Title: Entanglement, Bekenstein-Hawking Entropy and Spinfoams

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Abstract: I review recent developments on vacuum entanglement perturbations in perturbative quantum gravity and spinfoams, and discuss their relevance for understanding the nature of black hole entropy.

Entanglement, Bekenstein-Hawking entropy, and Spinfoams

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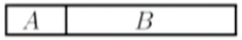
LOOPS '13

INTERNATIONAL CONFERENCE ON QUANTUM GRAVITY

July 22-26, 2013

at Perimeter Institute, Waterloo, Canada

At absolute zero temperature, a sub-system can behave as hot because of quantum correlations with the rest of the system, i.e. because of <u>entanglement</u>.



Examples:

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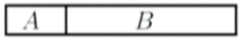
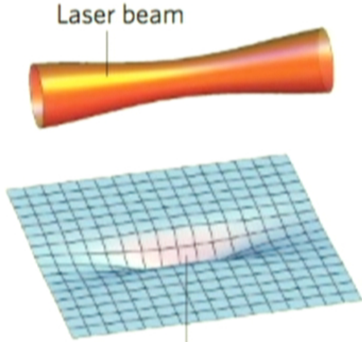
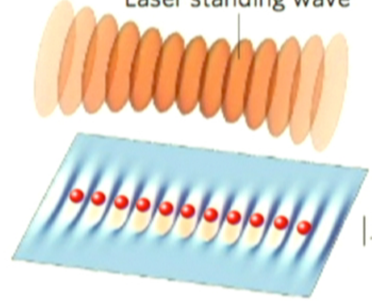


Example 1: Cold atoms in a optical lattice

- isolated quantum system in an energy-eigenstate $|E\rangle$
- subsystem: few atoms $\Rightarrow$ thermal behavior, $T \simeq E/Nk_B$

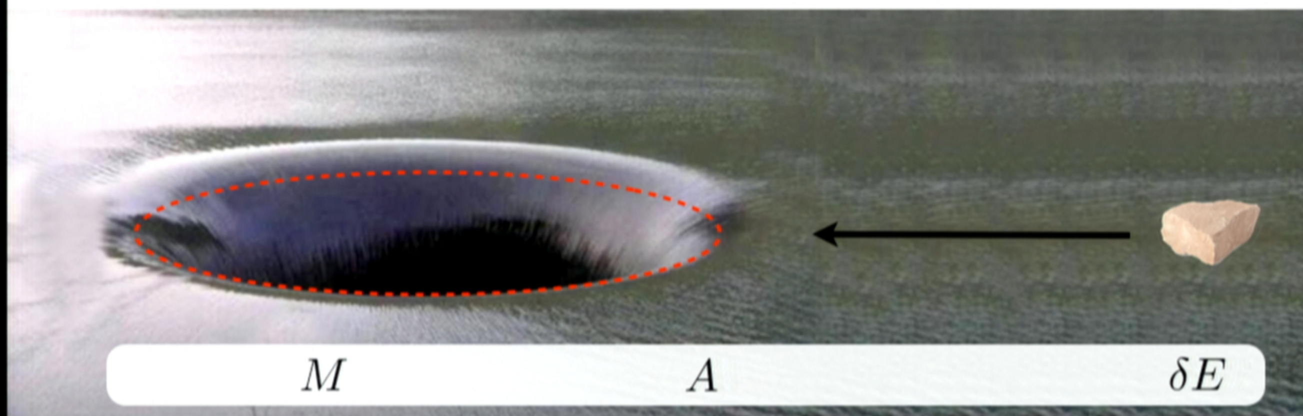




Eigenstate Thermalization / Canonical Typicality / Entanglement Thermodynamics

[Deutsch 1991, Srednicki 1994, Popescu-Short-Winter 2006, Rigol-Dunjko-Olshanii 2008]

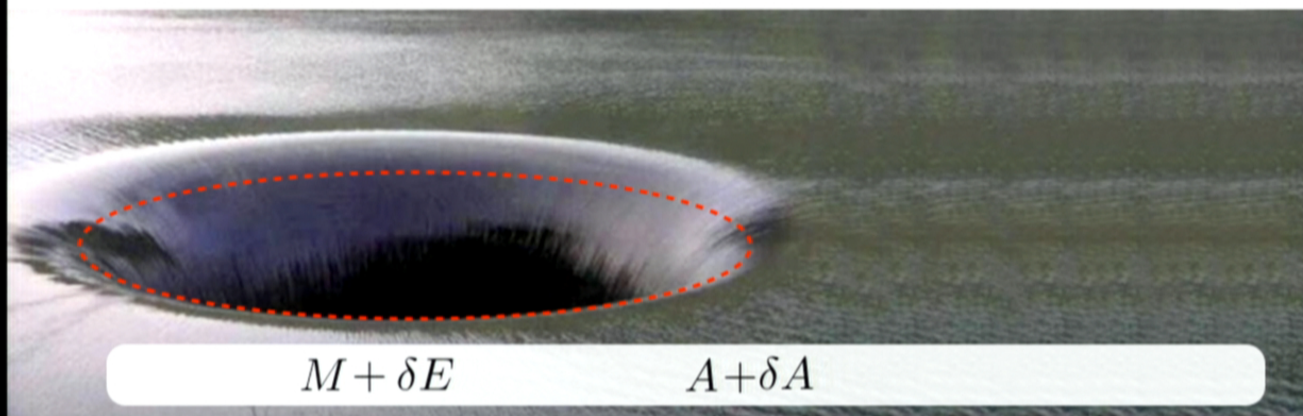
The Puzzle: what is the nature of Black Hole entropy?



Bekenstein, Hawking, 1974

The Puzzle: what is the nature of Black Hole entropy?

$$\delta S_{BH} = \frac{k_B\, c^3}{\hbar}\, \frac{\delta A}{4\, G}$$

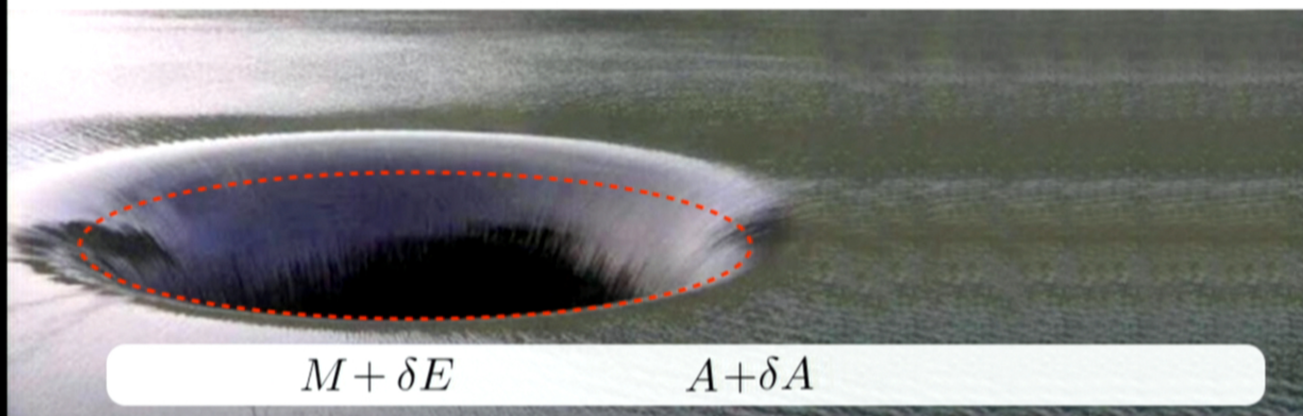


Bekenstein, Hawking, 1974

At absolute zero temperature, a sub-system can behave as hot because of quantum correlations with the rest of the system, i.e. because of entanglement.

A	B

This talk: Entanglement Entropy perturbations $\delta S_{\mathrm{ent}} = \delta S_{BH} = \frac{k_B\, c^3}{\hbar} \frac{\delta A}{4\, G}$



Bekenstein, Hawking, 1974
EB, 2012 "Entanglement thermodynamics and the nature of black hole entropy"

BH entropy: some of the new developments since Loops11 - Madrid

Near-Horizon physics: classical

- Mechanical Laws of the Rindler Horizon — Frodden-Ghosh-Perez 2011; Bianchi-Satz 2013
- Corner terms in the action — EB-Wieland 2012; Neiman-Bodendorfer 2013

Role of the Lorentz group: quantum, γ-independence

- Spinfoams, γ-simple reps, Quantum Rindler Horizon — Bianchi 2012
- Complex Ashtekar variables and the Lorentz group — Frodden-Geiller-Noui-Perez 2013; Pranzetti 2013

Entanglement entropy

- in perturbative QG and in Spinfoams — Bianchi 2012, EB-Satz 2013; EB-Myers 2013

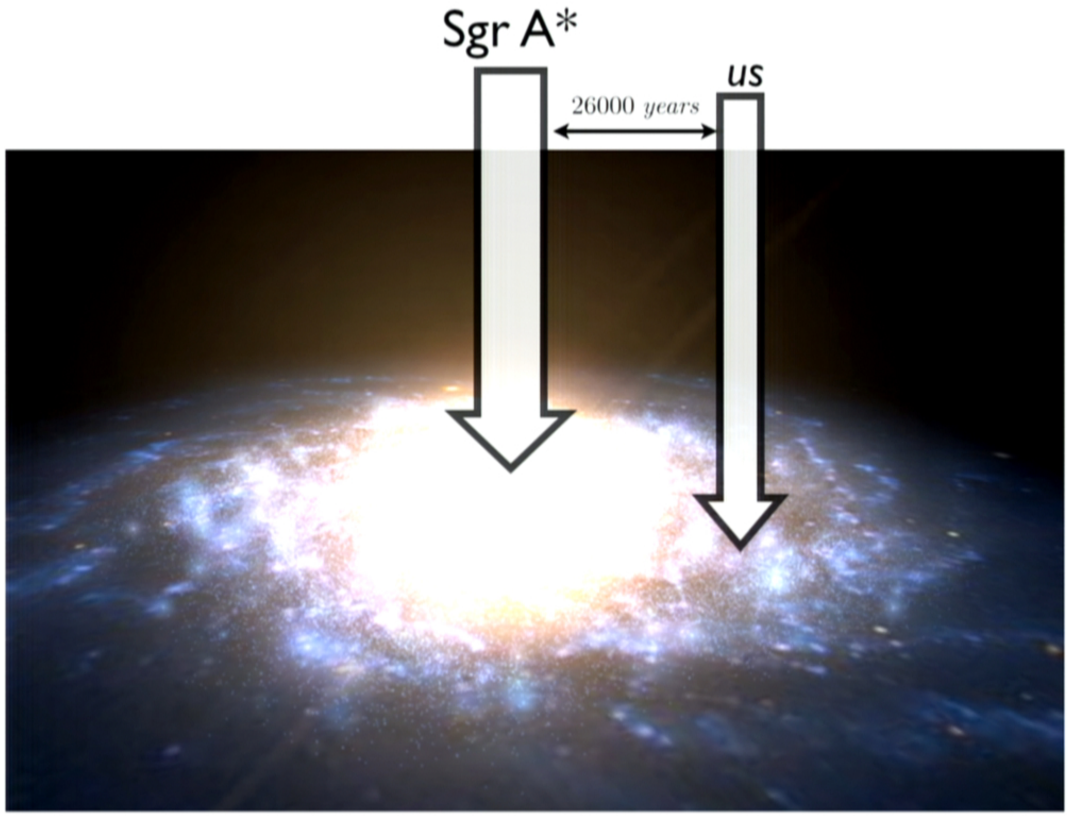
Plan of the talk

1. Near-Horizon Physics

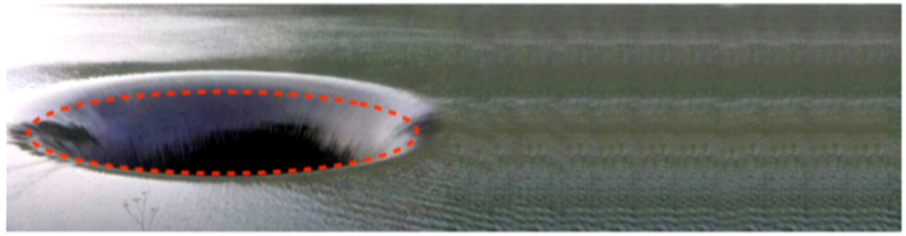
2. Entanglement Thermodynamics
   in perturbative QG and Spinfoams

3. Quantum fluctuations
   vs Statistical fluctuations

The black hole at the center of the Milky Way



Event horizon



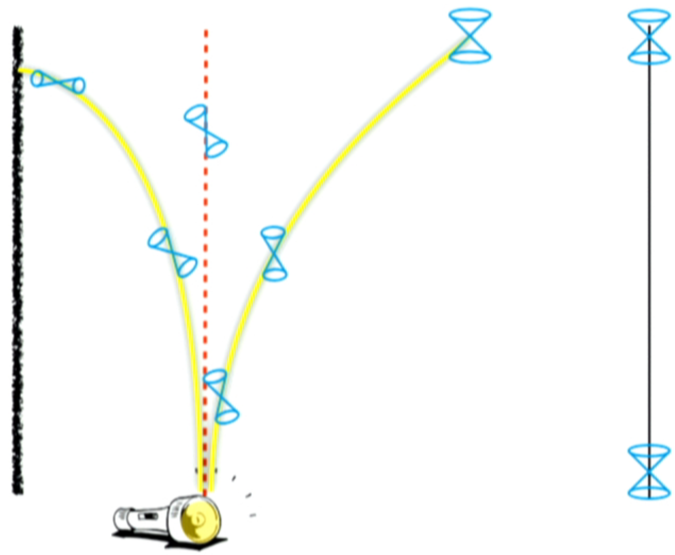
Horizon radius:

$$R_H = \frac{2GM}{c^2}$$

$$R_H(M_\odot) \simeq 3\ \mathrm{km}$$
$$R_H(10^6 M_\odot) \simeq 0.02\ \mathrm{AU}$$

Curvature at the horizon:

$$R^\mu{}_{\nu\rho\sigma} \sim \frac{1}{R_H^2}$$



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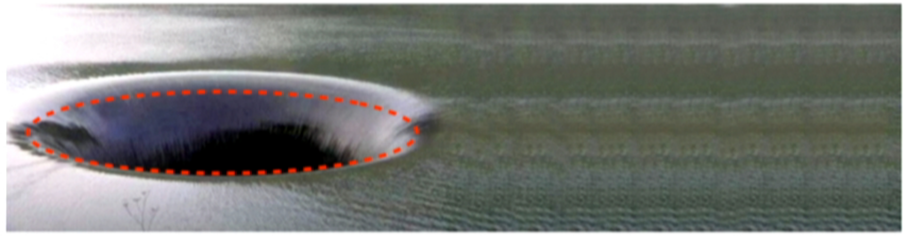

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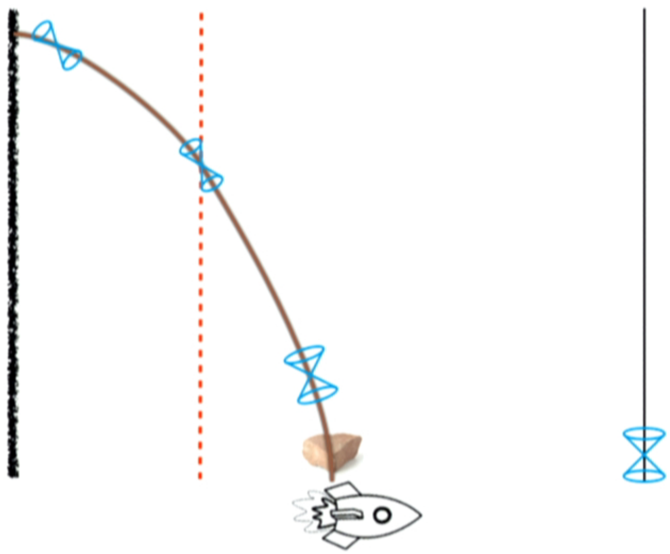

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⇨ 2. Entanglement Thermodynamics
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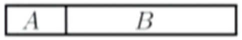
Bipartite quantum system



- Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

- Observables on *B* $\mathcal{O} = I_A \otimes \mathcal{O}_B$

$$\longrightarrow \quad \langle\psi|\mathcal{O}|\psi\rangle = \mathrm{Tr}_B(\mathcal{O}_B\,\rho_B)$$

- Reduced density matrix $\rho_B = \mathrm{Tr}_A(|\psi\rangle\langle\psi|)$

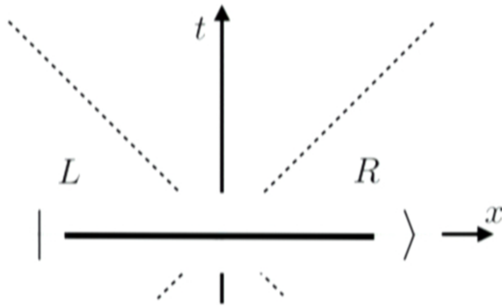
- Entanglement entropy $S_{\mathrm{ent}}(|\psi\rangle) = -\mathrm{Tr}_B(\rho_B \log \rho_B)$

Vacuum Entanglement in QFT

[Sorkin 1983, Srednicki 1993]

- 4d Minkowski space, $x^\mu = (t, x, y_1, y_2)$
- Regions: $L = \{x < |t|\}\,, \quad R = \{x > |t|\}$
- QFT $\mathcal{H} = \mathcal{H}_L \otimes \mathcal{H}_R$

- Minkowski Vacuum state $|0\rangle =$



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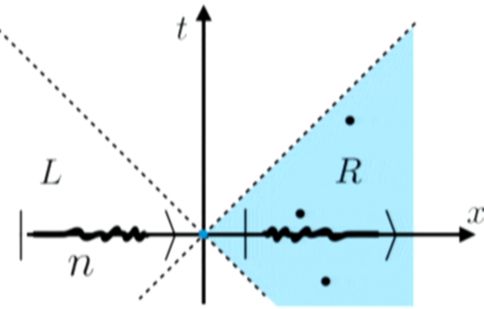

- Reduced density matrix $\rho_0 = \mathrm{Tr}_L(|0\rangle\langle 0|)$

$$= \sum_n e^{-2\pi\epsilon_n} |n\rangle\langle n|$$

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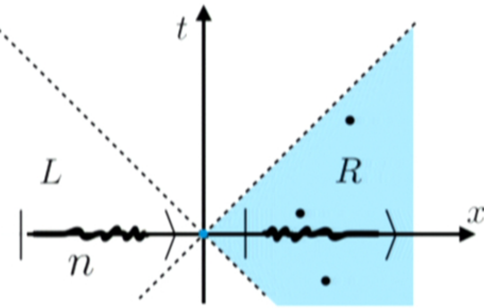

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- Vacuum Entanglement Entropy

$$S_{ent}(|0\rangle) = -\mathrm{Tr}_R(\rho_0 \log \rho_0)$$
$$= c_0\, A\, \Lambda^2 + \ldots$$

- UV cutoff $\Lambda \sim 1/\sqrt{G}$?

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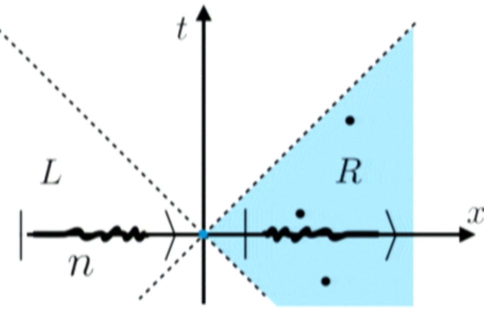

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- BH entropy? G_{IR}? $1/4$? Species problem?
  [Solodukhin Liv.Rev.Rel. 2011]

Entanglement perturbations

Bianchi 1211.0522

$|\psi\rangle$ = small perturbation of the vacuum

3 results about $\delta S_{ent} = S_{ent}(|\psi\rangle) - S_{ent}(|0\rangle)$

i. $\delta S_{ent} =$ finite

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3 results about $\delta S_{ent} = S_{ent}(|\psi\rangle) - S_{ent}(|0\rangle)$

i. $\delta S_{ent} =$ finite

ii. $$\delta S_{ent} = \frac{\delta E_H}{T}$$

iii. $$\delta S_{ent} = \frac{\delta A}{4G}$$

i. $\delta S_{ent} \; = $ finite UV divergences cancel

E.g.: 2d free massive scalar, 1-particle state in Unruh mode Ω (with A. Catuneanu and F. Mercati 2013)

$$|1\rangle = d_R^\dagger(\Omega)|0\rangle = \int_{-\infty}^{+\infty} dk_1 \, \frac{1}{\sqrt{2\pi E}} \left(\frac{E+k_1}{E-k_1}\right)^{+i\Omega/2} a^\dagger(k_1)|0\rangle$$

$$\rho_1 = \mathrm{Tr}_L |1\rangle\langle 1| \; = \; \frac{1}{\tilde{Z}} \left(\sum_{n=0}^{\infty} n\, e^{-2\pi\Omega n} |n,\Omega\rangle_R \langle n,\Omega|_R\right) \quad \bigotimes_{\Omega'\neq\Omega} \left(\sum_{n=0}^{\infty} e^{-2\pi\Omega' n} |n,\Omega'\rangle_R \langle n,\Omega'|_R\right)$$

Entanglement entropy

$$S_{ent}(|1\rangle) - S_{ent}(|0\rangle) \; =$$

ii. $\delta S_{ent} = \delta Q / T$ **thermodynamics of entanglement**

Basic notions:

- Noether current $J_\mu = (\mathcal{J}_\mu)_{x0} = x\, T_{\mu 0} - t\, T_{\mu x}$
- Generator of x-boosts $K = \int J_\mu d\Sigma^\mu \overset{t=0}{=} \int x\, T_{00}\, dx\, dy_1 dy_2 \;=\; K_R - K_L$
- Rindler energy

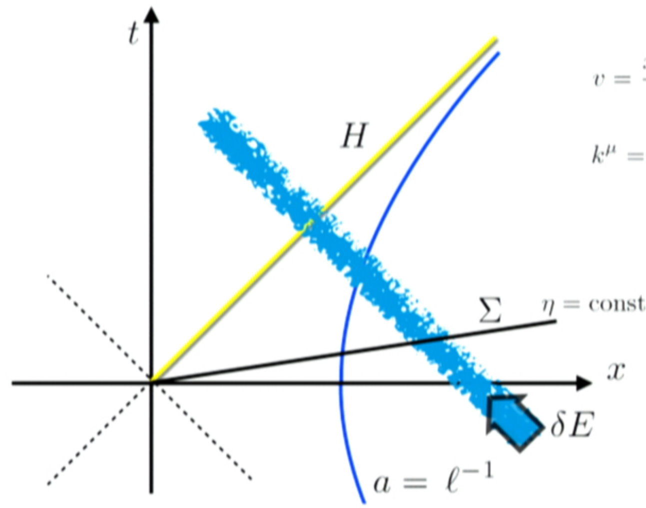
$$E_R = a\, K_R$$

- Energy crossing the Rindler horizon

$$E_H = a \int_H T_{\mu\nu} k^\mu k^\nu\, v dv\, dy_1 dy_2$$

- Energy conservation $\partial_\mu J^\mu = 0$

$$E_R = E_H + \not{E}_\infty$$



$$v = \frac{x+t}{2}$$

$$k^\mu = \frac{\partial x^\mu}{\partial v}$$

ii. $\delta S_{ent} = \delta Q / T$ — thermodynamics of entanglement

- Thermality of the vacuum

(*) $$\rho_0 = \mathrm{Tr}_L(|0\rangle\langle 0|) = \frac{e^{-2\pi K_R}}{Z}$$

Unruh temperature

$$T = \frac{a}{2\pi}$$

- Small perturbation $|\psi\rangle$

$$\rho_1 = \mathrm{Tr}_L(|\psi\rangle\langle\psi|) \qquad \delta\rho = \rho_1 - \rho_0 = \text{small}$$

- Energy crossing the Horizon

$$\delta E_H = \langle\psi|\hat{E}_H|\psi\rangle - \langle 0|\hat{E}_H|0\rangle = \mathrm{Tr}_R(\hat{E}_H\,\delta\rho)$$

- Entanglement perturbation:

$$\delta S_{\text{ent}} = -\delta \mathrm{Tr}(\rho \log \rho)$$

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$\mathrm{Tr}(\delta\rho) = 0$

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$$\overset{*}{=} 2\pi\mathrm{Tr}(K_R\,\delta\rho) + \log Z\,\mathrm{Tr}(\delta\rho)$$

$E_\infty = 0$

$$= \frac{2\pi}{a}\mathrm{Tr}(\hat{E}_H\,\delta\rho) = \frac{\delta E_H}{T}$$

ii. $\delta S_{ent} \; = \; \delta Q \,/T$ thermodynamics of entanglement

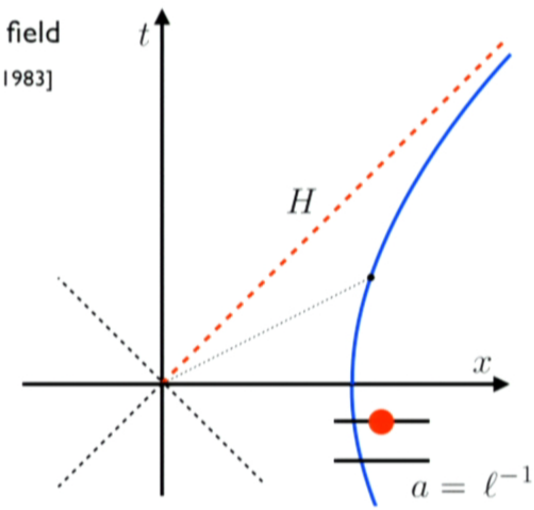
E.g.: decay of an excited two-level atom coupled to field

[cf Unruh-Wald 1983]

$$|\psi_{-\infty}\rangle = |0\rangle|\uparrow\rangle$$

$$\downarrow$$

$$|\psi_{+\infty}\rangle = (1-|\epsilon|^2/2)\,|0\rangle|\uparrow\rangle + \epsilon\,|1\rangle|\downarrow\rangle$$



- Density matrix for right modes of the field

$$\rho_{\pm\infty} = \mathrm{Tr}_L \mathrm{Tr}_A \Big(|\psi_{\pm\infty}\rangle\langle\psi_{\pm\infty}| \Big)$$

- Energy difference (field only)

$$\Delta E_H = \mathrm{Tr}(\hat{E}_H\,\rho_{+\infty}) - \mathrm{Tr}(\hat{E}_H\,\rho_0)$$
$$= |\epsilon|^2\,\frac{\Omega}{2}\,(1+\coth(\pi\Omega))\,a$$

- Entanglement entropy difference

$$\Delta S_{\mathrm{ent}} = S_{\mathrm{ent}}(|\psi_{+\infty}\rangle) - S_{\mathrm{ent}}(|0\rangle)$$
$$= \pi\Omega\,(|\epsilon|^2-2) + \pi\Omega|\epsilon|^2\coth(\pi\Omega) - \log\left(4|\epsilon|^2\sinh^2(\pi\Omega)\right) + \log\left(e^{2\pi\Omega}-1\right)$$
$$+\, 4|\epsilon|^2\sinh^2(\pi\Omega)\mathrm{LerchPhi}^{(0,1,0)}\Big(e^{-2\pi\Omega}, -1, -\frac{(|\epsilon|^2-1)\,(\coth(\pi\Omega)-1)}{2|\epsilon|^2}\Big)$$

Entanglement perturbations

arXiv:1211.0522

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ii. $\delta S_{ent} = \dfrac{\delta E_H}{T}$

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Dynamics of the Rindler Horizon

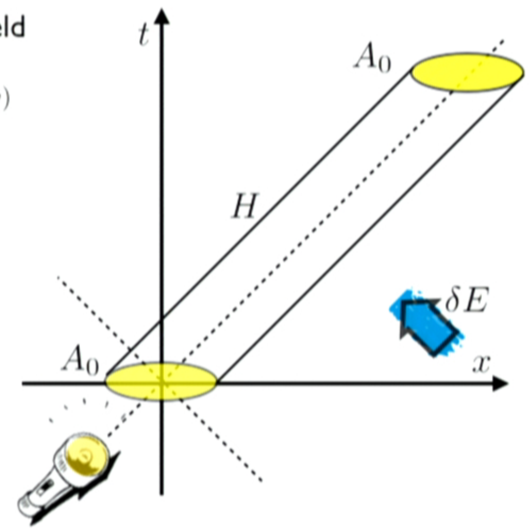
EB-Satz 1305.4986 [Jacobson-Parentani 2003]

Deflection of light rays by the gravitational field

$$L = \frac{1}{2}\eta_{\mu\nu}\dot{x}^\mu(v)\dot{x}^\nu(v) + \sqrt{8\pi G}h_{\mu\nu}(x(v))\,\dot{x}^\mu(v)\dot{x}^\nu(v)$$

Perturbation of the straight path

$$x^\mu(v) \;=\; x_0^\mu(v) + \xi^\mu(v)$$



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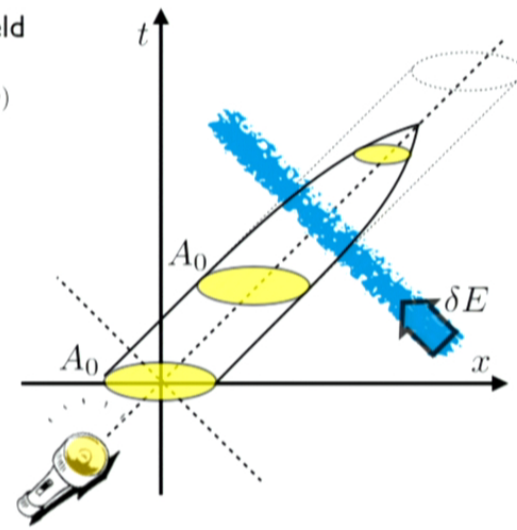


- Collimated beam of light rays $\dot{\xi}^\mu(v=0)=0$
- Energy crossing the beam

Gravity is attractive: it focuses light rays

Dynamics of the Rindler Horizon

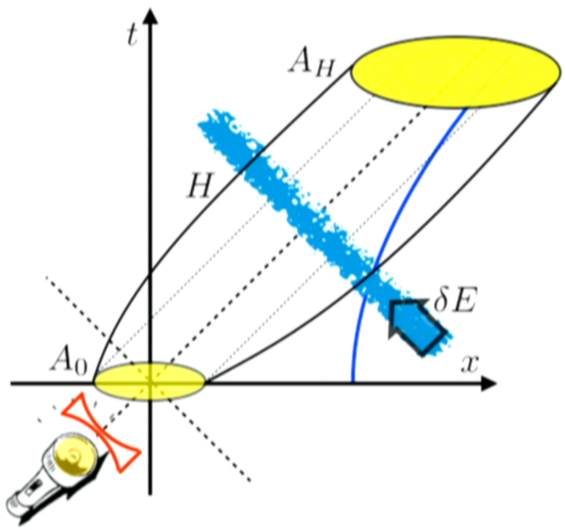
EB-Satz 1305.4986 [Jacobson-Parentani 2003]

Horizon:

Light rays finely balanced between → out of sight / → reach infinity

$v \to \infty$ $\qquad \dot{\xi}^{\mu}(v) \to 0$

$h_{\mu\nu}(x_0^{\mu}(v)) \to 0$



Gravity is attractive: it focuses light rays

⇨ diverging beam so that collimated at infinity

Area of the Rindler Horizon:

(******)

$$A_H = A_0 + \sqrt{8\pi G}\int_B d^2\vec{y}\int_0^{\infty} -\Box h_{\mu\nu}(x_0(v))\, k^{\mu}k^{\nu}\, v dv$$

iii. Bekenstein-Hawking area law for entanglement

EB, 1211.0522

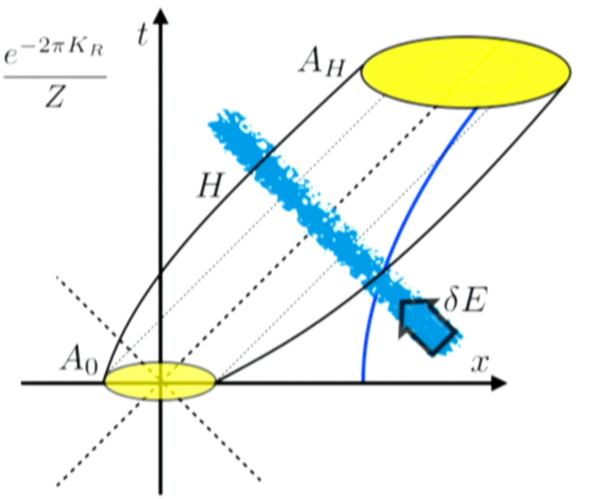
$|\psi\rangle$ = small perturbation of the vacuum

$\rho_1 = \mathrm{Tr}_L(|\psi\rangle\langle\psi|)$, $\rho_0 = \mathrm{Tr}_L(|0\rangle\langle 0|) \overset{(*)}{=} \frac{e^{-2\pi K_R}}{Z}$

$\delta\rho = \rho_1 - \rho_0 =$ small

$$\delta S_{ent} = S_{ent}(|\psi\rangle) - S_{ent}(|0\rangle)$$

$$\overset{(*)}{=} 2\pi\, \mathrm{Tr}(K_R\, \delta\rho)$$



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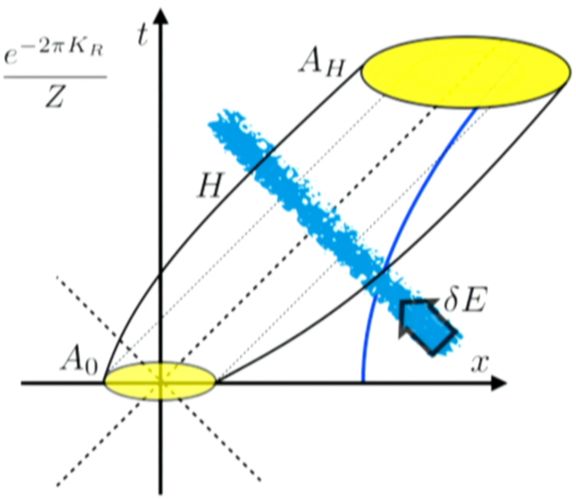

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$$\overset{E_\infty = 0}{=} 2\pi\, \mathrm{Tr}\Big(\int d^2\vec{y} \int_0^\infty T_{\mu\nu} k^\mu k^\nu\, v dv\, \delta\rho\Big)$$

$$\overset{\text{(e.o.m)}}{=} \frac{2\pi}{\sqrt{8\pi G}}\, \mathrm{Tr}\Big(\int d^2\vec{y} \int_0^\infty -\Box h_{\mu\nu}(x_0(v)) k^\mu k^\nu\, v dv\, \delta\rho\Big)$$

$$\overset{(**)}{=} \frac{2\pi}{8\pi G}\Big(\langle\psi|\hat{A}_H|\psi\rangle - \langle 0|\hat{A}_H|0\rangle\Big)$$

$$= \frac{\delta A_H}{4G}$$

- Universal: independent of the number and matter species
- No UV contribution
- Low-energy G

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EB, 1211.0522

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i. $\delta S_{ent} =$ finite

ii. $\delta S_{ent} = \dfrac{\delta E_H}{T}$

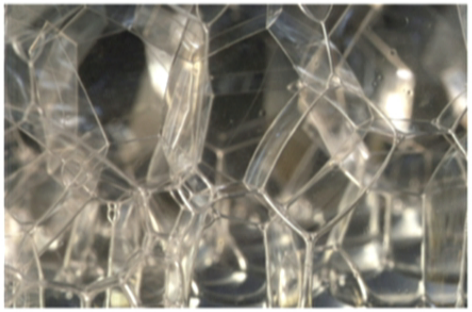
iii. $\delta S_{ent} = \dfrac{\delta A}{4G}$

Spin-foam path integral

- Realization of the path-integral over geometries for 4d Lorentzian gravity

$$Z = \int \mathcal{D}g_{\mu\nu} \; e^{\frac{i}{\hbar} S[g_{\mu\nu}]}$$

with gravitational d.o.f. unfrozen only on a 2d-foam



- Covariant formulation of loop quantum gravity (EPRL-FK)

$$Z_{\Delta_2} = \sum_{j_f\,,\, i_e} \prod_{f \in \Delta_2^*} (2j_f + 1) \prod_{v \in \Delta_2^*} \Big\{ \bigotimes_{e \in v} \mathcal{I}_\gamma(i_e) \Big\}$$

spin

foam

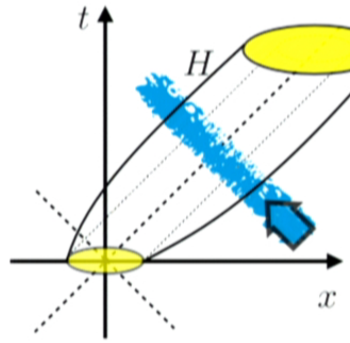
invariant of the Lorentz group SO(1,3)

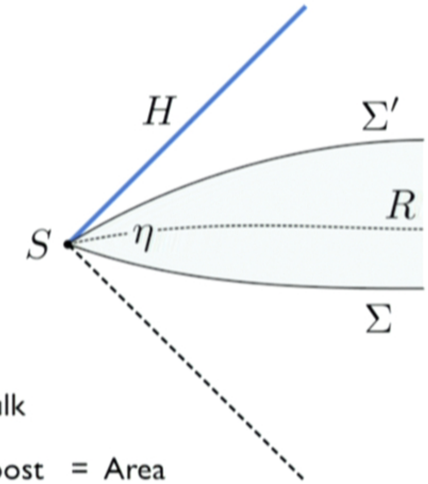
γ-simple reps $K = \gamma L$

cf. Wigner {*6j*}-*symbol*

See M.Han and F. Hellmann talks

Perturbative gravity vs Non-Perturbative GR





- GR action vanishes in the bulk
- Corner Hamiltonian = Boost = Area

$$H_{\text{corner}} = \int_S \epsilon^{IJ} \left(\frac{1}{2}\varepsilon_{IJKL}B^{KL} + \frac{1}{\gamma}B_{IJ}\right)$$

$$= \int_S K = \int_S \gamma\, L = \frac{A}{8\pi G}$$

[Carlip-Teitelboim '93, Bianchi-Wieland '12, Smolin '12]

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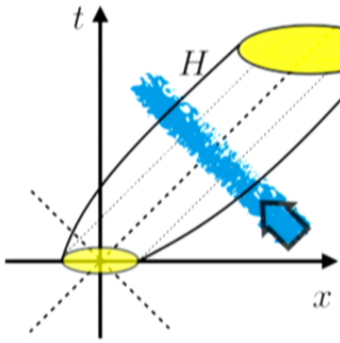

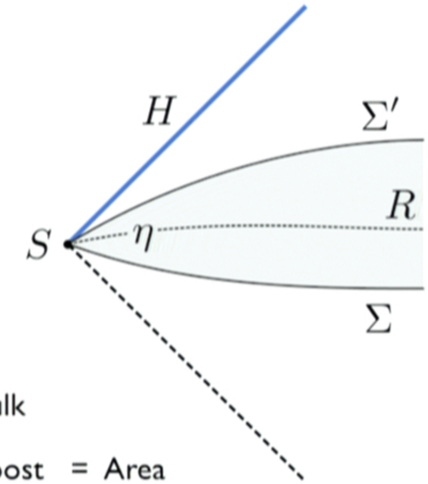

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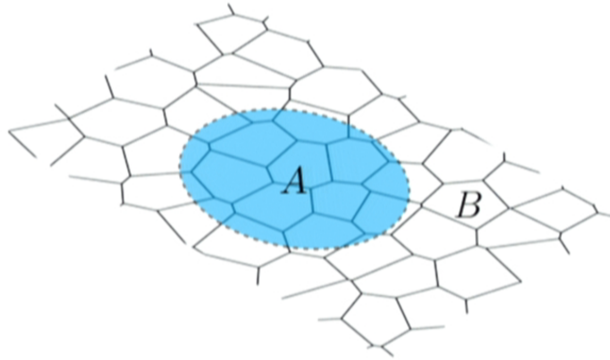
Entanglement entropy for a spin-network state

$$\mathcal{H} = \bigotimes_n \mathcal{H}_n^{\text{intertwiner}} \quad = \mathcal{H}_A \otimes \mathcal{H}_B$$

$$|\psi\rangle = \sum_{i_n} \psi_{i_1 \cdots i_N} |i_1, \ldots, i_N\rangle$$

$$\rho_B = \mathrm{Tr}_A |\psi\rangle\langle\psi|$$

$$S_{\text{ent}} = -\mathrm{Tr}_B(\rho_B \log \rho_B)$$



Donnelly 2008, Livine-Terno 2008

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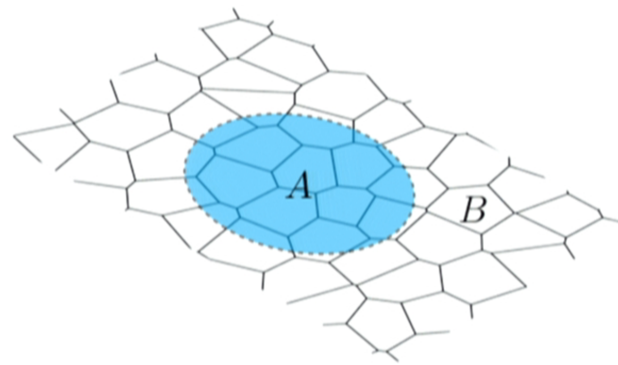

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Entanglement entropy in the Hartle-Hawking spin-network state

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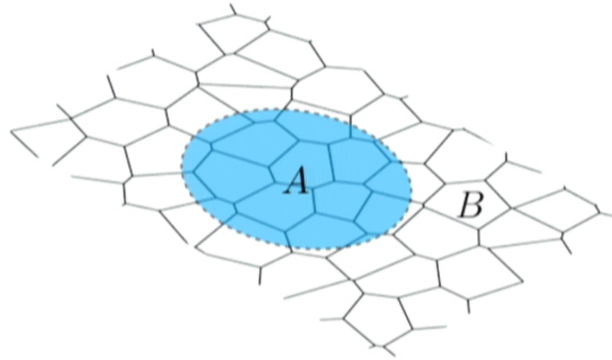

From Spinfoams: Tensor-Product-State

$$|\text{HH}\rangle = \sum_{i_n} \psi_{i_1 \cdots i_N} |i_1, \ldots, i_N\rangle$$

$$\psi_{HH}(h_l) = \langle h_l | HH \rangle = \int dg_n \prod_l \text{Tr}\big(Y^\dagger D^{\gamma(j_l+1), j_l}(e^{-\pi K}) Y D^{j_l}(g_s h_l g_t^{-1})\big)$$

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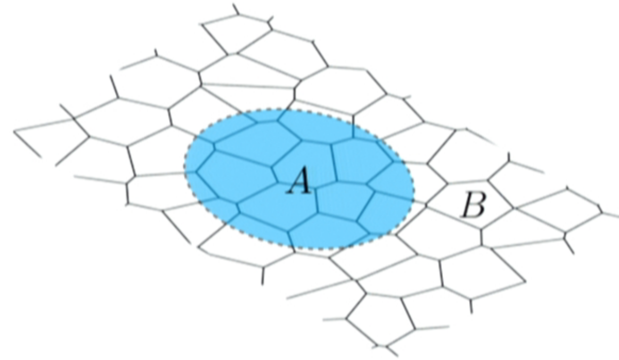

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$$\rho_0 = \text{Tr}_A \, |HH\rangle\langle HH| \quad = \prod_{l \in \partial B} \frac{Y^\dagger e^{-\pi K} Y Y^\dagger e^{-\pi K} Y}{Z_l}$$

EB 2012, EB-Myers 2013
cf Ghosh-Perez 2011-2013

Entanglement entropy

$$S_{\text{ent}} = -\text{Tr}(\rho_0 \log \rho_0) \quad = 2\pi \, \text{Tr}(\textstyle\sum_l K_l \, \rho_0) \, + \sum_l \log Z_l$$

$$= 2\pi\gamma \, \text{Tr}(\textstyle\sum_l L_l \, \rho_0) \, + \sum_l \log Z_l$$

$$= 2\pi\gamma \, \frac{A}{8\pi G \hbar \gamma} + \sum_l \log Z_l \qquad = \frac{A}{4G\hbar} + N\mu(\gamma)$$

Plan of the talk

1. Near-Horizon Physics

2. Entanglement Thermodynamics
   in perturbative QG and Spinfoams

⇨ 3. Quantum fluctuations
   vs Statistical fluctuations

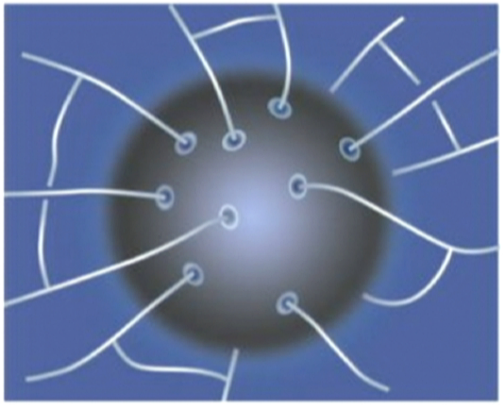
Quantum fluctuations vs Statistical fluctuations

Counting states assumes that the black hole is in a statistical ensemble

Ashtekar-Baez-Corichi-Krasnov 1998
Engle, Noui, Perez, Pranzetti 2010

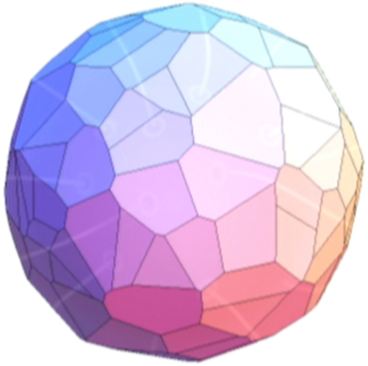
⇨ Statistical fluctuations of the shape of the horizon

Rovelli 1996, EB 2010



[Baez, Nature 2003]

Quantum fluctuations vs Statistical fluctuations



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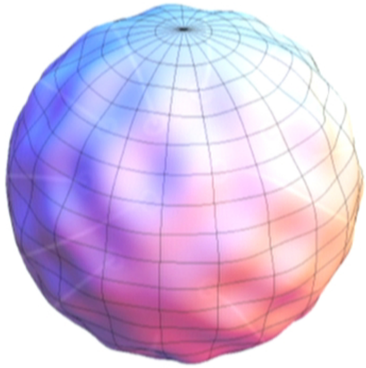

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⇨ Entanglement across the horizon results in a thermal ensemble for its shape at a universal temperature

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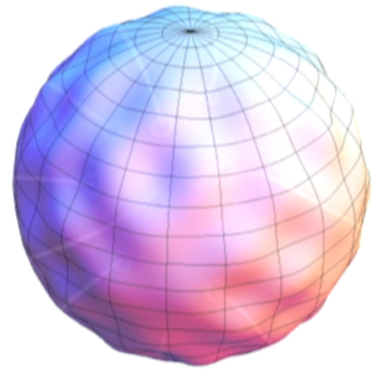

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Evaporation of the Rindler horizon

Evaporation by coupling the horizon to a cooler system:
accelerated two-level atom initially in the ground state

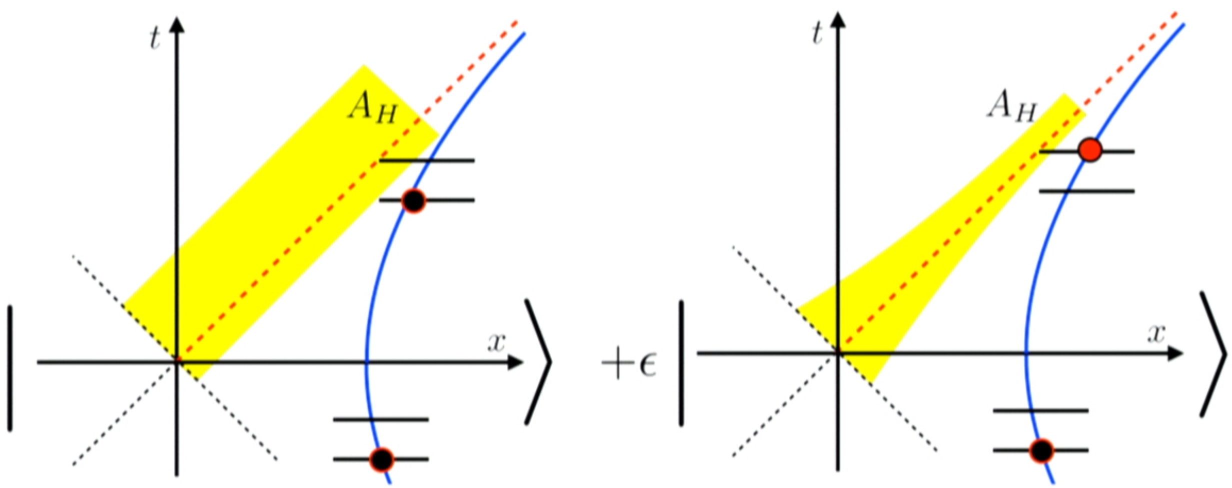
EB, to appear
[cf Unruh-Wald 1983]

$$|\psi_{-\infty}\rangle = |0\rangle|\downarrow\rangle$$

$$\downarrow$$

$$|\psi_{+\infty}\rangle = (1 - |\epsilon|^2/2)\,|0\rangle|\downarrow\rangle\ + \epsilon\,|1\rangle|\uparrow\rangle$$

Because of Horizon/Atom entanglement
tracing over the atom corresponds
to a thermal ensemble for the horizon



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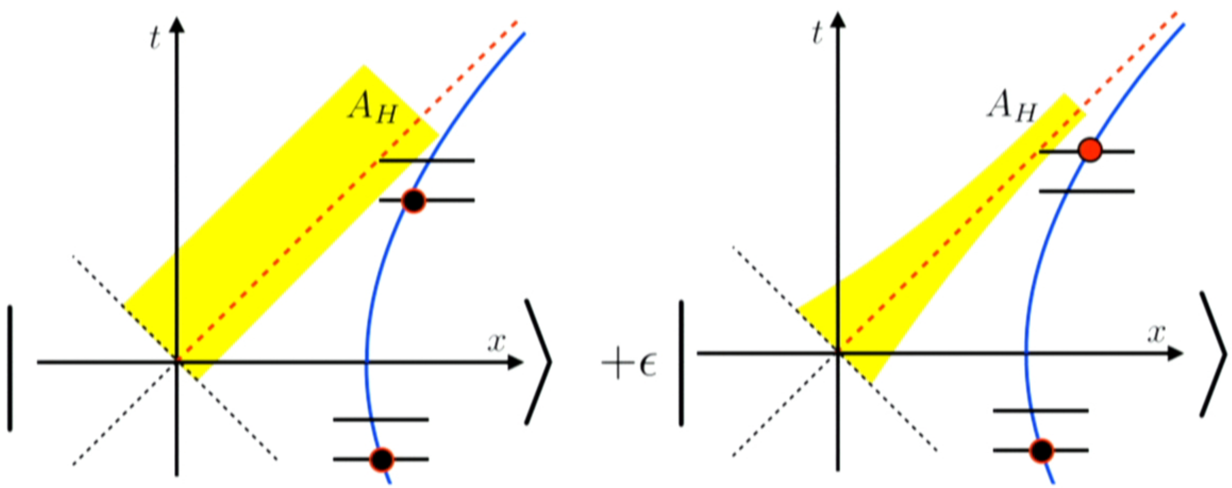
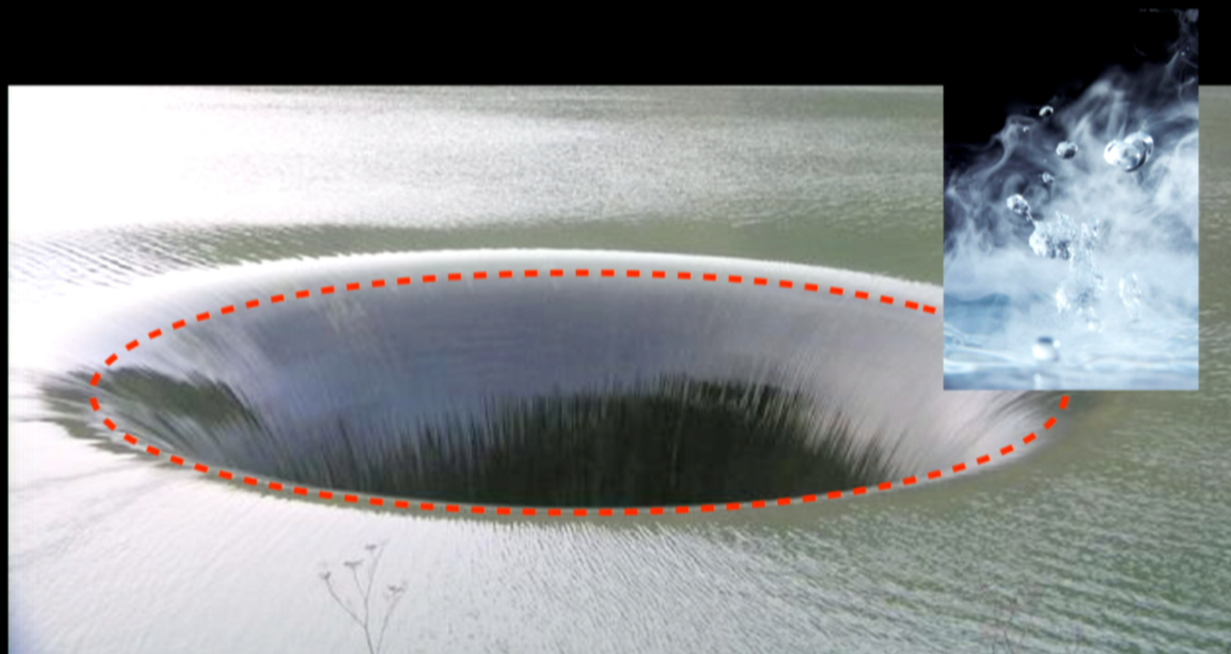


E. Bianchi   Entanglement, Bekenstein-Hawking entropy, and Spinfoams

Why are black holes hot ?



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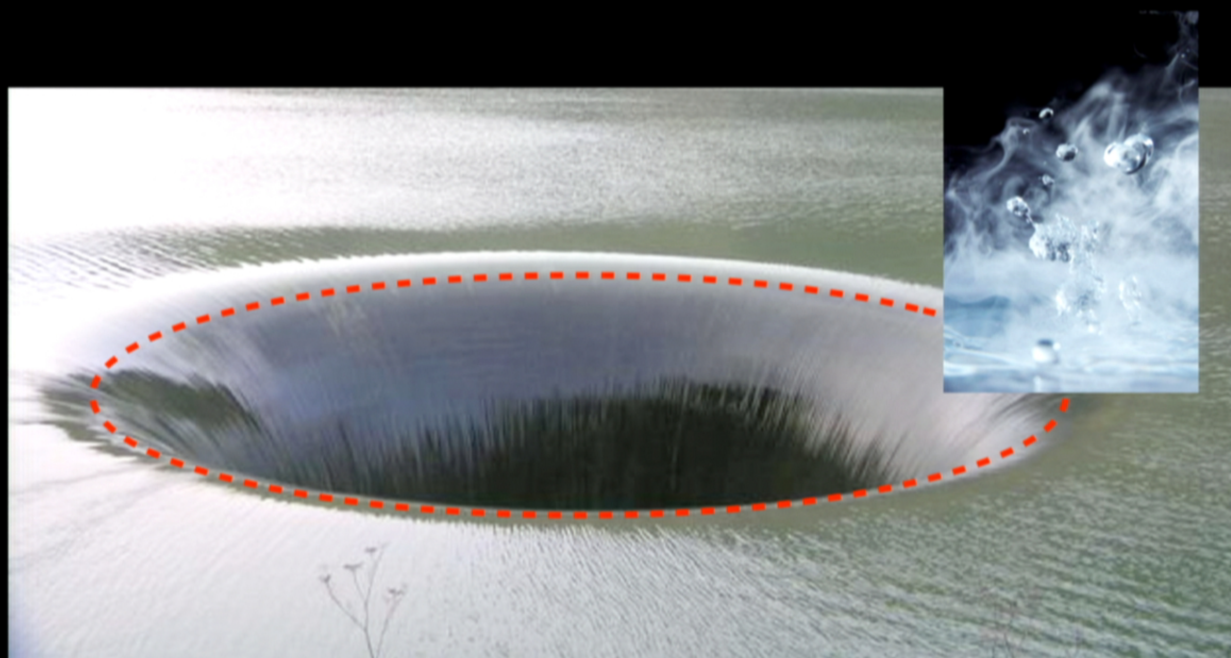
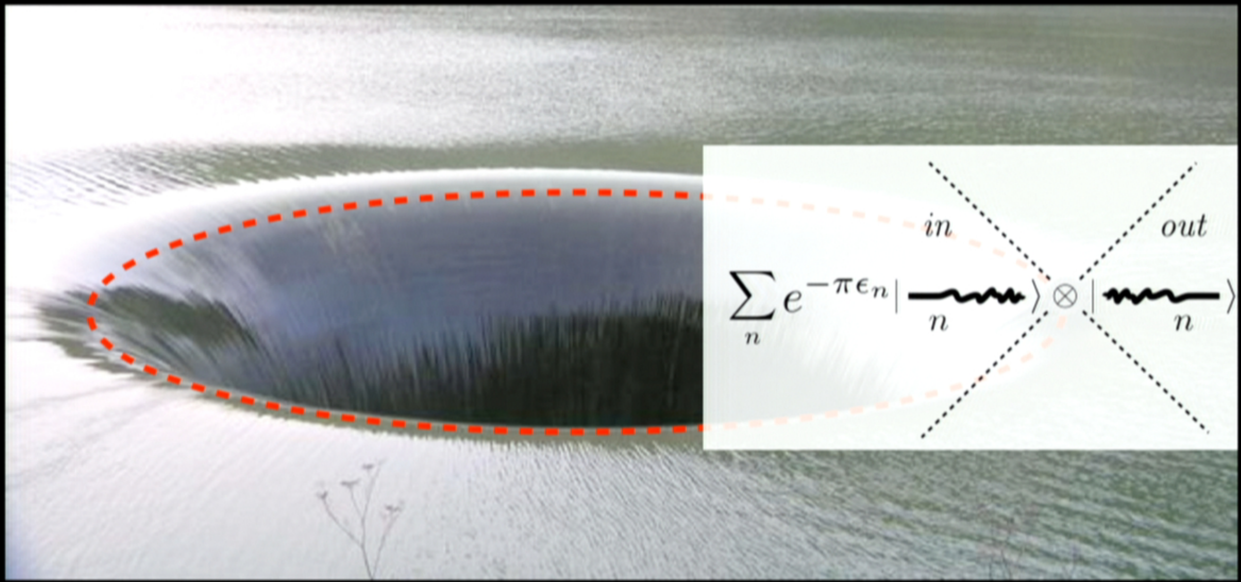


Why are black holes hot ?

because of entanglement across the horizon



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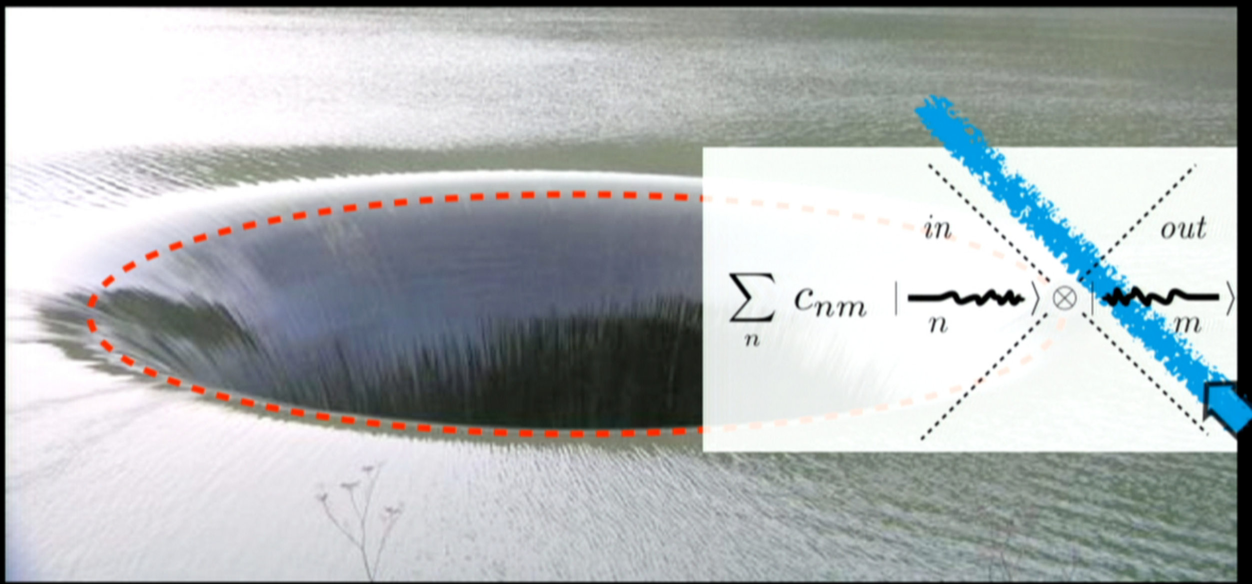

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