Title: Phenomenology - 1

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Abstract:

Searches for Quantum Gravity Signals using Gamma-Ray Bursts

Jonathan Granot

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on behalf of the Fermi LAT & GBM Collaborations

LOOPS13, Perimeter Institute, July 23, 2013

Outline of the Talk:

- Focus on vacuum energy dispersion (a form of LIV)
- Why do we use GRBs & how do we set the limits
- Limit from the bright long GRB 080916C at z ~ 4.35
- 3 different types of limits from the short bright GRB 090510 at z = 0.903: detailed description & results
- Summary of limits on LIV using Fermi LAT GRBs
- Future prospects: the Cherenkov Telescope Array
- **Conclusions**

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Vacuum energy dispersion: parameterization

- Some quantum-gravity (QG) models allow or even predict (e.g. Ellis et al. 2008) Lorentz invariance violation (LIV)
- We directly constrain a simple form of LIV - dependence of the speed of light on the photon energy: $v_{ph}(E_{ph}) \neq c$
- This may be parameterized through a Taylor expansion of the LIV terms in the dispersion relation:

$$c^2 p_{ph}^2 = E_{\mathrm{ph}}^2 \left[1 + \sum_{k=1}^{\infty} s_k \left(\frac{E_{\mathrm{ph}}}{E_{\mathrm{QG},k}} \right)^k \right] \quad , \quad \text{where } E_{\mathrm{QG},k} \le E_{\mathrm{Planck}} \text{ is naturally expected}$$

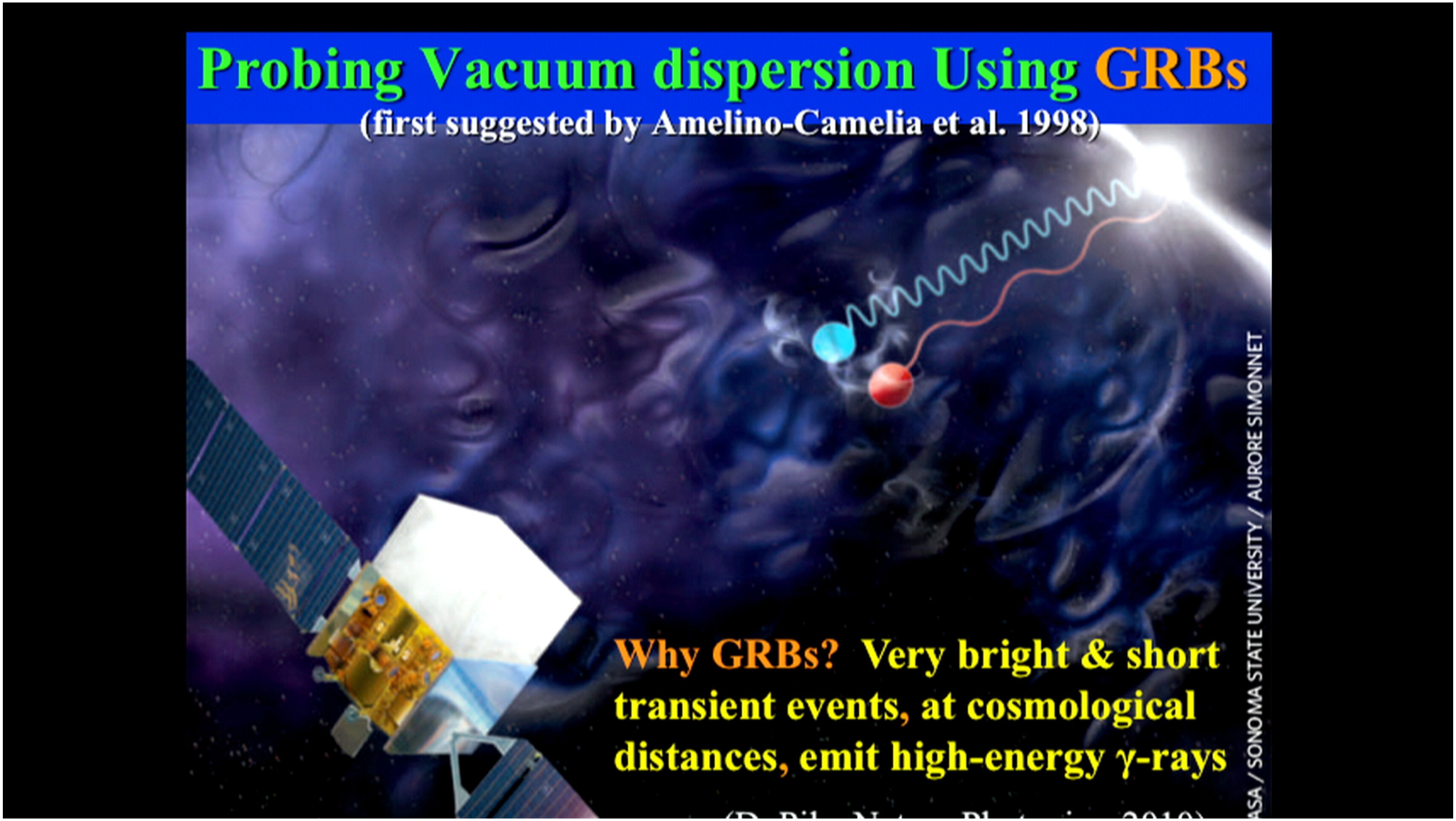
- $s_k = -1, 0, 1$ stresses the model dependent sign of the effect
- The most natural scale for LIV is the **Planck scale**
$l_{\mathrm{Planck}} \approx 1.62 \times 10^{-33}$ cm ; $E_{\mathrm{Planck}} = M_{\mathrm{Planck}} c^2 \approx 1.22 \times 10^{19}$ GeV

Vacuum energy dispersion: parameterization

- The photon propagation speed is given by the group velocity:

$$c^2 p_{ph}^2 = E_{\rm ph}^2\left[1+\sum_{k=1}^{\infty} s_k \left(\frac{E_{\rm ph}}{E_{\rm QG,k}}\right)^k\right] \quad , \quad v_{\rm ph} = \frac{\partial E_{\rm ph}}{\partial p_{\rm ph}} \approx c\left[1 - s_n \frac{(1+n)}{2}\left(\frac{E_{\rm ph}}{E_{\rm QG,n}}\right)^n\right]$$

- Since $E_{ph} \ll E_{QG,k} \lesssim E_{planck} \sim 10^{19}$ GeV the **lowest order** non-zero term, of order $n = \min\{k \mid s_k \neq 0\}$, **dominates**
- Usually n = 1 (linear) or 2 (quadratic) are considered
- We focus here on **n = 1**, since only in this case are our limits of the order of the Planck scale
- We try to constrain **both possible signs** of the effect:
  - $s_n = 1$, $v_{ph} < c$: higher energy photons propagate slower
  - $s_n = -1$, $v_{ph} > c$: higher energy photons propagate faster
- We stress: here $c = v_{ph}(E_{ph} \to 0)$ is the low energy limit of v_{ph}


Probing Vacuum dispersion Using GRBs
(first suggested by Amelino-Camelia et al. 1998)
Why GRBs? Very bright & short transient events, at cosmological distances, emit high-energy γ-rays
ASA / SONOMA STATE UNIVERSITY / AURORE SIMONNET

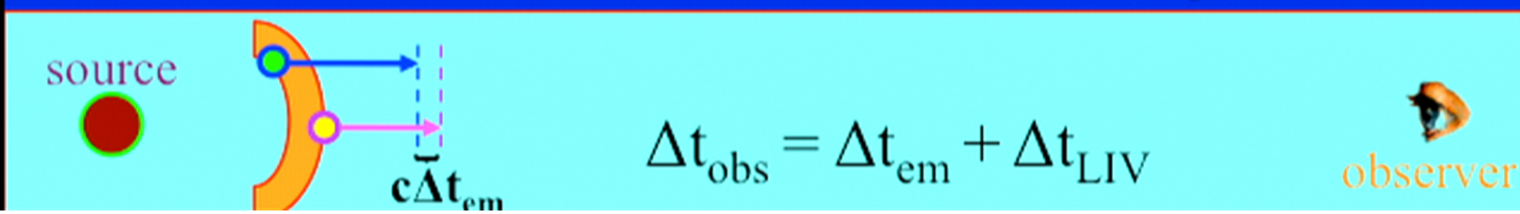
Constraining LIV Using GRBs

- A high-energy photon E_h would arrive after (in the sub-luminal case: $v_{ph} < c$, $s_n = 1$), or possibly before (in the super-luminal case, $v_{ph} > c$, $s_n = -1$) a low-energy photon E_l emitted together
- The time delay in the arrival of the high-energy photon is:

$$\Delta t_{\rm LIV} = s_n \frac{(1+n)}{2H_0} \frac{E_h^n - E_l^n}{E_{\rm QG,n}^n} \int_0^z \frac{(1+z')^n}{\sqrt{\Omega_m (1+z')^3 + \Omega_\Lambda}} dz'$$

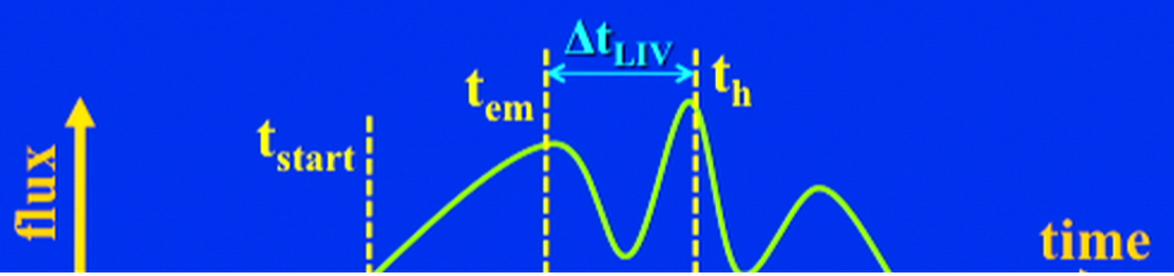
(Jacob & Piran 2008)

- The photons E_h & E_l do not have to be emitted at exactly the same time & place in the source, but we must be able to limit the difference in their effective emission times, i.e. in their arrival times to an observer near the GRB along our L.O.S



Method 1

- Limits only $s_n = 1$ - the sub-luminal case: $v_{ph} < c$, & positive time delay, $\Delta t_{LIV} = t_h - t_{em} > 0$ (here t_h is the actual measured arrival time, while t_{em} would be the arrival time if $v_{ph} = c$)
- We consider a single high-energy photon of energy E_h and assume that it was emitted after the onset time (t_{start}) of the relevant low-energy (E_l) emission episode: $t_{em} > t_{start}$



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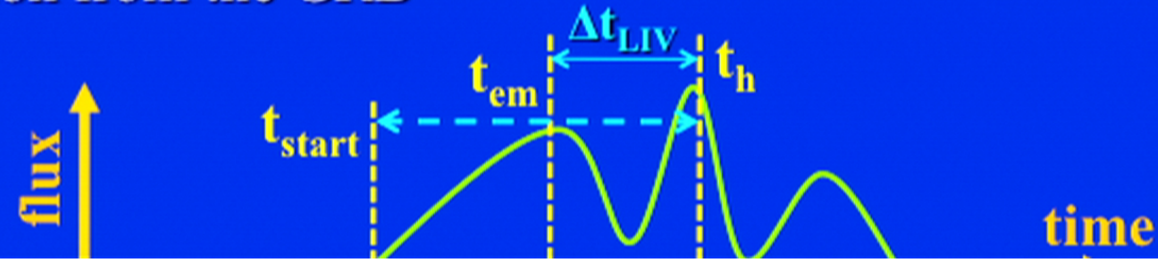
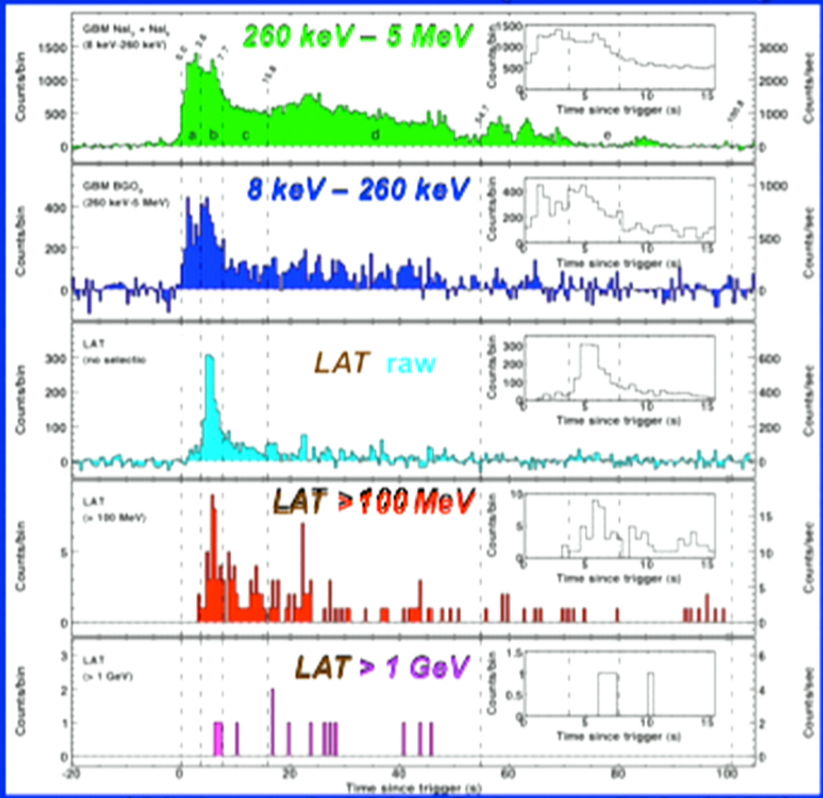
- ➔ $\Delta t_{LIV} = t_h - t_{em} < t_h - t_{start}$
- A conservative assumption: t_{start} = the onset of any observed emission from the GRB

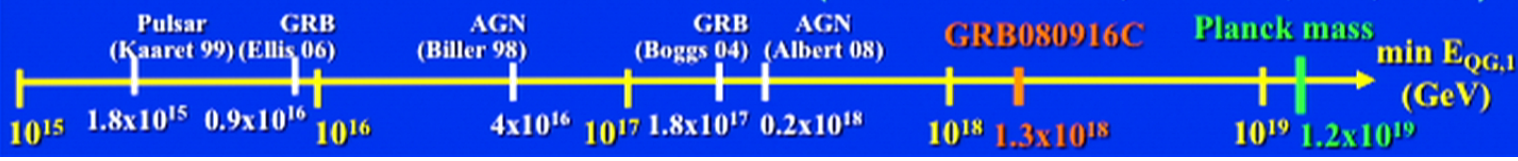


Limits on LIV: GRB080916C ($z \approx 4.35$)

- GRB080916C: highest energy photon (13 GeV) arrived 16.5 s after low-energy photons started arriving (=the GRB trigger)
  ➔ conservative lower limit:
  $E_{QG,1} > 1.3\times10^{18}$ GeV
  $\approx 0.11 E_{Planck}$
- This improved upon the previous limits of this type, reaching 11% of E_{Planck}

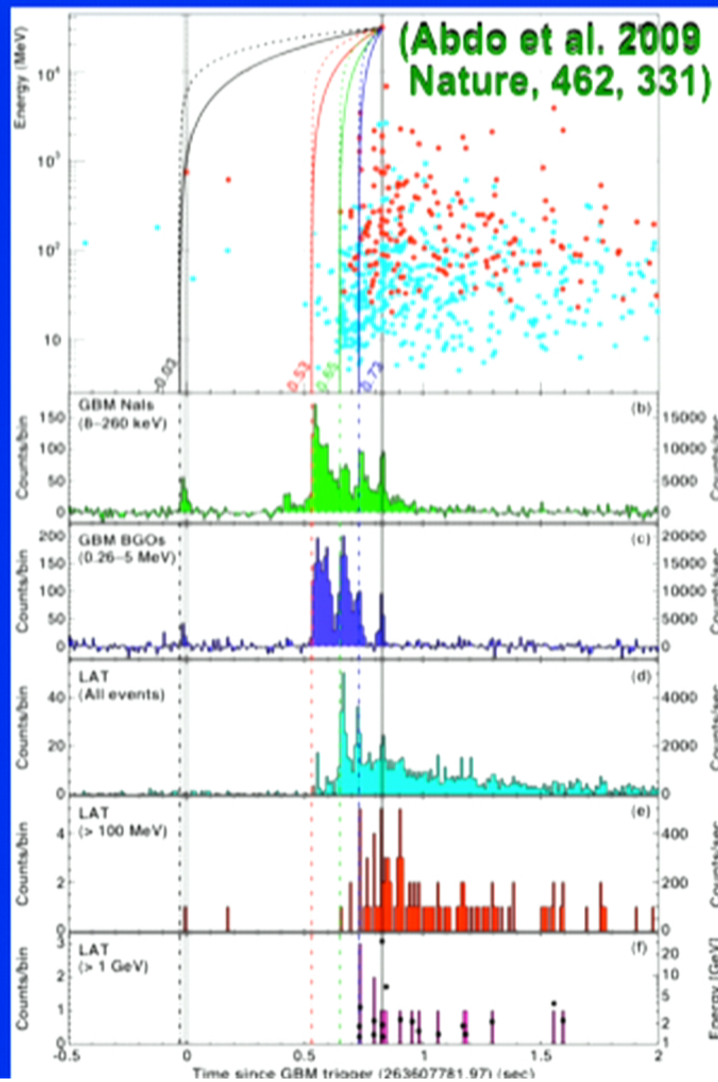


(Abdo et al. 2009, Science, 323, 1688)



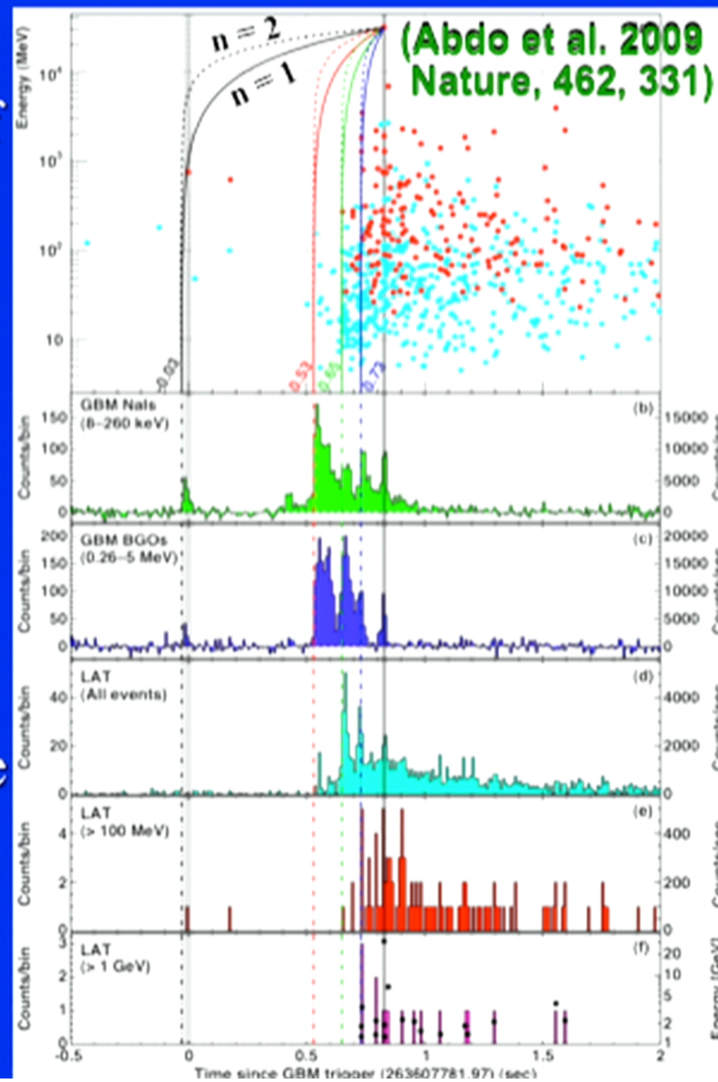
GRB090510: L.I.V

- A short GRB (duration ~1 s)
- Redshift: $z = 0.903 \pm 0.003$
- A ~31 GeV photon arrived at $t_h = 0.829$ s after the trigger
- We carefully verified it is a photon; from the GRB at $>5\sigma$
- We use the 1-σ lower bounds on the measured values of E_h (28 GeV) and z (0.900)
- Intrinsic spectral lags known on timescale of individual pulses: weak effect expected



GRB090510: L.I.V

- Method 1: different choices of t_{start} from the most conservative to the least conservative
- $t_{start} = -0.03$ s precursor onset
  ➔ $\xi_1 = E_{QG,1}/E_{Planck} > 1.19$
- $t_{start} = 0.53$ s onset of main emission episode ➔ $\xi_1 > 3.42$
- For any reasonable emission spectrum a ~31 GeV photon is accompanied by many γ's above 0.1 or 1 GeV that "mark" its t_{em}
- $t_{start} = 0.63$ s, 0.73 s onset of emission above 0.1, 1 GeV
  ➔ $\xi_1 > 5.12$, $\xi_1 > 10.0$

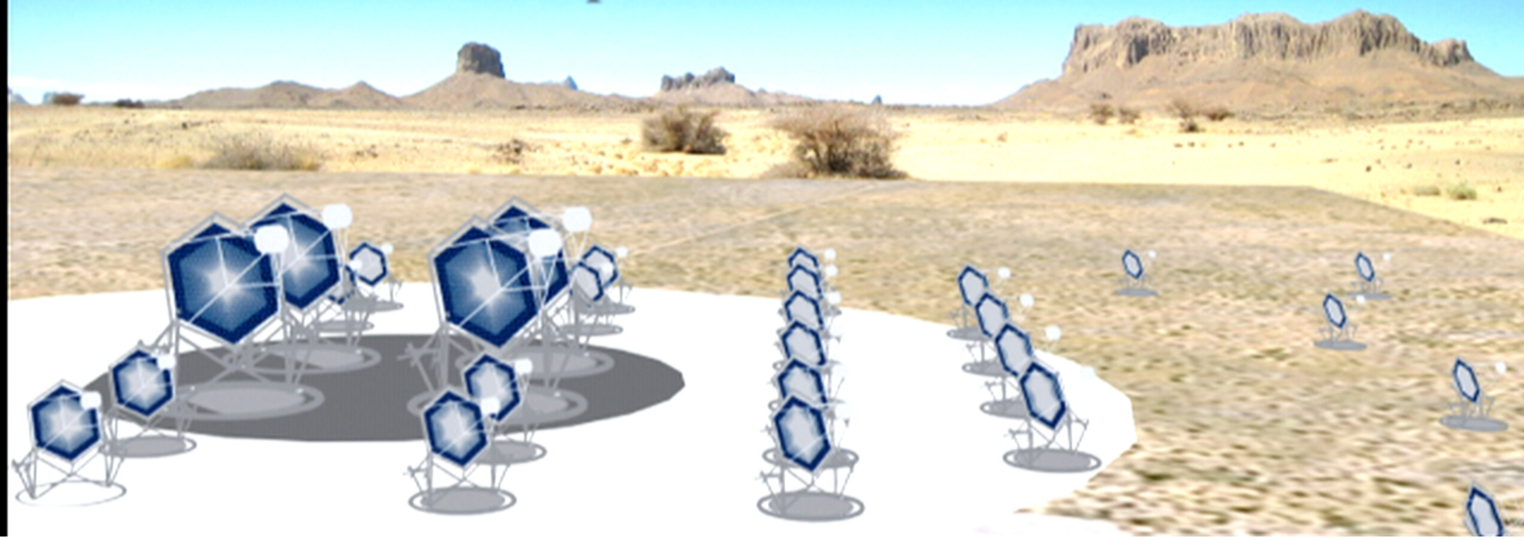


Method 3: DisCan (Scargle et al. 2008)

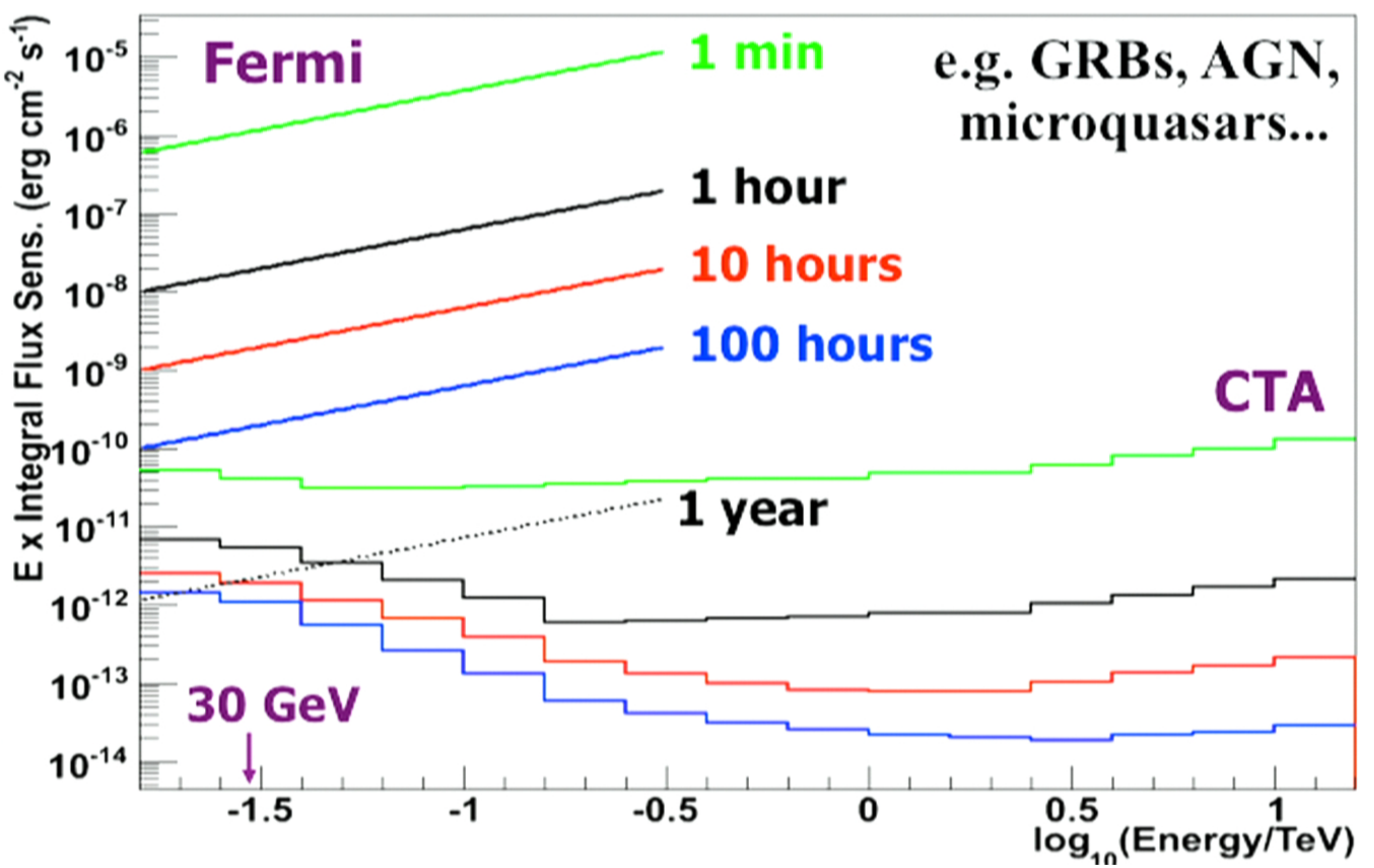
- Based on lack of smearing of the fine time structure (sharp narrow spikes in the lightcurve) due to energy dispersion
- Constrains both possible signs of the effect: $s_n = \pm 1$
- Uses all LAT photons during the brightest emission episode (obs. range 35 MeV – 31 GeV); no binning in time or energy
- Shifts the arrival time of photons according to a trail energy dispersion (linear in our case), finding the coefficient that maximizes a measure of the resulting lightcurve variability
- We found a symmetric upper limit on a linear dispersion: $|\Delta t/\Delta E| < 30$ ms/GeV (99% CL) ➔ $E_{QG,1} > 1.22 E_{Planck}$
- Remains unchanged when using only photons < 1 or 3 GeV (a very robust limit)

Future: Cherenkov Telescope Array (CTA)

- **Energy range: ~20 GeV to ~300 TeV**
  - an **order of magnitude more sensitive** than current instruments around 1 TeV (~150M€ price tag), better angular/energy resolution
  - >1000 members in 27 countries
  - Should become operational around ~2018
- **2 sites (southern + northern hemispheres)**
- **Hundreds of telescopes of 3 different sizes**



A bigger difference for transient sources



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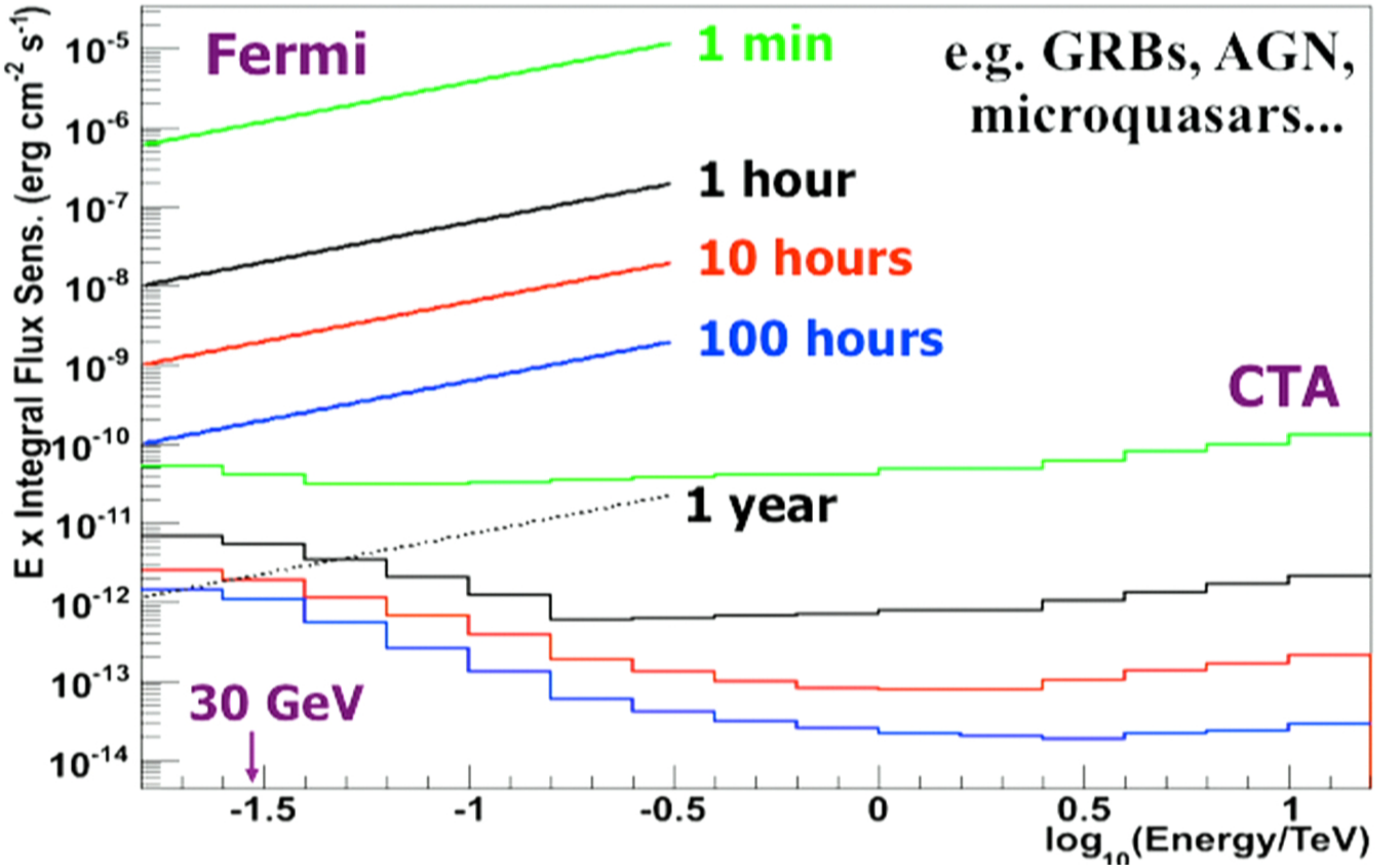

Prospects for LIV studies with CTA GRBs

- Method 1: it may be difficult to do much better
  - Our current limit $|\Delta t/\Delta E| < 30$ ms/GeV would require $E_h > 1$ TeV for a **response time** of 30 s
  - at > 1 TeV intrinsically fewer photons + EBL
- Method 3: might work best
  - Sharp bright spikes up to high energies exist also well within long GRBs
  - $t_{var} \sim 0.1$ s & $E_h \sim 0.1$ TeV could do ~30 times better
- A short GRB in CTA FoV (survey mode) would be great

10 ms, 1 TeV: $>10^3$ times better

Conclusions:

- GRBs are very useful for constraining LIV
- Bright **short** GRBs are more useful than long ones
- A very robust and conservative limit on a linear energy dispersion of either sign: $\mathbf{E_{QG,1} > 1.2 E_{Planck}}$
- Still conservative but somewhat less robust limits: $E_{QG,1}/E_{Planck} > 5.1, 10$ (onset of emission >0.1, 1 GeV)
- "Intuition builder" liberal limit: $E_{QG,1}/E_{planck} > 102$
- Quantum-Gravity Models with linear (n = 1) photon energy dispersion are disfavored

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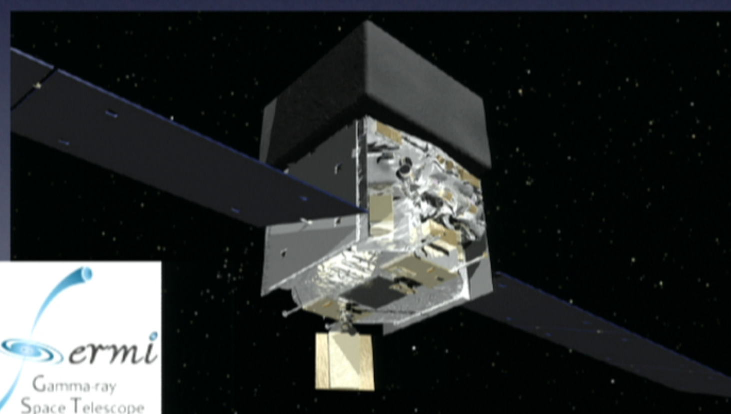
Lorentz Invariance Violation: the latest Fermi results

J. Bolmont
LPNHE - Université Pierre & Marie Curie



Fermi Gamma-ray Space Telescope

LOOPS 2013
July 22-26, 2013 - Waterloo

Contents

- Introduction
  - Brief reminder on the formalism
  - Propagation vs. intrinsic lags
- The latest Fermi results
  - The three methods in use
  - Acounting for intrinsic lags
  - Results
- Conclusions and prospects
  - GRB/AGN complementarity
  - Future developments

In brief: the formalism in use

- QG related effects should appear at $E_{QG} \sim O(E_P = 1.2\times10^{19}$ GeV)
- These effects include deformation or violation of Lorentz Invariance
- For E << E_{QG}, a series expansion is expected to be possible, giving:

$$E^2 \simeq p^2c^2 \times \left[1 - \sum_{n=1}^{\infty} s_\pm \left(\frac{E}{E_{QG}}\right)^n\right]$$

- $s_\pm$ = +1 for a subluminal propagation, -1 for a superluminal propagation
- We consider two photons with energy difference ΔE emitted at the same time and detected with a lag Δt.
  - We want to measure/constrain: $\tau_n \equiv \frac{\Delta t}{\Delta E} \simeq s_\pm \frac{(1+n)}{2H_0} \frac{1}{E^n_{\mathrm{QG}}} \times \kappa_n$

    where $$\kappa_n \equiv \int_0^z \frac{(1+z')^n}{\sqrt{\Omega_\Lambda + \Omega_\mathrm{M}(1+z')^3}} dz'$$
  - Cosmological parameters from WMAP and Hubble

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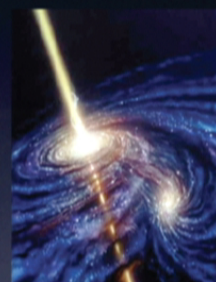
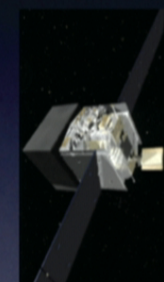
QG Effects vs. Source Effects

Emission processes can introduce a time lag too !

Propagation → LIV Effect





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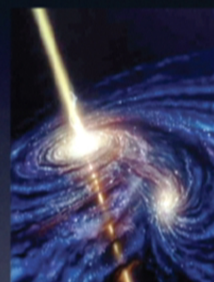

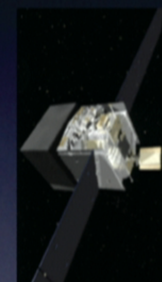

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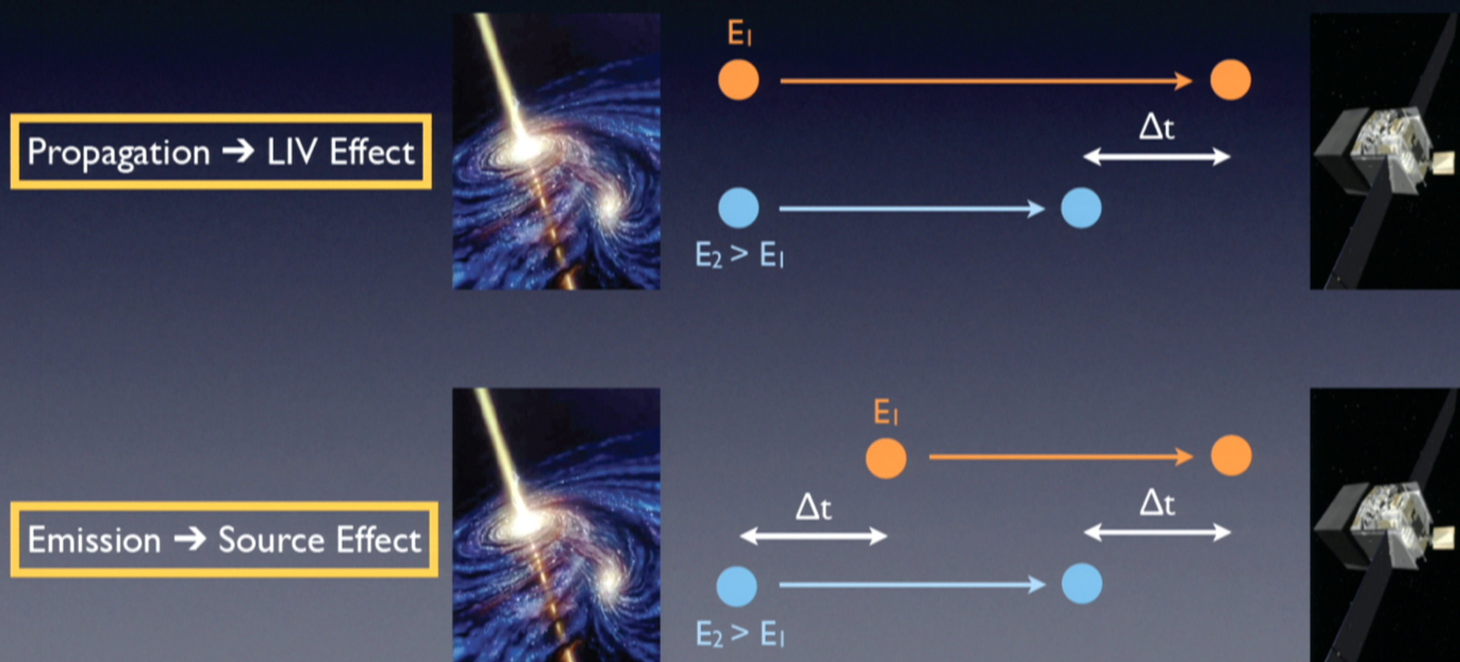


The latest Fermi results

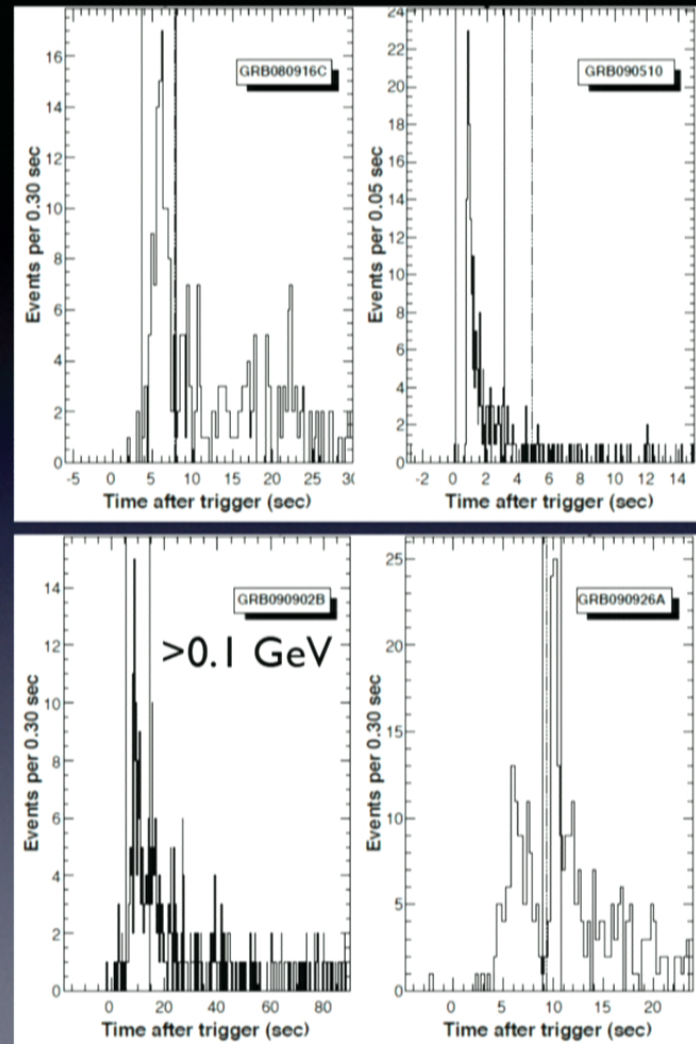
«Constraints on Lorentz Invariance Violation with Fermi-LAT observations of GRBs»

- V. Vasileiou, F. Piron, J. Cohen-Tanugi (LUPM Montpellier)
- A. Jacholkowska, JB, C. Couturier (LPNHE Paris)
- J. Granot (Open Univ. of Israel)
- F. Stecker (NASA GSFC)
- F. Longo (INFN Trieste)

Phys. Rev. D 87, 122001 (2013)
arXiv:1305.1553

Overview

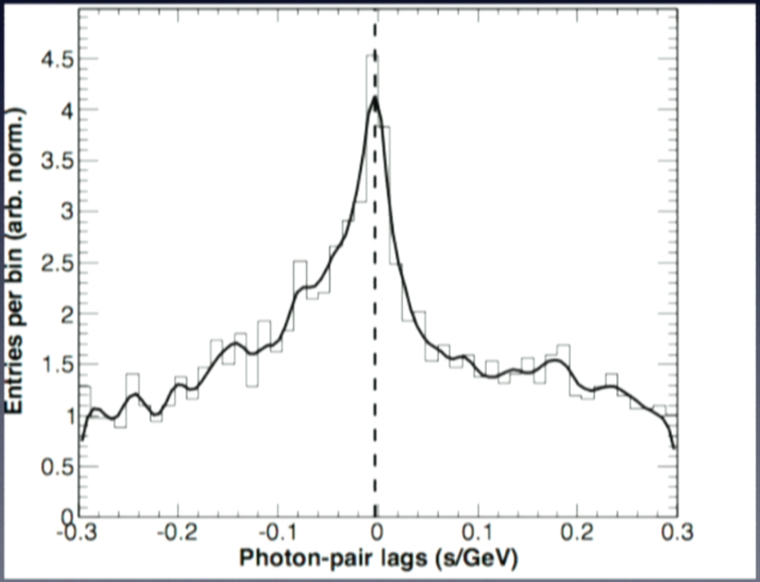
- Use of LAT data
  - 20 MeV - 300 GeV
  - High effective area
  - Low background
  - Good energy recontruction accuracy (~10 % at 10 GeV)
- 4 GRBs are analyzed
  - 090510, 090902B, 090926A, 080916C
  - Known redshifts (from 0.9 up to 4.3)
  - Variability time scale down to tens of ms
  - Maximum energy detected: ~30 GeV
  - ~100 events/GRB above 100 MeV
- 3 analysis methods



Method #1: PairView

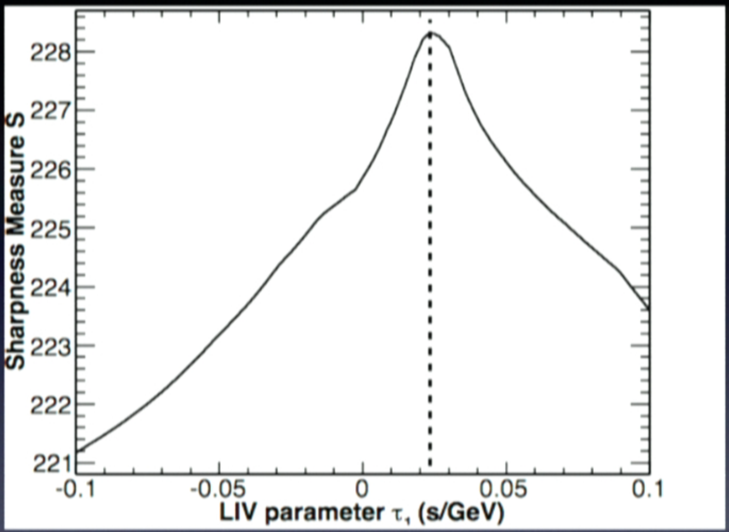
$$l_{i,j} \equiv \frac{t_i - t_j}{E_i^n - E_j^n}$$

- Calculate the spectral lags $l_{i,j}$ between all pairs of photons i and j < i in a dataset
- The distribution of $l_{i,j}$ values peaks approximatly at the true value of τ.
  - ➡ Histogram
- The peak position is determined using a Kernel Density Estimate of the distribution.
  - ➡ Smooth curve
- The KDE peak gives the estimate for τ.
  - ➡ Dashed line



Method #2: Sharpness Maximization Method

- LIV spectral dispersion smears light-curve structure and decrease sharpness
- Apply an inverse dispersion to the data to maximize the sharpness
  - ➡ Smooth curve
- The sharpness peak gives the estimate for τ.
  - ➡ Dashed line
- The sharpness S is defined by the formula on the right, where t'_i is the modified detection time of the i^{th} photon and ρ is a parameter selected using simulations



$$\mathcal{S}(\tau_n) = \sum_{i=1}^{N-\rho} \log\left(\frac{\rho}{t'_{i+\rho} - t'_i}\right)$$

Method #3: likelihood fit

- Study of the correlation between the arrival time and the energy of the photons

  (Biller et al. 99, Martinez & Errando 09, Abramowski et al. 11)

- We use the following form for the probability density function:

$$P(t,E) = N \int_0^\infty A(E_S)\,\Gamma(E_S)\,G(E - E_S, \sigma(E_S))\,F_S(t - \tau E_S)\,dE_S$$

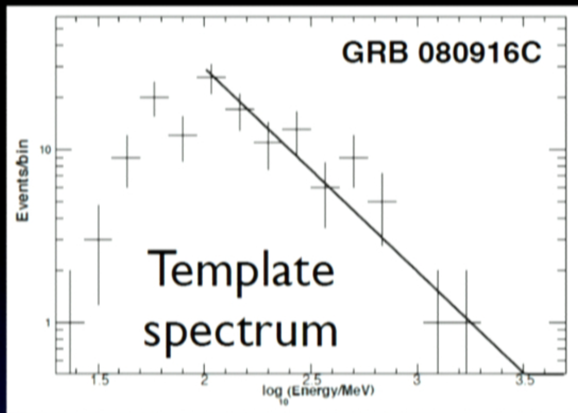
  where $\Gamma(E_S)$ is the emitted spectrum, $G(E\text{-}E_S, \sigma(E_S))$ is the smearing function in energy, $A(E_S)$ is the acceptance of the detector and F_S is the emission time distribution at the source

- The likelihood function is then given by the product

$$L = \prod_i P_i(t,E)$$

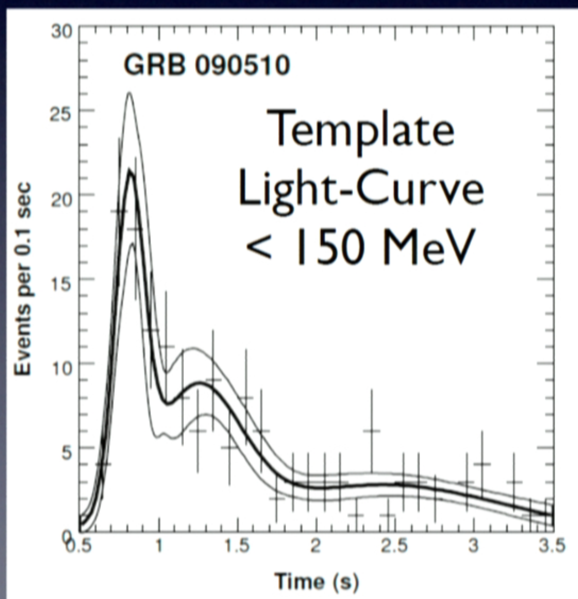
  over all photons in the studied sample

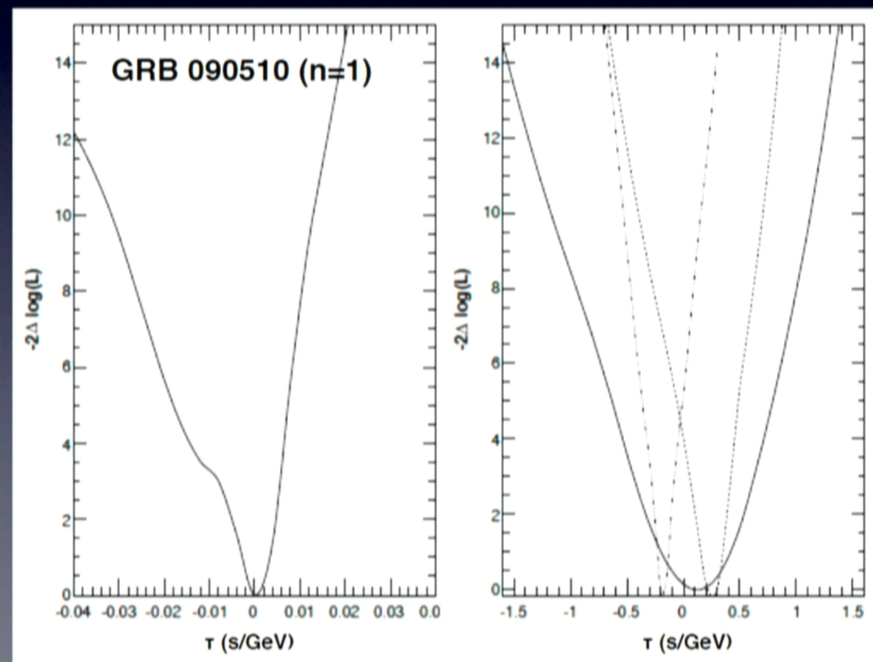
- The maximum of the likelihood gives the best estimate for τ

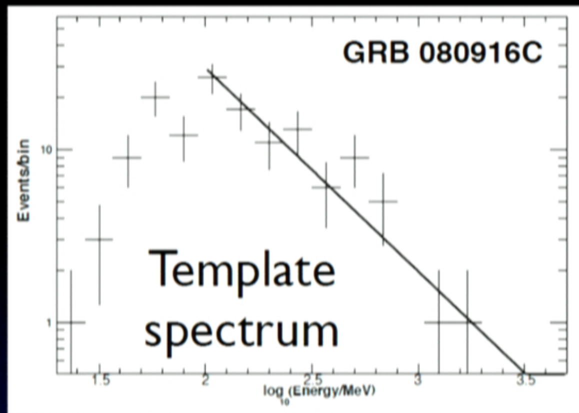


Method #3: Example

The minimum of the curve gives the best estimate of τ: on the right plot, 080916C (full line), 090902B (dotted line) and 090926A (dashed double-dotted line)







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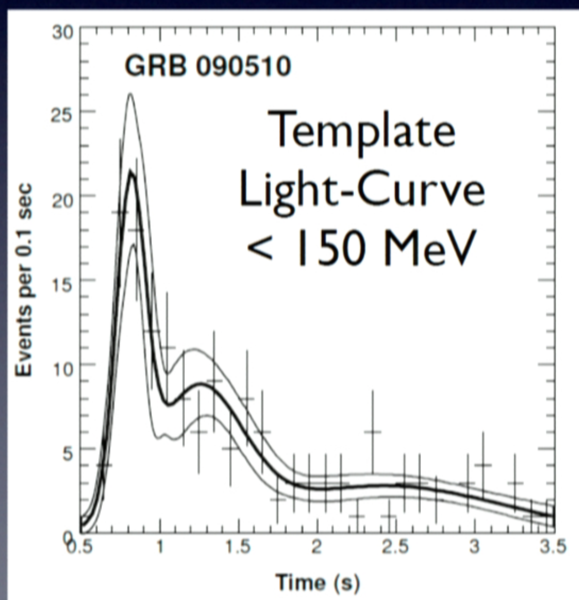

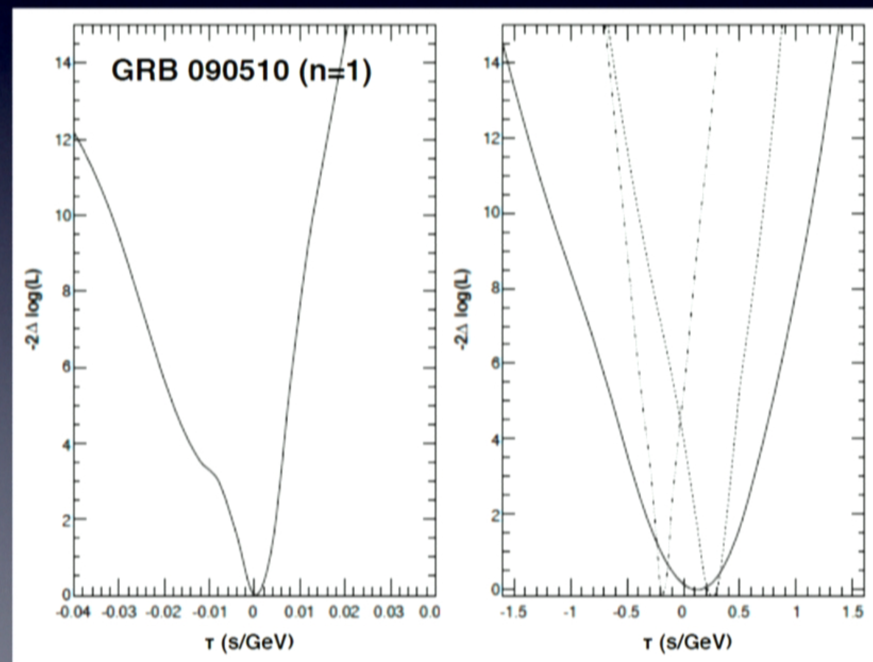

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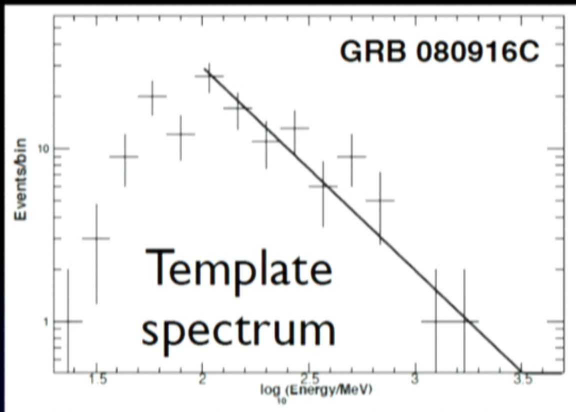
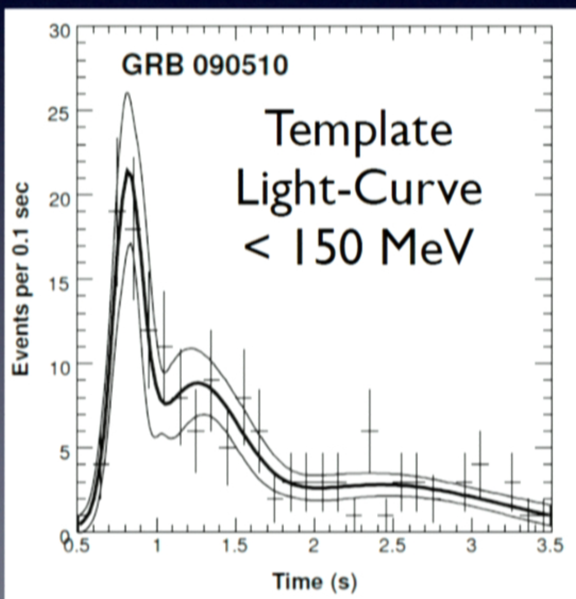
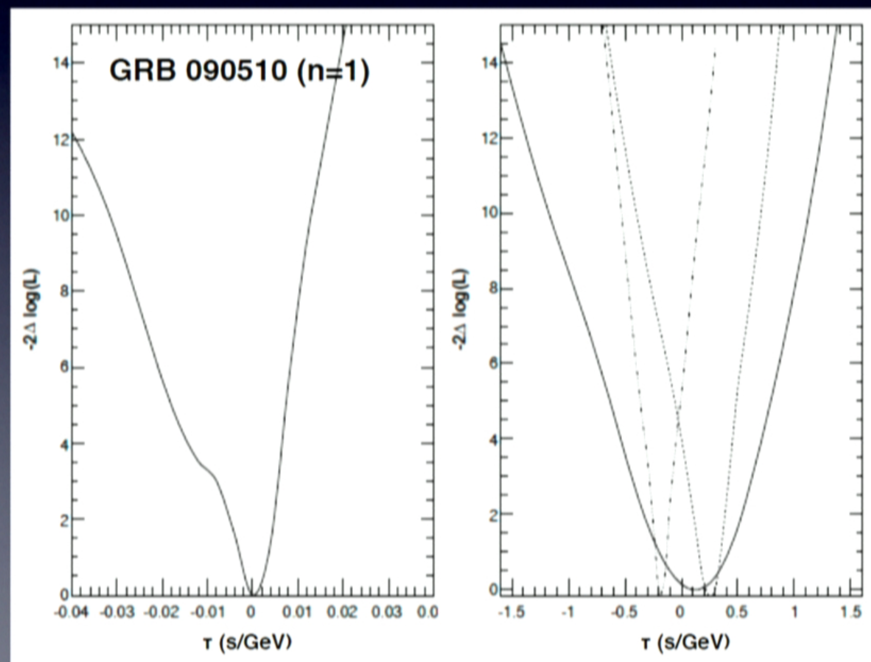
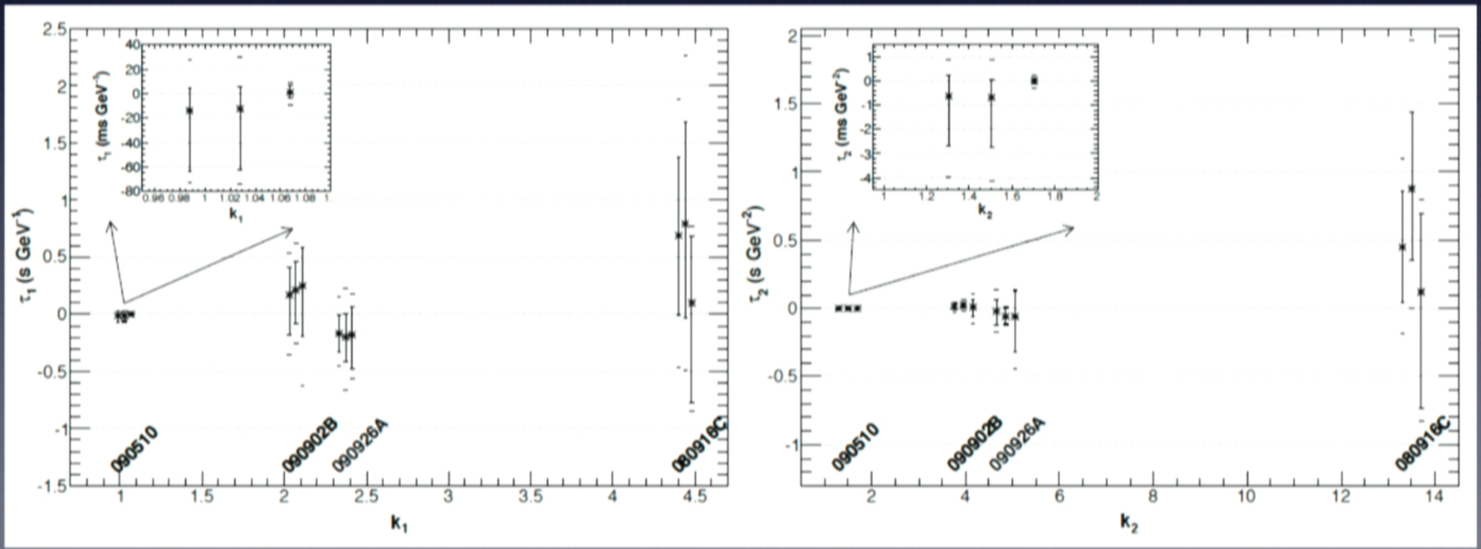






Results

- Three methods → three points for each GRB (PV, SMM, ML)
- Markers → best estimate of τ
- 90% (99%) CL intervals (two-sided)



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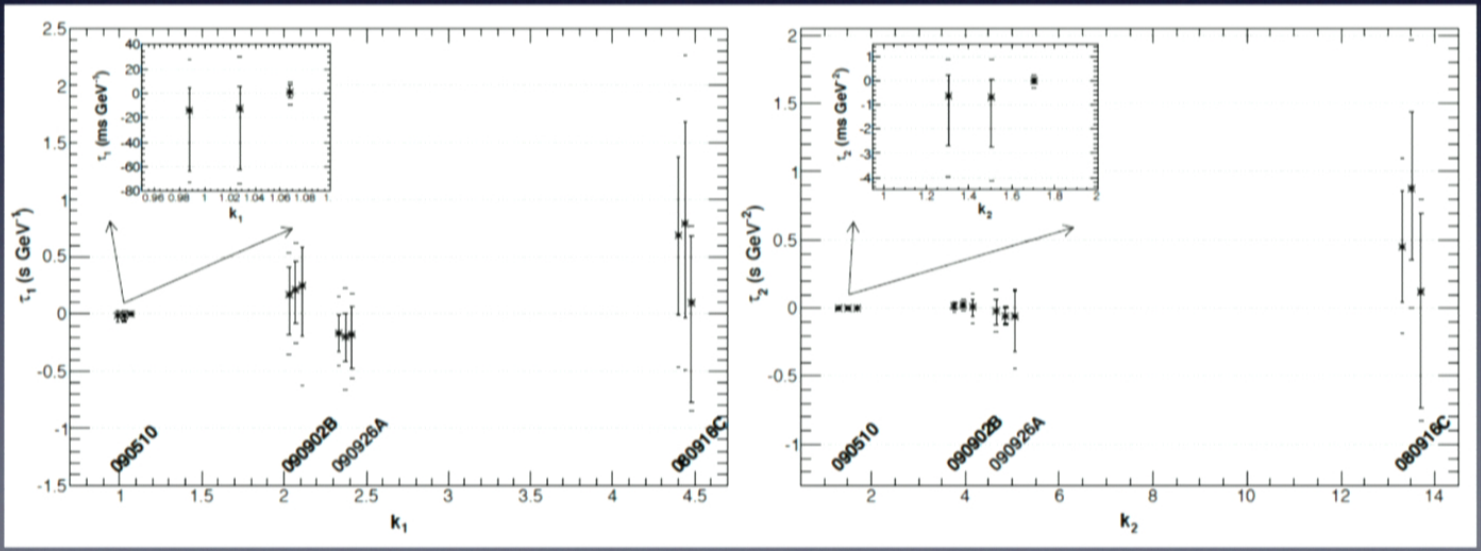

Results

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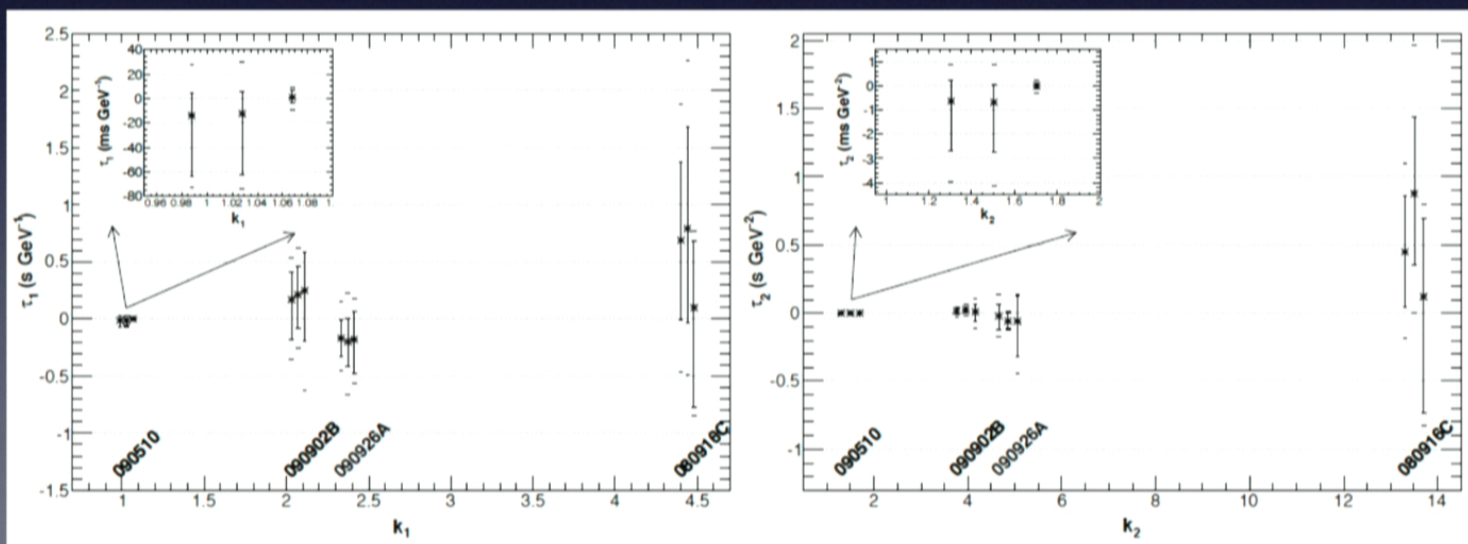
- 90% (99%) CL intervals (two-sided)

All confidence intervals are compatible with 0 dispersion

Constraints with the 3 methods are in good agreement



Accounting for Source-Intrinsic Effects

- It is probable the measured lag has two components:

$$\tau = \tau_{INT} + \tau_{LIV}$$

  where τ_{INT} is the intrinsic dispersion (due to the source) and τ_{LIV} is the LIV-induced dispersion

- There is no good model available to predict the value of τ_{INT}.

  ➡ A conservative modelization of τ_{INT} is used.

- We assume the observations are dominated by source effects
  - The PDF of τ_{INT} is chosen to match τ allowed by the data
    - Average of 0
    - Width matching the width of τ
  - τ_{INT} is modelled to reproduce the allowed range of possibilities for τ

    ➡ Worst case scenario

Most conservative limits on τ_{LIV}

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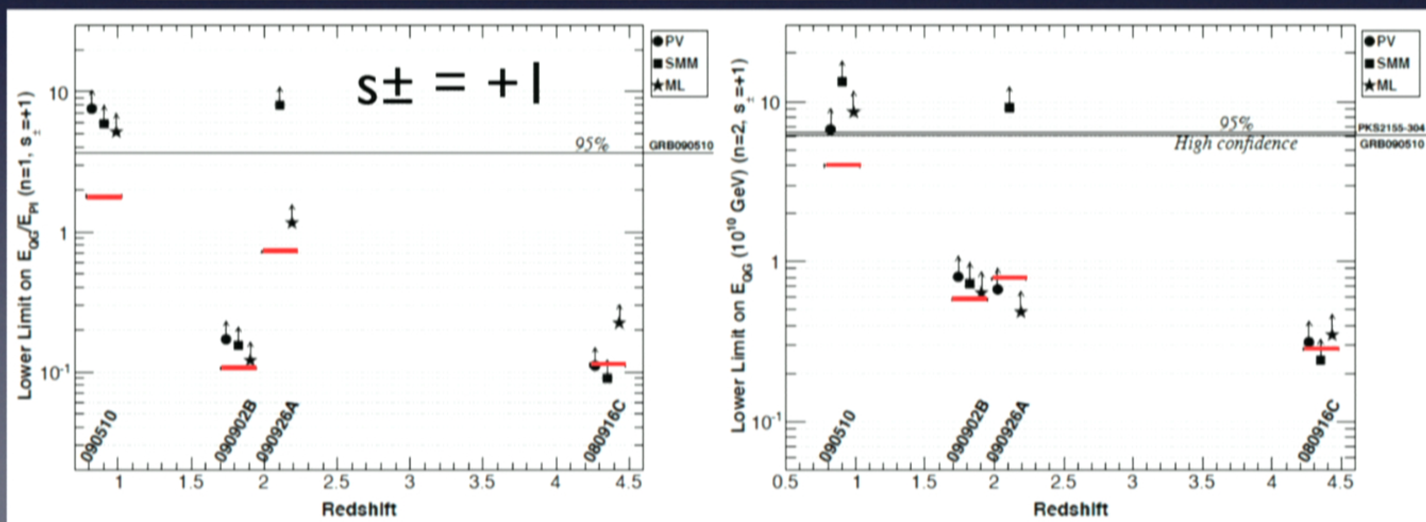
Most conservative limits on τ_{LIV}

95% CL lower limits on E_{QG}

- Left: linear LIV, Right: quadratic LIV
- Horizontal lines: previous published limits
- Bars: average constraint accounting for GRB-intrinsic effects
- Current limits improved by a factor 2-4

➡ $E_{QG} \gtrsim 8\ E_{Pl}$ for n=1
➡ $E_{QG} \gtrsim 1.3\times10^{11}$ GeV for n=2



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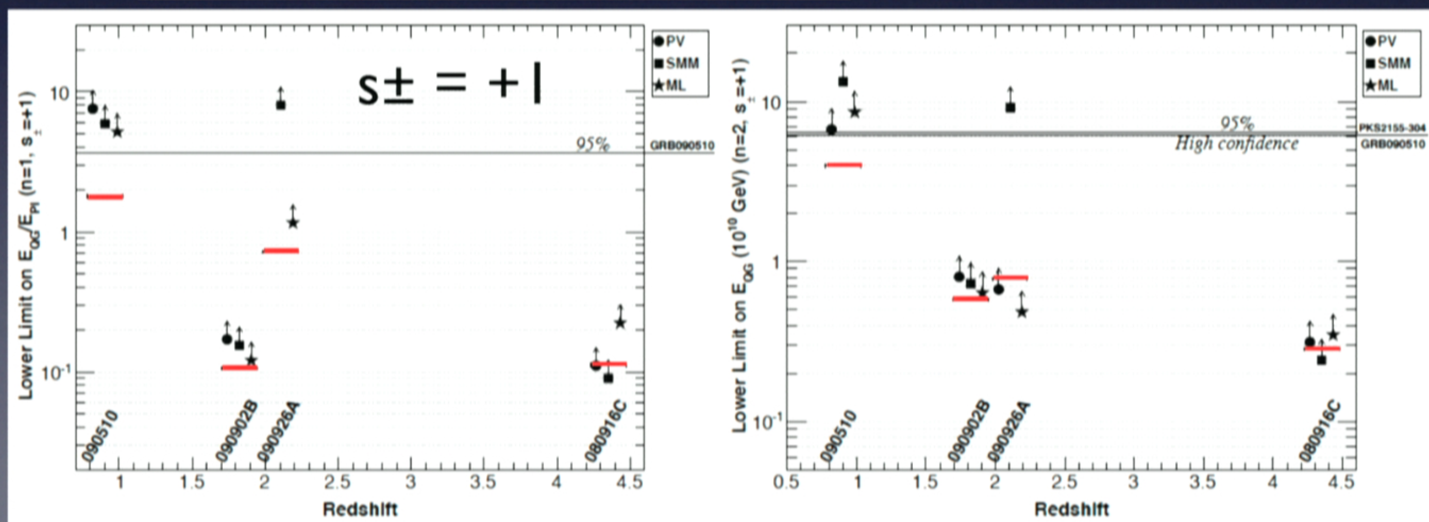
Over the Planck scale for 090510, even accounting for intrinsic effects



GRB/AGN Complementarity

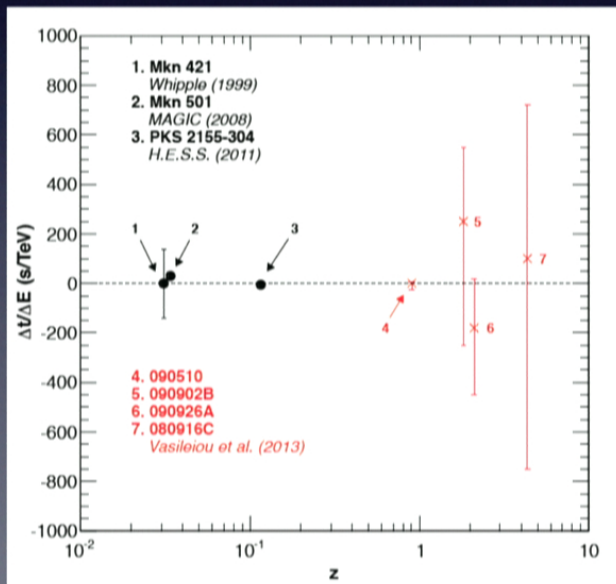
- AGNs → high statistics with ground-based instruments BUT low redshift (EBL) and low statistics with satellites

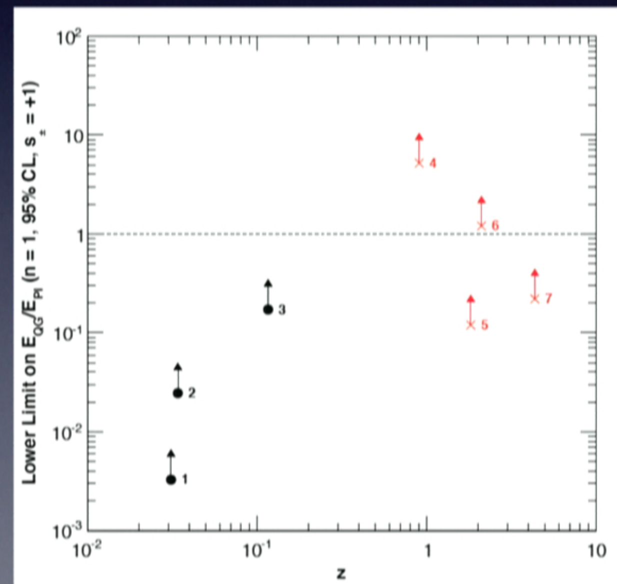
High energies, low distance

- GRBs → high statistics with space instruments BUT lower energies and no detection from the ground

Low energies, large distance

- Comparison between Vasileiou et al. results (ML) and previous results obtained with AGNs





What's next ?

- Linear LIV has reached the physicaly meaningful bound of the Planck scale
- In the future, the effort should be put on constraining the quadratic LIV !
  - ➡ Ground-based detectors and satellites will need to work together to make the energy range as large as possible (GeV - TeV)
  - ➡ Source effects need to be understood
- CTA will help !

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Relative locality
and
k-Poincaré

Giulia Gubitosi

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LOOPS13, Waterloo, CA

k-Poincaré Hopf algebra

dimensionful deformation of Poincaré algebra through an energy scale

$$\kappa \sim E_P$$

- commutators

$$[P, E] = 0, \quad [N, P] = \frac{\kappa}{2}\left(1 - e^{-2E/\kappa}\right) - \frac{1}{2\kappa}P^2, \quad [N, E] = P$$

- coproducts

$$\Delta E = E \otimes \mathrm{Id} + \mathrm{Id} \otimes E, \quad \Delta P = P \otimes \mathrm{Id} + e^{-E/\kappa} \otimes P, \quad \Delta N = N \otimes \mathrm{Id} + e^{-E/\kappa} \otimes N$$

- antipodes

$$S(E) = -E, \quad S(P) = -e^{E/\kappa}P, \quad S(N) = -e^{E/\kappa}N$$

Lukierski 1991

Bicrossproduct structure, where E and P close a Hopf-subalgebra

⟶ algebra of functions over a momentum space

J. Kowalski-Glikman and S. Nowak 2002-2003

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k-Poincaré momentum space - properties

In the basis where the k-Poincaré commutators are trivial

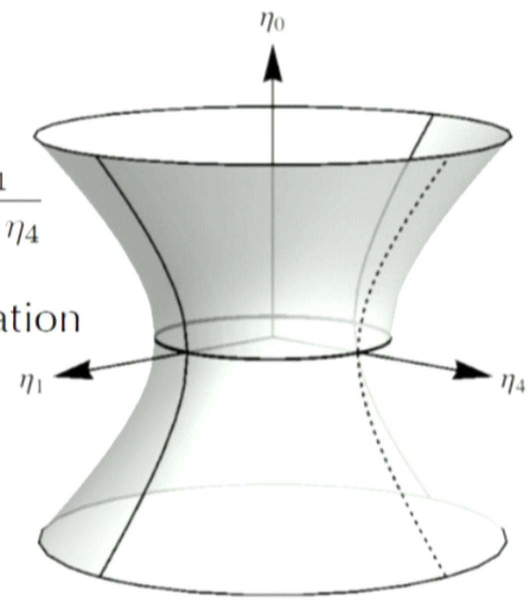
$$\eta_0 \equiv \kappa \sinh\left(\frac{E}{\kappa}\right) + e^{E/\kappa}\frac{P^2}{2\kappa} \qquad \eta_1 \equiv e^{E/\kappa} P$$

we can add a generator

$$\eta_4 \equiv \kappa \cosh\left(E/\kappa\right) - e^{e/\kappa}\frac{P^2}{2\kappa}$$

so that the inverse map is bi-univocal

$$E = \kappa \log\left(\frac{\eta_0 + \eta_4}{\kappa}\right), \quad P = \frac{\kappa \eta_1}{\eta_0 + \eta_4}$$



one finds out that the new generators satisfy the relation (de Sitter - embedding coordinates)

$$\eta_0^2 - \eta_1^2 - \eta_4^2 = -\kappa^2$$

which translates into the metric

$\longrightarrow$ $$ds^2 = dE^2 - e^{2E/\kappa} dp^2$$

Relative locality

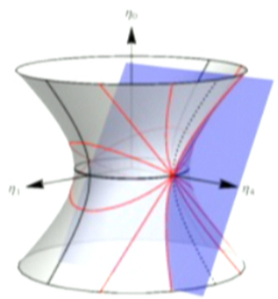
connection between the geometrical properties of a momentum space in terms of kinematical and dynamical properties of particles living on it, and vice-versa

Amelino-Camelia, Freidel, Kowalski-Glikman, Smolin 2011

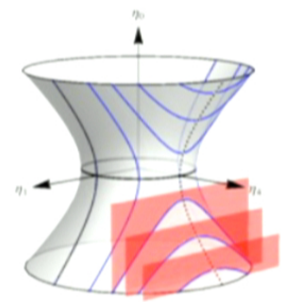
geodesic equation $$\frac{d^2\gamma_\lambda(t)}{dt^2} + A^{\mu\nu}{}_\lambda \frac{d\gamma_\mu(t)}{dt}\frac{d\gamma_\nu(t)}{dt} = 0$$	particle dispersion relation $$d^2(p,\vec{0}) = m^2$$
connection $$\Gamma^{\mu\nu}_\rho(k) = -\frac{\partial}{\partial p_\mu}\frac{\partial}{\partial q_\nu}(p \oplus_k q)_\rho\Big|_{p=q=k}$$	composition law of momenta $$(p \oplus q)_\mu$$
curvature $$R^{\mu\nu\rho}_\sigma(k) = \frac{\partial}{\partial p_{[\mu}}\frac{\partial}{\partial q_{\nu]}}\frac{\partial}{\partial r_\rho}((p \oplus_k q) \oplus_k r - p \oplus_k (q \oplus_k r))_\rho\Big|_{p=q=r=k}$$	associativity of composition law $$(p \oplus q) \oplus k = p \oplus (q \oplus k)$$
torsion $$T^{\mu\nu}_\rho(k) = -\frac{\partial}{\partial p_\mu}\frac{\partial}{\partial q_\nu}(p \oplus_k q - q \oplus_k p)_\rho\Big|_{p=q=k}$$	commutativity of composition law $$p \oplus q = q \oplus p$$

Particle dispersion relation

a particle of mass m lives on the curve of constant geodesic distance from the origin of momentum space



geodesics are given by intersection with planes passing through η_4



constant geodesic curves are given by intersection with planes orthogonal to η_4

in embedding coordinates the constant geodesic distance condition is

$$\eta_4 = \kappa \cosh\left(\frac{d}{\kappa}\right)$$

inverting the relation and going to bicrossproduct coordinates we get

$$m = d(p,0) = \kappa\, \mathrm{arcosh}\left(\cosh\left(E/\kappa\right) - e^{E/\kappa}\frac{P^2}{2\kappa}\right)$$

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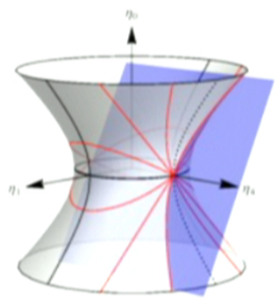

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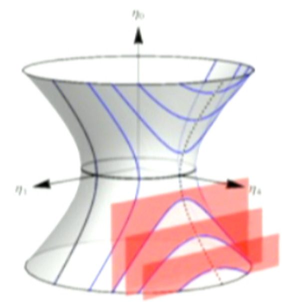

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Lorentz transformations

Composition law becomes covariant upon introducing a '*back-reaction*' of particle momentum on the boost

$$\xi \triangleleft q = 2\mathrm{arcsinh}\left(\frac{e^{-q_0/\kappa}\sinh(\xi/2)}{\sqrt{[\cosh(\xi/2) + \frac{q_1}{\kappa}\sinh(\xi/2)]^2 - e^{-2q_0/\kappa}\sinh^2(\xi/2)}} \right)$$

Majid 2006

$$\longrightarrow \quad \Lambda(\xi, p \oplus q) = \Lambda(\xi, q) \oplus \Lambda(\xi \triangleleft q, p)$$

GG, F. Mercati 2011

- keeps "covariance in form":

$$(q \oplus k)' = q' \oplus k'$$

with $q' = \Lambda(\xi, q)$ and $k' = \Lambda(\xi \triangleleft q, k)$

- keeps "covariance in substance":

$$\Lambda(\xi, q \oplus k) = \Lambda(\xi, \mathcal{P})\big|_{\mathcal{P}=q\oplus k}$$

G. Amelino-Camelia, GG, G. Palmisano 2013

Relativistic compatibility of Lorentz transformations

- on-shell momenta always stay on mass-shell

$$d\left(\Lambda(\xi \triangleleft q, k), 0\right) = d(k, 0)$$

- composition law is the same in every reference frame

- transformation rules are the same for all observers:

if $\Lambda(\xi, q \oplus k) = \Lambda(\xi, q) \oplus \Lambda(\xi \triangleleft q, k) \equiv q' \oplus k'$

$$\longrightarrow \quad \Lambda(-\xi, q' \oplus k') \equiv \Lambda(-\xi, q') \oplus \Lambda(-\xi \triangleleft q', k') = q \oplus k$$

G. Amelino-Camelia, GG, G. Palmisano 2013

Lorentz transformations in 3+1 dimensions

- in 3+1 dimensions also rotation generators play a role
- in k-Poincaré rotation generators have trivial algebra and coalgebra
- however they turn out to be affected by back-reaction, since

$$\Delta N_k = N_k \otimes 1 + e^{-P_0/\kappa} \otimes N_\kappa + \frac{1}{\kappa}\epsilon_{jkl} P_j \otimes R_l$$

$$\longrightarrow \quad \Lambda(\{\vec{\xi},\vec{\theta}\}, q \oplus k) = \Lambda(\{\vec{\xi},\vec{\theta}\}, q) \oplus \Lambda(\{\vec{\xi},\vec{\theta}\} \triangleleft q, k)$$

$$\{\vec{\xi},\vec{\theta}\} \triangleleft q = \{e^{-q_0/\kappa}\vec{\xi}, \vec{\theta} - \frac{\vec{\xi}\times\vec{q}}{\kappa}\}$$

GG, F. Mercati, Class. Quant. Grav. 2013

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GG, F. Mercati, Class. Quant. Grav. 2013

Conclusions

- within the relative locality framework k-Poincaré algebra becomes a coherent physical model for point particles with 'relatively local' interactions

- the model turns out to be compatible with the relativity principle, with a peculiar transformation law for particles participating to an interaction

- rotations are not classical in k-Poincaré framework, as they can be generated by back-reaction of momenta over boost transformations

Novel interpretation of deformed relativistic kinematics

Giovanni Palmisano
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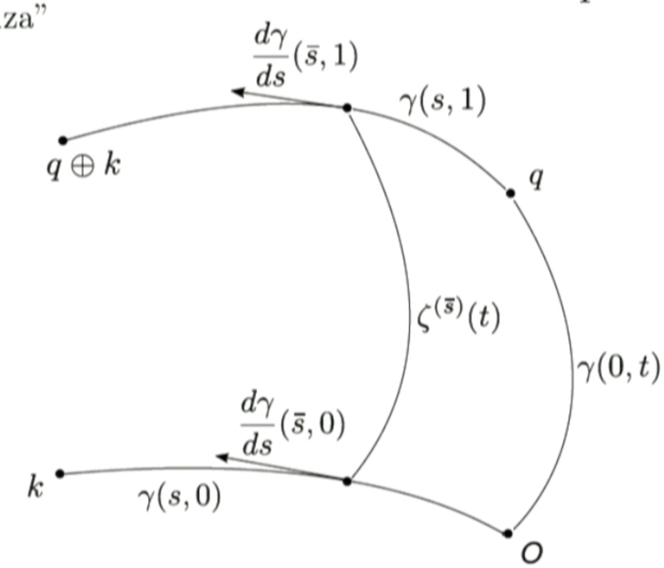
Loops 2013, Waterloo, CA



Giovanni Amelino Camelia, Giulia Gubitosi, GP

Contents

- New interpretation of deformed composition of momenta
- New curved momentum space with commutative and non associative composition law: Proper de Sitter

Standard Geometrical Interpretation

Special Relativity

$$\begin{cases} m^2 = E^2 - p^2 = \eta^{\mu\nu} p_\mu p_\nu \\ p = q + k \end{cases}$$

Curved Momentum Space

$$\begin{cases} m^2 = d_\ell^2 (p) \\ p = q \oplus_\ell k \end{cases}$$

$$d_\ell^2 (p) = \int dt \sqrt{g^{\mu\nu} (\gamma) \dot{\gamma}_\mu \dot{\gamma}_\nu}$$

$$\Gamma_\lambda^{\mu\nu} (p) = - \frac{\partial}{\partial q_\mu} \frac{\partial}{\partial k_\nu} \left(q \oplus_\ell^{[p]} k \right)_\lambda \Big|_{q=k=p}$$

$$\oplus_\ell \to \Gamma$$

$$q \oplus_\ell^{[p]} k = p \oplus_\ell ((\ominus_\ell p \oplus_\ell q) \oplus_\ell (\ominus_\ell p \oplus_\ell k))$$

The Cyclic Composition Law

$\Gamma \longrightarrow \oplus_\ell$

$\gamma(s,t)$ definition

$$\begin{cases} \gamma(s,0) = \gamma^{(k)}(s) \\ \gamma(0,t) = \gamma^{(q)}(t) \end{cases}$$

$$\frac{d}{dt}\frac{d}{ds}\gamma_\lambda(s,t) + \Gamma^{\mu\nu}{}_\lambda(\zeta^{(s)}(t))\frac{d\zeta^{(s)}_\mu(t)}{dt}\frac{d\gamma_\nu(s,t)}{ds} = 0$$

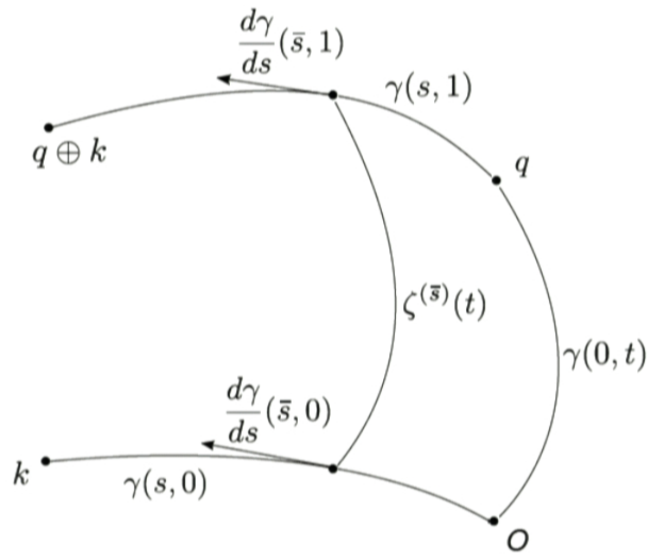
$\oplus_\ell$ definition

$$q \oplus_\ell k = \gamma(1,1)$$

translated composition law

$$q \oplus_\ell^{[p]} k$$

$$O \longrightarrow p$$



$\zeta^{(s)}(t)$ definition

$$\begin{cases} \frac{d^2}{dt^2}\zeta^{(s)}_\lambda(t) + \Gamma^{\mu\nu}{}_\lambda(\zeta^{(s)}(t))\frac{d\zeta^{(s)}_\mu(t)}{dt}\frac{d\zeta^{(s)}_\nu(t)}{dt} = 0 \\ \zeta^{(s)}(0) = \gamma^{(k)}(s) \\ \zeta^{(s)}(1) = \gamma(s,1) \end{cases}$$

Cyclic Composition Law Properties

Second Order Form

$$(q \oplus_\ell k)_\lambda = q_\lambda + k_\lambda - \ell \Gamma_\lambda^{\alpha\beta} q_\alpha k_\beta - \frac{\ell^2}{2} \partial^\rho \Gamma_\lambda^{\alpha\beta} q_\alpha k_\beta (q_\rho + k_\rho) + \frac{\ell^2}{2} \Gamma_\lambda^{\alpha\beta} \Gamma_\alpha^{\gamma\delta} q_\gamma k_\delta k_\beta + \frac{\ell^2}{2} \Gamma_\lambda^{\alpha\beta} \Gamma_\beta^{\gamma\delta} q_\alpha q_\gamma k_\delta$$

- $\Gamma_\lambda^{\mu\nu}(p) = \Gamma_\lambda^{\nu\mu}(p) \Longleftrightarrow q \oplus k = k \oplus q$
- $((q \oplus_\ell k) \oplus_\ell p)_\lambda - (q \oplus_\ell (k \oplus_\ell p))_\lambda = -\frac{\ell^2}{2} \left(F^{\rho\beta\alpha}{}_\lambda + D^\rho T^{\alpha\beta}{}_\lambda \right) k_\alpha p_\beta q_\rho$
- $Cycl_{\oplus_\ell} \{ (\oplus_\ell q \oplus_\ell (k \oplus_\ell q))_\lambda \} = Cycl_{\oplus_\ell} \{ ((\oplus_\ell q \oplus_\ell k) \oplus_\ell q)_\lambda \}$

$$Cycl_{\oplus_\ell} \{ (p \oplus_\ell (k \oplus_\ell q))_\lambda \} = (p \oplus_\ell (k \oplus_\ell q))_\lambda + (q \oplus_\ell (p \oplus_\ell k))_\lambda + (k \oplus_\ell (q \oplus_\ell p))_\lambda$$

Relativistic Compatibility Constraints

Behaviour Under Diffeomorfism

$$f\left(q \oplus_{\Gamma}^{[p]} k\right) = f\left(q\right) \oplus_{\Gamma'}^{[f(p)]} f\left(k\right)$$

Deformed Lorentz Covariance

$$d\left(\tilde{\Lambda}(p), 0\right) = d\left(p, 0\right) \quad \& \quad \tilde{\Lambda}\left(q \oplus_{\Gamma} k\right) = \tilde{\Lambda}(q) \oplus_{\Gamma} \tilde{\Lambda}(k)$$

$$\Uparrow$$

$$\Gamma'^{\mu\rho}{}_{\alpha}\left(p'\right) = \bar{J}^{\rho}_{\gamma} \bar{J}^{\mu}_{\nu} J^{\beta}_{\alpha} \Gamma^{\nu\gamma}{}_{\beta}(p) - \bar{J}^{\rho}_{\gamma} \bar{J}^{\mu}_{\nu} \left(\partial^{\nu} J^{\gamma}_{\alpha}\right) = \Gamma^{\mu\rho}{}_{\alpha}\left(p'\right)$$

depends on the metric only

Levi Civita case

$$\Gamma = A \equiv \text{Levi Civita connection}$$

$$\exists \tilde{\Lambda} : d\left(\tilde{\Lambda}(p), 0\right) = d\left(p, 0\right) \;\Rightarrow\; A' = A \;\Rightarrow\; \tilde{\Lambda}\left(q \oplus_{A} k\right) = \tilde{\Lambda}\left(q\right) \oplus_{A} \tilde{\Lambda}\left(k\right)$$

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- Equivalent for κ-dS and 1^{th} order
- Is different in general

1+1 D

General Setting

$$(q \oplus_\ell k)_\lambda = q_\lambda + k_\lambda + \ell X_\lambda^{\alpha\beta} q_\alpha k_\beta + \frac{\ell^2}{2} Y_\lambda^{\alpha\beta\gamma} q_\alpha q_\beta k_\gamma + \frac{\ell^2}{2} Z_\lambda^{\alpha\beta\gamma} q_\alpha k_\beta k_\gamma$$

8 12 12

$$\Gamma_\lambda^{\mu\nu}(p) = \Gamma_\lambda^{\mu\nu}(0) + p_\theta \partial^\theta \Gamma_\lambda^{\mu\nu}(p)\Big|_{p=0}$$

8 16

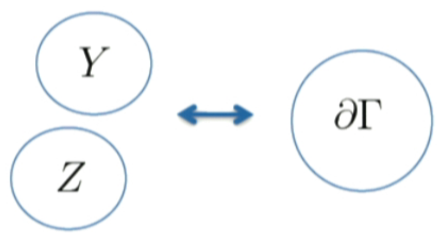
First Order Map

$$\Gamma_\lambda^{\mu\nu}(0) = -\ell X_\lambda^{\mu\nu}$$

→

$$Y, Z \leftrightarrow \partial\Gamma(0)$$

24 16



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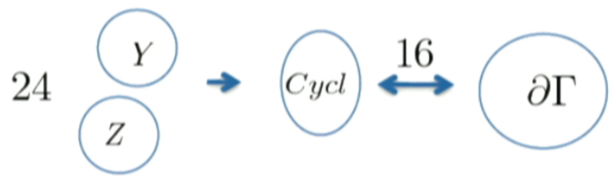
Cyclic Interpretation

- 16 dimensional $\quad Cycl_{\oplus_\ell}\{(\ominus_\ell q \oplus_\ell (k \oplus_\ell q))_\lambda\} = Cycl_{\oplus_\ell}\{((\ominus_\ell q \oplus_\ell k) \oplus_\ell q)_\lambda\}$



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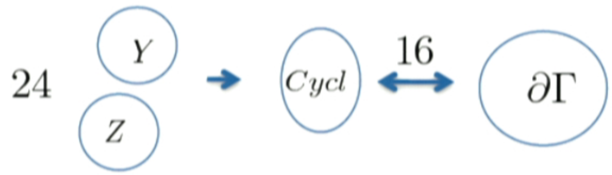
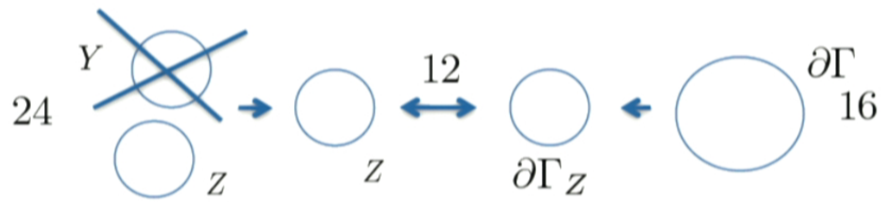


Standard Interpretation

$$\Gamma_\lambda^{\mu\nu}(p) = -\frac{\partial}{\partial q_\mu}\frac{\partial}{\partial k_\nu}\left(q \oplus_\ell^{[p]} k\right)_\lambda \Big|_{q=k=p} \longrightarrow \partial^\theta \Gamma_\lambda^{\mu\nu}(0) p_\theta = -Z_\lambda^{\alpha\mu\nu} p_\alpha + \Gamma_\lambda^{\mu\beta}\Gamma_\beta^{\gamma\nu} p_\gamma + \Gamma_\lambda^{\alpha\nu}\Gamma_\alpha^{\gamma\mu} p_\gamma - \Gamma_\lambda^{\alpha\beta} p_\alpha \Gamma_\beta^{\mu\nu}$$

- 12 dimensional $Y \to 0$ & $\partial^\theta \Gamma_\lambda^{[\mu\nu]} = \Gamma_\lambda^{[\mu\beta}\Gamma_\beta^{\theta\nu]} + \Gamma_\lambda^{\alpha[\nu}\Gamma_\alpha^{\theta\mu]} - \Gamma_\lambda^{\theta\beta}\Gamma_\beta^{[\mu\nu]}$



Standard VS Cyclic

Standard	Cyclic
12 dimensional map	16 (maximal) dimensional map
$\Gamma^{\mu\nu}_{\lambda}(p) = \Gamma^{\nu\mu}_{\lambda}(p) \nrightarrow q \oplus k = k \oplus q$	$\Gamma^{\mu\nu}_{\lambda} = \Gamma^{\nu\mu}_{\lambda} \Longleftrightarrow q \oplus k = k \oplus q$
$\Delta_\lambda = \frac{\ell^2}{2} F^{\rho\beta\alpha}{}_{\lambda} k_\alpha p_\beta q_\rho$	$\Delta_\lambda = \frac{\ell^2}{2}\left(F^{\rho\beta\alpha}{}_{\lambda} + D^{\rho} T^{\alpha\beta}{}_{\lambda}\right) k_\alpha p_\beta q_\rho$
?	$\Gamma' = \Gamma \Longrightarrow \tilde{\Lambda}(q \oplus_\ell k) = \tilde{\Lambda}(q) \oplus_\ell \tilde{\Lambda}(k)$

$$\Delta_\lambda = ((q \oplus_\ell k) \oplus_\ell p)_\lambda - (q \oplus_\ell (k \oplus_\ell p))_\lambda$$

De Sitter Proper Momentum Space

de Sitter metric

$$g^{\mu\nu}(p) = \begin{pmatrix} 1 & 0 \\ 0 & -e^{2\ell p_0} \end{pmatrix}$$

Levi Civita connection

$$A^{\lambda\mu}{}_{\nu} = \ell \left(\delta_0^\lambda \delta_1^\mu + \delta_0^\mu \delta_1^\lambda\right) \delta_\nu^1 + \ell \delta_1^\lambda \delta_1^\mu \delta_\nu^0 e^{\ell p_0}$$

On Shell Relation

$$m = d_\ell\,(p,0) = \frac{1}{\ell}\text{Arccosh}[\text{Cosh}[\ell p_0] - \frac{\ell^2}{2} p_1^2 e^{\ell p_0}]$$

2^{th} order composition law

$$\begin{cases} (q \oplus_\ell k)_0 = q_0 + k_0 - \ell q_1 k_1 + \frac{\ell^2}{2}\left[-q_1 k_1 (q_0 + k_0) + q_0 k_1^2 + q_1^2 k_0\right] \\ (q \oplus_\ell k)_1 = q_1 + k_1 - \ell\,(q_0 k_1 + q_1 k_0) + \frac{\ell^2}{2}\left[(q_0 k_1 + q_1 k_0)(q_0 + k_0) + q_1 k_1^2 + q_1^2 k_1\right] \end{cases}$$

commutative, non associative, relativistic

Standard VS Cyclic

Standard	Cyclic
12 dimensional map	16 (maximal) dimensional map
$\Gamma^{\mu\nu}_{\lambda}(p) = \Gamma^{\nu\mu}_{\lambda}(p) \nrightarrow q \oplus k = k \oplus q$	$\Gamma^{\mu\nu}_{\lambda} = \Gamma^{\nu\mu}_{\lambda} \Longleftrightarrow q \oplus k = k \oplus q$
$\Delta_\lambda = \frac{\ell^2}{2} F^{\rho\beta\alpha}{}_{\lambda} k_\alpha p_\beta q_\rho$	$\Delta_\lambda = \frac{\ell^2}{2} \left(F^{\rho\beta\alpha}{}_{\lambda} + D^{\rho} T^{\alpha\beta}{}_{\lambda} \right) k_\alpha p_\beta q_\rho$
?	$\Gamma' = \Gamma \Longrightarrow \tilde{\Lambda}(q \oplus_\ell k) = \tilde{\Lambda}(q) \oplus_\ell \tilde{\Lambda}(k)$

$$\Delta_\lambda = ((q \oplus_\ell k) \oplus_\ell p)_\lambda - (q \oplus_\ell (k \oplus_\ell p))_\lambda$$

De Sitter Proper Momentum Space

Commutativity

$$q \oplus_\ell k = k \oplus_\ell q$$

Curvature

$$F^{\rho\beta\alpha}{}_\lambda = 2\ell^2 \delta_0^{[\rho}\delta_1^{\beta]} \left(e^{2\ell p_0}\delta_1^\alpha \delta_\lambda^0 + \delta_\lambda^1 \delta_0^\alpha\right)$$

(Non) Associativity

$$((q \oplus_\ell k) \oplus_\ell p)_\lambda - (q \oplus_\ell (k \oplus_\ell p))_\lambda = \frac{\ell^2}{2}\left(\delta_\lambda^0 e^{2\ell p_0} k_1 + \delta_\lambda^1 k_0\right)(p_0 q_1 - p_1 q_0)$$

Relativistic Compatibility

$$d_\ell\left(\tilde\Lambda(p), 0\right) = d_\ell(p,0) \quad \Rightarrow \quad \begin{cases} \tilde\Lambda_0(p) = p_0 + \xi p_1 \\ \tilde\Lambda_1(p) = p_1 + \xi \frac{1}{2\ell}\left(1 - e^{-2\ell p_0}\right) - \xi \frac{\ell}{2} p_1^2 \end{cases}$$

$$\Downarrow$$

$$A' = A \quad \Rightarrow \quad \tilde\Lambda(q \oplus_A k) = \tilde\Lambda(q) \oplus_A \tilde\Lambda(k)$$

commutative, non associative, relativistic

De Sitter Proper Momentum Space

de Sitter metric

$$g^{\mu\nu}(p) = \begin{pmatrix} 1 & 0 \\ 0 & -e^{2\ell p_0} \end{pmatrix}$$

Levi Civita connection

$$A^{\lambda\mu}{}_{\nu} = \ell \left(\delta_0^\lambda \delta_1^\mu + \delta_0^\mu \delta_1^\lambda\right) \delta_\nu^1 + \ell \delta_1^\lambda \delta_1^\mu \delta_\nu^0 e^{\ell p_0}$$

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commutative, non associative, relativistic

Outlooks

- Find the exact inverse map $\oplus_\ell \rightarrow \Gamma$
- Compute Physical Predictions
- Find the finite form of the cyclic identity
- What non associative algebra is the one defined by the cyclic identity?

$$Cycl_{\oplus_\ell}\left\{(\ominus_\ell q \oplus_\ell (k \oplus_\ell q))_\lambda\right\} = Cycl_{\oplus_\ell}\left\{((\ominus_\ell q \oplus_\ell k) \oplus_\ell q)_\lambda\right\}$$

Is Relative Locality causal?

Lin-Qing Chen

Perimeter Institute for Theoretical Physics

Warm thanks to G.Amelino-Camelia, A.Banburski, L.Freidel & L.Smolin

Loops 13 July 23 2013

13年7月23日星期二

Outline

- Relative Locality and Kappa-Poincare momentum space in a nutshell
- Causal loop solutions and x-dependence
- Define orientability of loop processes, and prove

Causal $\Leftarrow$ The loop is orientable $\Leftrightarrow$ X-independent

$\Updownarrow$

Global momenta conserved

13年7月23日星期二

Relative Locality is a proposal for describing the Planck scale modifications to relativistic dynamics resulting from non-trivial momentum space geometry

- $\hbar \to 0 \qquad G_N \to 0$ BUT $M_p = \sqrt{\hbar c / G_N}$ fixed
- Taking momentum space $(\mathcal{P}, g^{ab}, \Gamma^{ab}_c)$ as primary, and formulating classical dynamics on the phase space $T^*(\mathcal{P})$
- There is no universal spacetime. Spacetimes are cotangent spaces attach on each momentum point $x_I \subset T^*_{p^I}$, canonical conjugate variable $\{x^a_I, p^J_b\} = \delta^a_b \delta^J_I$
- Nonlinear addition of momenta: $(\ominus p) \oplus p = 0$
  The form of vertex reflects "microscopic causal orders": $\mathcal{K}_a = (p \oplus q) \ominus k \equiv 0$
- Momentum space $\mathcal{P}$: Connection $\frac{\partial}{\partial p_a} \frac{\partial}{\partial q_b} (p \oplus_k q)_c \big|_{p=q=k} = -\Gamma^{ab}_c(k)$
  curvature: lack of associativity
  torsion: lack of commutativity of the momenta's combination

G.Amelino-Camelia, L.Freidel, J.Kowalski-Glikman and L.Smolin, PRD, 2011

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- The dynamics of particles are defined by the action:

$$S = \sum_J S^J_{free} + \sum_i S^i_{int}$$

$$= \sum_J \int_{s_i}^{s_i'} ds\,(x^a_J \dot{p}^J_a + \mathcal{N}_J \mathcal{C}^J(p^J)) + \sum \mathcal{K}^i_a(p^J(s_i)) z^a_i$$

mass shell — momenta conservation on vertex

$$\mathcal{C}^J(p) \equiv \mathcal{D}^2(p^J) - m_J^2$$

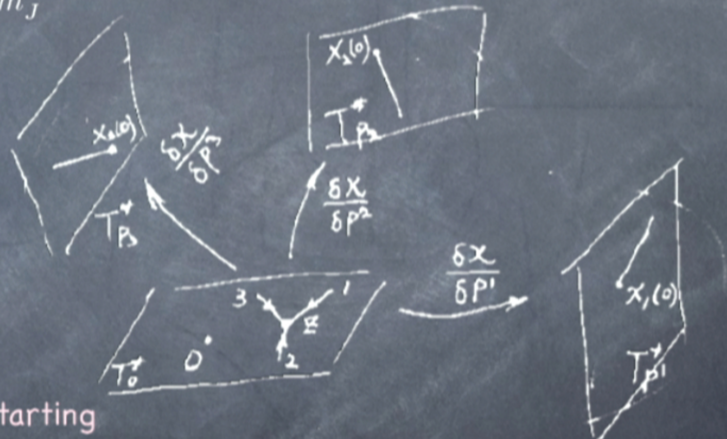
- e.o.m

$$u^a_J \equiv \dot{x}^a_J = \mathcal{N}_J \frac{\partial \mathcal{C}(k^J)}{\partial k^J_a}$$

$$x^a_J(s_i) = \pm z^b_i \frac{\partial \mathcal{K}^i_b}{\partial k^J_a}$$

$$\frac{\partial \mathcal{K}_b}{\partial k_a} : T^*_0 \to T^*_k$$

How an interaction connecting the end/starting points of different particles' worldlines



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13年7月23日星期二

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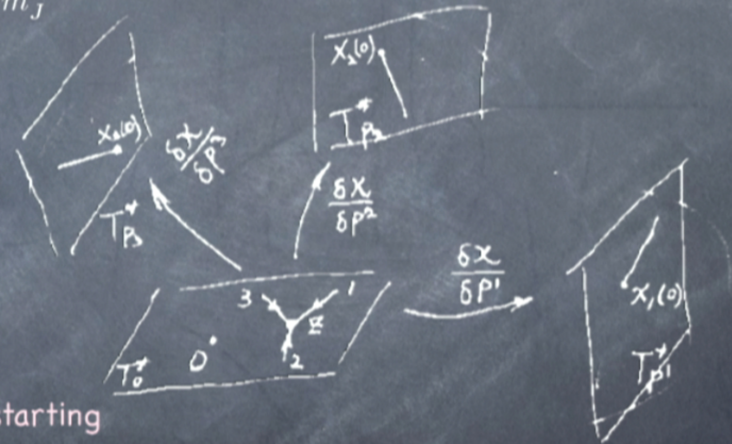

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Kappa Poincare Momentum Space

- Kappa-Poincare Hopf algebra, coming from a dimensionful deformation of the Poincare group, can describe a momentum space with de Sitter metric, torsion and nonmetricity.
[G.Gubitosi & F. Mercati 2011, G.Amelino-Camelia, M.Arzano, J.Kowalski-Glikman, G.Rosati & G.Trevisan 2011]

- Line element of the momentum space in comoving coordinates:
$$ds^2 = dp_0^2 - e^{2p_0/\kappa}\delta^{ij}dp_i dp_j \quad i,j = 1,2,3$$

- Mass-shell condition $m(p) = \kappa Arccosh(\cosh(p_0/\kappa) - e^{p_0/\kappa}|\vec{p}|^2/2\kappa^2)$

- Momenta addition $(p \oplus q)_0 = p_0 + q_0 \quad (p \oplus q)_i = p_i + e^{-p_0/\kappa}q_i$
$(p \ominus q)_0 = p_0 - q_0 \quad (p \ominus q)_i = p_i - e^{q_0 - p_0/\kappa}q_i$

- For notation convenience, define
$$\frac{\partial(p\oplus q)}{\partial q} := U^q_{p\oplus q}, \quad \frac{\partial(p\oplus q)}{\partial p} := V^p_{p\oplus q} \quad \frac{\partial(\ominus p)}{\partial p} := I_p$$

- Important properties

$(q \oplus p) \ominus p = q \longrightarrow U^0_q \cdot V^p_0 = V^{q\oplus p}_q U^p_{q\oplus p} = -U^{\ominus p}_q I_p$ Right inverse property

$(\ominus p) \oplus (p \oplus q) = q \longrightarrow V^0_p \cdot U^q_0 = U^{p\oplus q}_q V^p_{p\oplus q} = -V^{\ominus p}_q I_p$ Left inverse property

$U^p_k \cdot U^k_q = U^k_q \cdot U^p_k = U^p_q, \quad V^p_k \cdot V^k_q = V^k_q \cdot V^p_k = V^p_q$ Chain rule

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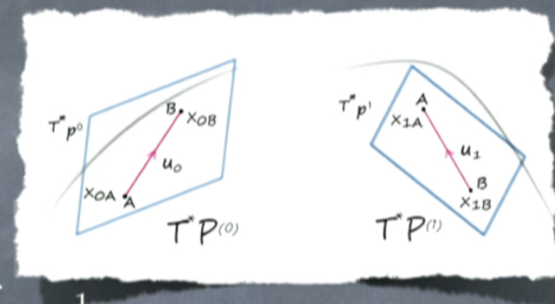
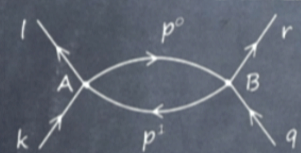
Causal loops



- The simplest case (two collisions) $A \prec B, B \prec A$

Particle 0 with momentum p^0 is created from event A and then collides with another particle at event B.
The twist is now to consider particle 1 with momentum p^1 created at event B and then colliding with another particle at event A, which creates the particle 0.

- Equations $(\mathcal{M}_A - \mathcal{M}_B)^\nu_\mu x^\mu_{0A} = \tau_0 (\mathcal{M}_B)^\nu_\mu u^\mu_0 + \tau_1 u^\nu_1, \ \tau_0, \tau_1 \in \mathbb{R}_+$



$$(\mathcal{M}_A)^\nu_\mu := (\partial \mathcal{K}^A_\alpha / \partial p^1_\nu) \cdot (-\partial \mathcal{K}^A_\alpha / \partial p^0_\mu)^{-1}$$

- In the limit of Special Relativity

$$\tau_0 u^a_0 + \tau_1 u^a_1 = 0$$

- Invariant under momentum space diffeomorphism

- Straightforward to generalize to loops that have more events $A \prec B, B \prec ...N, N \prec A$

$$(\mathcal{M}_A - \mathcal{M}_B \mathcal{M}_C ... \mathcal{M}_n)^\nu_\mu x^\mu_{1A} = \tau_1 \mathcal{M}_B ... \mathcal{M}_n u^\nu_1 +$$
$$+ \tau_2 \mathcal{M}_C ... \mathcal{M}_n u^\nu_2 + ... + \tau_{n-1} \mathcal{M}_n u^\nu_{n-1} + \tau_n u^\nu_n$$

L.Q.Chen ArXiv:1212.5233 [gr-qc]

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A lot of solutions

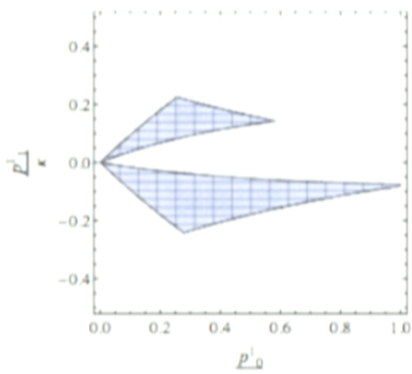
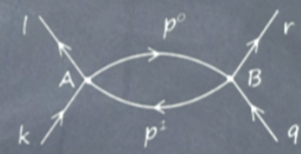


FIG.: The region of (p_0^1, p_1^1) that possiblly leads to a causal loop by events A & B when $p_0^0 = p_0^1, p_1^0 = -p_1^1, r_0 = 1.1k_0 = 0.88\kappa, k_1 = -r_1 = 0.1\kappa$.



$$\mathcal{K}_A = (k \oplus p^1) \ominus (p^0 \oplus l) \equiv 0$$

$$\mathcal{K}_B = (p^0 \oplus q) \ominus (r \oplus p^1) \equiv 0$$

- x-dependence? $$x_{0A}^1 \approx \frac{(p_0^1 p_1^0 - p_0^0 p_1^1)\tau_0 \kappa}{[p_0^1(k_0 - r_0) + p_1^1(r_1 - k_1)]m}$$

- QFT in Curved Spacetime: unitarity fails for interacting fields in Closed Timelike Curve and the subjective probabilities of events can be different for different observers. [J.L.Friedman, N.J.Papastamatiou & J.Z.Simon, PRD 1992 D.G.Boulware, PRD 1992,]

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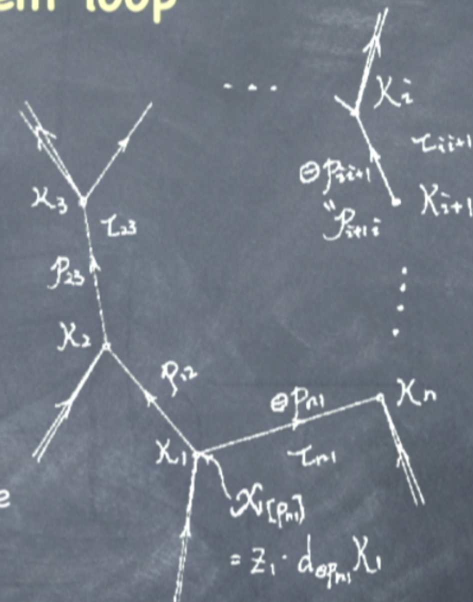
The general condition of x-independent loop

- Consider a loop with n vertices, in which each node is associated with an equation of the momenta conservation $\mathcal{K}_1, \mathcal{K}_2 ... \mathcal{K}_n \equiv 0$



- Define transport operator on vertex $\mathcal{K}_i$

$$H_i := \left(\frac{\partial \mathcal{K}_i}{\partial p_{i-1,i}}\right)^{-1} \left(-\frac{\partial \mathcal{K}_i}{\partial p_{i,i+1}}\right) \qquad T^*_{p_{i-1,i}} \to T^*_{p_{i,i+1}}$$

- Enforing equations of motion around the loop (from the $x_{1_{[p_{n,1}]}} \in T^*_{p_{n,1}}$ and finally coming back to the same point), we will get

$$x_{1_{[p_{n,1}]}} \cdot \left[\mathbb{1} - \prod_{i=1}^{n} H_i\right] = \sum_{i=1}^{n} \tau_{i,i+1} v_{i,i+1} \prod_{i<j\leq n} H_j \quad (i = n+1 := 1)$$

(Using $x_{1_{[p_{n,1}]}}$ to label the endpoint which corresponds to event $\mathcal{K}_1$ on particle $p_{n,1}$'s worldline.)

- The general condition of x-independence:

$$x - independent \Longleftrightarrow H_{tot} := \prod_{i=1}^{n} H_i = \mathbb{1}$$

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Orientable loops

- For the momenta conservation of a vertex $\mathcal{K}_i$ in a loop, if the order of adding internal momenta and external momenta has an orentation, i.e. clockwise or anti-clockwise, we say that the vertex is orientable.
- Three-vertices are always orientable.
- Above three-vertices (more than two external momenta), the vertices are orientable only when the external momenta can be grouped as a whole up to permutations.
  Non-orientable: $\mathcal{K} \equiv p_1 \oplus l_1 \oplus p_2 \oplus l_2 = 0$ ℓ: external momenta

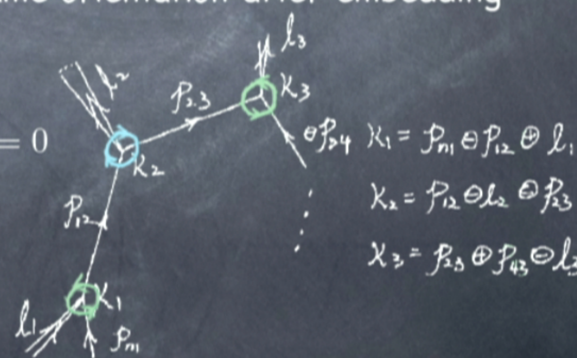
---

- A loop is orientable: if all the vertices have the same orientation after embedding the loop in a 2-d surface.
- A nice property:

$$\mathcal{K}_A \equiv p_1 \oplus l_1 \oplus p_2 \oplus l_2 = 0 \Rightarrow \mathcal{K}_{A'} \equiv l_1 \oplus p_2 \oplus l_2 \oplus p_1 = 0$$

though $d_{p_1}\mathcal{K}_A \neq d_{p_1}\mathcal{K}_{A'}$

$$H'_A = (d_{p_1}\mathcal{K}_{A'})^{-1}(-d_{p_2}\mathcal{K}_{A'}) = H_A \qquad H'_{tot} = H_{tot}$$



- Thus we just need to consider two cases

$$\mathcal{K}_i = l_i \oplus p_{i-1,i} \oplus (\ominus p_{i,i+1}) \qquad \tilde{\mathcal{K}}_i = l_i \oplus (\ominus p_{i,i+1}) \oplus p_{i-1,i}$$

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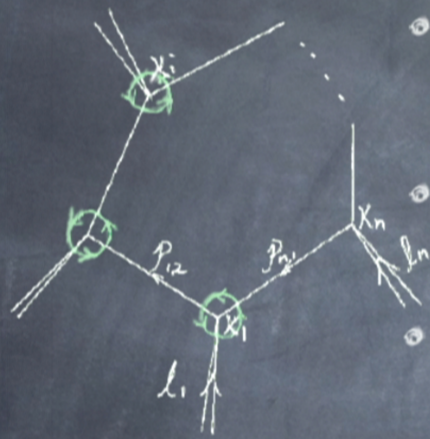
Orientable ⟷ x-independent



- Consider an orientable loop with n vertices and the conservation law on the vertex given by
$$\mathcal{K}_i = l_i \oplus p_{i-1,i} \ominus p_{i,i+1}$$
- Edge transport operator
$$H_i = -(U_0^{\ominus l_i} V_{\ominus l_i}^{p_{i-1,i}})^{-1} (U_0^{\ominus p_{i,i+1}} I_{p_{i,i+1}}) = U_{p_{i-1,i}}^{p_{i,i+1}}$$
- Around the whole loop:
$$H_{tot} = \prod_{i=1}^{n} U_{p_{i-1,i}}^{p_{i,i+1}} = \mathbb{1}$$

---

two kinds of non-orientable loops:

- E.g. the vertex m becomes of the opposite orientation compared with other vertices:
$$\tilde{H}_{tot} = U_{p_{n,1}}^{p_{m-1,m}} \cdot V_{p_{m-1,m}}^{p_{m,m+1}} \cdot U_{p_{m,m+1}}^{p_{n,1}}$$
$$\left(d_{p_\mu} \left[V^{q_\nu}_{(q \oplus p)_\rho} - U^{q_\nu}_{(p \oplus q)_\rho} \right] \Big|_{p,q=0} = T^{\mu\nu}_{\rho} = \frac{1}{\kappa} \delta_0^{[\mu} \delta_i^{\nu]} \delta^i_\rho, \quad i = 1,2,3 \right)$$
- Some of the vertices do not have orientation, e.g. $\mathcal{K}'_i = l_i \oplus p_{i-1,i} \oplus k_i \oplus (\ominus p_{i,i+1})$
$$H'_i = V_{p_{i-1,i}}^{p_{i-1,i} \oplus k_i} U^{p_{i,i+1}}_{p_{i-1,i} \oplus k_i} \quad \Rightarrow H'_{tot} \neq \mathbb{1}$$

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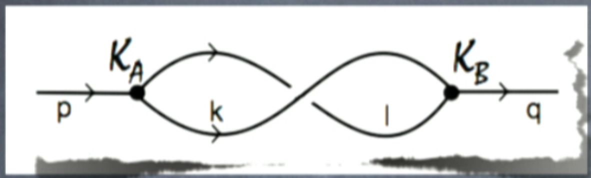
Orientable ⟷ Global momenta conserved



A twisting loop with $p \neq q$

$\mathcal{K}_A = p \ominus (k \oplus l) \quad \mathcal{K}_B = (l \oplus k) \ominus q$

- There are loop processes that locally momenta are conserved, but there is no global momentum conservation. [A.Banburski, 2013]
- Also has x-dependence

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Summary

- The theory of Relative Locality allows causal loops with x-dependence
- For loop processes in Kappa-Poincare momentum space,

Causal $\Leftarrow$ The loop is orientable $\Leftrightarrow$ X-independent

$\Updownarrow$

Global momenta conserved

- Non-orientable loops contain an "effective curvature" caused by a combination of nonlinear interactions, and these loops have strange features.

Outlook

- Understand the fundamental and testable reasons of choosing vertices' forms, make sense of "microscopic causal order".
- Understand the x-dependent loops, they are problems or new features? Try to find some testable predictions for x-dependent loops.

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