Title: Shape Dynamics - 1

Date: Jul 22, 2013 02:30 PM

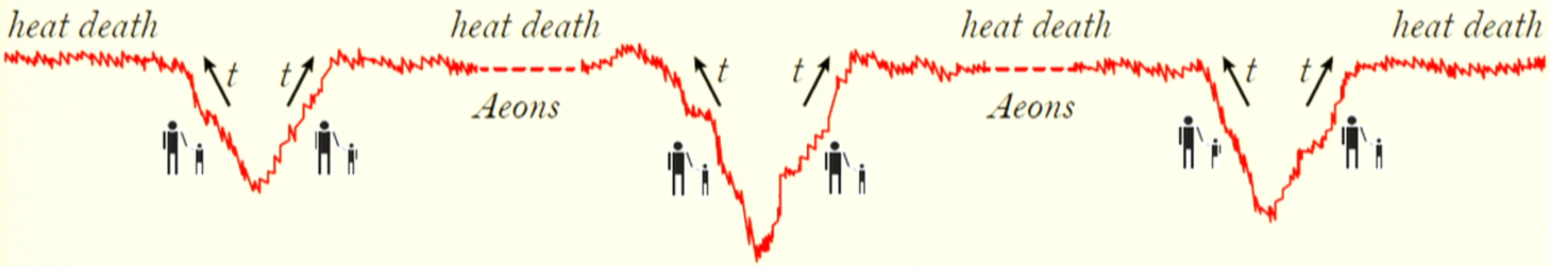
URL: http://pirsa.org/13070040

Abstract:


THE ARROW OF TIME: COMPLEXITY
AND BULK ENTROPY IN GRAVITY
JULIAN BARBOUR & FLAVIO MERCATI
TIM KOSLOWSKI

## POINCARÉ RECURRENCE

Boltzmann's resolution of time-reversal symmetry puzzle by Poincaré recurrence:




The direction of entropy growth defines the direction of time (to heat death)

Eternal recurrence of 'one-past–two-futures' scenario

# Poincaré recurrence

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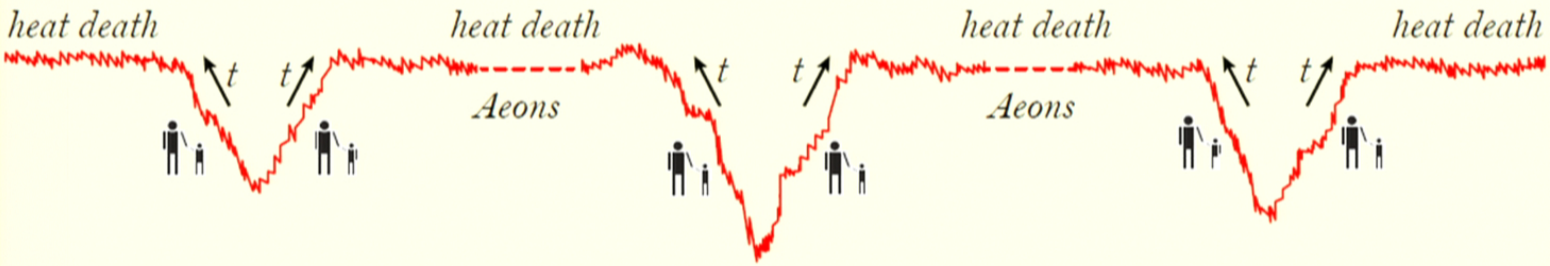


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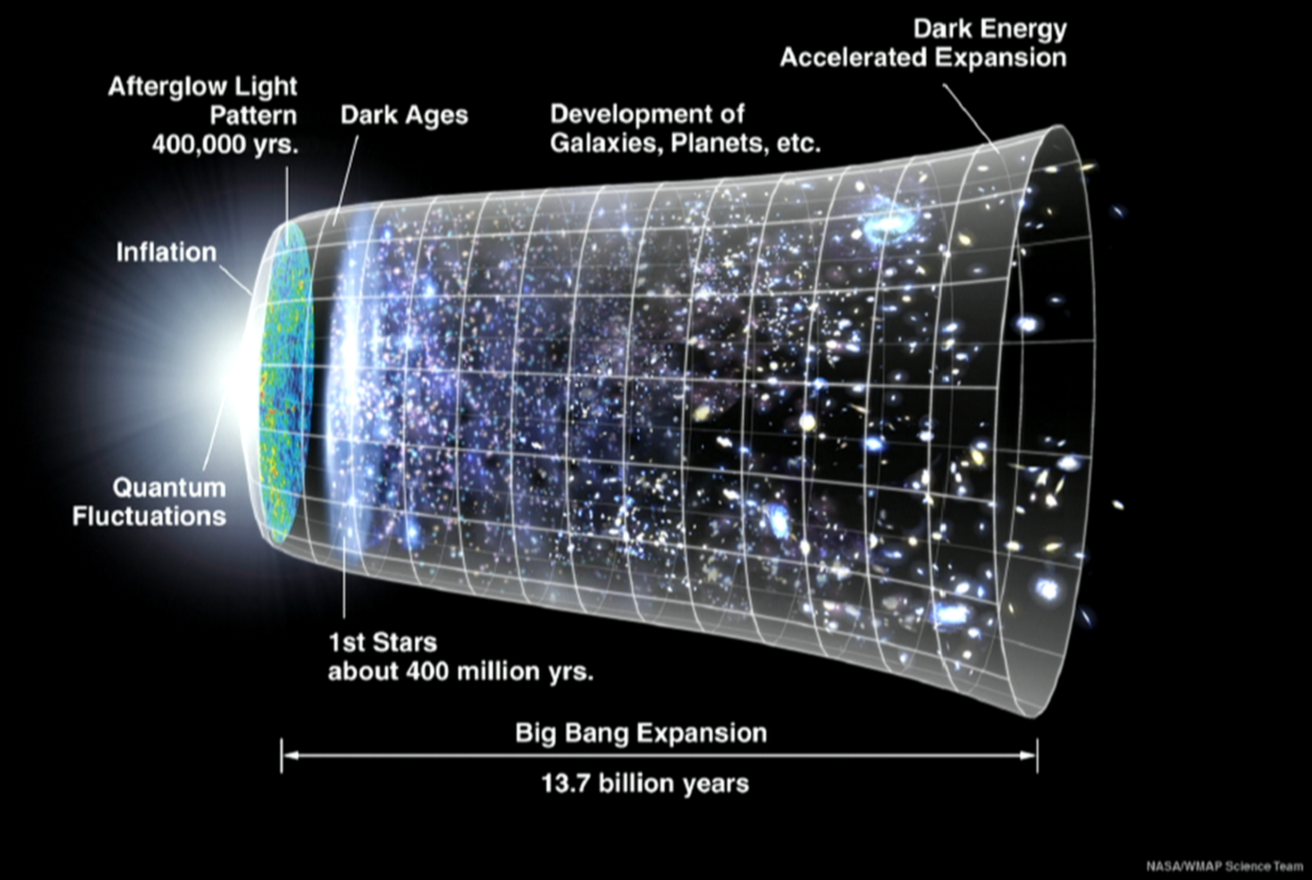
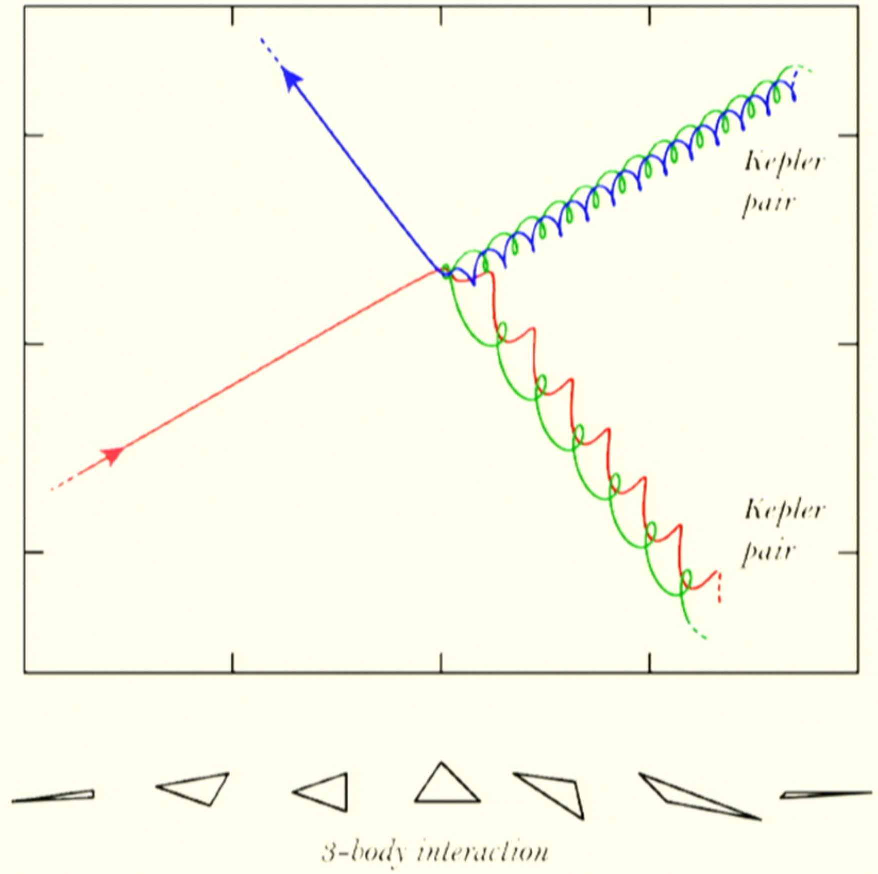



1. Requires very special pre-inflation state

2. Requires invisible expansion of space

# THE ZERO-ENERGY NEWTONIAN N-BODY PROBLEM

Toy model of the whole universe




Single dynamical ocurrence of 'one-past–two-futures' scenario


THE ZERO-ENERGY NEWTONIAN N-BODY PROBLEM
Toy model of the whole universe
Kepler pair
Kepler pair
3-body interaction
Single dynamical ocurrence of 'one-past–two-futures' scenario
4

THE ZERO-ENERGY NEWTONIAN N-BODY PROBLEM

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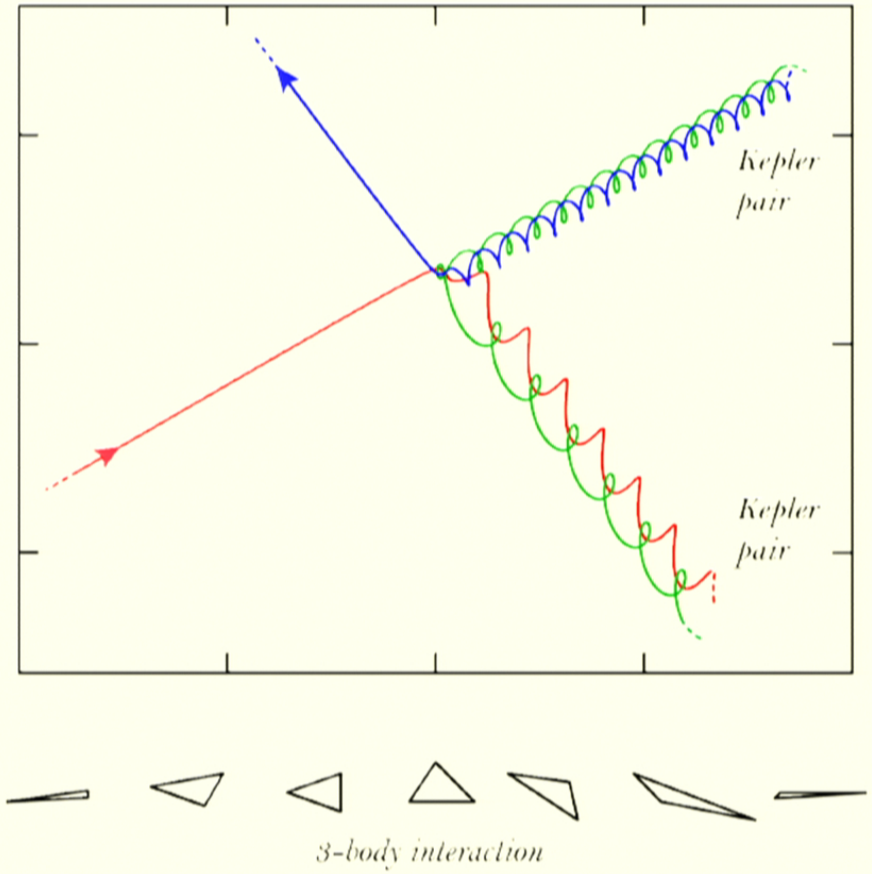


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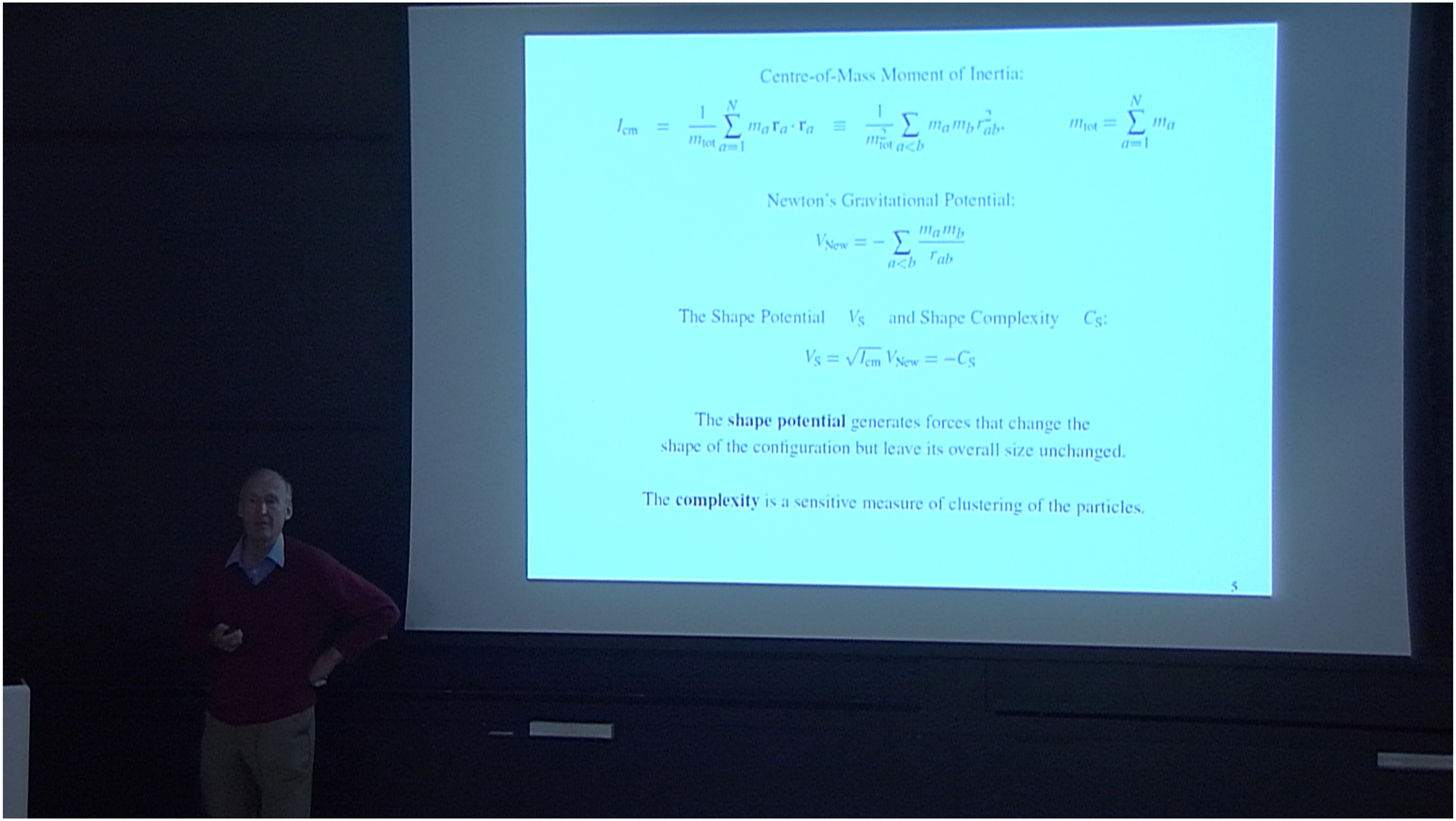
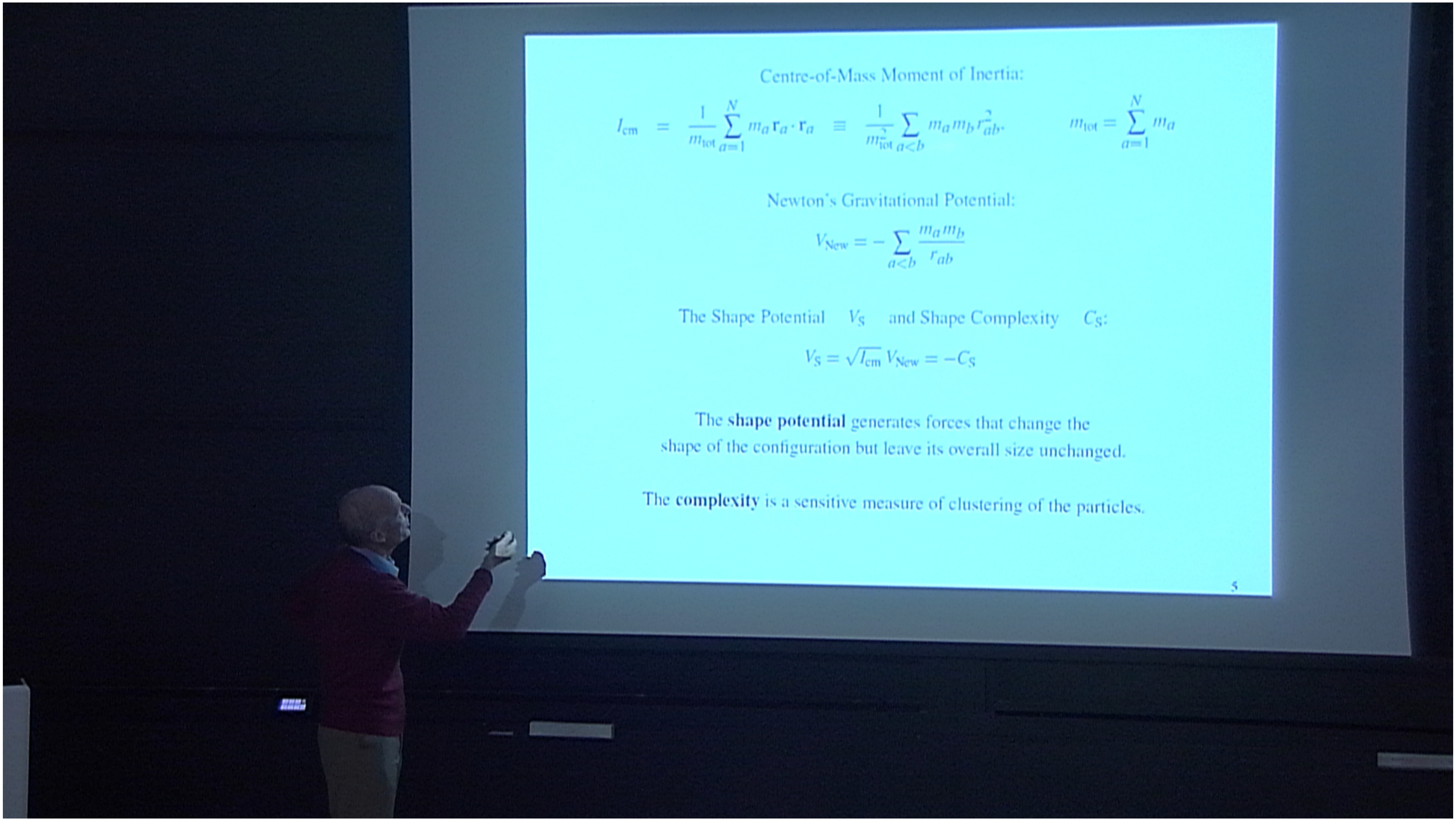

Centre-of-Mass Moment of Inertia:
$I_{\rm cm} = \frac{1}{m_{\rm tot}} \sum_{a=1}^{N} m_a \mathbf{r}_a \cdot \mathbf{r}_a \equiv \frac{1}{m_{\rm tot}^2} \sum_{a<b} m_a m_b r_{ab}^2, \qquad m_{\rm tot} = \sum_{a=1}^{N} m_a$
Newton's Gravitational Potential:
$V_{\rm New} = - \sum_{a<b} \frac{m_a m_b}{r_{ab}}$
The Shape Potential $V_S$ and Shape Complexity $C_S$:
$V_S = \sqrt{I_{\rm cm}}\, V_{\rm New} = -C_S$
The **shape potential** generates forces that change the shape of the configuration but leave its overall size unchanged.
The **complexity** is a sensitive measure of clustering of the particles.
5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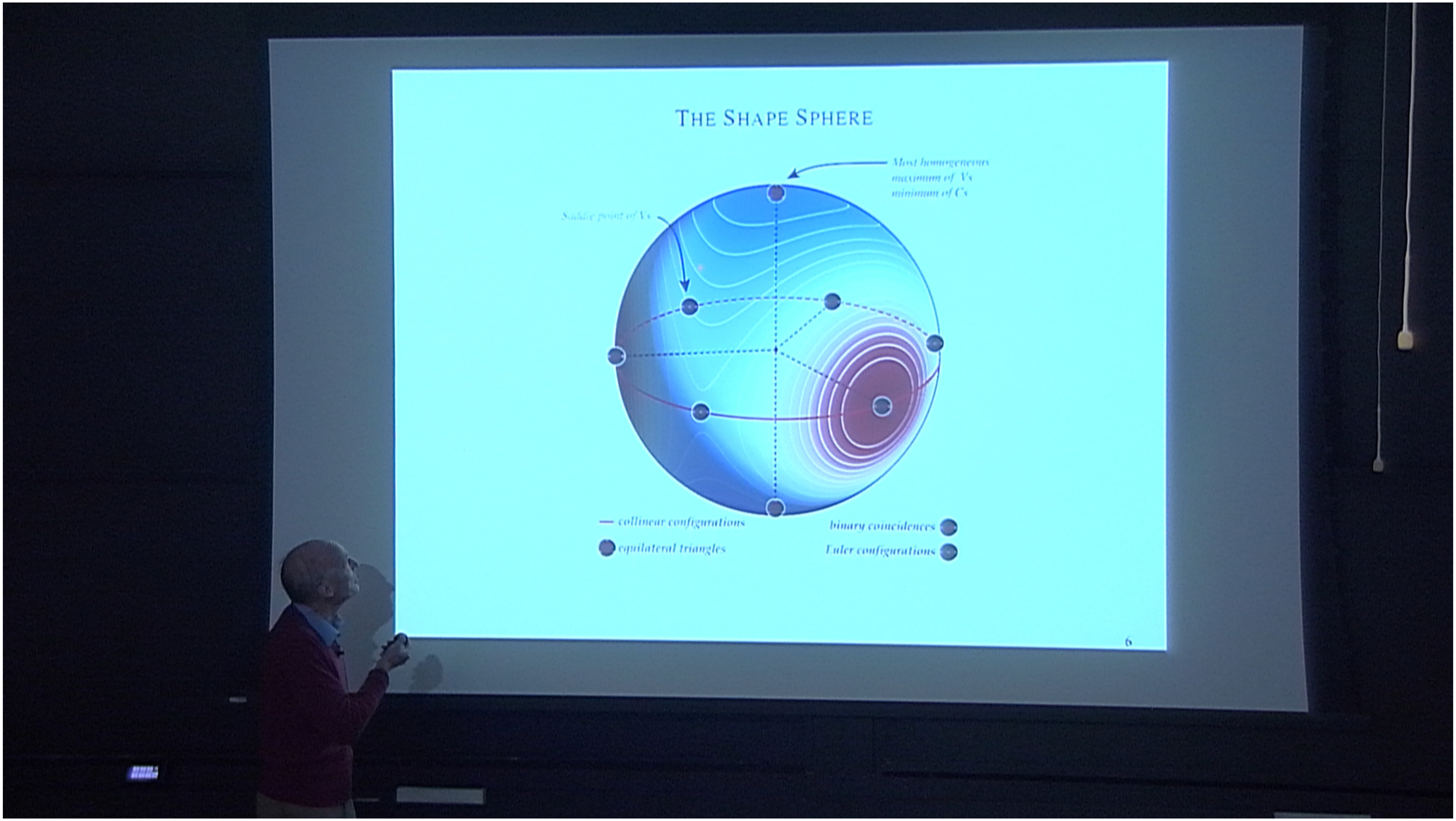
THE SHAPE SPHERE
Most homogeneous
maximum of Vs
minimum of Cs
Saddle point of Vs
collinear configurations
equilateral triangles
binary coincidences
Euler configurations
6



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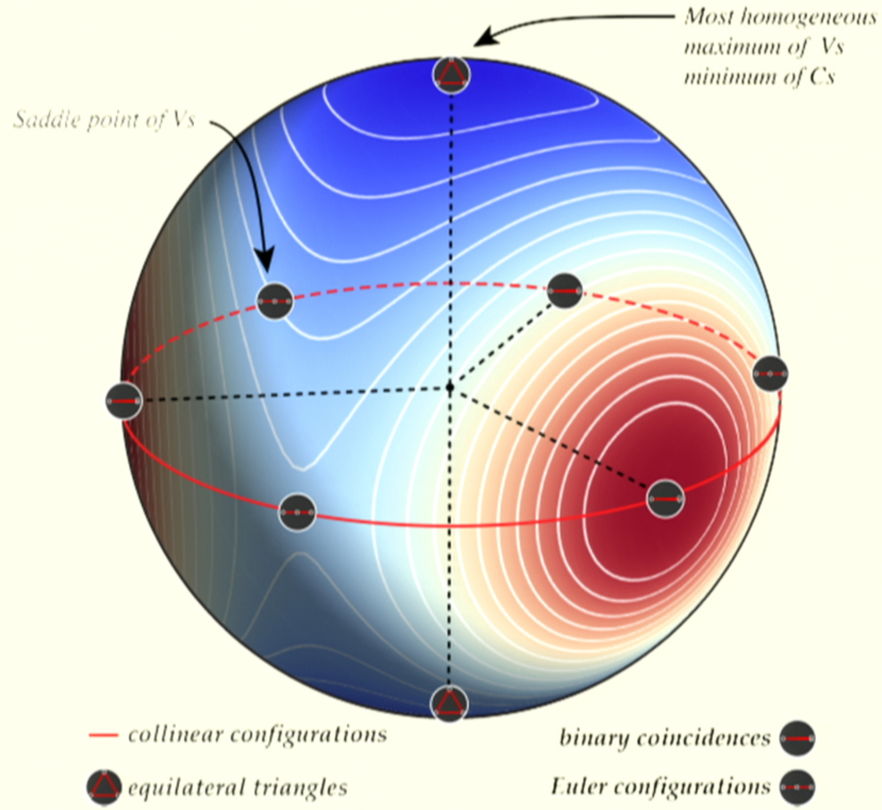


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## DYNAMICAL SIMILARITY & LAGRANGE-JACOBI RELATION

$$V_{\text{New}} = -\left(\frac{m_1 m_2}{r_{12}} + \frac{m_1 m_3}{r_{13}} + \frac{m_1 m_2}{r_{23}} + \dots\right)$$ is homogeneous of degree k = −1

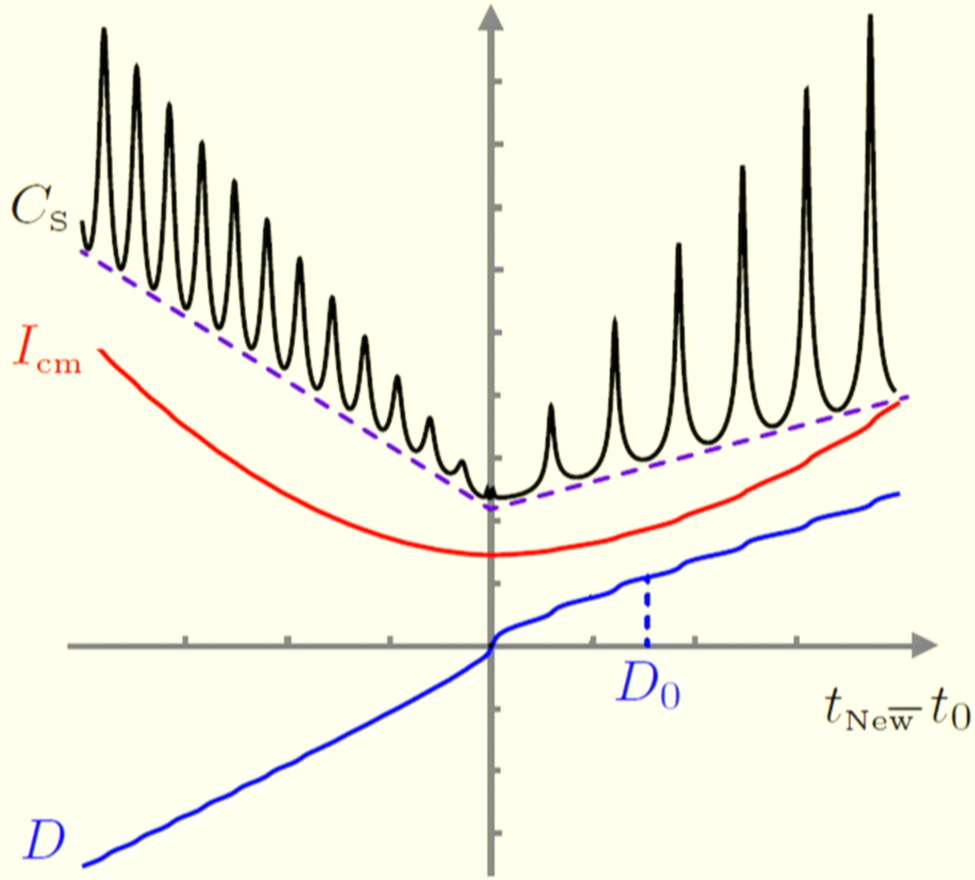
$$V(\alpha\, \mathbf{r}_a) \to \alpha^k\, V(\mathbf{r}_a) \qquad \mathbf{r}_a \to \alpha\, \mathbf{r}_a \qquad t \to \alpha^{1-\frac{k}{2}}\, t$$

$$\frac{dI_{\text{cm}}}{dt} = 2\,D, \qquad D = \sum_a \mathbf{r}_a \cdot \mathbf{p}^a$$ is the **dilatational momentum**

$$\frac{d^2 I_{\text{cm}}}{dt^2} = 4E - 2(k+2)V \qquad \Longrightarrow \qquad \frac{d^2 I_{\text{cm}}}{dt^2} = 4\,E - 2\,V_{\text{New}} > 0 \qquad \text{if } E \geq 0$$

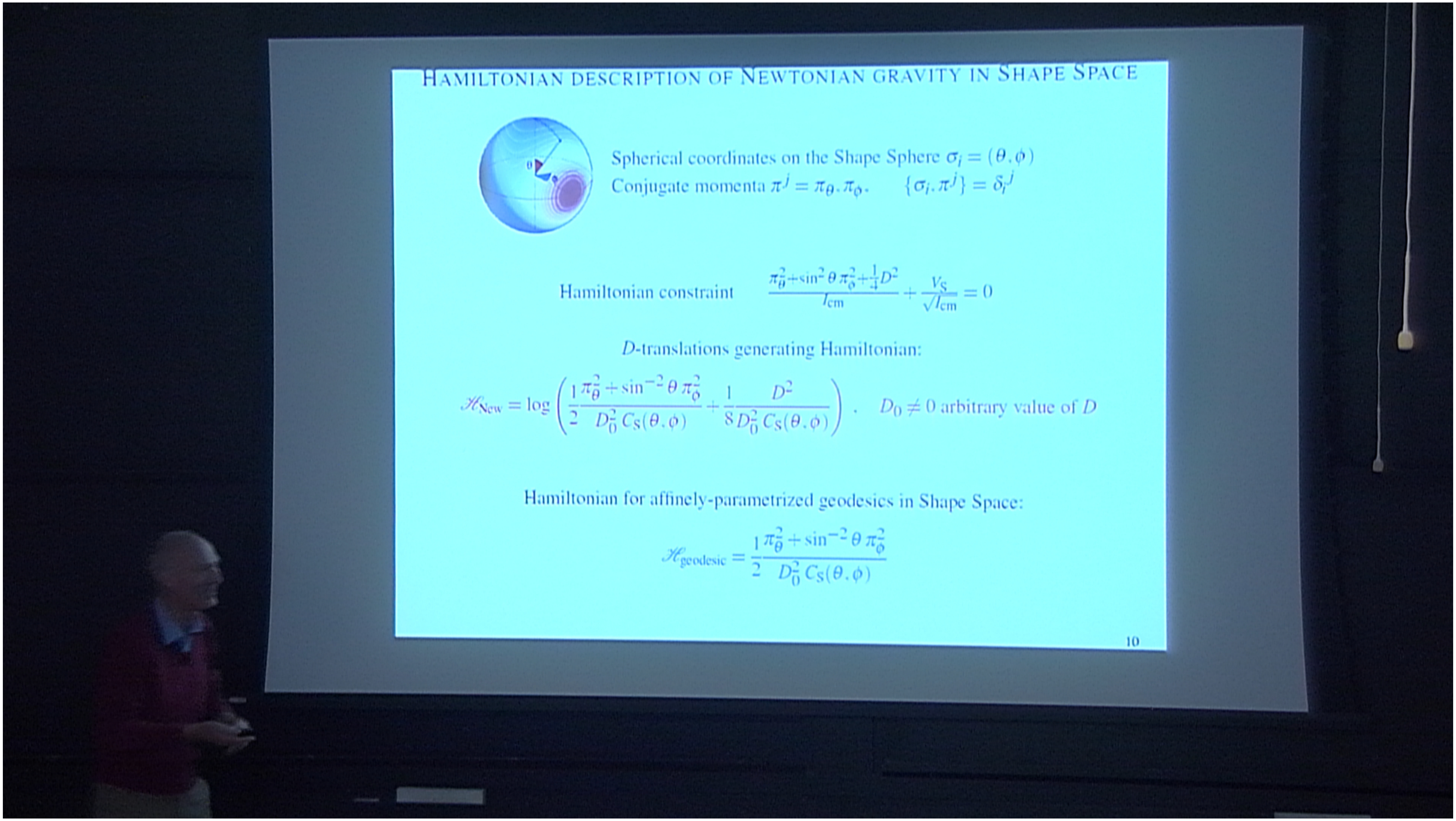
$$\Longrightarrow \quad \frac{dD}{dt} > 0$$ $D$ is **monotonic**

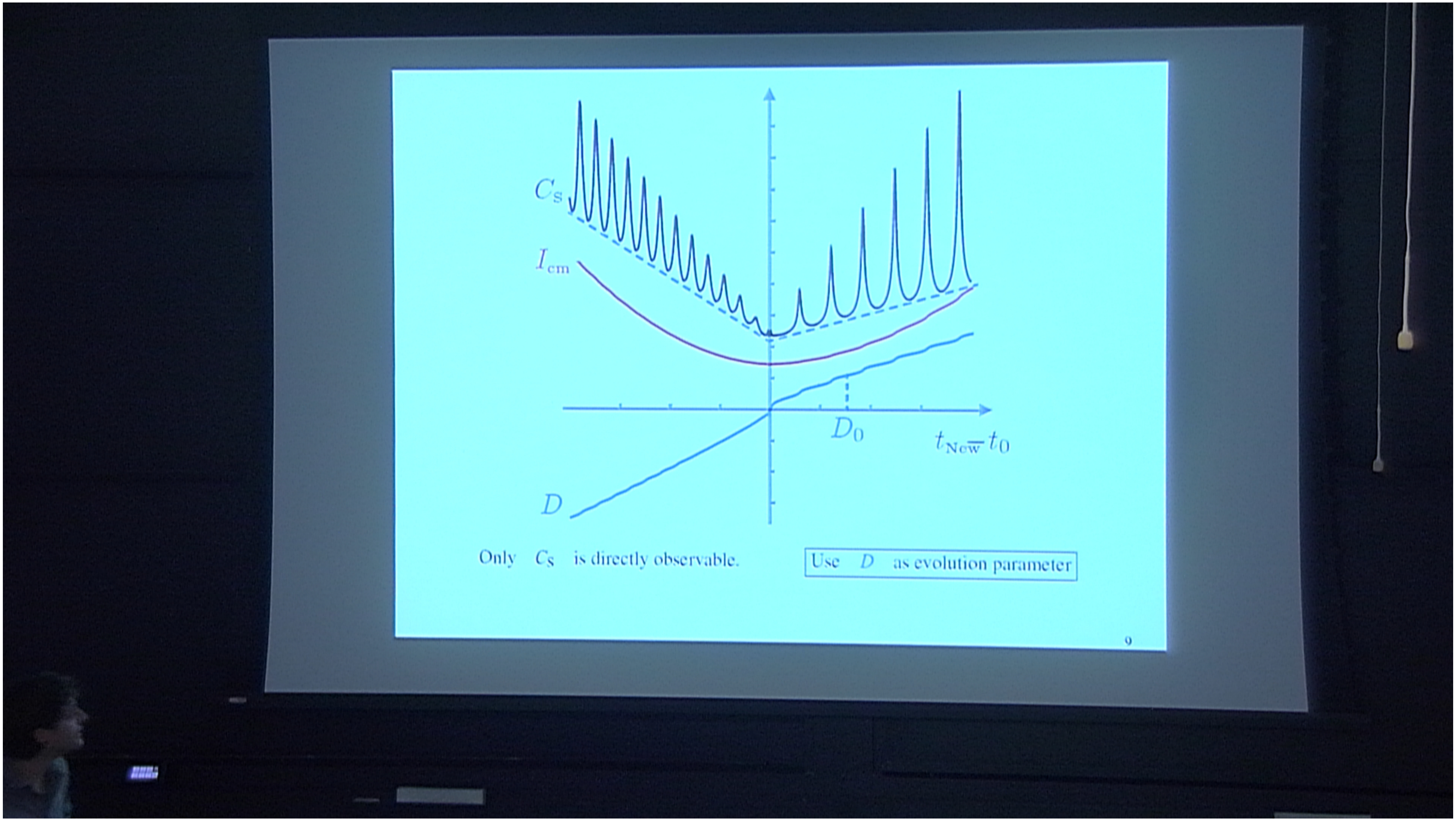
7




Only $C_S$ is directly observable.

Use $D$ as evolution parameter


HAMILTONIAN DESCRIPTION OF NEWTONIAN GRAVITY IN SHAPE SPACE
Spherical coordinates on the Shape Sphere $\sigma_i = (\theta, \phi)$
Conjugate momenta $\pi^j = \pi_\theta, \pi_\phi$, $\{\sigma_i, \pi^j\} = \delta_i^j$
Hamiltonian constraint $\frac{\pi_\theta^2 + \sin^2\theta\, \pi_\phi^2 + \frac{1}{4}D^2}{I_{cm}} + \frac{V_S}{\sqrt{I_{cm}}} = 0$
$D$-translations generating Hamiltonian:
$\mathscr{H}_{New} = \log\left(\frac{1}{2}\frac{\pi_\theta^2 + \sin^{-2}\theta\, \pi_\phi^2}{D_0^2\, C_S(\theta,\phi)} + \frac{1}{8}\frac{D^2}{D_0^2\, C_S(\theta,\phi)}\right)$, $D_0 \neq 0$ arbitrary value of $D$
Hamiltonian for affinely-parametrized geodesics in Shape Space:
$\mathscr{H}_{geodesic} = \frac{1}{2}\frac{\pi_\theta^2 + \sin^{-2}\theta\, \pi_\phi^2}{D_0^2\, C_S(\theta,\phi)}$
10



$C_s$
$I_{cm}$
$D$
$D_0$
$t_{Now}$ $t_0$
Only $C_s$ is directly observable.
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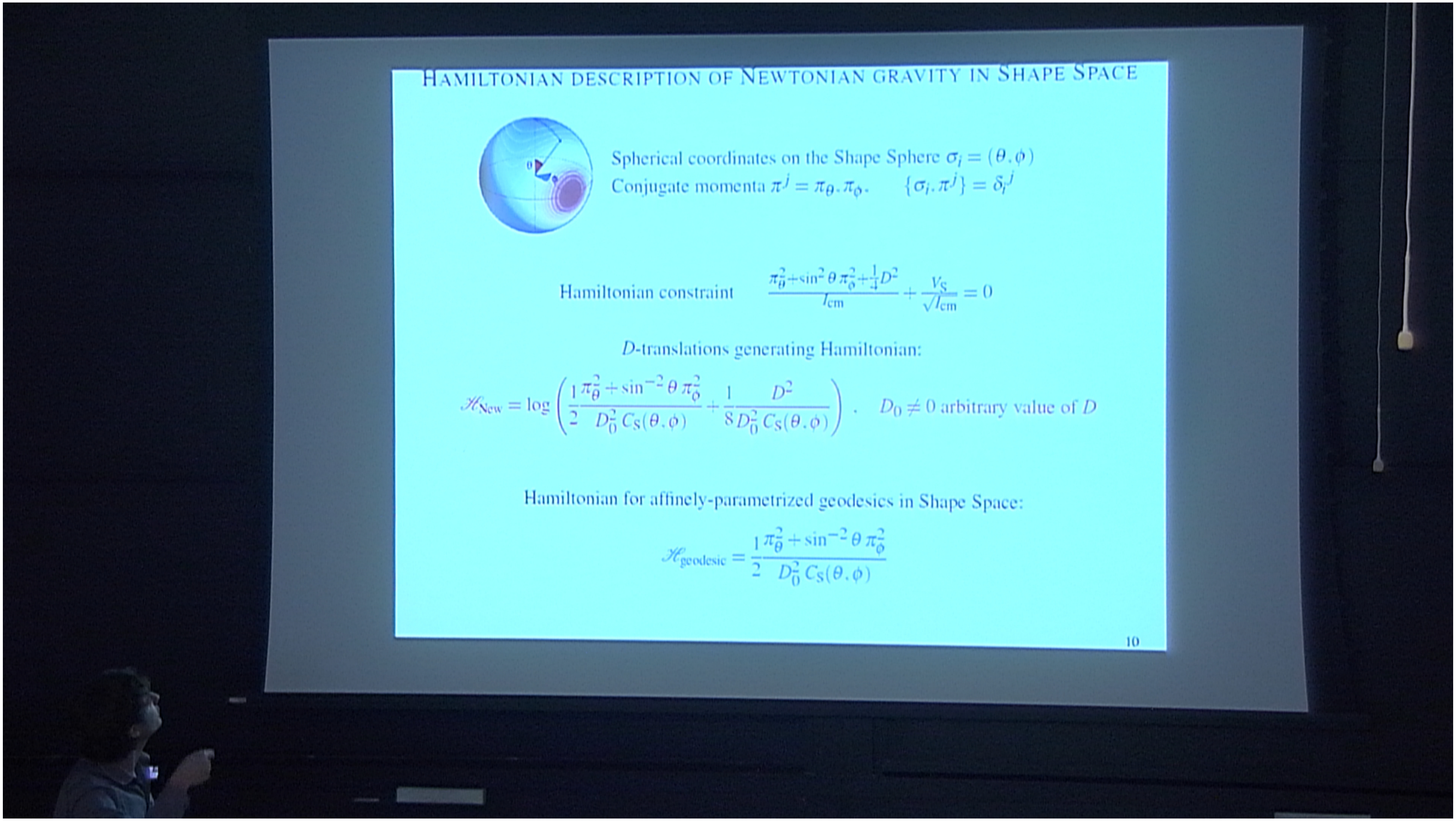

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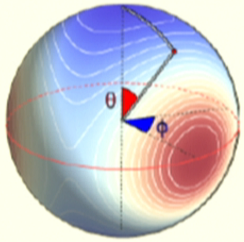


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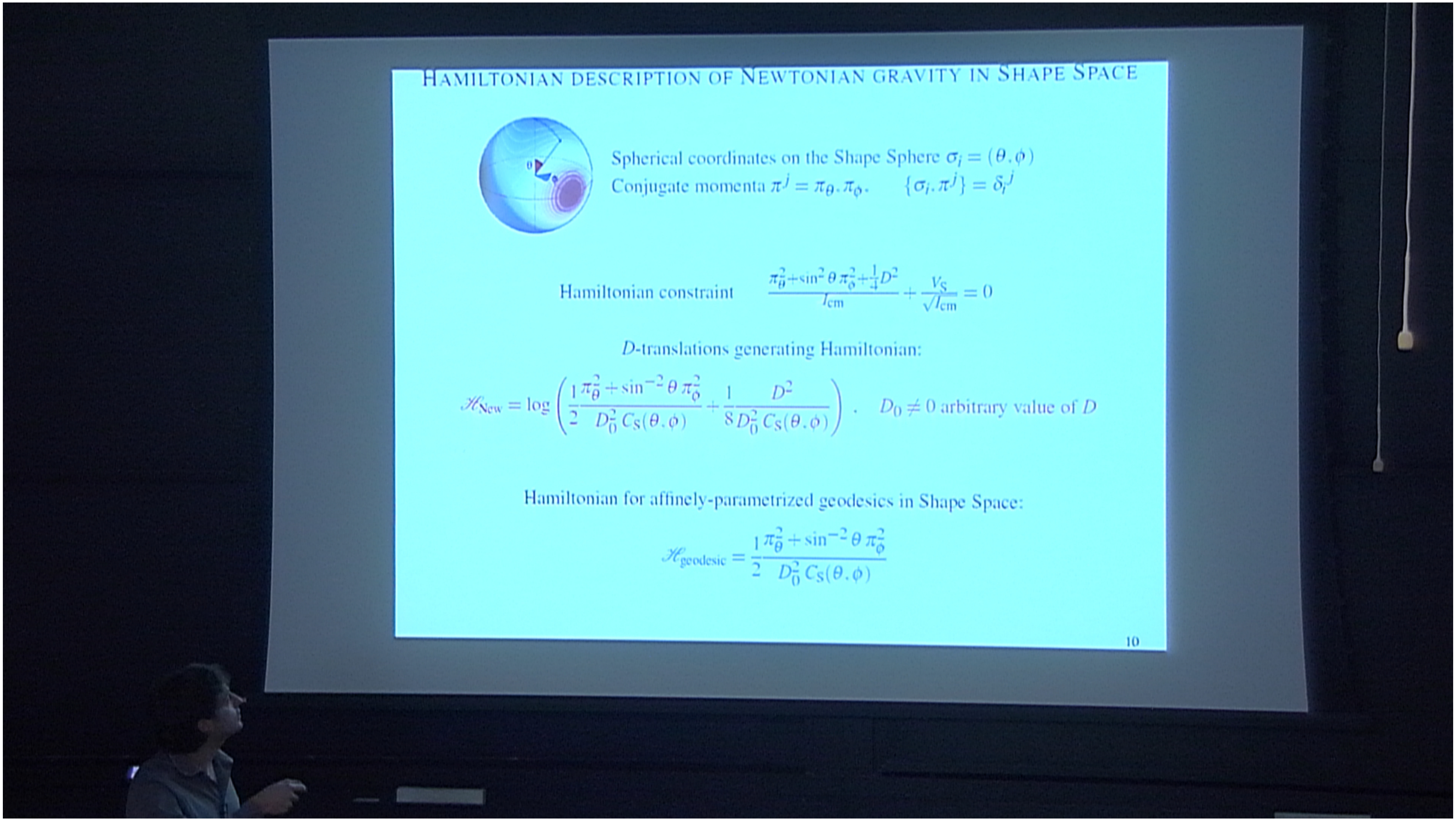
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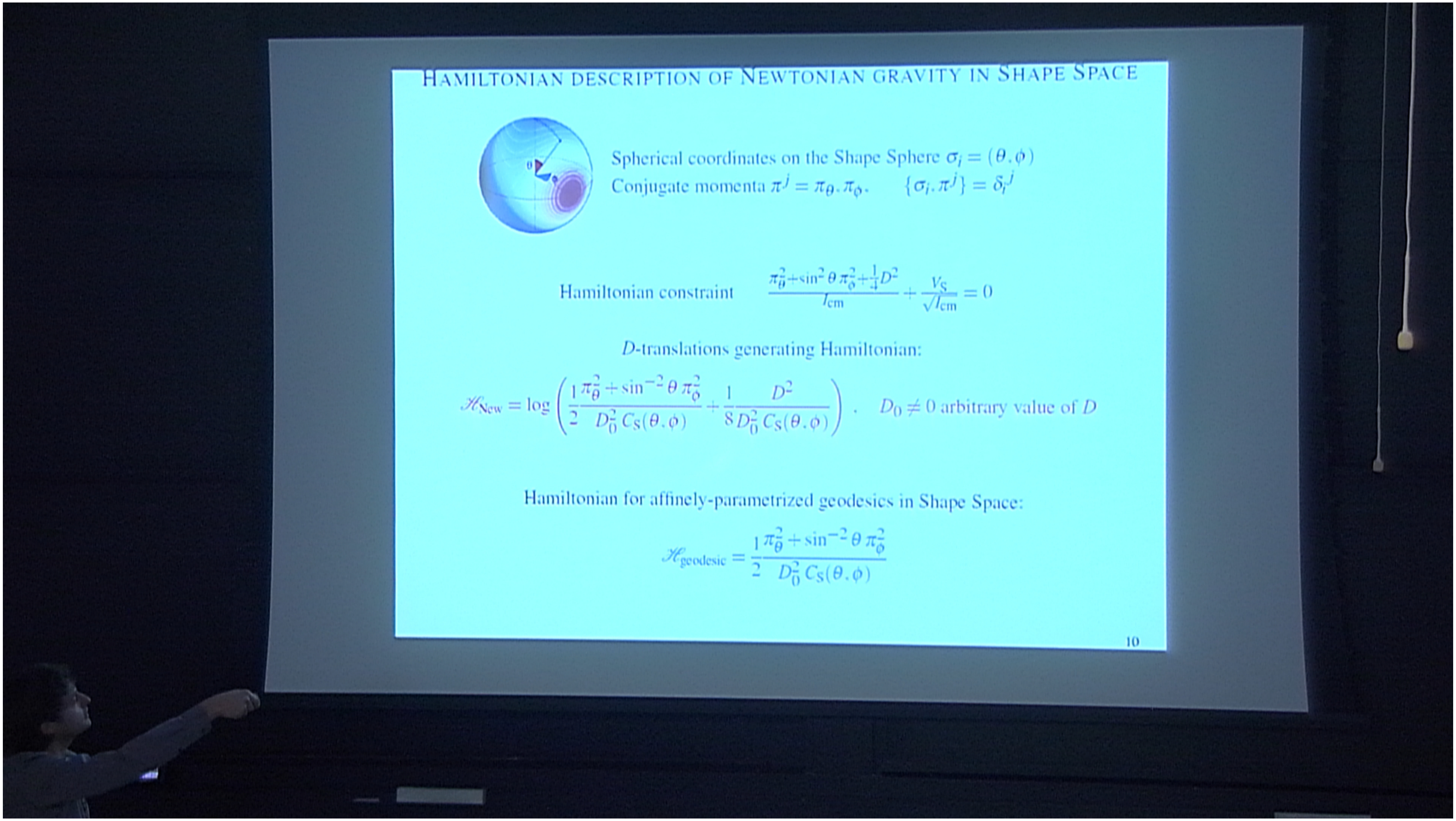
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## Equations of Motion for $\mathscr{H}_{\text{New}}$ with $D$ as evolution parameter

$$\frac{d\theta}{dD} = \frac{2\,\pi_\theta}{\pi_\theta^2 + \sin^{-2}\theta\,\pi_\phi^2 + \frac{1}{4}D^2} \qquad \frac{d\phi}{dD} = \frac{2\,\sin^{-2}\theta\,\pi_\phi}{\pi_\theta^2 + \sin^{-2}\theta\,\pi_\phi^2 + \frac{1}{4}D^2}$$

$$\frac{d\pi_\theta}{dD} = \frac{2\,\sin^{-3}\theta\,\cos\theta\,\pi_\phi^2}{\pi_\theta^2 + \sin^{-2}\theta\,\pi_\phi^2 + \frac{1}{4}D^2} + \frac{\partial \log C_S}{\partial\theta} \qquad \frac{d\pi_\phi}{dD} = \frac{\partial \log C_S}{\partial\phi}$$

Spherical coordinates on Shape Space are dimensionless $[\theta] = [\phi] = 1$.

Shape Momenta are still dimensionful $[\pi_\theta] = [\pi_\phi] = [D] = \ell^{\frac{1}{2}}$

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## Dimensionless momenta $\omega^i$ and evolution parameter $\lambda$

$$\lambda = \log\frac{D}{D_0}, \qquad \omega_\theta = \frac{\pi_\theta}{D}, \qquad \omega_\phi = \frac{\pi_\phi}{D}.$$

$$[\theta] = [\phi] = [\lambda] = [\omega_\theta] = [\omega_\phi] = 1$$

New equations of motion:

$$\frac{d\theta}{d\lambda} = \frac{2\,\omega_\theta}{\omega_\theta^2 + \sin^{-2}\theta\,\omega_\phi^2 + \frac{1}{4}} \qquad \frac{d\phi}{d\lambda} = \frac{2\,\sin^{-2}\theta\,\omega_\phi}{\omega_\theta^2 + \sin^{-2}\theta\,\omega_\phi^2 + \frac{1}{4}}$$

$$\frac{d\omega_\theta}{d\lambda} = -\omega_\theta + \frac{2\,\sin^{-3}\theta\,\cos\theta\,\omega_\phi^2}{\omega_\theta^2 + \sin^{-2}\theta\,\omega_\phi^2 + \frac{1}{4}} + \frac{\partial \log C_S}{\partial\theta} \qquad \frac{d\omega_\phi}{d\lambda} = -\omega_\phi + \frac{\partial \log C_S}{\partial\phi}$$

They include **dissipative** terms ($-\omega_\theta$ and $-\omega_\phi$)

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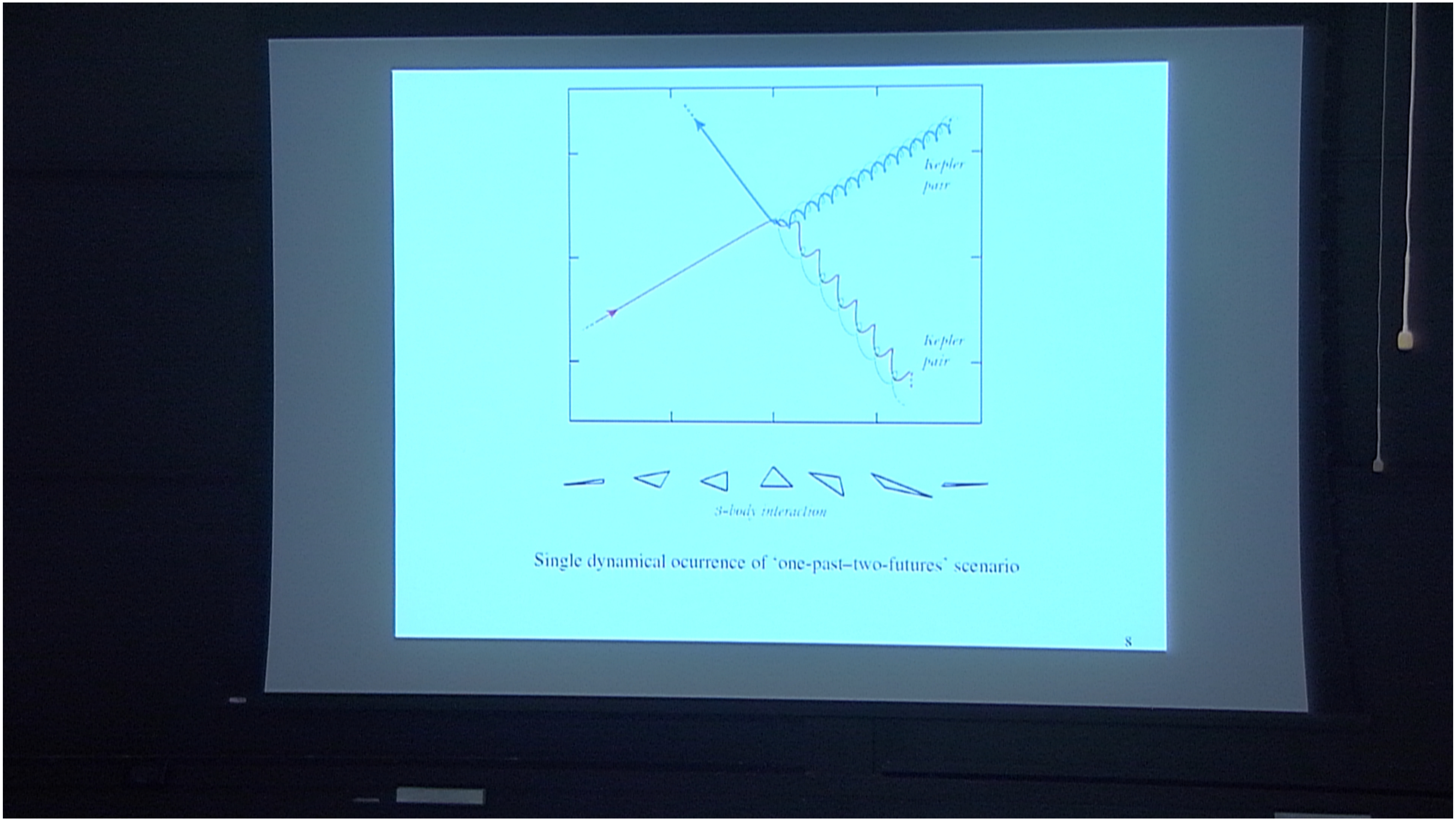
THE GENERIC ORBIT ON SHAPE SPACE
Two branches with irreversible dynamics
13


Kepler
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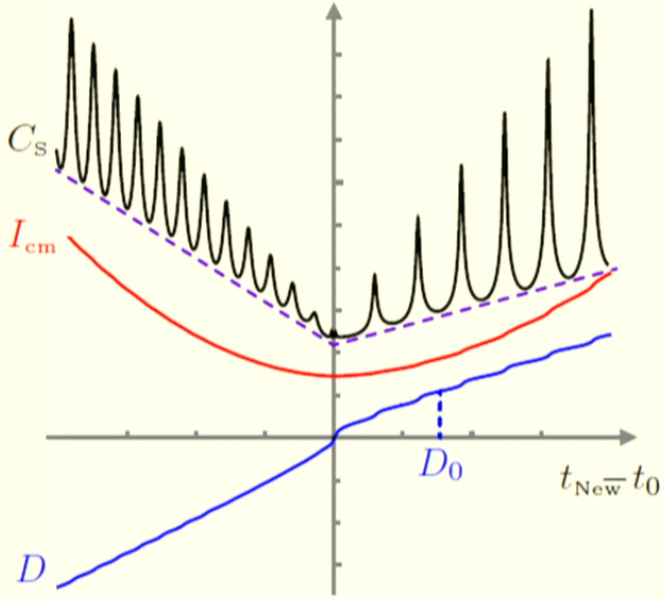
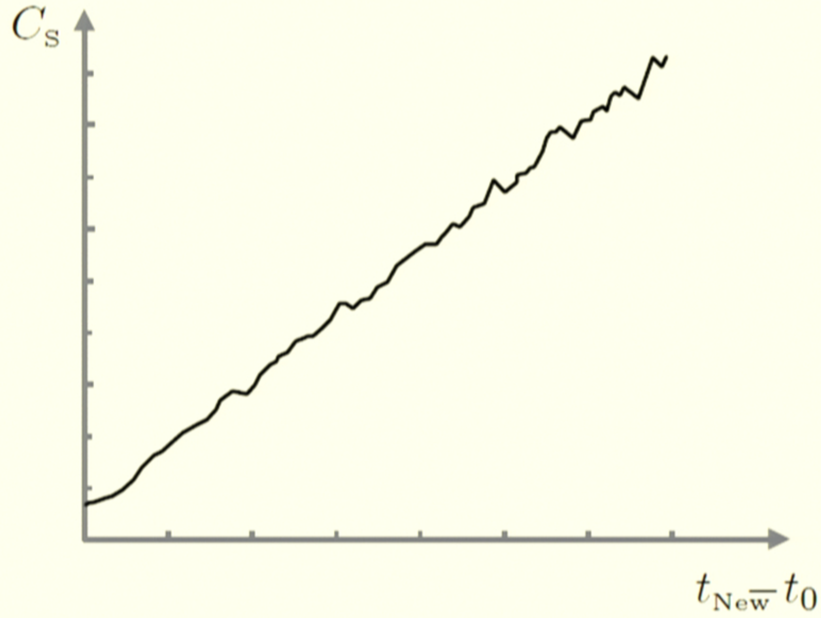
3-body interaction
Single dynamical ocurrence of 'one-past–two-futures' scenario
8

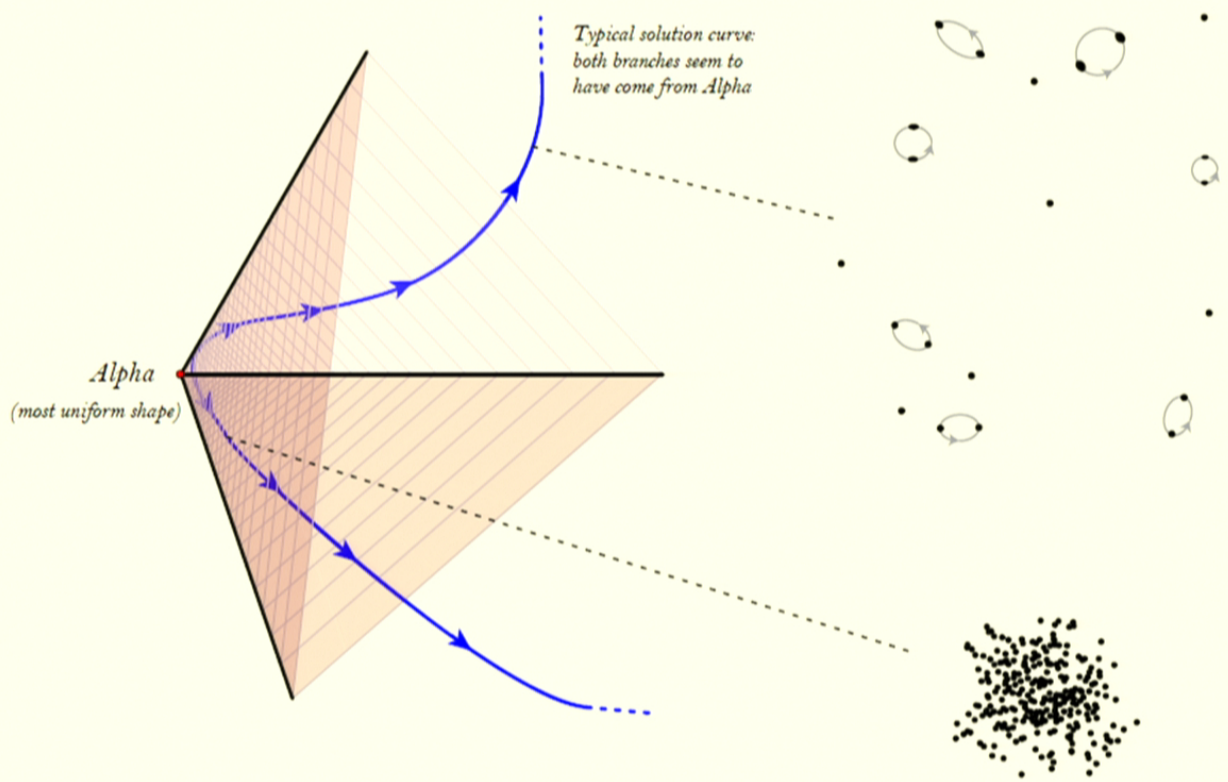



3 particles




1000 particles

# The Emergence of Complexity and Information

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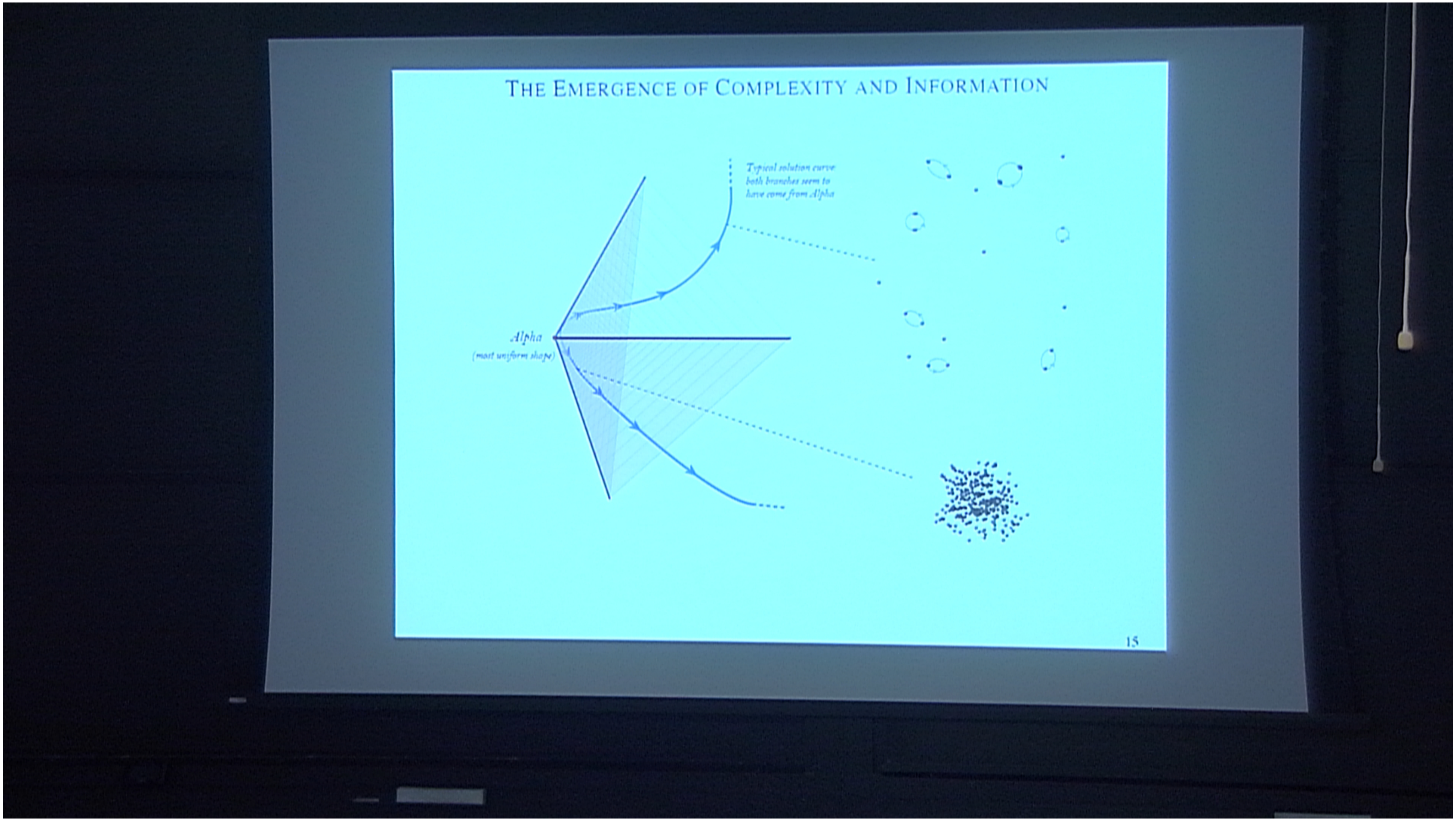
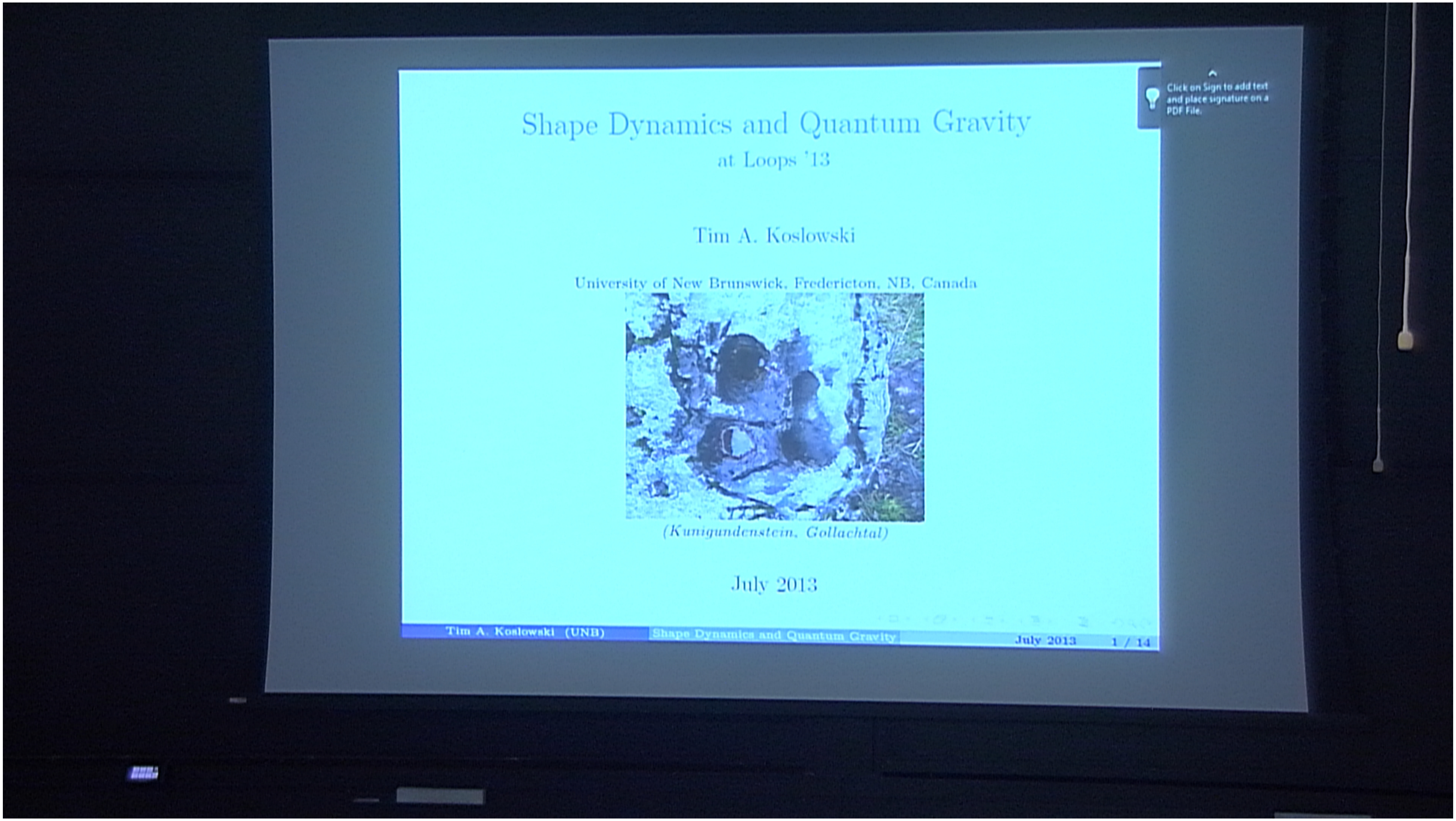

THE EMERGENCE OF COMPLEXITY AND INFORMATION
Typical solution curve
both branches seem to
have come from Alpha
Alpha
(most uniform shape)
15


Shape Dynamics and Quantum Gravity
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Shape Dynamics and Quantum Gravity
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# Shape Dynamics and Quantum Gravity

## at Loops '13

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# Outline

- Two explanations of classical gravity
  1. as spacetime geometry solving Einstein's equations (GR)
  2. as evolving conformal geometry (SD)
  - local dyn. equivalence, but global differences
  - differences important near spacetime singularities
- SD/GR equivalence reminiscent of AdS/CFT duality
  - SD/GR equivalence is bulk/bulk
  - boundary limit explains class. aspects of AdS/CFT
- New theory space for Quantum Gravity
  - SD from effective field theory
  - Loop–quantization attempts
  - Black Hole Thermodynamics

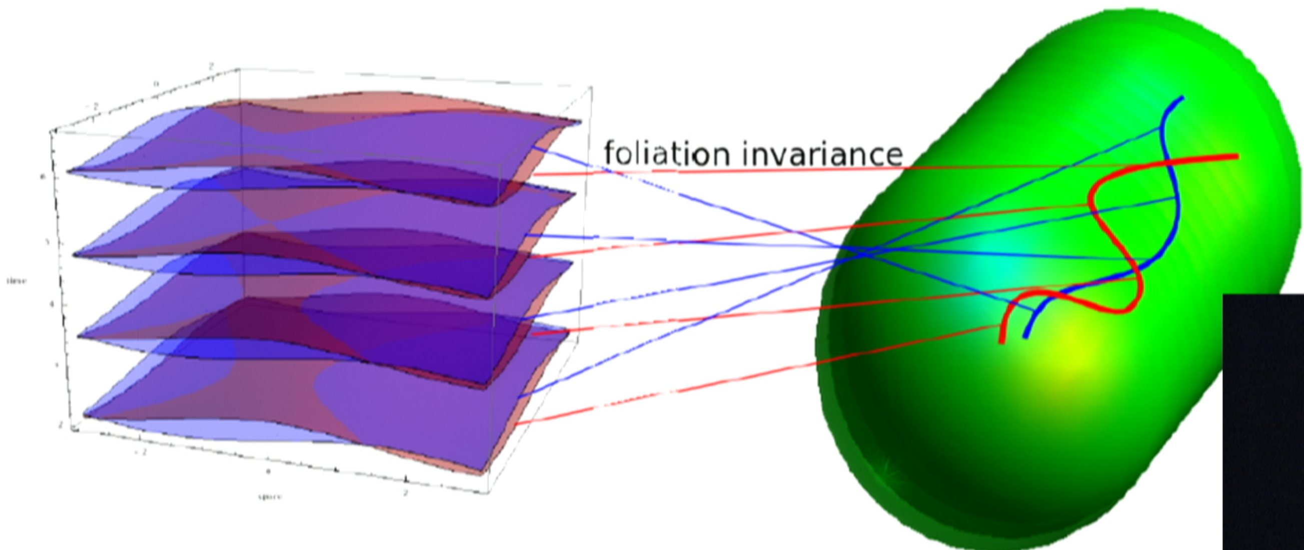


# Gravity explained as Spacetime Geometry

because spacetime "ties foliations together"

General Relativity:
gravity = spacetime geometry,
solves Einstein equ.

Hamiltonian GR:
gravity = equivalence class of
curves on superspace

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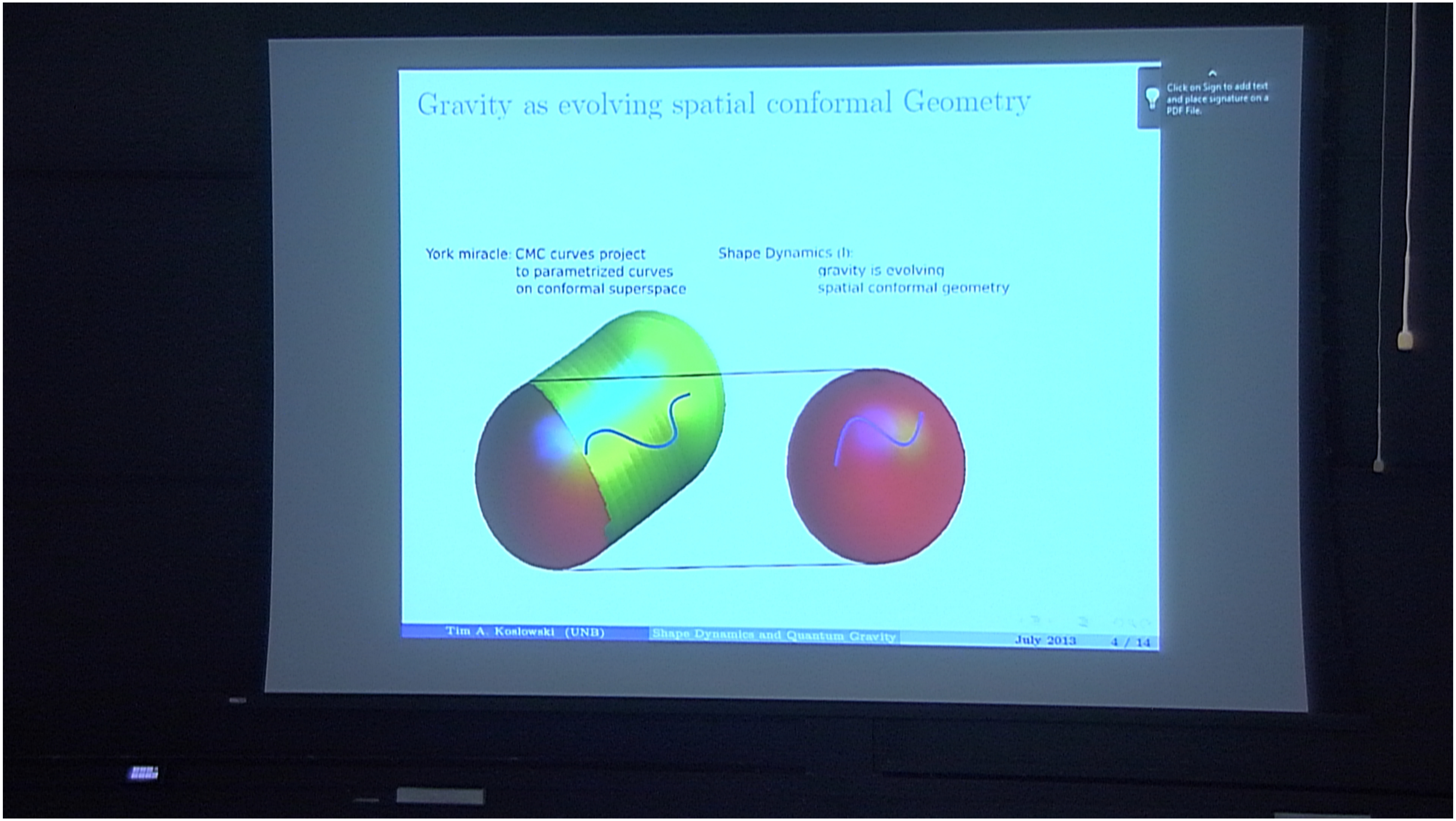
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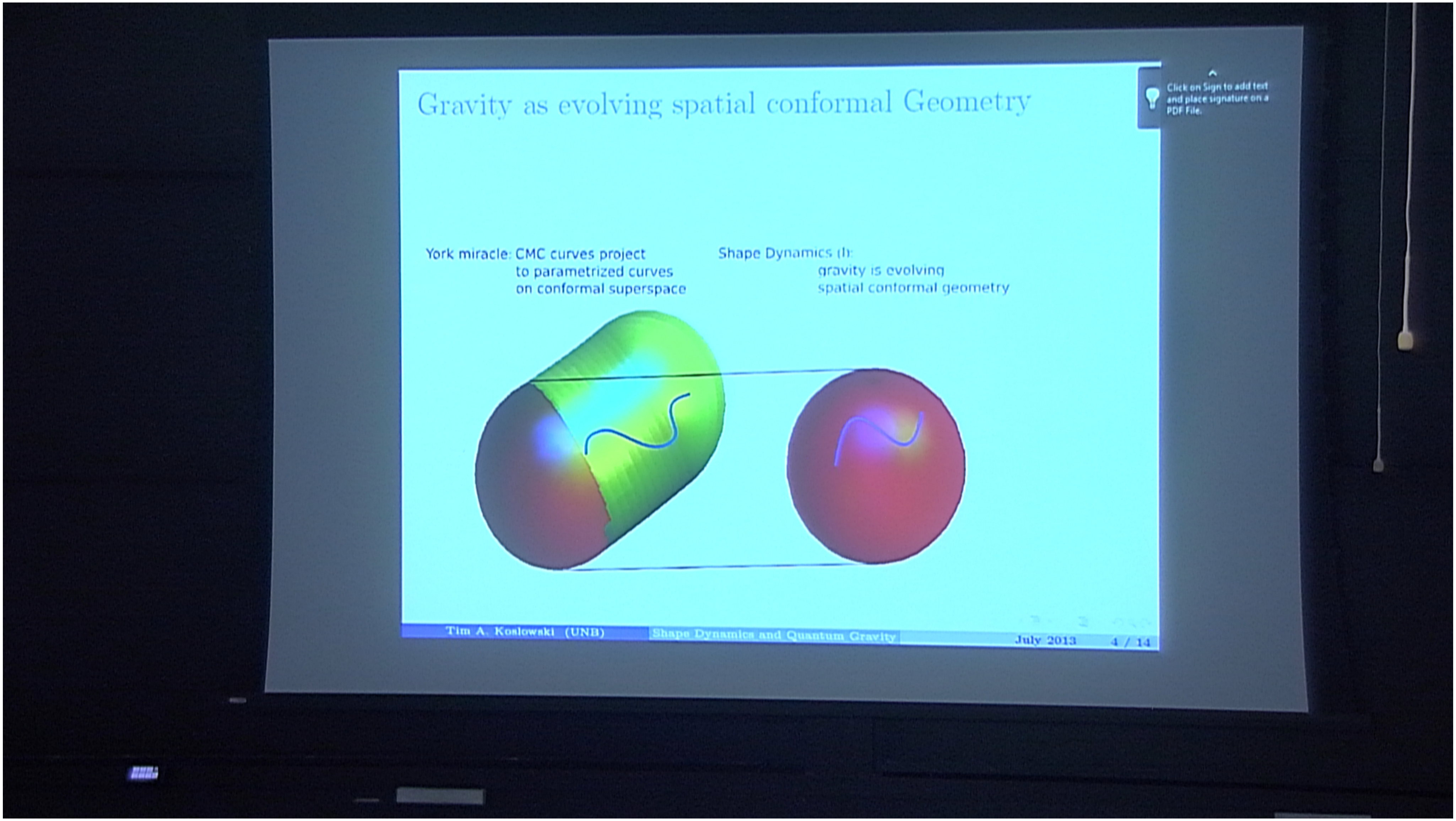

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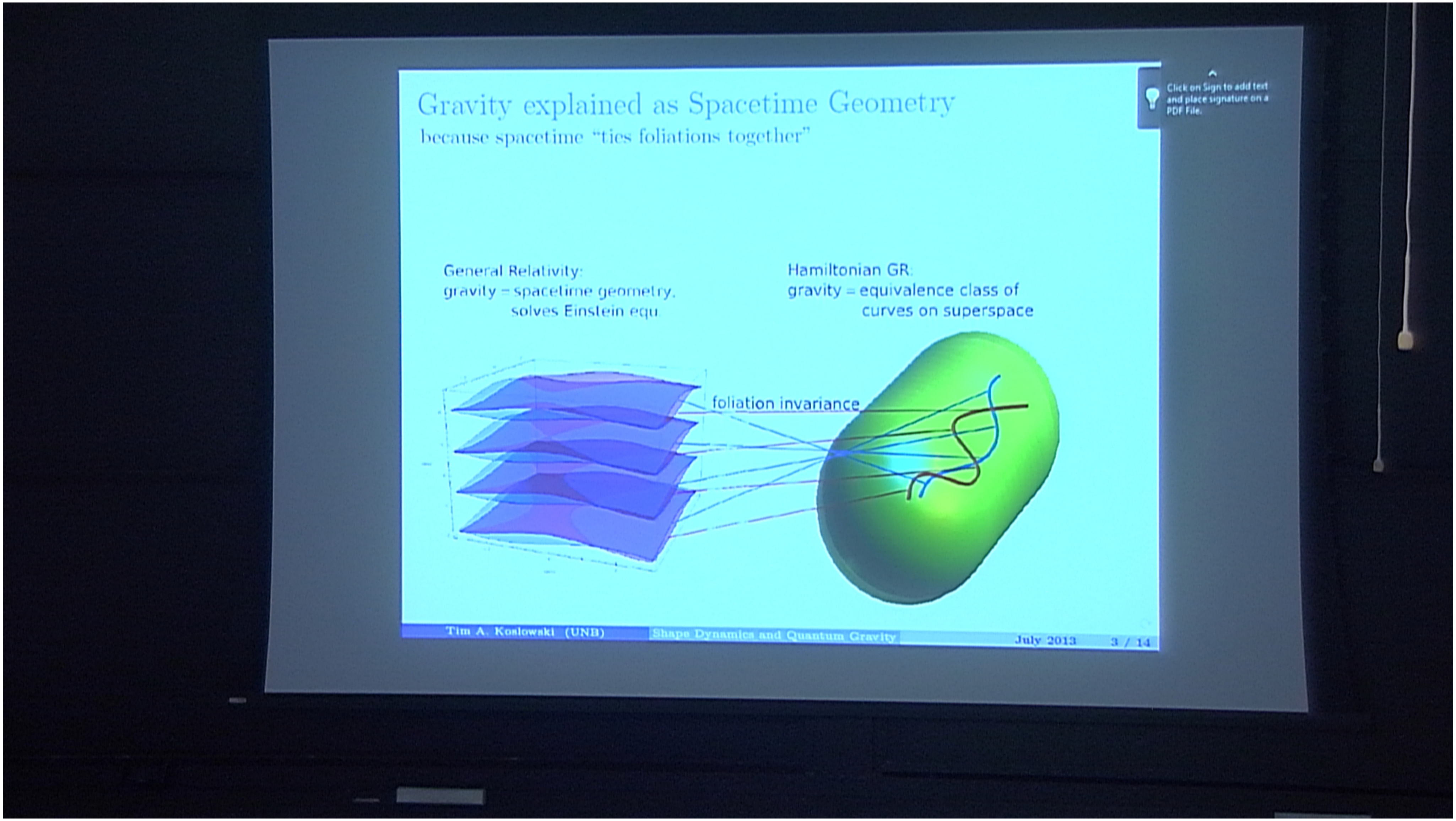
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# Gravity as dissipative spatial conformal Geometry



## Gauge–unfixed ADM in CMC gauge:

- gauge theory on ADM phase space, i.e. $\{g_{ab}(x), \pi^{cd}(y)\} = \delta_{ab}^{(cd)}(x, y)$
- gauge gen. $H(\xi) = \int d^3x \pi^{ab}(\mathcal{L}_\xi g)_{ab} \approx 0$, $Q(\rho) = \int d^3x \rho \, \pi \approx 0$
- Ham. $H_{SD} = \int d^3x \sqrt{|g|}\Omega[g, \pi]^6$; ($\Omega[g, \pi]$ solves Lichnerowicz-York equ.)

## Dynamical Similarity makes system autonomous and dimensionless

$H_{SD}(\rho, \sigma; \tau) = \tau^{-3} H_{SD}(\rho, \tau^2\sigma; 1) =: \tau^{-3} H_o(\rho, \tilde{\sigma})$,

where $\tilde{\sigma}^a_b$ denotes dimensionless tracefree metric momenta

$\{..\}_o := \tau^{-2}\{..\}$ is dimensionless

$\Rightarrow$ noncan. trf. introduces dissipation $\frac{\partial f}{\partial t} = \{f, H_o\}_o + 2\int \tilde{\sigma}\, \sigma f_{,\tilde{\sigma}}$

where $t = \ln(\tau/\tau_o)$ denotes dimensionless time and $(., S)$ a **bulk entropy**

## Shape Dynamics (II)

Gravity = dissipative evolution of spatial conformal geometry

# Bulk equivalence of SD and GR



## Local equivalence, global differences

E.o.m. of GR and SD coincide when:
(1) GR is evolved in CMC gauge ($g^{ab}K_{ab} = const.$, lapse $N = N_o[g, \pi]$)
(2) SD is evolved in Lichnerowicz–York gauge
$\Rightarrow$ local equivalence, global differences if CMC gauge breaks down

## Extendible curves on Shape Space can replace singularities

- CMC slices avoid many singularities

$\Rightarrow$ generic possible global difference whenever these singularity types occur
$\Rightarrow$ extensions of curve in Shape Space (not spacetime singularities)

## Example: Bianchi I cosmology (on 3–torus) in Shape Dynamics

dynamics = Teichmüller geodesics $\Rightarrow$ generically extendible
in GR: big bang = singularity in conformal factor (pure gauge)
freezeout = divergent lapse ("lost in translation" to spacetime)

# Bulk equivalence of SD and GR

## Local equivalence, global differences

E.o.m. of GR and SD coincide when:
(1) GR is evolved in CMC gauge ($g^{ab}K_{ab} = const.$, lapse $N = N_o[g, \pi]$)
(2) SD is evolved in Lichnerowicz–York gauge
$\Rightarrow$ local equivalence, global differences if CMC gauge breaks down

## Extendible curves on Shape Space can replace singularities

- CMC slices avoid many singularities

$\Rightarrow$ generic possible global difference whenever these singularity types occur
$\Rightarrow$ extensions of curve in Shape Space (not spacetime singularities)

## Example: Bianchi I cosmology (on 3–torus) in Shape Dynamics

dynamics = Teichmüller geodesics $\Rightarrow$ generically extendible
in GR: big bang = singularity in conformal factor (pure gauge)
freezeout = divergent lapse ("lost in translation" to spacetime)

# Bulk equivalence of SD and GR

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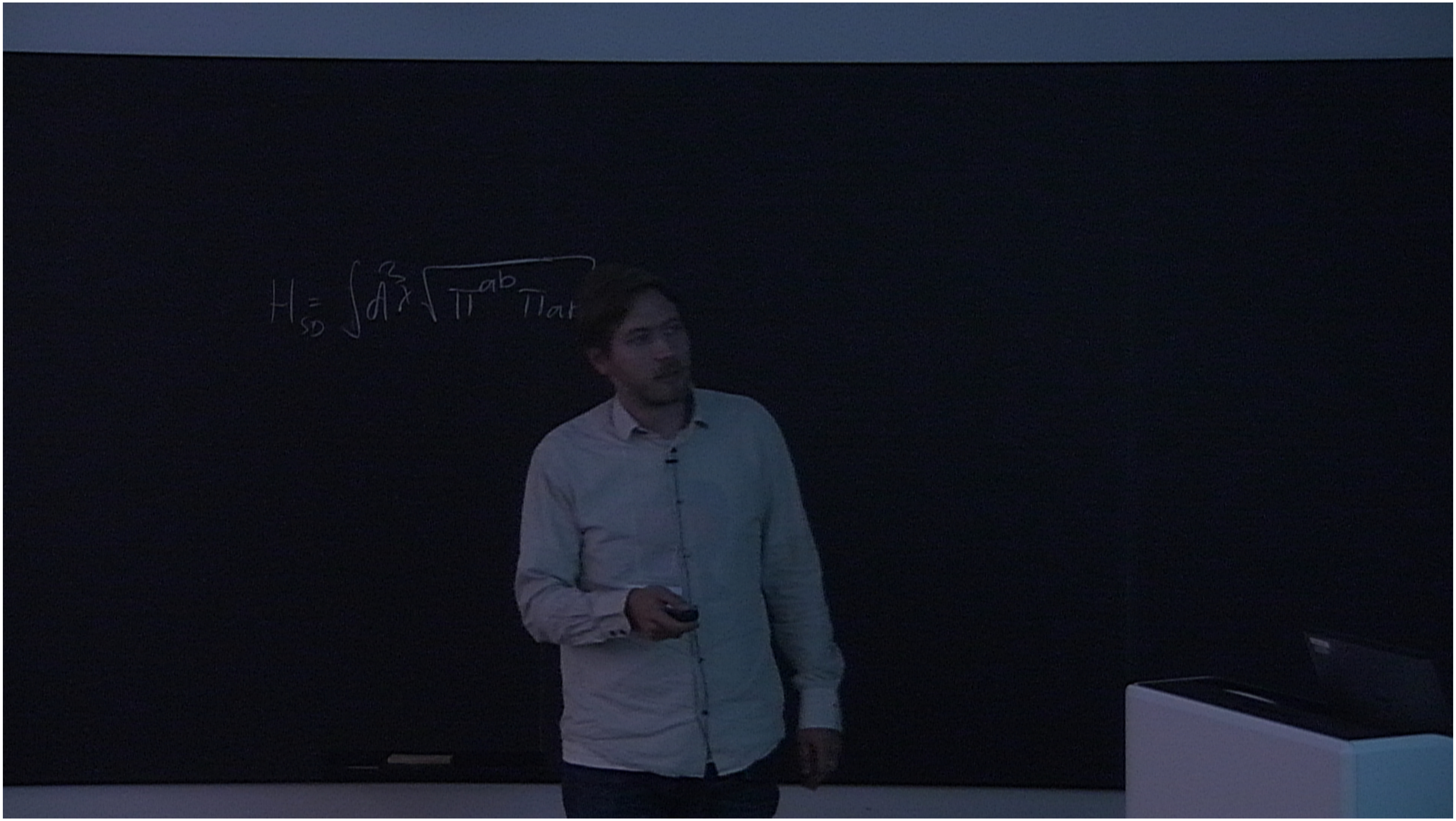

Gravity in d+1 dimensions

=

evolution of d-dimensional conformal theory

is like "bulk–bulk AdS/CFT"

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# Shape Dynamics explains classical AdS/CFT

### special CMC slices

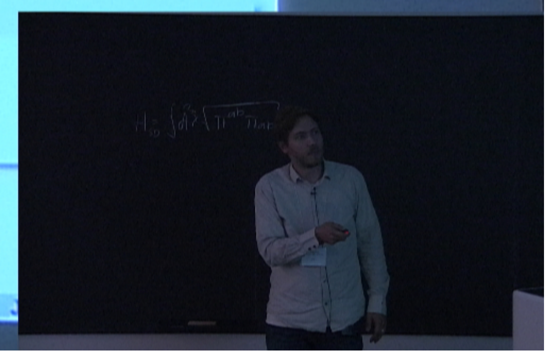
If $R$ is constant in a CMC slice, then $H(N \equiv 1)$ generates SD evolution.
$\Rightarrow$ conformal constraints are "emergent" gauge symmetry generators

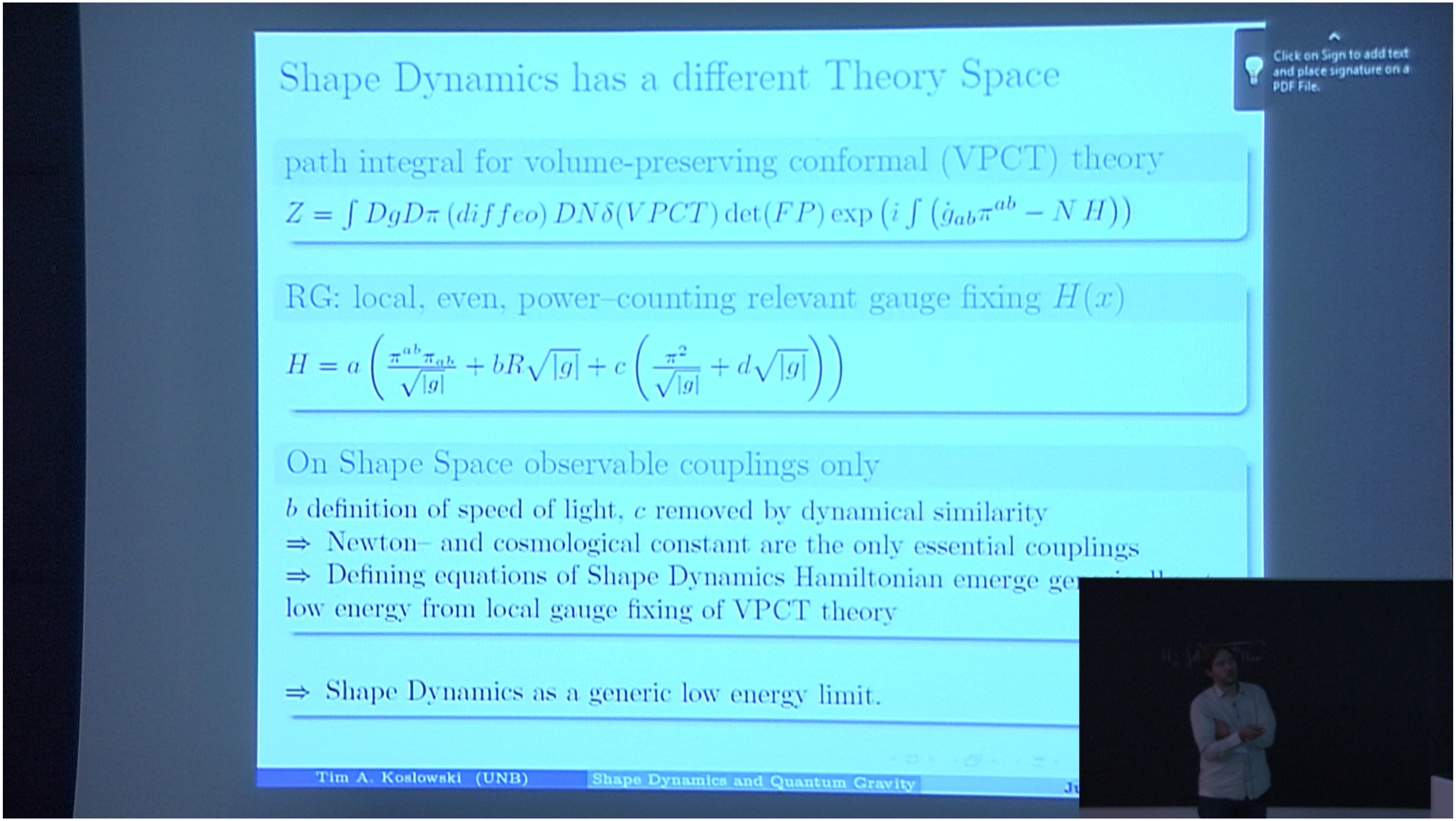
### asymptotically locally AdS (alAdS) boundary conds.

Euclidean alAdS conditions imply CMC and R=const. at boundary
$\Rightarrow$ boundary is special CMC slice
$\Rightarrow$ radial evolution at boundary $\equiv$ SD evolution
$\Rightarrow$ boundary CFT=restriction of SD to boundary

### More results:

- conformal symmetry at boundary is gauge symmetry of SD
- holographic RG equations are explicitly reproduced by SD





Shape Dynamics has a different Theory Space
path integral for volume-preserving conformal (VPCT) theory
$Z = \int DgD\pi\,(diffeo)\,DN\delta(VPCT)\det(FP)\exp\left(i\int\left(\dot{g}_{ab}\pi^{ab} - N\,H\right)\right)$
RG: local, even, power–counting relevant gauge fixing $H(x)$
$H = a\left(\frac{\pi^{ab}\pi_{ab}}{\sqrt{|g|}} + bR\sqrt{|g|} + c\left(\frac{\pi^2}{\sqrt{|g|}} + d\sqrt{|g|}\right)\right)$
On Shape Space observable couplings only
$b$ definition of speed of light, $c$ removed by dynamical similarity
⇒ Newton– and cosmological constant are the only essential couplings
⇒ Defining equations of Shape Dynamics Hamiltonian emerge ge
low energy from local gauge fixing of VPCT theory
⇒ Shape Dynamics as a generic low energy limit.
Tim A. Koslowski (UNB)
Shape Dynamics and Quantum Gravity
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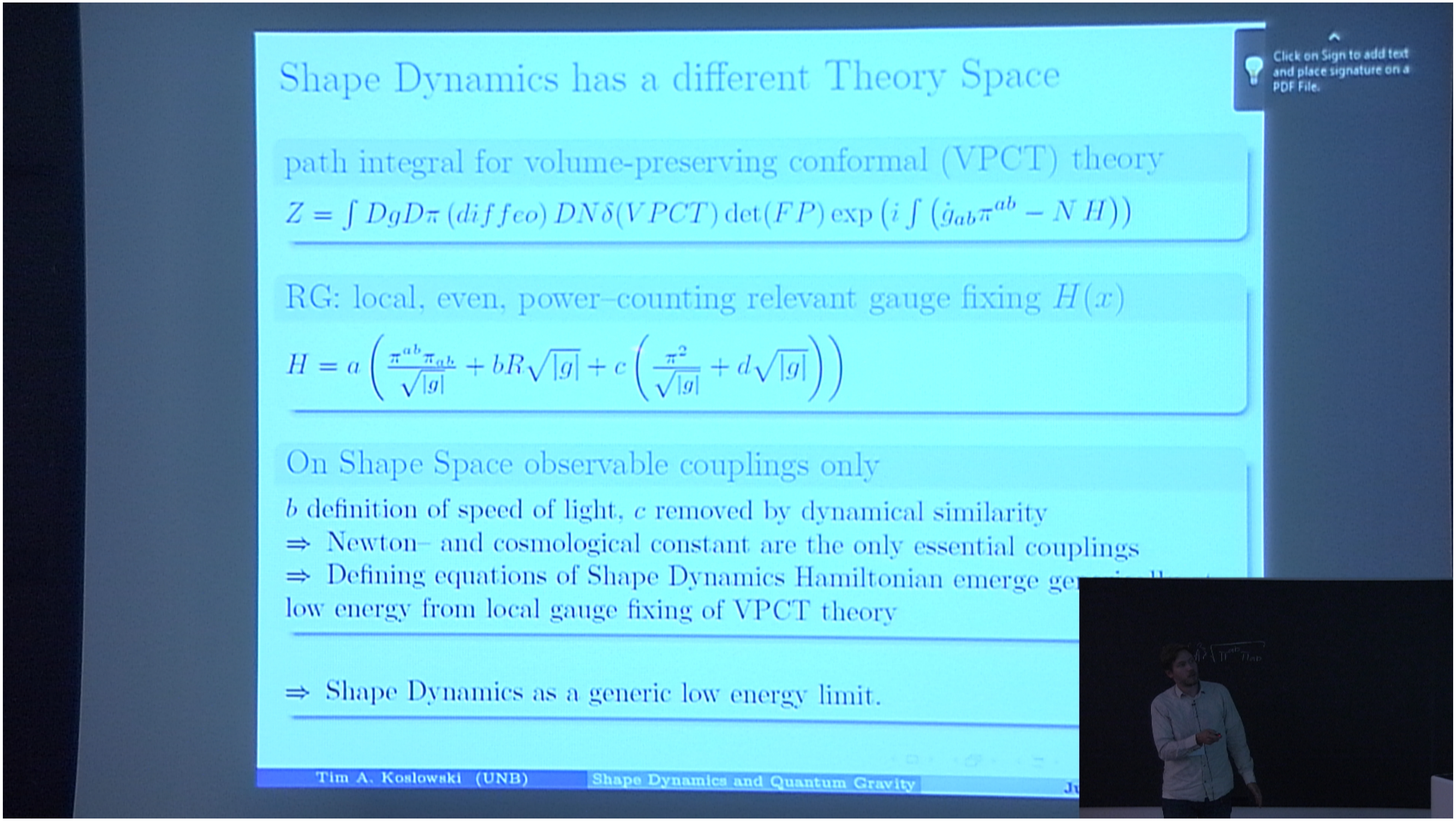
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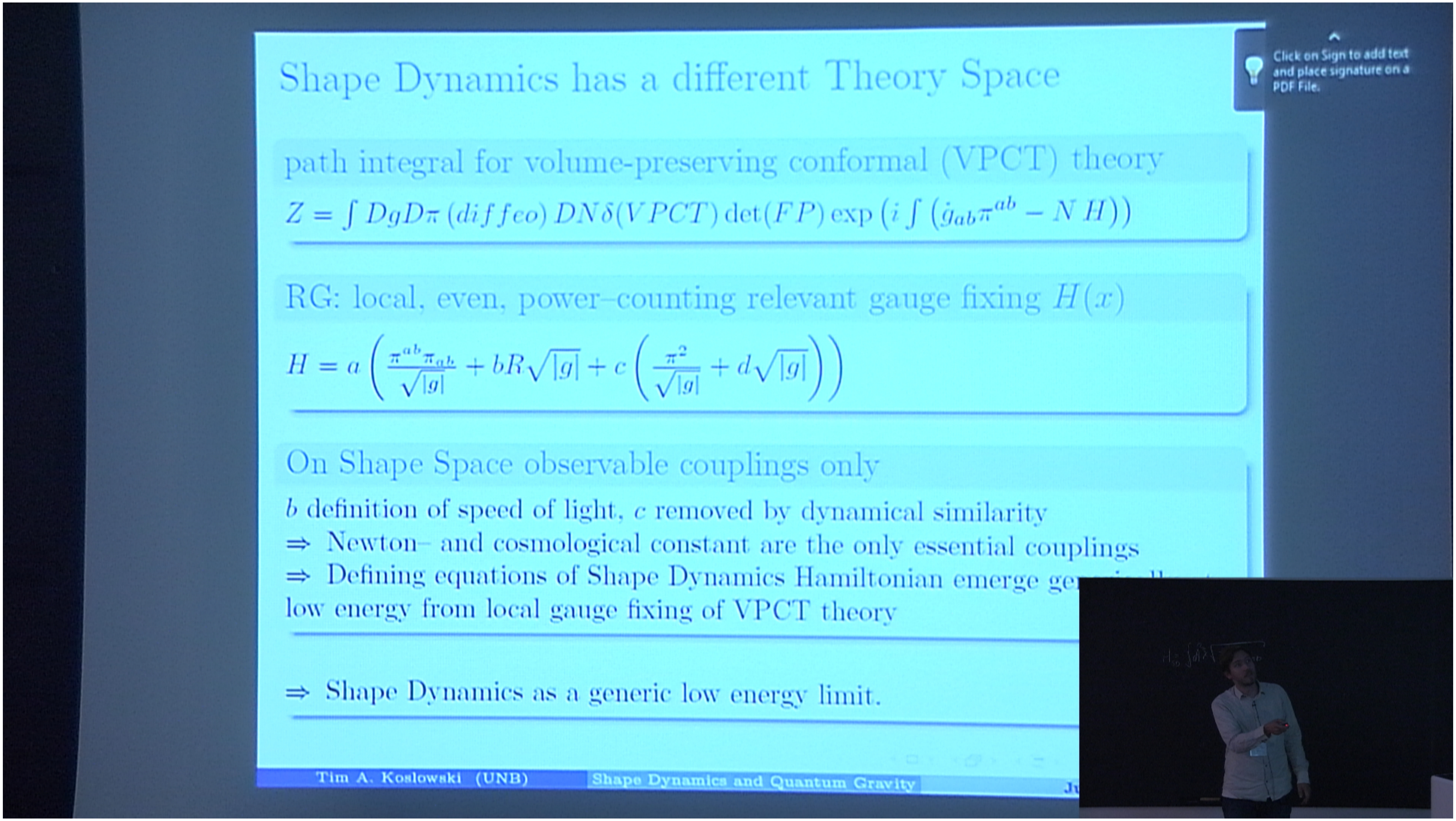
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# Highlights

- Properties of Shape Dynamics:
  1. explains gravity as dissipative evolution of spatial conformal geometry
     it is locally indistinguishable from GR
  2. evolution can be extended through some classical spacetime singularities
  3. explains some classical aspects of AdS/CFT
  4. emerges as a low energy limit on a VPCT theory space
- New approaches to Quantum Gravity:
  - heuristic physical Hilbert space in loop quantization
  - two hints that black hole entropy is due to entanglement in matter sector



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Gravity. arXiv:1303.7139

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# I. Hamiltonian constraints are not pure gauge generators

$$\mathscr{H} = \kappa \frac{\pi^{ab} G_{abcd} \pi^{cd}}{\sqrt{g}} - \frac{1}{\kappa}\left(R - \frac{d(d-1)k}{\ell^2}\right)\sqrt{g} \approx 0 \qquad (1)$$

$$V_a = 2 g_{ab} \nabla_c \pi^{bc} \approx 0, \qquad (2)$$

$$\{\mathscr{H}(N_1), \mathscr{H}(N_2)\} = V(\zeta^a(N_1, N_2, g^{ab})), \qquad (3)$$

$$\{V(N^a), \mathscr{H}(N)\} = \mathscr{H}(\mathscr{L}_{N^a} N), \qquad (4)$$

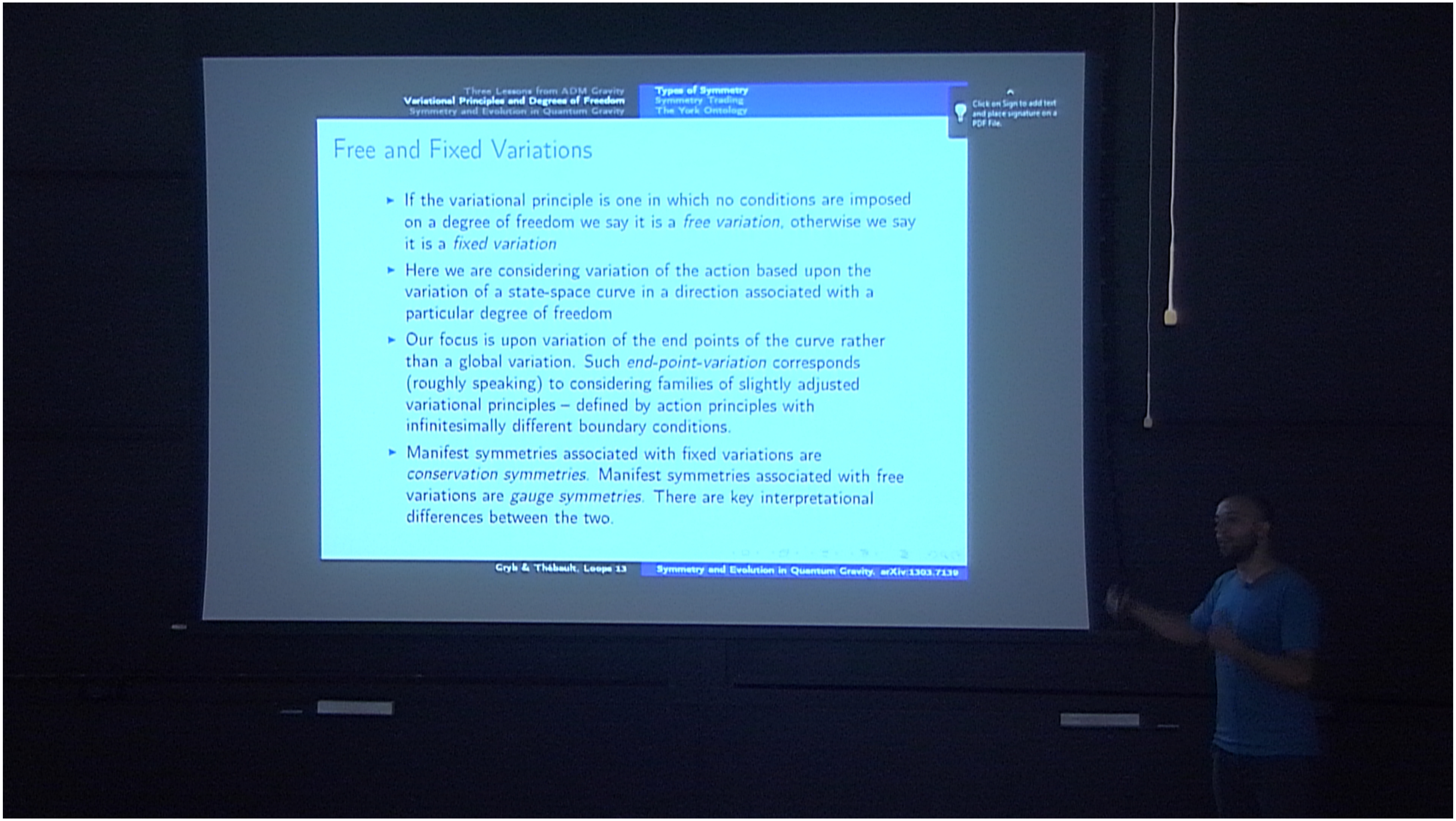
$$\{V(N_1^a), V(N_2^a)\} = V(\mathscr{L}_{N_1^a} N_2^a), \qquad (5)$$

$$\mathscr{H}(N) := \int_\Sigma d^d x N(x) \mathscr{H}(x), \; V(N^a) := \int_\Sigma d^d x N^a(x) V_a(x)$$

$$\zeta^a(f_1, f_2, g^{ab}) = g^{ab}(f_1 f_{2;b} - f_2 f_{1;b})$$

# Manifest and Hidden Symmetries

- A symmetry is associated with *some* form of redundancy occurring in the relationship between our mathematical formalism (configuration space, variational principle) and the *characteristic behaviour* of the system to which it corresponds (physical degrees of freedom, physical boundary conditions).
- If the action *does not* change when the sample paths are varied *globally* with respect to the function then this situation automatically implies a symmetry, and we will call a symmetry of this kind *manifest*.
- The other options are that there is no symmetry or that there is a hidden symmetry – hidden symmetries are important in GR and will return to them later in our discussion


Three Lessons from ADM Gravity
Variational Principles and Degrees of Freedom
Symmetry and Evolution in Quantum Gravity
Types of Symmetry
Symmetry Trading
The York Ontology
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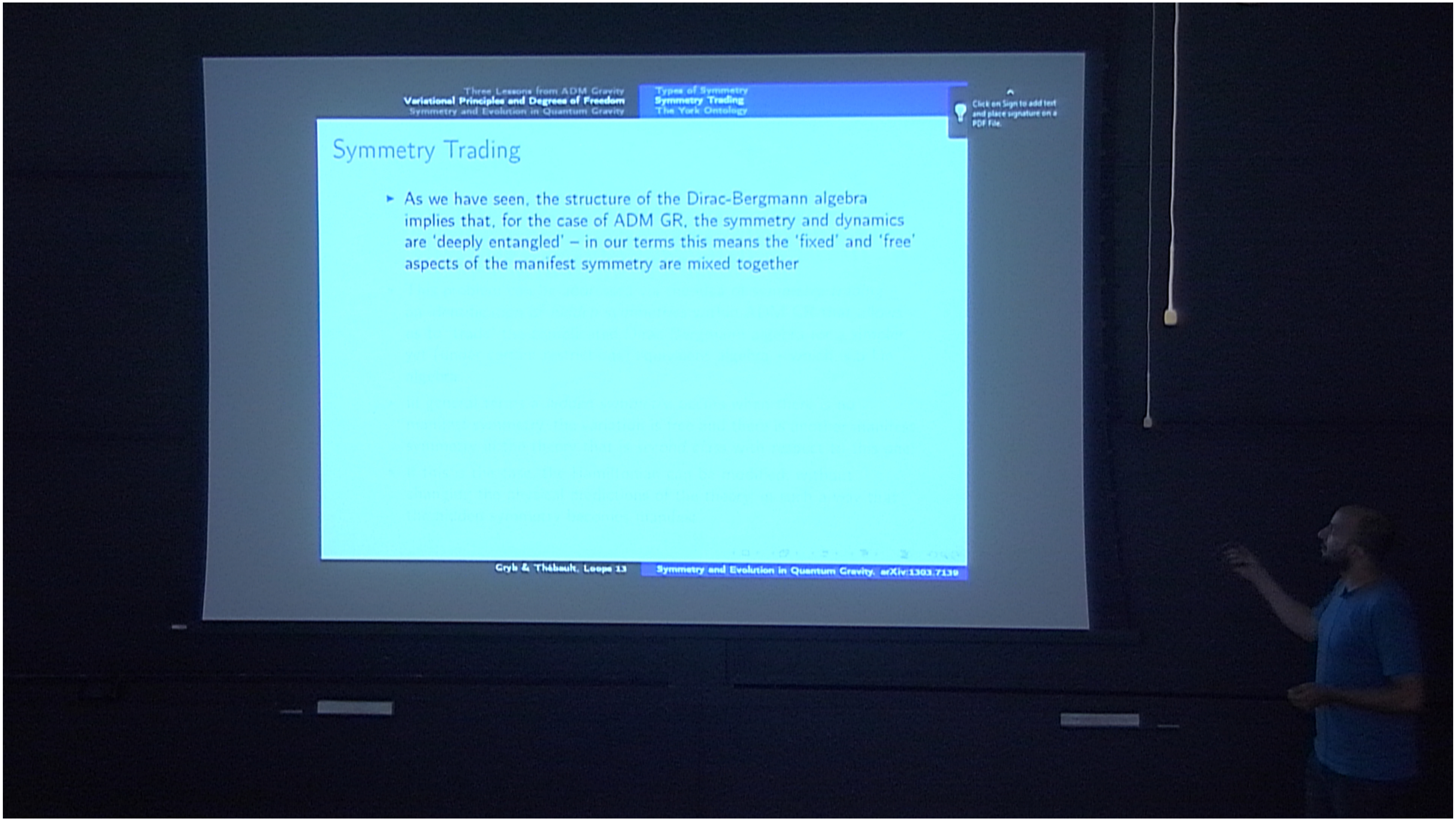
Free and Fixed Variations
If the variational principle is one in which no conditions are imposed on a degree of freedom we say it is a free variation, otherwise we say it is a fixed variation
Here we are considering variation of the action based upon the variation of a state-space curve in a direction associated with a particular degree of freedom
Our focus is upon variation of the end points of the curve rather than a global variation. Such end-point-variation corresponds (roughly speaking) to considering families of slightly adjusted variational principles – defined by action principles with infinitesimally different boundary conditions.
Manifest symmetries associated with fixed variations are conservation symmetries. Manifest symmetries associated with free variations are gauge symmetries. There are key interpretational differences between the two.
Gryb & Thébault, Loops 13
Symmetry and Evolution in Quantum Gravity. arXiv:1303.7139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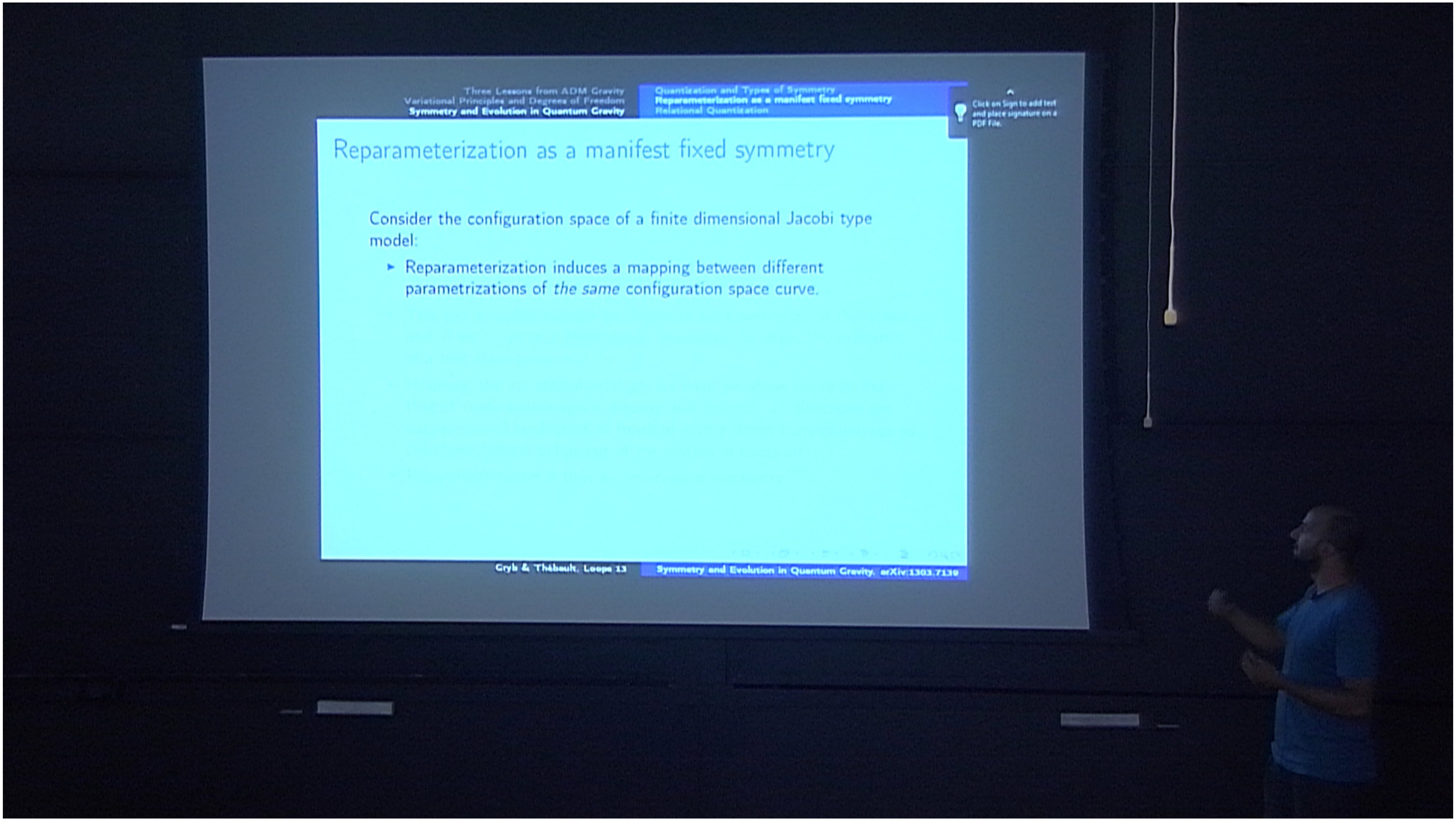
As we have seen, the structure of the Dirac-Bergmann algebra implies that, for the case of ADM GR, the symmetry and dynamics are 'deeply entangled' – in our terms this means the 'fixed' and 'free' aspects of the manifest symmetry are mixed together
Gryb & Thébault, Loops 13
Symmetry and Evolution in Quantum Gravity. arXiv:1303.7139

# Quantization and Types of Symmetry

- The physical basis of a symmetry is what should dictate its treatment within a faithful quantization procedure
- Since gauge symmetries result from surplus representational structure, the degrees of freedom to which they correspond must be eliminated at some stage in the construction of the physical Hilbert space
- Conservation symmetries on the other hand, reflect the existence of conserved charges and not redundant variables – quantization should preserve the number of degrees of freedom, and allow for superpositions of the relevant conserved charges.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Quantization and Types of Symmetry
Reparameterization as a manifest fixed symmetry
Relational Quantization
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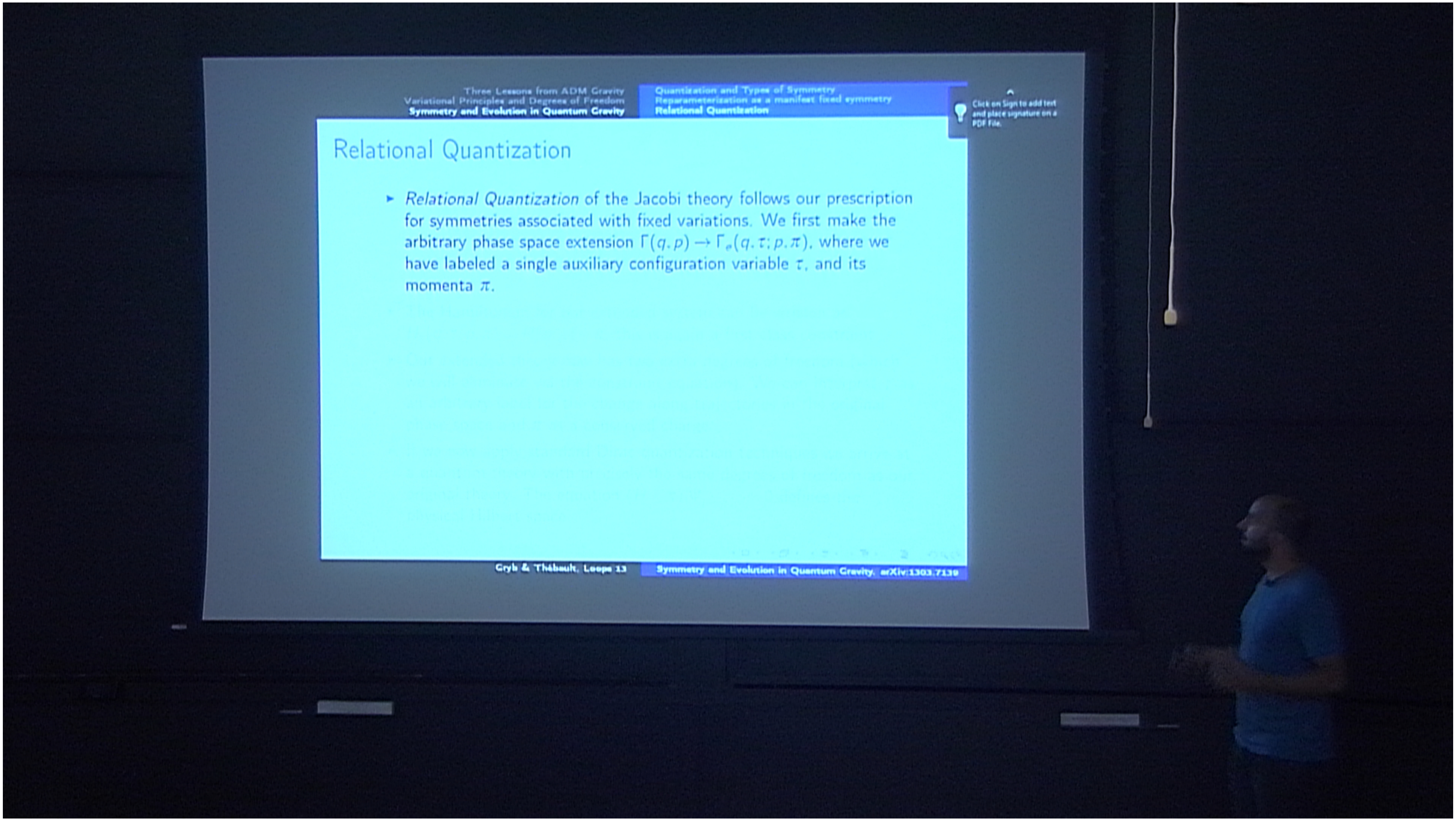
Reparameterization as a manifest fixed symmetry
Consider the configuration space of a finite dimensional Jacobi type model:
► Reparameterization induces a mapping between different parametrizations of *the same* configuration space curve.
Gryb & Thébault, Loops 13
Symmetry and Evolution in Quantum Gravity. arXiv:1303.7139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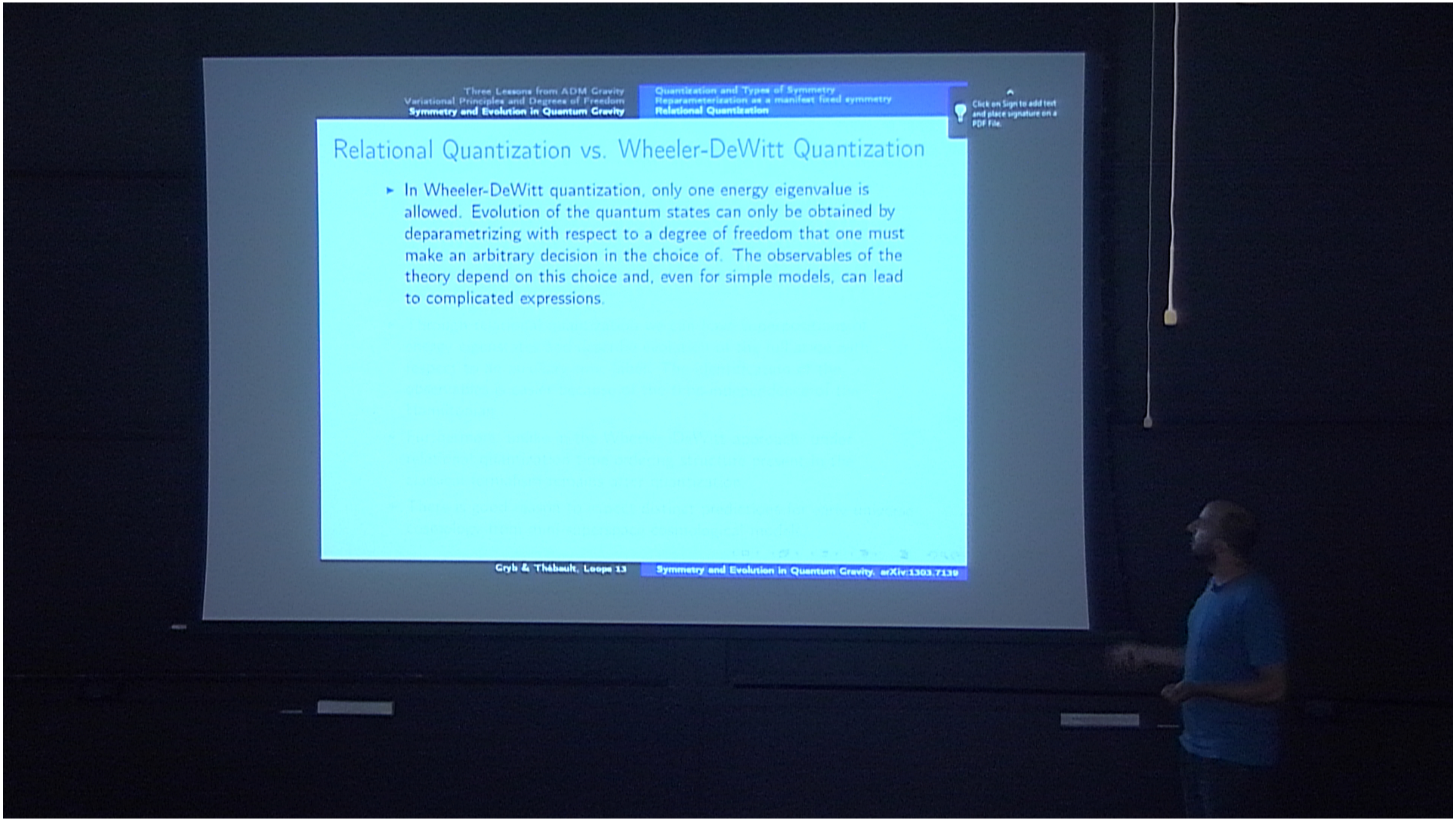
Relational Quantization
*Relational Quantization* of the Jacobi theory follows our prescription for symmetries associated with fixed variations. We first make the arbitrary phase space extension $\Gamma(q,p) \to \Gamma_e(q,\tau;p,\pi)$, where we have labeled a single auxiliary configuration variable $\tau$, and its momenta $\pi$.
Gryb & Thébault, Loops 13
Symmetry and Evolution in Quantum Gravity, arXiv:1303.7139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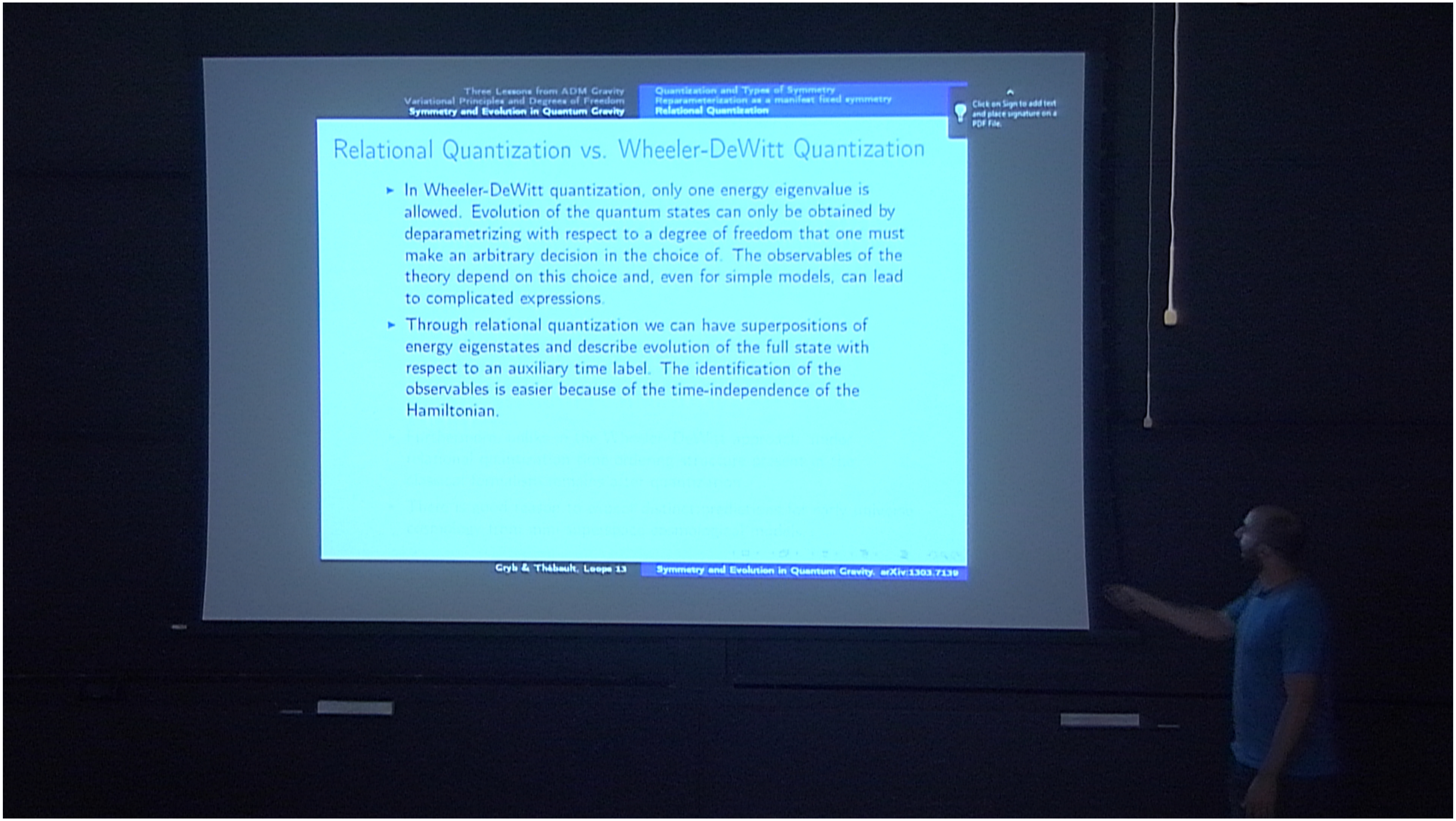
Relational Quantization vs. Wheeler-DeWitt Quantization
In Wheeler-DeWitt quantization, only one energy eigenvalue is allowed. Evolution of the quantum states can only be obtained by deparametrizing with respect to a degree of freedom that one must make an arbitrary decision in the choice of. The observables of the theory depend on this choice and, even for simple models, can lead to complicated expressions.
Gryb & Thébault, Loops 13
Symmetry and Evolution in Quantum Gravity. arXiv:1303.7139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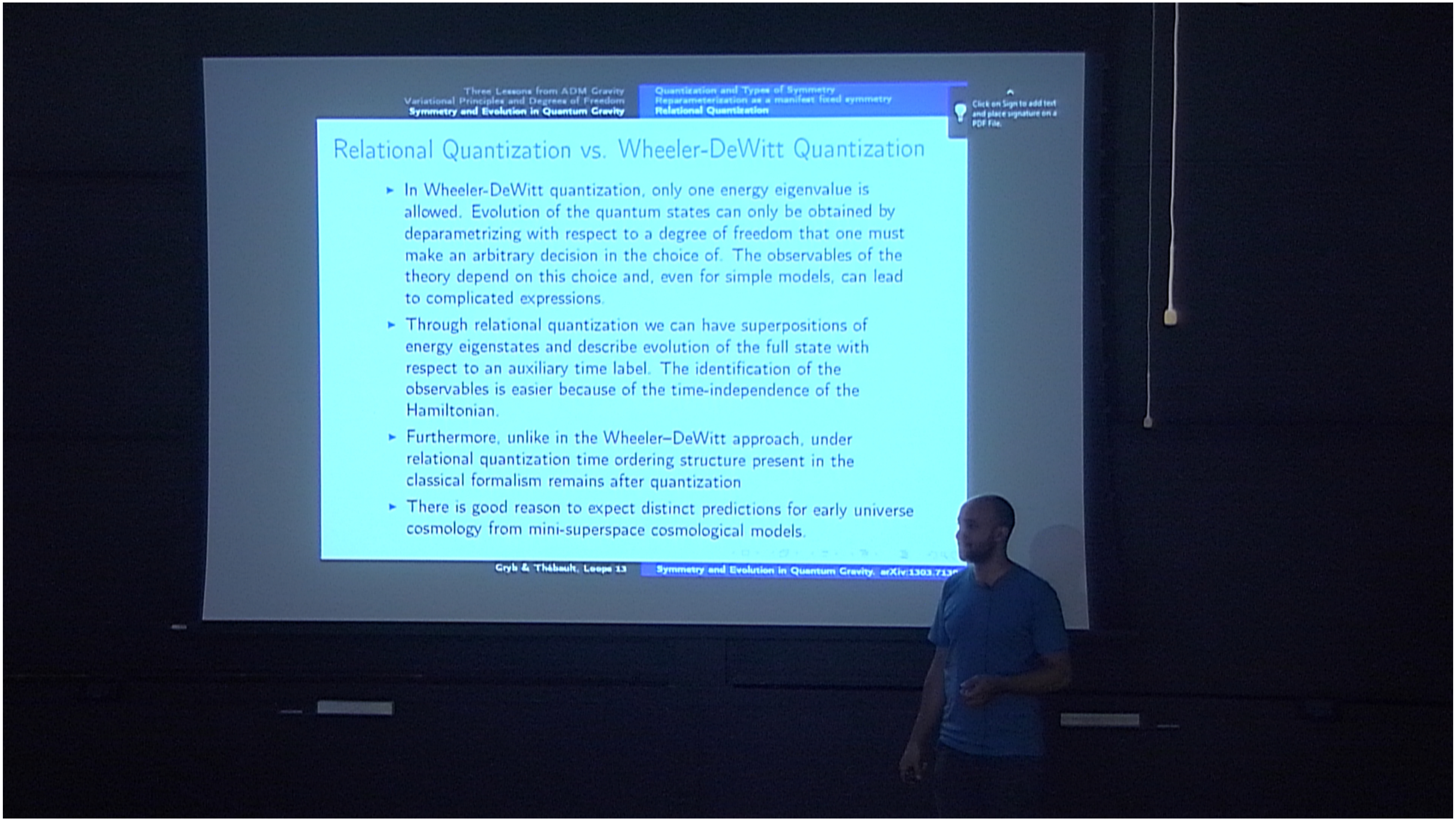
Through relational quantization we can have superpositions of energy eigenstates and describe evolution of the full state with respect to an auxiliary time label. The identification of the observables is easier because of the time-independence of the Hamiltonian.
Gryb & Thébault, Loops 13
Symmetry and Evolution in Quantum Gravity, arXiv:1303.7139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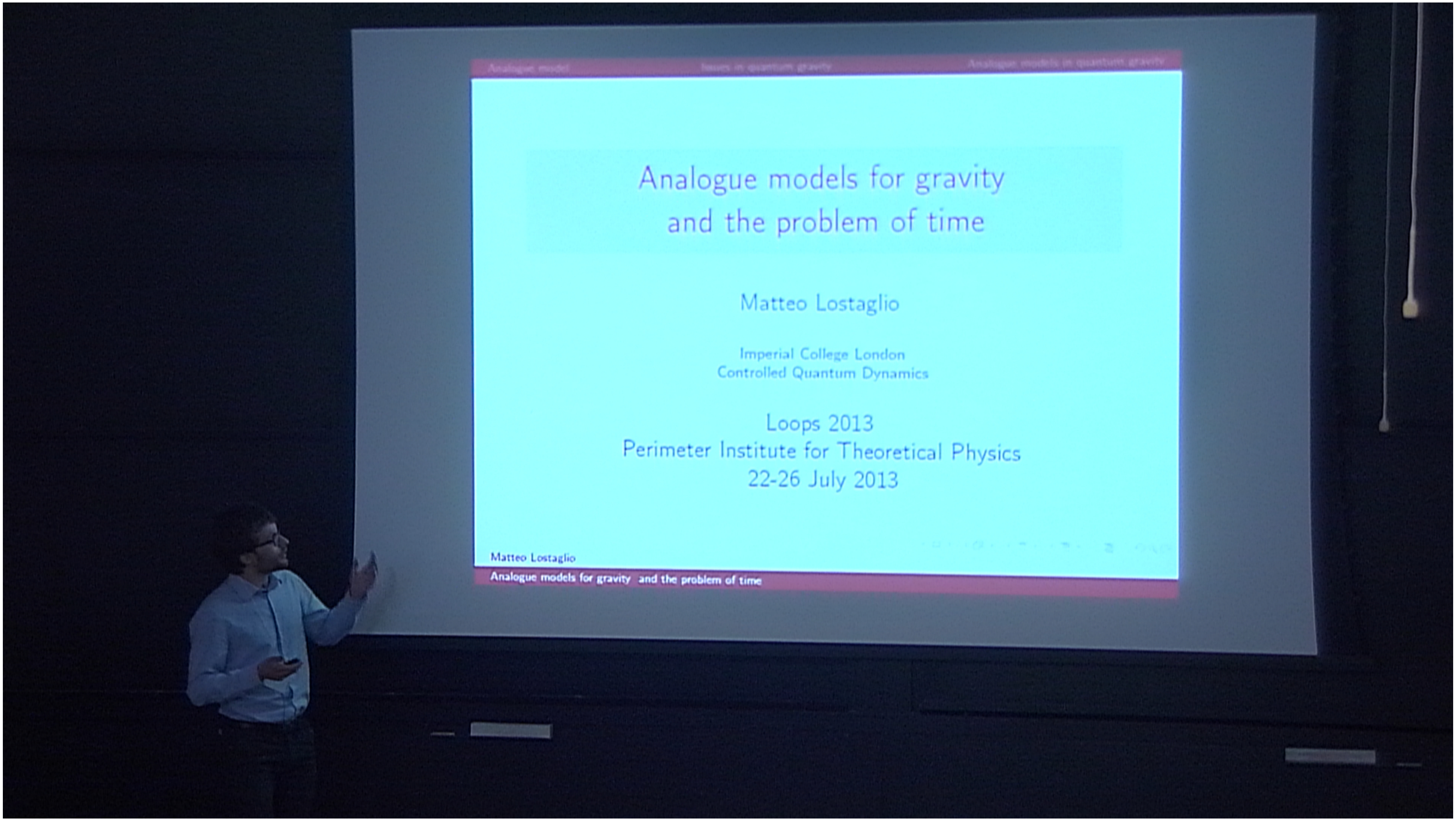
Furthermore, unlike in the Wheeler–DeWitt approach, under relational quantization time ordering structure present in the classical formalism remains after quantization
There is good reason to expect distinct predictions for early universe cosmology from mini-superspace cosmological models.
Gryb & Thébault, Loops 13
Symmetry and Evolution in Quantum Gravity.


EXIT


Analogue models for gravity
and the problem of time
Matteo Lostaglio
Imperial College London
Controlled Quantum Dynamics
Loops 2013
Perimeter Institute for Theoretical Physics
22-26 July 2013
Matteo Lostaglio
Analogue models for gravity and the problem of time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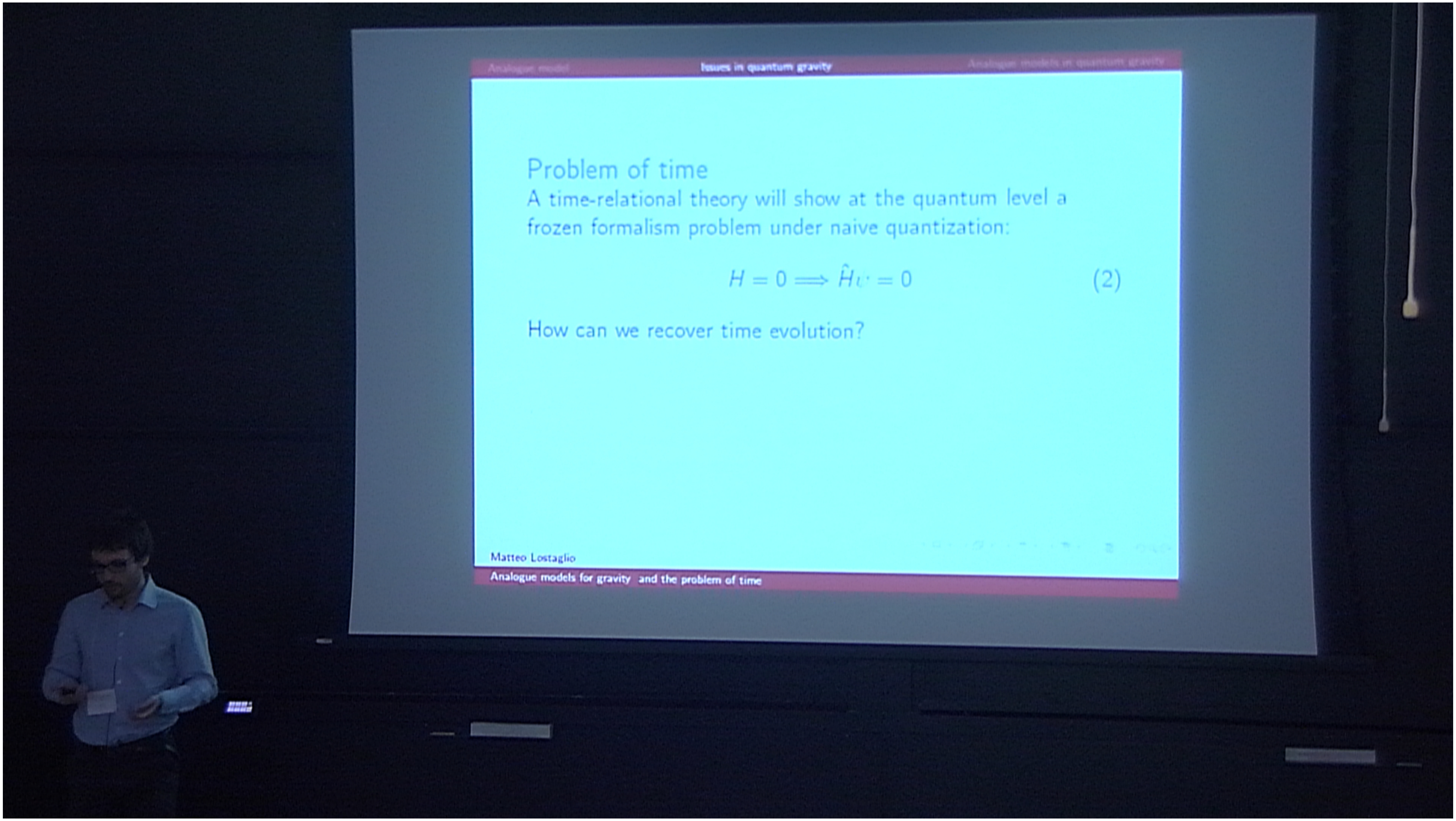
Issues in quantum gravity
Problem of time
A time-relational theory will show at the quantum level a frozen formalism problem under naive quantization:
$H = 0 \Longrightarrow \hat{H}\psi = 0$ (2)
How can we recover time evolution?
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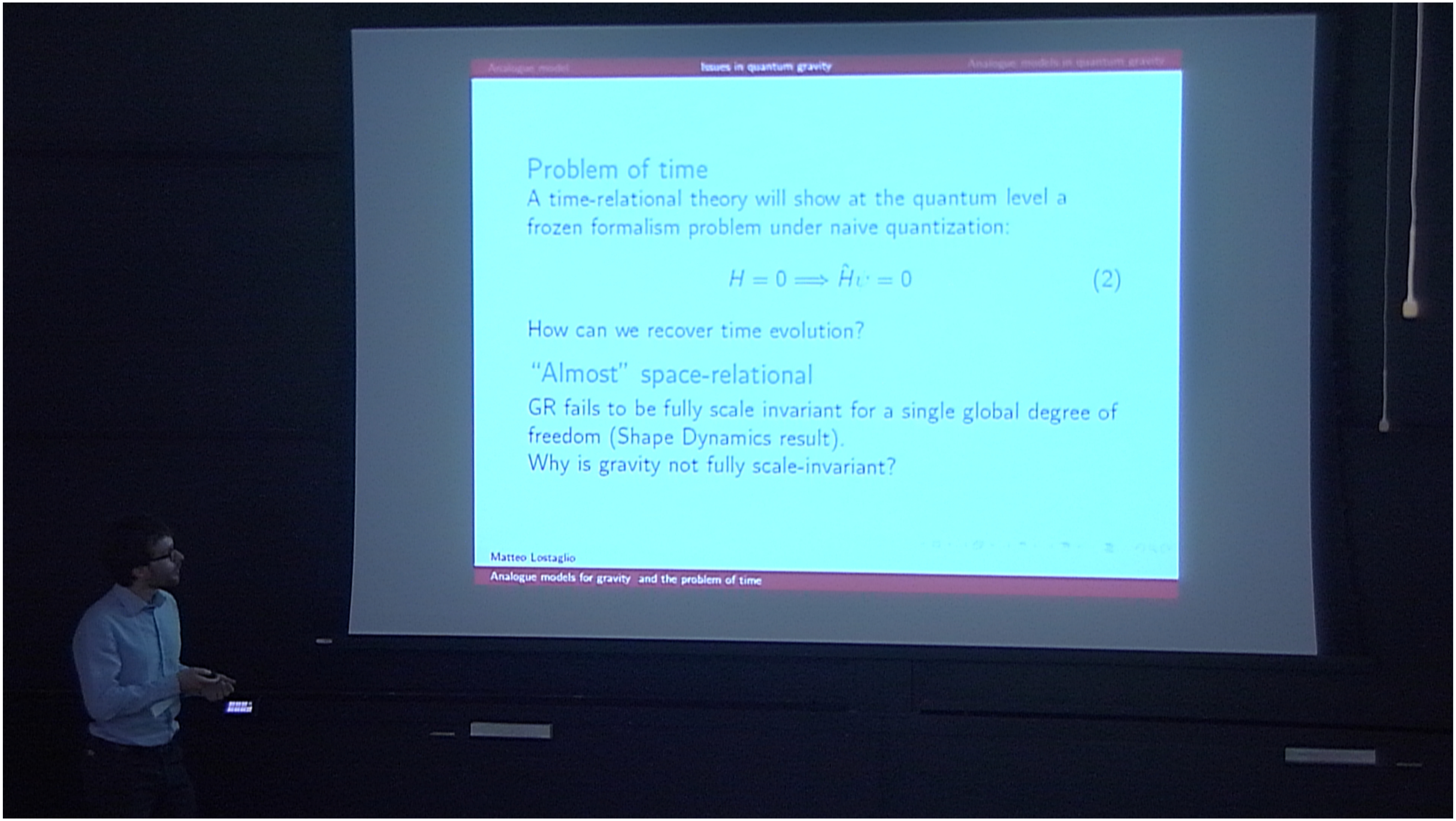
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How can we recover time evolution?
"Almost" space-relational
GR fails to be fully scale invariant for a single global degree of freedom (Shape Dynamics result).
Why is gravity not fully scale-invariant?
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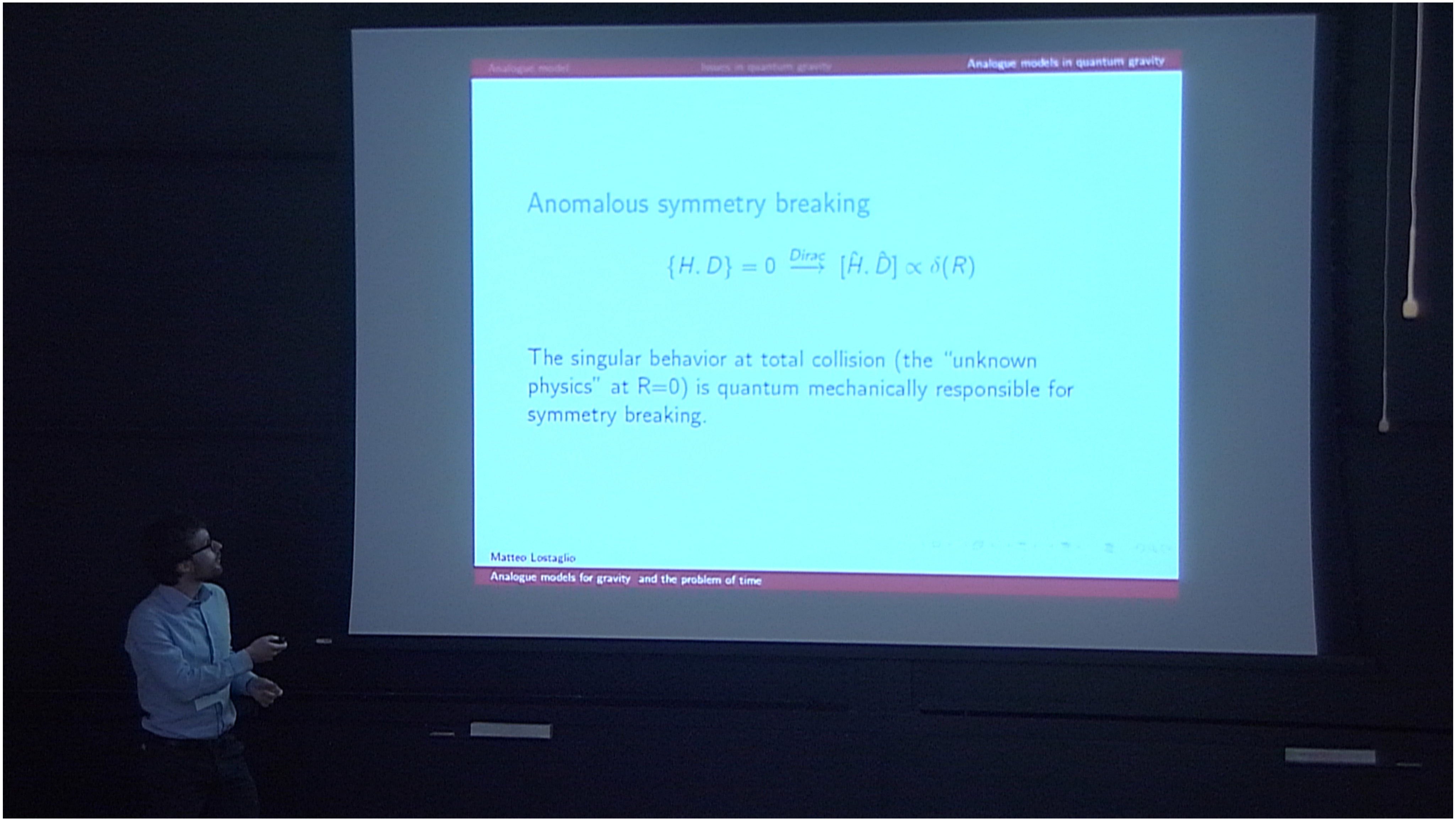
Analogue models for gravity and the problem of time

## The analogue model

The analogue model wants to capture the relational features of GR in the sense introduced above. So it is

1. Time-relational $\Rightarrow$ Hamiltonian constraint $H = 0$.
2. Space-relational $\Rightarrow$ No absolute space $P = 0$, $L = 0$ and scale invariant (invariant under global rescalings $x_i \to \lambda x_i, \ \lambda > 0$).


Analogue models in quantum gravity
Anomalous symmetry breaking
$\{H, D\} = 0 \xrightarrow{Dirac} [\hat{H}, \hat{D}] \propto \delta(R)$
The singular behavior at total collision (the "unknown physics" at R=0) is quantum mechanically responsible for symmetry breaking.
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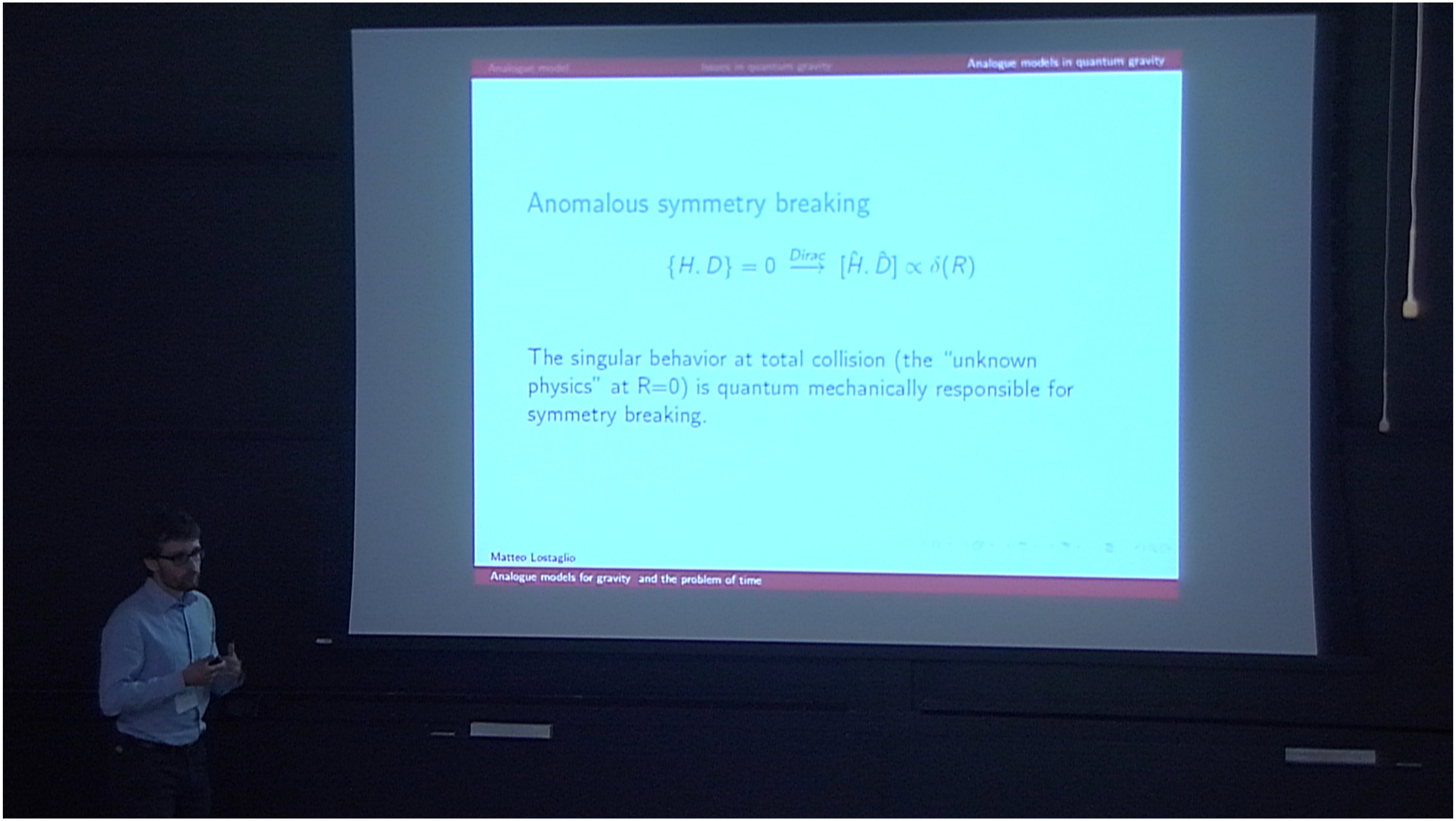
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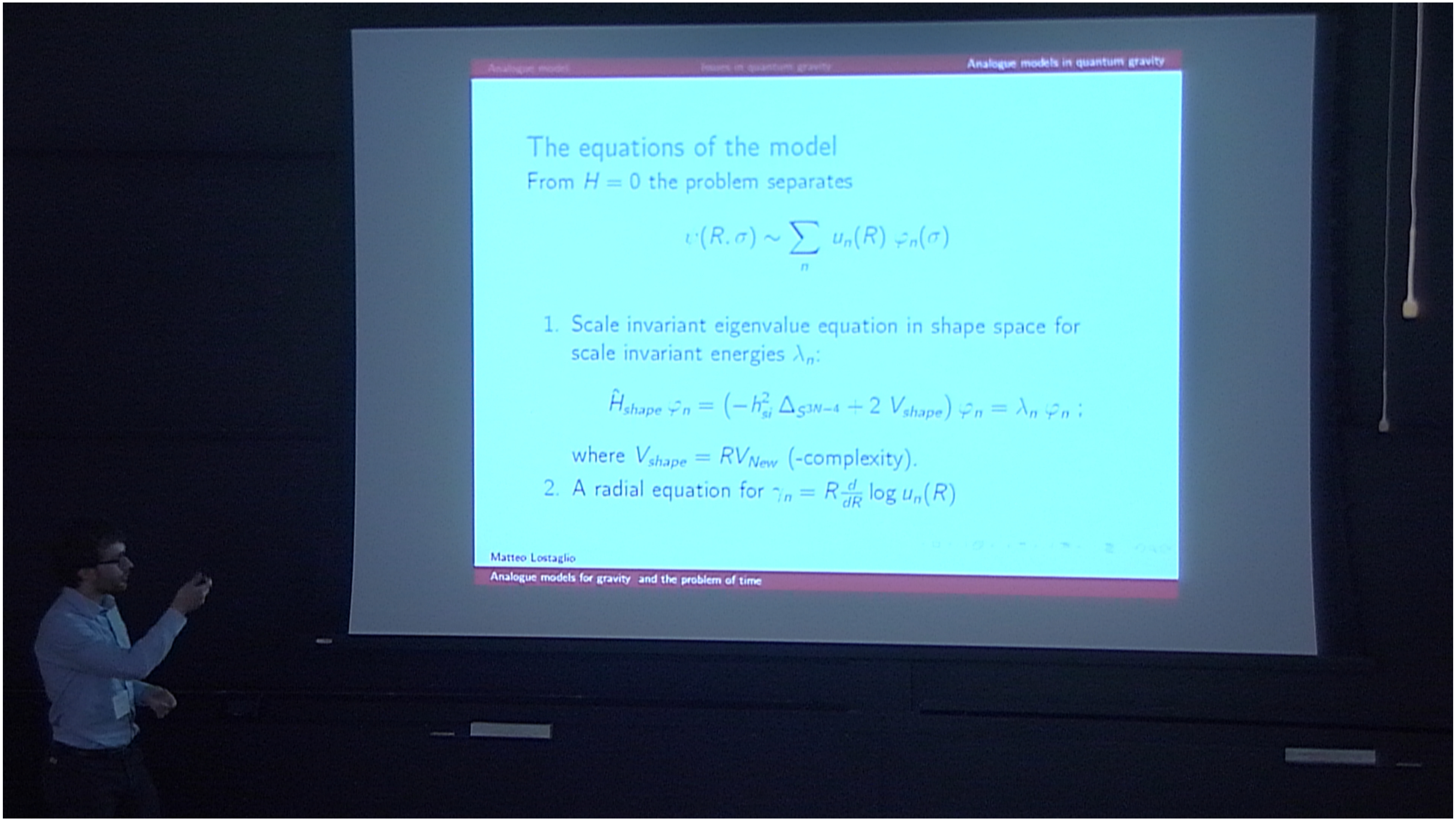
Analogue models in quantum gravity
The equations of the model
From $H = 0$ the problem separates
$\psi(R, \sigma) \sim \sum_n u_n(R)\, \varphi_n(\sigma)$
1. Scale invariant eigenvalue equation in shape space for scale invariant energies $\lambda_n$:
$\hat{H}_{shape}\, \varphi_n = \left(-\hbar_{si}^2\, \Delta_{S^{3N-4}} + 2\, V_{shape}\right) \varphi_n = \lambda_n\, \varphi_n\, ;$
where $V_{shape} = RV_{New}$ (-complexity).
2. A radial equation for $\gamma_n = R \frac{d}{dR} \log u_n(R)$
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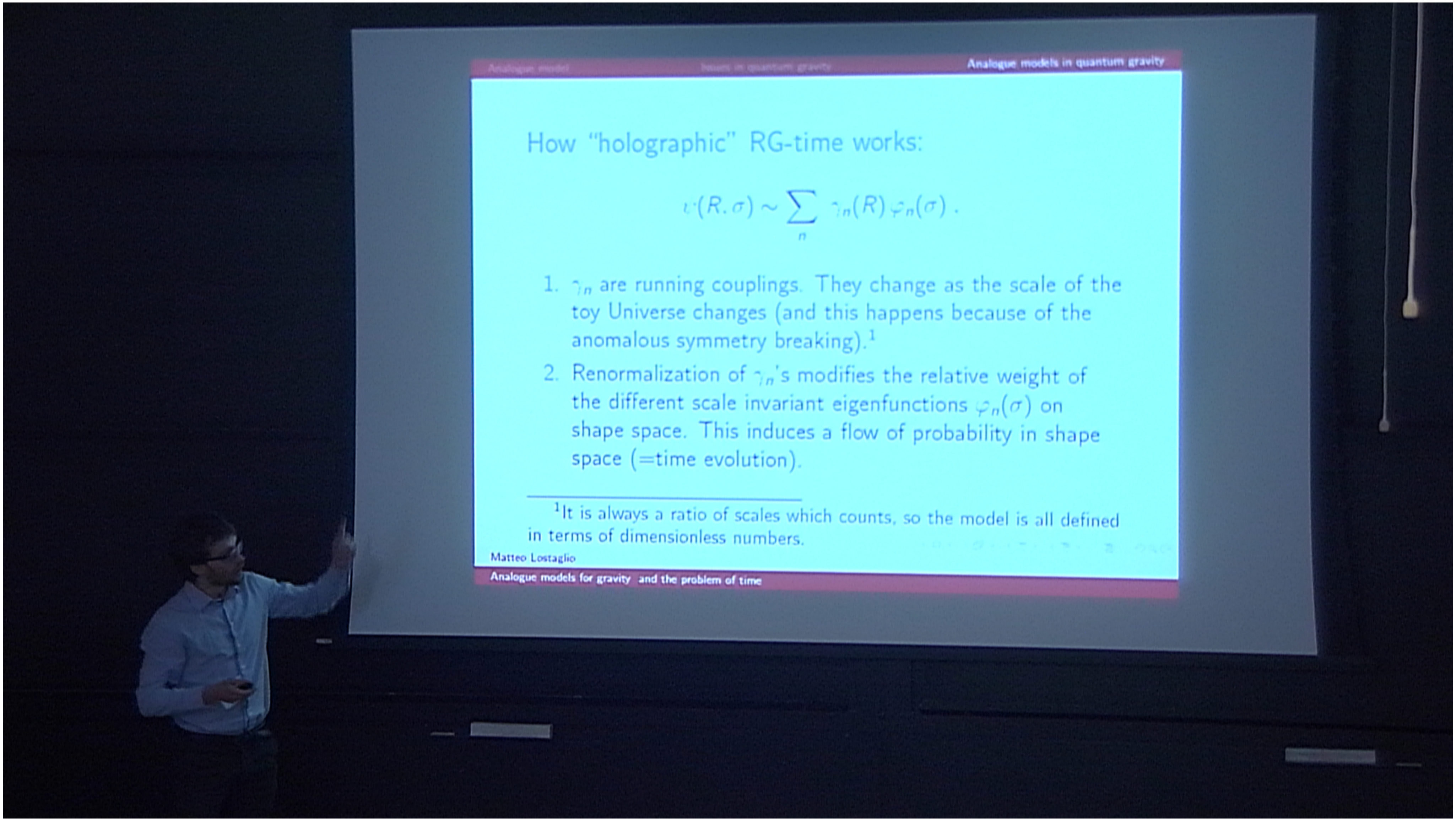
Analogue models in quantum gravity
How "holographic" RG-time works:
$\psi(R,\sigma) \sim \sum_n \gamma_n(R)\,\varphi_n(\sigma)$ .
1. $\gamma_n$ are running couplings. They change as the scale of the toy Universe changes (and this happens because of the anomalous symmetry breaking).[1]
2. Renormalization of $\gamma_n$'s modifies the relative weight of the different scale invariant eigenfunctions $\varphi_n(\sigma)$ on shape space. This induces a flow of probability in shape space (=time evolution).
[1] It is always a ratio of scales which counts, so the model is all defined in terms of dimensionless numbers.
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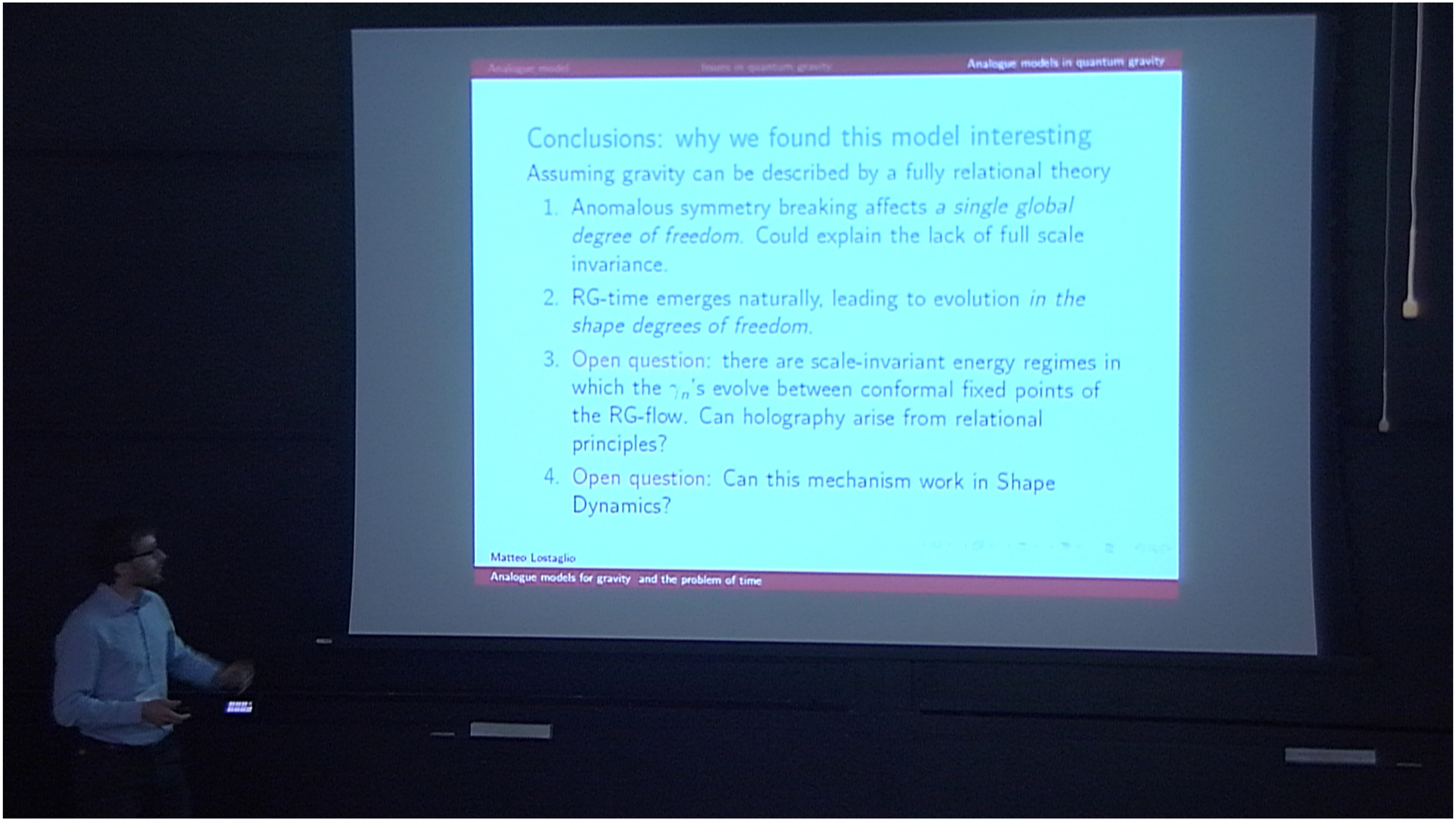
Analogue models in quantum gravity
Conclusions: why we found this model interesting
Assuming gravity can be described by a fully relational theory
1. Anomalous symmetry breaking affects *a single global degree of freedom*. Could explain the lack of full scale invariance.
2. RG-time emerges naturally, leading to evolution *in the shape degrees of freedom*.
3. Open question: there are scale-invariant energy regimes in which the $\gamma_n$'s evolve between conformal fixed points of the RG-flow. Can holography arise from relational principles?
4. Open question: Can this mechanism work in Shape Dynamics?
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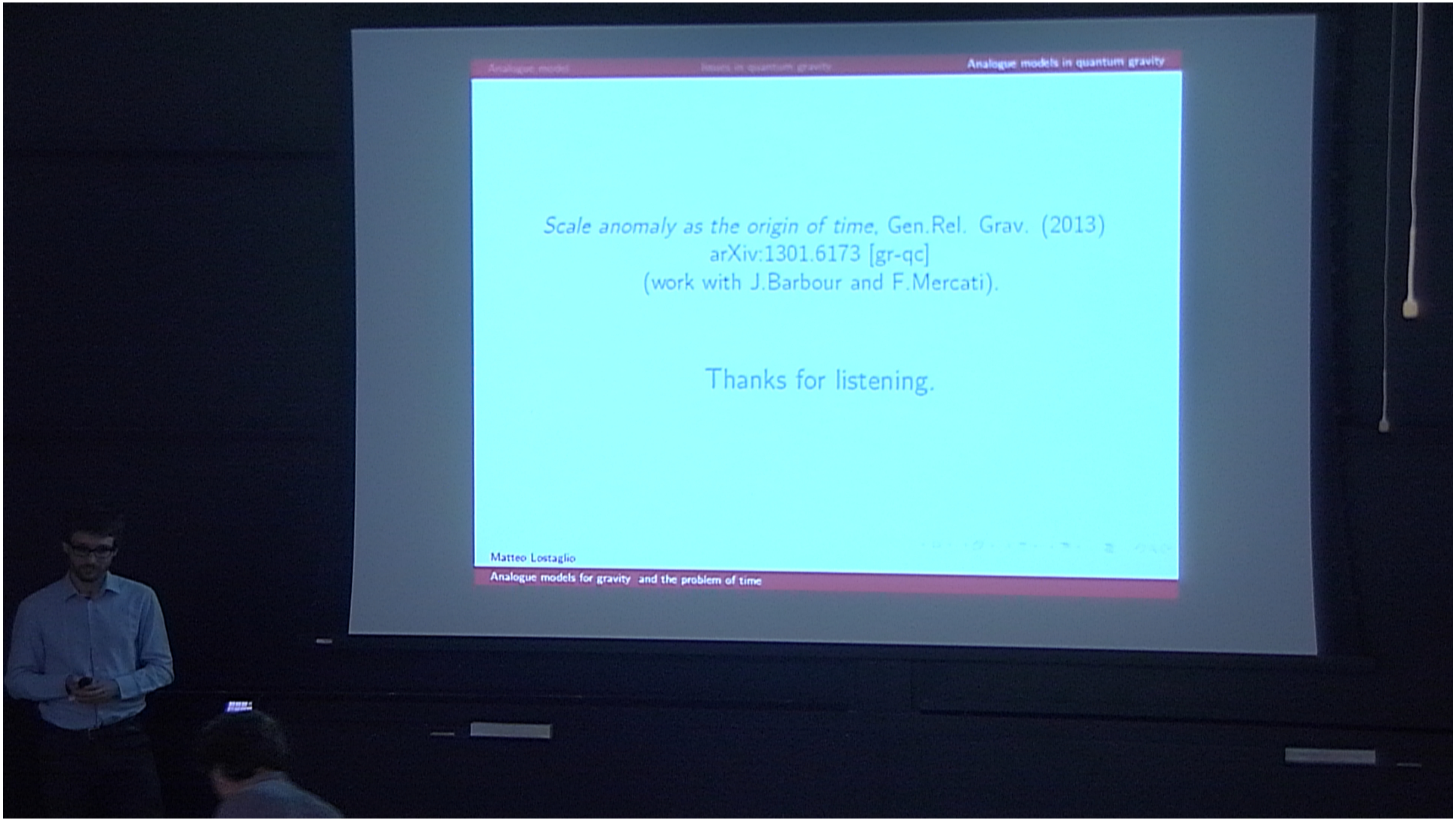
Analogue models in quantum gravity
Scale anomaly as the origin of time, Gen.Rel. Grav. (2013)
arXiv:1301.6173 [gr-qc]
(work with J.Barbour and F.Mercati).
Thanks for listening.
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Analogue models for gravity and the problem of time


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