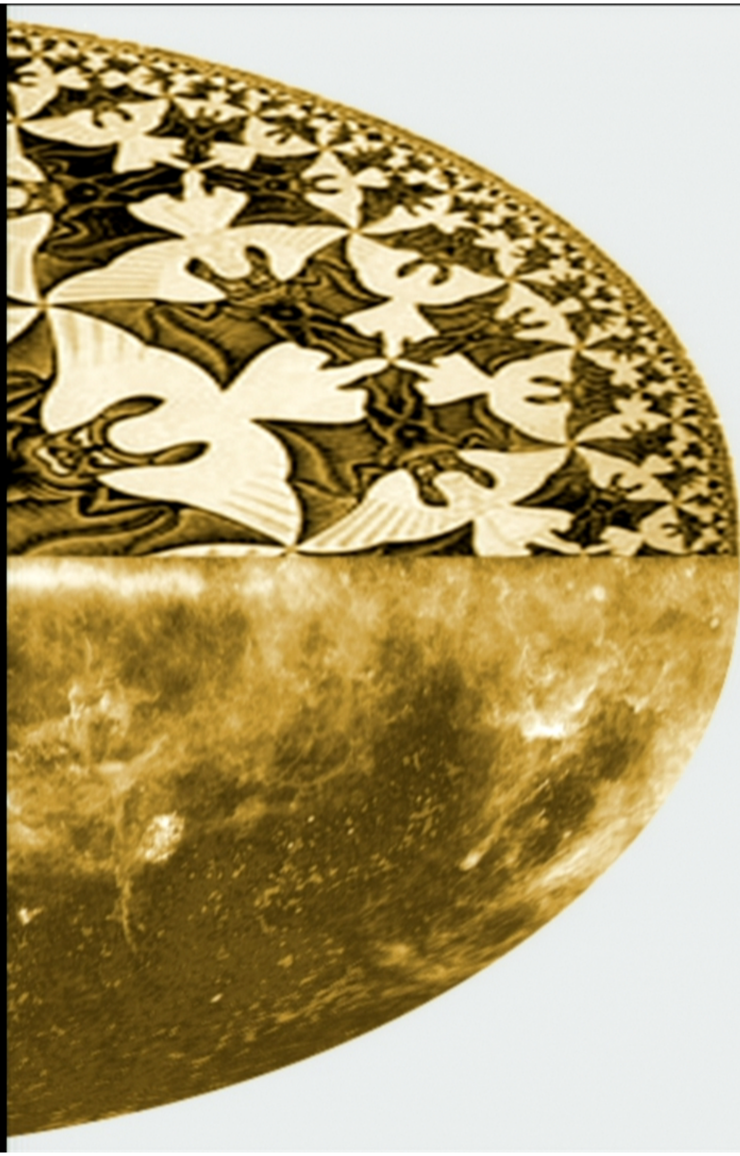
Title: Precision Holographic Cosmology

Date: Jul 08, 2013 10:50 AM

URL: http://pirsa.org/13070006

Abstract: We discuss holography for cosmology, focusing on a class of slow-roll inflationary spacetimes that are holographically dual to a perturbative RG flow between two nearby CFTs.   The cosmological power spectrum and non-Gaussianities may be calculated directly from the dual QFT using conformal perturbation theory, even when the dual QFT is strongly coupled. Holography thus offers new methods for computing cosmological observables. To illustrate, we show how to recover the power spectrum to second order in slow roll.



Precision holographic cosmology

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Cosmological Frontiers in Theoretical Physics 2013

Introduction

In this talk we present a precise and quantitative holographic decription of a class of inflationary slow-roll models.

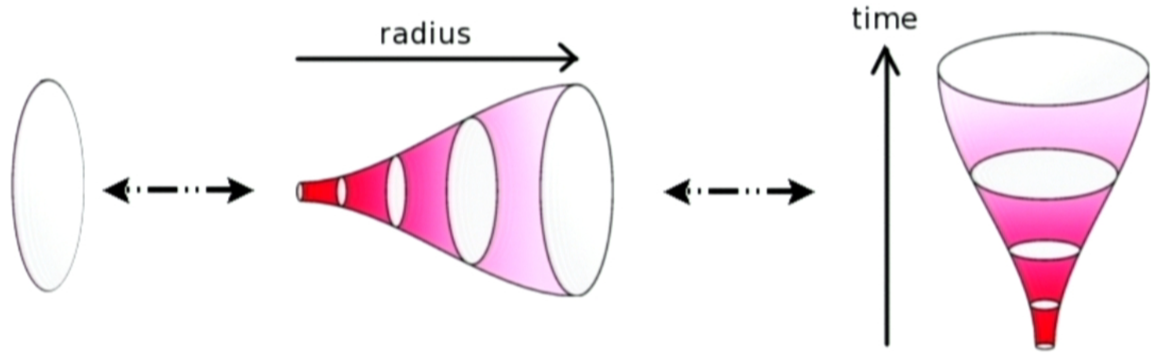
Through calculations in the three-dimensional dual QFT, we can derive

1. The inflationary power spectrum to second order in slow-roll
2. Non-Gaussianities

Introduction

Our holographic framework for cosmology is based on standard AdS/CFT plus the domain-wall/cosmology correspondence.

[PM & Skenderis '09-'11]



3d QFT

4d "domain-wall":

$$ds^2 = dr^2 + a^2(r)d\vec{x}^{\,2}$$

$$\varphi = \varphi(r)$$

4d cosmology:

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^{\,2}$$

$$\varphi = \varphi(t)$$

Framework

- This correspondence also holds for perturbations about the background solutions, and to non-linear order:

  One can set up a 1-1 map between solutions of the perturbed domain-wall equations of motion and those of the corresponding cosmology.

- Using this map, plus the standard holographic dictionary, one can construct holographic formulae linking quantities in the 3d QFT to cosmological observables.

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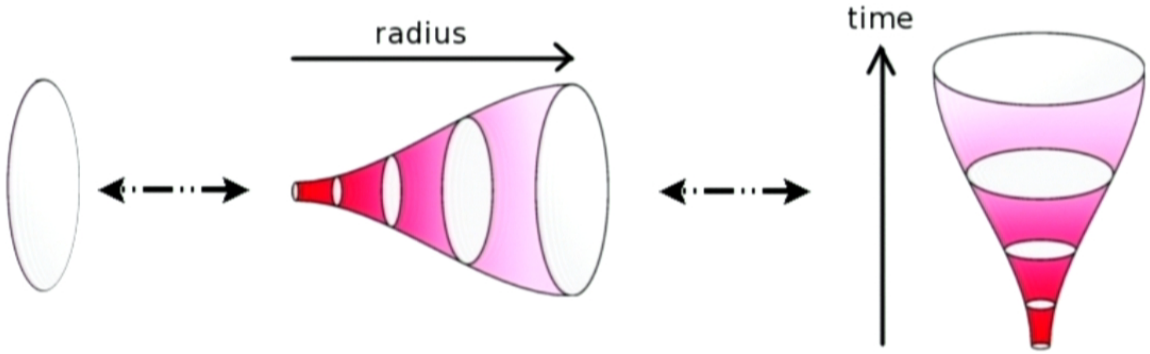

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For example, the primordial scalar power spectrum is given by

$$\Delta_S^2(q) = \frac{4}{\pi^4}\frac{1}{c(q)}$$

where $c(q)$ is the spectral density of the 2-point function $\langle TT \rangle$ in the dual QFT.

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Today

Our goal today is to apply this framework by recovering standard inflationary physics starting from the 3d QFT.

An apparent obstacle is that the QFT is strongly coupled: how do we compute?

One possibility is when the QFT is the deformation of a CFT by a scalar operator $\mathcal{O}$. We take $\mathcal{O}$ to be nearly marginal, $\Delta = 3 - \lambda$, and expand in $\lambda \ll 1$.

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Plan

1. The dual QFT
2. Properties: β function, spectral density
3. Holographic computation of power spectrum
4. Nature of corresponding inflationary cosmology
5. Non-Gaussianities

References

This talk is based on:

- Precision holographic cosmology: the power spectrum to second order in slow-roll.
  PM, [1307.xxxx]
- Holography for inflation using conformal perturbation theory.
  Bzowski, PM, Skenderis [1211.4550]

See also:

Schalm, Shiu, & van der Aalst [1211.2157]; Mata, Raju & Trivedi [1211.5482].

Conformal perturbation theory: Zamolodchikov '87; Cardy & Ludwig '87.

The dual QFT

We consider a dual QFT which is the deformation of a 3d CFT by a nearly marginal scalar operator,

$$S = S_{CFT} + \phi_0 \int \mathrm{d}^3x\, \mathcal{O}_0(x),$$

where $\mathcal{O}_0$ has scaling dimension $\Delta = 3 - \lambda$, with $0 < \lambda \ll 1$. We work in Euclidean signature.

Correlators in the deformed theory are given by

$$\langle \mathcal{O}_0(x_1)\mathcal{O}_0(x_2)\rangle = \langle \mathcal{O}_0(x_1)\mathcal{O}_0(x_2)\exp\left(-\phi_0 \int \mathrm{d}^3z\, \mathcal{O}_0(z)\right)\rangle_0$$

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$$\langle \mathcal{O}_0(q)\mathcal{O}_0(-q)\rangle = A_0 q^{3-2\lambda} - A_1\phi_0 q^{3-3\lambda} + \frac{1}{2}A_2\phi_0^2 q^{3-4\lambda} + O(\phi_0^3)$$

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Renormalisation

The theory has divergences in the limit $\lambda \to 0$ where $\mathcal{O}_0$ becomes marginal, e.g. $A_1 \sim 1/\lambda$.

- Cured by field renormalisation:

$$\mathcal{O} = \frac{\mathcal{O}_0}{\sqrt{Z(g)}}$$

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β-function

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$$\Rightarrow g(\mu) = \phi_0\mu^{-\lambda} - \frac{A_1}{4A_0}\phi_0^2\mu^{-2\lambda} + \frac{1}{12}\Big(\frac{A_2}{A_0} - \frac{A_1^2}{2A_0^2}\Big)\phi_0^3\mu^{-3\lambda} + O(\phi_0^4).$$

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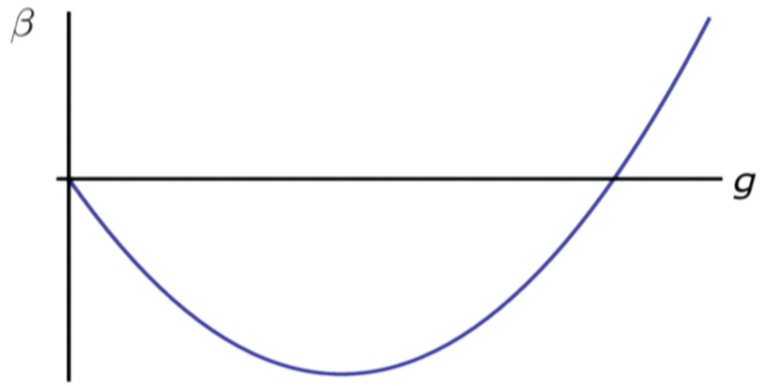
When $b_2 \sim 1$, this IR fixed point is close to the original UV fixed point.



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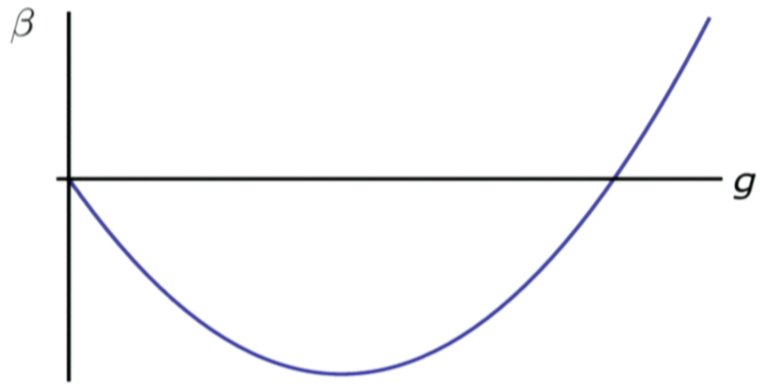

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Spectral density

The 2-point function of $T = T_i^i$ has the following spectral representation

$$\langle T(x)T(0)\rangle = \frac{\pi}{8}\int_0^\infty \mathrm{d}\rho\, c(\rho) \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\frac{q^4}{q^2+\rho^2}e^{iq\cdot x}.$$

The spectral density $c(\rho)$ encodes the contribution of propagating intermediate states of mass ρ.

$c(\rho)$ may be extracted via a dispersion relation:

$$c(\rho) = \frac{16}{\pi^2}\frac{1}{\rho^3}\,\mathrm{Im}\,\langle T(q)T(-q)\rangle\big|_{q^2=-\rho^2-i\epsilon}$$

[Capelli, Friedan, Latorre '91]

Spectral density

The 2-point function of $T = T_i^i$ has the following spectral representation

$$\langle T(x)T(0)\rangle = \frac{\pi}{8}\int_0^\infty \mathrm{d}\rho\, c(\rho) \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\frac{q^4}{q^2+\rho^2} e^{iq\cdot x}.$$

The spectral density $c(\rho)$ encodes the contribution of propagating intermediate states of mass ρ.

$c(\rho)$ may be extracted via a dispersion relation:

$$c(\rho) = \frac{16}{\pi^2}\frac{1}{\rho^3}\,\mathrm{Im}\,\langle T(q)T(-q)\rangle\big|_{q^2=-\rho^2-i\epsilon}$$

[Capelli, Friedan, Latorre '91]

Spectral density

To compute $c(\rho)$, we calculate $\langle T(q)T(-q)\rangle$ using RG improved perturbation theory. This uses the Callan-Symanzik equation to effectively resum the expansion in ϕ_0, giving correct scaling behaviour about the IR fixed point.

Running coupling:

$$\frac{\mathrm{d}\bar{\varphi}(q)}{\mathrm{d}\ln(q/\mu)} = \beta(\bar{\varphi}(q)), \qquad \bar{\varphi}(\mu) = \varphi.$$

Spectral density

With an appropriate choice of renormalisation scheme and operator normalisation, we find

$$c(\rho) = \frac{16}{\pi^2}\beta^2\Big[1 - 2b\beta' + \big(4 + 2b^2 - \frac{\pi^2}{2}\big)\beta'^2 + \big(b^2 - \frac{\pi^2}{12}\big)\beta''\beta + O(\lambda^3)\Big]$$

where $b \equiv 2 - \ln 2 - \gamma$ and $\beta = \beta(\bar{\varphi}(\rho))$.

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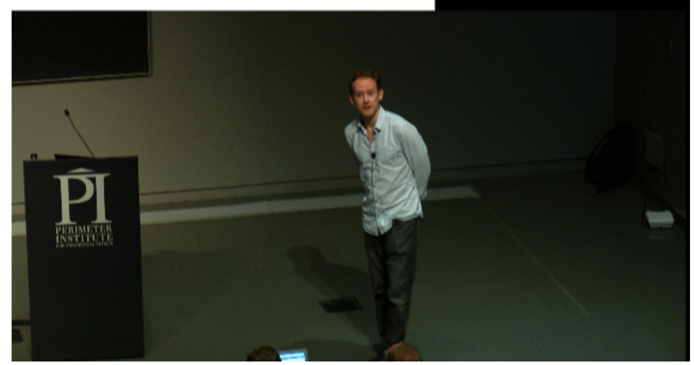

Holographic power spectrum

The primordial power spectrum for the dual cosmology is related to the spectral density by

$$\Delta_S^2(q) = \frac{4}{\pi^4}\frac{1}{c(q)}.$$

By design, in the renormalisation scheme we use, the running coupling $\bar{\varphi}(q)$ equals the value of the inflaton at horizon crossing $\varphi_*(q)$. This means

$$\epsilon_* = \frac{1}{2}\beta^2, \qquad \eta_* = \beta' + O(\lambda^4), \qquad \delta_{2*} = \beta'^2 + \beta''\beta + O(\lambda^4),$$

where the slow-roll parameters $\epsilon = \dot{\varphi}^2/2H^2$, $\eta = \ddot{\varphi}/H\dot{\varphi}$ and $\delta_2 = \dddot{\varphi}/H^2\dot{\varphi}$.

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Plugging in our results, we obtain the slow-roll power spectrum to second order:

$$\Delta_S^2(q) = \frac{H_*^2}{8\pi^2\epsilon_*}\Big[1 + 2b\eta_* + (3b^2 - 4 + \frac{5\pi^2}{12})\eta_*^2 + (-b^2 + \frac{\pi^2}{12})\delta_{2*} + O(\lambda^3)\Big].$$

cf. Gong & Stewart [astro-ph/0101225]

Note that since $\beta = -\lambda\varphi + B_2\varphi^2 + B_3\varphi^3 + O(\varphi^4)$, we have $\varphi \sim \lambda$, so

$$\beta \sim \lambda^2, \qquad \beta' \sim \lambda, \qquad \beta'' \sim 1,$$

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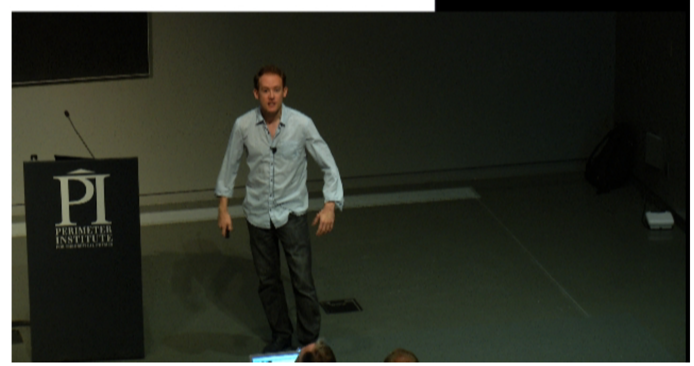

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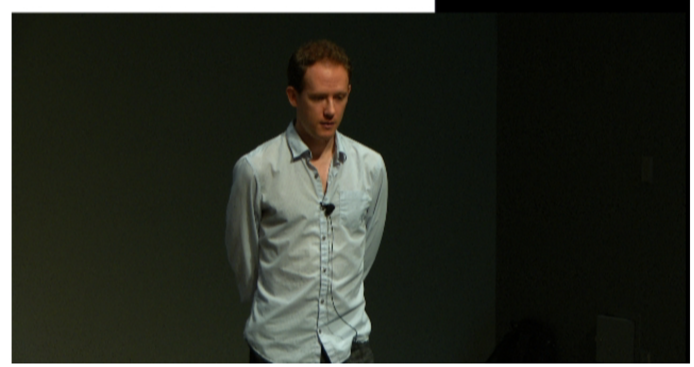


Inflaton potential

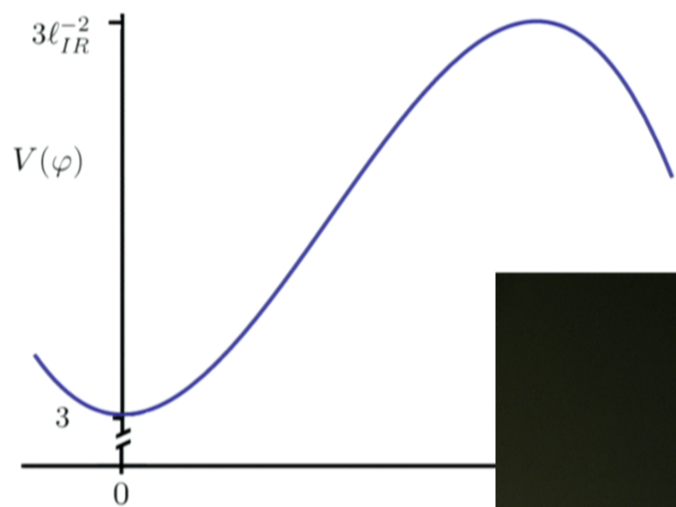
The inflaton potential may be derived from

$$V = \frac{1}{2}(6-\beta^2)\exp\Big(-\int_0^{\varphi}\beta \mathrm{d}\varphi\Big) = 3 + \frac{1}{2}m^2\varphi^2 - \frac{1}{3}g_3\varphi^3 - \frac{1}{4}g_4\varphi^4 + O(\varphi^5)$$

where

$$m^2 = \lambda(3-\lambda), \quad g_3 = 3B_2(1-\lambda), \qquad g_4 = (3-4\lambda)B_3 + 2B_2^2 + O(\lambda^2).$$

This describes a hilltop model: we roll from $\varphi_1 \sim \lambda/B_2$ to the origin.



Reheating

A more complete model would have to incorporate reheating.

Holographically, this amounts to changing the fate of the RG flow in the UV (= late times), e.g., by allowing other operators to enter.

The behaviour in the IR (= early times) is controlled by the most nearly marginal irrelevant operator (i.e., $\mathcal{O}$) and so is unchanged.

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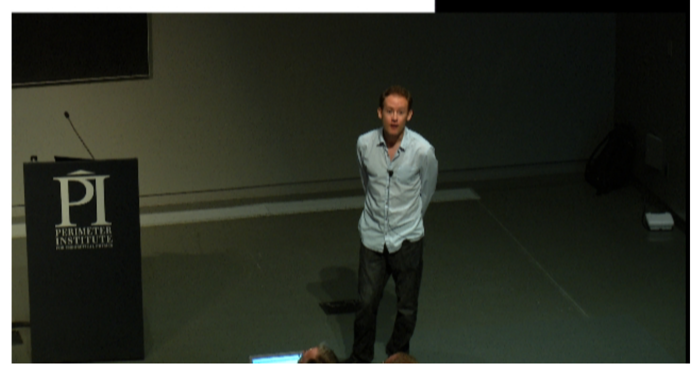
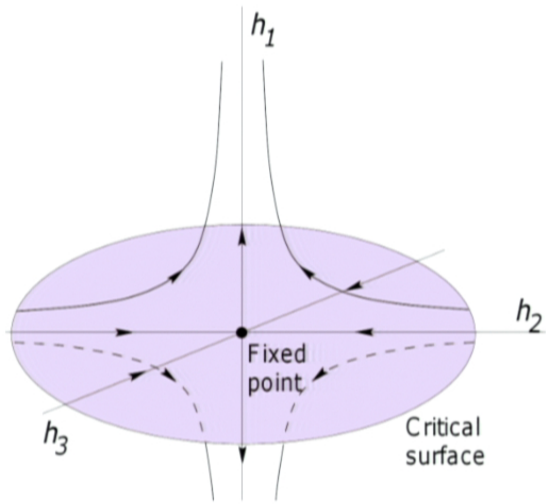
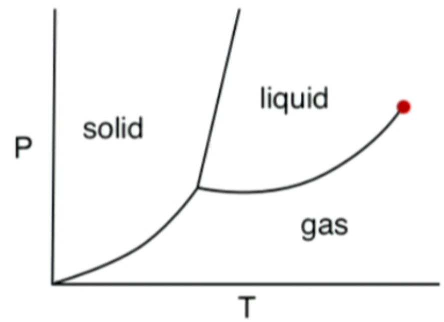


Universality

Analogy with critical phenomena: different UV Hamiltonians flow to same IR fixed point, giving universal long-wavelength behaviour.





$$(n_s - 1)\Big|_{IR} \approx -2\eta_*\Big|_{IR} \approx -2\beta'\Big|_{IR} \approx -2\lambda + O(\lambda^2)$$

Choice of scheme revisited

Let's return to our choice of renormalisation scheme.

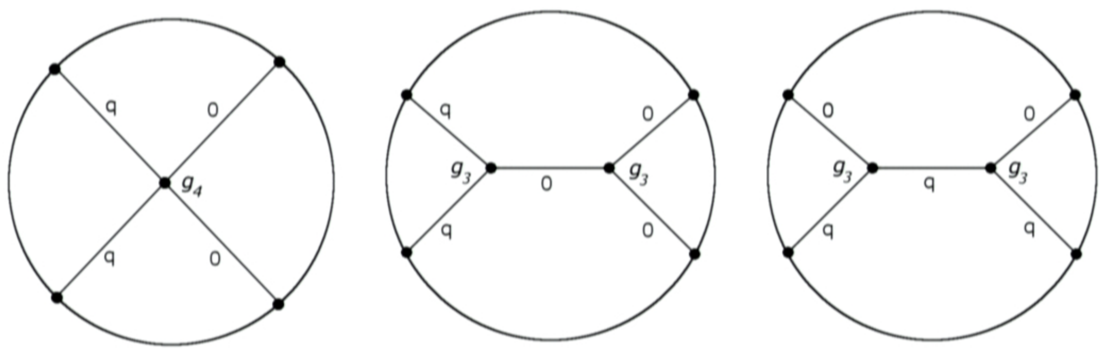
This choice does not affect the spectral density $c(q)$, but it does affect the β-function and the nature of the mapping to the bulk cosmology.

We selected our scheme so that the running coupling $\bar{\varphi}(q)$ equals the inflaton at horizon crossing $\varphi_*(q)$. How did we achieve this?

AdS/CFT

The coefficients A_0, A_1 and A_2 parametrising the UV CFT can be computed using standard AdS/CFT: we just solve $\Box\varphi = V'(\varphi)$ perturbatively on a fixed AdS background.

$$V_{DW}(\varphi) = -3 + \frac{1}{2}\lambda(\lambda-3)\varphi^2 + \frac{1}{3}g_3\varphi^3 + \frac{1}{4}g_4\varphi^4 + O(\varphi^5)$$



Choice of scheme

We then have $\beta(g)$ in terms of the coefficients λ, g_3, g_4 in the bulk potential.

We can similarly identify $\mathrm{d}\varphi_*(q)/\mathrm{d}\ln q$ in terms of these coefficients.

We now choose our field redefinition $g = \varphi(1 + a_1\varphi + a_2\varphi^2 + O(\varphi^3))$ to set the running coupling $\bar{\varphi}(q)$ equal to the inflaton at horizon crossing $\varphi_*(q)$:

$$\beta(\bar{\varphi}(q)) = \frac{\mathrm{d}\bar{\varphi}(q)}{\mathrm{d}\ln q} = \frac{\mathrm{d}\varphi_*(q)}{\mathrm{d}\ln q},$$

[Other schemes lead to a nontrivial mapping between $\bar{\varphi}(q)$ and $\varphi_*(q)$.]

Non-Gaussianities

One can also compute cosmological 3-point functions holographically, starting from 3-point functions in dual QFT.

See [1211.4550] for results at leading order in λ for $\zeta\zeta\zeta$, $\zeta\zeta\gamma$, $\zeta\gamma\gamma$, etc.

Find usual first order slow-roll expressions (for $\epsilon_* \ll \eta_*$)

e.g., $\langle\zeta(q_1)\zeta(q_2)\zeta(q_3)\rangle = \eta_* \sum_{i<j}\langle\zeta(q_i)\zeta(-q_i)\rangle\langle\zeta(q_j)\zeta(-q_j)\rangle$

$$\langle\zeta(q_1)\zeta(q_2)\gamma^{(\pm)}(q_3)\rangle = \frac{H_*^4}{16\sqrt{2\epsilon *}}\frac{1}{q_3^2 a c^3}(-a^3+ab+c)(a^3-4ab+8c)$$

where $a = \sum q_i$, $b = \sum_{i<j} q_i q_j$, $c = q_1 q_2 q_3$.

Conclusions

We constructed inflationary cosmology dual to 3d QFT which is deformation of a CFT by a nearly marginal scalar operator.

The small parameter λ controls (i) the dimension of $\mathcal{O}$, (ii) the separation of UV & IR fixed points, (iii) the spectral tilt $n_S - 1$.

- We computed the power spectrum to second order in slow roll using the holographic formula $\Delta_S^2 = 4/\pi^4 c(q)$.
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$\Rightarrow$ Striking test of holographic framework.

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