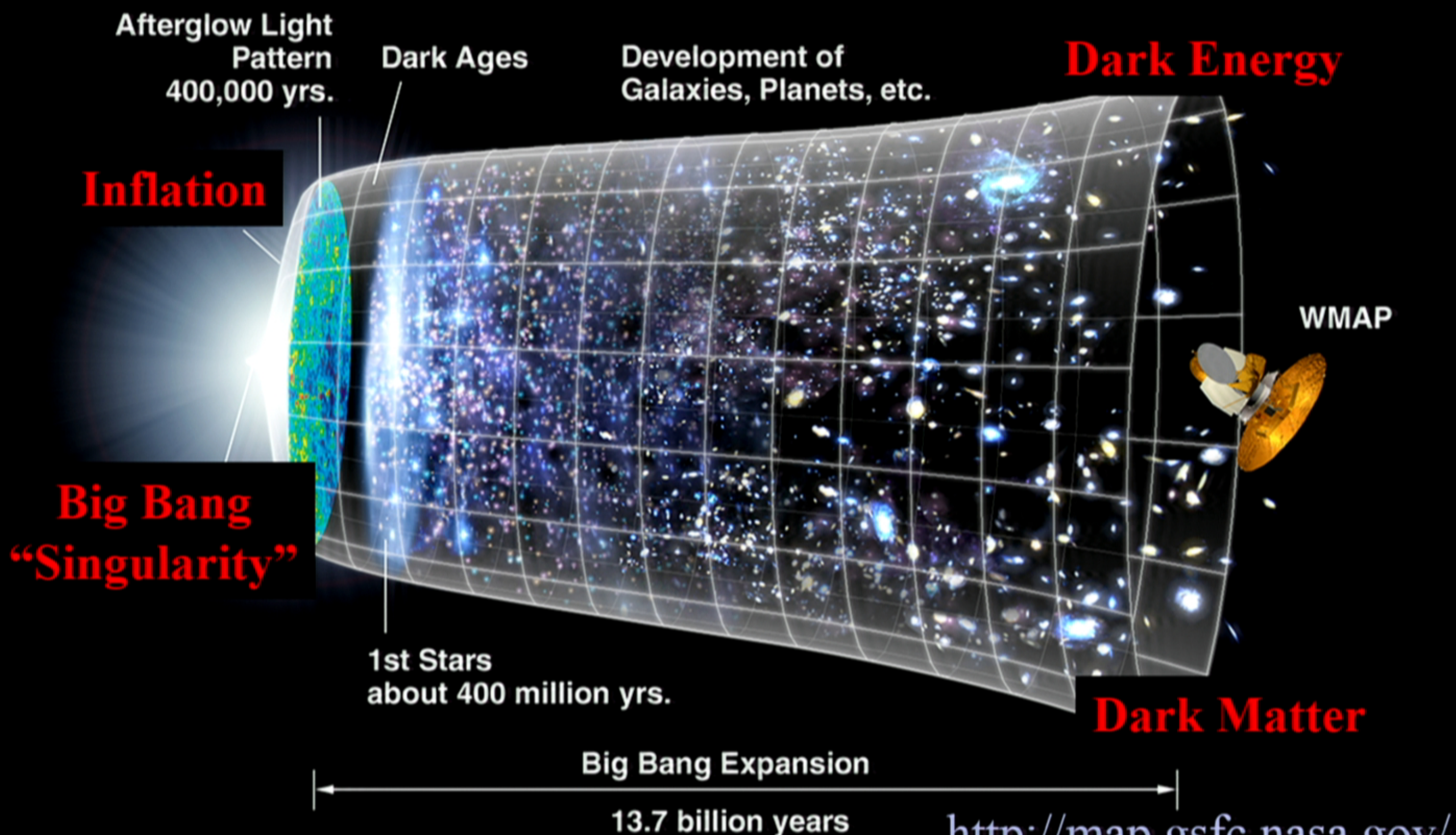
Title: Massive gravity and cosmology

Date: Jul 08, 2013 11:40 AM

URL: http://pirsa.org/13070002

Abstract:

Why alternative gravity theories?



http://map.gsfc.nasa.gov/

Three conditions for good alternative theories of gravity (my personal viewpoint)

1. Theoretically consistent
   e.g. no ghost instability
2. Experimentally viable
   solar system / table top experiments
3. Predictable
   e.g. protected by symmetry

Some examples

I. Ghost condensation
   IR modification of gravity
   motivation: dark energy/matter
II. Nonlinear massive gravity
   IR modification of gravity
   motivation: "Can graviton have mass?"
III. Horava-Lifshitz gravity
   UV modification of gravity
   motivation: quantum gravity
IV. Superstring theory
   UV modification of gravity
   motivation: quantum gravity, unified theory

A motivation for IR modification

- Gravity at long distances

  Flattening galaxy rotation curves

  extra gravity

  Dimming supernovae

  accelerating universe

A motivation for IR modification

- Gravity at long distances
  Flattening galaxy rotation curves
  extra gravity
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- Usual explanation: new forms of matter (DARK MATTER) and energy (DARK ENERGY).

Dark component in the solar system?

Precession of perihelion
observed in 1800's…

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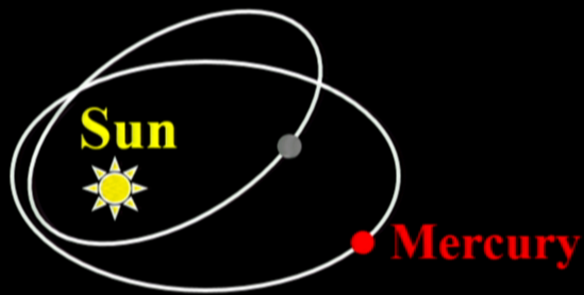
which people tried to explain with a "dark planet", Vulcan,



But the right answer wasn't "dark planet", it was "change gravity" from Newton to GR.

Can we change gravity in IR?

- **Change Theory?**

  Massive gravity Fierz-Pauli 1939

  DGP model Dvali-Gabadadze-Porrati 2000

Can we change gravity in IR?

- **Change Theory?**
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- **Change State?**
  Higgs phase of gravity
  The simplest: Ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004.

Linear massive gravity (Fierz-Pauli 1939)

- Simple question: Can spin-2 field have mass?
- $L = L_{EH}[h] + m_g^2[\eta^{\mu\rho}\eta^{\nu\sigma}h_{\mu\nu}h_{\rho\sigma} - (\eta^{\mu\nu}h_{\mu\nu})^2]$
  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- Unique linear theory without ghosts
- Broken diffeomorphism
  → no momentum constraint
  → 5 d.o.f. (2 tensor + 2 vector + 1 scalar)

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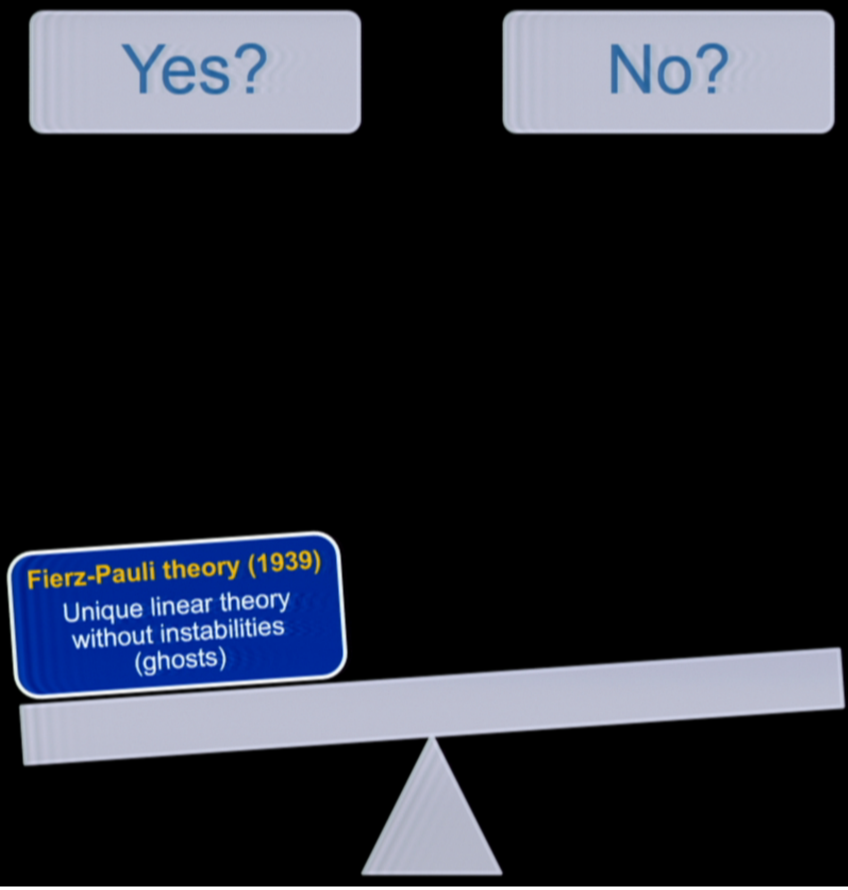
Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy



vDVZ vs Vainshtein

- van Dam-Veltman-Zhakharov (1970)
  Massless limit ≠ Massless theory = GR
  5 d.o.f remain → PPN parameter $\gamma = ½ \neq 1$
- Vainshtein (1972)
  Linear theory breaks down in the limit.
  Nonlinear analysis shows continuity and GR is recovered @ $r < r_V = (r_g/m_g^4)^{1/5}$.
  Continuity is not uniform w.r.t. distance.

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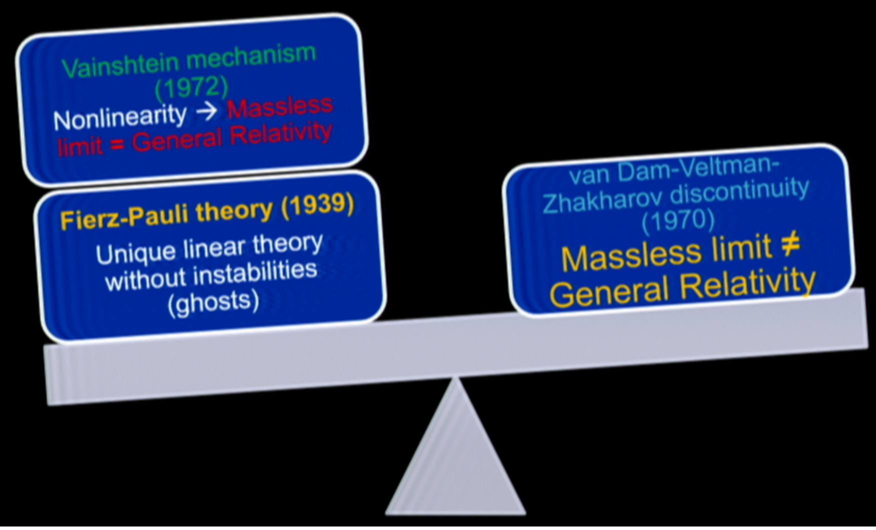
Yes?

No?



Naïve nonlinear theory and BD ghost

- FP theory with $\eta^{\mu\nu} \rightarrow g^{\mu\nu}$
  $L = L_{EH}[h] + m_g{}^2[g^{\mu\rho}g^{\nu\sigma}h_{\mu\nu}h_{\rho\sigma}-(g^{\mu\nu}h_{\mu\nu})^2]$
  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- Vainshtein effect (1972)
- Boulware-Deser ghost (1972)
  No Hamiltonian constraint @ nonlinear level
  $\rightarrow$ 6 d.o.f. = 5 d.o.f. of massive spin-2 + 1 ghost

Massive gravity: history

Simple question: Can graviton have mass?
May lead to acceleration without dark energy

Yes?	No?
Vainshtein mechanism (1972) Nonlinearity → Massless limit = General Relativity	Boulware-Deser ghost (1972) 6^{th} d.o.f.@Nonlinear level → Instability (ghost)
Fierz-Pauli theory (1939) Unique linear theory without instabilities (ghosts)	van Dam-Veltman-Zhakharov discontinuity (1970) Massless limit ≠ General Relativity

Nonlinear massive gravity

de Rham, Gabadadze 2010
de Rham, Gabadadze & Tolley 2010

- First example of fully nonlinear massive gravity without BD ghost since 1972!
- Purely classical
- Properties of 5 d.o.f. depend on background

- **4 scalar fields ϕ^a (a=0,1,2,3)**
- **Poincare symmetry in the field space:**

$$\phi^a \to \phi^a + c^a, \quad \phi^a \to \Lambda^a_b \phi^b$$

$$\to \quad \boxed{f_{\mu\nu} \equiv \eta_{ab}\partial_\mu \phi^a \partial_\nu \phi^b}$$

Pullback of
Minkowski metric in field space
to spacetime

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Systematic resummation

de Rham, Gabadadze & Tolley 2010

$$I_{mass}[g_{\mu\nu}, f_{\mu\nu}] = M_{Pl}^2 m_g^2 \int d^4x \sqrt{-g}\,(\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4)$$

$$\boxed{f_{\mu\nu} \equiv \eta_{ab}\partial_\mu \phi^a \partial_\nu \phi^b} \qquad \boxed{\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \left(\sqrt{g^{-1}f}\right)^\mu_{\ \nu}}$$

$$\mathcal{L}_2 = \frac{1}{2}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)$$

$$[\mathcal{A}] \equiv Tr\mathcal{A}$$

$$\mathcal{L}_3 = \frac{1}{6}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right)$$

$$\mathcal{L}_4 = \frac{1}{24}\left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]\right)$$

No helicity-0 ghost, i.e. no BD ghost, in decoupling limit

$$\mathcal{K}_{\mu\nu} = \partial_\mu \partial_\nu \pi \;\Longrightarrow\; \mathcal{L}_{2,3,4} = (\text{total derivative})$$

No BD ghost away from decoupling limit (Hassan&Rosen)

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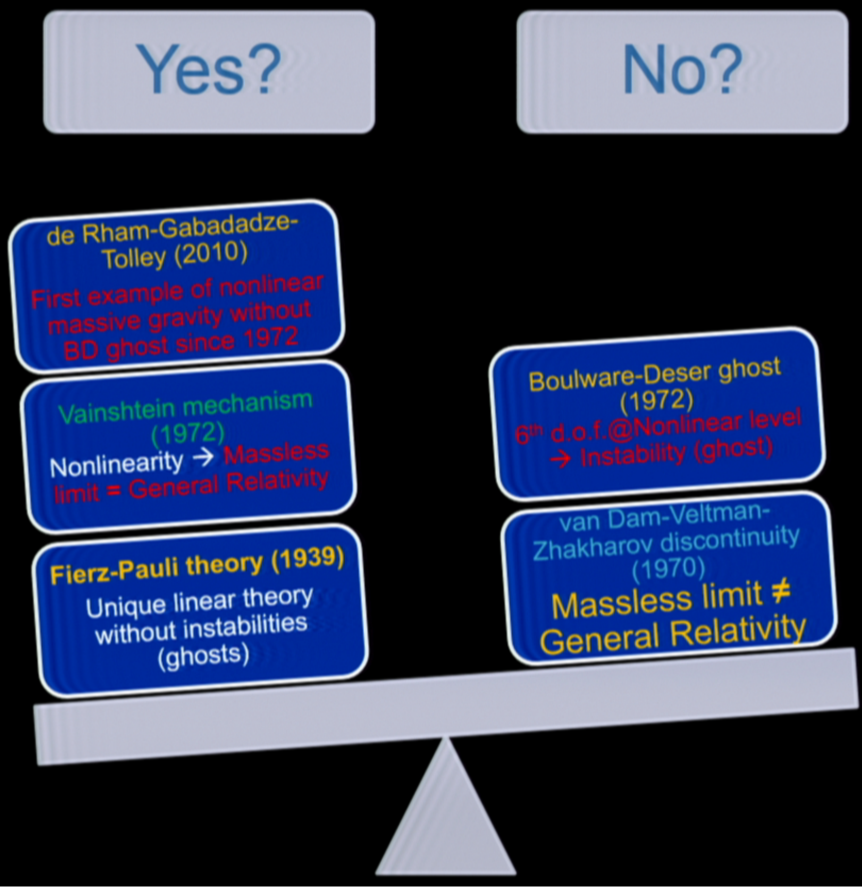

No FLRW universe?

D'Amico, de Rham, Dubovsky, Gabadadze, Pirtshalava, Tolley (2011)

- Flat FLRW ansatz in "Unitary gauge"

  $g_{\mu\nu}dx^{\mu}dx^{\nu} = -N^2(t)dt^2 + a^2(t)(dx^2+dy^2+dz^2)$

  $\phi^a = x^a \;\Rightarrow\; f_{\mu\nu} = \eta_{\mu\nu}$

- Bianchi "identity" → a(t) = const.

  c.f. $\nabla^{\mu}\left(\frac{2}{\sqrt{-g}}\frac{\delta I}{\delta g^{\mu\nu}}\right) = \frac{1}{\sqrt{-g}}\frac{\delta I_g}{\delta \phi^a}\partial_{\nu}\phi^a$

  → no non-trivial flat FLRW cosmology

- "Our conclusions on the absence of the homogeneous and isotropic solutions do not change if we allow for a more general maximally symmetric 3-space"

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Massive gravity: history
Simple question: Can graviton have mass?
May lead to acceleration without dark energy
Yes?
No?
Consistent Theory
found in 2010 but
No Viable Cosmology?
First example of nonlinear
massive gravity without
FRW (homogeneous
isotropic) universe!
(1972)
Nonlinearity →
(1972)
Fierz-Pauli theory (1939)
Unique linear theory
without instabilities
(ghosts)
Zhakharov discontinuity
(1970)
Massless limit ≠
General Relativity

Open FLRW solutions

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1109.3845 [hep-th]

- $f_{\mu\upsilon}$ spontaneously breaks diffeo.
- Both $g_{\mu\upsilon}$ and $f_{\mu\upsilon}$ must respect FLRW symmetry
- Need FLRW coordinates of Minkowski $f_{\mu\upsilon}$
- No closed FLRW chart
- Open FLRW ansatz

$$
\begin{aligned}
\phi^0 &= f(t)\sqrt{1+|K|(x^2+y^2+z^2)},\\
\phi^1 &= \sqrt{|K|}f(t)x,\\
\phi^2 &= \sqrt{|K|}f(t)y,\\
\phi^3 &= \sqrt{|K|}f(t)z.
\end{aligned}
$$

$$f_{\mu\nu}dx^\mu dx^\nu = -(\dot{f}(t))^2\,dt^2 + |K|\,(f(t))^2\,\Omega_{ij}(x^k)dx^i dx^j$$

$$
\begin{aligned}
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- EOM for ϕ^a (a=0,1,2,3)

$$(\dot{a}-\sqrt{|K|}N)\left[\left(3-\frac{2\sqrt{|K|}f}{a}\right)+\alpha_3\left(3-\frac{\sqrt{|K|}f}{a}\right)\left(1-\frac{\sqrt{|K|}f}{a}\right)+\alpha_4\left(1-\frac{\sqrt{|K|}f}{a}\right)^2\right]=0$$

- The first sol $\dot{a}=\sqrt{|K|}N$ implies $g_{\mu\upsilon}$ is Minkowski

  → we consider other solutions

$$f=\frac{a}{\sqrt{|K|}}X_\pm, \quad X_\pm \equiv \frac{1+2\alpha_3+\alpha_4 \pm \sqrt{1+\alpha_3+\alpha_3^2-\alpha_4}}{\alpha_3+\alpha_4}$$

- Latter solutions do not exist if K=0

- Metric EOM → self-acceleration

$$3H^2+\frac{3K}{a^2} = \Lambda_\pm + \frac{1}{M_{Pl}^2}\rho$$

$$\Lambda_\pm \equiv -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\alpha_3^2-3\alpha_4\right)\pm 2\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right]$$

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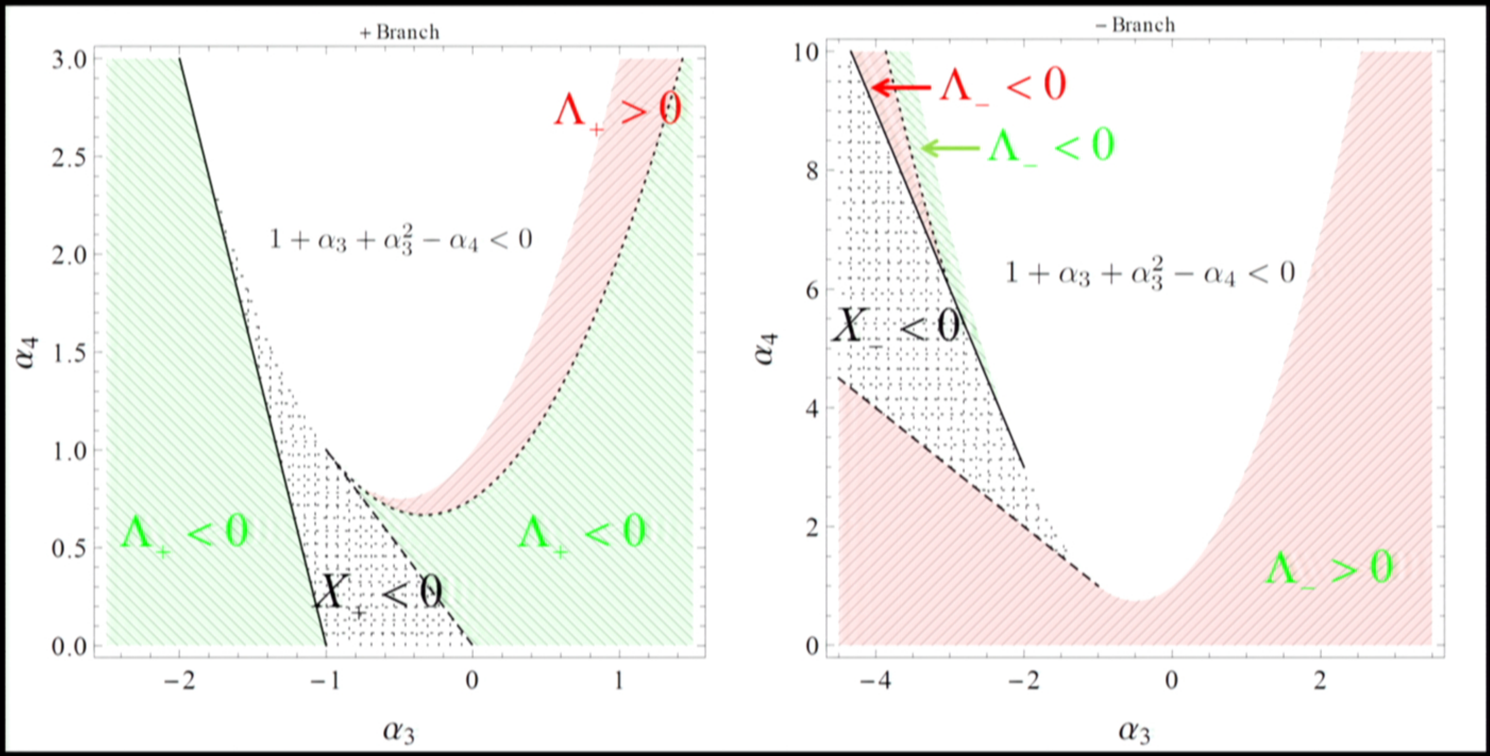
Self-acceleration



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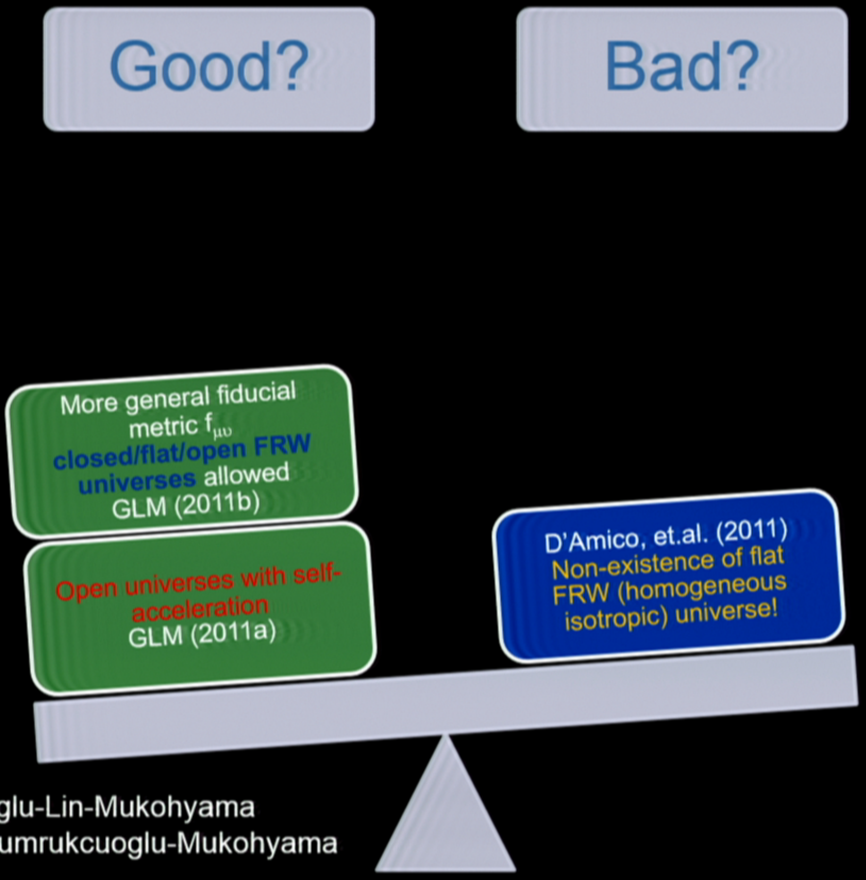
Our recent contributions

Cosmological solutions of nonlinear massive gravity



GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

Summary so far

- Nonlinear massive gravity
  free from BD ghost
- FLRW background
  No closed/flat universe
  Open universes with self-acceleration!
- More general fiducial metric $f_{\mu\nu}$
  closed/flat/open FLRW universes allowed
  Friedmann eq does not depend on $f_{\mu\nu}$
- Cosmological linear perturbations
  Scalar/vector sectors → same as in GR
  Tensor sector → time-dependent mass

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Nonlinear instability

DeFelice, Gumrukcuoglu, Mukohyama, arXiv: 1206.2080 [hep-th]

- de Sitter or FLRW fiducial metric
- Pure gravity + bare cc → FLRW sol = de Sitter
- Bianchi I universe with axisymmetry + linear perturbation (without decoupling limit)
- Small anisotropy expansion of Bianchi I + linear perturbation
  → nonlinear perturbation around flat FLRW
- Odd-sector:
  1 healthy mode + 1 healthy or ghosty mode
- Even-sector:
  2 healthy modes + 1 ghosty mode
- This is not BD ghost nor Higuchi ghost.

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Our recent contributions

Cosmological solutions of nonlinear massive gravity

Good?

Bad?

More general fiducial metric $f_{\mu\upsilon}$
closed/flat/open FRW universes allowed
GLM (2011b)

Open universes with self-acceleration
GLM (2011a)

NEW
Nonlinear instability of FRW solutions
DGM (2012)

D'Amico, et.al. (2011)
Non-existence of flat FRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama
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New class of cosmological solution

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1206.2723 [hep-th]
+ De Felice, arXiv: 1303.4154 [hep-th]

- Healthy regions with (relatively) large anisotropy
- Are there attractors in healthy region?
- Classification of fixed points
- Local stability analysis
- Global stability analysis

At attractors, physical metric is isotropic but fiducial metric is anisotropic.
→ Anisotropic FLRW universe!
statistical anisotropy expected
(suppressed by small m_g^2)

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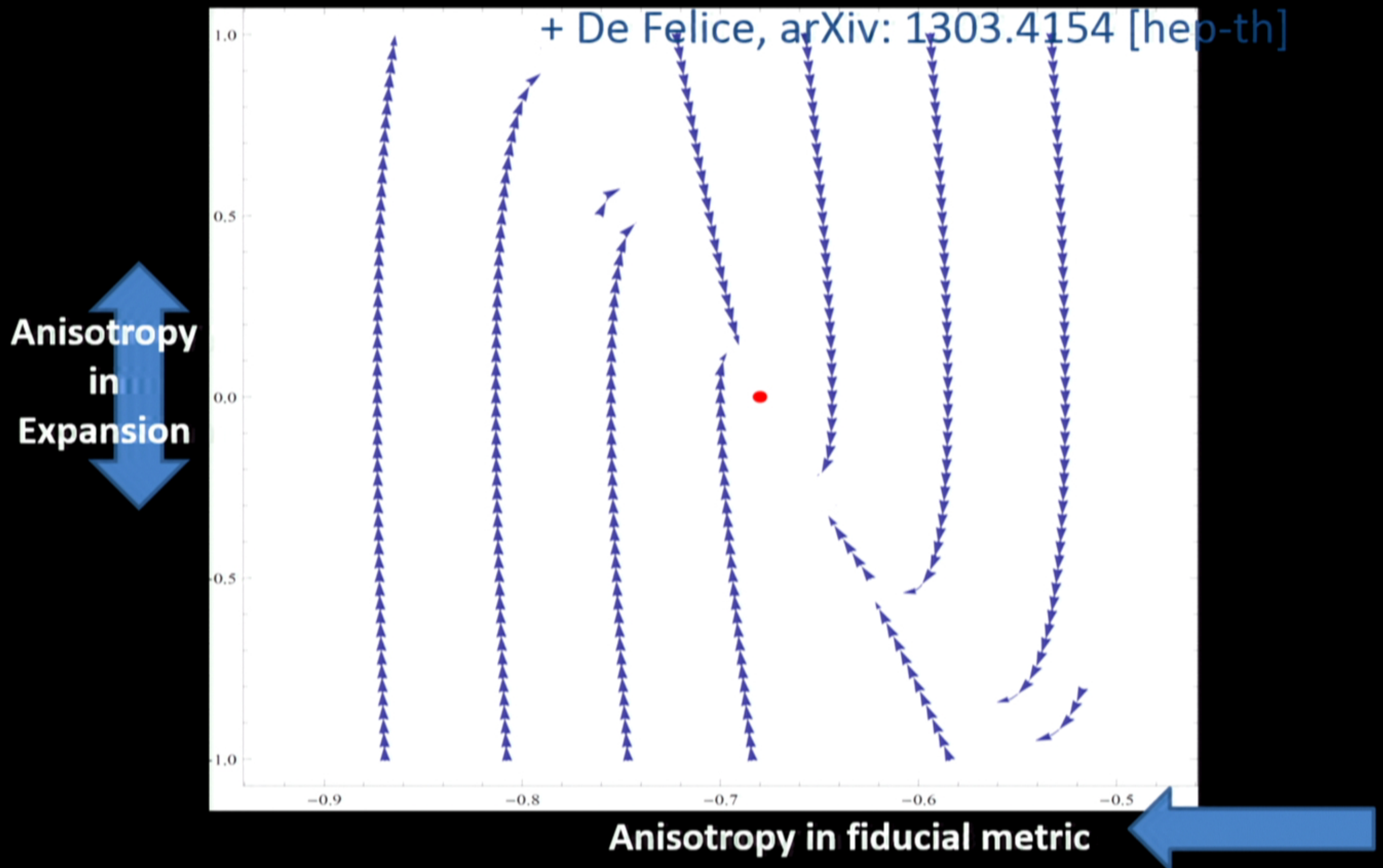

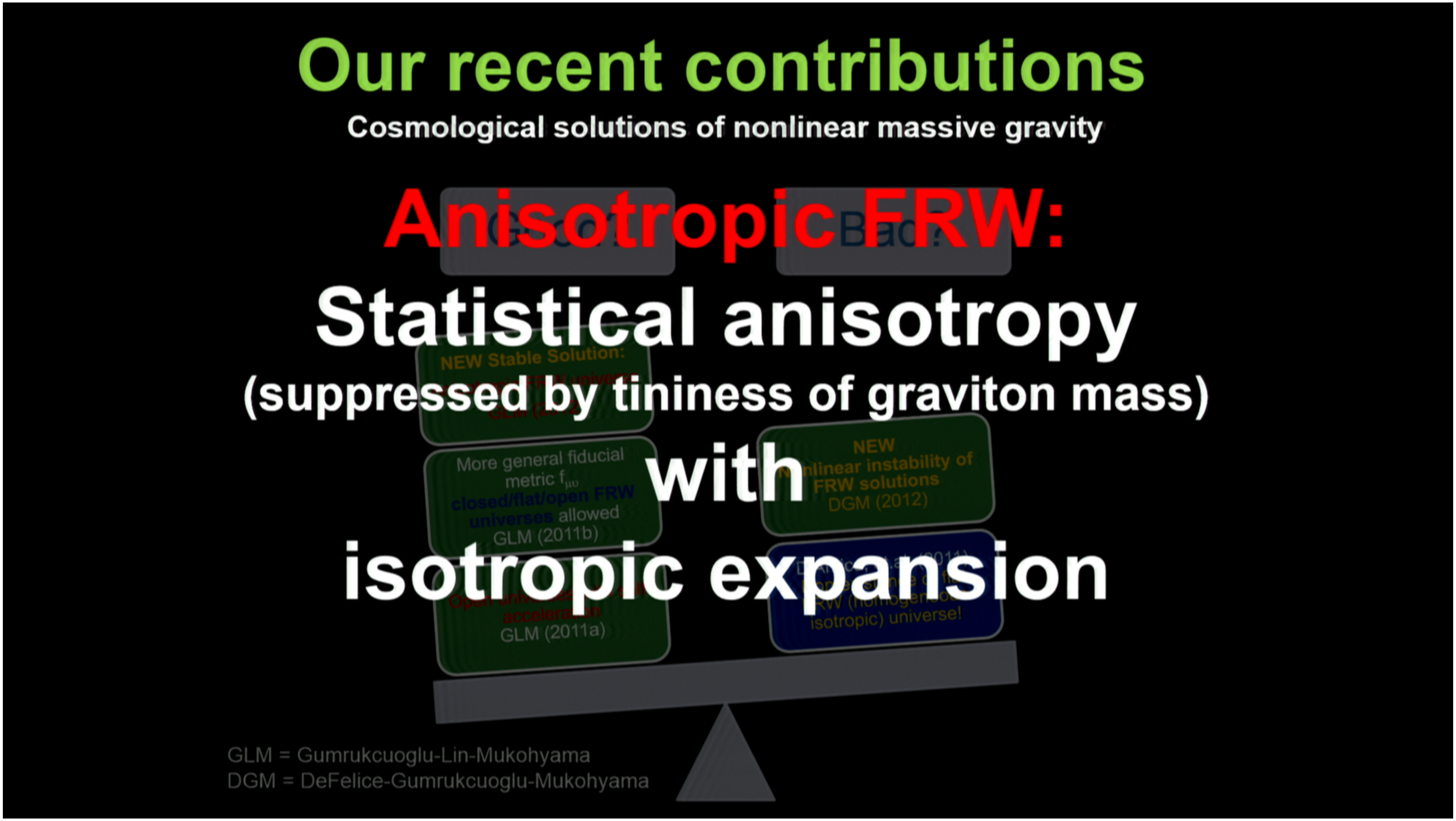

Our recent contributions
Cosmological solutions of nonlinear massive gravity
Anisotropic FRW:
Statistical anisotropy
(suppressed by tininess of graviton mass)
with
isotropic expansion
NEW Stable Solution:
More general fiducial
metric $f_{\mu\nu}$
closed/flat/open FRW
universes allowed
GLM (2011b)
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NEW
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Quasidilaton D'Amico, Gabadadze, Hui, Pirtskhalava, 2012

- New nonlinear instability [DeFelice, Gumrukcuoglu, Mukohyama 2012] → (i) new backgrounds, or (ii) extended theories
- Quasidilaton: scalar σ with global symmetry:

$$\sigma \to \sigma + \sigma_0 \qquad \phi^a \to e^{-\sigma_0/M_{\rm Pl}}\,\phi^a$$

- Action

$$S = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\Big[R - 2\Lambda - \frac{\omega}{M_{\rm Pl}^2}\partial_\mu\sigma\partial^\mu\sigma + 2m_g^2(\mathcal{L}_2 + \alpha_3\mathcal{L}_3 + \alpha_4\mathcal{L}_4)\Big]$$

$$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - e^{\sigma/M_{\rm Pl}}\left(\sqrt{g^{-1}f}\right)^\mu{}_\nu \qquad f_{\mu\nu} = \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$$

- Scaling solution = self-accelerating de Sitter (H = const > 0 with Λ = 0)

Stable extension of quasidilaton

arXiv: 1306.5502 [hep-th] /w A. De Felice

- Self-accelerating solution in the original quasidilaton theory has ghost instability
  [Gumrukcuoglu, Hinterbichler, Lin, Mukohyama, Trodden 2013; D'Amico, Gabadadze, Hui, Pirtskhalava 2013]
- Simple extension: $\boxed{f_{\mu\nu} \rightarrow \tilde{f}_{\mu\nu}}$

$$\tilde{f}_{\mu\nu} \equiv f_{\mu\nu} - \frac{\alpha_\sigma}{M_{\rm Pl}^2 m_g^2} e^{-2\sigma/M_{\rm Pl}} \partial_\mu\sigma\partial_\nu\sigma$$

- Self-accelerating solution is stable if

$$0 < \omega < 6 \qquad \boxed{X^2 < \frac{\alpha_\sigma H^2}{m_g^2} < r^2 X^2} \qquad X \equiv \frac{e^{\bar{\sigma}/M_{\rm Pl}}}{a}$$

$$M_{\rm GW}^2 \equiv \frac{(r-1)X^3 m_g^2}{X-1} + \frac{\omega H^2(rX + r - 2)}{(X-1)(r-1)} > 0 \qquad r \equiv \frac{n}{N} a$$

Stable extension of quasidilaton

arXiv: 1306.5502 [hep-th] /w A. De Felice

- Self-accelerating solution in the original quasidilaton theory has ghost instability
  [Gumrukcuoglu, Hinterbichler, Lin, Mukohyama, Trodden 2013; D'Amico, Gabadadze, Hui, Pirtskhalava 2013]
- Simple extension: $\boxed{f_{\mu\nu} \rightarrow \tilde{f}_{\mu\nu}}$

$$\tilde{f}_{\mu\nu} \equiv f_{\mu\nu} - \frac{\alpha_\sigma}{M_{\rm Pl}^2 m_g^2} e^{-2\sigma/M_{\rm Pl}} \partial_\mu\sigma\partial_\nu\sigma$$

- Self-accelerating solution is stable if

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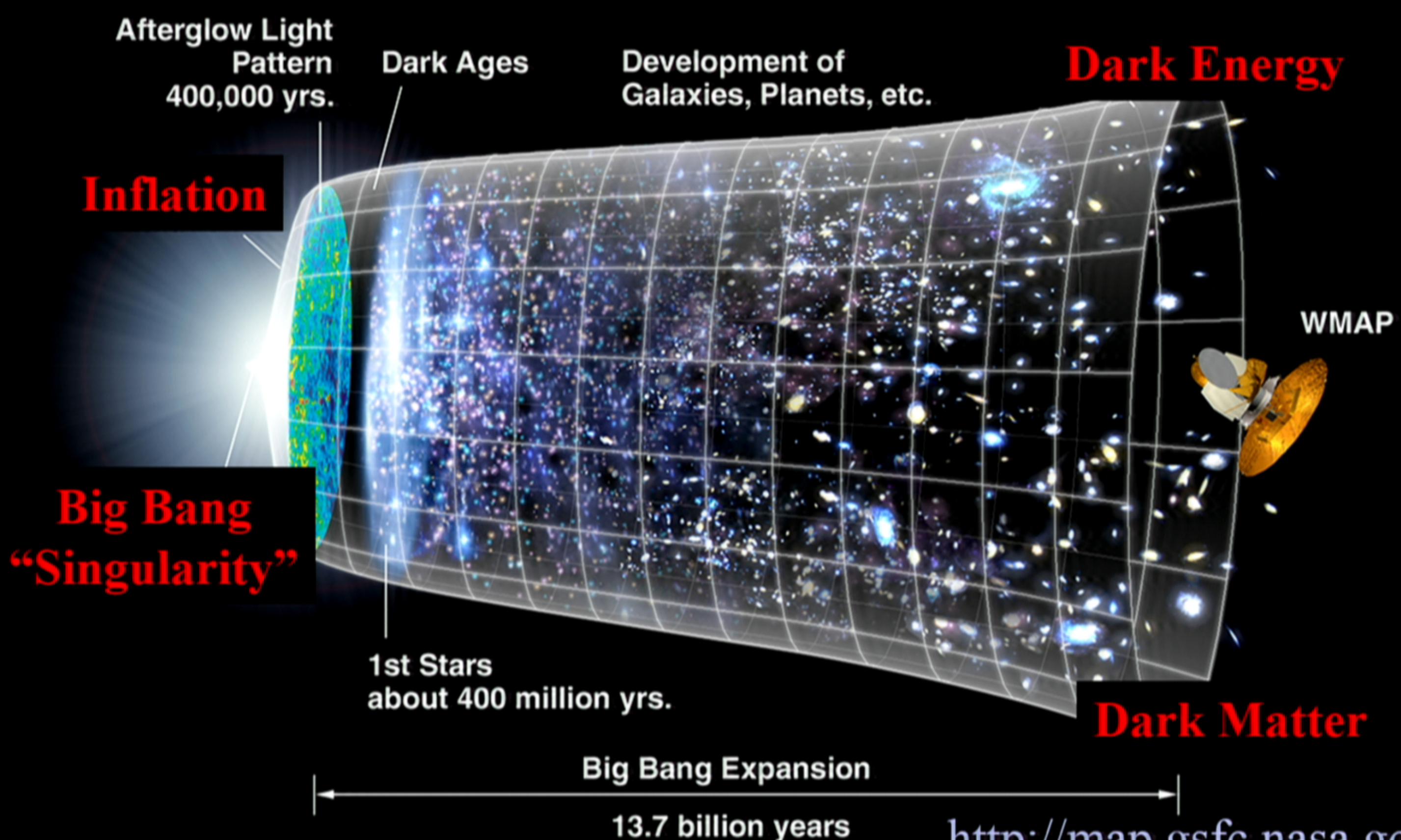
Summary

- Nonlinear massive gravity
  free from BD ghost
- FLRW background
  No closed/flat universe
  Open universes with self-acceleration!
- More general fiducial metric $f_{\mu\upsilon}$
  closed/flat/open FLRW universes allowed
  Friedmann eq does not depend on $f_{\mu\upsilon}$
- Cosmological linear perturbations
  Scalar/vector sectors → same as in GR
  Tensor sector → time-dependent mass
- All homogeneous and isotropic FLRW solutions have ghost
- New class of cosmological solution:
  anisotropic FLRW → statistical anisotropy
  (suppressed by small m_g^2)
  Analogue of Ghost Condensate!
- Extended quasidilaton: stable self-accelerating FLRW

Why alternative gravity theories?



http://map.gsfc.nasa.gov/

General fiducial metric

Appendix of Gumrukcuoglu, Lin, Mukohyama, arXiv: 1111.4107 [hep-th]

- Poincare symmetry in the field space
  → $$f_{\mu\nu} = (Minkowski)_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$$
- de Sitter symmetry in the field space
  → $$f_{\mu\nu} = (deSitter)_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$$
- FRW symmetry in the field space
  → $$f_{\mu\nu} = (FLRW)_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$$

Flat/closed/open FLRW cosmology allowed if "fiducial metric" $f_{\mu\nu}$ is de Sitter (or FRW)

→ **Friedmann equation with the same effective cc**

$$3H^2 + \frac{3K}{a^2} = \Lambda_\pm + \frac{1}{M_{Pl}^2}\rho$$

$$\Lambda_\pm \equiv -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\alpha_3^2-3\alpha_4\right) \pm 2\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right]$$