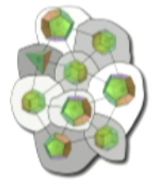
Title: Spin Foams - 1

Date: Jul 22, 2013 02:30 PM

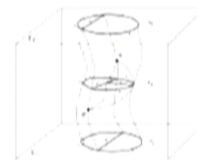
URL: http://pirsa.org/13070041

Abstract:

Spin foams: Dynamics for quanta of space



- spin network states $\quad s = |\Gamma, j_l, i_n\rangle$
- dynamics as state sum model



$$K[s] = \sum_{\sigma | \partial\sigma = s} A_\sigma[s] = \langle K | s \rangle$$

expected to project s on the kernel of the Hamiltonian constraint

- At fixed foam σ,

$$A_\sigma[s] = \sum_{j_f, i_e} \prod_f A_f[j_f] \prod_v A_v[j_f, i_e] \sim \int \mathcal{D}g_\sigma \, e^{\frac{i}{\hbar} S[g]}$$

Similar to LGT $\quad \Big(A_f = \dim j_f \, \exp\{-\beta S_f[j_f, a]\}, \quad A_v \text{ fixed by gauge invariance} \Big)$
but no background metric structure, no fixed lattice spacing
boundary dynamics in the dual cell Regge action to the vertex

a priori no continuum limit to take, ∞ dofs recovered by summing over all graphs

State of the art

- Main arena for testing: EPRL model
  Engle-Pereira-Rovelli '08, Engle-Pereira-Rovelli-Livine '09
  Related constructions: FK, KKL
  Freidel-Krasnov '09, Livine-S '09, Kaminski, Kisielowski and Lewandowski '09, Baratin and Oriti '11
  BO (same linear constraints, but based on non-commutative Fourier transform)
  Baratin and Oriti '11
- Explicit computations hard but possible
- Many active research groups worldwide

Many research directions

- semiclassical dynamics
- histories of fuzzy twisted geometries and intriguing link with twistors
- effect of quantum corrections
- coarse granining/continuum physics
- applications to cosmology and black holes
- matter coupling

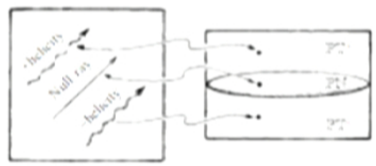
A large number of results!

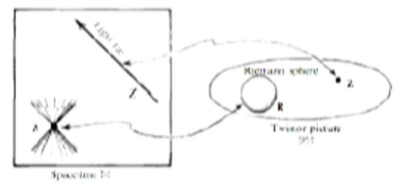
(More than 100 papers in 2011-2013)

Twistors and the Lorentz algebra

$$Z^\alpha = \begin{pmatrix} \omega^A \\ i\bar{\pi}_{\dot{A}} \end{pmatrix} \in \mathbb{T} := \mathbb{C}^2 \times \bar{\mathbb{C}}^{2*}, \qquad \bar{Z}_\alpha Z^\alpha = 2\mathrm{Im}(\pi\omega), \qquad \Theta_{\mathbb{T}} = \pi_A d\omega^A + cc$$

Incidence relation, $\omega^A = iX^{A\dot{A}}\bar{\pi}_{\dot{A}}$, $X \in M$ iff null twistor, $s = 0$





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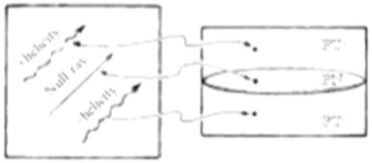

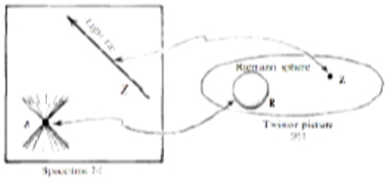

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  $\pi\omega$ Lorentz invariant, helicity $s = \mathrm{Im}(\pi\omega)$ conformal invariant
- both Lorentz generators and holonomies can be expressed as simple functions on a space of two twistors, provided they have the same complex helicity: $\pi\omega = \tilde\pi\tilde\omega$

$T^*\mathrm{SL}(2,\mathbb{C})$ is a symplectic submanifold of $\mathbb{T}^2$

- $\mathbb{PT}$ vs. $\mathbb{T}$: the scale of the twistor determines the value of the Casimir of the algebra
  $\Rightarrow$ the area of spin foam faces
- conformal invariance broken enforcing equal dilatations, $\mathrm{Re}(\pi\omega) = \mathrm{Re}(\tilde\pi\tilde\omega)$

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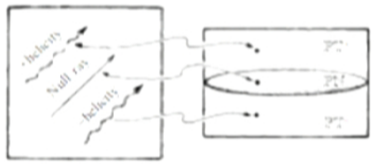

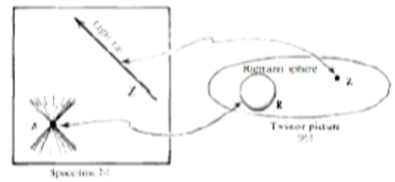

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Constraining the incidence relation and simple twistors

- Incidence relation, $\omega^A = i X^{A\dot{A}} \bar{\pi}_{\dot{A}}$, $\quad X \in M$ iff null twistor, $s = 0$
- simplicity constraints

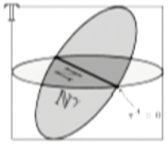
$$K^i + \gamma L^i = 0 \quad \Rightarrow \quad \omega^A = \frac{1}{r} n^{A\dot{A}} e^{i\frac{\theta}{2}} \bar{\pi}_{\dot{A}}, \qquad r \in \mathbb{R}, \gamma = \cot\frac{\theta}{2}$$

convenient to separe the norm r.
(Related to lapse and shift, the extrinsic curvature will later come from the variations of r)

- *simple twistors* $\quad Z^{\alpha}_{simple} := (\omega^A, r)$

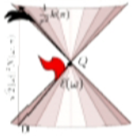
1. Not null: $\quad \mathrm{Re}(\pi\omega) = \gamma \mathrm{Im}(\pi\omega) \neq 0$
2. **isomorphic** to null twistors:

$$Z \;\mapsto\; Z_{\gamma} = (\omega^A, e^{i\theta/2}\bar{\pi}_{\dot{A}}), \quad s(Z_{\gamma}) = 0$$



Isomorphism depends on γ, reduces to the identity for $\gamma = \infty$
Corresponds to pure Einstein-Cartan, no Ashtekar formulation for purely null twistors

3. spinors' null poles aligned to the time normal

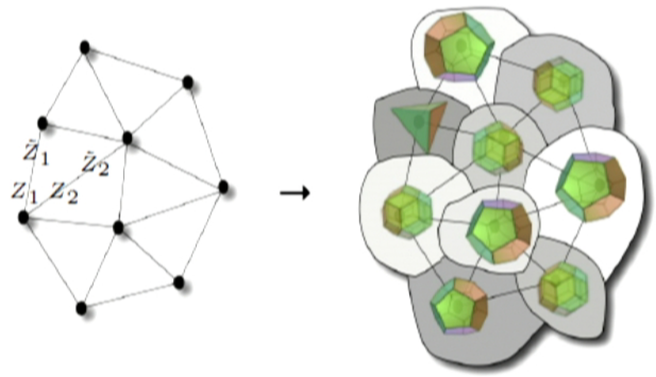


A simple twistor is a γ-null twistor with a time-like direction picked up

Defines a spacelike plane, associated with the simple bivector $(M = \Pi \oplus \bar{\Pi})$

$$B^{IJ} = (1 - \gamma\star) M^{IJ}, \qquad n_I B^{IJ} = 0, \qquad B^{IJ} = m^{[I}\bar{m}^{J]}$$

Schematic picture



- A set of twistors all satisfying the constraints
- This gives me a collection of **ruled** spacelike planes floating around
- Glue them together to form a discrete geometry
- How? local gauge-invariance defines a unique convex polytope around each node

  One obtains a collection of flat polyhedra plus information on their embedding

  a discrete version of the phase space (g_{ab}, K^{ab}) of GR, called twisted geometry
  Freidel-S '10

- Extrinsic geometry $\Xi := \ln \frac{\|\omega\|^2}{\|\tilde{\omega}\|^2}$
  Reduced $\mathrm{SU}(2)$ holonomy is the lattice equivalent of the AB connection:
  a non-trivial embedding able to probe the boost part of the Lorentz group

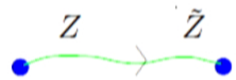
Quantization

- Quantize initial twistorial phase space, **à la Schrödinger**:

$$[\hat{\pi}_A, \hat{\omega}^B] = -i\hbar\delta_A^B, \qquad f(\omega) \in L^2(\mathbb{C}^2, d^4\omega) \qquad \left(\hat{\omega} = \omega,\ \hat{\pi} = -i\hbar\frac{\partial}{\partial\omega}\right)$$

- impose constraints: simplicity on half-links, then area matching on full link

$$(Z, \tilde{Z}) \in \mathbb{T}^2$$



- the result are homogeneous functions,

$$G^{(j)}_{m\tilde{m}}(\omega, \tilde{\pi}) := f^{(\gamma j, j)}_{jm}(\omega) f^{(\gamma j, j)}_{j\tilde{m}}(\tilde{\pi}), \qquad f^{(\gamma j, j)}_{jm}(\omega) = \|\omega\|^{2(i\gamma j - j - 1)} \langle j, m | j, \omega \rangle_{\text{Perelomov}}$$

- different representation of $\mathcal{H}^{LQG}_{\Gamma}$ than cylindrical functions
- carries a representation of the holonomy-flux algebra as ladder operators
  - Fluxes: Schwinger representation of $\mathrm{SL}(2, \mathbb{C})$
  - Holonomy: composite operator, need ordering prescription
    possibile also to consider angle operators corresponding to the spinor phases
- Note: functions are not holomorphic
  Is the quantization unitary equivalent to Penrose's?
- And the equivalence with the usual cylindrical functions?

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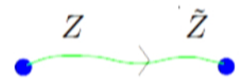

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Many research directions

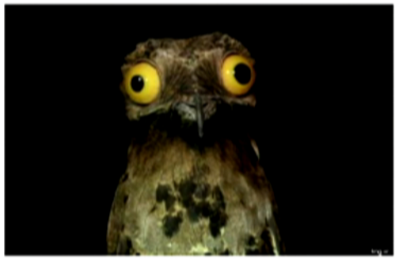
- semiclassical dynamics
- effect of quantum corrections
- coarse graining
- applications to cosmology and black holes
- matter coupling
- ...

A large number of results!

More than 100 papers in 2011-2013

Very hard to give justice to all the work that has been done

My bird's eye view...



Classical dynamics

- EPRL model leads to Regge action on a 4 simplex Barrett et al '09
  - Lorentzian propagator Bianchi-Ding '11
    (inverse square distance scaling of area correlations also for Lorentzian signature, completely spacelike boundary)
  - higher order correlations Rovelli-Zhang '11 (recovery of Regge 3-point function)
  - addressing the question of the cosine vs. exp projecting $\cos S_R$ to e^{iS_R} Engle '12
  - role of parity Wilson-Ewing and Rovelli
    (How to implement insensitivity to orientation, i.e. $|e| e^{\mu}_I e^{\nu}_J F^{IJ}_{\mu\nu}$)

Semiclassical behaviour

Plebanski action $S(e,\omega) = \int \mathrm{Tr}[(\star + \frac{1}{\gamma}\mathbb{1})B \wedge F(\omega)] + \Phi_{IJKL} B^{IJ} \wedge B^{KL}$

Simplicity constraints: $B^{IJ} \wedge B^{KL} - \frac{1}{12}\epsilon^{IJKL}\,\mathrm{Tr}(B \wedge \star B) = 0 \quad \Rightarrow B = e \wedge e$

Why does it work?

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SU(2) B-field: "cubic root" of the metric with internal symmetry $sl(3)$

Decomposition into irreps: $\quad \Phi \in (\mathbf{2},\mathbf{0}) \oplus (\mathbf{0},\mathbf{2}) \oplus (\mathbf{1},\mathbf{1}) \oplus (\mathbf{0},\mathbf{0})$

- $(\mathbf{1},\mathbf{1}) \oplus (\mathbf{0},\mathbf{0})$: Identify SD and ASD metrics
- $(\mathbf{2},\mathbf{0}) \oplus (\mathbf{0},\mathbf{2})$: Reduce internal symmetry $\quad sl(3) \times sl(3) \mapsto so(3) \times so(3) \cong so(3,1)$

Relaxing "Weyl components": larger internal gauge group
$\Rightarrow$ no extra dofs but modified dynamics Krasnov '07

Relaxing "Ricci components": the two metrics independent from each other
$\Rightarrow$ bi-metric theories, extra dofs

SD Plebanski action ($\gamma = \pm i$): only $(\mathbf{2},\mathbf{0})$ components present

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Quantum corrections

- q-deformation and **finiteness** M. Han '11, W. Fairbairn and C. Meusburger '11, see Han's talk
  interpretation as cosmological constant
  Regge action behaviour in appropriate limit $(1/\sqrt{\Lambda} \gg j \gg 1)$
- Radiative corrections Riello '13
  Self-energy diverges logarithmically in Λ (taking natural face weights)
- Relation between IR divergences and continuum gauge symmetries Bonzom-Dittrich '13
  bubble divergences and lack thereof in the Barrett-Crane model
- Definition of coarse graining Bahr-Dittrich-Hellmann-Kaminski '12
- Fixed points in 2d spin net toy models see Steinhaus's talk
- Effective action see Mikovic's talk

Various approaches to the key question of continuum physics

- Compute radiative corrections and find relevant graphs for given process
  see Riello's talk
- Study continuum limit as in lattice or tensor networks
  see Dittrich's and Steinhaus's talks
- Resummation of underlying GFT/tensor model
  see GFT's talks, Rivasseau, Oriti, Gurau

Quantum corrections

Various corrections present:

$\downarrow$ metric perturbative expansion, $S_R[\ell^0 + h] = S_0 + GS_2 + \dots$

$\left[e^{iS_R} + o(1/j)\right]_{\Gamma_1} +$	ℓ_P corrections	1st vs. 2nd order action
$\downarrow \quad \downarrow$		still metric in 3d, but in 4d?
$+\left[e^{iS_R} + o(1/j)\right]_{\Gamma_2} + \dots$	refined graph correction	
$\downarrow \quad \downarrow$	(λ expansion in GFT)	
$e^{iS_{EH}} \quad ?$		

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Quantum corrections:

1. Do they have a metric interpretation? Polyhedra picture still relevant?
2. Do they grow as we refine the graph, thus spoiling Regge's continuum limit?
3. What is their continuum interpretation? Can we match with EFT, in what framework? Modified Plebanski action?

Boundary states and symmetries

Usual QFT are defined with asymptotic boundary conditions
Spin foams are naturally defined on finite regions: general boundary formalism
Oeckl '04, Modesto and Rovelli '04

$$W_q(x,y) = \int_\Sigma \mathcal{D}g_{ab}\, \mathcal{O}(x)\, \mathcal{O}(y)\, K[g]\, \Psi_q[g]$$

$\Psi_q[g]$: semiclassical boundary state peaked on a classical configuration in phase space

- induces a background to expand around (graviton calculations)
- possible to analyse isometries

What is the specific property that encodes asymptotic flatness and Poincaré invariance?

- How about different properties, e.g. BHNHG?
- Analysis of boundary terms for Lorentzian general boundaries
  Bianchi-Wieland '12, Bodendorfer-Neiman '13
- exponential matter degeneracy? Frodden-Noui-Perez '13
- Treat generic Lorentzian 4-simplices see Immirzi's talk

Spin foam cosmology: some results

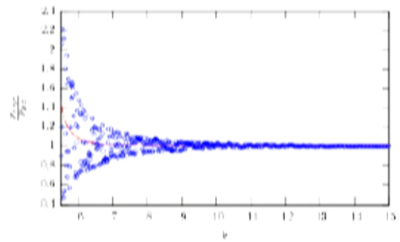
- Derivation of the Friedmann equation in dipole cosmology with Lorentzian signature
  Vidotto '13
- Dipole with N links and Friedmann equation
  Benito and Livine '11
- $U(N)$ symmetry guide to isotropic sectors
  see Livine's talk
- Beyond the dipole: some undesired contributions Hellmann '11
  turn out to be subdominant Kiselowski, Lewandowski and Puchta '12

- Possible extension to anisotropies
  Rennert and Sloan '13
- Studies of singularity resolution
  Rovelli and Vidotto '13
- Link with LQC
  Henderson et al '10, Ashtekar et al. '10

Spin foam cosmology: some research directions

- quantum corrections to the Friedmann equation
  their dependence on higher order graphs, on the choice of boundary state
- treat $k \neq 0$
- stregthen the comparison with LQC results
  match with the effective equation description
- Use as a toy model to investigate the role of the various different corrections at stake

  Find observables depending weakly on the specific choice of semiclassical boundary state, such as in LQC Agullo, Ashtekar and Nelson



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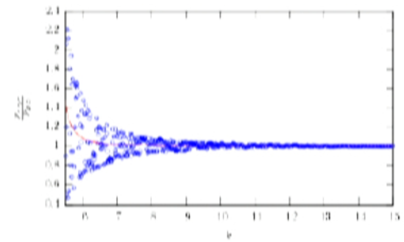

Conclusions

Spin foams with respect to Madrid:

- Improved understanding of the spin foam geometry
- Improved control over the quantum corrections
- Many new tools and results to deal with continuum physics
- Applications to cosmology and black hole physics

Many pieces are still missing!

- Semiclassical interpretation on arbitrary cellular decompositions?
- Geo or non-geo interpretation of the quantum corrections?
- Dynamical description of BHs?
- Matter coupling still to be developed
- Numerical evaluations still limited

Many research directions to explore, many young and motivated people

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- Matter coupling still to be developed
- Numerical evaluations still limited

Many research directions to explore, many young and motivated people

Questions

- What does (the most simple) radiative correction in a realistic 4d Lorentzian spin foam model look like?
- Does it introduce new elements in the theory?
- How divergent is the Lorentzian EPRL-FK model?
  (As much as the Euclidean one?)

Questions

- What does (the most simple) radiative correction in a realistic 4d Lorentzian spin foam model look like?
- Does it introduce new elements in the theory?
- How divergent is the Lorentzian EPRL-FK model?
  (As much as the Euclidean one?)

Motivations I

Divergent SF (and GFT) amplitudes

Tackle the issue of divergences in realistic 4d Spin Foam (SF) and Group Field Theory (GFT) models

- SF with unconstrained internal hypersurfaces generally diverge (bubbles) [Perez,Rovelli]
- These divergences are likely relevant for the continuum limit
  $\rightsquigarrow$ renormalization
- Does renormalization wash away the nice properties of our models?
  $\rightsquigarrow$ consistency issue
- Even in finite (q-deformed [Han, Fairbairn, Meusburger]) models some amplitudes will be very large, because the inverse cosmological constant *is* very large ($\lambda = (l_{\text{Planck}}^2 \Lambda)^{-1} \sim 10^{120}$)

Motvations II

Divergences are related to diffeos

In 3d: good understanding of the intimate relation among *bubble divergences* and *discrete diffeomorphisms*
[Noui,Perez,Freidel,Louapre,Bonzom,Smerlak,Baratin,Girelli,Oriti...]

Also, hints towards a connection with the sum over orientations
[Christodolou,Långvik,Röken,Rovelli,AR] [confirmed by melon analysis]

Why melons?

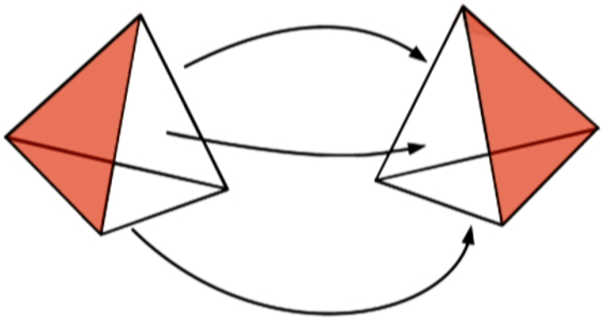
(+) The simplest bubble

Why melons?

(+) The simplest bubble

(+) Central in (coloured) Tensor Models and GFTs (most diverging building block $\Rightarrow$ role in $1/N$ expansion, renormalization) [Gureau,Bonzom,Rivasseau,AR,Carrozza,Oriti]

(−) Topological sphere, but dual to a degenerate triangulation



The Lorentzian EPRL-FK model - I

Vertex amplitude

The Lorentzian EPRL-FK vertex amplitude is obtained by restricting the $SL(2,\mathbb{C})$-BF one to the γ-simple representations, defined by

$$\begin{array}{rccc} \mathrm{Y}_\gamma : & \mathscr{H}^{j}_{SU(2)} & \to & \mathscr{H}^{\gamma j, j}_{SL(2,\mathbb{C})} \\ & |j;m\rangle & \mapsto & |\gamma j, j; j, m\rangle \end{array}$$

[Engle,Pereira,Rovelli,Livine,Speziale,Freidel,Krasnov]

The Lorentzian EPRL-FK model - II

Edge and face weights

Edge weight chosen to be 1 in spin representation (trivial gluing $\mathbb{P}$)
Renormalization engenders corrections
[cf. Bulatov model [Ben Geloun,Bonzom] and later in this talk]

Face weight fixed to the $SU(2)$-BF value of $2j+1$
[as required by SF composition invariance *à la* Atiyah] [Bianchi,Regoli,Rovelli]

IMPORTANT

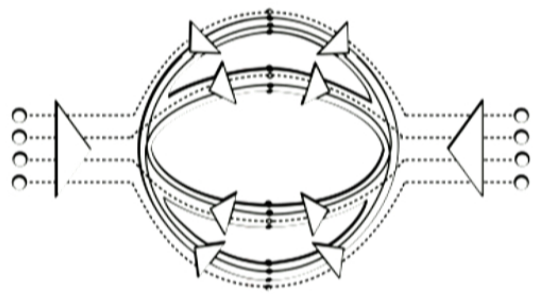
These choices of edge and face weights **strongly** influence the convergence (divergence) properties of the SF model

The amplitude of the melon - I

$$W_{\mathscr{M}}(n_a, \tilde{n}_a, j_a) = \left[\sum_{\{j_{ab}\}} \int \prod_a dg_a d\tilde{g}_a\right] \prod_{f\ \text{int}:ab} A_{ab}^{f\ \text{int}} \prod_{f\ \text{ext}:a} A_a^{f\ \text{ext}}$$

where for external and internal faces, respectively:

$$A_a^{f\ \text{ext}} = \langle j_a; n_a | \mathrm{Y}_\gamma^\dagger g_a^{-1} \mathrm{Y}_\gamma \mathrm{Y}_\gamma^\dagger \tilde{g}_a \mathrm{Y}_\gamma | j_a; \tilde{n}_a \rangle$$
$$A_{ab}^{f\ \text{int}} = (2j_{ab}+1)\mathrm{Tr}_{j_{ab}}^{SU(2)}\left[\left(\mathrm{Y}_\gamma^\dagger \tilde{g}_b^{-1} \tilde{g}_a \mathrm{Y}_\gamma\right)\left(\mathrm{Y}_\gamma^\dagger g_b^{-1} g_a \mathrm{Y}_\gamma\right)\right]$$



The amplitude of the melon - II

Source of divergences & regularization

$$W_{\mathscr{M}}(n_a, \tilde{n}_a, j_a) = \sum_{\{j_{ab}\}} \int \prod_a dg_a d\tilde{g}_a \prod_{f\ \text{int}:ab} A_{ab}^{f\ \text{int}} \prod_{f\ \text{ext}:a} A_a^{f\ \text{ext}}$$

- The integrals on $SL(2,\mathbb{C})$ at *fixed internal spins* are **finite** [Engle,Pereira]

Main techniques (& approximations)

- Large spin analysis within the internal faces (stationary phase technique) [Barrett,Dowdell,Pereira,Fairbairn,Gomes,Hellmann]
- Decoupling of the internal from the external faces
  $\rightsquigarrow$ effective 3d geometry (related to spike normal section - next slide)
- All spins scale together

Main techniques (& approximations)

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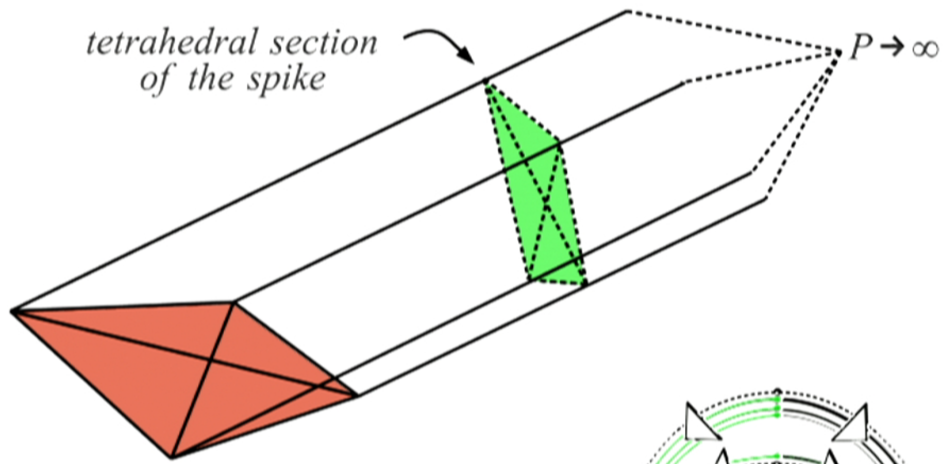
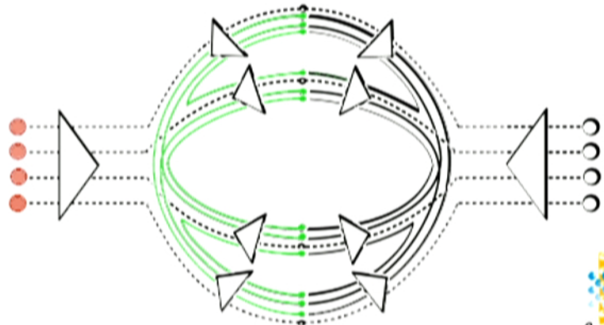
- All spins scale together

The geometrical interpretation

Going spiky or, better, going large (on the inside)





The calculation - I

The melon scaling

Careful analysis of space of stationary points and its symmetries

$\rightsquigarrow$ partial amplitude scaling

$\rightsquigarrow$ the melon diverges at most logarithmically in the cut-off

$$||W_{\mathscr{M}}|| \sim \log\left(\frac{K}{j_0}\right)$$

The calculation - I

The melon scaling

Careful analysis of space of stationary points and its symmetries

⇝ partial amplitude scaling

⇝ the melon diverges at most logarithmically in the cut-off

$$||W_{\mathscr{M}}|| \sim \log\left(\frac{K}{j_0}\right)$$

Scaling agrees with previous studies of the Euclidean version of the model [Perini,Rovelli,Speziale,Krajewski,Magnen,Tanasa,Rivasseau,Vitale]
(Obtained with different techniques)

Comments

- *Naively*, fixing $K = \lambda \sim 10^{120}$ (inverse cosmological constant) $\rightsquigarrow$ estimation of melon graph largeness at different scales
  - e.g., at Planck scale: $j_0 \sim 1 \rightsquigarrow ||W_{\mathscr{M}}|| \sim 280$

- The new gluing $\mathbb{T}^2_\gamma$:
  - $\mathbb{T}^2_\gamma \neq \mathbb{P}$, with $\mathbb{P}$ the bare EPRL-FK (and BF) trivial gluing. (Though, $\lim_{j_0 \to \infty} \mathbb{T}^2_\gamma = \mathbb{P}$ [Puchta])
  - is not a projector

- Face and edge weights strongly influence the divergence degree. However, provided the graph *is* divergent, the associated tensorial structure is given by $\mathbb{T}^2_\gamma$

What is the refinement limit of spin foam models?

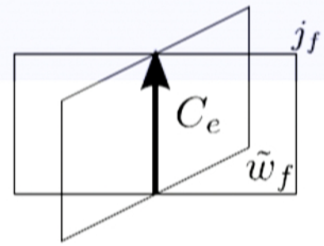
Spin foams: path integral approach related to Loop Quantum Gravity

[Barrett, Crane, Rovelli, Reisenberger, Engle, Livine, Pereira, Freidel, Krasnov, ...]

- Lattice as a regulator → relation to GR?
  - Semi-classical limit of large building blocks → Regge action (discrete gravity) [Barrett, Baez, Freidel, and many others]
- Renormalization / coarse-graining (refinement limit)?
  - What are the relevant d.o.f.?
  - No background scale [Bahr, Dittrich '09, Bahr, Dittrich, S.St. '11, Rovelli '11, Dittrich '12]
  - Do spin foams have more phases than lattice gauge theories?

Goal: Coarse-grain analogue spin foam models on Quantum groups.

Lattice gauge theories vs. spin foams



Partition function:

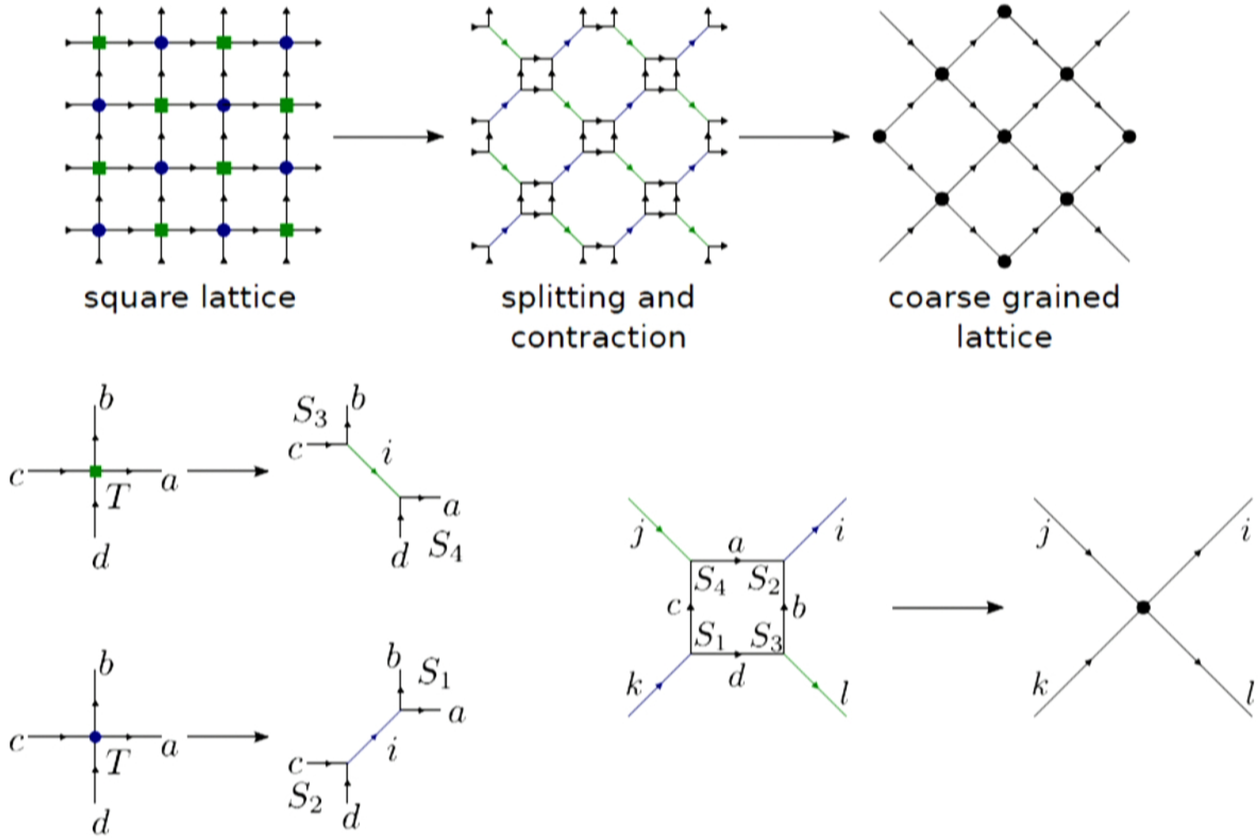
$$Z \sim \sum_{j_f} \left(\prod_f \tilde{w}_f(j_f) \right) \prod_e C_e(\{j_f\}_{f \supset e})$$

	Lattice gauge theory	Spin foams
Parameter space:	**face weights**: $\tilde{w}_f(j_f)$	replace **Haar projector**: $C_e(\{j_f\}_{f \supset e})$ smaller subspace [Bahr,Dittrich,Hellmann,Kamiński '12]
Phases / fixed points:	• BF / weak coupling • degenerate / strong coupling •BF on normal subgroups	Same phases as for lattice gauge theories **More phases?**

To apply numerical simulations, we have to make two simplifications:

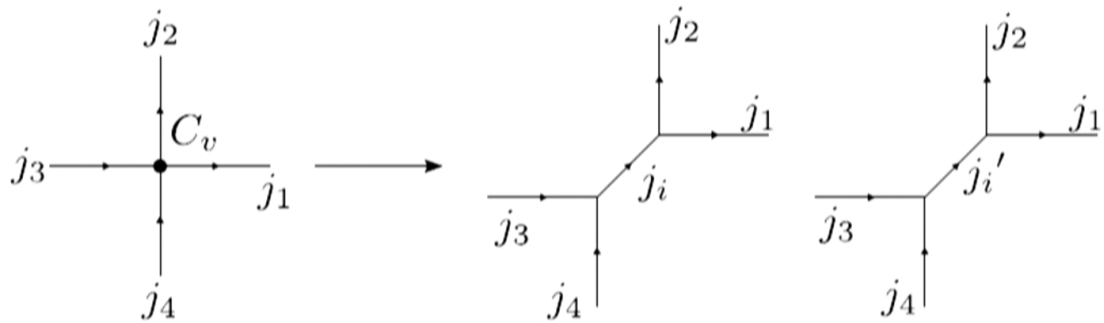
- Dimensional reduction
- 'Regularization' - finite amount of rep. labels

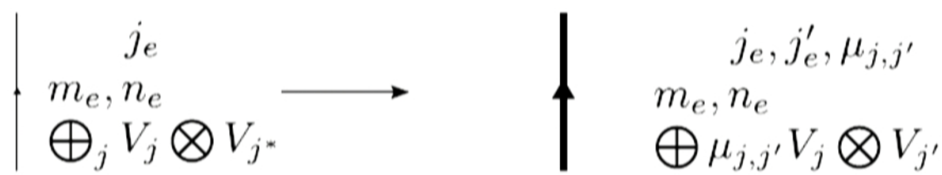
The algorithm [Levin,Nave '07, Gu, Wen '09]



Protecting the symmetries... [Dittrich, Martin-Benito,Schnetter '13]

Under coarse graining we get... [talk by M. Martin-Benito on Tuesday]





- effective edge with **doubling of rep. labels**.
- **enlarged space of models** $j' \neq j^*$ allowed.
- New C_v in **matrix block form**, labelled by **intertwiner labels**.

Fixed points and phases I see also [Dittrich, Martin-Benito, Schnetter '13]

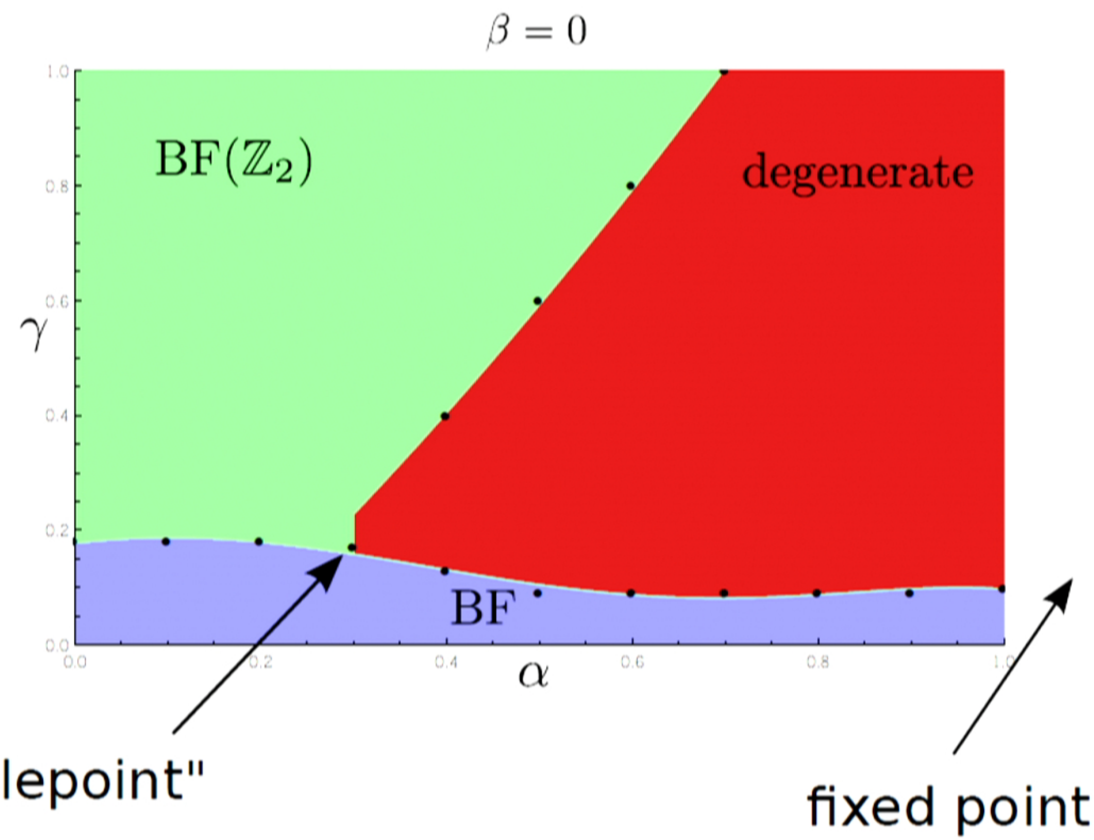
Conjecture:

Excitation of intertwiners is relevant and determines the phase.

standard models (lattice gauge theory) $j' = j^*$	simplicity constraints **some reps. forbidden** $j' \neq j^*$	factorizing j, j' or "mixed models"
degenerate (HT) $(0,0)$	$(0,0),(0,1),(1,0),(1,1)$ (also found for $k=6$)	$(0,0),(1,1),(0,1),(1,0),$ $(2,2),(0,2),(2,0),(1,2),(2,1)$
$SU(2)_k$ ordered (BF) $(0,0),(1,1),(2,2)$		$(0,0),(1,1),(2,2)$ $(0,2),(2,0)$
$\mathbb{Z}_2$ ordered $(0,0),(2,2)$		

Fixed points define **triangulation invariant** (three-valent) vertex model!

Fixed points and phases II see also [Dittrich, Martin-Benito, Schnetter '13]



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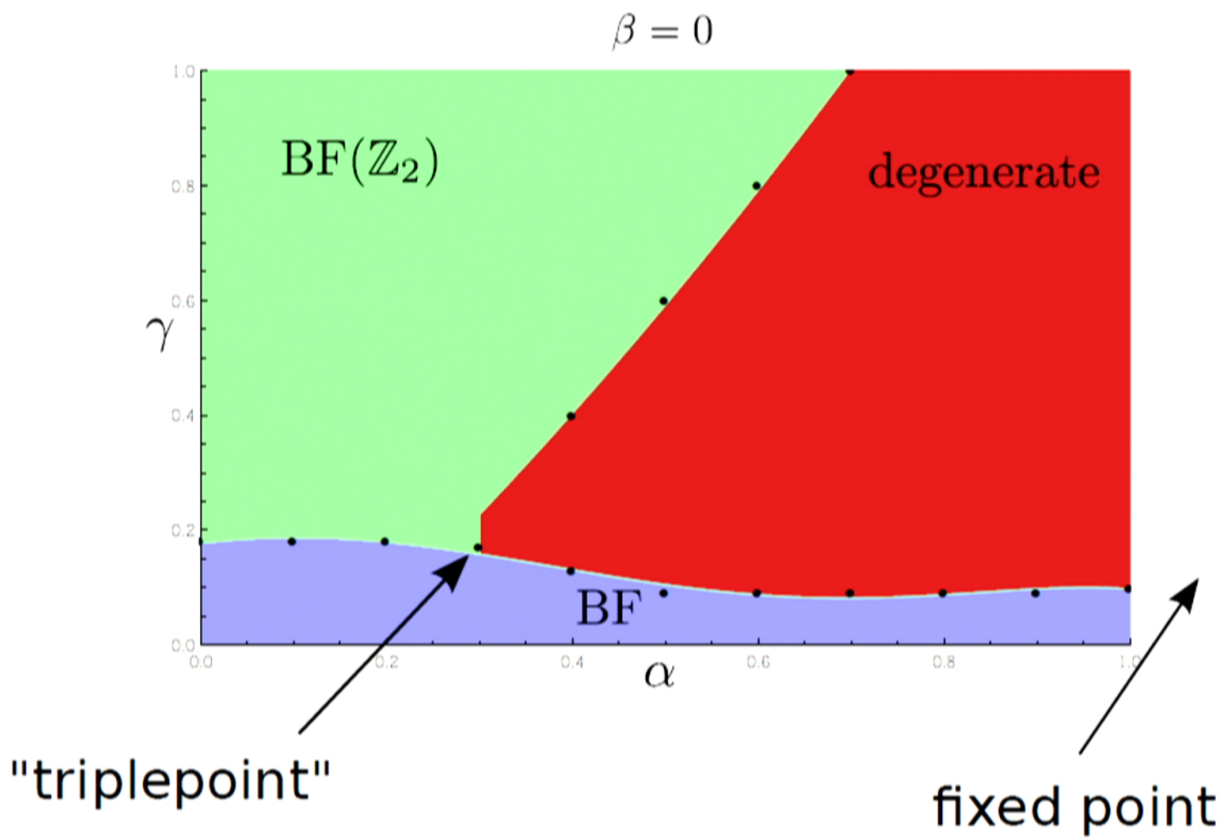


Conclusions

- Spin net models (of **Quantum Groups**) → analogue to spin foams
  - New class of (scale independent) models
  - Tensor Network Renormalization can be successfully applied!
  - Lots of structural information / **Embedding maps**
- **Conjecture: Intertwiner d.o.f. are relevant (also for spin foams)**
  - **Approximation method**: flow of models, keeps track of intertwiner d.o.f., (accuracy under control!)
  - Fine tuning necessary to avoid (fast) flow to ordered (BF) or degenerate phase [Christensen, Khavkine '07]
- **Potential: phases beyond standard lattice gauge theory**
- Although starting from a fixed lattice: Fixed points (under coarse-graining) describe **fully triangulation invariant models**
  - Supports strategy to construct models via coarse-graining [Bahr, Dittrich '09, Bahr, Dittrich, S.St. '11, Dittrich '12]

Outlook

- **TNR implies that intertwiner d.o.f. are relevant**
  - Systematic study of Quantum Group models
  - Structural information from **embedding maps**:
    - Examine (un)stable directions of the (new) fixed points.
    - Develop semi-analytical approximation schemes / analogue to Migdal-Kadanoff.
  - "Running" of the cosmological constant?
    - What happens to a fixed point of a Quantum Group with smaller k, if it is put into a Quantum Group with larger k?
  - Flow of **Simplicity constraints**?
- Apply what we learned to the full (analogue) Quantum Group models.

In reach: Refinement limit and phase diagram for spin foams!

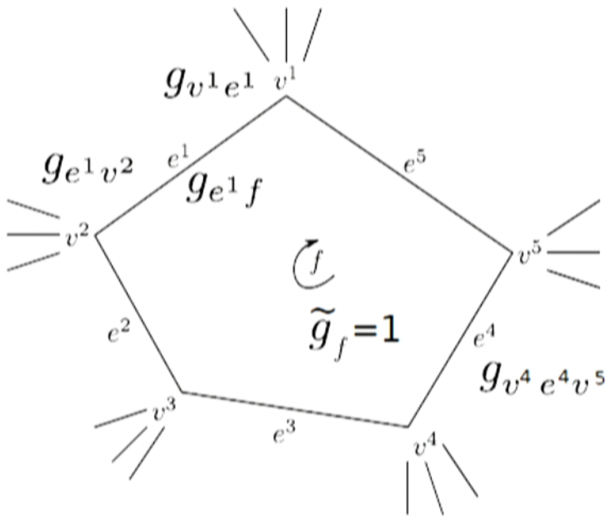
Thank you for your attention!

Statement of the problem

- **Setup:** $\mathcal{C}$ is two complex dual to simplicial decomposition with the boundary Γ. The state $\psi_{\lambda k_e}$ from projected Hilbert space is constructed from coherent states.
- **Goal:** Determine if the amplitude $\mathcal{Z}(\mathcal{C}, \psi_{\lambda k_e})$ is exponentially supressed in the limit $\lambda \to \infty$ (large spin $\overset{?}{=}$ semiclassical limit).
- **So far:** The known result are mainly about spinfoam with one vertex. Suppression unless boundary data coherent vectors form 4 simplex or $\mathrm{SU}(2)$ BF geometry. First sign of the flatness problem **[Bonzom]**
- **Method:** Wave front set analysis. It is not as precise as stationary phase approximation (phase is not determined), but results are more general.
- **See Frank Hellmann's plenary talk.**

The partition function $\mathcal{Z}_\star(\mathcal{C})$ in holonomy formulation

$$\mathcal{Z}_\star(\mathcal{C}) = \int \left(\prod_{e\subset f} \mathrm{d}g_{ef}\right)\left(\prod_{v\subset e} \mathrm{d}g_{ev}\right)\left(\prod_{e\subset f} E_\star(g_{ef})\right)\left(\prod_{f} \delta(\tilde{g}_f)\right).$$



Auxiliary holonomy
$\tilde{g}_f = g_{e^1 f} g_{e^1 v^2} g_{v^2 e^2} \dots g_{e^5 v^1} g_{v^1 e^1}$
Geometric holonomy
$g_f = g_{e^1 v^2} g_{v^2 e^2} \dots g_{e^5 v^1} g_{v^1 e^1}$

Boundary (projected Hilbert space) can be added ($g_{vev'}$ variables)

E is a simplicity function:
$E_{BF}(g) = \delta(g)$,
$E_{BC}(g) = \delta(g^+(g^-)^{-1})$

The formulation is based on previous works **[Pfeifer and Oeckl]**, **[Bahr]**

The wave front set and probing by coherent states

Wave front sets

The wave front set probes singular behaviour of the distribution (together with its direction). It is a cone subset of cotangent bundle. It has several nice geometric properties.

Coherent states

For the boundary (projected spin network)

$$\psi_\lambda(g_{v'ev},\ldots) = \prod_{v'ev\in\Gamma} \langle n_v^{e+}|g_{vev'}^{+}|n_e^{v'+}\rangle^{2\lambda j_{v'ev}^{+}} \langle n_v^{e-}|g_{v'ev}^{-}|n_e^{v'-}\rangle^{2\lambda j_{v'ev}^{-}} .$$

Let us introduce ($\mathbf{n}$ is $\mathfrak{su}(2)$ normalized element associated to n)

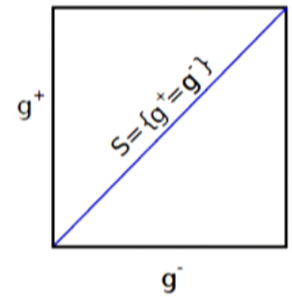
$$p_e^v = (2j_{v'ev}^{+}\mathbf{n}_e^{v+}, 2j_{v'ev}^{-}\mathbf{n}_e^{v-}), \quad p_{v'}^e = (2j_{v'ev}^{+}\mathbf{n}_{v'}^{e+}, 2j_{v'ev}^{-}\mathbf{n}_{v'}^{e-}) .$$

Condition for supression of the map $\lambda \to \langle\bar{\psi}_\lambda, \mathcal{Z}\rangle$ is

$$\nexists\ \{g_{v'ev}\colon (v'ev)\in\Gamma\}\colon\ p_{v'}^e = g_{v'ev} \triangleright p_e^v, \quad (g_{v'ev},\ldots; -p_e^v\ldots) \in WF(\mathcal{Z})$$

The WF set of the simplicity function E

The BC model



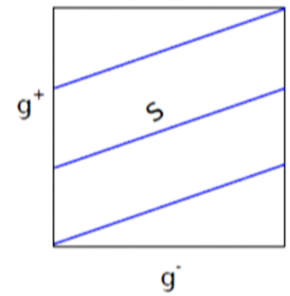
The singular support

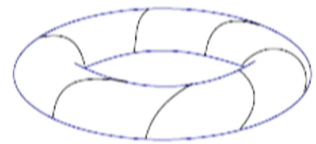
$$S = \{(g^+, g^-) : g^+ = g^-\}$$

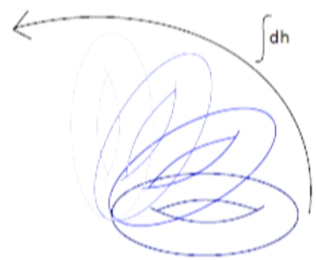
The wave front set: such (g, p)

$$g \in \mathrm{S} \text{ and } \underbrace{p \perp T\mathrm{S}}_{N^0 \cdot * p = 0}$$

The EPRL/FK model







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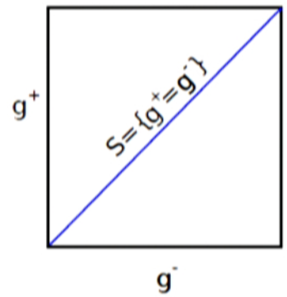

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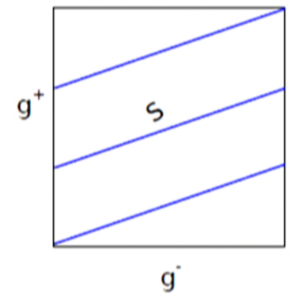

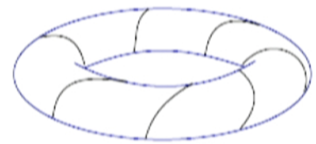

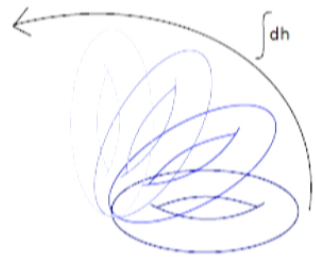

(Almost) uniqenesss of the wave front set for EPRL function.

The EPRL simplicity function satisfies the simplicity constraints

$$\left(\frac{1-\gamma}{2}\hat{j}^{+} - \frac{1+\gamma}{2}\hat{j}^{-}\right) E = 0$$
$$\left(\frac{1+\gamma}{2}\hat{k} - \hat{j}^{+}\right) E = 0$$

and $\mathrm{SU}(2)$ invariance

$$E(hgh^{-1}) = E(g), \quad h \in \mathrm{SU}(2)$$

Is the EPRL function the only solution? Not exactly, but...

The main results

The wave front set of the amplitude is supported on the points $\{g_{vev'}, p_{vev'}, \ldots\}$ for which there exists

$$\{g_{ev} = g_{ve}^{-1} \in Spin(4)\}, \ \{p^v_{ef} \in \bigwedge^2 \mathbb{R}^4\} \quad (1)$$

that together with $g_{vev'}$ and $p_{vev'}$ from the boundary satisfy

- **(parallel transport for geometric holonomy)** $\forall_f, \ p^v_{ef} = -g_{ev}g_{ve'} \triangleright p^v_{e'f}$ including $p_{vev'}$ on the boundary
- **(closure)** $\forall_{v,e} \ \sum_{\{ef\}\in v} p^v_{ef} = 0$
- **(simplicity)** $\forall_{v,e} \quad p^v_{ef}$ is twisted simple

$$p^v_{ef} = \chi(B^v_{ef} + \gamma * B^v_{ef}), \quad B^v_{ef} \text{ st. simple }, \ |B^v_{ef}| = 1$$

- if $p^v_{ef} \neq 0$ then holonomy $g_f = g_{ev}g_{ve'}g_{e'v'}\cdots$ is of the form

$$g_f = e^{\xi(*B^v_{ef} - \gamma B^v_{ef})}, \quad (2)$$

Condition: Complex $\mathcal{C}$ is non-tardis.

The wave front set and curvature constraints

For each vertex:
We have bivectors p^v_{ef} satisfying

- $g_{ve} \triangleright p^v_{ef} = -g_{ve'} \triangleright p^v_{e'f}$,
- $\sum_{f \ni e} p^v_{ef} = 0$

Together with twisted simplicity is the starting point for reconstruction theorem of 4-simplex if certain nondegeneracy conditions are satisfied.

In the nondegenerate case we have geometric constraint (Θ_f is deficiency angle)

$$g_f = \pm e^{\Theta_f * B^v_{ef}} \text{ for } B^v_{ef} \text{ st. simple }, \ |B^v_{ef}| = 1 \tag{3}$$

Together with condition from the wave front set

$$g_f = e^{\xi(*B^v_{ef} - \gamma B^v_{ef})} \tag{4}$$

this gives $\gamma\Theta_f = 0$ mod 2π.

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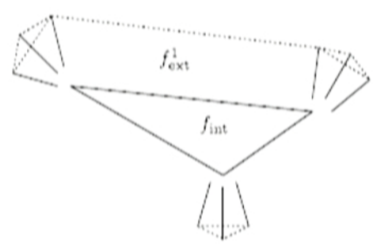
- $\gamma\Theta_f = 0 \text{ mod } 2\pi$ (finite set)
- Nontrivial example is $3-3$ move with one internal face (but no internal edge)



- Modification of face amplitude leading to lack of constraints on curvature exists.

Conclusions

Results

1. Analysis of the partition function without assumptions on the interior shows some problems leading to the curvature constraints
   - incompatibility between twisting and discretization,
   - lack of gluing constraints (improper use of area variables)

Other results:

1. Lorentzian (fate of curvature constraints) **[Perini]** and **[Han]**.
2. Divergencies **[Riello]**, **[Puchta]**, **[Bonzom, Dittrich]**

Thank you for your attention!