

Title: How much information is there in large scale structure?

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URL: <http://pirsa.org/13070018>

Abstract: <span>Large scale structure contains vastly more Fourier modes than the CMB, and is therefore a promising arena for studying the early universe.&nbsp; One obstacle to using these modes is the non-linearity of structure formation. The amount of weakly coupled information available is therefore very sensitive to scale at which non-linear effects become important and simulations become necessary.&nbsp; Using effective field theory techniques, I will present evidence that the perturbative description of dark matter is much better behaved than previously thought.&nbsp; I will discuss the implications for improving constraints on non-gaussian initial conditions.</span>



# How much information is there in large scale structure?



Courtesy of thecmb.org

Daniel Green  
Stanford

1304.4946 + to appear:  
with Carrasco, Foreman and Senatore

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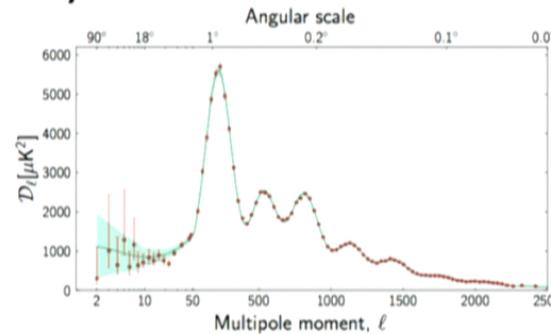
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## Life after Planck

For many quantities of interest

$$\left(\frac{S}{N}\right) \sim \frac{1}{\sqrt{N_{\text{modes}}}}$$

Planck has nearly saturated the modes in the CMB



$$l_{\max} \sim 1500 \rightarrow 2 \times 10^6 \text{ modes}$$

## Life after Planck

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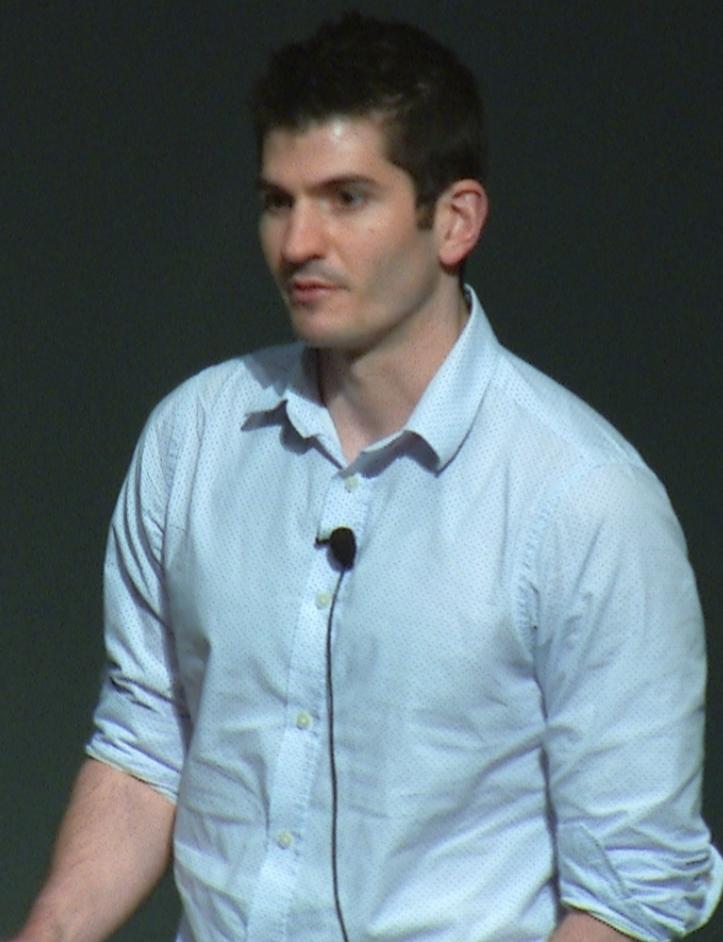
$$\left(\frac{S}{N}\right) \sim \frac{1}{\sqrt{N_{\text{modes}}}}$$

For significant improvements we need LSS:

$$N_{\text{linear}}^{\text{LSS}} \sim \left(\frac{k_{\text{max}}}{k_{\text{min}}}\right)^3 \sim \left(\frac{.1 h \text{ Mpc}^{-1}}{10^{-4} h \text{ Mpc}^{-1}}\right)^3 \sim 10^9$$

LSS contains a lot more information\*

\*if we measure the entire volume at low z



## Life after Planck



In practice, near term surveys:

$$N_{\text{linear modes}}^{\text{Euclid}} \sim \left( \frac{k_{\max}}{k_{\min}} \right)^3 \sim \left( \frac{0.1 h \text{ Mpc}^{-1}}{10^{-3} h \text{ Mpc}^{-1}} \right)^3 \sim 10^6$$

Just counting linear modes is comparable to CMB.

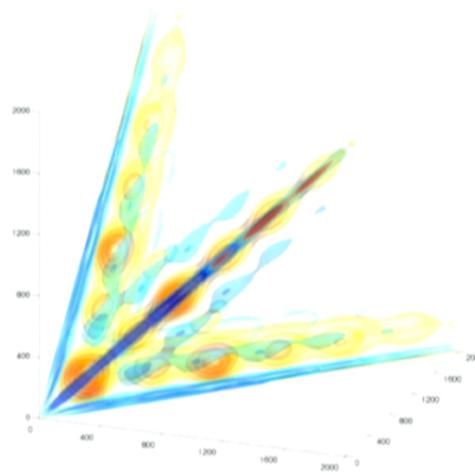
Can we do better than this? What is our goal?

I will focus on non-gaussianity  
(similar results apply to constraints on Dark Energy)



## Life after Planck

Planck reports limits on 3 templates:



$$f_{\text{NL}}^{\text{local}} = 2.7 \pm 5.8 \quad (68\% \text{ C.I.})$$

Courtesy of Fergusson & Shellard



## Life after Planck

The bound on equilateral seems especially weak  
Consider slow roll inflation + deformations [Creminelli](#)

$$\mathcal{L} = \mathcal{L}_{\text{slow roll}} + \frac{(\partial\phi)^4}{\Lambda^4}$$

For deformation to be under control  $\Lambda^2 > \dot{\phi}$

$$f_{\text{NL}}^{\text{equilateral}} \sim \frac{\dot{\phi}^2}{\Lambda^4} < 1$$

Any detection  $> 1$  rules out slow roll inflation

$$(\Delta f_{\text{NL}}^{\text{equilateral}})_{\text{Planck}} = 75 \ (1\sigma)$$

Many interesting models fall in between



## Life after Planck

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LSS constraints on equilateral require bispectra:

$$\langle \delta_{m,g}(\mathbf{k}_1) \delta_{m,g}(\mathbf{k}_2) \delta_{m,g}(\mathbf{k}_3) \rangle$$

Non-linearity will also generate a bispectrum.

Can we understand this well enough for  $\Delta f_{\text{NL}}^{\text{equi.}} = \mathcal{O}(1)$

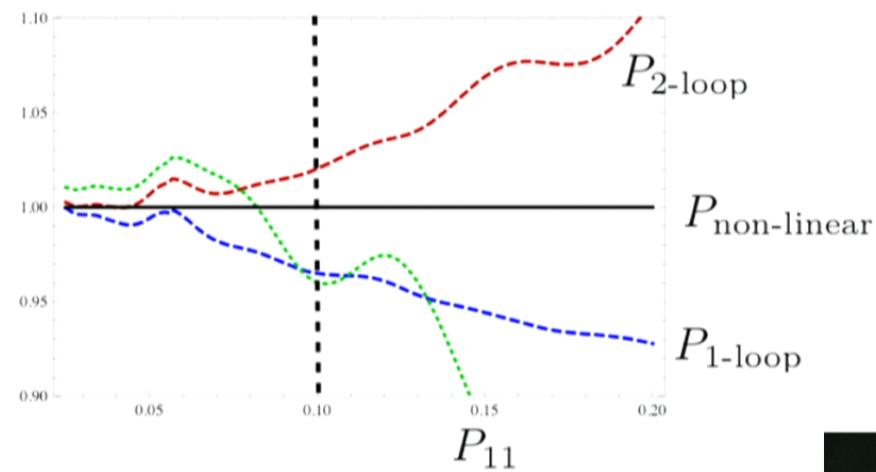
Often we use  $N_{\text{modes}}^{\text{max}} \sim \frac{k_{\text{NL}}^3}{k_{\text{min}}^3} \sim \frac{(0.1 h \text{ Mpc}^{-1})^3}{k_{\text{min}}^3}$

Is this really where non-linear effects come in?

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## Life after Planck

A common estimate is  $P_{\text{2-loop}} \gtrsim P_{\text{1-loop}} \gtrsim P_{11}$

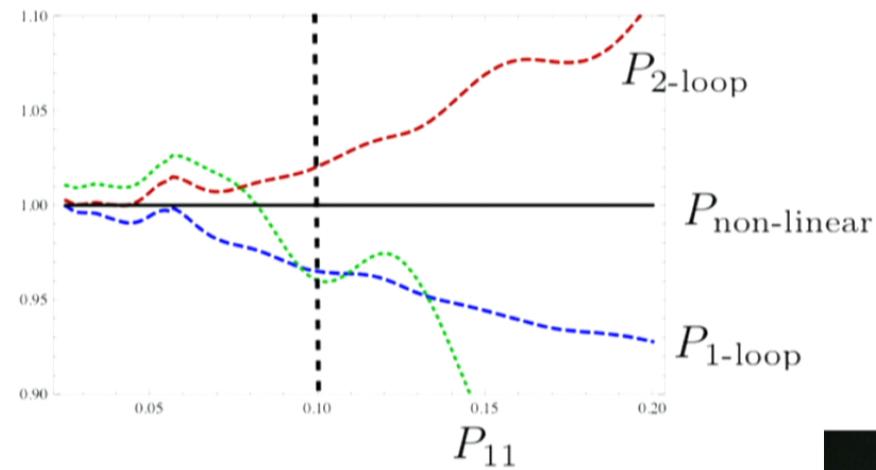


This would seem to give  $k_{\text{NL}} \sim .1 h \text{ Mpc}^{-1}$



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## Life after Planck



Is this really correct?

In many contexts:  $P_{\text{1-loop}}^{\text{STP}} = \infty$        $P_{\text{2-loop}}^{\text{STP}} = \infty$

Our perturbation theory is missing something:  
Dark matter is not a perfect fluid:

$$v^i + Hv_l^i + \frac{1}{a}v^j\partial_j v_l^i + \frac{1}{a}\partial^i\phi = -\frac{1}{a\rho}\partial_j\tau^{ij}$$

Many things will change when we include  $\tau^{ij} \neq 0$



## Outline

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Effective theory of LSS

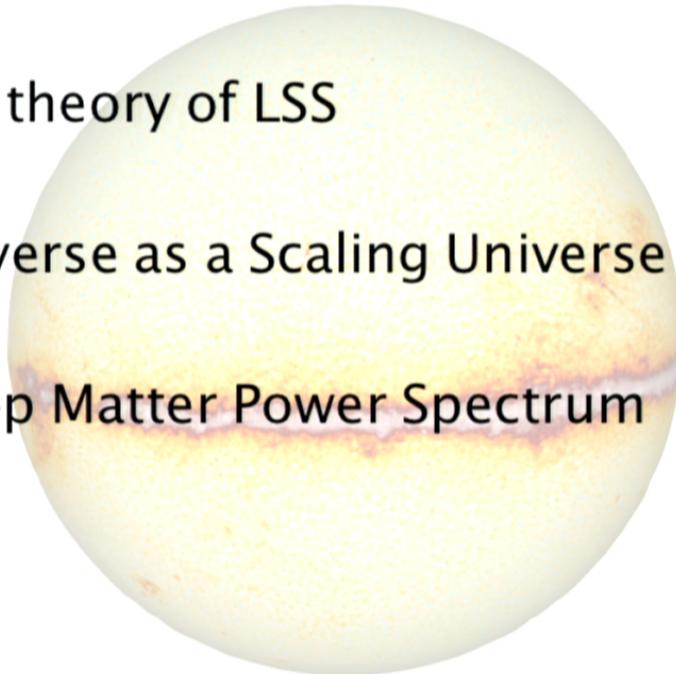
Real Universe as a Scaling Universe

Two-Loop Matter Power Spectrum

Outlook

Courtesy of thecmb.org

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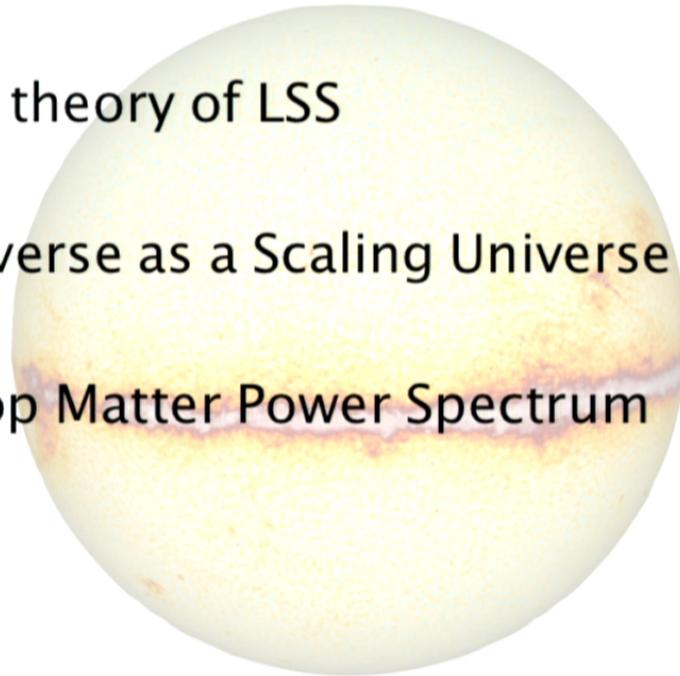
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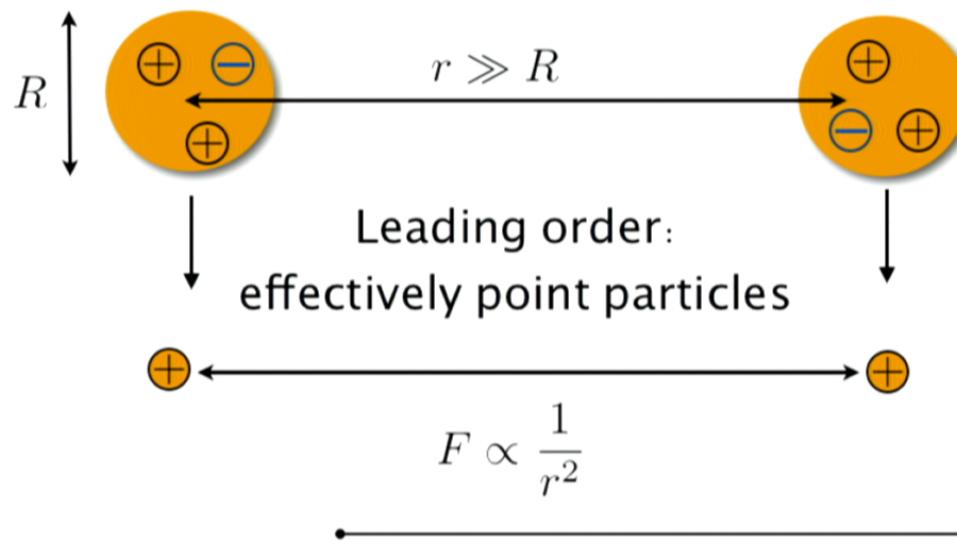
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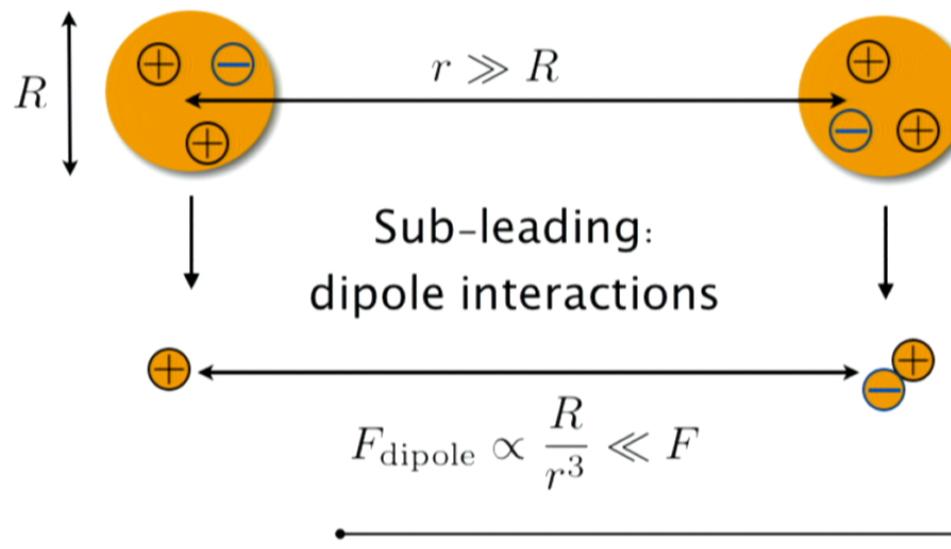
## Effective Field Theory

Often, EFT is a fancy term for normal physics  
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## Effective Field Theory

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E.g. Fluids

Start from the Boltzmann equation  $\frac{df[\mathbf{x}, \mathbf{p}, t]}{dt} = C[f]$

Take moments -  $\int d^3\mathbf{p} \mathbf{p}^n f[\mathbf{x}, \mathbf{p}, t]$

For perfect fluids, keep only  $n = 0, 1$

To describe viscosity, etc. need to keep  $n = 2, 3, \dots$

$$\dot{\mathbf{v}} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + c_b \nabla(\nabla \cdot \mathbf{v}) + c_v \nabla^2 \mathbf{v}$$

## Effective Field Theory

Small scale physics parameterized by a few numbers

However, in EFT, these “numbers” are not constant

Depends on: cutoff (regulator)  $\Lambda$   
renormalization scale  $\mu$

Then take  $\mu$  to match the scale of measurements

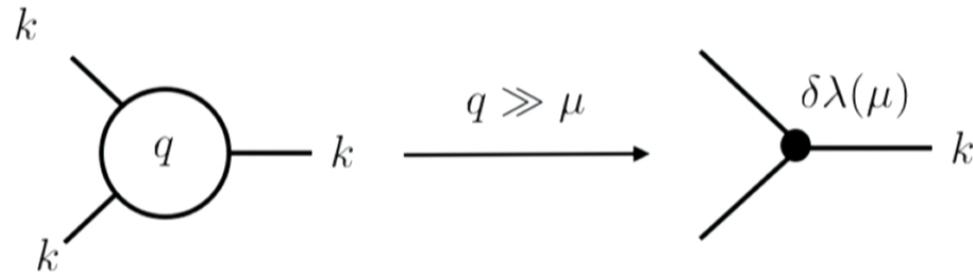
E.g. QED with massless electrons  $\alpha \propto \frac{1}{\log(\Lambda/\mu)}$

Potential from massive charge  $V(r) \propto \frac{1}{r \log(r\Lambda)}$

## Effective Field Theory

Same is true in classical field theory

Simply capturing the mixing between scales



Coupling changes by including  $\mu + \delta\mu > q > \mu$

## EFT of LSS

Dark matter is NOT a pressureless fluid

It is just a bunch of collision-less particles



On large scales it looks like a fluid (DM moves slow):

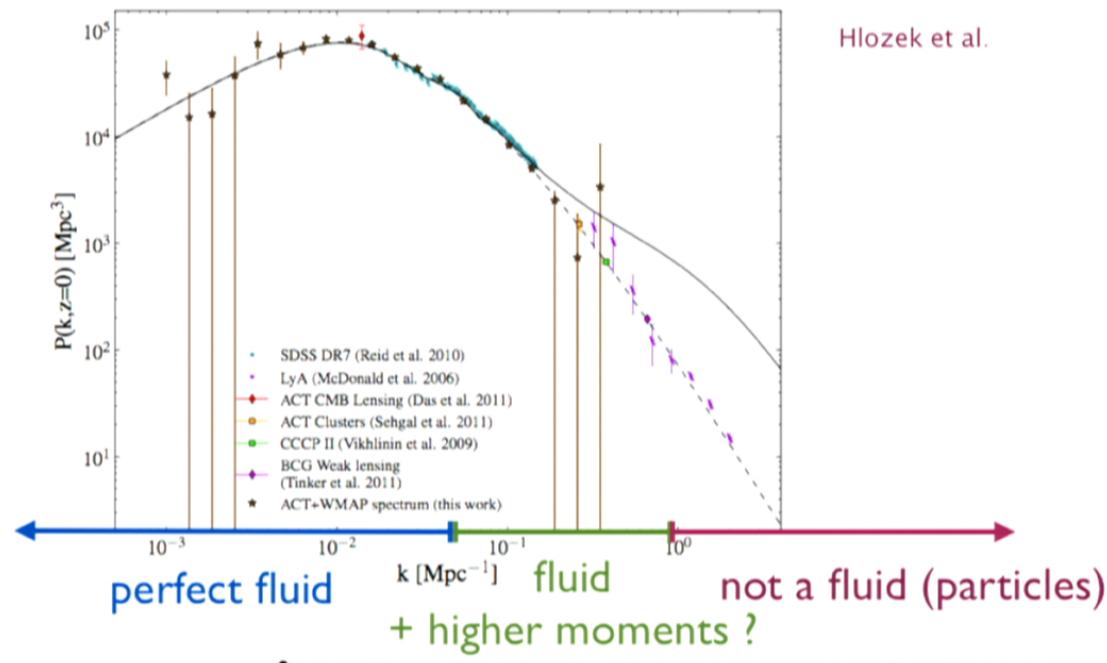
$$\int d^3p \left(\frac{\mathbf{p}}{m}\right)^n f(\mathbf{k}, \mathbf{p}, t) \sim (x_{\text{MFP}} k) \int d^3p \left(\frac{\mathbf{p}}{m}\right)^{n-1} f(\mathbf{k}, \mathbf{p}, t)$$

Like a perfect fluid when  $k \ll x_{\text{MFP}}^{-1}$

Baumann et al.  
Carrasco, Hertzberg & Senatore

# EFT of LSS

Dark matter is NOT a pressureless fluid



## EFT of LSS

Standard perturbation theory (SPT):

$$\nabla^2 \phi = \frac{3}{2} H_0^2 \Omega_m \frac{a_0^3}{a} \delta$$

$$\dot{\delta} = -\frac{1}{a} \partial_i ([1 + \delta] v^i)$$

$$\dot{v}^i + H v_l^i + \frac{1}{a} v^j \partial_j v_l^i + \frac{1}{a} \partial^i \phi = 0$$

EFT of LSS:

$$\dot{v}^i + H v_l^i + \frac{1}{a} v^j \partial_j v_l^i + \frac{1}{a} \partial^i \phi = -\frac{1}{a\rho} \partial_j \tau^{ij}$$

$$\tau^{ij} = \rho(c_s^2 \delta \delta^{ij} + \dots)$$

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## EFT of LSS

Standard perturbation theory (SPT):

Treat non-linear terms as perturbations ( $\theta \equiv \partial_i v^i$ )

$$a\mathcal{H}\delta' + \theta = - \int \frac{d^3q}{(2\pi)^3} \alpha(p, k-p) \delta(k-p) \theta(p) ,$$

$$a\mathcal{H}\theta' + \mathcal{H}\theta + \frac{3}{2}\mathcal{H}_0^2 \Omega_m \frac{a_0^3}{a} \delta = - \int \frac{d^3q}{(2\pi)^3} \beta(p, k-p) \theta(k-p) \theta(p)$$

EFT of LSS:

Also treat  $\partial_i \partial_j \tau^{ij}$  as a perturbation

$$a\mathcal{H}\theta' + \mathcal{H}\theta + \frac{3}{2}\mathcal{H}_0^2 \Omega_m \frac{a_0^3}{a} \delta = -\frac{1}{\rho} \partial^2 \tau^2 - \int \frac{d^3q}{(2\pi)^3} \beta \theta^2$$

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## What is the small number?



SPT is an expansion in  $\delta < 1$

Expect (hope?) loops are suppressed by  $\delta^L \ll \delta$

The EFT of LSS wants us to add:  $k^2\delta, k^2\delta^2, k^4\delta, \dots$

Problem: How do I compare  $\delta^L$  and  $k^{2p}\delta^q$  ?

We need a better understanding of  $\delta^L(k)$



## SPT in the Scaling Universe

The basic building block of perturbation theory is

$$\langle \delta^{(1)}(k) \delta^{(1)}(k') \rangle = P_{11}(k) (2\pi)^3 \delta^3(k + k')$$

We then solve for  $\delta = \sum_n \delta^{(n)} = \sum_n F_n(\{q_i\}) (\delta^{(1)})^n$

Simplest case to study is

$$P_{11}(k) = \frac{(2\pi)^3}{k_{\text{NL}}^3} \left( \frac{k}{k_{\text{NL}}} \right)^m$$

Only scale is  $k_{\text{NL}}$  : dim. analysis works

e.g. Jain & Bertschinger,  
Pajer & Zaldarriaga

## SPT in the Scaling Universe

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Finite parts ( $\Lambda$ - independent) are easy to estimate

$$P_{\text{L-loop}}^{\text{finite}} \sim (k^3 P_{11}(k))^L P_{11}(k) \sim \left(\frac{k}{k_{\text{NL}}}\right)^{(3+m)L} P_{11}(k)$$

E.g.:  $m = -\frac{3}{2}$  at one-loop:

$$P_{\text{1-loop}} = P_{31} + P_{22} \sim \left(\frac{k}{k_{\text{NL}}}\right)^{3/2} P_{11}(k)$$

There are also  $\Lambda$ -dependent contributions:

E.g.:  $m = -\frac{3}{2}$  at two-loops

$$P_{\text{2-loop}} \sim \left[ \frac{\Lambda}{k_{\text{NL}}} \frac{k^2}{k_{\text{NL}}^2} + \frac{k^3}{k_{\text{NL}}^3} \right] P_{11}(k) + \mathcal{O}\left(\frac{k}{\Lambda}\right)$$

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## SPT in the Scaling Universe

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All  $\Lambda$  - dependent terms must be removable

$$\partial_i \partial_j \tau^{ij} \sim (-\Lambda + c_0^2) \frac{k^2}{k_{\text{NL}}^2} \delta \rightarrow P_{c_s^2} = (-\Lambda + c_0^2) \frac{k^2}{k_{\text{NL}}^2} P_{11}(k)$$

These counter-terms also leave finite contributions:

$$P_{\text{2-loop}} + P_{c_s^2} \sim \left( c_0^2 \frac{k^2}{k_{\text{NL}}^2} + \frac{k^3}{k_{\text{NL}}^3} \right) P_{11}(k)$$

The finite part ( $c_0^2$ ) must be matched to simulations  
(not predicted by perturbation theory)

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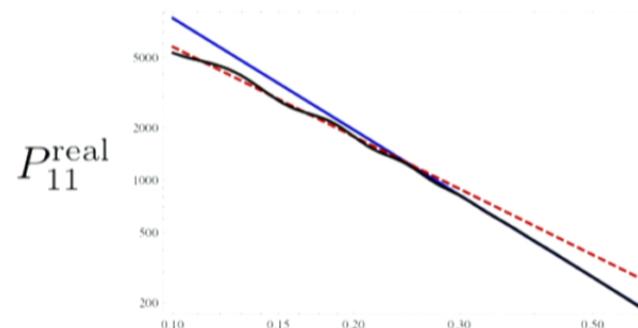
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## Scaling Behavior in the Real Universe

What does this have to do with the real universe?

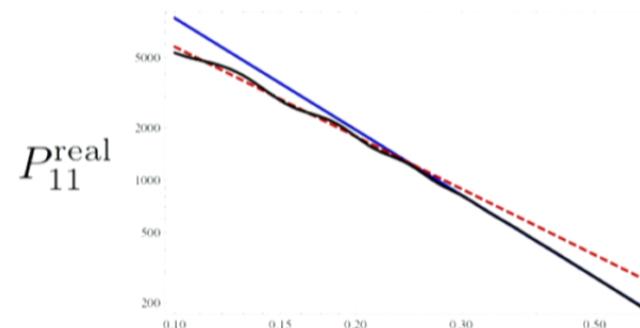


$$P_{11}^{\text{real}}(k) \sim (2\pi)^3 \begin{cases} \frac{1}{k_{\text{NL}}^3} \left(\frac{k}{k_{\text{NL}}}\right)^{-2.1} & \text{for } k > k_{\text{tr}} \\ \frac{1}{\tilde{k}_{\text{NL}}^3} \left(\frac{k}{\tilde{k}_{\text{NL}}}\right)^{-1.7} & \text{for } k < k_{\text{tr}} \end{cases}$$

$$k_{\text{NL}} \sim 4.6 h \text{ Mpc}^{-1} \quad k_{\text{tr}} \sim .25 h \text{ Mpc}^{-1}$$

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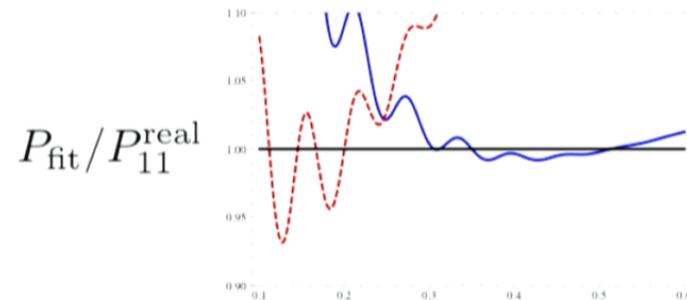


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## Scaling Behavior in the Real Universe

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What does this have to do with the real universe?

Above  $k \sim .25 h \text{ Mpc}^{-1}$ , we can use  $m = -2$

Estimate of error from 3-loop SPT

$$\frac{P_{\text{3-loop}}}{P_{\text{non-linear}}} (k = .5 h \text{ Mpc}^{-1}) \sim 0.02 - 0.04$$

Estimate of required “counter-terms”. Only need:

$$\partial_i \partial_j \tau^{ij} \sim [c_0^2 + c_{\text{2-loop}}(\Lambda)] \partial^2 \delta$$

All other counter-terms smaller than 3-loop SPT

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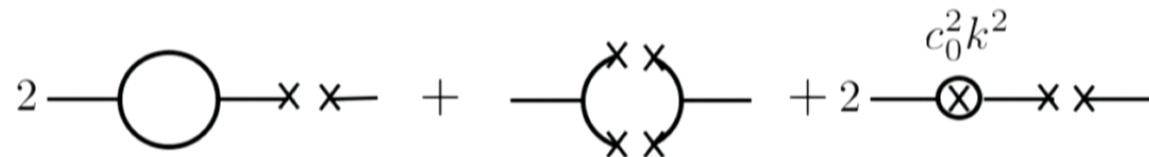
## “Measuring” parameters

From scaling universe, at 1-loop we have

$$\partial_i \partial_j \tau^{ij} = c_0^2 \frac{\partial^2}{k_{\text{NL}}^2} \delta$$

We can determine this using

$$P_{\text{1-loop}}^{\text{EFT}} = P_{\text{1-loop}}^{\text{STP}} + c_0^2 \frac{k^2}{k_{\text{NL}}} P_{11}$$



$$P_{31} \equiv \langle \delta^{(3)} \delta^{(1)} \rangle$$

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$$c_0^2 k^2 P_{11}$$

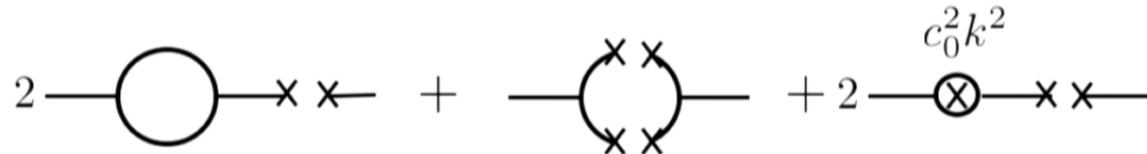
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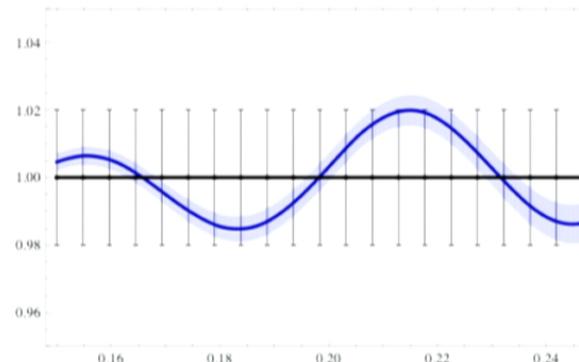
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Fit to non-linear data (Coyote):



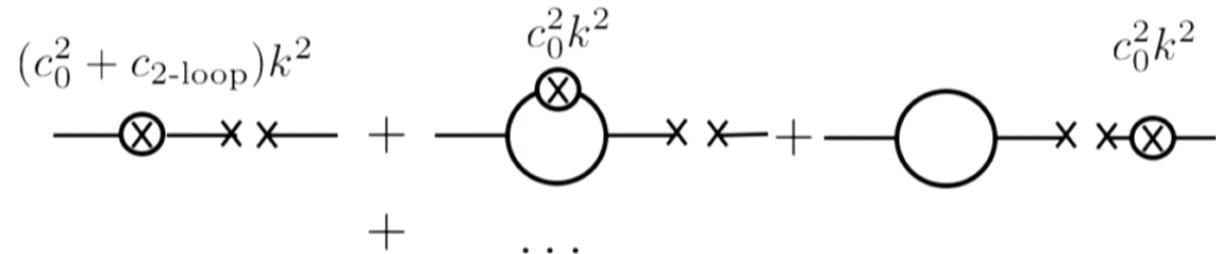
$$c_0^2 = (14.6 \pm 3.0) \left( \frac{k_{\text{NL}}}{h \text{ Mpc}^{-1}} \right)^2$$

## “Measuring” parameters

From scaling universe, at 2-loops we have

$$\partial_i \partial_j \tau^{ij} = (c_0^2 + c_{\text{2-loop}}) \frac{\partial^2}{k_{\text{NL}}^2} \delta$$

The two terms are evaluate at different orders:



$c_0^2$  counts as 1-loop and  $c_{\text{2-loop}}$  counts as 2-loops

## “Measuring” parameters

How do we determine  $c_{\text{2-loop}}$ ?

In the  $m=-2$  scaling universe:

$$P_{\text{2-loop}} = c^{\Lambda} \log(\Lambda/k) + \dots$$

The two loop “counter-term” should be

$$c_{\text{2-loop}} = -c^{\Lambda} \log(\Lambda/\mu)$$

This can be determined without non-linear data

Same idea works in real universe (but is more complicated)

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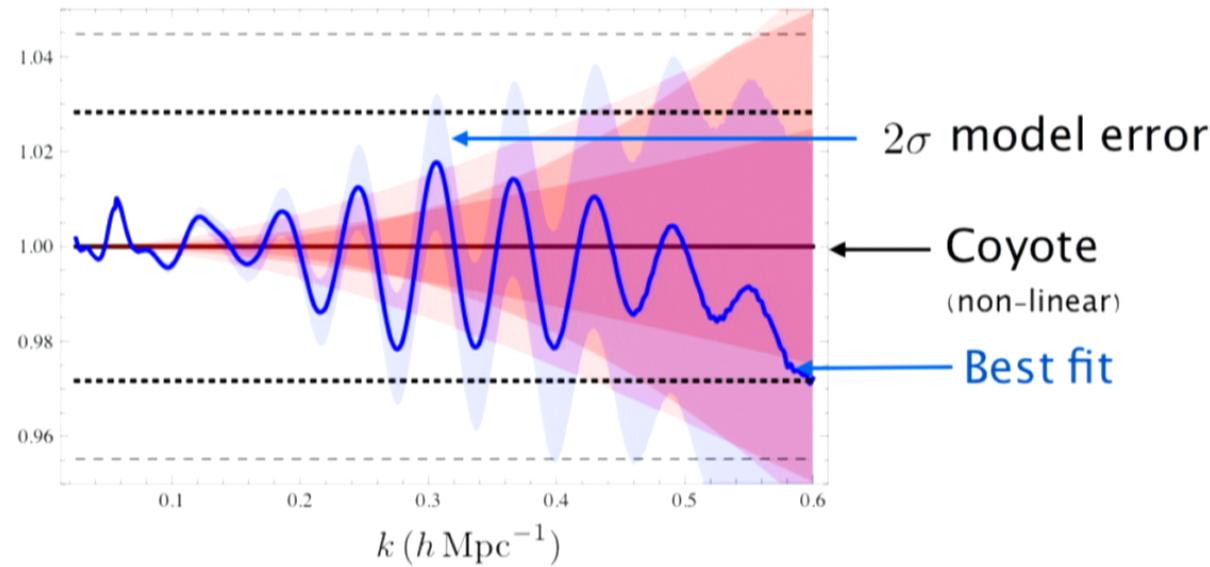
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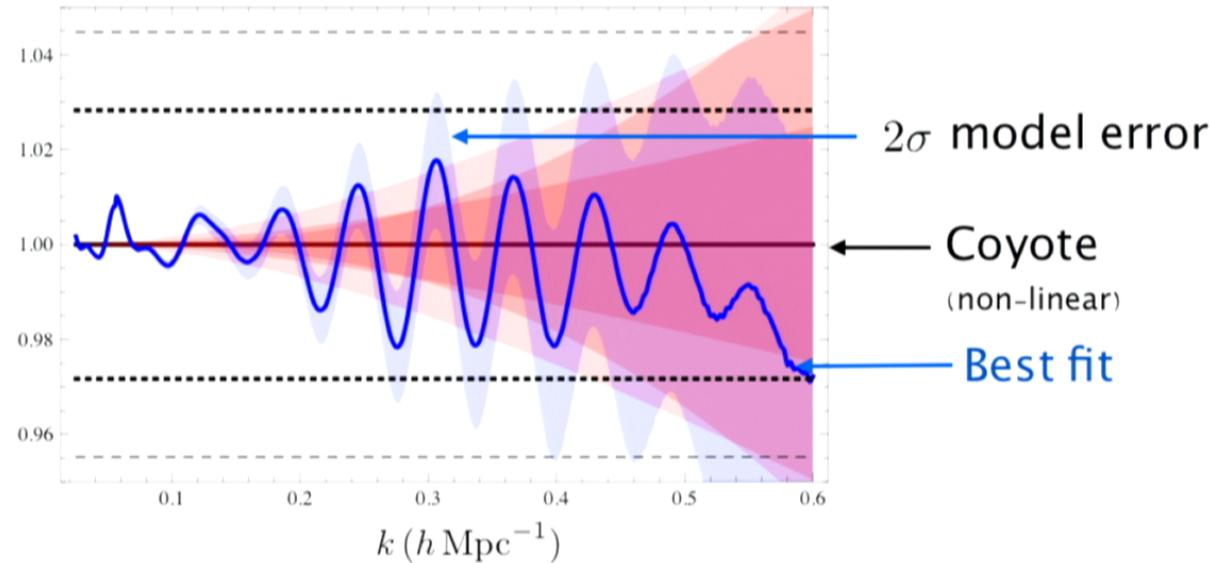
## Results

The 2-loop matter power spectrum:



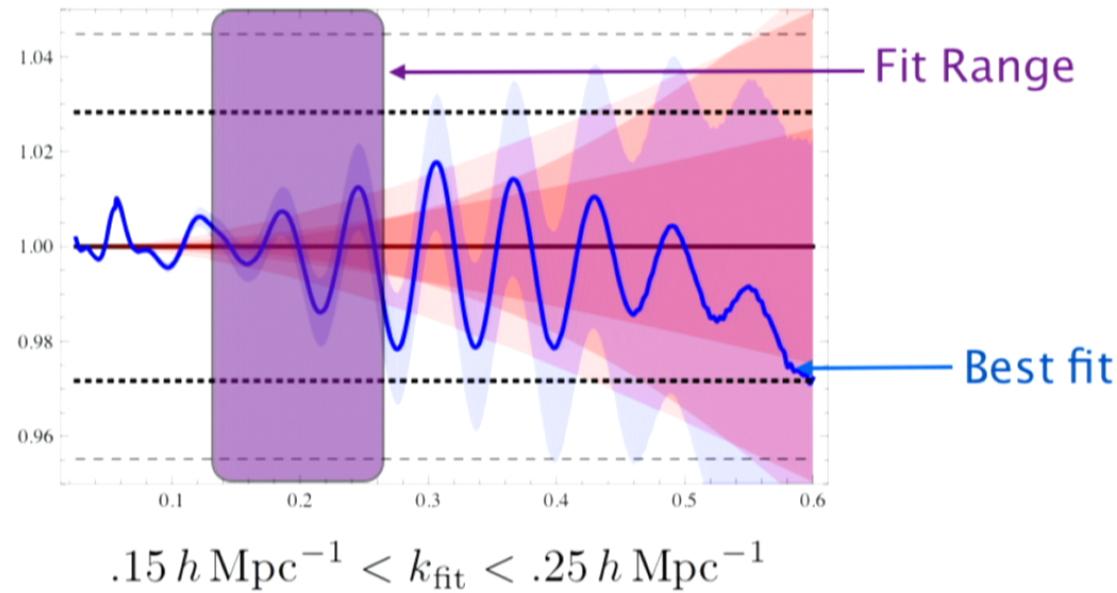
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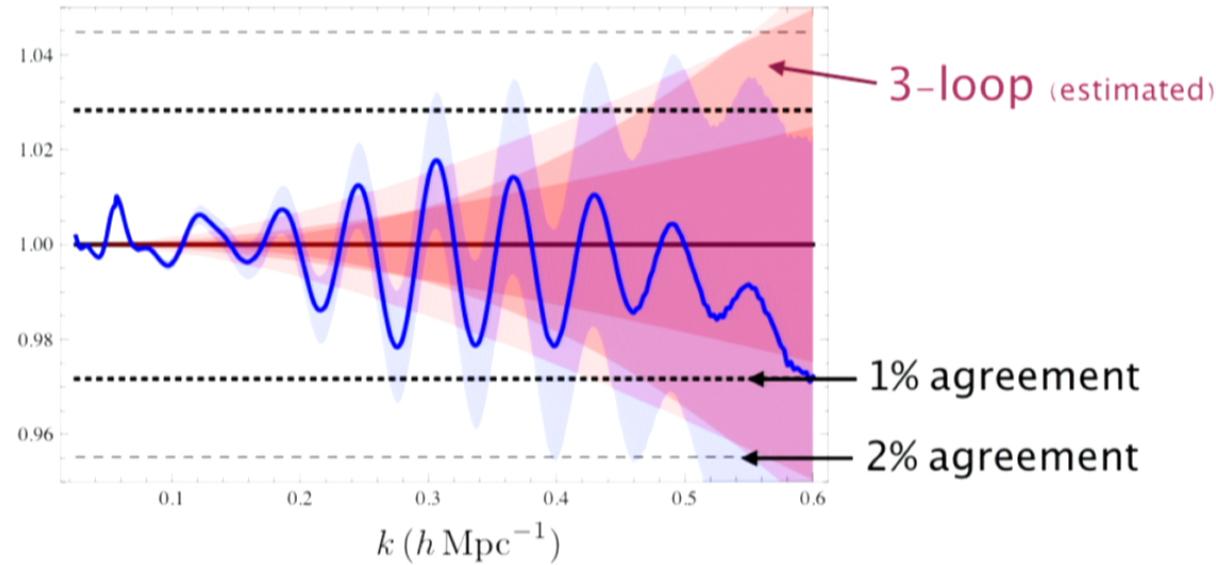
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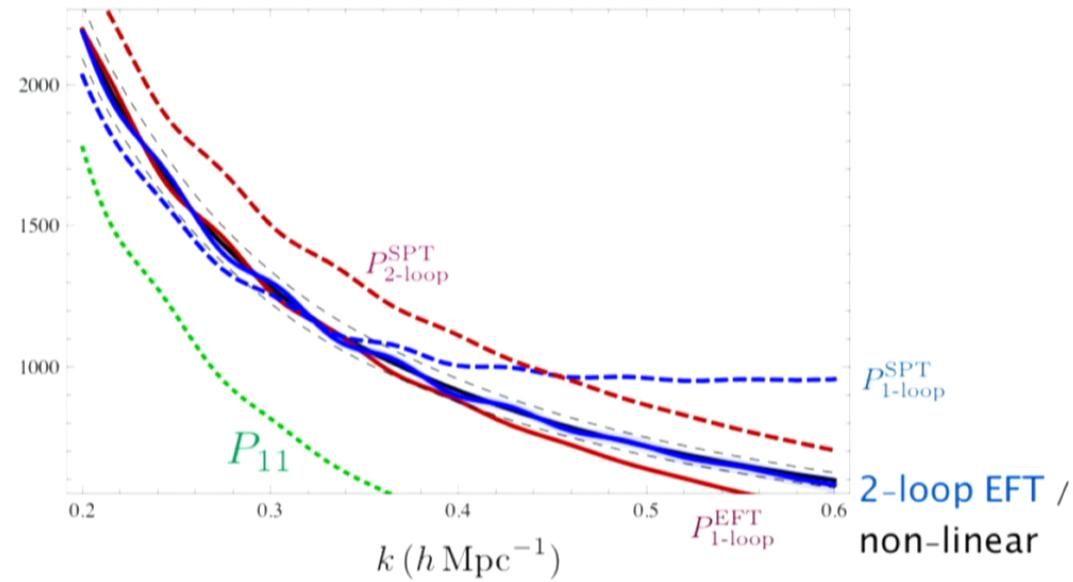
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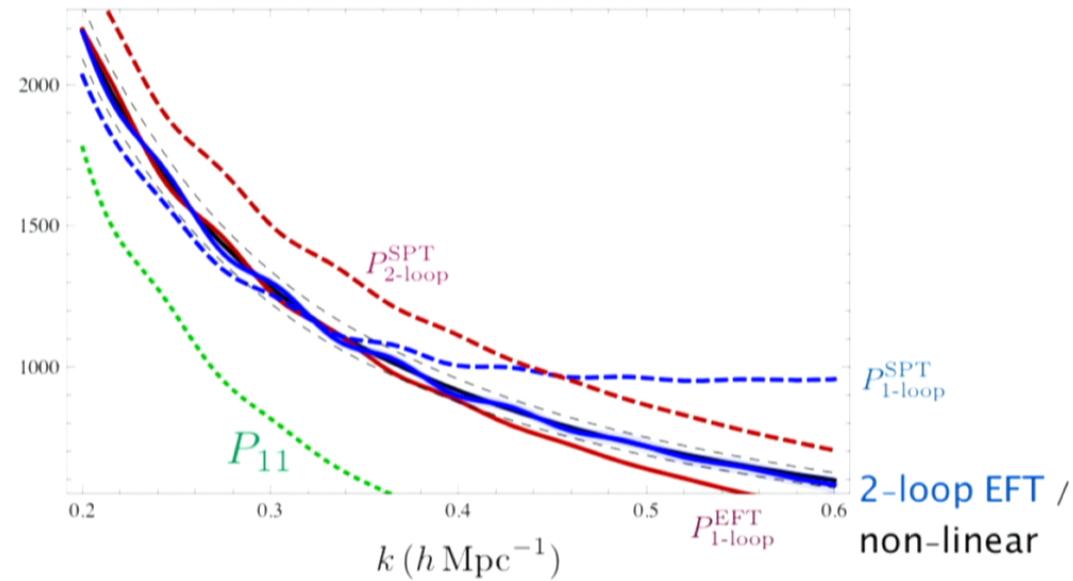
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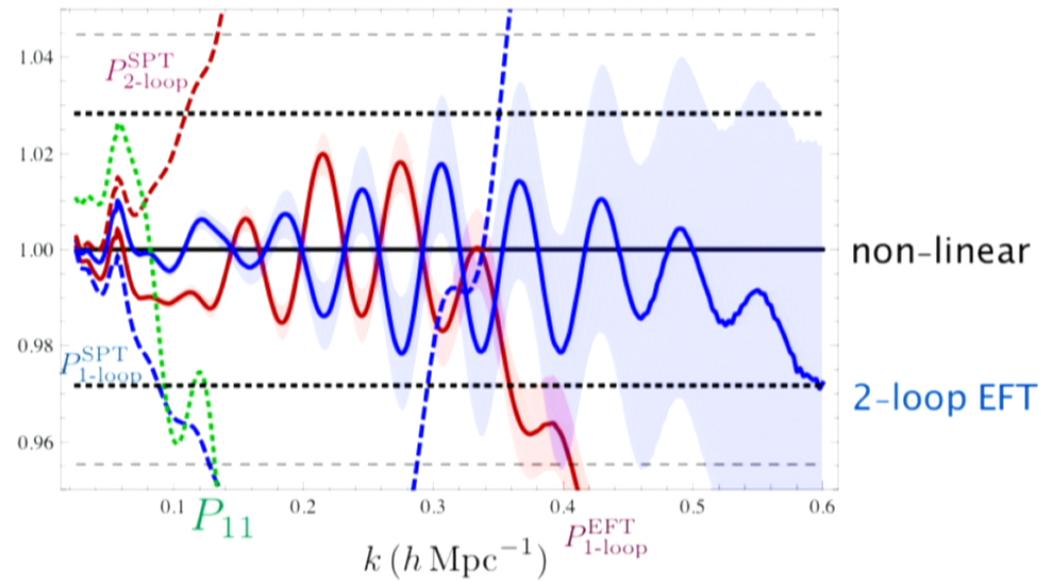
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## Implications for non-Gaussianity

Projections for future surveys give:

$$\Delta f_{\text{NL}}^{\text{equilateral}} \sim 20 - 30 \quad \text{for} \quad k_{\text{max}} = 0.1 h \text{ Mpc}^{-1}$$

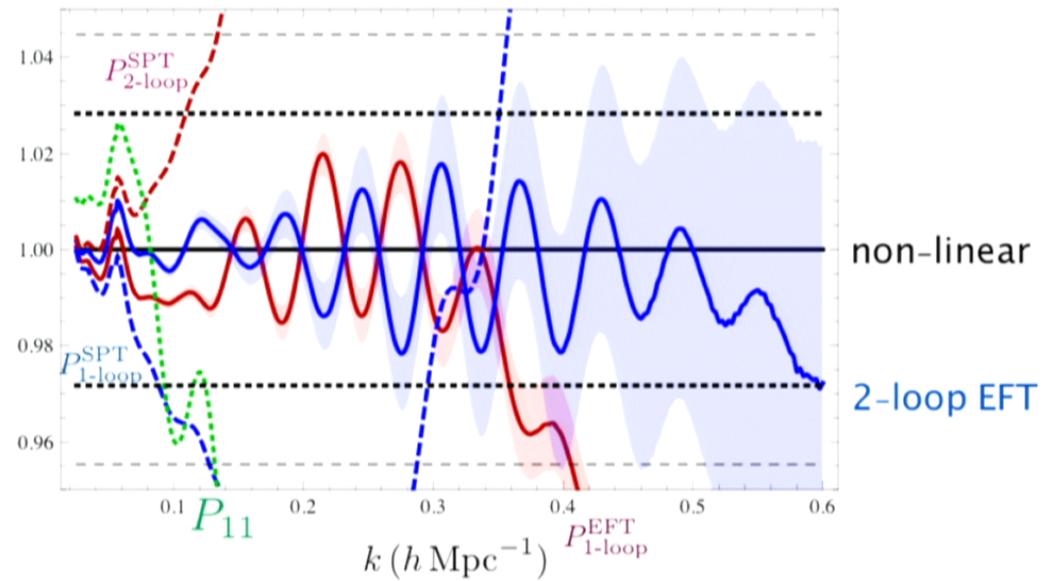
If we used the 2-loop EFT range of validity

$$\Delta f_{\text{NL}}^{\text{equilateral}} \sim 2 - 3 \quad \text{for} \quad k_{\text{max}} = 0.5 h \text{ Mpc}^{-1}$$

Equivalent to a survey >100x larger than LSST/Euclid

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## Outlook



## What we have shown

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Estimating the non-linear scale is non-trivial:

Previous estimates used  $P_{\text{2-loop}} \gtrsim P_{\text{1-loop}} \gtrsim P_{11}$

From the EFTofLSS we see this is not correct

Two loop EFT seems well behaved up to  $k \gtrsim .5 h \text{ Mpc}^{-1}$

Unfortunately, there is no rigorous definition:  
(there is no equivalent of perturbative unitarity)

## What is there to do?

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The real universe contains more than dark matter:

We don't observe DM: halo & galaxy biasing

Or observe in real space: redshift space distortions

Even if we measure DM directly (weak lensing):

Can we ignore or include baryons well enough?  
(is this an unmanageable mess?)

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