Title: Can Quantum correlations be Explained Casually

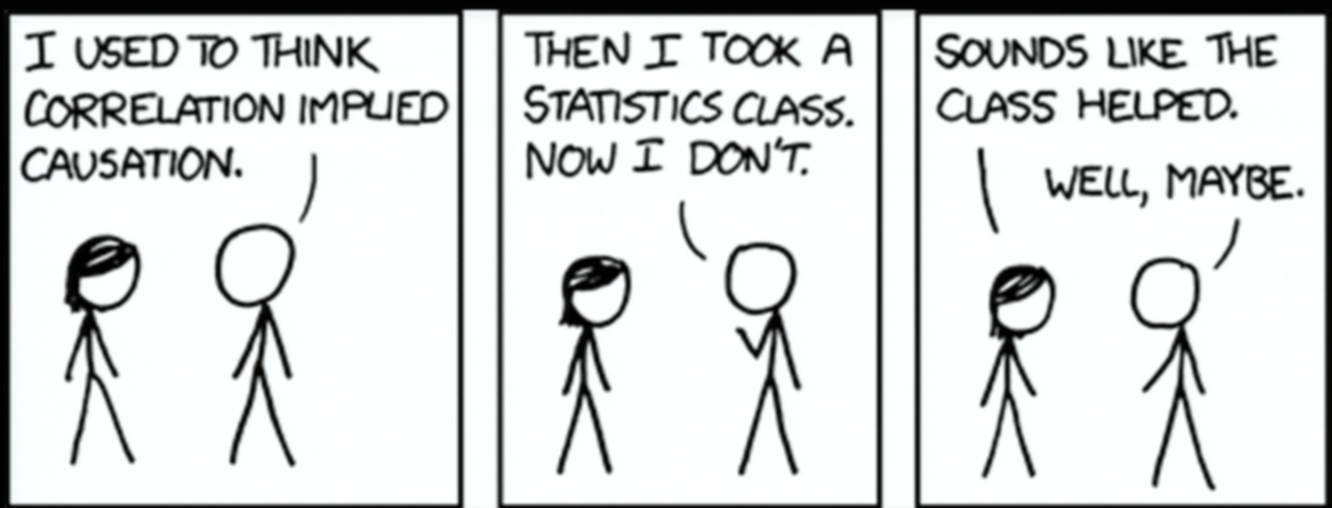
Date: Jul 15, 2013 10:30 AM

URL: http://pirsa.org/13070030

Abstract:

Can Quantum Correlations Be Explained Causally?

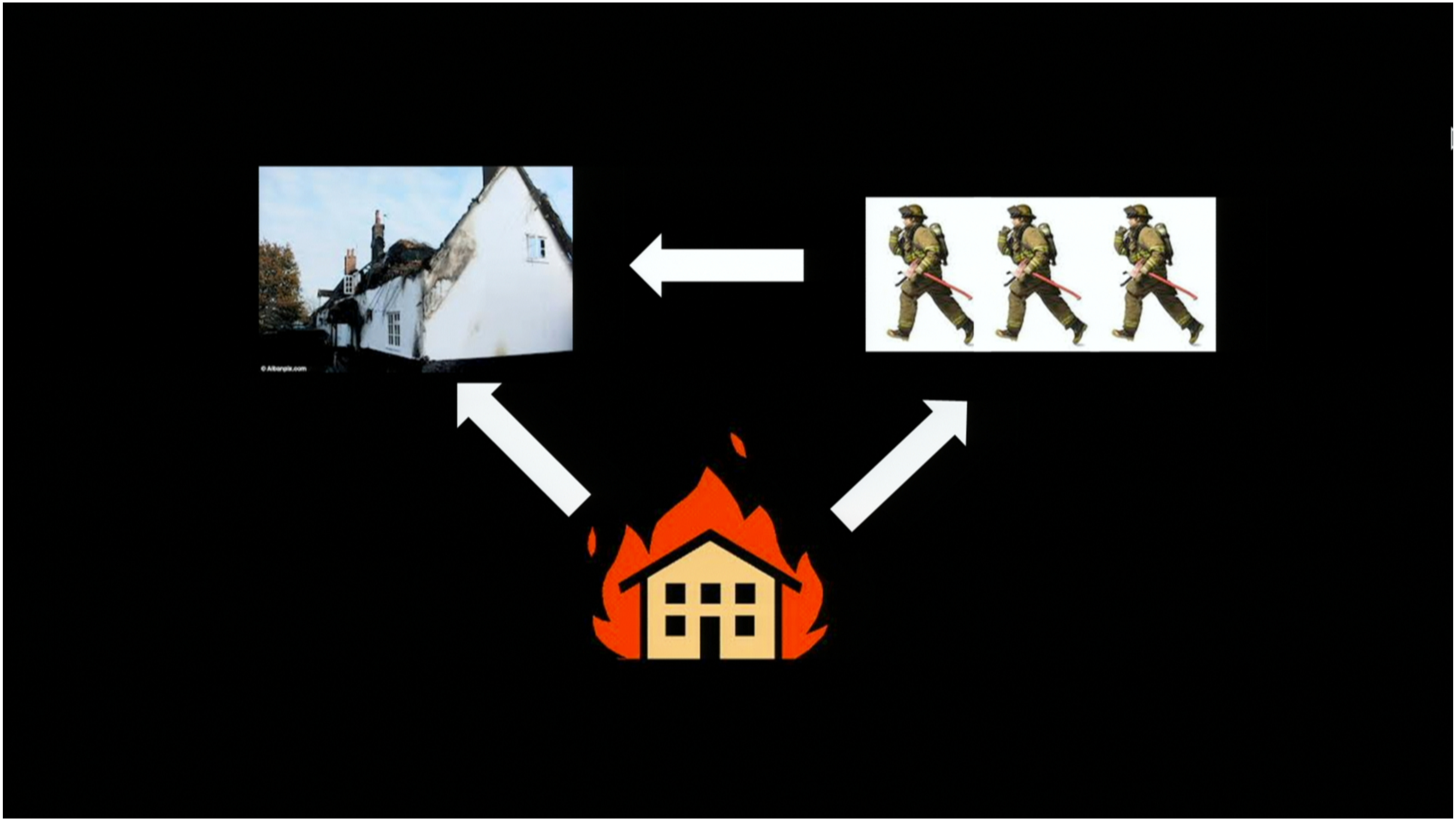
Rob Spekkens
Perimeter Institute



From XKCD comics

ISSYP 2013


© Albanpix.com



Simpson's Paradox

P(recovery | drug) > P(recovery | no drug)

Simpson's Paradox

P(recovery | drug) > P(recovery | no drug)

P(recovery | drug, male) < P(recovery | no drug, male)

P(recovery | drug, female) < P(recovery | no drug, female)

Recovery probability

	drug	no drug
male	180/300 = 60%	70/100 = 70%
female	20/100 = 20%	90/300 = 30%
combined	200/400 = 50%	160/400 = 40%

Simpson's Paradox

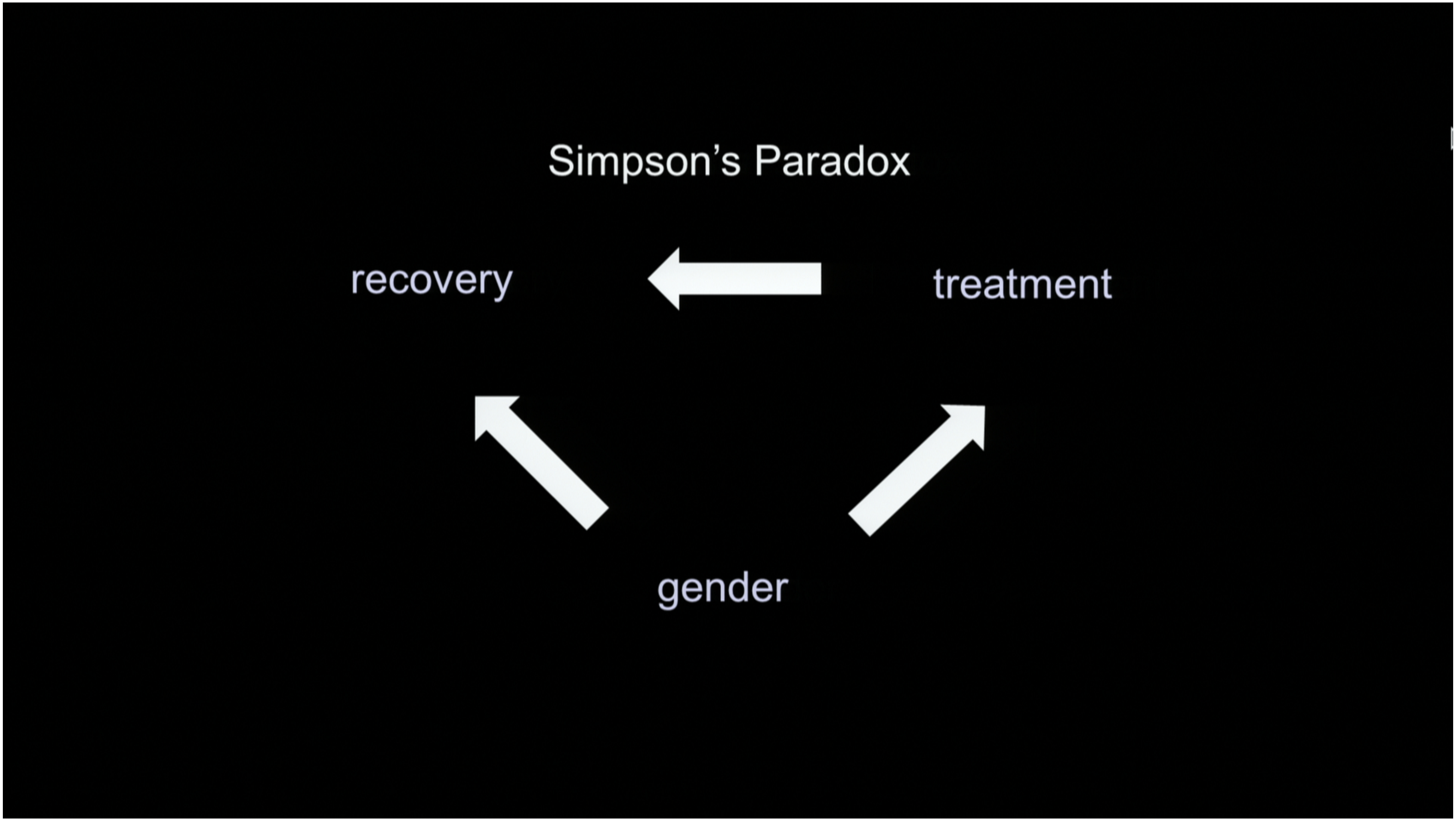
P(recovery | drug) > P(recovery | no drug)

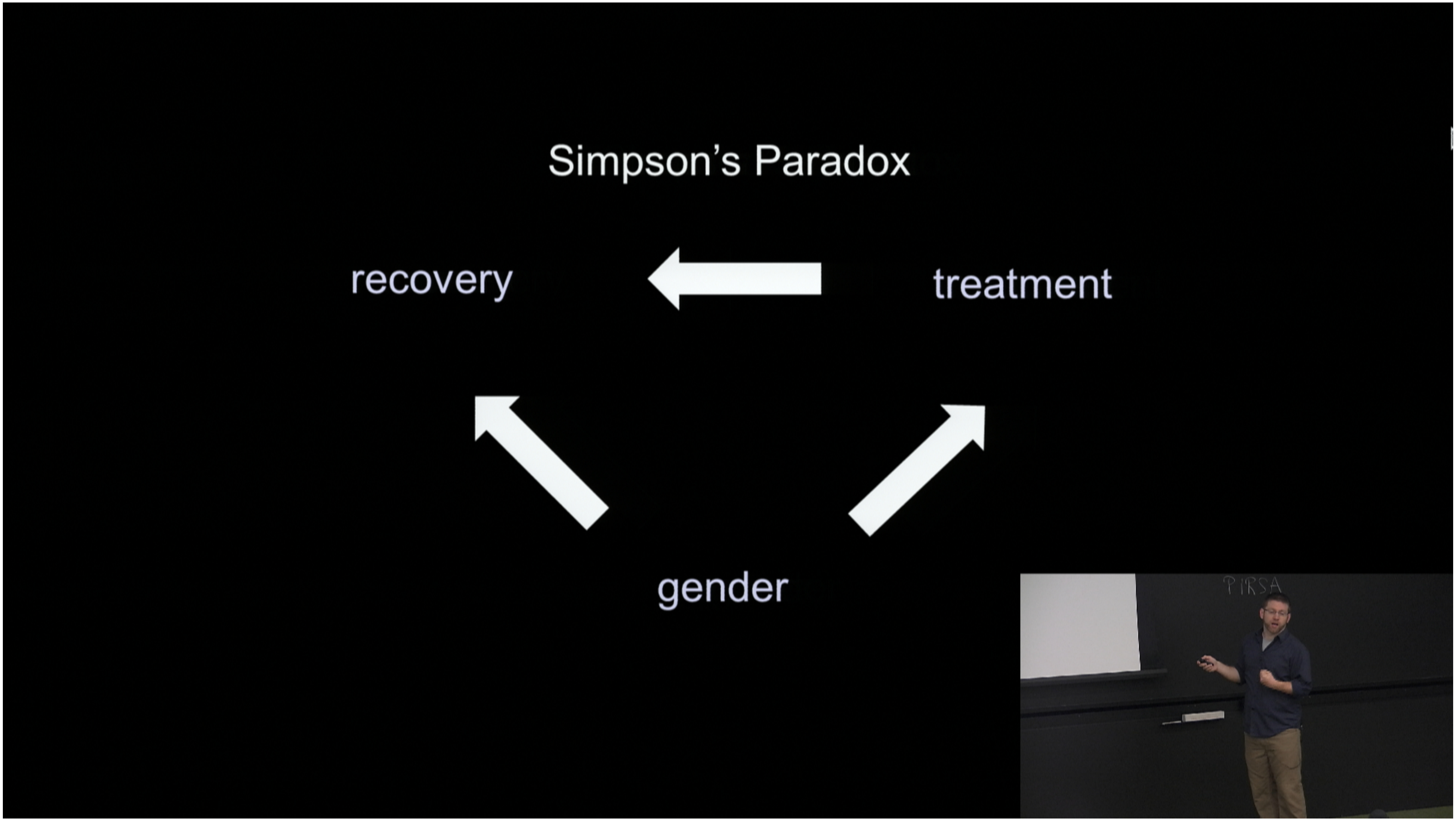
P(recovery | drug, male) < P(recovery | no drug, male)

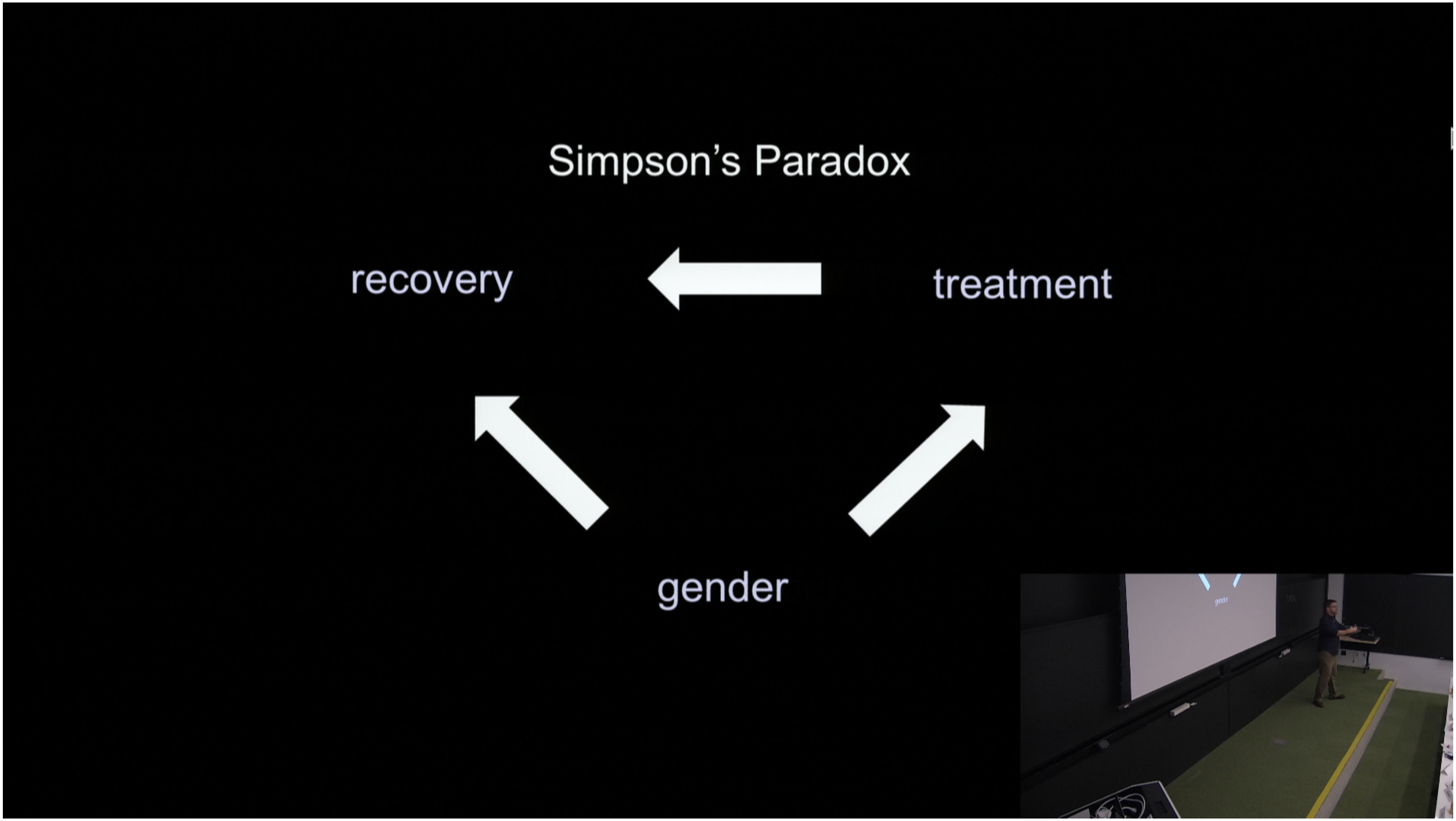
P(recovery | drug, female) < P(recovery | no drug, female)

Recovery probability

	drug	no drug
male	180/300 = 60%	70/100 = 70%
female	20/100 = 20%	90/300 = 30%
combined	200/400 = 50%	160/400 = 40%


Simpson's Paradox
recovery
treatment
gender

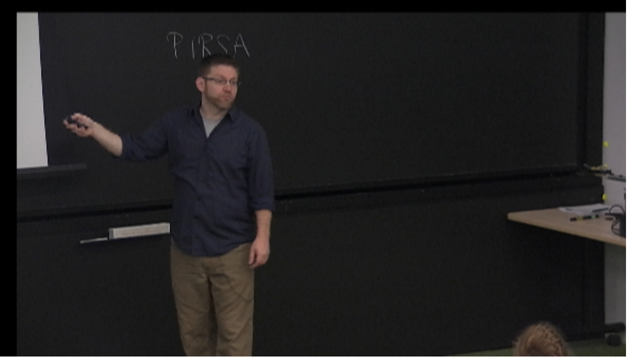

Simpson's Paradox
recovery
treatment
gender
PIRSA


Simpson’s Paradox
recovery
treatment
gender

Simpson's Paradox

P(recovery | do (drug)) ≠ P(recovery | observe (drug))

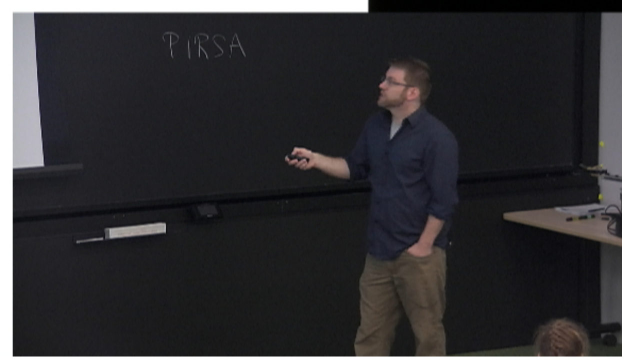
causation correlation

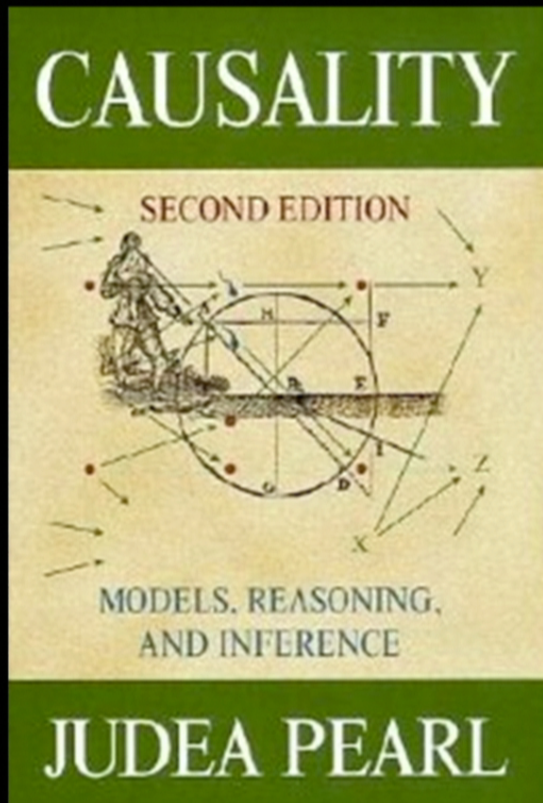


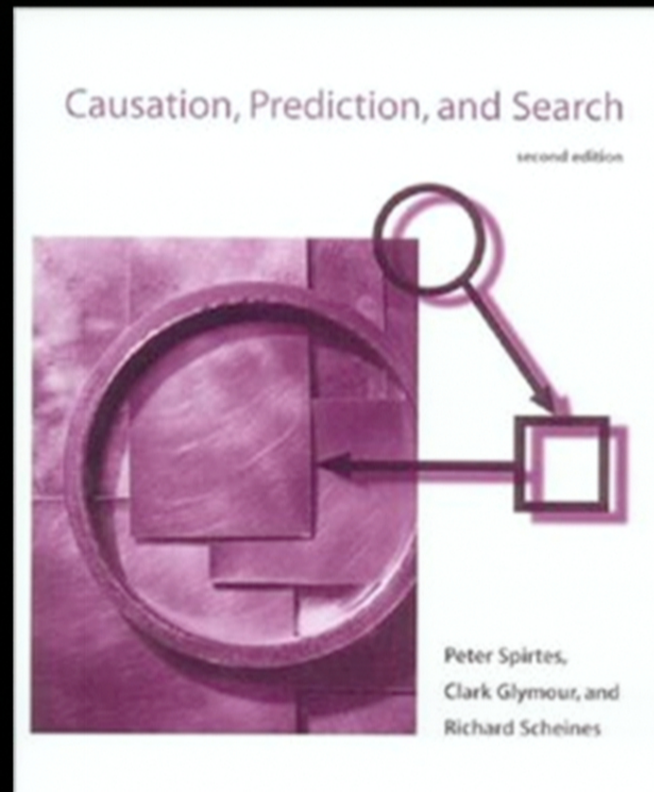
What formalism can we use to describe causal relations?

How do we come to have knowledge of causal relations?
(“we” = children, scientists, machine learning systems)

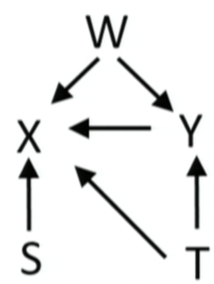
How do we come to have knowledge of causal relations in uncontrolled experiments?




CAUSALITY
SECOND EDITION
MODELS, REASONING,
AND INFERENCE
JUDEA PEARL


Causation, Prediction, and Search
second edition
Peter Spirtes,
Clark Glymour, and
Richard Scheines

Causal Model

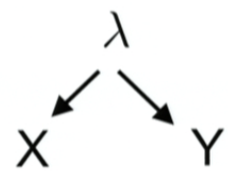
Causal Structure	Causal-Statistical Parameters
	$P(W)$ $P(S)$ $P(T)$ $P(X\|S,T,W,Y)$ $P(Y\|T,W)$

Reichenbach's principle

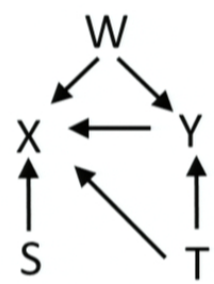
No correlation without causation!

If X and Y are correlated, then

$X \longrightarrow Y$ or $X \longleftarrow Y$ or

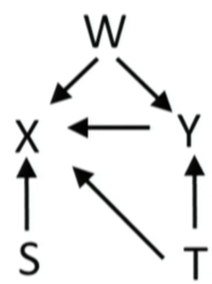


Causal Model

Causal Structure	Causal-Statistical Parameters
	$P(W)$ $P(S)$ $P(T)$ $P(X\|S,T,W,Y)$ $P(Y\|T,W)$

- Parentless variables are independently distributed

Given a causal model, what sorts of correlations can arise?

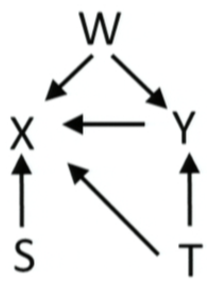


$P(W)$
$P(S)$
$P(T)$
$P(X|S,T,W,Y)$
$P(Y|T,W)$

$$P(X,Y,W,S,T) = P(X|S,T,W,Y)P(Y|T,W)P(W)P(S)P(T)$$

Causal inference algorithms seek to solve the inverse problem

Given a causal model, what sorts of correlations can arise?



$P(W)$

$P(S)$

$P(T)$

$P(X|S,T,W,Y)$

$P(Y|T,W)$

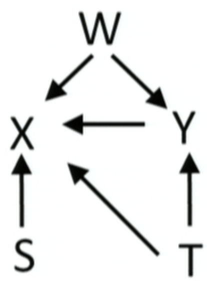
$$P(X,Y,W,S,T) = P(X|S,T,W,Y)P(Y|T,W)P(W)P(S)P(T)$$

Def'n: A and B are marginally independent

$$P(A|B) = P(A)$$
$$P(B|A) = P(B)$$
$$P(A,B) = P(A)P(B)$$

Denote this
$(A \perp B)$

Given a causal model, what sorts of correlations can arise?



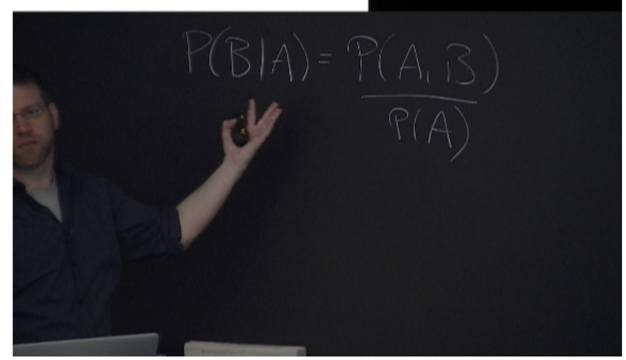
$$P(W)$$
$$P(S)$$
$$P(T)$$
$$P(X|S,T,W,Y)$$
$$P(Y|T,W)$$

$$P(X,Y,W,S,T) = P(X|S,T,W,Y)P(Y|T,W)P(W)P(S)P(T)$$

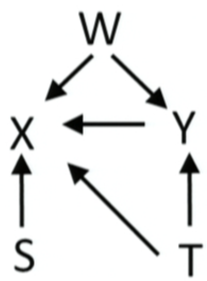
Def'n: A and B are marginally independent

$$P(A|B) = P(A)$$
$$P(B|A) = P(B)$$
$$P(A,B) = P(A)P(B)$$

Denote this
$$(A \perp B)$$



Given a causal model, what sorts of correlations can arise?



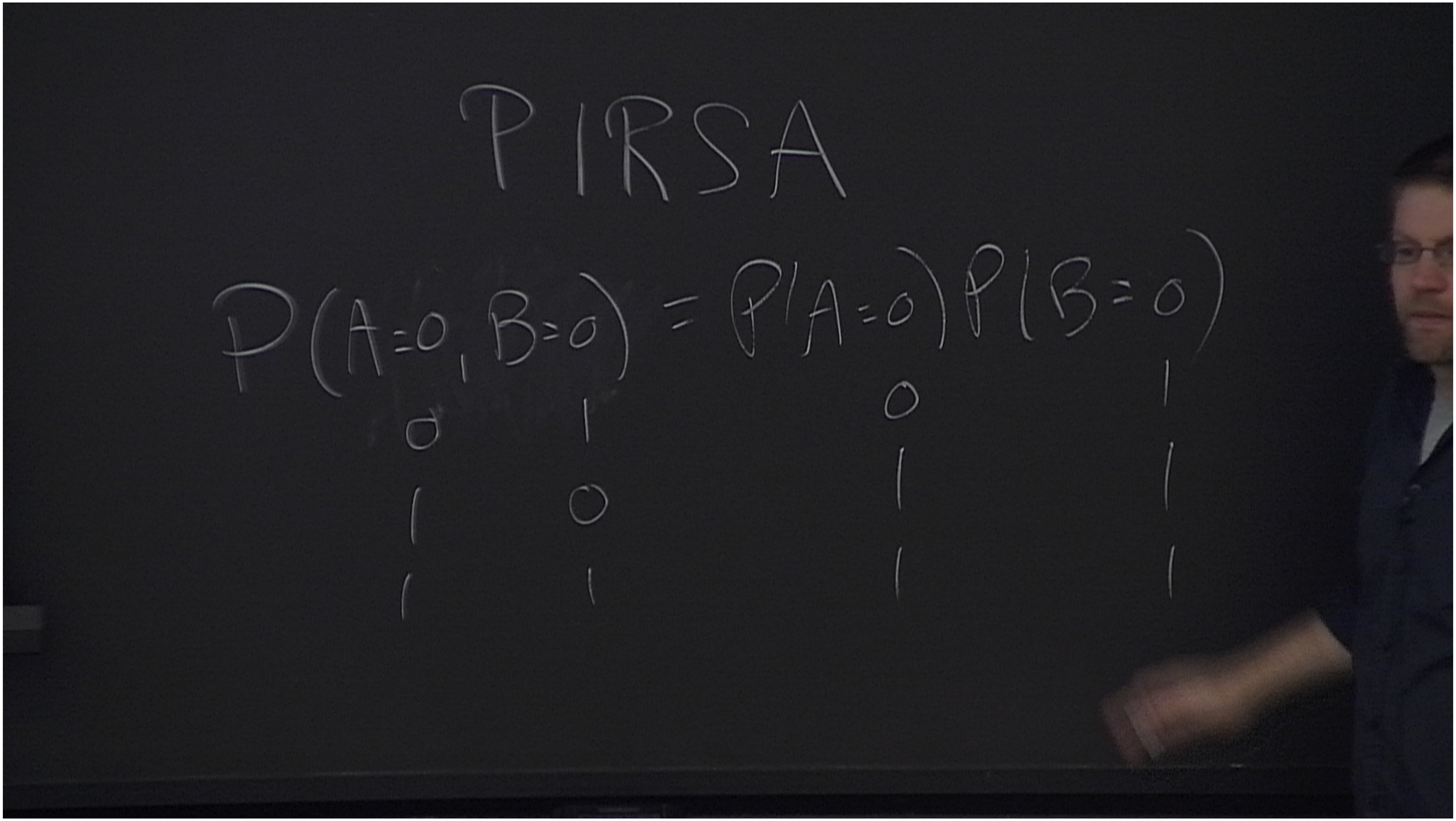
$P(W)$
$P(S)$
$P(T)$
$P(X|S,T,W,Y)$
$P(Y|T,W)$

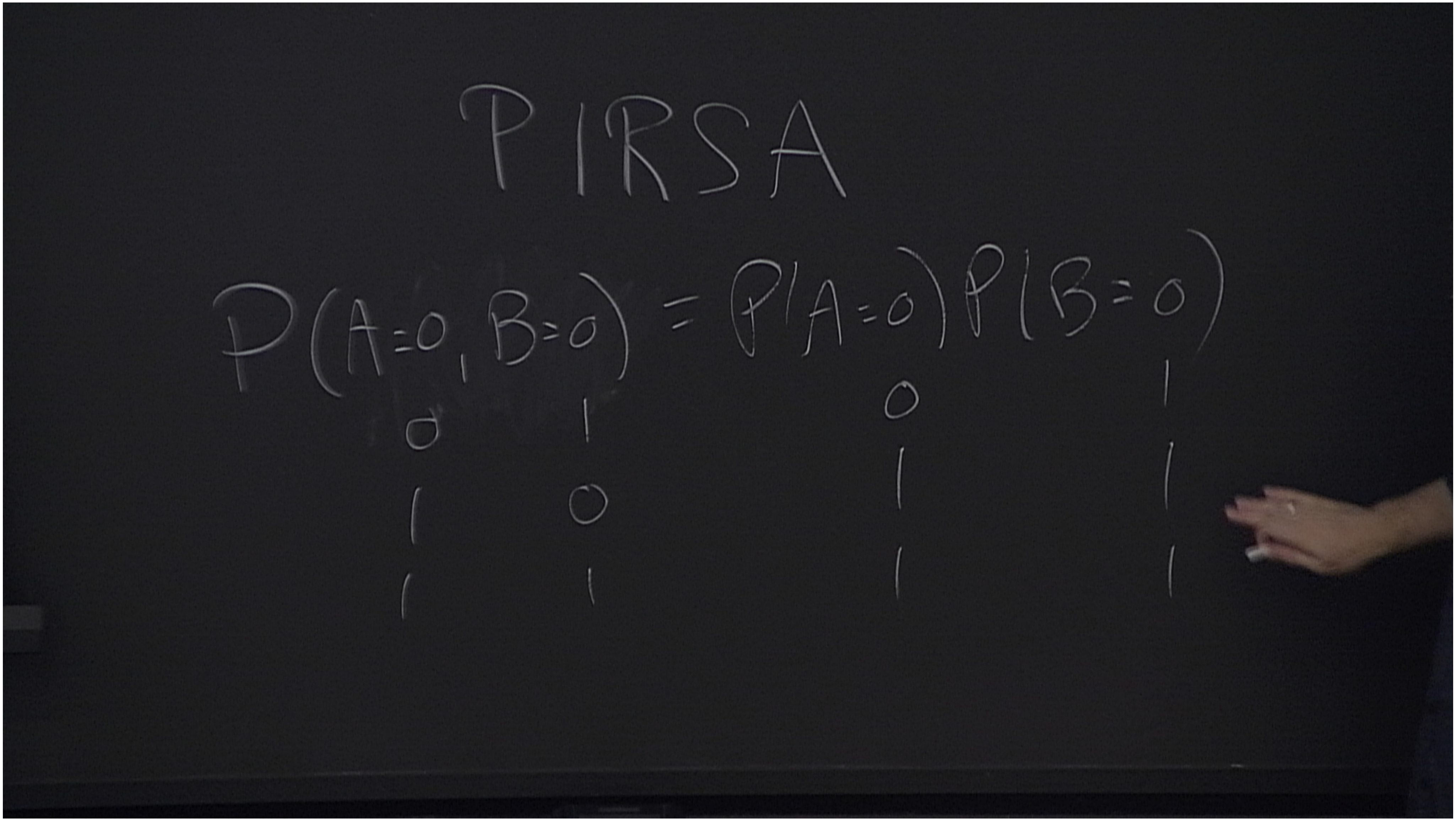
$$P(X,Y,W,S,T) = P(X|S,T,W,Y)P(Y|T,W)P(W)P(S)P(T)$$

Def'n: A and B are marginally independent

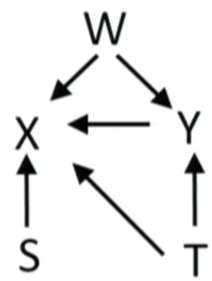
$$P(A|B) = P(A)$$
$$P(B|A) = P(B)$$
$$P(A,B) = P(A)P(B)$$

Denote this
$(A \perp B)$


PIRSA
P(A=0, B=0) = P(A=0) P(B=0)
0 1
1 0
1 1
0 1
1 1
1 1


PIRSA
P(A=0, B=0) = P(A=0) P(B=0)
0 1 0 1
1 0 1 1
1 1 1 1

Given a causal model, what sorts of correlations can arise?



$P(W)$
$P(S)$
$P(T)$
$P(X|S,T,W,Y)$
$P(Y|T,W)$

$$P(X,Y,W,S,T) = P(X|S,T,W,Y)P(Y|T,W)P(W)P(S)P(T)$$

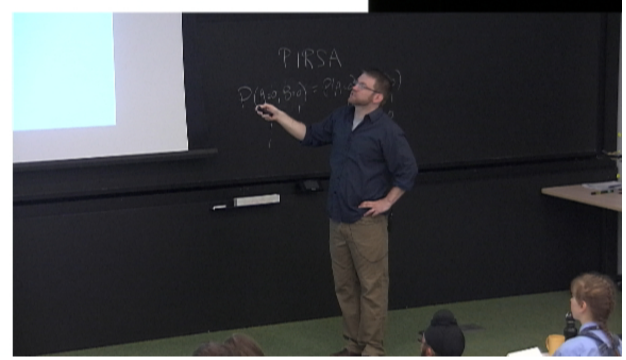
Def'n: A and B are conditionally independent given C

$$P(A|B,C) = P(A|C)$$
$$P(B|A,C) = P(B|C)$$
$$P(A,B|C) = P(A|C)P(B|C)$$

Denote this $(A \perp B|C)$

chain A → B → C ⇒ $A \not\perp C$

$(A \perp C | B)$

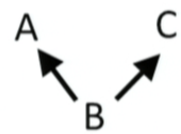


chain A → B → C ⇒

$A \not\perp C$

$(A \perp C|B)$

fork

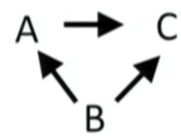


⇒

$A \not\perp C$

$(A \perp C|B)$

confounded cause



⇒

$A \not\perp C$

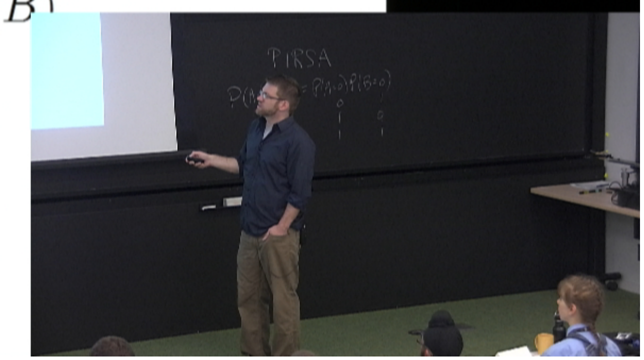
$(A \not\perp C|B)$

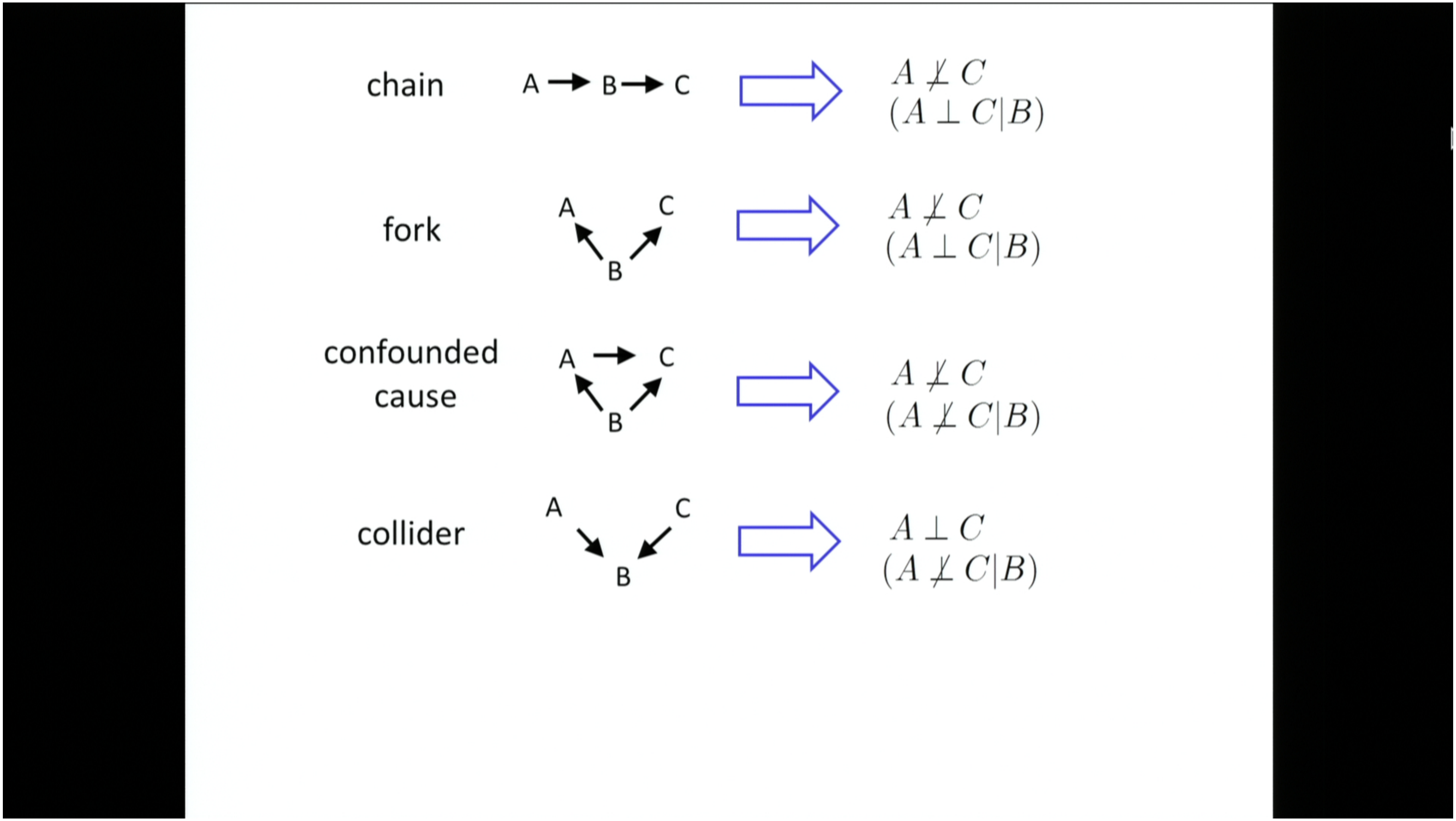
chain $A \rightarrow B \rightarrow C$ ⇒ $A \not\perp C$ $(A \perp C|B)$

fork $A \leftarrow B \rightarrow C$ ⇒ $A \not\perp C$ $(A \perp C|B)$

confounded cause $A \rightarrow C$, $A \leftarrow B \rightarrow C$ ⇒ $A \not\perp C$ $(A \not\perp C|B)$

collider $A \rightarrow B \leftarrow C$ ⇒ $A \perp C$ $(A \not\perp C|B)$

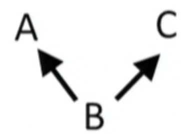



chain
A → B → C
$A \not\perp C$
$(A \perp C|B)$
fork
A C
B
$A \not\perp C$
$(A \perp C|B)$
confounded cause
A → C
B
$A \not\perp C$
$(A \not\perp C|B)$
collider
A C
B
$A \perp C$
$(A \not\perp C|B)$

chain $A \to B \to C$ ⇒

$$A \not\perp C$$
$$(A \perp C|B)$$

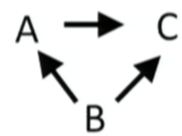
fork



⇒

$$A \not\perp C$$
$$(A \perp C|B)$$

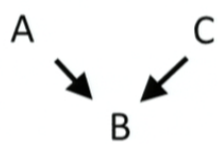
confounded cause



⇒

$$A \not\perp C$$
$$(A \not\perp C|B)$$

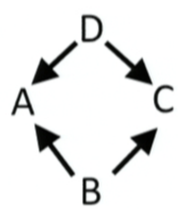
collider



⇒

$$A \perp C$$
$$(A \not\perp C|B)$$

Pair of forks

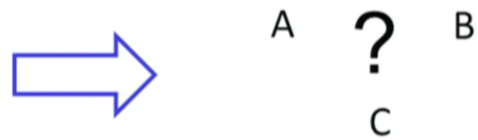


⇒

$$A \not\perp C$$
$$(A \not\perp C|B)$$
$$(A \perp C|B, D)$$

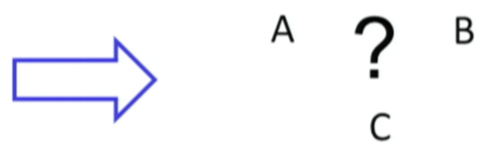
$A \perp B$

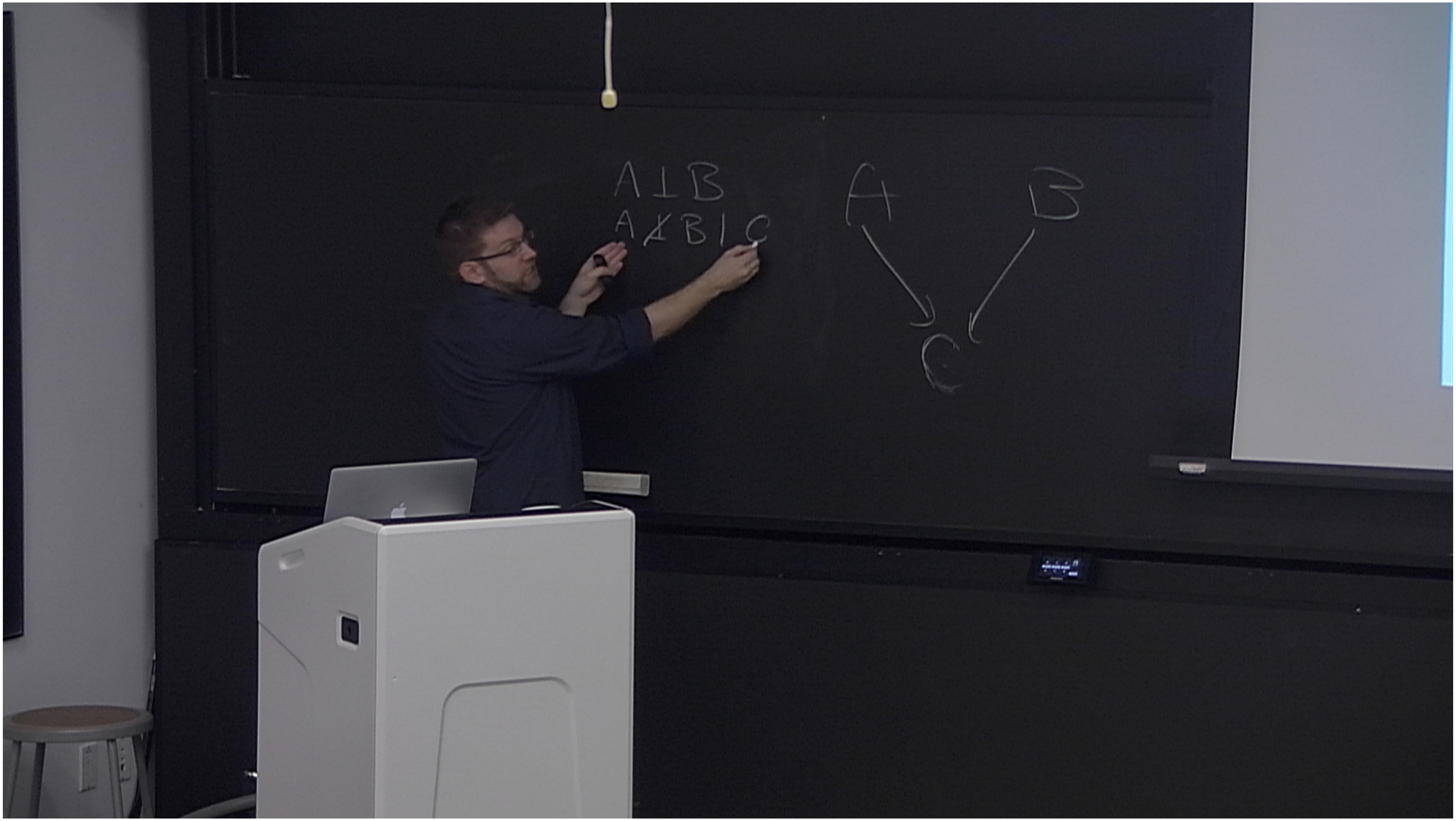
and no other independence relations



$A \perp B$

and no other independence relations

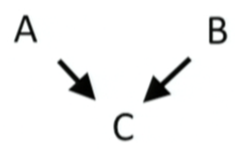



A⊥B
A⊥̸B|C
A
B
C

$A \perp B$

and no other independence relations

⇒

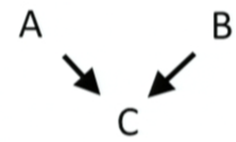


$P(A)$
$P(B)$
$P(C|A,B)$

✓

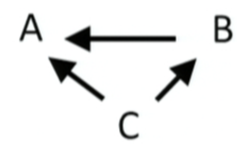
$A \perp B$

and no other independence relations



$P(A)$
$P(B)$
$P(C|A, B)$

✓

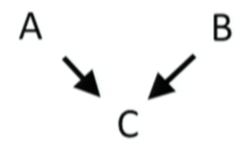


$P(C)$
$P(B|C)$
$P(A|B, C)$

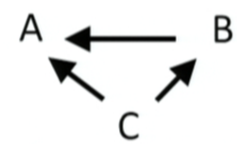
?

$A \perp B$

and no other independence relations



$P(A)$
$P(B)$
$P(C|A,B)$



$P(C)$
$P(B|C)$
$P(A|B,C)$

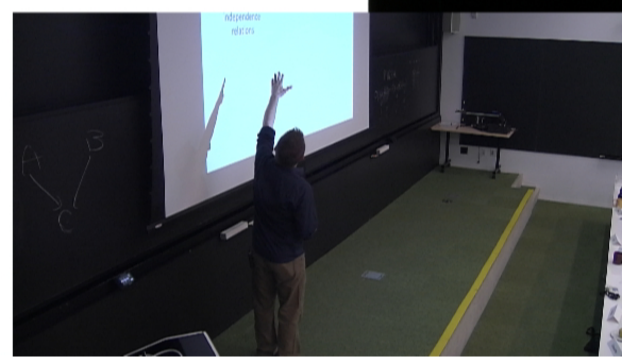
No Fine-tuning!

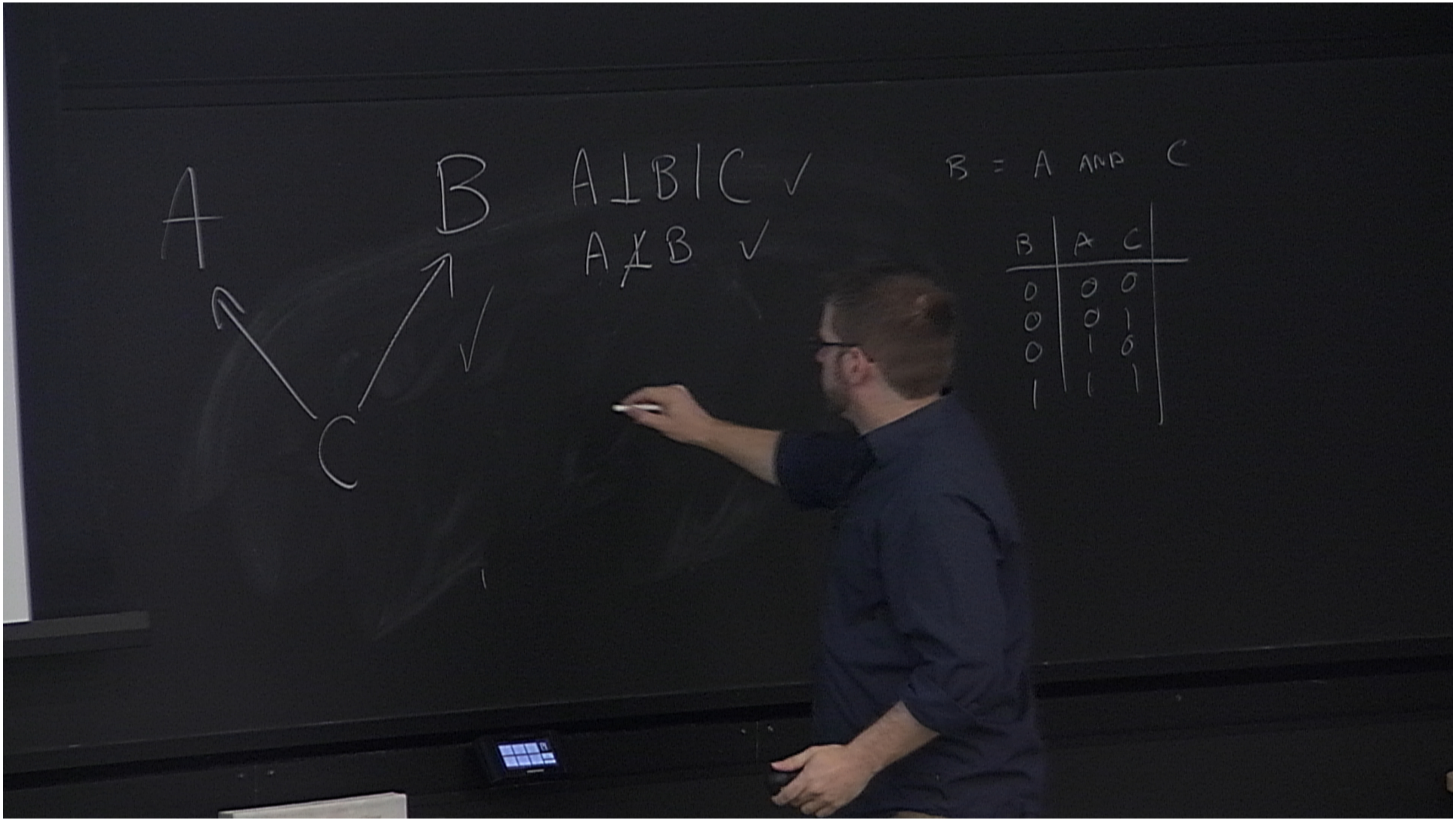
$(A \perp B | C)$

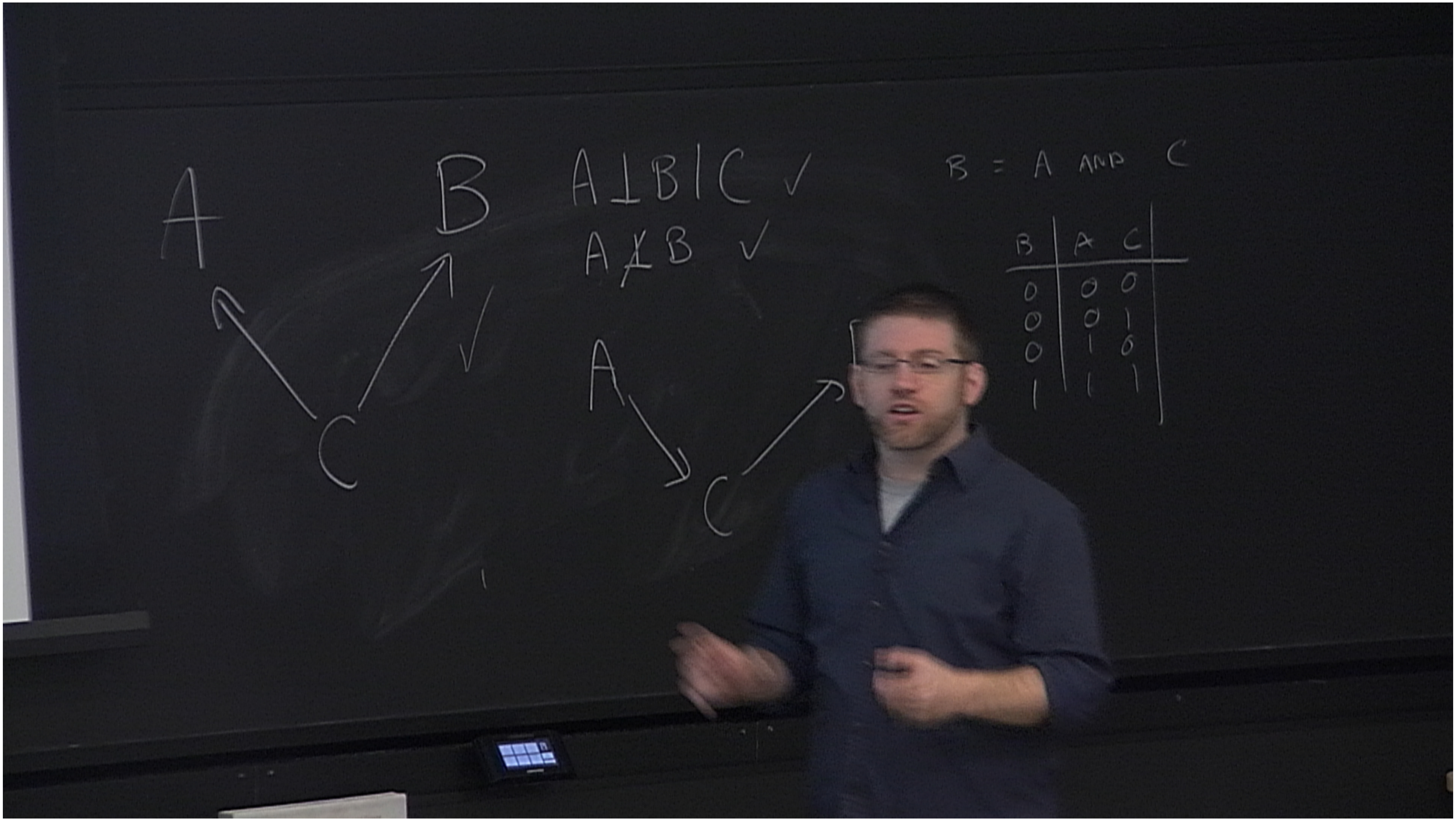
and no other independence relations

A ? B

C




A
B
C
A⊥B|C ✓
A⊥̸B ✓
B = A AND C
B | A C
0 | 0 0
0 | 0 1
0 | 1 0
1 | 1 1

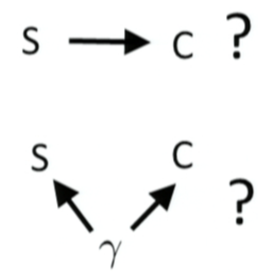

A ⊥ B | C ✓
A ⊥̸ B ✓
B = A AND C
B | A C
0 | 0 0
0 | 0 1
0 | 1 0
1 | 1 1

Allowing latent variables in the causal structure

Notational Convention
Observed variables: A, B, C,...
Latent variables: λ, μ, ν, ...

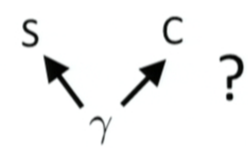
Does smoking cause lung cancer?

$S \not\perp C$ ⇒



Does smoking cause lung cancer?

$S \not\perp C$ ⇨ S → C ?



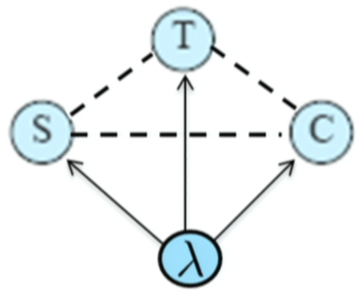
Suppose you also observe

$$S \perp C \mid T$$

and no other independences

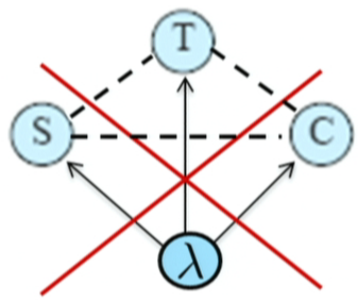
$$(S \perp C \mid T)$$

Latent common cause for S, C and T?



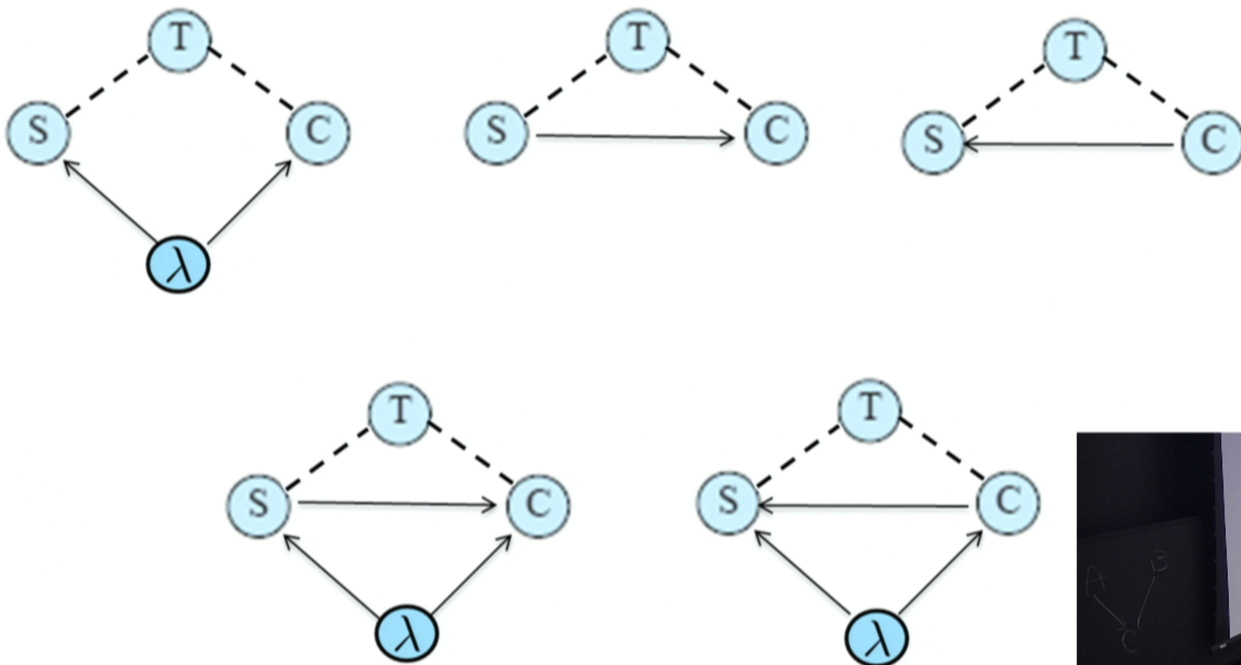
$(S \perp C \mid T)$

Latent common cause for S, C and T?



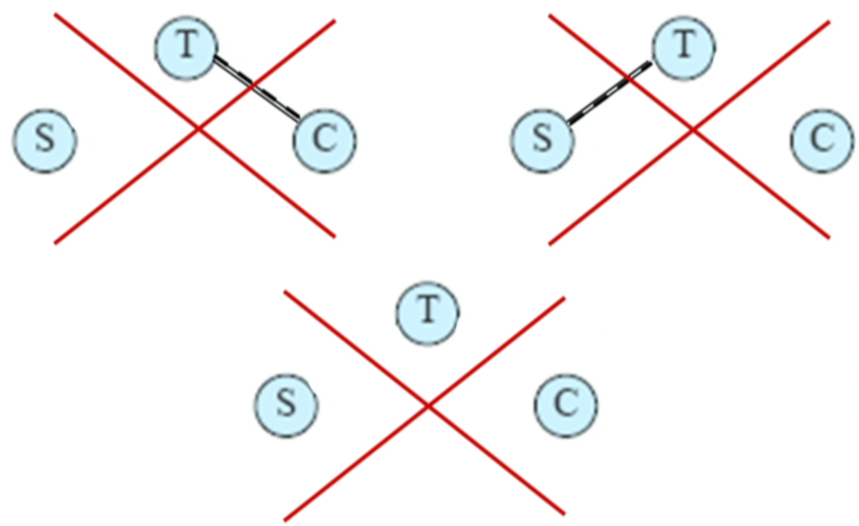
$$(S \perp C \mid T)$$

Latent common cause or direct causal relation (or both) between S and C?



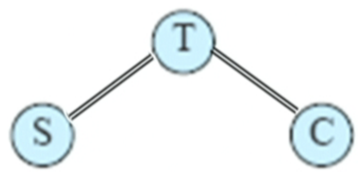
$(S \perp C \mid T)$

Marginal independence between remaining pairs?



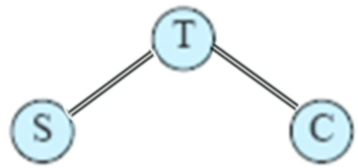
$(S \perp C \mid T)$

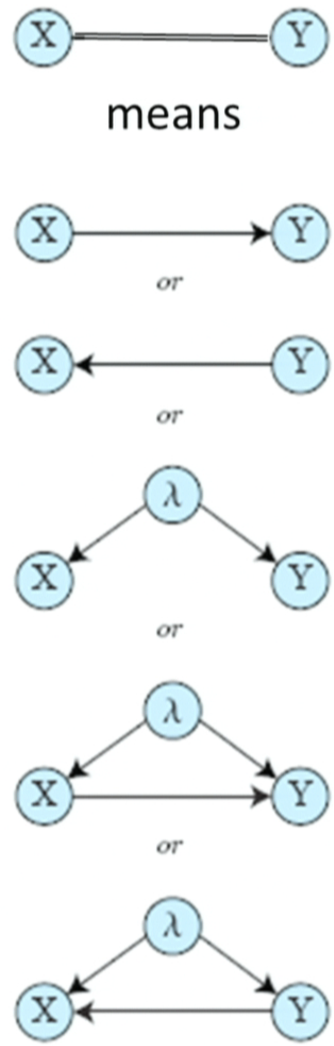
So the causal structure
must be of the form



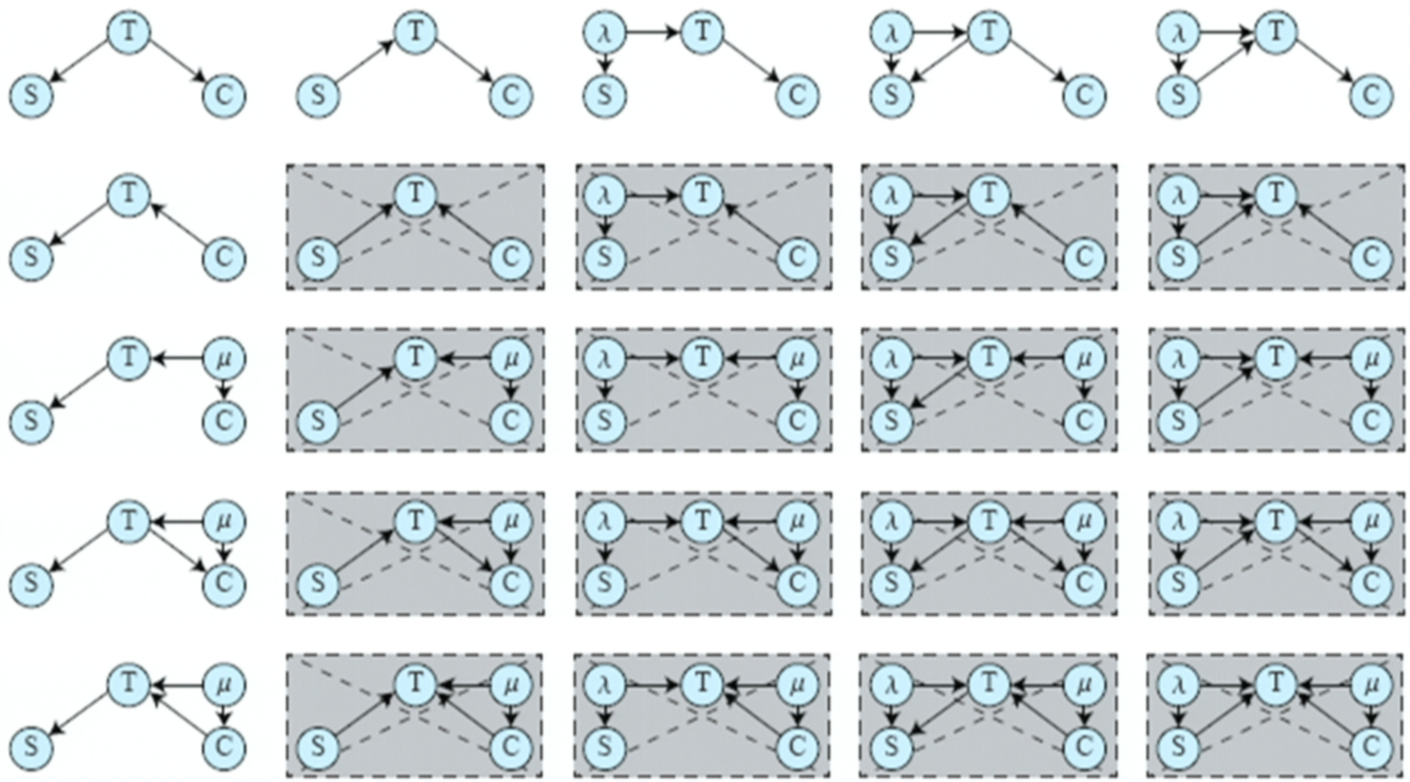
$(S \perp C \mid T)$

So the causal structure must be of the form

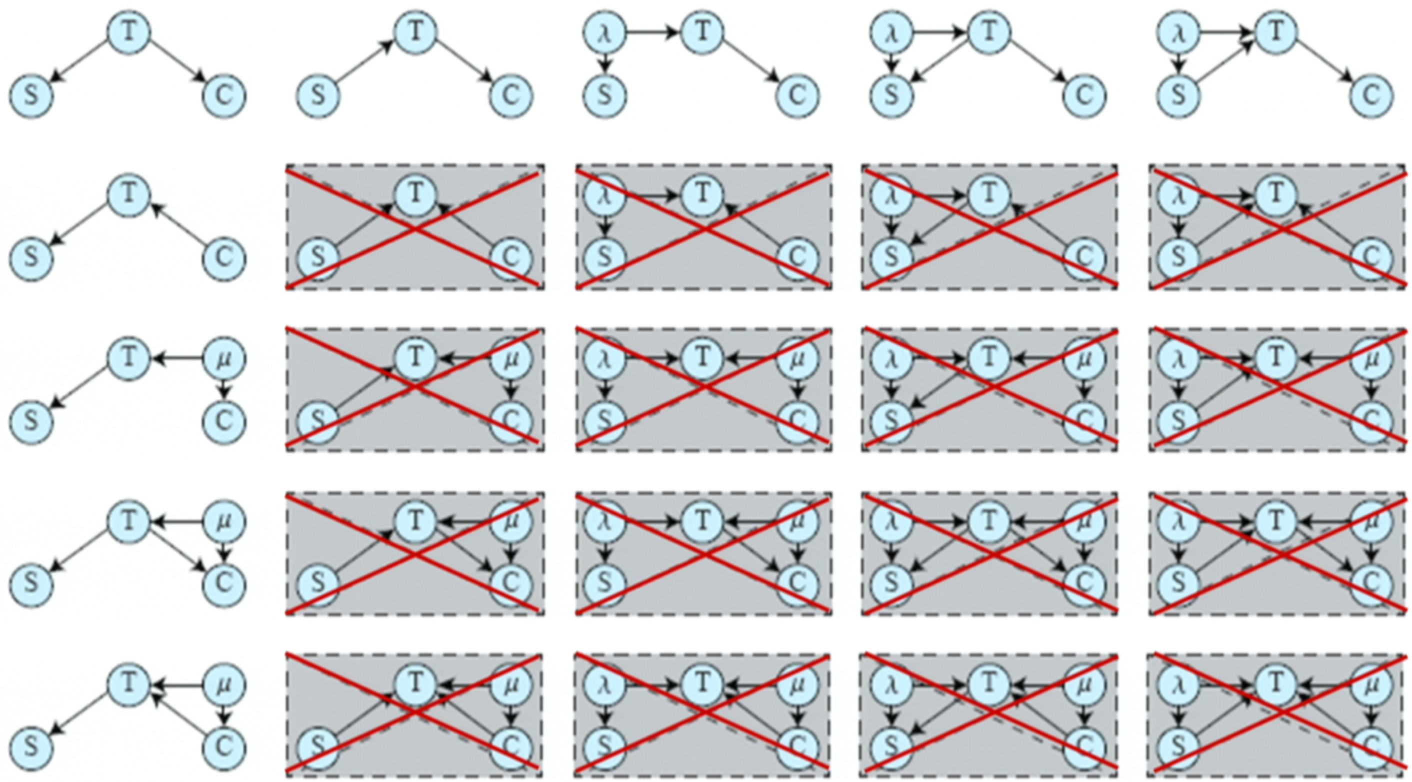




$(S \perp C \mid T)$

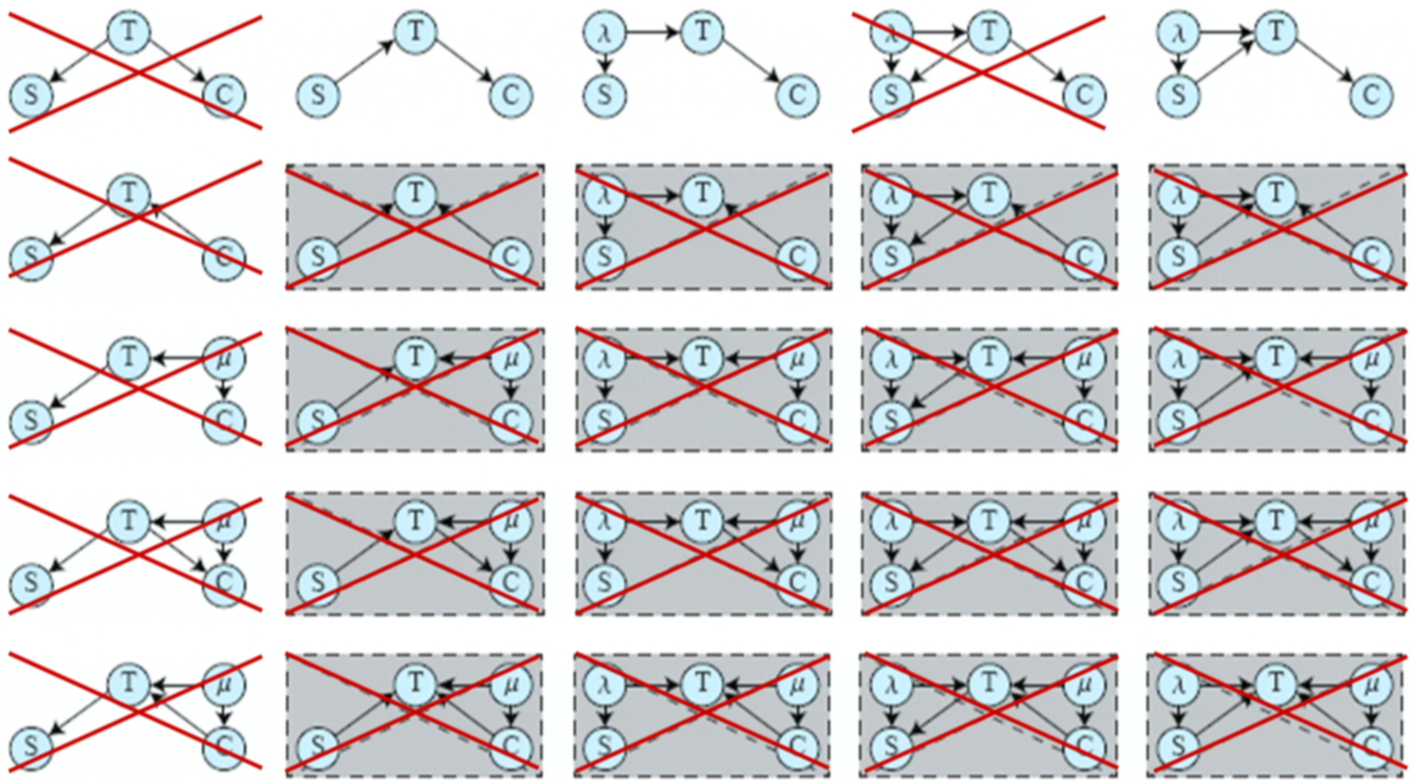


$(S \perp C \mid T)$



$(S \perp C \mid T)$

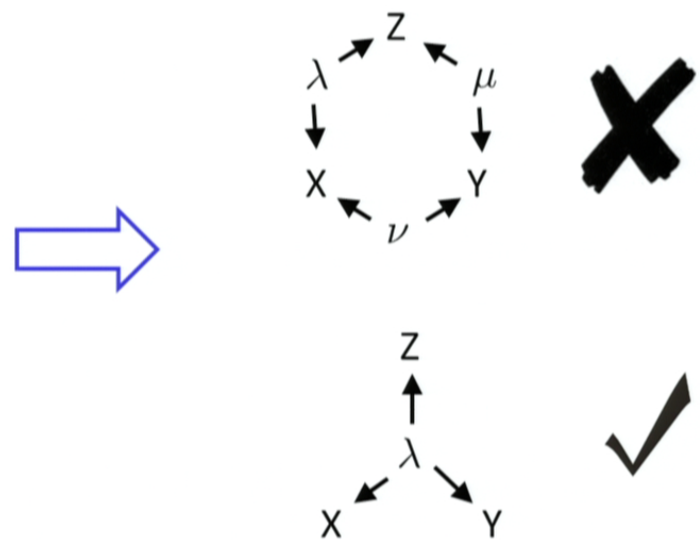
Assume one extra piece of data: S always precedes T

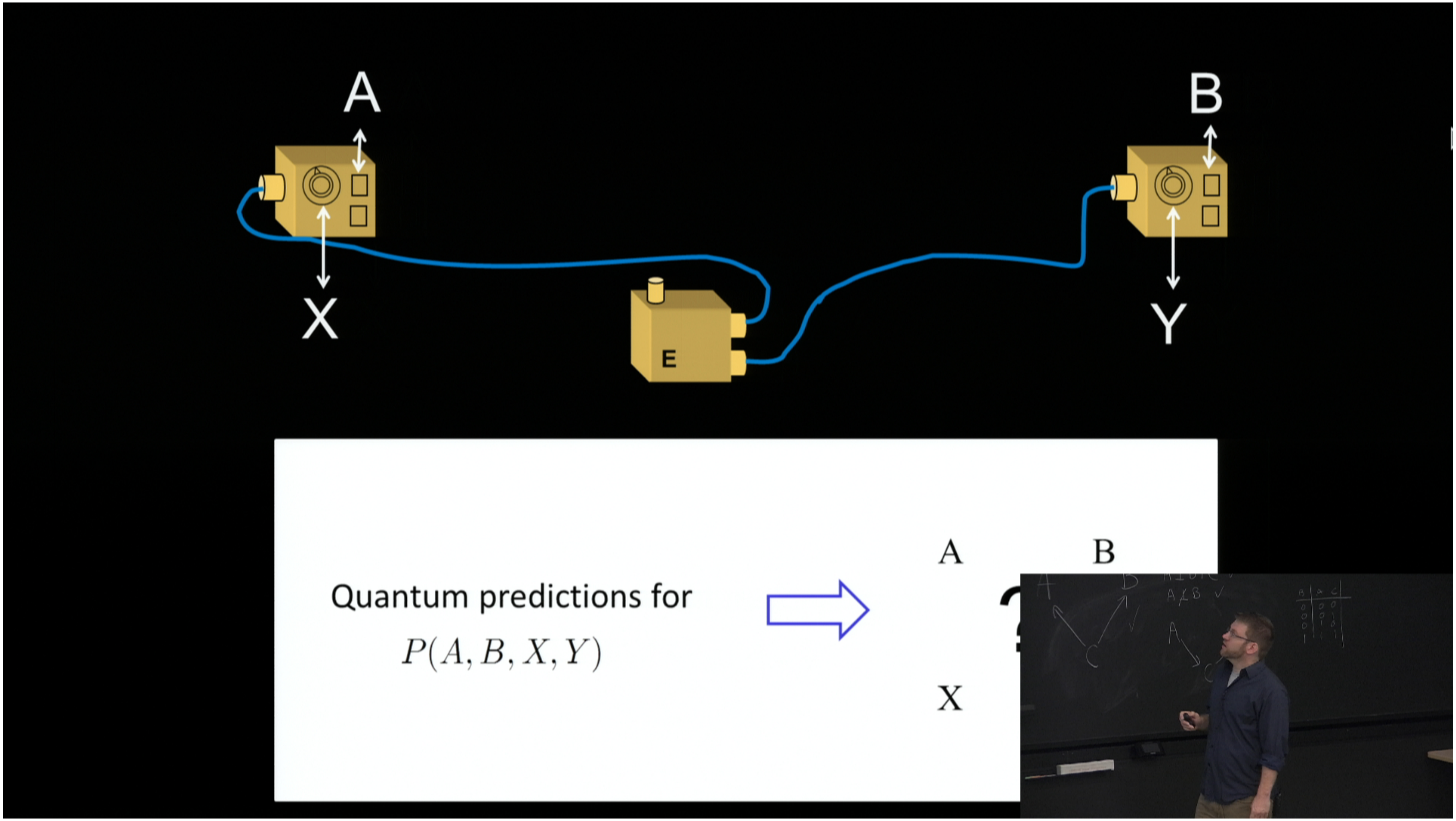


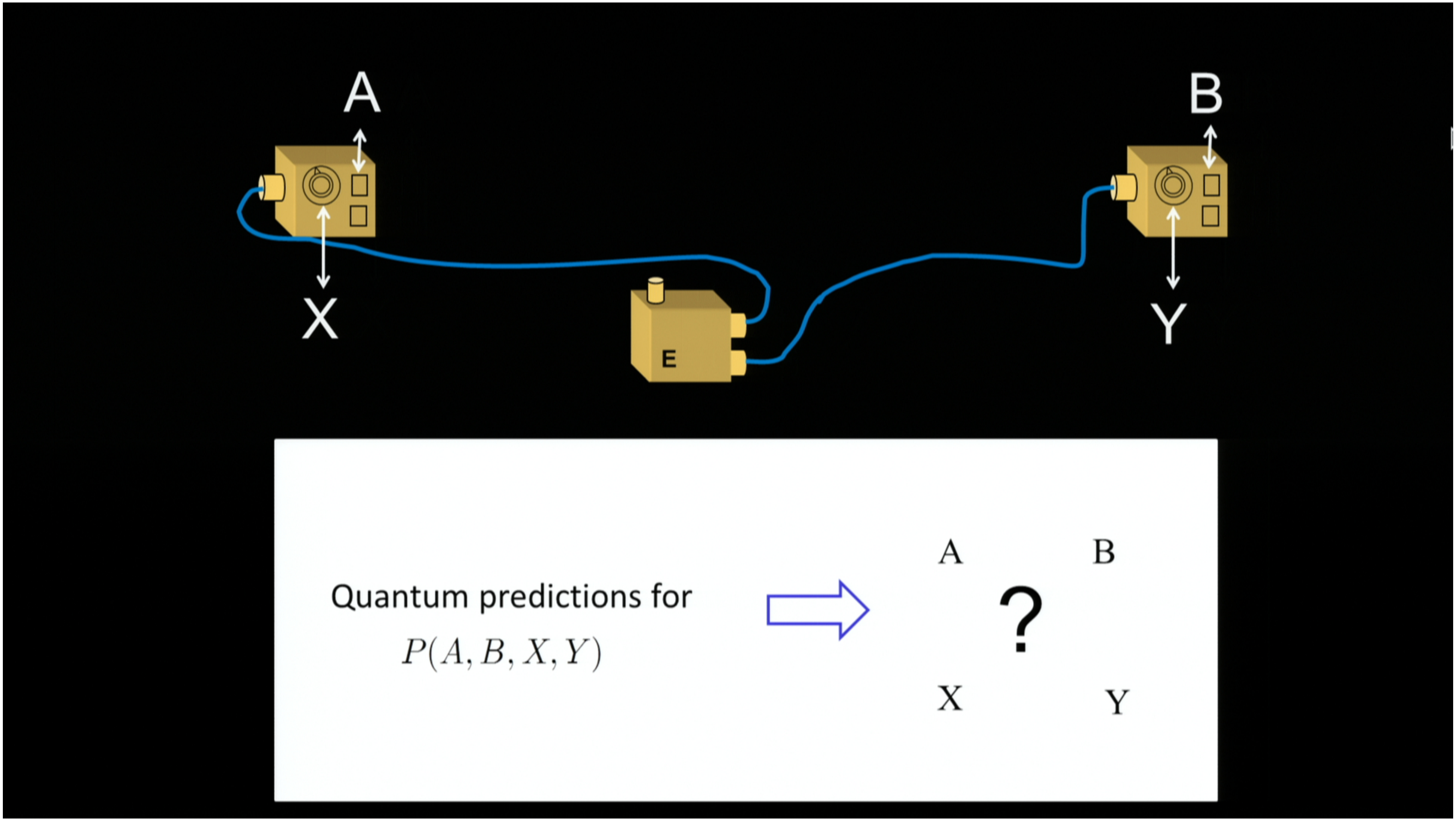
Inferring facts about the causal structure from the strength of correlations

Strength of Correlations

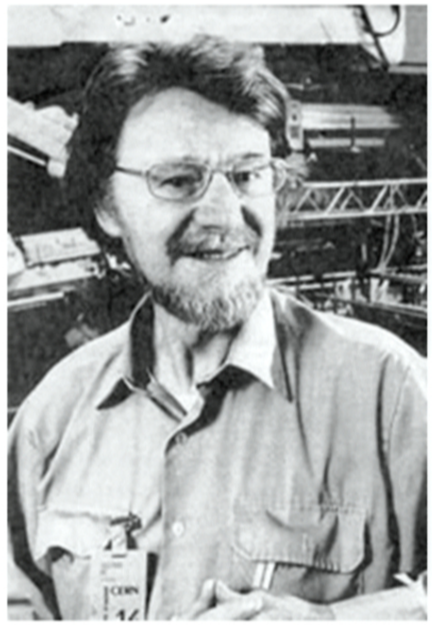
$$P(X, Y, Z) = \tfrac{1}{2}[000] + \tfrac{1}{2}[111]$$




A
B
X
Y
E
Quantum predictions for
P(A, B, X, Y)
A
B
X

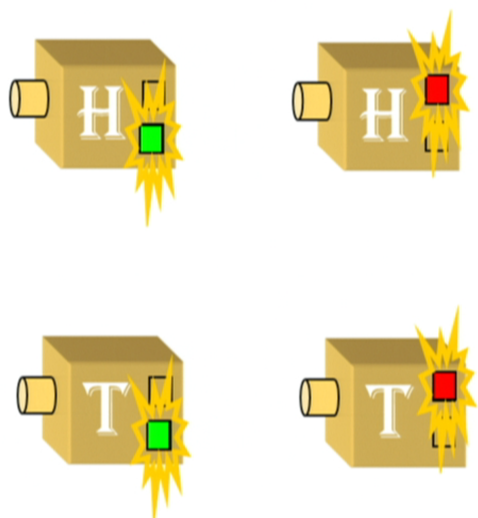

A
B
X
Y
E
Quantum predictions for
P(A, B, X, Y)
A
B
?
X
Y

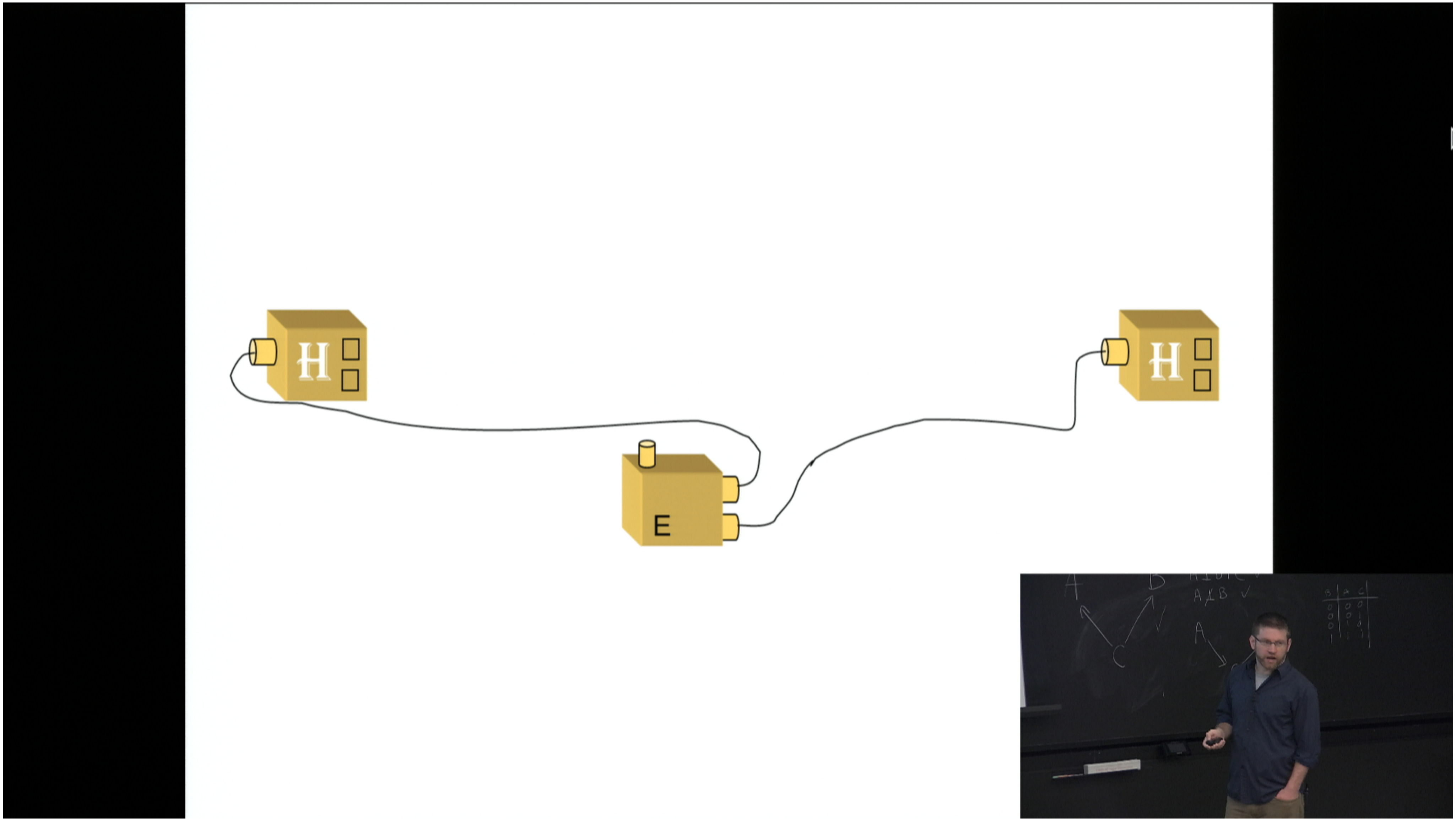
Bell's theorem



John S. Bell
(1928-1990)

A pair of two-outcome measurements




H
H
E

There are two possible measurements, H and T, with two outcomes each: green or red

Suppose which of H or T occurs at each wing is chosen at random

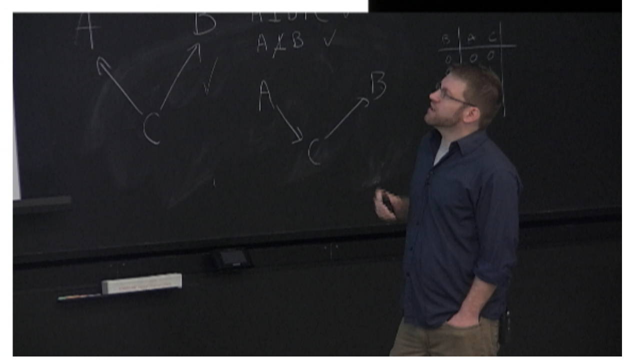
Scenario 1

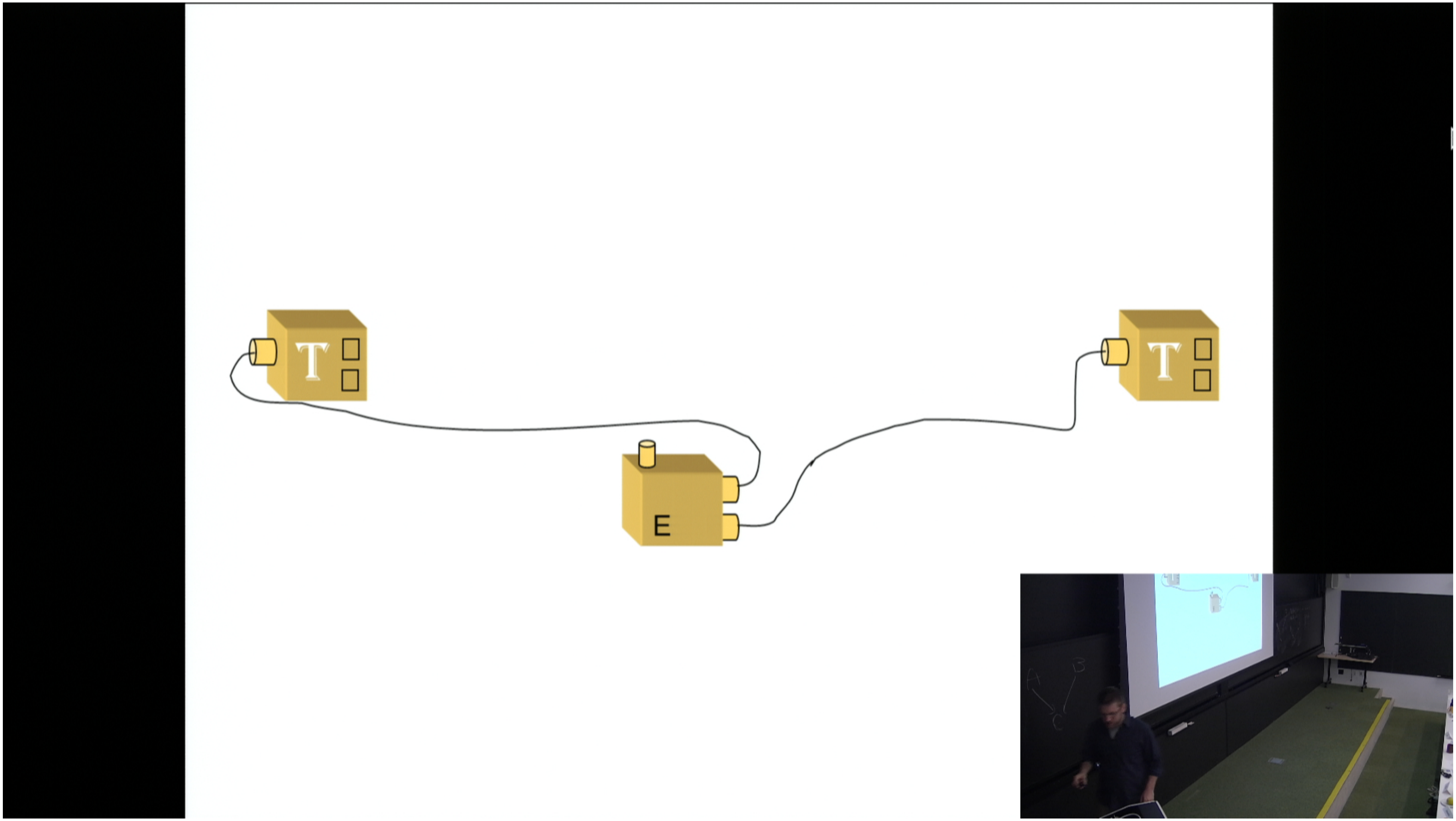
1. Whenever the same measurement is made on A and B, the outcomes always agree

   H and H
   or
   T and T

2. Whenever different measurements are made on A and B, the outcomes always disagree

   H and T
   or
   T and H




T
T
E

There are two possible measurements, H and T, with two outcomes each: green or red

Suppose which of H or T occurs at each wing is chosen at random

Scenario 1

1. Whenever the same measurement is made on A and B, the outcomes always agree — H and H or T and T

2. Whenever different measurements are made on A and B, the outcomes always disagree — H and T or T and H

There are two possible measurements, H and T,
with two outcomes each: green or red

Suppose which of H or T occurs at each wing is chosen at random

Scenario 2

1. Whenever the same measurement is made on A and B, the outcomes always disagree	H and H or T and T
2. Whenever different measurements are made on A and B, the outcomes always agree	H and T or T and H


H
H
T
T
outcomes disagree
outcomes agree
outcomes agree
outcomes disagree

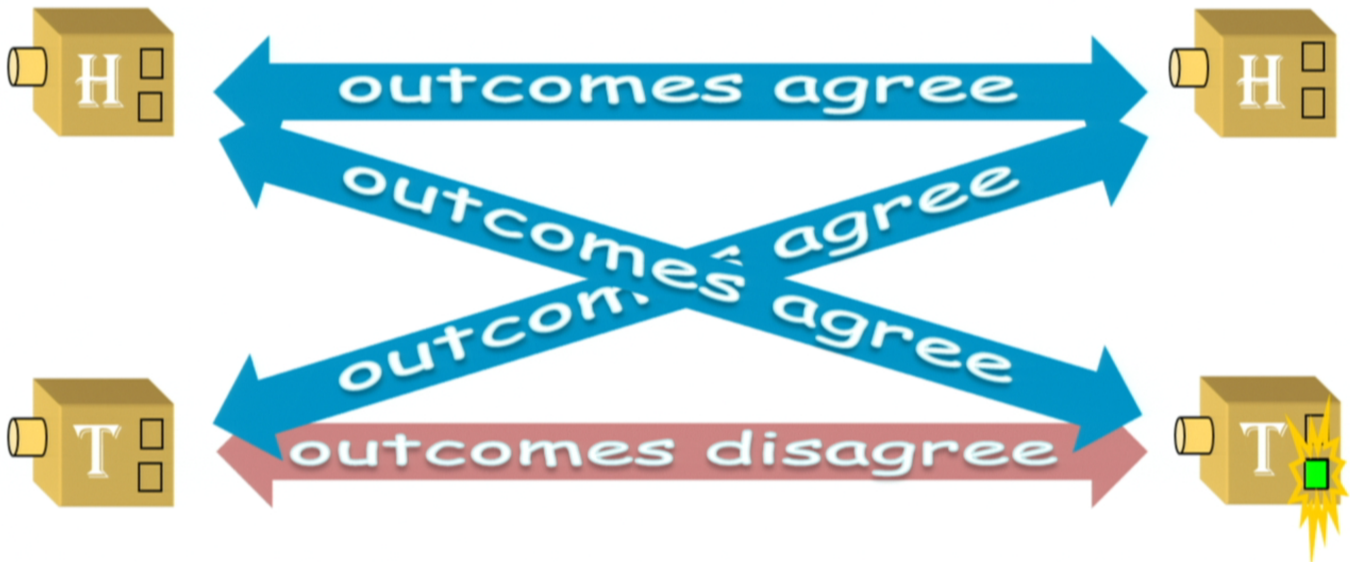
There are two possible "measurements", H and T,
with two outcomes each: green or red

Suppose which of H or T occurs at each wing is chosen at random

Scenario 3

1. Whenever the measurement T is made on both A and B, the outcomes always disagree — T and T

2. Otherwise, the outcomes always agree — H and H or H and T or T and H


H
H
outcomes agree
outcomes agree
outcomes agree
outcomes disagree
T
T

The game can be won at most 75% of the time by local strategies

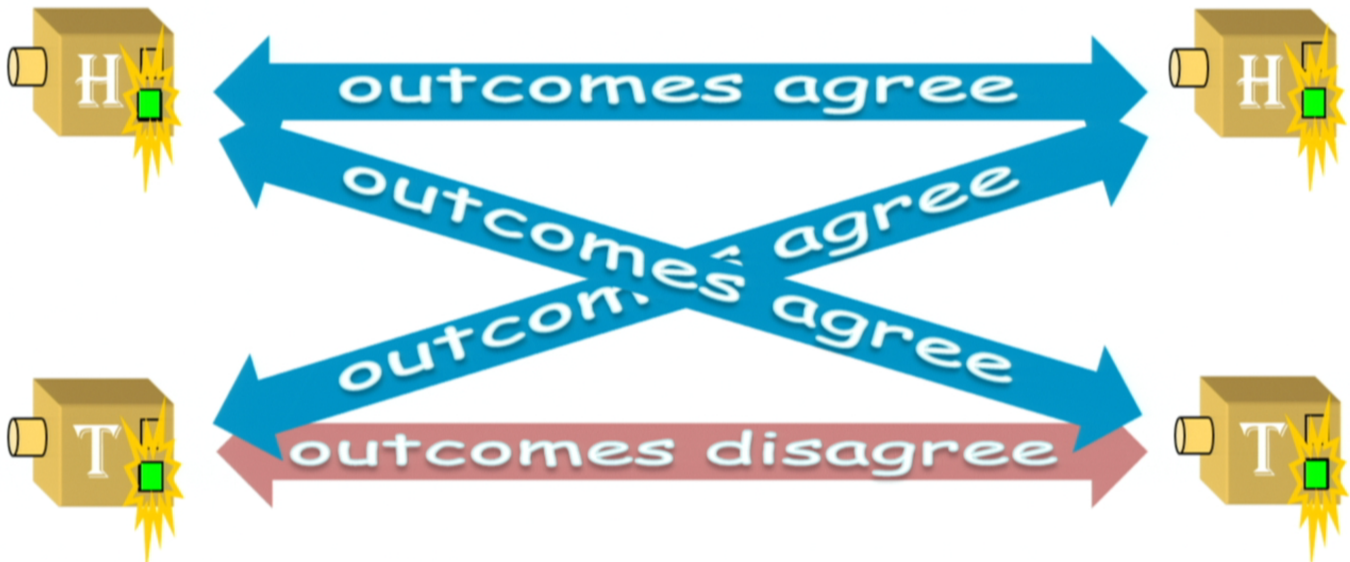
The game can be won at most 75% of the time by local strategies

Using quantum theory, it can be won 85% of the time!

The game can be won at most 75% of the time by local strategies

Using quantum theory, it can be won 85% of the time!

Experiments corroborate quantum theory

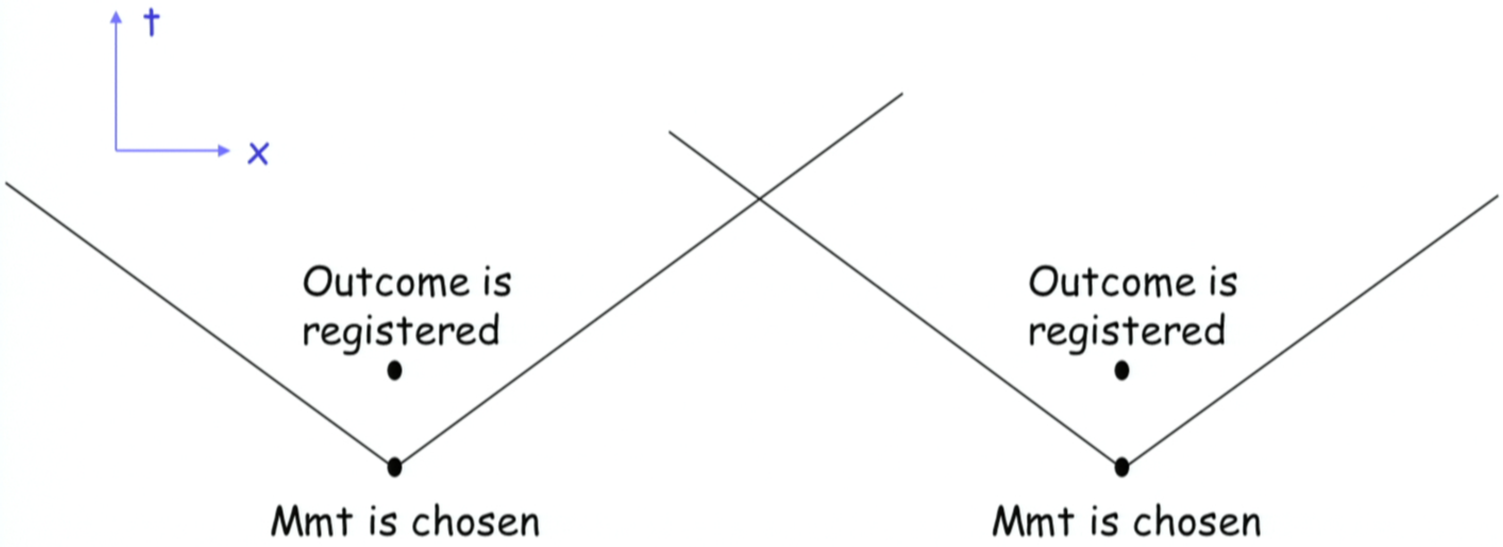

H
H
outcomes agree
outcomes agree
outcomes agree
outcomes agree
T
T
outcomes disagree

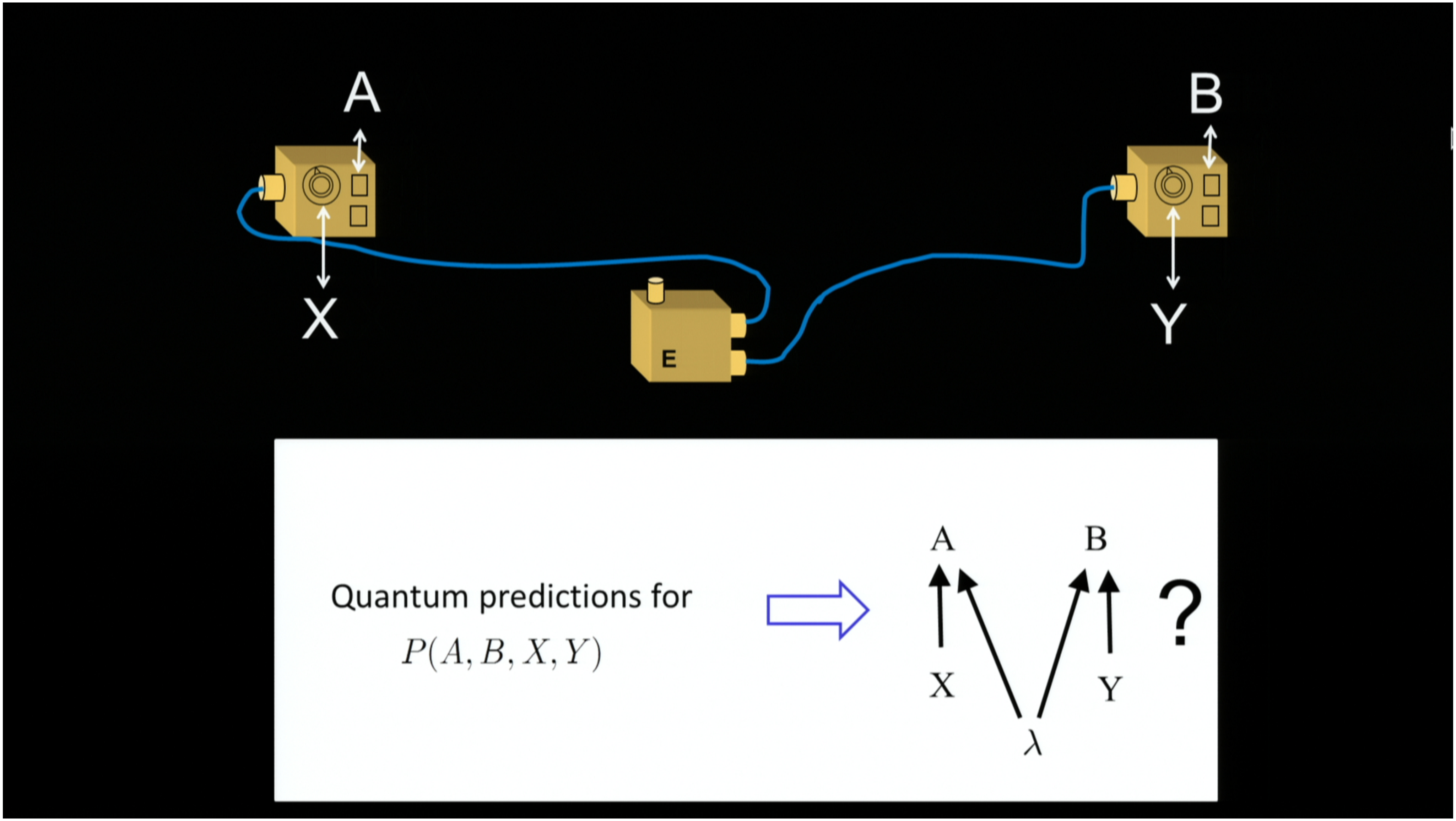
A: Rig the game so that the choices of settings are not random but instead are correlated with the local strategies

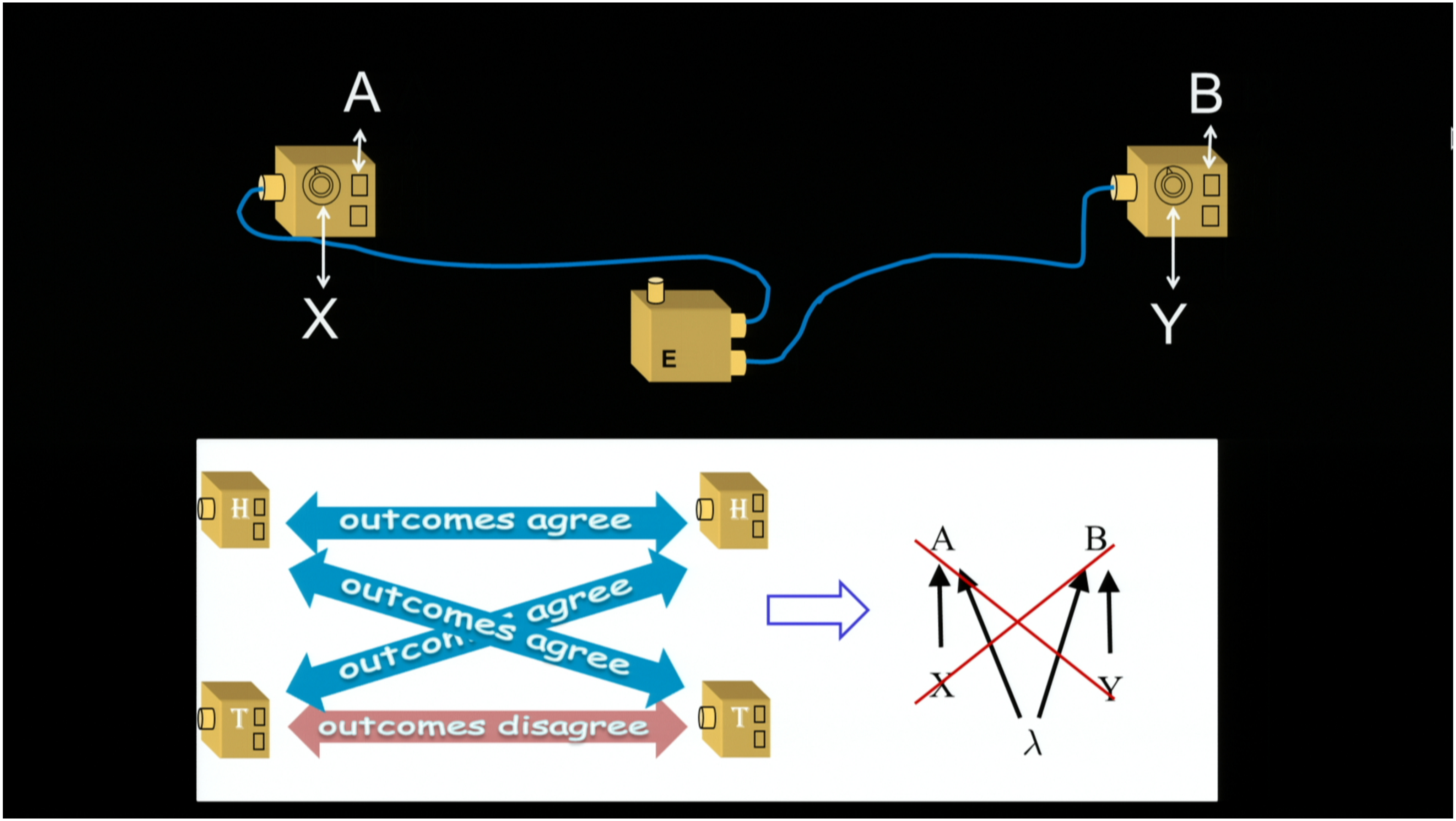
A: Rig the game so that the choices of settings are not random but instead are correlated with the local strategies

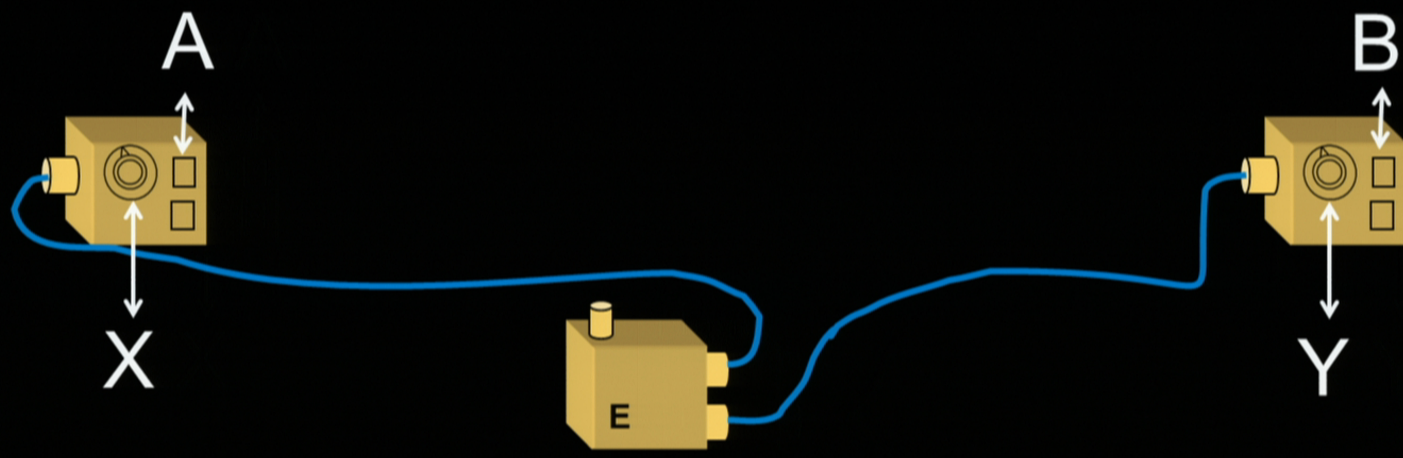
But for the quantum experiments, this would require nature to be conspiratorial and would require us to deny free will

Tension with the theory of relativity

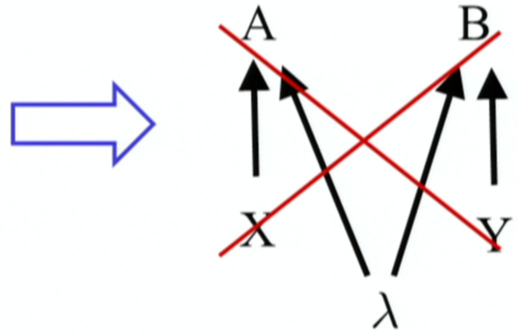


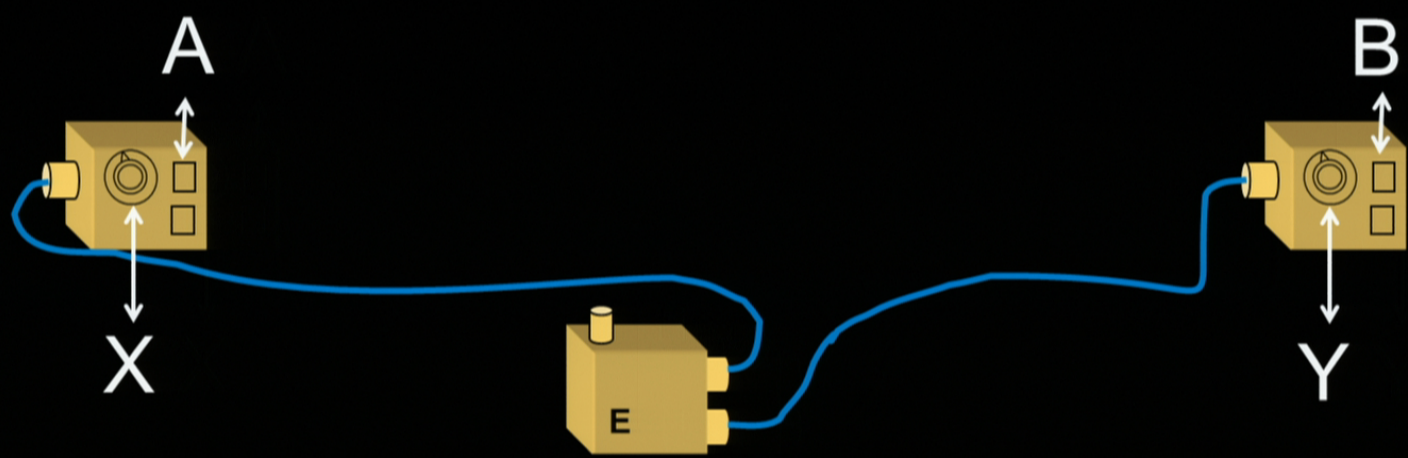

A
B
X
Y
E
Quantum predictions for
$P(A, B, X, Y)$
A
B
X
Y
λ
?


A
B
X
Y
E
H
T
outcomes agree
outcomes agree
outcomes agree
outcomes disagree
A
B
X
Y
λ

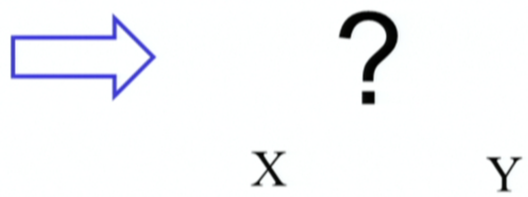


$$P(A,B|X,Y)$$
$$= \tfrac{1}{2}[00] + \tfrac{1}{2}[11] \quad \text{if } XY = 0$$
$$= \tfrac{1}{2}[01] + \tfrac{1}{2}[10] \quad \text{if } XY = 1$$

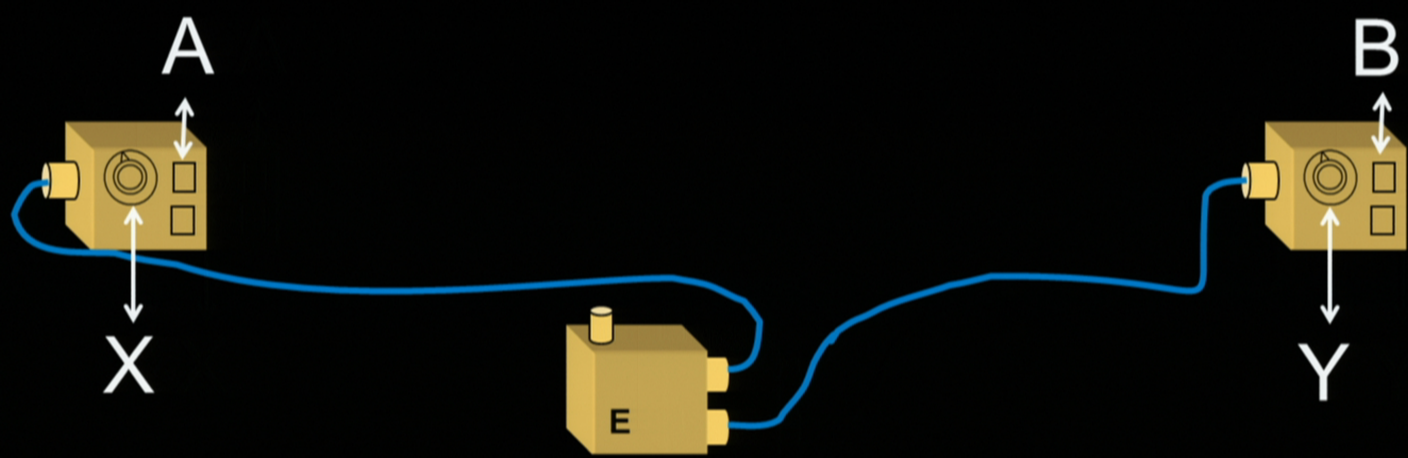




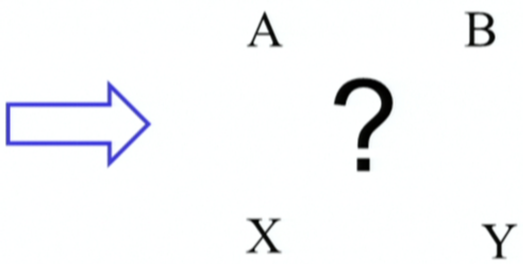
$$P(A, B|X, Y)$$
$$= \tfrac{1}{2}[00] + \tfrac{1}{2}[11] \quad \text{if } XY = 0$$
$$= \tfrac{1}{2}[01] + \tfrac{1}{2}[10] \quad \text{if } XY = 1$$



The statistical independences are:
$(X \perp Y), (A \perp Y \mid X), (B \perp X \mid Y)$



$$P(A,B|X,Y)$$
$$= \tfrac{1}{2}[00] + \tfrac{1}{2}[11] \quad \text{if } XY = 0$$
$$= \tfrac{1}{2}[01] + \tfrac{1}{2}[10] \quad \text{if } XY = 1$$



The statistical independences are:

$(X \perp Y)$, $(A \perp Y \mid X)$, $(B \perp X \mid Y)$

- Reichenbach’s principle
(no correlation without causation)

- No fine-tuning

- The “hidden common causes” are variables and our knowledge of them is described by probability theory

⇩

Contradiction with quantum theory and experiment!

- Reichenbach's principle
(no correlation without causation)

- No fine-tuning

- The "hidden common causes" are variables and our knowledge of them is described by probability theory

⇩

Contradiction with quantum theory and experiment!