixed on $\partial\Omega$ not $g_{\mu\nu}$ or e^I, but only the intrinsic



The imaginary part of corner contributions

There is a loophole in the variational principle

- Can redefine corner angles by a constant $\theta \to \theta + \theta_0$.
- Modifies the action by $A\theta_0$.

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Ridiculous! We know what angles are!

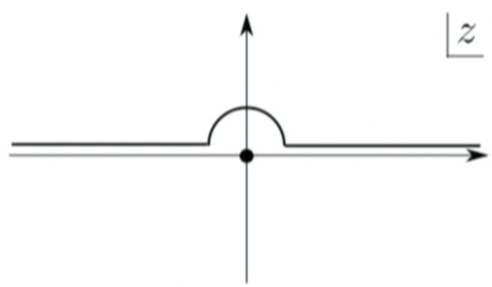
- E.g. the angle between the x and y axes is $\theta = \pi/2$.

Well, what's the angle between the x and t axes?

- Answer usually used for the action: $\theta = 0$.
- Correct answer: $\theta = \pi i/2$. [Sorkin, 1974]

An argument for $\theta = \pi i/2$: rotation & rescaling

- Write your rotating vector as: $n^\mu \sim L^\mu + z\ell^\mu \; ; \; z \in [-1, 1]$.
- Boost angle is: $d\theta = \dfrac{dz}{2z}$.
- When n^μ flips signature, z crosses through zero.
- Must deform the integration contour to avoid the pole!



$$\theta = \int_{-1}^{1} \frac{dz}{2z} = \pm\frac{\pi i}{2}$$

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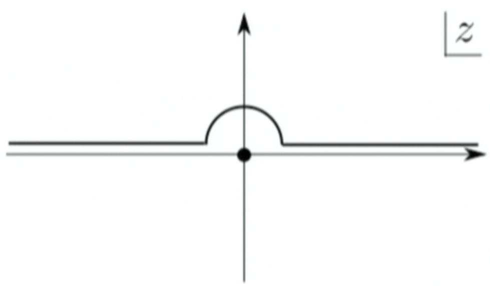


$$\theta = \int_{-1}^{1} \frac{dz}{2z} = \pm \frac{\pi i}{2}$$

The imaginary part of the GR action

Corner angles with signature flips:

$$\operatorname{Im}\theta = \sum_{\text{flips}} \frac{\pi}{2}$$

$$\Downarrow$$

In the GR action:

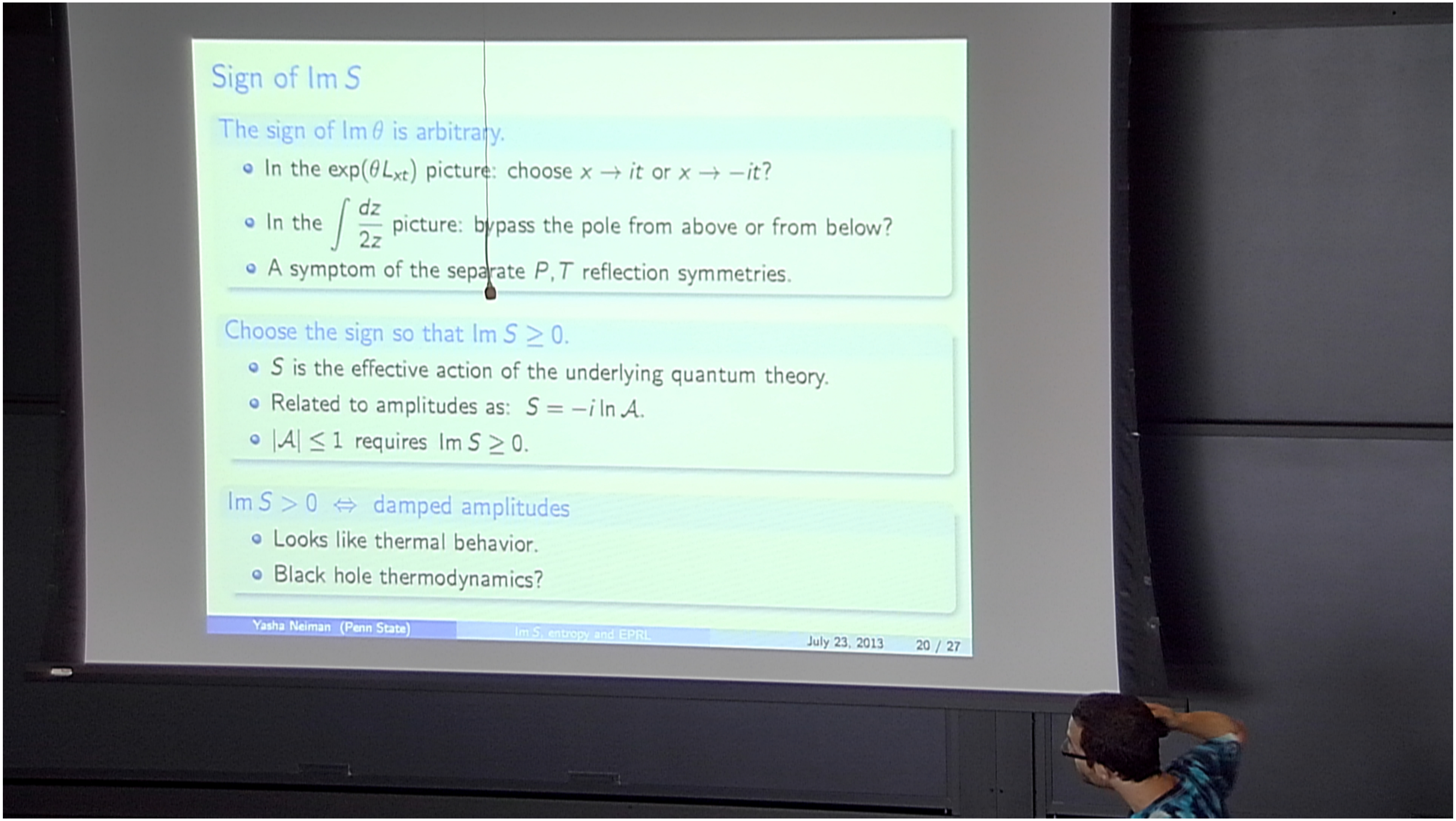
$$\operatorname{Im} S = \frac{\pi}{2} \sum_{\text{flips}} A$$

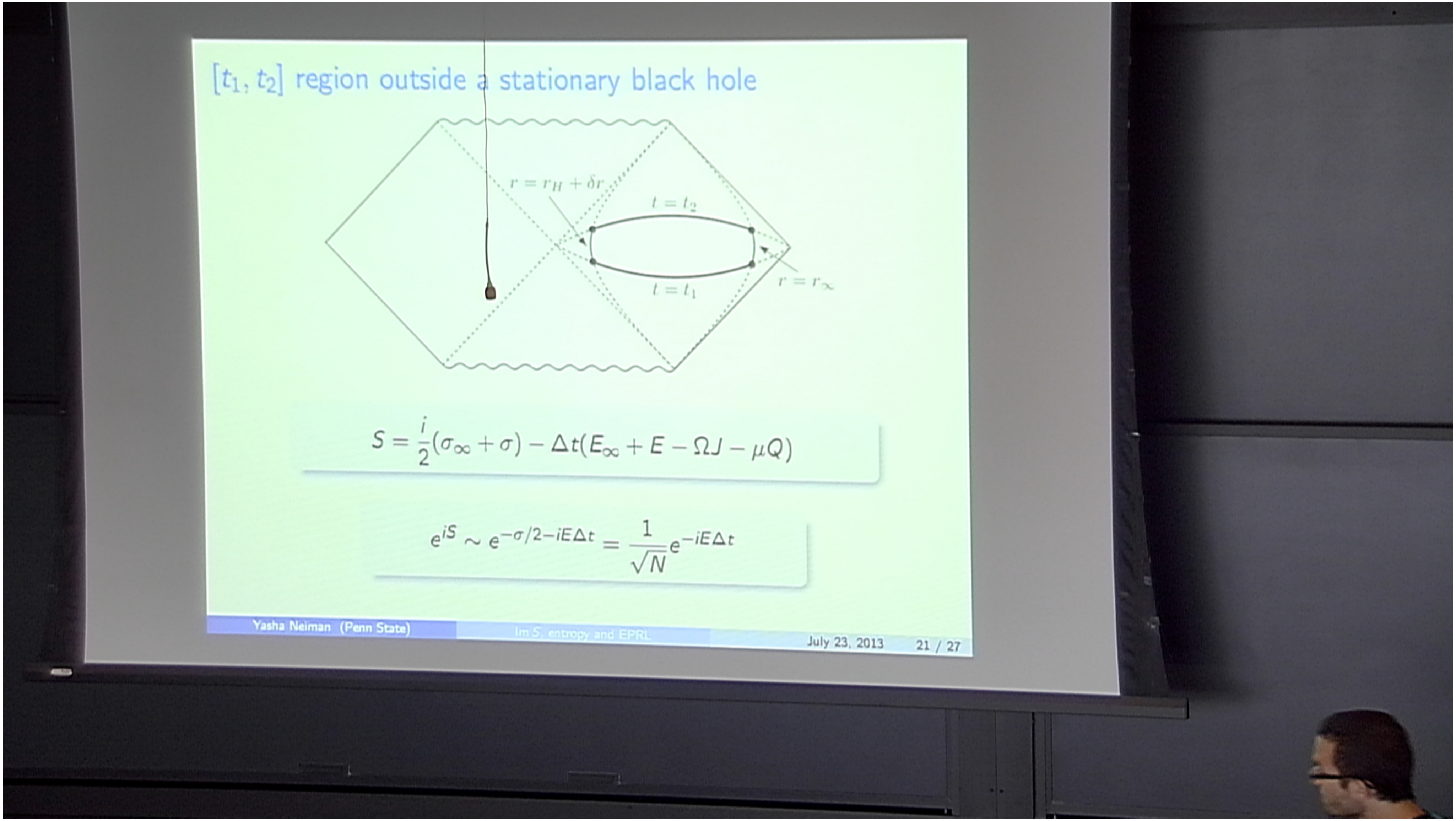
(A - area)

More generally:

$$\operatorname{Im} S = \frac{1}{4} \sum_{\text{flips}} \sigma$$

(σ - Wald entropy functional)


Sign of Im S
The sign of Im θ is arbitrary.
In the $\exp(\theta L_{xt})$ picture: choose $x \to it$ or $x \to -it$?
In the $\int \frac{dz}{2z}$ picture: bypass the pole from above or from below?
A symptom of the separate P, T reflection symmetries.
Choose the sign so that Im $S \geq 0$.
S is the effective action of the underlying quantum theory.
Related to amplitudes as: $S = -i \ln \mathcal{A}$.
$|\mathcal{A}| \leq 1$ requires Im $S \geq 0$.
Im $S > 0 \Leftrightarrow$ damped amplitudes
Looks like thermal behavior.
Black hole thermodynamics?
Yasha Neiman (Penn State)
Im S, entropy and EPRL
July 23, 2013
20 / 27


[t_1, t_2] region outside a stationary black hole
$r = r_H + \delta r$
$t = t_2$
$t = t_1$
$r = r_\infty$
$$S = \frac{i}{2}(\sigma_\infty + \sigma) - \Delta t(E_\infty + E - \Omega J - \mu Q)$$
$$e^{iS} \sim e^{-\sigma/2 - iE\Delta t} = \frac{1}{\sqrt{N}} e^{-iE\Delta t}$$
Yasha Neiman (Penn State)
Im S, entropy and EPRL
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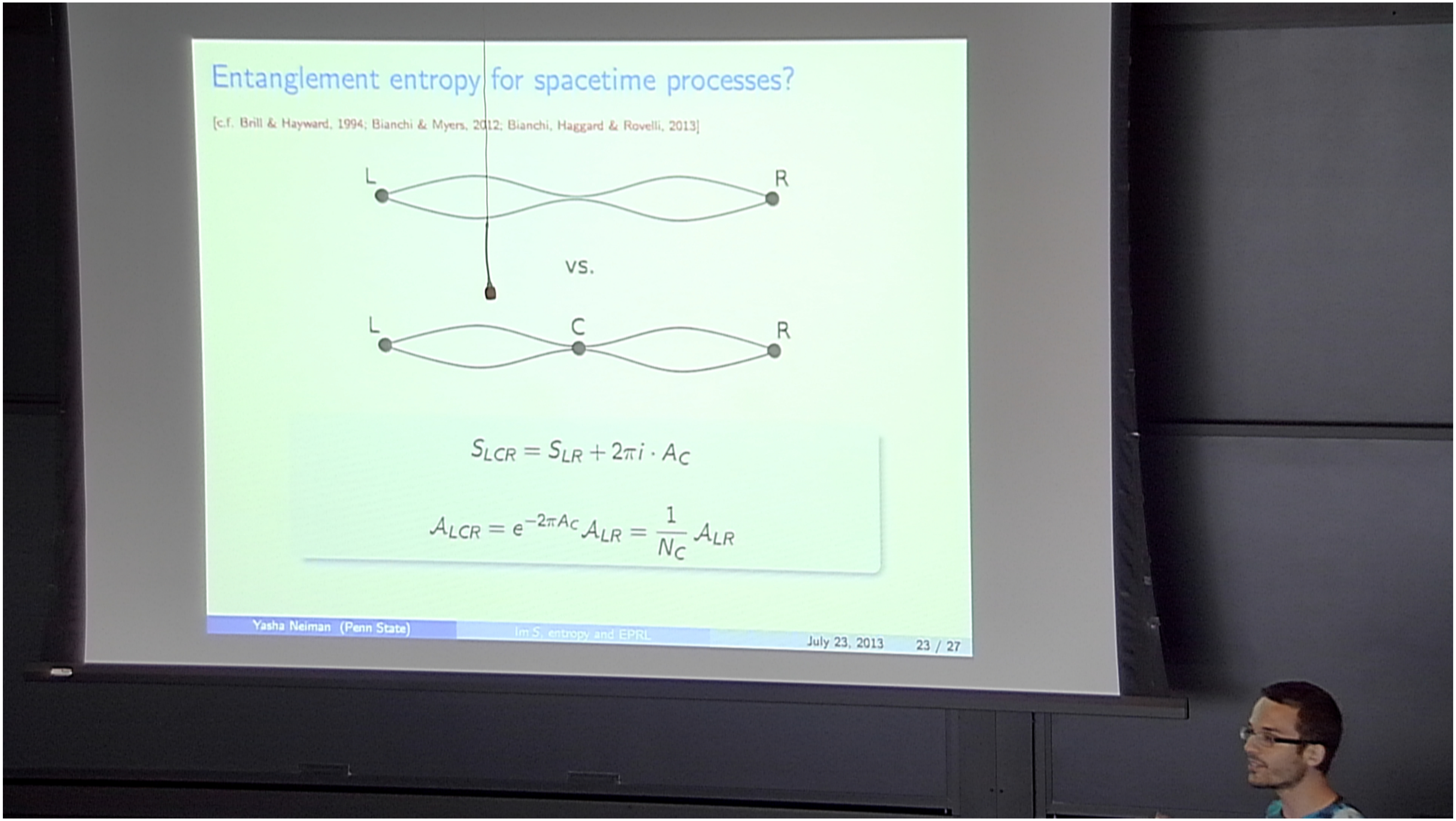
21 / 27


Entanglement entropy for spacetime processes?
[c.f. Brill & Hayward, 1994; Bianchi & Myers, 2012; Bianchi, Haggard & Rovelli, 2013]
L
R
vs.
L
C
R
$S_{LCR} = S_{LR} + 2\pi i \cdot A_C$
$\mathcal{A}_{LCR} = e^{-2\pi A_C} \mathcal{A}_{LR} = \frac{1}{N_C} \mathcal{A}_{LR}$
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23 / 27

Entanglement entropy for spacetime processes?

[c.f. Brill & Hayward, 1994; Bianchi & Myers, 2012; Bianchi, Haggard & Rovelli, 2013]

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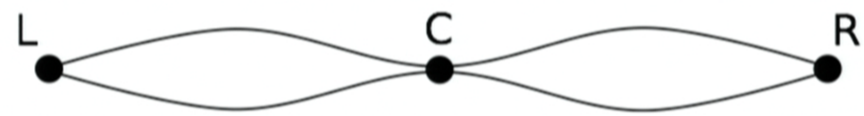

$$S_{LCR} = S_{LR} + 2\pi i \cdot A_C$$

$$\mathcal{A}_{LCR} = e^{-2\pi A_C}\mathcal{A}_{LR} = \frac{1}{N_C}\mathcal{A}_{LR}$$

Does the EPRL spinfoam know about $\mathrm{Im}\,\theta$?

- Not at first glance.
- The effective action of a large-spin 4-simplex is: [Barrett et. al., 2010]

$$\boxed{S_{\text{EPRL}} = \pm \sum_{\ell} A_\ell \,\mathrm{Re}\,\theta_\ell}$$

Open questions

Understand the physical meaning of $\mathrm{Im}\, S$.

- Proper relation to black hole entropy?
- Proper relation to entanglement entropy?
- Implications on unitarity?

The large-spin limit of EPRL wants $\gamma = \pm i$.

- Must be a way to quantize with $\gamma = \pm i$ directly!

Plan of the talk

1. Entropy calculation: Basic ingredients
2. Conformally coupled scalar: Ambiguity in the variables
3. Generalized theories and Wald entropy
4. Higher dimensions

Entropy calculation: Basic ingredients

[Smolin '95; Krasnov '96; Rovelli '96; Ashtekar, Baez, Corichi, Krasnov '97-; ...; Engle, Noui, Perez '09-; ...]

Isolated horizon boundary of spacetime

Connection variables $\to$ boundary degrees of freedom

Ingredients on horizon slice H

- Boundary symplectic structure $\int_H \delta_1 A^i \wedge \delta_2 A_i$
- Boundary condition $F^i(A) = \Sigma^i(e)$
- Area spectrum $8\pi G \gamma \sqrt{j(j+1)}$ (measures $\sqrt{h}$ = area density on H)

Important observation for BH entropy from LQG:

$$S_{\mathrm{BH}} \propto \frac{1}{\gamma} \int_H \sqrt{h(e)}$$

γ = Barbero-Immirzi parameter

Generalized gravity theories [Wald gr-qc/9307038]

$$S_{\mathrm{Wald}} = \frac{1}{4G} \int_H \sqrt{h} \frac{-\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu} \epsilon_{\rho\sigma} \neq \frac{A}{4G}$$

$\mathcal{L}$ = Lagrangian, ϵ_{μ}

Conformally coupled scalar field [NB, Stottmeister, Thurn 1203.6525]

Setting: Entropy calculation based on physical states

Solve Hamiltonian constraint classically with a constant mean curvature gauge fixing.

$\rightarrow$ Quantize Dirac bracket $\rightarrow$ physical states in state counting

$$S = \int_M d^4X \sqrt{g} R\, a(\Phi) - \frac{1}{2} \int_M d^4X \sqrt{g} g^{\mu\nu} (\nabla_\mu \Phi)(\nabla_\nu \Phi), \qquad a(\Phi) = (1 - \phi^2/6)$$

Norbert Bodendorfer (IGC, PennState) LQG and the Wald entropy formula

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Basic issue: Ambiguity in the choice of variables

$$K_a^i \dot{E}_i^a + \pi_\Phi \dot{\Phi} = \left(K_a^i / f(\Phi)\right)(f(\dot{\Phi}) E_i^a) + \left((\pi_\Phi - K_a^i E_i^a f'(\Phi)\right)\dot{\Phi}$$

Leads to different results for the entropy

$$S \propto \frac{A}{\gamma} \qquad \neq \qquad S \propto \frac{A}{\gamma} f(\Phi) \qquad\qquad (\Phi = \text{const on } H)$$

Entropy based on physical states: $S \propto \frac{A}{\gamma}\Phi^2 \not\propto S_{\text{Wald}} = \frac{A}{4G} a(\Phi)$

Conformally coupled scalar field

Two possible resolutions:

- There is a "correct" set of variables (used in [Ashtekar, Corichi gr-qc/0305082])
- Comparing with the Wald entropy formula is not straight forward (e.g. setting $\gamma = \gamma_0$ too naive)

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- We argue for the second choice! See also [Jacobson 0707.4026]
- However, there is a "preferred" set of variables (more discussion later)

preferred = leads to correct proportionality of the entropy

Possible way of comparison: $\gamma \to$ selfdual, $j \to \infty$ regime

EPRL/FK 4-Simplex amplitude semiclassical for pure GR [NB, Neiman 130

Entropy $S = A/4$ in same regime [Frodden, Geiller, Noui, Perez 1212.4060]

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Norbert Bodendorfer (IGC, PennState) LQG and the Wald entropy formula

Generalized gravity theories

Restrict to GR phase space plus standard matter (no higher time derivatives):

- Lovelock gravity plus non-minimally coupled scalars (+ standard matter)
- Works also in higher dimensions

Pure GR

The connection and the momentum both have standard geometric interpretation!

$$A_{ai} = \Gamma_{ai} + \gamma K_{ai}, \qquad q q^{ab} = E^{ai} E^{b}_{i}, \qquad \{A_{ai}, E^{bj}\} = \gamma\, \delta^b_a \delta^j_i$$

Γ_{ai} = spin connection, K_{ai} = extrinsic curvature, γ = Barbero-Immirzi parameter, q^{ab} = spatial metric

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Generalized theory - (preferred variables $\rightarrow$ next slide)

The momentum P^{ai} conjugate to $A_{ai} = \Gamma_{ai} + \gamma K_{ai}$ is not the densitized triad E^{ai}!

$$P^{ai} \propto \frac{\partial \mathcal{L}}{\partial \dot{A}_{ai}} \qquad \Rightarrow \quad \{A_{ai}, P^{bj}\} = \gamma\, \delta^b_a \delta^j_i$$

e.g. conf. coupled scalar: $\mathcal{L} = a(\Phi) R + \ldots \quad \Rightarrow \quad P^{ai} = a(\Phi) E^{ai}$

Area $\to$ Wald entropy

$\sqrt{(s_a P^a)^2}$ = Wald entropy density on horizon slice H

$$s_a\, P^{ai} \;\propto\; \frac{\partial \mathcal{L}}{\partial R_{\mu\nu\rho\sigma}} \epsilon^{\mu\nu}\epsilon^{\rho\sigma} \times s_a\, E^{ai}$$
[NB, Neiman arXiv:1304.3025]

$$\propto\; \frac{S_{\text{Wald}}}{\text{Area}} \times s_a\, E^{ai}$$
[Wald gr-qc/9307038]

$\epsilon^{\mu\nu}$ = binormal on horizon slice, $\mathcal{L}$ undensitized Lagrangian, s_a = horizon slice co-normal

Generalized area operator

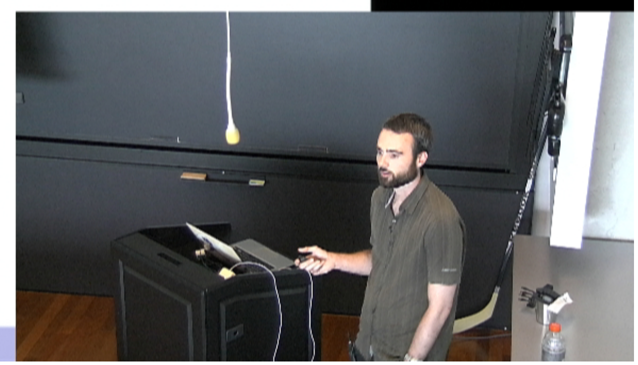
Idea: $\widetilde{\text{Area}} \propto \int \sqrt{|P|^2}$ $\boxed{\Rightarrow \text{Spec}(\widehat{\widetilde{\text{Area}}}) = \gamma\sqrt{j(j+1)}, \quad j \in \mathbb{N}_0/2}$

Higher dimensions: Chern-Simons theory [NB, Thiemann, Thurn 1304.2679]

Bulk variables:

SO$(D+1)$ connection $\boxed{A_{aIJ}}$, densitized hybrid vielbein $\boxed{\pi^{bKL}}$, $\{A_{aIJ}, \pi^{bKL}\} = \delta_a^b \delta_{IJ}^{KL}$

n^I = internal normal, $\pi^{aIJ} \approx 2/\beta n^{[I} E^{aJ]}$, $\beta \in \mathbb{R}$ free parameter



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Boundary symplectic structure 3+1: $\int_H \delta_1 e^i \wedge \delta_2 e_i \propto \int_H \delta_1 A^i \wedge \delta_2 A_i$

$$\int_H \delta_{[1} \tilde{s}^I \delta_{2]} n_I \quad \propto \quad \frac{A_H}{\chi\, \beta} \int_H \epsilon^{IJKLM_2N_2 \ldots M_nN_n} \left(\delta_{[1} {\Gamma^0}_{IJ}\right) \wedge \left(\delta_{2]} {\Gamma^0}_{KL}\right) \wedge {R^0}_{M_2N_2} \wedge \ldots \wedge {R^0}_{M_nN_n}$$

$Dn^I = Ds^I = D \underleftarrow{e}^I = 0$, $D = \partial + \Gamma^0$, $R^0_{IJ} = R(\Gamma^0)_{IJ}$, A_H = horizon area, χ = Euler characteristic of H, $n = (D-1)/2$

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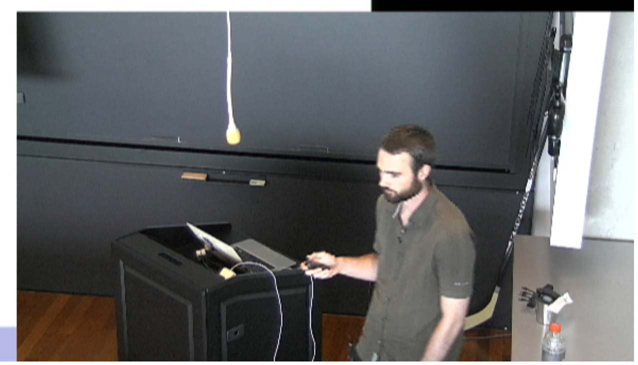
Norbert Bodendorfer (IGC, PennState)   LQG and the Wald entropy formula

Higher dimensions: quantization [NB 1307.5029]

Smeared binormals: Algebra of fluxes

Use $\boxed{L_S^{IJ} = \int_S 2/\beta\, n^{[I}\tilde{s}^{J]}}$ $\quad \{L_S^{IJ}, L_S^{KL}\} = 4\,\delta^{L][J} L_S^{I][K}$ $\quad \to \mathrm{so}(D+1)$ Lie algebra

SU(2) case: [Engle, Noui, Perez, Pranzetti 1006.0634]



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Off-diagonal horizon simplicity constraints

Non-rotating isolated horizon $\rightarrow$ Off-diagonal simplicity constraints $\boxed{L_{S_1}^{[IJ} L_{S_2}^{KL]} \approx 0}$

$\rightarrow$ Breaks local gauge invariance to global invariance on H.

Boundary Hilbert space

Product of SO$(D+1)$ representation spaces

$\rightarrow$ Non-trivial at points where bulk spin network punctures boundary

$\rightarrow$ Related to bulk rep. by boundary condition $\boxed{\hat{s}_a \pi^{aIJ} = L^{IJ}}$

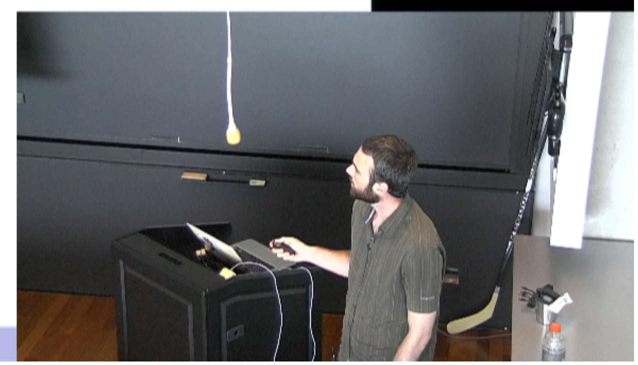
Higher dimensions: quantization [NB 1307.5029]

Restrictions on horizon Hilbert space

Gauge invariance / tracing: SO$(D+1)$ intertwiner See also [Rovelli, Krasnov 0905.4916]

Bulk simplicity: Simple representations of SO$(D+1)$ on H
(Highest weight $\vec{\Lambda} = (\lambda, 0, \ldots, 0)$, $\lambda \in \mathbb{N}_0$)

Off-diagonal simplicity: $\boxed{\text{Simple intertwiner}}$ (Intertwining repr. simple)



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1 to 1 mapping of simple $SO(D+1)$ and $SU(2)$ intertwiners

$\rightarrow$ Using dimension formulas from $SU(2)$ counting: $S = \text{const(D)}\frac{A}{\beta} - \frac{3}{2}\text{Log}\, A$

$\rightarrow$ (Up to β) Same result as Carlip and Solodukhin using CFT metho

[Carlip hep-th/9812013, gr-qc/0005017; Solodukhin hep-th/9812056; log correct.: Kaul and

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Summary

1. In general, **different choices of connection variables** possible
   - Can lead to different entropies before dealing with γ
   - Setting $\gamma = \gamma_0$ inappropriate in general

2. **Generalized theory** of gravity:
   - Preferred: Connection A_{ai} retains geometric interpretation
     $\Rightarrow \; \{A_{ai}, P^{bj}\} = \gamma \, \delta_a^b \delta_i^j$
   - $\sqrt{|P|^2} \propto$ Wald entropy density
     $\Rightarrow$ Entropy $S \propto S_{\mathrm{Wald}}$

3. **Higher dimensions**
   - Chern-Simons theory description possible, but hard to quantize
   - Largely dimension-independent result from using densitized bi-normals

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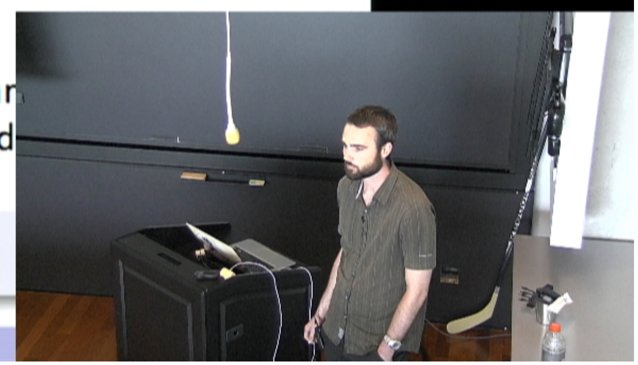

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- ⋆ Intriguing role played by the Barbero-Immirzi parameter γ:
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- ⋆ What point of view to adopt about γ?
  - New physical parameter of quantum gravity, fixed by BH entropy calculation or cosmology?
  - Should be abandoned classically (work with self-dual value $\gamma = \mathrm{i}$)?
  - A regulator that should eventually be removed in quantum theory?

Motivations

- ⋆ Intriguing role played by the Barbero-Immirzi parameter γ:
  - Irrelevant at the Lagrangian level,
  - A bit more subtle at the Hamiltonian level (connection ambiguity),
  - Central in quantum theory (geometric spectra, EPRL/FK).
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- ⋆ Hiding deeper questions: Lorentz symmetry, choice of gauge group, imposition of constraints, Hamiltonian constraint,...

Outline

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   2. **A regulator that should eventually be removed in quantum theory.**

⋆ Hiding deeper questions: Lorentz symmetry, choice of gauge group, imposition of constraints, Hamiltonian constraint,...

⋆ We are going to illustrate 1. with 3d gravity, and then argue for 2. in the context of LQG black hole entropy.

The Barbero-Immirzi parameter in 3d Lorentzian gravity

Actions for $\mathrm{SL}(2,\mathbb{C})$ connection ω

$$S[B,\omega]=\int_{\mathcal{M}_3}\mathrm{d}^3x\,\varepsilon^{\mu\nu\rho}(\star+\gamma^{-1})B^{IJ}_\mu F^{IJ}_{\mu\nu}+\phi^{\mu\nu}B^{IJ}_\mu B^{IJ}_\nu \tag{1}$$

$$S[e,x,\omega]=\int_{\mathcal{M}_3}\mathrm{d}^3x\,\varepsilon^{\mu\nu\rho}(\star+\gamma^{-1})x^I e^J_\mu F^{IJ}_{\nu\rho} \tag{2}$$

- ⋆ γ disappears on half shell (zero torsion).
- ⋆ Spin foam quantization of (1) shows disappearance of γ if all the Plebanski constraints are imposed [MG, Noui, 2012 | Alexandrov, 2012].
- ⋆ What about canonical quantization of (2) [Ben Achour, MG, Noui, Yu, 2013]?

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Two gauge choices

- ⋆ SU$(1,1)$ gauge: fix $x^I=(0,0,0,1)$, then (2) becomes SU$(1,1)$ BF theory, i.e. 3d Lorentzian gravity. Known γ-independent quantization.

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SU(2) kinematics

- ⋆ Discrete γ-dependent area spectrum: $a_j = 8\pi\gamma\ell_{\mathrm{Pl}}\sqrt{j(j+1)}$.
- ⋆ BTZ black hole entropy via Turaev-Viro model on solid torus:

$$W = \frac{\mathcal{Z}(\mathbf{T},\mathcal{O})}{\mathcal{Z}(\mathbf{T})}, \qquad S = \log W = \frac{L}{4\ell_{\mathrm{Pl}}} \quad \text{if} \quad \gamma = \frac{\log(2j+1)}{2\pi\sqrt{j(j+1)}}.$$

- ⋆ Contrast with the canonical calculation of [Frodden, MG, Noui, Perez, 2013].

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Reality conditions on $\mathbf{A}$

- ⋆ Physical motivation: the underlying theory is $\mathrm{SU}(1,1)$ BF.
- ⋆ Two possibilities:

$$J\pm\gamma^{-1}P=0 \quad\Rightarrow\quad \mathbf{A}=A^iJ_i,\ \text{the } \mathfrak{su}(2) \text{ Ashtekar-Barbero connection},$$
$$G\pm\gamma^{-1}F=0 \quad\Rightarrow\quad \mathbf{A}=\pm\mathrm{i}\gamma^{-1}A^iJ_i,\ \text{an } \mathfrak{su}(1,1) \text{ connection},$$

where J spans $\mathfrak{su}(2)$, F spans $\mathfrak{su}(1,1)$, and P and G the complements in $\mathfrak{sl}(2,\mathbb{C})$.
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Length operator and physical states

- ⋆ 3d geometric operators induced from the 4d ones.
- ⋆ For the $\mathfrak{su}(1,1)$ connection $\{E^a_i, \mathbf{A}^j_b\} = \pm \mathrm{i}\delta^a_b \delta^j_i$, so γ-independent spectrum

$$(\vec{X}_e)^2 \triangleright \mathbf{D}^{(s)}(h_{e'}) = \ell^2_{\mathrm{Pl}} \delta_{e,e'} Q^{(s)} \mathbf{D}(h_e),$$

and $Q^{(s)} = F_1^2 + F_2^2 - F_0^2$ positive for the continuous series $j = \mathrm{i}s - 1/2$, $s \in \mathbb{R}$.
Consistent with [Freidel, Livine, Rovelli, 2003].

- ⋆ Discrete spectra and γ-dependent theory if we work with the $\mathfrak{su}(2)$ connection.

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4d black hole entropy

Usual $\gamma \in \mathbb{R}$ counting

- ⋆ Horizon degrees of freedom in terms of SU(2) Chern-Simons with defects:

$$k = \frac{a_{\mathrm{H}}}{2\pi \ell_{\mathrm{Pl}}^2 \gamma (1-\gamma^2)}, \qquad S = \log\big(\dim\left[\mathcal{H}_{\mathrm{CS}}(j_1, \dots, j_p)\right]\big)$$

$$= \log\big(\dim\left[\mathcal{H}_{\mathrm{inv}}(j_1 \otimes \cdots \otimes j_p)\right]\big) = \frac{a_{\mathrm{H}}}{4\ell_{\mathrm{Pl}}^2} \frac{\gamma}{\gamma_0}.$$

- ⋆ Local observer's point of view [Frodden, Gosh, Perez, 2011].

4d black hole entropy

Insights delivered by $\gamma = \mathrm{i}$ calculations

- ⋆ Initial observation: large j behavior of $\dim\left[\mathcal{H}_{\mathrm{CS}}(j_1, \ldots, j_p)\right]$ for $\gamma = \mathrm{i}$ leads to $S = a_{\mathrm{H}}/(4\ell_{\mathrm{Pl}}^2)$ [Frodden, MG, Noui, Perez, 2013].
- ⋆ Asymptotics of spin foam amplitudes and imaginary part of the Lorentzian action [Bodendorfer, Neiman, 2013].
- ⋆ Solutions of the self-dual constraint $J \pm \mathrm{i}P$ of the type $(p, k) = \big(\mathrm{i}(j+1), \mathrm{i}j\big)$, with $\mathrm{SU}(1,1)$ labels $j = \mathrm{i}s - 1/2$ satisfy $A^2 = B_{\pm} \cdot B_{\pm} > 0$.
- ⋆ Holographic behavior

$$D[\{n_j\}] \approx \exp\left(\frac{a_{\mathrm{H}}}{4\ell_{\mathrm{Pl}}^2}\right) = \prod_j \exp\left(\frac{a_j n_j}{4\ell_{\mathrm{Pl}}^2}\right)$$

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for (indistinguishable) puncture weight leads to large spins and many punctures when

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Conclusion

- ⋆ The 3d model illustrates aspects that were so far suspected in 4d.
- ⋆ The passage $\gamma \rightarrow \mathrm{i}$ is not going to be inoffensive, so it is quite surprising that there is a setup in which it leads to consistent results.
- ⋆ One should look for more occurrences of such a possibility.
- ? Apply to more general class of black holes?
- ? Fate of logarithmic corrections?
- ? Spin foam model for self-dual theory?

- ? (Unrelated) Use 3d Plebanski action for 3d massive gravity models?

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Thank you for your attention.