Title: Resolution of Cosmic Singularities and Bounces

Date: Jul 09, 2013 03:50 PM

URL: http://pirsa.org/13070008

Abstract: The AdS/CFT correspondence provides new insights and tools to answer previously inaccessible questions in quantum gravity. Among the most interesting is whether it is possible to describe a cosmological "bounce" in a mathematically complete and consistent way. In the talk, I'll discuss joint work with M. Smolkin, developing the dual description of the simplest possible 4d M-theory cosmology in the stringy regime, employing the full quantum dynamics of its dual CFT. I'll also present evidence that the description extends to the Einstein-gravity regime.<strongr>

</strongr>

Resolving Cosmological Singularities with AdS/CFT

Neil Turok

work with M. Smolkin
hep-th/1211.1322; in preparation, 2013

Developing earlier work with B. Craps and T. Hertog

Planck, WMAP and other CMB experiments, as well as the LHC, seem to have revealed surprising simplicity in the universe

The challenge is to find the physical principles which can predict such extreme simplicity in nature

Mathematical explorations are needed, to develop more powerful principles and methods

A compelling theory must explain and resolve:

1) the cosmological singularity, from which everything we observe seemingly emerged

2) the measure on the space of cosmologies, showing why a universe like ours is likely

These problems are related to profound issues in quantum gravity:

The kinetic energy for the scale factor of the universe has the "wrong sign"

The Liouville measure $\Pi dpdq$ is divergent and must be carefully regulated

The Euclidean action for gravity is unbounded below

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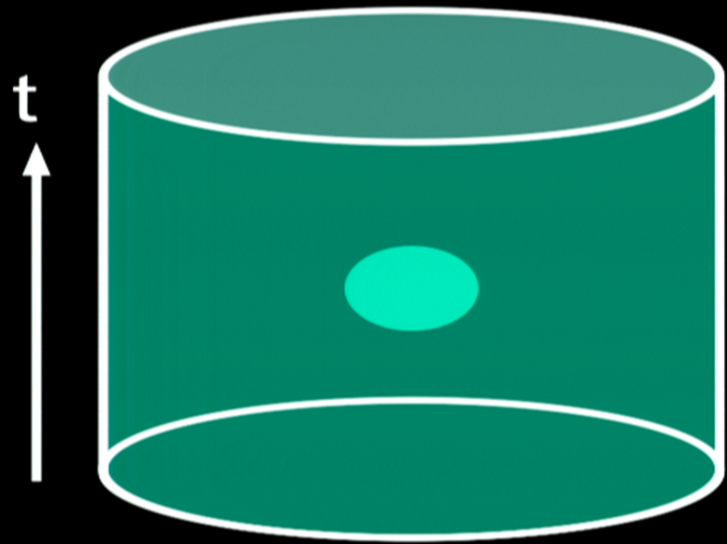
This talk will try to tackle these questions within a mathematical laboratory:

M-theory in asymptotically-AdS spacetime

Again and again, when I have been at a loss how to proceed, I have just had to wait until I have felt the mathematics lead me by the hand. It has led me along an unexpected path, a path where new vistas open up…

Paul Dirac

AdS/CFT



a theory with gravity in

asymptotically AdS spacetime

is dual to

a theory with no gravity living on the boundary

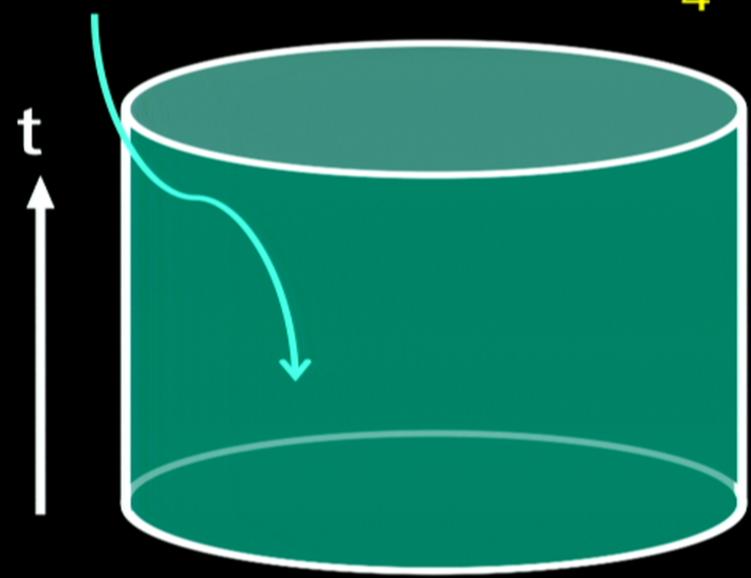
- a conformal field theory

a marvellous theoretical laboratory!

Best understood example:

M theory on $AdS_4 \times (S_7 / \mathbb{Z}_k)$

- fibration over $\mathbb{CP}_3$

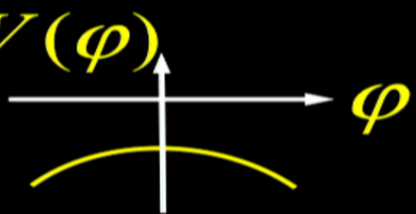


CFT_3 on $S_2 \times \mathbb{R}$

For purposes of choosing AdS-invariant bcs, truncate to gravity + scalar

$$S_{bulk} = \int_{bulk} \left(\frac{1}{2} R - \frac{1}{2} (\partial \varphi)^2 + R_{AdS}^{-2} \left(3 \cosh \sqrt{\tfrac{2}{3}} \varphi \right) \right)$$

↑ quadrupole of S_7 traceless bilinear under $SO(8)$



$$\varphi = 0 : m_\varphi^2 = -2 R_{AdS}^{-2} > -\frac{9}{4} R_{AdS}^{-2} \equiv m_{BF}^2$$

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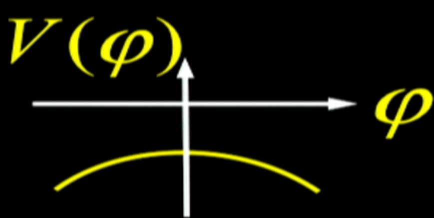

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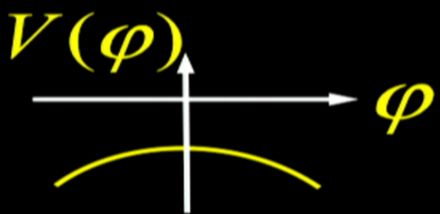

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general asymptotically AdS solutions:

$$\varphi \sim \alpha(t,\Omega)e^{-r} + \beta(t,\Omega)e^{-2r} + \ldots$$

AdS isometries act as conformal gp on CFT

-> identify $\alpha(t,\Omega) \sim O,\ \beta(t,\Omega) \sim J_O,\ O \sim \phi^2$ in D=3

SUSY bcs: $\alpha = 0$ or $\beta = 0$ -> static and stable

Generalised AdS-invariant bcs:

$$\beta = \lambda\alpha^2$$

- correspond to adding deformation to CFT

$$S_{CFT} \to S_{CFT} + \tfrac{\lambda}{3}\int O^3, \text{ i.e., } V \to V + \tfrac{\lambda}{3}\phi^6$$

Hertog, Maeda,
Horowitz, Witten

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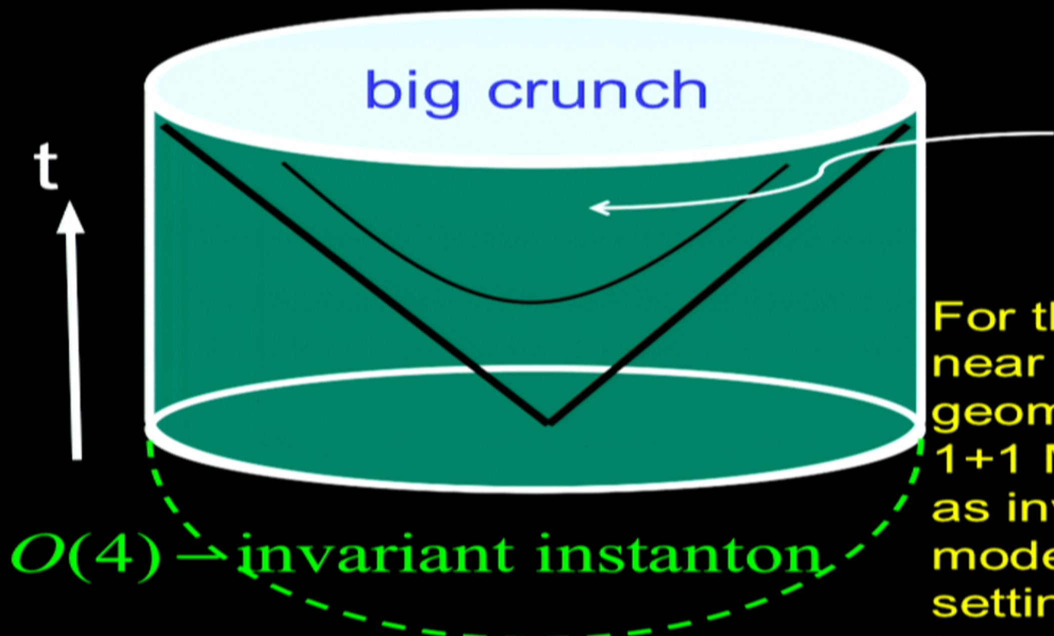
AdS cosmology

Hertog+Horowitz

For each $\lambda \neq 0$, $\exists$ a cosmological instanton solution:



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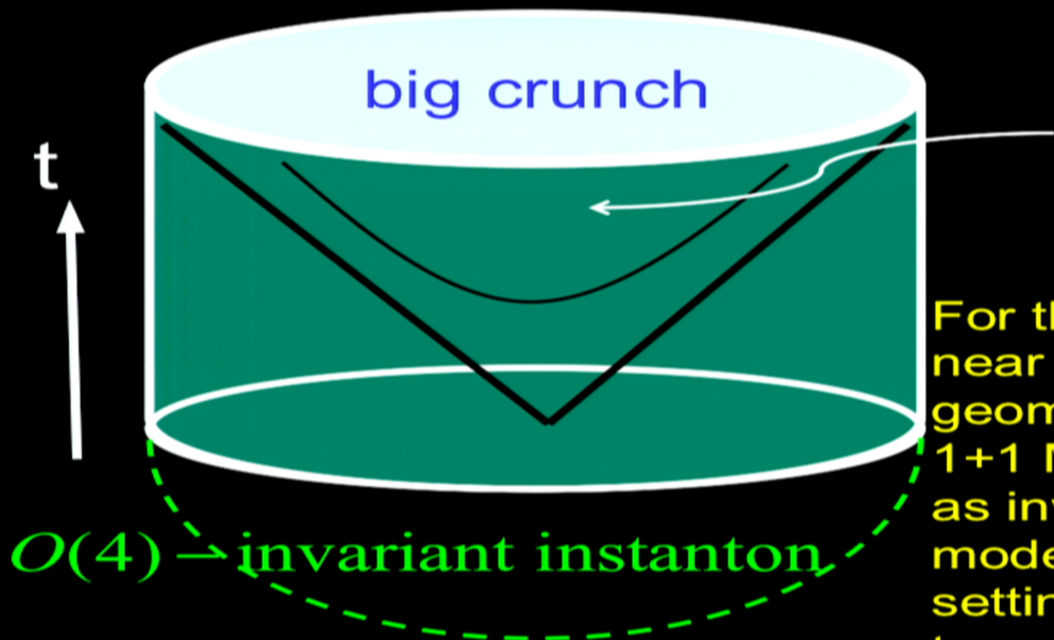

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Dual CFT identified by Aharony et al (ABJM):

$U(M)\times U(M)$ Chern-Simons theory with 4 bi-fundamental Higgs fields $Y^{I}_{a\bar{a}}$

$$S_{CFT}:\ \int_3 \left(k(A\wedge F + A\wedge A\wedge A) - |DY|^2 - k^{-2}(Y^*Y)^3 + fermions\right)$$

't Hooft limit

$$M, k \to \infty, \text{ at fixed } g_t \equiv M/k; \quad \frac{R_{AdS}}{l_{Pl}} \sim (Mk)^{1/4},\ \frac{R_{AdS}}{l_s} \sim \left(\frac{M}{k}\right)^{1/4}$$

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However, this interpretation is problematic since AdS is infinite and there would be an infinite total rate of decay, with bubble collisions ruining any simple cosmological picture. (D. Harlow)

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We shall sum over all of these instantons and show that they collectively define a stable, nonsingular “cosmological phase” of the boundary CFT.

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Along the way we shall encounter (and resolve) many strange phenomena:

An apparently unbounded-below Hamiltonian.

Infinite numbers of bubbles, each containing a singularity. Nevertheless, when correctly summed, they combine to give a completely regular state.

We are required to deal with the AdS bulk in global coordinates.The dual theory makes no sense in flat space, but is well-defined on a de Sitter boundary.

Conformal symmetry lies at the heart of our analysis. It is spontaneously broken but there is no Goldstone mode. Conformal symmetry shall completely determine the propagation of modes across the singularity.

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Dual theory has a UV fixed point

Craps,Hertog, NT
(2009)

Pisarski 80's - for O(N) model in 3d ϕ_3^6

$$\beta_6 = \frac{3}{\pi^2 N}\left(\lambda_6 - \frac{1}{192}\lambda_6^3\right)$$

* suppressed by 1/N
* In deformed ABJM, same calculation shows UV fixed point, $\lambda_{6*} = 192$ at weak 't Hooft coupling
* AdS/CFT calculation also indicates a UV fixed point at strong 't Hooft coupling
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* Historical remark: at N=1 this is the model exhibiting the famous Wilson-Fisher IR fixed point

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Although λ_6 is positive, Hamiltonian is unbounded below!

Trial wavefunction: free field mass m

$$\langle \phi^2 \rangle = \int_0^\Lambda \frac{d^2k}{(2\pi)^2} \frac{1}{2\omega_k} = \tfrac{1}{4\pi}(\Lambda - m);$$

$$\langle \phi^2 \rangle_{ren} = \tfrac{1}{4\pi}(-m) \quad \Rightarrow \langle H \rangle = N \frac{m^3}{24\pi}\left(1 - \frac{\lambda_6}{16\pi^2}\right)$$

For $\lambda > 16\pi^2$, in flat spacetime, theory has no ground state Bardeen,Moshe,Bander

Gain more insight by studying time-evolution towards the singularity

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Gain more insight by studying time-evolution towards the singularity

Scaling: $\left\langle \vec{\phi}^{\,2} \right\rangle_{ren} = -\frac{C}{|t|} N$ (at $N = \infty$; at finite N, exponent gets N^{-1} corrns)

$$\phi = \sum_{\vec{k}} \chi_{\vec{k}}(t)\, e^{i\vec{k}\cdot\vec{x}},\ \vec{\phi}^{\,2} \longrightarrow \left\langle \vec{\phi}^{\,2} \right\rangle \text{ in } N \longrightarrow \infty \text{ limit}$$

Field eq $\Rightarrow \ddot{\chi}_{\vec{k}} = -k^2 \chi_{\vec{k}} - \lambda_6 \frac{C^2}{t^2} \chi_{\vec{k}}$, Bessel $\nu^2 = \frac{1}{4} - \lambda_6 C^2$

Gap equation $\frac{1}{4} - \nu^2 = \frac{\lambda_6 \nu^2}{16\pi^2} \left(\cot \nu\pi\right)^2$

$$\lambda_6 \longrightarrow \lambda_{6*} = 192 \Rightarrow \nu \longrightarrow \nu_* = iN_*,\ N_* = 1.061..$$

Instability dynamically breaks Poincare invariance

BUT the theory is Weyl invariant

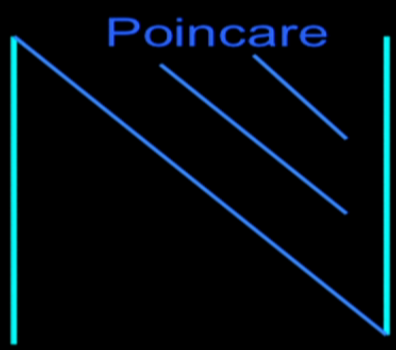
1) UV fixed pt
2) No trace anomaly in D=3

so the "singularity" can be removed via a Weyl transformation

$$\left\langle \vec{\phi}^{\,2} \right\rangle = -\frac{C}{|t|} N,$$

$$\vec{\phi} \to |t|^{\frac{1}{2}} \vec{\phi}, \;\; \eta_{\mu\nu} \to \frac{1}{|t|^2} \eta_{\mu\nu}$$

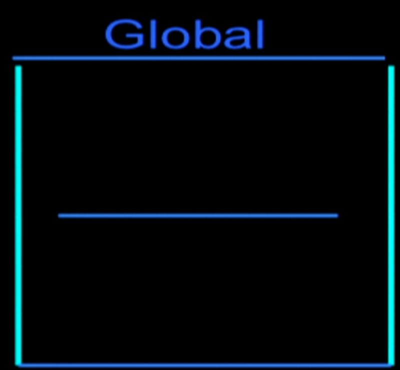
constant $\left\langle \vec{\phi}^{\,2} \right\rangle = -CN$ in de Sitter!



$$\left\langle \vec{\phi}^2 \right\rangle = -\frac{C}{|t|}N,\ \vec{\phi} \rightarrow |t|^{\frac{1}{2}}\vec{\phi},\ \eta \rightarrow \frac{1}{|t|^2}\eta$$

Minkowski

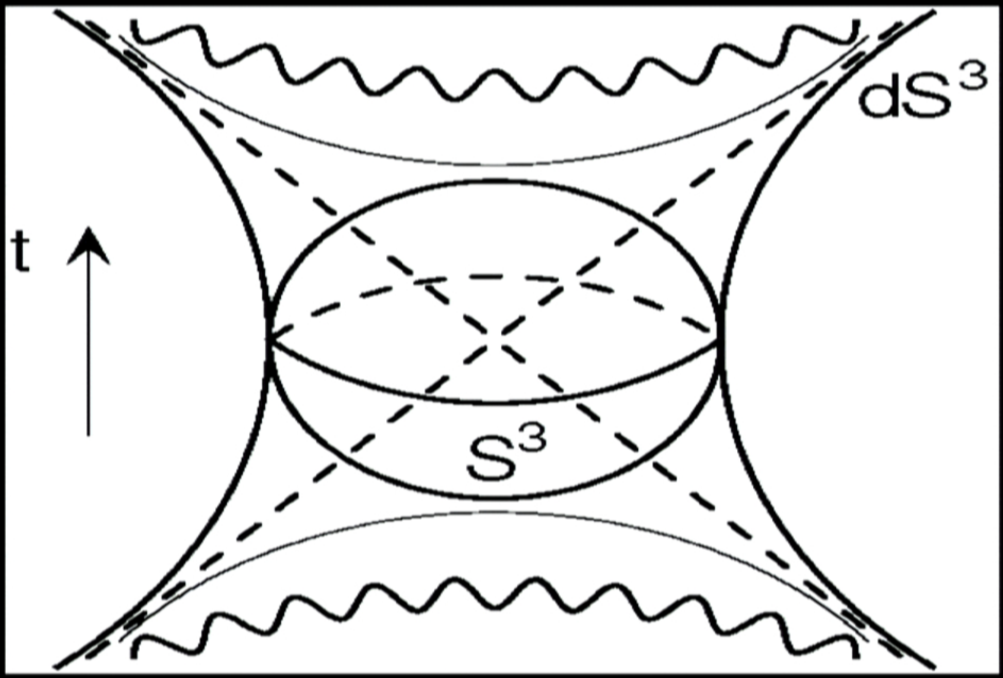
de Sitter!



dS^3

$$\mathrm{ds}^2 = \frac{1}{(\cos\tau)^2}(-\mathrm{d}\tau^2 + \mathrm{d}\Omega_2^2),\ \left\langle \vec{\phi}^2 \right\rangle = -CN$$

Global Picture



By choosing boundary to be dS^3, we avoid the singularity!

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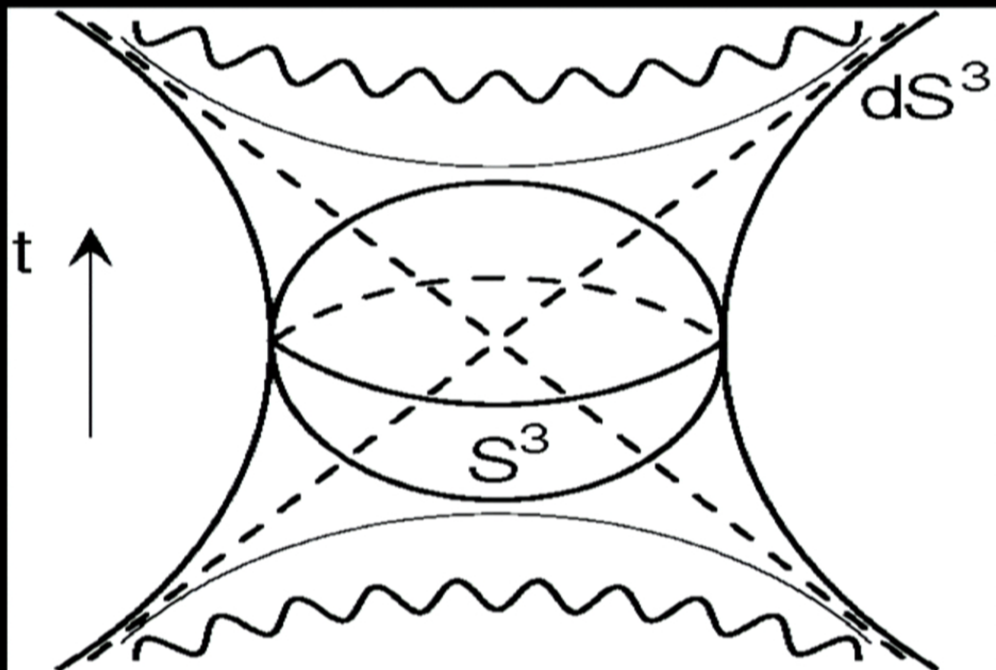

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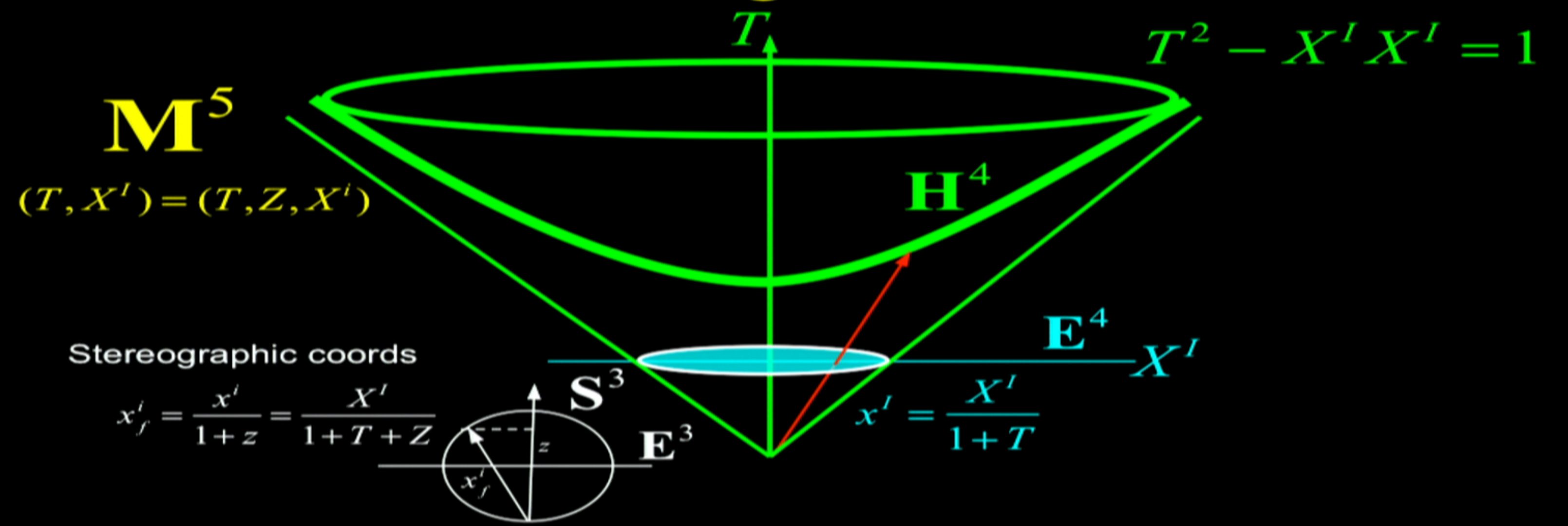
AdS/CFT in global coordinates



$M_{MN} = X_M P_N - X_N P_M = SO(4,1) \Rightarrow$ Special Conformal Generators on x^i_f

Dual CFT

$$S_E = \int_{\mathbf{S}^3} \Big[(\partial \vec{\phi})^2 + R\vec{\phi}^{\,2} + \frac{\lambda_6}{N^2} (\vec{\phi}^{\,2})^3 \Big]$$

$$Z_{\vec{J}} = \int DsD\rho D\vec{\phi}\, e^{-\int \left[N(\lambda_6 \rho^3 - s\rho) + \vec{\phi}\hat{O}_s\vec{\phi} + \vec{J}.\vec{\phi} \right]}$$

$$= \int DsD\rho e^{-\int \left[N(\lambda_6 \rho^3 - s\rho + \mathrm{Tr}\ln \hat{O}_s) + \vec{J}\hat{O}_s^{-1}\vec{J} \right]}, \quad \hat{O}_s = -\Box + \frac{R}{8} + s$$

Formally, use

$$e^{-\int \frac{\lambda_6}{N^2}(\vec{\phi}^2)^3} \propto \int DsD\rho e^{-\int \left[N(g_6 \rho^3 + s(\vec{\phi}^2 - \rho) \right]}$$

Prove by differentiating, integrating by parts.
True for any (s, ρ) contour for which the integral converges.

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Consider theory on $\mathbf{S}^3$ of radius r_0

Saddle point equations $\rho = \langle x | O_s^{-1} | x \rangle, \qquad s = \lambda_6 \rho^2$

Defining $N = \sqrt{s r_0^2 - \frac{1}{4}}$, we obtain the "gap equations"

$$\rho r_0 = -\frac{N \mathrm{Coth} N\pi}{4\pi}, \qquad s r_0^2 = N^2 + \frac{1}{4} = \lambda_6 \left(\frac{N \mathrm{Coth} N\pi}{4\pi} \right)^2$$

For $\lambda_6 > 16\pi^2$ there is a nontrivial solution, with

$$\bar{s} = r_0^{-2} C(\lambda_6), \;\; \infty > C > 0 \text{ as } 16\pi^2 < \lambda_6 < \infty;$$

$$C(\lambda_{6*}) = 1.38..$$

This is the simplest, homogeneous instanton on $\mathbf{S}^3$.
It breaks conformal symmetry SO(4,1) to SO(4)

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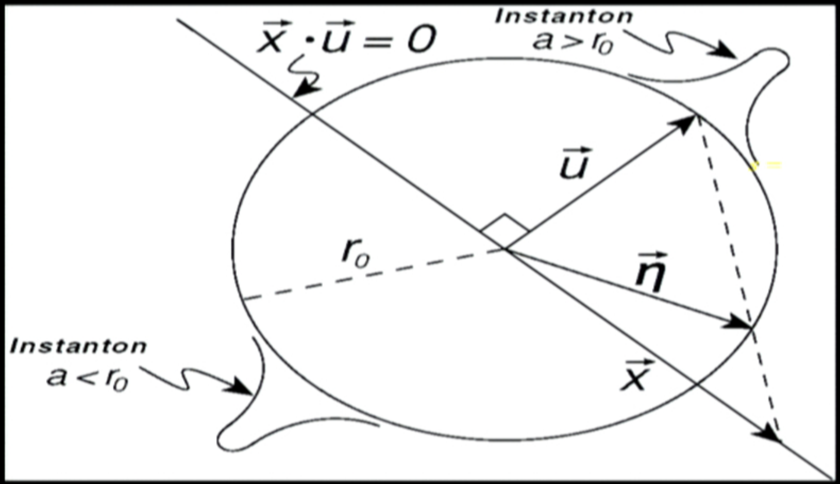
There is a 4-parameter family of such instantons, parameterized by a size modulus a and centre $\hat{u}$



In flat space they are known as "Fubini" instantons
They may be obtained by Weyl-transforming between spheres of different sizes.

Each Fubini instanton represents the boundary image of a bulk cosmological instanton. If analytically continued to real time, each describes a "bubble" with the field rolling downhill towards a finite-time singularity.

We are able to regulate and perform the (coherent) sum over all of these instantons in the Euclidean region, before continuation to real time. The resulting field theory on the boundary is completely regular and stable.

Note in particular the moduli space turns out to be Euclidean AdS^4 i.e. H^4. We regularise and integrate, either by dimensional reg or by adding counterterms:

$$\Omega^{d-1}\int_0^\infty dr\ \sinh^{d-1} r = \pi^{\frac{d-1}{2}}\Gamma\left(-\frac{(d-1)}{2}\right) = \frac{4\pi^2}{3}\ \text{in } d = 4$$

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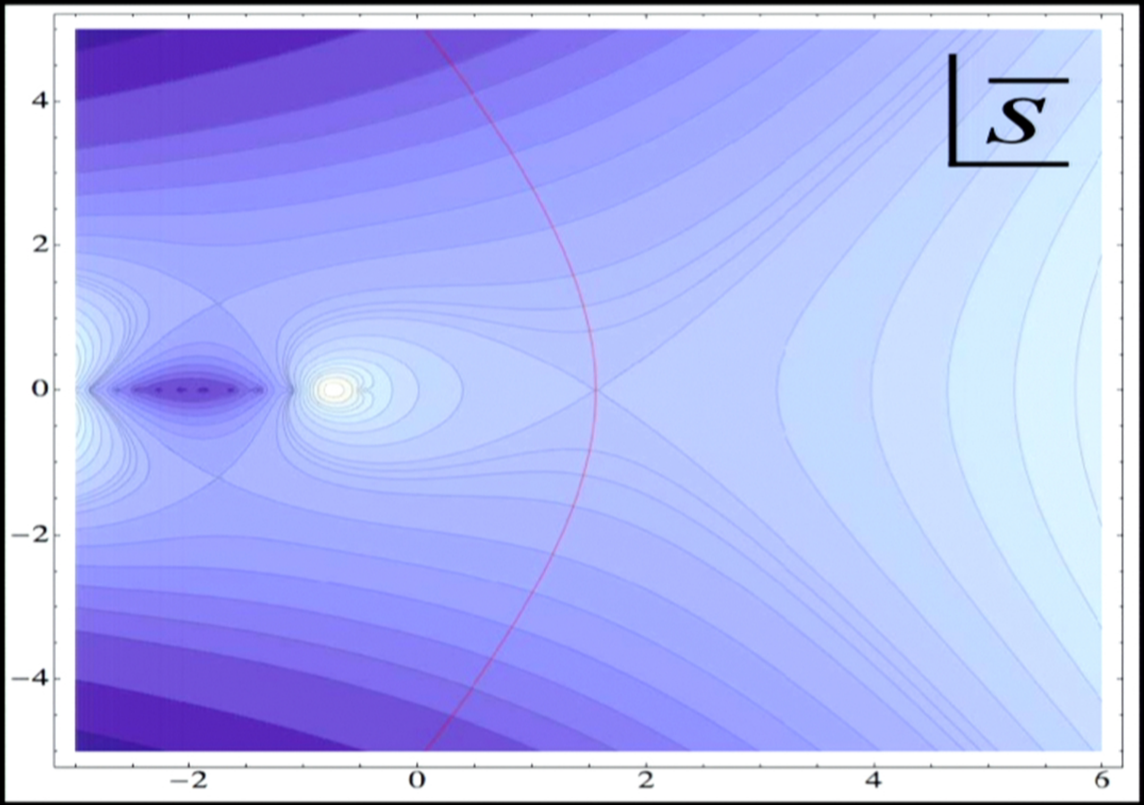
Each instanton defines a saddle point for the path integral; we must still choose a contour in (s, ρ) along which the path integral will converge.

Careful analysis shows that for $\lambda_6 > 16\pi^2$, spontaneous breakdown of conformal symmetry occurs. The homogeneous modes of both s and ρ must be integrated along imaginary contours (contours which reverse under complex conjugation).

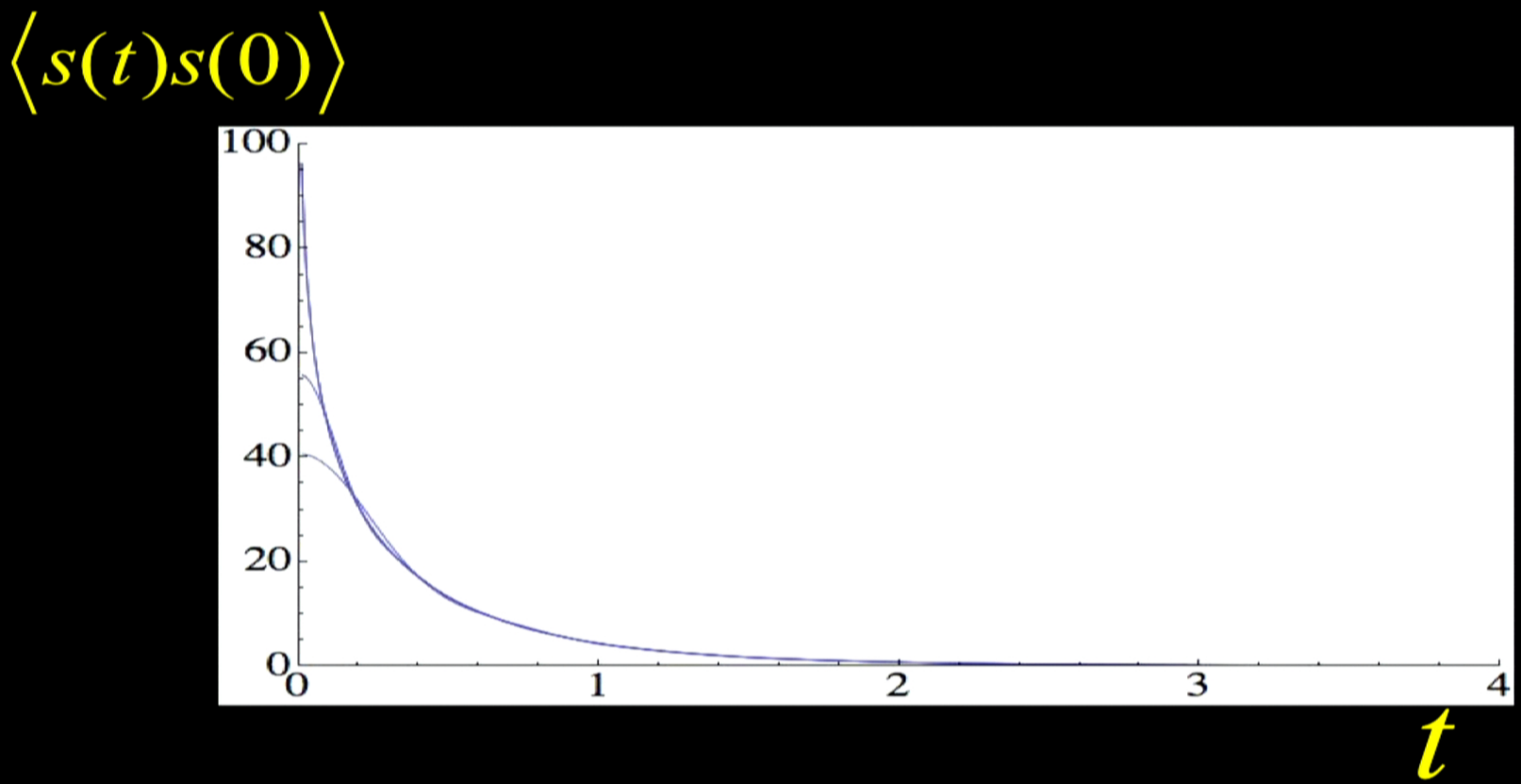
Nevertheless, the partition function Z is real and the two-point correlation function satisfies reflection positivity (which is required for unitarity).



4
2
0
−2
−4
−2
0
2
4
6
$\lfloor\overline{S}$

Our claim is that the CFT defined by this choice of contour is well-defined and stable when continued to $\mathbf{dS}^3$

e.g. s-s correlator constructed via Sommerfeld-Watson transform and analytic continuation from Euclidean region


$\langle s(t)s(0)\rangle$
100
80
60
40
20
0
0
1
2
3
4
t

In contrast, one can easily show that the same procedure, applied to a Hawking-Moss instanton for example, results in an exponentially growing correlator, i.e., an unstable theory.

So, in that case, the instability associated with the Euclidean negative mode cannot be cured by a contour rotation).

Integrating over moduli space, we explicitly obtain the full 2-point function for $\vec{\phi}$ on $\mathbf{dS}^3$, with good short and long distance behaviour:

Instanton sum shifts vev away from free field of mass $N^2+\frac{1}{4}$

$$\left\langle \phi^m(\hat{\eta}^1)\phi^m(\hat{\eta}^2)\right\rangle \sim \delta^{mn}\left(\frac{1}{4\pi\delta\eta} - \frac{9N\coth\pi N}{16\pi r_0} + \ldots\right) \qquad \text{short distances}$$

$$\sim (\hat{\eta}^1.\hat{\eta}^2)^{-\frac{1}{2}}, \qquad \text{large distances}$$

where $\hat{\eta}$, obeying $\hat{\eta}^2 = r_0^2$, is a point on $\mathbf{dS}^3$

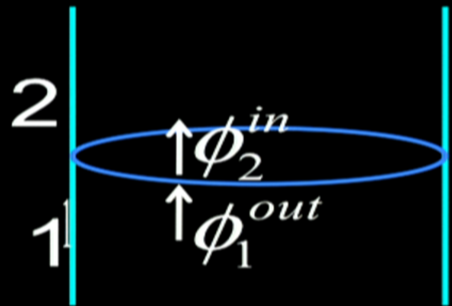
In summary:

We have shown that the dual CFT exists and is stable on $\mathbf{S}^3 / \mathbf{dS}^3$. It cannot be defined on flat space because the moduli space measure cannot be regulated in a conformal invariant manner.

In the cosmological phase of the theory, we have a well-defined vacuum and S-matrix

Crossing the singularity

dS^3 conformal to Einstein cylinder



need "S matrix": ϕ_1^{out} to ϕ_2^{in}

demand $SO(3,1)$ invariance

conformal weight

$$\phi_1 \sim (\tfrac{\pi}{2} - \tau)^{1+iN_*} f_1^{out}(\Omega) + \text{h.c.}, \ \tau \longrightarrow \tfrac{\pi}{2}$$

Factor out dependence -> correlators take CFT form, weight $h=1+iN_*$

At small g_t, compute in boundary theory
At large g_t, compute in bulk theory
Find qualitative agreement for large λ_6 :

$g_t \ll 1$	$g_t \gg 1$
$\langle \rho \rangle \sim \langle \vec{\phi}^2 \rangle \sim \lambda_6^{-1}$	$\alpha \sim \lambda_6^{-1}$
$\langle s \rangle \sim \langle (\vec{\phi}^2)^2 \rangle \sim \lambda_6^{-1}$	$\beta \sim \lambda_6^{-1}$
$S_E \sim N\lambda_6^{-2}$	$S_E \sim N\lambda_6^{-2}$

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