Title: Bimetric theory, Conformal Gravity and Partial Masslessness

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Abstract: Ghost-free bimetric theories can be used to describe gravitational interactions in the presence of an extra neutral massive
spin-2 field that can modify gravity in non-trivial ways. They also
provide a natural framework for a possible non-linear extension of
partially masslessness known to arise in linear Fierz-Pauli theory.
This talk will describe bimetric theories and a procedure that
identifies a unique bimetric action as a candidate for a nonlinear
partially massless theory. We then show that in the low curvature
limit, the candidate partial massless theory is related to Conformal
Gravity.

Collaborators:

- SFH, Angnis Schmidt-May, Mikael von Strauss
  arXiv:1203.5283, 1204.5202,1208:1515, 1208:1797, 1212:4525, 1303.6940, 1307.xxxx
- SFH, Rachel A. Rosen,
  arXiv:1103.6055, 1106.3344, 1109.3515, 1109.3230, 1111.2070

Outline of the talk

Review: Linear and Nonlinear massive spin-2 fields

Ghost-free bimetric theory

Mass spectrum of bimetric theory

Partially Massless bimetric theory

Conformal gravity

Relation between CG and PM bimetric theory

Perturbative construction of the complete symmetry transformation

Linear massive spin-2 fields

The Fierz-Pauli equation:

Linear spin-2 field $h_{\mu\nu}$ in background $\bar{g}_{\mu\nu}$

$$\bar{\mathcal{E}}^{\rho\sigma}_{\mu\nu}\, h_{\rho\sigma} - \Lambda\Big(h_{\mu\nu} - \tfrac{1}{2}\bar{g}_{\mu\nu}h^{\rho}_{\rho}\Big) + \frac{m^2_{\rm FP}}{2}\, \left(h_{\mu\nu} - \bar{g}_{\mu\nu}h^{\rho}_{\rho}\right) = 0$$

[Fierz-Pauli, 1939]

5 propagating modes (massive spin-2)

- Massive gravity ?
- What determines $\bar{g}_{\mu\nu}$? (flat, dS, AdS, $\cdots$)
- Nonlinear generalizations?

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Nonlinear massive spin-2 fields

- "Massive gravity" (fixed $f_{\mu\nu}$):

$$\mathcal{L} = m_p^2 \sqrt{-g} \left[R - m^2 \, V(g^{-1} f) \right]$$

- Interacting spin-2 fields (dynamical g and f):

$$\mathcal{L} = m_p^2 \sqrt{-g} \left[R - m^2 \, V(g^{-1} f) \right] + \mathcal{L}(\nabla f)(?)$$

Counting modes:

Generic massive gravity:

- Linear theory: 5 modes (massive spin-2)
- Non-linear theory : 5 + 1 (ghost)

Generic bimetric theory:

- Linear theory: 5 ($\delta g - \delta f$) + 2 ($\delta g + \delta f$) modes
- Non-linear theory: 7 + 1 (ghost)

Complication: Since the ghost shows up nonlinearly, its absence needs to be established nonlinearly


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Construction of ghost-free nonlinear theories

Do ghost-free massive gravity & bimetric theories exist?

"Decoupling limit" analysis:

- Massive gravity potential: $V_{dRGT}(\sqrt{g^{-1}\eta})$
- Shown to be ghost-free in "decoupling limit", also perturbatively in $h = g - \eta$

[de Rham, Gabadadze, 2010; de Rham, Gabadadze, Tolley, 2010]

(For details see the talks by C. de Rham and S. Mukohyama)

Earlier work:

[Creminelli, Nicolis, Papucci, Trincherini, (hep-th/0505147)]

Questions beyond decoupling limit ....

Construction of ghost-free nonlinear theories [cont]

Non-linear Hamiltonian methods (non-perturbative):

Addresses questions beyond "decoupling limit":

- Is massive gravity with $V(\sqrt{g^{-1}\eta})$ ghost-free nonlinearly?
  [SFH, Rosen (1106.3344, 1111.2070)]
- Is it ghost-free for generic fixed $f_{\mu\nu} \neq \eta_{\mu\nu}$?
  [SFH, Rosen, Schmidt-May (1109.3230)]
- Can $f_{\mu\nu}$ be given ghost-free dynamics?
  [SFH, Rosen (1109.3515)]
- Ghost-free multivielbein/multimetric interactions?
  [Hinterbichler, Rosen (arXiv:1203.5783)]

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Ghost-free bimetric theory

Digression: Elementary symmetric polynomials of $\mathbb{X}$ with eigenvalues $\lambda_1, \cdots, \lambda_4$:

$$e_0(\mathbb{X}) = 1, \qquad e_1(\mathbb{X}) = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4,$$
$$e_2(\mathbb{X}) = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4,$$
$$e_3(\mathbb{X}) = \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4,$$
$$e_4(\mathbb{X}) = \lambda_1\lambda_2\lambda_3\lambda_4 = \det\mathbb{X}.$$

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$$e_4(\mathbb{X}) = \lambda_1\lambda_2\lambda_3\lambda_4 = \det\mathbb{X}.$$

$$e_0(\mathbb{X}) = 1, \qquad e_1(\mathbb{X}) = [\mathbb{X}],$$
$$e_2(\mathbb{X}) = \tfrac{1}{2}([\mathbb{X}]^2 - [\mathbb{X}^2]),$$
$$e_3(\mathbb{X}) = \tfrac{1}{6}([\mathbb{X}]^3 - 3[\mathbb{X}][\mathbb{X}^2] + 2[\mathbb{X}^3]),$$
$$e_4(\mathbb{X}) = \tfrac{1}{24}([\mathbb{X}]^4 - 6[\mathbb{X}]^2[\mathbb{X}^2] + 3[\mathbb{X}^2]^2 + 8[\mathbb{X}][\mathbb{X}^3] - 6[\mathbb{X}^4]),$$
$$e_k(\mathbb{X}) = 0 \quad \text{for} \quad k > 4,$$

$$[\mathbb{X}] = \mathrm{Tr}(\mathbb{X}), \qquad e_n(\mathbb{X}) \sim (\mathbb{X})^n$$

- The $e_n(\mathbb{X})$'s and $\det(\mathbb{1} + \mathbb{X})$:

$$\det(\mathbb{1} + \mathbb{X}) = \sum_{n=0}^{4} e_n(\mathbb{X})$$

- Introduce "deformed determinant" :

$$\widehat{\det}(\mathbb{1} + \mathbb{X}) = \sum_{n=0}^{4} \beta_n \, e_n(\mathbb{X})$$

Ghost-free bi-metric theory

Ghost-free combination of kinetic and potential terms for g & f:

$$\mathcal{L} = m_g^2 \sqrt{-g} R_g - 2m^4 \sqrt{-g} \sum_{n=0}^{4} \beta_n \, e_n(\sqrt{g^{-1} f}) + m_f^2 \sqrt{-f} \, R_f$$

[SFH, Rosen (1109.3515,1111.2070)]

Symmetry under $f \leftrightarrow g$,

$$\sqrt{-g} \sum_{n=0}^{4} \beta_n \, e_n(\sqrt{g^{-1} f}) = \sqrt{-f} \sum_{n=0}^{4} \beta_{4-n} \, e_n(\sqrt{f^{-1} g})$$

Hamiltonian analysis: 7 nolinear propagating modes, no ghost!

$$C(\gamma, \pi) = 0 \,, \quad C_2(\gamma, \pi) = \tfrac{d}{dt} C(x) = \{H, C\} = 0$$

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Mass spectrum of bimetric theory

[SFH, A. Schmidt-May, M. von Strauss 1208:1515, 1212:4525]

$$S_{gf} = -\int d^d x \left[m_g^{d-2} \sqrt{g} R_g - 2m^d \sqrt{g} \sum_{n=0}^{d} \beta_n \, e_n(S) + m_f^{d-2} \sqrt{f} R_f \right]$$

$$\begin{aligned} R_{\mu\nu}(g) - \tfrac{1}{2} g_{\mu\nu} R(g) + V^g_{\mu\nu} &= T^g_{\mu\nu} \\ R_{\mu\nu}(f) - \tfrac{1}{2} f_{\mu\nu} R(f) + V^f_{\mu\nu} &= T^f_{\mu\nu} \end{aligned}$$

Questions:

- Q1: When are the 7 fluctuations in $\delta g_{\mu\nu}$, $\delta f_{\mu\nu}$ good mass eigenstates? (FP mass)
- Q2: In what sense is this Massive spin-2 field + gravity ?
- Q3: How to characterize deviations from General Relativity?

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Proportional backgrounds

A1: FP masses exist only around,

$$\bar{f}_{\mu\nu} = c^2 \bar{g}_{\mu\nu}$$

g and f equations:

$$R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) + \begin{pmatrix}\Lambda_g \\ \Lambda_f\end{pmatrix}\bar{g}_{\mu\nu} = 0 \text{ or } \begin{pmatrix}T^g_{\mu\nu} \\ T^f_{\mu\nu}\end{pmatrix}$$

$$\Lambda_g = \frac{m^4}{m_g^2}\sum_{k=0}^{3}\binom{3}{k}c^k\beta_k, \quad \Lambda_f = \frac{m^4}{m_f^2}\sum_{k=1}^{4}\binom{3}{k-1}c^{k-2}\beta_k$$

Implication:

$$\Lambda_g = \Lambda_f \quad \Rightarrow \quad c = c(\beta_n, \alpha \equiv m_f/m_g)$$

(Exception: Partially massless (PM) theory)

Mass spectrum around proportional backgrounds

Linear modes: Massless mode:

$$\delta G_{\mu\nu} = \left(\delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu}\right)$$

Massive mode:

$$\delta M_{\mu\nu} = \tfrac{1}{2c}\left(\delta f_{\mu\nu} - c^2 \delta g_{\mu\nu}\right) \ ,$$

The FP mass of δM:

$$m_{\mathrm{FP}}^2 = \frac{m^4}{m_g^2}\left(1 + (\alpha c)^{-2}\right) \sum_{k=1}^{3} \binom{2}{k-1} c^k \beta_k$$

Non-linear extensions:

$$G_{\mu\nu} = g_{\mu\nu} + \alpha^2 f_{\mu\nu} \ , \quad M^G_{\mu\nu} = G_{\mu\rho}(\sqrt{g^{-1}f})^{\rho}{}_{\nu} - cG_{\mu\nu}$$

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Bimetric as massive spin-2 field + gravity

A2: The massless mode is not gravity! $G_{\mu\nu} = g_{\mu\nu} + \alpha^2 f_{\mu\nu}$ has no ghost-free matter coupling!

Hence:

- Gravity: $g_{\mu\nu}$
- Massive spin-2 field: $M_{\mu\nu} = g_{\mu\rho}(\sqrt{g^{-1}f})^{\rho}{}_{\nu} - cg_{\mu\nu}$
- $m_f \to \infty$: $g_{\mu\nu}$ becomes massive gravity
- $m_g >> m_f$: $g_{\mu\nu}$ mostly massless (opposite to massive gravity)

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A3: $M_{\mu\nu} = 0 \Rightarrow$ GR.

$M_{\mu\nu} \neq 0 \Rightarrow$ deviations from GR, driven by matter couplings

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Partial masslessness in FP theory

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma}\, h_{\rho\sigma} - \Lambda(h_{\mu\nu} - \tfrac{1}{2}\bar{g}_{\mu\nu}h^{\rho}_{\rho}) + \frac{m^2_{\mathrm{FP}}}{2}\,(h_{\mu\nu} - \bar{g}_{\mu\nu}h^{\rho}_{\rho}) = 0$$

dS/Einstein backgrounds:

$$\bar{g}_{\mu\nu}: \qquad R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 0$$

Higuchi Bound:

$$m^2_{FP} = \tfrac{2}{3}\Lambda$$

New gauge symmetry:

$$\Delta h_{\mu\nu} = (\nabla_\mu\nabla_\nu + \tfrac{\Lambda}{3})\xi(x)$$

Gives 5-1=4 propagating modes (no troublesome helicity-0 mode)

[Deser, Waldron, $\cdots$ (1983-2012)]

Can a nonlinear extension of PM theory exist?

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Non-linear PM theory = Nonlinear spin-2 fields with a gauge invariance!

Does it exist? Independent of dS/Einstein backgrounds?

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We will identify a specific bimetric theory as the candidate nonlinear PM theory

Partial masslessness in Bimetric theory

[SFH, Schmidt-May, von Strauss, 1208:1797, 1212:4525]

Around $\bar{f} = c^2 \bar{g}$, $\delta M_{\mu\nu} \sim \delta f_{\mu\nu} - c^2 \delta g_{\mu\nu}$ satisfies the FP equation. When $m_{FP}^2 = \frac{2}{3}\Lambda$ there is a PM symmetry:

$$\delta M_{\mu\nu} \to \delta M_{\mu\nu} + \left(\nabla_\mu \nabla_\nu + \frac{\Lambda}{3}\, \bar{g}_{\mu\nu}\right)\xi(x)\,, \qquad \delta G_{\mu\nu} \to \delta G_{\mu\nu}$$

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$$\delta M_{\mu\nu} \rightarrow \delta M_{\mu\nu} + \left(\nabla_\mu \nabla_\nu + \tfrac{\Lambda}{3}\,\bar{g}_{\mu\nu}\right)\xi(x)\,, \qquad \delta G_{\mu\nu} \rightarrow \delta G_{\mu\nu}$$

- Find the transformation of $\delta g_{\mu\nu}$ & $\delta f_{\mu\nu}$.
- The dS-preserving subset $\xi = \xi_0$ (const), can be integrated to finite transformations,

$$\bar{g}'_{\mu\nu} = (1 + a\xi_0\,)\bar{g}_{\mu\nu}\,, \quad \bar{f}'_{\mu\nu} = (1 + b\xi_0\,)\bar{f}_{\mu\nu}$$

$$\bar{f}' = c'^2(\xi_0)\,\bar{g}' \qquad c' \neq c$$

A symmetry can exist only if $\Lambda_g = \Lambda_f$ does not determine c

Candidate PM bimetric theory in d=4

The necessary condition for the existence of PM symmetry is that c is not determined by $\Lambda_g = \Lambda_f$, or

$$\beta_1 + \left(3\beta_2 - \alpha^2\beta_0\right) c + \left(3\beta_3 - 3\alpha^2\beta_1\right) c^2 + \left(\beta_4 - 3\alpha^2\beta_2\right) c^3 + \alpha^2\beta_3 c^4 = 0$$

This gives the candidate nonlinear PM theory (d=4)

$$\alpha^2\beta_0 = 3\beta_2\,, \qquad 3\alpha^2\beta_2 = \beta_4\,, \qquad \beta_1 = \beta_3 = 0$$

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Nonlinear PM bimetric theory

Checks:

- $$m_{\rm FP}^2 = 2\frac{m^4}{m_g^2}\left(\alpha^{-2} + c^2\right)\beta_2 = \frac{2}{3}\Lambda_g$$
- Nonlinear PM bimetric can exist only for $d = 3, 4$.
- In $d > 4$ PM could be restored by Lanczos-Lovelock terms
- Realization of the ξ_0 gauge transformation in the nonlinear theory on dS, gauge invariant variables.

If the candidate PM theory really has a gauge symmetry, if will propagate 6=7-1 modes

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Digression: Conformal gravity

HD gravity:

$$S^{\mathrm{HD}}_{(2)}[g] = m_g^2 \int d^4x \sqrt{g} \left[\Lambda + c_R R(g) - \frac{c_{RR}}{m^2} \left(R^{\mu\nu} R_{\mu\nu} - \frac{1}{3} R^2 \right) \right]$$

7 modes: massless spin-2 + massive spin-2 (ghost) *[Stelle (1977)]*

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Conformal Gravity:

$$S^{\mathrm{CG}}[g] = -c \int d^4x \sqrt{g} \left[R^{\mu\nu} R_{\mu\nu} - \frac{1}{3} R^2 \right] ,$$

EoM (Bach tensor):

$$B_{\mu\nu} \equiv -\nabla^2 P_{\mu\nu} - \nabla_\mu \nabla \nu P \cdots = 0$$

Invariance:
$g_{\mu\nu} \rightarrow e^{\phi} g_{\mu\nu} \Rightarrow$ 6 modes: 2 (massless spin-2) + 4 ghosts

[Riegert (1984), Maldacena (2011)]

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Curvature expansion of bimetric equations

[SFH, Schmidt-May, von Strauss, 1303:6940]

Define

$$P_{\mu\nu} = R_{\mu\nu} - \frac{1}{2(d-1)} g_{\mu\nu} R$$

Relation between CG and PM bimetric theory

- In PM bimetric theory, the g and f-equations yield,

$$B_{\mu\nu} + \mathcal{O}(R^3/m^2) = 0$$

Thus in the low curvature limit, PM bimetric theory has a gauge symmetry even away from dS and definitely propagates $7 - 1 = 6$ modes! None is a ghost

- Obtaining the bimetric PM transfromations:
$\Delta g_{\mu\nu} = \phi g_{\mu\nu} \quad \Rightarrow \Delta f_{\mu\nu} = -\phi g_{\mu\nu}/\alpha^2 - 1/(m^2\beta_2)\nabla_\mu\nabla_\nu\phi$
So that,

$$\Delta\delta M_{\mu\nu} = \nabla_\mu\nabla_\nu\phi + (\Lambda/3)\phi g_{\mu\nu}$$

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  So that,

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- CG eom is the low curvature limit of PM bimetric eom. Conversely, PM bimetric is a ghost-free completion of CG
- Does the symmetry survive at higher orders? Yes! The construction can be extended to any number of derivatives

Relation between CG and PM bimetric theory

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Perturbative construction of the complete symmetry transformation

Define

$$g'_{\mu\nu} = g_{\mu\nu}/\alpha \qquad\qquad f'_{\mu\nu} = \alpha f_{\mu\nu}$$

g-eom: $\frac{\alpha}{m^2\beta_2} R_{\mu\nu}(g') - 3g'_{\mu\nu} + [g'_{\mu\rho} S'^{2\rho}_{\nu} - Tr(S') g_{\mu\rho} S'^{\rho}_{\nu}] = 0$

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Complete $g' \leftrightarrow f'$ interchange symmetry!

Trivial invariance of g-eom: $\Delta g \Rightarrow \Delta f$.

(Is this also a symmetry of the f-eom?)

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Starting with $\Delta g_{\mu\nu} = \phi g_{\mu\nu} + \cdots$, one can iteratively construct $\Delta f_{\mu\nu}$ to any order such that

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At the 2-derivative level,

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If true at 2n derivative level, the construction can be extended to 2n+2 derivative level. Hence, the existence of an on-shell symmetry if proved by induction.

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