Title: Cosmological Magnetic Fields

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Abstract:

Cosmological Magnetic Fields

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9 July 2013

Outline

- Magneto-genesis.
- Link with matter-genesis.
- Chiral effects.
- Observations.

Primordial Origin of B

Inflationary origin: Turner & Widrow;...

<u>*Pros:*</u> large coherence scale.

<u>*Cons:*</u> speculative theoretical models (e.g. violate gauge invariance), strong coupling, extremely weak fields.

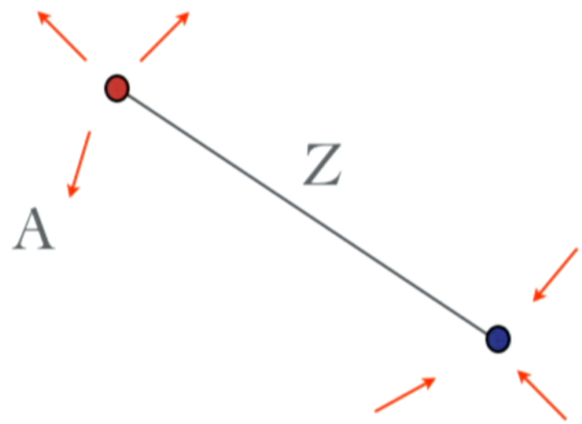
Phase transition origin: TV

<u>*Pros:*</u> based on reasonable particle physics models, B fields are strong.

<u>*Cons:*</u> small coherence scale at outset... but helicity & chirality can make a difference.

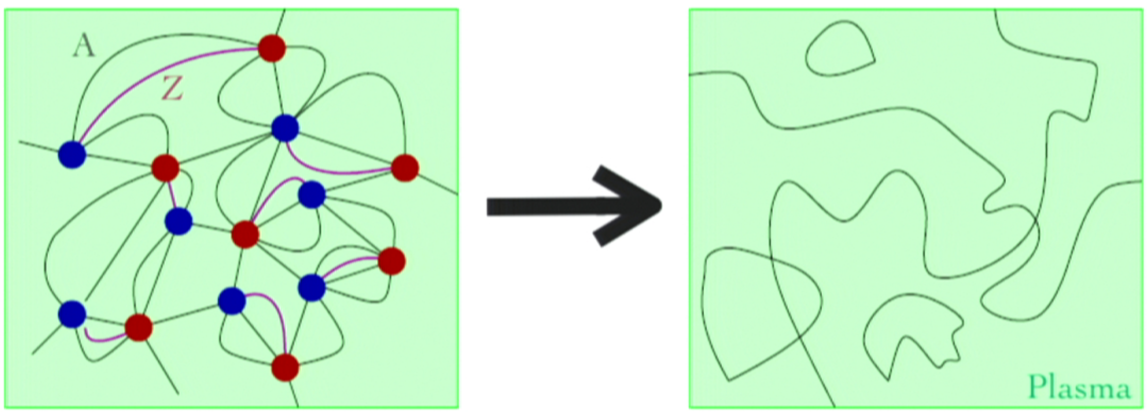
Magnetic monopoles in Standard Model

Nambu 1977
Achucarro & TV (Phy. Rep.)



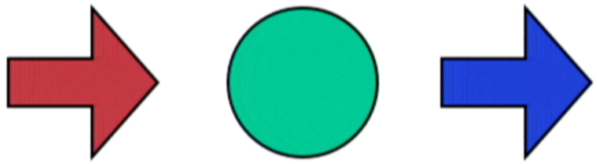
Magnetic fields from the electroweak phase transition

TV, 1991, 1994



Baryon Number Production

baryon #=0

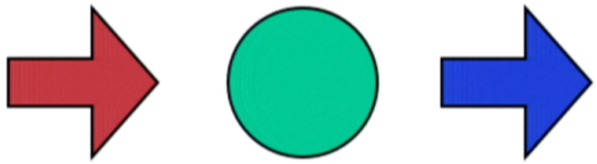


baryon number≠0

$$\Delta(\text{baryon number}) = \Delta(\text{Chern} - \text{Simons number})$$

$$CS = \frac{N_F}{32\pi^2}\int d^3x \epsilon_{ijk}\left[g^2\left(W^{aij}W^{ak} - \frac{g}{3}\epsilon_{abc}W^{ai}W^{bj}W^{ck}\right) - g'^2 Y^{ij}Y^k\right]$$

Baryon Number Production

baryon #=0  baryon number≠0

~Sphaleron

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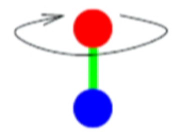
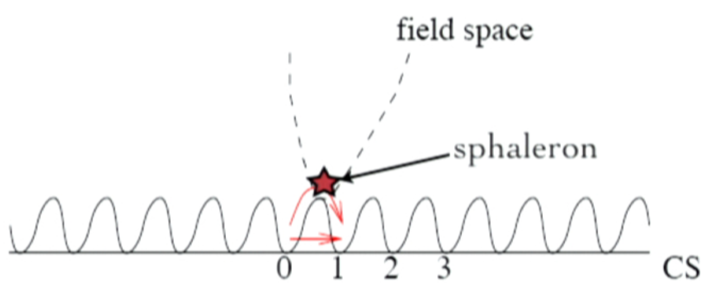
Sphaleron

= monopole-antimonopole bound state solution



Taubes
Manton
Manton & Klinkhamer
TV & Field
James & Hindmarsh

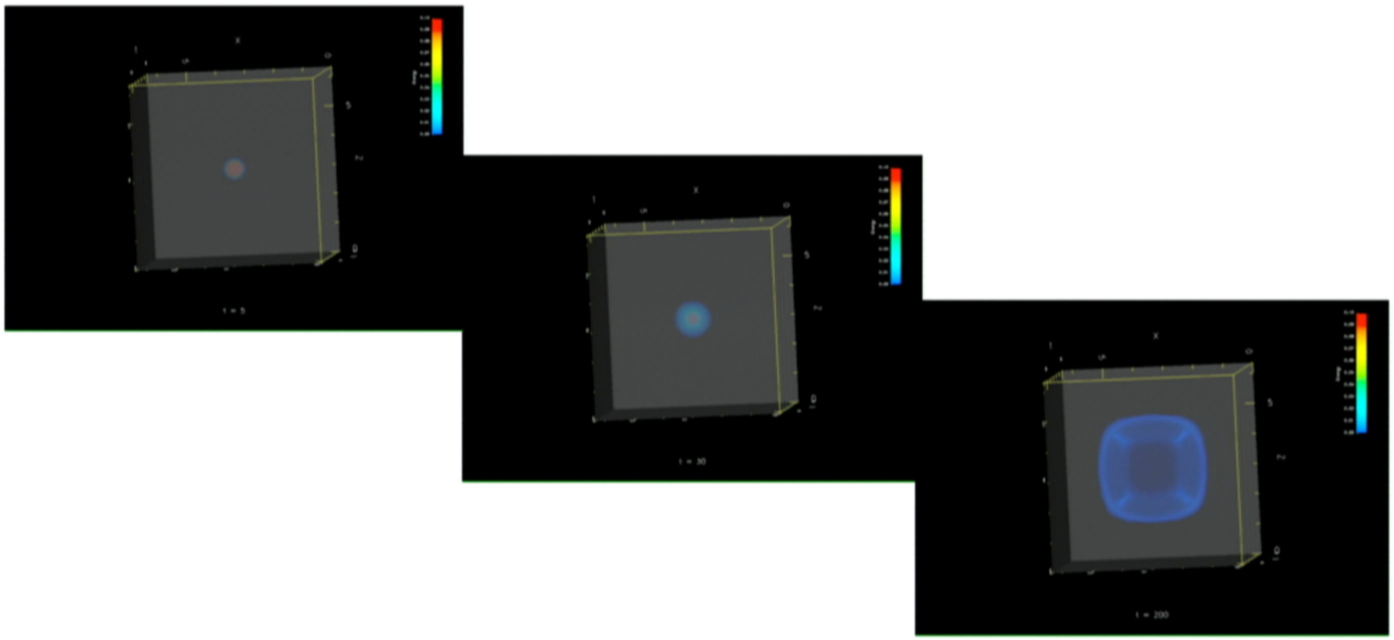
$$\mu_{\rm sphaleron} \approx 0.314 \text{ GeV}^{-1} \quad \mu_W = \frac{e}{m_W} \approx 0.0038 \text{ GeV}^{-1}$$



Sphaleron Decay

Copi, Ferrer, TV & Achucarro, 2008

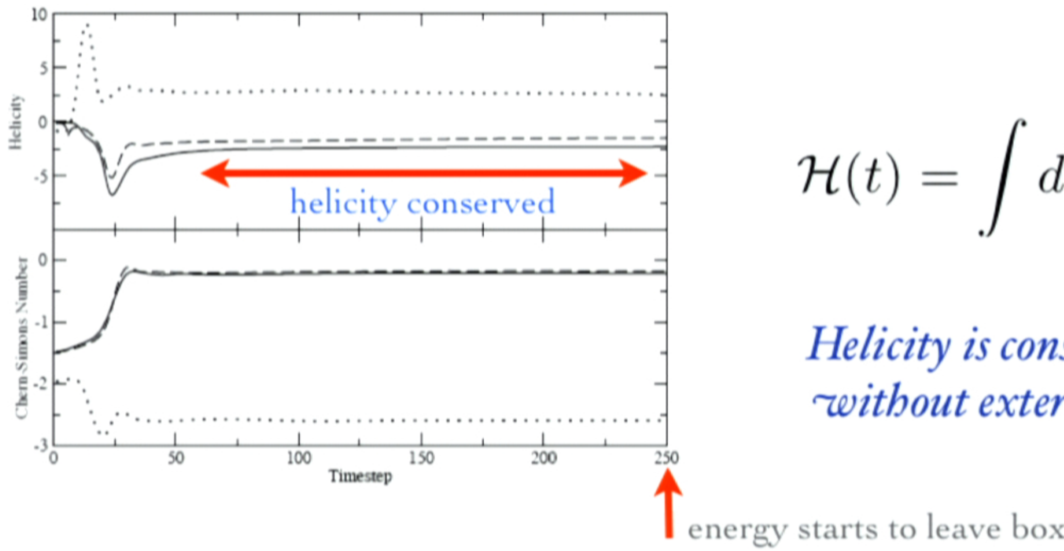
Evolve "latticized" classical electroweak equations with perturbed sphaleron initial conditions.



Magnetic Helicity in Sphaleron Decay

Copi, Ferrer, TV & Achucarro, 2008
Diaz-Gil, Garcia-Bellido, Perez & Gonzalez-Arroyo, 2008

Track helicity in numerical evolution of sphaleron decay.



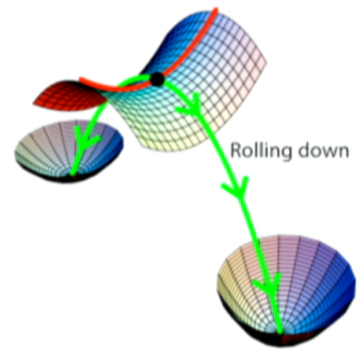
$$\mathcal{H}(t) = \int d^3x \ \mathbf{A} \cdot \mathbf{B}$$

Helicity is conserved even without external plasma.

Sphaleron decay - 2

Chu, Dent & TV

A decay path for the sphaleron is known.



Therefore currents can be calculated up to one function (flow velocity).

$$\Box A^\mu = j^\mu_{\text{sphaleron decay}}$$

Image- http://spie.org/x31524.xml?ArticleID=x31524

Calculate magnetic helicity generation along this path:

$$\mathcal{H}(t) = \int d^3x \ \mathbf{A} \cdot \mathbf{B}$$

(Asymptotic helicity is independent of flow velocity.)

Cosmological magnetic helicity

Every $\Delta B \implies \Delta\mathcal{H}$

Cornwall
TV

$$\implies \quad \boxed{h \approx -\#\frac{n_b}{\alpha}}$$

Independent of details of electroweak baryogenesis scenario ("topological").

CP Violation

Both baryons and antibaryons are produced during baryogenesis and CP violation biases transitions in favor of baryons.

$$h \approx -\# \frac{(\mathcal{N}_b - \bar{\mathcal{N}}_b)}{\alpha}$$

Energy density injected in magnetic fields is: $\rho_B \approx \#' \frac{(\mathcal{N}_b + \bar{\mathcal{N}}_b)}{\alpha \xi}$

(ξ = length scale)

Energy density enhancement factor: $$\frac{\mathcal{N}_b + \bar{\mathcal{N}}_b}{\mathcal{N}_b - \bar{\mathcal{N}}_b} \equiv \frac{1}{\delta_{\rm CP}^2}$$

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Inverse Cascade

Coherence scale grows due to Hubble expansion and inverse cascade.

$$\frac{\xi_p(t)}{t} \approx \frac{\xi_i}{t_i} \left(\frac{T_{\rm eq}}{T_i}\right)^{1-\alpha} \left(\frac{T_0}{T_{\rm eq}}\right)^{(1-\alpha)/2}$$

where simulations give $\alpha = \frac{2}{3}$.

Kahniashvili, Tevzadze, Brandenburg & Neronov

Coherence & Field Strength

Hubble expansion + inverse cascade: $\xi \sim \text{kpc}$

Conservation of helicity: $h \sim (10^{-21}\ \text{G})^2 - \text{kpc}$

CP enhanced: $B \sim \frac{10^{-21}}{\delta_{\text{CP}}} \sqrt{\frac{\text{kpc}}{\xi}}\ G$

Chirality in Cosmology

Joyce & Shaposhnikov

Left- and right-handed particles have different weak interactions.

As in $SU(2)_{\mathrm{Left}} \times U(1)_{\mathrm{hypercharge}}$.

Then it is possible that the number density of left-handed particles is not equal to the number density of right-handed particles in the early universe (before electroweak symmetry breaking).

Introduce independent chemical potentials for left- and right-handed particles.

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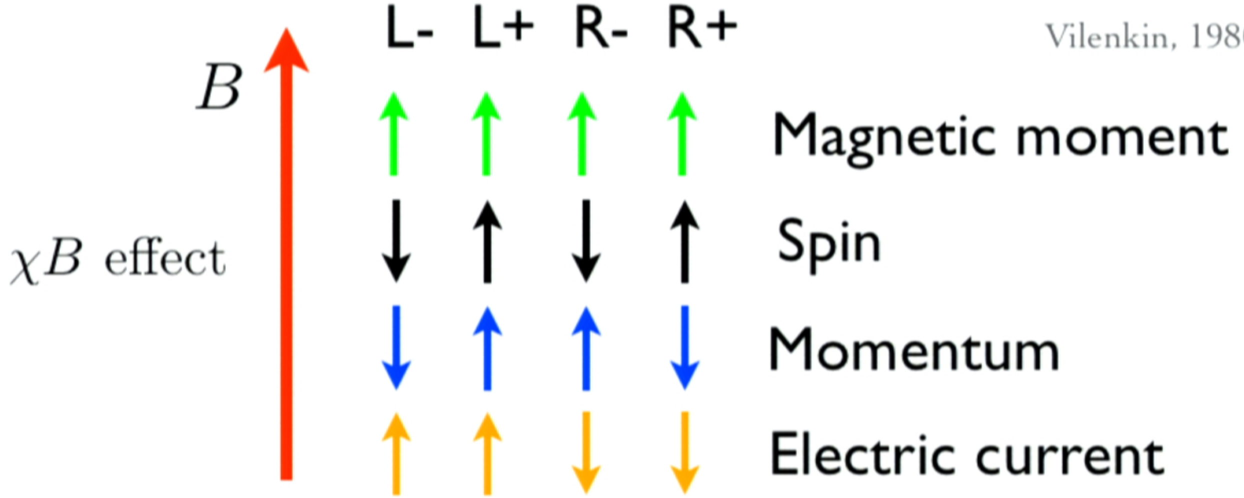
Chiral-Magnetic Effect

Vilenkin, 1980



$$J_{\chi B} \propto [n(e_L^-) - n(e_R^+)] - [n(e_R^-) - n(e_L^+)]$$

$$\mathbf{J}_{\chi B} = \frac{e^2}{2\pi^2} \Delta\mu \, \mathbf{B}$$

(Similarly -- the chiral-vorticity effect.)

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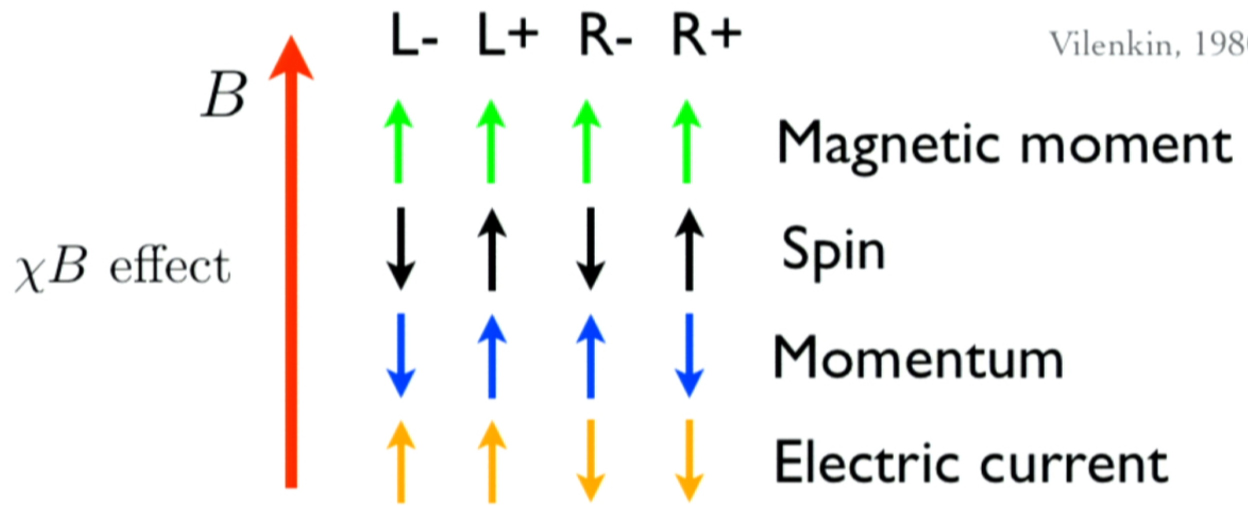

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$\chi - \mathbf{B}$ Effect above Electroweak

Joyce & Shaposhnikov

Boyarsky, Frohlich & Ruchayskiy

Chiral anomaly:

$$\frac{d(n_L - n_R)}{dt} = -\frac{\alpha}{\pi}\frac{dh}{dt} = \frac{2\alpha}{\pi V}\int_V d^3x\ \mathbf{E}\cdot\mathbf{B}$$

Maxwell & Ohm & chi-B:

$$\begin{aligned} \nabla \times \mathbf{B} &= \mathbf{J}_{\text{total}} \\ &= \mathbf{J}_{\text{Ohm}} + \mathbf{J}_{\chi\text{B}} \\ &= \sigma\mathbf{E} + \frac{\alpha}{\pi}\Delta\mu(t)\mathbf{B} \end{aligned}$$

Chiral MHD

Tashiro, TV & Vilenkin

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \gamma_D \nabla^2 \mathbf{B} + \gamma_\omega \nabla \times \omega + \gamma_B \nabla \times \mathbf{B}$$

$$\gamma_D = \frac{1}{4\pi\sigma} \ , \quad \gamma_\omega = \frac{e\Delta\mu^2}{4\pi^2\sigma} \ , \quad \gamma_B = \frac{e^2\Delta\mu}{2\pi^2\sigma}$$

$$\frac{d(\Delta\mu)}{dt} = -\frac{c_\Delta \alpha}{T^2} \frac{d}{dt} \left[\frac{1}{V} \int_V d^3x \mathbf{A} \cdot \mathbf{B} \right] - \Gamma_F \Delta\mu$$

Initial conditions: **B**=0. Therefore ignore advection term.
Reasonable for some duration that we can determine a posteriori.

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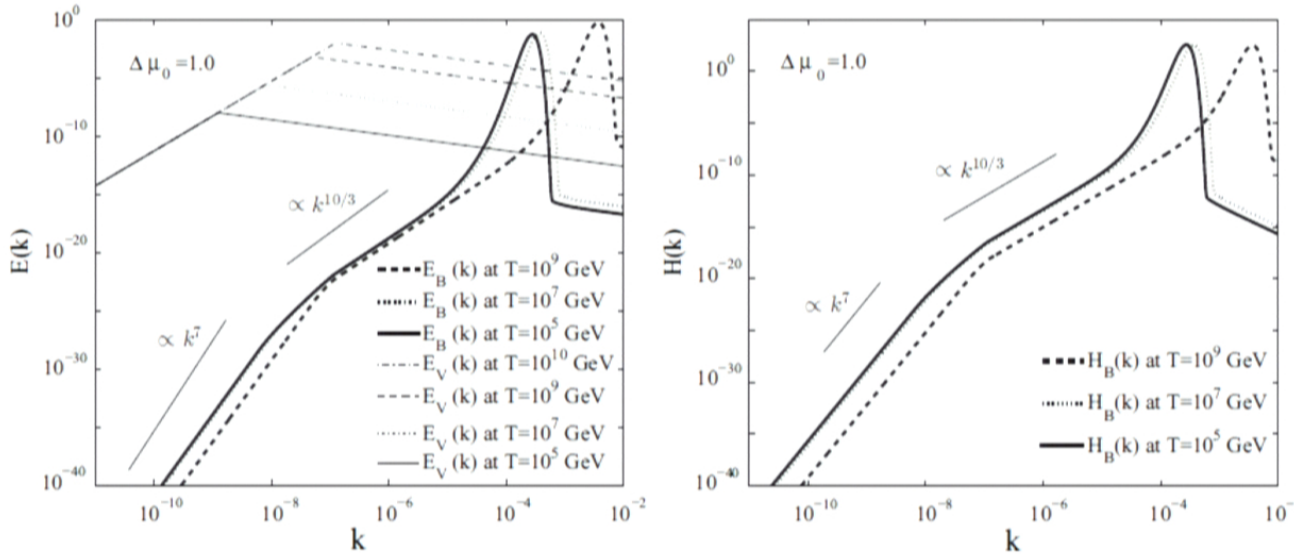
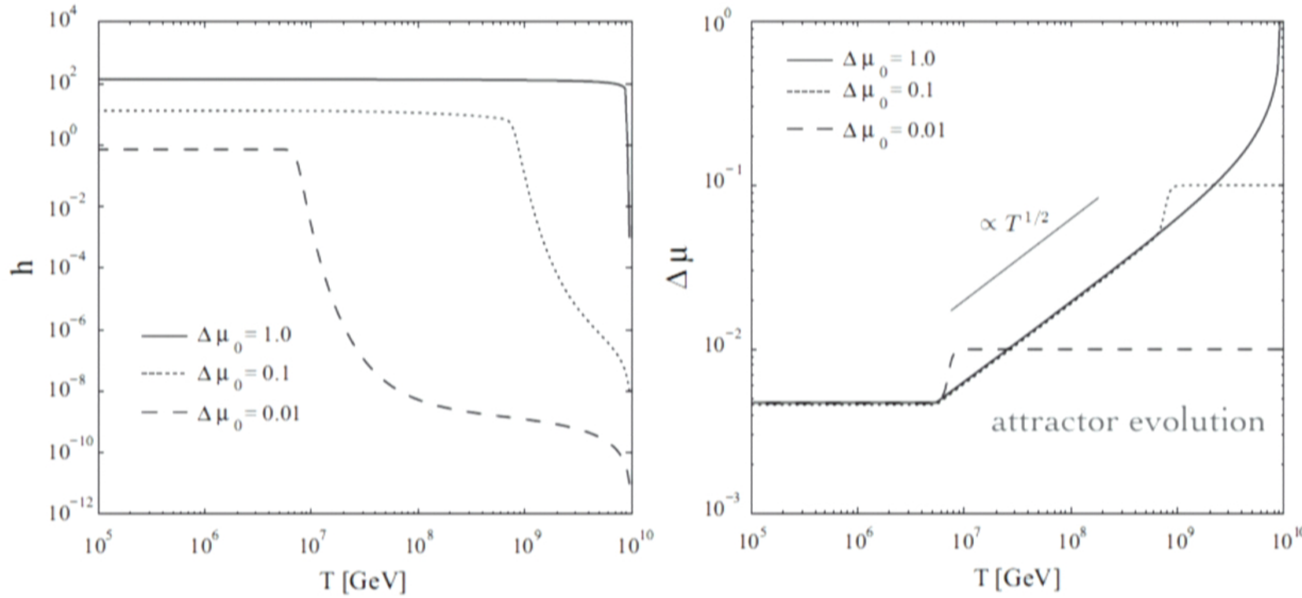
Evolution without advection



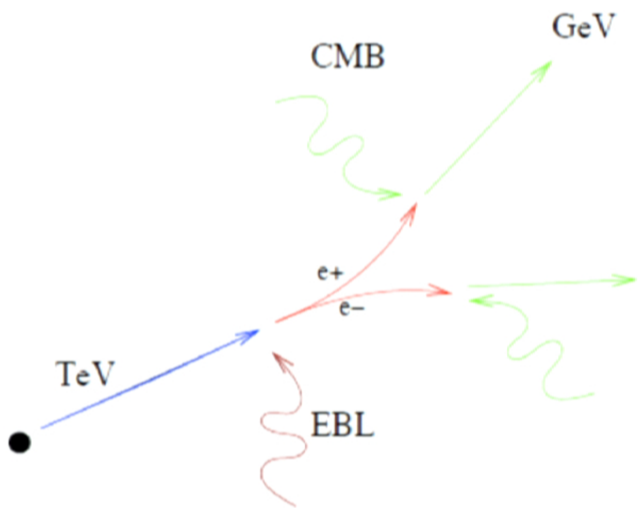
Peak shifts to larger length scales with time.

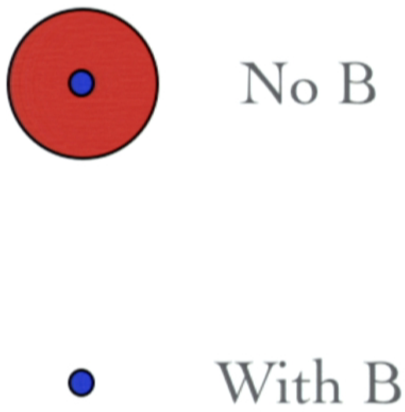
Helicity & chemical potential



Chirality dissipates "wrong" sign helicity.

Blazar observations





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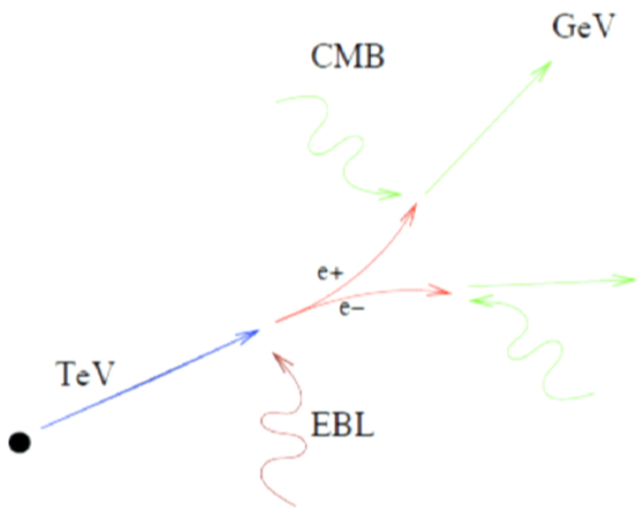
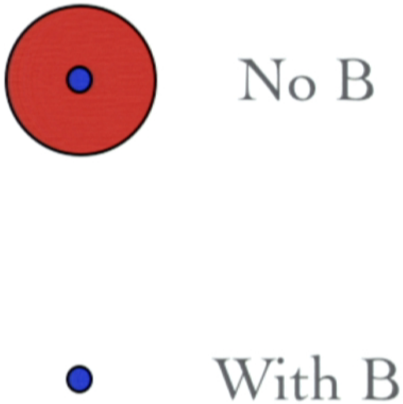




Observational Status

FERMI data gives *lower* bound of $\sim 10^{-16}$ G inter-galactic B.

Neronov & Vovk, 2010

Further discussion by --

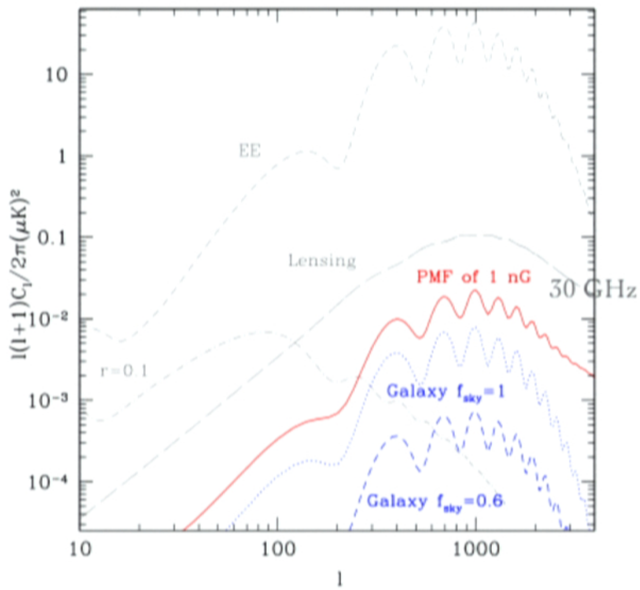
Ando & Kusenko, 2010
Neronov, Semikoz, Tinyakov & Tkachev, 2010
Broderick, Chang & Pfrommer, 2011
Miniati & Elyiv, 2012
Arlen, Vassilev, Weisgarber, Wakeley & Shafi, 2012
...

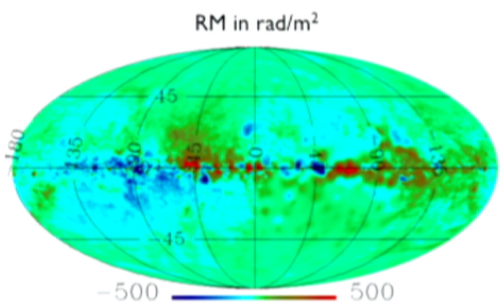
CMB Observations

Loeb & Kosowsky; Harari, Hayward & Zaldarriaga; Kamionkowski; Kahniashvili et al....

CMB Faraday Rotation:
BB, EB correlators; through the Milky Way.

{Pogosian, Yadav, De, Ng, TV}





Oppermann et al

Temperature anisotropy

TT (vector contributions).

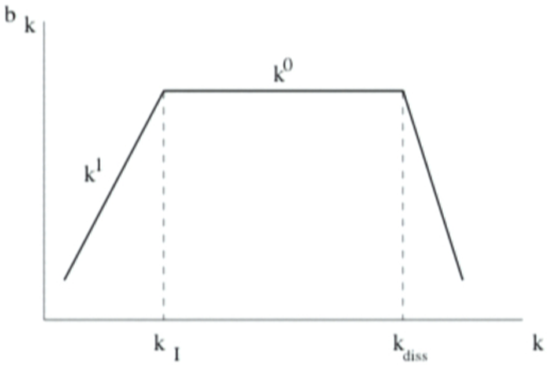
Barrow, Seshadri & Subramanian; Paoletti & Finelli; Lewis; Giovannini; Yamazaki et al...

Useful for nG strength B on large scales (Mpc).

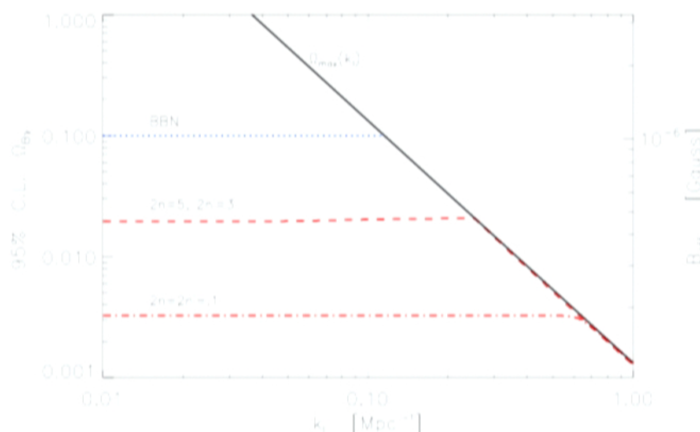
More detailed recombination with small-scale magnetic fields -- promising to 0.01 nG. Jedamzik, Katalinic, Olinto

Upper Bounds

Quasars (Faraday rotation), CMB, BBN provide upper bounds.



Jedamzik & Sigl; Banerjee& Jedamzik; Durrer & Caprini; Jedamzik, Katalinic & Olinto; Kahniashvili et al; Tevzadze et al.



$$B_{\rm eff} \equiv \sqrt{8\pi\rho_{\rm B}}$$

Pogosian, Yadav, Ng & TV

$$\boxed{B_{\rm 1Mpc} \lesssim 1~{\rm nG}}$$

...from several different observables.

Current Bounds

$$\boxed{10^{-16}\ \mathrm{G} \lesssim B \lesssim 10^{-9}\ \mathrm{G}^*}$$ ⚑

Would like spectral information including helicity.

°Assumes correlation length, typically 1 Mpc, or some power spectrum.

⚑ Helicity does not play a (direct) role.

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Helicity spectrum

Related to the linking and twisting of magnetic field lines.

Direct measures should probe the magnetic field in 3D.
e.g. cosmic rays Kahniashvili & TV

$$\langle \tilde{B}_i^*(\mathbf{k})\tilde{B}_j(\mathbf{k}')\rangle = (2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')\left[\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)S(k)+i\epsilon_{ijl}\frac{k_l}{k}A(k)\right]$$

Indirect measures can utilize the affect of helicity on magnetic field evolution.

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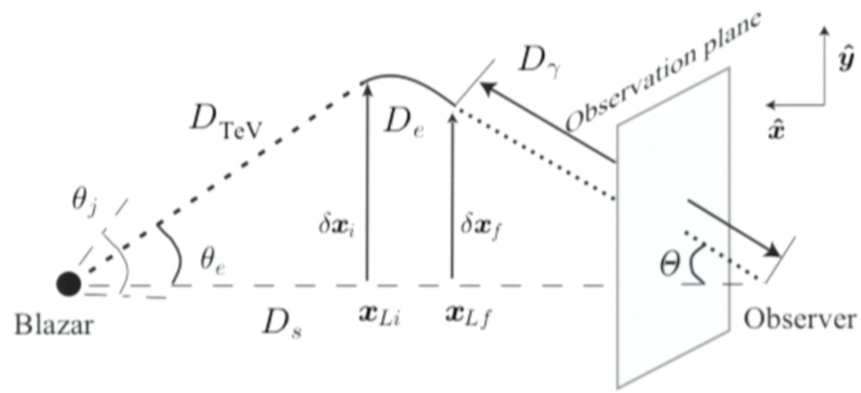
Spectrum of B from blazars

Tashiro & TV



$$\theta_j \sim \text{few degrees}$$

$$D_{\mathrm{TeV}} \sim 100 \text{ Mpc}, \quad D_e \sim 30 \text{ kpc}, \quad D_\gamma \sim 1 \text{ Gpc}$$

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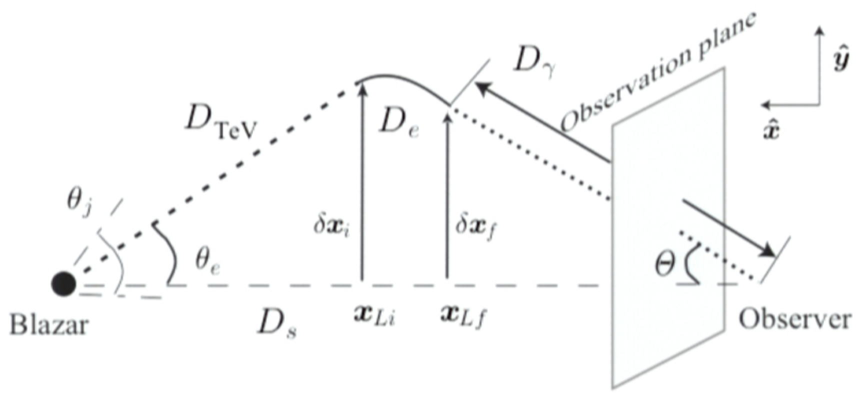

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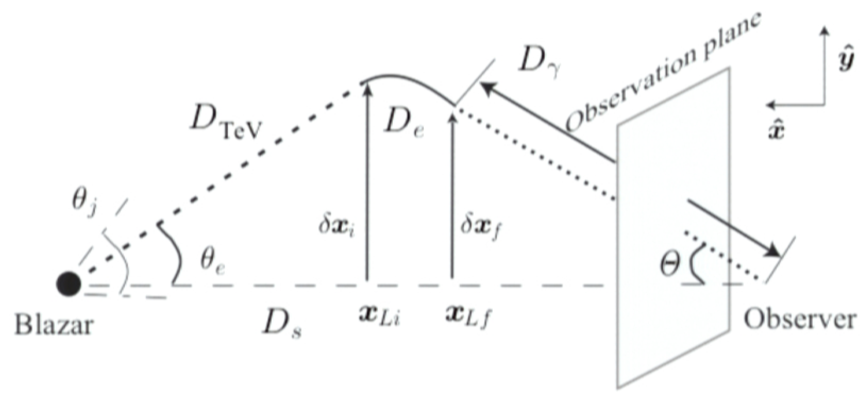

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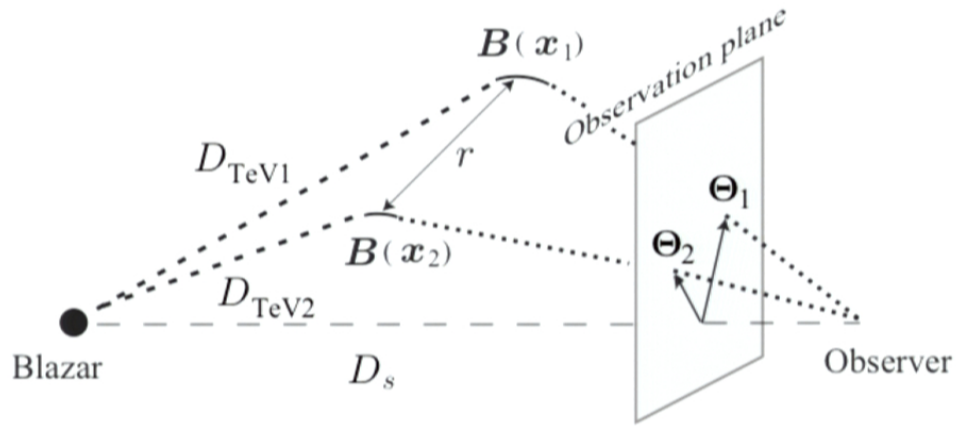
B-Correlators



$$\langle B_i(\mathbf{x}+\mathbf{r})B_j(\mathbf{x})\rangle = M_N(r)\left[\delta_{ij} - \frac{r_i r_j}{r^2}\right] + M_L(r)\frac{r_i r_j}{r^2} + M_H(r)\epsilon_{ijl}r^l,$$

$$M_N(|r_{12}|) = 2\kappa\langle\mathbf{\Theta}(E_1)\cdot\mathbf{\Theta}(E_2)\rangle$$

$$\kappa = \frac{E_{e1}E_{e2}D_s^2}{e^2 D_{\mathrm{TeV1}}D_{\mathrm{TeV2}}D_{e1}D_{e2}}$$

$$M_H(|r_{12}|) = \frac{2\kappa}{r_{12}}\langle\mathbf{\Theta}(E_1)\times\mathbf{\Theta}(E_2)\cdot\hat{\mathbf{x}}\rangle$$

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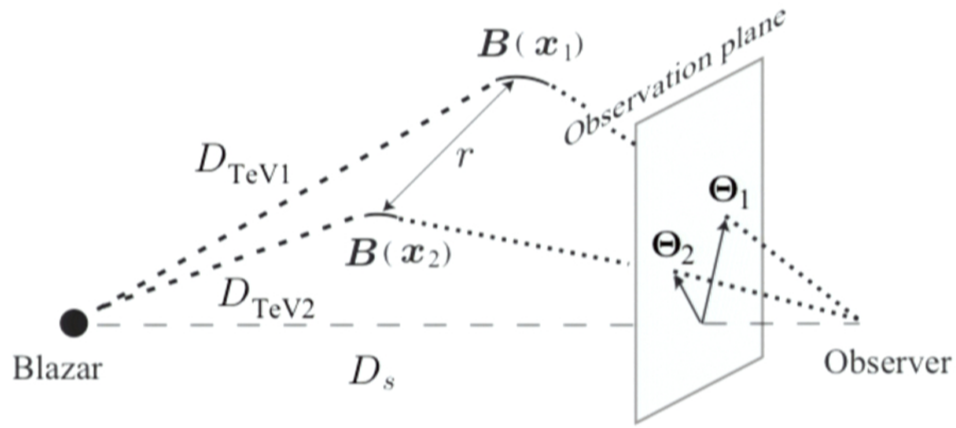

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Conclusions

- Tight connection between baryogenesis and magnetogenesis.
- Fields today can have "interesting" amplitude on astrophysical scales.
- Properties of cosmic magnetic fields can be a window to high energy particle physics and the early universe.
- TeV gamma rays, cosmic rays, and CMB can detect primordial magnetic fields and characterize their properties.

NORDITA program on
"Origin, Evolution and Observation of
Cosmological Magnetic Fields"
in Stockholm
15 June - 10 July, 2015