Title: Quantum Cosmology - 2

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Abstract:

Confronting Loop Quantum Cosmology with Observations

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Loops '13

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WE, Class.Quant.Grav. 29 (2012) 215013,
WE, JCAP 1303 (2013) 026,
WE, JCAP (at press), arXiv:1306.6582 [gr-qc].

Motivation

It is generally expected that quantum gravity effects will only become important when

- the space-time curvature becomes very large,
- or at very small scales / very high energies.

Since we cannot probe sufficiently small distances with accelerators, or even with cosmic rays, the best chance of testing any theory of quantum gravity appears to be by observing regions with high space-time curvature.

The two obvious candidates are black holes and the early universe. However, since the strong gravitational field near the center of astrophysical black holes is hidden by a horizon, it seems that observations of the early universe are the best option.

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Cosmological Observables

The variables $\mathcal{R}_k$ and h_k are the variables that represent the scalar and tensor perturbations, in Fourier space. The scalar power spectrum is

$$\Delta^2_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2 \sim k^{n_s-1},$$

and if the perturbations are scale-invariant, $n_s = 1$.

The relative amplitudes of tensor and scalar modes are given by the tensor-to-scalar ratio r,

$$r = \frac{\Delta^2_h(k)}{\Delta^2_{\mathcal{R}}(k)}.$$

Observations of temperature anisotropies in the CMB by the Planck collaboration, and of its polarization by WMAP, indicate that

$$n_s = 0.9603 \pm 0.0073, \qquad (68\%),$$

$$r < 0.120, \qquad (95\%).$$

The Goal

Use cosmological observations to test LQC.

There exist several cosmologies that predict scale-invariant scalar perturbations and a small r, among them

- inflation,
- the matter bounce,
- the ekpyrotic universe.

As observations becomes more precise, one scenario is likely to become favoured.

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- inflation,
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As observations becomes more precise, one scenario is likely to become favoured.

- How does LQC change the predictions in these scenarios when quantum gravity effects are included?
- Are there LQC corrections (that may be small) to the predictions that can be tested observationally?

I will focus on holonomy corrections.

Outline

1. Perturbations in Loop Quantum Cosmology
2. Predictions for Two Loop Cosmologies
   - The Matter Bounce Scenario
   - The Ekpyrotic Universe
3. Conclusion

Loop Quantum Cosmology

In loop quantum cosmology (LQC), we use the same variables as in loop quantum gravity: holonomies of A_a^i and areas.
So, in the definition of the Hamiltonian constraint operator, the field strength operator is expressed in terms of holonomies of A_a^i around the smallest loop possible: a loop that has an area of Δ, the smallest area eigenvalue in loop quantum gravity.

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So, in the definition of the Hamiltonian constraint operator, the field strength operator is expressed in terms of holonomies of A_a^i around the smallest loop possible: a loop that has an area of Δ, the smallest area eigenvalue in loop quantum gravity.

This procedure can be followed for homogeneous cosmologies.
[Bojowald; Ashtekar, Lewandowski, Martín-Benito, Mena Marugán, Olmedo, Pawłowski, Singh, Szulc, Vandersloot, WE, ...].

However, it is hard to use loop techniques to treat inhomogeneities. Instead, inhomogeneities are usually treated by a Fock quantization on a loop background; these are called 'hybrid' approaches, [Garay, Martín-Benito, Martín-de Blas, Mena Marugán, WE, ...] and can be used for perturbations too.
[Fernández-Méndez, Mena Marugán, Olmedo; Agulló, Ashtekar, Nelson]

Perturbations in Loop Quantum Cosmology

One approach that makes it possible to perform a loop quantization of both the background and the perturbations is to generalize the 'separate universes' approach to LQC. [Wands, Malik, Lyth, Liddle]

a(1) φ(1)	a(2) φ(2)	a(3) φ(3)
a(4) φ(4)	a(5) φ(5)	a(6) φ(6)
a(7) φ(7)	a(8) φ(8)	a(9) φ(9)

The idea is to work on a lattice, where each cell is homogeneous, and the gravitational and matter fields vary from one cell to another. Then, after some gauge-fixing, the standard LQC techniques can be used in each homogeneous cell. It turns out that interaction terms are easy to handle.

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Then, for states where the perturbations are small, the scalar and diffeomorphism constraints for each cell, $\widehat{\mathcal{H}}(\vec{z})$ and $\widehat{\mathcal{H}_a}(\vec{z})$, weakly commute with the Hamiltonian $\widehat{\mathcal{C}_H}$.

Effective Equations of Lattice LQC

In the FLRW models, the effective equations provide an excellent approximation to the dynamics of sharply peaked states, throughout the entire evolution including the bounce.

We will assume that the effective equations will be a good approximation for the perturbations too.

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Defining

$$z = a\sqrt{\rho + P}/H,$$

the effective equation for $v = z\mathcal{R}$ coming from lattice LQC is

$$v'' - \left(1 - \frac{2\rho}{\rho_c}\right)\nabla^2 v - \frac{z''}{z} v = 0,$$

where ρ_c is the critical energy density of LQC.

The Anomaly Freedom Approach

Another way to obtain effective equations for perturbations in LQC is to follow the anomaly freedom approach.

Working at the effective level, correction functions are added to the classical constraints in order to represent holonomy or inverse triad effects. Then the constraint algebra typically no longer closes, so it is necessary to add some counterterms in order to ensure that the constraint algebra remains free of any anomalies. [Bojowald, Hossain, Kagan, Shankaranarayanan]

When this is procedure is followed with holonomy corrections for perturbations on a flat FLRW background, the same effective equation is found for v as in lattice LQC, [Cailleateau, Mielczarek, Barrau, Grain]

$$v'' - \left(1 - \frac{2\rho}{\rho_c}\right) \nabla^2 v - \frac{z''}{z} v = 0.$$

An Aside: The Constraint Algebra

A modification to the constraint algebra for the truncated constraints appears in the Poisson bracket of the two scalar constraints:

$$\{\mathcal{H}[N_1], \mathcal{H}[N_2]\} = \left(1 - \frac{2\rho}{\rho_c}\right) \mathcal{H}_a[N_1 \partial^a N_2 - N_2 \partial^a N_1].$$

Recall that

$$\{\mathcal{H}[N_1], \mathcal{H}[N_2]\} = -s\, \mathcal{H}_a[N_1 \partial^a N_2 - N_2 \partial^a N_1],$$

where $s = -1$ for Lorentzian space-times and $s = 1$ for Euclidean space-times.

Does this mean that the space-time becomes Euclidean near the bounce? The results of Hojman, Kuchař and Teitelboim seem to argue that it does. [Bojowald, Paily]

An Aside: The Euclidean Scalar Constraint

The scalar constraint in loop variables is

$$\mathcal{H}_g = s\frac{E^a_i E^b_j \epsilon^{ij}{}_k F_{ab}{}^k}{\gamma^2\sqrt{q}} - \left(1 - \frac{s}{\gamma^2}\right)\sqrt{q}\,{}^{(3)}R + 2\sqrt{q}\,\Lambda.$$

Do the s terms appearing in the action change signs?

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Do the s terms appearing in the action change signs?

This is most easily checked in the flat $\Lambda \neq 0$ FLRW model, where ${}^{(3)}R = 0$. The LQC effective scalar constraint is [Pawłowski, Ashtekar]

$$\mathcal{H}_g = \frac{3p^{3/2}}{8\pi G}\left(-\frac{\sin^2\bar{\mu}c}{\gamma^2\Delta} + \frac{\Lambda}{3}\right),$$

and we see that the sign between the two terms never changes. So it seems as though the space-time does not become Euclidean at the bounce point after all.

Maybe the results of Hojman, Kuchař and Teitelboim do not hold here because these are truncated constraints?

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Predictions from LQC

Let's return now to the main focus of this talk, and use the holonomy-corrected Mukhanov-Sasaki equation in order to determine whether LQC effects may modify the standard predictions.

There exist several possible cosmologies, among them:

- Inflation [Bojowald, Calcagni, Tsujikawa; Agulló, Ashtekar, Nelson; Linsefors, Cailleteau, Barrau, Grain, ...],
- Matter Bounce,
- Ekpyrotic Universe.

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Example: The Matter Bounce Scenario

The matter bounce is an alternative to inflation where a contracting dust-dominated universe with

$$P = 0, \qquad \Rightarrow \qquad \rho \sim a^{-3},$$

turns quantum vacuum fluctuations into scale-invariant fluctuations, just as an exponentially expanding universe does. [Wands]

If there is a bounce, continuity arguments give some hope that the perturbations in the expanding post-bounce era will also be scale-invariant. [Finelli, Brandenberger]

However, the specifics of how the perturbations travel through the bounce depend on the detailed dynamics of the bounce, and also whether the equations of motion for the perturbations are modified as they go through the bounce.

Perturbations in the Matter Bounce

The classical solution to the Mukhanov-Sasaki equation is

$$v_k(\eta) = \frac{\sqrt{\pi\hbar(-\eta)}}{2} H^{(1)}_{\frac{3}{2}}(-k\eta),$$

where the numerical pre-factor is chosen so that v_k are quantum vacuum fluctuations at early times ($\eta \to -\infty$),

$$v_k \sim \sqrt{\frac{\hbar}{2k}} e^{-ik\eta}.$$

Then, when the modes exit the Hubble radius as the bounce is approached, $\mathcal{R}_k = v_k/z$ contains a growing scale-invariant term,

$$\mathcal{R}_k \sim k^{3/2} + \frac{k^{-3/2}}{\eta^3}.$$

Evolution Through the Bounce

In the long wavelength limit, the LQC Mukhanov-Sasaki equation is

$$v_k'' - \frac{z''}{z} v_k = 0, \qquad v_k = A_1 z + A_2 z \int^{\eta} \frac{\mathrm{d}\bar{\eta}}{z^2},$$

where $z \sim (t^2/t_{\rm Pl}^2 + 1)^{5/6}/t$. The solution for $\mathcal{R}_k = v_k/z$ is

$$\mathcal{R}_k \sim k^{3/2} + k^{-3/2} \left[t_{\rm Pl} \left(\arctan(t/t_{\rm Pl}) + \frac{\pi}{2} \right) - \frac{t}{(t/t_{\rm Pl})^2 + 1} \right].$$

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The k-dependence is determined by demanding that for $t \ll -t_{\rm Pl}$,

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in agreement with the classical result. After the bounce for $t \gg t_{\rm Pl}$,

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Results for the Matter Bounce in LQC

The same procedure can be followed for the tensor perturbations (although the LQC corrections to the Mukhanov-Sasaki equation for tensor modes are slightly different), and we find that:

- Initial vacuum fluctuations give scale-invariant perturbations in the post-bounce expanding branch,
- The scalar and tensor perturbations have the same tilt,
- The tensor-to-scalar ratio is small, $r \sim 10^{-3}$,
- In order to match observations, $\rho_c \sim 10^{-9} \rho_{\rm Pl}$.

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The Ekpyrotic Universe

The ekpyrotic universe is a slowly contracting space-time

$$a(t) = (-t)^p, \qquad 0 < p \ll 1,$$

with one or two scalar fields φ with some potentials $V(\varphi)$ that mimic a perfect fluid

$$P = \omega\rho, \qquad \omega = \frac{2}{3p} - 1.$$

In the contracting branch, the Bardeen potential becomes scale-invariant, though not the comoving curvature perturbations $\mathcal{R}$.
[Khoury, Ovrut, Steinhardt, Turok; Lyth]

If there is more than one scalar field, then the entropy perturbations δs are also scale-invariant. [Finelli; Lehners, McFadden, Turok, Steinhardt]

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The Single Scalar Field Case

The Bardeen potential contains a growing mode in the contracting branch that is scale-invariant,

$$\Phi \sim k^{-1/2} + \frac{k^{-3/2}}{\eta}.$$

What happens on the other side of the bounce?

By solving the LQC Mukhanov-Sasaki equation in the ekpyrotic universe, we find that in the expanding branch of the universe

$$\Phi \sim k^{-1/2} + k^{1/2} + \frac{k^{-3/2}}{\eta}.$$

Since the last term will decay rapidly in the expanding universe, the dominant term for long wavelengths goes as $k^{-1/2}$ and has a blue spectrum.

The Two Scalar Field Case

If there are two scalar fields, then entropy perturbations can become important. Assuming initial quantum vacumm perturbations, as the bounce is approached the growing mode of δs_k is scale-invariant.

Entropy perturbations can source curvature perturbations. From this entropic mechanism, the curvature perturbations become [Lehners, McFadden, Turok, Steinhardt; Buchbinder, Khoury, Ovrut, ...]

$$\mathcal{R}_k \sim k^{-1/2} + \frac{k^{1/2}}{\eta} + k^{3/2} + k^{-3/2},$$

already in the contracting branch. Now we can use LQC to determine the form of the comoving curvature perturbations in the expanding branch,

$$\mathcal{R}_k \sim k^{-3/2} + k^{-1/2} + k^{1/2} + k^{3/2} + \frac{k^{1/2}}{\eta},$$

and we find that the dominant term is scale-invariant.

Results for Ekpyrotic LQC

- The ekpyrotic scenario with a single scalar field is not viable in LQC.
- The two scalar field realization of the ekpyrotic universe agrees with observations: a nearly scale-invariant power spectrum, and no tensor modes.
- The predicted values for the tilt and amplitude of the scalar perturbations depend on the specific form of the potentials for the scalar fields.
- The dominant term in the constant mode of $\mathcal{R}_k$ can be determined in the classical regime before the bounce; LQC corrections do not affect it.

Conclusions

- LQC effective equations with holonomy corrections for perturbations can be obtained by working with a lattice of homogeneous cosmologies.
- These effective equations can be used to see how perturbations travel through the bounce, e.g. in the matter bounce scenario and the ekpyrotic universe.
- The matter bounce and the ekpyrotic universe with two scalar fields are both in agreement with the results of Planck.
- Observations of tensor perturbations with $r \gtrsim 0.01$ would rule both of these cosmologies out.
- Next steps: inflationary models going beyond the $V(\varphi) \sim \varphi^2$ potentials, non-Gaussianities, anisotropies, and more.

LOOP QUANTUM COSMOLOGY IN THE COSMIC MICROWAVE BACKGROUND

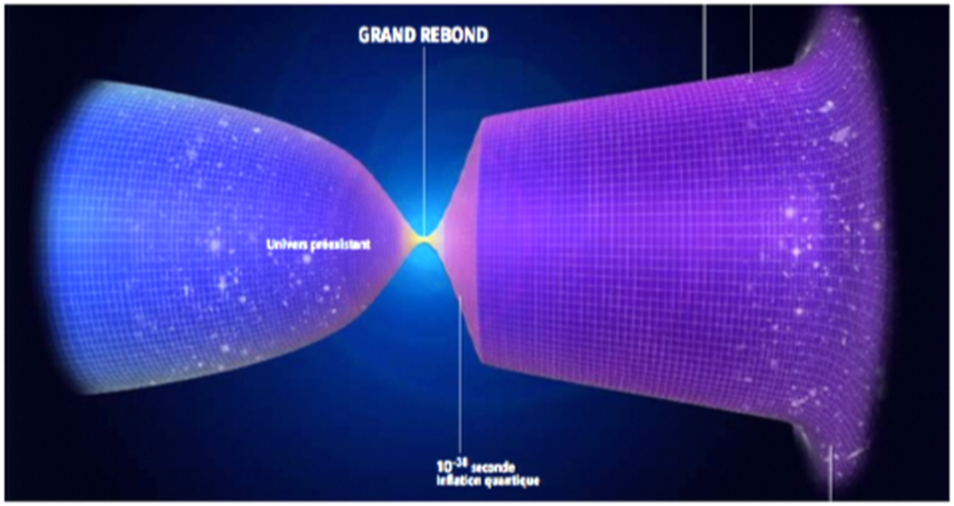
-J. Grain (IAS, Orsay)-

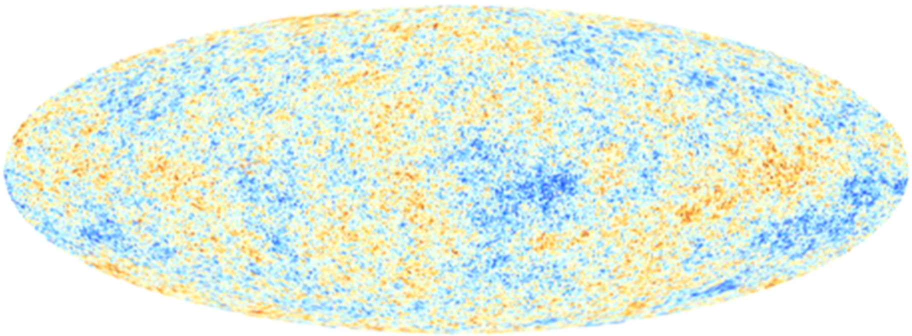
-B. Bolliet, L. Linsefors, A. Barrau (LPSC, Grenoble)-

-T. Cailleteau (Penn State)-

-C. Stahl (Univ. Paris Sud)-

-J. Mielczarek (Cracaw University)-

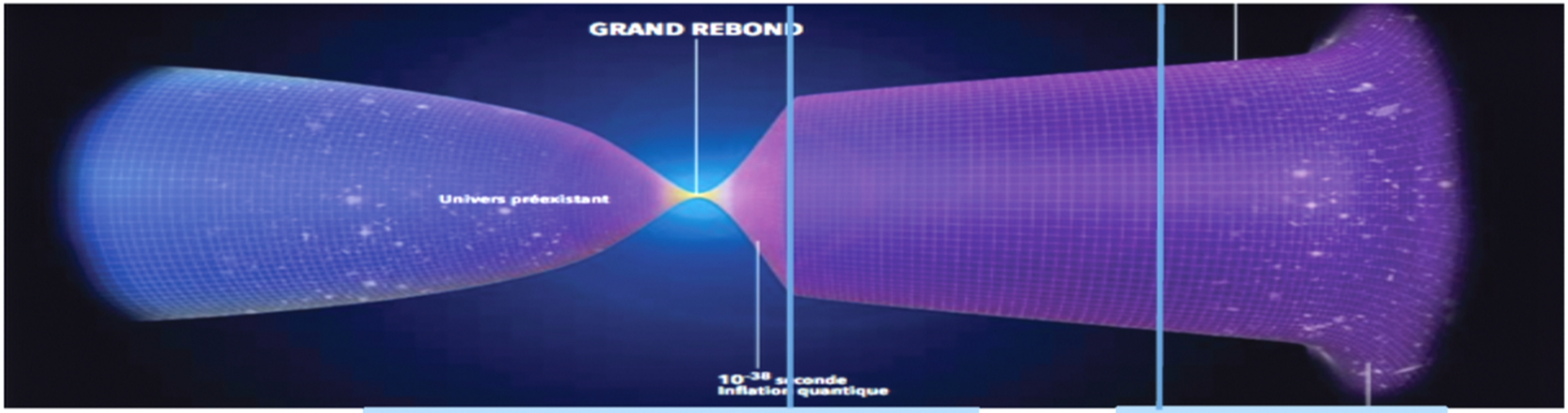




THE STRATEGY

LQC-model of the Universe :

- ✓ Background evolution : Bounce and inflation
- ✓ Perturbations (scalar and tensor) : the observables !



Power spectrum for primordial perturbations (at the end of inflation) → **CMB angular power spectra**

- ✓ **Based on holonomy corrections** (inverse volume not considered here)
- ✓ **Assume the following small scale limite** $P_{S/T}^{LQC}(k \to \infty) \approx P_{S/T}^{STD}(k)$

SEE M. BOJOWALD'S TALK

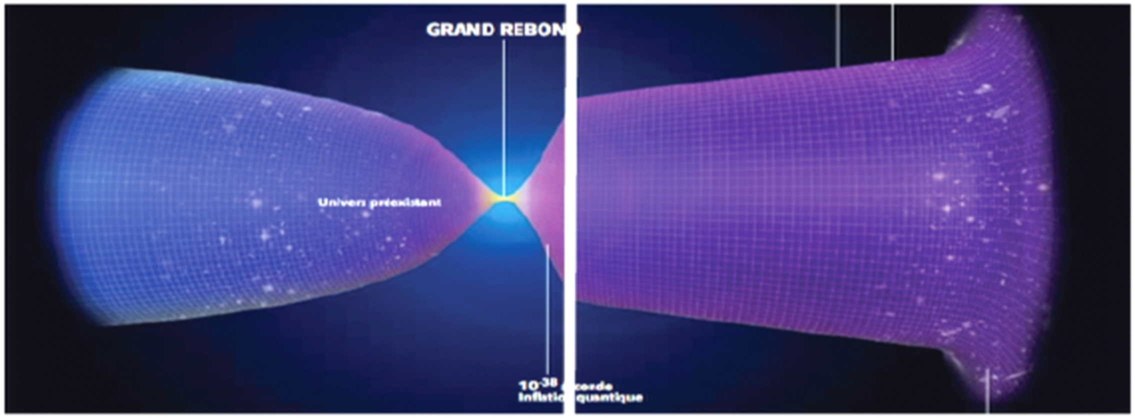
SEE B. BOLLIET'S POSTER

FROM *PRIMORDIAL* SPECTRUM TO *ANGULAR* SPECTRUM (I)

The line-of-sight solution : $C_\ell^{BB} = \langle a_{B,\ell m} a^*_{B,\ell m} \rangle = \int \left(\Delta_\ell^B (k/k_H) \right)^2 P_T(k) dk$

Δ_L transfer functions : probes from the end of inflation to today

P_T primordial power spectrum : probes the (contraction, bounce and) inflation



Described by P_T Described by Δ_L

Inapplicability of L.O.S. : break the history of the Universe in two parts
✓ After the end of inflation : the standard cosmic history
✓ Before the end of inflation : the Universe is dominated by the scalar field (no radiative transfer, particles production)

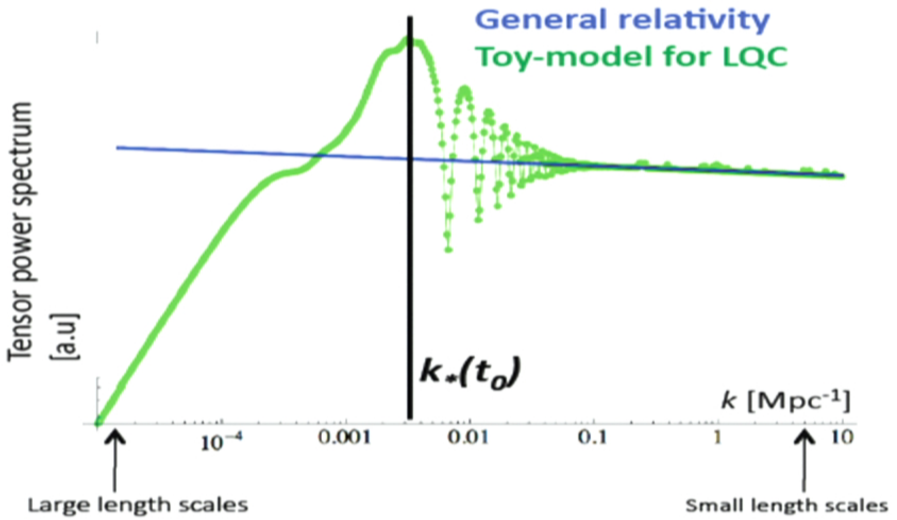
FROM *PRIMORDIAL* SPECTRUM TO *ANGULAR* SPECTRUM (III)

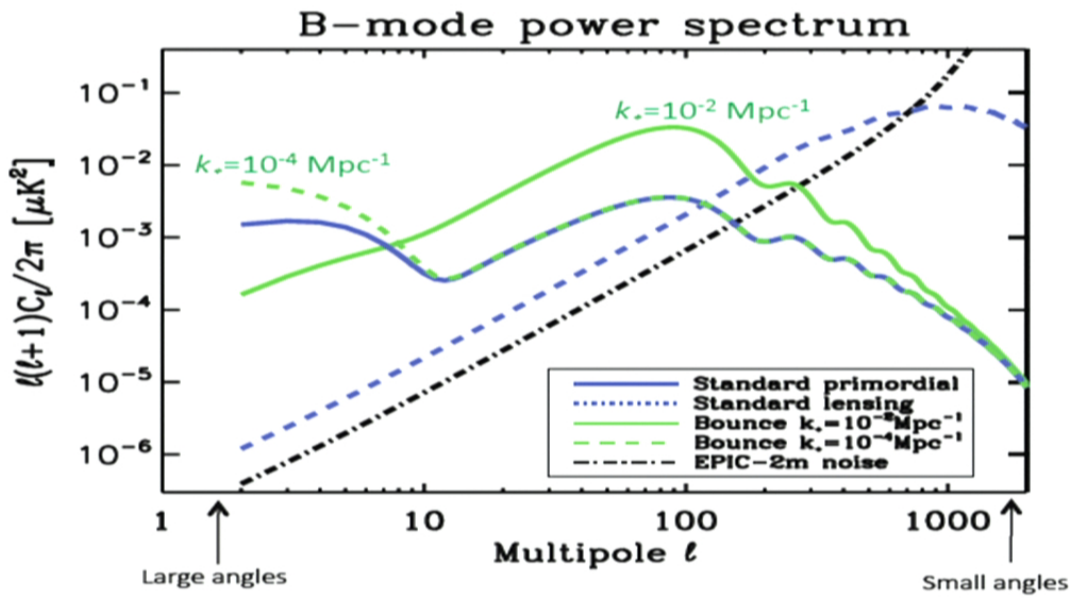
The Hubble scale *today* : $k_H \sim 2.3\times10^{-4}$ Mpc^{-1}
if $k<k_H$ the mode cannot be observed

Typical scale of LQC at the *bounce* : $k_\star \sim (8\pi G\rho_c)^{1/2}$
if $k<k_\star$ LQC affects the power spectrum

LQC observed in CMB if $k_\star > k_H$ *today* : $(k_*(t_B)/k_H(t_0)) \geq \exp(N(t_B,t_0))$
$k_\star(t_B) \sim 1\, L_{Pl}^{-1}$: $N_{inf} < 70\text{-}90$ (depending on the reheating and superinflation)

$$\rightarrow \ell_* \approx k_*(t_0)/k_H(t_0) \text{ assuming } \Delta_\ell(k/k_H) \propto \delta(k/k_H - \ell)$$





MIELCZAREK, CAILLETEAU, GRAIN, BARRAU, *Phys. Rev. D 81 104049 (2010)*
GRAIN, BARRAU, CAILLETEAU, MIELCZAREK, *Phys. Rev. D 82 123520 (2010)*

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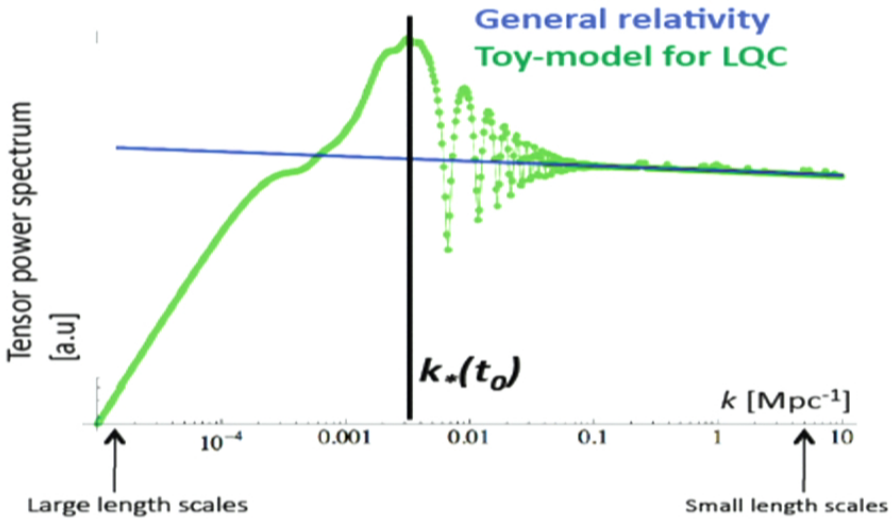
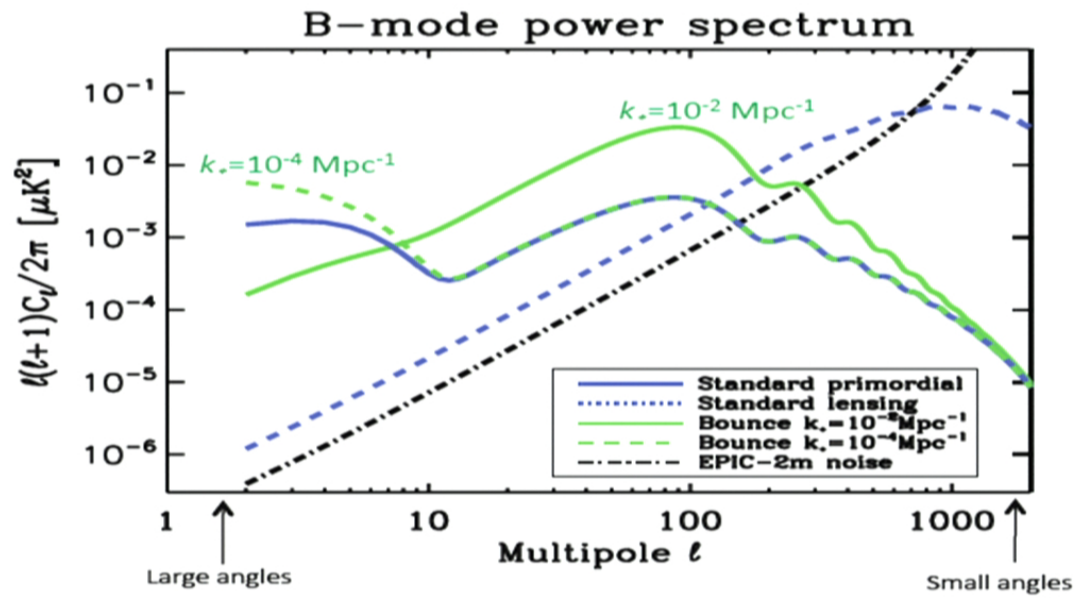
$$\rightarrow \ell_* \approx k_*(t_0)/k_H(t_0) \;\; \text{assuming } \Delta_\ell(k/k_H) \propto \delta(k/k_H - \ell)$$



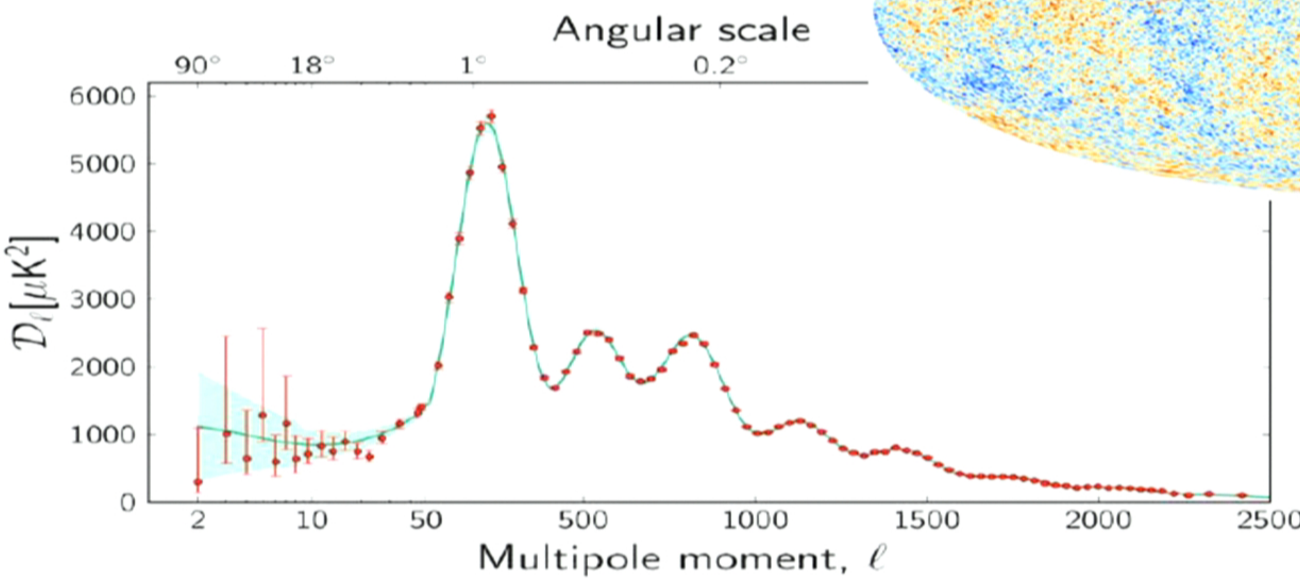


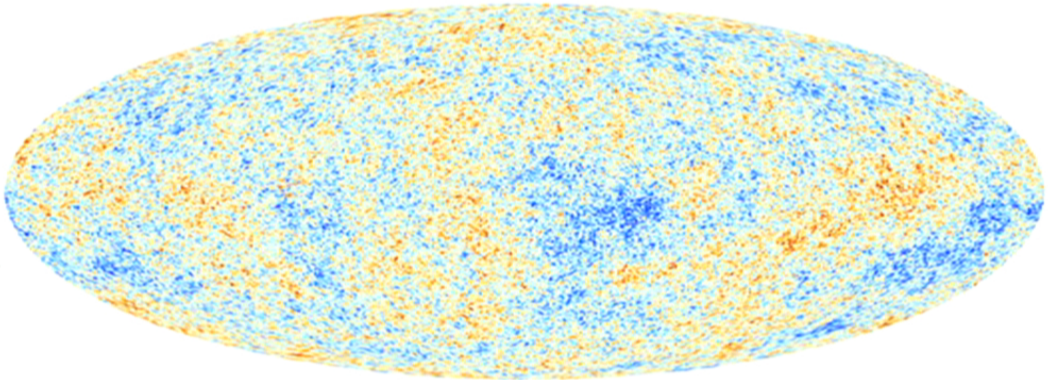
Mielczarek, Cailleteau, Grain, Barrau, *Phys. Rev. D 81 104049 (2010)*
Grain, Barrau, Cailleteau, Mielczarek, *Phys. Rev. D 82 123520 (2010)*

FROM *ANGULAR* SPECTRUM TO *PRIMORDIAL* SPECTRUM (I)

What are the major outputs of a CMB experiment ?

PLANCK temperature map and C_L's





CMB maps, CMB angular power spectra are mandatory… but not *SUFFICIENT* !!!

FROM *ANGULAR* SPECTRUM TO *PRIMORDIAL* SPECTRUM (II)

A final output LIKELIHOOD FUNCTION: $L(\theta_i) = \mathrm{Prob}(C_\ell^{obs} \mid \theta_i)$

$$P(\theta_i \mid C_\ell^{obs}) = \frac{L(\theta_i) \times \mathrm{Prob}(\theta_i)}{\mathrm{Prob}(C_\ell^{obs})}$$

dual object: relates observed spectra to predicted spectra *assuming* a theoretical framework

A simple example: $L(\theta_i) = \frac{1}{\sqrt{2\sigma_\ell[\theta_i]}} \times \exp\left(\frac{(C_\ell^{obs} - C_\ell^{th}[\theta_i])}{2\sigma_\ell^2[\theta_i]}\right)$

with $C_\ell^{th}[\theta_i]$ given by the line-of-sight solution

LOS assumes a specific shape for the theoretical ***C***$_L$**'s**, that is a specific shape for ***P***$_T$***(k)***

➔ any constraint on *e.g.* the inflaton potential assumes *pure inflation*

➔ the prior probability for LQC model is set to **zero**

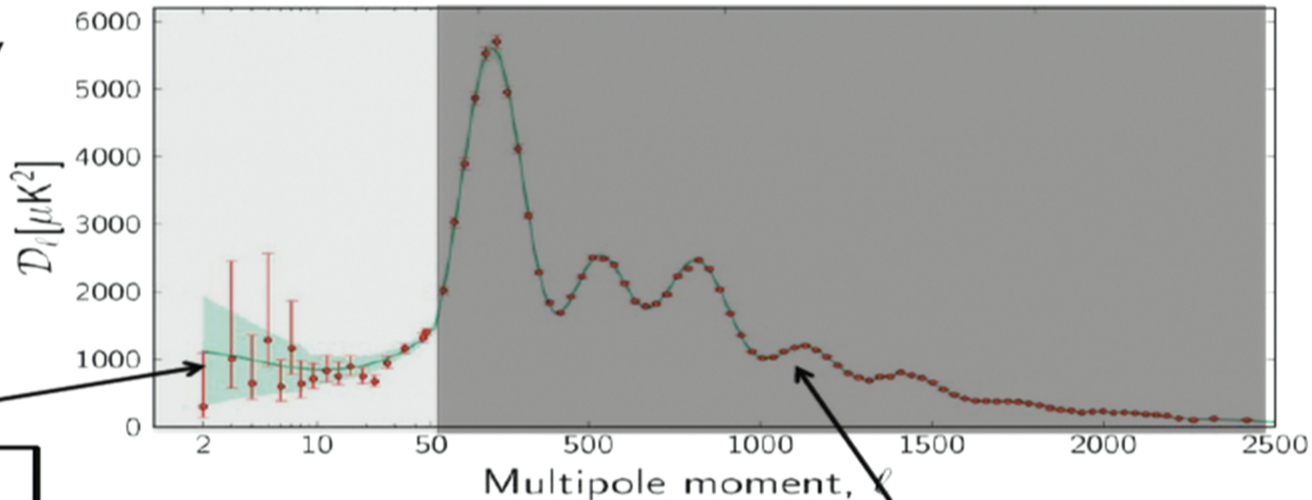
2 steps process:

✓ With CMB, we directly constrain the ***primordial power spectrum***

✓ Translated into constraints on a **specific model** *e.g.* inflaton potential

FROM *ANGULAR* SPECTRUM TO *PRIMORDIAL* SPECTRUM (IV)

Temperature C_L's as measured by PLANCK



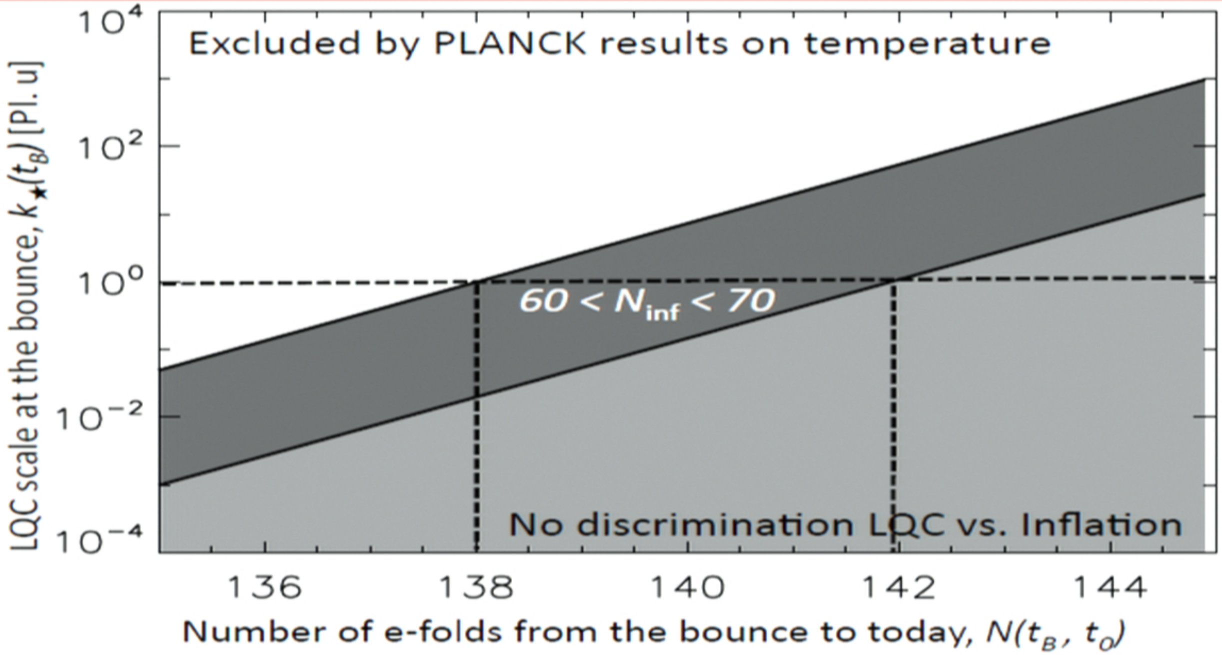
Tension with inflation at ~2.5σ

Very good agreement with a inflationnary *P(k)*

Constraint on the typical scale of LQC : $k_*(t_0)/k_H(t_0) \approx 50$

- ✓ For $k(t_0) > 50 \times k_H$: primordial spectrum should be identical to inflationnary prediction
- ✓ For $k(t_0) < 50 \times k_H$: primordial spectrum should be suppressed and/or oscillations

FROM *ANGULAR* SPECTRUM TO *PRIMORDIAL* SPECTRUM (V)



$k_*(t_0)/k_H(t_0) \approx 50$

$k_*(t_0)/k_H(t_0) \approx 1$

$k_*(t_B) = 1\ L_{Pl}^{-1}$

This assumes that :

$$P_S^{LQC}(k > k_*) \approx P_S^{STD}(k > k_*)$$

CONCLUSION

- ✓ LQC could be observed if $k_*(t_0)/k_H(t_0) > 1$
- ✓ From PLANCK results, LQC parametr space regions not excluded if $k_*(t_0)/k_H(t_0) < 50$
- ✓ Current prediction of LQC shows that $k_\star(t_0)$ falls in that range
- ✓ CMB data could be used to constrain LQC model of the Universe

---

- ✓ **CMB firstly probes the *primordial power spectra*** (asuming classes of models)
- ✓ **One should go through the computation of the LIKELIHOOD for any quantitative conclusions**

  Done for *inverse volume* corrections

  BOJOWALD, CALCAGNI, TSUJIKAWA, *JCAP 11 046 (2011)*
  BOJOWALD, CALCAGNI, TSUJIKAWA, *Phys. Rev. Lett. 107 211302 (2011)*

  Still to be done for *holonomy* correction

Duration of slow-roll inflation as a prediction of effective LQC

Linda Linsefors
Laboratoire de Physique Subatomique et de Cosmologie

July 2013

Modified Friedmann equation: $H^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right)$, $\kappa := 8\pi G$

Inflaton field: $V(\phi) = \frac{m^2}{2}\phi^2 \quad \Rightarrow \quad \ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$

⟹ Predict length of slow-roll inflation

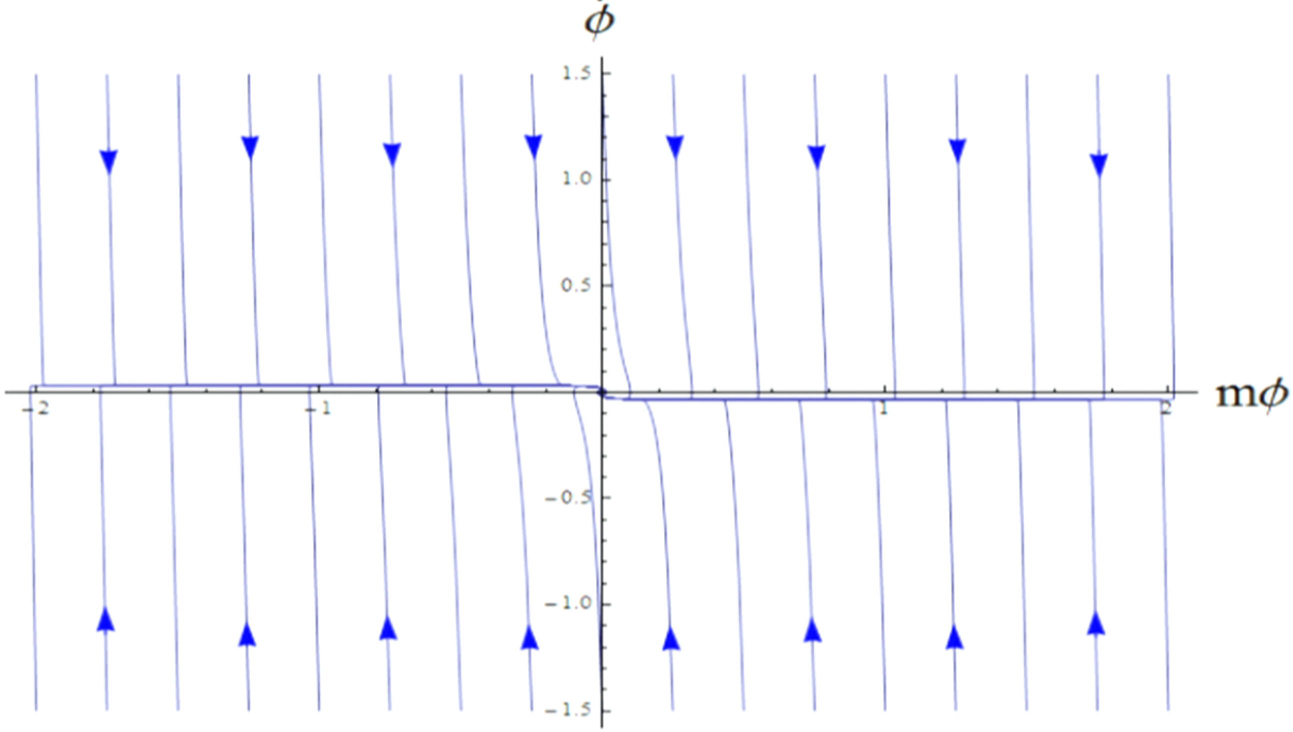
Modified Friedmann equation: $$H^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \qquad \kappa := 8\pi G$$

Inflaton field: $$V(\phi) = \frac{m^2}{2}\phi^2 \quad \Rightarrow \quad \ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

⟹ Predict length of slow-roll inflation

Classical inflation

$$H^2 = \frac{\kappa}{3}\rho, \qquad \ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0, \qquad m = 0.2\, m_{Pl}, \qquad \kappa := 8\pi G$$



1.1

Classical inflation

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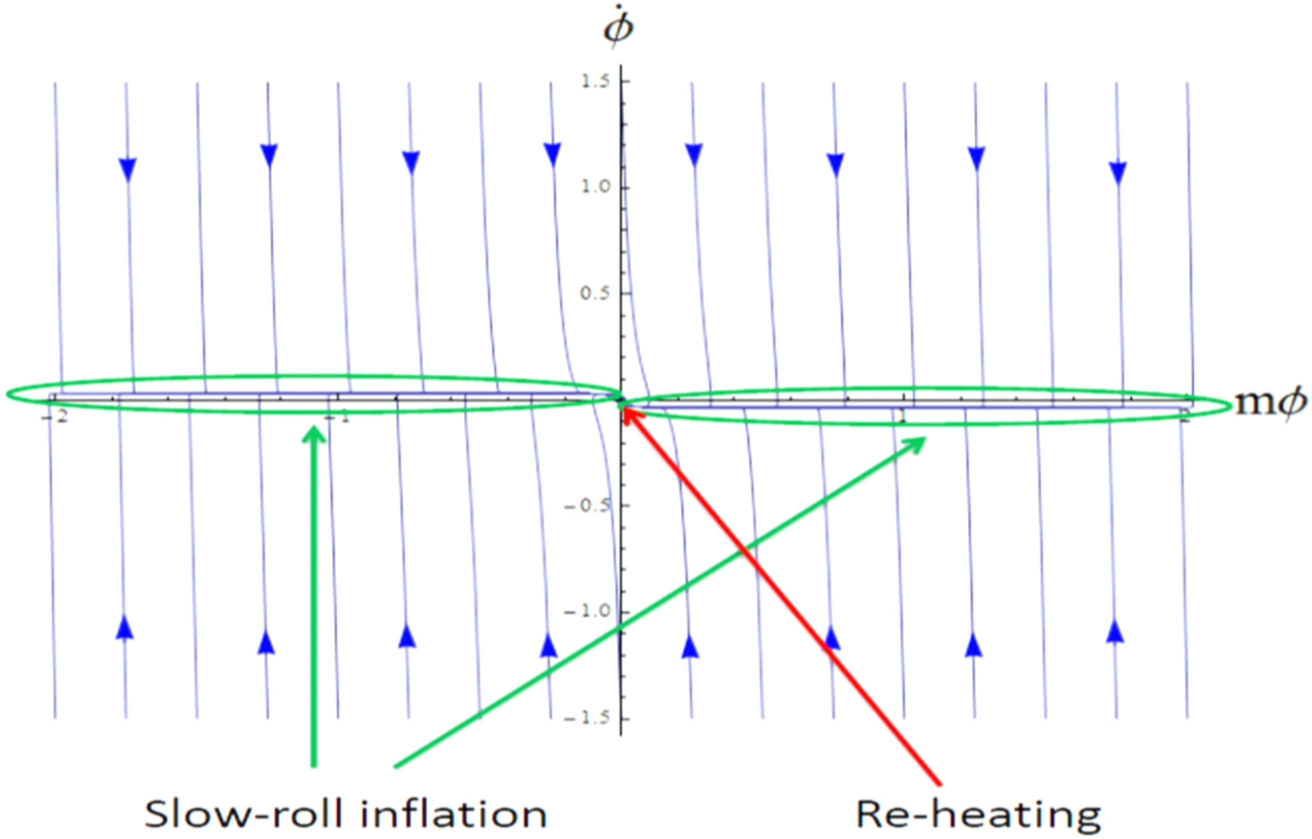
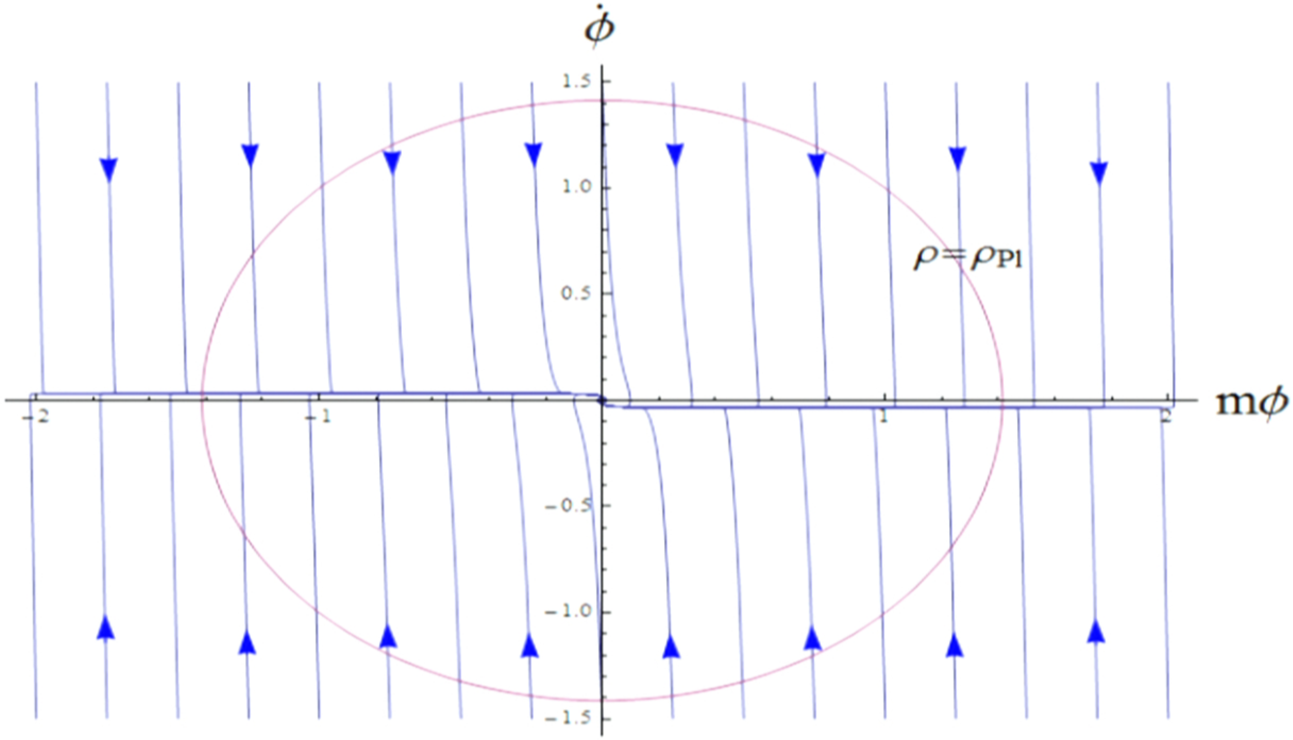


1.2

Classical inflation

$$H^2 = \frac{\kappa}{3}\rho, \qquad \ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0, \qquad m = 0.2\, m_{Pl}, \qquad \kappa := 8\pi G$$



1.3

Classical inflation

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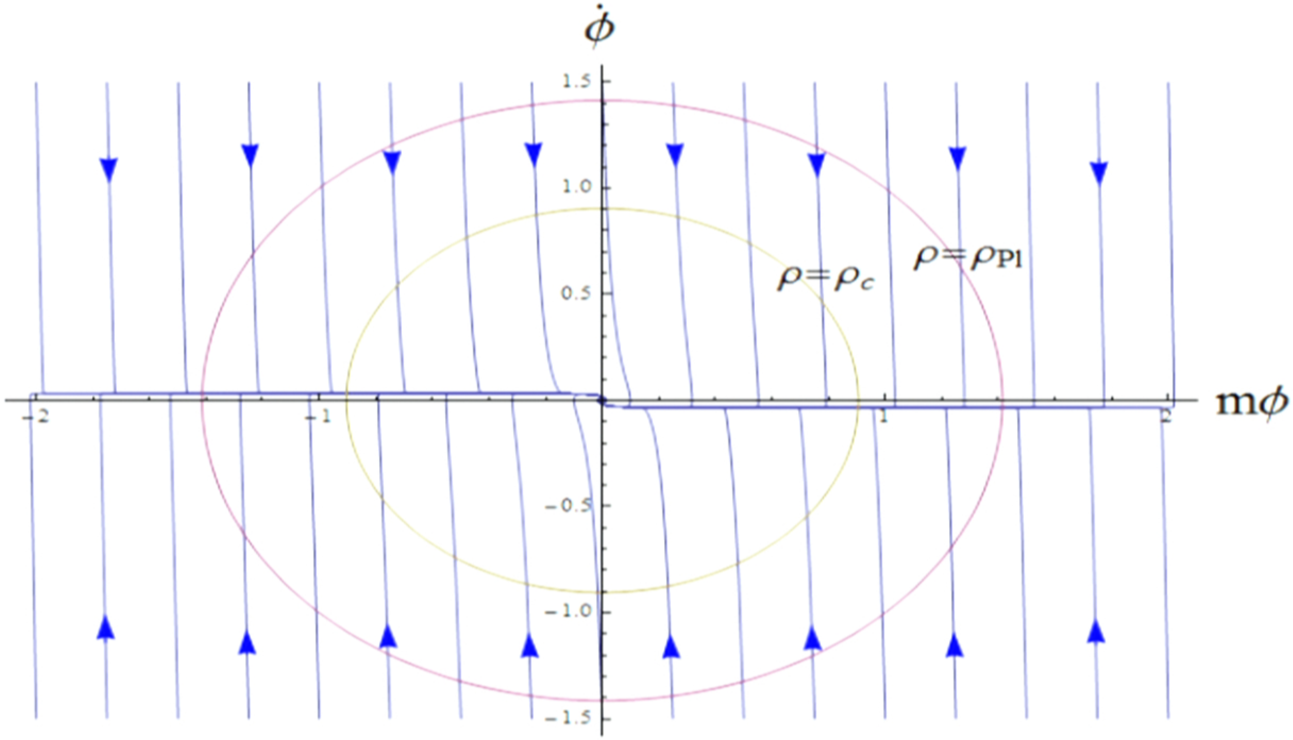
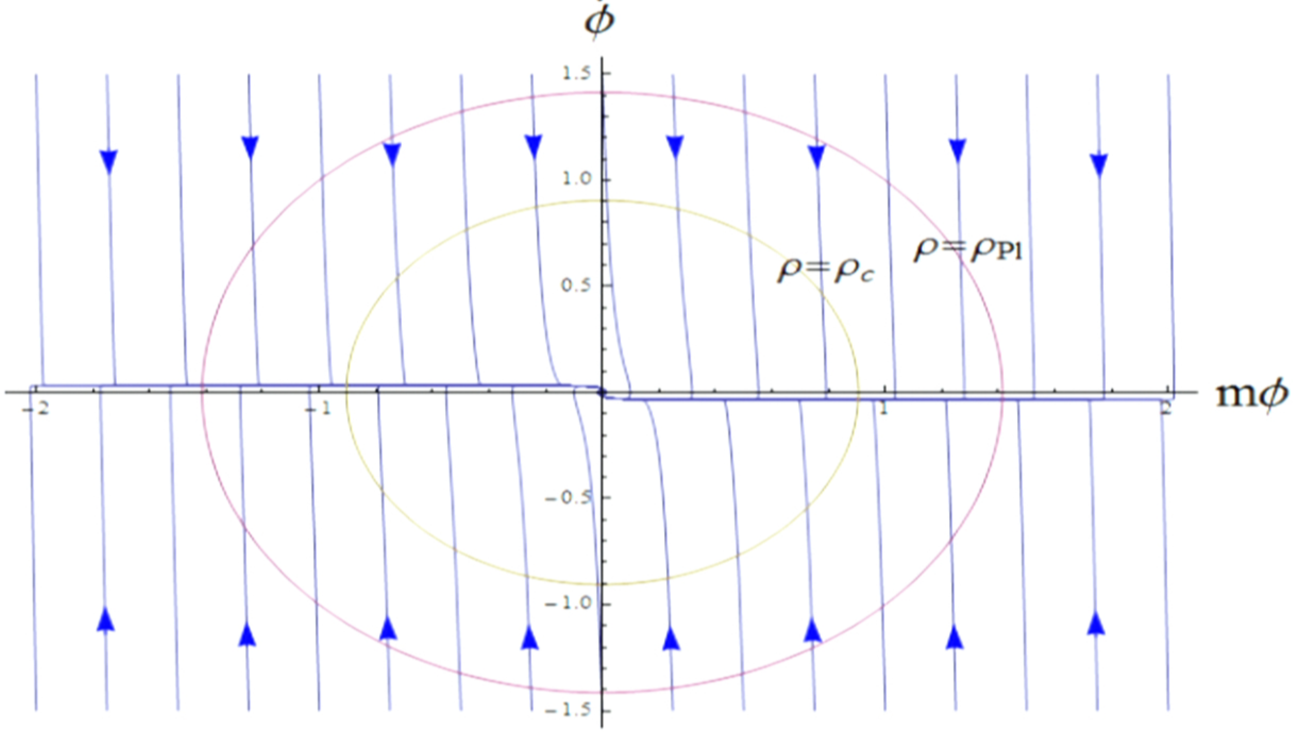


$\rho_c = 0.41\, \rho_{Pl}$, 1.4

Classical inflation

$H^2 = \frac{\kappa}{3}\rho, \qquad \ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0, \qquad m = 0.2\, m_{Pl}, \qquad \kappa := 8\pi G$

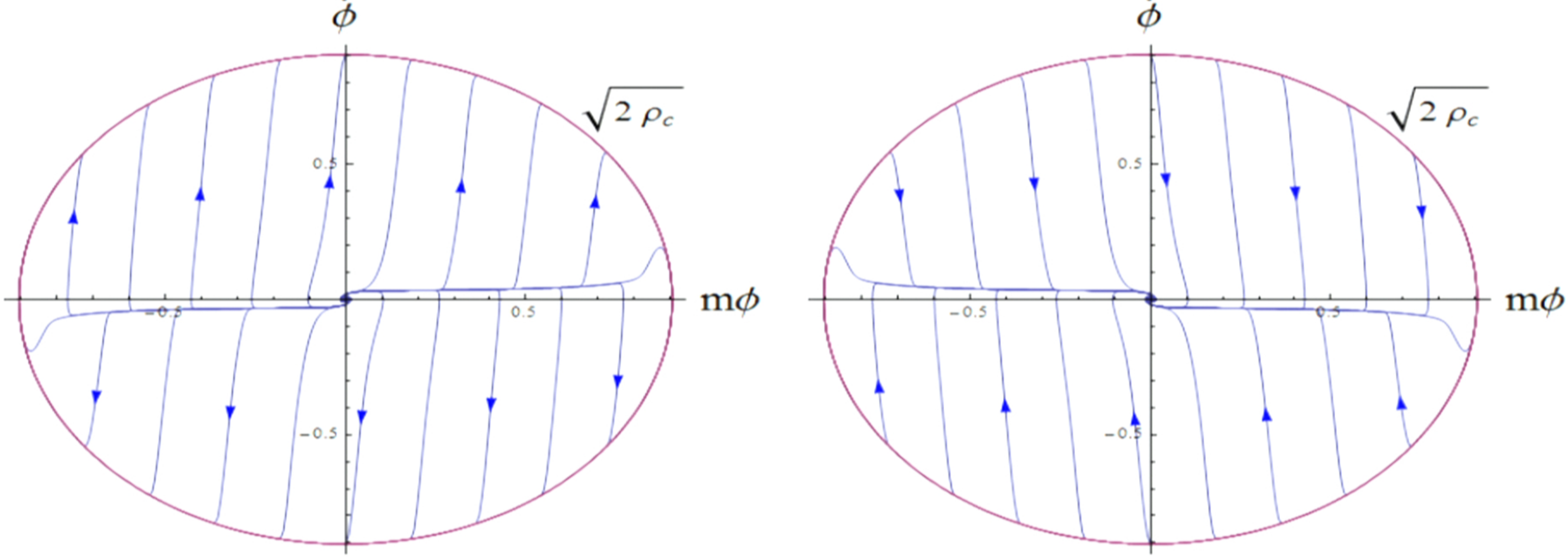


$\rho_c = 0.41\ \rho_{Pl}, \qquad N = 2\pi(\phi_{start})^2 = \frac{2\pi}{m^2}(m\phi_{start})^2$

1.5

LQC inflation

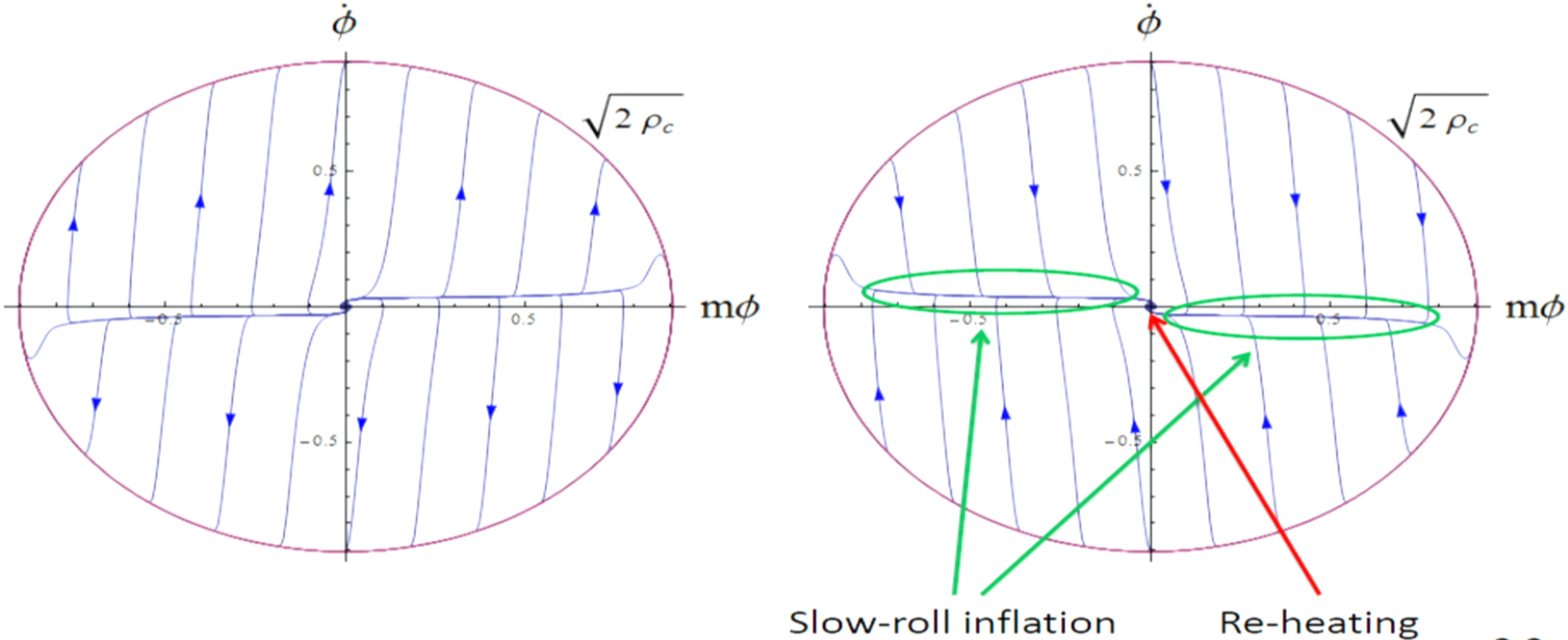
$$H^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \qquad \ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0, \qquad m = 0.2\, m_{Pl}, \qquad \kappa := 8\pi G$$



2.1

LQC inflation

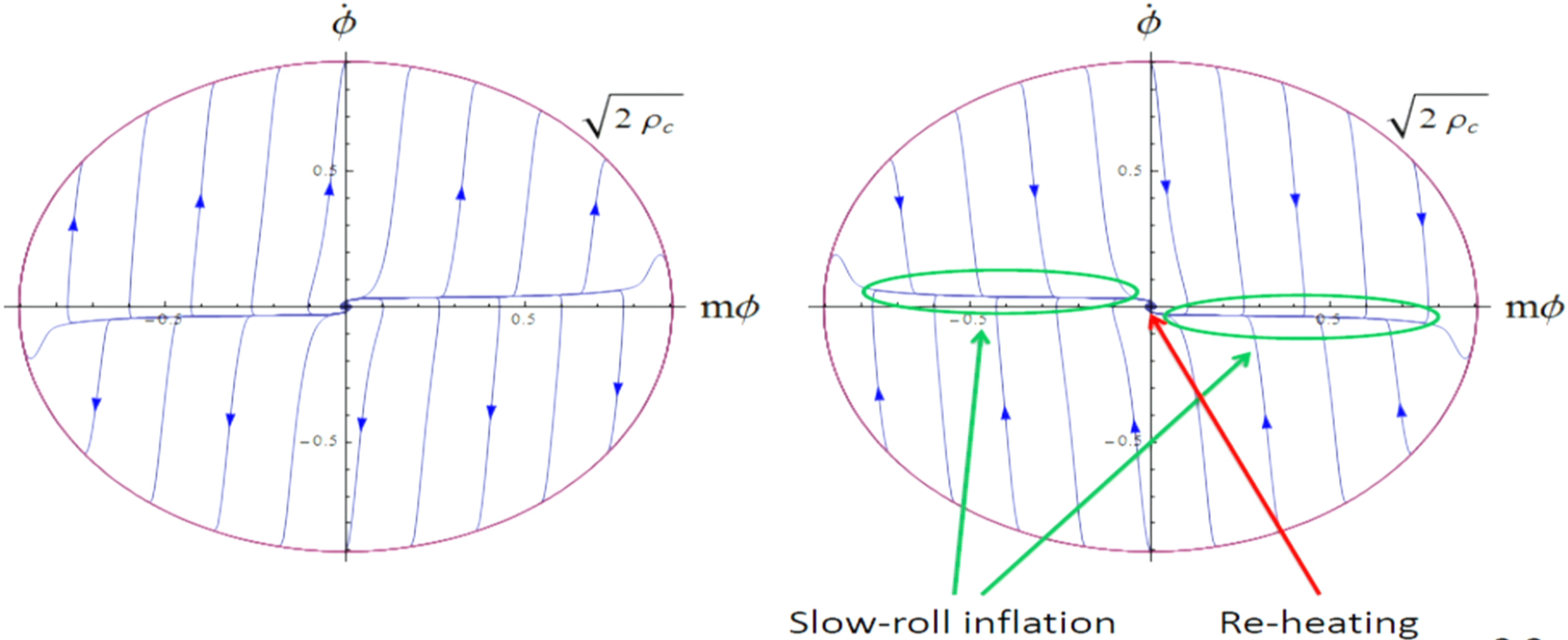
$$H^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \qquad \ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0, \qquad m = 0.2\, m_{Pl}, \qquad \kappa := 8\pi G$$



2.2

LQC inflation

$$H^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \qquad \ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0, \qquad m = 0.2\, m_{Pl}, \qquad \kappa := 8\pi G$$



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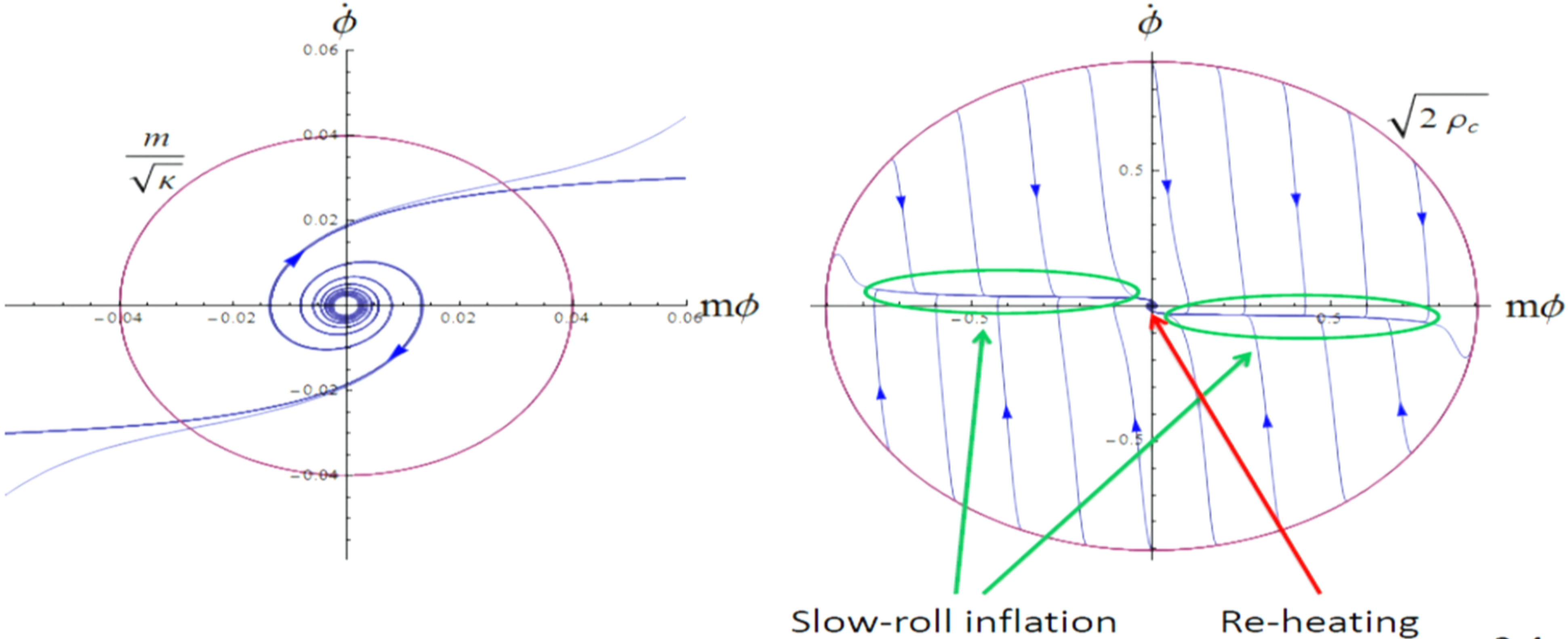
LQC inflation

$$H^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \qquad \ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0, \qquad m = 0.2\, m_{Pl}, \qquad \kappa := 8\pi G$$



2.4

LQC inflation

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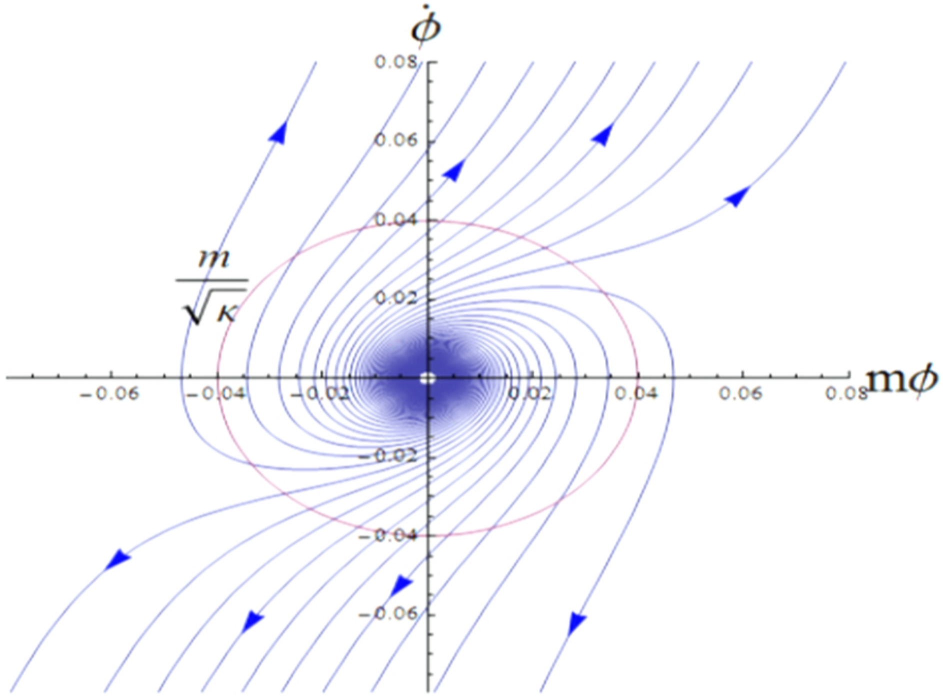

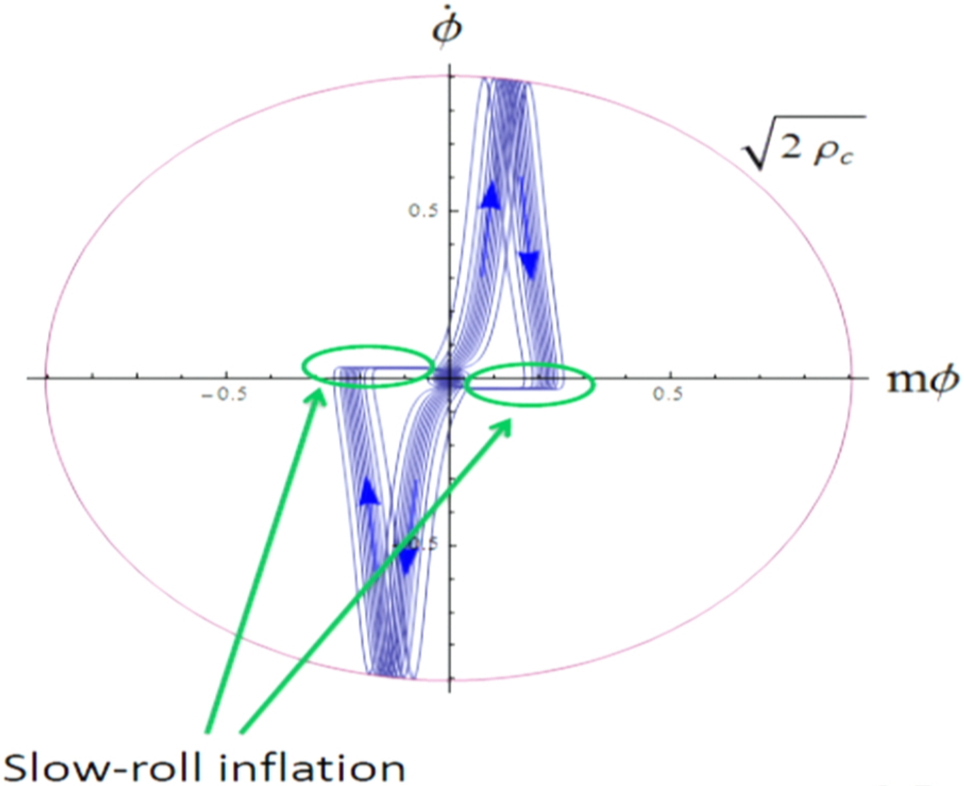

2.5

LQC inflation

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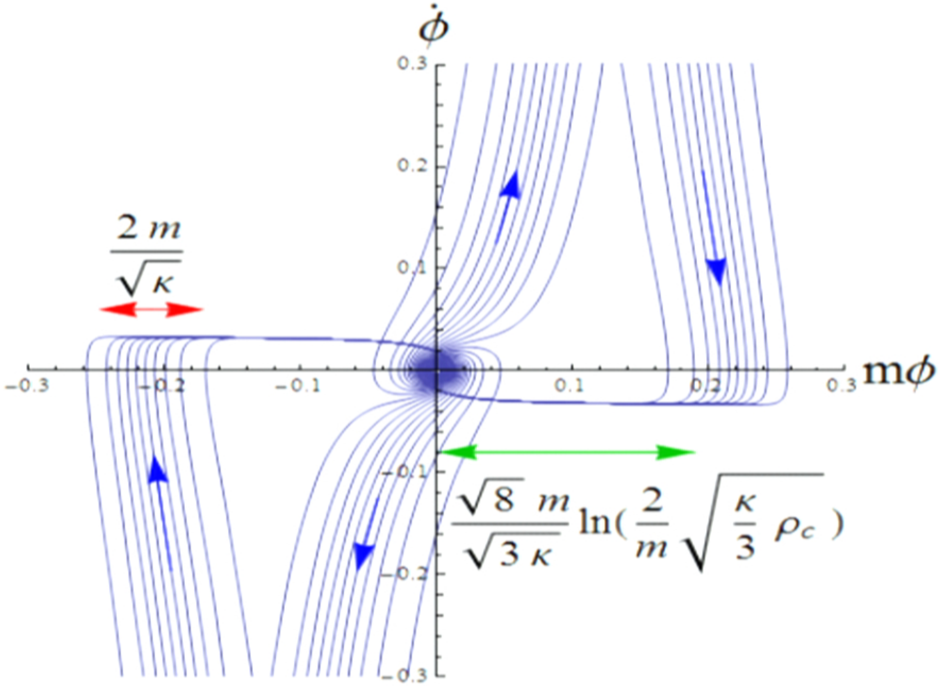
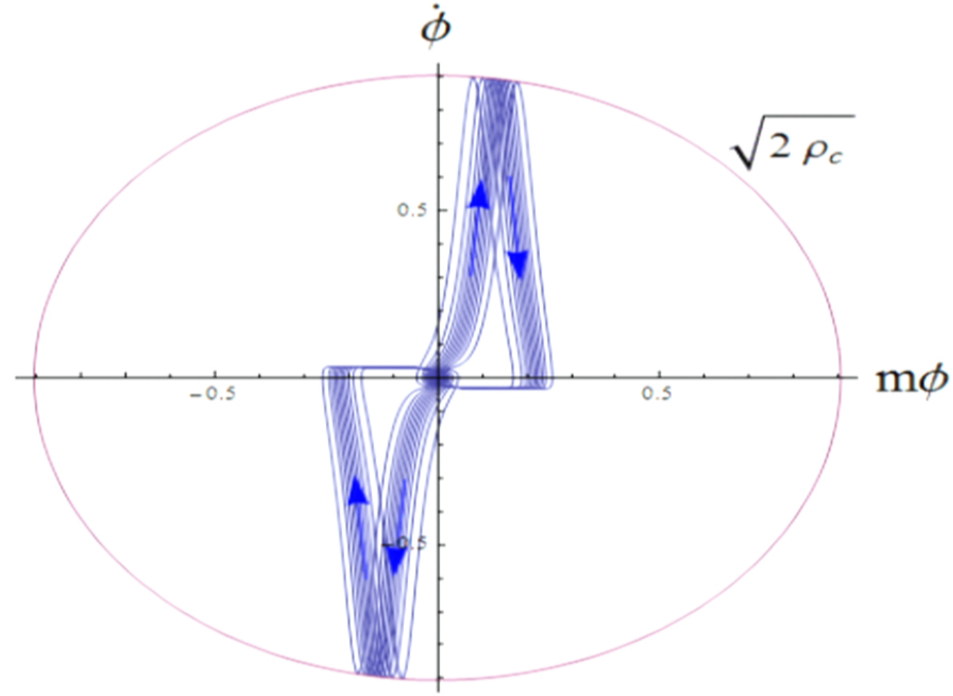


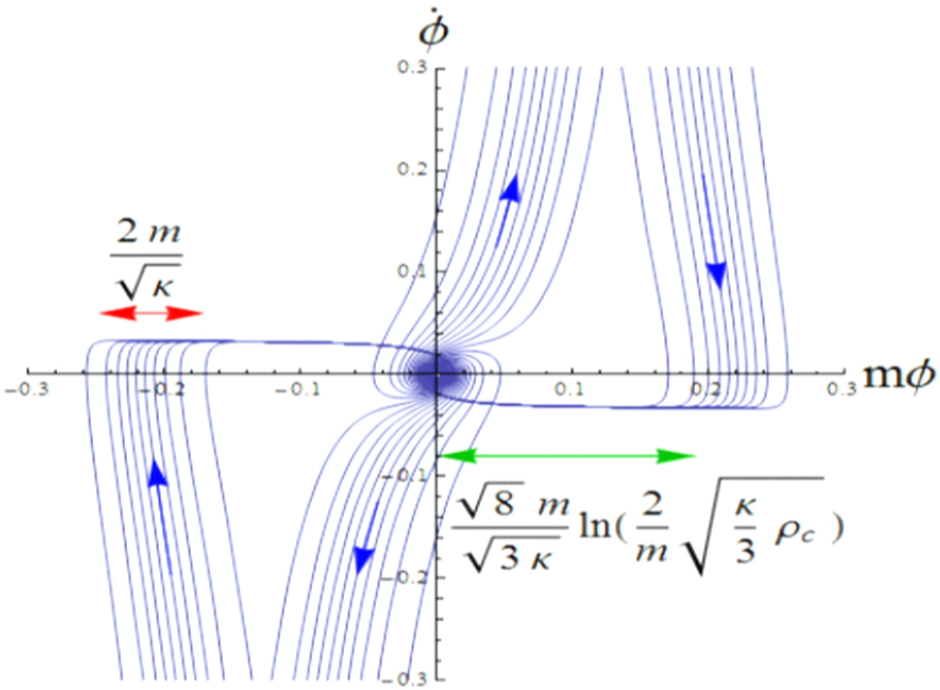


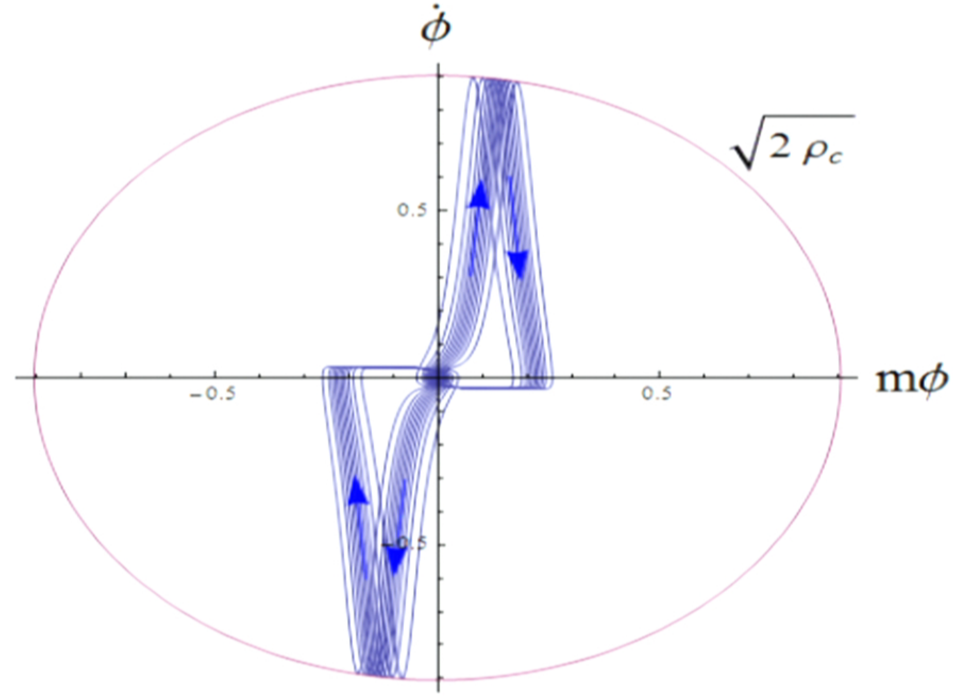
$$\langle N\rangle = \langle 2\pi(\phi_{start})^2\rangle \approx \frac{2}{3}\ln\left(\frac{4}{m}\sqrt{\frac{\kappa}{3}\rho_c}\right), \qquad \Delta N \approx \sqrt{\langle N\rangle}$$

2.6

LQC inflation

$$H^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \qquad \ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0, \qquad m = 0.2\, m_{Pl}, \qquad \kappa := 8\pi G$$





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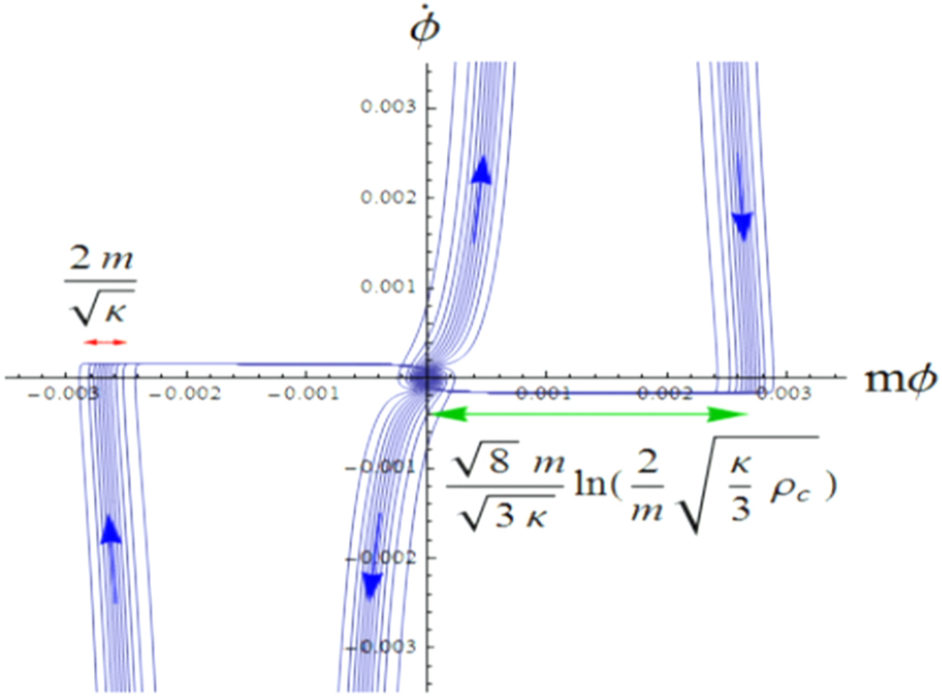
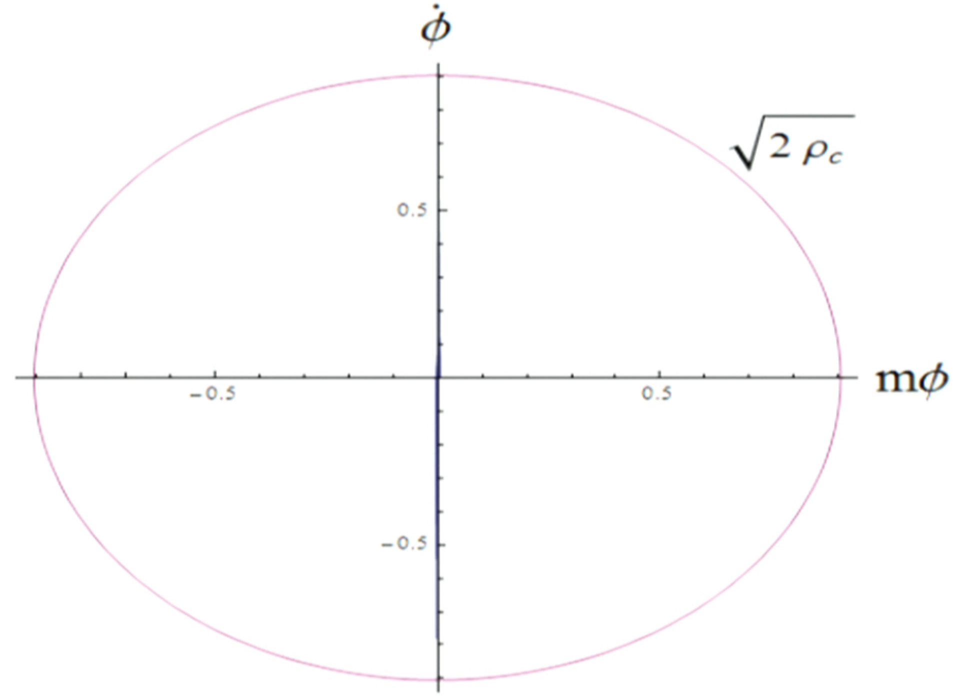
LQC inflation

$$H^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \qquad \ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0, \qquad m = 10^{-3}\, m_{Pl}, \qquad \kappa := 8\pi G$$





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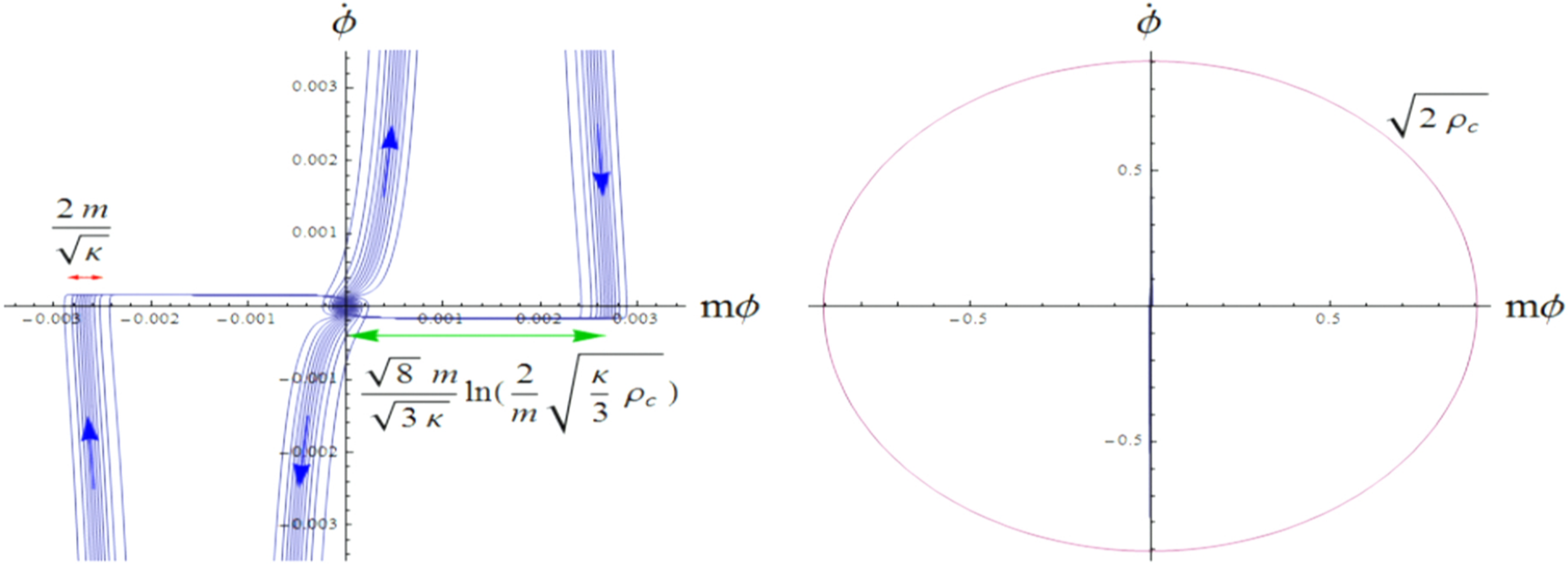

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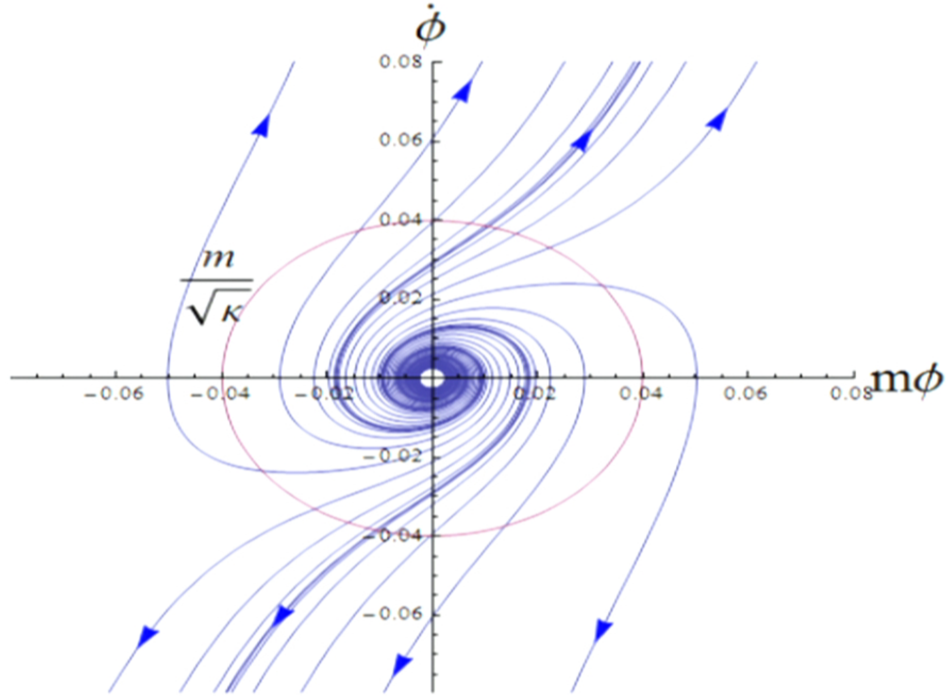
2.7

Initial conditions: $\begin{cases} m\phi(t=0) = \sqrt{2\rho_0}\ \sin(\delta) \\ \dot{\phi}(t=0) = \sqrt{2\rho_0}\cos(\delta) \end{cases}$

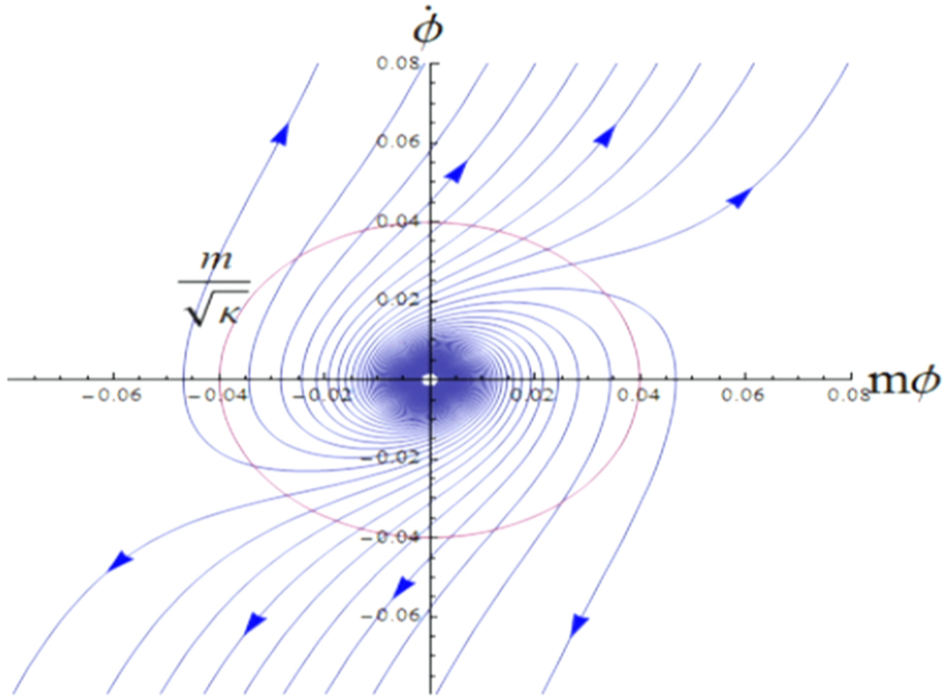
3.1

Initial conditions: $\begin{cases} m\phi(t=0) = \sqrt{2\rho_0}\ \sin(\delta) \\ \dot{\phi}(t=0) = \sqrt{2\rho_0}\cos(\delta) \end{cases}$



3.2

Initial conditions: $$\begin{cases} m\phi(t=0) = \sqrt{2\rho_0}\left(1 - \frac{\sqrt{3\kappa\rho_0}}{4m}\sin(2\delta)\right)^{-1}\sin(\delta) \\ \dot{\phi}(t=0) = \sqrt{2\rho_0}\left(1 - \frac{\sqrt{3\kappa\rho_0}}{4m}\sin(2\delta)\right)^{-1}\cos(\delta) \end{cases}$$



3.3

Initial conditions: $$\begin{cases} m\phi(t=0) = \sqrt{2\rho_0}\left(1 - \frac{\sqrt{3\kappa\rho_0}}{4m}\sin(2\delta)\right)^{-1}\sin(\delta) \\ \dot{\phi}(t=0) = \sqrt{2\rho_0}\left(1 - \frac{\sqrt{3\kappa\rho_0}}{4m}\sin(2\delta)\right)^{-1}\cos(\delta) \end{cases}$$

$$H^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \qquad \ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0, \qquad m = 1.21 \times 10^{-6} m_{Pl}$$

3.4

Initial conditions: $\begin{cases} m\phi(t=0) = \sqrt{2\rho_0}\left(1 - \frac{\sqrt{3\kappa\rho_0}}{4m}\sin(2\delta)\right)^{-1}\sin(\delta) \\ \dot{\phi}(t=0) = \sqrt{2\rho_0}\left(1 - \frac{\sqrt{3\kappa\rho_0}}{4m}\sin(2\delta)\right)^{-1}\cos(\delta) \end{cases}$

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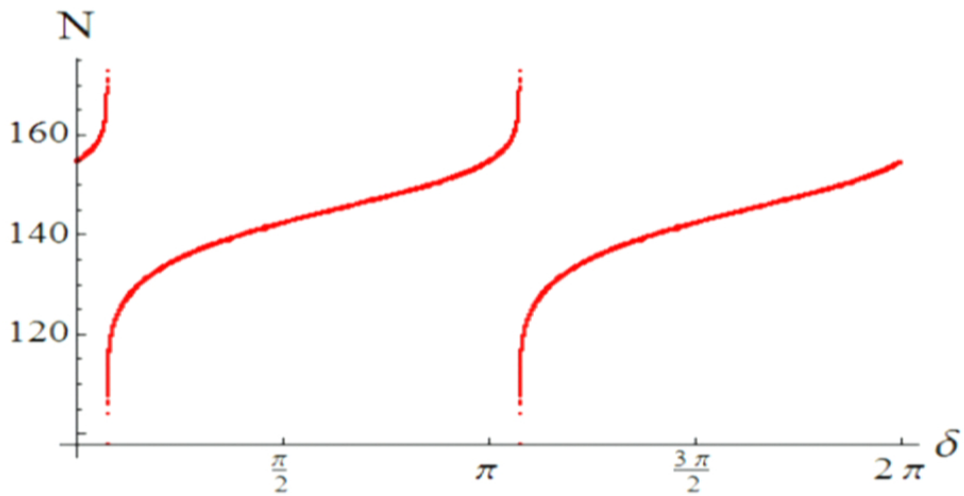

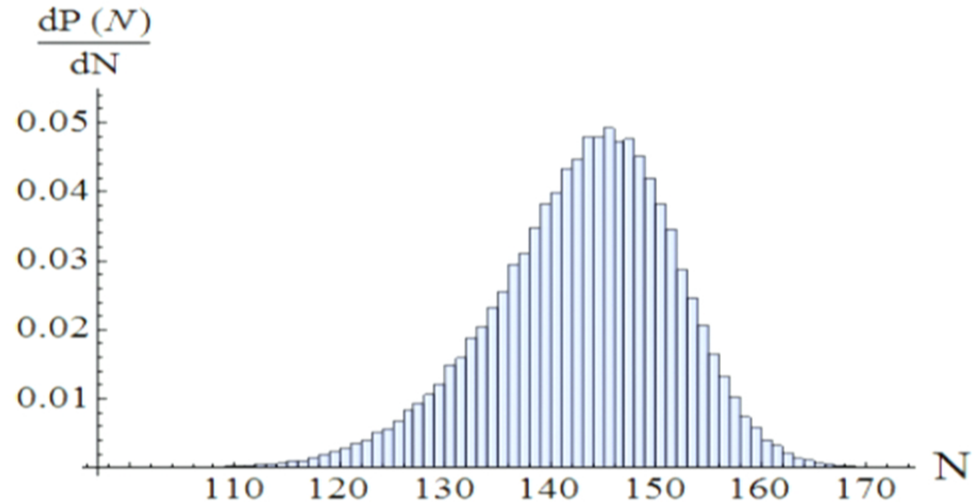

3.5

Initial conditions: $\begin{cases} m\phi(t=0) = \sqrt{2\rho_0}\left(1 - \frac{\sqrt{3\kappa\rho_0}}{4m}\sin(2\delta)\right)^{-1}\sin(\delta) \\ \dot{\phi}(t=0) = \sqrt{2\rho_0}\left(1 - \frac{\sqrt{3\kappa\rho_0}}{4m}\sin(2\delta)\right)^{-1}\cos(\delta) \end{cases}$

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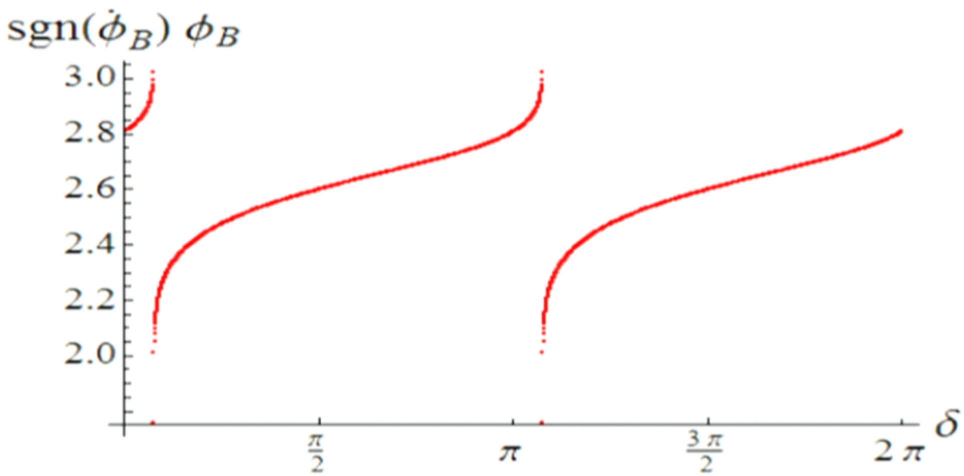

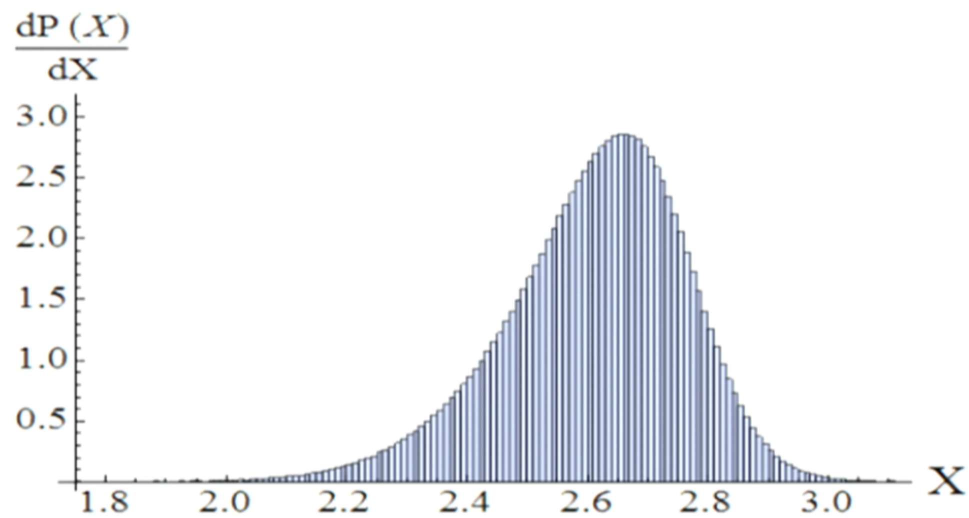

$X = sgn(\dot{\phi}_B)\phi_B$

3.5

Barbero-Immirzi parameter, from black hole entropy: $\gamma = 0.2375$

Critical density, from minimum area: $\rho_c = \frac{\sqrt{3}}{32\,\pi^2\,\gamma^3} = 0.41$

4.1

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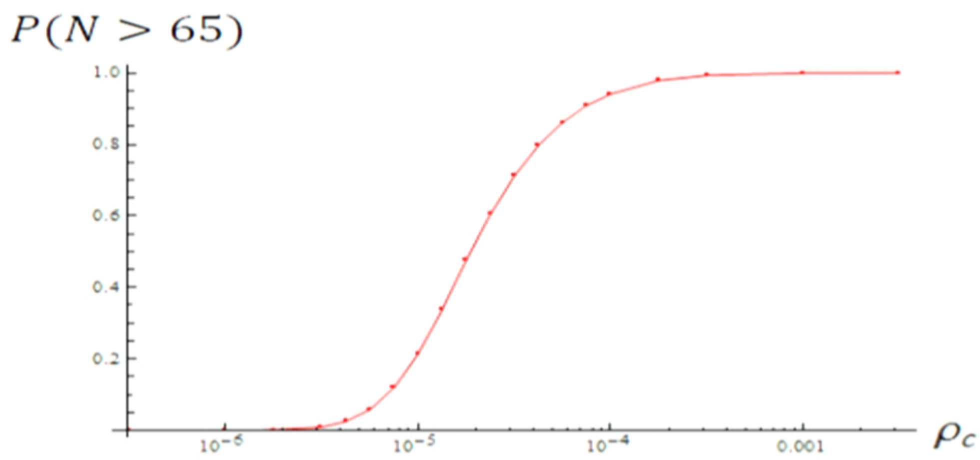


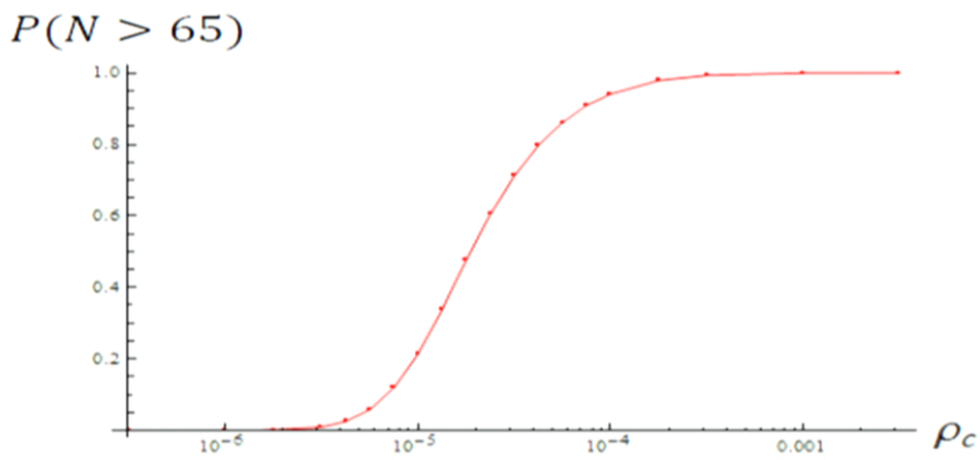
$P(N>65)$	ρ_c	γ
0.5	1.9×10^{-5}	6.6
0.95	5.4×10^{-6}	10.1
0.99	3.2×10^{-6}	11.9

$$m = 1.21 \times 10^{-6} m_{Pl}$$

4.2

Barbero-Immirzi parameter, from black hole entropy: $\gamma = 0.2375$

Critical density, from minimum area: $\rho_c = \frac{\sqrt{3}}{32\,\pi^2\,\gamma^3} = 0.41$



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0.99	3.2×10^{-6}	11.9

$$m = 1.21 \times 10^{-6} m_{Pl}$$

4.2

Conclusions

- If we put arbitrary initial conditions at about plank density, then we generally get a lot of inflation. This is not unique for LQC!
- If we put arbitrary initial conditions in the early pre-bounce universe, we get enough inflation, and a very peaked probability distribution for the number of e-folds

QFT in Quantum Spacetime

A. Dapor

University of Warsaw

in collaboration with J. Lewandowski and J. Puchta

PI, 22 July 2013

Phys. Rev. D **87**, 063512 (2013) [arXiv:1211.0161]
Phys. Rev. D **87**, 104038 (2013) [arXiv:1302.3038]

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outline

1. motivation
2. construction
3. first question: which canonical variables?
4. second question: which dressed metric?
5. conclusions

motivation

Answer a couple of questions about QFT on (cosmological) QS:

1. Which phase space variables should we quantize?
2. Which dressed metric should we use in the case of massive fields (inflaton)?

full theory

action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} R - \frac{1}{2} g^{\mu\nu} \partial_\mu T \partial_\nu T - V_T(T) - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V_\phi(\phi) \right]$$

full theory

action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} R - \frac{1}{2} g^{\mu\nu} \partial_\mu T \partial_\nu T - V_T(T) - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V_\phi(\phi) \right]$$

canonical analysis $\Rightarrow \Gamma = \Gamma_G \times \Gamma_T \times \Gamma_\phi$

full theory

action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} R - \frac{1}{2} g^{\mu\nu} \partial_\mu T \partial_\nu T - V_T(T) - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V_\phi(\phi) \right]$$

canonical analysis $\Rightarrow \Gamma = \Gamma_G \times \Gamma_T \times \Gamma_\phi$

- $g_{\mu\nu} \longrightarrow (q_{ab}, \pi^{ab})$
- $T \longrightarrow (T, p_T)$
- $\phi \longrightarrow (\phi, \pi_\phi)$

plus C and C_a.

outline

1. motivation
2. construction
3. first question: which canonical variables?
4. second question: which dressed metric?
5. conclusions

kinematical splitting

cauchy surface $\Sigma = \mathbb{T}^3 \Rightarrow$ cosmological coordinates $(x^a) \in [0, 1)^3$

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the "rest"

$$\begin{cases} \delta q_{ab}(x) & = & q_{ab}(x) - e^{2\alpha}\delta_{ab} \\ \delta\pi^{ab}(x) & = & \pi^{ab}(x) - \frac{\pi_\alpha}{6}e^{-2\alpha}\delta^{ab} \\ \delta T(x) & = & T(x) - T^{(0)} \\ \delta p_T(x) & = & p_T(x) - p_T^{(0)} \\ \delta\phi(x) & = & \phi(x) \\ \delta\pi_\phi(x) & = & \pi_\phi(x) \end{cases}$$

gauge-fixing

F-transform the rest ($\delta\gamma(x) \longrightarrow \delta\check{\gamma}(k)$); and projection of geometry modes in scalar, vector and tensor sectors

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T-expansion of C and C_a (1st order):

$$C^{(0)}, \quad E(k) := \breve{C}^{(1)}(k), \quad M(k) := k^a \breve{C}_a^{(1)}(k), \quad V(k) := v^a \breve{C}_a^{(1)}(k), \quad W(k) := w^a \breve{C}_a^{(1)}(k)$$

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reduction to Γ_C:

$$\begin{array}{rcl} C^{(0)} \approx 0 & \Rightarrow & p_T^{(0)} = \sqrt{\kappa\pi_\alpha^2/6 - 2e^{6\alpha} V_T(T^{(0)})} \\ E(k) \approx 0,\ M(k) \approx 0 & \Rightarrow & p^1 = p^1(\gamma_{free}),\ p^2 = p^2(\gamma_{free}) \\ V(k) \approx 0,\ W(k) \approx 0 & \Rightarrow & p^3 = p^3(\gamma_{free}),\ p^4 = p^4(\gamma_{free}) \end{array}$$

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G-fixing to $\overline{\Gamma}$:

$$T^{(0)} - \tau = 0, \qquad q_1 = q_2 = q_3 = q_4 = 0$$

hamiltonian(s)

(γ_{free}) and $T^{(0)}$ uniquely define the dynamics:

$$\frac{d}{d\tau}O(\gamma_{free}) = \left\{O(\gamma_{free}), h_P\right\}$$

where

$$h_P = H_{\text{hom}} - \sum_{k\neq 0, m=5,6} H^G_{m,k} - \sum_{k\neq 0} H^T_k - \sum_k H^\phi_k$$

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In particular

$$H^T_k = H^{-1}_{\rm hom}\left[\frac{1}{2}\left(\delta\breve{p}_T(k) - \frac{\kappa\pi_\alpha}{2}\delta\breve{T}(k)\right)^2 + \frac{1}{2}e^{4\alpha}k^2\delta\breve{T}(k)^2 + \frac{1}{2}e^{6\alpha}V''_T\delta\breve{T}(k)^2\right]$$

$$H^\phi_k = H^{-1}_{\rm hom}\left[\frac{1}{2}\delta\breve{\pi}_\phi(k)^2 + \frac{1}{2}e^{4\alpha}k^2\delta\breve{\phi}(k)^2 + \frac{1}{2}e^{6\alpha}V''_\phi\delta\breve{\phi}(k)^2\right]$$

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the problem

focus on H_k^T:

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$\Rightarrow$ not a canonical transformation if the background is dynamical!

$$\{\alpha, P(k)\} = -\frac{\kappa}{2}Q(k) \neq 0$$

a solution

canonical transformation

$$\bar{\alpha} := \alpha + \frac{\kappa}{4}\delta\breve{T}(k)^2, \qquad \bar{\pi}_\alpha := \pi_\alpha, \qquad Q_k := \delta\breve{T}(k), \qquad P_k := \delta\breve{p}_T(k) - \frac{\kappa\pi_\alpha}{2}\delta\breve{T}(k)$$

produces

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hybrid quantization:

- Polymeric G sector: $L_2(\bar{\mathbb{R}}, d\mu_{\text{Bohr}})$
- Schroedinger T sector: $L_2(\mathbb{R}, dQ_k)$, with $\hat{Q}_k = Q_k$ and $\hat{P}_k = -i\partial/\partial Q_k$

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the Hamiltonian has the form (also for test field ϕ)

$$H = H_{\text{hom}} - H_{\text{hom}}^{-1}\frac{1}{2}\left[p_k^2 + k^2 a^4 q_k^2 + V'' a^6 q_k^2\right]$$

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compare with Schroedinger eq for QFT on classical FLRW spacetime $d\tilde{s}^2 = -\tilde{N}(\tau)^2d\tau^2 + \tilde{a}(\tau)^2\left(dx^2+dy^2+dz^2\right)$:

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3 eq's for 2 unknowns $\Rightarrow$ no sol for generic k and V!

a solution

consider a Bianchi I effective metric:

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with axes oriented such that $\vec{k} = (0, 0, k)$

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$$d\tilde{s}^2 = -\tilde{N}^2 d\tau^2 + \tilde{a}^2 \left(dx^2 + dy^2\right) + \tilde{b}^2 dz^2$$

with axes oriented such that $\vec{k} = (0, 0, k)$

QFT on $\langle QS \rangle$:

$$i \frac{d}{d\tau} \psi_\tau(q_k) = \frac{\langle \hat{H}_{\text{hom}}^{-1} \rangle}{2} \left[\hat{p}_k^2 + k^2 \frac{\langle \hat{H}_{\text{hom}}^{-1/2} \hat{a}^4 \hat{H}_{\text{hom}}^{-1/2} \rangle}{\langle \hat{H}_{\text{hom}}^{-1} \rangle} \hat{q}_k^2 + V'' \frac{\langle \hat{H}_{\text{hom}}^{-1/2} \hat{a}^6 \hat{H}_{\text{hom}}^{-1/2} \rangle}{\langle \hat{H}_{\text{hom}}^{-1} \rangle} \hat{q}_k^2 \right] \psi_\tau(q_k)$$

QFT on $\tilde{g}_{\mu\nu}$:

$$i \frac{d}{d\tau} \psi_\tau(q_k) = \frac{\tilde{N}}{2\tilde{a}^2 \tilde{b}} \left[\hat{p}_k^2 + k^2 \tilde{a}^4 \hat{q}_k^2 + V'' \tilde{a}^4 \tilde{b}^2 \hat{q}_k^2 \right] \psi_\tau(q_k)$$

a solution

consider a Bianchi I effective metric:

$$d\tilde{s}^2 = -\tilde{N}^2 d\tau^2 + \tilde{a}^2 \left(dx^2 + dy^2\right) + \tilde{b}^2 dz^2$$

with axes oriented such that $\vec{k} = (0, 0, k)$

QFT on $\langle QS \rangle$:

$$i\frac{d}{d\tau}\psi_\tau(q_k) = \frac{\langle \hat{H}_{\text{hom}}^{-1} \rangle}{2} \left[\hat{p}_k^2 + k^2 \frac{\langle \hat{H}_{\text{hom}}^{-1/2} \hat{a}^4 \hat{H}_{\text{hom}}^{-1/2} \rangle}{\langle \hat{H}_{\text{hom}}^{-1} \rangle} \hat{q}_k^2 + V'' \frac{\langle \hat{H}_{\text{hom}}^{-1/2} \hat{a}^6 \hat{H}_{\text{hom}}^{-1/2} \rangle}{\langle \hat{H}_{\text{hom}}^{-1} \rangle} \hat{q}_k^2 \right] \psi_\tau(q_k)$$

QFT on $\tilde{g}_{\mu\nu}$:

$$i\frac{d}{d\tau}\psi_\tau(q_k) = \frac{\tilde{N}}{2\tilde{a}^2\tilde{b}} \left[\hat{p}_k^2 + k^2 \tilde{a}^4 \hat{q}_k^2 + V'' \tilde{a}^4 \tilde{b}^2 \hat{q}_k^2 \right] \psi_\tau(q_k)$$

3 eq's for 3 unknowns $\Rightarrow$ unique sol $\tilde{g}_{\mu\nu}$!

conclusions

- M-S variables are not suited for canonical quantization of the perturbations *and* the background
- a good choice of variables exist, and quantization is at hand
- once the quantum theory is reached, the concept of dressed metric can be applied in the case of a massive field, but the emergent metric "felt" by the field modes is of the Bianchi I type rather than FLRW

Niá:wen!