

Title: Shape Dynamics - 2

Date: Jul 22, 2013 04:40 PM

URL: <http://pirsa.org/13070044>

Abstract:





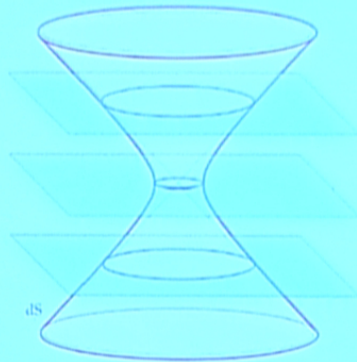
Motivation

Using a toy model of cosmology

- Give explicit scheme for global symmetry trading in SD.
- Study “shape observers” in de Sitter (dS) and clock synchronization.
- Compute explicit holographic model.



The Model: de Sitter Spacetime



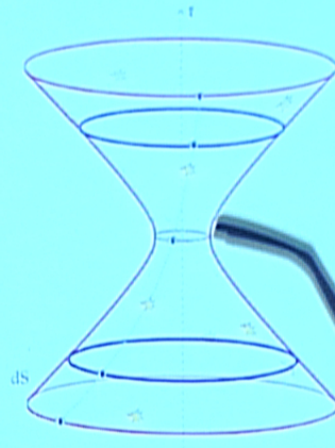
$t = \text{const.} \Rightarrow 3\text{-spheres}$

- Definition: $-t^2 + w^2 + \vec{x}^2 = \ell^2$
($\Lambda = \frac{d(d-1)}{2\ell^2}$ = cosmological constant)
- SD: closed CMC slices.
 $t = \text{const} = \ell \sinh \alpha$
(α = proper time)



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The Model: de Sitter Spacetime



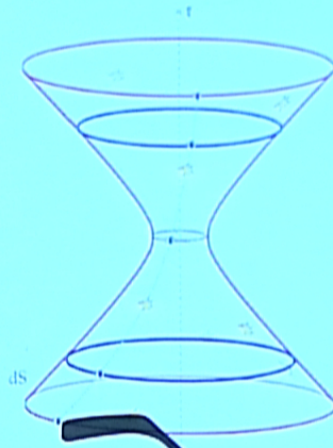
Timelike Geodesics

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 $t = \text{const} = \ell \sinh \alpha$
 $(\alpha = \text{proper time})$
- Geodesics: Intersection of plane spanned by past and future directed null vectors, $\xi_{\text{out}}, \xi_{\text{in}}$
- Stationary Observer:
 $w = -\ell \cosh \alpha, x = y = z = 0$
 (Source: 'O')



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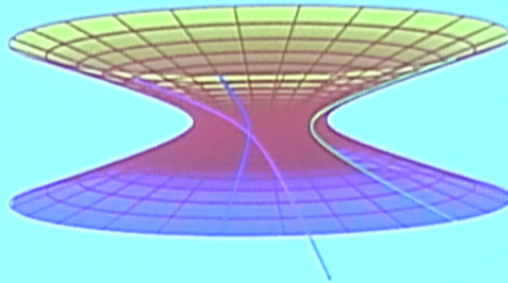
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Shape Description: Stereographic Projection

Shapes: several inertial observers viewed by 'O'.



Instantaneous Shapes (e.g., Triangle)

'O' = green line

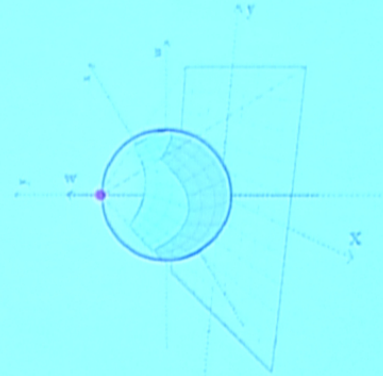
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Stereographic Projection

$$X^i = \frac{x_i / (\ell \cosh \alpha)}{1 - w / (\ell \cosh \alpha)}$$

"Shadow" of points onto plane \Rightarrow map from S^3 - (North Pole) to \mathbb{R}^3 .



\Rightarrow Projection defines a **shape** on the plane $\mathbb{R}^3 / \text{conf}(3)$.



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Holographic Shape Observers

Define Particle SD Holographically

Use bulk geodesic principle ($\mu = 0 \dots 4$)

$$S_{\text{bulk}}(x_{\text{in}}^{\mu I}, x_{\text{out}}^{\mu I}) = \ell \sum_I \int_{x_{\text{in}}^{\mu I}}^{x_{\text{out}}^{\mu I}} d\lambda \sqrt{\dot{\alpha}^2 - \cosh^2 \alpha \left(\dot{w}_I^2 + \dot{x}_I^2 \right)}$$

take the limit ($i = 1 \dots 3$)

$$S_{\text{boundary}}^{\text{on-shell}}(X_{\text{in}}^{iI}, X_{\text{out}}^{iI}) = \lim_{\alpha_0 \rightarrow \infty} S(X_{\text{in}}^{iI}(-\alpha_0), X_{\text{out}}^{iI}(\alpha_0)) \quad (1)$$

$$\sim 2N\alpha_0 - \sum_I \log(\xi_{\text{in}}^{\mu I} \xi_{\text{out}, \mu I}) \quad (2)$$

This defines the Hamilton-Jacobi functional for a boundary SD the

$\Rightarrow S_{\text{boundary}}^{\text{on-shell}}(X_{\text{in}}^i, X_{\text{out}}^i)$ is invariant under $\text{conf}(3)!$



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Remarks / Extensions

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- Very simple, completely explicit holographic model.
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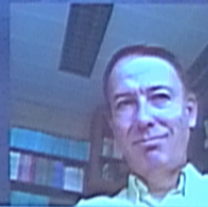


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Black Holes in Shape Dynamics

Niayesh Afshordi

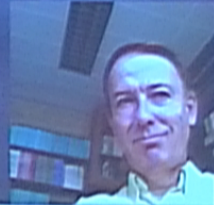
(with ~~Michael Susskind~~ & Robert Mann)
University of Waterloo, Perimeter Institute



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Incompressible fluids in the (theory) Universe

- Shape Dynamics
- Horava-Lifshitz gravity
- Cuscuton gravity $\mathcal{L} = \sqrt{\partial^\mu \varphi \partial_\mu \varphi} - V(\varphi)$
- Empty Black Holes (Membrane Paradigm)
- superluminality in DGP/massive gravity, early universe (a la Magueijo), Einstein-Aether

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Constant Mean Curvature (CMC) Foliation

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- Gravity in CMC foliation = GR+ incompressible aether

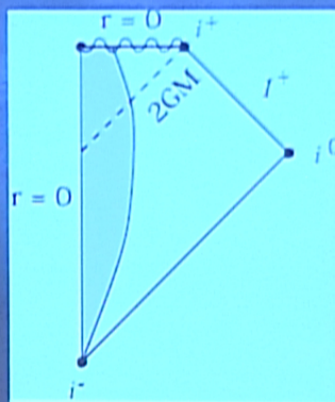
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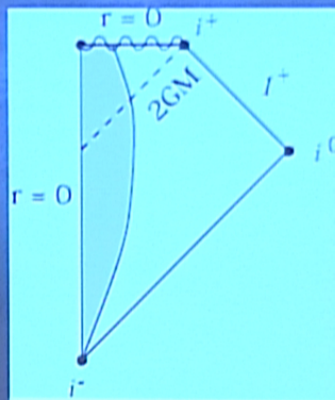
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Forming Black Holes



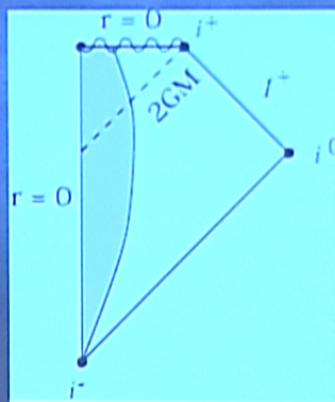
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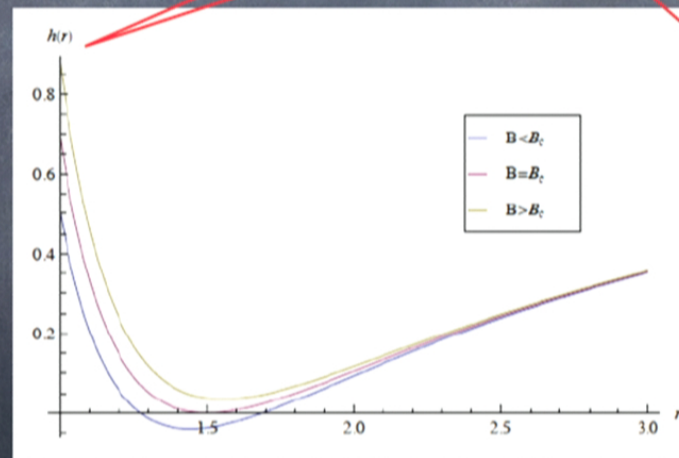


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The boundary of Shape Dynamics "Space-Time"!

$$t_{CMC}(r) = t_{shell}(R) + \int_R^r dx \frac{\frac{K}{3}x - \frac{B}{x^2}}{f(x) \sqrt{f(x) + \left(\frac{K}{3}x - \frac{B}{x^2}\right)^2}}, \quad r > R$$

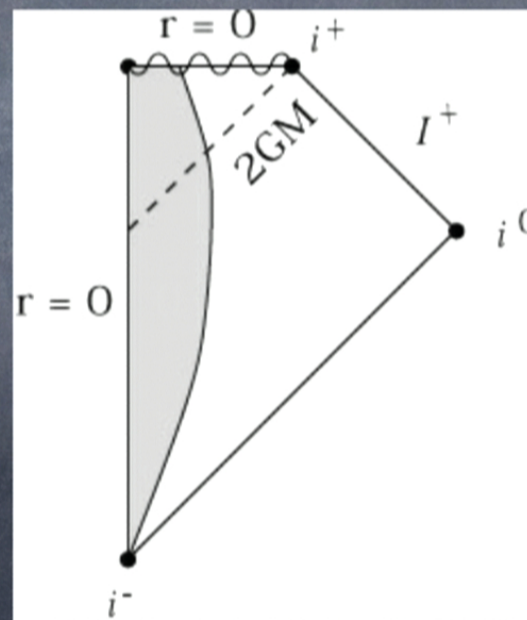
$$f(r) = 1 - \frac{2M}{r}$$



Aether Flux: fixed by
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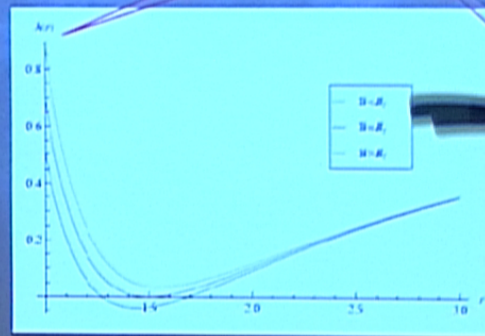


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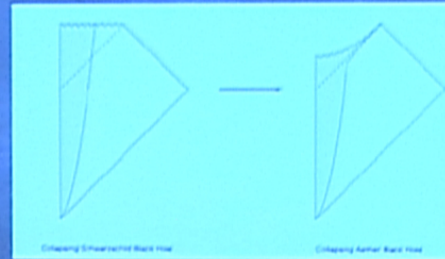
Universal Horizons and Cosmic Censorship

- Static/Stationary Black Hole solutions in Einstein have a “universal horizon”, which trap even arbitrarily superluminal aether disturbances (Barausse, Jacobson & Sotiriou 2011)
- We believe we have a dynamical realization of that
- “Generalized Cosmic Censorship” Conjecture: Space-like singularities are preceded by Universal Horizons, where “classical” evolution ceases

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From BH formation to BH entropy?

- An incompressible membrane at BH horizon, where classical space-time ends (e.g. fuzzball, firewall) has Bekenstein-Hawking entropy (S. W. Hawking, NA, & Mann 2011)
- Can we move the boundary of space-time from $0.75 R_s$ to R_s ?



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Final Thoughts

- Many motivations for preferred-frame, superluminal theories of gravity
- Maximal Superluminality = Incompressibility = CMC foliation
- Black Hole Space-times seems to end @ a **Universal Horizon = $0.75 R_s$**
- Quantum Evolution? Microstates of Black Holes? Observables? Censorships?!

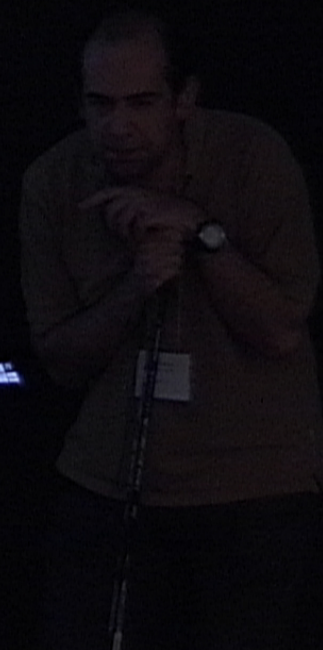
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Microstates of Black
Holes?



Classical BRST

- We introduce Ghosts and Their momenta corresponding to $H_a(\xi)$ and $\mathcal{C}(\rho)$ -

$$(\eta, \eta^a; P, P_a)$$

- Correspondingly we have the (minimal) BRST charge

$$Q_{SD} = \int d^3x [\eta \text{tr} \pi + \eta^a \gamma_{ac} \nabla_b \pi^{cb} + \eta^b \eta_{,b}^a P_a + \frac{1}{2} \eta^a \eta_{,a} P]$$

- The BRST differential is given by

$$\delta_{Q_{SD}} = \langle \cdot, X_{Q_{SD}} \rangle$$

$$\text{where } \Omega(X_{Q_{SD}}) = \delta Q_{SD}$$

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Classical BRST (contd.)

- By construction

$$\delta_{Q_{SD}} Q_{SD} = 0$$

- The Observables all belong to:

$$H^0(\delta_{Q_{SD}}) = \left(\frac{\text{Ker} \delta_{Q_{SD}}}{\text{Im} \delta_{Q_{SD}}} \right)^0$$



Polarization of Extended Phase Space

- The Phase Space $\Gamma \ni (\pi^{ab}, \gamma_{ab})$ is extended to accomodate the ghosts and their momenta by taking the product of the above with $\Gamma_\Lambda \ni (P_i, \eta^i)$ i.e. $\Gamma_{ext} = \Gamma \times \Gamma_\Lambda$
- We choose the (real) vertical polarization for both phase spaces which are given by $P = \text{span} \left\{ \frac{\delta}{\delta \pi^{ab}} \right\}$ and $P_\Lambda = \text{span} \left\{ \frac{\delta}{\delta P_i} \right\}$, note that $P^\perp = P$, similarly with P_Λ .

The Space of Extended Geometric Quantum States

- The prequantum line bundle $\pi : L \rightarrow \Gamma_{ext}$ over Γ_{ext} is one for which

$$-i\mathcal{D}_{\Theta_{ext}}\mathcal{D}_{\Theta_{ext}} = \frac{1}{h}\Omega_{ext},$$

where $\mathcal{D}_{\Theta_{ext}} = d_\delta - \frac{i}{h}\Theta_{ext}$ and $\Theta_{ext} = \Theta_{SD} + \Theta_{\Gamma_\Lambda}$

- The extended geometric quantum states are sections ψ of the bundle $\pi_P : L \rightarrow (\Gamma/P) \times (\Gamma_\Lambda/P_\Lambda)$ that satisfy

$$\mathcal{D}_{\Theta_{ext}}(X)\psi = 0 \forall X \in P \cup P_\Lambda$$

and can be written as

$$\psi = \psi_0 + \psi_\eta + \psi_a \eta^a + \psi_{ab} \eta^a \eta^b + \psi_{abc} \eta^a \eta^b \eta^c$$

Ghost Number $\pm m/2$ (Here ± 2) States

- The Ghost number operator is given by

$$\hat{G} = \frac{i}{2} \sum_{k=0}^{k=4} (k+1) [\hat{\eta}^{i_k} \hat{P}_{i_k} + \hat{P}_{i_k} \hat{\eta}^{i_k}]$$

Clearly

$$\hat{G}\psi_0 = -2$$

and

$$\hat{G}\psi_I = 2$$

- Ghost number 0 singlets can be attained from the above according to the following formula:

$$H^0(\delta_{Q_{SD}^{NM}}) = (H^{-2}(\delta_{Q_{SD}}) \otimes H^2(\delta_{Q'_{SD}})) \otimes (H^2(\delta_{Q_{SD}}) \otimes H^{-2}(\delta_{Q'_{SD}}))$$

Ghost Number 0 States

- Ghost number 0 states can be formed by tensoring the plus and minus two states:

$$\psi[\gamma]_{IJ=0, \pi_\lambda=0, \bar{P}=0} = \psi[\gamma]_{IJ=0} \otimes \psi_{\pi_\lambda=0, \bar{P}=0}$$

- The action of the various operators on said states are:

$$\hat{\gamma}^{ab} \psi[\gamma]_{IJ=0, \pi_\lambda=0, \bar{P}=0} = \gamma^{ab} \psi[\gamma]_{IJ=0, \pi_\lambda=0, \bar{P}=0}$$

$$\hat{\eta} \psi[\gamma]_{IJ=0, \pi_\lambda=0, \bar{P}=0} = 0 = \hat{\eta}^a \psi[\gamma]_{IJ=0, \pi_\lambda=0, \bar{P}=0}$$

$$\hat{\pi}_\lambda \psi[\gamma]_{IJ=0, \pi_\lambda=0, \bar{P}=0} = 0 = \hat{\bar{P}}_C \psi[\gamma]_{IJ=0, \pi_\lambda=0, \bar{P}=0}$$

$$= \hat{\bar{P}}_{C_a}^a \psi[\gamma]_{IJ=0, \pi_\lambda=0, \bar{P}=0}$$

$$Q_{SD} \psi[\gamma]_{IJ=0, \pi_\lambda=0, \bar{P}=0} = 0$$

Vasudev Shyam

On Geometric Quantization and Shape Dynamics

Conclusions

- Due to the simple nature of the first class constraints and presence of a true Hamiltonian, the problems of the quantum kinematics and dynamics are separated
- Up to some indefinite scalar product between $\psi[\gamma]$'s a physical (regularized) inner product is feasible to construct.
- Outlook
 - Identifying the correct rigorous form of $(\psi[\gamma], \psi'[\gamma])$ (also to identify the correct integration measure on $Met(\Sigma)$)
 - The Dynamics of the theory and making sense of the non local Hamiltonian

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$$\begin{aligned}
 & \hat{G} \gamma_4 \gamma_5 = 0 \\
 & (\hat{G}_a, \hat{G}_b) \gamma_4 = -C_{ab} \gamma_4 \\
 & (4, 4) \rightarrow C.S(0) \leftarrow 4/4 \rightarrow \boxed{4}
 \end{aligned}$$

$$117\eta \quad \psi^a$$

$$\frac{1}{k\epsilon} [F_a]$$

Background: Quotient Manifold Method

Attempts to write general relativity as a gauge theory led to an inversion of the standard construction. (Ne'emann and Regge, 1978)

- The quotient of a Lie group by one of its Lie subgroups is a manifold.

$$\mathcal{G} / \mathcal{H} = \mathcal{M}$$

$$\dim(\mathcal{M}) = \dim(\mathcal{G}) - \dim(\mathcal{H})$$

- The Maurer-Cartan structure equations allow us to write the Lie algebra in terms of a connection on \mathcal{M} .

$$d\omega^A = -\frac{1}{2}c_{BC}^A \omega^B \wedge \omega^C$$

- Generalize the connection to allow curvature on \mathcal{M} .
- The subgroup, \mathcal{H} , becomes the local symmetry of the final geometry.
- Build an action from the curvatures (Now tensors with respect to \mathcal{H}).

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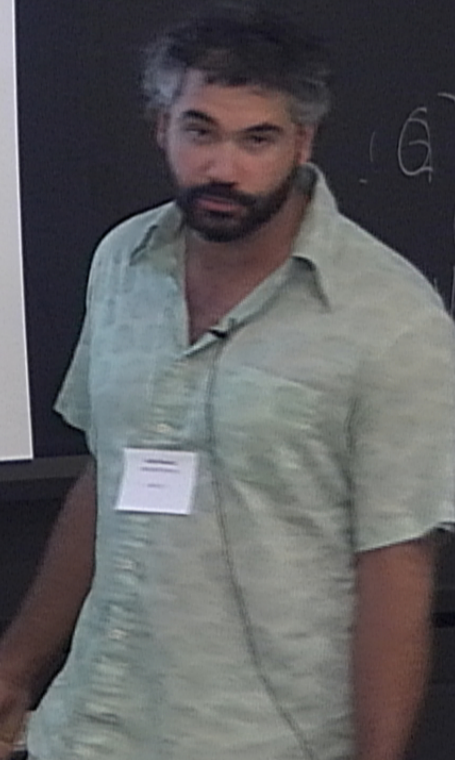
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$$\begin{aligned} & \{ \hat{G}_1, \hat{G}_2 \} = 0 \\ & \{ \hat{G}_1, \hat{G}_3 \} = -C_{ab}^c \hat{G}_c \\ & \{ \hat{G}_2, \hat{G}_3 \} = -C_{ab}^c \hat{G}_c \end{aligned}$$

Biconformal Space

Special Characteristics of the biconformal gauging

$$\text{Conformal}(p, q) / \text{Weyl}(p, q)$$



Properties

- ① **Symplectic Form.** The only Lorentz covariant quotient of $SO(p+1, q+1)$ for which the dilatation equation describes a symplectic form. $d\omega = e^a \wedge f_a$
- ② **Metric Space.** The only Lorentz covariant quotient of $SO(p+1, q+1)$ that has a non-degenerate Killing metric on \mathcal{M} .

$$K_{AB} = \begin{bmatrix} \frac{1}{2}(\delta_d^a \delta_b^c - \delta^{ac} \delta_{db}) & 0 & 0 & 0 \\ 0 & 0 & \delta_b^a & 0 \\ 0 & \delta_a^b & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Jeffrey Hazboun (USU)

GR in signature changing phase space

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Curved Biconformal Space

Action Linear in Curvatures.¹ Since the basis forms of the sub-manifolds have opposite conformal weight we can write an action linear in the curvatures *in any dimension*.

Torsion-free solutions with the biconformal action and involute configuration space generically reproduce general relativity on the cotangent bundle.²

$$S = \int (\alpha \Omega^a_b + \beta \delta^a_b \Omega + \gamma e^a f_b) \varepsilon_{ac\dots d} \varepsilon^{be\dots f} e^c \dots e^d f_e \dots f_f$$

¹Wehner, Wheeler, Nucl.Phys.B 557(1999)380-406, arXiv:hep-th/9812099.

²Wheeler (2001) <http://www.physics.usu.edu/Wheeler/GaugeTheory/VanishingT03May12.pdf>

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Biconformal Space as Relativistic Phase Space

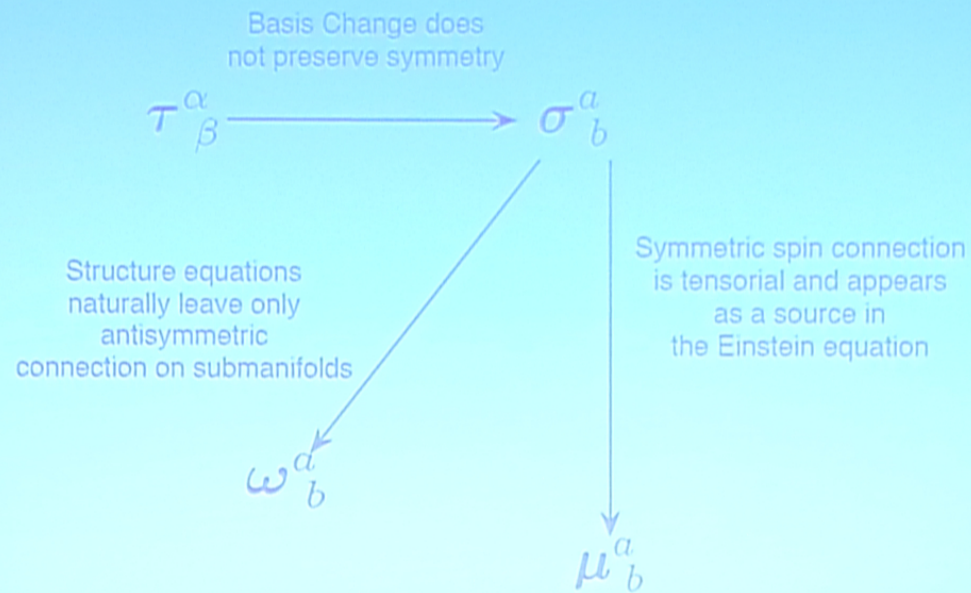
Looking at general starting signatures and imposing the structure of a phase space lead to:

Lorentzian Signature Theorem⁵

Flat 8-dim (2n-dim) biconformal space is a phase space with canonically conjugate, orthogonal, metric sub-manifolds if and only if the initial 4-dim (n-dim) space we gauge is Euclidean or signature zero. In either of these cases the resulting configuration sub-manifold is necessarily Lorentzian.

$$\eta_{ab} = \delta_{ab} - \frac{2v_a v_b}{v^2}$$

⁵Spencer, Wheeler, Int. Jour. Geom. Meth. Mod. Ph. Vol 8, 2(2011), pg 273-303

Curved Biconformal Space: *Current Work*

Curved Biconformal Space: *Current Work*

Simplified symmetric spin connection:

Combining the field equations from the action linear in the curvatures with the integrability conditions on the submanifolds (First Bianchi identity):

- **General Relativity on submanifolds.** (Almost)
- **Dilatational curvature vanishes** on the submanifolds.
- Submanifold Weyl vector structure equations.

$$d\omega|_x = 0 \rightarrow v \wedge dv|_x = 0 \quad \} \text{Hypersurface orthogonal}$$

Curved Biconformal Space: *Current Work*

Simplified symmetric spin connection:

Combining the field equations from the action linear in the curvatures with the integrability conditions on the submanifolds (First Bianchi identity):

- **General Relativity on submanifolds.** (Almost)
- **Dilatational curvature vanishes** on the submanifolds.
- Submanifold Weyl vector structure equations.

$$d\omega|_x = 0 \rightarrow v \wedge dv|_x = 0 \quad \} \text{Hypersurface orthogonal}$$

Biconformal Space and Shape Dynamics

Like Shape Dynamics

- Based on ideas about what can be measured.
- Scale Invariant theory
- The scale invariance is connected to the “flow of time”.
- Time is an emergent phenomenon.
- Time is connected with dynamics.
- Agrees with “time-foliated” GR . (Also in CMC gauge)

Extra

- Full conformal symmetry
- Reference matter.
- Gravity in n dimensions

Curved Biconformal Space: *Current Work*

The current goal is to reproduce general relativity from Euclidean biconformal gauge theory in such a way that time arises as part of the solution.

Start with:

- Biconformal gauging
- Canonically conjugate metric sub-manifolds
- Action linear in curvatures *in orthonormal symplectic basis.*

$$S = \int (\alpha \Omega_b^a + \beta \delta_b^a \Omega + \gamma e^a f_b) \varepsilon_{ac\dots d} \varepsilon^{be\dots f} e^c \dots e^d f_e \dots f_f$$