Title: Black Holes - 2

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Abstract:


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July 22-26, 2013

Horace Hearne Jr.
Laboratory for Theoretical Physics
Louisiana State University

Complete quantization of vacuum spherically symmetric gravity

Jorge Pullin
Horace Hearne Institute
for Theoretical Physics
Louisiana State University

With Rodolfo Gambini
University of the Republic of Uruguay

LSU

There has been some progress in the past:

Kastrup and Thiemann (NPB399, 211 (1993)) using the "old" (complex) Ashtekar variables were able to quantize through a series of gauge fixings. The resulting quantization has waveforms $\Psi(M)$, with M being a Dirac observable. There is no sense in which the singularity is "resolved".

Kuchař (PRD50, 3961 (1994)) through a series of canonical transformation using the traditional metric variables isolated the single degree of freedom of the model (the ADM mass). Results similar to Kastrup and Thiemann's

Campiglia, Gambini and JP (CQG24, 3649 (2007)) using modern Ashtekar variables gauge fixed the diffeomorphism constraint and rescaled the Hamiltonian constraint to make it Abelian. The quantization ends up being equivalent to those of Kastrup, Thiemann and Kuchař.

Various authors (Modesto, Boehmer and Vandersloot, Ashtekar and Bojowald, Campiglia, Gambini, JP) studied the quantization of the interior of a black hole using the isometry to Kantowski-Sachs and treating it as a LQC. The singularity is resolved.

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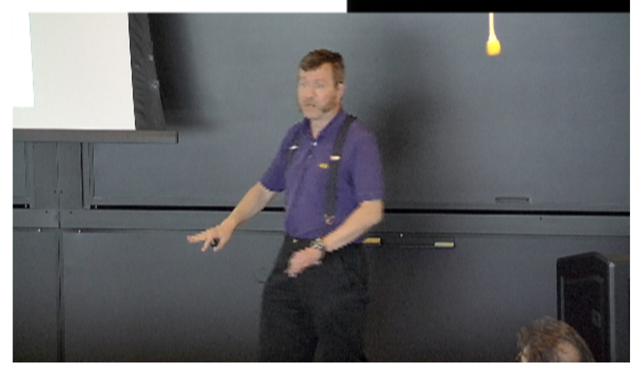

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Summary:

The main point today: one can **rescale the Hamiltonian without gauge fixing the diffeomorphism constraint**. The resulting constraint algebra is a Lie algebra.

$[D,D]=D, \quad [D,H]=H, \quad [H,H]=0$

The Dirac quantization using loop quantum gravity techniques can be completed in exact form, finding the space of physical states H_{phys}.

The metric can be represented as an operator corresponding to an evolving constant of the motion on H_{phys} and the singularity is resolved

We use the variables adapted to spherical symmetry developed by Bojowald and Swiderski (CQG23, 2129 (2006)). One ends up with two canonical pairs, E^x, E^φ, K_x, K_φ.

$$g_{xx} = \frac{(E^\varphi)^2}{|E^x|}, \qquad g_{\theta\theta} = |E^x|,$$

$$K_{xx} = -\text{sign}(E^x)\frac{(E^\varphi)^2}{\sqrt{|E^x|}} K_x \qquad K_{\theta\theta} = -\sqrt{|E^x|}\frac{A_\varphi}{2\gamma},$$

$$H_T = N\left[-\frac{E^\varphi}{2\sqrt{E^x}} - 2K_\varphi\sqrt{E^x}K_x - \frac{E^\varphi K_\varphi^2}{2\sqrt{E^x}} + \frac{((E^x)')^2}{8\sqrt{E^x}E^\varphi} - \frac{\sqrt{E^x}(E^x)'(E^\varphi)'}{2(E^\varphi)^2} + \frac{\sqrt{E^x}(E^x)''E^x}{2E^\varphi}\right] + N_r\left[-(E^x)'K_x + E^\varphi(K_\varphi)'\right].$$

Rescaling the lapse and shift:

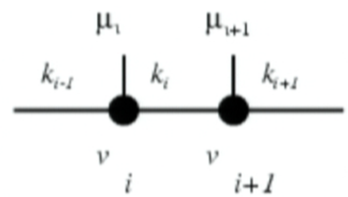
$$N_r^{\rm old} = N_r^{\rm new} - 2N^{\rm old}\frac{K_\varphi\sqrt{E^x}}{(E^x)'} \text{ and } N^{\rm old} = N^{\rm new}\frac{(E^x)'}{E^\varphi},$$

Yields the constraints with the Lie algebra structure:

$$H_T = \int dx \left[-N' \left(-\sqrt{E^x}\left(1+K_\varphi^2\right) + \frac{\left((E^x)'\right)^2 \sqrt{E^x}}{4\left(E^\varphi\right)^2} + 2GM \right) + N_r \left[-(E^x)' K_x + E^\varphi (K_\varphi)' \right] \right]$$

To proceed to quantize we again follow Bojowald and Swiderski and define suitable one-dimensional "spin networks"

$$T_{g,\vec{k},\vec{\mu}}(K_x, K_\varphi) = \langle K_x, K_\varphi \,\Big|$$



$$\Big\rangle$$

$$= \prod_{e_j \in g} \exp\left(\frac{i}{2} k_j \int_{e_j} K_x(x) dx \right) \prod_{v_j \in g} \exp\left(\frac{i}{2} \mu_j \gamma K_\varphi(v_j) \right)$$

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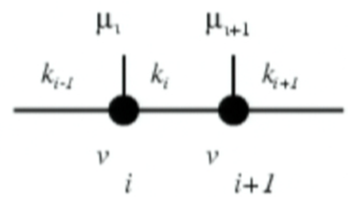

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On such states the triads are well defined

$$\begin{aligned}\hat{E}^x(x)T_{g,\vec{k},\vec{\mu}}(K_x,K_\varphi) &= \ell^2_{\rm Planck} k_i(x)T_{g,\vec{k},\vec{\mu}}(K_x,K_\varphi),\\ \hat{E}^\varphi(x)T_{g,\vec{k},\vec{\mu}}(K_x,K_\varphi) &= \ell^2_{\rm Planck}\sum_{v_i\in g}\delta(x-x(v_i))\mu_i T_{g,\vec{k},\vec{\mu}}(K_x,K_\varphi),\end{aligned}$$

And we proceed to polymerize and factor order the rescaled Hamiltonian constraint,

$$\hat{H}(N)=\int dx N(x)\left(2\left\{\sqrt{\sqrt{\hat{E}^x}\left(1+\frac{\sin^2\left(\rho\hat{K}_\varphi\right)}{\rho^2}\right)-2GM}\right\}\hat{E}^\varphi-\sqrt[4]{\hat{E}^x}\left(\hat{E}^x\right)'\right),$$

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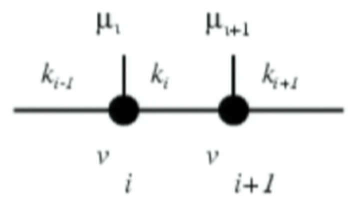

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And its action is well defined on the spin network states,

$$\hat{H}(N)T_{g,\vec{k},\vec{\mu}}(K_x,K_\varphi) = \sum_{v_i\in g} N(v_i)\left(k_i\ell^2_{\rm Planck}\right)^{\frac{1}{4}}\left[2\sqrt{1+\frac{\sin^2\left(\rho K_\varphi(v_i)\right)}{\rho^2}-\frac{2GM}{\sqrt{k_i\ell^2_{\rm Planck}}}}\ell^2_{\rm Planck}\mu_i - \left(k_i-k_{i-1}\right)\ell^2_{\rm Planck}\right]T_{g,\vec{k},\vec{\mu}}(K_x,K_\varphi).$$

And one can exactly solve it,

$$\Psi\left(K_\varphi,K_x,g,\vec{k}\right)=\Psi(M)\exp\left(f\left(K_\varphi,g,\vec{k}\right)\right)\Pi_{e_j\in g}\exp\left(\frac{i}{2}k_j\int_{e_j}K_x(x)dx\right),$$

$$f = \sum_{v_j\in g} -\tfrac{i}{2}\Delta K_j m_j F\left(\sin\left(\rho K_\varphi(v_j)\right), im_j\right),$$

$$\text{with}\quad \Delta K_j = K_\varphi(v_j) - K_\varphi(v_{j-1}),$$

$$m_j = \left[\rho\sqrt{1-2GM/\sqrt{k_j}\ell_{\rm Planck}}\right]^{-1}$$

$F(\phi,m) = \int_0^\phi \left(1-m^2\sin^2 t\right)^{-1/2}dt$ the Jacobi elliptic function of the first kind.

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The model has quantum observables without classical counterparts.

Since the basis of the physical space H_{phys} have a well defined number of vertices, one can construct a Dirac observable operator $\hat{V}$ with eigenvalue V, the number of vertices.

E^x is not well defined on H_{phys} as an operator, since it is not invariant under diffeomorphisms. However, since it must be a monotonous function of x, there is a portion of it that can be isolated as diffeo invariant.

One starts by noticing that the sequence $\vec{k}$ is well defined in H_{phys}

One defines a Dirac observable O(z) z in [0,1]

$$\hat{O}(z)|\vec{k},\tilde{g}\rangle_{\rm phys} = \ell^2_{\rm Planck} k_{{\rm Int}(Vz)}|\vec{k},\tilde{g}\rangle_{\rm phys},$$

And in terms of it and an arbitrary function from the real line to [0,1] z(x) one can define an action for E^x in H_{phys},

$$\hat{E}^x(x)|\vec{k},\tilde{g}\rangle_{\rm phys} = \hat{O}(z(x))|\vec{k},\tilde{g}\rangle_{\rm phys}.$$

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Recalling the form of the space-time metric, e.g., g_{tx},

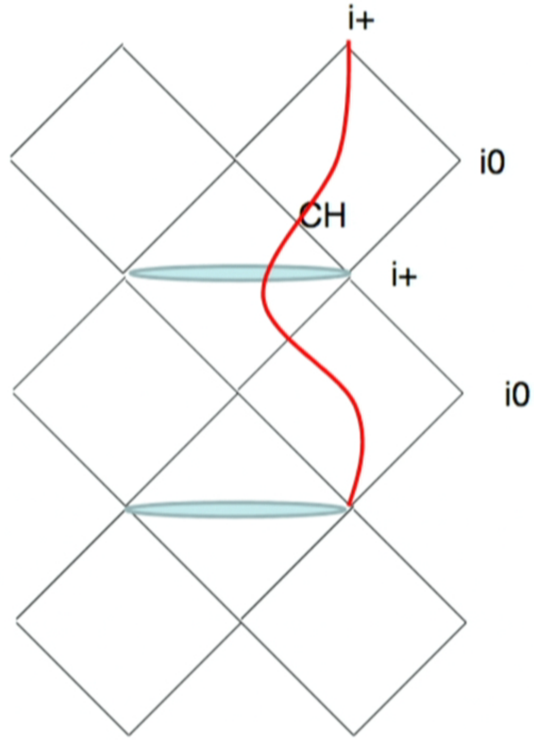
$$g_{tx} = g_{xx} N_r = -\frac{(E^x)' K_\varphi}{2\sqrt{E^x}\sqrt{1 + K_\varphi^2 - \frac{2GM}{\sqrt{E^x}}}},$$

One can straightforwardly write it as an evolving constant of motion acting on Hphys parameterized by the functional parameters Kφ and z(x).

In order to be a self-adjoint operator the radical should be Positive. This imposes limitations on the values of $\vec{k}$. The limitations imply that the metric is not singular where the classical singularity should be.

What kind of space-time emerges? It depends on the state.

If one wants a semi-classical space-time, one will have to choose $\Psi(M)$ peaked around some value of the mass, and one will need small jumps between k_i and k_{i+1}. The resulting geometry is distributional since Ex is only non-vanishing at vertices. One would be approximating a smooth function with
Dirac deltas.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Summary

- Rescaling the Hamiltonian constraint leads to a Lie algebra of constraints without the need to gauge fix.
- The Dirac quantization can be completed and the physical space of states found exactly.
- New quantum observables appear without classical counterpart.
- The metric can be realized on the space of physical states as an evolving constant of the motion.
- It is non-singular in the black hole interior and the space-time can be extended.
- It may open new possibilities for the “firewall” problem


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Falling into a black hole: the light from above

Matteo Smerlak

Max-Planck-Institut für Gravitationsphysik
(Albert-Einstein-Institut)

Loops 13
July 22, 2013

MS, S. Singh, "New perspectives on Hawking radiation", arXiv:1304.2858.

A very brief history of Hawking radiation

- Hawking 1974: black holes as black bodies

  "Any black hole will create and emit particles [...] at just the rate that one would expect if the black hole was a body with a temperature of $\kappa/2\pi$."

- Unruh 1976: response of infalling detectors

  "A geodesic detector near the horizon will not see the Hawking flux of particles"

- Almheiri *et al.* 2012: firewall argument

  "Perhaps the most conservative resolution is that the infalling observer burns up at the horizon."

Back to semiclassical Hawking radiation, and surprises

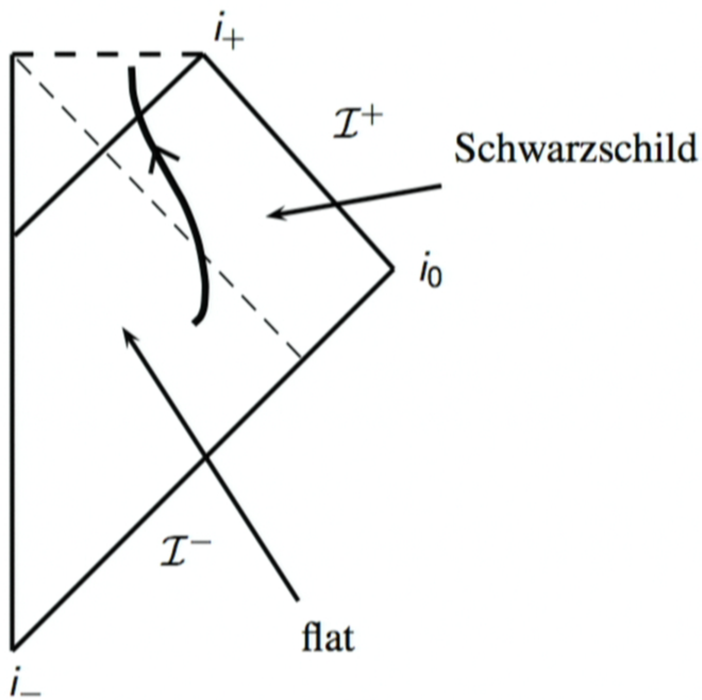
In this talk I wish to

- reconsider the semiclassics, à la Hawking and Unruh
- study non-asymptotic, non-stationary trajectories
- in particular, geodesics with orbital parameters (E, L)

I will show that

- geodesic detectors near the horizon do see Hawking radiation (actually more than at in infinity)
- in the $E \to 0$ limit, horizon radiation is both hot and intense
- the vacuum energy density does not have a definite sign

Setup and approximations



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  - no charge
  - flat in the past
- Field
  - massless
  - scalar
  - spherically symmetric (s-wave sector)
- Detector
  - point-like
  - monopole
  - weak coupling

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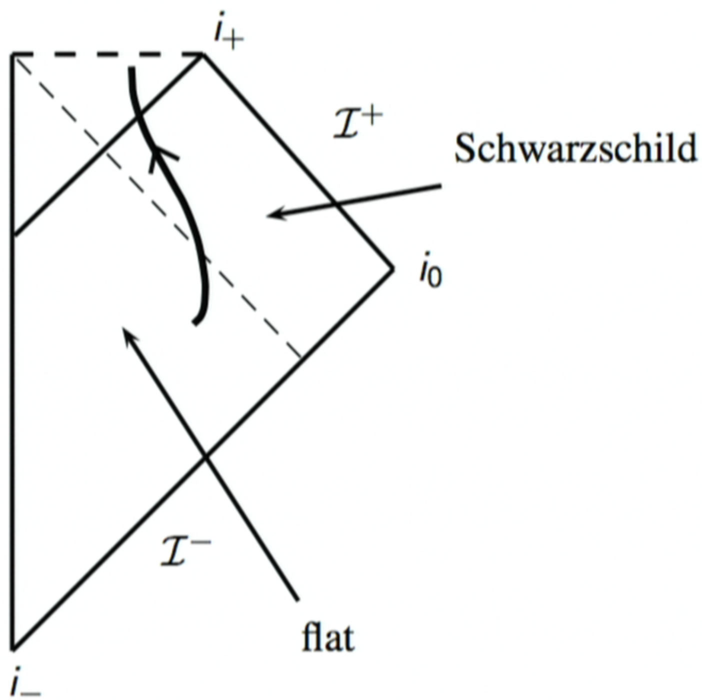

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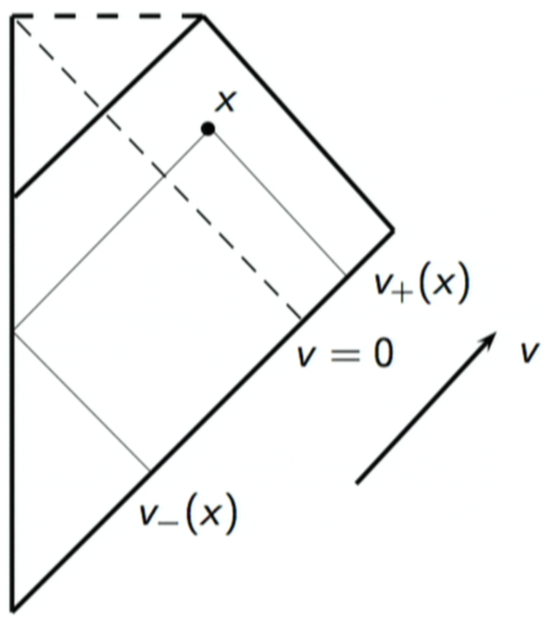
Collapse geometry: Vaidya ingoing shell



- Eddington-Finkelstein:

$$ds^2 = -\left(1 - \frac{r_s}{r}\Theta(v)\right) dv^2 + 2dvdr$$

- Null coordinates:

$$ds^2 = -C(v_+, v_-)\, dv_+ dv_-$$

  - constant v_+: incoming fronts
  - constant v_-: outgoing fronts
  - $v_+ = 0$: shell
  - $v_- = -2r_s$: horizon

- Vacuum state (in the s-wave sector):

$$G(x, y) \propto \ln\left((\Delta v_+ - i0)(\Delta v_- - i0)\right)$$

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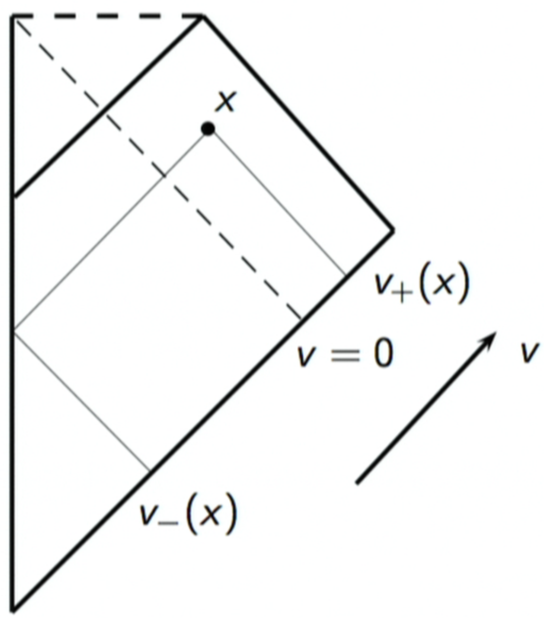

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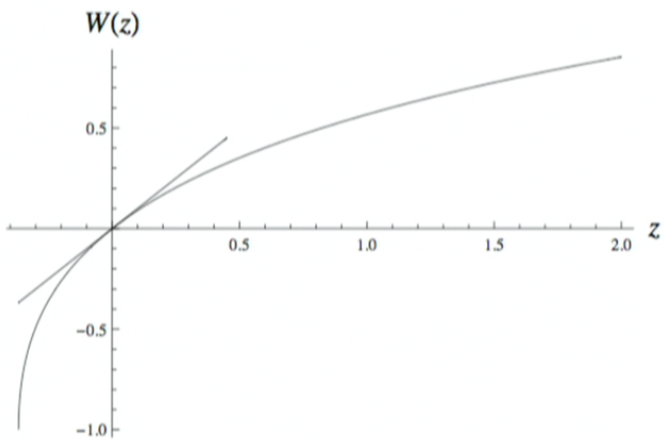
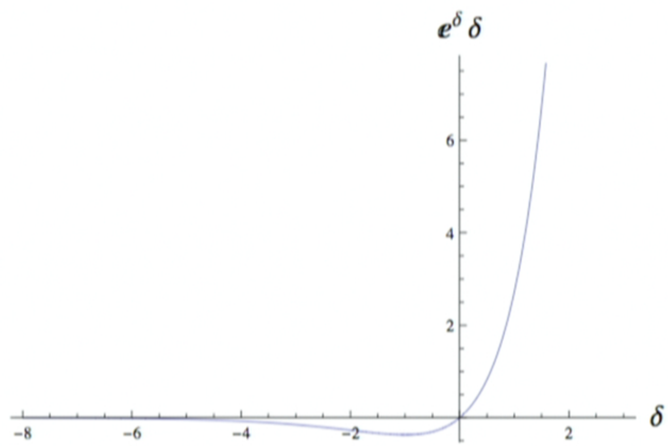
Structure of outgoing fronts I: the v_- coordinate

Setting $\delta = r/r_s - 1$,

$$v_-(v,r) = \begin{cases} v - 2r & \text{for } v < 0 \\ -2r_s\left[1 + W\left(\delta\, e^{\delta - \kappa v}\right)\right] & \text{for } v \geq 0, \end{cases}$$





Structure of outgoing fronts II: portrait of the vacuum

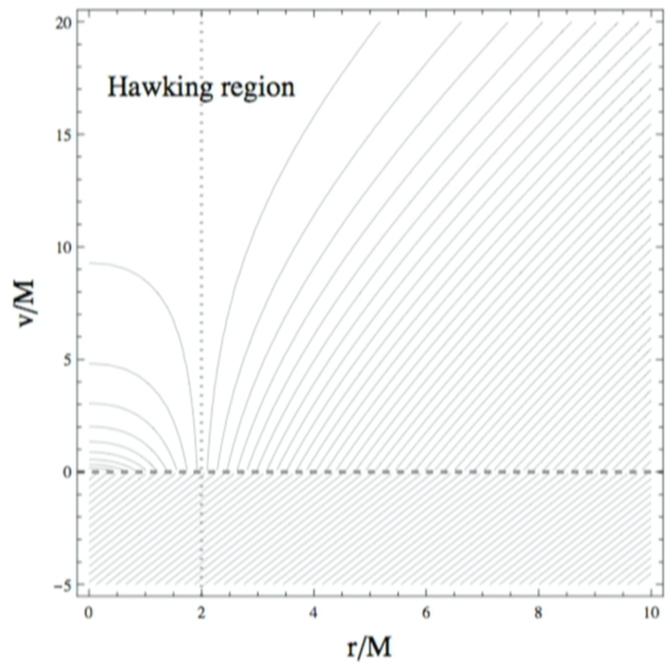


Figure: Level curves of $v_-(v, r)$.

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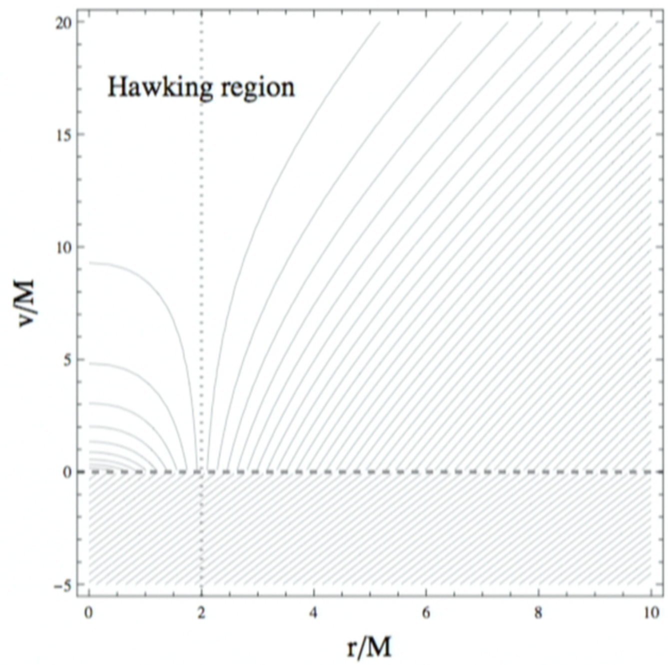


Figure: Level curves of $v_{-}(v, r)$.

Outline

Unruh-DeWitt response function

The response of an Unruh-DeWitt detector at time τ reads

$$\mathcal{R}(\tau,\Omega) = 2\,\mathrm{Re}\int_{-\infty}^{\infty} du\,\chi_\tau(u)\int_0^{\infty} ds\,\chi_\tau(u-s)\,e^{-i\Omega s}\,G\big(\gamma(u),\gamma(u-s)\big).$$

where

- $\gamma(s)$ is a timelike trajectory
- Ω is the energy gap (frequency) of the detector
- χ_τ a non-negative switching function such that

$$\int ds\,\chi_\tau(s) = 1 \qquad \text{and} \qquad \chi_\tau(s) \simeq 0 \quad \text{for} \quad s \geq \tau$$

Decoupling of incoming and outgoing

The splitting of the Wightman function

$$G(x,y) \propto \ln\Big((\Delta v_+ - i0)(\Delta v_- - i0)\Big) = \ln\Big(\Delta v_+ - i0\Big) + \ln\Big(\Delta v_- - i0\Big)$$

implies

$$\mathcal{R}(\tau,\Omega) = \mathcal{R}_+(\tau,\Omega) + \mathcal{R}_-(\tau,\Omega).$$

Incoming and outgoing modes decouple.

Thermal and quasi-thermal spectra

A stationary spectrum is thermal if the detailed balance relation holds:

$$\mathcal{R}(-\Omega) = e^{\Omega/T}\mathcal{R}(\Omega) \qquad \text{for any} \qquad \Omega.$$

I call a non-stationary spectrum quasi-thermal if

$$\mathcal{R}(\tau,-\Omega) \sim e^{\Omega/T(\tau)}\mathcal{R}(\tau,\Omega) \qquad \text{for} \qquad |\Omega| \gg T(\tau).$$

In this case I call T the quasi-temperature of the spectrum; it becomes a proper temperature in the adiabatic limit

$$\left|\frac{\dot{T}}{T^2}\right| \ll 1.$$

Sufficient condition for thermality

Since

$$\int_0^\infty ds\,\cos(\Omega s)\ln(s - i0) = 0 \qquad \text{for} \qquad \Omega > 0,$$

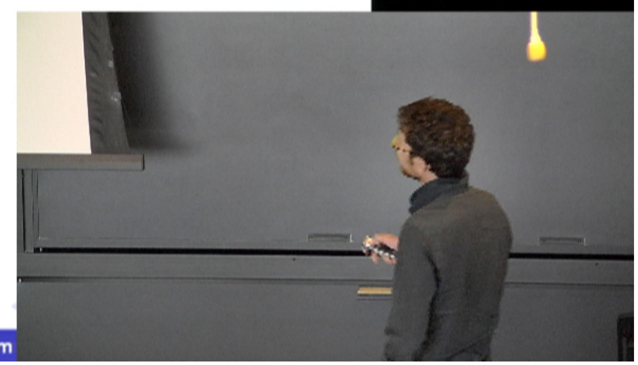
the first relevant term in $\ln(\Delta v_\pm - i0)$ is the second-order derivative of $\Delta v_\pm$ wrt s. In fact, when

$$T_\pm = \frac{1}{2\pi}\left|\frac{\ddot{v}_\pm}{\dot{v}_\pm}\right|$$

is constant, so that $v_\pm(\tau)$ is exponential in τ, the spectrum

$$\int_0^\infty ds\, e^{-i\Omega s}\log(\Delta v_\pm - i0) = 0$$

is thermal, at temperature $T_\pm$.



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Since

$$\int_0^\infty ds\, \cos(\Omega s) \ln(s - i0) = 0 \qquad \text{for} \qquad \Omega > 0,$$

the first relevant term in $\ln(\Delta v_\pm - i0)$ is the second-order derivative of $\Delta v_\pm$ wrt s. In fact, when

$$T_\pm = \frac{1}{2\pi}\left|\frac{\ddot{v}_\pm}{\dot{v}_\pm}\right|$$

is constant, so that $v_\pm(\tau)$ is exponential in τ, the spectrum

$$\int_0^\infty ds\, e^{-i\Omega s} \log(\Delta v_\pm - i0) = 0$$

is thermal, at temperature $T_\pm$.

Thermality for static observers

This is what happens with static trajectories outside the hole, where

$$\delta = \frac{r}{r_s} - 1 = \text{constant} > 0.$$

Indeed, from

$$v_-(v,r) = -2r_s\left[1 + W\left(\delta\, e^{\delta-\kappa v}\right)\right] \simeq -2r_s\left[1 + \delta\, e^{\delta-\kappa v}\right]$$

we see that

$$\left|\frac{\ddot{v}_-}{\dot{v}_-}\right| = \dot{v}\kappa = \frac{\kappa}{\sqrt{1 - r_s/r}}.$$

This gives the standard results

$$T_- = \frac{T_H}{\sqrt{1 - r_s/r}} \qquad \text{and} \qquad T_+ = 0.$$

Quasi-temperature formalism

For a more general, non-static trajectory, the quantities

$$T_{\pm}(\tau) = \frac{1}{2\pi}\left|\frac{\ddot{v}_{\pm}(\tau)}{\dot{v}_{\pm}(\tau)}\right|$$

are the quasi-temperatures of incoming and outgoing modes. The corresponding adiabaticity parameters are

$$\eta_{\pm}(\tau) = \frac{\dot{T}_{\pm}(\tau)}{T_{\pm}(\tau)^2}.$$

Advantages of this approach:

- easy to compute: just evaluate $\dot{v}_{\pm}$ and $\ddot{v}_{\pm}$ along the trajectory
- straightforward interpretation: ultraviolet decay rate of detector spectra $\mathcal{R}_{\pm}(\tau, \Omega)$

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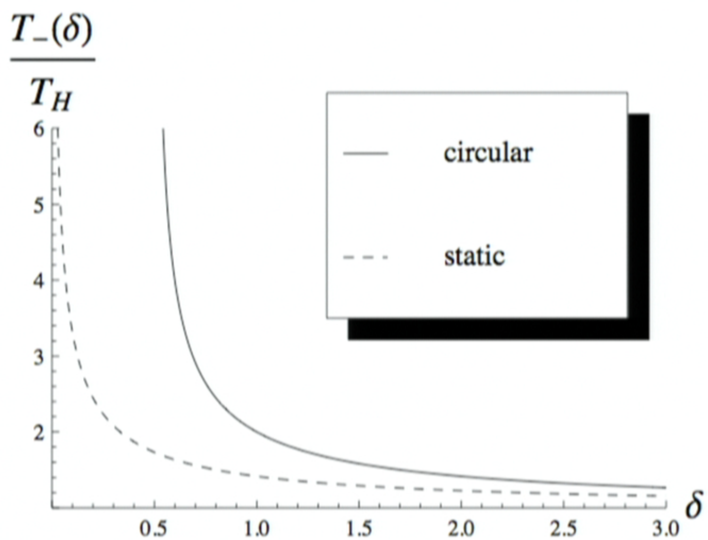
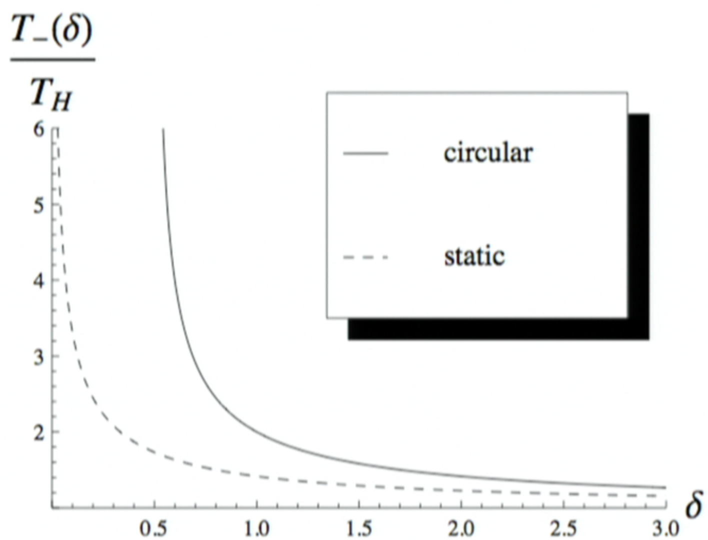
Schwarzschild geodesics I: circular orbits



$$T_- = T_H \left(\frac{1+\delta}{\delta - 1/2}\right)^{1/2}$$
$$T_+ = 0.$$

Temperature of outgoing modes higher than on the static trajectory, although $a = 0$.

Schwarzschild geodesics I: circular orbits



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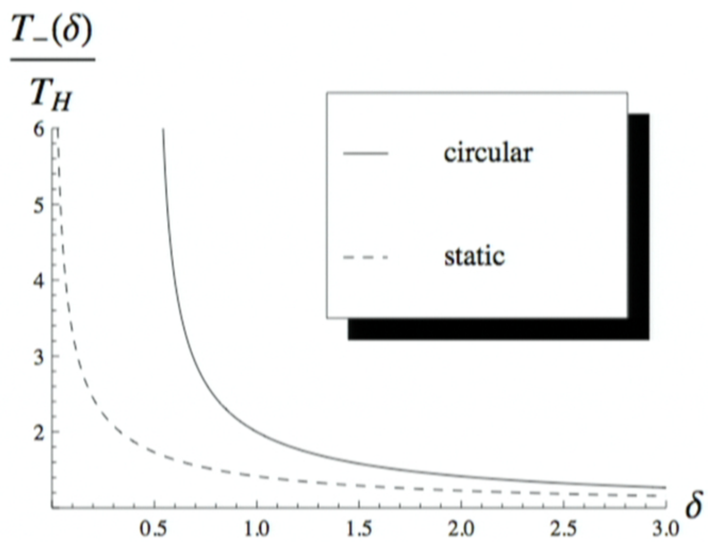

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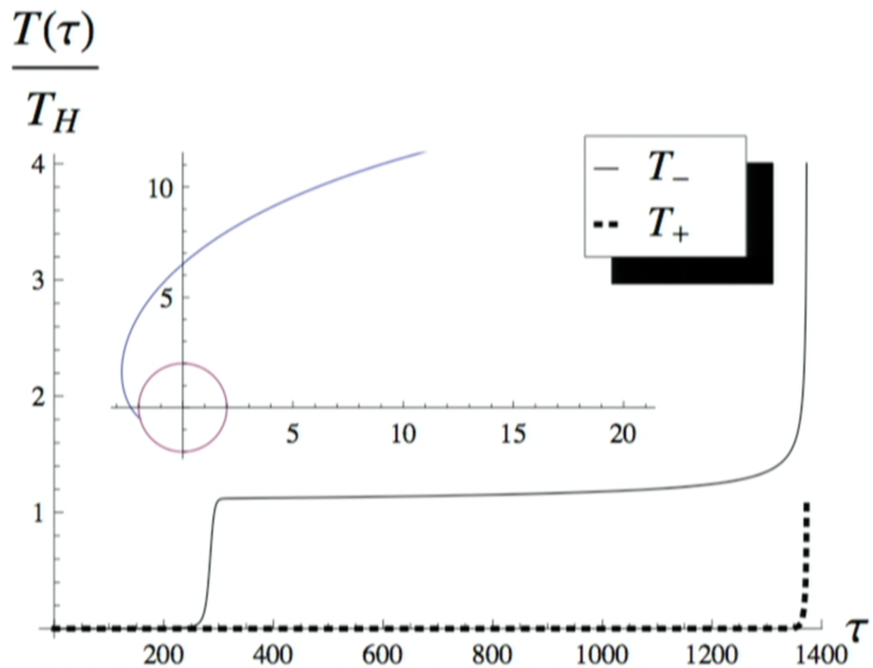
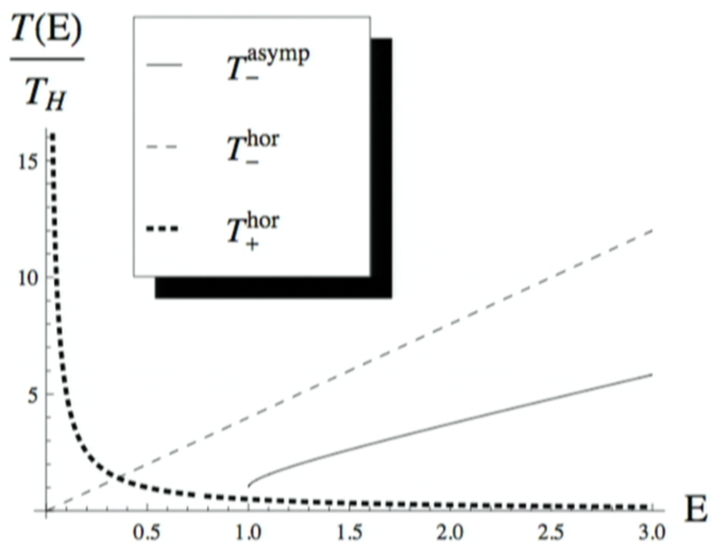
Schwarzschild geodesics II: infalling trajectories



Observe that ingoing modes couple near the horizon.

Schwarzschild geodesics III: radial trajectories



At horizon-crossing, we find

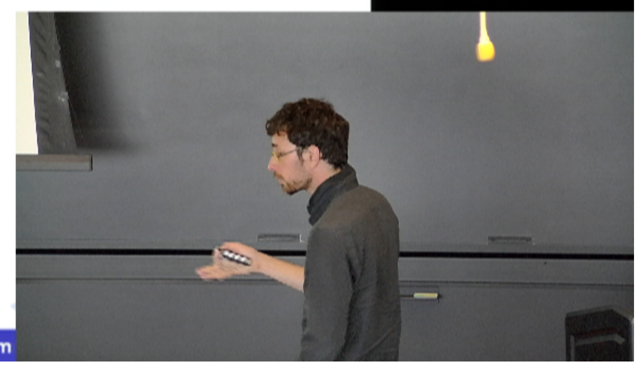
$$T_-^{\text{hor}} = 4ET_H$$
$$T_+^{\text{hor}} = \frac{T_H}{2E}$$

together with

$$\eta_-^{\text{hor}}(E) = \frac{\pi}{4}\left(2 + \frac{1}{E^2}\right)$$
$$\eta_+^{\text{hor}}(E) = 2\pi|1 - 8E^2|$$

- Large quasi-temperature for highly bound states ($E \ll 1$).
- Never actually thermal.

Outline

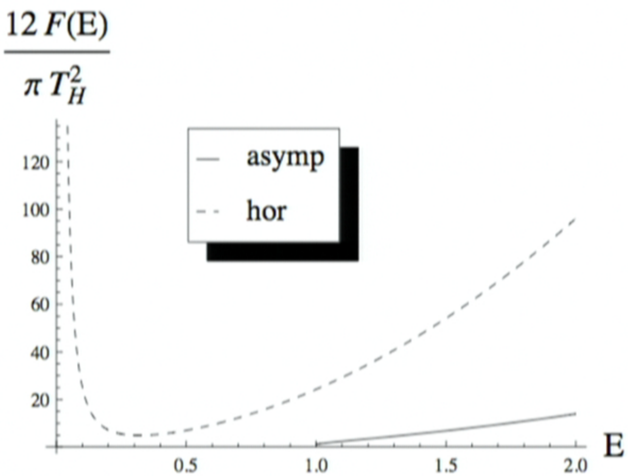


Flux

The high intensity of Hawking radiation as perceived by $E \to 0$ observers is confirmed by a flux computation:

$$\mathcal{F}(E) = -\langle T_{ab} \rangle \, u^a n^b$$

where u^a geodesic 4-velocity and n^b unit normal to u^a.



At horizon-crossing,

$$\mathcal{F}^{\rm hor}(E) = \pi T_H^2 \left(2E^2 + \frac{1}{48E^2} \right).$$

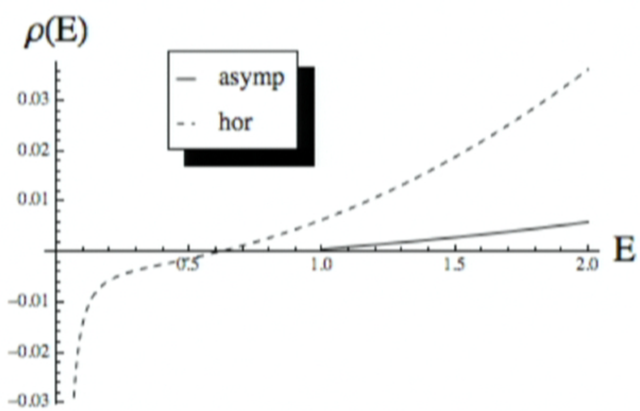
Large ingoing flux in the $E \to 0$ limit.

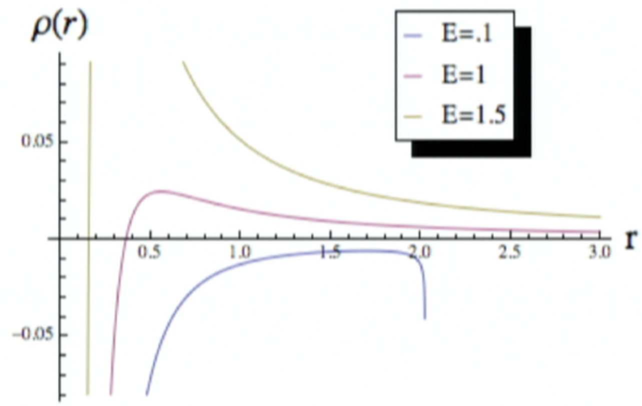
Energy density

We can also compute the energy density measured by infalling observers:

$$\rho(E) = \langle T_{ab} \rangle \, u^a u^b$$

It does not have a definite sign.





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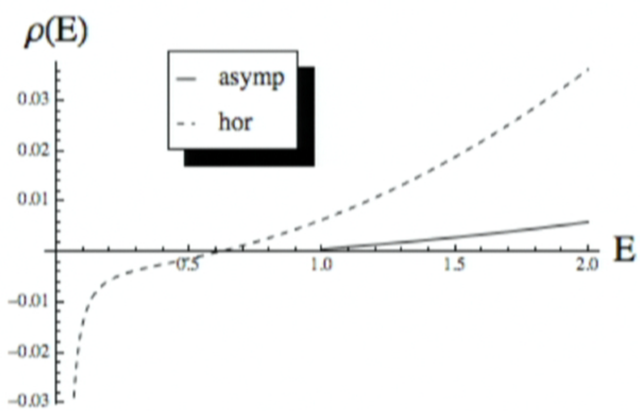

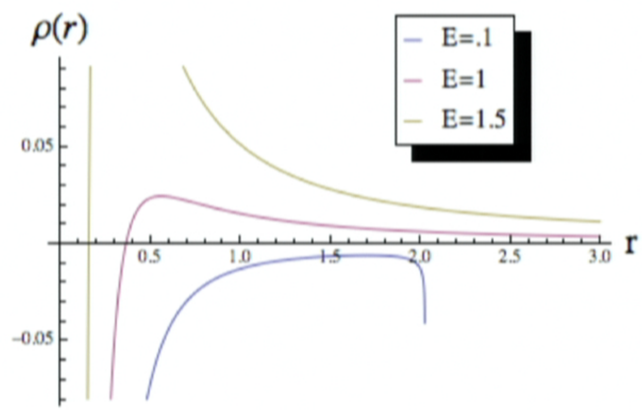

Punchlines

- For highly bound trajectories, Hawking radiation is dominated by ingoing modes and becomes arbitrarily hot as $E \to 0$
- In this $E \to 0$ limit,
  - the outgoing flux goes to positive infinity
  - the energy density goes to negative infinity
- But not all observers measure a negative energy density close to the horizon. Observers rushing into the hole at high velocity see a high positive energy density.
- This is a purely semiclassical effect. No conceptual connection with any "firewall" argument whatsoever.

Thank you!


Loops13.Poster.pdf (page 1 of 1)
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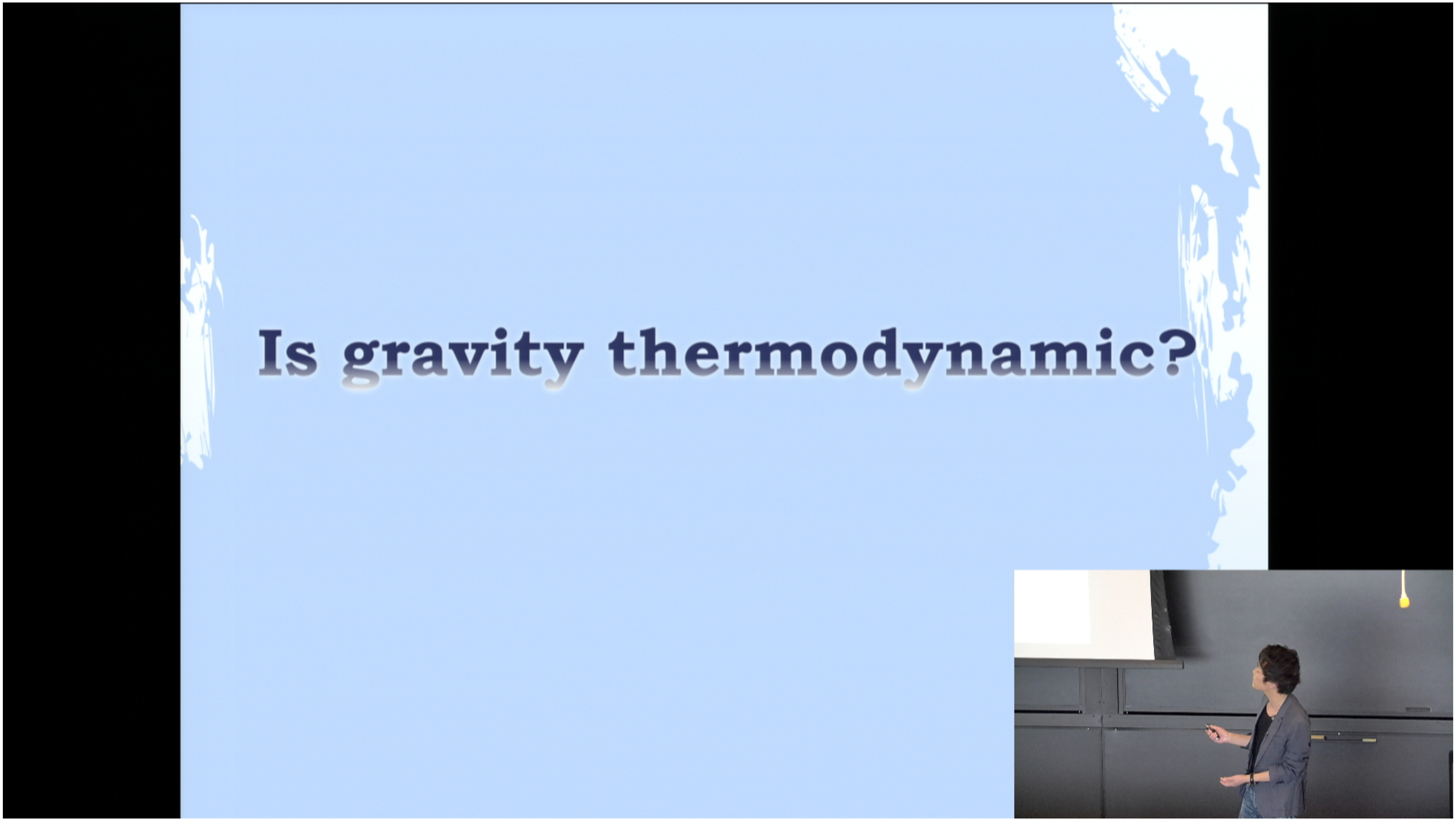
July 22-26, 2013

Fluid-Gravity Duality for a General Screen

Kyoto University
Yuki Yokokura

(with Laurent Freidel, Perimeter Institute)

In Loops13 @ Perimer Institute


Is gravity thermodynamic?

Dynamical “object”

Described by curved spacetime

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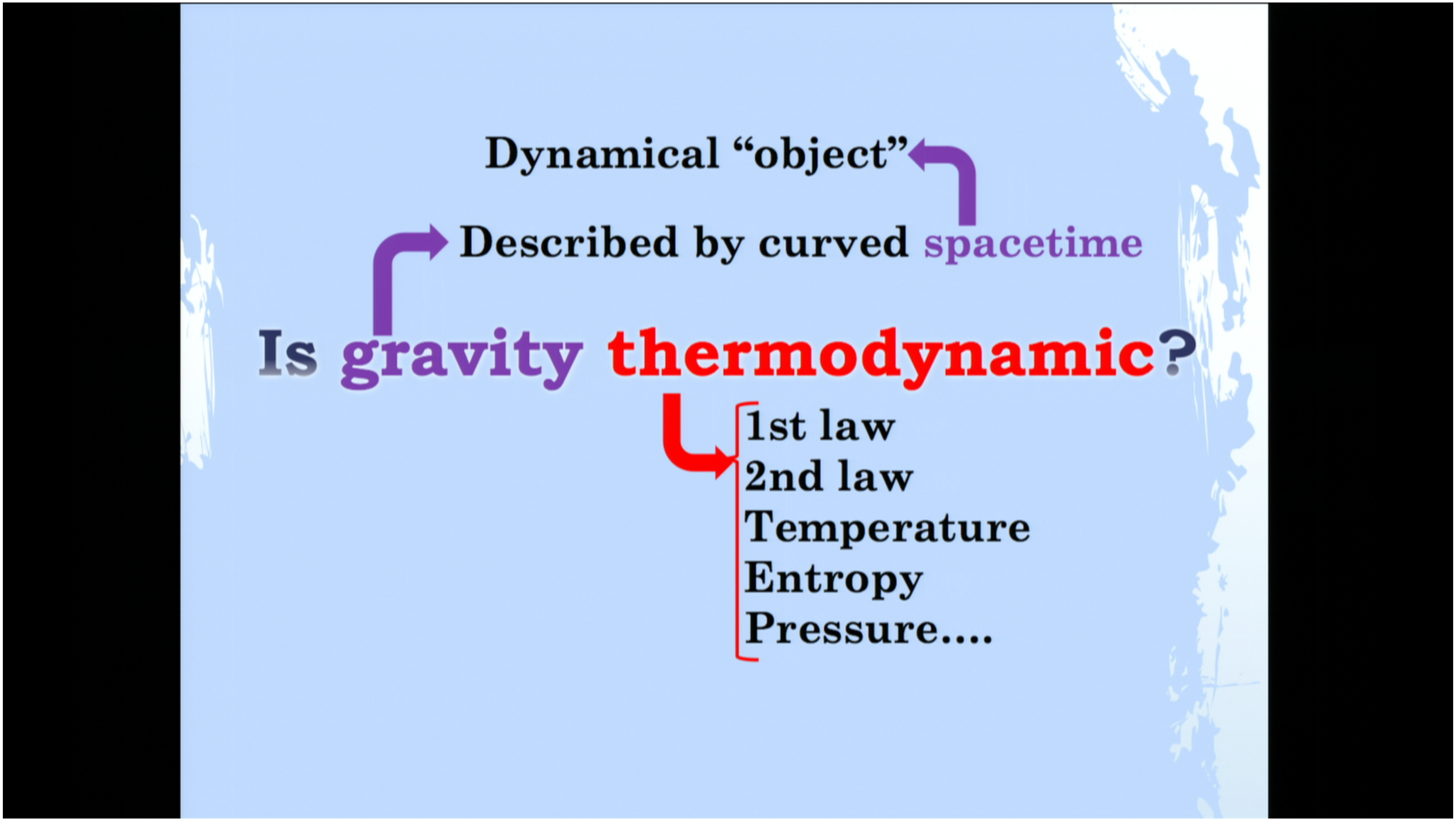
Dynamical "object"
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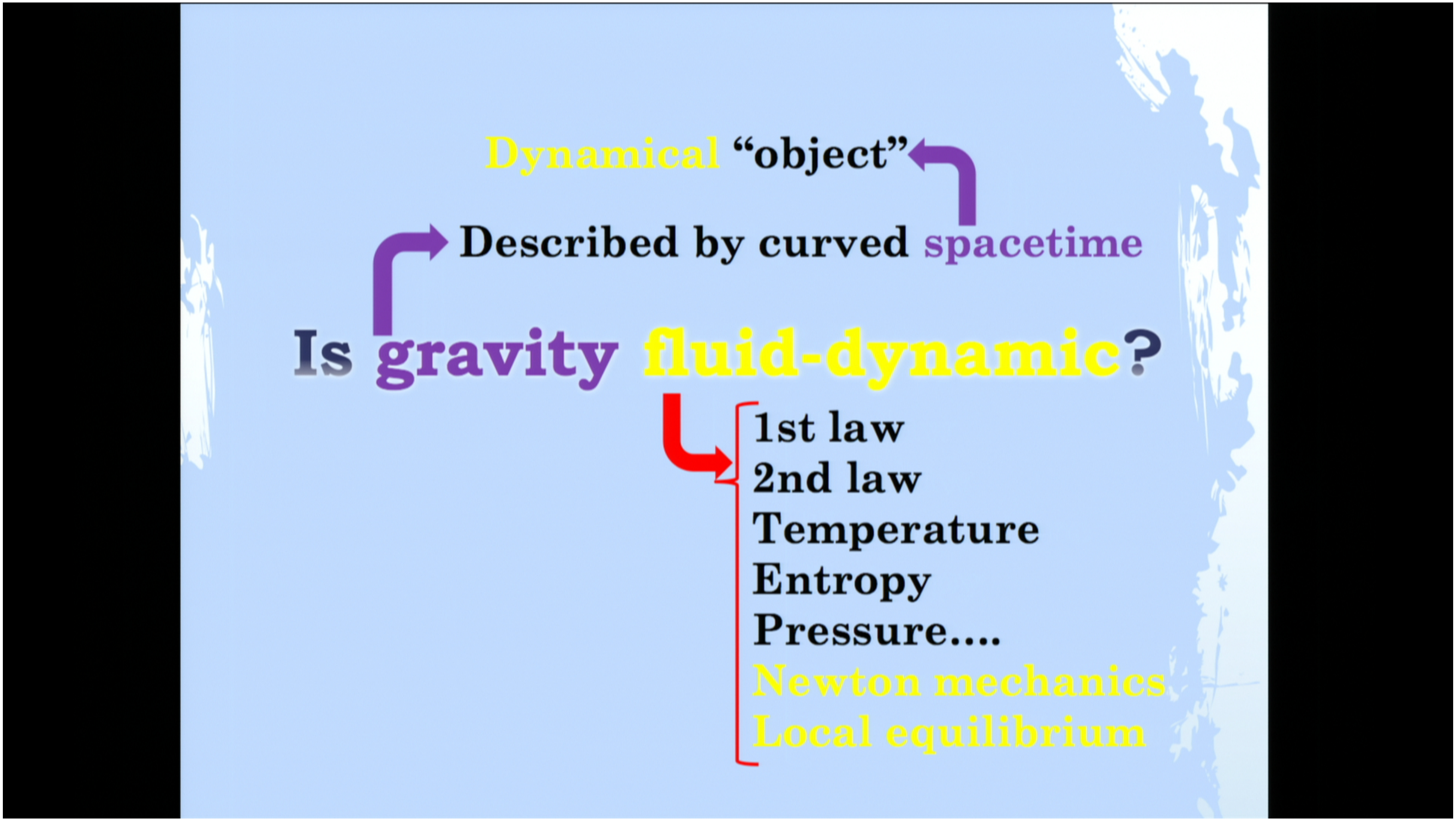
1st law
2nd law
Temperature
Entropy
Pressure....



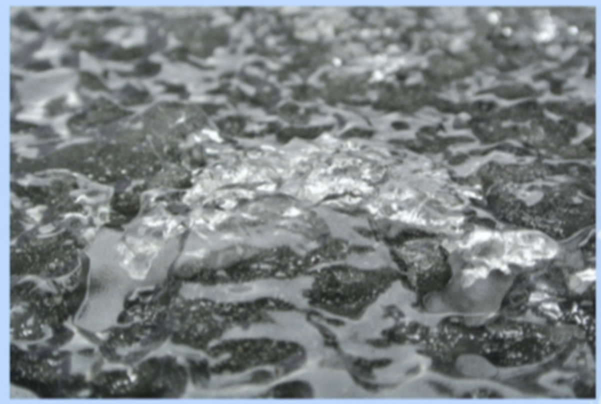
Dynamical "object"
Described by curved spacetime
Is gravity fluid-dynamic?
1st law
2nd law
Temperature
Entropy
Pressure....
Newton mechanics
Local equilibrium

From macroscopic viewpoint

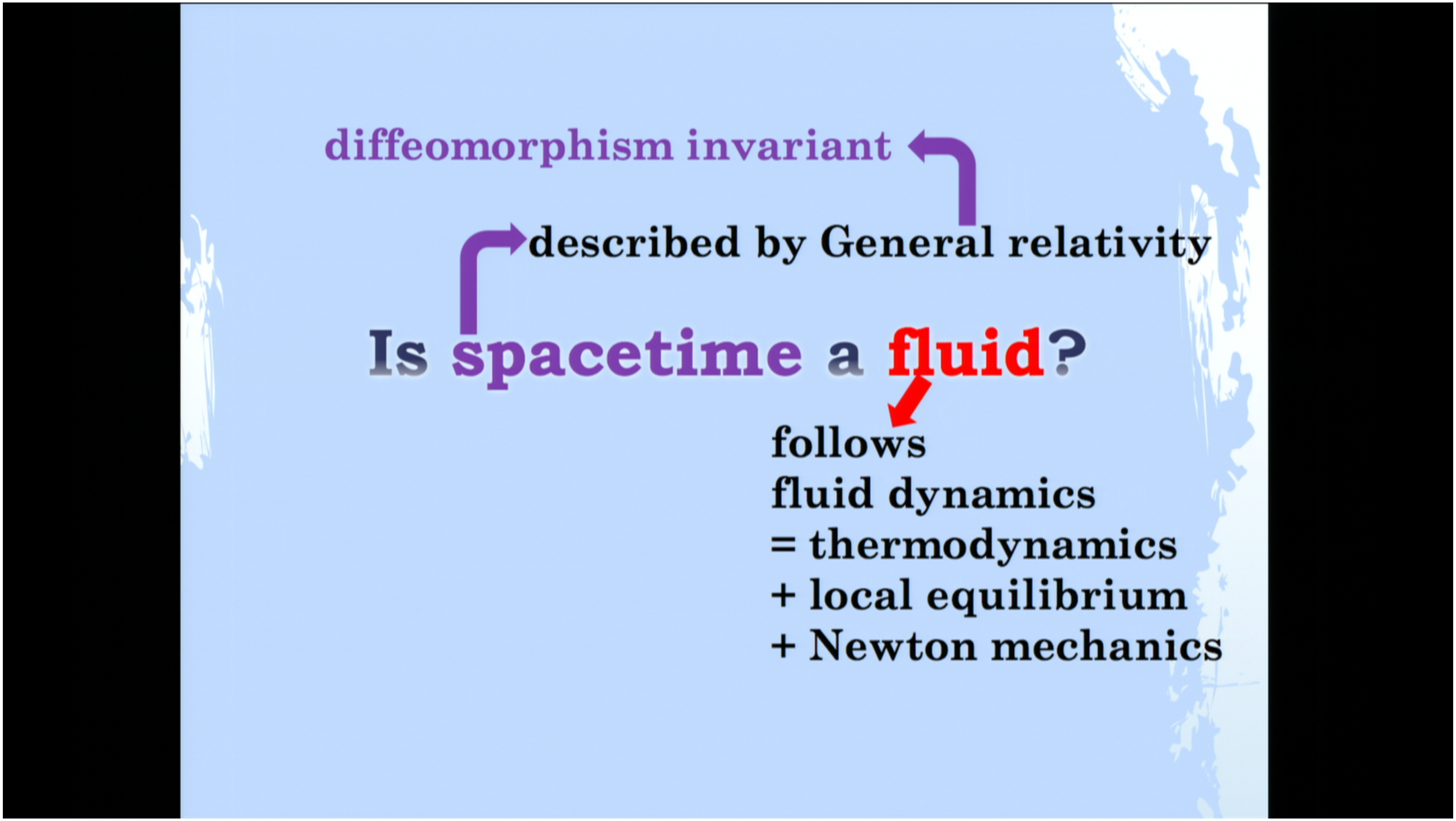
・Does spacetime follow fluid dynamics, that is, the Navier-Stokes eq, the 1st law and ,mass conservation? (⇒ Today's topic)

・Dose 2nd law hold for spacetime?

?
=



Is spacetime a fluid?


diffeomorphism invariant
described by General relativity
Is spacetime a fluid?
follows
fluid dynamics
= thermodynamics
+ local equilibrium
+ Newton mechanics

diffeomorphism invariant

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Is spacetime a fluid?

follows
fluid dynamics
= thermodynamics
+ local equilibrium
+ Newton mechanics

based on
a preferred time *t* and
a laboratory frame

Review of Fluid Dynamics 1

- Mass conservation law

$$\partial_t \rho + \partial_a(\rho v^a) = 0$$

- Momentum conservation law

= the Navier-Stokes equation

momentum
$\pi^a = \rho v^a$

$$\rho\left(\partial_t v_a + v^b \partial_b v_a\right) = -\partial_a p + \partial_b \tau^b{}_a$$

- Internal energy balance law

=1st law of thermodynamics

$$\partial_t \epsilon + v^b \partial_b \epsilon + \epsilon \partial_b v^b = -p \partial_b v^b + \tau^{ab} \partial_a v_b - \partial_b q^b$$

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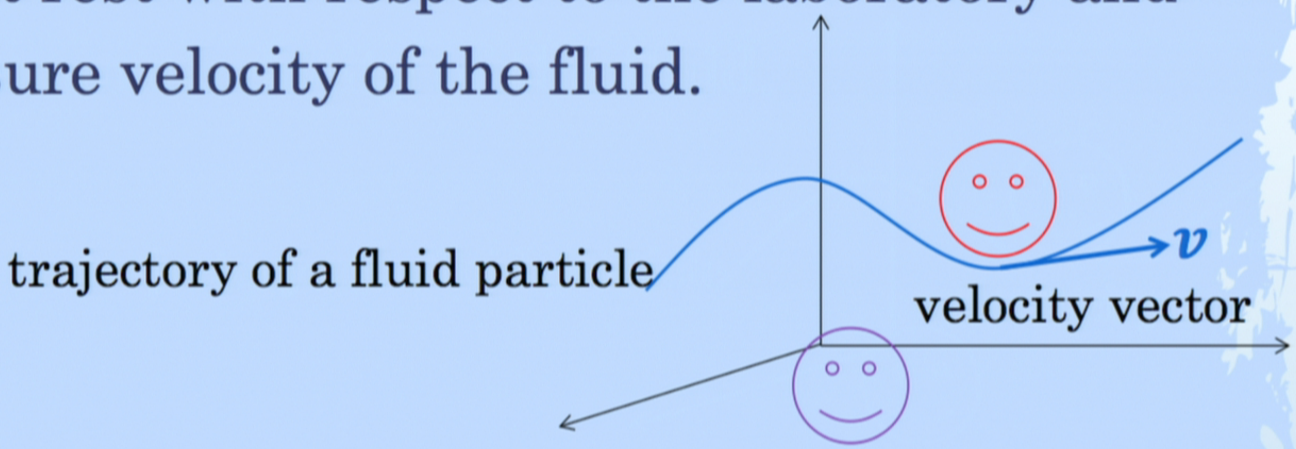
Review of Fluid Dynamics 2

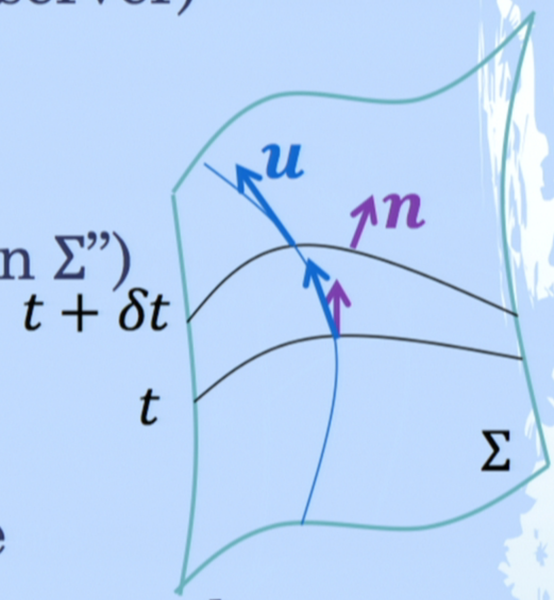
There are two kinds of observers in fluid dynamics:

- ***Comoving observers*** who follow a fluid particle at the fluid velocity and measure purely thermodynamic quantities.
- ***Laboratory observers*** who are at rest with respect to the laboratory and measure velocity of the fluid.



What corresponds to this situation in general relativity?

- Fluid velocity (=comoving observer)
  = a timelike vector $\boldsymbol{u}$
- 2-dim spatial fluid system
  =A timelike surface (= "screen Σ")



- choice of laboratory and time
  = choice of foliation t and frame on the screen
- Laboratory observer
  = timelike normal vector $\boldsymbol{n} \propto -\boldsymbol{dt}$

The question is reduced to…

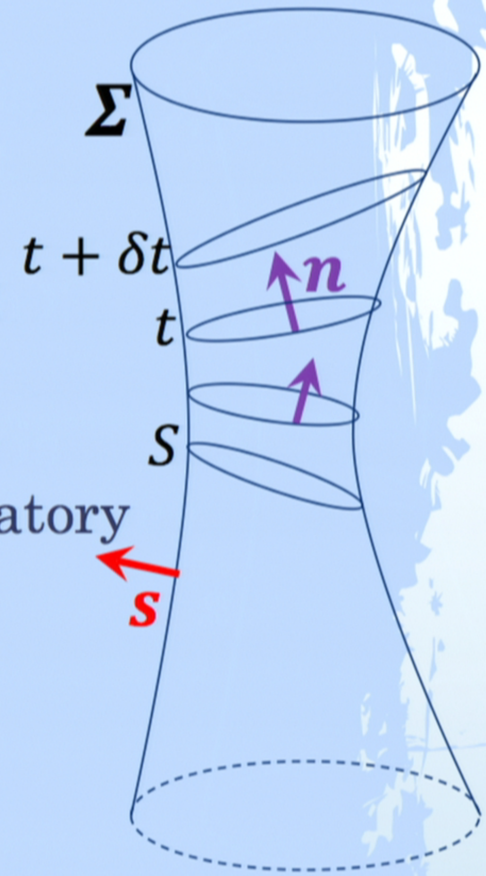
Can we choose a timelike vector, foliation, and frame on the screen such that the vector follows the Navier-Stokes eqation?

Let's start construction of fluid dynamics for spacetime!

1: set up of Screen

- $g_{\mu\nu}$ is given.
- A timelike surface Σ (=screen) is determined by the spacelike unit vector $\boldsymbol{s}$ orthogonal to Σ.
- $h_{\mu\nu} = g_{\mu\nu} - s_\mu s_\nu$: 3-dim metric on Σ
- Consider a time foliation t on Σ
- The normal timelike unit vector is given by $\boldsymbol{n} \propto -dt$, which is the laboratory observer.
- The metric of 2-dim spacelike "laboratory" S is given by

$$q_{\mu\nu} = n_\mu n_\nu + h_{\mu\nu}$$
$$= n_\mu n_\nu + s_\mu s_\nu + g_{\mu\nu}$$



2-1: Surface Energy-momentum tensor

- Use Israel's junction condition for the screen as in BH membrane paradigm, and then obtain the surface energy-momentum tensor:

$$S_{\mu\nu} = -\frac{1}{8\pi G}\left(H_{\mu\nu} - h_{\mu\nu}H\right)$$

(extrinsic curvature: $H_{\mu\nu} \equiv h_\mu{}^\alpha h_\nu{}^\beta \nabla_\alpha s_\beta$)

2-2: Surface Energy-momentum tensor

- Decompose $S_{\mu\nu}$ by $h_{\mu\nu} = -n_\mu n_\nu + q_{\mu\nu}$ as

$$\boxed{S_{\mu\nu} = \epsilon n_\mu n_\nu + \pi_\mu n_\nu + n_\mu \pi_\nu + \Pi_{\mu\nu}}$$

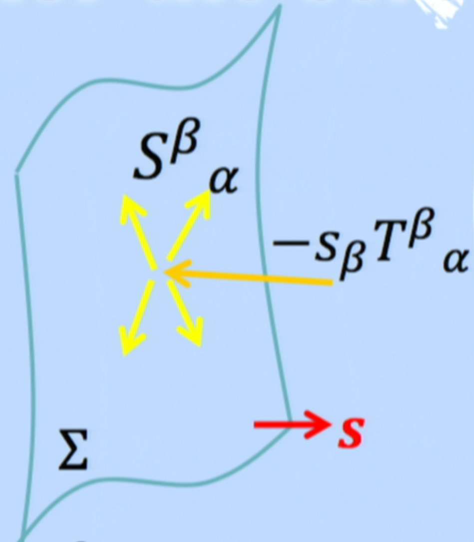
Each quantity is measured by the laboratory observer ***n*.**

- Energy density: $\epsilon \equiv S_{\mu\nu} n^\mu n^\nu$
- Momentum density: $\pi_\mu \equiv -q_\mu{}^\alpha S_{\alpha\nu} n^\nu$
- stress tensor: $\Pi_{\mu\nu} \equiv q_\mu{}^\alpha q_\nu{}^\beta S_{\alpha\beta} = p q_{\mu\nu} - \Theta_{\mu\nu}$
- $p \equiv \frac{1}{8\pi G} n^\mu \boldsymbol{s} \cdot \nabla_\mu \boldsymbol{n}$
- $\Theta_{\mu\nu} \equiv \frac{1}{8\pi G} (q_\mu{}^\alpha q_\nu{}^\beta - q_{\mu\nu} q^{\alpha\beta}) \nabla_\alpha s_\beta$

3: Conservation laws for the screen

- Consider the conservation laws for the screen as the membrane paradigm.

$$D_\beta S^\beta{}_\alpha = -s_\beta T^\beta{}_\alpha$$



(D_μ: derivative on Σ)

- For example, $q_A{}^\alpha D_\beta S^\beta{}_\alpha = -q_A{}^\alpha s_\beta T^\beta{}_\alpha$ can become

$$\mathcal{L}_{\hat{t}}\pi_A + \theta_{\hat{t}}\pi_A = -d_A p + d_B \widehat{\Theta}^B{}_A - \epsilon d_A \phi - T_{\hat{r}A}$$

Newton gravity emerges!

$\hat{t} = \rho \boldsymbol{n}$, $\hat{r} = \rho \mathbf{s}$, ($\rho$:redshift factor)
d_A: derivative on S, $\theta_{\hat{t}} \equiv q^{\alpha\beta}\nabla_\alpha \hat{t}_\beta$
$\phi = log\rho$: Newton potential

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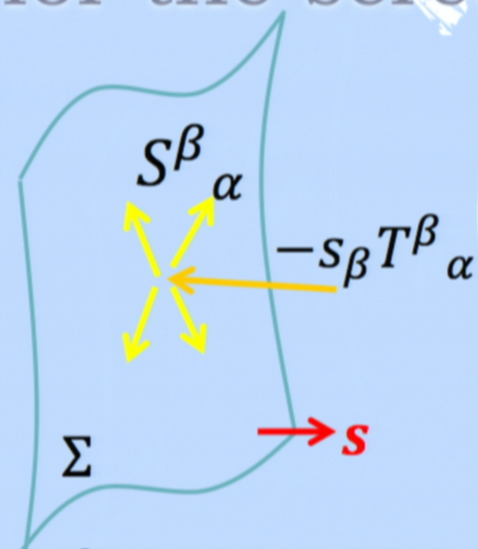

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$\phi = log\rho$: Newton potential

4: 2+2 decomposition of the Einstein eq

- The equations can be derived directly from the Einstein equation.
- Decompose the Einstein equation by

$$g_{\mu\nu} = \underbrace{-n_\mu n_\nu + s_\mu s_\nu}_{\text{2-dim}} + \underbrace{q_{\mu\nu}}_{\text{2-dim}}$$

- For example,

$$q_A{}^\mu G_{\mu\nu} s^\nu = 8\pi G q_A{}^\mu T_{\mu\nu} s^\nu$$

$$\Rightarrow \boxed{\mathcal{L}_{\hat{t}} \pi_A + \theta_{\hat{t}} \pi_A = -d_A p + d_B \widehat{\Theta}^B{}_A - \epsilon d_A \phi - T_{\hat{r}A}}$$

⇒The equations are the dynamical equation of motion for the screen.

5: Let's compare these with fluid equations

- the Navier-Stokes eq (in another form)

$$d_t \pi_A + \theta \pi_A = -\partial_A p + \partial_B \tau^B{}_A - \rho \partial_A \phi + f_A$$

$d_t \equiv \frac{\partial}{\partial t} + v^A \frac{\partial}{\partial x^A}$, $\theta \equiv \partial_A v^A$, $\pi_A = \rho v_A$, f_A: external force

- The Equation of motion for the screen

$$\mathcal{L}_{\hat{t}} \pi_A + \theta_{\hat{t}} \pi_A = -d_A p + d_B \widehat{\Theta}^B{}_A - \epsilon d_A \phi - T_{\hat{r}A}$$

Problems

(1)Fluid velocity v^A does not appear.

(2) $p, \widehat{\Theta}^B{}_A, \epsilon$ are not measured by comoving observer.

⇒This equation is *not* the Navier-Stokes eq yet.

(Note: The eq of G_{sn} is also similar to 1st law.)

How can we identify the physical velocity of the screen fluid?

6-1: Relativistic fluid picture

- Use a condition for the velocity vector in relativistic fluid dynamics:

$$u^{\mu} \propto S^{\mu}{}_{\nu} u^{\nu}$$

velocity ∝ energy flow

- This $\boldsymbol{u}$ can define the physical spatial velocity $\boldsymbol{v}$, in1+2 formalism with $(N, \boldsymbol{V})$ in the screen, as

$$\boldsymbol{u} \equiv \frac{\boldsymbol{n} + \boldsymbol{v}}{\sqrt{1 - \boldsymbol{v}^2}}, \qquad \boldsymbol{v} \equiv \frac{\boldsymbol{V}}{N}$$

for a given foliation $\boldsymbol{n}$.

6-2: Relativistic fluid picture

- The $\boldsymbol{u}$ and $\boldsymbol{v}$ relate the quantities for the comoving observers and those for the laboratory ones:

$$\epsilon = \frac{\epsilon'}{1-\boldsymbol{v}^2} + \Pi'_{\boldsymbol{vv}}, \qquad \pi_\mu = \frac{\epsilon'}{1-\boldsymbol{v}^2} v_\mu + q_\mu{}^\alpha \Pi'_{\alpha \boldsymbol{v}},$$

$$\Pi_{\mu\nu} = \frac{\epsilon'}{1-\boldsymbol{v}^2} v_\mu v_\nu + q_\mu{}^\alpha q_\nu{}^\beta \Pi'_{\alpha\beta}$$

Here "′" is for the comoving observer, as

$$\epsilon' \equiv S_{\alpha\beta} u^\alpha u^\beta, \ \pi'_\mu \equiv -q'_\mu{}^\alpha S_{\alpha\beta} u^\beta, \Pi'_{\mu\nu} \equiv q'_\mu{}^\alpha q'_\nu{}^\beta S_{\alpha\beta}.$$

- These relations are the same as ones for a usual relativistic fluid. Especially, $\pi'_\mu = 0$.

⇒Thus, the screen can be considered as a relativistic fluid, except for entropy production.

How can we obtain the exact Navier-Stokes eq from here?

- We still can use

diffeomorphism for the 2-dim spatial space S and diffeomorphism for the "radial" direction, $\boldsymbol{s}$, in 4-dim spacetime.

- We have to consider a constituent equation, which relates viscous stress tensor $\Theta_{\mu\nu}$ and deformation tensor $\partial_A v_B$,

  to satisfy 2nd law.

- The condition

$$\partial_t q_{\mu\nu} = 0$$

might correspond to the physical laboratory.

⇒We are now trying this problem from the above point of view!

Conclusions

- For arbitrary timelike screen, we made hydro-dynamic like equations from the Einstein equation by using 2+2 decomposition formalism, or, by using conservation laws for the screen.
- By identifying the physical velocity, we constructed the energy-momentum tensor which takes relativistic-fluid-dynamic form.
- These results strongly suggest that spacetime itself behaves as a fluid!


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July 22-26, 2013

Correction to the area law for Loop Black holes

Kinjalk Lochan[a], Cenalo Vaz[b]

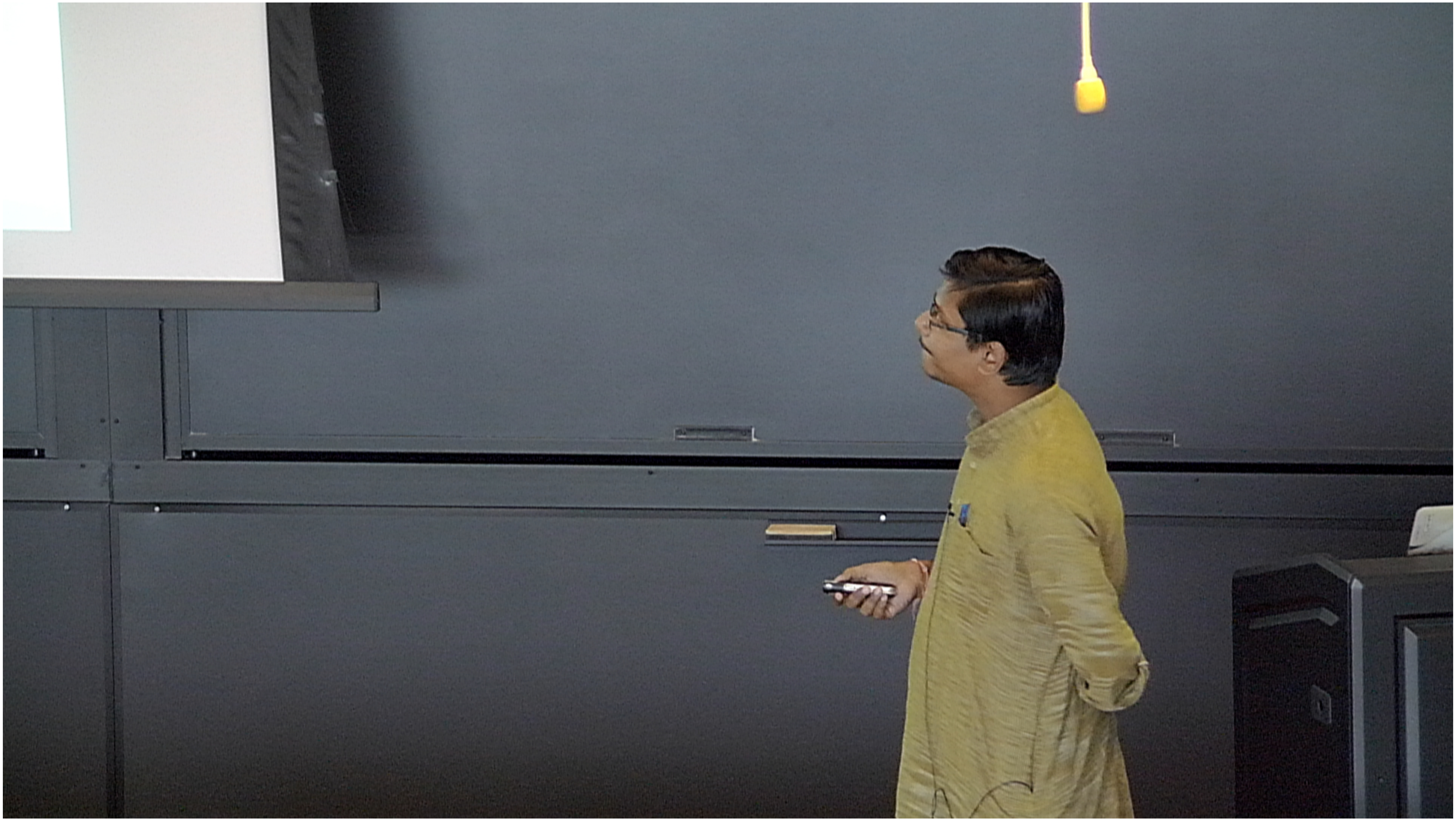
[a]Tata Institute of Fundamental Research, Mumbai, India
[b]University of Cincinnati, USA

22 July 2013

Phys. Rev. D 85 (2012) 104041
Phys. Rev. D 86 (2012) 044035

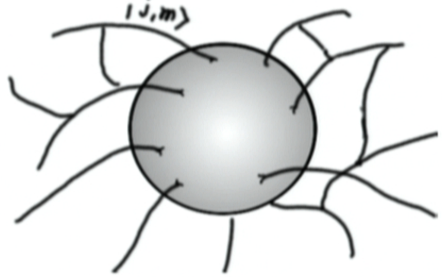
Plan

- Punctures as *Quantum Hair*
- Statistical analysis
- Correction to the Area law
- Discussion



I. Introduction

- Black hole in LQG : *Spacetime with inner boundary*: Isolated horizon with punctures.
- Chern Simons theory on the horizon. Edges of spin network in the bulk thread the horizon.
- Punctures contribute area elements to the horizon and construct the microstates accounting for the entropy.



- Area of the horizon is an observable. Statistical analysis for area of the horizon.

I. Introduction

- Microcanonical studies have been done GM, DL, ENP, .. , characterizing the horizon as

$$A = 8\pi\gamma l_P^2 \sum_P \sqrt{j_P(j_P+1)},$$

$$\sum_P m_P = 0.$$

and counting the number of such configurations

$$\Omega \sim \frac{e^{\lambda A}}{\sqrt{A}},$$

and

$$S \sim \lambda A - \frac{\mathbf{1}}{\mathbf{2}} \log \mathbf{A}.$$

- However number of puctures can not be held fixed, horizon can exchange the number of area quanta with the bulk.

- Does this situation corresponds to entropy calculation of a photon gas?

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I. Introduction: Punctures as Quantum hair

- Major Class. Quant. Grav. **17** (2000), Ghose and Perez Phys. Rev. Lett. **107** (2011): Punctures as a *quantum hair*.

- Chemical Potential associated with a puncture.

- Horizon as a gas of punctures.

- Modified version of the first law.

$$dE = TdS + \mu dN$$

- Statistical analysis suggests Bekenstein-Hawking area law recovered at leading order.

- Implications for subleading corrections ?

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II. Canonical ensemble analysis

- We first fix a graph Γ and calculate the (*canonical*) partition function as first step

$$Z_\Gamma(\beta, N) = \sum_{\{n_{jm_j}\}} \frac{N!}{\prod_{jm_j} n_{jm_j}!} \, \delta_{p,0} \, e^{-\beta \sum_{jm_j} n_{jm_j} a_j},$$

with

$$N = \sum_{j,m_j} n_{jm_j}, \quad \text{and} \quad 2\sum_{j,m_j} n_{jm_j} m_j = p.$$

- We use a suitable representation of the delta function to turn the partition function into

$$Z_\Gamma(\beta, N) = \frac{1}{2\pi} \sum_{\{n_{jm_j}\}} \frac{N!}{\prod_{jm_j} n_{jm_j}!} \int_0^{2\pi} \mathbf{dk e^{2ik \sum_{jm_j} n_{jm_j} m_j}} e^{-\beta \sum_{jm_j} n_{jm_j} a_j}.$$

- On simplification,

$$Z_\Gamma(\beta, N) = \frac{1}{2\pi} \int_0^{2\pi} dk \left(\sum_{jm_j} e^{(2ikm_j - \beta a_j)} \right)^N$$

- If we work with Flux area operator [Barbero, Lewandowski,Vilsenor], the Unitary representation of Area operator [Livine], or the semiclassical limit

$$a_j = (j+1) \qquad\qquad j \in \mathbb{N}$$

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II. Canonical ensemble analysis

- In this case

$$Z_\Gamma(\beta, N) \approx \frac{1}{2\pi}\int_0^{2\pi} dk \left(\frac{1}{e^{2ik}-1}\sum_{l=1}^{\infty} e^{-\sigma(l+1)}\{e^{ik(l+2)} - e^{-ikl}\}\right)^N$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} dk \left(\frac{2\cos k - e^{-\sigma}}{e^{2\sigma} - 2e^{\sigma}\cos k + 1}\right)^N ,$$

with

$$\sigma = 4\pi\gamma l_p^2 \beta,$$

which *might* be evaluated in the thermodynamic limit $N >> 1$.

- With a transformation

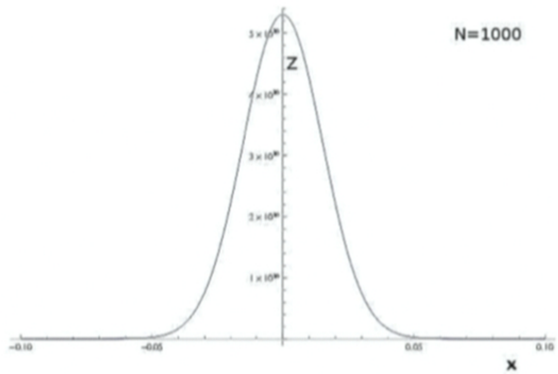
$$k = 2\tan^{-1}(x/2)$$

- the partition function

$$Z_\Gamma(\beta, N) = \int_{-\infty}^{\infty} dx \frac{1}{2\pi(1 + x^2/4)} \left(\frac{2\cos k(x) - e^{-\sigma}}{e^{2\sigma} - 2e^{\sigma}\cos k(x) + 1}\right)^N$$

II. Canonical ensemble analysis

- Partition unction is a unimodal symmetric distribution



- We would like it to approximate as accurately as possible.

II. Canonical ensemble analysis: Approximation schemes

- Moment generating function for a (Non-normalized) Gaussian with a zero mean

$$C\int_{-\infty}^{\infty} dx e^{-\frac{1}{2}\frac{x^2}{\sigma^2}},$$

is given by

$$M(t) = Ce^{\frac{t^2\sigma^2}{2}}\int_{-\infty}^{\infty} dx e^{-\frac{1}{2}\frac{(x-t\sigma^2)^2}{\sigma^2}}.$$

- With the substitution $x - t\sigma^2 = x'$ we have

$$M(t) = Ce^{\frac{t^2\sigma^2}{2}}\int_{-\infty}^{\infty} dx' e^{-\frac{1}{2}\frac{(x')^2}{\sigma^2}} = Af(i\sigma^2 t),$$

where $\mathbf{f(x) = Ce^{-\frac{1}{2}\frac{x^2}{\sigma^2}}}$ and $A = \sqrt{2\pi\sigma^2}$.

- In a non-normalized gaussian distribution (with zero mean), the $n-th$ moment is given by

$$\mu_n = \frac{C\int_{-\infty}^{\infty} dx x^n e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}}{C\int_{-\infty}^{\infty} dx e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}}$$

II. Canonical ensemble analysis: Approximation schemes

- Now,

$$M(t) = C\int_{-\infty}^{\infty} dx e^{tx} e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}$$
$$= C\int_{-\infty}^{\infty} dx(1 + tx + \frac{(tx)^2}{2!} + ... + \frac{(tx)^n}{n!} + ...)e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}.$$

Thus,

$$\mu_n = \frac{M^{(n)}(t)|_0}{M(t)|_0}.$$

- Now,

$$M^{(n)}(t)|_0 = A(i\sigma^2)^n f^{(n)}(i\sigma^2 t)|_0 = A(i\sigma^2)^n f^{(n)}(0),$$

Therefore the $n-th$ moment is

$$\mu_n = \frac{(i\sigma^2)^n f^{(n)}(0)}{f(0)}.$$

- **Variance**

$$\sigma^2 = -\frac{f(0)}{f''(0)},$$

II. Canonical ensemble analysis: Approximation schemes

- **Kurtosis**
  The 4-th moment is again obtained as

$$\mu_4 = \frac{(i\sigma^2)^4 f^{(4)}(0)}{f(0)}.$$

  Therefore the kurtosis is given by

$$\beta_2 = \frac{\mu_4}{\sigma^4} = \frac{f(0) f^{(4)}(0)}{(f''(0))^2}.$$

- The kurtosis for the distribution becomes

$$\frac{\mu_4}{\tilde{\sigma}^4} = \frac{f(x)|_0 f^{(4)}(x)|_0}{[f''(x)|_0]^2} =$$

$$\frac{6[(1-2e^{\sigma})^2(e^{\sigma}-1)^4 + 8e^{3\sigma}(-1+2e^{\sigma}+e^{3\sigma}-e^{2\sigma})N + 8e^{6\sigma}N^2]}{[-1+e^{\sigma}(4+e^{\sigma}(-5+e^{\sigma}(2+4N)))]^2}$$

II. Canonical ensemble analysis: Approximation schemes

- The "excess kurtosis" in the thermodynamic limit vanishes

$$\lim_{N\to\infty} \frac{\mu_4}{\tilde{\sigma}^4} - 3 \to 0.$$

enabling us to approximate the distribution as gaussian and evaluate the partition function as

$$Z_\Gamma(\beta, N) \approx \left[e^{-\sigma}\sqrt{\frac{2\log 4}{N}}\right]\left(\frac{2-e^{-\sigma}}{(e^{\sigma}-1)^2}\right)^N .$$

- Corresponding canonical entropy

$$S = \ln Z_\Gamma + \beta A = N[\ln z(\sigma) + \sigma q] - \frac{1}{2}\ln N + \text{const.},$$

with $q = -\partial \log z/\partial\sigma$.
The entropy is extremized w.r.t. the number of constituents [photon gas] to get

$$S \approx \frac{\sigma(q_0)A}{4\pi\gamma l_p^2} - \frac{\mathbf{1}}{\mathbf{2}}\ln\left(\frac{\mathbf{A}}{\mathbf{4\pi\gamma l_p^2 q_0}}\right) + \text{const.}$$

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II. Canonical ensemble analysis

- We recover the B-H area law for the leading order if we take $\gamma = 0.258$

	Analysis	γ
Ghosh et. al. Ghosh, Mitra, Phys. Rev. D. **71** (2005)	Microcanonical LQG	0.274
Ling, Zhang Ling, Zhang, Phys. Rev. D. **68** (2003)	N=1 SUSY LQG	0.247
KL, CV KL & Vaz, Phys. Rev. D. **85** (2012)	Canonical LQG	0.258

- Recent proposals suggest fixation of Immirizi parameter is not core to obtaining the area-law when the problem is posed in terms of local observers

- We also obtain sub-leading logarithmic corrections with a negative signature.

- Next we allow the number of punctures to vary.

III. Grand-canonical ensemble analysis

- The corresponding *grand-canonical* treatment gives

$$\Xi(\beta,\alpha)=\sum_{N=0}^{\infty}\sum_{n_j=0}^{N}\frac{N!}{\prod_j n_j!}\prod_j(2j+1)^{n_j}e^{-(8\pi\gamma\beta a_j-\alpha)n_j}$$

The average quantities will be given by

$$\langle N\rangle=\frac{\partial\ln\Xi}{\partial\alpha}=\sum_j\langle n_j\rangle=\frac{\lambda z}{1-\lambda z}.$$

$$A=-\frac{\partial\ln\Xi}{\partial\beta}=\frac{\partial}{\partial\beta}\ln(1-\lambda z)=-N\frac{\partial\ln z}{\partial\beta},$$

where $\lambda(\alpha)=e^{\alpha}$ is the fugacity, and

$$z(\beta)=\sum_j(2j+1)e^{-8\pi\gamma\beta a_j}.$$

III. Grand-canonical ensemble analysis

- Using the relation

$$\Xi = \sum_N Z^N e^{\alpha N}$$

and using the canonical partition function we get

$$\Xi(\sigma, \alpha) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{dk}{1 - \lambda(\alpha) \sum_{l=1}^{\infty} z_l(\sigma) \left(\frac{\sin k(l+1)}{\sin k} \right)},$$

with $z_l(\sigma) = e^{-\sigma(l+1)}$ and $\lambda(\alpha) = e^{\alpha}$.

- Again, the partition function can be approximated (saddle-point) in the thermodynamic limit

$$\Xi(\sigma, \alpha) \approx \sqrt{2\pi} f(0) \tilde{\sigma} = \frac{1}{\sqrt{\pi \{1 - \lambda z(\sigma)\}\{1 + \lambda b(\sigma)\}}},$$

where

$$z(\sigma) = \sum_{l=1}^{\infty} z_l(\sigma)(l+1)$$

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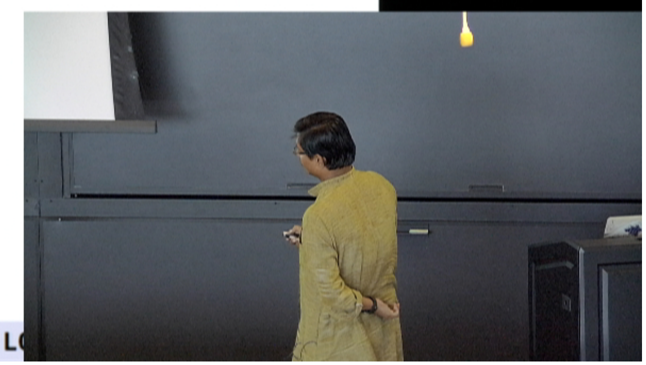

III. Grand-canonical ensemble analysis

- Large N limit is given by
$$\lambda z \to 1$$
- **In this limit the ratio A/N depends on the chemical potential and is constant for isothermal cases : Good intensive variable to use.**
- Legendre transform of $\ln \Xi$, which is the entropy, becomes
$$S(A,N) = \ln \Xi + \beta A - \alpha N = (N+1)\ln(N+1) - N\ln N + Na\sigma(a) + N\ln z(a)$$
and simplifies, in the limit of large N, to
$$S(A,N) \approx \ln N + N[a\sigma(a) + \ln z(a)] = \frac{\sigma(a)}{\pi\gamma}\frac{A}{4l_p^2} + N\ln z(a) + \ln N.$$
- **At some fixed value of the temperature, σ_0, or of the chemical potential, α_0, we find that $a(\sigma_0) = a_0$ then**
$$N = \frac{A}{4\pi\gamma l_p^2 a_0}$$
can be used to eliminate N
$$S(A) \approx \frac{1}{\pi\gamma}\left[\sigma_0 + \frac{\ln z(a_0)}{a_0}\right]\frac{A}{4l_p^2} + \ln\frac{A}{4l_p^2} + \text{const.},$$

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- Inclusion of the projection constraint and the fluctuation in N, in large N limit gives

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- Therefore for isothermal case B-H law is obtained upto fixing the Immirizi parameter.
- The logarithmic correction now becomes of positive signature and differs from microcanonical results.
- Holds true for "photon-gas sceario" as well.
- Difference of ensembles related to taking thermodynamic limits ? Introduction of quantum hair N does not seem helping.
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- In general, the Immirizi parameter is a function of the temperature/chemical potential.
- Canonical/grand-canonical analysis suggests correction to area law, logarithmic in nature but with opposite signatures.
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