Title: Bootstrapping N=4 super Yang-Mills

Date: Jun 11, 2013 02:00 PM

URL: http://pirsa.org/13060017

Abstract: I will describe recent results obtained for N=4 superconformal field
theories in four dimensions by means of the conformal bootstrap.

This talk will be related to the content of arXiv:1304.1803, as well as
some additional work in progress.

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$\mathcal{N} = 4$ Bootstrapping

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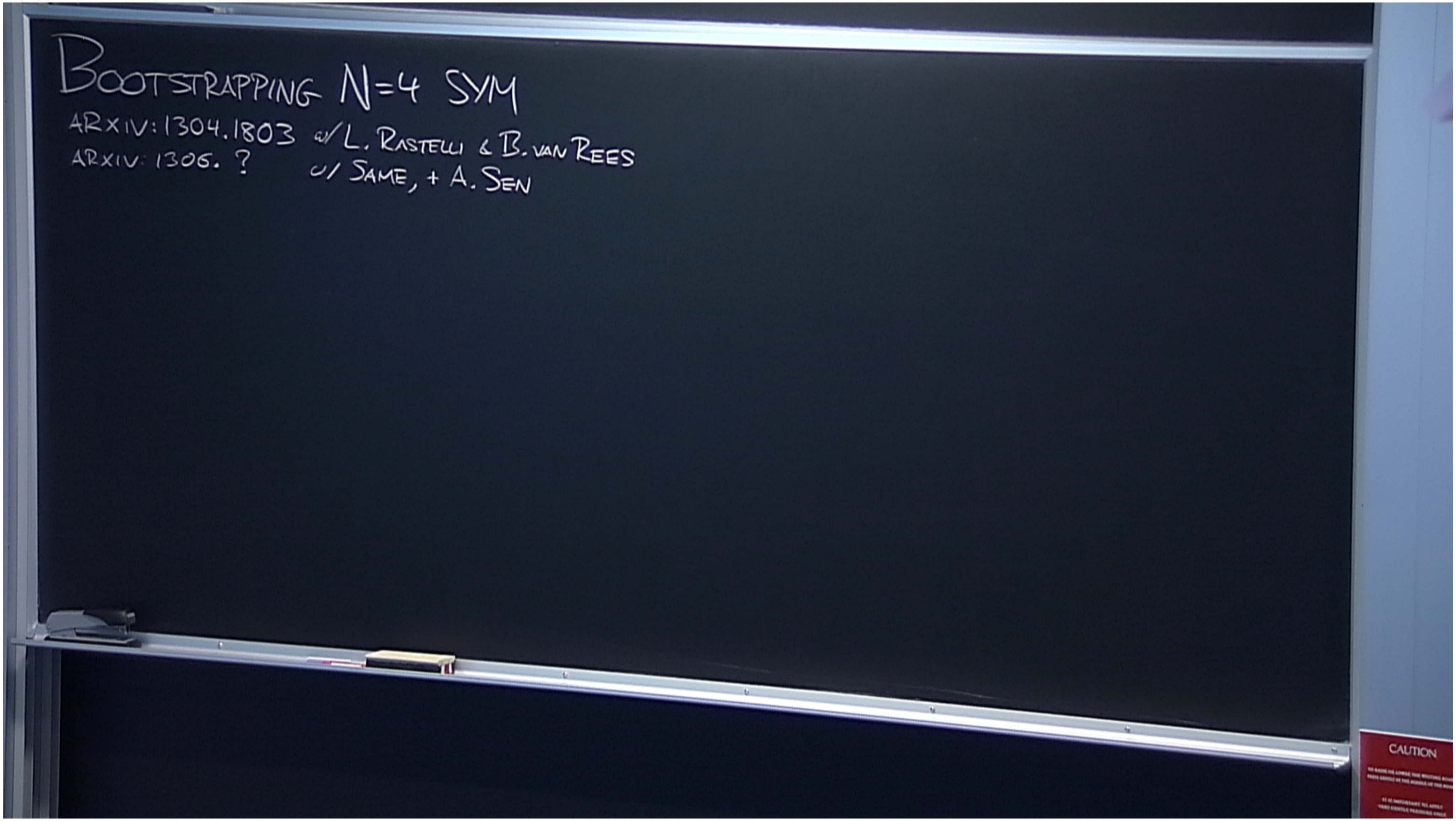
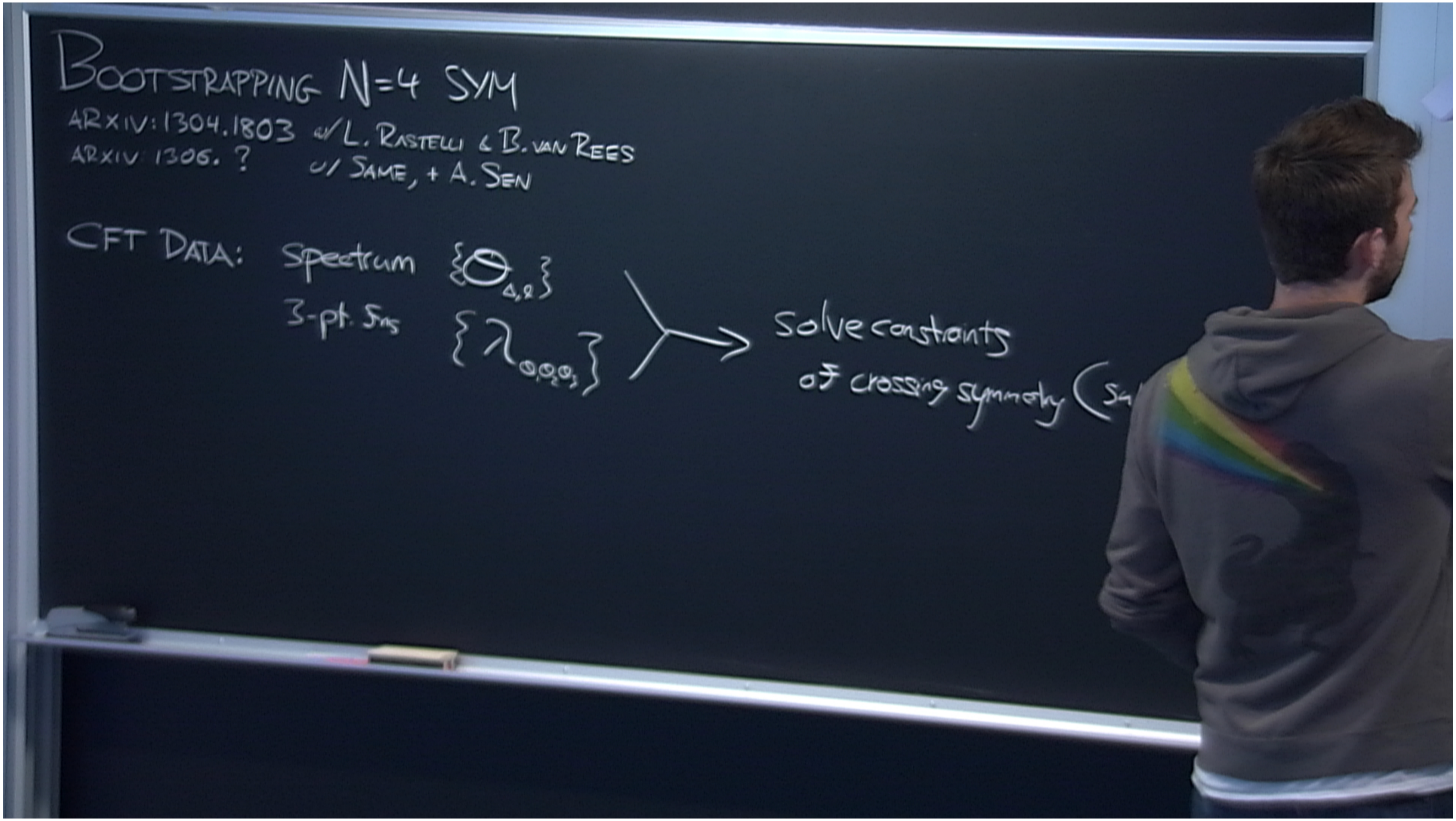
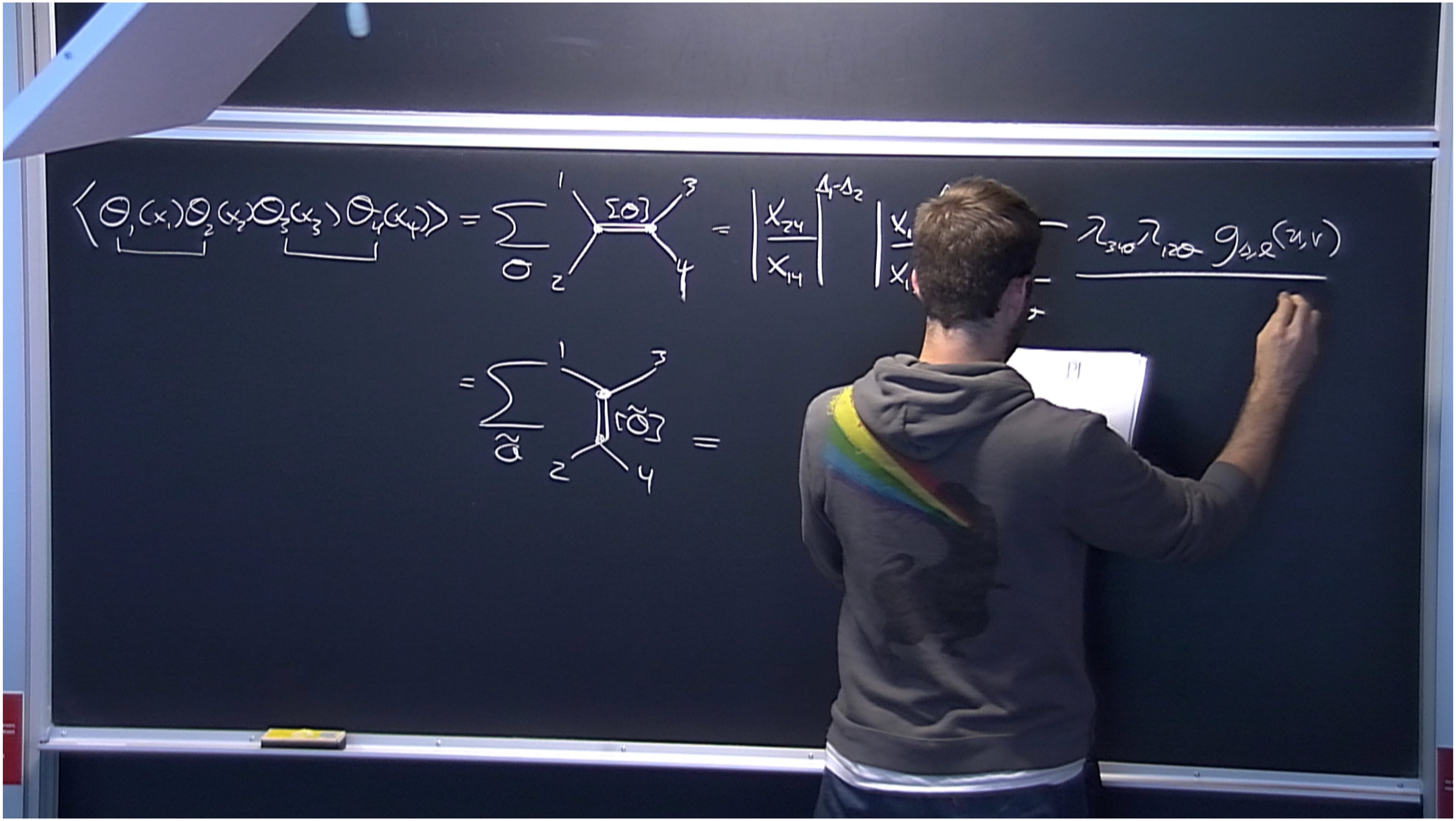
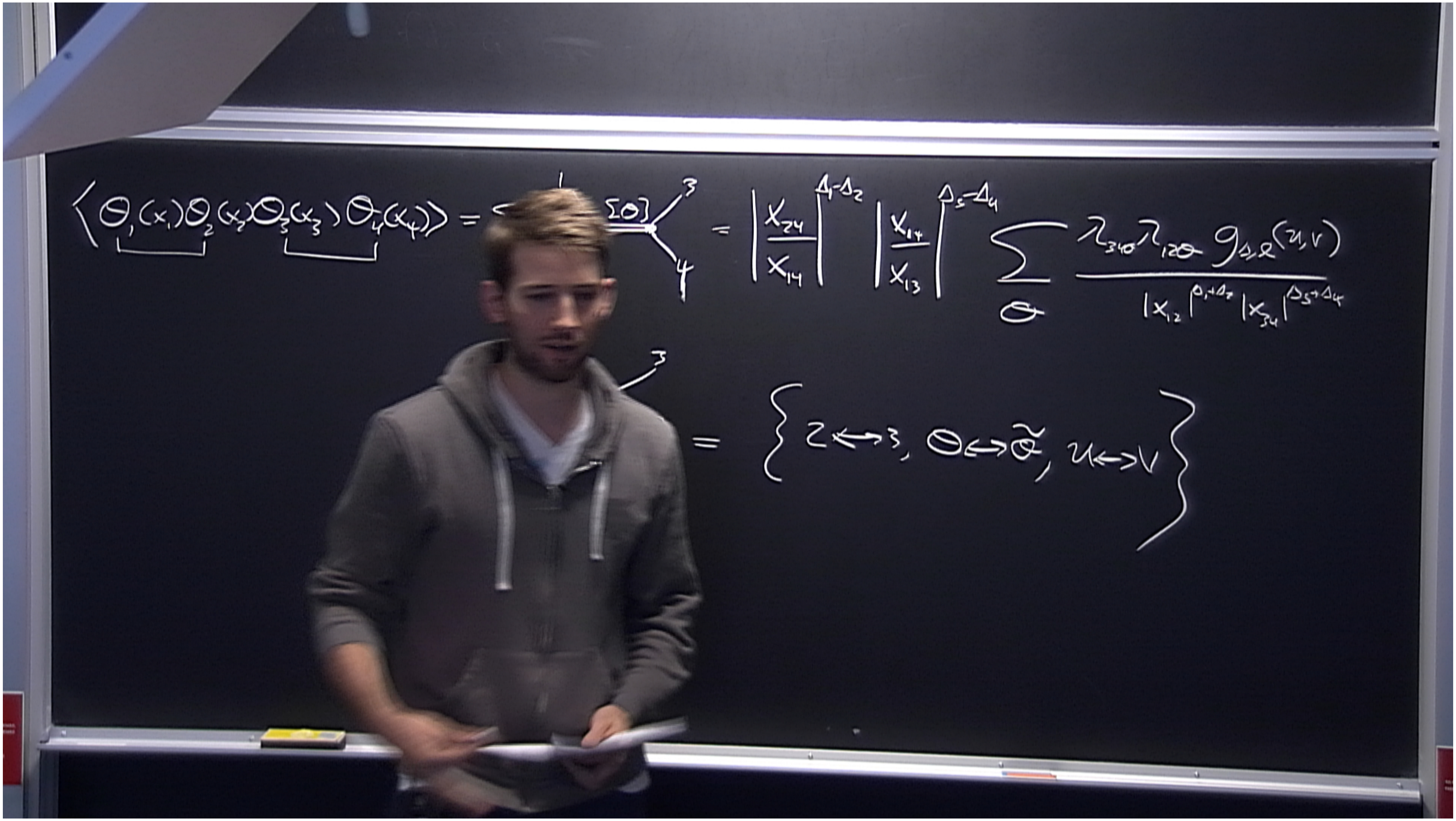
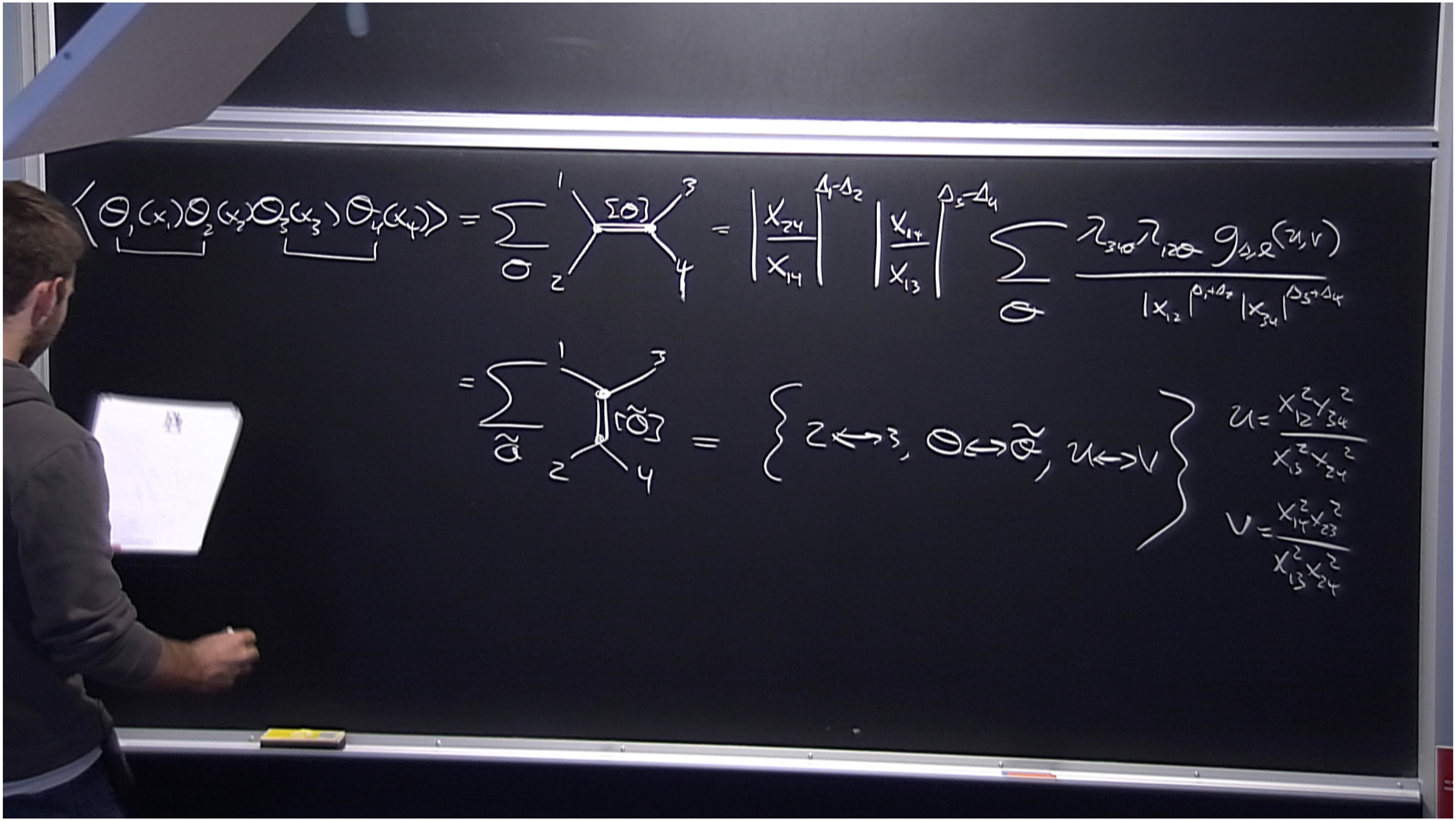
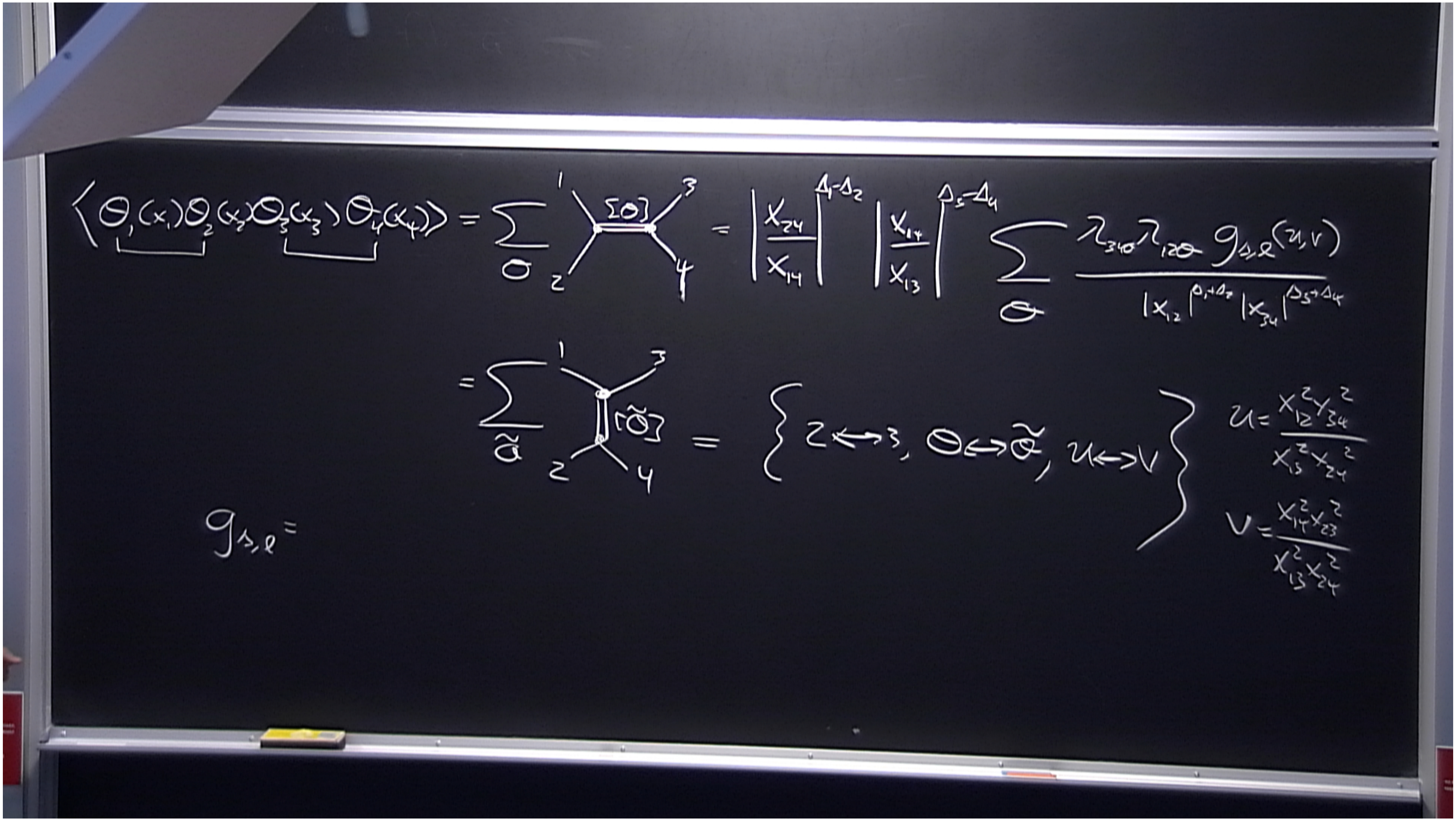

BOOTSTRAPPING N=4 SYM
ARXIV:1304.1803 w/ L. RASTELLI & B. VAN REES
ARXIV: 1306. ? w/ SAME, + A. SEN
CAUTION

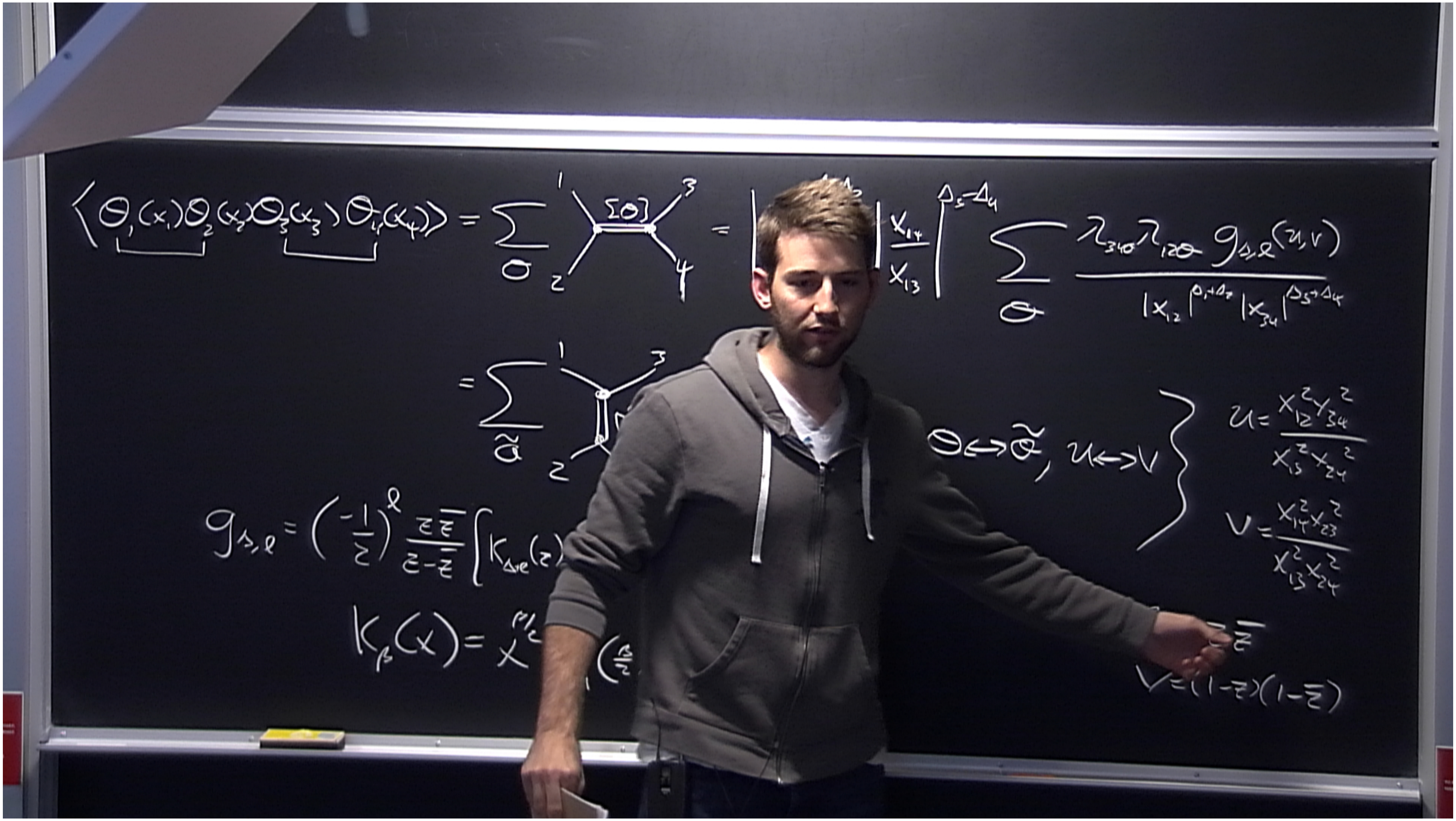

BOOTSTRAPPING N=4 SYM
ARXIV:1304.1803 w/ L. RASTELLI & B. VAN REES
ARXIV: 1306. ? w/ SAME, + A. SEN
CFT DATA: Spectrum {$O_{\Delta,\ell}$}
3-pt. fns {$\lambda_{O_1 O_2 O_3}$}
solve constraints
of crossing symmetry

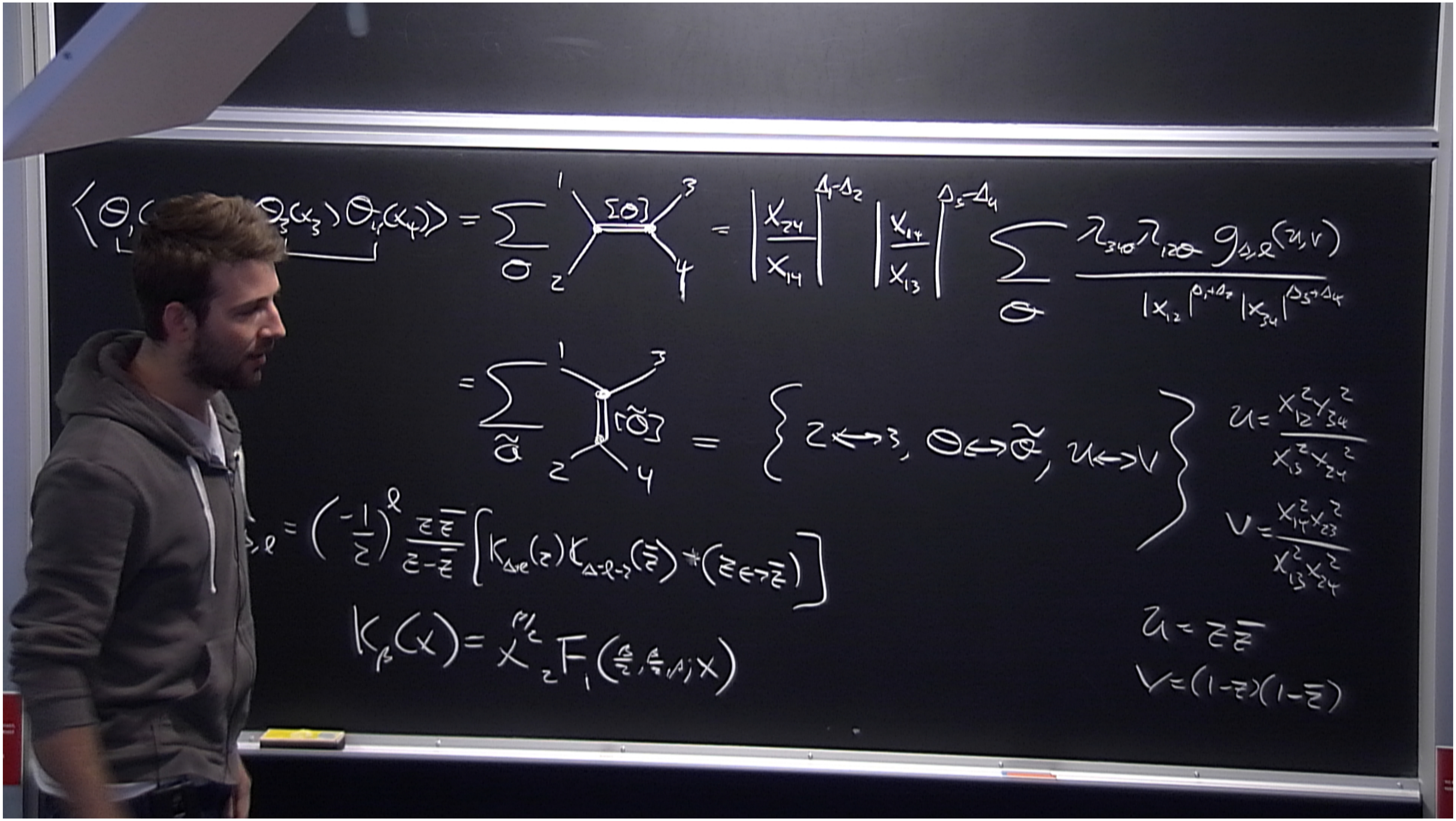

⟨O1(x1)O2(x2)O3(x3)O4(x4)⟩ = Σ_O
[O]
= Σ_Õ
[Õ]
=

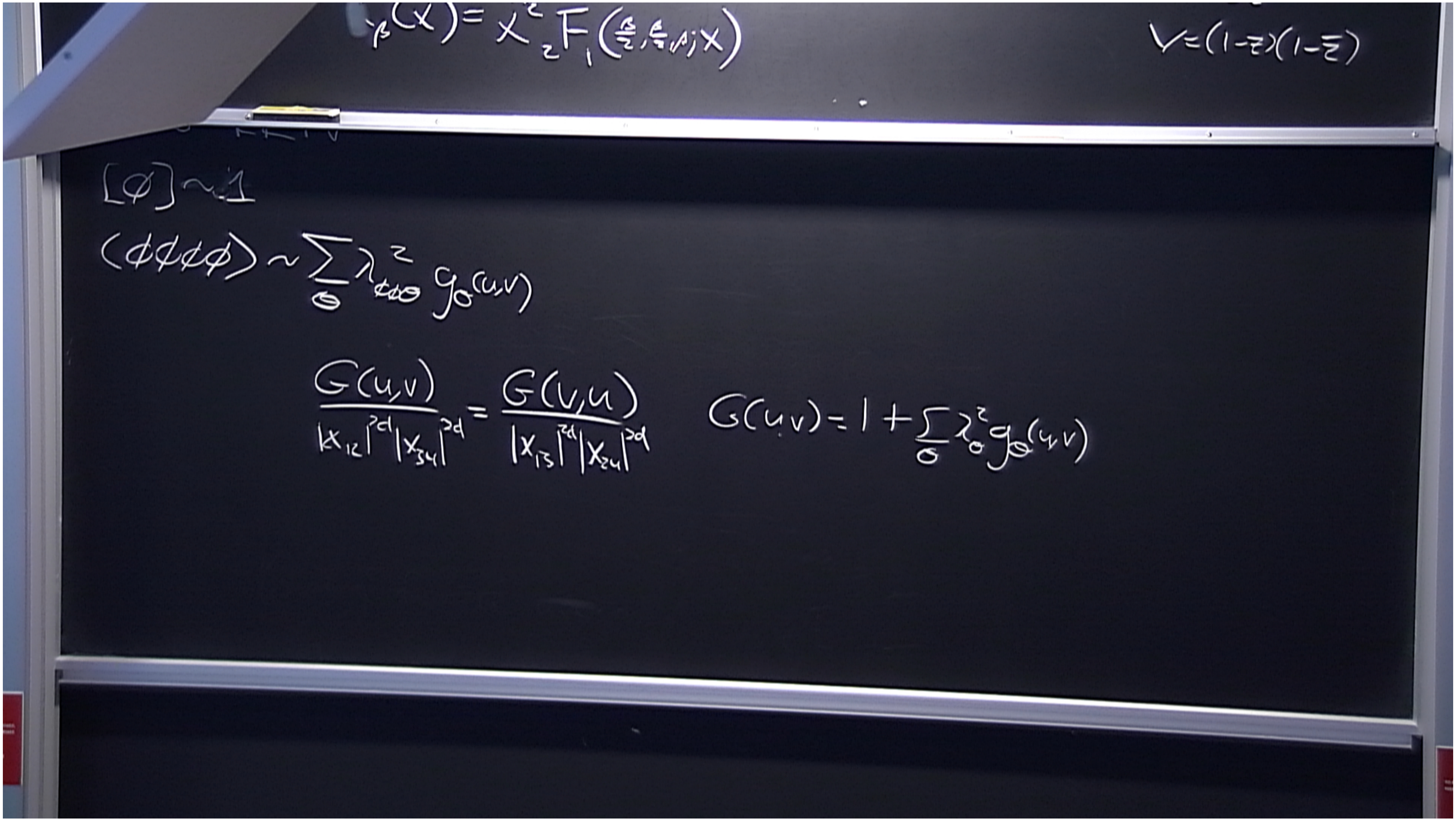

= {2 ↔ 3, O ↔ Õ, u ↔ v}

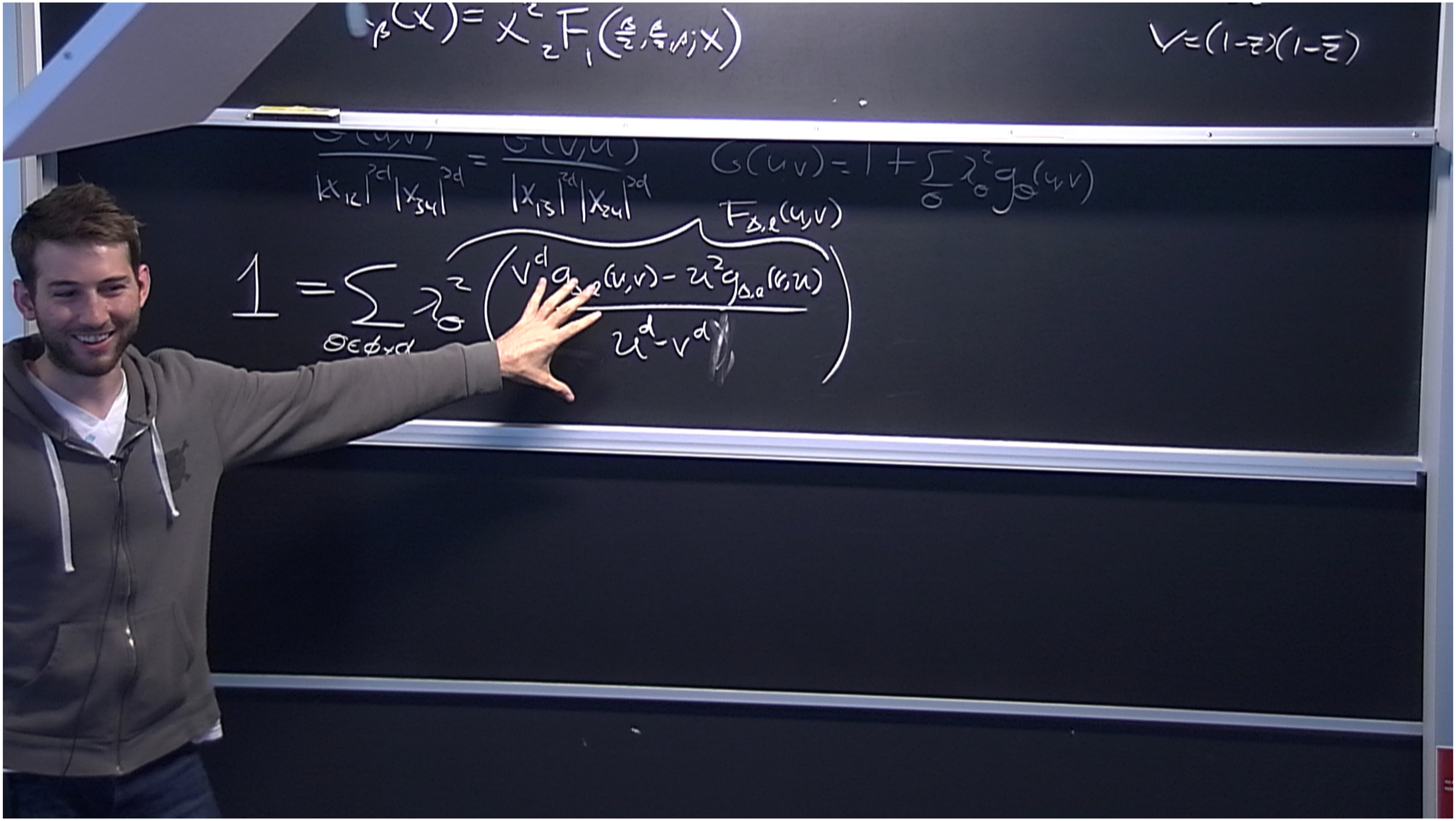

$\langle O_1(x_1) O_2(x_2) O_3(x_3) O_4(x_4) \rangle = \sum_O$ [O] $= \left|\frac{x_{24}}{x_{14}}\right|^{\Delta_1-\Delta_2} \left|\frac{x_{14}}{x_{13}}\right|^{\Delta_3-\Delta_4} \sum_O \frac{\lambda_{34O}\lambda_{12O}\, g_{\Delta,\ell}(u,v)}{|x_{12}|^{\Delta_1+\Delta_2}|x_{34}|^{\Delta_3+\Delta_4}}$
$= \sum_{\tilde{O}}$ $[\tilde{O}]$ $= \{ 2 \leftrightarrow 3,\ O \leftrightarrow \tilde{O},\ u \leftrightarrow v \}$
$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$
$v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$


⟨O1(x1)O2(x2)O3(x3)O4(x4)⟩ = Σ_O [O] = |x24/x14|^(Δ1−Δ2) |x14/x13|^(Δ3−Δ4) Σ_O λ34O λ12O gΔ,ℓ(u,v) / |x12|^(Δ1+Δ2) |x34|^(Δ3+Δ4)
= Σ_Õ [Õ] = {2↔3, O↔Õ, u↔v}
u = x12² x34² / x13² x24²
v = x14² x23² / x13² x24²
gΔ,ℓ =


⟨O1(x1)O2(x2)O3(x3)O4(x4)⟩ = Σ_O
[O]
O ↔ Õ, u ↔ v
u = x12^2 x34^2 / x13^2 x24^2
v = x14^2 x23^2 / x13^2 x24^2
v = (1−z)(1−z̄)


= Σ_O [O]
= |x24/x14|^(Δ1-Δ2) |x14/x13|^(Δ3-Δ4) Σ_O λ34O λ12O g_Δ,ℓ(u,v) / |x12|^(Δ1+Δ2) |x34|^(Δ3+Δ4)
= Σ_Õ [Õ] = {2 ↔ 3, O ↔ Õ, u ↔ v}
u = x12² x34² / x13² x24²
v = x14² x23² / x13² x24²
u = z z̄
v = (1−z)(1−z̄)
K_β(x) = x^(β/2) ₂F₁(β/2, β/2, β; x)


$g_\beta(x) = x^{\beta}\,{}_2F_1(\frac{\beta}{2},\frac{\beta}{2},\beta;x)$
$v=(1-z)(1-\bar{z})$
$[\phi]\sim 1$
$\langle\phi\phi\phi\phi\rangle \sim \sum_{O} \lambda^2_{\phi\phi O}\, g_O(u,v)$
$\frac{G(u,v)}{|x_{12}|^{2d}|x_{34}|^{2d}} = \frac{G(v,u)}{|x_{13}|^{2d}|x_{24}|^{2d}}$
$G(u,v) = 1 + \sum_{O} \lambda_O^2\, g_O(u,v)$


V=(1-z)(1-z̄)
G(u,v) = 1 + Σ λ_O^2 g_O(u,v)
F_{Δ,ℓ}(u,v)

$$g_\beta(x) = x^{2}\,{}_2F_1\left(\tfrac{\beta}{2},\tfrac{\beta}{2},\beta;x\right)$$

$$v = (1-z)(1-\bar{z})$$

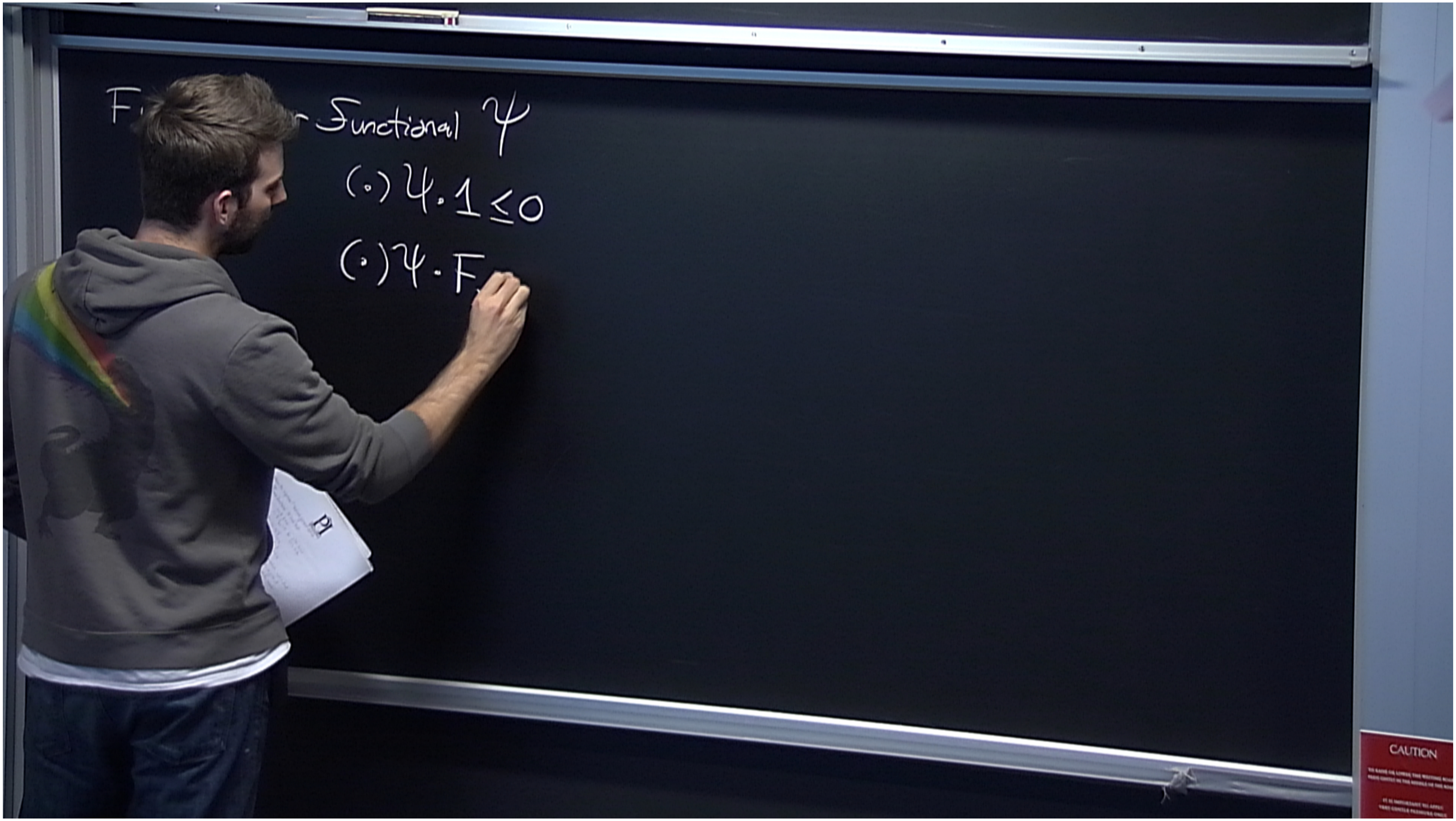
$$\frac{G(u,v)}{|x_{12}|^{2d}|x_{34}|^{2d}} = \frac{G(v,u)}{|x_{13}|^{2d}|x_{24}|^{2d}} \qquad G(u,v) = 1 + \sum_{O} \lambda_O^2 g_O(u,v)$$

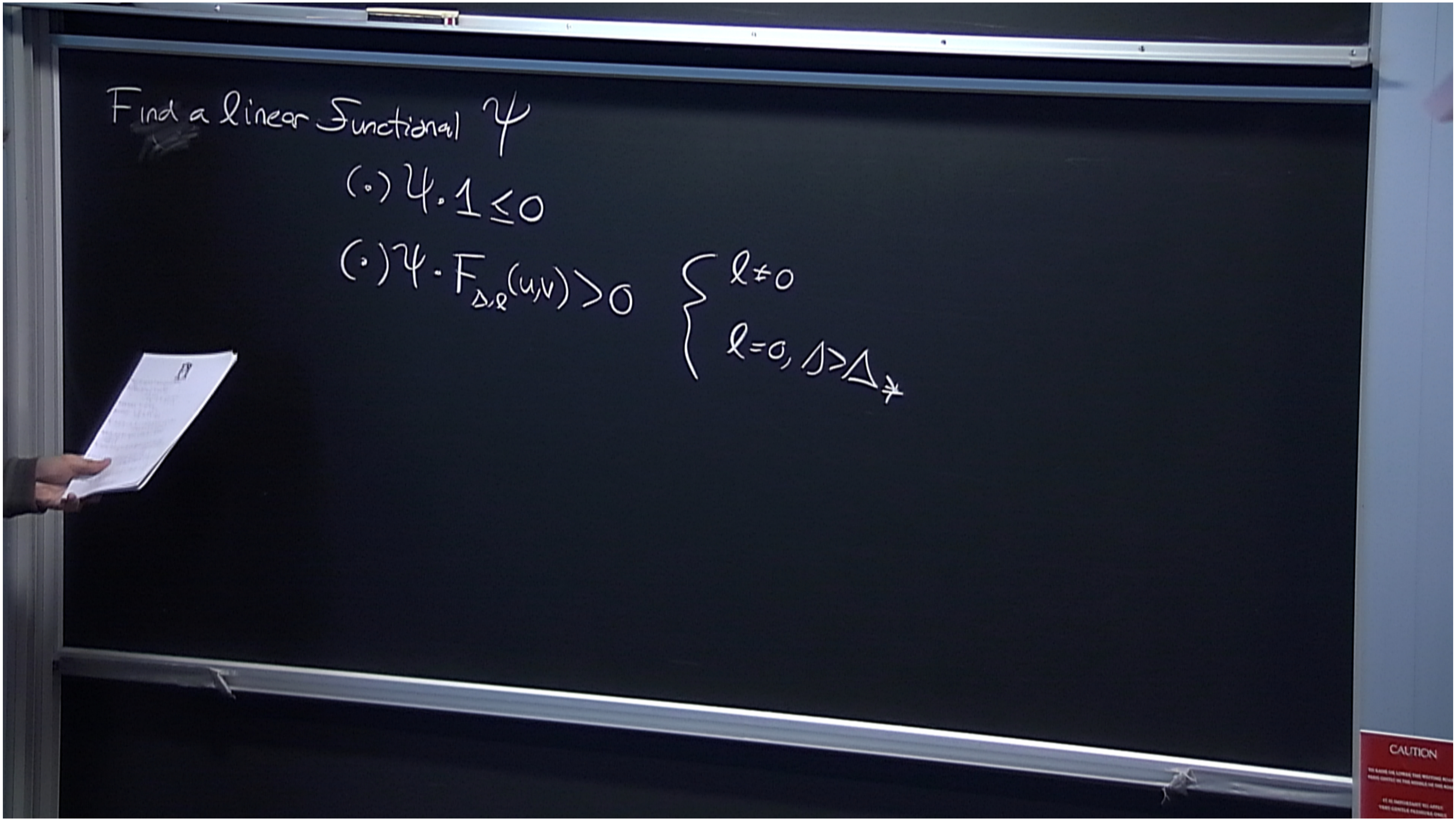
$$1 = \sum_{O\in\phi\times\phi} \lambda_O^2 \overbrace{\left(\frac{v^d g_{\Delta,l}(u,v) - u^2 g_{\Delta,l}(v,u)}{u^d - v^d}\right)}^{F_{\Delta,l}(u,v)}$$

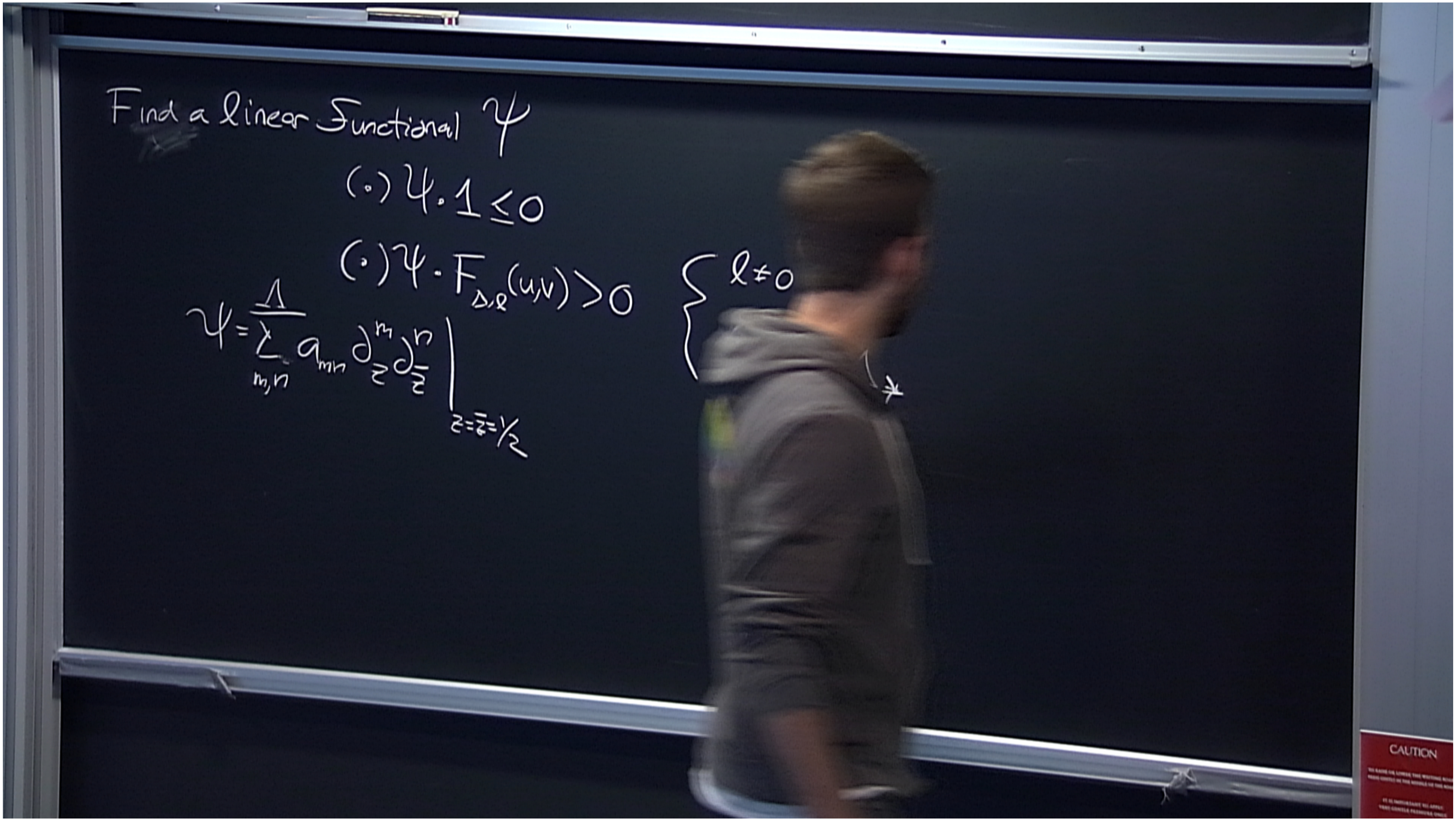
$z=\bar{z}$, $F''_{\Delta,l}\left(\tfrac{1}{2}\right) > 0$ $\quad l \neq 0$

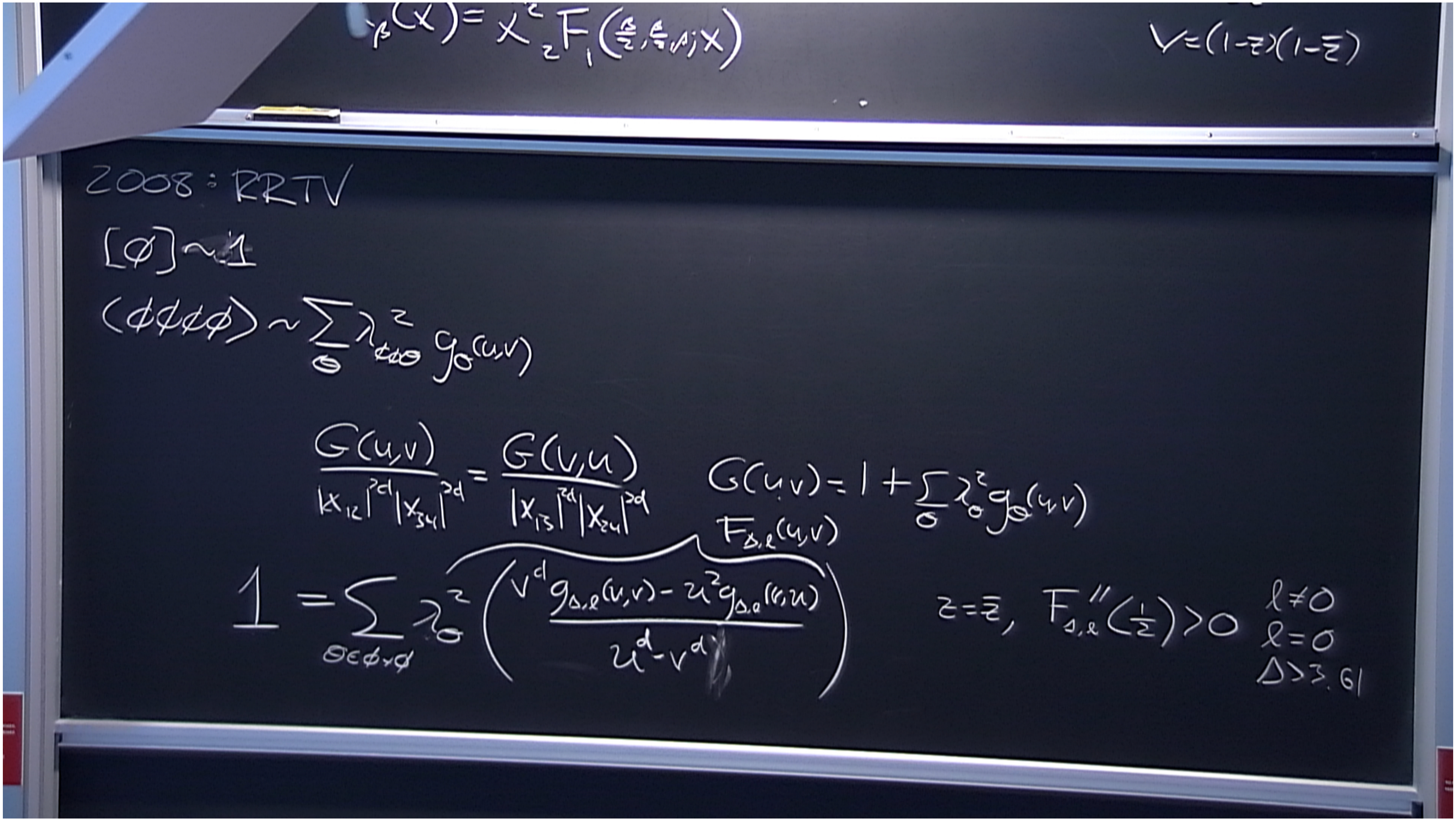
$l = 0$

$\Delta > 3.61$


Functional ψ
(·) ψ·1 ≤ 0
(·) ψ·F

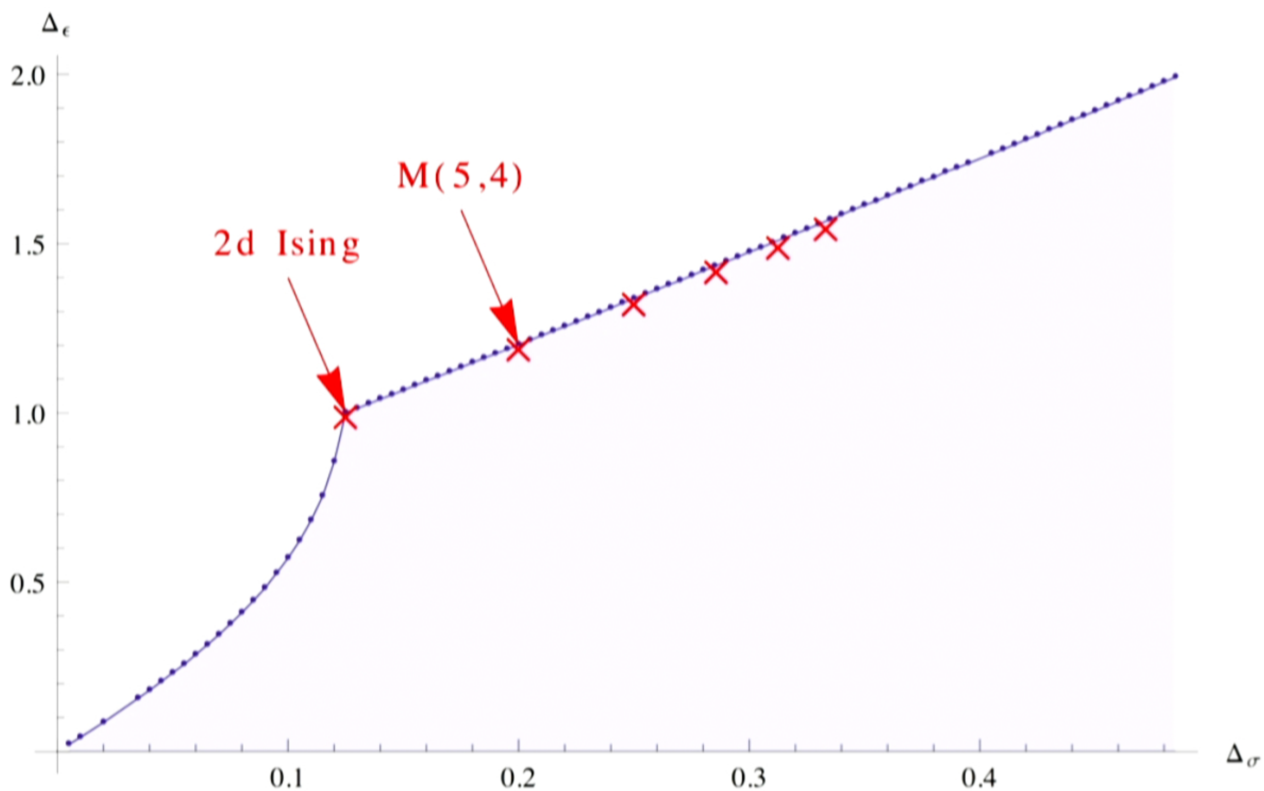

Find a linear functional ψ
(・) $\psi \cdot 1 \le 0$
(・) $\psi \cdot F_{\Delta,\ell}(u,v) > 0$
$\ell \neq 0$
$\ell = 0, \Delta > \Delta_*$
CAUTION


Find a linear functional ψ
(·) ψ·1 ≤ 0
(·) ψ·F_{Δ,ℓ}(u,v) > 0
ℓ≠0
ψ = Σ_{m,n}^{Λ} a_{mn} ∂_z^m ∂_z̄^n |_{z=z̄=1/2}



$_{\beta}(x) = x^{2} {}_2F_1(\frac{\beta}{2}, \frac{\beta}{2}, \beta; x)$
$v = (1-z)(1-\bar{z})$
2008: RRTV
$[\phi] \sim 1$
$\langle \phi\phi\phi\phi \rangle \sim \sum_{O} \lambda^2_{\phi\phi O}\, g_O(u,v)$
$\frac{G(u,v)}{|x_{12}|^{2d}|x_{34}|^{2d}} = \frac{G(v,u)}{|x_{13}|^{2d}|x_{24}|^{2d}}$
$G(u,v) = 1 + \sum_{O} \lambda_O^2 g_O(u,v)$
$F_{\Delta,\ell}(u,v)$
$1 = \sum_{O \in \phi \times \phi} \lambda_O^2 \left(\frac{v^d g_{\Delta,\ell}(u,v) - u^2 g_{\Delta,\ell}(v,u)}{u^d - v^d} \right)$
$z = \bar{z},\ F''_{\Delta,\ell}(\frac{1}{2}) > 0$
$\ell \neq 0$
$\ell = 0$
$\Delta > 3.61$

El-Showk, Paulos (2012)



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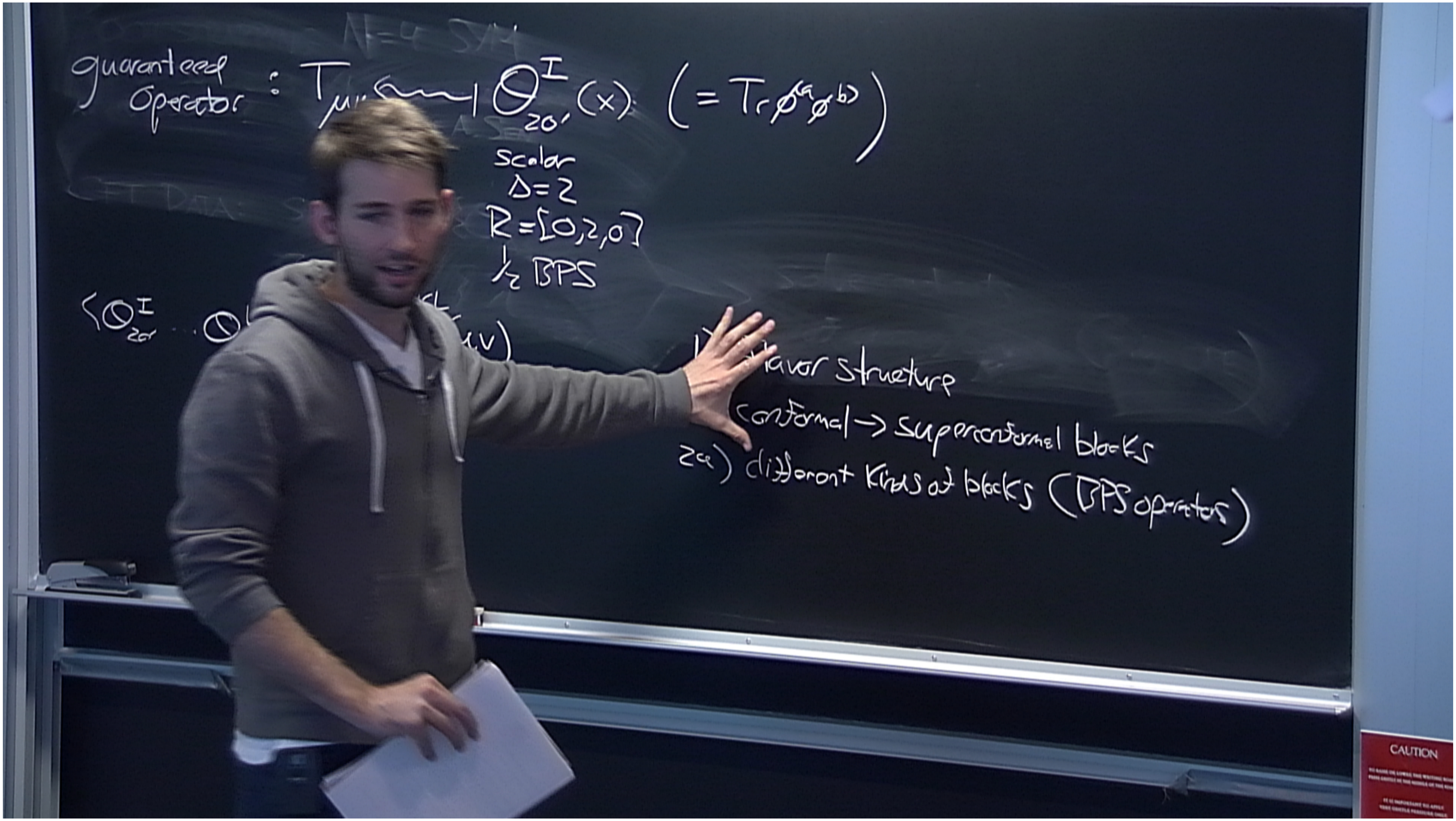
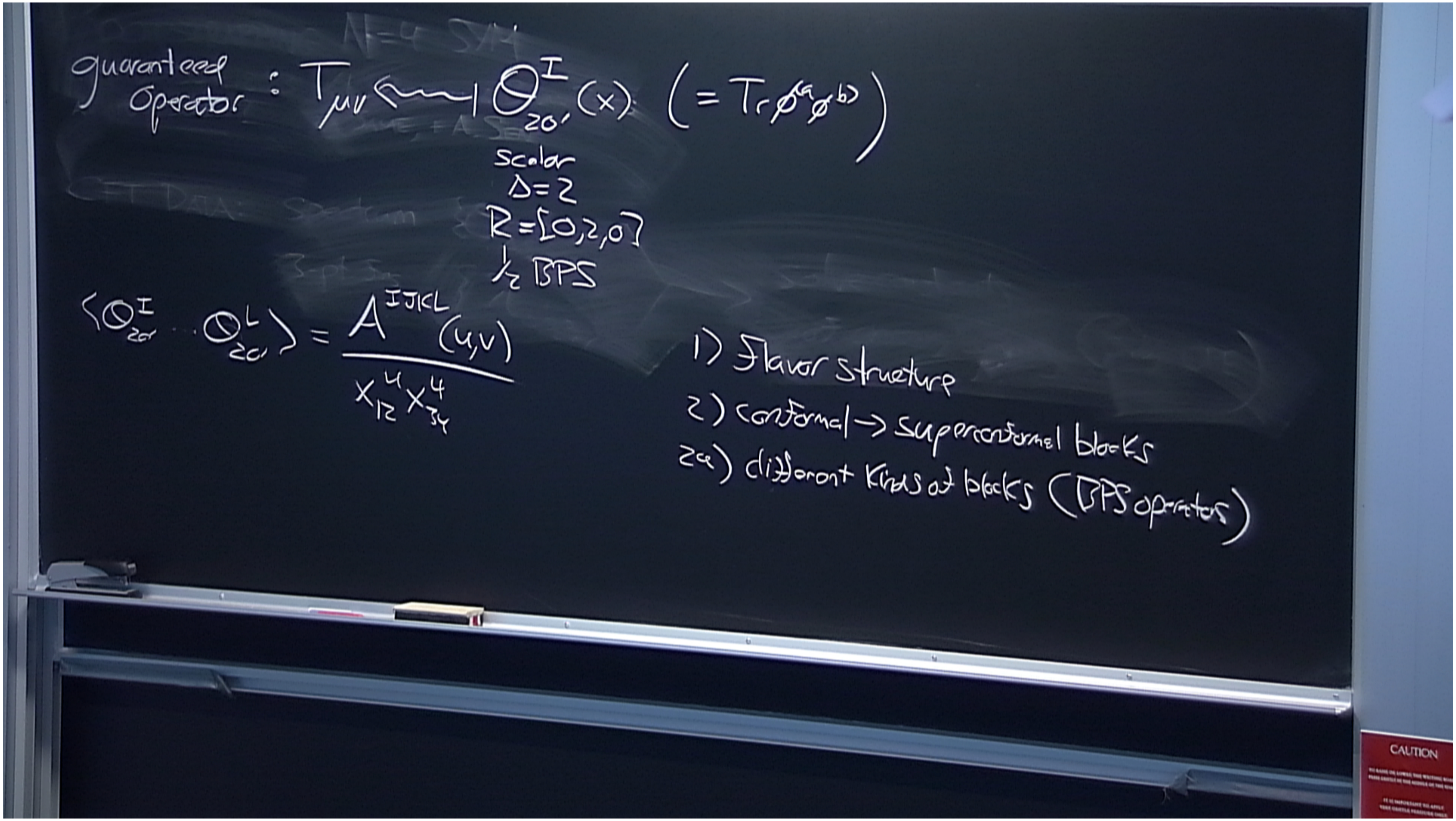
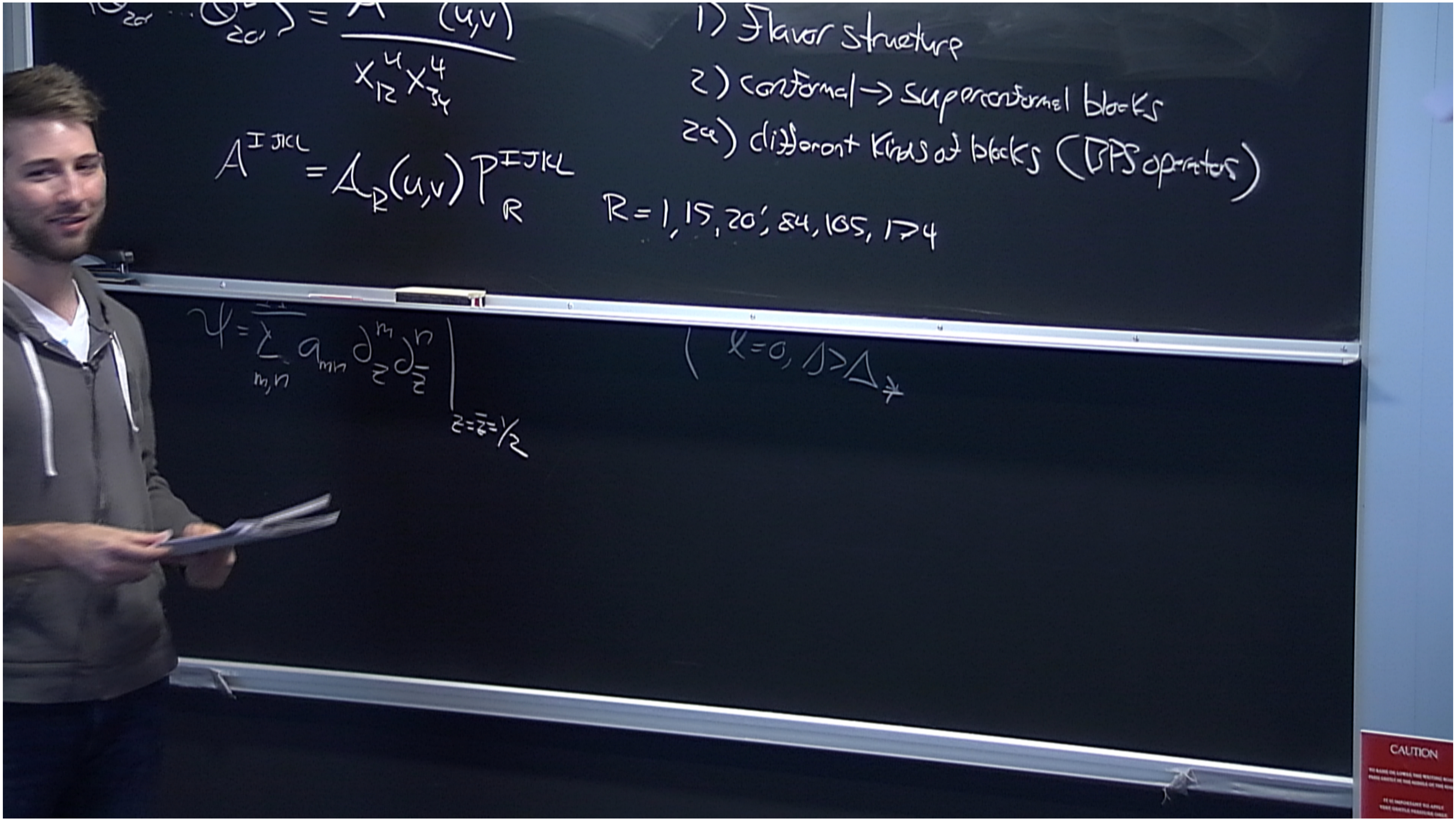
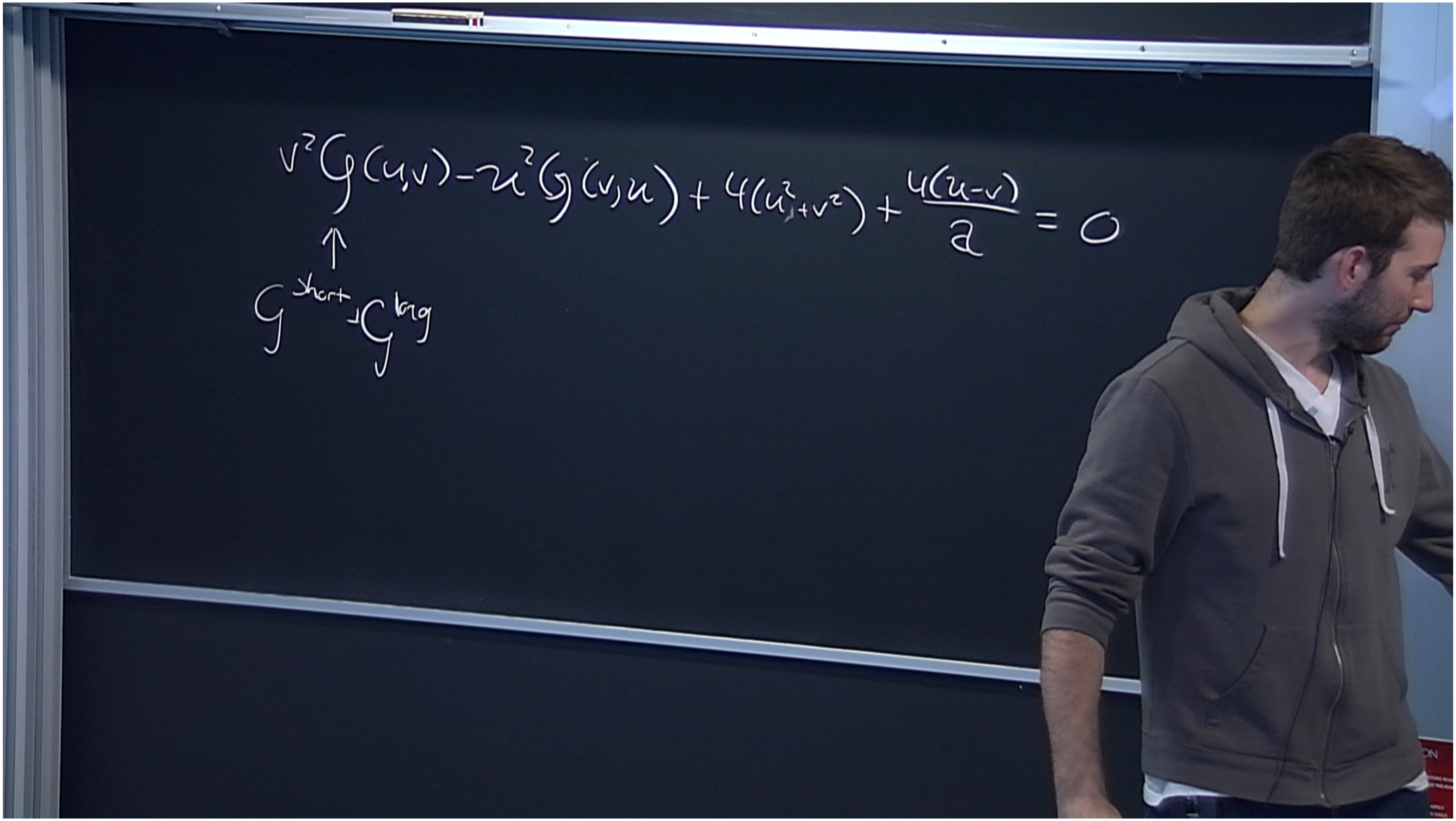

guaranteed Operator : $T_{\mu\nu}$ ~ $\mathcal{O}^{I}_{20'}(x)$ $(= \mathrm{Tr}\,\phi^{a}\phi^{b})$
scalar
$\Delta = 2$
$R = [0,2,0]$
$\frac{1}{2}$ BPS
flavor structure
conformal → superconformal blocks
2a) different kinds of blocks (BPS operators)
CAUTION

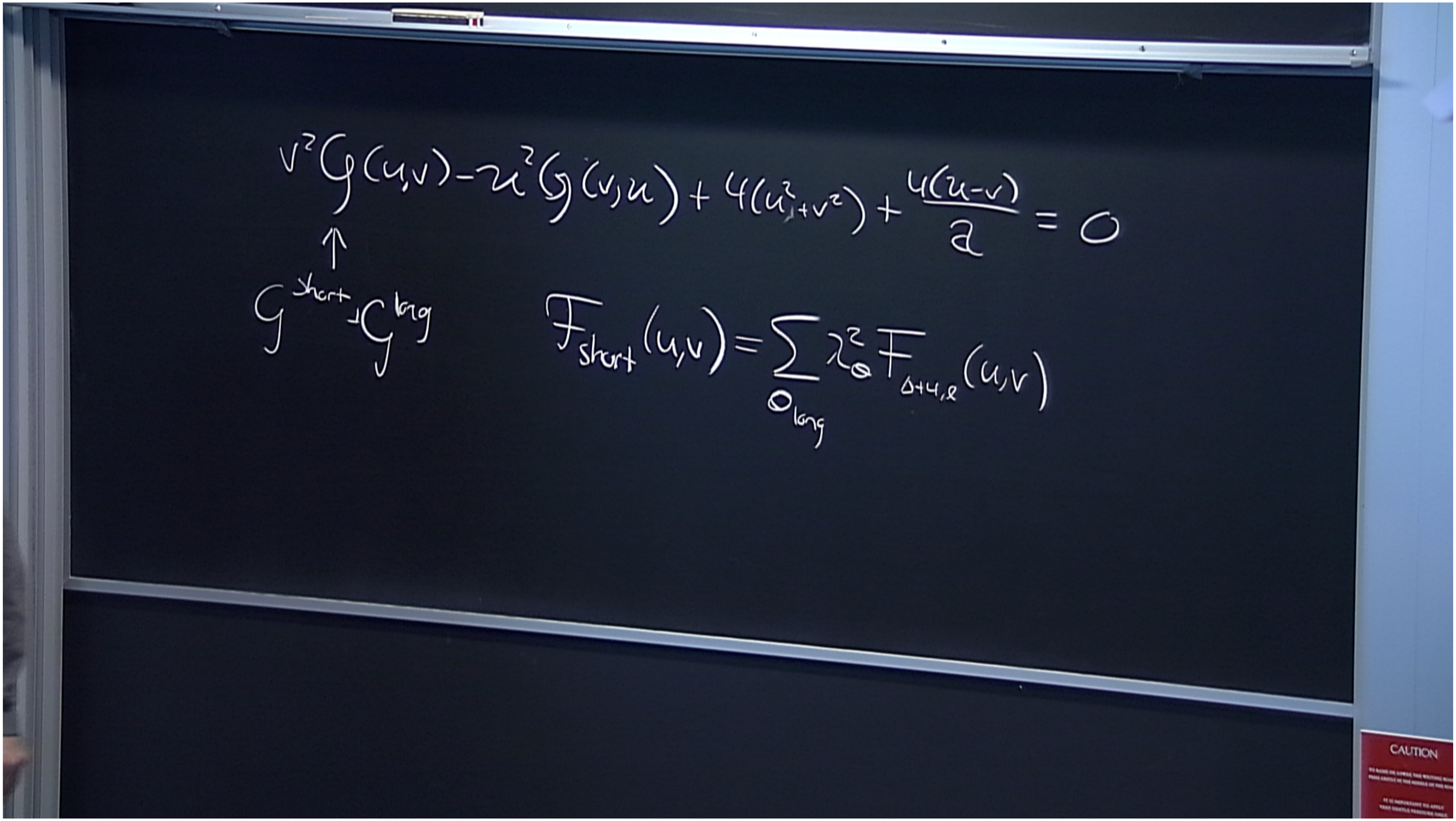

guaranteed Operator : Tμν ⟿ O^I_20'(x) (= Tr φ^(a φ^b))
scalar
Δ=2
R=[0,2,0]
½ BPS
⟨O^I_20' ⋯ O^L_20'⟩ = A^IJKL(u,v) / x_12^4 x_34^4
1) flavor structure
2) conformal → superconformal blocks
2a) different kinds of blocks (BPS operators)


1) Flavor structure
2) conformal → superconformal blocks
2a) different kinds of blocks (BPS operators)
CAUTION

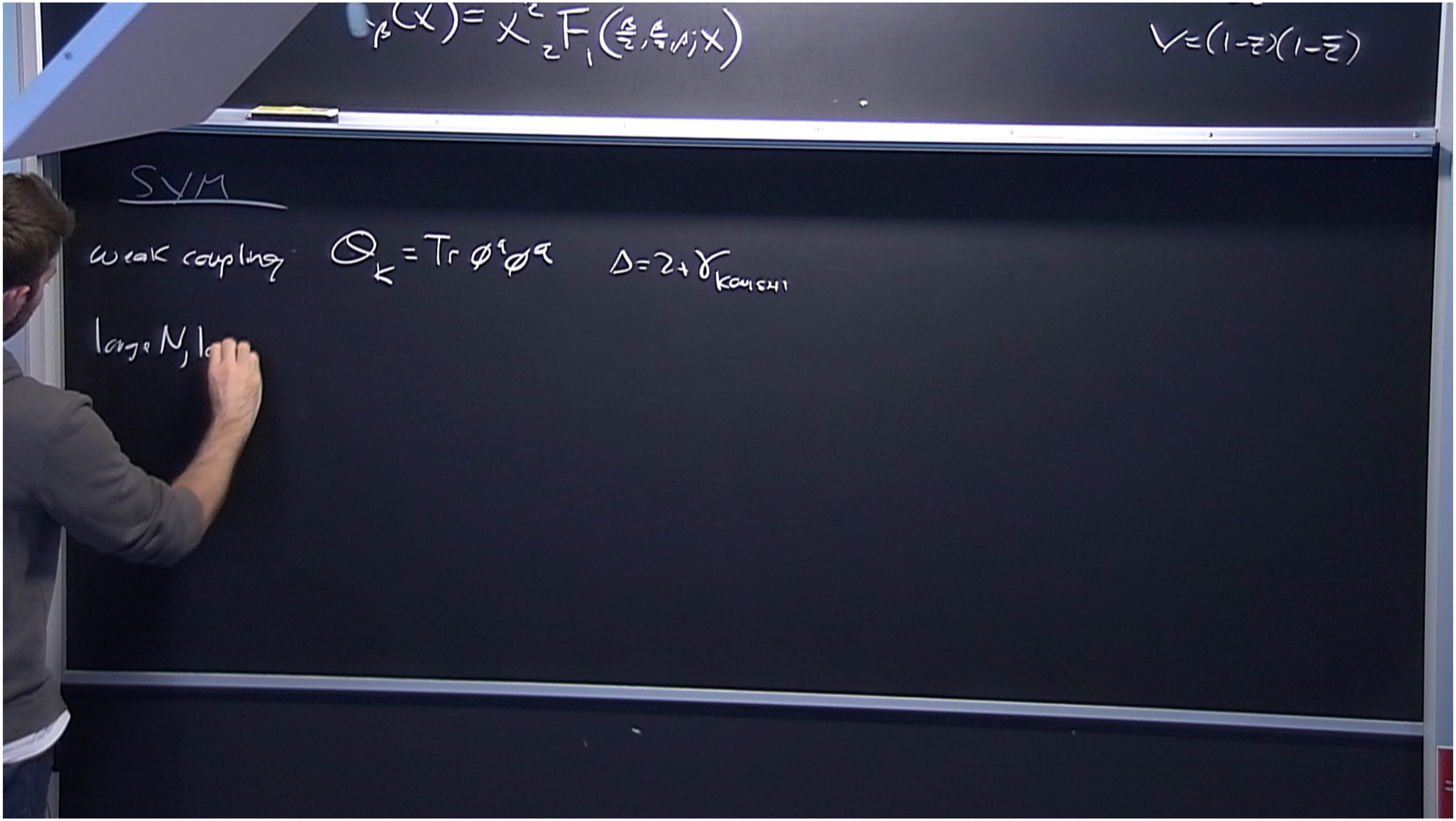
Dolan, Osborn (2001)

$$
\begin{aligned}
A_1 &= \tfrac{1}{3}\big(u^2 + 10(1-v)^2 - 8u(1+v) + 60v\big)\mathcal{G} \\
&\quad - \frac{10}{3}(1-v)\,\frac{\tilde{f}_2(z) - \tilde{f}_2(x)}{z-x} + \frac{8}{3}u\,\frac{\frac{2-z}{z}\tilde{f}_2(z) - \frac{2-x}{x}\tilde{f}_2(x)}{z-x} \\
&\quad + 5u\,\frac{\big(\frac{2-z}{z}\big)^2\tilde{f}(z) - \big(\frac{2-x}{x}\big)^2\tilde{f}(x)}{z-x} - \frac{5}{3}u\,\frac{\tilde{f}(z) - \tilde{f}(x)}{z-x} \\
&\quad - 20\big(f_2(z) + f_2(x)\big) + 20k\,, \\
A_{20} &= \tfrac{1}{6}\big(u^2 + 10(1-v)^2 - 5u(1+v)\big)\mathcal{G} \\
&\quad - \frac{5}{3}(1-v)\,\frac{\tilde{f}_2(z) - \tilde{f}_2(x)}{z-x} + \frac{5}{6}u\,\frac{\frac{2-z}{z}\tilde{f}_2(z) - \frac{2-x}{x}\tilde{f}_2(x)}{z-x} \\
&\quad + \frac{5}{3}u\,\frac{\tilde{f}(z) - \tilde{f}(x)}{z-x}\,, \\
A_{84} &= \tfrac{1}{2}u\big(3(1+v) - u\big)\mathcal{G} - \tfrac{3}{2}u\,\frac{\frac{2-z}{z}\tilde{f}_2(z) - \frac{2-x}{x}\tilde{f}_2(x)}{z-x}\,, \\
A_{105} &= u^2\mathcal{G}\,, \\
A_{15} &= -(1-v)\big(2(1+v) - u\big)\mathcal{G} + 2(1-v)\,\frac{\frac{2-z}{z}\tilde{f}_2(z) - \frac{2-x}{x}\tilde{f}_2(x)}{z-x} \\
&\quad - u\,\frac{\big(\frac{2-z}{z}\big)^2\tilde{f}_2(z) - \big(\frac{2-x}{x}\big)^2\tilde{f}_2(x)}{z-x} - 2u\,\frac{\frac{2-z}{z}\tilde{f}(z) - \frac{2-x}{x}\tilde{f}(x)}{z-x}\,, \\
A_{175} &= -u(1-v)\mathcal{G} + u\,\frac{\tilde{f}_2(z) - \tilde{f}_2(x)}{z-x}\,.
\end{aligned}
\tag{6.14}
$$


$v^2 G(u,v) - u^2 G(v,u) + 4(u^2+v^2) + \frac{4(u-v)}{2} = 0$
$G^{short} + G^{long}$


$v^2 G(u,v) - u^2 G(v,u) + 4(u^2+v^2) + \frac{4(u-v)}{a} = 0$
$G^{short} + G^{long}$
$F_{short}(u,v) = \sum_{O_{long}} \lambda_O^2 F_{\Delta+4,\ell}(u,v)$
CAUTION

The new "identity" contribution


(x) = x 2F1(
v=(1-z)(1-z̄)
SYM
weak coupling
O_k = Tr φ φ
Δ = 2 + γ_Konishi
large N,

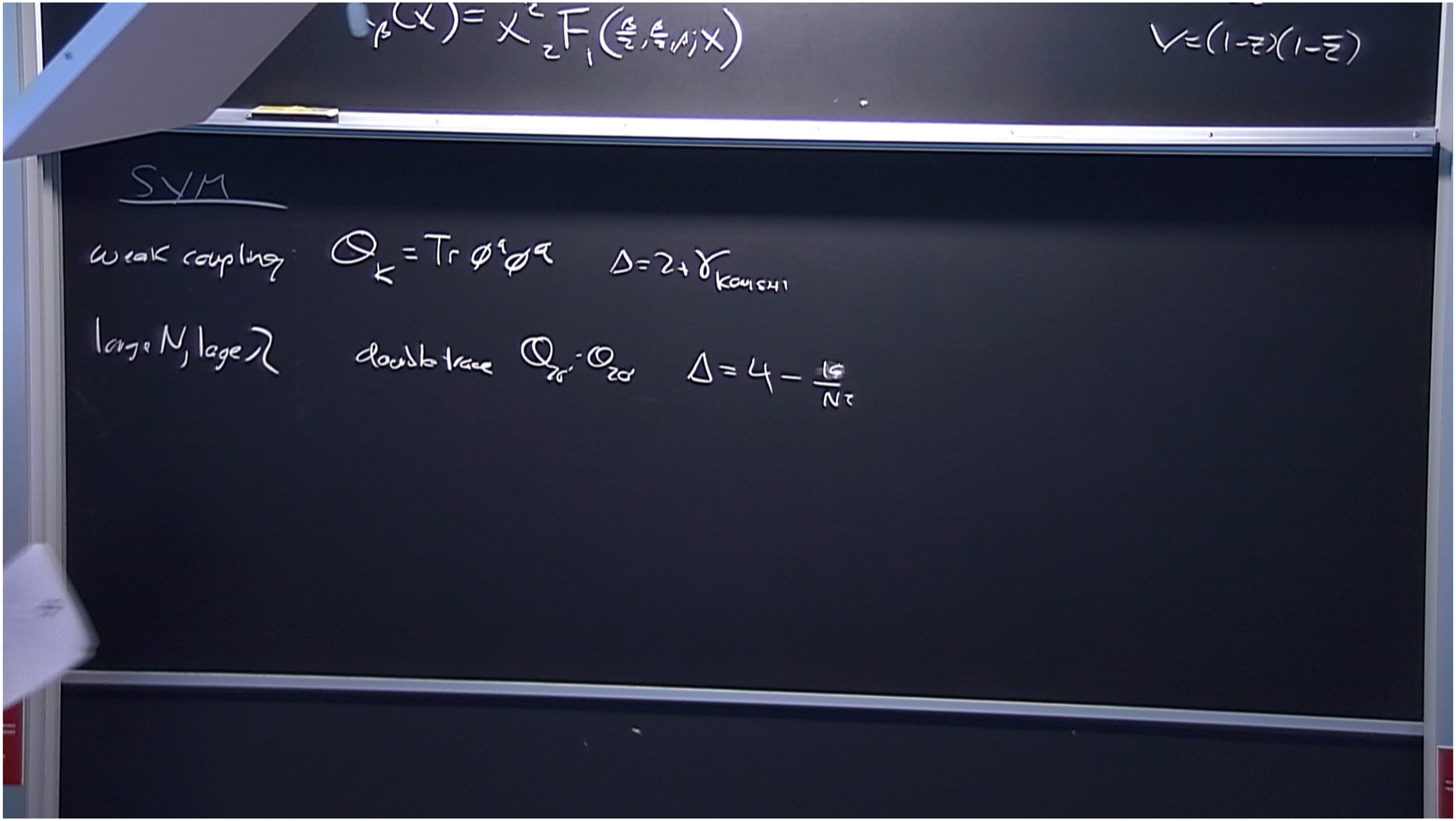
$_\beta(x) = x^2 \, {}_2F_1\left(\frac{\beta}{2},\frac{\beta}{2},\beta; x\right)$

$v = (1-z)(1-\bar{z})$

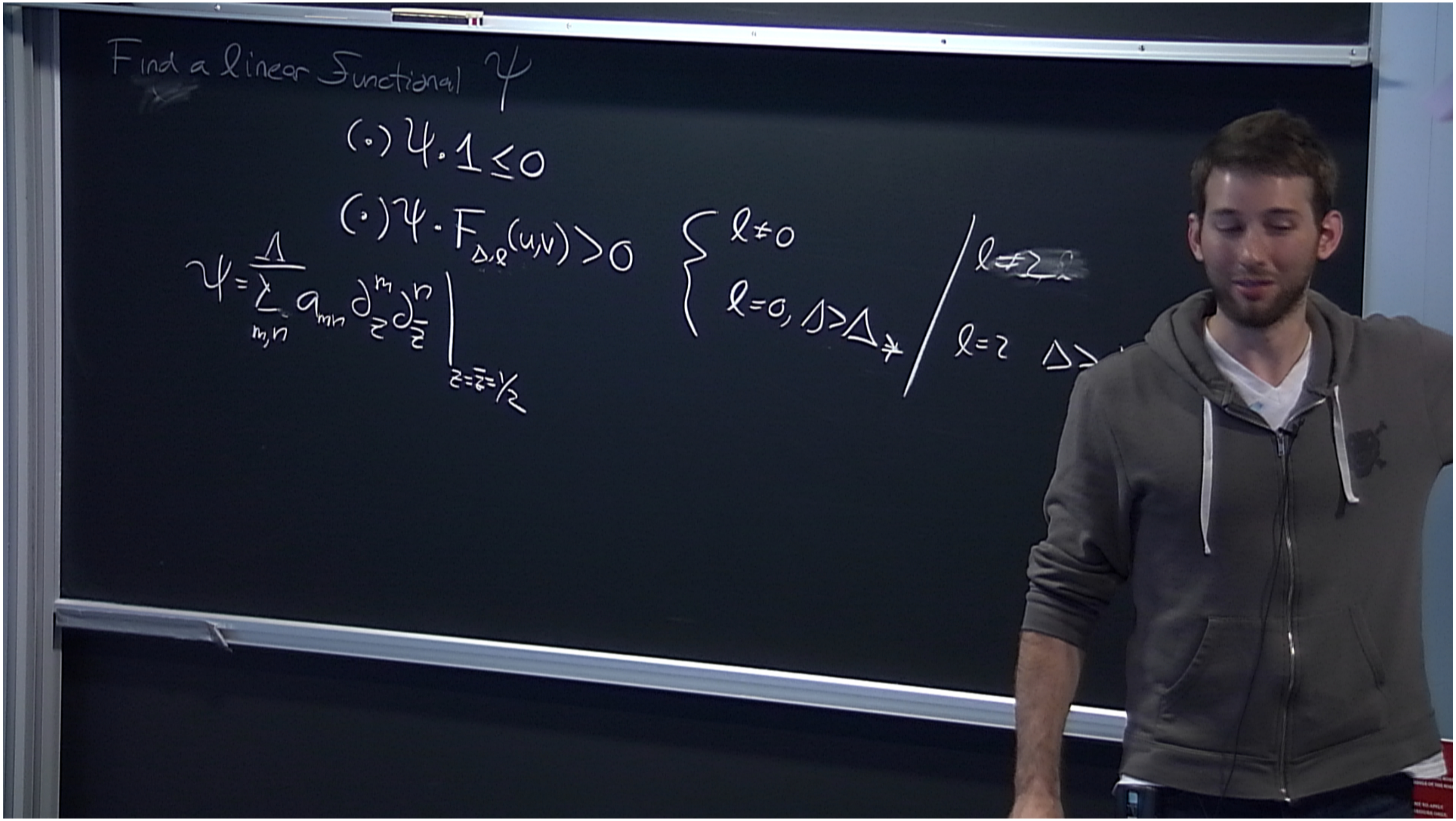
SYM

weak coupling: $O_K = \mathrm{Tr}\, \phi^q \phi^q$ $\Delta = 2 + \gamma_{Konishi}$

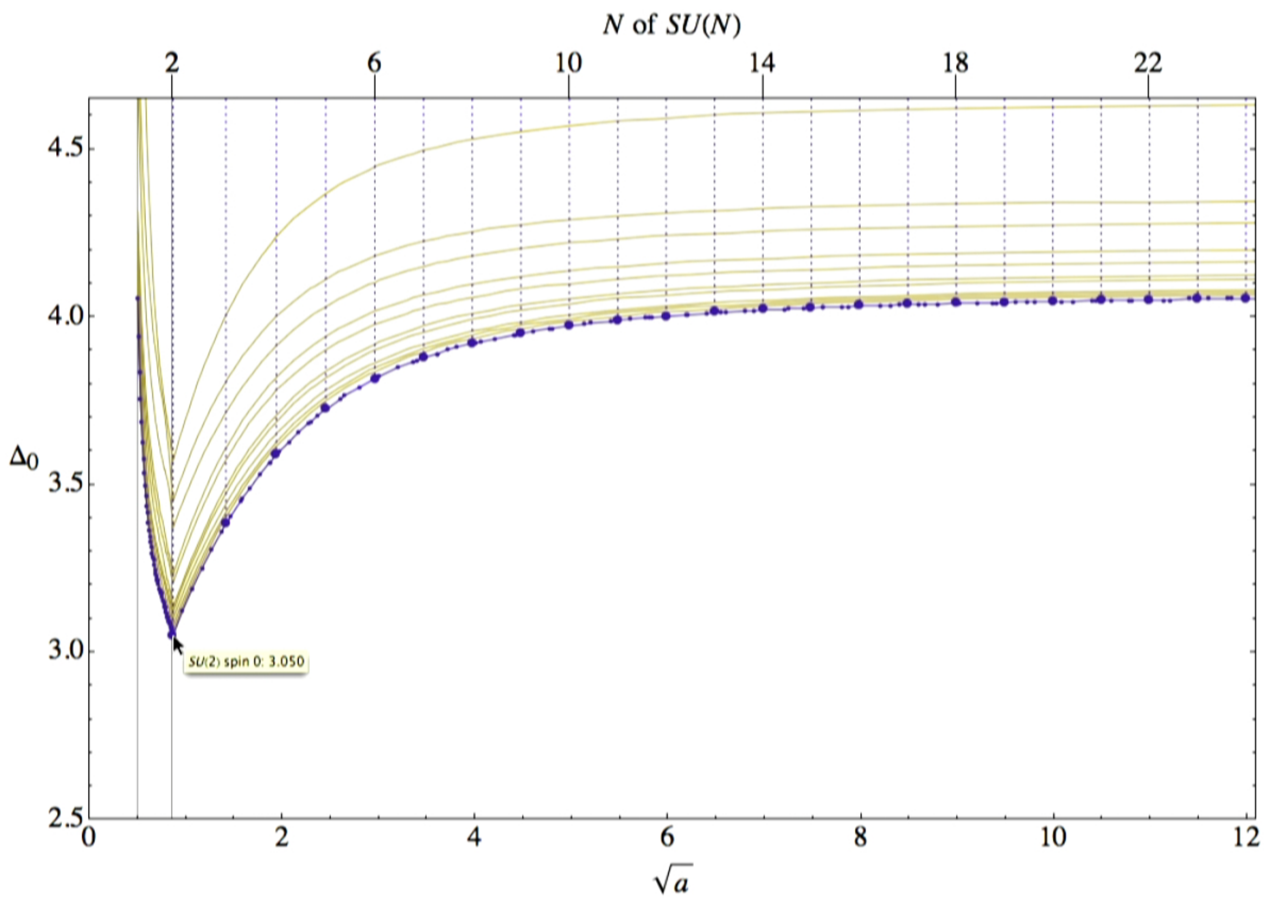
large N, large λ double trace $O_{20'} \cdot O_{20'}$ $\Delta = 4 - \frac{16}{N^2}$



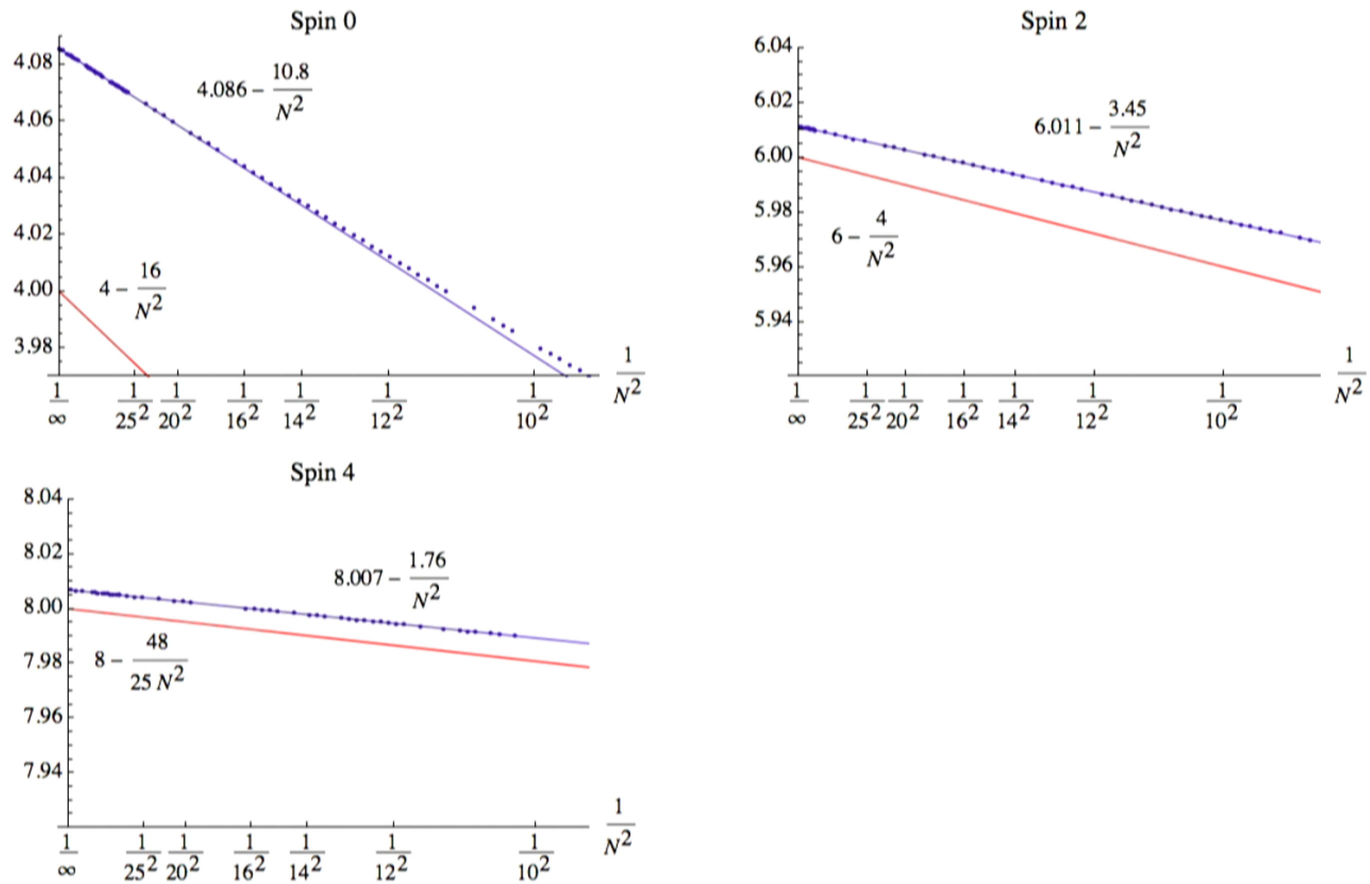
β(x) = x² ₂F₁(β/2, β/2, β; x)
v = (1−z)(1−z̄)
SYM
weak coupling: O_k = Tr φ^q φ^q   Δ = 2 + γ_Konishi
large N, large λ   double trace O_20'·O_20'   Δ = 4 − 16/N²


Find a linear Functional ψ
(·) ψ·1 ≤ 0
(·) ψ·F_{Δ,ℓ}(u,v) > 0
ℓ≠0
ℓ=0, Δ>Δ*
ψ = Σ^Λ_{m,n} a_{mn} ∂_z^m ∂_z̄^n |_{z=z̄=1/2}
ℓ=2 Δ>

Bounding first unprotected scalar



Bounds at large central charge



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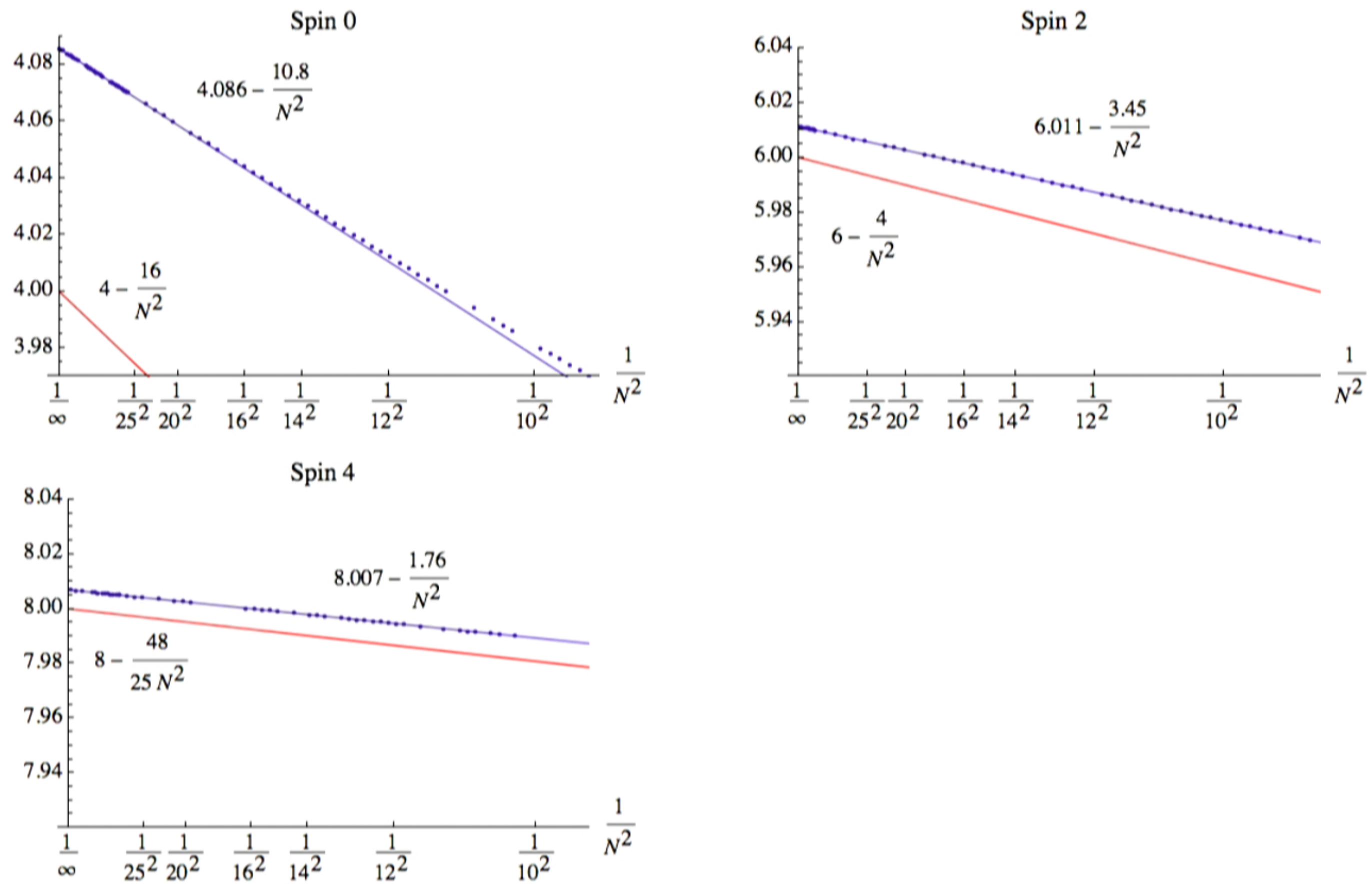
Bounds at large central charge



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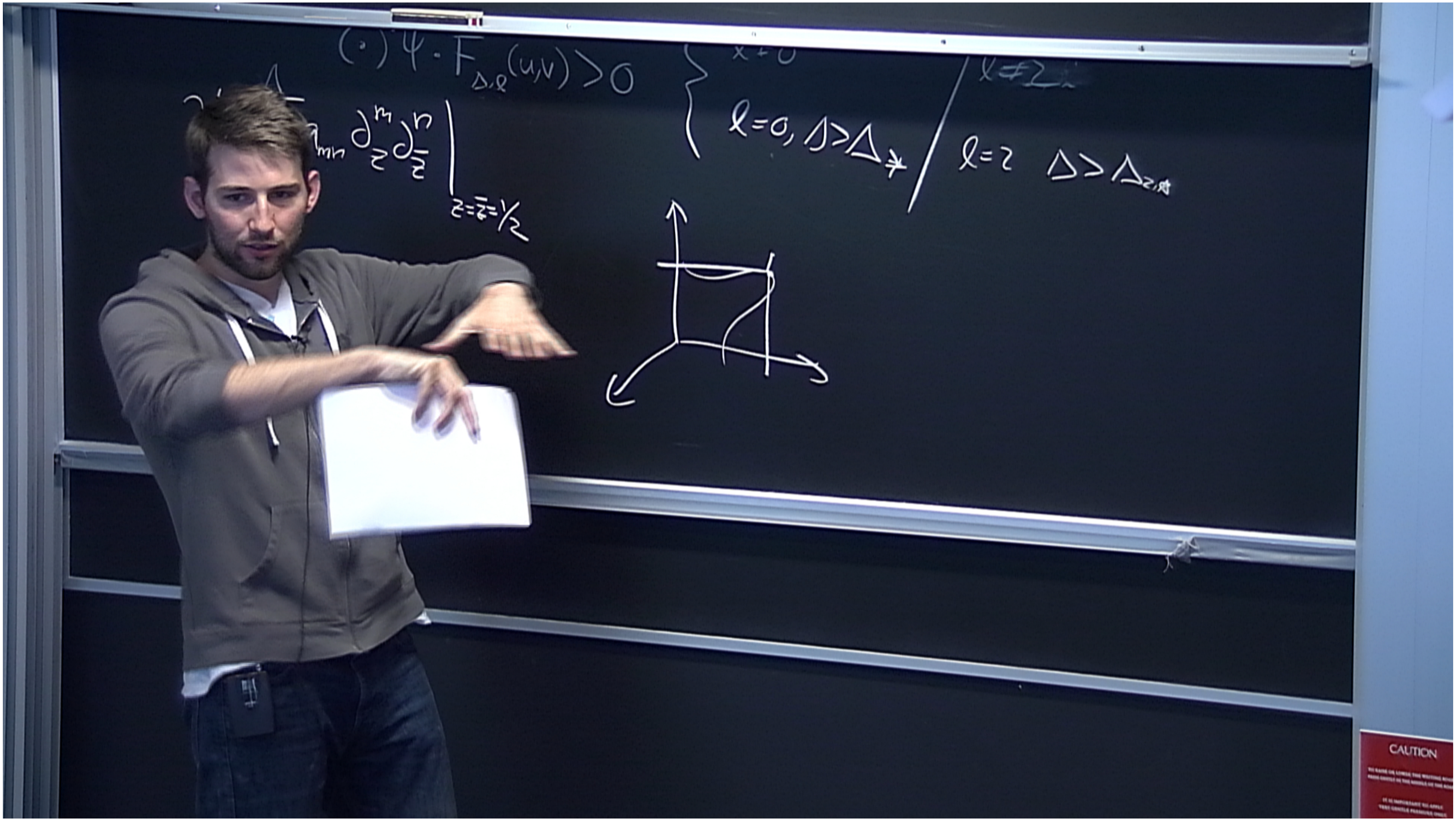
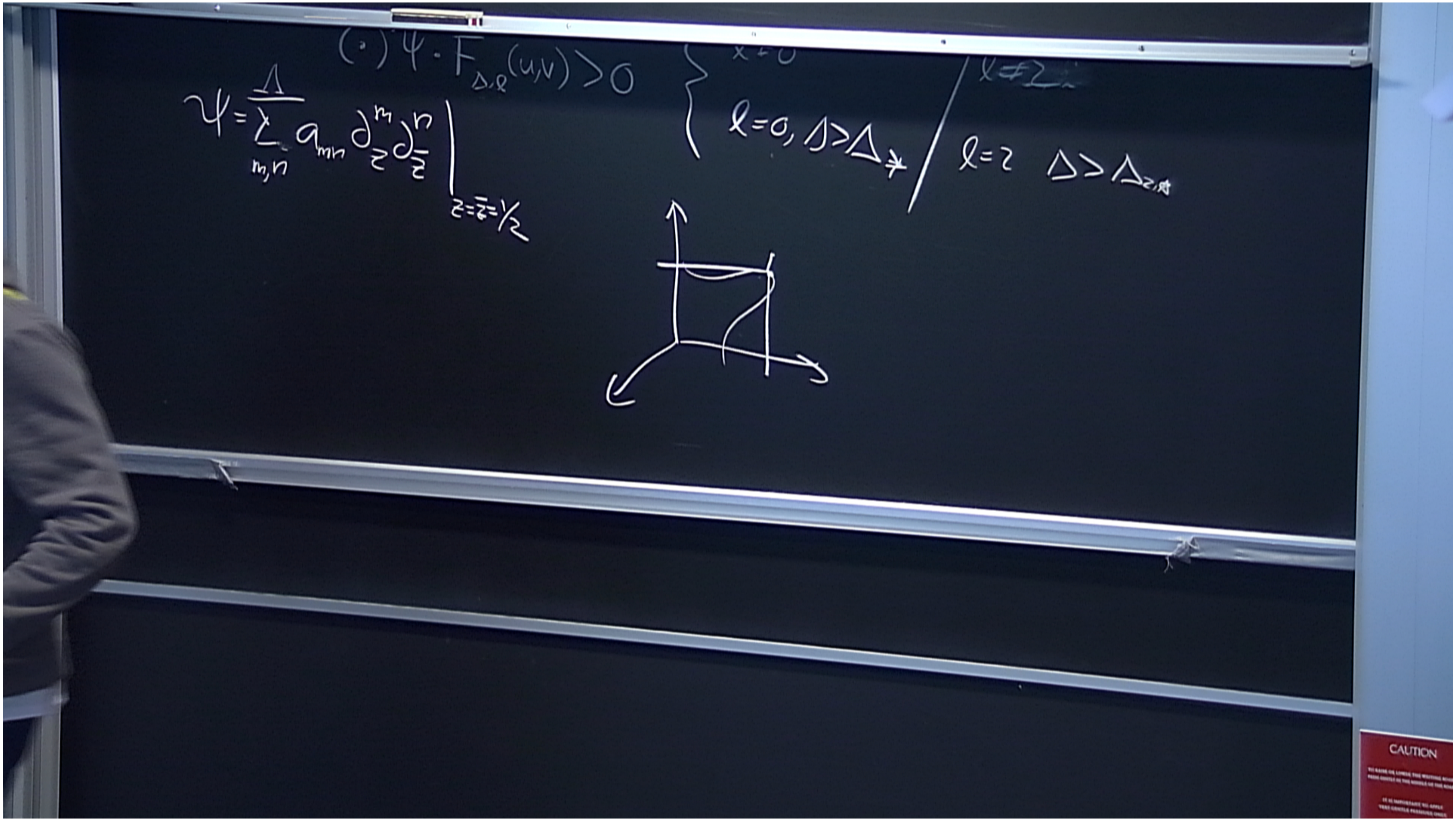
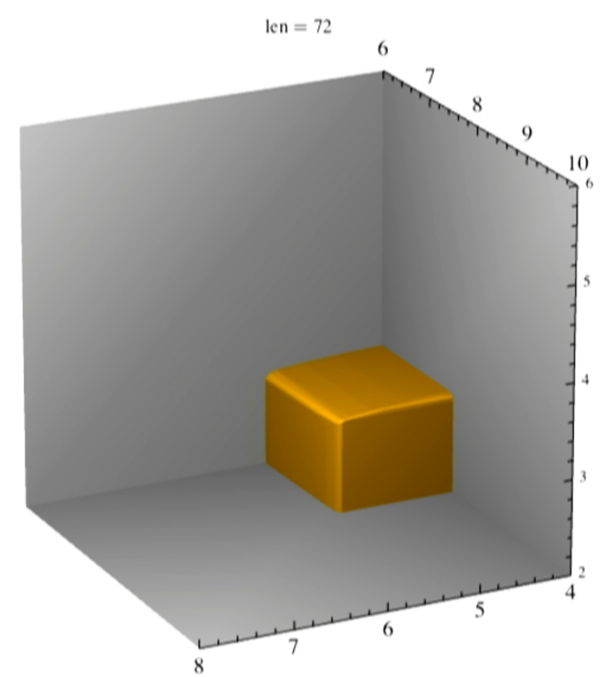
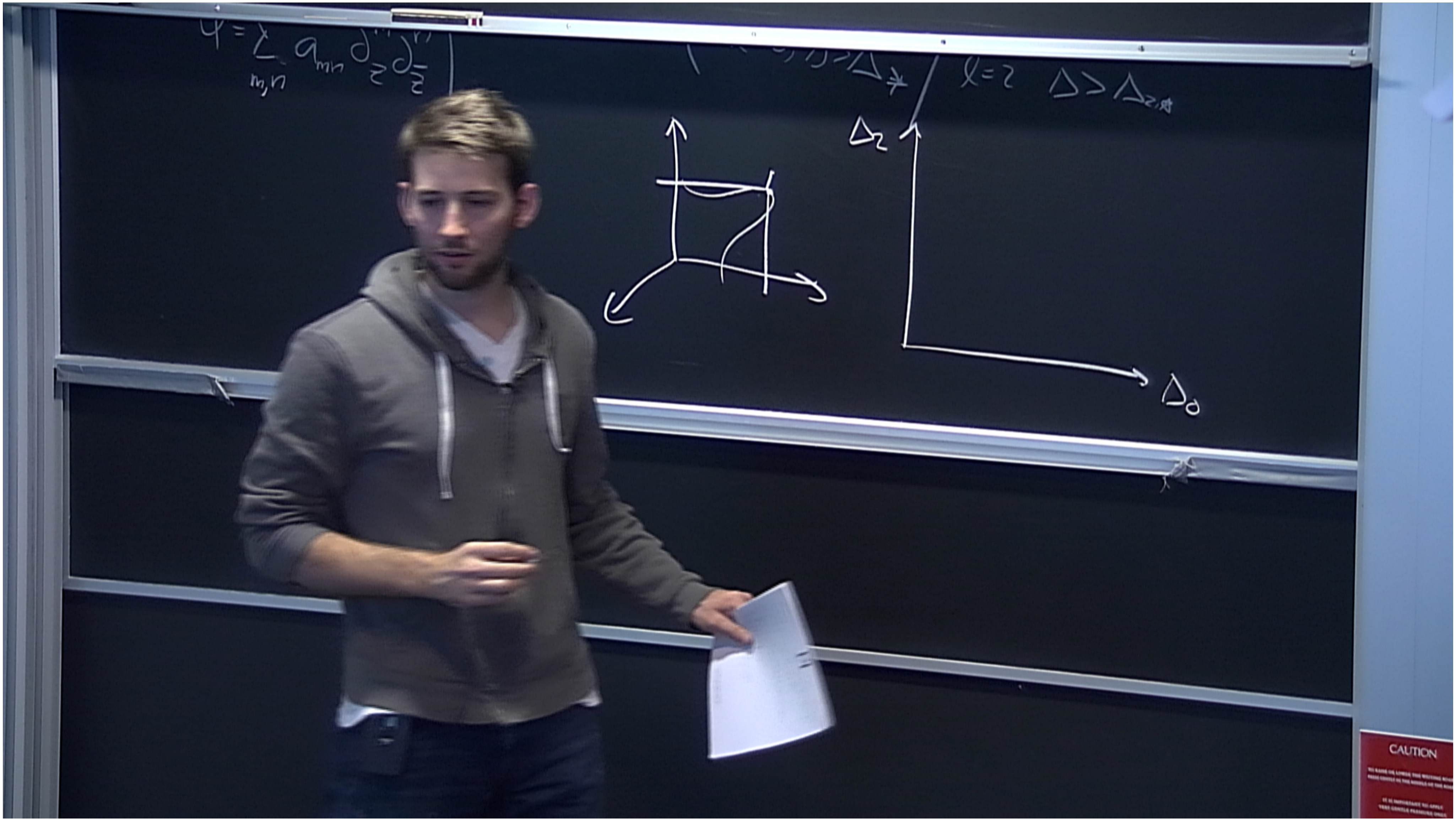


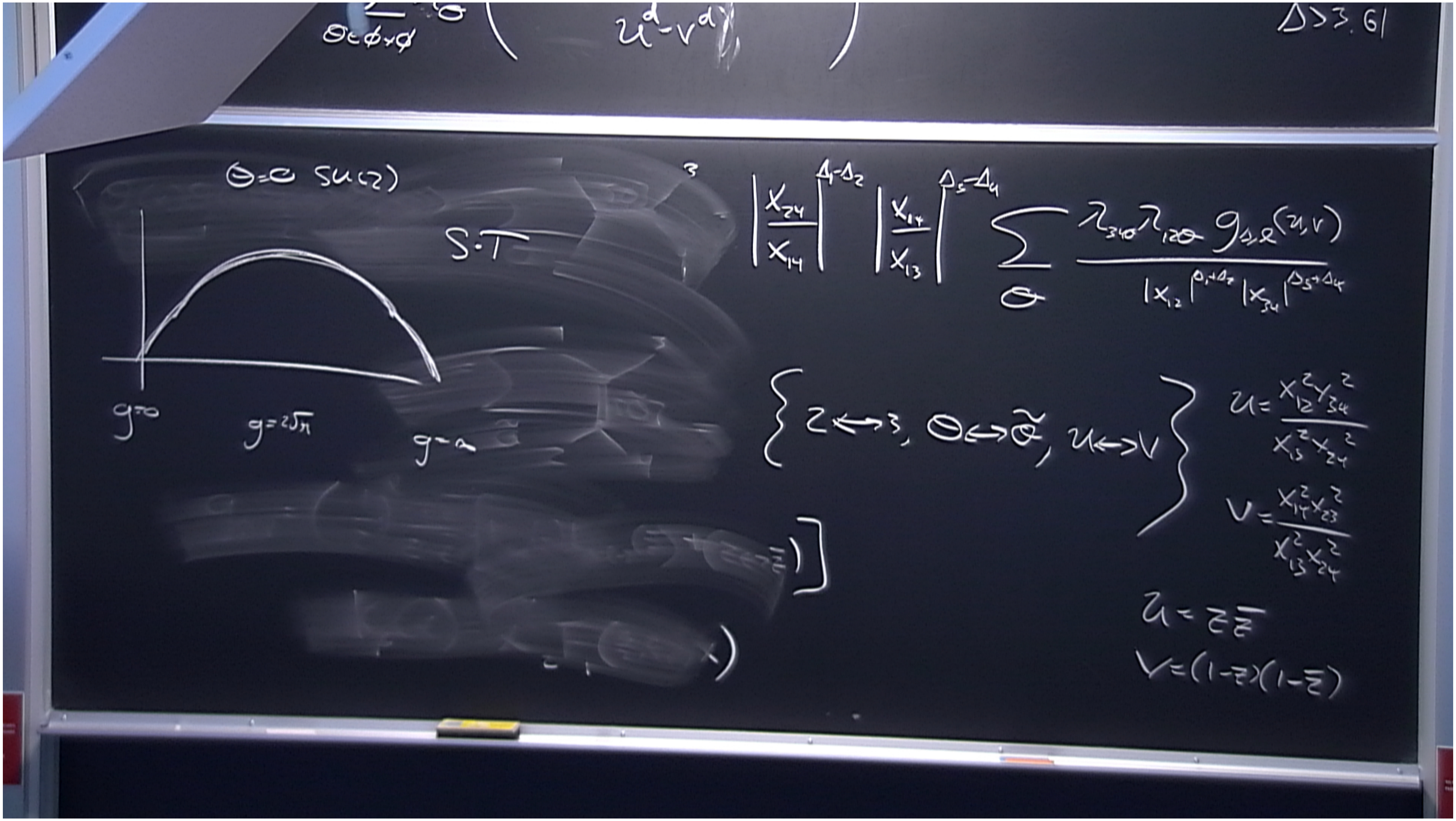
l=0, Δ>Δ*
l=2 Δ>Δ2,*
z=z̄=1/2
CAUTION


(·) Ψ · F_{Δ,ℓ}(u,v) > 0
ℓ = 0, Δ > Δ_*
ℓ = 2 Δ > Δ_{2,*}
ℓ ≠ 2
Ψ = Σ_{m,n}^{Λ} a_{mn} ∂_z^m ∂_z̄^n |_{z=z̄=1/2}

Carving out the allowed operator spectra

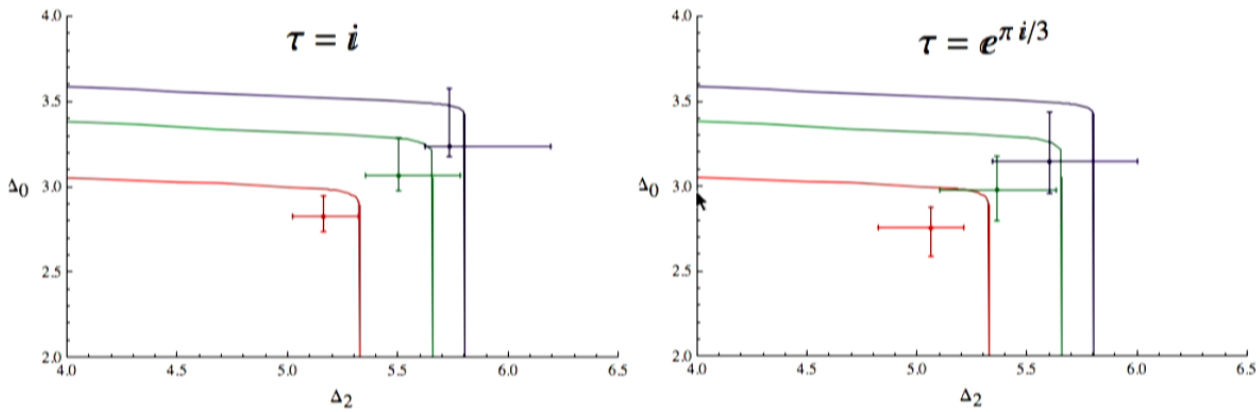



l=2 Δ > Δ2,*
Δ2
Δ0


Δ>3.61
O=O SU(2)
S·T
g=0
g=2√π
g=∞
|x24/x14|^(Δ1−Δ2) |x14/x13|^(Δ3−Δ4) Σ_O λ34O λ12O gΔ,l(u,v) / |x12|^(Δ1+Δ2) |x34|^(Δ3+Δ4)
{2↔3, O↔Õ, u↔v}
u = x12² x34² / x13² x24²
v = x14² x23² / x13² x24²
u = z z̄
v = (1−z)(1−z̄)

Resummation

(work in progress with A. Sen)



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