Title: Cosmological Consistency Relations as Ward Identities

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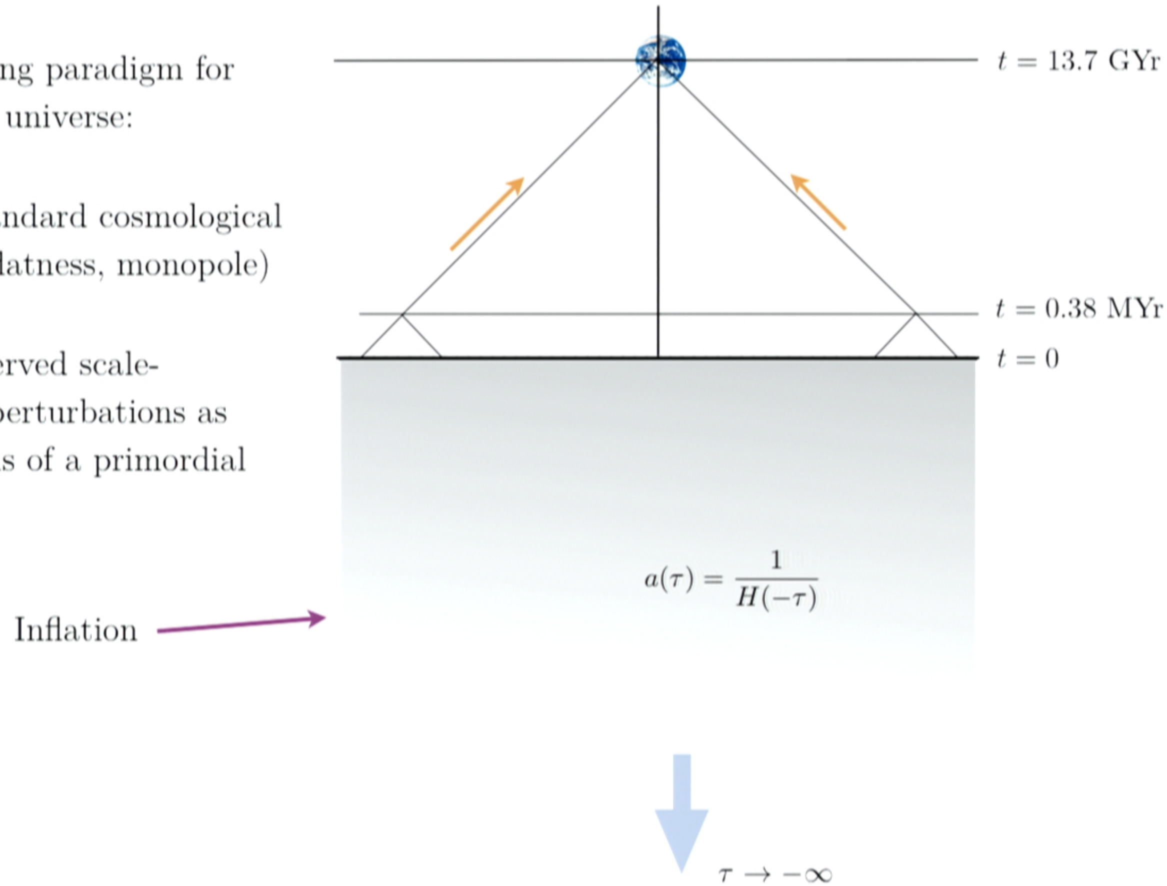
Abstract:

Inflation

Inflation is the leading paradigm for explaining the early universe:

- Addresses the standard cosmological problems (horizon, flatness, monopole)

- Explains the observed scale-invariant Gaussian perturbations as quantum fluctuations of a primordial field



Inflationary correlation functions

We're interested in computing N-point functions of ζ's and γ's at late times

$$\langle \zeta(x_1)\zeta(x_2)\cdots\gamma_{ij}(y_1)\gamma_{kl}(y_2)\cdots\rangle$$

Some notation:

$\langle\cdots\rangle'$ denotes momentum space correlator with overall momentum conserving delta function removed

$\langle\cdots\rangle_c$ denotes the connected correlator

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Power spectrum comes from two point function:

$$\langle\zeta(k)\zeta(k')\rangle = \int d^3x d^3x' e^{ik\cdot x} e^{ik'\cdot x'} \langle\zeta(x)\zeta(x')\rangle = (2\pi)^3\delta^3(k+k')P_\zeta(k)$$

$$\langle\gamma_k^s\gamma_{k'}^{s'}\rangle = \sum_{ij}\sum_{i'j'}\int d^3x d^3x' e^{ik\cdot x} e^{ik'\cdot x'} \epsilon_{ij}^{s'}(k)\epsilon_{i'j'}^{s}(k')\langle\gamma_{ij}(x)\gamma_{i'j'}(x')\rangle = (2\pi)^3\delta^3(k+k')P_\gamma(k)\delta^{ss'}$$

How inflationary correlation functions are computed

Maldacena (2002)

Starting point is GR plus a scalar (diff invariant):

$$\int d^4x\sqrt{-G}\left[\frac{M_P^2}{2}R - \frac{1}{2}\left(\partial\phi\right)^2 - V(\phi)\right]$$

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Go to ADM variables (3+1 split): $G_{\mu\nu}=\left(\begin{array}{c|c}-N^2+N^iN_i & N_i \\ \hline N_i & g_{ij}\end{array}\right)$

$$\frac{1}{2}\int d^4xN\sqrt{g}\left[M_P^2\left(R^{(3)}+K_{ij}^2-K^2\right)+\frac{1}{N^2}\left(\dot\phi-N^i\partial_i\phi\right)^2-(\nabla\phi)^2-2V\right]$$

Extrinsic curvature $K_{ij}=\frac{1}{2N}\left(\dot g_{ij}-\nabla_iN_j-\nabla_jN_i\right)$

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$$\frac{1}{2}\int d^4x N\sqrt{g}\left[M_P^2\left(R^{(3)} + K_{ij}^2 - K^2\right) + \frac{1}{N^2}\left(\dot\phi - N^i\partial_i\phi\right)^2 - (\nabla\phi)^2 - 2V\right]$$

Extrinsic curvature $K_{ij} = \frac{1}{2N}\left(\dot g_{ij} - \nabla_i N_j - \nabla_j N_i\right)$

Lapse and shift appear non-linearly and without time derivatives $\rightarrow$ auxiliary variables $\rightarrow$ can be eliminated via their own equations of motion:

$$M_P^2\nabla^j\left(K_{ij} - K g_{ij}\right) - \frac{1}{N}\left(\dot\phi - N^j\partial_j\phi\right)\partial_i\phi = 0,$$

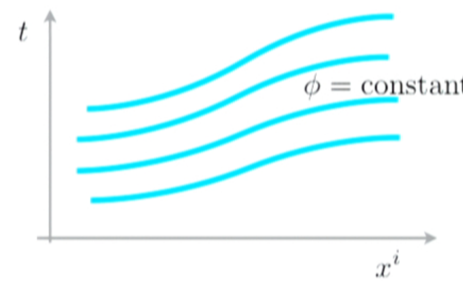
$$\frac{1}{2}\left[M_P^2\left(R^{(3)} - K_{ij}^2 + K^2\right) - \frac{1}{N^2}\left(\dot\phi - N^i\partial_i\phi\right)^2 - (\nabla\phi)^2 - 2V\right] = 0.$$

How inflationary correlation functions are computed

Expand around background solution: $3M_p^2H^2 = \frac{1}{2}\dot{\phi}^2 + V, \quad \ddot{\phi} + 3H\dot{\phi} + V' = 0$

Use the diff symmetry to choose co-moving gauge:

$$\phi = \bar{\phi}, \quad g_{ij} = a^2 e^{2\zeta} (\exp \gamma)_{ij}\,, \quad \gamma^i{}_i = 0, \quad \partial_i \gamma^i{}_j = 0,$$



DOF are carried by ζ and γ

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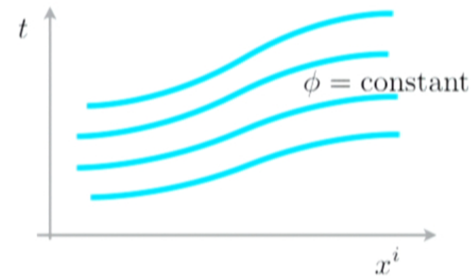

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DOF are carried by ζ and γ

Lapse and shift are expanded and solved order by order:

$$N = 1 + N_{(1)} + N_{(2)} + \cdots\,, \qquad N^i = N^i_{(1)} + N^i_{(2)} + \cdots\,,$$

$$N_{(1)i} = \partial_i \psi + N^T_{(1)i}, \qquad \partial^i N^T_{(1)i} = 0,$$

How inflationary correlation functions are computed

Plug the solution back into the action:

$$S = S_2 + S_3 + \cdots$$

Scalar modes:

$$\begin{aligned} S_2 &= M_P^2 \int \mathrm{d}^3x\mathrm{d}t \; a^3\epsilon \left[\dot{\zeta}^2 - a^{-2}(\vec{\partial}\zeta)^2\right] \\ S_3 &= M_P^2 \int \mathrm{d}^3x\mathrm{d}t \; a^3 \left\{ -a^{-2}\epsilon\zeta(\vec{\partial}\zeta)^2 + 3\epsilon\zeta\dot{\zeta}^2 - \epsilon\frac{\dot{\zeta}^3}{H} \right. \\ &+ \left. \frac{1}{2M_P^4}\left(3\zeta - \frac{\dot{\zeta}}{H}\right)\left(\partial_i\partial_j\psi\partial^i\partial^j\psi - (\vec{\nabla}^2\psi)^2\right) - \frac{2}{M_P^4}\vec{\nabla}^2\psi\partial_i\psi\partial^i\zeta \right\} \\ &\vdots \end{aligned}$$

Tensor modes:

$$S_2 = \frac{M_P^2}{8}\int d^4x \; a^3\left(\dot{\gamma}_{ij}\dot{\gamma}^{ij} - a^{-2}\partial_k\gamma_{ij}\partial^k\gamma^{ij}\right)$$

$$\vdots$$

How inflationary correlation functions are computed

Plug the solution back into the action:

$$S = S_2 + S_3 + \cdots$$

Scalar modes:

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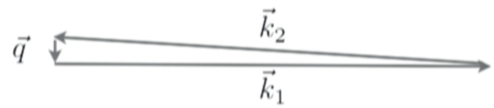
$$\vdots$$

Power spectrum comes from S_2
Higher point functions come from S_3, S_4, ...

The consistency relation

Maldacena (2002)

$$\lim_{\vec{q}\to 0} \frac{1}{P_\zeta(q)} \langle \zeta_{\vec{q}} \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle' = -\vec{k}_1 \cdot \frac{\partial}{\partial \vec{k}_1} \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle'$$



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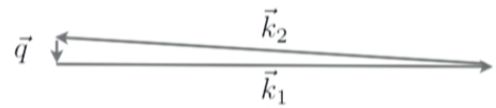

Holds in all single-field models, under the assumptions:

- constant growing mode (background is an attractor)
- adiabatic (Bunch-Davies) vacuum

Measuring a large 3-point function in this limit would automatically r
all standard single-field models (Planck: $f_{NL}^{\rm local} = 2.7 \pm 5.8$)

Extending the consistency relation

- Higher point functions

Creminelli & Zaldarriaga (2004);
Cheung, Fitzpatrick, Kaplan & Senatore (2007);
Assassi, Baumann & Green (2012);
Goldberger, Hui & Nicolis (2013)

$$\lim_{\vec{q}\to 0}\frac{1}{P_\zeta(q)}\langle\zeta(\vec{q})\mathcal{O}(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c = -\left(3(N-1)+\sum_{a=1}^{N}\vec{k}_a\cdot\frac{\partial}{\partial\vec{k}_a}\right)\langle\mathcal{O}(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c$$

- "Special conformal" consistency relations

Creminelli, Norena & Simonovic, 1203.4595;
Goldberger, Hui & Nicolis, 1303.1193;
Creminelli, D'Amico, Musso & Norena, 1104.1462

$$\lim_{\vec{q}\to 0}\frac{\partial}{\partial q^i}\left(\frac{1}{P_\zeta(q)}\langle\zeta(\vec{q})\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c\right) = -\frac{1}{2}\sum_{a=1}^{N}\left(6\frac{\partial}{\partial k_a^i}-k_a^i\frac{\partial^2}{\partial k_a^j\partial k_a^j}+2k_a^j\frac{\partial^2}{\partial k_a^j\partial k_a^i}\right)\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c + \ldots.$$

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These look like the soft pion theorems of chiral perturbation theory.

Suggests they should be derivable as the Ward identity of some spontaneously broken symmetry.

The consistency relation as a Ward identity

See also Assassi, Baumann & Green (2012);
Goldberger, Hui & Nicolis (2013).

Our goal is to find the most general such relation, and interpret it as a Ward identity for a spontaneously broken symmetry:

$$\lim_{\vec{q}\to 0} \frac{\partial^n}{\partial q^n} \left(\frac{1}{P_\zeta(q)} \langle \zeta(\vec{q}) \mathcal{O} \rangle + \frac{1}{P_\gamma(q)} \langle \gamma(\vec{q}) \mathcal{O} \rangle \right) \sim \quad \frac{\partial^n}{\partial k^n} \langle \mathcal{O} \rangle$$

- Constrains q^n behavior of correlators in the soft limit

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These extended relations involving higher-point functions and the sub-leading behavior of soft limits are likely not observable anytime soon.

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"So far, loop corrections to correlation functions appear to be much too small ever to be observed. The present work is motivated by the opinion that we ought to understand what our theories entail, even where in practice its predictions cannot be verified experimentally, just as field theorists in the 1940s and 1950s took pains to understand quantum electrodynamics to all orders of perturbation theory, even though it was only possible to verify results in the first few orders."

- Steven Weinberg, hep-th/0506236

Symmetries of zeta modes

Creminelli, Norena & Simonovic, 1203.4595; KH, Hui & Khoury, 1203.6351

Co-moving gauge choice (without the tensors):

$$\phi = \bar{\phi}, \qquad g_{ij} = a^2 e^{2\zeta} \delta_{ij}$$

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Any transformation on zeta which looks like a spatial coordinate transformation of the metric,

$$\delta\left(a^2(t) e^{2\zeta(\vec{x},t)} \delta_{ij}\right) = \mathcal{L}_{\vec{\xi}}\left(a^2(t) e^{2\zeta(\vec{x},t)} \delta_{ij}\right)$$

will be a symmetry of the gauge fixed action, with

$$\delta N^i = \mathcal{L}_\xi N^i + \dot{\xi}^i \quad , \qquad \delta N = \mathcal{L}_\xi N$$

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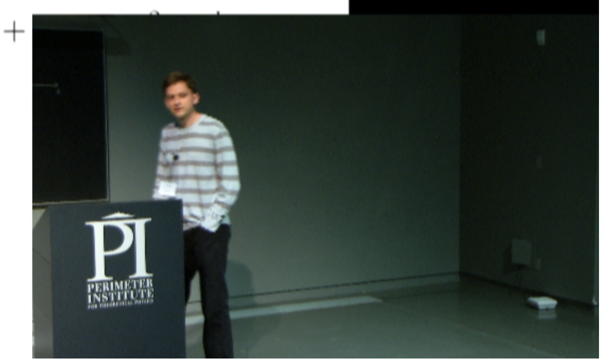
These are precisely the conformal transformations of spatial slices: $\partial_i\xi_j +$

Zeta transforms with a shift (like a dilaton): $\delta\zeta = \frac{1}{3}\partial^i\xi_i + \xi_i\partial^i\zeta$



Symmetries of zeta modes

Ordinary spatial translations and rotations are linearly realized.

$$\delta_i \zeta = -\partial_i \zeta, \quad \delta_{ij}\zeta = (x_i \partial_j - x_j \partial_i)\zeta$$

The dilation/SCT symmetries are non-linearly realized.

$$\begin{aligned} \delta\zeta &= 1 + \vec{x} \cdot \vec{\nabla}\zeta \\ \delta_{\vec{b}}\zeta &= 2\vec{b} \cdot \vec{x} + \left(2\vec{b} \cdot \vec{x}\, x^i - \vec{x}^2 b^i\right) \partial_i \zeta \end{aligned}$$

They close to form the de Sitter algebra $so(4,1)$

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Symmetry breaking pattern is: $so(4,1) \rightarrow iso(3)$

Extending to tensors

Co-moving gauge choice:

$$\phi = \bar{\phi}, \quad g_{ij} = a^2 e^{2\zeta} \left(\exp \gamma\right)_{ij}, \quad \gamma^i{}_i = 0, \quad \partial_i \gamma^i{}_j = 0,$$

Look for transformations of ζ and γ which make the spatial metric transform as a tensor:

$$\delta\left(e^{2\zeta}\left(e^{\gamma}\right)_{ij}\right) = \mathcal{L}_{\xi}\left(e^{2\zeta}\left(e^{\gamma}\right)_{ij}\right)$$

will be a symmetry of the full gauge fixed action, with

$$\delta N^i = \mathcal{L}_{\xi} N^i + \dot{\xi}^i \quad , \qquad \delta N = \mathcal{L}_{\xi} N$$

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$$\delta N^i = \mathcal{L}_\xi N^i + \dot{\xi}^i \quad , \qquad \delta N = \mathcal{L}_\xi N$$

We look for solutions order by order in powers of the tensor

$$\begin{aligned} \delta\gamma_{ij} &= \delta\gamma_{ij}^{(\gamma^0)} + \delta\gamma_{ij}^{(\gamma^1)} + \ldots \\ \delta\zeta &= \delta\zeta^{(\gamma^0)} + \delta\zeta^{(\gamma^1)} + \ldots \\ \xi_i &= \xi_i^{(\gamma^0)} + \xi_i^{(\gamma^1)} + \ldots \end{aligned}$$

At each order we will impose that $\delta\gamma_{ij}$ be transverse and traceless.

Extending to tensors

Zero-th order in tensors:

$$\delta\left(e^{2\zeta}\left(e^{\gamma}\right)_{ij}\right)=\mathcal{L}_{\xi}\left(e^{2\zeta}\left(e^{\gamma}\right)_{ij}\right) \quad \Longrightarrow \quad 2\delta\zeta^{(\gamma^0)}\delta_{ij}+\delta\gamma_{ij}^{(\gamma^0)}=2\xi_k^{(\gamma^0)}\partial^k\zeta\delta_{ij}+\partial_i\xi_j^{(\gamma^0)}+\partial_j\xi_i^{(\gamma^0)}$$

Take the trace, solve for the zeta transformation:

$$\delta\zeta^{(\gamma^0)}=\frac{1}{3}\partial^i\xi_i^{(\gamma^0)}+\xi_i^{(\gamma^0)}\partial^i\zeta$$

Plug back, solve for the tensor transformation:

$$\delta\gamma_{ij}^{(\gamma^0)}=\partial_i\xi_j^{(\gamma^0)}+\partial_j\xi_i^{(\gamma^0)}-\frac{2}{3}\partial^k\xi_k^{(\gamma^0)}\delta_{ij}$$

Take a divergence, get an equation for the gauge parameter:

$$\vec{\nabla}^2\xi_i^{(\gamma^0)}+\frac{1}{3}\partial_i\partial^j\xi_j^{(\gamma^0)}=0$$

Extending to tensors

Zero-th order in tensors:

$$\delta\left(e^{2\zeta}\left(e^{\gamma}\right)_{ij}\right) = \mathcal{L}_{\xi}\left(e^{2\zeta}\left(e^{\gamma}\right)_{ij}\right) \quad \Longrightarrow \quad 2\delta\zeta^{(\gamma^0)}\delta_{ij} + \delta\gamma_{ij}^{(\gamma^0)} = 2\xi_k^{(\gamma^0)}\partial^k\zeta\delta_{ij} + \partial_i\xi_j^{(\gamma^0)} + \partial_j\xi_i^{(\gamma^0)}$$

Take the trace, solve for the zeta transformation:

$$\delta\zeta^{(\gamma^0)} = \frac{1}{3}\partial^i\xi_i^{(\gamma^0)} + \xi_i^{(\gamma^0)}\partial^i\zeta$$

Plug back, solve for the tensor transformation:

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Take a divergence, get an equation for the gauge parameter:

$$\vec{\nabla}^2\xi_i^{(\gamma^0)} + \frac{1}{3}\partial_i\partial^j\xi_j^{(\gamma^0)} = 0$$

- No solutions which vanishes at infinity
- No dependence on zeta

Extending to tensors

First order in tensors:

$$\delta\left(e^{2\zeta}\left(e^{\gamma}\right)_{ij}\right)=\mathcal{L}_{\xi}\left(e^{2\zeta}\left(e^{\gamma}\right)_{ij}\right)\quad\Longrightarrow$$

$$2\delta\zeta^{(\gamma^1)}\delta_{ij}+2\delta\zeta^{(\gamma^0)}\gamma_{ij}+\delta\gamma_{ij}^{(\gamma^1)}=2\xi_k^{(\gamma^1)}\partial^k\zeta\delta_{ij}+2\xi_k^{(\gamma^0)}\partial^k\zeta\gamma_{ij}+\partial_i\xi_j^{(\gamma^1)}+\partial_j\xi_i^{(\gamma^1)}+\mathcal{L}_{\xi^{(\gamma^0)}}\gamma_{ij}$$

Take the trace, solve for the zeta transformation:

$$\delta\zeta^{(\gamma^1)}=\xi_i^{(\gamma^1)}\partial^i\zeta+\frac{1}{3}\partial^i\xi_i^{(\gamma^1)}+\frac{1}{3}\partial_i\xi_k^{(\gamma^0)}\gamma^{ik}$$

Plug back, solve for the tensor transformation:

$$\begin{aligned}\delta\gamma_{ij}^{(\gamma^1)}= \quad & \partial_i\xi_j^{(\gamma^1)}+\partial_j\xi_i^{(\gamma^1)}-\frac{2}{3}\partial^k\xi_k^{(\gamma^1)}\delta_{ij}\\ & -\frac{2}{3}\partial_l\xi_k^{(\gamma^0)}\gamma^{lk}\delta_{ij}-\frac{2}{3}\partial^k\xi_k^{(\gamma^0)}\gamma_{ij}+\mathcal{L}_{\xi^{(\gamma^0)}}\gamma_{ij}\end{aligned}$$

Take a divergence, get an equation for the gauge parameter:

$$\nabla^2\xi_i^{(\gamma^1)}+\frac{1}{3}\partial_i\left(\partial^j\xi_j^{(\gamma^1)}\right)=\partial^j\left(\frac{2}{3}\partial_l\xi_k^{(\gamma^0)}\gamma^{lk}\delta_{ij}+\frac{2}{3}\partial^k\xi_k^{(\gamma^0)}\gamma_{ij}-\mathcal{L}_{\xi^{(\gamma^0)}}\gamma_{ij}\right)$$

$$\Longrightarrow\quad\xi_i^{(\gamma^1)}=-\frac{\partial^k}{\nabla^2}\left(\delta_i^{\ \ell}-\frac{1}{4}\frac{\partial_i\partial^\ell}{\nabla^2}\right)\left(\mathcal{L}_{\xi^{(\gamma^0)}}\gamma_{k\ell}-\frac{2}{3}\partial_m\xi_q^{(\gamma^0)}\gamma^{mq}\delta^{k\ell}-\frac{2}{3}\partial^m\xi_m^{(\gamma^0)}\gamma^{k\ell}\right)$$

Extending to tensors

There is no obstruction to continuing this procedure to all orders in tensors.

The n-th order transformations and parameter $\delta\zeta^{(\gamma^n)},\ \delta\gamma^{(\gamma^n)},\ \xi_1^{(\gamma^n)}$ are all determined in terms of the zero-th order parameter:

$$\vec{\nabla}^2\xi_i^{(\gamma^0)} + \frac{1}{3}\partial_i\partial^j\xi_j^{(\gamma^0)} = 0$$

This is the divergence of the conformal Killing equation on R^3

Solutions

$$\vec{\nabla}^2 \xi_i^{(\gamma^0)} + \frac{1}{3}\partial_i \partial^j \xi_j^{(\gamma^0)} = 0$$

• Any conformal Killing vector is a solution.
Call these the conformal symmetries

$$\xi_i^{\text{dilation}} = \lambda x_i$$
$$\xi_i^{\text{SCT}} = 2b^j x_j x_i - \vec{x}^2 b_i \,.$$

• There will also be solutions which are not conformal Killing vectors.
Call these tensor symmetries

Solutions

$$\vec{\nabla}^2 \xi_i^{(\gamma^0)} + \frac{1}{3}\partial_i \partial^j \xi_j^{(\gamma^0)} = 0$$

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• There will also be solutions which are not conformal Killing vectors.
Call these tensor symmetries

The dilation symmetry happens to be *exact* (no corrections higher order in the tensors)

$$\delta^{\text{dilation}}\zeta(\vec{x}) = \lambda\left(1 + x^i \partial_i \zeta(\vec{x})\right) ,$$
$$\delta^{\text{dilation}}\gamma_{ij}(\vec{x}) = \lambda x^k \partial_k \gamma_{ij}(\vec{x}) \,.$$

All other symmetries receive corrections higher order in the tensors.

An infinite number of global symmetries

Taylor expand the parameter in powers of x

$$\xi_i(x) = \sum_{n=0}^{\infty} \frac{1}{(n+1)!} M_{i\ell_0 \ldots \ell_n} x^{\ell_0} \cdots x^{\ell_n}$$

M is symmetric in the l indices

Imposing $\vec{\nabla}^2 \xi_i + \frac{1}{3}\partial_i \partial^j \xi_j = 0$ fixes

$$M_{i\ell\ell\ell_2 \ldots \ell_n} = -\frac{1}{3} M_{\ell i \ell \ell_2 \ldots \ell_n} \qquad (n \geq 1)$$

To ensure transversality is preserved in Fourier space at finite momentum $\hat{q}^i \delta\gamma_{i\ell_0}(\vec{q}) = 0$

$$\hat{q}^i \left(M_{i\ell_0\ell_1 \ldots \ell_n}(\hat{q}) + M_{\ell_0 i \ell_1 \ldots \ell_n}(\hat{q}) - \frac{2}{3}\delta_{i\ell_0} M_{\ell\ell\ell_1 \ldots \ell_n}(\hat{q}) \right) = 0$$

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Symmetries indexed by n=0,1,2,... will constrain q^n in the soft limit

Deriving the consistency relation

Start with the statement that the charge generates a symmetry transformation:

L.H.S. R.H.S.

$$\langle\Omega|[Q,\mathcal{O}]|\Omega\rangle = -i\langle\Omega|\delta\mathcal{O}|\Omega\rangle$$

Q is the Noether charge corresponding to the symmetry:

$$Q = \frac{1}{2}\int \mathrm{d}^3x\left(\{\Pi_\zeta(x),\delta\zeta(x)\} + \{\Pi^{ij}_\gamma(x),\delta\gamma_{ij}(x)\}\right)$$

The operator is any equal-time product of ζ's and γ's.

The state is the in-vacuum, related to the free Bunch-Davies vacuum $|0\rangle$ by: $|\Omega\rangle = \Omega(-\infty)|0\rangle$

$$\Omega(t_i) \equiv U^\dagger(t_i,0)U_0(t_i,0)$$

Deriving the consistency relation

The right hand side:

The operator is a product of product of equal time ζ's and γ's

$$\mathcal{O}(\vec{k}_1,\ldots,\vec{k}_N) = \mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_M)\cdot\mathcal{O}^\gamma_{i_{M+1}j_{M+1},\ldots,i_Nj_N}(\vec{k}_{M+1},\ldots,\vec{k}_N)\,,$$

$$\mathcal{O}^\zeta \equiv \textstyle\prod_{a=1}^M \zeta(\vec{k}_a,t) \qquad \mathcal{O}^\gamma_{i_{M+1}j_{M+1},\ldots,i_Nj_N} \equiv \textstyle\prod_{b=M+1}^N \gamma_{i_bj_b}(\vec{k}_b,t)$$

Here is its transformation under our symmetries (in momentum space, first order in tensors):

$$\begin{aligned}\langle\bar{\delta}^{(n)}\mathcal{O}\rangle_c = &-\frac{(-i)^n}{n!}M_{i\ell_0\ldots\ell_n}\Bigg\{\sum_{a=1}^N\left(\delta^{i\ell_0}\frac{\partial^{n-1}}{\partial k^a_{\ell_1}\cdots\partial k^a_{\ell_n}}+\frac{k^i_a}{n+1}\frac{\partial^{n+1}}{\partial k^a_{\ell_0}\cdots\partial k^a_{\ell_n}}\right)\langle\mathcal{O}(\vec{k}_1,\ldots,\vec{k}_N)\rangle_c\\
&-\sum_{a=1}^M\Upsilon^{i\ell_0 i_a j_a}(\hat{k}_a)\frac{\partial^n}{\partial k^a_{\ell_1}\cdots\partial k^a_{\ell_n}}\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_{a-1},\vec{k}_{a+1},\ldots\vec{k}_M)\gamma_{i_aj_a}(\vec{k}_a)\mathcal{O}^\gamma(\vec{k}_{M+1},\ldots,\vec{k}_N)\rangle_c\\
&-\sum_{b=M+1}^N\Gamma^{i\ell_0}{}_{i_bj_b}{}^{k_b\ell_b}(\hat{k}_b)\frac{\partial^n}{\partial k^b_{\ell_1}\cdots\partial k^b_{\ell_n}}\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_M)\mathcal{O}^\gamma_{i_{M+1}j_{M+1},\ldots,k_b\ell_b,\ldots i_Nj_N}(\vec{k}_{M+1},\ldots,\vec{k}_N)\rangle_c\Bigg\}+\ldots\end{aligned}$$

where

$$\Upsilon_{rsij}(\hat{k}) \equiv \frac{1}{2}\delta_{s(i}\delta_{j)r}-\frac{1}{4}\hat{k}_s\hat{k}_{(i}\delta_{j)r}+\frac{5}{12}\hat{k}_i\hat{k}_j\delta_{rs}\,;$$

$$\Gamma_{rsijk\ell}(\hat{k}) \equiv 2\left(\delta_{s(i}-\hat{k}_r\hat{k}_{(i}\right)\delta_{j)(k}\delta_{\ell)r}-\left(\delta_{ij}-\hat{k}_i\hat{k}_j\right)\delta_{r(k}\delta_{\ell)s}-\frac{2}{3}\delta_{i(k}\delta_{\ell)j}\delta_{rs}$$

Deriving the consistency relation

The left hand side:

Intertwining relation: $Q\Omega(-\infty) = \Omega(-\infty)Q_0 \quad \Longrightarrow \quad Q|\Omega\rangle = \Omega(-\infty)Q_0|0\rangle$

We need to calculate $Q_0|0\rangle$

The "free" charge Q_0 is a symmetry of the free theory. It generates the non-linear shift in the fields:

$$Q_0 \sim \int d^3x \; x^n \left(\Pi_\zeta(x) + \Pi_\gamma(x)\right) \sim \lim_{q\to 0} \frac{\partial^n}{\partial q^n} \left(\Pi_\zeta(q) + \Pi_\gamma(q)\right)$$

$$\delta_0\zeta, \delta_0\gamma \sim x^n$$

Deriving the consistency relation

The left hand side:

Insert a complete set of free-field eigenstates: $|\zeta_0, \gamma_0\rangle \equiv |X_0\rangle$

$$Q_0|0\rangle \sim \lim_{q\to 0} \frac{\partial^n}{\partial q^n} \int DX_0 |X_0\rangle\langle X_0|\Pi(q)|0\rangle \sim \lim_{q\to 0} \frac{\partial^n}{\partial q^n} \int DX_0 |X_0\rangle \frac{\delta}{\delta X_0(q)} \langle X_0|0\rangle$$

The free-vacuum wavefunctional is a Gaussian (here is the assumption that the initial state is Bunch-Davies)

$$\langle X_0|0\rangle \sim \exp\left[-\int \frac{d^3k}{(2\pi)^3} \frac{1}{P_X(k)} X_0(k) X_0(-k)\right]$$

$$Q_0|0\rangle \sim \lim_{q\to 0} \frac{\partial^n}{\partial q^n} \left(\frac{1}{P_X(q)} X_0(q)\right)|0\rangle$$

Now use the assumption that ζ, γ have constant growing modes:

$$\zeta_0|0\rangle = \zeta|\Omega\rangle$$
$$\gamma_0|0\rangle = \gamma|\Omega\rangle$$

$$\langle\Omega|[Q, \mathcal{O}]|\Omega\rangle \sim \lim_{q\to 0} \frac{\partial^n}{\partial q^n} \left(\frac{1}{P_\zeta(q)} \langle\zeta(q)\mathcal{O}\rangle + \frac{1}{P_\gamma(q)} \langle\gamma(q)\mathcal{O}\rangle\right)$$

The full consistency relation

$$\lim_{\vec{q}\to 0} P_{i\ell_0\ldots\ell_n jm_0\ldots m_n}(\hat{q})\frac{\partial^n}{\partial q_{m_1}\cdots\partial q_{m_n}}\left(\frac{1}{P_\gamma(q)}\langle\gamma^{jm_0}(\vec{q})\mathcal{O}(\vec{k}_1,\ldots,\vec{k}_N)\rangle_c' + \frac{\delta^{jm_0}}{3P_\zeta(q)}\langle\zeta(\vec{q})\mathcal{O}(\vec{k}_1,\ldots,\vec{k}_N)\rangle_c'\right)$$

$$= -P_{i\ell_0\ldots\ell_n jm_0\ldots m_n}(\hat{q})\left\{\sum_{a=1}^{N}\left(\delta^{jm_0}\frac{\partial^n}{\partial k^a_{m_1}\cdots\partial k^a_{m_n}} - \frac{\delta_{n0}}{N}\delta^{jm_0} + \frac{k^j_a}{n+1}\frac{\partial^{n+1}}{\partial k^a_{m_0}\cdots\partial k^a_{m_n}}\right)\langle\mathcal{O}(\vec{k}_1,\ldots,\vec{k}_N)\rangle_c'\right.$$

$$-\sum_{a=1}^{M}\Upsilon^{jm_0 i_a j_a}(\hat{k}_a)\frac{\partial^n}{\partial k^a_{m_1}\cdots\partial k^a_{m_n}}\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_{a-1},\vec{k}_{a+1},\ldots\vec{k}_M)\gamma_{i_a j_a}(\vec{k}_a)\mathcal{O}^\gamma(\vec{k}_{M+1},\ldots,\vec{k}_N)\rangle_c'$$

$$\left.-\sum_{b=M+1}^{N}\Gamma^{jm_0}{}_{i_b j_b}{}^{k_b\ell_b}(\hat{k}_b)\frac{\partial^n}{\partial k^b_{m_1}\cdots\partial k^b_{m_n}}\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_M)\mathcal{O}^\gamma_{i_{M+1}j_{M+1},\ldots,k_b\ell_b,\ldots i_N j_N}(\vec{k}_{M+1},\ldots,\vec{k}_N)\rangle_c'\right\} + \ldots$$

where

$$\Upsilon_{rsij}(\hat{k}) \equiv \frac{1}{2}\delta_{s(i}\delta_{j)r} - \frac{1}{4}\hat{k}_s\hat{k}_{(i}\delta_{j)r} + \frac{5}{12}\hat{k}_i\hat{k}_j\delta_{rs}\,;$$

$$\Gamma_{rsijk\ell}(\hat{k}) \equiv 2\left(\delta_{s(i} - \hat{k}_r\hat{k}_{(i}\right)\delta_{j)(k}\delta_{\ell)r} - \left(\delta_{ij} - \hat{k}_i\hat{k}_j\right)\delta_{r(k}\delta_{\ell)s} - \frac{2}{3}\delta_{i(k}\delta_{\ell)j}\delta_{rs}$$

The full consistency relation (schematically)

$$\lim_{\vec{q}\to 0} \frac{\partial^n}{\partial q^n}\left(\frac{1}{P_\zeta(q)}\langle\zeta(\vec{q})\mathcal{O}\rangle + \frac{1}{P_\gamma(q)}\langle\gamma(\vec{q})\mathcal{O}\rangle\right) \sim \quad \frac{\partial^n}{\partial k^n}\langle\mathcal{O}\rangle$$

- Constrains q^n behavior of correlators in the soft limit
- Holds on any spatially-flat FRW background (no slow-roll conditions required)
- Holds to all loops

The first few cases

<u>n=0</u> $\xi_i^{(n=0)} = M_{i\ell_0} x^{\ell_0}$

Decompose into a trace part, a symmetric traceless part and an anti-symmetric part:

$$M_{i\ell_0} = \lambda\delta_{i\ell_0} + S_{i\ell_0} + \omega_{i\ell_0}$$

- Anti-symmetric part is just spatial rotations (linearly realized)

- Trace part is dilations

- Symmetric traceless part gives two tensor symmetries (anisotropic rescaling) $\qquad \hat{q}^i S_{i\ell_0}(\vec{q}) = 0$

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(anisotropic rescaling)

Ward identity reproduces the original consistency relations:

Maldacena (2002); Creminelli & Zaldarriaga (2004); Cheung, Fitzpatrick, Kaplan & Senatore (2007); Assassi, Baumann & Green (2012); Goldberger, Hui & Nicolis (2013)

$$\lim_{\vec{q}\to 0} \frac{1}{P_\zeta(q)} \langle \zeta(\vec{q}) \mathcal{O}(\vec{k}_1, \ldots, \vec{k}_N)\rangle'_c = -\left(3(N-1) + \sum_{a=1}^{N} \vec{k}_a \cdot \frac{\partial}{\partial \vec{k}_a}\right) \langle \mathcal{O}(\vec{k}_1, \ldots, \vec{k}_N)\rangle'_c$$

$$\lim_{\vec{q}\to 0} \frac{1}{P_\gamma(q)} \langle \gamma^s(\vec{q}) \mathcal{O}^\zeta(\vec{k}_1, \ldots, \vec{k}_N)\rangle'_c = -\frac{1}{2}\epsilon^s_{i\ell_0}(\hat{q}) \sum_{a=1}^{N} \left\{ k_a^i \frac{\partial}{\partial k_a^{\ell_0}} \langle \mathcal{O}^\zeta(\vec{k}_1, \ldots, \vec{k}_N)\rangle'_c - \frac{1}{2} \langle \mathcal{O}^\zeta(\vec{k}_1, \ldots, \vec{k}_{a-1}, \vec{k}_{a+1}, \ldots, \vec{k}_N) \gamma_{i\ell_0}(\vec{k}_a)\rangle'_c \right\} + \ldots$$

The first few cases

n=1 $\quad \xi_i^{(n=1)} = M_{i\ell_0\ell_1} x^{\ell_0} x^{\ell_1}$

- 3 special conformal symmetries (both γ and ζ transform non-linearly)
  $M^{\rm SCT}_{i\ell_0\ell_1} = b_{\ell_1}\delta_{i\ell_0} + b_{\ell_0}\delta_{i\ell_1} - b_i\delta_{\ell_0\ell_1}$
- 4 tensor symmetries (only γ transforms non-linearly)

Ward identity reproduces the special conformal consistency relation and linear-gradient tensor relation:

Creminelli, Norena & Simonovic, 1203.4595;
Goldberger, Hui & Nicolis, 1303.1193;
Creminelli, D'Amico, Musso & Norena, 1104.1462

$$\lim_{\vec{q}\to 0}\frac{\partial}{\partial q^i}\left(\frac{1}{P_\zeta(q)}\langle\zeta(\vec{q})\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c\right) = -\frac{1}{2}\sum_{a=1}^N\left(6\frac{\partial}{\partial k_a^i} - k_a^i\frac{\partial^2}{\partial k_a^j\partial k_a^j} + 2k_a^j\frac{\partial^2}{\partial k_a^j\partial k_a^i}\right)\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c + \ldots .$$

$$\lim_{\vec{q}\to 0} q^{\ell_1}\frac{\partial}{\partial q^{\ell_1}}\left(\frac{1}{P_\gamma(q)}\langle\gamma^s(\vec{q})\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c\right) = -\frac{1}{2}q^{\ell_1}\epsilon^s_{i\ell_0}(\vec{q})\sum_{a=1}^N\Bigg\{\left(k_a^i\frac{\partial}{\partial k_a^{\ell_1}} - \frac{k_a^{\ell_1}}{2}\frac{\partial}{\partial k_i^a}\right)\frac{\partial}{\partial k^a_{\ell_0}}\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c - \left(2\Upsilon^{i\ell_0 i_a j_a}(\hat{k}_a)\frac{\partial}{\partial k^a_{\ell_1}} - \Upsilon^{\ell_1 i i_a j_a}(\hat{k}_a)\frac{\partial}{\partial k^a_{\ell_0}}\right) \times \langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_{a-1},\vec{k}_{a+1},\ldots,\vec{k}_N)\gamma_{i\ell_0}(\vec{k}_a)\rangle'_c\Bigg\} + \ldots$$

The first few cases

n=2 $\quad \xi_i^{(n=2)} = M_{i\ell_0\ell_1\ell_2} x^{\ell_0} x^{\ell_1} x^{\ell_2}$ First genuinely new consistency relations.

See however Creminellia, Perkob, Senatore, Simonovic, Trevisan 1307.0503

Six new symmetries for each n≥2:

- 4 tensor symmetries (only γ transforms non-linearly)
- 2 mixed symmetries (both γ and ζ transform non-linearly)

Example: n=2 tensor symmetry consistency relation with two zeta insertions:

$$\lim_{\vec{q}\to 0} P^{\mathrm{T}}_{i\ell_0\ell_1\ell_2 j m_0 m_1 m_2}(\hat{q}) \frac{\partial^2}{\partial q_{m_1}\partial q_{m_2}} \left(\frac{1}{P_\gamma(q)} \langle \gamma^{jm_0}(\vec{q}) \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle' \right) = -P^{\mathrm{T}}_{i\ell_0\ell_1\ell_2 j m_0 m_1 m_2}(\hat{q}) \sum_{a=1}^{2} \frac{k_a^j}{3} \frac{\partial^3 \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle'}{\partial k^a_{m_0} \partial k^a_{m_1} \partial k^a_{m_2}} .$$

Explicit check: Maldacena's 3-point function: ✓

$$\frac{1}{P_\gamma(q)} \langle \gamma_{i\ell_0}(\vec{q}) \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle' = P^{\mathrm{T}}_{i\ell_0 j m_0}(\hat{q}) \frac{H^2}{4\epsilon k_1^3 k_2^3} k_1^j k_2^{m_0} \left(-K + \frac{(k_1+k_2)q + k_1 k_2}{K} + \frac{q k_1 k_2}{K^2} \right)$$

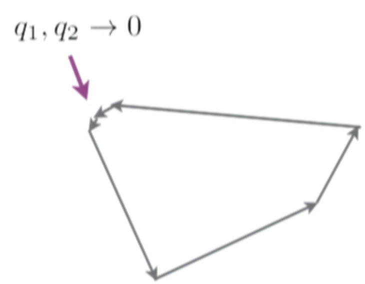
where $K \equiv q + k_1 + k_2$,

$$P^{\mathrm{T}}_{i\ell_0 j m_0} = P_{ij} P_{\ell_0 m_0} + P_{i m_0} P_{j\ell_0} - P_{i\ell_0} P_{j m_0} \, , \qquad P_{i\ell_0} = \delta_{i\ell_0} - \hat{q}_i \hat{q}_{\ell_0}$$

Further directions

- Multiple soft limits should expose non-trivial commutation relations of the symmetry group

Creminelli, Joyce, Khoury & Simonovic, in progress



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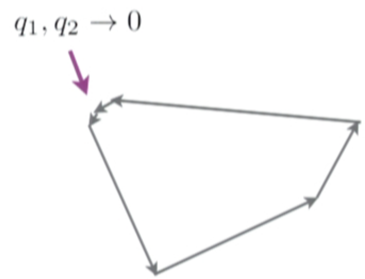
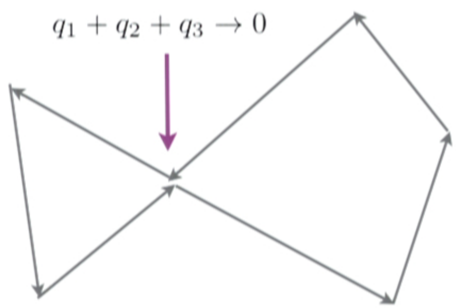


- Collapsed limits

Seery, Sloth, Vernizzi (2008)
Tasinato, Byrnes, Nurmi, Wands (2013)



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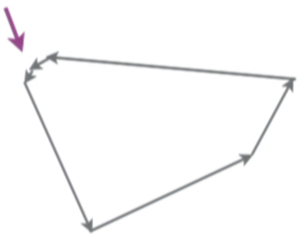
Creminelli, Joyce, Khoury & Simonovic, in progress

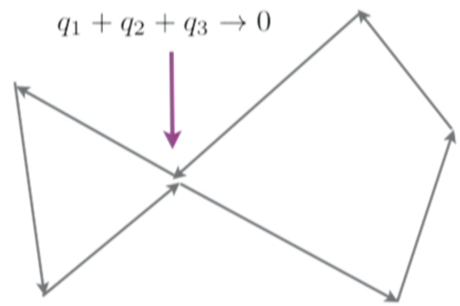
$q_1, q_2 \to 0$



- Collapsed limits

Seery, Sloth, Vernizzi (2008)
Tasinato, Byrnes, Nurmi, Wands (2013)

$q_1 + q_2 + q_3 \to 0$



- Modified initial state consistency relations

Agarwal, Holman, Tolley & Lin, 1212.1172
Flauger, Green & Porto, 1303.1430
Aravind, Lorshbough & Paban, 1303.1440