Title: The Information Paradox and an an Infalling Observer in AdS/CFT

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Abstract:

An Infalling Observer and the Black Hole Information Paradox in AdS-CFT

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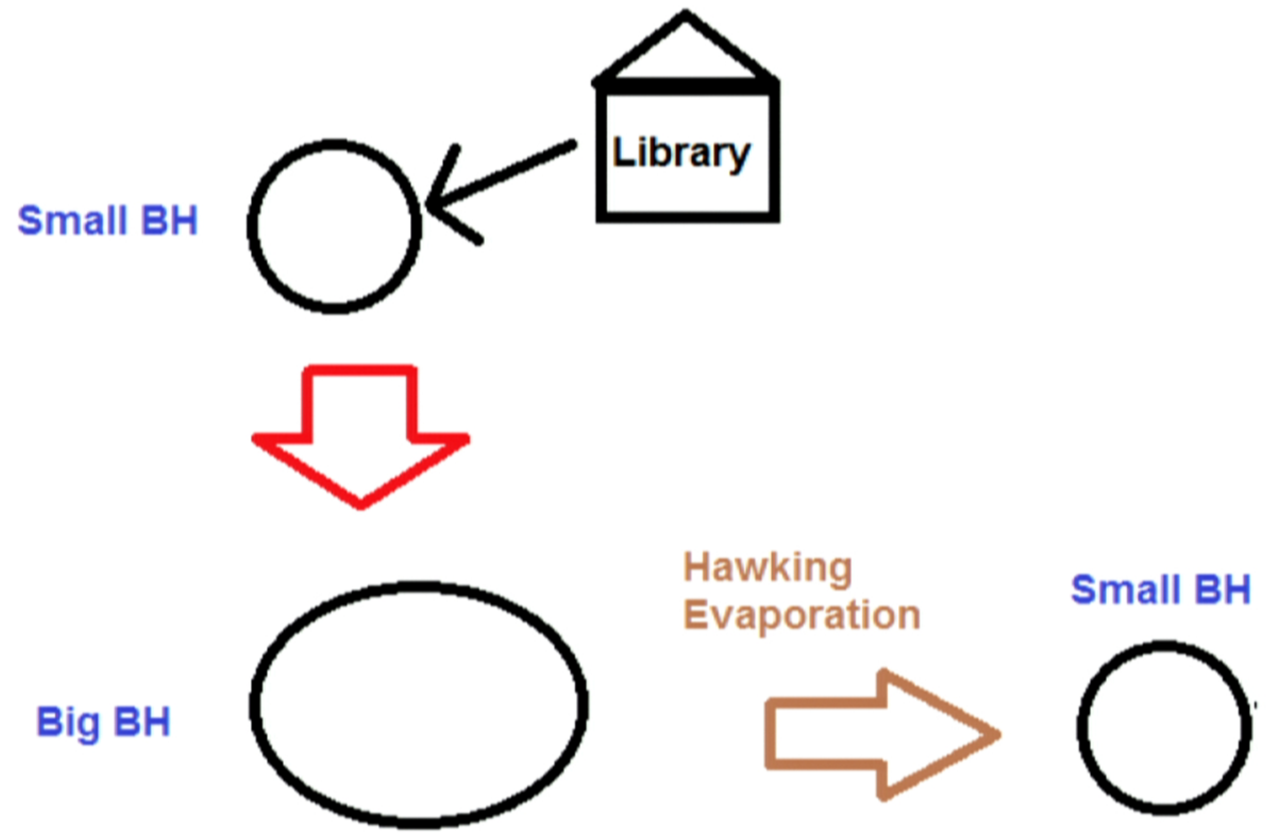
The Information Paradox

- Consider the collapse of matter into a big Schwarzschild black hole, which then evaporates via Hawking radiation.
- The radiation is black body radiation, with information only about the temperature!
- Where has the information about the initial state of the matter gone?

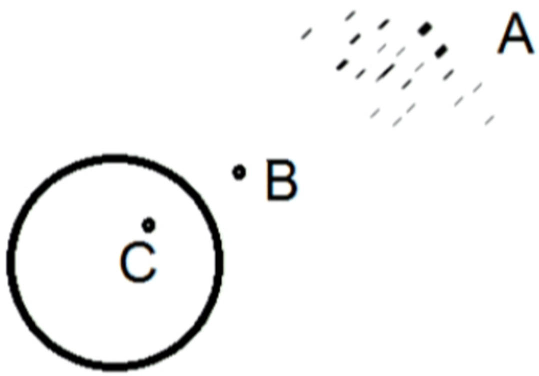
Where is the Information?



Recent sharpening of the information paradox

- The standard belief is that small corrections to Hawking radiation can restore unitarity.
- But, the info paradox was sharpened by Mathur in 2009.
- This argument has recently been expanded upon by Almheiri, Marolf, Polchinski, and Sully, and has attracted much attention.
- The claim is that for quantum gravity to be unitary, quantum corrections must be so large that they violently alter the structure of the horizon!
- Contradicts effective field theory intuition.

Three Subsystems



They key point is to think of three subsystems

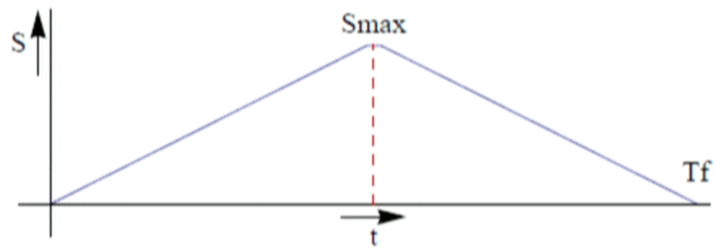
1. The radiation emitted long ago – A
2. The Hawking quanta just being emitted – B
3. Its partner falling into the BH – C

Entropy of A

- Say the Black Hole is formed by the collapse of a pure state.
- Consider the entropy of system A

$$S_A = -\mathrm{Tr}\rho_A \ln \rho_A$$

- Very general arguments due to Page tell us this must eventually start decreasing.



Strong Subadditivity contradiction?

- Now, consider an old black hole, beyond its "Page time" where S_A is decreasing. We must have

$$S_{AB} < S_A$$

since B is purifying A.

- Second, the pair B, C is related to the Bogoliubov transform of the vacuum of the infalling observer, we have

$$S_{BC} = 0$$

- Finally, both B and C are thermal, so

$$S_B = S_C > 0$$

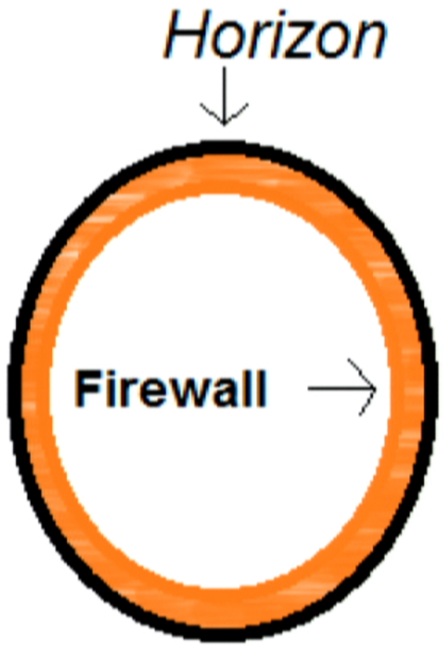
- However, a very general theorem tells us that for any three distinct systems A, B, C, we have

$$S_A + S_C < S_{AB} + S_{BC}$$

The Firewall Proposal

- The firewall proposal is the suggestion that we should drop $S_{BC} = 0$.

- Once we do this, it is very hard to prevent the infalling observer from burning up at the horizon — a firewall.



Use AdS/CFT?

Can we test this proposal using the AdS/CFT correspondence?

- Can we describe the results of local experiments in AdS using the boundary theory?
- Can we use the CFT to look beyond a black hole horizon?
- Can we use these answers to say something about the information paradox?

Generalized Free Fields

- The most generic feature of the AdS/CFT correspondence is that in some regime, the boundary theory has generalized free fields, $O(x)$ of low dimension.

- Correlators of these fields factorize:

$$\langle 0|\mathcal{O}(x_1)\ldots\mathcal{O}(x_{2n})|0\rangle = \langle 0|\mathcal{O}(x_1)\mathcal{O}(x_2)|0\rangle\ldots\langle 0|\mathcal{O}(x_{2n-1})\mathcal{O}(x_{2n})|0\rangle + \frac{1}{N}\cdots,$$

- However, O does not obey an equation of motion.

- For example, O could be $\mathrm{Tr}(F^2)$ in $\mathcal{N}=4$ SYM, but in general for our discussion it could be any scalar primary operator of dimension Δ.

Local Observables in empty AdS

- We can recast dynamics of O using a one-to-one map to another operator $\phi_{\rm CFT}$

[Banks et. al., 98]

.

$$O \Leftrightarrow \phi_{\rm CFT}$$

- The precise definition is:

$$\phi_{\rm CFT}(t, x, z) = \int_{\omega>0} \frac{d\omega d^{d-1}k}{(2\pi)^d} \left[\mathcal{O}_{\omega,k}\xi_{\omega,k}(t, x, z) + \text{h.c.}\right]$$

where ξ are appropriately chosen functions.

- $\phi_{\rm CFT}$ behaves like a free-field in AdS. For example:

$$[\phi_{\rm CFT}(t, x, z), \dot{\phi}_{\rm CFT}(t, x', z')] = \frac{i}{(2\pi)^d}\delta^{d-1}(x - x')\delta(z - z')z^{d-1}.$$

Transfer Function

- We can write this in position space also

[Bena, Kabat et al.,]

$$\phi_{\text{CFT}}(t, x, z) = \int O(t', x')K(t', x', t, x, z)dtd^{d-1}x$$

The Kernel K is called a transfer function

- This construction can be extended to higher orders in perturbation theory in $\frac{1}{N}$
- So, this method tells us how AdS emerges from the CFT.

Local Observables outside a Black Hole

- Consider the CFT in a pure state $|\Psi\rangle$ that is "close" to a thermal state.
- The same generalized free-fields O have different correlators in the state $|\Psi\rangle$.
- However, we can still construct perturbative local fields

$$\phi_{\mathrm{CFT}}(t, x, z) = \int_{\omega>0} \frac{d\omega d^{d-1}k}{(2\pi)^d} \left[\mathcal{O}_{\omega,k} f_{\omega,k}(t, x, z) + \mathrm{h.c.}\right]$$

- The mode functions f now solve the wave equation in front of the horizon of an AdS black brane.

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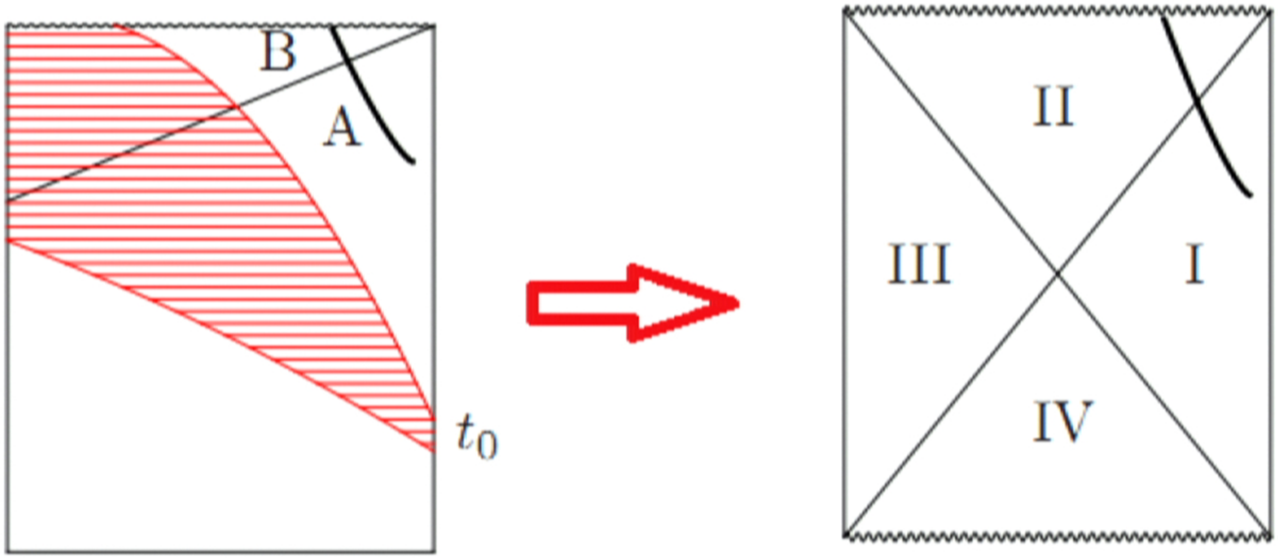
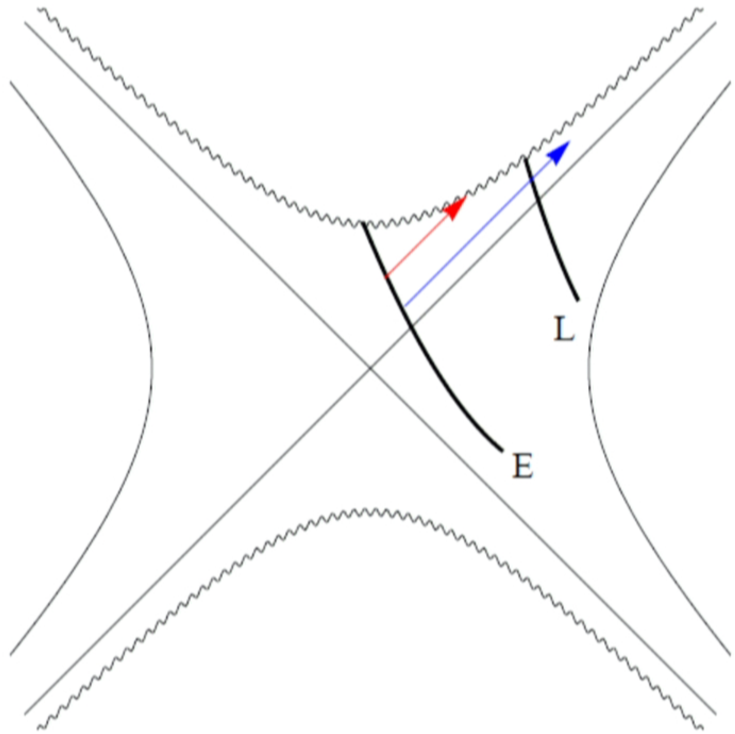
Behind the Horizon: Semi-Classical Expectations



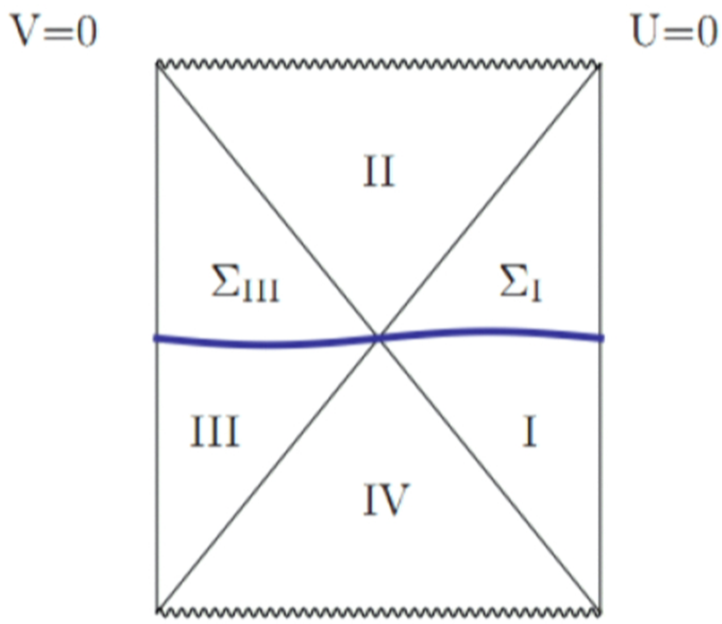
A collapsing geometry can be replaced by an eternal black hole for a late-enough observer

Intuitive Justification for No-Hair



It is harder and harder for an early observer to send a signal to a late observer. The previous statement follows by extrapolating this observation.

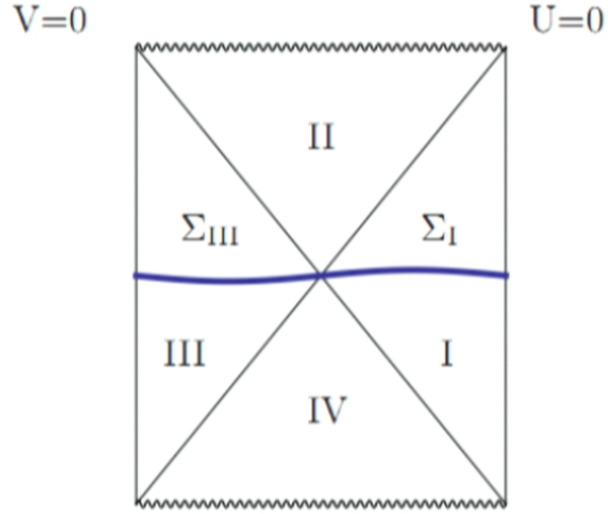
Dual of an eternal black hole



- Maldacena proposed that the dual of an eternal black hole consists of two CFTs — one on each boundary — placed in a thermofield doubled state

$$|\Psi\rangle_{\rm tfd} = e^{\frac{-\beta E}{2}} |E\rangle |\widetilde{E}\rangle$$

Mirrored Operators



- To construct local operators behind the horizon in this state, we need the operators O, and mirror operators $\widetilde{O}$.

What is the interpretation of the $\widetilde{O}$ operators in a pure state?

Coarse Graining the CFT

- The CFT in a pure state can look like a thermal state if we coarse grain the system.

- For example, we may choose to measure operators like $\int c_{\text{smooth}}(\omega)\hat{\mathcal{O}}^{\dagger}_{\omega,k}\hat{\mathcal{O}}_{\omega,k}$ for some smooth function c peaked about a particular frequency. But this measurement does not fix the state of the theory.

- It seems natural to divide the CFT Hilbert space into

$$\mathcal{H}_{CFT} = \mathcal{H}_{\text{coarse}} \otimes \mathcal{H}_{\text{fine}}.$$

  We will return to this later in the talk.

- The fine-grained degrees of freedom act like a thermal bath for the coarse grained d.o.f.

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Entanglement between coarse and fine d.o.f.

- Any state in the CFT can be written

$$|\Psi\rangle = \sum_{i,j} \alpha_{ij} |\Psi_i^c\rangle \otimes |\Psi_j^f\rangle,$$

where i runs over an orthonormal basis in $\mathcal{H}_{\text{coarse}}$ and j over an orthonormal basis in $\mathcal{H}_{\text{fine}}$.

- We can perform a singular value decomposition of the matrix α

$$\alpha_{ij} = \sum_m U_{im} D_{mm} V_{mj}$$

where D is a rectangular diagonal matrix

$$D = \begin{pmatrix} D_{11} & 0 & 0 & 0 & \dots \\ 0 & D_{22} & 0 & 0 & \dots \\ 0 & 0 & D_{33} & 0 & \dots \end{pmatrix}$$

Mirrored Operators

- In this basis, the state becomes
$$|\Psi\rangle = \sum_i D_{ii} |\hat{\Psi}_i^c\rangle \otimes |\hat{\Psi}_i^f\rangle$$
- What our low energy observer really measures is
$$\mathcal{O}_c(t,x) = \mathcal{P}_{\text{coarse}}\left(\mathcal{O}(t,x)\right)$$
where $\mathcal{P}_{\text{coarse}}$ traces out the fine-grained states.
- For some matrix elements ω_{i_1,i_2}
$$\mathcal{O}_c(t,x) = \sum_{i_1,i_2} \omega_{i_1 i_2}(t,x) |\hat{\Psi}_{i_1}^c\rangle\langle\hat{\Psi}_{i_2}^c|.$$
- Define a mirrored operator on the fine-grained space:
$$\widetilde{\mathcal{O}}(t,x) = \sum_{i_1,i_2} \omega^*_{i_1 i_2}(t,x) |\hat{\Psi}_{i_1}^f\rangle\langle\hat{\Psi}_{i_2}^f|.$$

Relation to Analytic Continuation

- For mixed-expectation values within $|\Psi\rangle$, $\widetilde{O}_c$ actually acts like an analytically continued version of O_c within a thermal trace

$$\langle\Psi|\mathcal{O}_c(t_1, x_1)\widetilde{\mathcal{O}}_c(t_2, x_2)|\Psi\rangle = \frac{1}{Z_\beta^c}\mathrm{Tr}_c\left[e^{-\beta H}\mathcal{O}_c(t_1, x_1)\mathcal{O}_c(t_2 + \frac{i\beta}{2}, x_2)\right]$$

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Local operators Behind the Horizon

- We can now easily construct local operators behind the black hole horizon

$$\phi^{\mathrm{II}}_{\mathrm{CFT}}(t,x,z) = \int_{\omega>0} \frac{d\omega d^{d-1}k}{(2\pi)^d} \Big[\mathcal{O}_{\omega,k}\, g^{(1)}_{\omega,k}(t,x,z) + \widetilde{\mathcal{O}}_{\omega,k}\, g^{(2)}_{\omega,k}(t,x,z) + \mathrm{h.c.}\Big]$$

- Here, roughly, we can think of $g^{(1)}$ as analytic continuations of left-moving solutions to the KG equation from region I to region II, and $g^{(2)}$ as analytic continuations of right-moving solutions from region III to region II.

Summary

- We have now constructed local operators both outside the black hole and behind the horizon.
- We can compute bulk correlators using this construction, and they are perfectly regular across the horizon.
- How do we reconcile this with indirect arguments from the information paradox, which suggest that the horizon is modified?

Non-perturbative corrections

- We believe that it should be possible to correct this prescription order by order in $\frac{1}{N}$.
- However, it may not be possible to interpret non-perturbative physics in the CFT in terms of local bulk physics.

Efficacy of small corrections

Exponentially small corrections of the order of e^{-S} can resolve the information paradox.

Path Integral Perspective

- Imagine formulating quantum gravity through the Feynman path integral
$$\mathcal{Z} = \int e^{-S} \mathcal{D} g_{\mu\nu}$$
- A semi-classical spacetime is a saddle point of this path-integral.
- Perturbative effective field theory (used to derive the Hawking answer) is an asymptotic series expansion of this path-integral.
- Non-perturbatively, the notion of spacetime breaks down.
- We expect non-local corrections of order e^{-S}.

An Aside on Numerical Magnitudes

- One could wonder how non-locality could be important at distances like the Schwarzschild radius of a solar-mass black hole.
- The entropy of a solar-mass black hole is approximately 10^{77}.
- So, exponentially suppressed corrections are of the order $e^{-10^{77}}$!

Form of the Corrections

- Start by consider the radiation outside. (Will return to strong-subadditivity paradox later.)
- We need:

$$\rho_{\text{exact}} = \rho_{\text{hawk}} + e^{-S}\rho_{\text{corr}},$$

- The condition is that in a natural basis of observables, ρ_{corr} has elements that are $O(1)$.

A Toy Model

- It is possible to show that this is possible.
- Also easy to produce a toy-model where the density matrix has these properties.
- Consider a system of N spin-(1/2) spins. This has 2^N states. We can label these states by numbers and read off the individual spins using the binary expansion of the number.

$$
\begin{aligned}
&|+++\ldots\ldots\ldots++\rangle \equiv |0\rangle \\
&|+++\ldots\ldots\ldots+-\rangle \equiv |1\rangle \\
&|+++\ldots\ldots\ldots-+\rangle \equiv |2\rangle \\
&|+++\ldots\ldots\ldots--\rangle \equiv |3\rangle \\
&\ldots
\end{aligned}
$$

Pure States and Hawking Evaporation

- Consider a generic pure state in this spin-model

$$|\Psi\rangle = \frac{1}{2^{\frac{N}{2}}} \sum_{i=0}^{2^N-1} a_i |i\rangle$$

where the a_i are chosen to be either 1 or -1 with probability $\frac{1}{2}$.

- Consider breaking off the spins one by one.

Thermal Density Matrices with Small Corrections

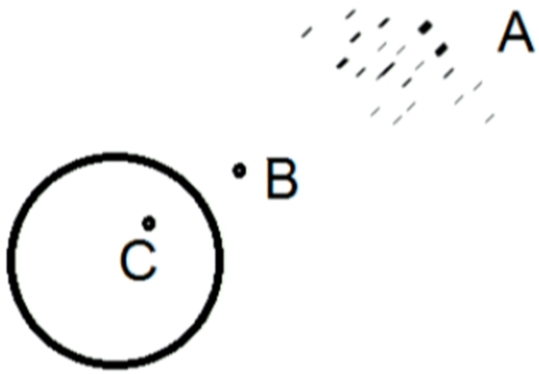
- Even though the full density matrix is pure, if we consider K-spins for $K << \frac{N}{2}$, their density matrix will look "thermal" (proportional to the identity) up to exponentially small corrections.

- For example,

$$\rho_1 = \frac{1}{2^N}\left(\sum_{j=0}^{2^{N-1}-1} a_{2j}^2|0\rangle\langle 0| + a_{2j+1}^2|1\rangle\langle 1| + a_{2j}a_{2j+1}\Big(|0\rangle\langle 1| + |1\rangle\langle 0|\Big)\right)$$
$$= \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1| + \mathrm{O}\left(2^{-\frac{N}{2}}\right)\Big(|0\rangle\langle 1| + |1\rangle\langle 0|\Big)\right).$$

- But if we start looking at $\frac{N}{2}$ spins or more, the exponentially small corrections become important.

Strong Subadditivity and Exponentially Small Corrections



- Can exponentially small corrections resolve the strong subadditivity paradox.
- Our construction leads us to expect exponentially small commutators between operators outside and inside the black hole.

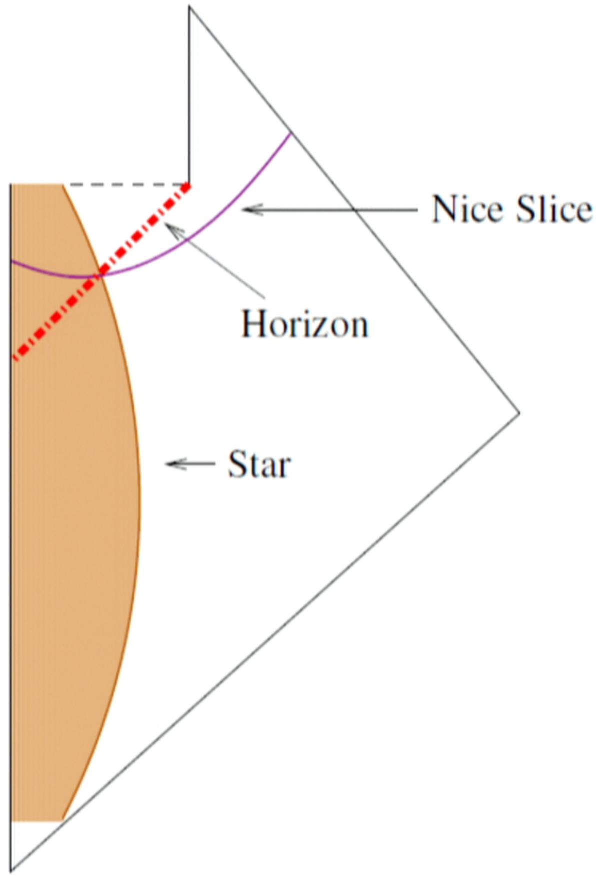
Resolution to the Strong Subadditivity Paradox

The resolution of the strong-subadditivity paradox is through

Black Hole Complementarity: The interior and exterior of a black hole are not independent. The interior is a scrambled version of (part of the) exterior!

This resolves the strong subadditivity paradox because A and C are not independent.

Original Motivation for BH Complementarity



Objections to Black Hole Complementarity

- To explain the objection, let us go back to our spin-chain model. [AMPSS, 13]
- We can model the Hawking quanta outside the black-hole, as spins "breaking off" from the spin chain. [WARNING: May be misleading]
- What about the Hawking quanta that falls into the black hole? Where do we see that in this toy model?

Infalling Quanta in the Toy Model

- Let us make a simple coarse-graining of the system:

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_{N-1}$$

  The coarse-grained d.o.f. is the first spin, and the fine-grained d.o.fs are all the other spins.

- Let us write out pure state as

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|+\rangle|\phi_+\rangle + |-\rangle|\phi_-\rangle \right)$$

- We measure

$$S_1 = |-\rangle\langle-| - |+\rangle\langle+|$$

  We can define

$$\widetilde{S}_1 = |\phi_+\rangle\langle\phi_+| - |\phi_-\rangle\langle\phi_-|$$

- Measurement of $\widetilde{S}_1$ are precisely anti-correlated with measurements of S_1.

Large commutators in the Toy Model

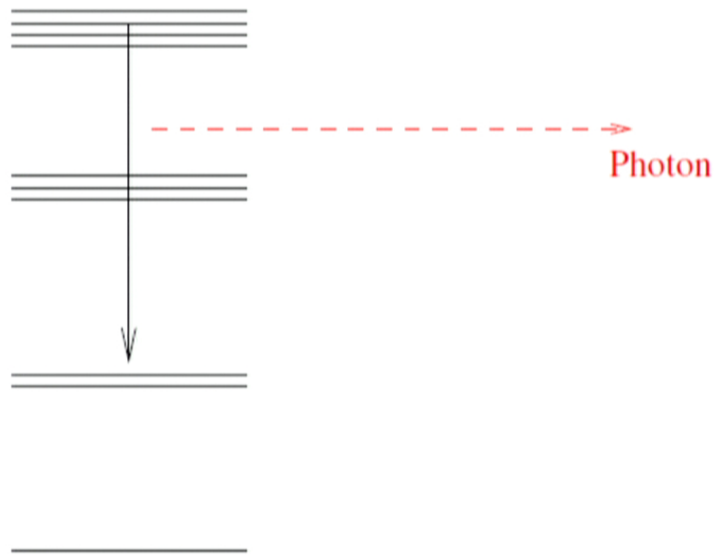
- We could, for example, choose the any p-bits to correspond to the coarse d.o.fs and the other $N - p$ to correspond to the fine d.o.fs
- However, if we take $p > \frac{N}{2}$, then there is no $\widetilde{S}_p$ that has small commutators with $S_1, S_2, \ldots S_{p-1}$.
- The naive translation of this fact is: once more than half the black hole has evaporated, we are forced to have large commutators between operators outside and inside the black hole.

The Abstract Problem of BH Complementarity

Perhaps the spins-breaking-off model is not a good model. Abstractly, we need a setup with the following property:

1. A large Hilbert space $\mathcal{H}_{\text{full}}$ and a subspace $\mathcal{H}_{\text{in}}$.
2. A "natural basis" of operators O_n of $\mathcal{H}_{\text{full}}$ and a basis of operators $\widetilde{O}_n$ for $\mathcal{H}_{\text{in}}$.
3. These have the property that $[\widetilde{O}_n, O_m] \sim \frac{c_{nm}}{\dim(\mathcal{H}_{\text{full}})}$
4. Also, $\widetilde{O}_n$ and O_n are perfectly correlated (up to exponentially suppressed corrections) in some given state.

A Toy Model with Natural Coarse Graining



- Imagine a system, with fine-spacing in its energy levels.
- Transitions between these energy levels lead to the emission of a photon.

Coarse Graining the Photon Field

- The photon field outside can be quantized in terms of

$$A_\mu(x,t) = \sum_n a_{n,\mu} e^{i\omega_n(t-x)} + a'_{n,\mu} e^{i(\omega_n+\epsilon_n)(t-x)} + h.c$$

- However, an observer with limited resolving power will see an effective coarse grained field

$$A_\mu^{\text{coarse}}(x,t) = \sum_n (a_{n,\mu} + a'_{n,\mu}) e^{i\omega_n(t-x)} + h.c$$

- If we consider a microcanonical configuration of photons, with large total energy E then half the degrees of freedom are in excitations of the oscillators $\frac{1}{\sqrt{2}}(a_{n,\mu} - a'_{n,\mu})$.

Summary: proposed resolution of the strong subadditivity paradox

- The information may be outside the black-hole, but is not accessible to a coarse-grained observer:

$$H_{\text{coarse}} \neq H_{\text{out}}$$

- We can use the fine structure of the emitted radiation to reconstruct the interior of the black hole.

- This leads to commutators

$$[\phi_{\text{out}}, \phi_{\text{in}}] \sim e^{-S},$$

which is acceptable.

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Summary

- The firewall proposal suggests that quantum effects can violently alter the structure of the horizon of even a large black hole.
- We constructed local operators outside and inside a black hole in AdS/CFT, and found no such phenomenon.
- Our construction leads us to expect non-local corrections of order e^{-S}.
- These corrections are sufficient, *in principle*, to resolve the information paradox.
- However, it is necessary to understand the structure of the CFT Hilbert space better, and show that our postulated "coarse $\times$ fine" splitting actually exists.