Title: Primordial non-Gaussianity in the CMB and large-scale structure

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Abstract:

Primordial non-Gaussianity in CMB and large-scale structure

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Perimeter, July 2013

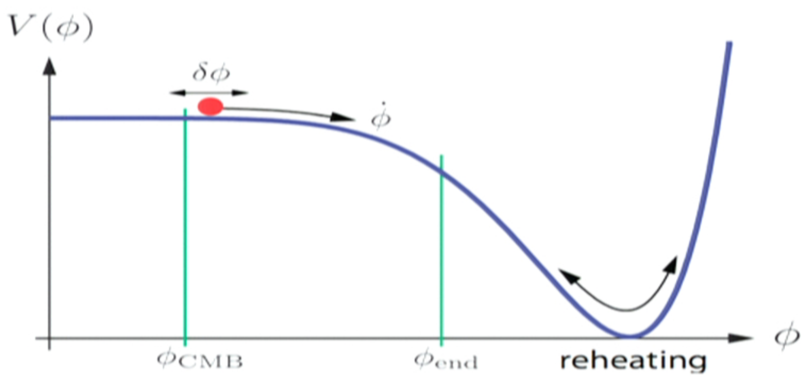
Inflation

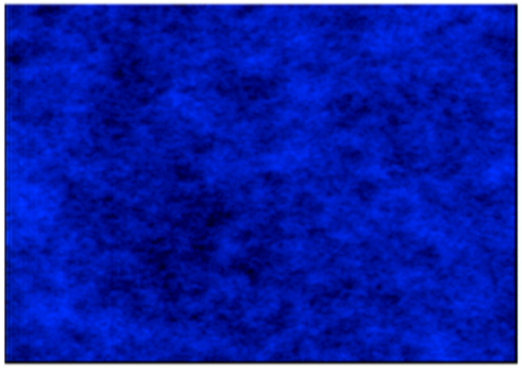
Consider single-field slow-roll inflation

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \right)$$

If we start with all modes $\phi_{\mathbf{k}}$ in their ground state (Bunch-Davies vacuum), fluctuations are generated at horizon crossing $k \approx aH$

At the end of inflation, we get an adiabatic curvature perturbation with power-law spectrum $P_\zeta(k) \propto k^{n_s - 4}$





Inflation

Single-field slow roll:

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \right)$$

To an excellent approximation ($f_{NL} = \mathcal{O}(10^{-2})$), the curvature perturbation ζ at the end of inflation is Gaussian

This calculation assumes:

- Curvature perturbations are generated by the inflaton
- Standard kinetic term with minimal coupling to gravity
- No self-interactions of the inflaton, beyond those included in the slow-roll potential $V(\phi)$

Deviations from these assumptions can generate non-Gaussian ζ

A NG model: modulated reheating

Suppose that the decay rate Γ of the inflaton is not constant, but controlled by an auxiliary field σ :

$$\Gamma(\mathbf{x}) = \Gamma_0 + \Gamma_1\,\sigma(\mathbf{x}) + \Gamma_2\,\sigma(\mathbf{x})^2 + \cdots$$

This generates an adiabatic curvature fluctuation after the inflaton decays (regions which decay later undergo more expansion):

$$\zeta(\mathbf{x}) = A(\delta\sigma(\mathbf{x})) + B(\delta\sigma(\mathbf{x}))^2 + \cdots$$

Suppose σ is a Gaussian field (via the standard quantum mechanical mechanism). After a change of variables:

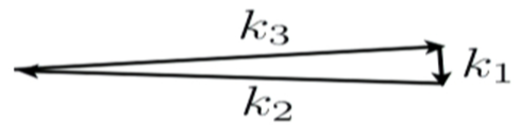
$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{NL}^{\rm loc} \zeta_G(\mathbf{x})^2 + \cdots$$

where ζ_G is a Gaussian field and $f_{NL}^{\rm loc}$ is a free parameter.

“Local” NG: $f_{NL}, g_{NL}, \tau_{NL}$

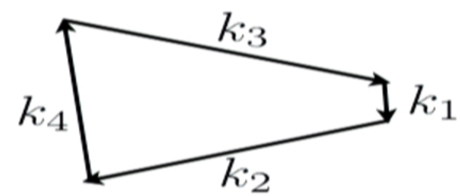
f_{NL} model: signal is in the squeezed 3-point function

$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{NL}^{\text{loc}} \zeta_G(\mathbf{x})^2$$



$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = \frac{6}{5} f_{NL} P_\zeta(k_1) P_\zeta(k_2) + 2 \text{ perm.}$$

g_{NL} model: signal is in the squeezed 4-point function



$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{9}{25} g_{NL} \zeta_G(\mathbf{x})^3$$

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle = \frac{54}{25} g_{NL} P_\zeta(k_1) P_\zeta(k_2) P_\zeta(k_3) + 4 \text{ perm.}$$

"Local" NG: $f_{NL}, g_{NL}, \tau_{NL}$

3-point and 4-point functions in the model $\zeta = \zeta_G + \frac{3}{5} f_{NL} \zeta_G^2$:

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = \frac{6}{5} f_{NL} P_\zeta(k_1) P_\zeta(k_2) + 2 \text{ perm.}$$

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle = \frac{36}{25} f_{NL}^2 P_\zeta(k_1) P_\zeta(k_2) P_\zeta(|\mathbf{k}_1 + \mathbf{k}_3|) + 12 \text{ perm.}$$

Generalization: allow coefficient of 4-point function to be an independent parameter τ_{NL}

"Local" NG: $f_{NL}, g_{NL}, \tau_{NL}$

"τ_{NL} model":

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = \frac{6}{5} f_{NL} P_\zeta(k_1) P_\zeta(k_2) + 2 \text{ perm.}$$

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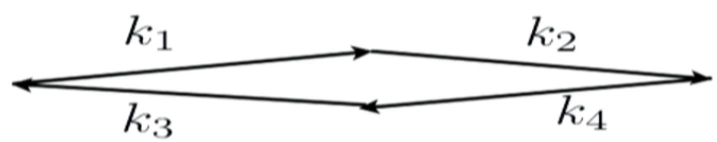
"f_{NL} model" is the special case $\tau_{NL} = \frac{36}{25} f_{NL}^2$

Example showing that f_{NL}, τ_{NL} can generally be independent:

$\zeta = \zeta_G + \epsilon \zeta_G \phi_G$ (ζ_G, ϕ_G independent Gaussian fields)

In this model, $f_{NL} = 0$ and $\tau_{NL} = \epsilon^2$

τ_{NL} signal is in
"collapsed quadrilaterals"



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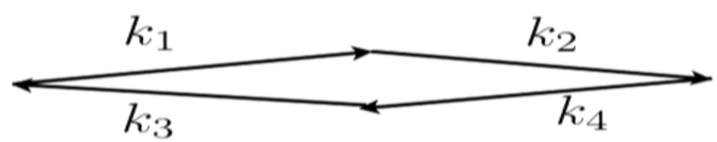
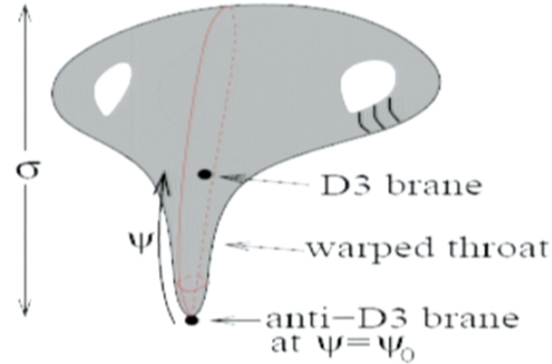


“Non-local” NG: DBI inflation

String-motivated model of inflation (Alishahiha, Silverstein & Tong)

$$\mathcal{L} = -\frac{1}{g_s}\left(\frac{\sqrt{1+f(\phi)(\partial\phi)^2}}{f(\phi)} + V(\phi)\right)$$



After a suitable change of variables, the effective action can be approximated as a massless scalar with a $\dot{\sigma}^3$ interaction

$$S = \frac{1}{2}\int d\tau\, d^3x\, a(\tau)^2 \left[\left(\frac{\partial\sigma}{\partial\tau}\right)^2 - (\partial_i\sigma)^2\right] + f\, a(\tau)\left(\frac{\partial\sigma}{\partial\tau}\right)^3$$

small coupling constant

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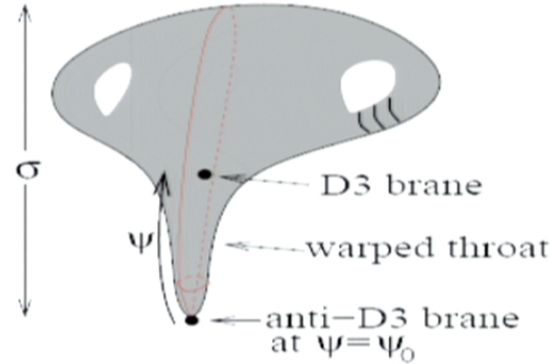

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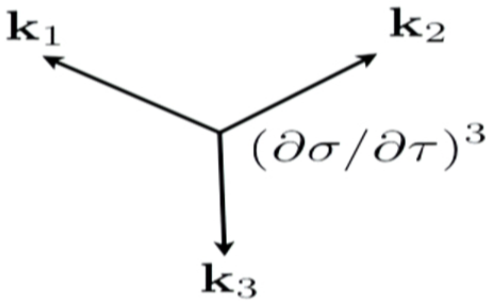
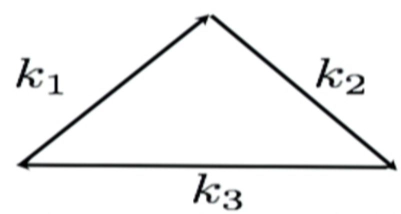
To first order in f, non-Gaussianity shows up in the 3-point function



$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle \propto f\int_{-\infty}^{0} d\tau\, \frac{\tau^2 e^{(k_1+k_2+k_3)\tau}}{k_1k_2k_3}$$
$$= \frac{2f}{k_1k_2k_3(k_1+k_2+k_3)^3}$$

Signal-to-noise comes from equilateral triangles

Cosmologists' terminology: $f = f_{NL}^{\text{equilateral}}$



Non-local NG: DBI inflation

DBI example:

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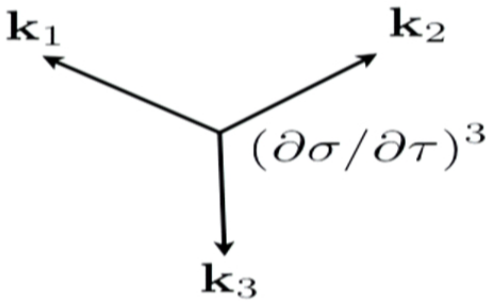

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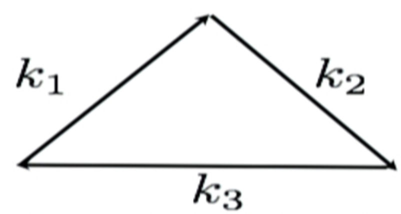


EFT of inflation

π = Goldstone boson of spontaneously broken time translations

1-1 correspondence between operators in S_π and f_{NL}-like parameters
(Degree-N operator shows up in N-point CMB correlation function)

$$S_\pi = \int d^4x\,\sqrt{-g}(-\dot{H}M_{\rm pl}^2)\Bigg[\frac{\dot{\pi}^2}{c_s^2} - \frac{(\partial_i\pi)^2}{a^2}$$

$$+\frac{A}{c_s^2}\pi_t^3 + \frac{1-c_s^2}{c_s^2}\frac{\pi_t(\partial_i\pi)^2}{a^2}$$

Equilateral+orthogonal 3-point shapes
(Senatore, KMS & Zaldarriaga 2009)

$$+B\pi_{ttt}^3 + C\pi_{ttt}\pi_{ijk}^2 + \cdots$$

Higher-derivative 3-point shapes
(Behbahani, Mirbabayi, Senatore & KMS to appear)

$$+D\dot{\pi}^4 + E\dot{\pi}^2(\partial_i\pi)^2 + F(\partial_i\pi)^2(\partial_j\pi)^2 + \cdots$$

4-point shapes
(Senatore & Zaldarriaga 2009)

$$+\frac{1}{2}\dot{\sigma}^2 - \frac{1}{2}(\partial_i\sigma)^2 - \frac{m^2}{2}\sigma^2 + \rho\dot{\pi}\sigma + G\sigma^3\Bigg]$$

Quasi single-field inflation
(Chen & Wang 2009,
Baumann & Green 2011)

Outline

1. Non-Gaussian models

2. CMB data analysis

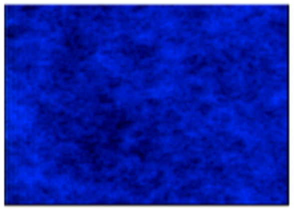
3. Large-scale structure

CMB data analysis

Degree-N operator $\mathcal{O}$ (e.g. $\mathcal{O} = \dot{\pi}^3$ or $\mathcal{O} = \dot{\pi}^4$)

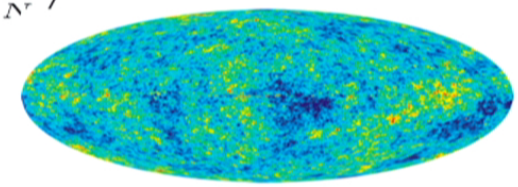
↓

Curvature N-point function $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \cdots \zeta_{\mathbf{k}_N} \rangle$



↓

CMB N-point function $\langle a_{\ell_1 m_1} a_{\ell_2 m_2} \cdots a_{\ell_N m_N} \rangle$



↓

CMB estimator

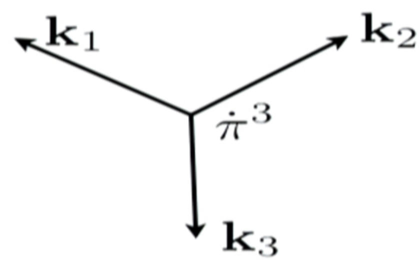
$$\mathcal{E} = \sum_{\ell_i m_i} \langle a_{\ell_1 m_1} a_{\ell_2 m_2} \cdots a_{\ell_N m_N} \rangle \prod_{i=1}^{N} \tilde{a}_{\ell_i m_i} + \cdots$$

Computational difficulties

Example: $\dot{\pi}^3$ interaction

Computing the curvature 3-point function is straightforward....

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \propto \int_{-\infty}^{0} d\tau \, \frac{\tau^2 e^{(k_1+k_2+k_3)\tau}}{k_1 k_2 k_3}$$
$$= \frac{2}{k_1 k_2 k_3 (k_1 + k_2 + k_3)^3}$$



Computational difficulties

...but subsequent steps look intractable in full generality:

CMB three-point function: 4D oscillatory integral for each (ℓ_i, m_i)

$$\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle = \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \times \int dr\, dk_1\, dk_2\, dk_3 \left(\prod_{i=1}^{3} \frac{2k_i^2}{\pi} j_{\ell_i}(k_i r) \Delta_{\ell_i}(k_i) \right) \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle$$

CMB transfer function (computed numerically)

CMB estimator: number of terms in sum is $\mathcal{O}(\ell_{\max}^5)$

$$\mathcal{E} = \sum_{\ell_i m_i} \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle\, \tilde{a}_{\ell_1 m_1} \tilde{a}_{\ell_2 m_2} \tilde{a}_{\ell_3 m_3} + \cdots$$

observed CMB multiples (appropriately filtered)

Factorizability = computability

Suppose the curvature 3-point function is factorizable

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = f_1(k_1) f_2(k_2) f_3(k_3) + 5 \text{ perm.}$$

Define (and precompute) $\alpha_\ell^{(i)}(r) = \int \frac{2k^2\, dk}{\pi} f_i(k) j_\ell(kr)$

CMB three-point function is fast to compute:

$$\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle = \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \times \int dr\, r^2\, \alpha_{\ell_1}^{(1)}(r) \alpha_{\ell_2}^{(2)}(r) \alpha_{\ell_3}^{(3)}(r) + 5 \text{ perm.}$$

CMB estimator is fast to evaluate:

$$\mathcal{E} = \int r^2 dr \int d^2\mathbf{n} \prod_{i=1}^{3} \left(\sum_{\ell m} \alpha_\ell^{(i)} \tilde{a}_{\ell m} Y_{\ell m}(\mathbf{n}) \right)$$

Komatsu, Spergel & Wandelt 2003
Creminelli, Nicolis, Senatore, Tegmark & Zaldarriaga 2005
KMS & Zaldarriaga 2006

Making shapes factorizable

Two possibilities for making shape factorizable

e.g. $\dot{\pi}^3$ shape: $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = \dfrac{2}{k_1 k_2 k_3 (k_1 + k_2 + k_3)^3}$

1. approximate by a factorizable shape ("template shape")

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \approx \frac{-\sum_i k_i^3 + \sum_{i \neq j} k_i k_j^2 - 2k_1 k_2 k_3}{k_1^3 k_2^3 k_3^3}$$

equilateral template

2. perform an algebraic magic trick, e.g. find integral representation

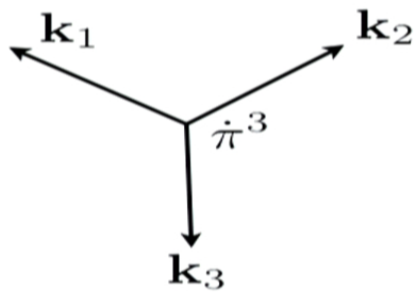
$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = \int_{-\infty}^{0} t^2 dt \left(\frac{e^{tk_1}}{k_1} \right) \left(\frac{e^{tk_2}}{k_2} \right) \left(\frac{e^{tk_3}}{k_3} \right)$$

KMS & Zaldarriaga 2006

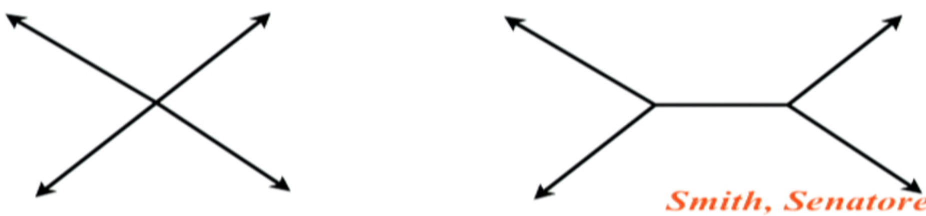
Factorizability + Feynman diagrams

Observation: for $\dot{\pi}^3$ shape, the integral representation is just undoing the last step of the Feynman diagram calculation

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \propto \int_{-\infty}^{0} d\tau \, \frac{\tau^2 e^{(k_1+k_2+k_3)\tau}}{k_1 k_2 k_3} = \frac{2}{k_1 k_2 k_3 (k_1 + k_2 + k_3)^3}$$



Generalizes to any tree diagram, e.g. 4-point estimators:



Smith, Senatore & Zaldarriaga, to appear

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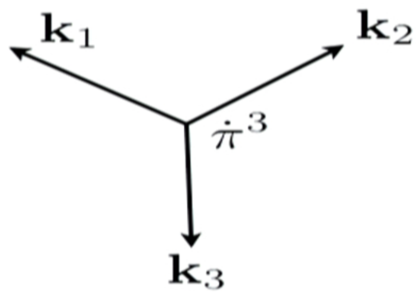

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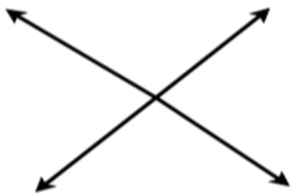

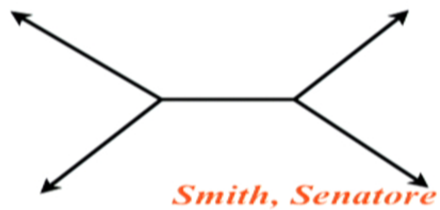

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Example: "resonant" model

(e.g. Pajer & Flauger 2010)

Just an example to illustrate the power of this method in finding a factorizable representation...

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \propto \frac{1}{k_1^2 k_2^2 k_3^2} \left[\sin\left(A \log \frac{k_1 + k_2 + k_3}{k_*} \right) + A^{-1} \sum_{i \neq j} \frac{k_i}{k_j} \cos\left(A \log \frac{k_1 + k_2 + k_3}{k_*} \right) \right]$$

Hard to see how this could ever be made factorizable, but going back to the physics gives the following factorizable representation!

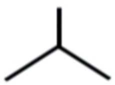
$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \propto \mathrm{Re} \left[\frac{e^{(1+iA)\log(i+iA)+iA\log k_*}}{A\Gamma(1+iA)} \int_{-\infty}^{\infty} dx\, e^{(1+iA)x} g(k_1, x) g(k_2, x) g(k_3, x) \right.$$
$$\left. \times \left(\left(1 + \frac{iA}{2} \right) \frac{1}{k_1 k_2^2 k_3^2} + \frac{1}{k_2^2 k_3^3} + \frac{1}{k_1 k_2 k_3^3} + 5 \text{ perm.} \right) \right]$$

$$g(k, x) = \exp[-(1 + iA) k e^x]$$

Factorizability + Feynman diagrams

Ultimate generalization of KSW construction: "Estimator" Feynman rules which go directly from the diagram to the CMB estimator

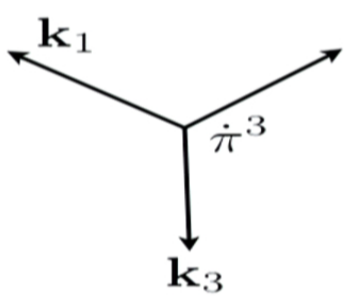
→ external line = CMB + harmonic-space factor $\alpha_\ell(r,t)\tilde{a}_{\ell m}$



vertex = $\int r^2 dr\, dt \left(\text{ N-way real-space product } \right)$

— internal line = harmonic-space factor $A_\ell(r,t,r',t')$

e.g.



$$\mathcal{E} = \int r^2 dr\, dt \left(\sum_{\ell m} \alpha_\ell(r,t)\tilde{a}_{\ell m} Y_{\ell m}(\mathbf{n}) \right)^3$$

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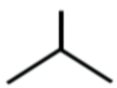
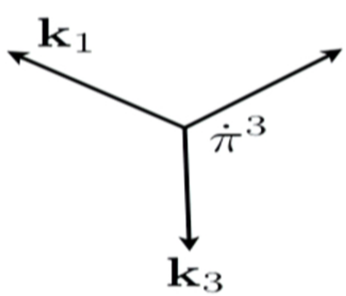


vertex = $\int r^2 dr\, dt \Big($ N-way real-space product $\Big)$

internal line = harmonic-space factor $A_\ell(r,t,r',t')$

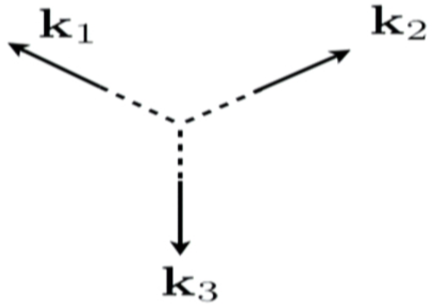
e.g.



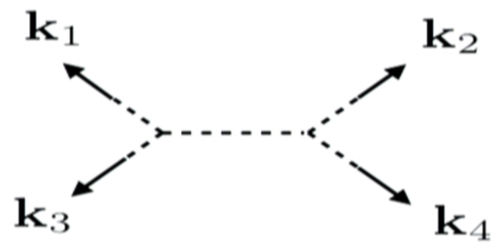
$$\mathcal{E} = \int r^2 dr\, dt \left(\sum_{\ell m} \alpha_\ell(r,t)\tilde{a}_{\ell m} Y_{\ell m}(\mathbf{n}) \right)^3$$

Example: quasi single field inflation

$$S_\pi = \int d^4x\,\sqrt{-g}\left(\frac{1}{2}(\partial\pi)^2 + \frac{1}{2}(\partial\sigma)^2 - \frac{M^2}{2}\sigma^2 + \rho\dot{\pi}\sigma - \frac{g}{3!}\sigma^3\right)$$



$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle \propto g\rho^3$$



$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle \propto g^2\rho^4$$

Because 3-point function and 4-point function depend on different combinations of parameters, either one can have larger signal-to-noise in different parts of the QSFI parameter space

Planck: current status

Blue = constraint reported in recent Planck papers
Red = no constraint reported

3-point

local { $f_{NL}^{\rm local}$

nonlocal { $f_{NL}^{\rm equil}$, $f_{NL}^{\rm orthog}$; Quasi-single field inflation ; Higher-derivative shapes

4-point

local { τ_{NL} ; g_{NL}

nonlocal { Single-field shapes $(\dot\pi^4, \dot\pi^2(\partial\pi)^2, (\partial\pi)^4)$; Quasi-single field inflation

In all cases, no deviation from Gaussian statistics is found (errors 1σ)

$$f_{NL}^{\rm local} = 2.7 \pm 5.8 \qquad f_{NL}^{\rm equil} = -42 \pm 75 \qquad f_{NL}^{\rm orthog} = -25 \pm 39$$

Planck: current status

The Planck non-Gaussianity constraints "put pressure" on certain models, but it's hard to rule out qualitative classes of models without getting to $f_{NL} \sim 1$

E.g. modulated reheating model or cyclic model are not ruled out by Planck measurement $f_{NL}^{\rm local} = 2.7 \pm 5.8$

There are interesting exceptions, e.g. DBI

$$-r^2 f_{NL}^{\rm equil} \gtrsim \mathcal{O}(10^1) \times (1 - n_s)$$

is consistent with Planck constraints, but if we get to $r \sim \mathcal{O}(10^{-2})$ without a detection, it would presumably be ruled out

Can we ever get to $f_{NL} \sim 1$?

Not in the CMB, but maybe in large-scale structure.

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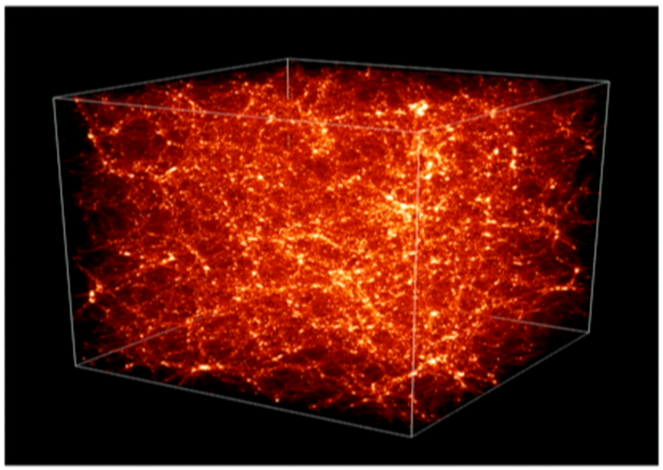
Outline

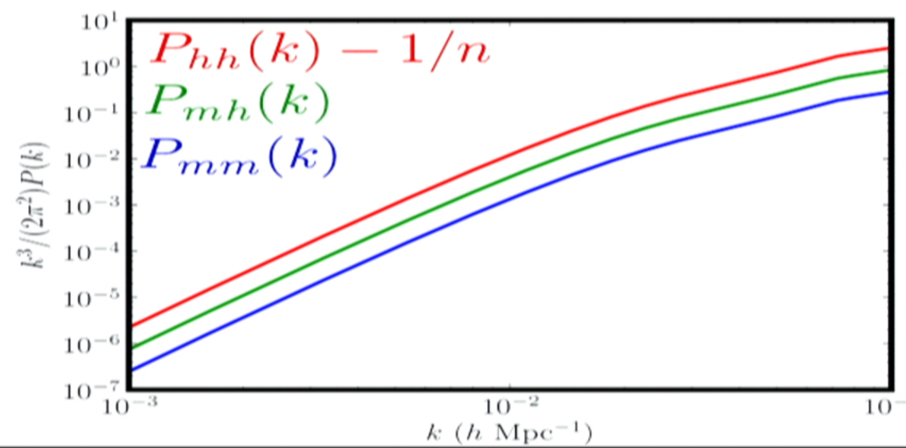
1. Non-Gaussian models
2. CMB data analysis
3. Large-scale structure

Large-scale structure

In a Gaussian cosmology, halo number density is proportional to the dark matter density on large scales: $\frac{\delta\rho_h}{\bar{\rho}_h} \approx b \frac{\delta\rho_m}{\bar{\rho}_m}$ b = "halo bias"

Matter-halo power spectrum P_{mh} and halo-halo power spectrum P_{hh} are proportional to matter power spectrum $P_{mm}(k)$





Large-scale structure

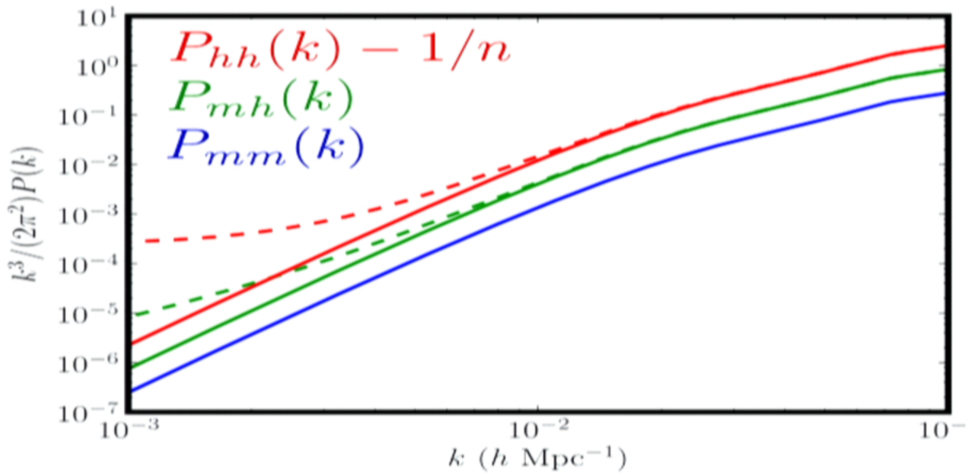
Local model: $\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{NL} \zeta_G(\mathbf{x})^2$

Non-Gaussian contribution to halo bias on large scales:

$$b(k) \approx b_0 + f_{NL} \frac{b_1}{(k/aH)^2} \quad \text{as } k \longrightarrow 0$$

(Dalal et al 2007)

$$P_{mh}(k) \approx b(k) P_{mm}(k)$$
$$P_{hh}(k) \approx b(k)^2 P_{mm}(k)$$



Constraints are ultimately better than the CMB

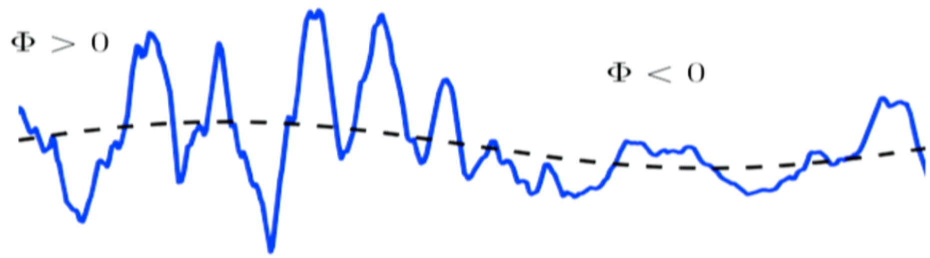
Planck: $\sigma(f_{NL}) = 5$

LSST: $\sigma(f_{NL}) \sim 1$

NG halo bias: interpretation

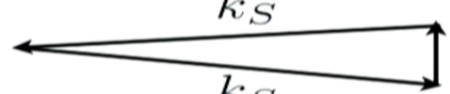
Three equivalent descriptions of an f_{NL} cosmology:

1. Correlation between long-wavelength mode and small-scale power



2. Three-point function is large in squeezed triangles

$$\langle \zeta_{\mathbf{k_L}} \zeta_{\mathbf{k_S}} \zeta_{\mathbf{k_S}} \rangle \propto f_{NL} \frac{1}{k_L^3 k_S^3}$$

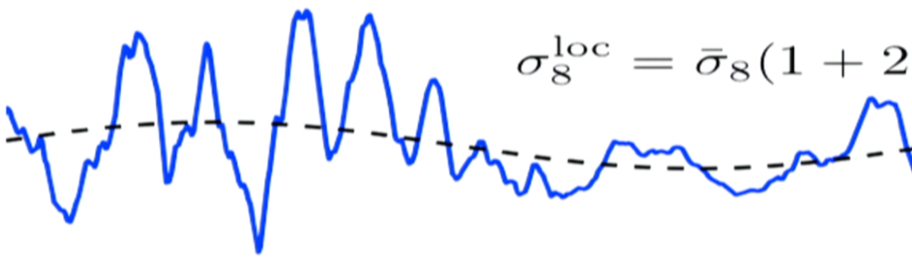


3. Locally measured fluctuation amplitude σ_8^{loc} near a point x depends on value of Newtonian potential $\Phi(\mathbf{x})$

$$\sigma_8^{\mathrm{loc}} = \bar{\sigma}_8 (1 + 2 f_{NL} \Phi)$$

NG halo bias: interpretation

This picture naturally leads to enhanced large-scale clustering



$$\sigma_8^{\rm loc} = \bar{\sigma}_8(1 + 2 f_{NL}\Phi)$$

$$\frac{\delta n_h}{\bar{n}_h} = \underbrace{\frac{\partial \log n_h}{\partial \log \rho_m}}_{b_0} \frac{\delta \rho_m}{\bar{\rho}_m} + \underbrace{\frac{\partial \log n_h}{\partial \log \sigma_8}}_{b_1/2} \frac{\delta \sigma_8}{\bar{\sigma}_8}$$

$$= b_0 \frac{\delta \rho_m}{\bar{\rho}_m} + b_1 f_{NL} \Phi$$

$$= \left(b_0 + b_1 \frac{f_{NL}}{(k/aH)^2} \right) \frac{\delta \rho_m}{\bar{\rho}_m}$$

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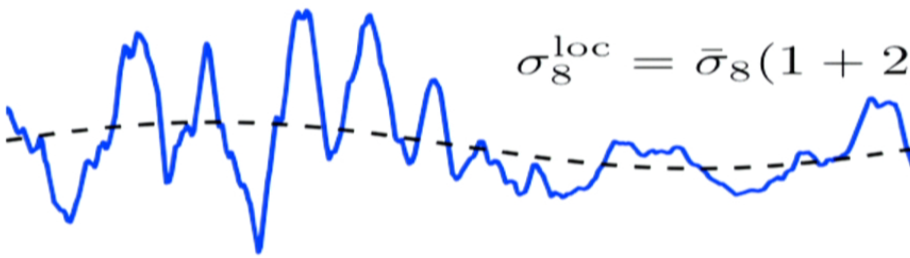

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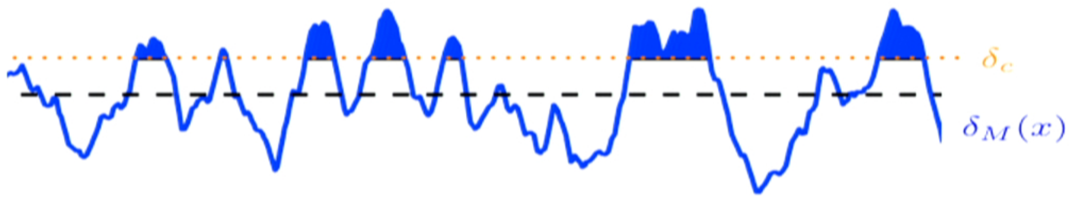
NG halo bias: general calculation

Let's try to generalize to an arbitrary NG model, parametrized by its cumulants $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \cdots \zeta_{\mathbf{k}_n} \rangle_c$

Model halos as follows ("barrier crossing model")

- Start with linearly evolved density field $\delta_{\rm lin}(\mathbf{k}, z) = \alpha(k, z)\zeta(\mathbf{k})$
- Smooth on mass scale M to obtain field δ_M
- Identify (halos of mass $\geq M$) $\Leftrightarrow$ (points where $\delta_M(x) \geq \delta_c$)



In this model, the halo field is a completely specified nonlinear function of the initial curvature ζ ; we can expand $P_{mh}(k), P_{hh}(k)$ in cumulants by a formal procedure (the Edgeworth expansion)

Baumann, Green, Ferraro & KMS 2013

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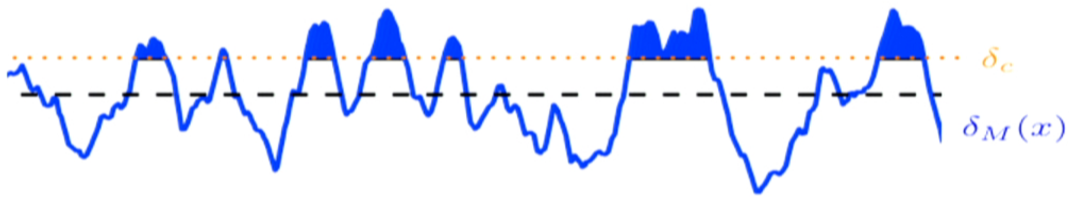

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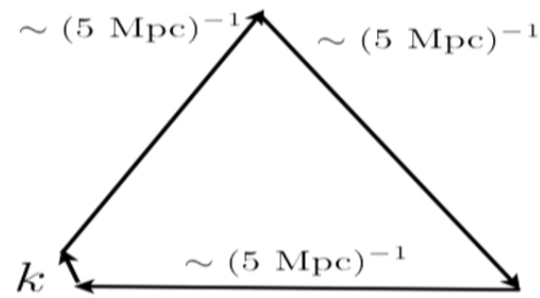
Baumann, Green, Ferraro & KMS 2013

NG halo bias: general expression

Schematic form:

$$P_{mh}(k) = \left(b_0 + \sum_{N=1}^{\infty} b_N f_{N+2}(k) \right) P_{mm}(k)$$

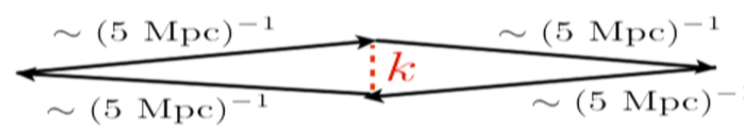
$$f_N(k) = \int_{\mathbf{k}_i} \langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}_1} \cdots \zeta_{\mathbf{k}_{N-1}} \rangle$$



"squeezed limit"

$$P_{hh}(k) = \left(b_0^2 + 2 \sum_{N=1}^{\infty} b_0 b_N f_{N+2}(k) + \sum_{MN} b_M b_N g_{M+1,N+1}(k) \right) P_{mm}(k)$$

$$g_{MN}(k) = \int_{\substack{\sum \mathbf{k}_i = \mathbf{k} \\ \sum \mathbf{k}'_j = -\mathbf{k}}} \langle \zeta_{\mathbf{k}_1} \cdots \zeta_{\mathbf{k}_M} \zeta_{\mathbf{k}'_1} \cdots \zeta_{\mathbf{k}'_N} \rangle$$



Baumann, Ferraro, Green & KMS 2012

"collapsed limit"

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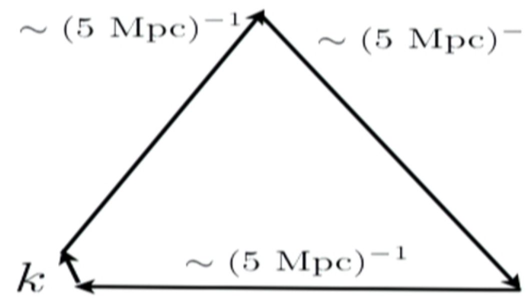

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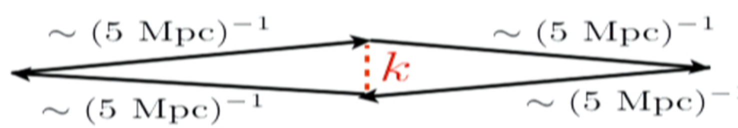

Baumann, Ferraro, Green & KMS 2012

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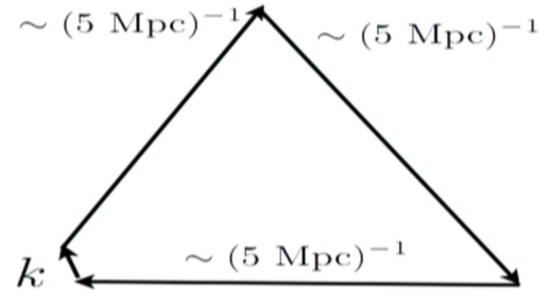

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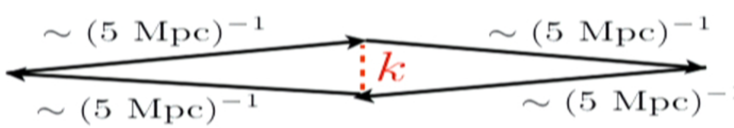


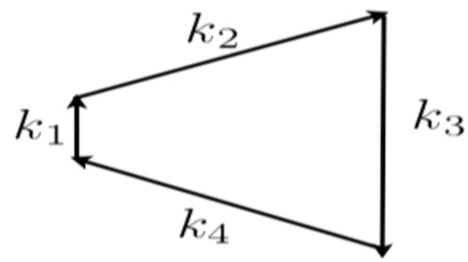
"collapsed limit"

Baumann, Ferraro, Green & KMS 2012

Example 1: bias from squeezed 4-point function

g_{NL} model:

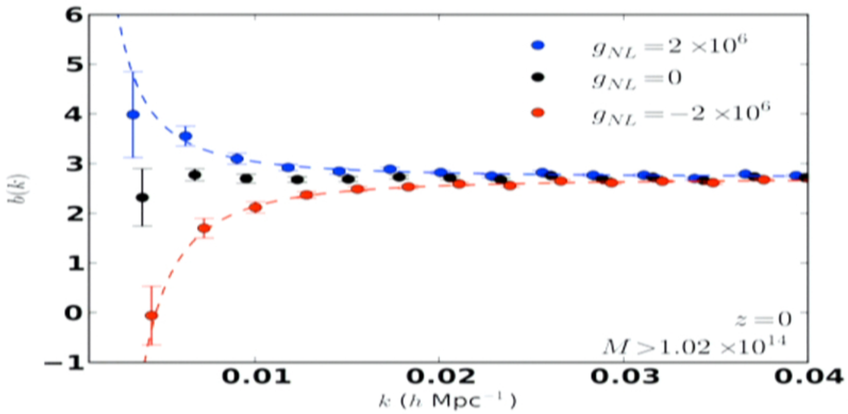
$$\zeta = \zeta_G + g_{NL}\zeta_G^3$$



Simple example of non-Gaussian model whose 4-point function $\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle$ is large in the squeezed limit $k_1 \longrightarrow 0$.

Scale dependence of bias is same as $f_{NL}^{\rm loc}$ model

Mass and redshift dependence are different, but hard in practice to discriminate $f_{NL}^{\rm loc}$ and g_{NL}



Smith, Ferraro & LoVerde 2011

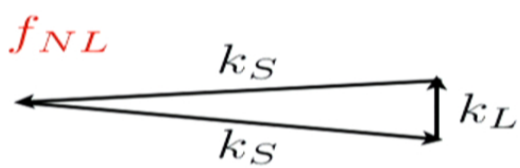
Example 2: stochastic bias from collapsed 4-pt

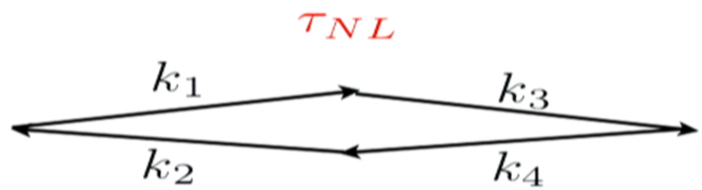
τ_{NL} model:

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = \frac{6}{5} f_{NL} (P_\zeta(k_1) P_\zeta(k_2) + 2 \text{ perm.})$$

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle = 2\tau_{NL} \Big[P_\zeta(k_1) P_\zeta(k_2) P_\zeta(|\mathbf{k}_1 + \mathbf{k}_3|) + 23 \text{ perm.} \Big]$$

In simple local model ($\zeta = \zeta_G + \frac{3}{5} f_{NL} \zeta_G^2$) one has
$\tau_{NL} = (\frac{6}{5} f_{NL})^2$ but in general f_{NL}, τ_{NL} can be independent





Example 2: stochastic bias from collapsed 4-pt

Our general expression predicts the following:

$$P_{mh}(k) = \left(b_0 + b_1 \frac{f_{NL}}{(k/aH)^2}\right) P_{mm}(k)$$

$$P_{hh}(k) = \left(b_0^2 + 2b_0 b_1 \frac{f_{NL}}{(k/aH)^2} + b_1^2 \frac{\frac{25}{36}\tau_{NL}}{(k/aH)^4}\right) P_{mm}(k)$$

Qualitative prediction of τ_{NL} model: "stochastic" halo bias

Matter and halo fields are not proportional on large scales

Gives some scope for distinguishing $f_{NL},\ \tau_{NL}$:

- Different bias values inferred from $P_{mh}(k),\ P_{hh}(k)$
- Different tracer populations are not 100% correlated
- Even with a single population, can separate k^{-2}, k^{-4} terms

Smith & LoVerde 2010

Example 3: quasi-single field inflation

$$S_\pi = \int d^4x\,\sqrt{-g}\left(\frac{1}{2}(\partial\pi)^2 + \frac{1}{2}(\partial\sigma)^2 - \frac{M^2}{2}\sigma^2 + \rho\dot{\pi}\sigma - \frac{g}{3!}\sigma^3\right)$$

Squeezed/collapsed limits (where $\alpha = \frac{3}{2} - \sqrt{\frac{9}{4} - \frac{M^2}{H^2}}$)

$$\lim_{k_L\to 0}\langle\zeta_{\mathbf{k}_L}\zeta_{\mathbf{k}_S}\zeta_{\mathbf{k}_S}\rangle \propto g\rho^3\left(\frac{1}{k_L^{3-\alpha}k_S^{3+\alpha}}\right)$$

$$\lim_{|\mathbf{k}_1+\mathbf{k}_2|\to 0}\langle\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle \propto g^2\rho^4\left(\frac{1}{|\mathbf{k}_1+\mathbf{k}_2|^{3-2\alpha}\;k_1^{3+\alpha}k_3^{3+\alpha}}\right)$$

Prediction: non-Gaussian bias has spectral index given by

$$b(k) = b_0 + b_1\frac{g\rho^3}{(k/aH)^{2-\alpha}}$$ ← exponent sensitive to mass M

Prediction: bias is mostly stochastic ("τ_{NL}" $= g^2\rho^4$ is enhanced relative to the square of "f_{NL}" $= g\rho^3$)

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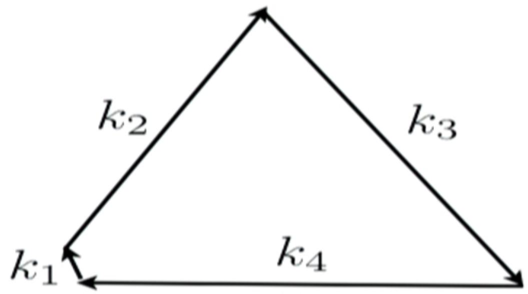
Prediction: bias is mostly stochastic ("τ_{NL}"$= g^2\rho^4$ is enhanced relative to the square of "f_{NL}"$= g\rho^3$)

LSS: general picture

Large-scale structure is not sensitive to all NG models, only those which have large squeezed/collapsed limits



"Squeezed" 4-point function

"Collapsed" (2+2)-point function

Comment: there are theorems which say that all squeezed/collapsed limits are unobservably small in single-field models of inflation

Single-field consistency relations

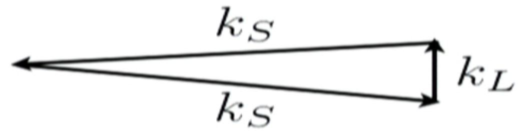
Suppose inflation is single-field (ζ is sourced only by the inflaton)

Simplest consistency relation gives squeezed limit of 3-point function

$$\lim_{k_L \to 0} \langle \zeta_{k_L} \zeta_{k_S} \zeta_{k_S} \rangle = (1 - n_s) P_\zeta(k_L) P_\zeta(k_S)$$

or equivalently

$$f_{NL} = \frac{5}{12}(1 - n_s) \approx 0.02$$



Physical picture: in single-field inflation, the value of the inflaton is the only physical clock. After a long-wavelength inflaton fluctuation exits the horizon, it is physically unobservable and only changes the scaling between physical distances and coordinate distances.

(Maldacena 2002; Creminelli & Zaldarriaga 2004)

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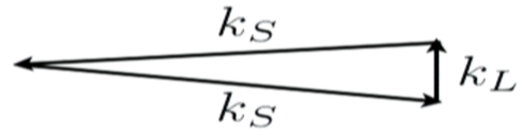

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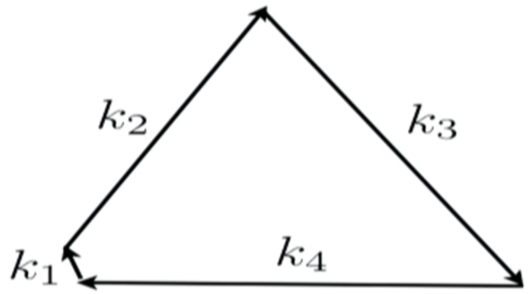
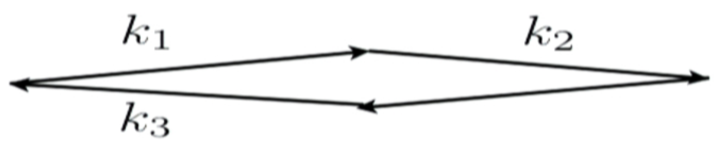
Single-field consistency relations

This has been generalized by many authors to show that in all single-field models, all squeezed/collapsed limits are unobservably small.



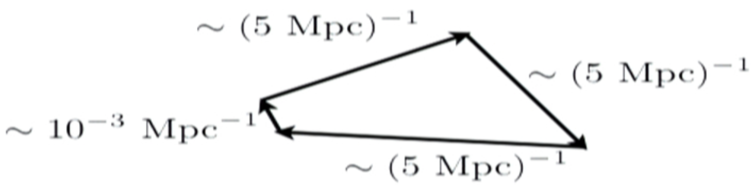


Therefore, large-scale clustering is a very sensitive probe of multifield inflation, but insensitive to single-field inflation

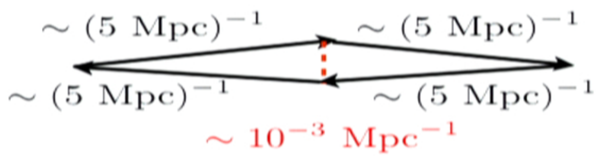
More on squeezed/collapsed limits

Large-scale structure constraints are best understood as precise tests of statistical homogeneity of the universe on large scales

Non-Gaussian models with large squeezed limits can be interpreted as large-scale inhomogeneity in statistics of small-scale modes, e.g:



large-scale correlation between density and small-scale skewness



large-scale inhomogeneity in small-scale power, uncorrelated to density ("stochastic")

An amusing side note

"Sugiyama-Yamaguchi inequality": $\tau_{NL} \geq \left(\frac{6}{5} f_{NL}\right)^2$

$$f_{NL} = \frac{5}{12} \lim_{k_1 \to 0} \frac{\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle}{P_\zeta(k_1) P_\zeta(k_2)}$$

$$\tau_{NL} = \frac{1}{4} \lim_{|\mathbf{k}_1 + \mathbf{k}_2| \to 0} \frac{\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle_c}{P_\zeta(k_1) P_\zeta(k_3) P_\zeta(|\mathbf{k}_1 + \mathbf{k}_2|)}$$

2011: conjectured to be general, proven in special cases

Intuitive interpretation of squeezed/collapsed limits makes general proof essentially trivial: if we define a field $\sigma(x)$ which represents "locally observed small-scale power near x", then the correlation coefficient between σ and ζ on large scales is

$$r = \frac{6}{5} \frac{f_{NL}}{\tau_{NL}^{1/2}}$$

so S-Y inequality is just $-1 \leq r \leq 1$

KMS, Loverde & Zaldarriaga 2011

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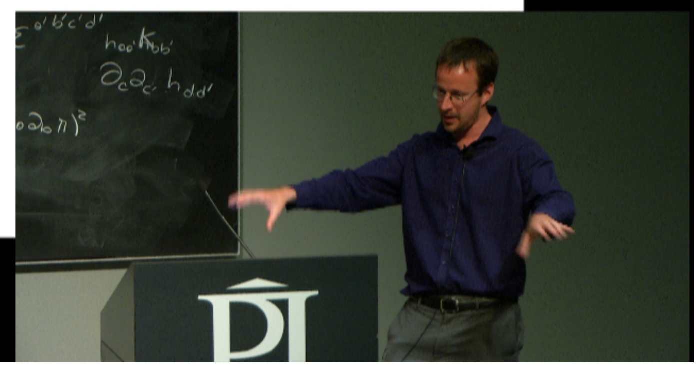

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Large-scale structure:

- Future constraints should eventually beat the CMB, but only for models with large squeezed/collapsed limits (=> multifield)
- Difficult to separate different N-point shapes (or different values of N) but some scope for discriminating models based on k-dependence of halo bias and stochastic vs non-stochastic bias

Factorizability + Feynman diagrams

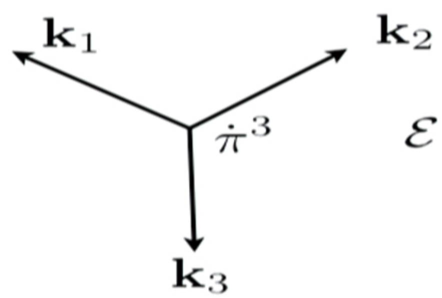
Ultimate generalization of KSW construction: "Estimator" Feynman rules which go directly from the diagram to the CMB estimator

external line = CMB + harmonic-space factor $\alpha_\ell(r,t)\tilde{a}_{\ell m}$

vertex = $\int r^2 dr\, dt \left(\text{ N-way real-space product } \right)$

internal line = harmonic-space factor $A_\ell(r,t,r',t')$

e.g.



$$\mathcal{E} = \int r^2 dr\, dt \left(\sum_{\ell m} \alpha_\ell(r,t)\tilde{a}_{\ell m} Y_{\ell m}(\mathbf{n}) \right)^3$$

Example: "resonant" model

(e.g. Pajer & Flauger 2010)

Just an example to illustrate the power of this method in finding a factorizable representation...

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \propto \frac{1}{k_1^2 k_2^2 k_3^2} \left[\sin\left(A \log \frac{k_1 + k_2 + k_3}{k_*} \right) + A^{-1} \sum_{i \neq j} \frac{k_i}{k_j} \cos\left(A \log \frac{k_1 + k_2 + k_3}{k_*} \right) \right]$$

Hard to see how this could ever be made factorizable, but going back to the physics gives the following factorizable representation!

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \propto \mathrm{Re} \left[\frac{e^{(1+iA)\log(i+iA) + iA \log k_*}}{A\Gamma(1+iA)} \int_{-\infty}^{\infty} dx \, e^{(1+iA)x} g(k_1, x) g(k_2, x) g(k_3, x) \right.$$
$$\left. \times \left(\left(1 + \frac{iA}{2}\right) \frac{1}{k_1 k_2^2 k_3^2} + \frac{1}{k_2^2 k_3^3} + \frac{1}{k_1 k_2 k_3^3} + 5 \text{ perm.} \right) \right]$$

$$g(k, x) = \exp[-(1 + iA) k e^x]$$

Large-scale structure

Local model: $\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{NL} \zeta_G(\mathbf{x})^2$

Non-Gaussian contribution to halo bias on large scales:

$$b(k) \approx b_0 + f_{NL} \frac{b_1}{(k/aH)^2} \quad \text{as } k \longrightarrow 0$$

(Dalal et al 2007)

$$P_{mh}(k) \approx b(k) P_{mm}(k)$$
$$P_{hh}(k) \approx b(k)^2 P_{mm}(k)$$

Constraints are ultimately better than the CMB

Planck: $\sigma(f_{NL}) = 5$

LSST: $\sigma(f_{NL}) \sim 1$