Title: Black Holes in Loop Quantum Gravity: New Insights and Perspectives from Semiclassical Consistency

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Abstract: I will argue that the recently introduced quasilocal framework for black hole mechanics (based on the form of the near horizon geometry of stationary black holes (BHs)) together with an additional assumption on the degeneracy of the area spectrum in quantum gravity (holography for non geometric degrees of freedom) leads to agreement between the statistical mechanical treatment of quantum black holes and standard semiclassical results in BH thermodynamics. More precisely, up to small quantum corrections, quantum black holes satisfy the following properties: Entropy is Bekenstein-Hawking entropy, and fluctuations of the horizon area are small. Moreover, under the above assumption, an explicit correspondence between the statistical mechanical treatment of the fundamental LQG degrees of freedom and the semiclassical Euclidean path integral formulation can be explicitly established.

Black holes in loop quantum gravity: new insights and perspectives from semiclassical consistency

Alejandro Perez
Centre de Physique Théorique, Marseille, France

LOOPS13
PI

based on recent results obtained in collaboration with
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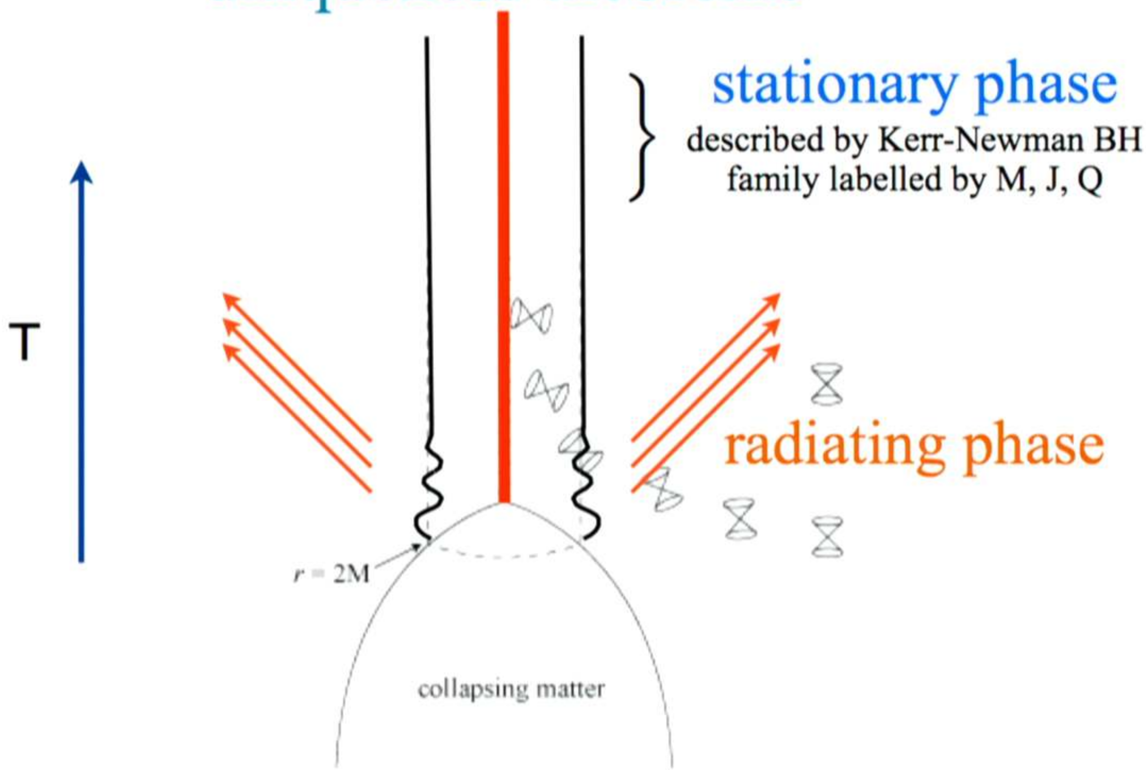
Agullo, Alesci, Alexandrov, Ashtekar, Baez, Barbero, Bianchi, Borja, Corichi, Diaz-Polo, Domagala, Engle, Frodden, Ghosh, Krasnov, Kaul, Lewandowski, Livine, Majumdar, Meissner, Mitra, Pranzetti, Rovelli, Sahlmann, Smolin, Terno, Thiemann, Villasenor...

Black Hole Thermodynamics

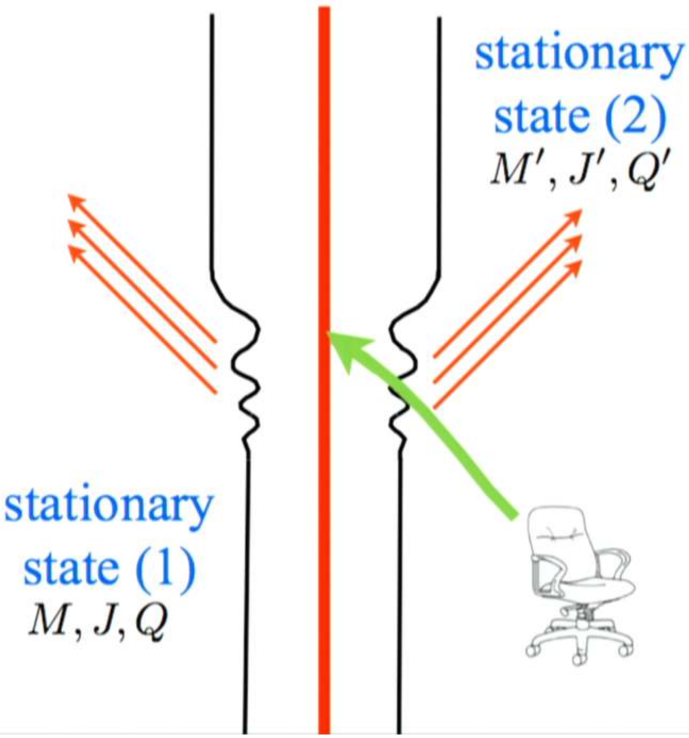
The Robinson-Carter-Hawking-Israel uniqueness theorems



Black Hole Thermodynamics

The 0th, 1st, 2nd and 3rd laws of BH

Some definitions
- $\Omega \equiv$ horizon angular velocity
- $\kappa \equiv$surface gravity ('grav. force' at horizon)
  If ℓ^a =killing generator, then $\ell^a \nabla_a \ell^b = \kappa \ell^b$.
- $\Phi \equiv$electromagnetic potential.



0th law: the surface gravity κ is constant on the horizon.

1st law:

$$\delta M = \frac{\kappa}{8\pi}\delta A + \underbrace{\Omega \delta J + \Phi \delta Q}_{\text{work terms}}$$

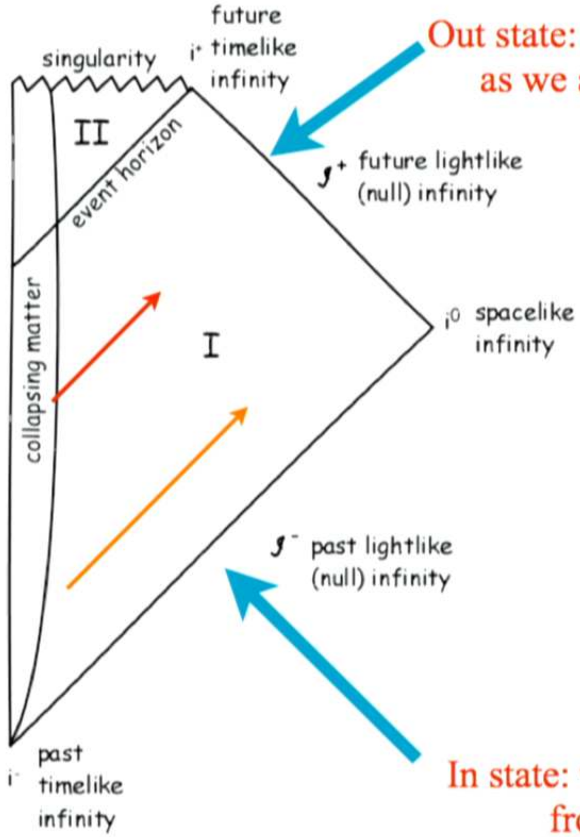
2nd law:

$$\delta A \geq 0$$

3rd law: the surface gravity value $\kappa = 0$ (extremal BH) cannot be reached by any physical process.

Black Hole Thermodynamics

Hawking Radiation: QFT on a BH background



Out state: thermal flux of particles as we approach the point **i**+

In state: vacuum far from **i**⁻

Temperature at infinity

$$T_\infty = \frac{\kappa}{2\pi}$$

From the first law

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega\delta J + \Phi\delta Q$$

One infers the ENTROPY

$$S = \frac{A}{4\ell_p^2}$$

Black Hole Thermodynamics

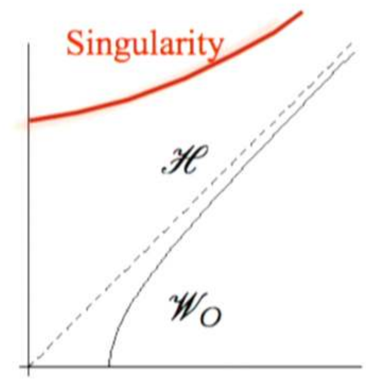
A local perspective

$\ell^2 << A$

Introduce a family of
local stationary observers
~ZAMOS

$$\chi = \xi + \Omega\,\psi = \partial_t + \Omega\,\partial_\phi$$

$$u^a = \frac{\chi^a}{\|\chi\|}$$

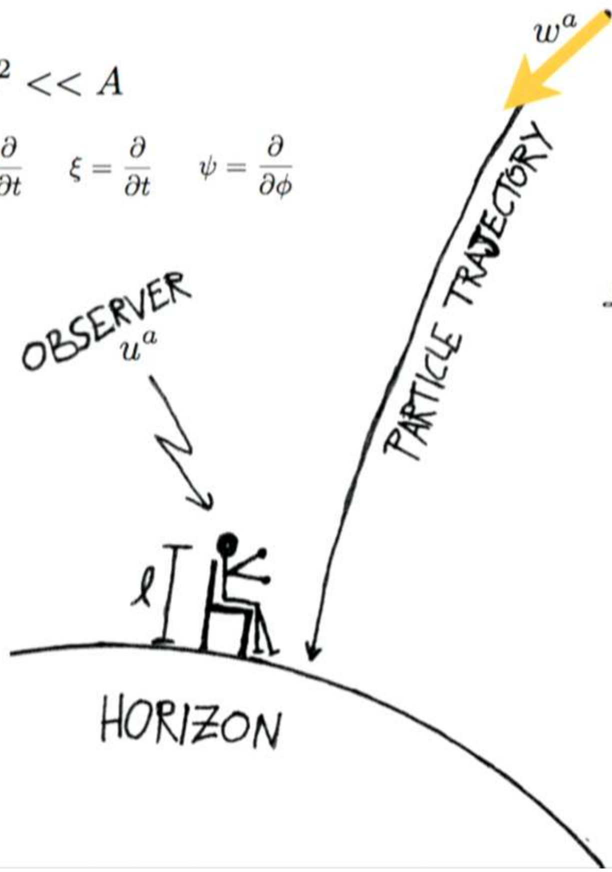


A thought experiment

throwing a test particle from infinity

$$\ell^2 << A$$

$$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \qquad \xi = \frac{\partial}{\partial t} \qquad \psi = \frac{\partial}{\partial \phi}$$



Particle's equation of motion

$$w^a \nabla_a w_b = q\, F_{bc} w^c$$

Symmetries of the background

$$\mathscr{L}_\xi g_{ab} = \mathscr{L}_\psi g_{ab} = \mathscr{L}_\xi A_a = \mathscr{L}_\psi A_a = 0$$

Conserved quantities

$$\mathcal{E} \equiv -w^a \xi_a - q A^a \xi_a$$

$$L \equiv w^a \psi_a + q A^a \psi_a$$

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throwing a test particle from infinity

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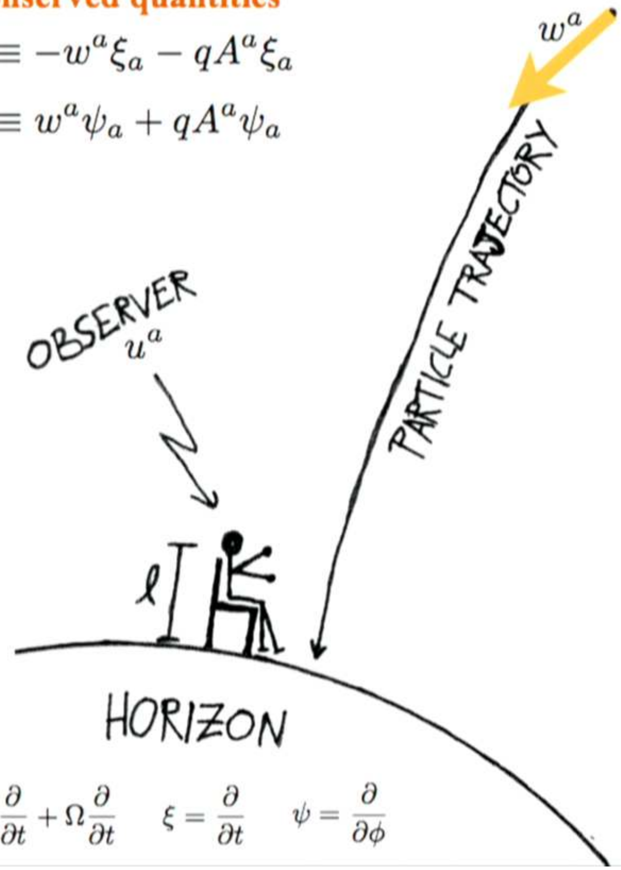


Particle at infinity

$$\mathcal{E} = -w^a \xi_a|_\infty \equiv \text{energy}$$

$$L = w^a \Psi_a|_\infty \equiv \text{angular momentum}$$

$$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \quad \xi = \frac{\partial}{\partial t} \quad \psi = \frac{\partial}{\partial \phi}$$

After absorption
seen by a local observer

$$\frac{\kappa}{8\pi}\delta A = \mathcal{E} - \Omega L - \Phi q$$

$$\mathcal{E}_{\ell oc} = \frac{\mathcal{E} - \Omega L - q\Phi}{\|\chi\|}$$

$$\mathcal{E}_{loc} = \frac{\kappa}{8\pi\|\chi\|}\delta A$$

$$\mathcal{E}_{loc} = \frac{\overline{\kappa}}{8\pi}\delta A$$

$$\overline{\kappa} \equiv \frac{\kappa}{\|\chi\|}$$

The appropriate local energy notion must be the one such that:

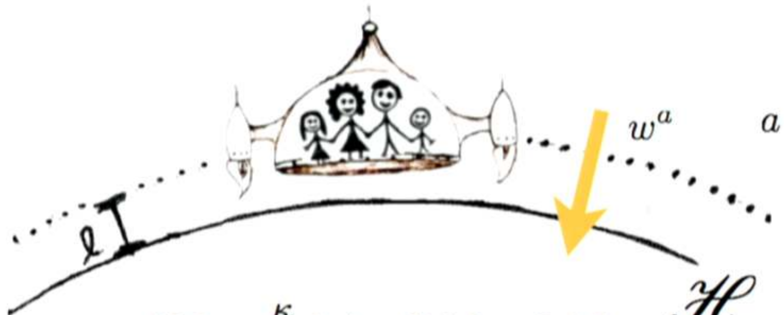
$$\delta E = \mathcal{E}_{\ell oc}$$

$$\boxed{\delta E = \frac{\overline{\kappa}}{8\pi}\delta A}$$

Local first law

Main classical result

$\ell^2 << A$



$a = ||u^a \nabla_a u^b|| = \frac{1}{\ell}$

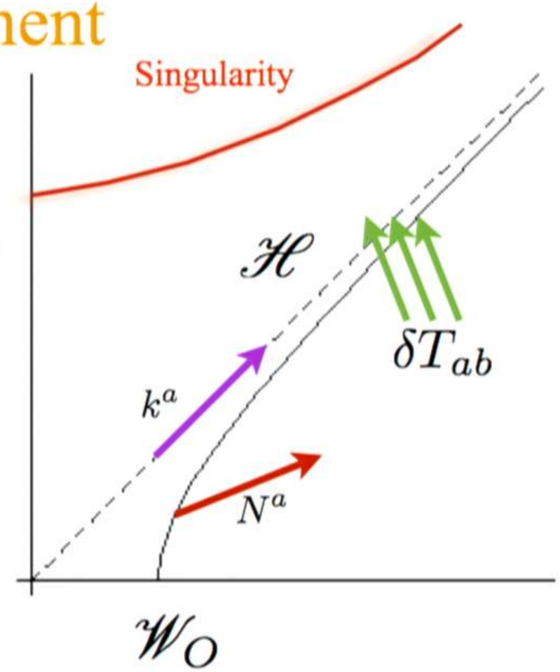
$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega\,\delta J + \Phi\,\delta Q,$$

$$\delta E = \frac{\overline{\kappa}}{8\pi}\delta A \qquad \overline{\kappa} \equiv \frac{\kappa}{||\chi||} = \frac{1}{\ell} + o(\ell)$$

$$E = \frac{A}{8\pi\ell} + E_0$$

Local first law

A refined argument



$J^a = \delta T^a{}_b \chi^b$ is conserved thus

$$\int_{\mathscr{H}} dV\, dS\ \delta T_{ab} \chi^a k^b = \int_{W_{\mathscr{O}}} J_b N^b$$

$$\int_{\mathscr{H}} dV\, dS\ \delta T_{ab} \underbrace{\kappa V k^a}_{\chi^a} k^b = \int_{W_{\mathscr{O}}} \| \chi \|\, \delta T_{ab} u^a N^b$$

The Raychaudhuri equation

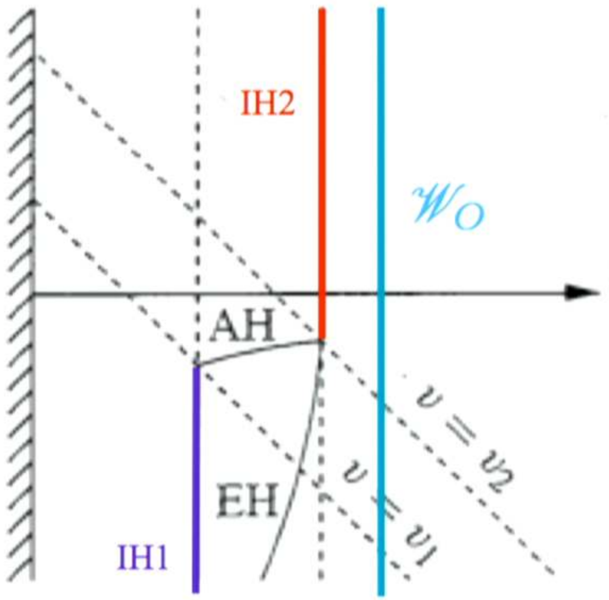
$$\frac{d\theta}{dV} = -8\pi \delta T_{ab} k^a k^b$$

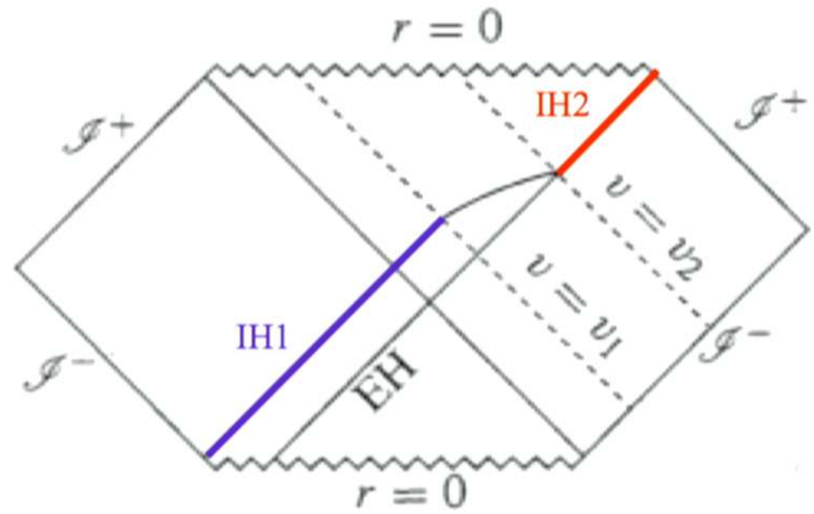
$$\int_{\mathscr{H}} dV\, dS\ V \frac{d\theta}{dV} = -\frac{8\pi \| \chi \|}{\kappa} \delta E,$$

$$\delta E = \frac{\overline{\kappa}}{8\pi} \delta A$$

The Local first law is dynamical

Simple example: Vaidya spacetime





The same holds in non symmetric situations (detailed proof in progress AP, O. Moreschi, E. Gallo)

$$\delta E = \frac{\overline{\kappa}}{8\pi}\delta A$$

Implications for the quantum theory

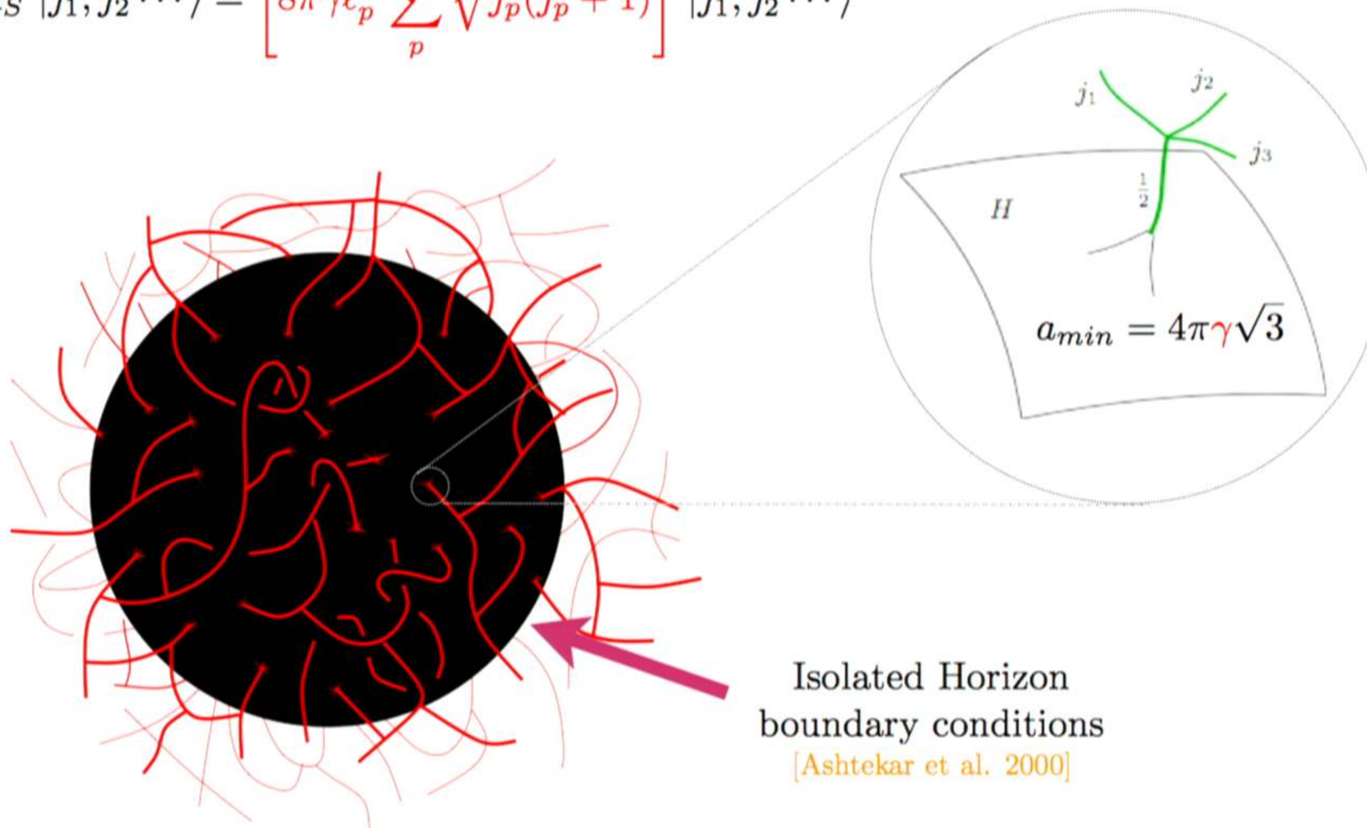
The quasilocal approach: insights into the statistical mechanical origin of BH entropy

[Ghosh, Perez, 2011 PRL]
[Frodden, Ghosh, Perez, to appear]

The black hole area spectrum

The area gap

$$\widehat{A}_S \; |j_1, j_2 \cdots\rangle = \left[8\pi\gamma\ell_p^2 \sum_p \sqrt{j_p(j_p+1)} \right] |j_1, j_2 \cdots\rangle$$



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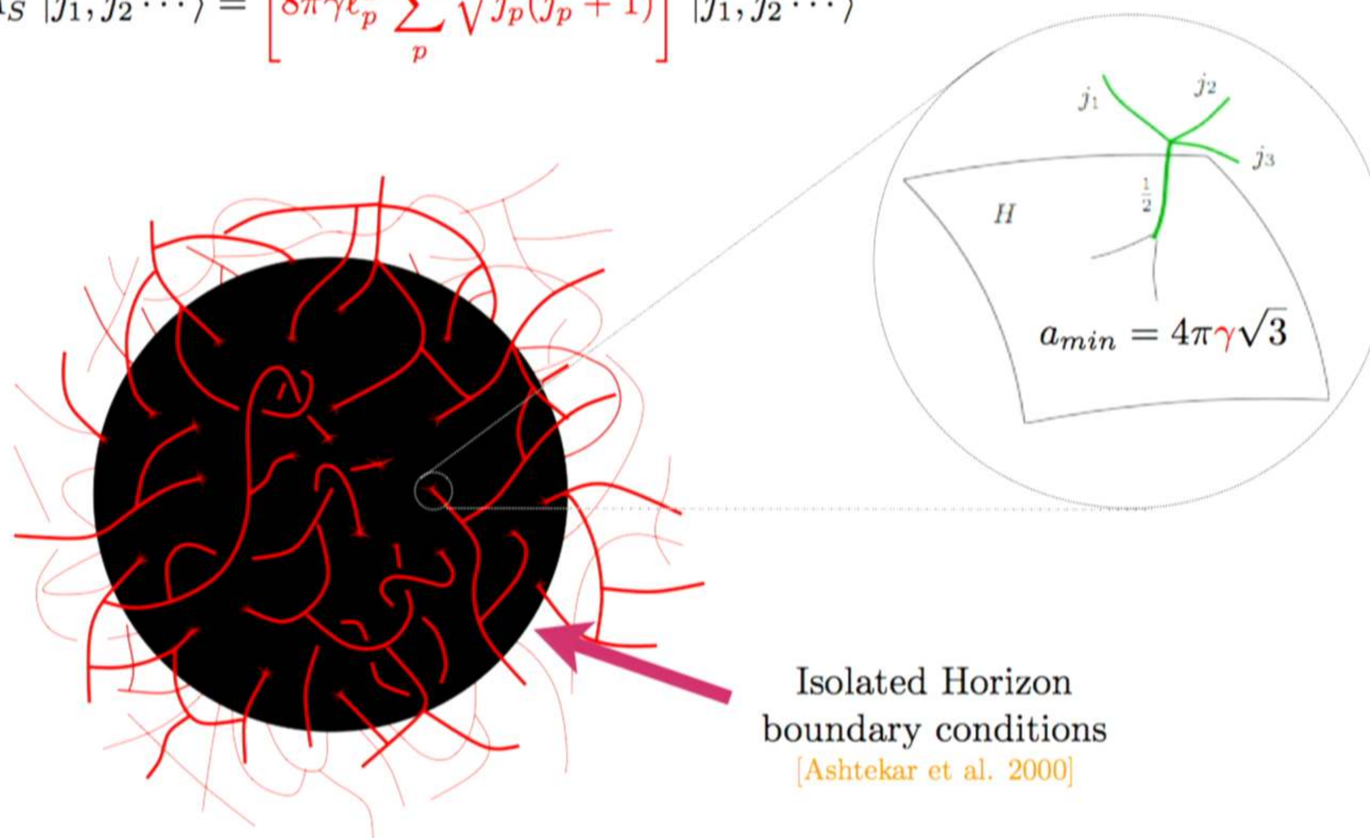
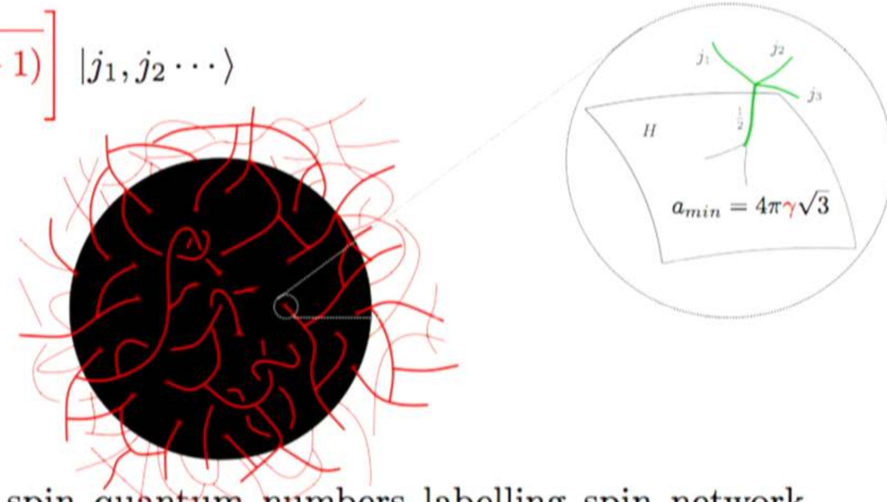


Is the number of punctures an important observable?

Energy Spectrum vs. Chemical Potential

$$\hat{H}|j_1, j_2 \cdots\rangle = \left[\frac{\gamma \ell_p^2}{\ell} \sum_p \sqrt{j_p(j_p+1)}\right] |j_1, j_2 \cdots\rangle$$



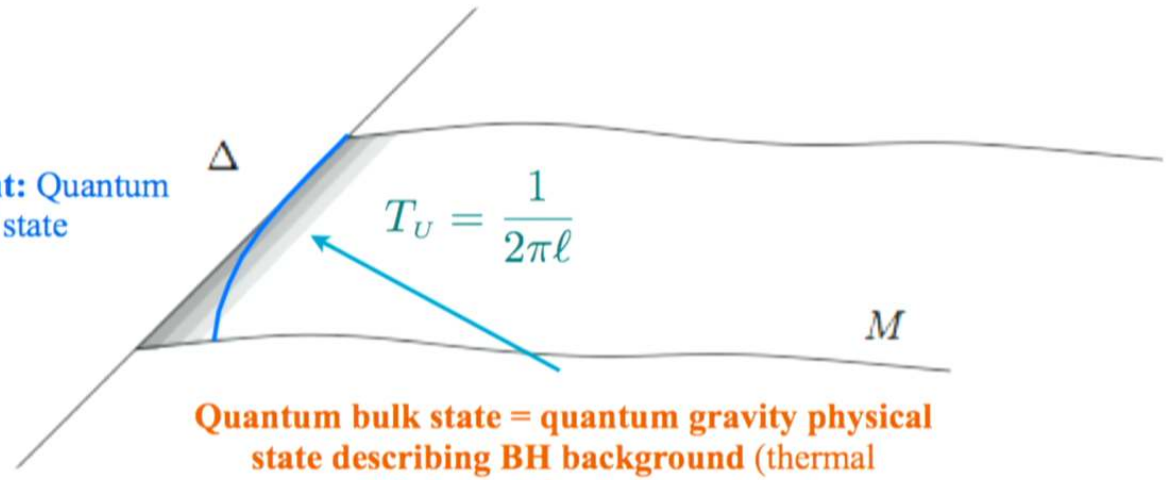
a) By a rearrangement of the spin quantum numbers labelling spin network links ending at punctures on the horizon without changing the number of punctures N (in the large area regime this kind of transitions allows for area jumps as small as one would like as the area spectrum becomes exponentially dense in $\mathbb{R}^+$ [Rovelli 96]

b) By the emission or absorption of punctures with arbitrary spin (such transitions remain discrete at all scales and are responsible for a modification of the first law: a chemical potential arises and encodes the mean value of the area change in the thermal mixture of possible values of spins j).

Computation of entropy in LQG

pure geometry calculation

Present ingredient: Quantum IH physical state



Quantum bulk state = quantum gravity physical state describing BH background (thermal equilibrium at Unruh temperature)

[for argument about how to perhaps avoid this assumption see Bianchi 2012 and Pranzetti 2013]

$\ell \equiv$ arbitrary fixed proper distance to the horizon

Black Hole Entropy from LQG

Pure gravity calculation
(neglecting matter contributions);
distinguishable punctures

Number of punctures contribute to S

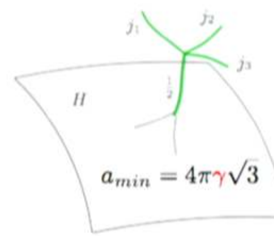
Distinguishability

The canonical partition function is given by

$$Z(N,\beta) = \sum_{\{s_j\}} \prod_j \frac{N!}{s_j!} [(2j+1)]^{s_j} e^{-\beta s_j E_j} \qquad \Longrightarrow \qquad \log Z = N \log[\sum_j [(2j+1)] e^{-\beta E_j}]$$

where $E_j = \ell_g^2 \sqrt{j(j+1)}/\ell$. A simple calculation gives

$$S = -\,\beta^2 \frac{\partial}{\partial \beta}(\frac{1}{\beta}\log Z)\Big|_{\beta=2\pi\ell} = \frac{A}{4\ell_p^2} + \log Z$$



more precisely

$$S = \frac{A}{4\ell_p^2} + \sigma(\gamma)N \qquad \text{where} \qquad \sigma(\gamma) \equiv \log[\sum_j (2j+1) e^{-2\pi\gamma\sqrt{j(j+1)}}].$$

The (thermodynamical) local first law versus the (geometric) local first law

$$\delta M = \frac{\kappa}{2\pi}\,\delta S + \Omega\,\delta J + \Phi\,\delta Q + \mu\,\delta N \qquad \Longleftrightarrow \qquad \delta M = \frac{\kappa}{2\pi}\,\delta A + \Omega\,\delta J + \Phi\,\delta Q$$

$$\boxed{\mu = -T\frac{\partial S}{\partial N}|_A = -\frac{\kappa}{2\pi}\sigma(\gamma)}$$

Number of punctures: an important observable

The canonical partition function is given by

$$Z(N,\beta) = \sum_{\{s_j\}} \prod_j \frac{N!}{s_j!}\,[(2j+1)]^{s_j} e^{-\beta s_j E_j} \qquad \Longrightarrow \qquad \log Z = N \log[\sum_j [(2j+1)] e^{-\beta E_j}]$$

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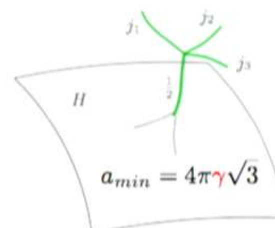

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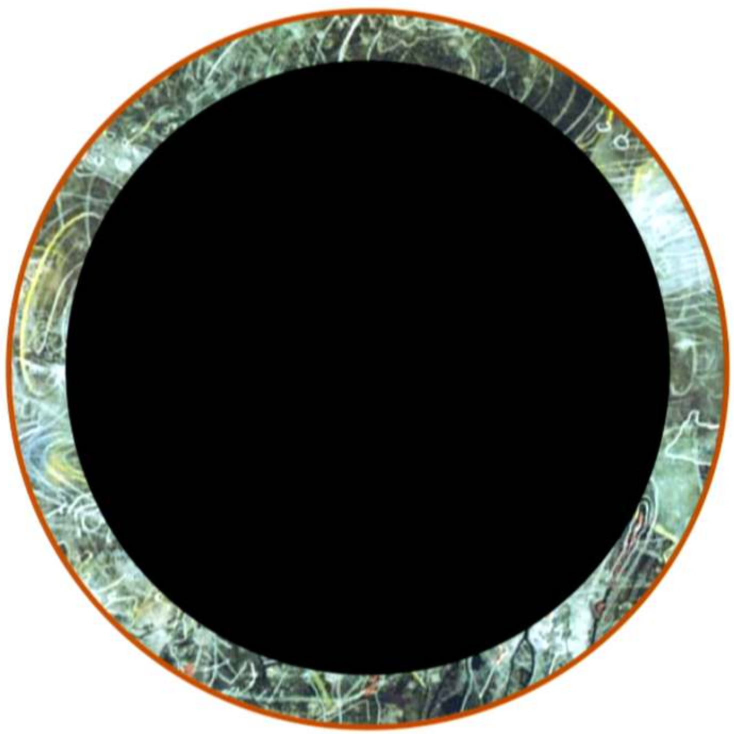
What about matter?

Matter entanglement, t'Hooft brick wall model, etc



$$S_{matter} = \lambda \frac{A}{\epsilon^2} + \text{corrections}$$

$\lambda =$ undertermined constant
(UV regularization dependent
species problem)

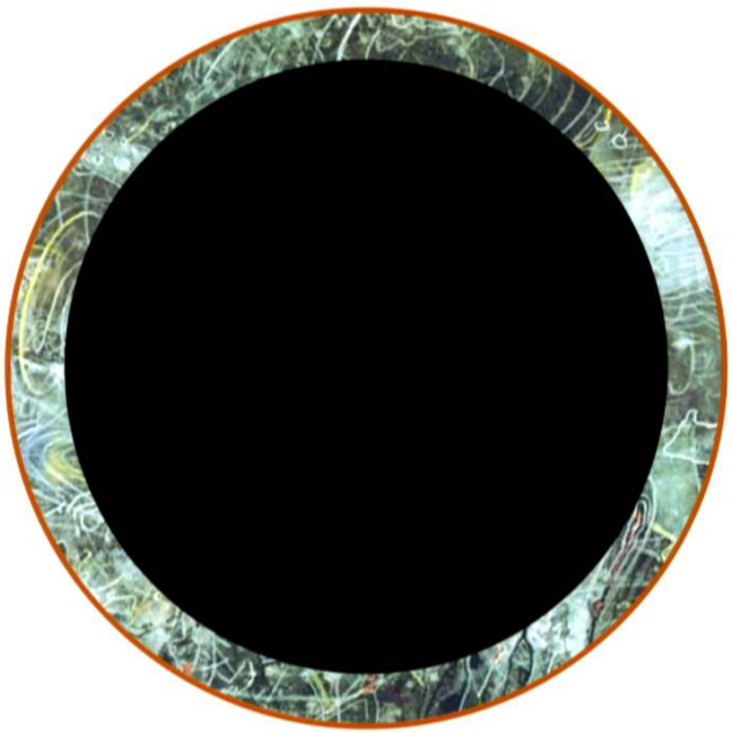
$\epsilon =$ UV cut-off

Number of d.o.f. dominated by boundary contribution

$$D \approx \exp(\lambda A/(4\ell_p^2))$$

What about matter?

Matter entanglement, t'Hooft brick wall model, etc



$$S_{matter} = \lambda \frac{A}{\epsilon^2} + \text{corrections}$$

In LQG: Energy=Area

Matter d.o.f. = degeneracy of area spectrum

$$\epsilon = \ell_p$$

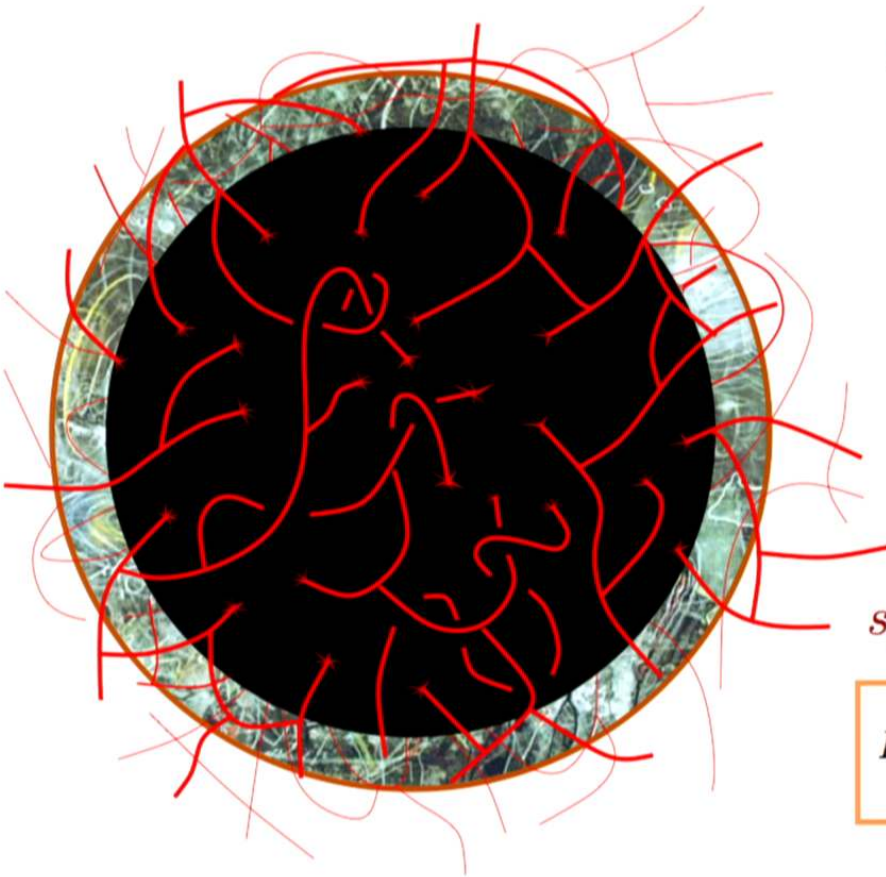
Just a new notation $\lambda = \frac{1-\delta}{4}$

Degeneracy grows exponentially with A

$$D \approx \exp\left[\lambda \frac{A}{\epsilon^2}\right]$$

What about matter?

Matter entanglement, t'Hooft brick wall model, etc



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In LQG: Energy=Area

Matter d.o.f. = degeneracy of area spectrum

$$\epsilon = \ell_p$$

Just a new notation $\lambda = \frac{1-\delta}{4}$

$s_j \equiv$ **number of punctures with spin j**

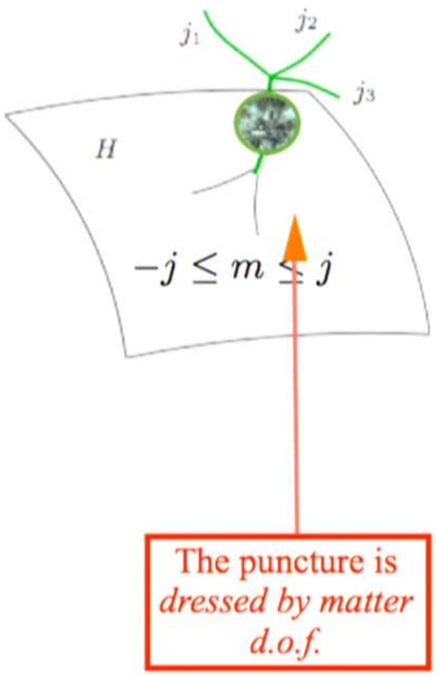
$$D[\{s_j\}] \approx \prod_j \exp \frac{(1-\delta) a_j s_j}{4\ell_p^2}$$

Black Hole Entropy from LQG

Gravity+Matter; indistinguishable punctures

$$Z[\beta] = \sum_{\{s_j\}} \prod_j e^{-(\beta - \beta_U + \delta\beta_U) s_j E_j}$$

$$\beta_U = 2\pi\ell$$



$$Z[\beta_U] = \prod_j [1 \pm \exp(-2\pi\gamma\delta j)]^{\pm 1}$$

$$U = \frac{A}{8\pi\ell} = -\partial_\beta \log Z$$

$$= \frac{\gamma \ell_p^2}{\ell} \sum_{j=1/2}^{\infty} \frac{j}{\exp(2\pi\gamma\delta j) \pm 1}$$

$$A \approx \frac{4\ell_p^2}{\pi\gamma\delta^2} \int_0^\infty \frac{x\,dx}{e^x \pm 1} = \frac{\epsilon_\pm \pi \ell_p^2}{3\gamma\delta^2}$$

$$\delta = \sqrt{\frac{\pi\epsilon_\pm \ell_p^2}{3\gamma A}} \ll 1$$

Black Hole Entropy from LQG

Thermal state is a semiclassical low energy state

$$Z[\beta] = \sum_{\{s_j\}} \prod_j e^{-(\beta - \beta_U + \delta\beta_U) s_j E_j}$$

$\tilde{\beta} \equiv \beta - \beta_U(1-\delta) \ll 1$

Semiclassical and low energy regime

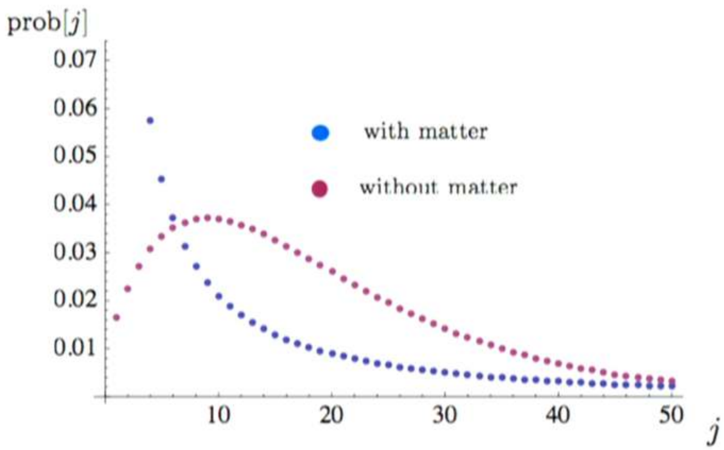
$$\ell_p^2 \ll \bar{a} = \ell_p^2 \langle j \rangle \ll A$$

Thiemann, Sahlmann, Winkler (2001)
Ashtekar et al. (2001)

Han et al. (2012) see spin foam talk

$$\langle N \rangle \approx \sqrt{A/\ell_p^2}, \quad \langle j \rangle \approx \sqrt{A/\ell_p^2},$$

$$\Delta j \approx \sqrt{A/\ell_p^2}$$



Q: can we get a more explicit manifestation that we are indeed in the semiclassical regime?

Black Hole Entropy from LQG

Thermal state is a semiclassical low energy state

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$\tilde{\beta} \equiv \beta - \beta_U(1-\delta) \ll 1$

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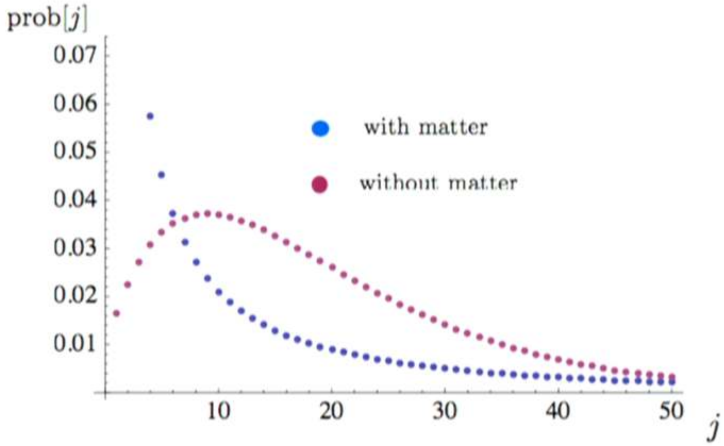

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$$\tilde{\beta} \equiv \beta - \beta_U(1-\delta) \ll 1$$

Semiclassical and low energy regime

$$\ell_p^2 \ll \bar{a} = \ell_p^2 \langle j \rangle \ll A$$

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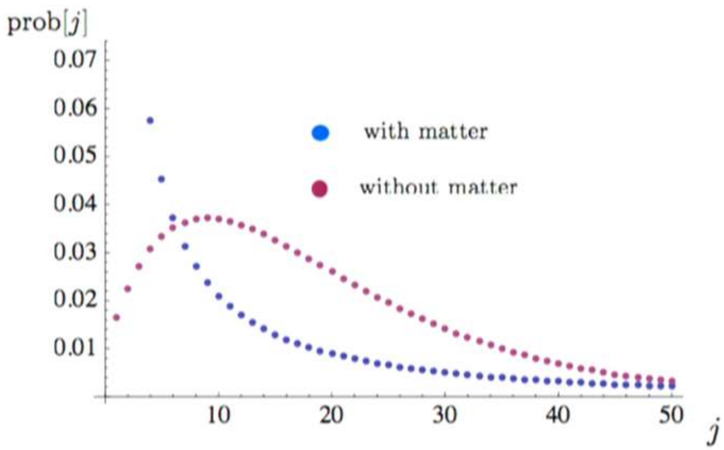


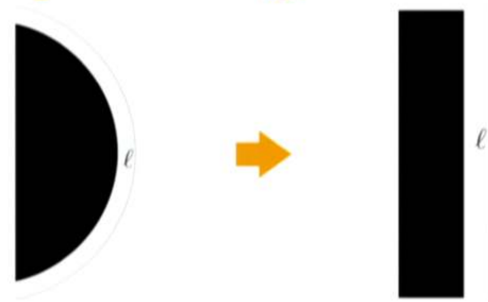
Q: can we get a more explicit manifestation that we are indeed in the semiclassical regime?

Semiclassical correspondence

relationship with the Euclidean path integral

$$Z[\beta] = \sum_N \sum_{\{s_j\}} e^{-\sum_j (\beta - \beta_U + \delta\beta_U) s_j E_j}$$

$$\approx \sum_N \sum_{\{s_j\}} e^{-(\beta - 2\pi\ell) \sum_j s_j \frac{a_j}{8\pi\ell_p^2 \ell}}$$



$$Z_{PI}[\beta] \equiv \int Dg_\beta^{(4)} \exp\Big[- \frac{1}{16\pi\ell_p^2} \int_M R[g^{(4)}] - \frac{1}{8\pi\ell_p^2} \int_{\partial M} (K - K_0) \Big]$$

$$\approx \exp\Big[- (\beta - 2\pi\ell) \frac{A[g^{(2)}]}{8\pi\ell_p^2 \ell} \Big]$$

[Banados-Teitelboim-Zanelli, Carlip-Teitelboim, etc]

[E. Frodden thesis]

$$Z[\beta] \approx Z_{PI}[\beta]$$

Semiclassical correspondence

relationship with the Euclidean path integral

$$Z[\beta] = \sum_N \sum_{\{s_j\}} e^{-\sum_j (\beta - \beta_U + \delta\beta_U) s_j E_j}$$

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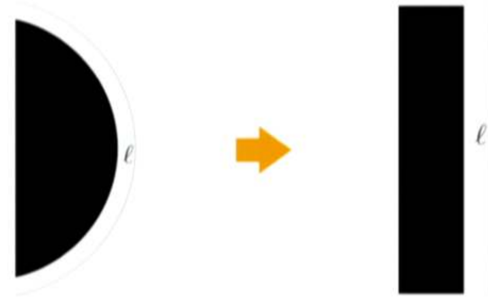

$$\Updownarrow$$

$$Z_{PI}[\beta] \equiv \int Dg_\beta^{(4)} \exp\Big[-\frac{1}{16\pi\ell_p^2} \int_M R[g^{(4)}] - \frac{1}{8\pi\ell_p^2} \int_{\partial M} (K - K_0) \Big]$$

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[Banados-Teitelboim-Zanelli, Carlip-Teitelboim, etc]

[E. Frodden thesis]

$$Z[\beta] \approx Z_{PI}[\beta]$$

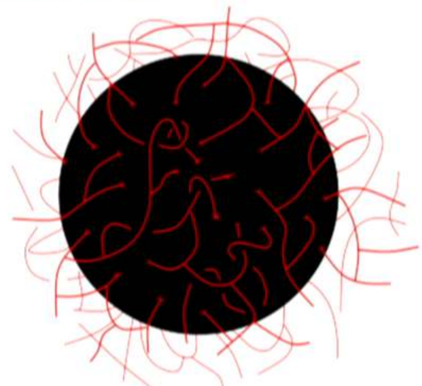
Conclusions

- The quasilocal approach captures the relevant physics for black hole thermodynamics.
- It is complementary to the *isolated horizon framework* of Ashtekar et. al.
- It provides an effective energy notion proportional to the horizon area.
- It holds for stationary horizons that are not necessarily asymptotically flat (no need to normalize killing fields in the derivation of the quasilocal first law)
- This energy notion can be used in the statistical mechanical description of the quantum horizon degrees of freedom.

Conclusions

If matter contributions are neglected, the quantum geometry degeneracy of the area spectrum implies *low spin dominance*.

Punctures taken as distinguishable.
[Rovelli 96, Ashtekar-Baez-Corichi-Krasnov 98, Pithis 2012]



$$S = \frac{A}{4\ell_p^2} + \sigma(\gamma)N$$

The chemical potential $\mu \neq 0$.

No clear how to establish the correspondence with the semiclassical low energy limit.

Outlook

➡ Can we have a microscopic description of the *holographic* matter degeneracy in LQG?

- Studies of matter coupling in LQG and the relationship with QFT
  [Ashtekar et al. and Thiemann et al. $\approx$ 2001]

- Chern-Simons quantum horizon for self dual gravity?
  [Smolin (1995), Krasnov (1996), Ashtekar-Baez-Corichi-Krasnov (2000), Engle-Noui-AP (2010)]

Analytic continuation to self dual variables

[Frodden-Geiller-Noui-AP (2012), Bodendorfer-Stottmeister-Thurn (2012), Pranzetti (2013)]

$$j \quad \Rightarrow \quad is - \frac{1}{2}$$

self dual representations satisfying reality condition $\hat{\Sigma} \cdot \hat{\Sigma} > 0$

$$D_k(j_1, \ldots, j_N) \quad \Rightarrow \quad i^{-p} D_k(is_1 - \frac{1}{2}, \ldots, is_p - \frac{1}{2}) = \frac{2}{k+2} \sum_{d=1}^{k+1} \sin^2\left(\frac{\pi d}{k+2}\right) \prod_{\ell=1}^{p} \frac{\sinh\left(\frac{2\pi d s_\ell}{k+2}\right)}{\sin\left(\frac{\pi d}{k+2}\right)}$$

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What about matter?

The vacuum in QFT

"We conclude that one has to attribute the black hole entropy not to the space-time metric itself but to the *quantized fields* present there [+gravity]... In short, the black hole entropy includes the entropy of the *quantized fields in its neighborhood*"

t'Hooft (1993)