Title: Some Possible Ways to Observe Consequences of Loop Quantum Gravity

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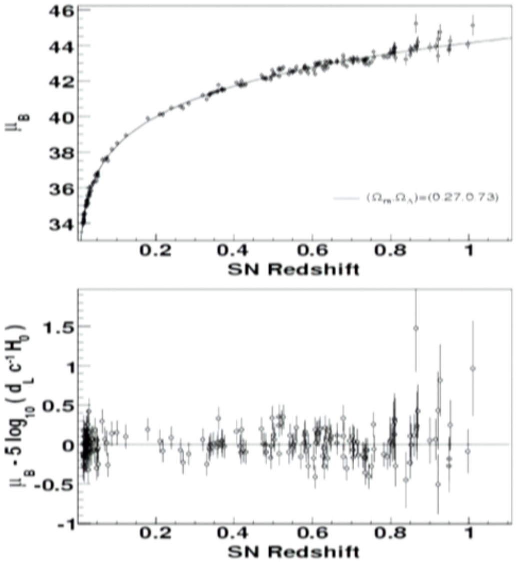
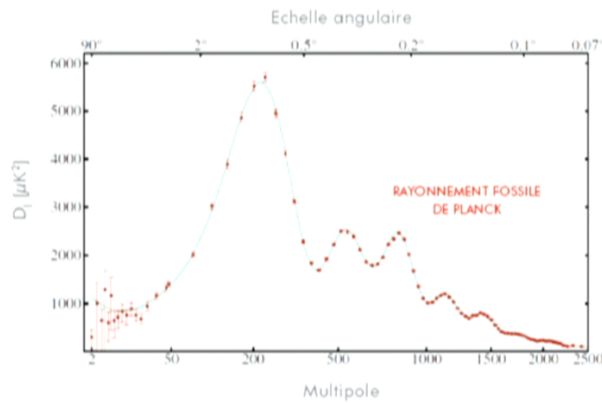
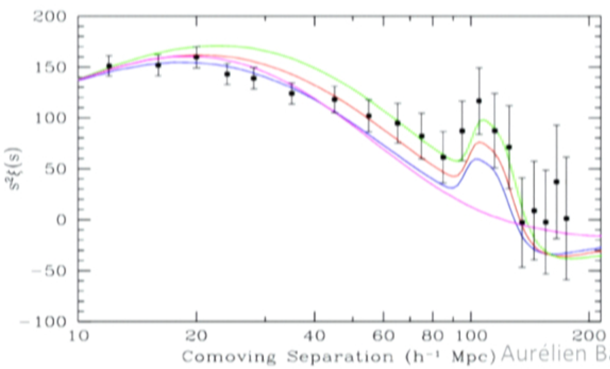
Abstract: In this talk, I'll briefly review some possible observational consequences of loop quantum gravity. I will first address the issue of the closure of the algebra of constraints in holonomy-corrected effective loop quantum cosmology for tensor, vector, and scalar modes. I will underline some unexpected features like a possible change of signature. The associated primordial power spectrum and the basics of the related CMB analysis will be presented. The "asymptotic silence" hypothesis will be mentioned as a promising alternative. Then, I'll address the issue of the probability for inflation and the prediction of its duration from a new perspective. Finally, I'll present some prospect about the evaporation of black holes in LQG.

0. Testing "beyond GR" models. Things are easy in the IR







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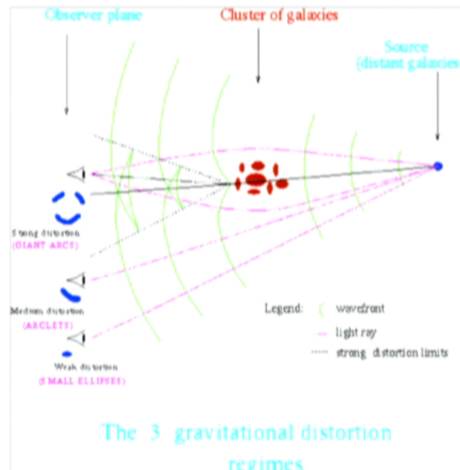


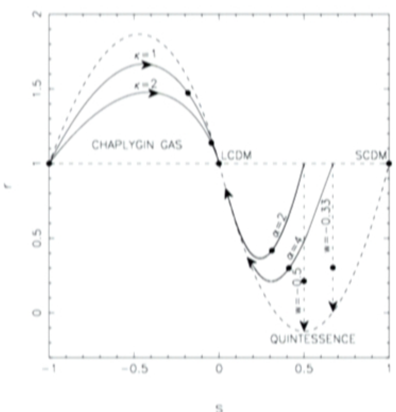
$$\Lambda / 8\pi G \sim 10^{-47}\, GeV^4$$

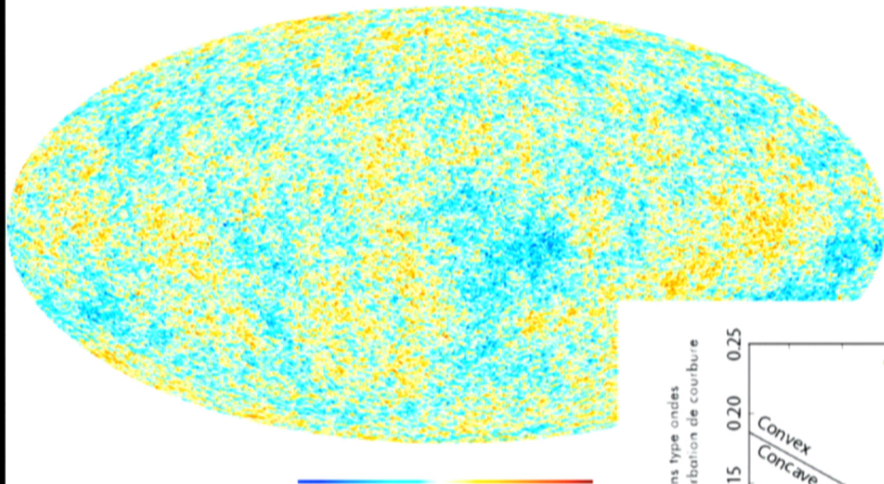
$$H^2 = \frac{8\pi G}{3}\left(\sum_a \rho_a + \rho_{DE}\right) - \frac{k}{a^2}\ ,$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\sum_a (\rho_a + 3p_a) + \rho_{DE} + 3p_{DE}\right)$$

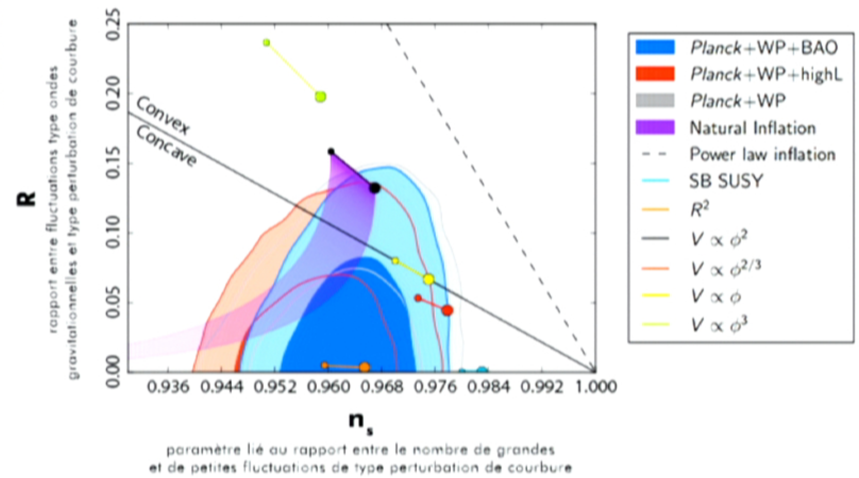
$$a(t) = a(t_0) + \dot{a}\big|_0 (t - t_0) + \frac{\ddot{a}\big|_0}{2}(t - t_0)^2 + \frac{\dddot{a}\ \big|_0}{6}(t - t_0)^3 + \ldots\ .$$

Level	Geometrical Parameter	Physical Parameter
1	$H(z) \equiv \frac{\dot{a}}{a}$	$\rho_m(z) = \rho_{0m}(1+z)^3$, $\rho_{\rm DE} = \frac{3H^2}{8\pi G} - \rho_m$
2	$q(z) \equiv -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{d \log H}{d \log (1+z)}$ $q(z)\Big\vert_{\Lambda\rm CDM} = -1 + \frac{3}{2}\Omega_m(z)$	$V(z)$, $T(z) \equiv \frac{\dot{\phi}^2}{2}$, $w(z) = \frac{T-V}{T+V}$, $\Omega_V = \frac{8\pi G V}{3H^2}$, $\Omega_T = \frac{8\pi G T}{3H^2}$
3	$r(z) \equiv \frac{\dddot{a}\, a^2}{\dot{a}^3}$, $s \equiv \frac{r-1}{3(q-1/2)}$ $\{r,s\}\Big\vert_{\Lambda\rm CDM} = \{1,0\}$	$\Pi(z) \equiv \dot{V} = \dot{\phi}V'$, $\Omega_\Pi = \frac{8\pi G \dot{V}}{3H^3}$





Things are difficult in the UV. But, in the Planck era…



Reverse engineer's Friedmann's equations

$$\phi(t) = \sqrt{2} M_p \int \sqrt{-\dot{H}} \qquad V(t) = M_p^2 (3H^2 + \dot{H})$$

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Line of thought chosen here

At this stage, testing LQG is extremely difficult.

The points raised by I. Agullo, A. Ashtekar, and W. Nelson are of course important and their approach is very exciting.

My « cosmologist » view is that it is now mandatory to explore all the possible paths. It is too early to know which one will lead to really observable effects.

In the following I will *not* choose a specific scheme or require full consistency with the mother theory. I will *not* choose between different schools of thought within the LQG community. I will *not* pick-up a single effective theory.

Instead, I will try to explore, as many probes as possible (studied in Grenoble since the Madrid Loop conference). Finding one which could be observed is already so difficult. Many will be in the spirit of deformed algebra studied in particular by M. Bojowald and G. Pailly.

I. Reminder on first attemps

There are 2 main corrections in effective LQC :
- the holonomy correction
- the inverse-volume correction

We have first investigated how they modify the propagation of GW in a standard inflationnary background

Example of effective potential :

I.V.

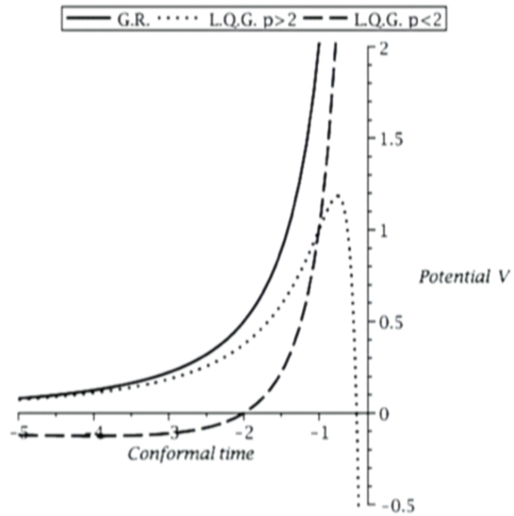
$$E_k(\eta) = \left[1 + 2\lambda\left(\frac{\ell_{\rm Pl}}{\ell_0}\right)^{\kappa}|\eta|^{\kappa(1+\epsilon)}\right]k^2,$$

$$V(\eta) = \frac{2+3\epsilon}{\eta^2} + \lambda\kappa(1+2\epsilon)\left(\frac{\ell_{\rm Pl}}{\ell_0}\right)^{\kappa}|\eta|^{\kappa(1+\epsilon)-2},$$

Holo

$$E_k(\eta) = k^2,$$

$$V(\eta) = \frac{2+3\epsilon}{\eta^2} - 2\sqrt{\pi}\gamma^3(1+4\epsilon)\left(\frac{\ell_{\rm Pl}}{\ell_0}\right)^2|\eta|^{2\epsilon-2}.$$



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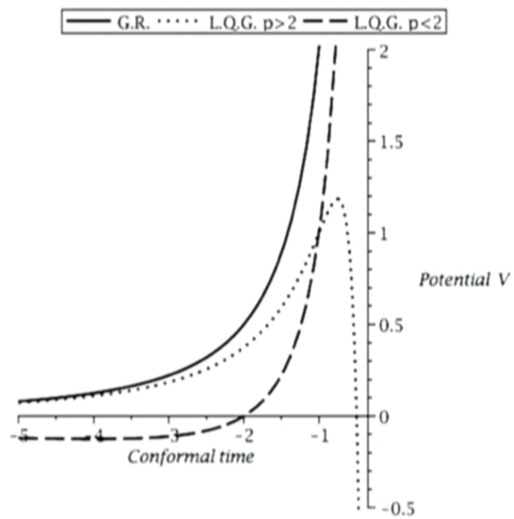

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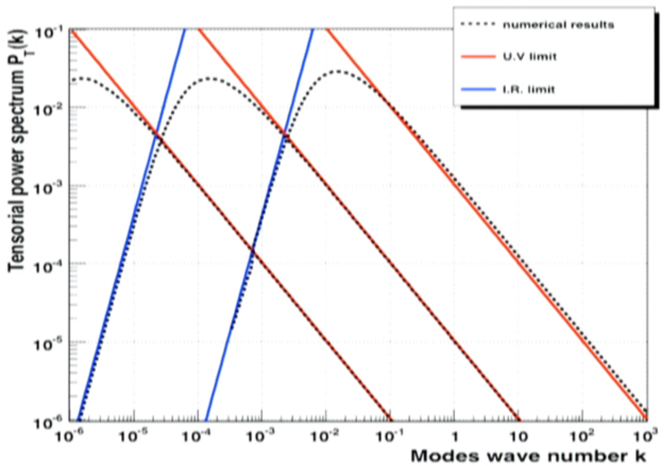
Holo



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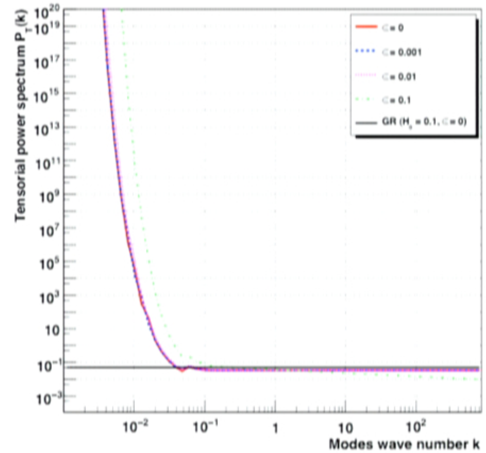


Holo+I.V.

$$P_T^{IR}(k) = 16\pi^3 \left(\frac{l_{PL}}{l_0}\right)^2 (Z(1-4\omega))^{-\frac{3}{2}} k^3 e^{\pi\sqrt{\frac{Z}{8}}\frac{(1-4\omega)}{k}}$$

$$H_{\mathrm{G}}^{\mathrm{Phen}}[N] = \frac{1}{2\kappa}\int_\Sigma \mathrm{d}^3x \bar{N}\alpha\left[-6\sqrt{\bar{p}}\left(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma}\right)^2 - \frac{1}{2\bar{p}^{3/2}}\left(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma}\right)^2(\delta E^c_j\delta E^d_k\delta^k_c\delta^j_d)\right.$$
$$\left.+\sqrt{\bar{p}}(\delta K^j_c\delta K^k_d\delta^c_k\delta^d_j) - \frac{2}{\sqrt{\bar{p}}}\left(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma}\right)(\delta E^c_j\delta K^j_c) - \frac{1}{\bar{p}^{3/2}}(\delta_{cd}\delta^{jk}E^c_j\delta^{ef}\partial_e\partial_f E^d_k)\right]$$

$$H_{matter}[\bar{N}] = \int_\Sigma d^3x\left(\frac{1}{2}D(q)\frac{p_\Phi^2}{\bar{p}^{\frac{3}{2}}} + \bar{p}^{\frac{3}{2}}V(\Phi)\right).$$

J. Grain & A.B., Phys. Rev. Lett. 102, 081301, 2009

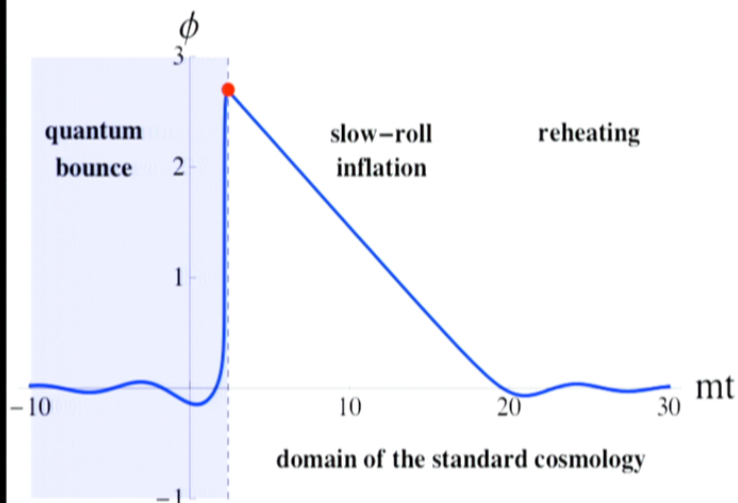
J. Grain, T. Cailleteau, A.B., A. Gorecki, Phys. Rev. D. , 2009

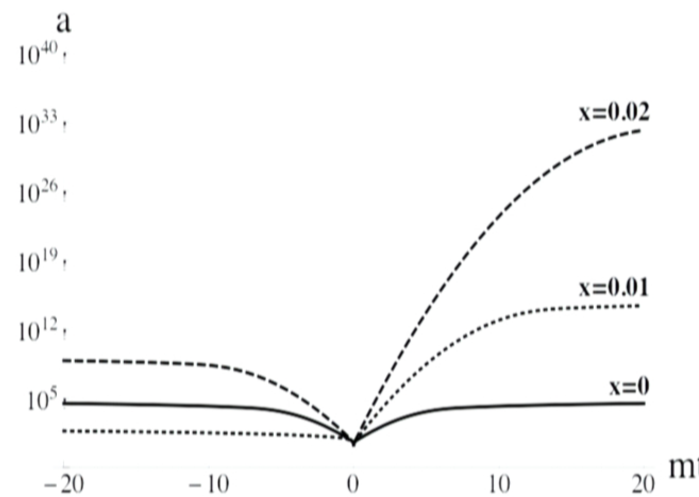
J. Grain, A.B., A. Gorecki, Phys. Rev. D , 79, 084015, 2009

With holonomy corrections for the background also

H changes sign in the KG equation ϕ’’+3Hϕ’+m2ϕ=0

→ Inflation occurs !





Mielczarek, Cailleteau, Grain, A.B., Phys. Rev. D, 81, 104049, 2010

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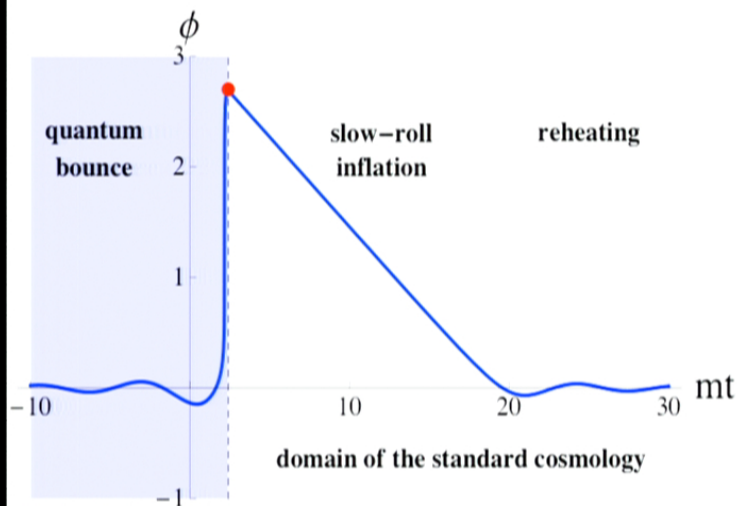

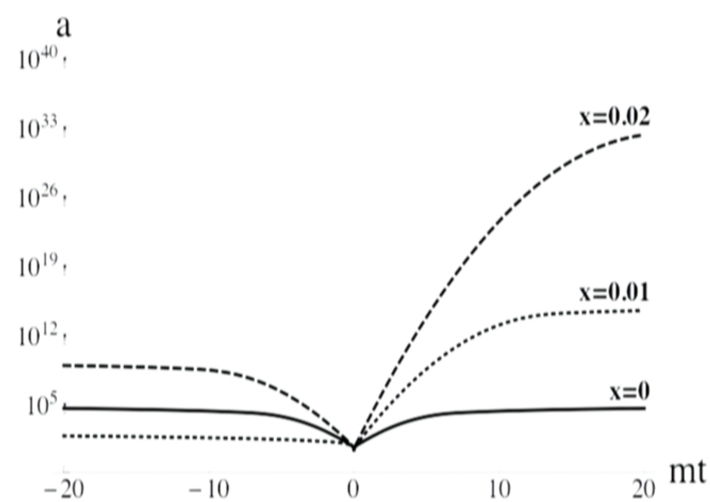

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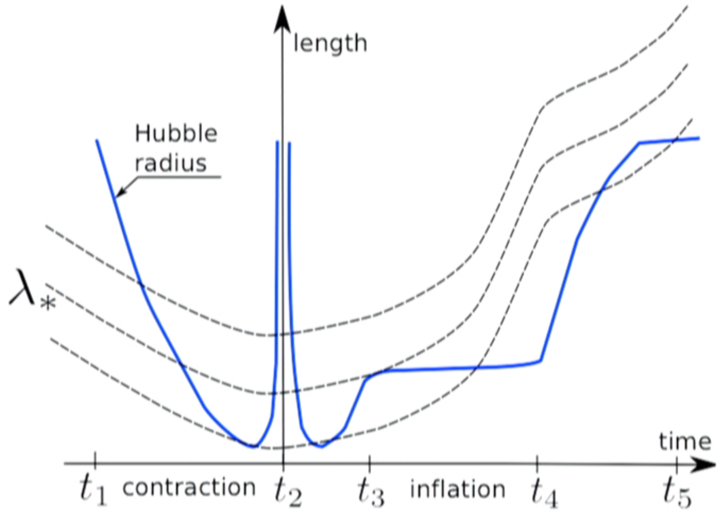
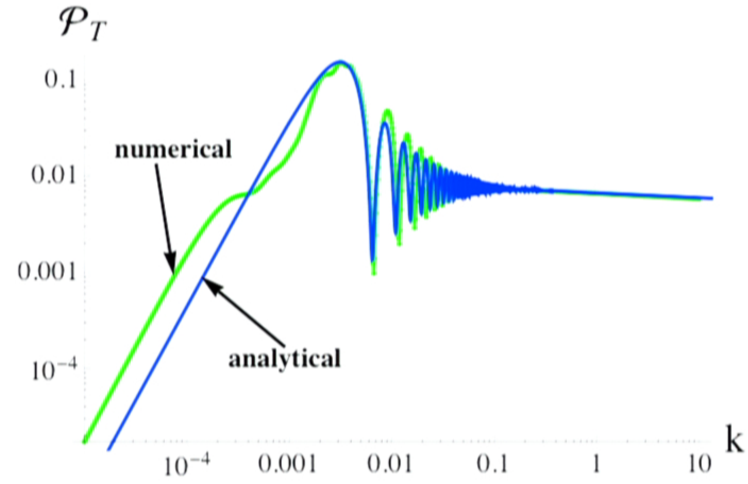
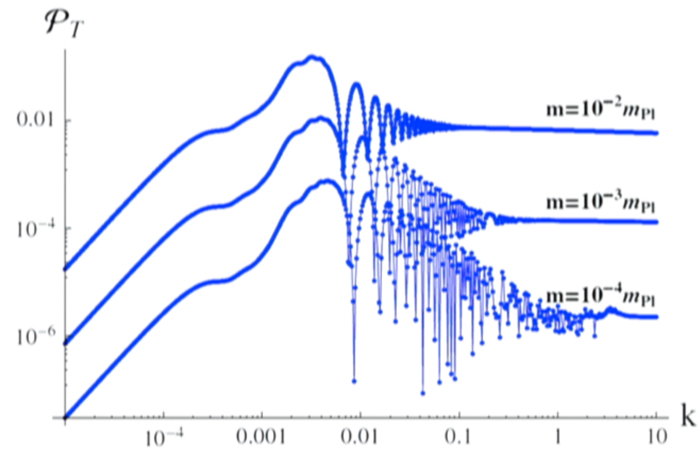


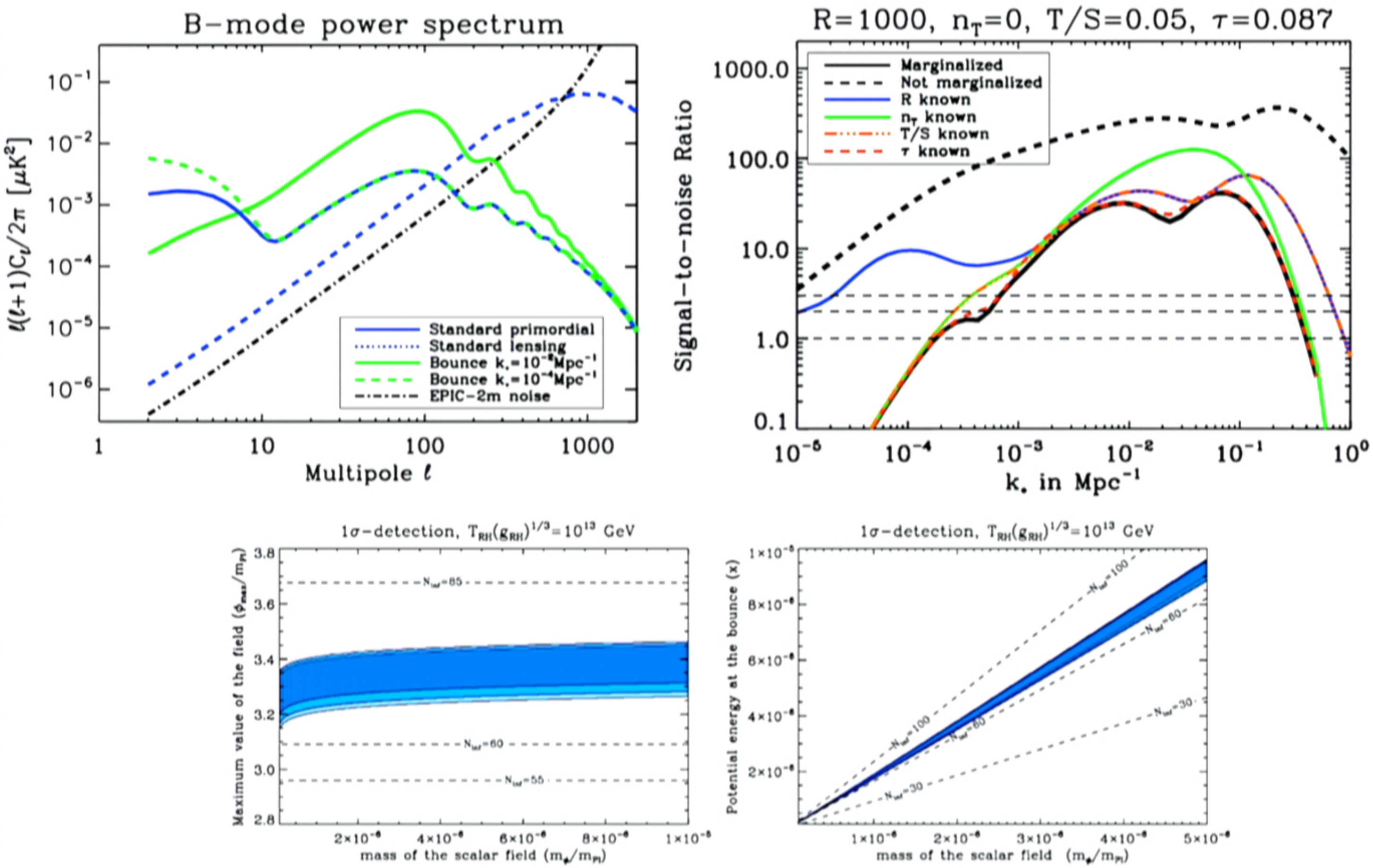


-Critical scale position depends on the fraction of potential energy at the bounce.

-The amplitude of the bump depends on the mass



Fisher analysis



Grain, A.B., Cailleteau, Mielczarek, Phys. Rev. D, 82, 123520, 2010

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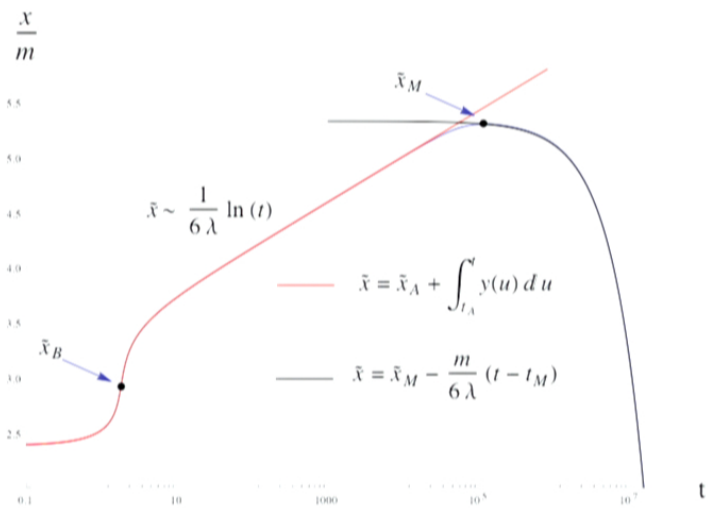
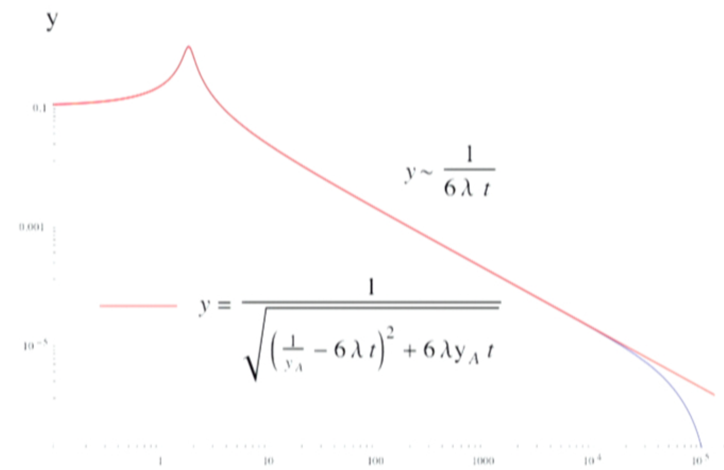
talk by Julien Grain at this conference

Analytical approximations





Poster by Boris Bolliet at this conference

$$x \equiv \frac{m\phi}{\sqrt{2\rho_c}}; y = \frac{\dot{\phi}}{\sqrt{2\rho_C}}$$

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II. Algebra closure issue

As soon as one wants to deal with other degrees of freedom, the anomalies must be taken into account.

Basically this means that the evolution generated by the constraints has to be « compatible » with the constraints themselves.

→ Strong requirement.

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Vector modes (holonomy corrections)

Smeared constraints for GR :

$$\begin{aligned}\mathcal{C}_1 &= G[N^i] = \frac{1}{2\kappa}\int_\Sigma d^3x\ N^i C_i,\\ \mathcal{C}_2 &= D[N^a] = \frac{1}{2\kappa}\int_\Sigma d^3x\ N^a C_a,\\ \mathcal{C}_3 &= S[N] = \frac{1}{2\kappa}\int_\Sigma d^3x\ NC,\end{aligned}$$

$$\left\{G[N^i]+D[N^a]+S[N], G[M^i]+D[M^a]+S[M]\right\}\approx 0.$$

$$\{\mathcal{C}_I,\mathcal{C}_J\} = f^K{}_{IJ}(A^j_b, E^a_i)\mathcal{C}_K.$$

← First class algebra. However, when going to the quantum version anomalies usually appear.

$$\{\mathcal{C}^Q_I,\mathcal{C}^Q_J\} = f^K{}_{IJ}(A^j_b, E^a_i)\mathcal{C}^Q_K + \mathcal{A}_{IJ}.$$

Vector modes (holonomy corrections)

Smeared constraints for GR :

$$
\begin{aligned}
\mathcal{C}_1 &= G[N^i] = \frac{1}{2\kappa}\int_\Sigma d^3x\; N^i C_i, \\
\mathcal{C}_2 &= D[N^a] = \frac{1}{2\kappa}\int_\Sigma d^3x\; N^a C_a, \\
\mathcal{C}_3 &= S[N] = \frac{1}{2\kappa}\int_\Sigma d^3x\; N C,
\end{aligned}
$$

$$\left\{G[N^i] + D[N^a] + S[N], G[M^i] + D[M^a] + S[M]\right\} \approx 0.$$

$$\{\mathcal{C}_I, \mathcal{C}_J\} = f^K{}_{IJ}(A^j_b, E^a_i)\mathcal{C}_K .$$

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$$\{\mathcal{C}^Q_I, \mathcal{C}^Q_J\} = f^K{}_{IJ}(A^j_b, E^a_i)\mathcal{C}^Q_K + \mathcal{A}_{IJ}.$$

This issue is especially important when dealing with perturbations around the cosmological background.

$$A_a^i = \gamma \bar{k} \delta_a^i + \delta A_a^i \quad \text{and} \quad E_i^a = \bar{p} \delta_i^a + \delta E_i^a,$$

In bouncing cosmologies vector modes can be important.

Already derived for inverse-triad corrections and for holonomies up to fourth order in k. This is not enough to go through the bounce.

We follow the usual prescription but with an arbitrary n integer. We don't restrict a prioris the mu dependance upon p.

$$\bar{k} \rightarrow \frac{\sin(n\bar{\mu}\gamma\bar{k})}{n\bar{\mu}\gamma}$$

→ Defined as K[n]

One has to write down vector perturbations in the canonical formulation. Then the quantum holonomy corrected hamiltonian constraint

$$S^Q[N] = \frac{1}{2\kappa}\int_\Sigma d^3x \left[\bar{N}(C^{(0)} + C^{(2)})\right],$$

where

$$\begin{aligned} C^{(0)} &= -6\sqrt{\bar{p}}\,(\mathbb{K}[1])^2\,, \\ C^{(2)} &= -\frac{1}{2\bar{p}^{3/2}}\,(\mathbb{K}[1])^2\,(1+\alpha_1)(\delta E^c_j \delta E^d_k \delta^k_c \delta^j_d) \\ &+ \sqrt{\bar{p}}(\delta K^j_c \delta K^k_d \delta^c_k \delta^d_j) \\ &- \frac{2}{\sqrt{\bar{p}}}\,(\mathbb{K}[v_1])\,(1+\alpha_2)(\delta E^c_j \delta K^j_c). \end{aligned}$$

Quantum holonomy corrected diffeomorphism constraint.

$$\begin{aligned} D^Q[N^a] &= \frac{1}{\kappa}\int_\Sigma d^3x \delta N^c \left[-\bar{p}(\partial_k \delta K^k_c)\right. \\ &- \left.(\mathbb{K}[v_2])\,\delta^k_c(\partial_d \delta E^d_k)\right], \end{aligned}$$

In LQG, it keeps its standard expression.

In order to investigate the algebra of constraints, the Poisson brackets have to be calculated.

$$\{S^Q[N_1], S^Q[N_1]\} = 0, \quad (18)$$

$$\{D^Q[N_1^a], D^Q[N_2^a]\} = 0, \quad (19)$$

$$\begin{aligned}\{S^Q[N], D^Q[N^a]\} &= \frac{\bar{N}}{\sqrt{\bar{p}}}\mathcal{B}D^Q[N^a] \\ &+ \frac{\bar{N}}{\kappa\sqrt{\bar{p}}}\int_\Sigma d^3x\delta N^c\delta_c^k(\partial_d\delta E_k^d)\delta E_k^d\mathcal{A},\end{aligned}$$

$$\mathcal{B} := (1+\alpha_2)\mathbb{K}[v_1] + \mathbb{K}[v_2] - 2\mathbb{K}[2]$$

$$\begin{aligned}\mathcal{A}_1 &= \mathcal{B}\mathbb{K}[v_2], \\ \mathcal{A}_2 &= 2\mathbb{K}[2]\bar{p}\frac{\partial\mathbb{K}[v_2]}{\partial\bar{p}} - \frac{1}{2}(\mathbb{K}[1])^2\cos(v_2\bar{\mu}\gamma\bar{k}) \\ &- 2\mathbb{K}[1]\bar{p}\frac{\partial\mathbb{K}[1]}{\partial\bar{p}}\cos(v_2\bar{\mu}\gamma\bar{k}) \\ &+ (1+\alpha_2)\mathbb{K}[v_1]\mathbb{K}[v_2] - \frac{1}{2}\mathbb{K}[1]^2(1+\alpha_1).\end{aligned}$$

In order to investigate the algebra of constrains, the Poisson brackets have to be calculated.

1) **Without counterterms. One is led to a (Pell-type) Diophantine equations in v1 and v2. Infinite number of solutions, but up to 4th order only.**

2) **General case. A=0 →**

$$\alpha_1 = -1 + 4(1+\alpha_2)\frac{\mathbb{K}[v_1]\mathbb{K}[v_2]}{\mathbb{K}[1]^2} - 4(1+\beta)\frac{\mathbb{K}[2]\mathbb{K}[v_2]}{\mathbb{K}[1]^2} + 2\frac{\mathbb{K}[v_2]^2}{\mathbb{K}[1]^2} + (4\beta - 1)\cos(v_2\bar{\mu}\gamma\bar{k}).$$

$$H_{\rm m}[N] = \bar{H}_{\rm m} + \delta H_{\rm m} = \int_\Sigma d^3x \bar{N}(C^{(0)}_{\rm m} + C^{(2)}_{\rm m}), \quad (37)$$

where

$$C^{(0)}_{\rm m} = \bar{p}^{3/2}\left[\frac{1}{2}\frac{\bar{\pi}^2}{\bar{p}^3} + V(\bar{\varphi})\right]. \quad (38)$$

The value of $C^{(2)}_{\rm m}$ is given by

$$C^{(2)}_{\rm m} = \frac{1}{2}\frac{\delta\pi^2}{\bar{p}^{3/2}} + \frac{1}{2}\sqrt{\bar{p}}\delta^{ab}\partial_a\delta\varphi\partial_b\delta\varphi + \frac{1}{2}\bar{p}^{3/2}V_{,\varphi\varphi}(\bar{\varphi})\delta\varphi^2 + \left(\frac{1}{2}\frac{\bar{\pi}^2}{\bar{p}^{3/2}} - \bar{p}^{3/2}V(\bar{\varphi})\right)\frac{\delta^k_c\delta^j_d\delta E^c_j\delta E^d_k}{4\bar{p}^2}, \quad (39)$$

where we have used the condition $\delta^i_a\delta E^a_i = 0$. The matter diffeomorphism constraint is given by:

$$D_{\rm m}[N^a] = \int_\Sigma d^3x\delta N^a\bar{\pi}(\partial_a\delta\varphi). \quad (40)$$

At this stage, ambiguities remain. Matter has to be introduced

←

The matter Ham. does not depend on the Ashtekar connection : no holo cor.

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$$\{S_{\rm tot}[N_1], S_{\rm tot}[N_1]\} = 0,$$
$$\{D_{\rm tot}[N_1^a], D_{\rm tot}[N_2^a]\} = 0,$$
$$\{S_{\rm tot}[N], D_{\rm tot}[N^a]\} = \frac{\bar{N}}{\sqrt{\bar{p}}}\mathcal{B}D^Q[N^a]$$
$$+\frac{\bar{N}}{\kappa\sqrt{\bar{p}}}\int_\Sigma d^3x\delta N^c\delta_c^k(\partial_d\delta E_k^d)\delta E_k^d\mathcal{A}$$
$$+[\cos(v_2\bar{\mu}\gamma\bar{k})-1]\frac{\sqrt{\bar{p}}}{2}\left(\frac{\bar{\pi}^2}{2\bar{p}^3}-V(\bar{\varphi})\right)\times$$
$$\times\int_\Sigma d^3x\bar{N}\partial_c(\delta N^a)\delta_a^j\delta E_j^c$$
$$+\frac{\bar{\pi}}{\bar{p}^{3/2}}\int_\Sigma d^3x\bar{N}(\partial_a\delta N^a)\delta\pi$$
$$-\bar{p}^{3/2}V_\varphi(\bar{\varphi})\int_\Sigma d^3x\bar{N}(\partial_a\delta N^a)\delta\varphi.$$

←The total Poisson brackets can now be calculated.

The algebra can be closed without ambiguity.

The free Hamiltonian reads as :

$$S^Q_{\rm free}[N] = \frac{1}{2\kappa}\int_\Sigma d^3x\left[\bar{N}(C^{(0)}_{\rm free}+C^{(2)}_{\rm free})\right]. \qquad (46)$$

where

$$C^{(0)}_{\rm free} = -6\sqrt{\bar{p}}\left(\mathbb{K}[1]\right)^2, \qquad (47)$$
$$C^{(2)}_{\rm free} = -\frac{1}{2\bar{p}^{3/2}}\left[4(1-\beta)\mathbb{K}[2]\bar{k}-2\bar{k}^2+(4\beta-1)\mathbb{K}[1]^2\right]\times$$
$$\times\ (\delta E_j^c\delta E_k^d\delta_c^k\delta_d^j)+\sqrt{\bar{p}}(\delta K_c^j\delta K_d^k\delta_k^c\delta_j^d)$$
$$-\ \frac{2}{\sqrt{\bar{p}}}\left(2\mathbb{K}[2]-\bar{k}\right)(\delta E_j^c\delta K_c^j). \qquad (48)$$

By studying effects of an infinitesimal change of coordinates on dE and dK, it is possible to define a gauge-invariant variable.
The equations of motion for the background are

$$\begin{aligned}\dot{\bar{p}} &= \bar{N} 2\sqrt{\bar{p}}(\mathbb{K}[2]),\\ \dot{\bar{k}} &= -\frac{\bar{N}}{\sqrt{\bar{p}}}\left[\frac{1}{2}(\mathbb{K}[1])^2 + \bar{p}\frac{\partial}{\partial \bar{p}}(\mathbb{K}[1])^2\right]\\ &+ \frac{\kappa}{3V_0}\left(\frac{\partial \bar{H}_{\rm m}}{\partial \bar{p}}\right),\end{aligned}$$

For a free scalar field, this can be solved analytically. Introducing a symmetrized gauge-invariant variable, one is led to:

$$-\frac{1}{2}\frac{d}{d\eta}\mathfrak{S}^i_a - \frac{1}{2}\underbrace{(2\mathbb{K}[2])}_{=\frac{1}{\bar{p}}\frac{d\bar{p}}{d\eta}}\mathfrak{S}^i_a = 0,$$

III. Scalar modes (holonomy corrections)

Scalar perturbations are the more important ones from the observational viewpoint. Developping and anomaly-free and gauge-invariant framework for holonomy corrections has been an open issue.

→FLRW background with scalar perturbations. 4 background variables (k,p, phi, pi) and 4 perturbed variables (dK, dE, dPhi, dPi).

→The full Poisson Bracket can be decomposed in 4 main terms

→The usual replacement k→sin is performed

→The beta parameter is kept free (mu_bar=p^beta)

Cailleteau, Mielzcarek, A.B., Grain, CQG 2012

III. Scalar modes (holonomy corrections)

Scalar perturbations are the more important ones from the observational viewpoint. Developping and anomaly-free and gauge-invariant framework for holonomy corrections has been an open issue.

→FLRW background with scalar perturbations. 4 background variables (k,p, phi, pi) and 4 perturbed variables (dK, dE, dPhi, dPi).

→The full Poisson Bracket can be decomposed in 4 main terms

→The usual replacement k→sin is performed

→The beta parameter is kept free (mu_bar=p^beta)

Cailleteau, Mielzcarek, A.B., Grain, CQG 2012

The holonomy-modified Hamiltonian constraint reads as

$$H_G^Q[N] = \frac{1}{2\kappa}\int_\Sigma d^3x \left[\bar{N}(\mathcal{H}_G^{(0)} + \mathcal{H}_G^{(2)}) + \delta N \mathcal{H}_G^{(1)}\right],$$

where

$$\mathcal{H}_G^{(0)} = -6\sqrt{\bar{p}}(\mathbb{K}[1])^2,$$

$$\mathcal{H}_G^{(1)} = -4\sqrt{\bar{p}}\left(\mathbb{K}[s_1] + \alpha_1\right)\delta_j^c \delta K_c^j - \frac{1}{\sqrt{\bar{p}}}\left(\mathbb{K}[1]^2 + \alpha_2\right)\delta_c^j \delta E_j^c$$
$$+\frac{2}{\sqrt{\bar{p}}}(1+\alpha_3)\partial_c\partial^j \delta E_j^c,$$

$$\mathcal{H}_G^{(2)} = \sqrt{\bar{p}}(1+\alpha_4)\delta K_c^j \delta K_d^k \delta_k^c \delta_j^d - \sqrt{\bar{p}}(1+\alpha_5)(\delta K_c^j \delta_j^c)^2$$
$$-\frac{2}{\sqrt{\bar{p}}}\left(\mathbb{K}[s_2] + \alpha_6\right)\delta E_j^c \delta K_c^j - \frac{1}{2\bar{p}^{3/2}}\left(\mathbb{K}[1]^2 + \alpha_7\right)\delta E_j^c \delta E_k^d \delta_c^k \delta_d^j$$
$$+\frac{1}{4\bar{p}^{3/2}}\left(\mathbb{K}[1]^2 + \alpha_8\right)(\delta E_j^c \delta_c^j)^2 - \frac{1}{2\bar{p}^{3/2}}(1+\alpha_9)\delta^{jk}(\partial_c \delta E_j^c)(\partial_d \delta E_k^d).$$

The standard holo corrections are parametrized by 2 integers s1 and s2. The alpha_i are counter-terms, which are introduced to remove anomalies (vanishing in the mu→0 limit). The diffeo constraint holds its classical form :

$$D_G[N^a] = \frac{1}{\kappa}\int_\Sigma d^3x \delta N^c \left[\bar{p}\partial_c(\delta_k^d \delta K_d^k) - \bar{p}(\partial_k \delta K_c^k) - \bar{k}\delta_c^k(\partial_d \delta E_k^d)\right].$$

Poisson Brackets.

$$\left\{H_G^Q[N], D_G[N^a]\right\} = -H_G^Q\left[\delta N^a \partial_a \delta N\right] + \mathcal{B}\; D_G[N^a]$$
$$+\frac{\sqrt{\bar{p}}}{\kappa}\int_\Sigma d^3x \delta N^a (\partial_a \delta N)\mathcal{A}_1 + \frac{\bar{N}\sqrt{\bar{p}}\bar{k}}{\kappa}\int_\Sigma d^3x \delta N^a (\partial_i \delta K_a^i)\mathcal{A}_2$$
$$+\frac{\bar{N}}{\kappa\sqrt{\bar{p}}}\int_\Sigma d^3x \delta N^i (\partial_a \delta E_i^a)\mathcal{A}_3 + \frac{\bar{N}}{2\kappa\sqrt{\bar{p}}}\int_\Sigma d^3x (\partial_a \delta N^a)(\delta E_i^b \delta_b^i)\mathcal{A}_4,$$

where

$$\mathcal{B} = \frac{\bar{N}}{\sqrt{\bar{p}}}\left[-2\mathbb{K}[2] + \bar{k}(1+\alpha_5) + \mathbb{K}[s_2] + \alpha_6\right],$$

and

$$\mathcal{A}_1 = 2\bar{k}(\mathbb{K}[s_1] + \alpha_1) + \alpha_2 - 2\mathbb{K}[1]^2,$$
$$\mathcal{A}_2 = \alpha_5 - \alpha_4,$$
$$\mathcal{A}_3 = -\mathbb{K}[1]^2 - \bar{p}\frac{\partial}{\partial \bar{p}}\mathbb{K}[1]^2 - \frac{1}{2}\alpha_7$$
$$+\bar{k}(-2\mathbb{K}[2] + \bar{k}(1+\alpha_5) + 2\mathbb{K}[s_2] + 2\alpha_6),$$
$$\mathcal{A}_4 = \alpha_8 - \alpha_7.$$

{H,H} introduces 4 more anomalies
{D,D} is vanishing

Including matter

$$D_M[N^a] = \int_\Sigma \delta N^a \bar{\pi}(\partial_a \delta\varphi).$$

The scalar matter Hamiltonian can be expressed as:

$$H_M^Q[N] = H_M[\bar{N}] + H_M[\delta N],$$

where

$$H_M[\bar{N}] = \int_\Sigma d^3x \bar{N} \left[\left(\mathcal{H}_\pi^{(0)} + \mathcal{H}_\varphi^{(0)}\right) + \left(\mathcal{H}_\pi^{(2)} + \mathcal{H}_\nabla^{(2)} + \mathcal{H}_\varphi^{(2)}\right) \right],$$

$$H_M[\delta N] = \int_\Sigma d^3 \delta N \left[\mathcal{H}_\pi^{(1)} + \mathcal{H}_\varphi^{(1)}\right].$$

$$\mathcal{H}_\pi^{(0)} = \frac{\bar{\pi}^2}{2\bar{p}^{3/2}},$$

$$\mathcal{H}_\varphi^{(0)} = \bar{p}^{3/2} V(\bar{\varphi}),$$

$$\mathcal{H}_\pi^{(1)} = \frac{\bar{\pi}\delta\pi}{\bar{p}^{3/2}} - \frac{\bar{\pi}^2}{2\bar{p}^{3/2}} \frac{\delta_c^j \delta E_j^c}{2\bar{p}},$$

$$\mathcal{H}_\varphi^{(1)} = \bar{p}^{3/2} \left[V_{,\varphi}(\bar{\varphi})\delta\varphi + V(\bar{\varphi}) \frac{\delta_c^j \delta E_j^c}{2\bar{p}} \right],$$

$$\mathcal{H}_\pi^{(2)} = \frac{1}{2}\frac{\delta\pi^2}{\bar{p}^{3/2}} - \frac{\bar{\pi}\delta\pi}{\bar{p}^{3/2}} \frac{\delta_c^j \delta E_j^c}{2\bar{p}} + \frac{1}{2}\frac{\bar{\pi}^2}{\bar{p}^{3/2}} \left[\frac{(\delta_c^j \delta E_j^c)^2}{8\bar{p}^2} + \frac{\delta_c^k \delta_d^j \delta E_j^c \delta E_k^d}{4\bar{p}^2} \right],$$

$$\mathcal{H}_\nabla^{(2)} = \frac{1}{2}\sqrt{\bar{p}}(1+\alpha_{10})\delta^{ab}\partial_a\delta\varphi\partial_b\delta\varphi,$$

$$\mathcal{H}_\varphi^{(2)} = \frac{1}{2}\bar{p}^{3/2} V_{,\varphi\varphi}(\bar{\varphi})\delta\varphi^2 + \bar{p}^{3/2} V_{,\varphi}(\bar{\varphi})\delta\varphi \frac{\delta_c^j \delta E_j^c}{2\bar{p}}$$
$$+ \bar{p}^{3/2} V(\bar{\varphi}) \left[\frac{(\delta_c^j \delta E_j^c)^2}{8\bar{p}^2} - \frac{\delta_c^k \delta_d^j \delta E_j^c \delta E_k^d}{4\bar{p}^2} \right].$$

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One can now compute the total constraints

$$\{D_{tot}[N_1^a], D_{tot}[N_2^a]\} = 0.$$

$$\begin{aligned}\{H_{tot}[N], D_{tot}[N^a]\} &= \left\{H_M^Q[N], D_{tot}[N^a]\right\} + \left\{H_G^Q[N], D_G[N^a]\right\} \\ &+ \left\{H_G^Q[N], D_M[N^a]\right\}.\end{aligned}$$

$$\begin{aligned}\{H_{tot}[N_1], H_{tot}[N_2]\} &= \left\{H_G^Q[N_1], H_G^Q[N_2]\right\} + \{H_M[N_1], H_M[N_2]\} \\ &+ \left[\left\{H_G^Q[N_1], H_M[N_2]\right\} - (N_1 \leftrightarrow N_2)\right].\end{aligned}$$

$$\begin{aligned}&\left\{H_G^Q[N_1], H_M[N_2]\right\} - (N_1 \leftrightarrow N_2) = \\ &= \frac{1}{2}\int_\Sigma d^3x \bar{N}(\delta N_2 - \delta N_1)\left(\frac{\bar{\pi}^2}{2\bar{p}^3} - V(\bar{\varphi})\right)(\partial_c\partial^j \delta E_j^c)\mathcal{A}_9 \\ &+3\int_\Sigma d^3x \bar{N}(\delta N_2 - \delta N_1)\left(\frac{\bar{\pi}\delta\pi}{\bar{p}^2} - \bar{p}V_\varphi(\bar{\varphi})\delta\varphi\right)\mathcal{A}_{10} \\ &+\int_\Sigma d^3x \bar{N}(\delta N_2 - \delta N_1)(\delta_j^c \delta K_j^c)\left(\frac{\bar{\pi}^2}{2\bar{p}^3} - V(\bar{\varphi})\right)\bar{p}\mathcal{A}_{11} \\ &+\frac{1}{2}\int_\Sigma d^3x \bar{N}(\delta N_2 - \delta N_1)(\delta_c^j \delta E_j^c)\left(\frac{\bar{\pi}^2}{2\bar{p}^3}\right)\mathcal{A}_{12} \\ &+\frac{1}{2}\int_\Sigma d^3x \bar{N}(\delta N_2 - \delta N_1)(\delta_c^j \delta E_j^c)V(\bar{\varphi})\mathcal{A}_{13},\end{aligned}$$

This introduces 5 more anomalies.

where

$$\begin{aligned}\mathcal{A}_9 &= \frac{\partial \alpha_3}{\partial k}, \\ \mathcal{A}_{10} &= \mathbb{K}[2] - \mathbb{K}[s_1] - \alpha_1, \\ \mathcal{A}_{11} &= -\frac{\partial}{\partial k}(\mathbb{K}[s_1] + \alpha_1) + \frac{3}{2}(1 + \alpha_5) - \frac{1}{2}(1 + \alpha_4), \\ \mathcal{A}_{12} &= -\frac{1}{2}\frac{\partial}{\partial k}(\mathbb{K}[1]^2 + \alpha_2) + 5(\mathbb{K}[s_1] + \alpha_1) - 5\mathbb{K}[2] + \mathbb{K}[s_2] + \alpha_6, \\ \mathcal{A}_{13} &= \frac{1}{2}\frac{\partial}{\partial k}(\mathbb{K}[1]^2 + \alpha_2) + \mathbb{K}[s_1] + \alpha_1 - \mathbb{K}[2] - \mathbb{K}[s_2] - \alpha_6.\end{aligned}$$

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Anomaly freedom : A_i=0
It is indeed possible ! And uniquely determined under reasonable assumptions. In addition, it requires beta=-1/2.

$$\begin{aligned}
\alpha_1 &= \mathbb{K}[2] - \mathbb{K}[s_1], & \alpha_6 &= 2\mathbb{K}[2] - \mathbb{K}[s_2] - \bar{k}\Omega,\\
\alpha_2 &= 2\mathbb{K}[1]^2 - 2\bar{k}\mathbb{K}[2], & \alpha_7 &= -4\mathbb{K}[1]^2 + 6\bar{k}\mathbb{K}[2] - 2\bar{k}^2\Omega,\\
\alpha_3 &= 0, & \alpha_8 &= -4\mathbb{K}[1]^2 + 6\bar{k}\mathbb{K}[2] - 2\bar{k}^2\Omega,\\
\alpha_4 &= \Omega - 1, & \alpha_9 &= 0,\\
\alpha_5 &= \Omega - 1, & \alpha_{10} &= \Omega - 1.
\end{aligned}$$

Omega = dK[2]/dk = cos(2 mu gamma k) = 1-2*rho/rho_c
The final Hamiltonian does *not* depend on s1 and s2.
Full algebra of constraints:

$$\begin{aligned}
\{D_{tot}[N_1^a], D_{tot}[N_2^a]\} &= 0,\\
\{H_{tot}[N], D_{tot}[N^a]\} &= -H_{tot}[\delta N^a \partial_a \delta N],\\
\{H_{tot}[N_1], H_{tot}[N_2]\} &= D_{tot}\left[\Omega \frac{\bar{N}}{\bar{p}} \partial^a (\delta N_2 - \delta N_1)\right]
\end{aligned}$$

26
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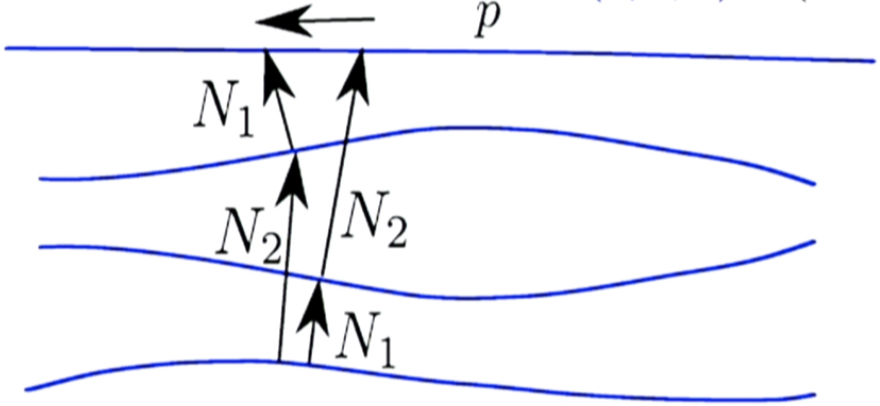
Although the algebra is closed, there are modifications with respect to the classical case, due to the presence of the Omega factor. Not only the dynamics (as a result of the modification of the Hamiltonian constraint) is modified but the very structure of spacetime itself is deformed.

$$N^a = \frac{\bar{N}}{\bar{p}}\cos(2\bar{\mu}\gamma\bar{k})\partial^a(\delta N_2 - \delta N_1)$$



The equations of motion can now be derived

1) For the background (as usual)

2) For the perturbed variables

$$
\begin{aligned}
\delta \dot{E}^a_i &= -\bar{N}\left[\sqrt{\bar{p}}\Omega\delta K^j_c\delta^c_i\delta^a_j - \sqrt{\bar{p}}\Omega(\delta K^j_c\delta^c_j)\delta^a_i - \frac{1}{\sqrt{\bar{p}}}(2\mathbb{K}[2]-\bar{k}\Omega)\delta E^a_i\right] + \\
&+ \delta N\left(2\mathbb{K}[2]\sqrt{\bar{p}}\delta^a_i\right) - \bar{p}(\partial_i\delta N^a - (\partial_c\delta N^c)\delta^a_i), \qquad (87)\\
\delta \dot{K}^i_a &= \bar{N}\left[-\frac{1}{\sqrt{\bar{p}}}(2\mathbb{K}[2]-\bar{k}\Omega)\delta K^i_a\right.\\
&- \frac{1}{2\bar{p}^{3/2}}(-3\mathbb{K}[1]^2 + 6\bar{k}\mathbb{K}[2] - 2\bar{k}^2\Omega)\delta E^c_j\delta^j_a\delta^i_c\\
&+ \left.\frac{1}{4\bar{p}^{\frac{3}{2}}}(-3\mathbb{K}[1]^2 + 6\bar{k}\mathbb{K}[2] - 2\bar{k}^2\Omega)(\delta E^c_j\delta^j_c)\delta^i_a + \frac{\delta^{ik}}{2\bar{p}^{\frac{3}{2}}}\partial_a\partial_d\delta E^d_k\right]\\
&+ \frac{1}{2}\left[-\frac{1}{\sqrt{\bar{p}}}(3\mathbb{K}[1]^2 - 2\bar{k}\mathbb{K}[2])\delta^i_a\delta N + \frac{2}{\sqrt{\bar{p}}}(\partial_a\partial^i\delta N)\right]\\
&+ \delta^i_c(\partial_a\delta N^c) + \kappa\delta N\frac{\sqrt{\bar{p}}}{2}\left[-\frac{\bar{\pi}^2}{2\bar{p}^3} + V(\bar{\varphi})\right]\delta^i_a\\
&+ \kappa\bar{N}\left[-\frac{\bar{\pi}\delta\pi}{2\bar{p}^{5/2}}\delta^i_a + \frac{\sqrt{\bar{p}}}{2}\delta\varphi\frac{\partial V(\bar{\varphi})}{\partial\bar{\varphi}}\delta^i_a + \left(\frac{\bar{\pi}^2}{2\bar{p}^{3/2}} + \bar{p}^{3/2}V(\bar{\varphi})\right)\frac{\delta^j_c\delta E^c_j}{4\bar{p}^2}\delta^i_a\right.\\
&+ \left.\left(\frac{\bar{\pi}^2}{2\bar{p}^{3/2}} - \bar{p}^{3/2}V(\bar{\varphi})\right)\frac{\delta^i_c\delta^j_a\delta E^c_j}{2\bar{p}^2}\right], \qquad (88)\\
\delta\dot{\varphi} &= \delta N\left(\frac{\bar{\pi}}{\bar{p}^{3/2}}\right) + \bar{N}\left(\frac{\delta\pi}{\bar{p}^{3/2}} - \frac{\bar{\pi}}{\bar{p}^{3/2}}\frac{\delta^j_c\delta E^c_j}{2\bar{p}}\right), \qquad (89)\\
\delta\dot{\pi} &= -\delta N\left(\bar{p}^{3/2}V_{,\varphi}(\bar{\varphi})\right) + \bar{\pi}(\partial_a\delta N^a)\\
&- \bar{N}\left[-\sqrt{\bar{p}}\Omega\delta^{ab}\partial_a\partial_b\delta\varphi + \bar{p}^{3/2}V_{,\varphi\varphi}(\bar{\varphi})\delta\varphi + \bar{p}^{3/2}V_{,\varphi}(\bar{\varphi})\frac{\delta^j_c\delta E^c_j}{2\bar{p}}\right]. \qquad (90)
\end{aligned}
$$

$$\ddot{\phi} + 2\left[\mathcal{H} - \left(\frac{\ddot{\bar{\varphi}}}{\dot{\bar{\varphi}}} + \epsilon\right)\right]\dot{\phi} + 2\left[\dot{\mathcal{H}} - \mathcal{H}\left(\frac{\ddot{\bar{\varphi}}}{\dot{\bar{\varphi}}} + \epsilon\right)\right]\phi - c_s^2\nabla^2\phi = 0,$$

with the quantum correction

$$\epsilon = \frac{1}{2}\frac{\dot{\Omega}}{\Omega} = 3\mathbb{K}[2]\left(\frac{\rho + P}{\rho_c - 2\rho}\right),$$

and the squared velocity

$$c_s^2 = \Omega.$$

Interestingly the perturbations « do not propagate anymore » for rho>rho_c/2. This corresponds to an effective *euclidean* space-time.

Several strategies are possible.

IV. General Hamilton-Jacobi and Mukhanov-Sasaki equations for scalar perturbations

This extends the method introduced by Langlois.

This approach exhibits several advantages:
- the treatment is purely Hamiltonian with easy calculations,
- the Mukhanov variables v and R are obtained directly and the equation of motion is easily found without using Bardeen Potentials,
- it helps to construct an anomaly-free algebra by imposing relations on the Poisson brackets,
- the z variable can be found without ambiguity in a quite simple way,
- the generating functions are clearly defined, easy to handle and allow one to trace back deeply the origin of gauge invariance,
- it works for any kind of constraint theory.

Cailleteau, A.B., Phys.Rev. D, 85, 123534, 2012

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Cailleteau, A.B., Phys.Rev. D, 85, 123534, 2012

Gauge-invariant variables for scalar perturbations.

Writing the equations for Ψ and $\delta\varphi^{GI}$, which are

$$\ddot{\Psi} + 2\left[\mathcal{H} - \left(\frac{\ddot{\bar{\varphi}}}{\dot{\bar{\varphi}}} + \epsilon\right)\right]\dot{\Psi} + 2\left[\dot{\mathcal{H}} - \mathcal{H}\left(\frac{\ddot{\bar{\varphi}}}{\dot{\bar{\varphi}}} + \epsilon\right)\right]\Psi - c_s^2\nabla^2\Psi = 0 \qquad (126)$$

and

$$\delta\ddot{\varphi}^{GI} + 2\mathbb{K}[2]\delta\dot{\varphi}^{GI} - \Omega\nabla^2\delta\varphi^{GI} + \bar{p}V_{,\varphi\varphi}(\bar{\varphi})\delta\varphi^{GI} + 2\bar{p}V_{,\varphi}(\bar{\varphi})\Psi - 4\dot{\bar{\varphi}}^{GI}\dot{\Psi} = 0, \quad (127)$$

one obtains equation for the variable (125):

$$\ddot{v} - \Omega\nabla^2 v - \frac{\ddot{z}}{z}v = 0, \qquad (128)$$

$$z = \sqrt{\bar{p}}\frac{\dot{\bar{\varphi}}}{\mathbb{K}[2]}, \qquad (129)$$

The material needed to compute the power spectrum is available.

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V. Consistency

Initially, tensor, vector and scalar perturbations were studied independently, leading to different algebras of constraints. It can be shown that the algebra can be modified by a simple quantum correction, ***holding for all types*** of perturbations.

→ consistency of the theory

→ lessons from the study of scalar perturbations should be taken into account when studying tensor modes.

$$\delta A_a^i = \delta\Gamma_a^i + \gamma \delta K_a^i$$

$$\delta\Gamma_a^i = \frac{1}{2\bar{p}} X_{ca}^{ijb} \partial_b \delta E_j^c + \frac{1}{2\bar{p}^2} Y_{abc}^{ijkl} \delta E_j^b \partial_k \delta E_l^c ,$$

$$X_{ca}^{ijb} = \epsilon_c^{ij} \delta_a^b - \epsilon_c^{ib} \delta_a^j + \epsilon^{ijb} \delta_{ca} + \epsilon_a^{ib} \delta_c^j .$$

$$\delta E_i^a = \bar{p} \left[-2\psi \delta_i^a + (\delta_i^a \partial^d \partial_d - \partial^a \partial_i) E - c_1 \partial^a F_i - c_2 \partial_i F^a - \tfrac{1}{2} h_i^a \right]$$

Cailleteau, A.B., Grain, Vidotto, Phys.Rev. D, 2012

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One then write down the constraints for each kind of perturbation. Quite a lot of algebra. In the classical case:

$$\{H_{(m+g)}[N_1], H_{(m+g)}[N_2]\} = D_{(m+g)}\left[\frac{\bar{N}}{\bar{p}}\partial^a(\delta N_2 - \delta N_1)\right]$$

Then, quantum corrections (holonomy) are turned on: $\bar{k} \rightarrow \frac{\sin(\bar{\mu}\gamma\bar{k})}{\bar{\mu}\gamma}$

Remarkably, the resulting quantum-corrected algebra valid for all kinds of perturbations is obtained with a single structure modification:

$$\{H_{(m+g)}[N_1], H_{(m+g)}[N_2]\} = \mathbf{\Omega}\, D_{(m+g)}\left[\frac{\bar{N}}{\bar{p}}\partial^a(\delta N_2 - \delta N_1)\right] \qquad \mathbf{\Omega} = \cos(2\bar{\mu}\gamma\bar{k}) = 1 - 2\frac{\rho}{\rho_c}$$

One can then derive the correction to the Mukhanov-Sasaki equation of motion for gauge-invariant perturbations of scalar and tensor type :

$$v''_{\mathrm{S(T)}} - \mathbf{\Omega}\,\nabla^2 v_{\mathrm{S(T)}} - \frac{z''_{\mathrm{S(T)}}}{z_{\mathrm{S(T)}}} v_{\mathrm{S(T)}} = 0$$

For scalar perturbations, the Mukhanov variables in the quantum case are given by

$$v_{\mathrm{S}} = \sqrt{\bar{p}}\left(\delta\phi + \frac{\bar{\varphi}'}{H}\phi\right) \qquad z_{\mathrm{S}} = \sqrt{\bar{p}}\,\frac{\bar{\varphi}'}{H}$$

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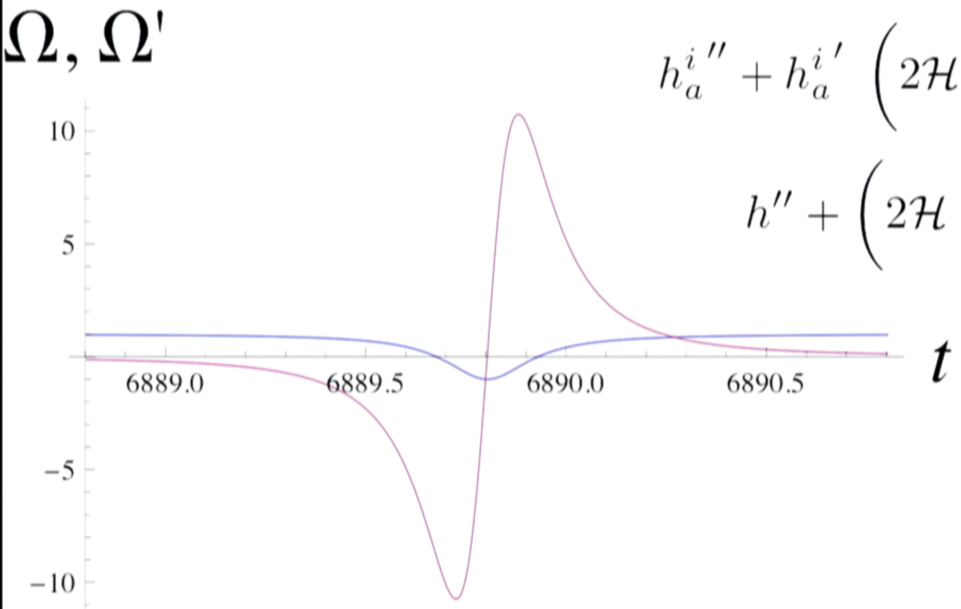
VI. The tensor spectrum in Omega-LQC (1st approach)

The algebra has to be deformed even for tensor modes :

$$v_{\mathrm{T}} = z_{\mathrm{T}} \times h^i_a$$

$$\mathbf{\Omega} = \cos(2\bar{\mu}\gamma\bar{k}) = 1 - 2\frac{\rho}{\rho_c}$$

$$v''_{\mathrm{T}} - \mathbf{\Omega}\,\nabla^2 v_{\mathrm{T}} - \frac{z''_{\mathrm{T}}}{z_{\mathrm{T}}} v_{\mathrm{T}} = 0 \; ; \; z_{\mathrm{T}} = \frac{a}{\sqrt{\mathbf{\Omega}}}$$



$$h^{i\,\prime\prime}_a + h^{i\,\prime}_a \left(2\mathcal{H} - \frac{\mathbf{\Omega}'}{\mathbf{\Omega}}\right) - \mathbf{\Omega}\,\nabla^2 h^i_a = 0$$

$$h'' + \left(2\mathcal{H} - \frac{\mathbf{\Omega}'}{\mathbf{\Omega}}\right) h' + \mathbf{\Omega} k^2 h = 0$$

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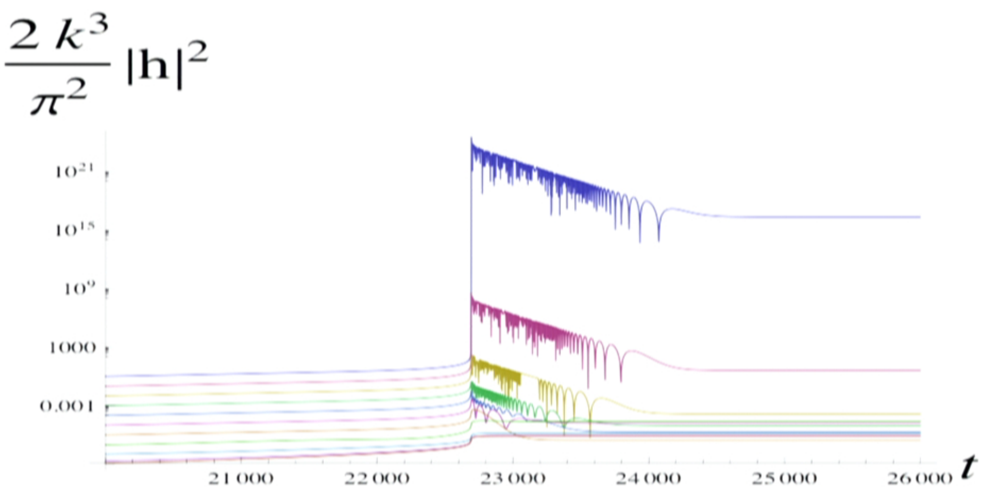
Can be made regular:

$$h' = \mathbf{\Omega} g \; ; \; g' = -2\mathcal{H} g - k^2 h$$

$$g'' + 2\mathcal{H} g' + (2\mathcal{H}' + \mathbf{\Omega} k^2) g = 0$$

In cosmic time : $$\dot{h} = \frac{\mathbf{\Omega}}{a} g \; ; \; \dot{g} = -2Hg - \frac{k^2}{a} h$$



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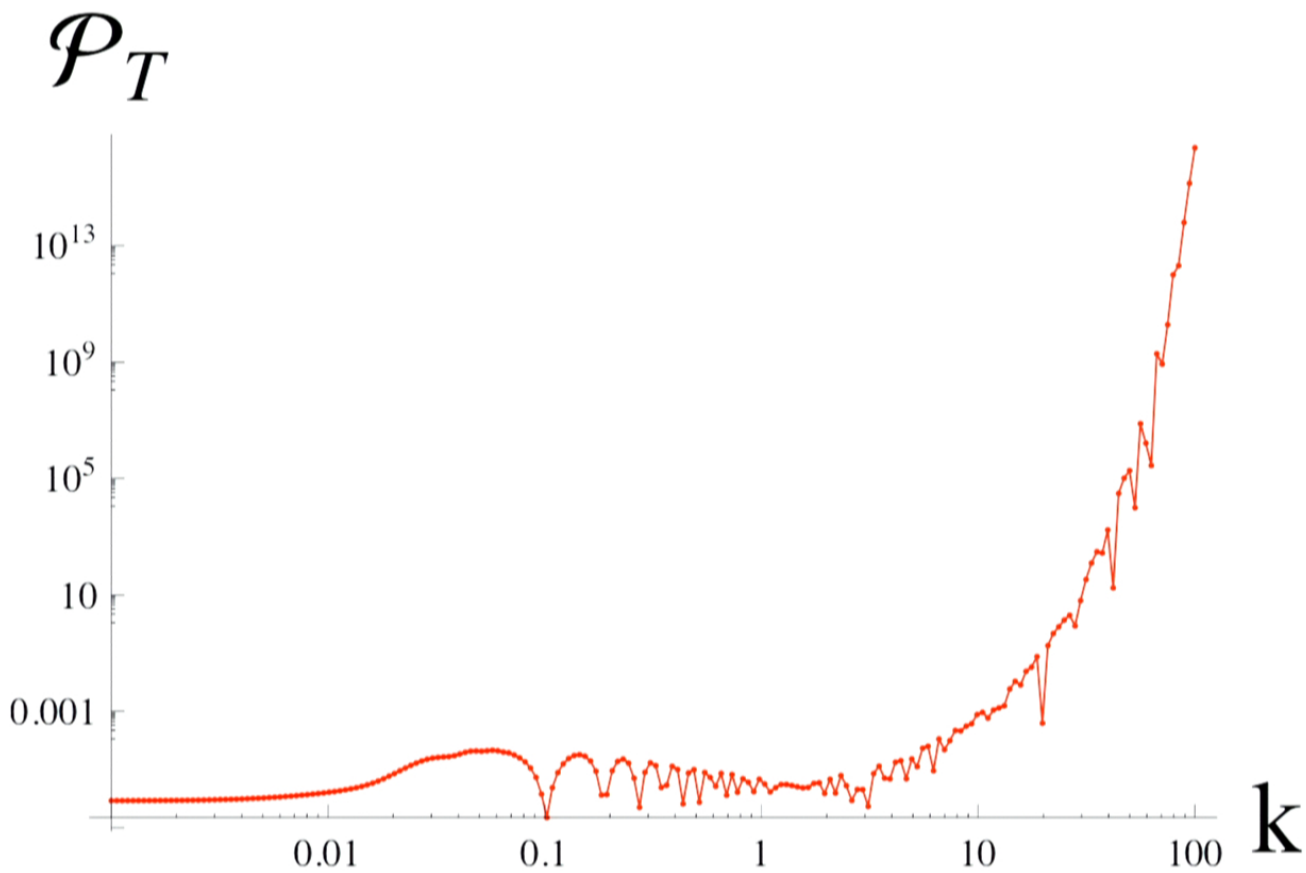

Lisenfors, Cailleteau, A.B., Cailleteau, Grain, Phys. Rev. D, 87, 107503, 2013

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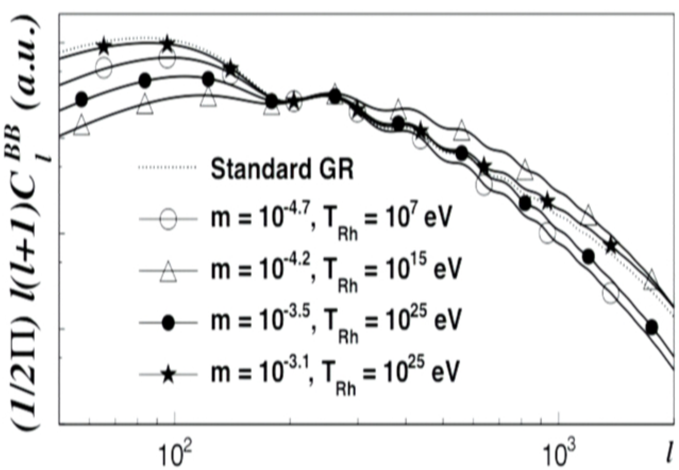
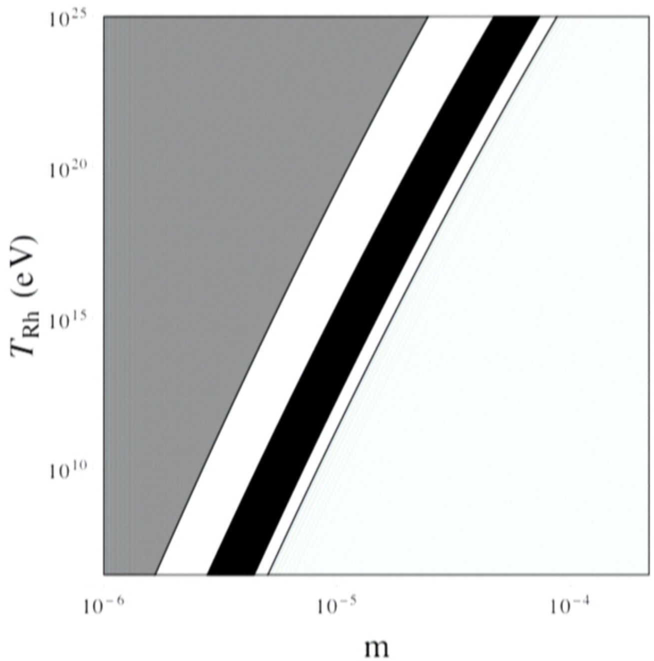
VII. Associated phenomenology

Using the CLASS modular code, one can propagate this P(k) primordial spectrum to obtain an « observable » c(l) spectrum. In progress.

Two preliminary results (from B. Bolliet)





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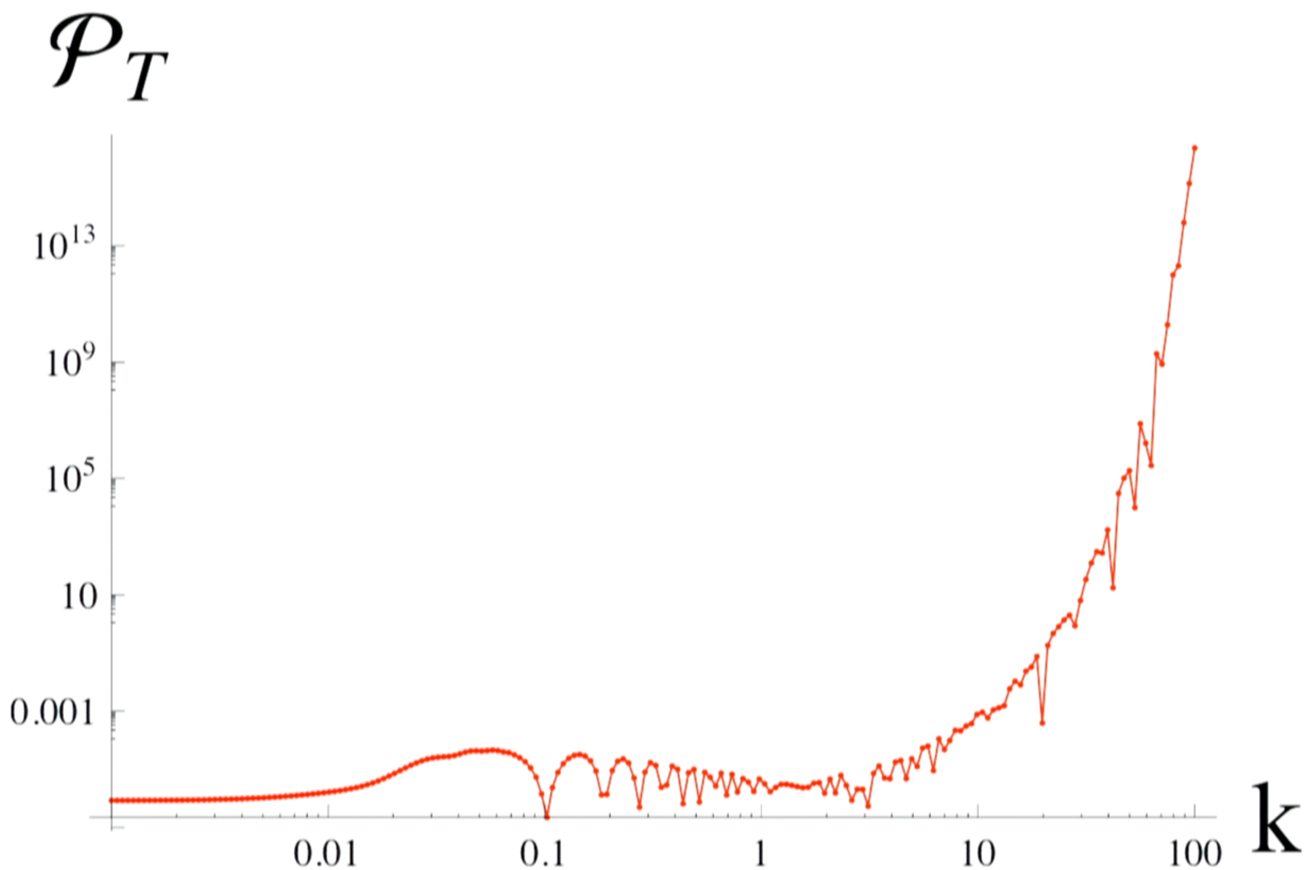

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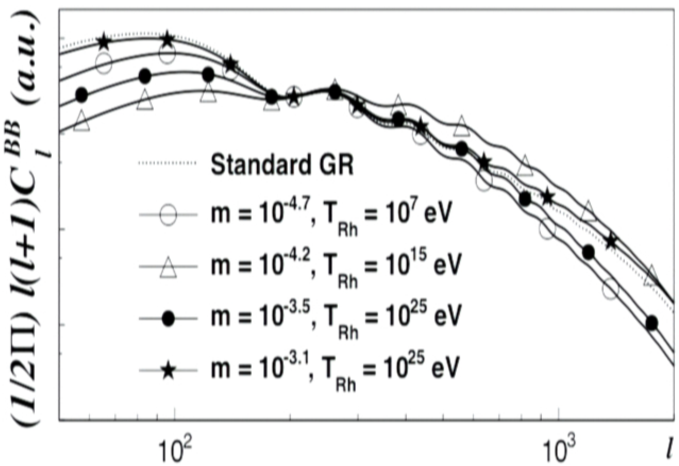
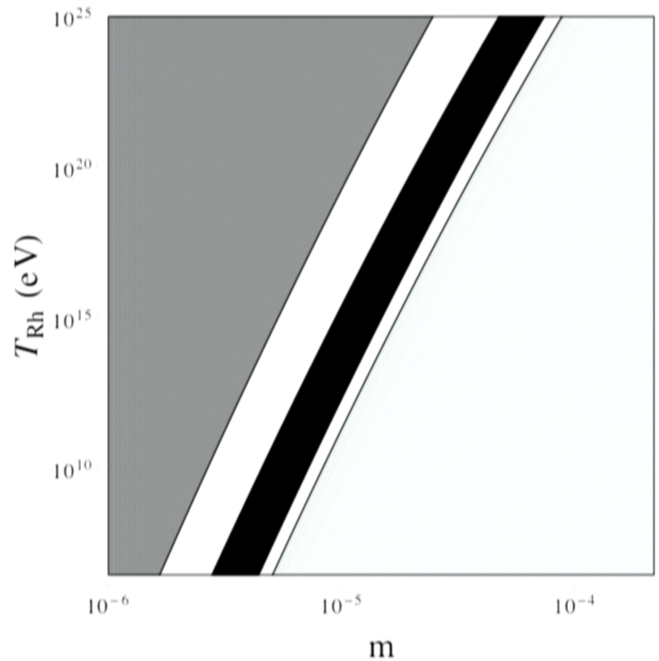




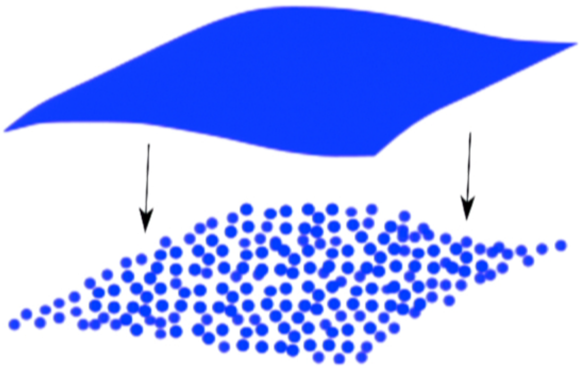
VIII. Asymptotic silence (2nd hypothesis)

asymptotic silence → causal disconnection of the space points.

Anticipated by the BKL conjecture. Although BKL is classical, one can expect that asymptotic silence should have a quantum gravity origin.

Asymptotic silence can be recovered in different limit:

- $c \to 0$ (Carrollian limit).
- $G \to \infty$. The scalar constraint can be schematically written as ($G \cdot$ *kinetic* + $1 \cdot$ *potential)*. Only the potential term G contains spatial derivatives, which relate the neighboring points.



In the limit $G \to \infty$ only the kinetic term remains and the theory becomes ultralocal.

Mielczarek, arXiv:1212.3527

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The classical algebra of constraints is :

$$\{D[N_1^a], D[N_2^a]\} = D[N_1^b \partial_b N_2^a - N_2^b \partial_b N_1^a],$$
$$\{S[N], D[N^a]\} = -S[N^a \partial_a N],$$
$$\{S[N_1], S[N_2]\} = sD\left[g^{ab}(N_1 \partial_b N_2 - N_2 \partial_b N_1)\right]$$

s=1 corresponds to the Lorentzian signature and s=-1 to the Euclidean one. In the ultralocal limit the algebra simplifies to the Lie algebra:

$$\{D[N_1^a], D[N_2^a]\} = D[N_1^b \partial_b N_2^a - N_2^b \partial_b N_1^a],$$
$$\{S[N], D[N^a]\} = -S[N^a \partial_a N],$$
$$\{S[N_1], S[N_2]\} = 0.$$

This happens in LQC for rho=rho_c/2, but also Horava-Lifshitz and in CDT !

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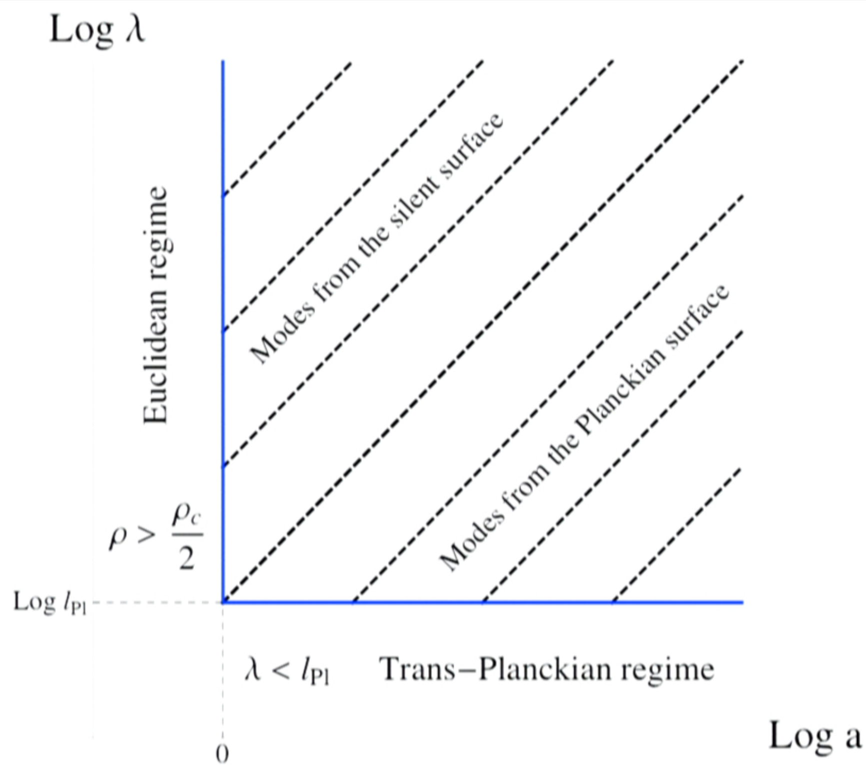
Reminiscent of the Hartle-Hawking Wick rotation. Sudden rotation → smooth one. (Close to what happens with nanowires : SO(3) → SO(1,2))

SO(4) → SO(1,3). Symmetry breaking at the fundamental level of spacetime ? The time direction would be an seen as the order parameter of the broken phase. Glodstone bosons as Inflaton field ?

Natural initial conditions for pertubations: white noise.

$$\mathcal{P}(k) = \frac{2}{3}\frac{G_0}{\pi}(k\xi)^3 + \mathcal{O}((k\xi)^5)$$

$$\frac{d^2\mathcal{P}}{d\eta^2} - \frac{1}{2\mathcal{P}}\left(\frac{d\mathcal{P}}{d\eta}\right)^2 + 2\beta k^2\mathcal{P} + 2\frac{d\mathcal{P}}{d\eta}\frac{z'}{z} - \frac{1}{2z^4\mathcal{P}}\left(\frac{k^3}{2\pi^2}\right)^2 = 0.$$



1) Considering « trans-planckian » modes. With a deformed Poincaré algebra,

$$\beta = 1 - 2\frac{\rho}{\rho_c} - \frac{1}{m_{\rm Pl}^2}\left(\frac{k}{a}\right)^2 \qquad \frac{d}{d\eta^2}v_k - \frac{z''}{z}v_k = 0$$

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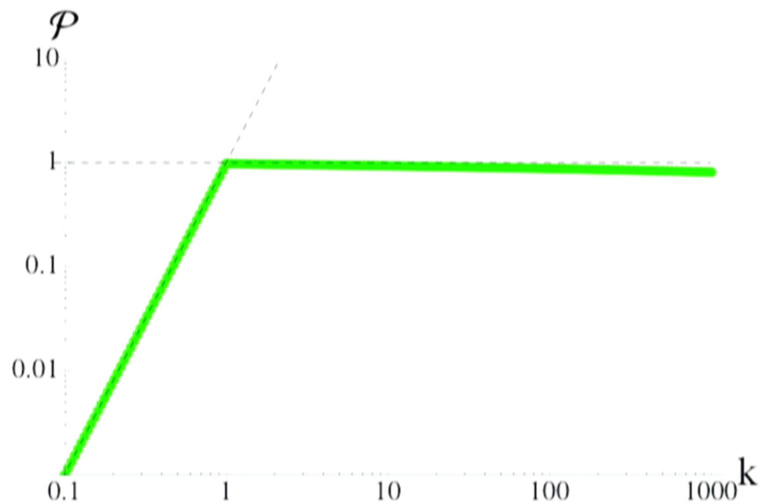

2) Considering the « silent » surface : on needs to different stages with different eq. of states. Works without inflation.

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IX Fully corrected algebra (holo + inverse-volume)

-Inverse-volume case: important results from Bojowald, Hossain, Kagan, Shankaranarayanan. Have been generalized by including non-linear effects.

-Holonomy case: also generalized by underlying implicit assumptions in previous works. This makes the Euclidean phase more speculative although the asymptotic silence remains.

-Combined case: some freedom remains but the main picture becomes to be quite clear.

Basically, corrections acts in a « multiplicative » way. But some subtelties are underlined, in particular, the inverse-volume correction has to depend on the holonomy one.

-Gauge-invariant EOM derived

$$\Omega = \frac{\alpha}{\sqrt{\alpha_E}}\left[\left(-\cos(2\bar{\mu}\gamma\bar{k})(1-H-H^2)+(H^2-1)\right)+2\frac{\sin(2\bar{\mu}\gamma\bar{k})}{2\bar{\mu}\gamma\bar{k}}\frac{\partial H}{\partial\bar{k}}\right]$$

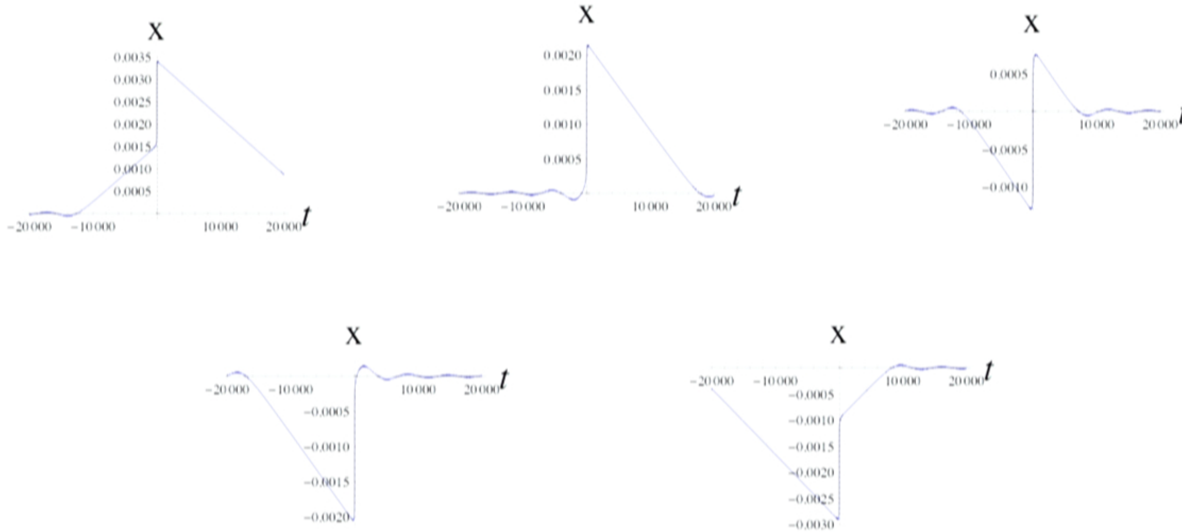
with

$$H(K,E) = 1+\frac{1}{2}\frac{\tan(\bar{\mu}\gamma\bar{k})}{\gamma\bar{\mu}}\frac{1}{\alpha_{E,K}}\frac{\partial\alpha_{E,K}}{\partial K}$$

Talk by T. Cailleteau et this confernce

Cailleteau, Linsefors, AB, submitted to CGQ 2013

X Indirect probe : Probability for inflation



Those situations are not equally probable. In the pre-bounce oscillatory phase:

$$\rho = \rho_0 \left(1 - \frac{1}{2}\sqrt{3\kappa\rho_0}\left(t + \frac{1}{2m}\sin(2mt + 2\delta)\right)\right)^{-2}$$

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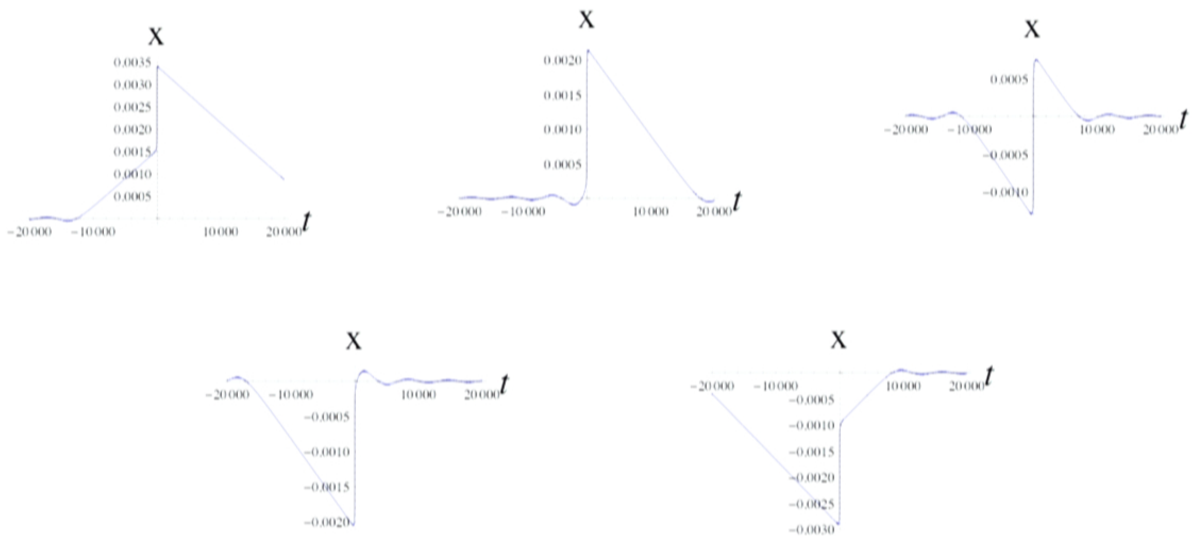
Linsefors, A.B., Phys. Rev. D, 2013

X Indirect probe : Probability for inflation



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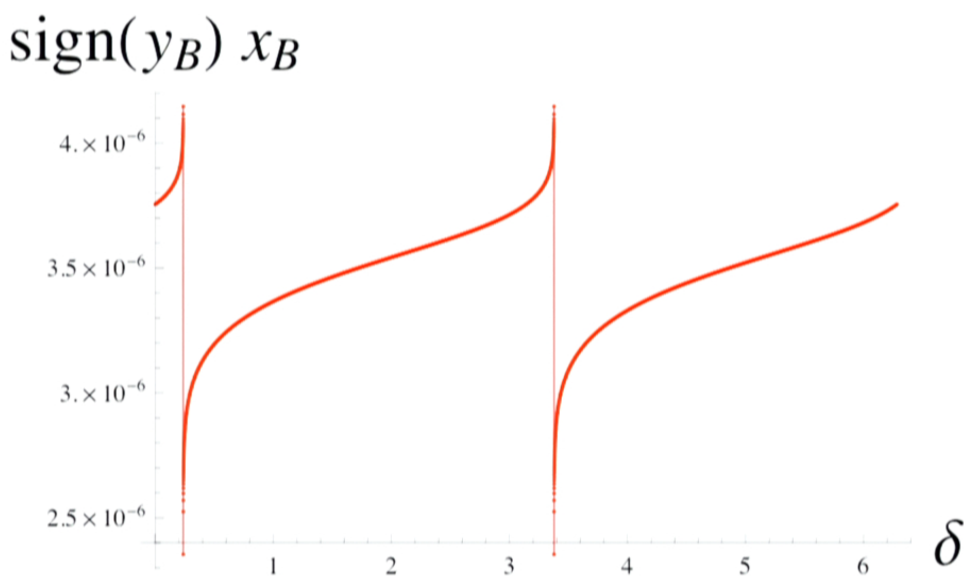
Linsefors, A.B., Phys. Rev. D, 2013

$$x = \sqrt{\frac{\rho}{\rho_c}} \sin(mt + \delta) \ , \ y = \sqrt{\frac{\rho}{\rho_c}} \cos(mt + \delta)$$

In addition of being the obviously expected distribution for any oscillatory process of this kind, a flat proability for δ will be preserved over time within the pre-bounce oscillation phase, making it a very natural choice for initial conditions.



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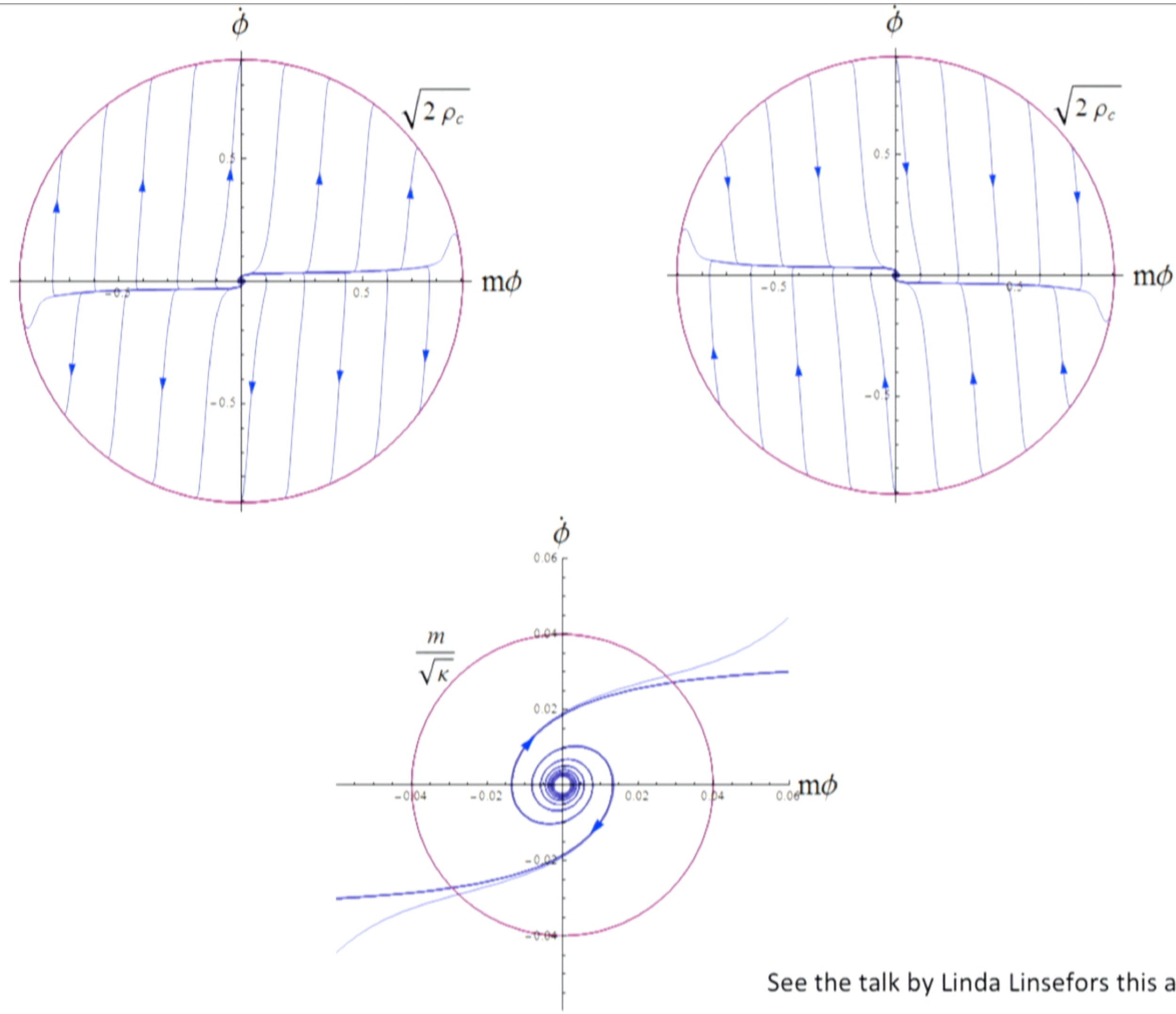

See the talk by Linda Linsefors this arfternoon

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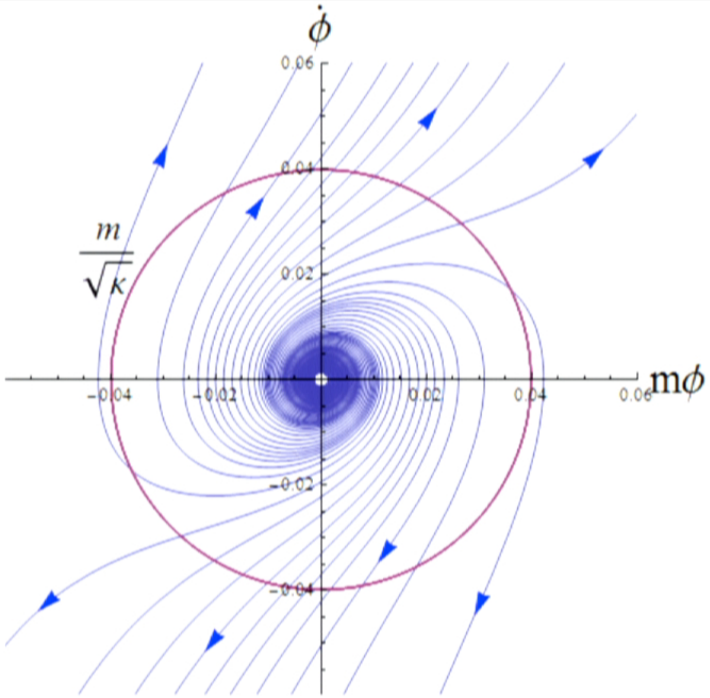

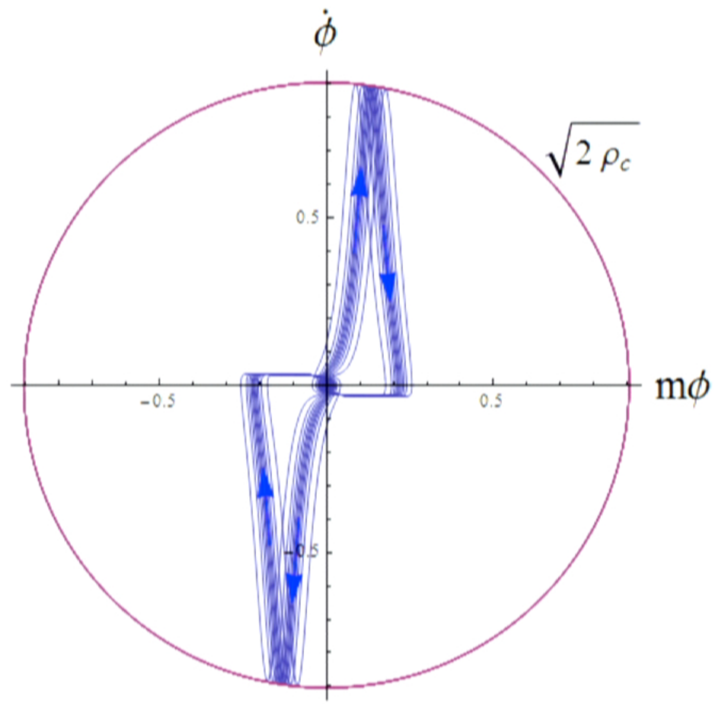

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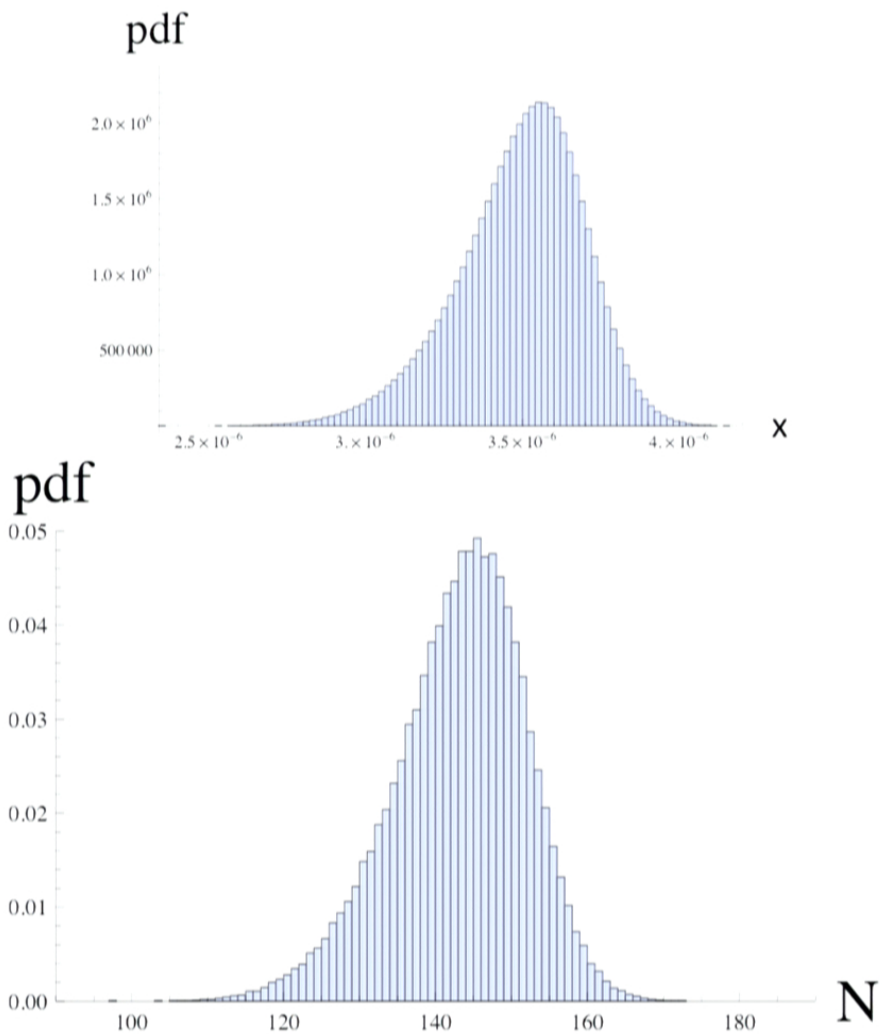

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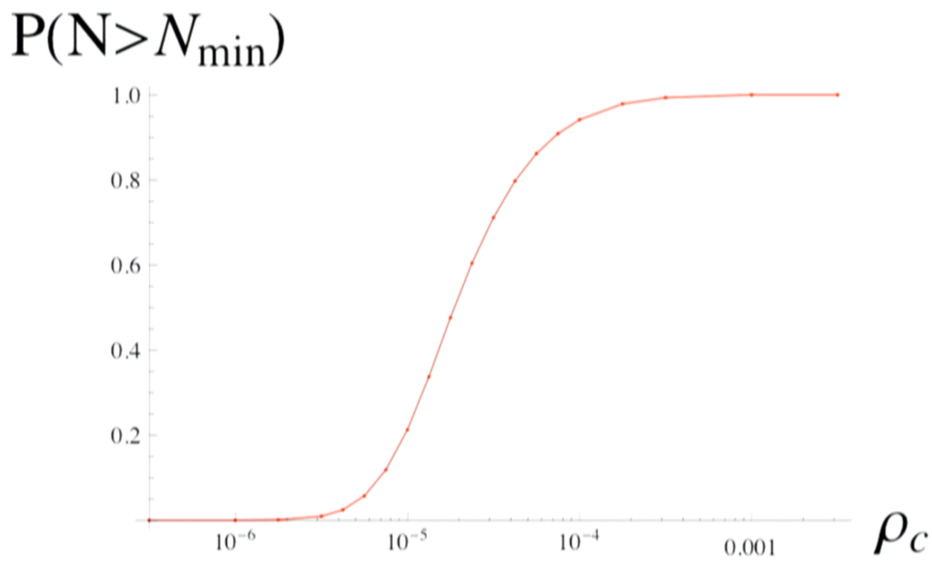
One can also constrain rho_c and therefore gamma to have a long enough period of inflation:



The main results of this analysis are that gamma<10.1 at 95% confidence level and gamma<11.9 at 99% confidence level.

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XI. Anisotropies

the shear term basically scales as 1/a^6.

$$ds := -N^2 d\tau^2 + a_1^2 dx^2 + a_2^2 dy^2 + a_3^2 dz^2,$$

$$\mathcal{H}_G = \frac{N}{\kappa\gamma^2}\left(\sqrt{\frac{p_1p_2}{p_3}}c_1c_2 + \sqrt{\frac{p_2p_3}{p_1}}c_2c_2 + \sqrt{\frac{p_3p_1}{p_2}}c_2c_3\right)$$

$$\{c_i, p_j\} = \kappa\gamma\delta_{ij} \quad , \quad \{\phi_n, \pi_m\} = \delta_{mn}$$

$$a_1 = \sqrt{\frac{p_2p_3}{p_1}} \quad \text{and cyclic expressions.}$$

Quantization $$c_i \rightarrow \frac{\sin(\bar{\mu}_i c_i)}{\bar{\mu}_i} \quad \bar{\mu}_1 = \lambda\sqrt{\frac{p_1}{p_2p_3}}$$

$\lambda = \sqrt{\Delta}$ LQG area gap

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Linsefors, A.B., arXiv:1305.4516

Defininng the quantum shear as

$$\sigma_Q^2 := \frac{1}{3\lambda^2\gamma^2}\left(1 - \frac{1}{3}\Big[\cos(\bar{\mu}_1 c_1 - \bar{\mu}_2 c_2) + \cos(\bar{\mu}_2 c_2 - \bar{\mu}_3 c_3) + \cos(\bar{\mu}_3 c_3 - \bar{\mu}_1 c_1)\Big]\right)$$

We are led to the generalized Friedmann equation :

$$H^2 = \sigma_Q^2 + \frac{\kappa}{3}\rho - \lambda^2\gamma^2\left(\frac{3}{2}\sigma_Q^2 + \frac{\kappa}{3}\rho\right)^2$$

With

$$\rho \le \rho_c := \frac{3}{\kappa}\frac{1}{\lambda^2\gamma^2} \qquad \sigma_Q^2 \le \sigma_{Q\,c}^2 := \frac{4}{9}\frac{1}{\lambda^2\gamma^2}$$

Defining $h_1 := \bar{\mu}_1 c_1 = \lambda\sqrt{\frac{p_1}{p_2 p_3}}c_1$ and cyclic expressions.

The dynamics can be studied into the details

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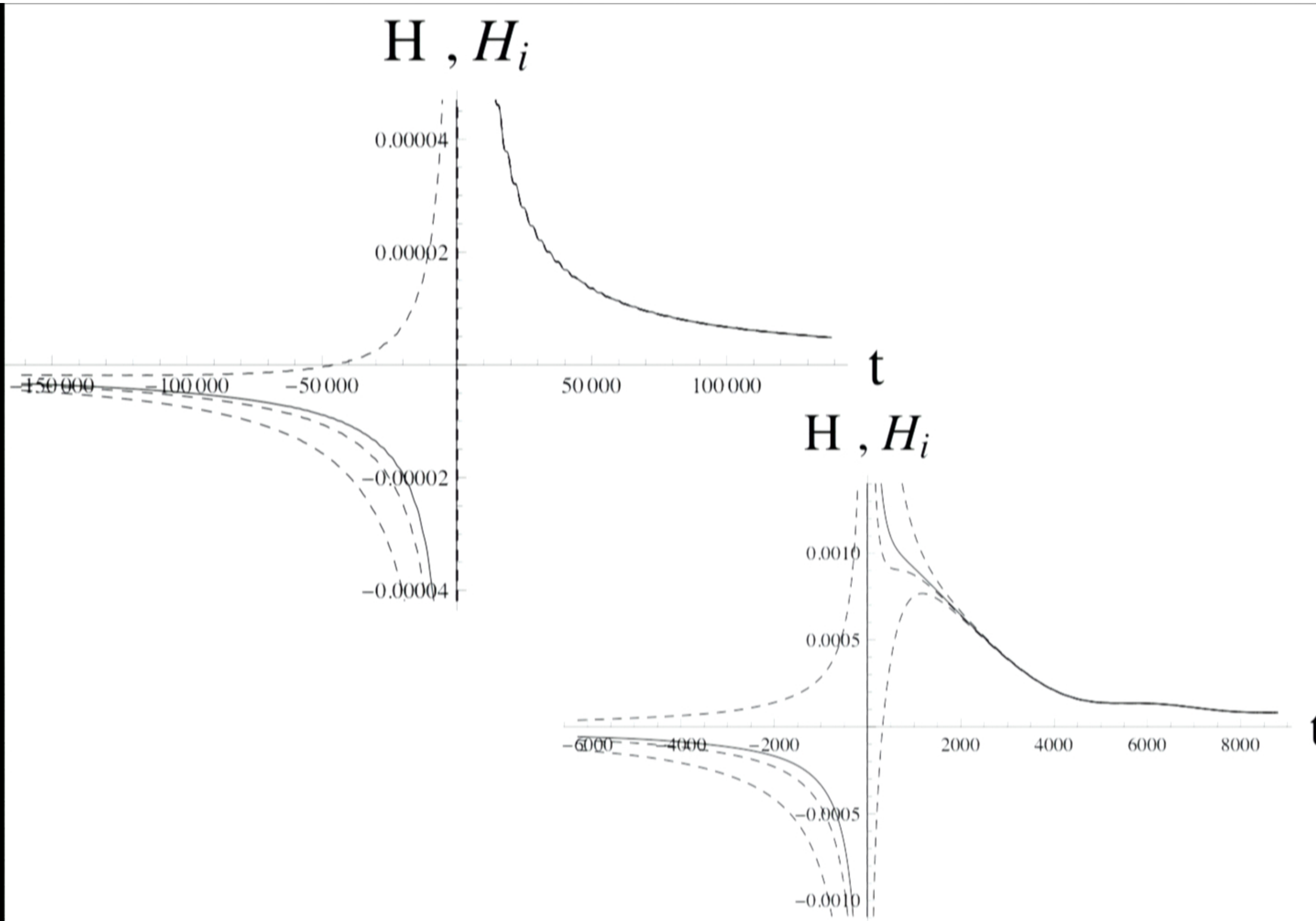

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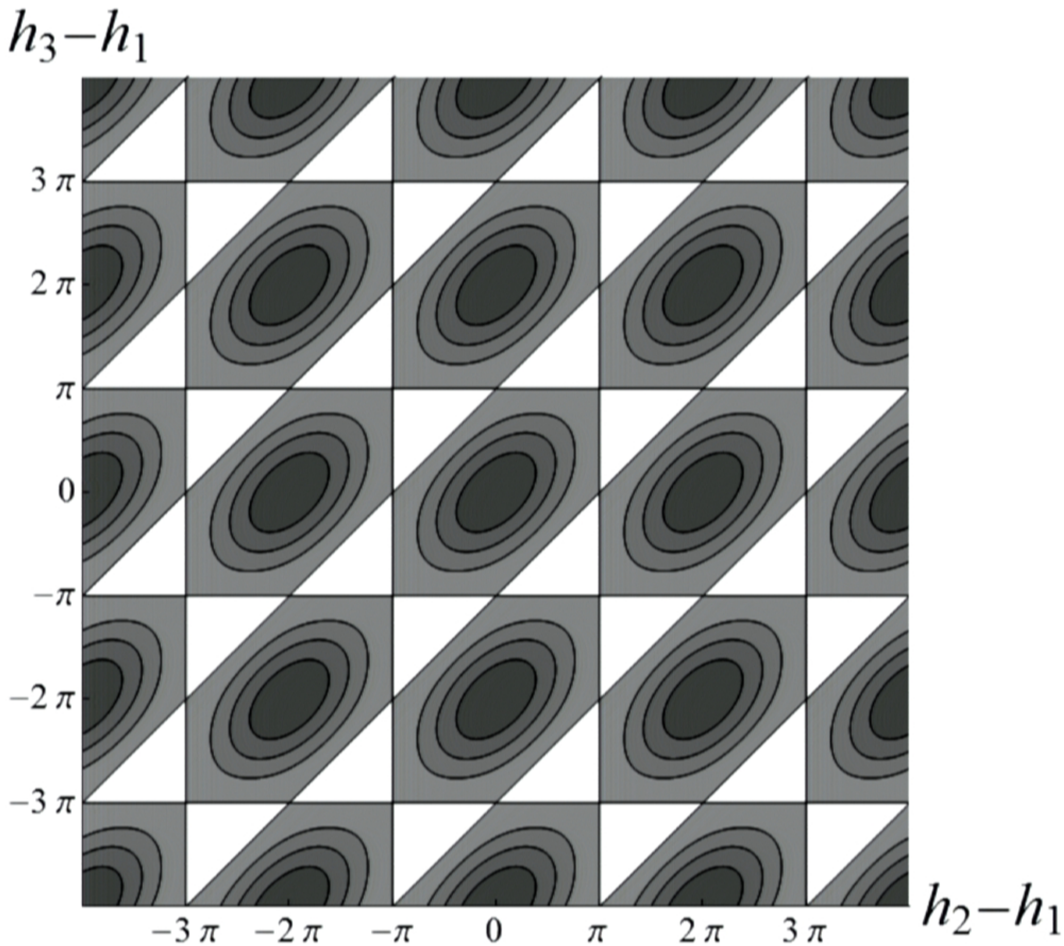
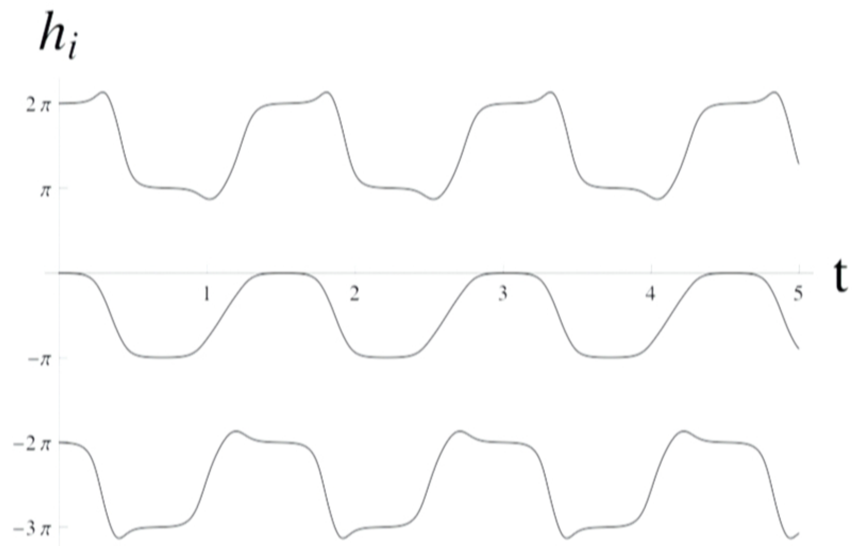
White areas are forbidden



Linsefors, A.B., arXiv:1305.4516

there are infinitely many solutions that never reach a classical limit.



Important issue for LQC.
- If initial conditions at the bounce → there are infinitely many more cases leading to universes that do not resemble ours than cases leading to a classically expanding universe.
- If initial conditions in the classically contracting phase, this problem is evaded. But we face another one: what is the ”natural” initial shear?

→ What about inflation ?

XII. Black holes

State couting through isolated horizons, introduced as a boundaries of the manifold before quantization.

For a given area A, physical states arise from a punctured sphere whose punctures carry quantum labels. J is the spin carrying information about the area and m is its projection carrying information about the curvature. They satisfy:

$$A - \Delta \leq 8\pi\gamma\ell_P^2 \sum_{p=1}^{N} \sqrt{j_p(j_p+1)} \leq A + \Delta \qquad \sum_{p=1}^{N} m_p = 0$$

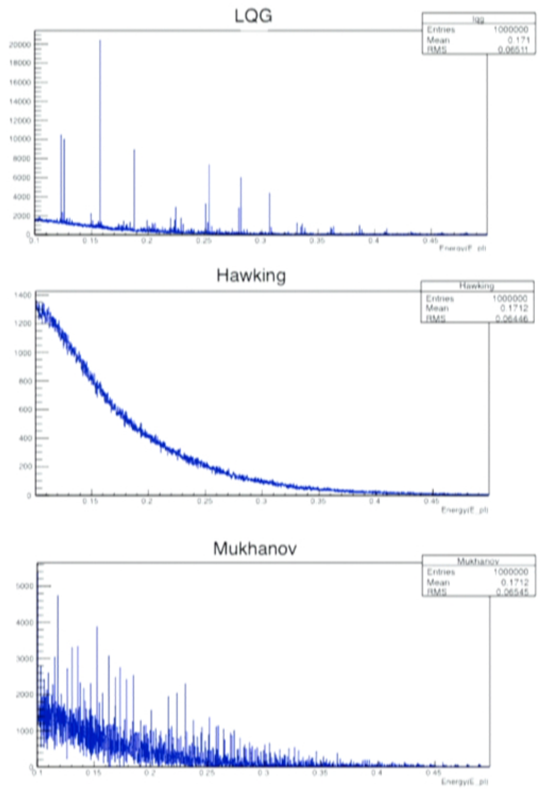
Can we use evaporating BHs as a probe of LQG ?

3 approaches can be considered.

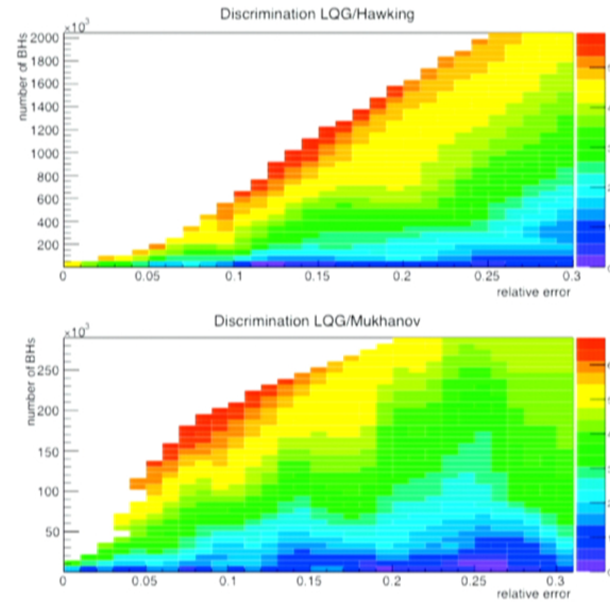
A.B., Cao, Diaz-Polo, Grain, Cailleteau, Phys. Rev. Lett., 107, 261301, 2011
Cao, A.B., arXiv:1111.1975

A) Emission Lines in the Planck Regime.

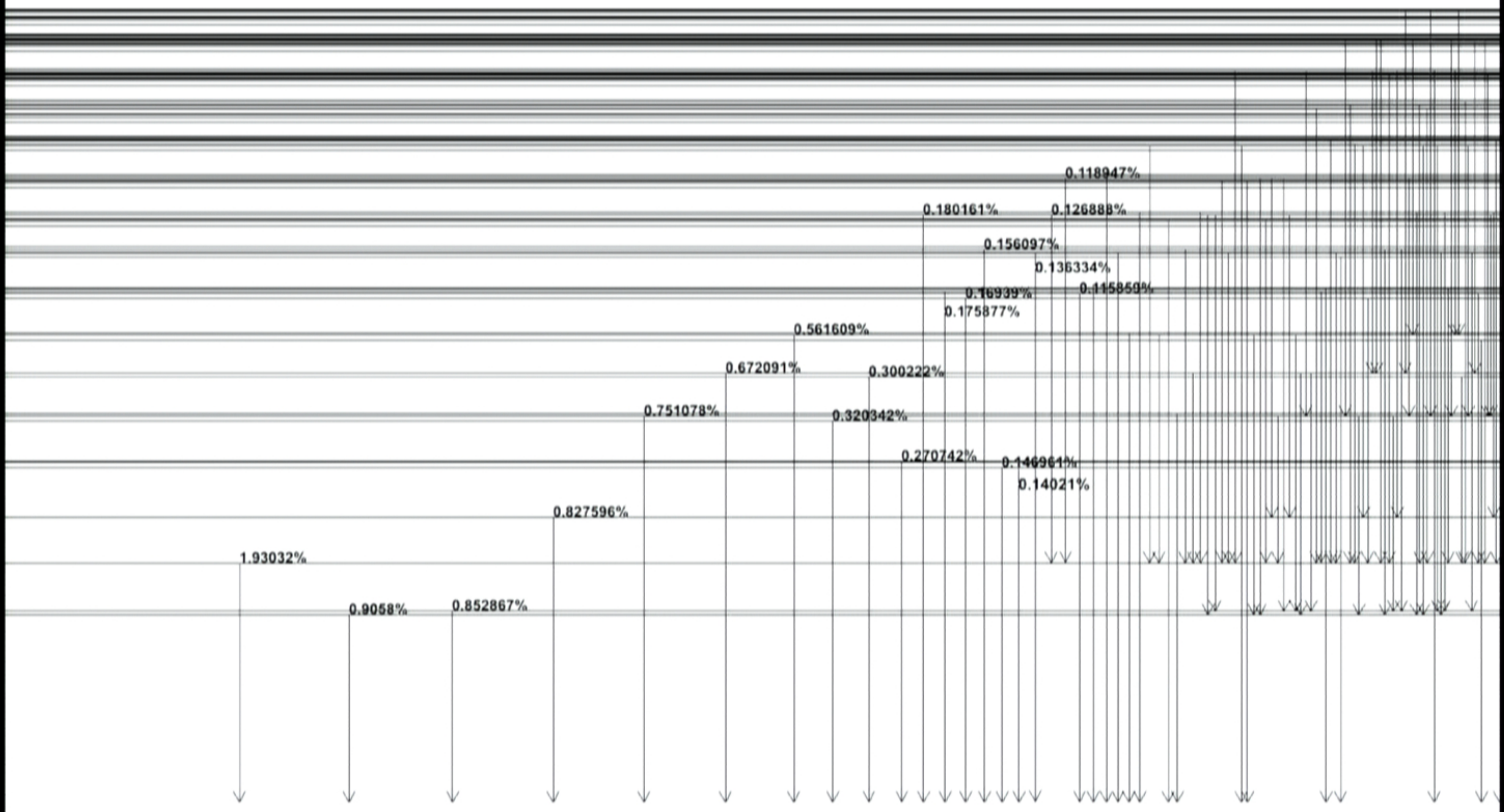
We used the full greybody factors and same endpoint mass



spectra



KS test



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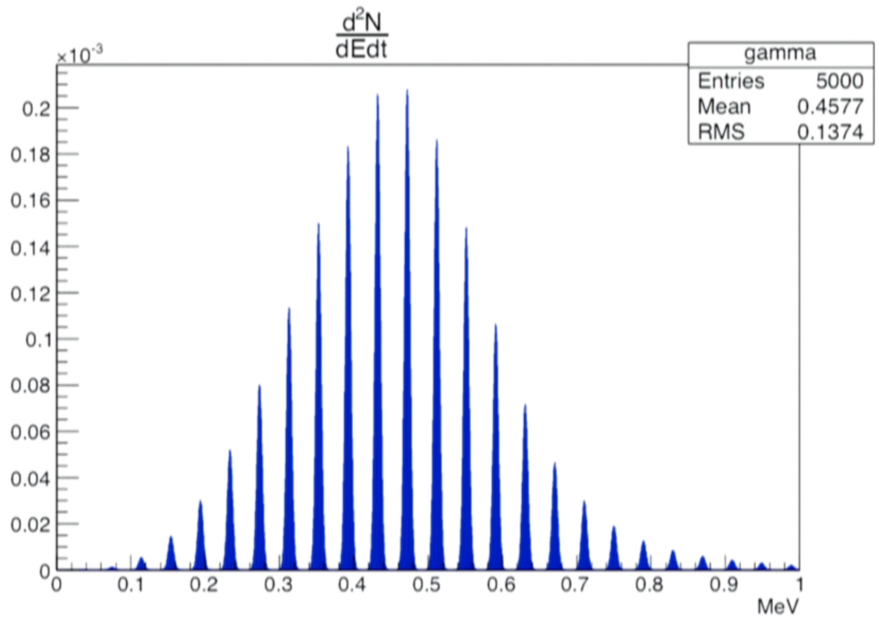
C) Peaks in the Higher-Mass Regime

Some LQG specific features (large scale area periodicity) lead to broader peaks in the spectrum.

Realistic models for PBH formation are associated with phase transitions → black holes are formed at the same mass Mc. If Mc > M∗, their mass has not changed.
The number of photons received would be a macroscopic signal for a large range of masses and densities.



Prospects

A little story…

→ universal effects ?

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