Title: From empirical practice to observables and the action principle

Date: Jun 25, 2013 03:30 PM

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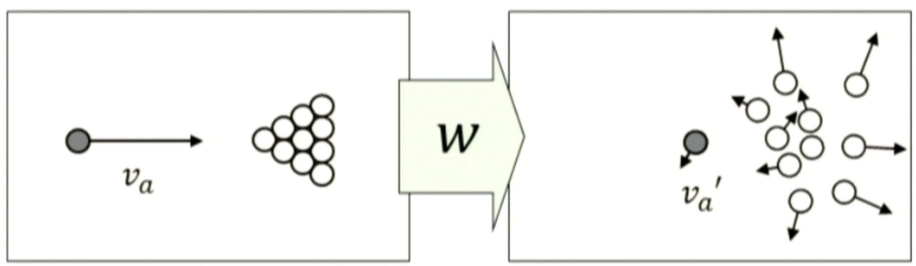
Abstract: Physical theories ought to be built up from colloquial notions such as â€™long bodiesâ€™, â€™energetic sourcesâ€™ etc. in terms of which one can define pre-theoretic ordering relations such as â€™longer thanâ€™, â€™more energetic thanâ€™. One of the questions addressed in previous work is how to make the transition from these pre-theoretic notions to quantification, such as making the transition from the ordering relation of â€™longer thanâ€™ (if one body covers the other) to the notion of how much longer. In similar way we introduce dynamical notions â€™more impulseâ€™ (if in a collision one object overruns the other) and â€™more energeticâ€™ (if the effect of one source exceeds the effect of the other). In a physical model - built by coupling congruent standard actions - those basic pre-theoretic notions become measurable. We derive all (classical and relativistic) equations between basic physical quantities of Energy, Momentum and Inertial Mass and ultimately the principle of least action.

Action Functional

$$S_{\mathrm{Ham}}[\gamma] := \int dt \left(\frac{1}{2} \cdot m_I \cdot \boldsymbol{v}_I{}^2 - V_{\mathrm{pot}}\right)$$

...

Physical Interaction



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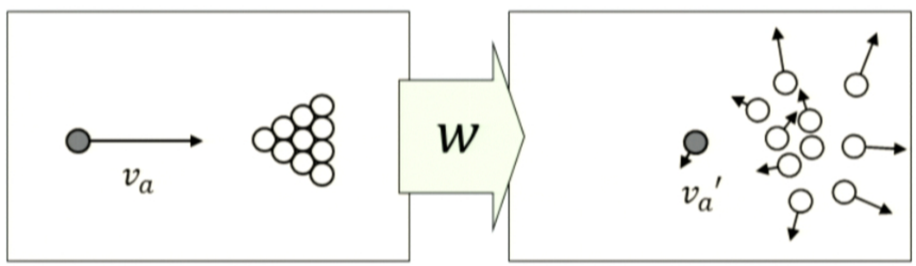
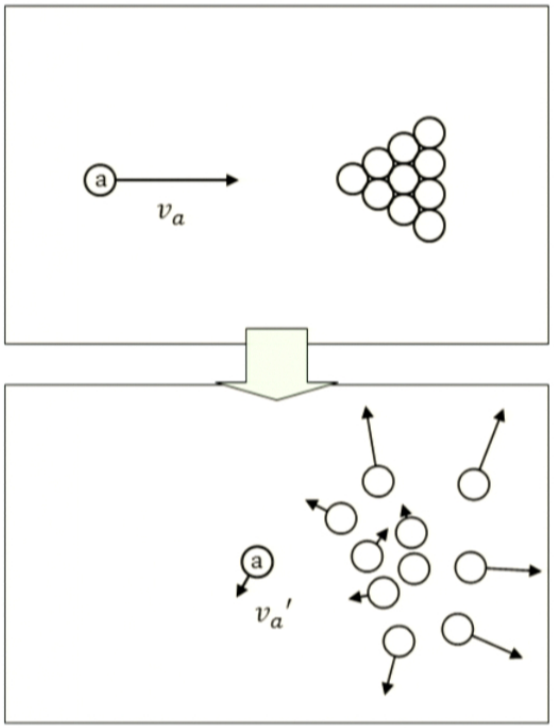


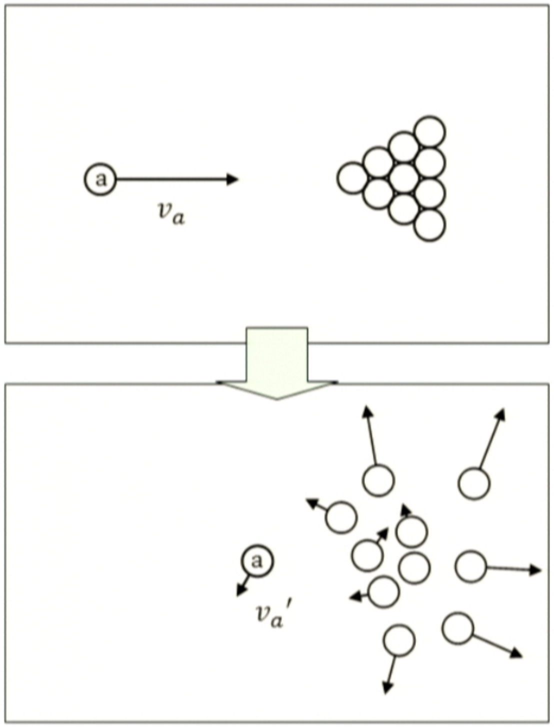
generic Billiard collision

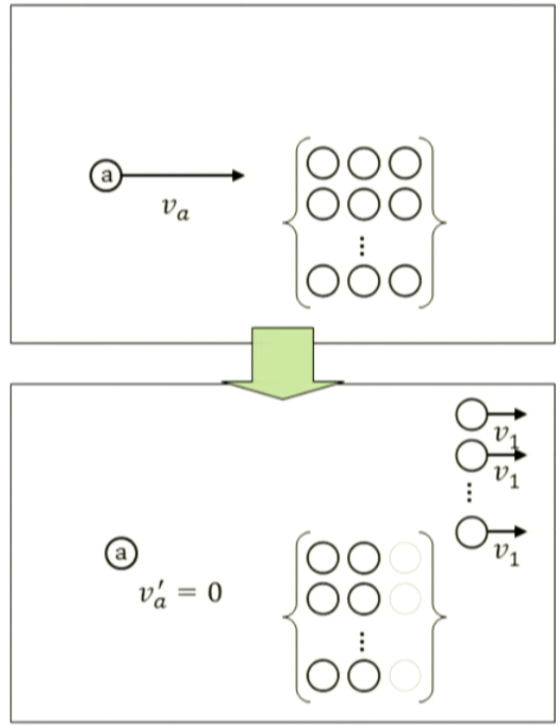


Interaction of Motion

generic Billiard collision

controlled replacement process

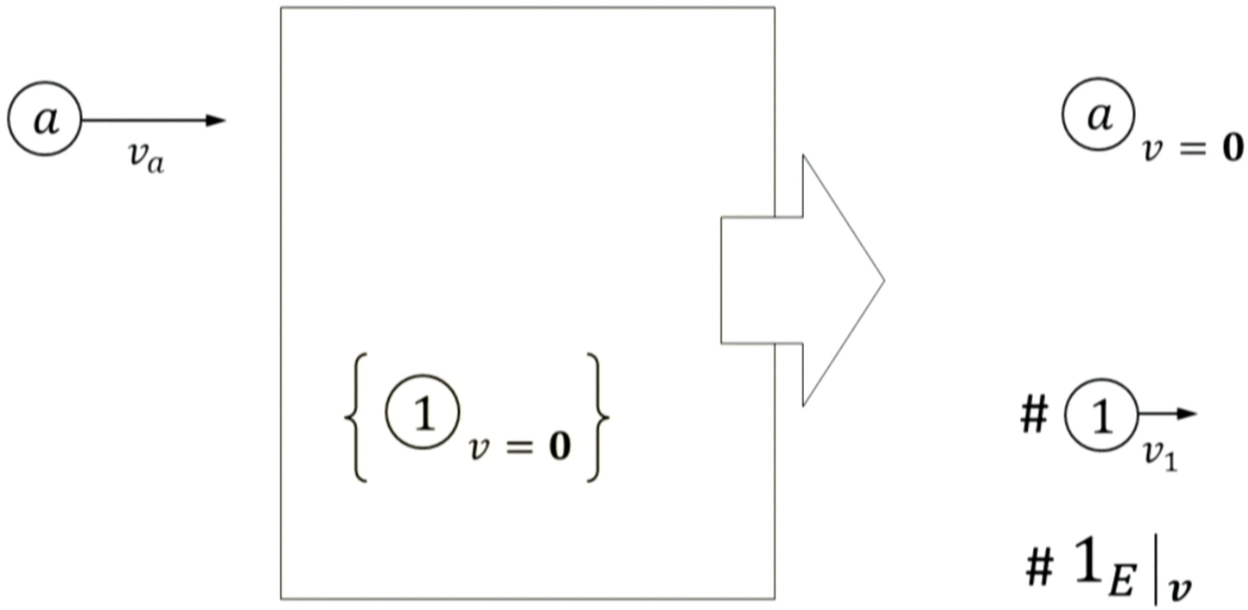




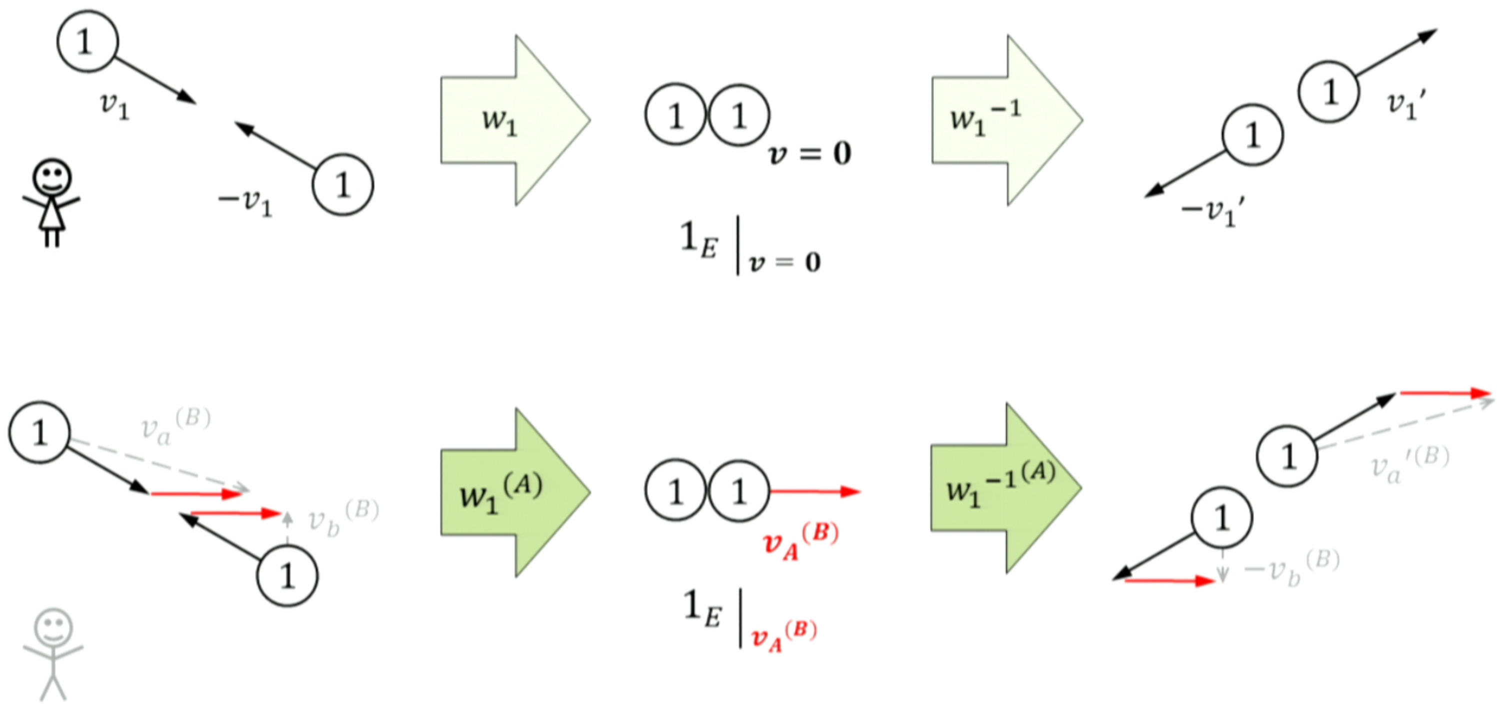
Interaction of Motion:

- 'potential to cause action'
- 'striking power' or 'impulse'

Calorimeter Model



Consecutive Association (concatenation ∗)



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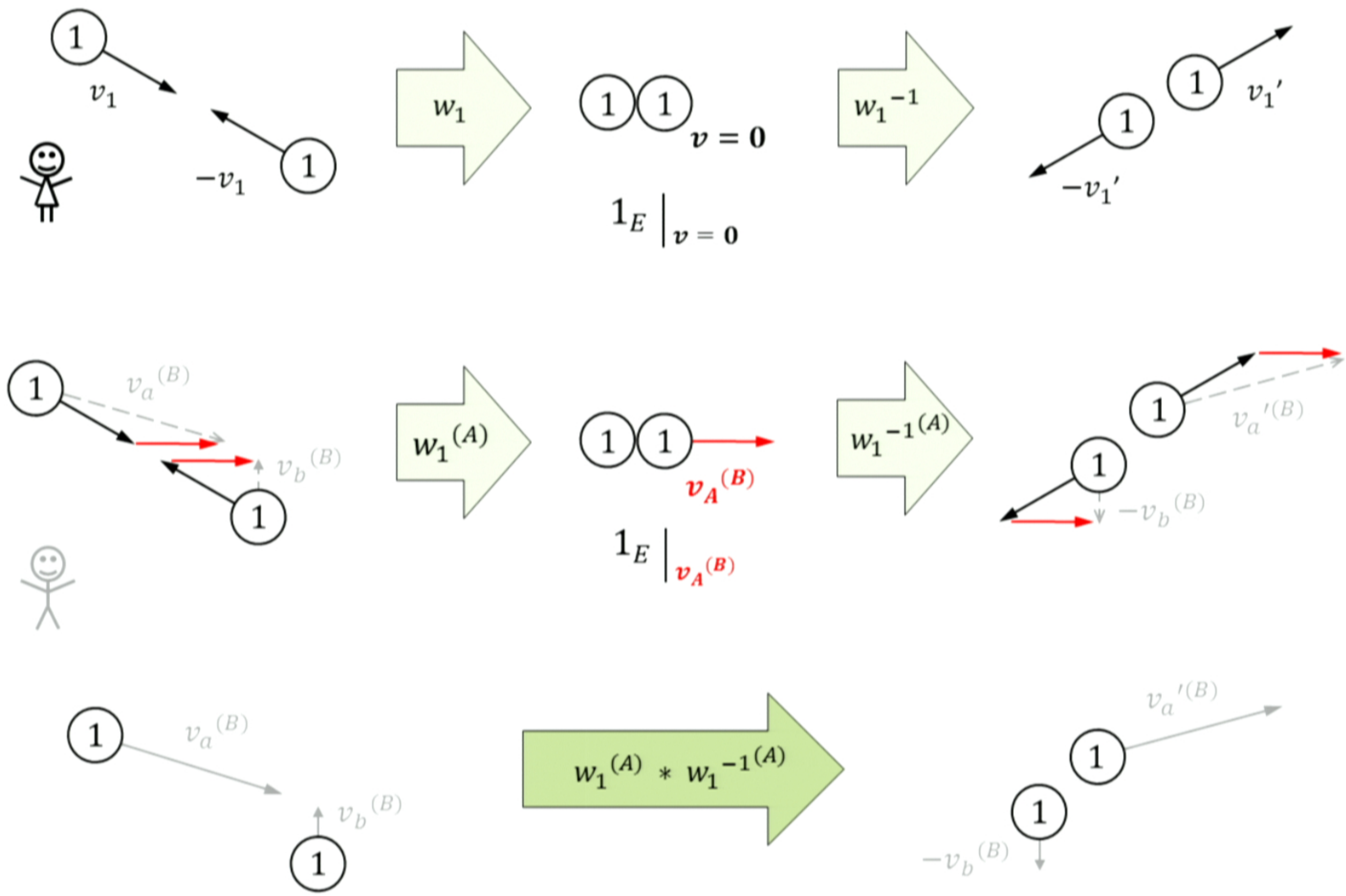
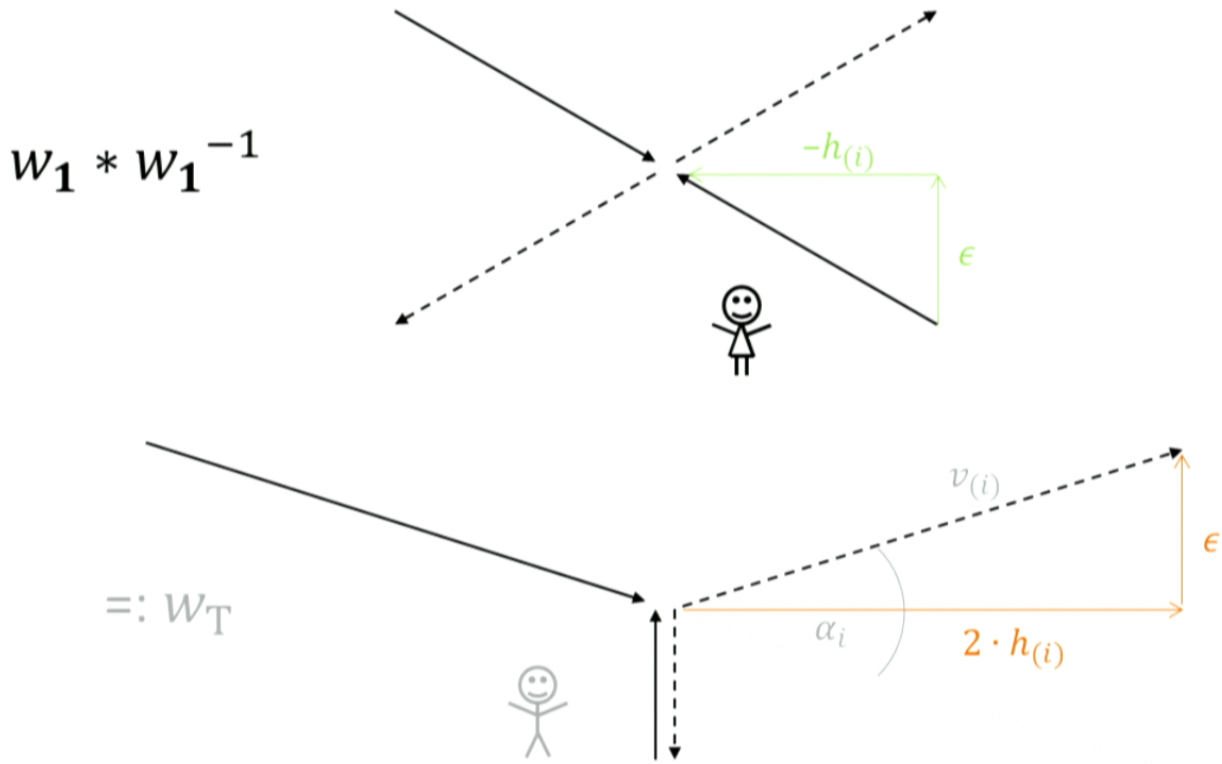
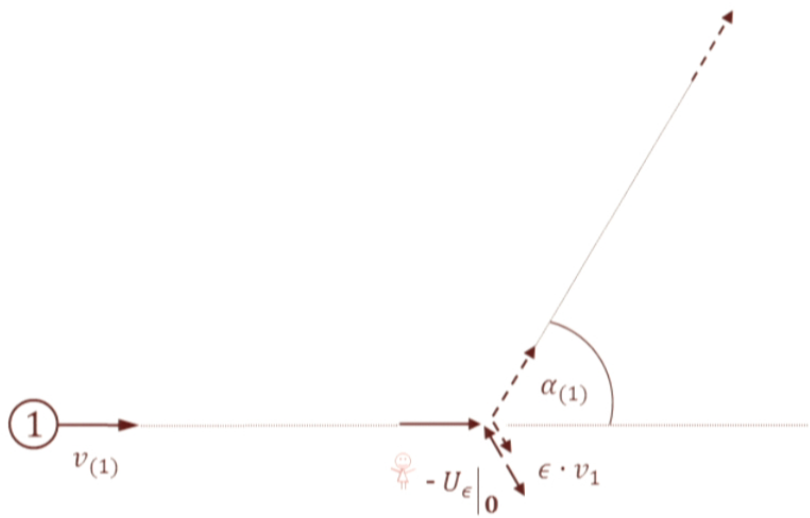


Elastic Transversal Collision

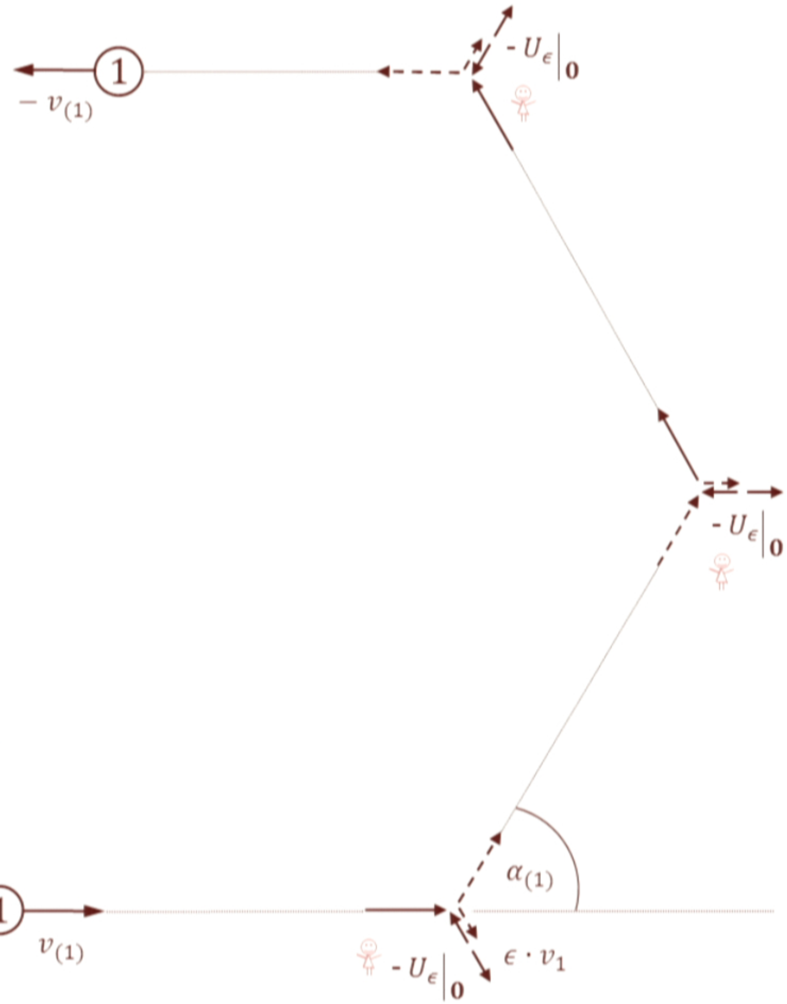


Impulse Reversion Process

w_{T}

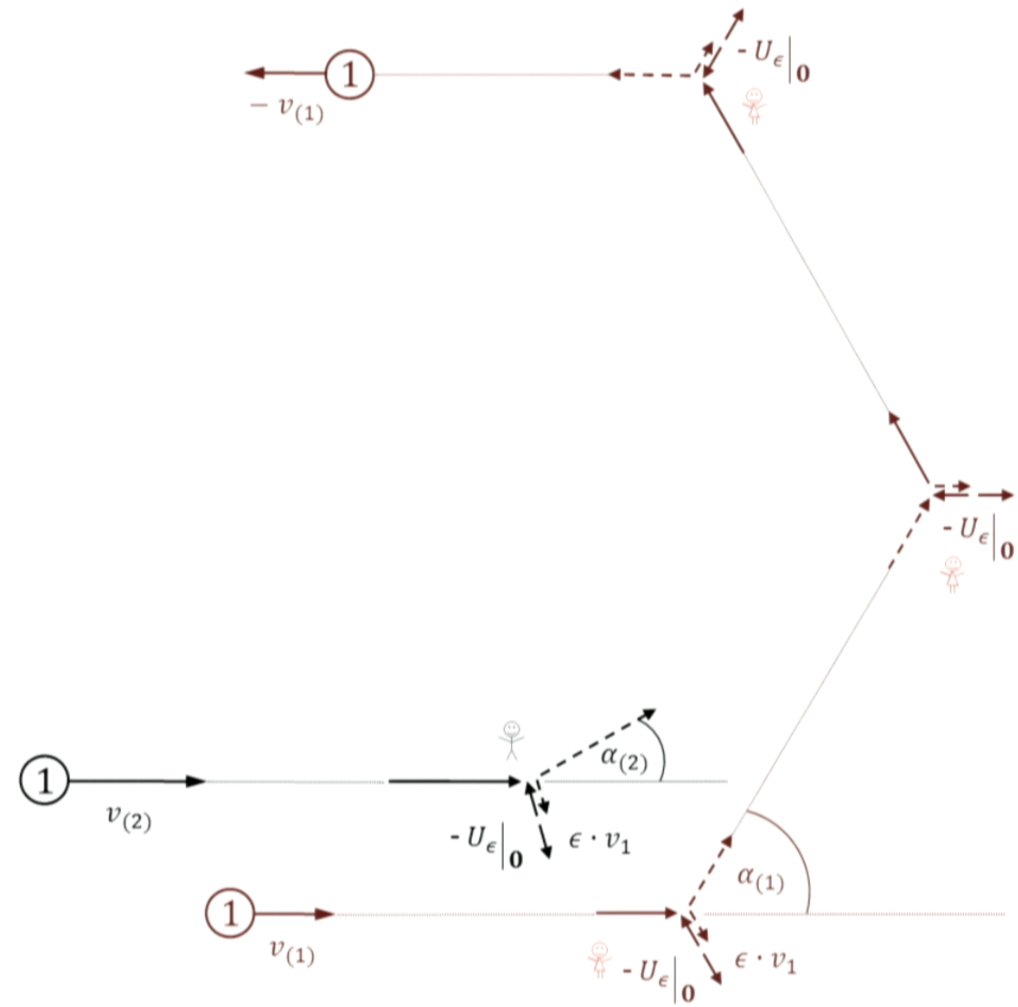


Impulse Reversion Process

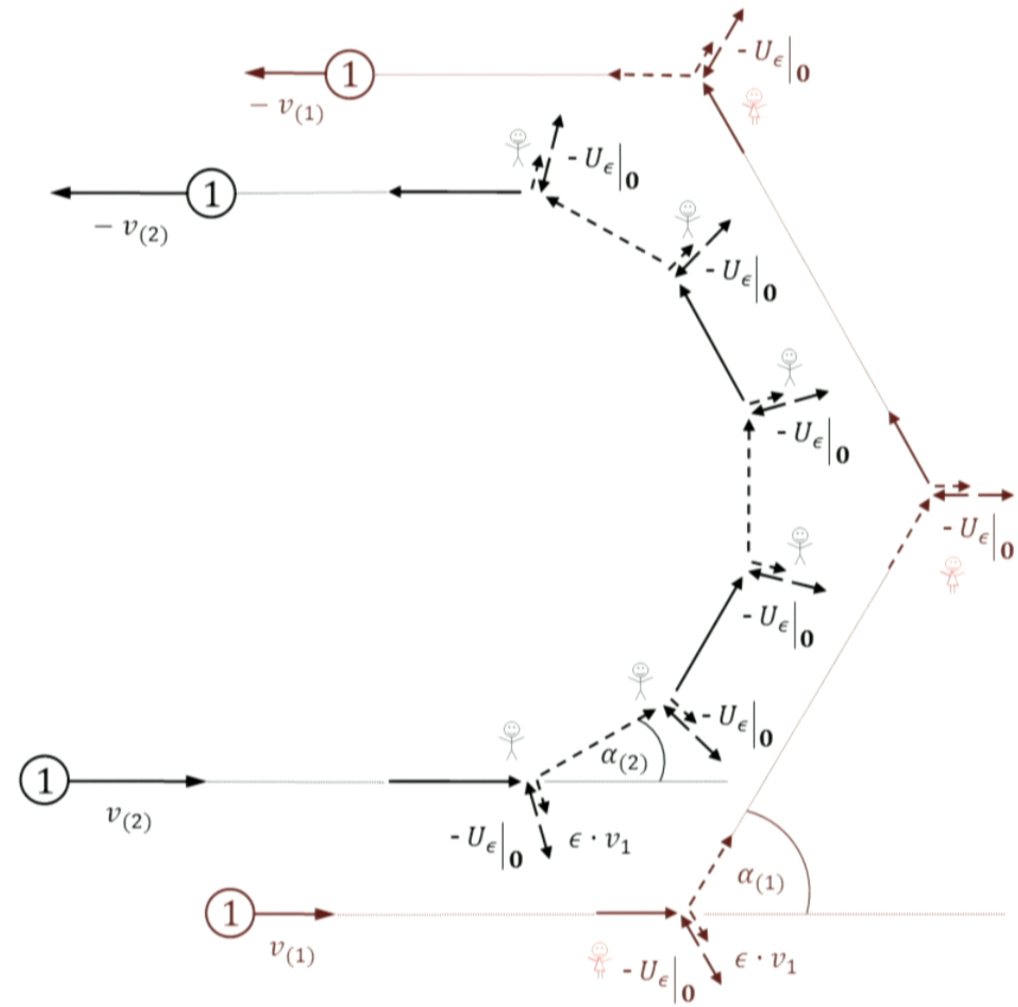


$$w_T * w_T * w_T$$

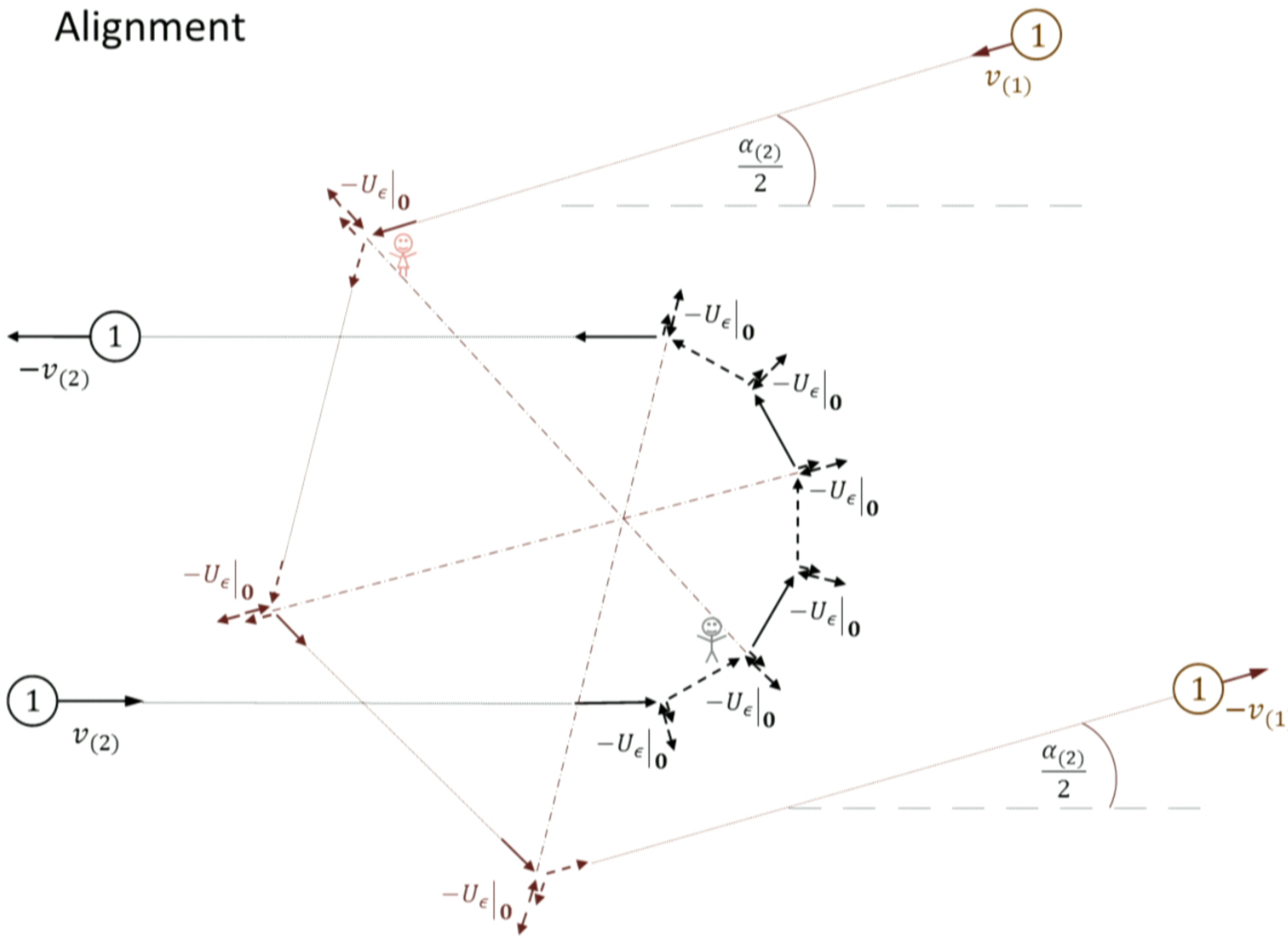
Impulse Reversion Process



Impulse Reversion Process

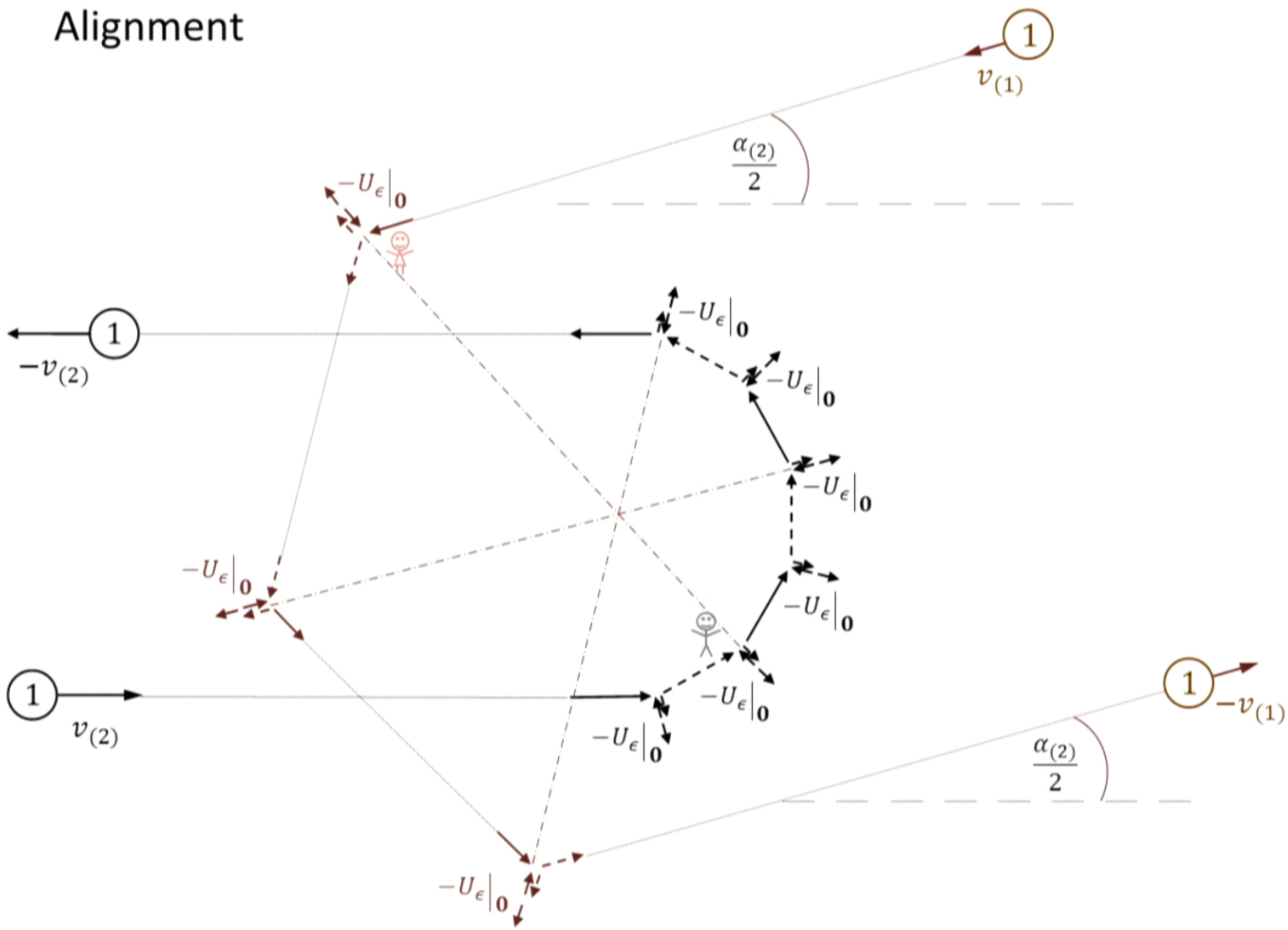


Alignment



1
$v_{(1)}$
$\frac{\alpha_{(2)}}{2}$
$-U_\epsilon|_0$
1
$-v_{(2)}$
$-U_\epsilon|_0$
$-U_\epsilon|_0$
$-U_\epsilon|_0$
$-U_\epsilon|_0$
$-U_\epsilon|_0$
1
$v_{(2)}$
$-U_\epsilon|_0$
$-U_\epsilon|_0$
1
$-v_{(1)}$
$\frac{\alpha_{(2)}}{2}$
$-U_\epsilon|_0$

Alignment



1
$v_{(1)}$
$\frac{\alpha_{(2)}}{2}$
$-U_\epsilon|_0$
1
$-v_{(2)}$
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$-U_\epsilon|_0$
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1
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$-U_\epsilon|_0$
1
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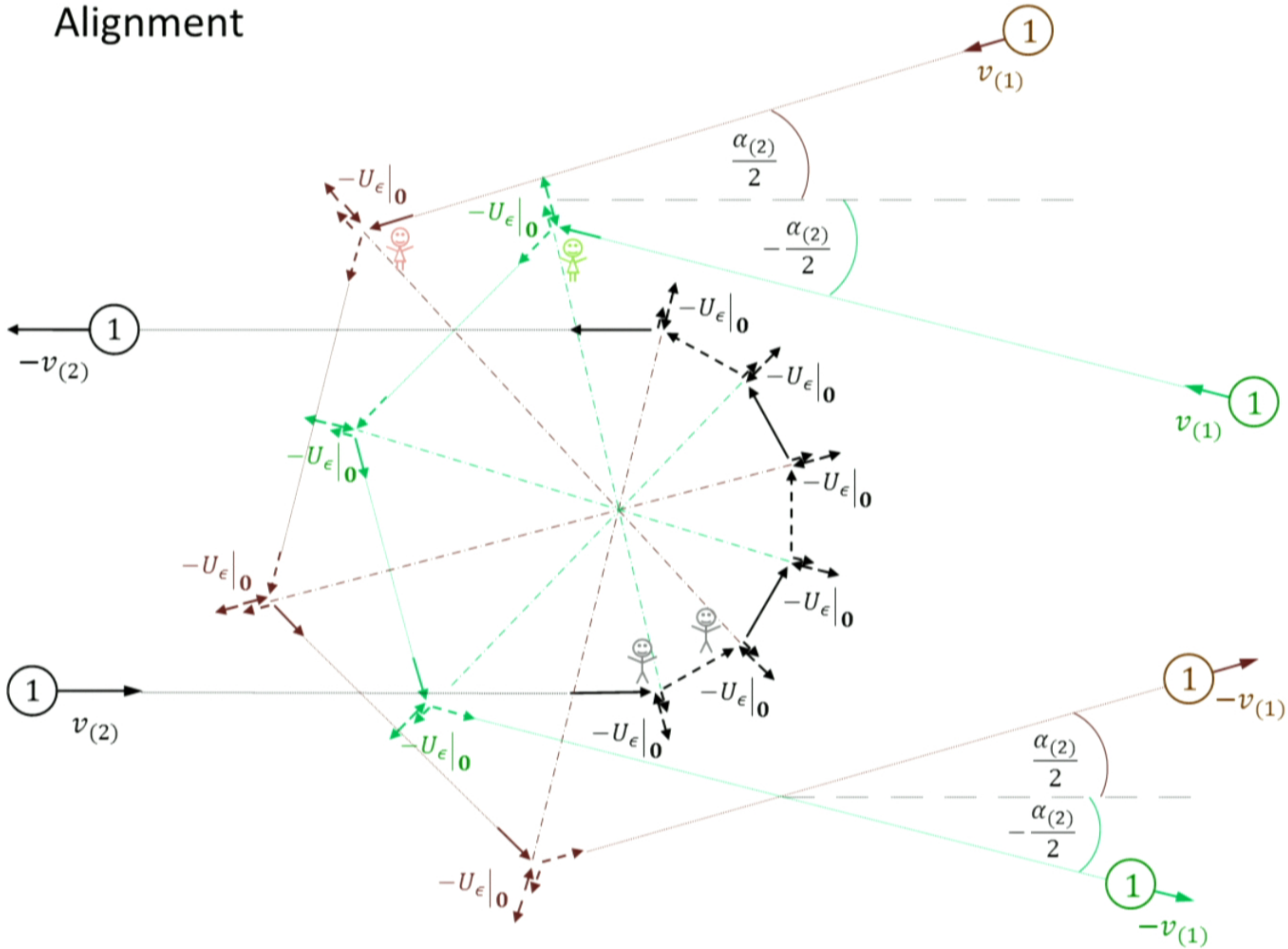
Alignment



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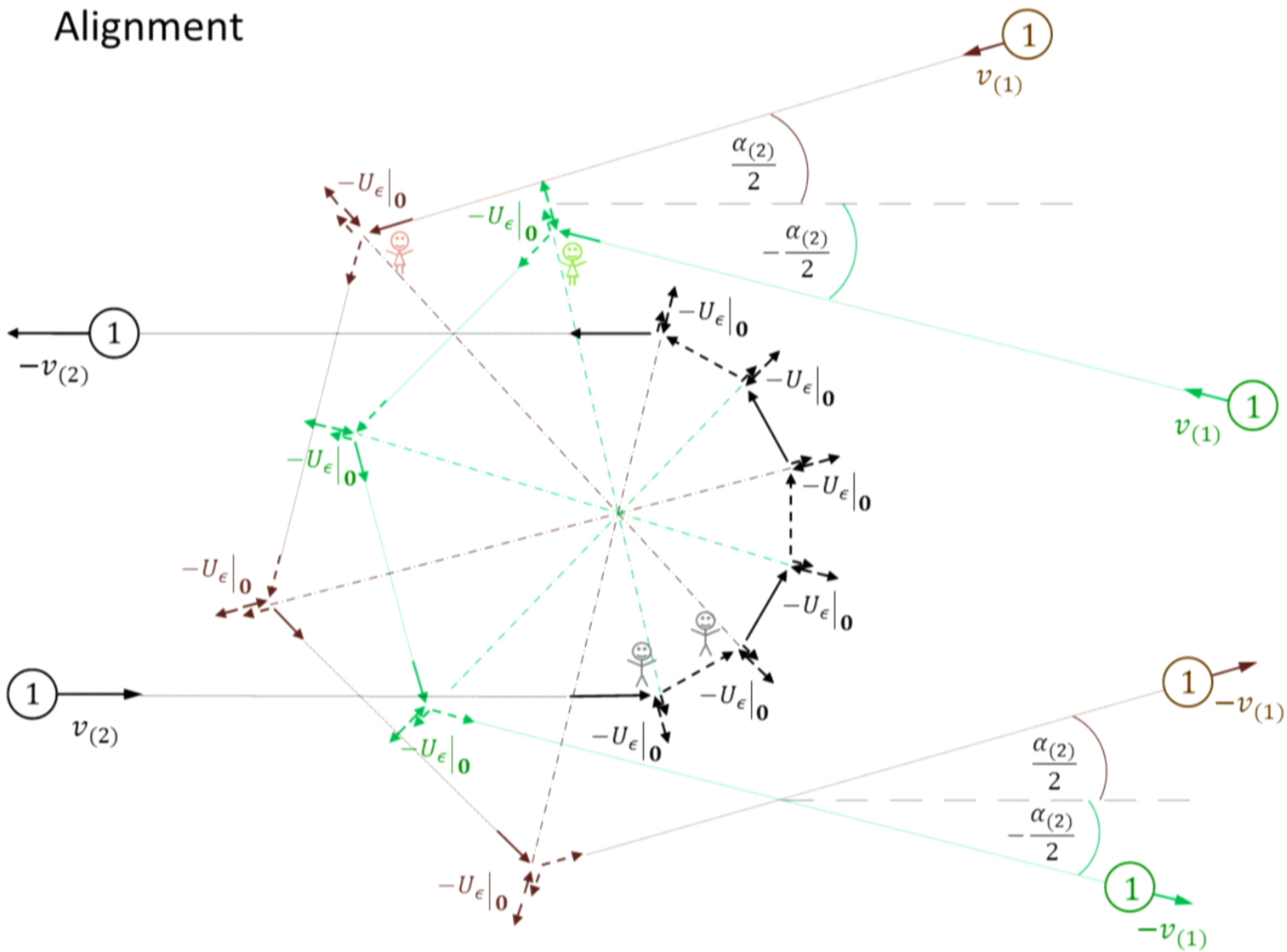
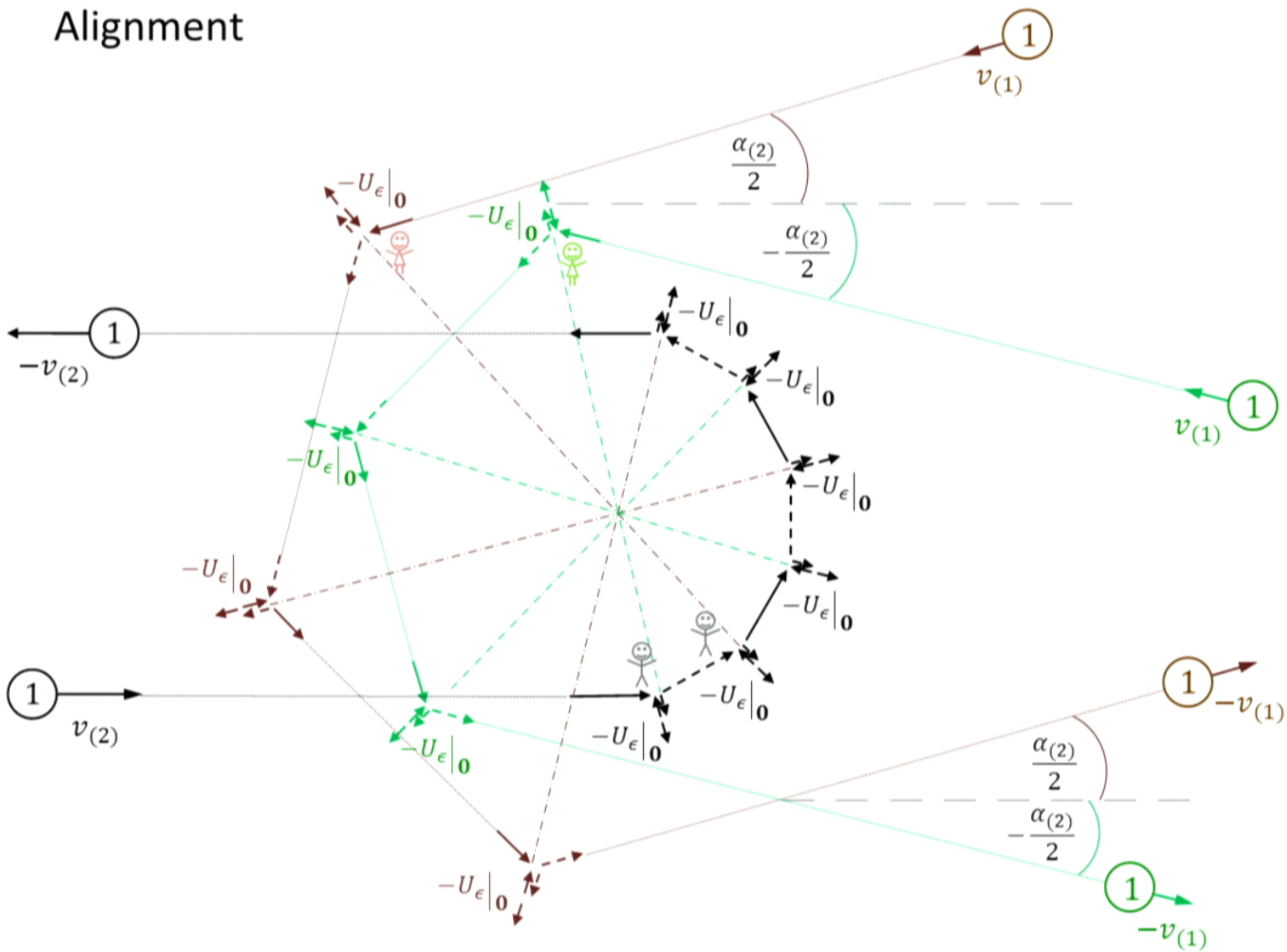
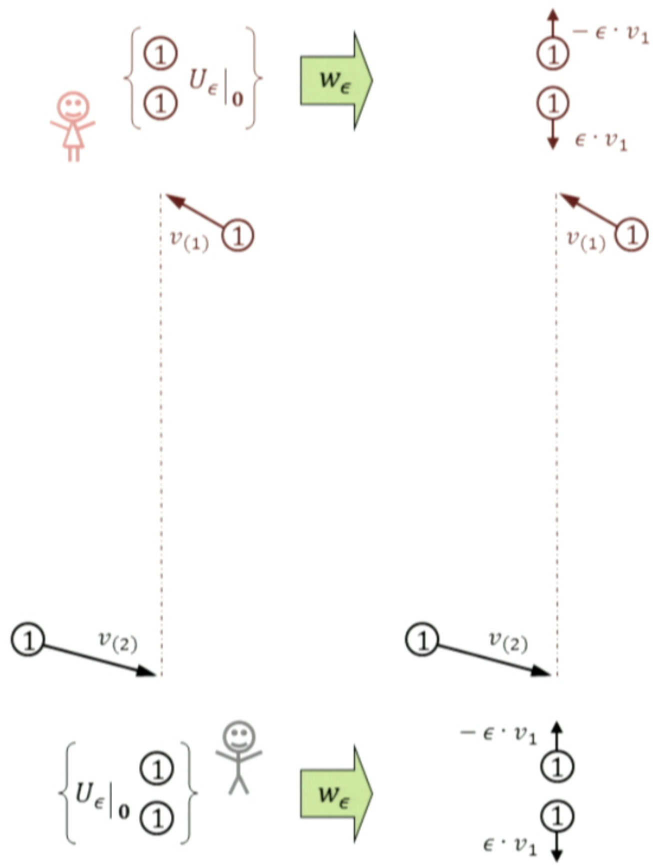


Alignment



Recycling at diametrically opposed positions



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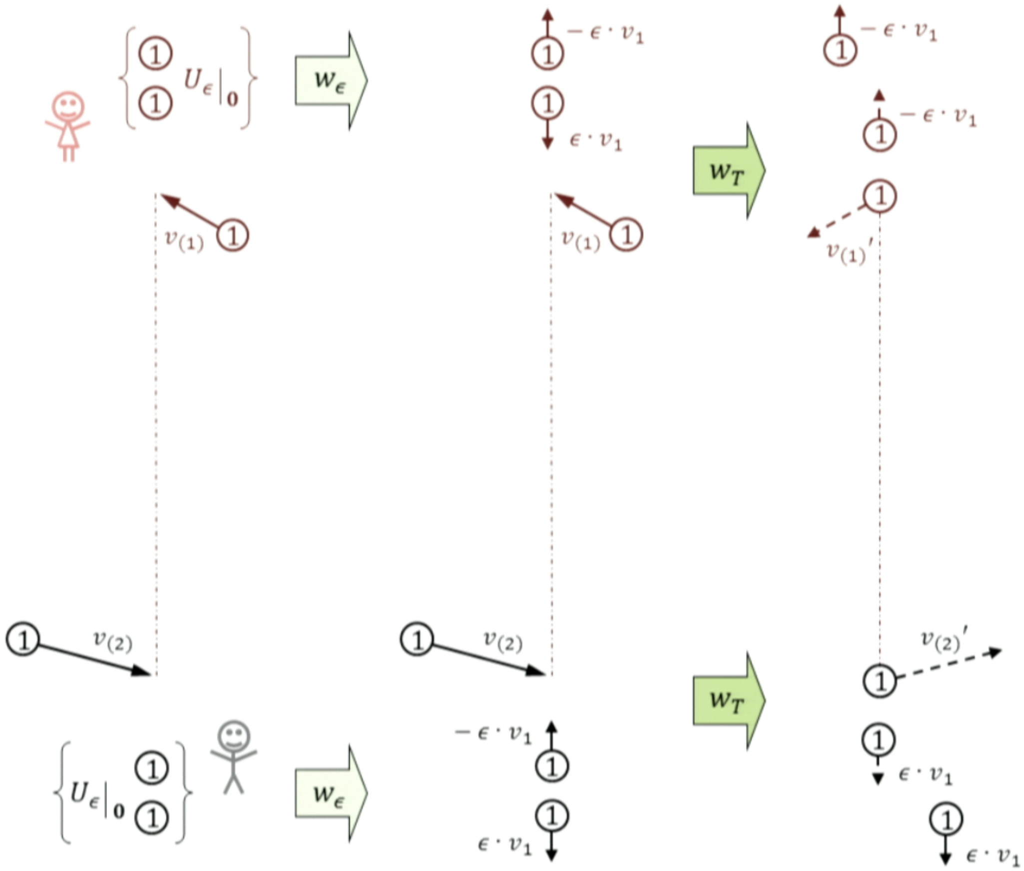

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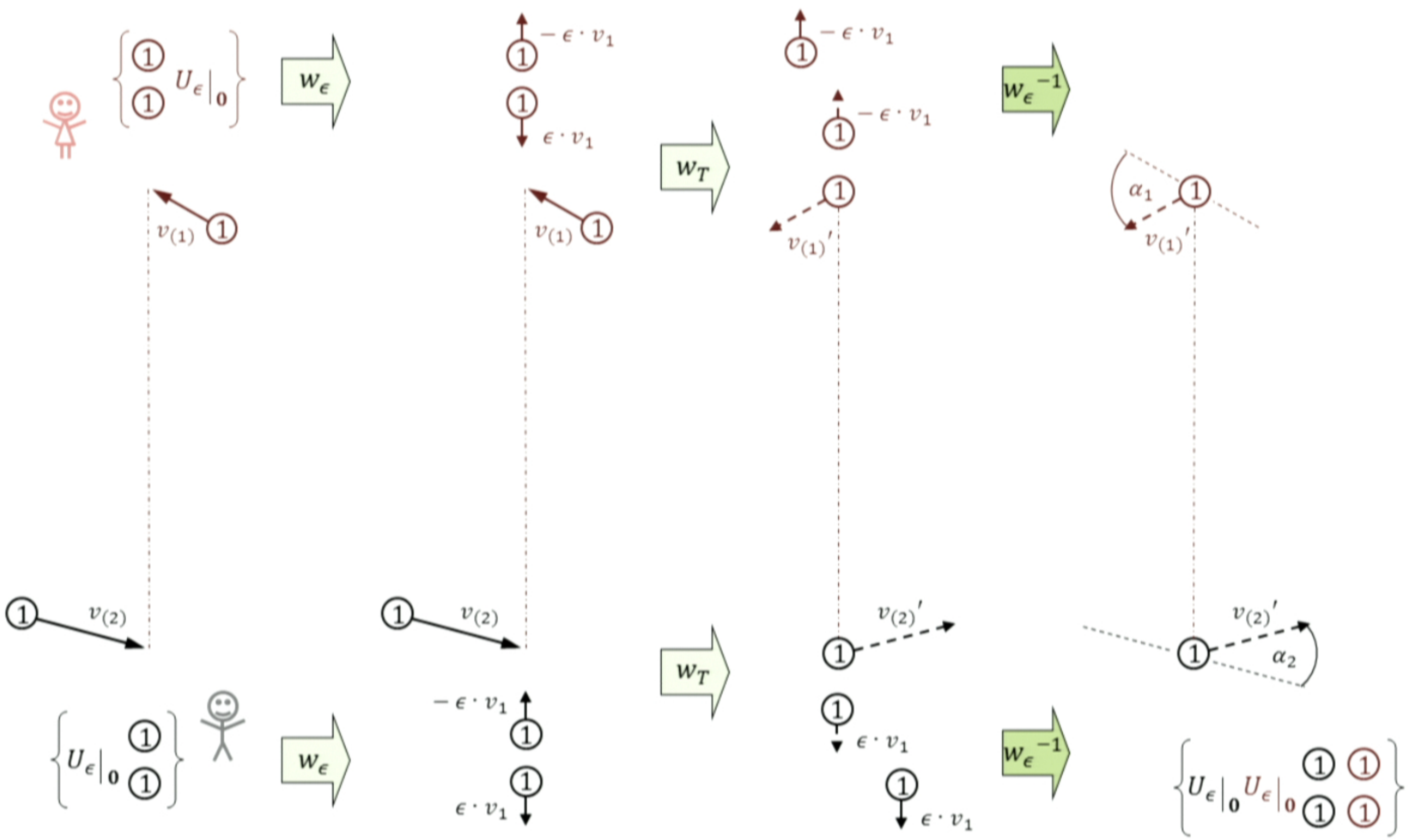

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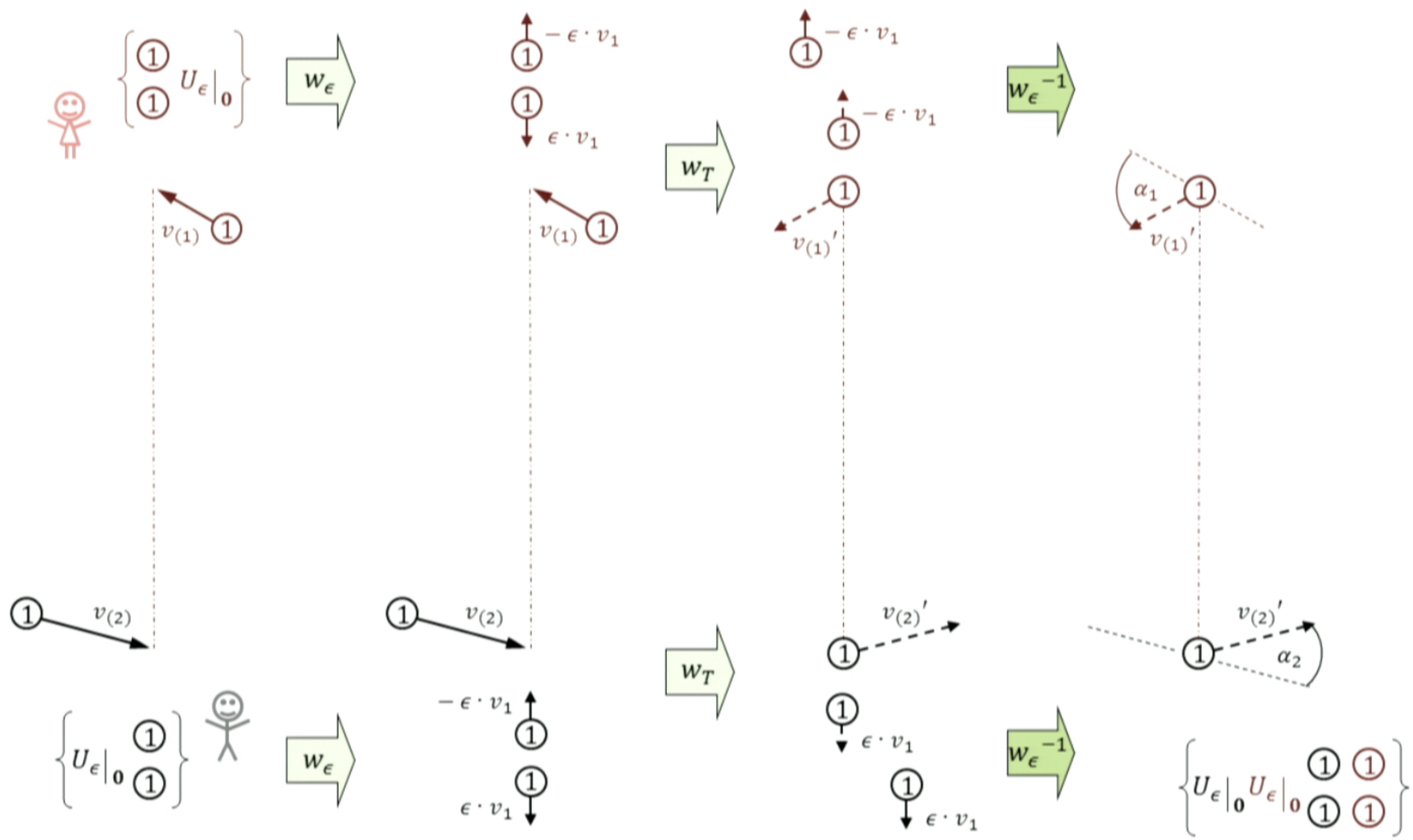
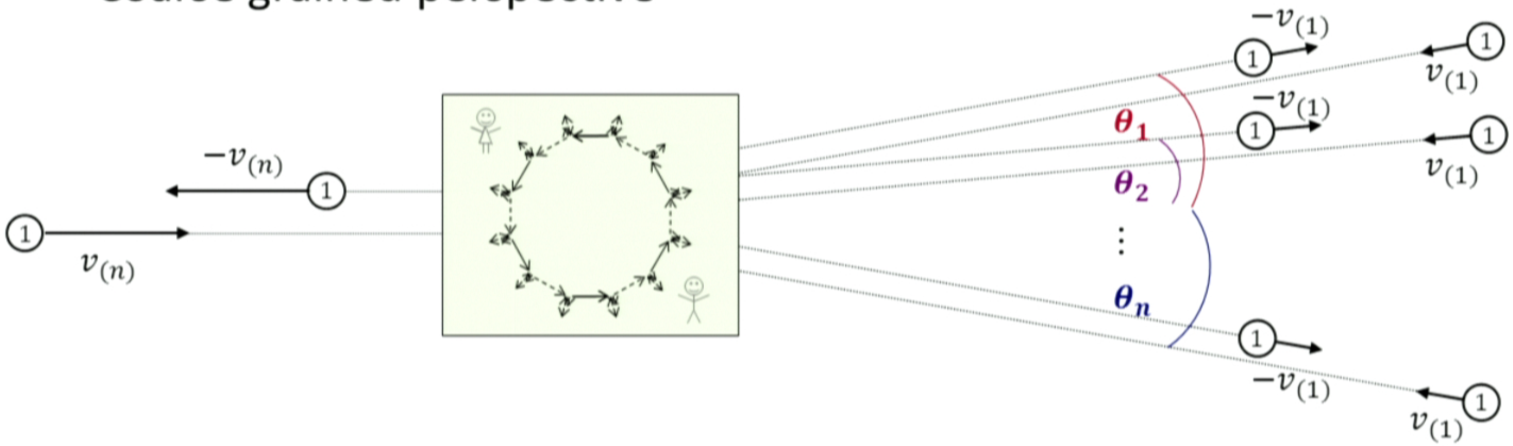
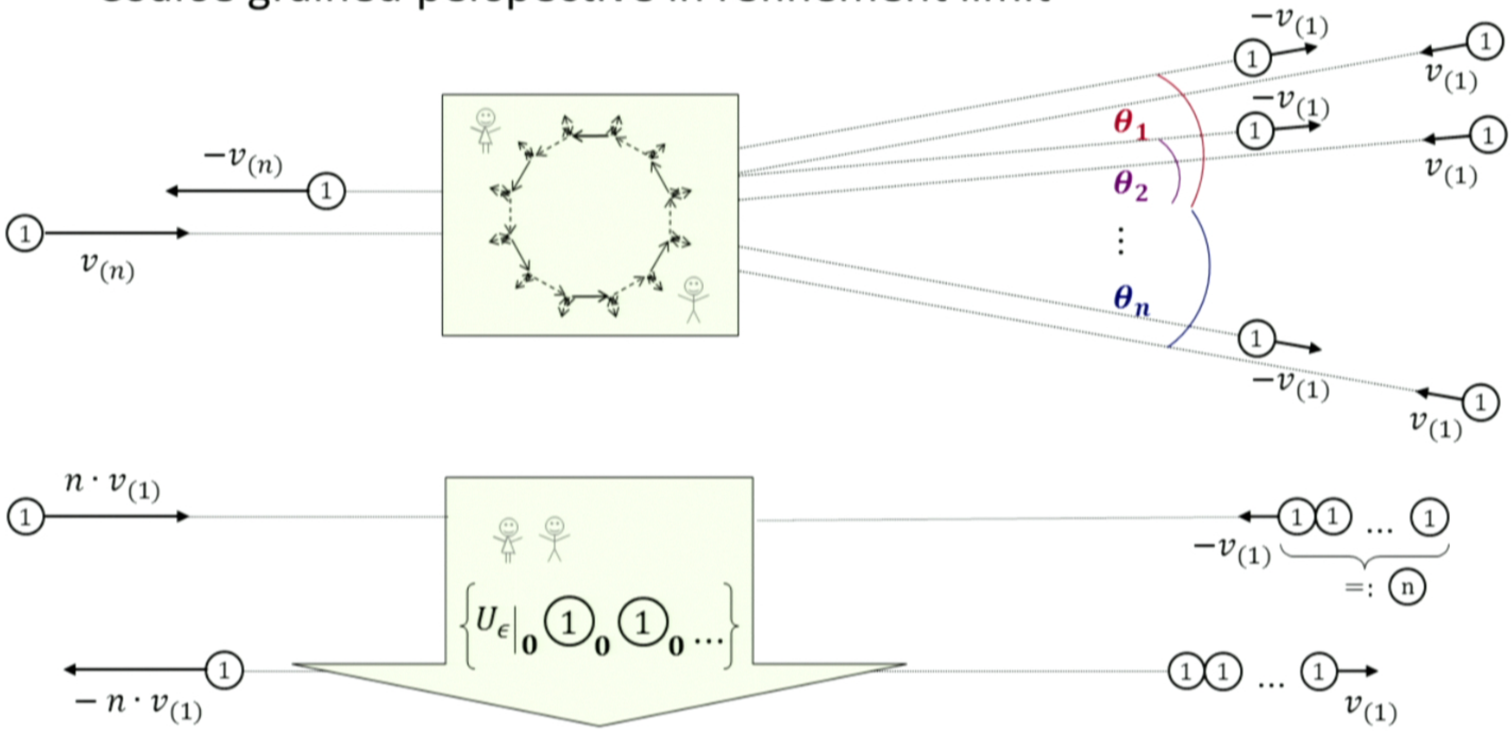


Coarse grained perspective



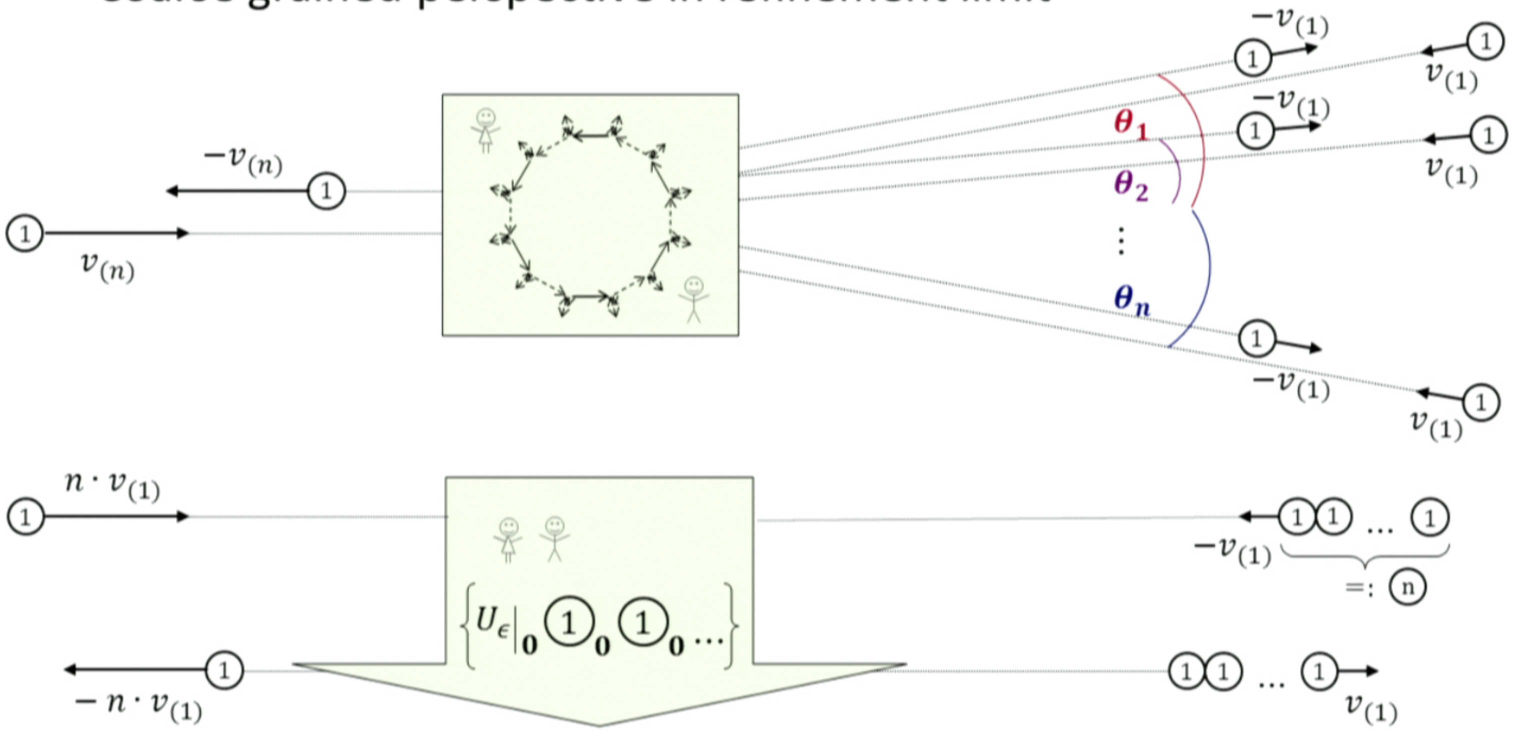
Coarse grained perspective in refinement limit



Quantification of Elastic Collision

$$w : ⓝ_{1\cdot v},\ ①_{-n\cdot v} \Rightarrow ⓝ_{-1\cdot v},\ ①_{n\cdot v}$$

Coarse grained perspective in refinement limit



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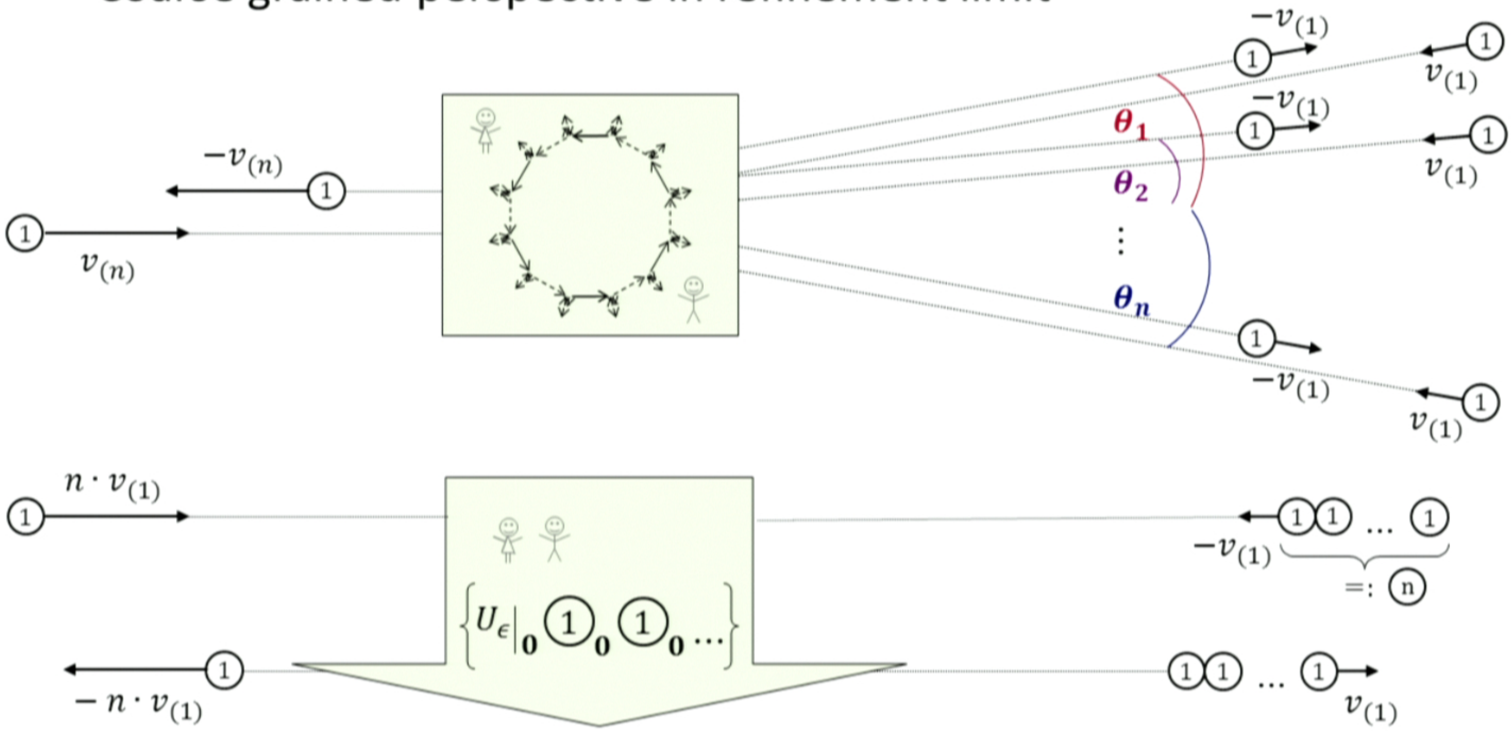
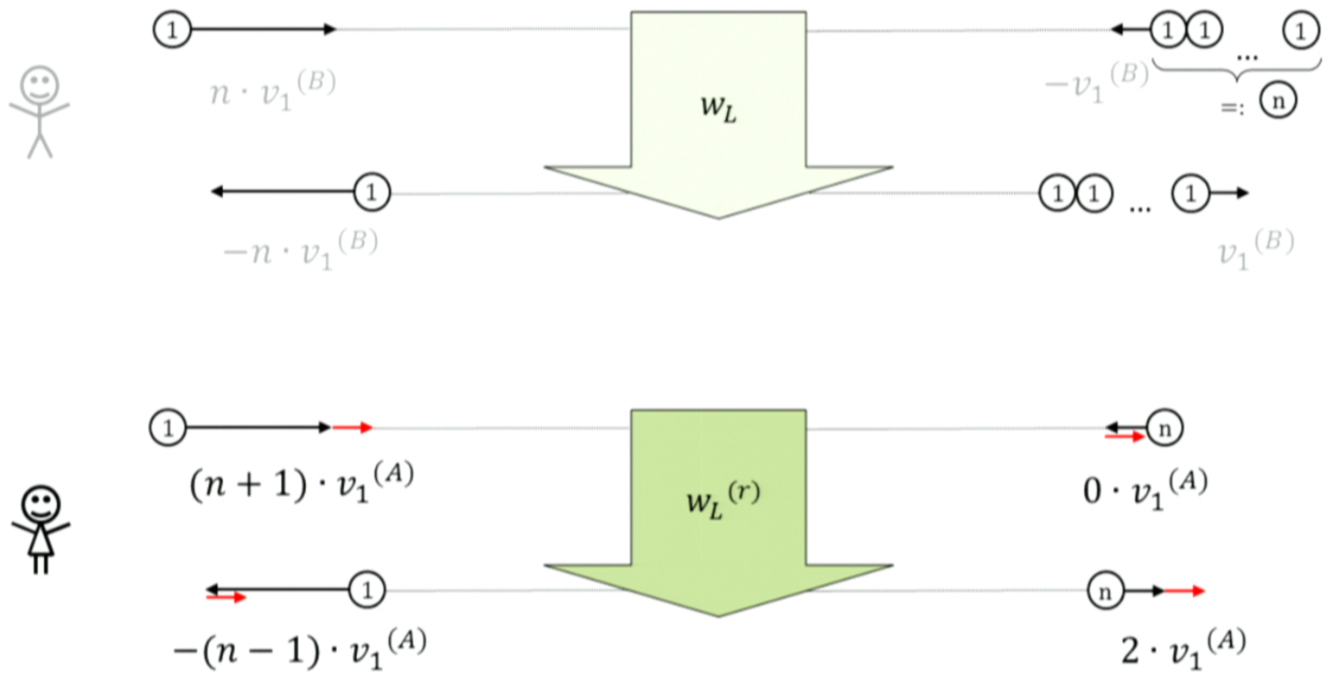
Coarse grained perspective in refinement limit



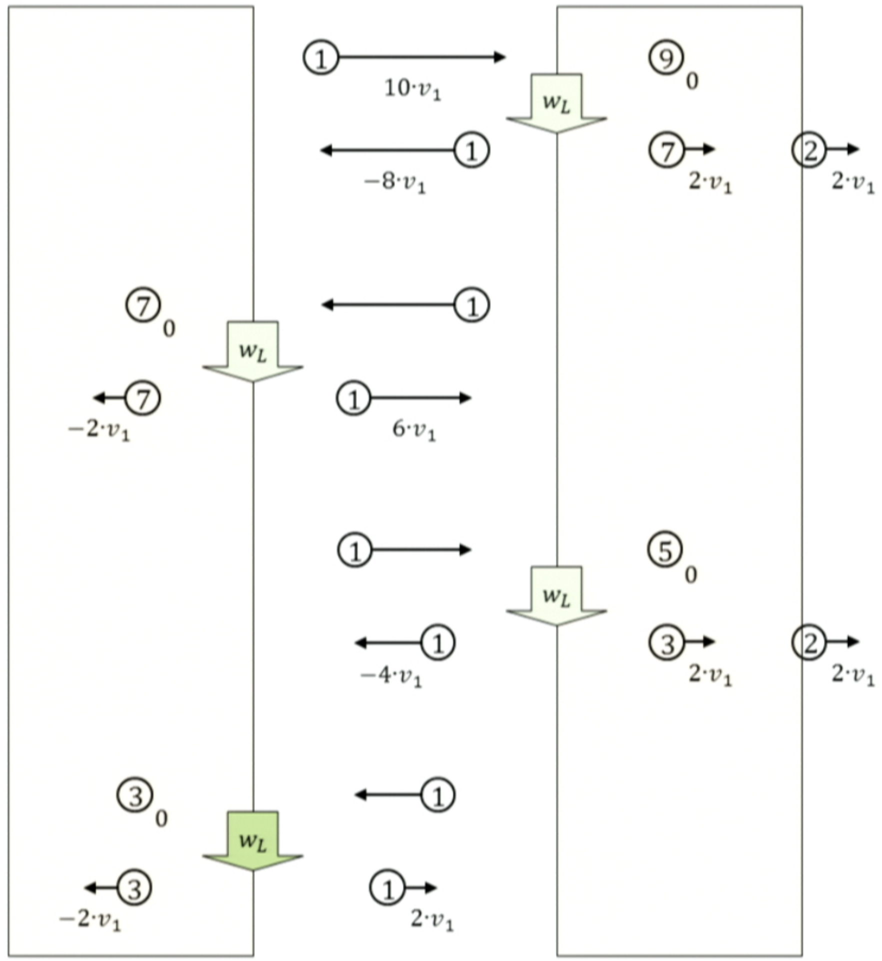
Quantification of Elastic Collision

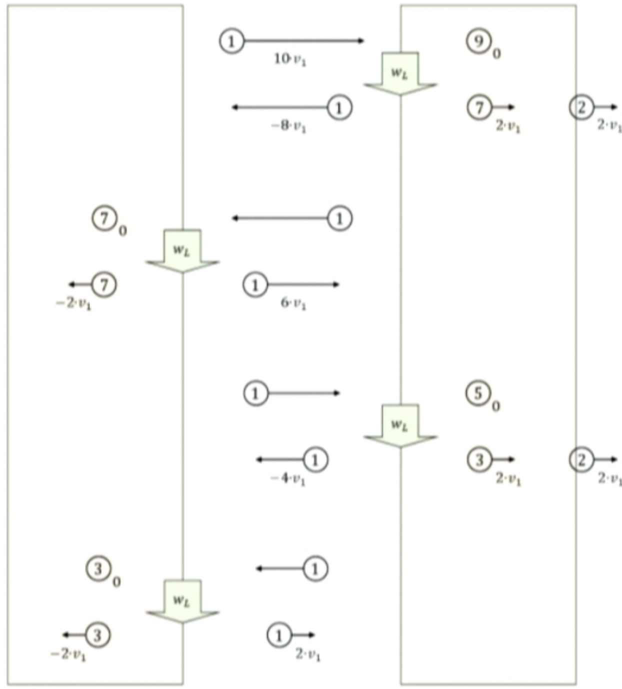
$$w : ⓝ_{1\cdot v},\ ①_{-n\cdot v} \Longrightarrow ⓝ_{-1\cdot v},\ ①_{n\cdot v}$$

Elastic Longitudinal Collision



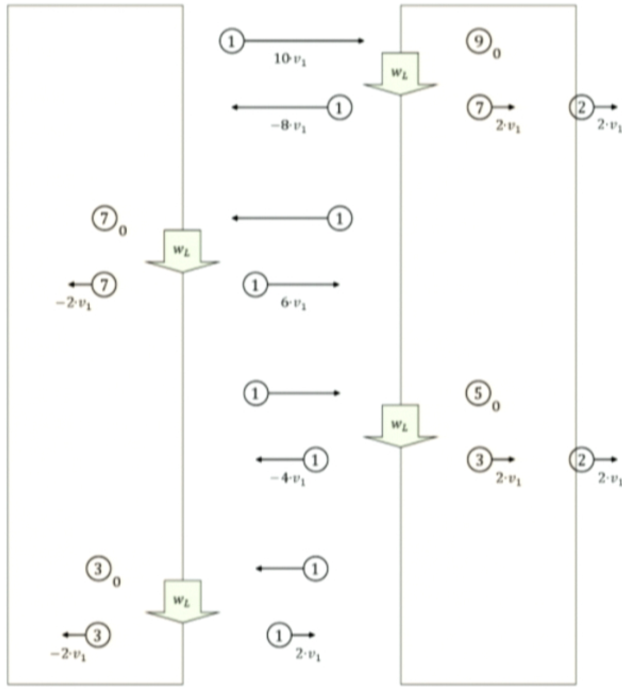
Absorption Action





Quantification of Calorimeter Action

$$W_{\text{cal}} : \textcircled{1}_{10\cdot\mathrm{v}},\ 25\cdot\textcircled{1}_{\mathbf{0}} \Rightarrow \textcircled{1}_{\mathbf{0}}\ ,\underbrace{10\cdot\{\textcircled{1}_{2\cdot\mathrm{v}},\textcircled{1}_{-2\cdot\mathrm{v}}\},5\cdot\textcircled{1}_{2\cdot\mathrm{v}}}_{\text{Calorimeter Extract}}$$



Quantification of Calorimeter Action

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Abstraction

Definition: *In an <u>abstraction</u> we regard the common quality of two empirical objects for itself without needing to consider the dissimilarity (of both objects in other regards).*

$\sim_l$ if two extended objects lie on top of each other: one will *cover* the other

$\sim_t$ if two processes begin simultaneously: one will *outlast* the other

$\sim_E$ if coupled against same system $\{G_I\}$: the effect of one source *exceeds* the other

$\sim_{\boldsymbol{p}}$ in an inelastic collision against one another: one object *overruns* the other

$$\sim_m \;:=\; \sim_{\boldsymbol{p}} |_{\boldsymbol{v}_a=-\boldsymbol{v}_b}$$

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Quantification

Momentum:

$\sim \boldsymbol{p}$ $\quad\quad \mathbf{1}_{\boldsymbol{p}} := ①_{v_1} \quad\quad *$

$ⓐ_{v_a} \quad \sim \boldsymbol{p} \quad \underbrace{①_{v_1} * \cdots * ①_{v_1}}_{\text{Calorimeter Extract}}$

$\boldsymbol{p}\,[\,ⓐ_{v_a}\,] = \boldsymbol{p}_a{}^{(A)} \cdot \boldsymbol{p}_{\mathbf{1}^{(A)}}$

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$\sim \boldsymbol{p}$ $\quad$ $\mathbf{1}_{\boldsymbol{p}} :=$ ①$_{v_1}$ $\quad$ $*$

ⓐ$_{v_a}$ $\sim \boldsymbol{p}$ $\quad$ $\underbrace{\text{①}_{v_1} * \cdots * \text{①}_{v_1}}_{\text{Calorimeter Extract}}$

$\boldsymbol{p}$ [ⓐ$_{v_a}$] = $\boldsymbol{p}_a{}^{(A)} \cdot \boldsymbol{p}_{\mathbf{1}^{(A)}}$

Quantification

Momentum:

$\sim_{\boldsymbol{p}}$ $\qquad \mathbf{1}_{\boldsymbol{p}} := ①_{v_1}$ $\qquad *$

$ⓐ_{v_a} \sim_{\boldsymbol{p}} \underbrace{①_{v_1} * \cdots * ①_{v_1}}_{\text{Calorimeter Extract}}$

$\boldsymbol{p}\,[\,ⓐ_{v_a}\,] = \boldsymbol{p}_a{}^{(A)} \cdot \boldsymbol{p}_{\mathbf{1}}{}^{(A)}$

Energy:

$\sim_E$ $\qquad \mathbf{1}_E|_{\mathbf{0}}$ $\qquad *$

$E\,[\,ⓐ_{v_a}\,] = E_a{}^{(A)} \cdot E_{\mathbf{1}}{}^{(A)}$

Quantification

quantified (physical) measure

(basic) physical quantities

$$E_a = E_a^{(A)} \cdot E_{\mathbf{1}^{(A)}} \qquad \boldsymbol{p}_a = \boldsymbol{p}_a^{(A)} \cdot \boldsymbol{p}_{\mathbf{1}^{(A)}}$$

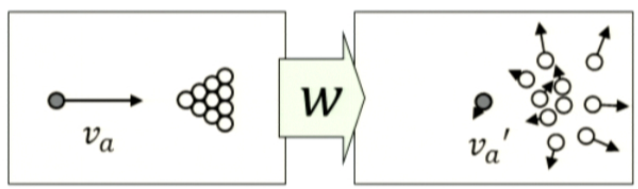
$$w \sim_{E,\boldsymbol{p}} W_{\text{cal}} := w_{\mathbf{1}} * \ldots * w_{\mathbf{1}}$$

observable/ physical measure

$$E_{ⓐ_{v_a}} \qquad \boldsymbol{p}_{ⓐ \cup ⓑ_{\boldsymbol{v}_I}}$$

$$\sim_E \qquad \sim_{\boldsymbol{p}}$$

empirical basis



Physical Principles

Principle of Causality

Principle of Inertia

Impossibility of a Perpetuum Mobile

Principle of Sufficient Reason

Equivalence Principle

Superposition Principle

Methodical Principles

Basic measurement: as doubling of physical measures

Congruence Principle: for reliable quantification

Equipollence Principle: of measuring the cause of potential action by its (kinetic) effect

Fundamental Equations

count equivalent elements in calorimeter Model W_{cal}

$$\#\{\mathbf{1}_E|_{\mathbf{0}}\} \qquad \#\{\mathbf{1}_{\boldsymbol{p}}\} \qquad \#\{①\} \qquad \#\{\boldsymbol{v}_{\mathbf{1}}\}$$

(tailored) quantitative equations

$$E_a{}^{(A)} = \tfrac{1}{2} \cdot m_a{}^{(A)} \cdot \boldsymbol{v}_a{}^{(A)\,2} \qquad \boldsymbol{p}_a{}^{(A)} = m_a{}^{(A)} \cdot \boldsymbol{v}_a{}^{(A)}$$

numerical values in the form

$$E_a{}^{(A)} = \frac{E\,[\,ⓐ_{\boldsymbol{v}_a}]}{E_{1(A)}} \quad \boldsymbol{p}_a{}^{(A)} = \frac{\boldsymbol{p}\,[\,ⓐ_{\boldsymbol{v}_a}]}{\boldsymbol{p}_{1(A)}} \quad m_a{}^{(A)} = \frac{m\,[\,ⓐ_{\boldsymbol{v}_a}]}{m_{1(A)}} \quad \boldsymbol{v}_a{}^{(A)} = \frac{\boldsymbol{v}\,[\,ⓐ_{\boldsymbol{v}_a}]}{\boldsymbol{v}_{1(A)}}$$

Fundamental Equations

count equivalent elements in calorimeter Model W_{cal}

$$\#\{\mathbf{1}_E|_\mathbf{0}\} \qquad \#\{\mathbf{1}_p\} \qquad \#\{①\} \qquad \#\{\boldsymbol{v}_\mathbf{1}\}$$

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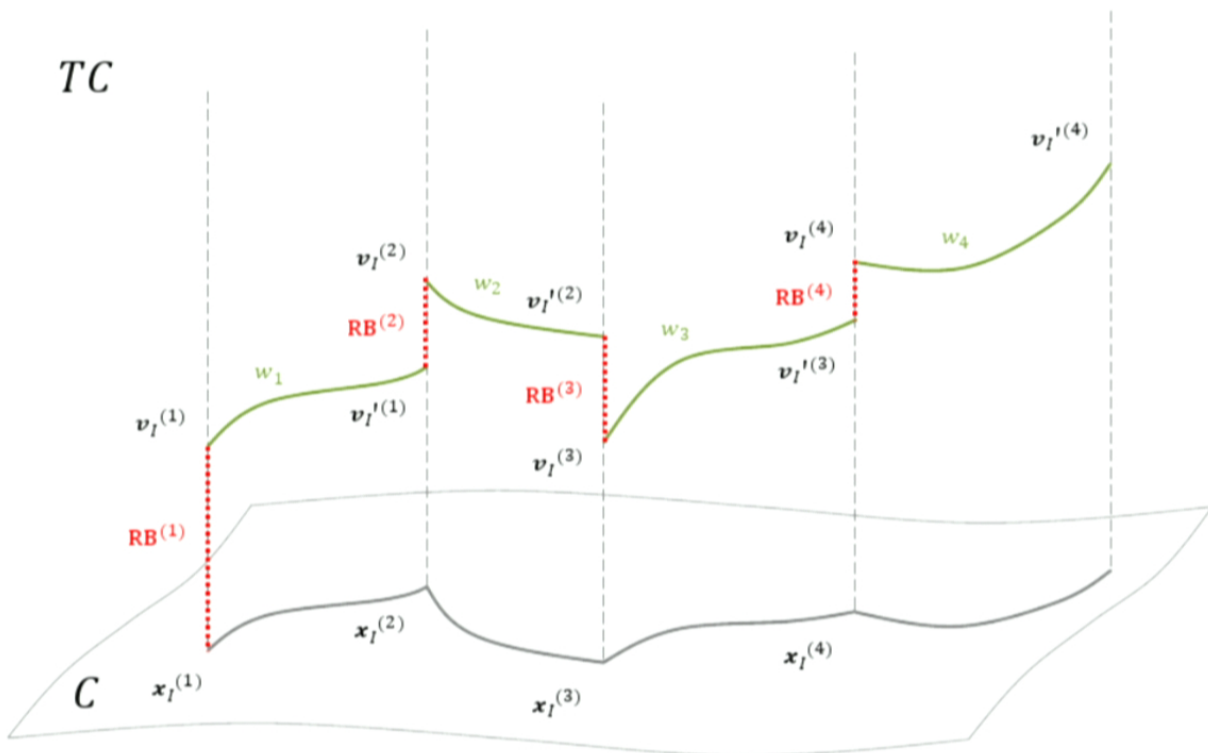
numerical values in the form

$$E_a{}^{(A)} = \frac{E\,[ⓐ_{\boldsymbol{v}_a}]}{E_{1(A)}} \quad \boldsymbol{p}_a{}^{(A)} = \frac{\boldsymbol{p}\,[ⓐ_{\boldsymbol{v}_a}]}{\boldsymbol{p}_{1(A)}} \quad m_a{}^{(A)} = \frac{m\,[ⓐ_{\boldsymbol{v}_a}]}{m_{1(A)}} \quad \boldsymbol{v}_a{}^{(A)} = \frac{\boldsymbol{v}\,[ⓐ_{\boldsymbol{v}_a}]}{\boldsymbol{v}_{1(A)}}$$

Derived Quantities



Fundamental Equations

count equivalent elements in calorimeter Model W_{cal}

$$\#\{\mathbf{1}_E|_{\mathbf{0}}\} \qquad \#\{\mathbf{1}_{\boldsymbol{p}}\} \qquad \#\{①\} \qquad \#\{\boldsymbol{v}_{\mathbf{1}}\}$$

(tailored) quantitative equations

$$E_a{}^{(A)} = \frac{1}{2} \cdot m_a{}^{(A)} \cdot \boldsymbol{v}_a{}^{(A)^2} \qquad \boldsymbol{p}_a{}^{(A)} = m_a{}^{(A)} \cdot \boldsymbol{v}_a{}^{(A)}$$

numerical values in the form

$$E_a{}^{(A)} = \frac{E\,[ⓐ_{\boldsymbol{v}_a}]}{E_{1(A)}} \qquad \boldsymbol{p}_a{}^{(A)} = \frac{\boldsymbol{p}\,[ⓐ_{\boldsymbol{v}_a}]}{\boldsymbol{p}_{1(A)}} \qquad m_a{}^{(A)} = \frac{m\,[ⓐ_{\boldsymbol{v}_a}]}{m_{1(A)}} \qquad \boldsymbol{v}_a{}^{(A)} = \frac{\boldsymbol{v}\,[ⓐ_{\boldsymbol{v}_a}]}{\boldsymbol{v}_{1(A)}}$$

Derived Quantities

meaningful derived physical quantities & EOM

$$V_{\text{pot}}[\boldsymbol{x}_I \rightarrow \boldsymbol{x}'_I] := V_{\text{pot}}[\gamma] \big/_{\text{mod } \gamma}$$

$$\boldsymbol{F}_a := \frac{\Delta \boldsymbol{p}_a}{\Delta t_a} \left[w|_{\boldsymbol{x}_I, \boldsymbol{v}_I} \right] \big/_{\text{mod } \boldsymbol{v}_I}$$

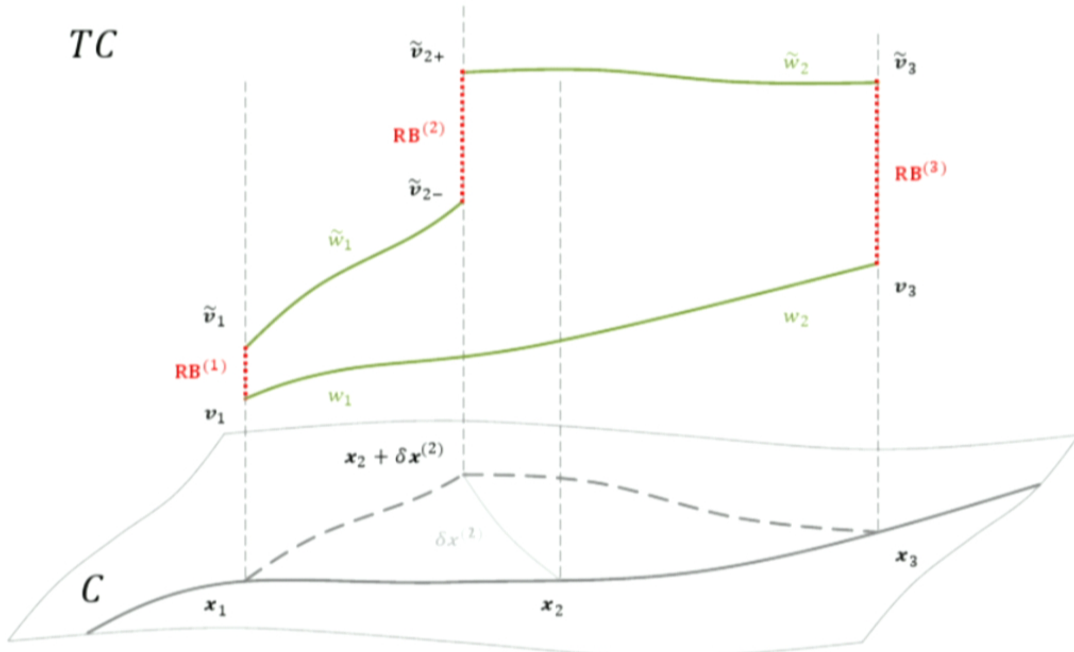
$$m_i \cdot \frac{\mathrm{d}^2 \boldsymbol{s}_i}{\mathrm{d}t^2} \left[w|_{\boldsymbol{x}_I, \boldsymbol{v}_I} \right] = -\nabla^{(i)} V_{\text{pot}} \quad \forall\, i \in I$$

analyse course of intrinsic action w
by external steering action $\mathrm{RB}^{(i)}$

quantitative equations

$$E_a{}^{(A)} = \frac{1}{2} \cdot m_a{}^{(A)} \cdot \boldsymbol{v}_a{}^{(A)\,2} \qquad \boldsymbol{p}_a{}^{(A)} = m_a{}^{(A)} \cdot \boldsymbol{v}_a{}^{(A)}$$

Principle of Least Action



steer Hamilton type variation $\delta\gamma^{(\mathrm{Ham})}$ of free course γ of intrinsic action w

$0 < \delta S_{\mathrm{Ham}}[\gamma]$

Principle of Least Action
(external steering effort)

steer Hamilton type variation $\delta\gamma^{(\mathrm{Ham})}$
of free course γ of intrinsic action w

meaningful derived physical quantities & EOM

$V_{\mathrm{pot}}[\boldsymbol{x}_I \to \boldsymbol{x}'_I] := V_{\mathrm{pot}}[\gamma] \big/ \mathrm{mod}\ \gamma$ $\quad \boldsymbol{F}_a := \frac{\Delta \boldsymbol{p}_a}{\Delta t_a}[w|_{\boldsymbol{x}_I,\boldsymbol{v}_I}] \big/ \mathrm{mod}\ \boldsymbol{v}_I$ $\quad m_i \cdot \frac{\mathrm{d}^2 \boldsymbol{s}_i}{\mathrm{d}t^2}[w|_{\boldsymbol{x}_I,\boldsymbol{v}_I}] = -\nabla^{(i)} V_{\mathrm{pot}}$

$\forall\, i \in I$

analyse course of intrinsic action w
by external steering action $\mathrm{RB}^{(i)}$

$E_a{}^{(A)} = \frac{1}{2} \cdot m_a{}^{(A)} \cdot \boldsymbol{v}_a{}^{(A)2}$ $\quad \boldsymbol{p}_a{}^{(A)} = m_a{}^{(A)} \cdot \boldsymbol{v}_a{}^{(A)}$ quantitative equations

$\#\{\mathbf{1}_E|_0\}$ $\quad \#\{\mathbf{1}_p\}$ $\quad \#\{①\}$

equivalent elements in calorimeter model W_{cal}

quantified (physical) measure $\quad E_a = E_a{}^{(A)} \cdot E_{\mathbf{1}(A)}$ $\quad \boldsymbol{p}_a = \boldsymbol{p}_a{}^{(A)} \cdot \boldsymbol{p}_{\mathbf{1}(A)}$

(basic) physical quantities

basic physical measurement/ quantification

$w \sim_{E,p} W_{\mathrm{cal}} := w_{\mathbf{1}} * \ldots * w_{\mathbf{1}}$

observable/ physical measure $\quad E_{ⓐ_{\boldsymbol{v}_a}}$ $\quad \boldsymbol{p}_{ⓐ \cup ⓑ_{\boldsymbol{v}_I}}$

Abstraction: $\sim_E$ $\quad \sim_p$

empirical basis