Title: Bimetric theory, Conformal Gravity and Partial Masslessness

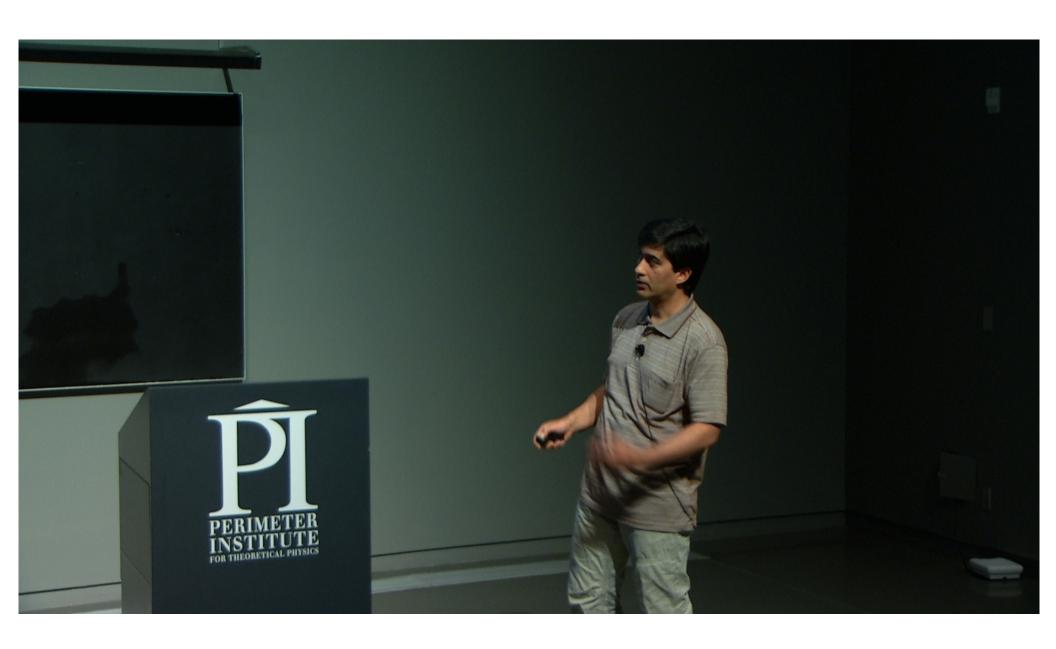
Date: Jul 09, 2013 11:10 AM

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Abstract: Ghost-free bimetric theories can be used to describe gravitational interactions in the presence of an extra neutral massive

 chr>spin-2 field that can modify gravity in non-trivial ways. They also
 chr>provide a natural framework for a possible non-linear extension of
 chr>partially masslessness known to arise in linear Fierz-Pauli theory.
 chr>This talk will describe bimetric theories and a procedure that
 chr>identifies a unique bimetric action as a candidate for a nonlinear
 chr>partially massless theory. We then show that in the low curvature
 chr>limit, the candidate partial massless theory is related to Conformal
 chr>c/span>

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Collaborators:

- SFH, Angnis Schmidt-May, Mikael von Strauss arXiv:1203.5283, 1204.5202,1208:1515, 1208:1797, 1212:4525, 1303.6940, 1307.xxxx
- SFH, Rachel A. Rosen, arXiv:1103.6055, 1106.3344, 1109.3515, 1109.3230, 1111.2070



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Outline of the talk

Review: Linear and Nonlinear massive spin-2 fields

Ghost-free bimetric theory

Mass spectrum of bimetric theory

Partially Massless bimetric theory

Conformal gravity

Relation between CG and PM bimetric theory

Perturbative construction of the complete symmetry transformation



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Linear massive spin-2 fields

The Fierz-Pauli equation:

Linear spin-2 field $h_{\mu
u}$ in background $ar{g}_{\mu
u}$

$$ar{\mathcal{E}}_{\mu
u}^{
ho\sigma}\,h_{
ho\sigma}^{}-igwedge^{}\left(h_{\mu
u}^{}-rac{1}{2}ar{g}_{\mu
u}^{}h_{
ho}^{
ho}
ight)+rac{m_{ ext{FP}}^{2}}{2}\,\left(h_{\mu
u}^{}-ar{g}_{\mu
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ho}^{
ho}
ight)=0$$

[Fierz-Pauli, 1939]

5 propagating modes (massive spin-2)

- Massive gravity ?
- What determines $\bar{g}_{\mu\nu}$? (flat, dS, AdS, · · ·)
- Nonlinear generalizations?



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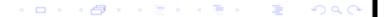
Nonlinear massive spin-2 fields

• "Massive gravity" (fixed $f_{\mu\nu}$):

$$\mathcal{L} = m_p^2 \sqrt{-g} \left[R - m^2 V(g^{-1} f) \right]$$

Interacting spin-2 fields (dynamical g and f):

$$\mathcal{L} = m_p^2 \sqrt{-g} \left[R - m^2 V(g^{-1}f) \right] + \mathcal{L}(\nabla f)$$
 (?)



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Counting modes:

Generic massive gravity:

- Linear theory: 5 modes (massive spin-2)
- Non-linear theory : 5 + 1 (ghost)

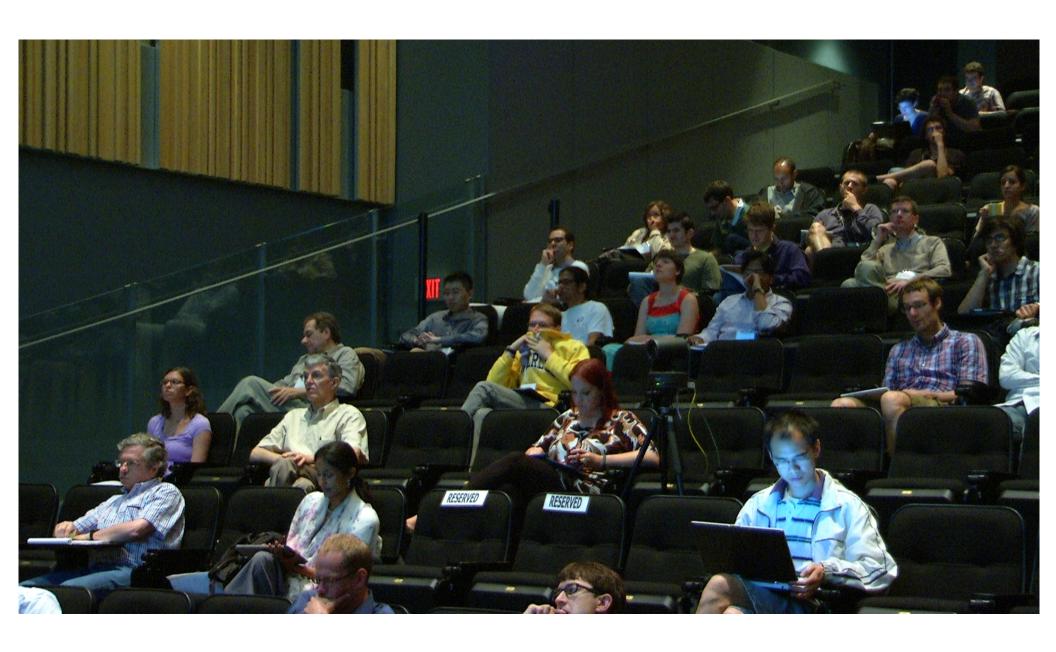
Generic bimetric theory:

- ▶ Linear theory: $5(\delta g \delta f) + 2(\delta g + \delta f)$ modes
- Non-linear theory: 7 + 1 (ghost)

Complication: Since the ghost shows up nonlinearly, its absence needs to be established nonlinearly



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Construction of ghost-free nonlinear theories

Do ghost-free massive gravity & bimetric theories exist?

"Decoupling limit" analysis:

- ▶ Massive gravity potential: $V_{dRGT}(\sqrt{g^{-1}\eta})$
- Shown to be ghost-free in "decoupling limit", also perturbatively in $h = g \eta$

[de Rham, Gabadadze, 2010; de Rham, Gabadadze, Tolley, 2010]

(For details see the talks by C. de Rham and S. Mukohyama)

Earlier work:

[Creminelli, Nicolis, Papucci, Trincherini, (hep-th/0505147)]

Questions beyond decoupling limit



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Construction of ghost-free nonlinear theories [cont]

Non-linear Hamiltonian methods (non-perturbative):

Addresses questions beyond "decoupling limit":

- ▶ Is massive gravity with $V(\sqrt{g^{-1}\eta})$ ghost-free nonlinearly? [SFH, Rosen (1106.3344, 1111.2070)]
- ▶ Is it ghost-free for generic fixed $f_{\mu\nu} \neq \eta_{\mu\nu}$?

 [SFH, Rosen, Schmidt-May (1109.3230)]
- ► Can $f_{\mu\nu}$ be given ghost-free dynamics?

 [SFH, Rosen (1109.3515)]
- ► Ghost-free multivielbein/multimetric interactions?

 [Hinterbichler, Rosen (arXiv:1203.5783)]



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Ghost-free bimetric theory

Digression: Elementary symmetric polynomials of \mathbb{X} with eigenvalues $\lambda_1, \dots, \lambda_4$:

$$e_0(\mathbb{X}) = 1$$
, $e_1(\mathbb{X}) = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$,
 $e_2(\mathbb{X}) = \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_1 \lambda_4 + \lambda_2 \lambda_3 + \lambda_2 \lambda_4 + \lambda_3 \lambda_4$,
 $e_3(\mathbb{X}) = \lambda_1 \lambda_2 \lambda_3 + \lambda_1 \lambda_2 \lambda_4 + \lambda_1 \lambda_3 \lambda_4 + \lambda_2 \lambda_3 \lambda_4$,
 $e_4(\mathbb{X}) = \lambda_1 \lambda_2 \lambda_3 \lambda_4 = \det \mathbb{X}$.



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Ghost-free bimetric theory

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$$\begin{split} e_0(\mathbb{X}) &= 1, & e_1(\mathbb{X}) = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4, \\ e_2(\mathbb{X}) &= \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_1 \lambda_4 + \lambda_2 \lambda_3 + \lambda_2 \lambda_4 + \lambda_3 \lambda_4, \\ e_3(\mathbb{X}) &= \lambda_1 \lambda_2 \lambda_3 + \lambda_1 \lambda_2 \lambda_4 + \lambda_1 \lambda_3 \lambda_4 + \lambda_2 \lambda_3 \lambda_4, \\ e_4(\mathbb{X}) &= \lambda_1 \lambda_2 \lambda_3 \lambda_4 = \det \mathbb{X}. \end{split}$$

$$\begin{split} e_0(\mathbb{X}) &= 1 \;, \qquad e_1(\mathbb{X}) = [\mathbb{X}] \;, \\ e_2(\mathbb{X}) &= \frac{1}{2}([\mathbb{X}]^2 - [\mathbb{X}^2]), \\ e_3(\mathbb{X}) &= \frac{1}{6}([\mathbb{X}]^3 - 3[\mathbb{X}][\mathbb{X}^2] + 2[\mathbb{X}^3]) \;, \\ e_4(\mathbb{X}) &= \frac{1}{24}([\mathbb{X}]^4 - 6[\mathbb{X}]^2[\mathbb{X}^2] + 3[\mathbb{X}^2]^2 + 8[\mathbb{X}][\mathbb{X}^3] - 6[\mathbb{X}^4]) \;, \\ e_k(\mathbb{X}) &= 0 \qquad \text{for} \quad k > 4 \;, \\ [\mathbb{X}] &= \text{Tr}(\mathbb{X}) \;, \qquad e_n(\mathbb{X}) \sim (\mathbb{X})^n \end{split}$$

► The $e_n(\mathbb{X})$'s and $det(\mathbb{1} + \mathbb{X})$:

$$\det(\mathbb{1}+\mathbb{X})=\sum
olimits_{n=0}^4e_n(\mathbb{X})$$

Introduce "deformed determinant" :

$$\widehat{\det}(\mathbb{1}+\mathbb{X})=\sum_{n=0}^4 \frac{\beta_n}{\beta_n} e_n(\mathbb{X})$$



Ghost-free bi-metric theory

Ghost-free combination of kinetic and potential terms for g & f:

$$\mathcal{L} = m_g^2 \sqrt{-g} R_g - 2m^4 \sqrt{-g} \sum_{n=0}^4 \beta_n \, e_n (\sqrt{g^{-1} f}) + m_f^2 \sqrt{-f} \, R_f$$

[SFH, Rosen (1109.3515,1111.2070)]

Symmetry under $f \leftrightarrow g$,

$$\sqrt{-g}\sum_{n=0}^{4}\beta_{n}\,e_{n}(\sqrt{g^{-1}f})=\sqrt{-f}\sum_{n=0}^{4}\beta_{4-n}\,e_{n}(\sqrt{f^{-1}g})$$

Hamiltonian analysis: 7 nolinear propagating modes, no ghost!

$$C(\gamma,\pi)=0$$
, $C_2(\gamma,\pi)=\frac{d}{dt}C(x)=\{H,C\}=0$



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Mass spectrum of bimetric theory

[SFH, A. Schmidt-May, M. von Strauss 1208:1515, 1212:4525]

$$S_{gf} = -\int d^dx \Big[m_g^{d-2} \sqrt{g} R_g - 2 m^d \sqrt{g} \sum_{n=0}^d \beta_n \, e_n(S) + m_f^{d-2} \sqrt{f} R_f \Big]$$

$$egin{aligned} R_{\mu
u}(g) - rac{1}{2}g_{\mu
u}R(g) + V^g_{\mu
u} &= T^g_{\mu
u} \ R_{\mu
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Questions:

- ▶ Q1: When are the 7 fluctuations in $\delta g_{\mu\nu}$, $\delta f_{\mu\nu}$ good mass eigenstates? (FP mass)
- Q2: In what sense is this Massive spin-2 field + gravity ?
- Q3: How to characterize deviations from General Relativity?



Mass spectrum of bimetric theory

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Proportional backgrounds

A1: FP masses exist only around,

$$ar{f}_{\mu
u}=c^2ar{g}_{\mu
u}$$

g and f equations:

$$R_{\mu
u}(ar{g}) - rac{1}{2}ar{g}_{\mu
u}R(ar{g}) + inom{\Lambda_g}{\Lambda_f}ar{g}_{\mu
u} = 0 \,\, ext{or} \,\, inom{T_{\mu
u}^g}{T_{\mu
u}^f}$$

$$\Lambda_{g} = \frac{m^{4}}{m_{g}^{2}} \sum_{k=0}^{3} {3 \choose k} c^{k} \beta_{k}, \quad \Lambda_{f} = \frac{m^{4}}{m_{f}^{2}} \sum_{k=1}^{4} {3 \choose k-1} c^{k-2} \beta_{k}$$

Implication:

$$\Lambda_g = \Lambda_f \quad \Rightarrow \quad c = c(\beta_n, \alpha \equiv m_f/m_g)$$

(Exception: Partially massless (PM) theory)



Mass spectrum around proportional backgrounds

Linear modes: Massless mode:

$$\delta G_{\mu\nu} = \left(\delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu}\right)$$

Massive mode:

$$\delta M_{\mu\nu} = \frac{1}{2c} \left(\delta f_{\mu\nu} - c^2 \delta g_{\mu\nu} \right) ,$$

The FP mass of δM :

$$m_{\text{FP}}^2 = \frac{m^4}{m_g^2} \left(1 + (\alpha c)^{-2} \right) \sum_{k=1}^3 \binom{2}{k-1} c^k \beta_k$$

Non-linear extensions:

$${m G}_{\mu
u} = g_{\mu
u} + lpha^2 f_{\mu
u} \ , \quad {m M}_{\mu
u}^{m G} = G_{\mu
ho} ig(\sqrt{g^{-1} f} ig)^{
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ho}_{
u} - m{c} m{G}_{\mu
u}$$



Bimetric as massive spin-2 field + gravity

A2: The massless mode is not gravity! $G_{\mu\nu} = g_{\mu\nu} + \alpha^2 f_{\mu\nu}$ has no ghost-free matter coupling!

Hence:

- Gravity: $g_{\mu\nu}$
- ▶ Massive spin-2 field: $M_{\mu\nu} = g_{\mu\rho} (\sqrt{g^{-1}f})^{
 ho}_{\nu} c g_{\mu\nu}$
- $m_f \rightarrow \infty$: $g_{\mu\nu}$ becomes massive gravity
- $m_g >> m_f$: $g_{\mu\nu}$ mostly massless (opposite to massive gravity)



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A3:
$$M_{\mu\nu}=0\Rightarrow$$
 GR. $M_{\mu\nu}\neq0\Rightarrow$ deviations from GR, driven by matter couplings



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Partial masslessness in FP theory

$$ar{\mathcal{E}}_{\mu
u}^{
ho\sigma}\,h_{
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u} - {1\over2}ar{g}_{\mu
u}h_{
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ho}ig) + {\color{blue} m_{ ext{FP}}^2\over2}\,ig(h_{\mu
u} - ar{g}_{\mu
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dS/Einstein backgrounds:

$$ar{g}_{\mu
u}: \qquad R_{\mu
u}-rac{1}{2}g_{\mu
u}R+\Lambda g_{\mu
u}=0$$

Higuchi Bound:

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$$m_{FP}^2 = \frac{2}{3}\Lambda$$

New gauge symmetry:

$$\Delta h_{\mu\nu} = (\nabla_{\mu}\nabla_{\nu} + \frac{\Lambda}{3})\xi(x)$$

Gives 5-1=4 propagating modes (no troublesome helicity-0 mode)

[Deser, Waldron, · · · (1983-2012)]

Can a nonlinear extension of PM theory exist?

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Partial masslessness beyond FP theory

Non-linear PM theory = Nonlinear spin-2 fields with a gauge invariance!

Does it exist? Independent of dS/Einstein backgrounds?



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Partial masslessness beyond FP theory

Non-linear PM theory = Nonlinear spin-2 fields with a gauge invariance!

Does it exist? Independent of dS/Einstein backgrounds?

We will identify a specific bimetric theory as the candidate nonlinear PM theory



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Partial masslessness in Bimetric theory

[SFH, Schmidt-May, von Strauss, 1208:1797, 1212:4525]

Around $\bar{f}=c^2\bar{g}$, $\delta M_{\mu\nu}\sim\delta f_{\mu\nu}-c^2\delta g_{\mu\nu}$ satisfies the FP equation. When $m_{FP}^2=\frac{2}{3}\Lambda$ there is a PM symmetry:

$$\delta \emph{M}_{\mu
u}
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u} + \left(
abla_{\mu}
abla_{
u} + rac{\Lambda}{3} \, ar{g}_{\mu
u}
ight) \xi(\emph{x}) \,, \qquad \delta \emph{G}_{\mu
u}
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u}$$



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ight) \xi(\emph{x}) \,, \qquad \delta \emph{G}_{\mu
u}
ightarrow \delta \emph{G}_{\mu
u}$$

- Find the transformation of $\delta g_{\mu\nu}$ & $\delta f_{\mu\nu}$.
- ► The dS-preserving subset $\xi = \xi_0$ (const), can be integrated to finite transformations,

$$ar{g}'_{\mu
u} = (1+a\xi_0\,)ar{g}_{\mu
u}\,, \quad ar{f}'_{\mu
u} = (1+b\xi_0\,)ar{f}_{\mu
u} \ ar{f}' = c'^2(\xi_0)\,ar{g}' \qquad c'
eq c$$

A symmetry can exist only if $\Lambda_g = \Lambda_f$ does not determine c

(D) (B) (E) (E) E 900

Candidate PM bimetric theory in d=4

The necessary condition for the existence of PM symmetry is that c is not determined by $\Lambda_g = \Lambda_f$, or

$$\begin{split} \beta_1 + \left(3\beta_2 - \alpha^2\beta_0\right)c + \left(3\beta_3 - 3\alpha^2\beta_1\right)c^2 \\ + \left(\beta_4 - 3\alpha^2\beta_2\right)c^3 + \alpha^2\beta_3c^4 = 0 \end{split}$$

This gives the candidate nonlinear PM theory (d=4)

$$\alpha^2 \beta_0 = 3\beta_2$$
, $3\alpha^2 \beta_2 = \beta_4$, $\beta_1 = \beta_3 = 0$



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Nonlinear PM bimetric theory

Checks:

$$m_{\text{FP}}^2 = 2 \frac{m^4}{m_g^2} \left(\alpha^{-2} + c^2 \right) \beta_2 = \frac{2}{3} \Lambda_g$$

- ▶ Nonlinear PM bimetric can exist only for d = 3, 4.
- In d > 4 PM could be restored by Lanczos-Lovelock terms
- ▶ Realization of the ξ_0 gauge transformation in the nonlinear theory on dS, gauge invariant variables.

If the candidate PM theory really has a gauge symmetry, if will propagate 6=7-1 modes



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Digression: Conformal gravity

HD gravity:

$$S_{(2)}^{\mathrm{HD}}[g] = m_g^2 \int d^4x \sqrt{g} \left[\Lambda + c_R R(g) - rac{c_{RR}}{m^2} \left(R^{\mu
u} R_{\mu
u} - rac{1}{3} R^2
ight)
ight]$$

7 modes: massless spin-2 + massive spin-2 (ghost) [Stelle (1977)]



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Digression: Conformal gravity

HD gravity:

$$S_{(2)}^{\mathrm{HD}}[g] = m_g^2 \int d^4x \sqrt{g} \left[\Lambda + c_R R(g) - rac{c_{RR}}{m^2} \left(R^{\mu
u} R_{\mu
u} - rac{1}{3} R^2
ight)
ight]$$

7 modes: massless spin-2 + massive spin-2 (ghost) [Stelle (1977)] Conformal Gravity:

$$\mathcal{S}^{\mathrm{CG}}[g] = -c \int d^4 x \sqrt{g} \left[R^{\mu
u} R_{\mu
u} - rac{1}{3} R^2
ight] \, ,$$

EoM (Bach tensor):

$$B_{\mu\nu} \equiv -\nabla^2 P_{\mu\nu} - \nabla_{\mu} \nabla_{\nu} P \cdots = 0$$

Invariance:

 $g_{\mu\nu} o e^\phi g_{\mu\nu} \Rightarrow$ 6 modes: 2 (massless spin-2) + 4 ghosts [Riegert (1984), Maldacena (2011)]

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Outline of the talk

Review: Linear and Nonlinear massive spin-2 fields

Ghost-free bimetric theory

Mass spectrum of bimetric theory

Partially Massless bimetric theory

Conformal gravity

Relation between CG and PM bimetric theory

Perturbative construction of the complete symmetry transformation



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Curvature expansion of bimetric equations

[SFH, Schmidt-May, von Strauss, 1303:6940]

Define

$$P_{\mu
u}=R_{\mu
u}-rac{1}{2(d-1)}g_{\mu
u}R$$



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In PM bimetric theory, the g and f-equations yield,

$$B_{\mu\nu} + \mathcal{O}(R^3/m^2) = 0$$

Thus in the low curvature limit, PM bimetric theory has a gauge symmetry even away from dS and definitely propagates 7 - 1 = 6 modes! None is a ghost

Obtaining the bimetric PM transfromations:

$$\Delta g_{\mu\nu} = \phi g_{\mu\nu} \ \Rightarrow \Delta f_{\mu\nu} = -\phi g_{\mu\nu}/\alpha^2 - 1/(m^2\beta_2)\nabla_{\mu}\nabla_{\nu}\phi$$
 So that,

$$\Delta \delta M_{\mu\nu} = \nabla_{\mu} \nabla_{\nu} \phi + (\Lambda/3) \phi g_{\mu\nu}$$



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 Conversely, PM bimetric is a ghost-free completion of CG
- Does the symmetry survive at higher orders? Yes! The construction can be extended to any number of derivatives



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Some recent debate:

Arguments against non-linear PM symmetry in massive gravity

[de Rham, Hinterbichler, Rosen, Tolley, (2013)] [Deser, Sandora, Waldron (2013)]

(the construction does not apply to bimetric theory)

Arguments against non-linear PM symmetry in Bimetric theory?

[Deser, Sandora, Waldron, "No consistent bimetric theory?" (2013)]

No correct arguments provided. Misquotes the relation between PM-bimetric theory and conformal gravity explained above as part of the argument



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Define

$$g'_{\mu\nu} = g_{\mu\nu}/\alpha$$
 $f'_{\mu\nu} = \alpha f_{\mu\nu}$

g-eom:
$$rac{lpha}{m^2eta_2}R_{\mu
u}(g')-3g'_{\mu
u}+[g'_{\mu
ho}S'^{2
ho}_{
u}- agrack Tr(S')g_{\mu
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u}]=0$$

f-eom:
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Complete $g' \leftrightarrow f'$ interchange symmetry!

Trivial invariance of g-eom: $\Delta g \Rightarrow \Delta f$.

(Is this also a symmetry of the f-eom?)

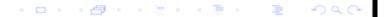
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(with $\tilde{\Delta}$ obtained from Δ by $g' \leftrightarrow f'$ interchange).

lf

$$\Delta f = \tilde{\Delta} f$$

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At the 2-derivative level,

$$\Delta g'_{\mu\nu} = \phi g'_{\mu\nu} - \frac{1}{2m^2\beta_2} \phi P_{\mu\nu}(g') - \frac{1}{2m^2\beta_2} \nabla^g_{\mu} \nabla^g_{\nu} \phi + \cdots$$

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If true at 2n derivative level, the construction can be extended to 2n+2 derivative level. Hence, the existence of an on-shell symmetry if proved by induction.



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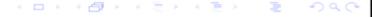
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