

Title: Bootstrapping N=4 super Yang-Mills

Date: Jun 11, 2013 02:00 PM

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Abstract: I will describe recent results obtained for N=4 superconformal field theories in four dimensions by means of the conformal bootstrap. This talk will be related to the content of arXiv:1304.1803, as well as some additional work in progress.

# $\mathcal{N} = 4$ Bootstrapping



# BOOTSTRAPPING $N=4$ SYM

ARXIV: 1304.1803 w/ L. RASTELLI & B. VAN REES  
ARXIV: 1306. ? w/ SAME, + A. SEN

CAUTION  
NO BATTERIES FOR LAMP  
PLEASE CONTACT THE SUPPORT  
TECHNICAL STAFF FOR ASSISTANCE



# BOOTSTRAPPING $N=4$ SYM

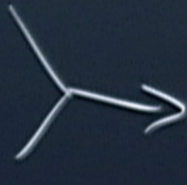
ARXIV: 1304.1803 w/ L. RASTELLI & B. VAN REES  
ARXIV: 1306. ? w/ SAME, + A. SEN

CFT DATA: Spectrum

$$\{\mathcal{O}_{\Delta, \ell}\}$$

3-pt. Sns

$$\{\lambda_{\mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3}\}$$



solve constraints  
of crossing symmetry (S)



$$\langle \Theta_1(x_1) \Theta_2(x_2) \Theta_3(x_3) \Theta_4(x_4) \rangle = \sum_0 \int \text{[Diagram: A central square with vertices labeled 1, 2, 3, 4 and a box labeled } \Sigma_0 \text{ inside]} = \frac{\left| \frac{X_{24}}{X_{14}} \right|^{\Delta_1 - \Delta_2} \left| \frac{X_{13}}{X_{14}} \right|^{\Delta_1 - \Delta_2}}{\mathcal{N}_{310} \mathcal{N}_{120} g_{\Delta, R}(z_1, v)}$$

$$= \sum_{\alpha_2} \int \text{[Diagram: A central square with vertices labeled 1, 2, 3, 4 and a box labeled } \Sigma_0 \text{ inside]} =$$



$$\langle \theta_1(x_1) \theta_2(x_2) \theta_3(x_3) \theta_4(x_4) \rangle = \langle \text{[0]} \rangle = \left| \frac{x_{24}}{x_{14}} \right|^{\Delta_1 - \Delta_2} \left| \frac{x_{14}}{x_{13}} \right|^{\Delta_5 - \Delta_4} \sum_{\mathcal{O}} \frac{\lambda_{310} \lambda_{120} g_{\Delta, \mathcal{R}}(z, v)}{|x_{12}|^{\Delta_1 + \Delta_2} |x_{34}|^{\Delta_3 + \Delta_4}}$$

$$= \left\{ z \leftrightarrow \tilde{z}, \mathcal{O} \leftrightarrow \tilde{\mathcal{O}}, u \leftrightarrow v \right\}$$







$$\langle \Theta_1(x_1) \Theta_2(x_2) \Theta_3(x_3) \Theta_4(x_4) \rangle = \sum_{\sigma} \begin{array}{c} 1 \\ \diagdown \\ \text{---} \text{---} \text{---} \\ \diagup \\ 2 \end{array} \begin{array}{c} 3 \\ \diagup \\ \text{---} \text{---} \text{---} \\ \diagdown \\ 4 \end{array} = \left| \frac{x_{24}}{x_{14}} \right|^{\Delta_1 - \Delta_2} \left| \frac{x_{14}}{x_{13}} \right|^{\Delta_3 - \Delta_4} \sum_{\sigma} \frac{\lambda_{310} \lambda_{120} g_{\Delta, \mathcal{R}}(u, v)}{|x_{12}|^{\Delta_1 + \Delta_2} |x_{34}|^{\Delta_3 + \Delta_4}}$$

$$= \sum_{\alpha} \begin{array}{c} 1 \\ \diagdown \\ \text{---} \text{---} \text{---} \\ \diagup \\ 2 \end{array} \begin{array}{c} 3 \\ \diagup \\ \text{---} \text{---} \text{---} \\ \diagdown \\ 4 \end{array} = \left\{ 2 \leftrightarrow 3, \Theta \leftrightarrow \tilde{\Theta}, u \leftrightarrow v \right\}$$

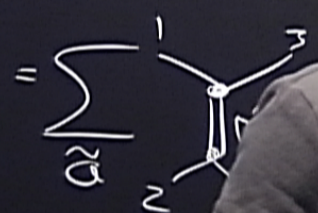
$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$$

$$v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

$g_{\Delta, \mathcal{R}} =$



$$\langle \theta_1(x_1) \theta_2(x_2) \theta_3(x_3) \theta_4(x_4) \rangle = \sum_{\sigma} \left[ \text{Diagram} \right] = \left| \frac{x_{14}}{x_{13}} \right|^{\Delta_5 - \Delta_4} \sum_{\sigma} \frac{\lambda_{310} \lambda_{120} g_{\Delta, \mathcal{R}}(z, \bar{z})}{|x_{12}|^{\Delta_1 + \Delta_2} |x_{34}|^{\Delta_3 + \Delta_4}}$$



$$\left. \begin{aligned} \theta \leftrightarrow \tilde{\theta}, \quad u \leftrightarrow v \\ u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \\ v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} \\ \sqrt{v} = (1 - \bar{z})(1 - \bar{z}) \end{aligned} \right\}$$

$$g_{\Delta, \mathcal{R}} = \left( \frac{-1}{z} \right)^{\Delta} \frac{z \bar{z}}{z - \bar{z}} \left[ K_{\Delta, \mathcal{R}}(z) \right]$$

$$K_{\beta}(x) = x^{\beta/2}$$



$$\langle \Theta_L(\alpha_1) \Theta_L(\alpha_2) \Theta_L(\alpha_3) \Theta_L(\alpha_4) \rangle = \sum_{\sigma} \int \mathcal{D}\phi \mathcal{D}\psi = \left| \frac{x_{24}}{x_{14}} \right|^{\Delta_1 - \Delta_2} \left| \frac{x_{14}}{x_{13}} \right|^{\Delta_3 - \Delta_4} \sum_{\sigma} \frac{\lambda_{310} \lambda_{120} g_{\Delta, \mathcal{R}}(z, \bar{z})}{|x_{12}|^{\Delta_1 + \Delta_2} |x_{34}|^{\Delta_3 + \Delta_4}}$$

$$= \sum_{\tilde{\sigma}} \int \mathcal{D}\tilde{\phi} \mathcal{D}\tilde{\psi} = \left\{ z \leftrightarrow \tilde{z}, \Theta \leftrightarrow \tilde{\Theta}, u \leftrightarrow \tilde{v} \right\}$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$$

$$v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

$$g_{\Delta, \mathcal{R}} = \left( \frac{-1}{z} \right)^{\Delta} \frac{z \bar{z}}{z - \bar{z}} \left[ K_{\Delta, \mathcal{R}}(z) K_{\Delta - \mathcal{R}}(\bar{z}) + (\bar{z} \leftrightarrow z) \right]$$

$$K_{\beta}(x) = x^{\beta/c} {}_2F_1\left(\frac{\beta}{2}, \frac{\beta}{2}, 1, x\right)$$

$$z = z \bar{z}$$

$$v = (1 - z)(1 - \bar{z})$$



$$\gamma_\beta(x) = x^2 {}_2F_1\left(\frac{\alpha}{2}, \frac{\alpha}{2}; \alpha; x\right)$$

$$V = (1-z)(1-\bar{z})$$

$$[\phi] \sim \Delta$$

$$(\phi\phi\phi) \sim \sum_{\sigma} \lambda_{\sigma}^2 g_{\sigma}(u,v)$$

$$\frac{G(u,v)}{|x_{12}|^{2d} |x_{34}|^{2d}} = \frac{G(v,u)}{|x_{13}|^{2d} |x_{24}|^{2d}}$$

$$G(u,v) = 1 + \sum_{\sigma} \lambda_{\sigma}^2 g_{\sigma}(u,v)$$



$$\psi(x) = x^2 F_1\left(\frac{z}{2}, \frac{z}{2}, 1; x\right)$$

$$v = (1-z)(1-\bar{z})$$

$$\frac{G(u,v)}{|x_{12}|^{2d} |x_{34}|^{2d}} = \frac{G(v,u)}{|x_{13}|^{2d} |x_{24}|^{2d}}$$

$$G(u,v) = 1 + \sum_{\sigma} \lambda_{\sigma}^2 g_{\sigma}(u,v)$$

$$F_{\Delta, \ell}(u,v)$$

$$1 = \sum_{\sigma \in \Phi_{2d}} \lambda_{\sigma}^2 \left( \frac{v^d g_{\sigma}(u,v) - u^2 g_{\Delta, \sigma}(u,v)}{u^d - v^d} \right)$$



$$\gamma_\beta(x) = x^2 F_1\left(\frac{\Delta}{2}, \frac{\Delta}{2}, 1; x\right)$$

$$v = (1-z)(1-\bar{z})$$

$$\frac{G(u,v)}{|x_{12}|^{2d} |x_{34}|^{2d}} = \frac{G(v,u)}{|x_{13}|^{2d} |x_{24}|^{2d}}$$

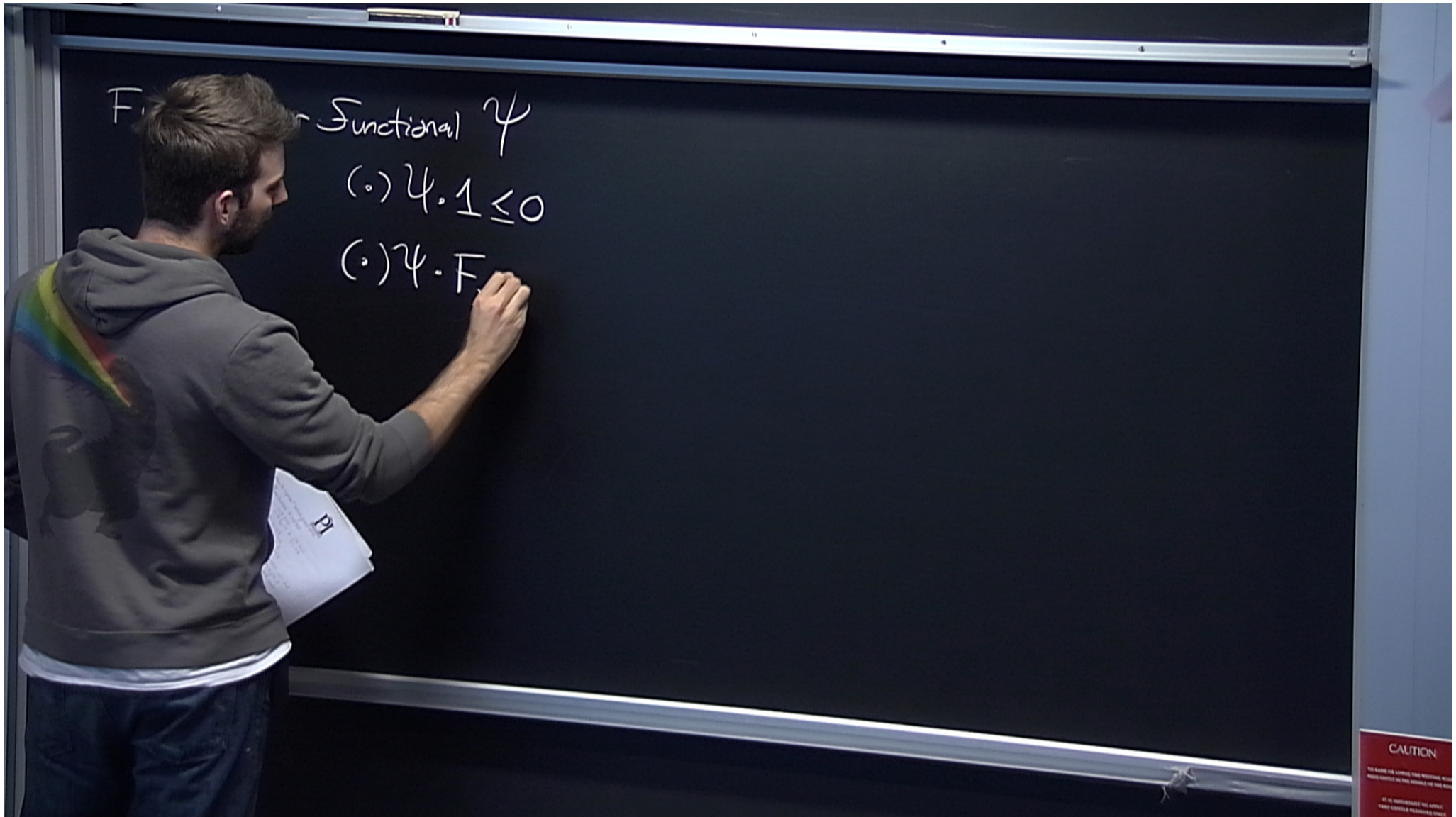
$$G(u,v) = 1 + \sum_{\sigma} \lambda_{\sigma}^2 g_{\sigma}(u,v)$$

$F_{\Delta, \ell}(u,v)$

$$1 = \sum_{\sigma \in \phi \times \phi} \lambda_{\sigma}^2 \left( \frac{v^d g_{\Delta, \ell}(u,v) - z^2 g_{\Delta, \ell}(v,u)}{z^d - v^d} \right)$$

$$z = \bar{z}, F_{\Delta, \ell}''\left(\frac{1}{2}\right) > 0 \quad \begin{array}{l} \ell \neq 0 \\ \ell = 0 \\ \Delta > 3.61 \end{array}$$







Find a linear functional  $\psi$

$$(\cdot) \psi \cdot 1 \leq 0$$

$$(\cdot) \psi \cdot F_{\Delta, \ell}(u, v) > 0 \quad \begin{cases} \ell \neq 0 \\ \ell = 0, \Delta > \Delta^* \end{cases}$$



Find a linear functional  $\psi$

$$(\cdot) \psi \cdot 1 \leq 0$$

$$(\cdot) \psi \cdot F_{\Delta, \ell}(u, v) > 0$$

$$\psi = \frac{1}{\sum_{m, n} a_{mn}} \left. \begin{matrix} \sum_{m, n} a_{mn} d_{\frac{m}{z}}^m d_{\frac{n}{z}}^n \\ z = \bar{z} = 1/2 \end{matrix} \right\}$$

$\ell \neq 0$







$$\gamma(x) = x^2 F_1\left(\frac{x}{2}, \frac{x}{2}, 1; x\right)$$

$$V = (1-z)(1-\bar{z})$$

2008: RRTV

$$[\phi] \sim 1$$

$$(\phi\phi\phi) \sim \sum_{\sigma} \lambda_{\sigma}^z g_{\sigma}(u,v)$$

$$\frac{G(u,v)}{|x_{12}|^{2d} |x_{34}|^{2d}} = \frac{G(v,u)}{|x_{13}|^{2d} |x_{24}|^{2d}}$$

$$G(u,v) = 1 + \sum_{\sigma} \lambda_{\sigma}^z g_{\sigma}(u,v)$$

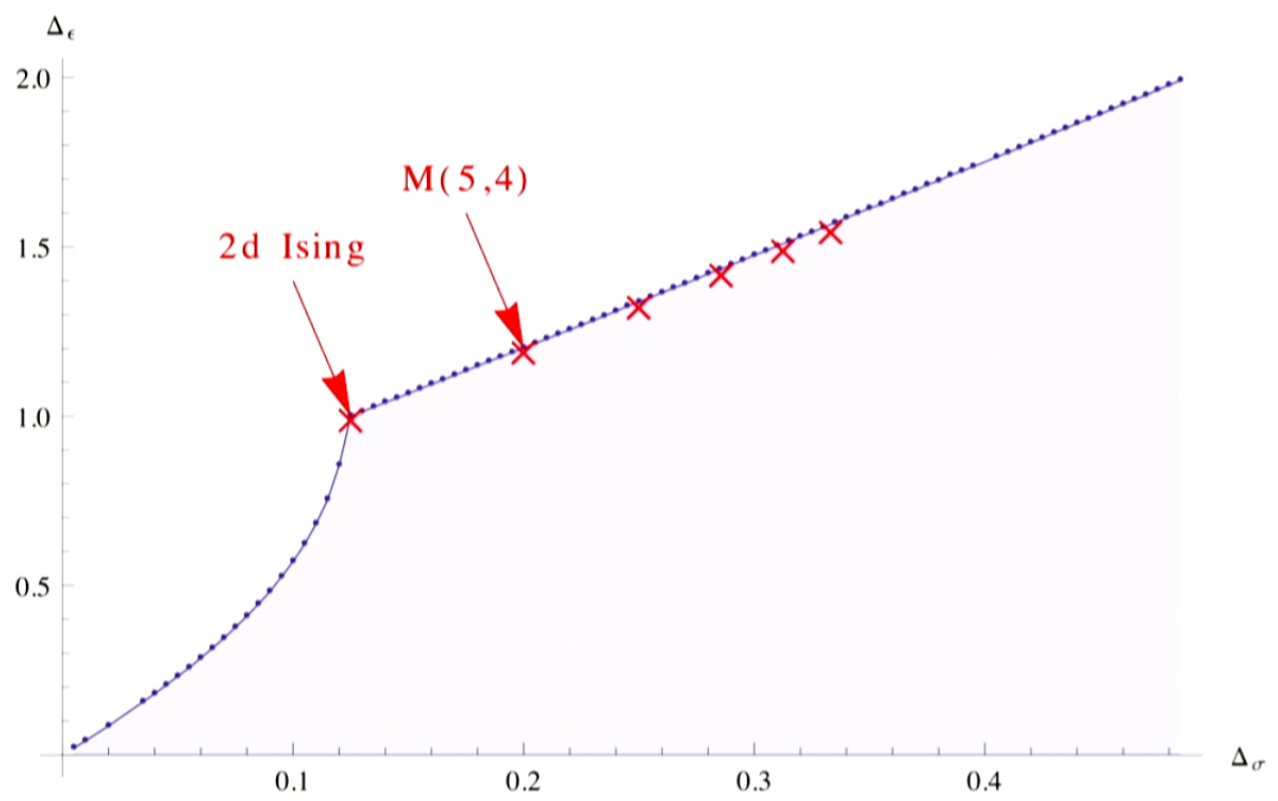
$F_{\Delta, \ell}(u,v)$

$$1 = \sum_{\sigma \in \phi \vee \phi} \lambda_{\sigma}^z \left( \frac{v^d g_{\Delta, \ell}(u,v) - z^2 g_{\Delta, \ell}(v,u)}{z^d - v^d} \right)$$

$$z = \bar{z}, F_{\Delta, \ell}''\left(\frac{1}{2}\right) > 0 \quad \begin{array}{l} \ell \neq 0 \\ \ell = 0 \\ \Delta > 3, 6 \end{array}$$



# El-Showk, Paulos (2012)



guaranteed operator :  $T_{\mu\nu} \sim \mathcal{O}_{20'}^I(x) (= \text{Tr} \phi^a \phi^b)$

scalar  
 $\Delta=2$   
 $R=[0,2,0]$   
 $\frac{1}{2}$  BPS

$\langle \mathcal{O}_{20'}^I \rangle$   $\langle \mathcal{O}_{20'}^I \rangle$

flavor structure  
conformal  $\rightarrow$  superconformal blocks  
2cc) different kinds of blocks (BPS operators)



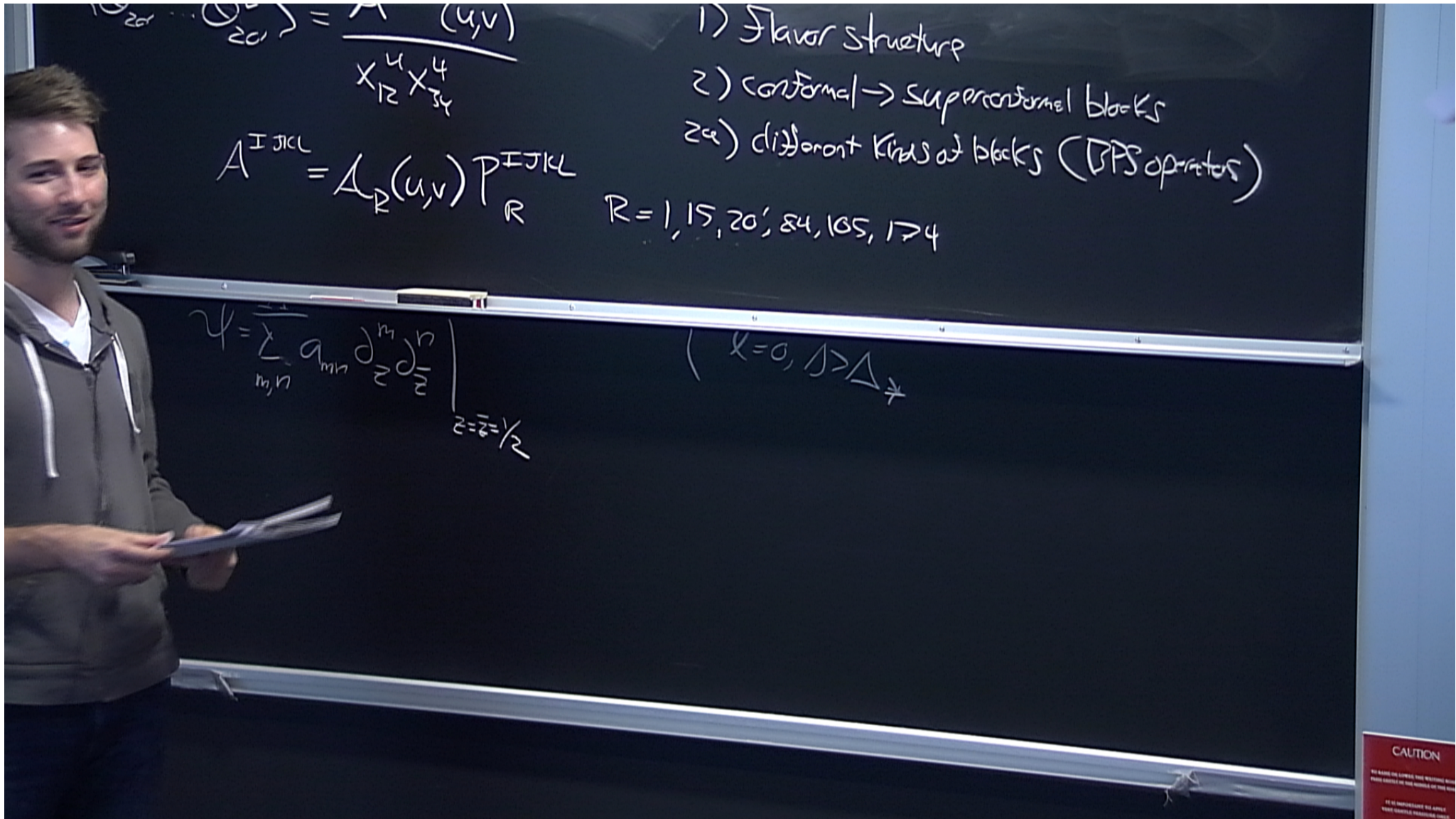
guaranteed operator:  $T_{\mu\nu} \sim \mathcal{O}_{20'}^I(x) (= \text{Tr} \phi^a \phi^b)$

scalar  
 $\Delta=2$   
 $R=[0,2,0]$   
 $\frac{1}{2}$  BPS

$$\langle \mathcal{O}_{20'}^I \cdot \mathcal{O}_{20'}^L \rangle = \frac{A^{IJKL}(u,v)}{x_{12}^4 x_{34}^4}$$

- 1) Flavor structure
- 2) conformal  $\rightarrow$  superconformal blocks
- 2a) different kinds of blocks (BPS operators)





$$\sum_{20} = \frac{A(u,v)}{x_{12}^4 x_{34}^4}$$

$$A^{IJKL} = \Delta_R(u,v) P_R^{IJKL}$$

$$R = 1, 15, 20, 24, 105, 174$$

- 1) Flavor structure
- 2) conformal  $\rightarrow$  superconformal blocks
- 3a) different kinds of blocks (DPS operators)

$$\psi = \sum_{m,n} a_{mn} d_z^m d_{\bar{z}}^n$$

$z = \bar{z} = 1/2$

$$(x=0, \Delta > \Delta^*)$$

**CAUTION**  
 THE BOARD IS HOTTER THAN THE SUN  
 PLEASE HANDLE WITH CARE  
 IF IT BURNES YOU PLEASE  
 TELL YOUR PROFESSOR IMMEDIATELY

## Dolan, Osborn (2001)

$$\begin{aligned}
 A_1 &= \frac{1}{3}(u^2 + 10(1-v)^2 - 8u(1+v) + 60v)\mathcal{G} \\
 &\quad - \frac{10}{3}(1-v) \frac{\tilde{f}_2(z) - \tilde{f}_2(x)}{z-x} + \frac{8}{3}u \frac{\frac{2-z}{z}\tilde{f}_2(z) - \frac{2-x}{x}\tilde{f}_2(x)}{z-x} \\
 &\quad + 5u \frac{(\frac{2-z}{z})^2\tilde{f}_2(z) - (\frac{2-x}{x})^2\tilde{f}_2(x)}{z-x} - \frac{5}{3}u \frac{\tilde{f}_2(z) - \tilde{f}_2(x)}{z-x} \\
 &\quad - 20(f_2(z) + f_2(x)) + 20k, \\
 A_{20} &= \frac{1}{6}(u^2 + 10(1-v)^2 - 5u(1+v))\mathcal{G} \\
 &\quad - \frac{5}{3}(1-v) \frac{\tilde{f}_2(z) - \tilde{f}_2(x)}{z-x} + \frac{5}{6}u \frac{\frac{2-z}{z}\tilde{f}_2(z) - \frac{2-x}{x}\tilde{f}_2(x)}{z-x} \\
 &\quad + \frac{5}{3}u \frac{\tilde{f}_2(z) - \tilde{f}_2(x)}{z-x}, \\
 A_{84} &= \frac{1}{2}u(3(1+v) - u)\mathcal{G} - \frac{3}{2}u \frac{\frac{2-z}{z}\tilde{f}_2(z) - \frac{2-x}{x}\tilde{f}_2(x)}{z-x}, \\
 A_{105} &= u^2\mathcal{G}, \\
 A_{15} &= -(1-v)(2(1+v) - u)\mathcal{G} + 2(1-v) \frac{\frac{2-z}{z}\tilde{f}_2(z) - \frac{2-x}{x}\tilde{f}_2(x)}{z-x} \\
 &\quad - u \frac{(\frac{2-z}{z})^2\tilde{f}_2(z) - (\frac{2-x}{x})^2\tilde{f}_2(x)}{z-x} - 2u \frac{\frac{2-z}{z}\tilde{f}_2(z) - \frac{2-x}{x}\tilde{f}_2(x)}{z-x}, \\
 A_{175} &= -u(1-v)\mathcal{G} + u \frac{\tilde{f}_2(z) - \tilde{f}_2(x)}{z-x}.
 \end{aligned} \tag{6.14}$$



$$v^2 G(u, v) - u^2 G(v, u) + 4(u^2 + v^2) + \frac{4(u-v)}{a} = 0$$

↑  
 $G^{\text{short}} \quad G^{\text{long}}$



$$v^2 G(u,v) - u^2 G(v,u) + 4(u^2 + v^2) + \frac{u(u-v)}{a} = 0$$

$\uparrow$   
 $G^{\text{short}} \rightarrow G^{\text{long}}$

$$F_{\text{short}}(u,v) = \sum_{O_{\text{long}}} \lambda_{\theta}^2 F_{\Delta+4,2}(u,v)$$

CAUTION  
 DO NOT USE LENSES THAT EXCEED THE MAXIMUM  
 POWER SPECIFIED ON THE HANDLE OF THIS TOOL  
 IF IS NECESSARIO USAR Oculos  
 NÃO EXCELA O POTENCIAL MÁXIMO





$$\gamma(x) = x^2 F_1\left(\frac{q}{2}, \frac{q}{2}; q; x\right)$$

$$V = (1-z)(1-\bar{z})$$

SYM

weak coupling

$$\mathcal{O}_K = \text{Tr} \phi^r \phi^q$$

$$\Delta = 2 + \gamma_{\text{KONISHI}}$$

large  $N$ , large 't Hooft coupling



$$\gamma(x) = x^2 F_1\left(\frac{q}{2}, \frac{q}{2}, 1; x\right)$$

$$V = (1-z)(1-\bar{z})$$

SYM

weak coupling  $\mathcal{O}_K = \text{Tr} \phi^a \phi^a \quad \Delta = 2 + \gamma_{\text{KOSHI}}$

large  $N$ , large  $\lambda$  double trace  $\mathcal{O}_{2\alpha} - \mathcal{O}_{2\alpha} \quad \Delta = 4 - \frac{16}{N^2}$



$$\gamma(x) = x^2 F_1\left(\frac{c}{2}, \frac{c}{2}, 1; x\right)$$

$$V = (1-z)(1-\bar{z})$$

SYM

weak coupling  $\mathcal{O}_K = \text{Tr} \phi^a \phi^a \quad \Delta = 2 + \gamma_{\text{KOHLSCH}}^1$

large  $N$ , large  $\lambda$  double trace  $\mathcal{O}_{2\alpha} \cdot \mathcal{O}_{2\alpha} \quad \Delta = 4 - \frac{16}{N^2}$



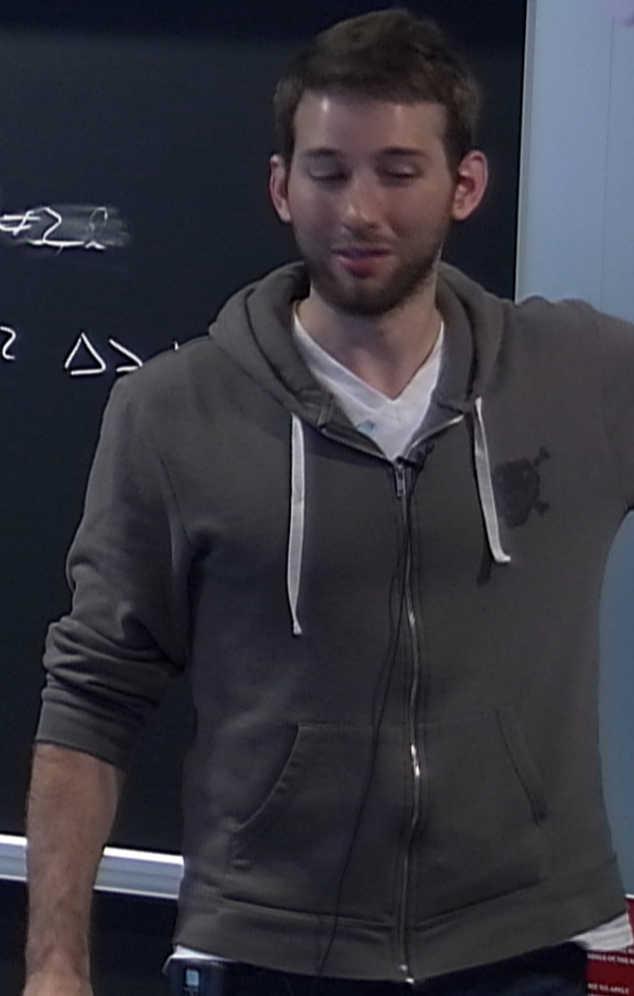
Find a linear functional  $\psi$

$$(\cdot) \psi \cdot 1 \leq 0$$

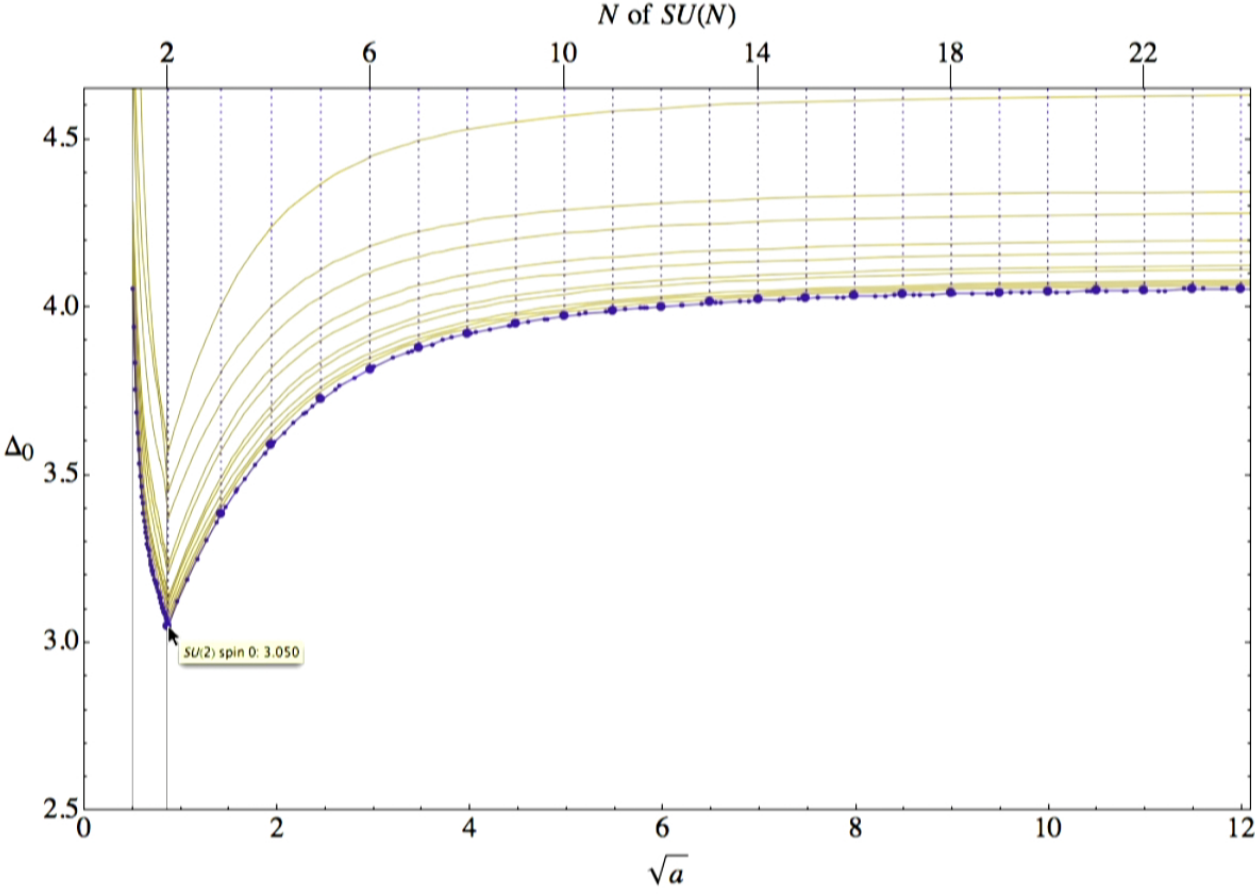
$$(\cdot) \psi \cdot F_{\Delta, \rho}(u, v) > 0$$

$$\psi = \frac{1}{\Delta} \sum_{m, n} a_{mn} d_{\frac{z}{2}}^m d_{\frac{\bar{z}}{2}}^n \Big|_{z = \bar{z} = 1/2}$$

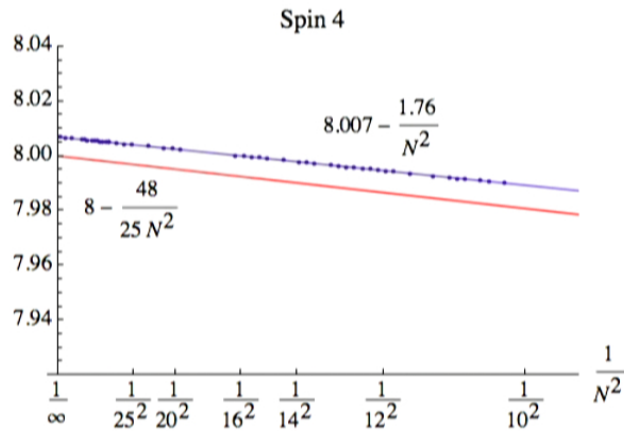
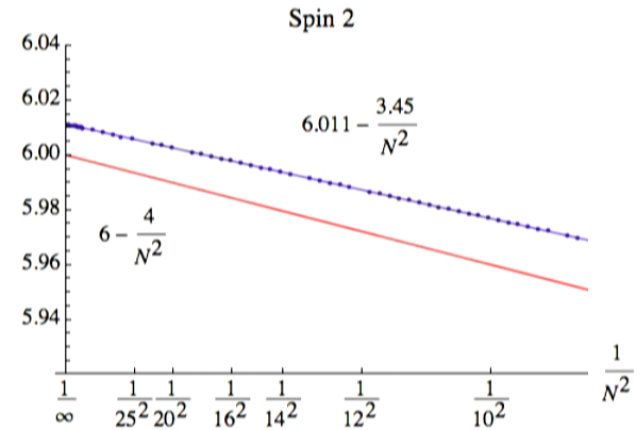
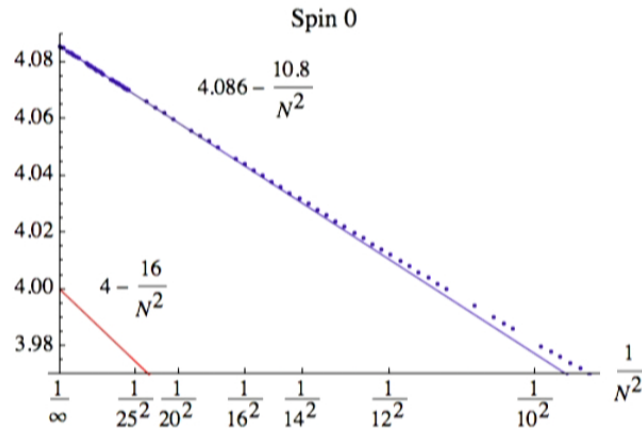
$$\begin{cases} l \neq 0 \\ l = 0, \Delta > \Delta \neq \end{cases} / \begin{matrix} l \neq 2 \\ l = 2 \end{matrix}$$



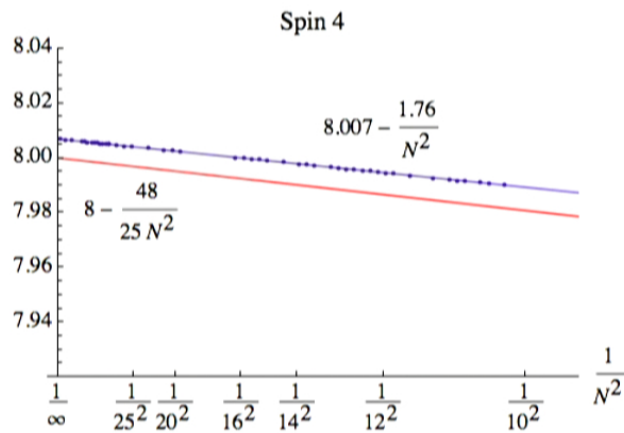
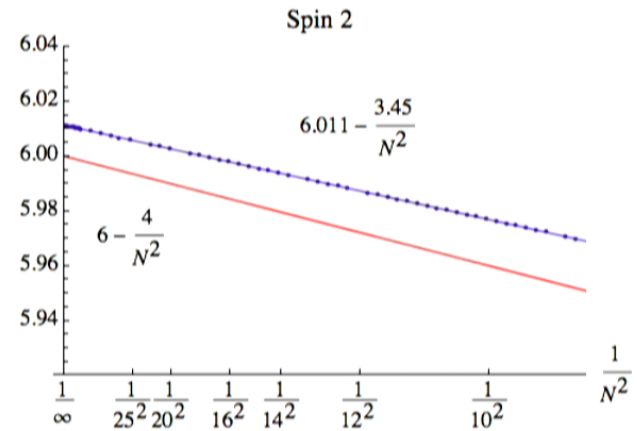
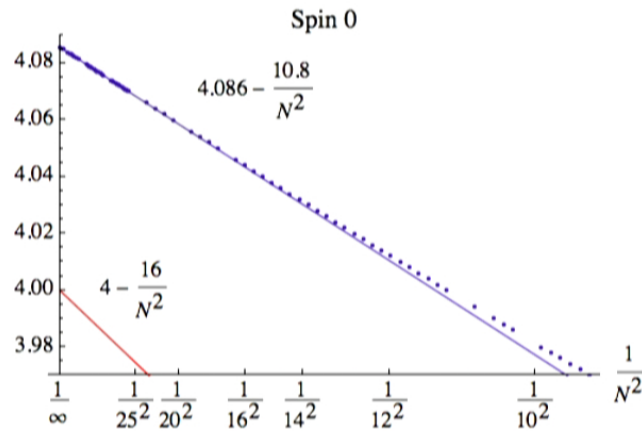
# Bounding first unprotected scalar



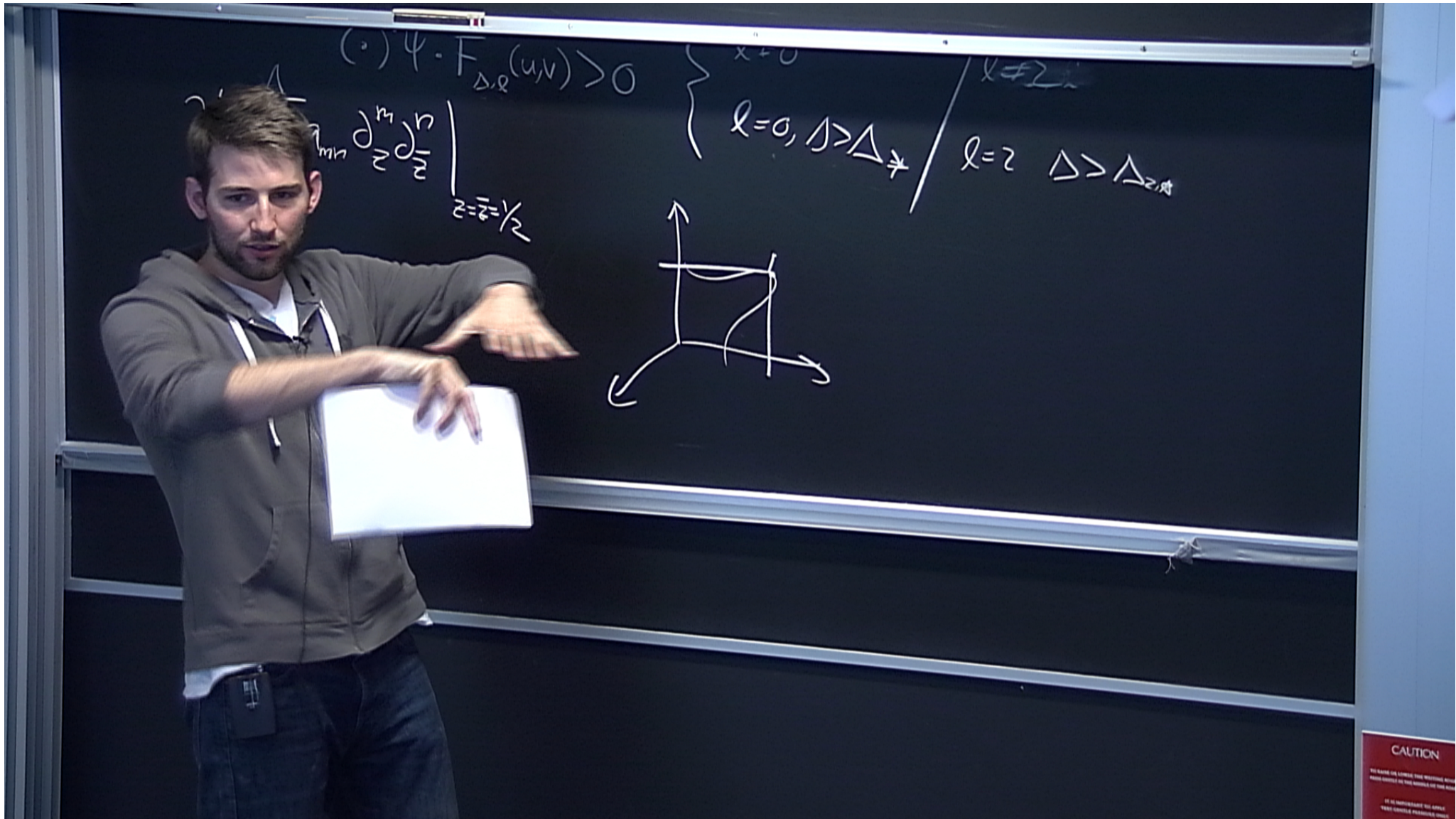
## Bounds at large central charge



## Bounds at large central charge







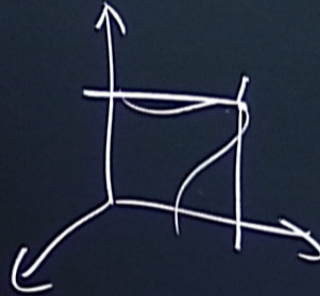


$$\psi = \sum_{m,n} a_{mn} d_{z, \frac{1}{2}}^m d_{\frac{1}{2}}^n$$

$$(\cdot) \psi \cdot F_{\Delta, R}(u, v) > 0$$

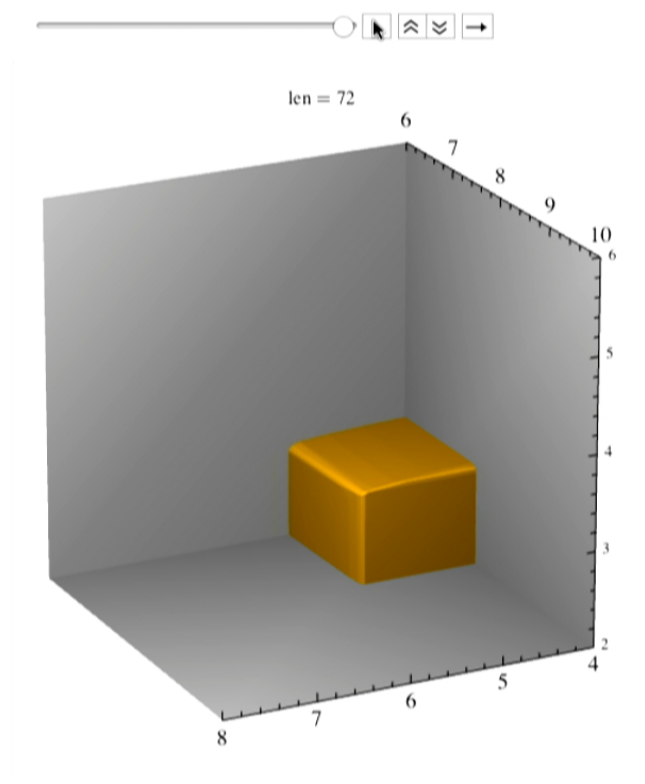
$$\left. \begin{array}{l} x \neq 0 \\ l=0, \Delta > \Delta_{z, \star} \\ l=2, \Delta > \Delta_{z, \star} \end{array} \right\} \quad l \neq 2$$

$$z = \bar{z} = \frac{1}{2}$$

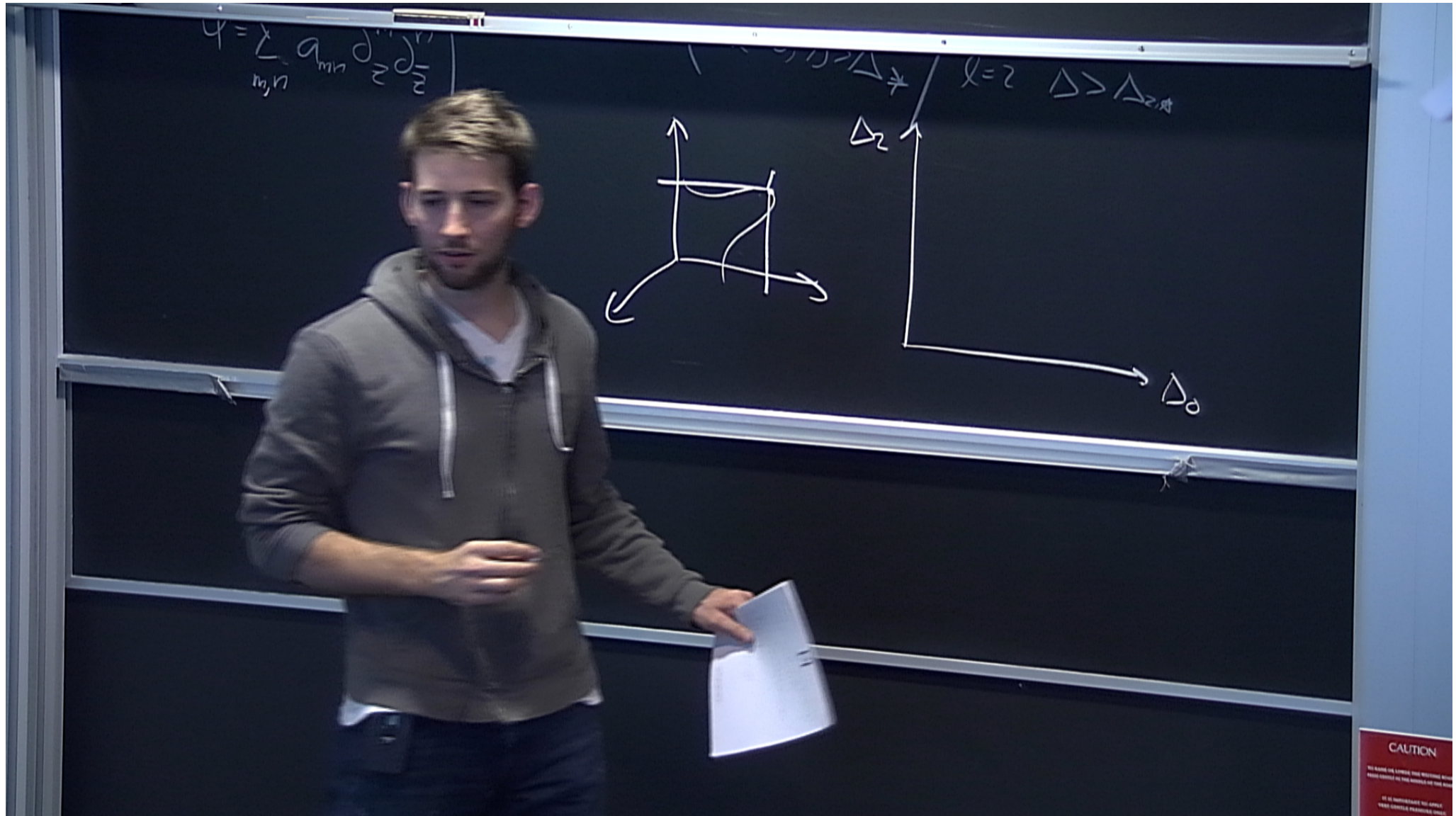


CAUTION  
 DO NOT REACH FOR THE HANDLE OF THE TOOL  
 IF IT IS NEARBY THE SPINNING TOOL  
 USE CARE & PROTECTIVE GEAR

# Carving out the allowed operator spectra





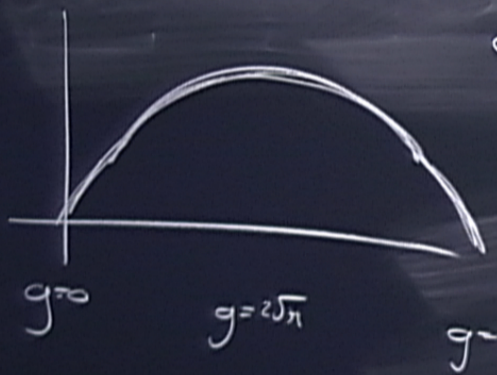




$\theta \in \phi \times \phi$   $(u-v)$

$\Delta \gg 3.61$

$\theta = 0 \text{ } \text{su}(2)$



S-T

$$\sum_{\sigma} \frac{\lambda_{310} \lambda_{120} g_{\Delta, 2}(z, v)}{|x_{12}|^{\Delta_1 + \Delta_2} |x_{34}|^{\Delta_3 + \Delta_4}} \left| \frac{x_{24}}{x_{14}} \right|^{\Delta_1 - \Delta_2} \left| \frac{x_{14}}{x_{13}} \right|^{\Delta_3 - \Delta_4}$$

$$\left\{ z \leftrightarrow \bar{z}, \theta \leftrightarrow \bar{\theta}, u \leftrightarrow v \right\}$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$$

$$v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

$$u = z \bar{z}$$

$$v = (1 - \bar{z})(1 - z)$$



## Resummation

(work in progress with A. Sen)

