Title: Many-body Entanglement :a quantum information key to unconventional condensed matter phases

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Abstract: When a large number of quantum mechanical particles are put together and allowed to interact, various condensed matter phases emerge with macroscopic quantum properties. While conventional quantum phases like superfluids or quantum magnets can be understood as a simple collection of
single particle quantum states, recent discoveries of fractional quantum Hall or spin liquids states contain intrinsic entanglement among all the particles. To understand such unconventional phases requires unconventional methods. In this talk, I will discuss how the quantum information insights about many-body entanglement gives us a unique perspective and a powerful tool to study these
unconventional phases. In particular, starting from simple entanglement building blocks, we are able to construct new gapped quantum phases, classify all possible gapped phases in certain cases and obtain a better understanding of the structure of the phase diagram. With these progress, we expect the many-body entanglement point of view to play an important role in our effort to map the full quantum phase diagram, leading to breakthroughs in our understanding of gapless phases and phase transitions and in the development of numerical tools to simulate such systems.

Many-body Entanglement
-- a quantum information key to unconventional condensed matter phases

Xie Chen, UC Berkeley

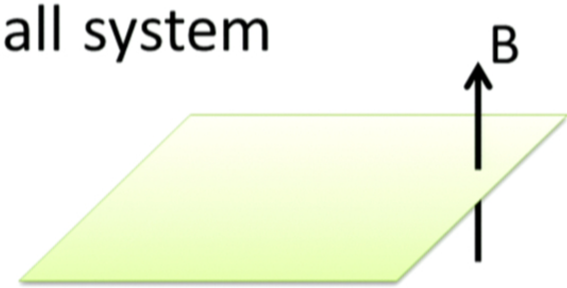
Perimeter Institute, June, 2013

Outline

- Background:
  conventional vs. unconventional quantum phases in condensed matter systems
- Result:
  Understanding unconventional gapped quantum phases using quantum information tools
- Conclusion and outlook:
  mapping the quantum phase diagram

Unconventional phases

- Fractional quantum Hall system
  2D electron gas
  low T
  large B



- Fractional degrees of freedom
  Each excitation is a fraction (e.g. 1/3) of an electron
  fractional charge
  fractional statistics
  topological order

Laughlin, Stormer, Tsui, 1982-1983

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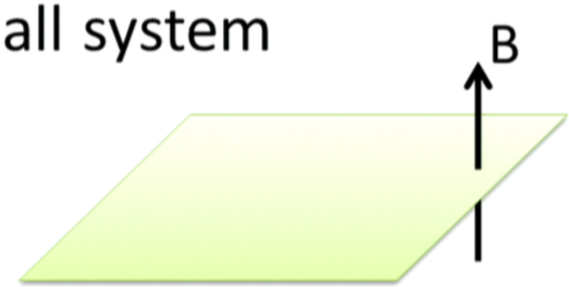

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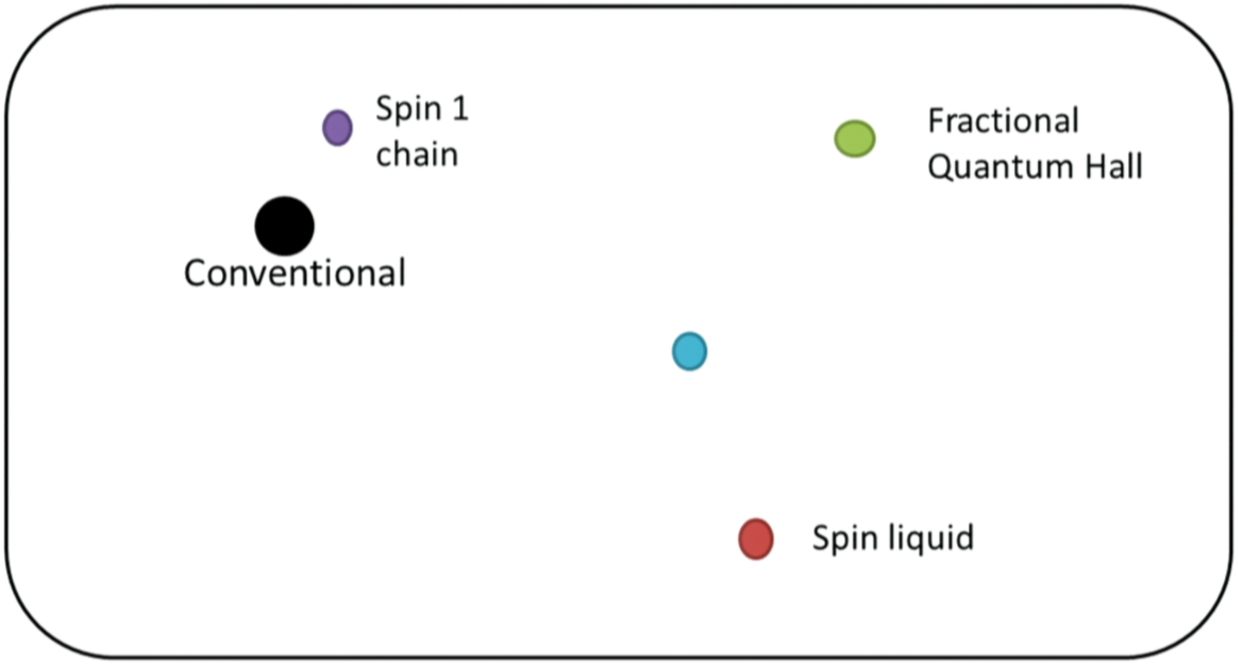
Why interesting

- Conceptually challenging
- Important for understanding of experiments under extreme quantum conditions
  - Fractional quantum Hall
  - High temperature superconductivity
- Application to highly error resisting quantum computation

Why interesting

A whole new world to be explored!



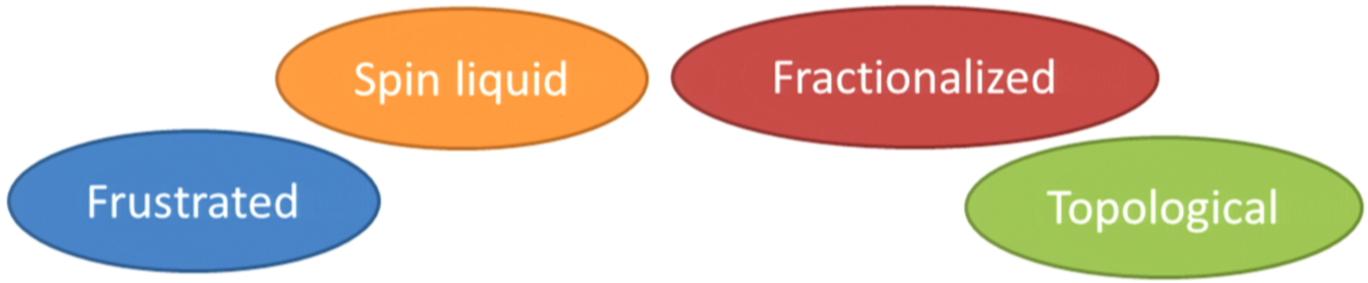
Unconventional phases

- ? What unconventional phases are possible
- ? What universal properties do they have
- ? What is the relation between them
- ? What are phase transition like

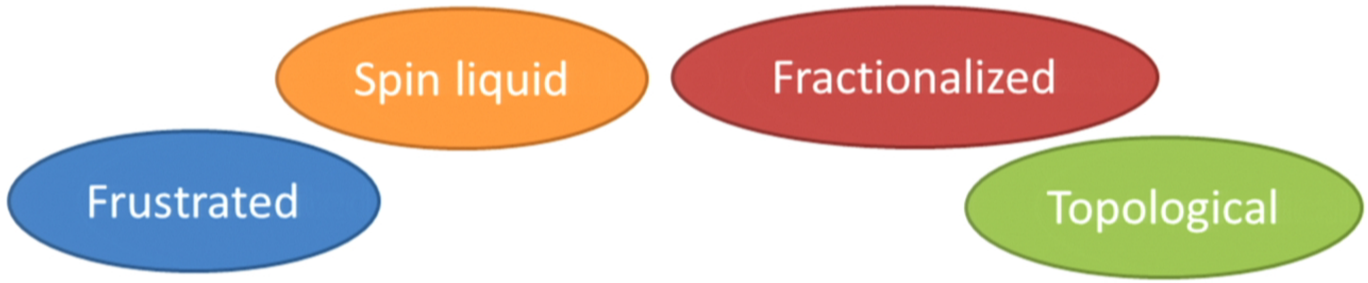
Method in use

- Mean field approximation
- Treat many-body problems as collection of single-body problems
- Anti-ferromagnetism → spin coupled to alternating fields
- When is this approximation valid?
- Is it general enough?
- Are we missing anything?

Key feature



Key feature



$$|\Psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_N\rangle$$

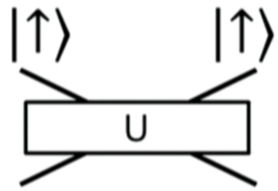
$$|\Psi\rangle = \sum_i c_i \, |\psi_1^i\rangle \otimes |\psi_2^i\rangle \otimes \cdots \otimes |\psi_N^i\rangle$$

Outline

- Background: conventional vs. unconventional quantum phases in condensed matter systems
- Result: Many-body entanglement approach to unconventional gapped quantum phases
  - Construct new phases
  - Classify all possible phases (in certain cases)
  - Structure of phase diagram
- Conclusion and outlook:
  mapping the quantum phase diagram

Short range entanglement (SRE)

- Two spin ½'s



- Singlet

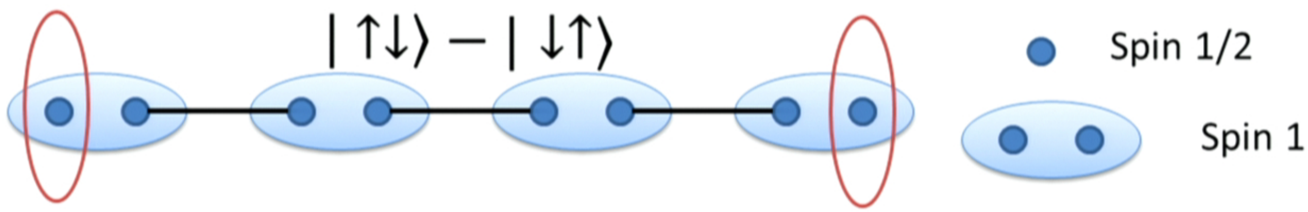
$$|s\rangle = |\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle$$

- Put together into a many-body entangled state
- $|\Psi\rangle = |s\rangle \otimes |s\rangle \otimes \cdots \otimes |s\rangle$

- Short range entanglement

SRE in Haldane chain

- Spin 1 anti-ferromagnetic chain (Haldane chain)

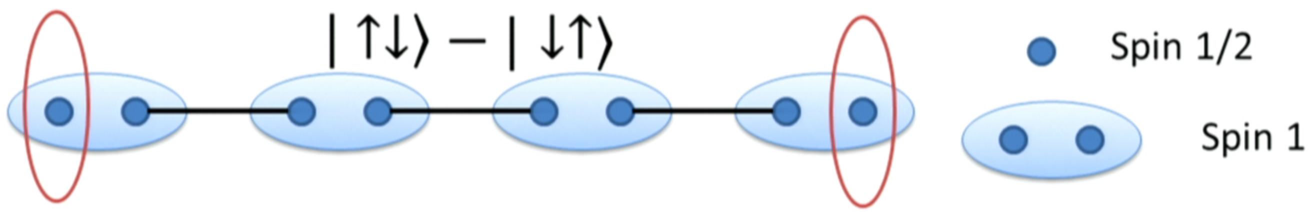


- Gapped
- No magnetic order
- spin ½ at edge

Affleck, Kennedy, Lieb, Tasaki, 1987

SRE in Haldane chain

- Spin 1 anti-ferromagnetic chain (Haldane chain)



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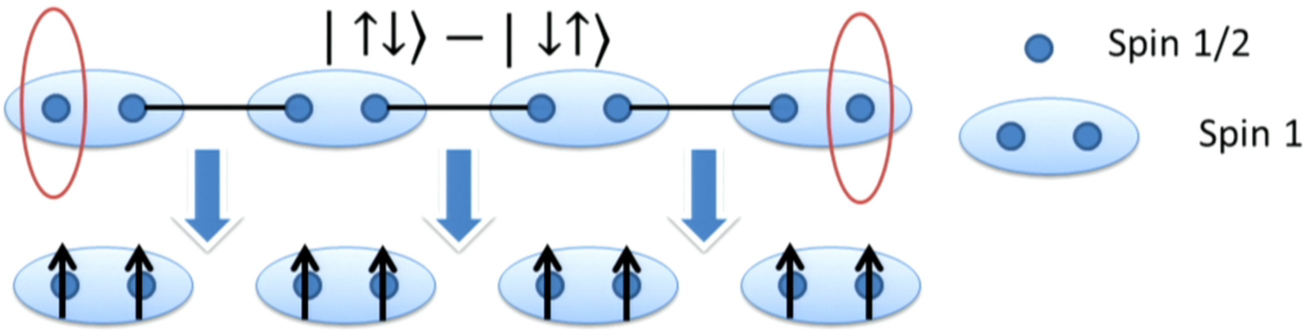


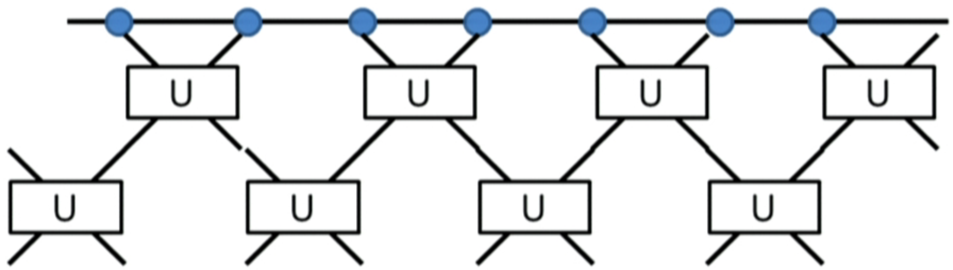
Not possible with spin rotation symmetry
Symmetry protected topological order

Pollmann, Berg, Turner, Oshikawa 2009; Gu, Wen, 2009

SRE in all gapped 1D spin chain

XC, Gu, Wen, Phys. Rev. B 83, 035107 (2011)

1D Gapped spin chain, finite correlation length



Any symmetry of group G

- Bulk composed of symmetry singlets
- Degenerate edge state carrying projective representation of G

Verstraete, Cirac, Latorre, Rico, Wolf, 2005; Turner, Pollmann, Berg, 2010;

Symmetry Protected Topological Order

- Classification of symmetry protected topological order in 1D XC, Gu, Wen, Phys. Rev. B 83, 035107 (2011)

Symmetry	None	Spin rotation	Time reversal	Parity	Translation + parity
No. of different SPT orders	1	2	2	2	4

Schuch, Perez-Garcia, Cirac 2011; Fidkowski, Kitaev, 2011

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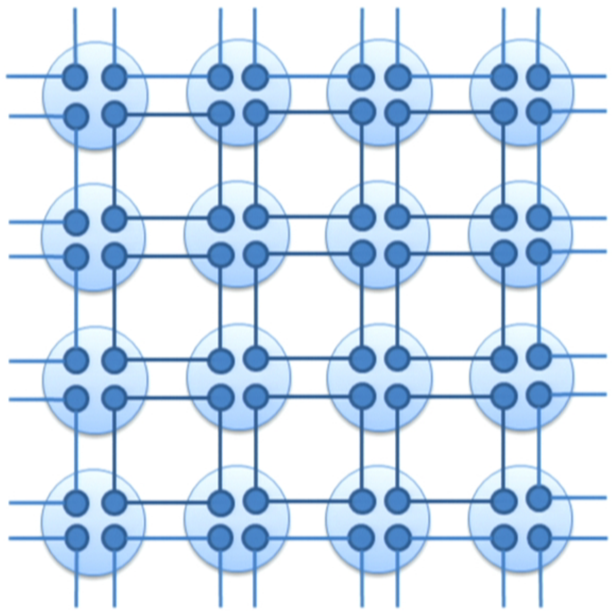
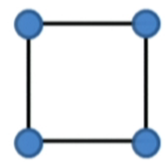
SPT order in > 1D

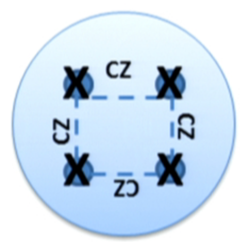
- 2D short range entanglement and symmetry protected topological order

XC, Liu, Wen, Phys. Rev. B 84, 235141 (2011)





$|GHZ\rangle =$
$|0\rangle \otimes |0\rangle \otimes |0\rangle \otimes |0\rangle +$
$|1\rangle \otimes |1\rangle \otimes |1\rangle \otimes |1\rangle$



$X = |0\rangle\langle 1| + |1\rangle\langle 0|$

$CZ = |00\rangle\langle 00| + |01\rangle\langle 01| +$
$|10\rangle\langle 10| - |11\rangle\langle 11|$

SPT order in >1D

- Exactly solvable model with symmetry protected topological order in spin (bosonic) system
- Short range entanglement in the bulk
- Gapless edge state
- Previously known in free fermion systems (topological insulators / superconductors)
- Not known to exist in bosonic / spin systems (product bosonic / spin states trivial)
- Effect of strong interaction not known

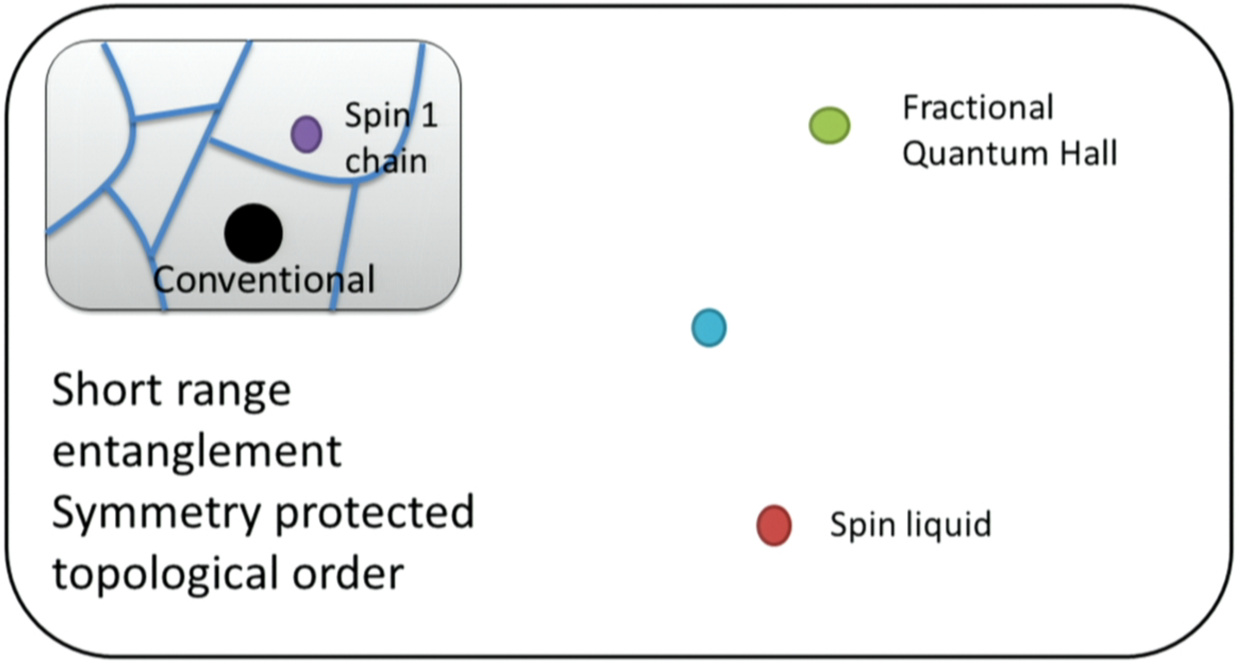
SPT order in >1D

XC, Gu, Liu, Wen, Science 338, 1604 (2012)

Symmetry	Z2	Time reversal	SO(3)	U(1)	U(1) and time reversal
d=2	2	1	∞	∞	2
d=3	1	2	1	1	4

- Discovered many new phases
- Systematic construction of SPT order in strongly interacting bosonic / spin systems related to group cohomology
- Proof of stability under arbitrary interaction

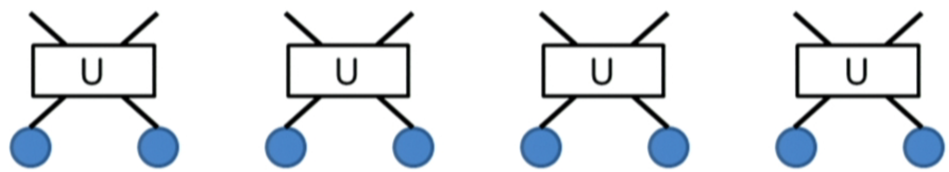
From short range to long range entanglement



Moving on to phases with longer range entanglement.
How far away and how much more complicated are they?

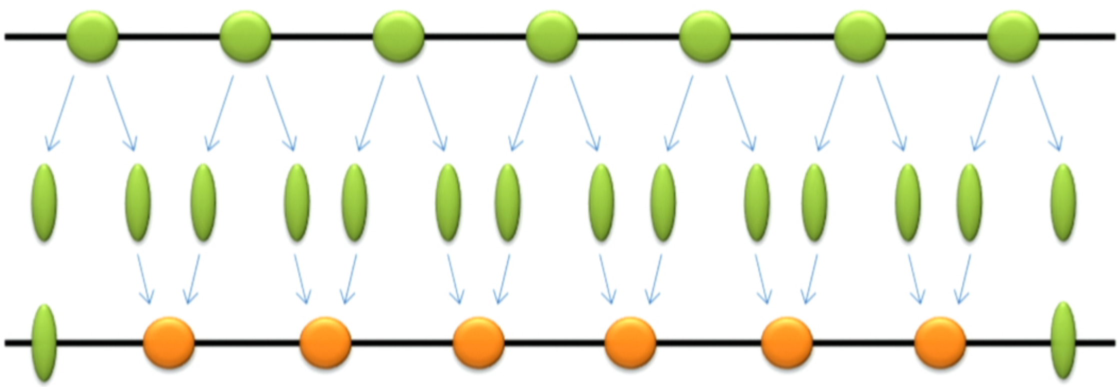
Local unitary circuit depth

A measure of complexity for many-body entanglement



Vidal, 2007; Verstraete, Cirac, Latorre, Rico, Wolf, 2005; XC, Wen, 2010; Bravyi, Hastings, Verstraete, 2006

Majorana chain



Fermion mode c

Majorana mode $\eta_l = \frac{c+c^\dagger}{2}, \eta_r = \frac{c-c^\dagger}{2i}$

- Gapped
- Majorana mode on edge, dimension $\sqrt{2}$

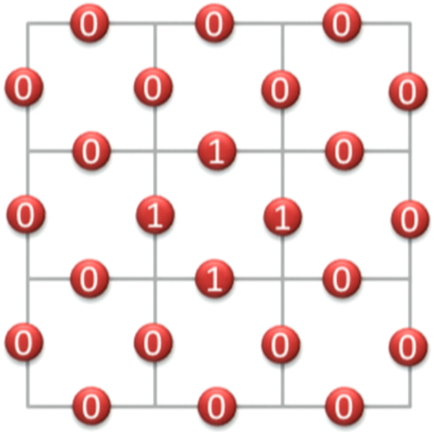
Kitaev, 2000; Lieb, Robinson, 1972

Symmetry + Topology

Option 1: bring a short range entangled state with symmetry and attach

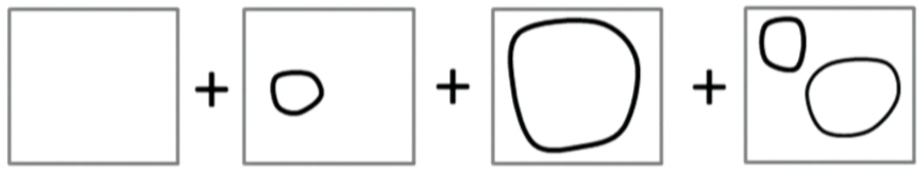
Option 2: go inside the long range entanglement pattern and allow closer interplay between short and long range entanglement patterns

Z_2 gauge theory (Toric code)

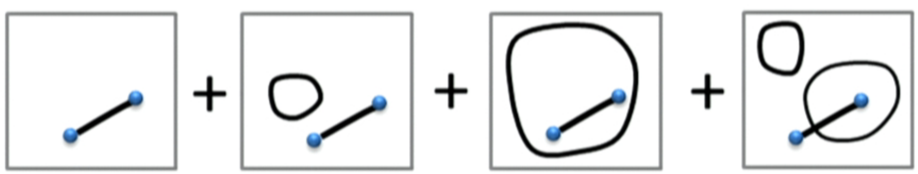


$|0\rangle$ no string, $|1\rangle$ string

$|\Psi\rangle = \sum |loop\ configurations\ \rangle$



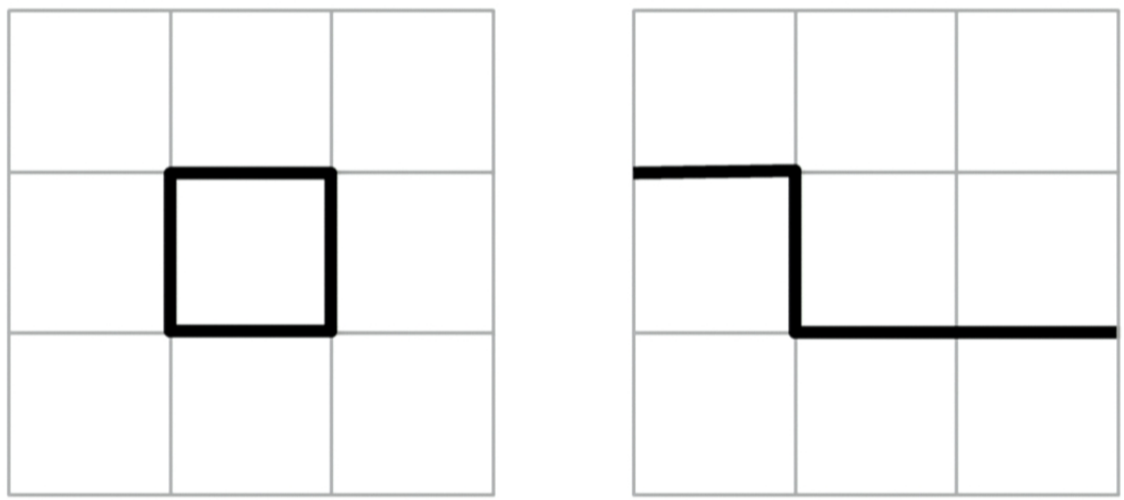
Fractional excitation: end of string



Kitaev, 2003

Z_2 gauge theory + spin rotation symmetry= Z_2 spin liquid

- Decorate strings with Haldane chains
- End of string carrying spin ½

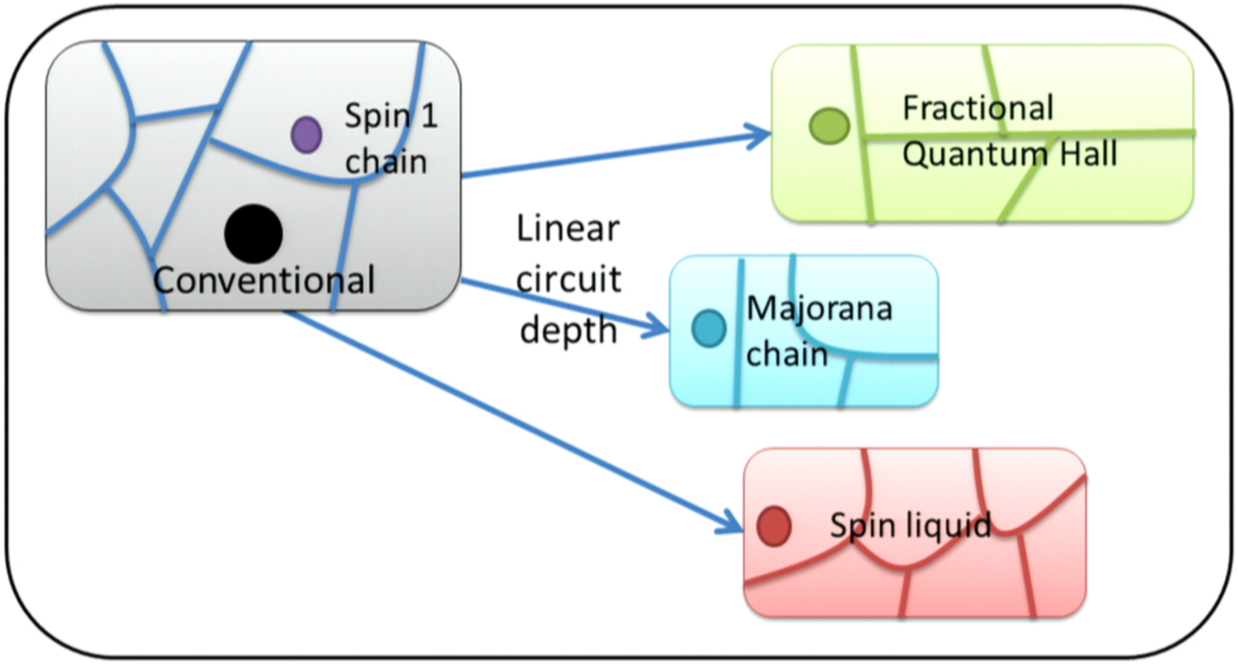


Yao, Fu and Qi, 2010

Symmetry + Topology

- Find new Z_2 gauge theory with flux carrying symmetry protected degeneracies (in 2D) or gapless excitations (3D) XC, Lu, Vishwanath, arXiv:1303.4301
- Find 2D topological state with symmetry on the surface of 3D symmetry protected topological phases
  - Bosonic topological superconductor Burnell, XC, Fidkowski, Vishwanath, arXiv:1302.7072
  - Fermionic topological superconductor / insulator XC, Fidkowski, Vishwanath, arXiv: 1306.XXXX; Fidkowski, XC, Vishwanath, arXiv:1305.5851;

Mapping the quantum phase diagram



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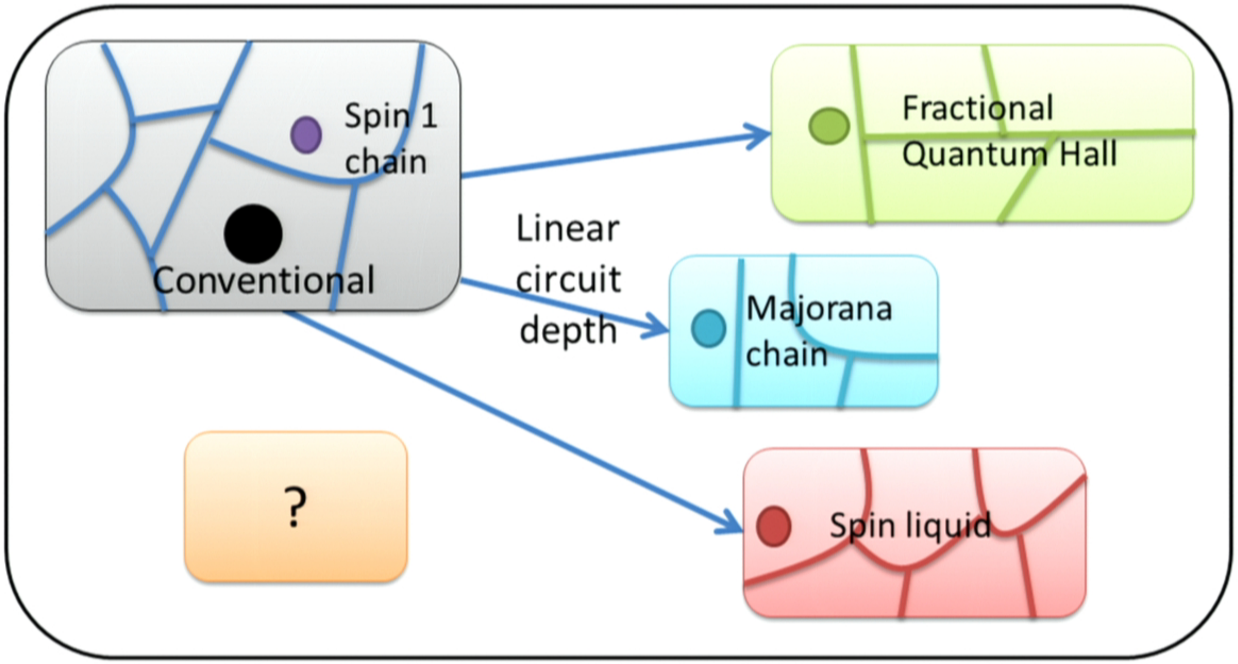

Where do we go from here?

- Analytic understanding
  - What gapless phases exist?
  - What are phase transitions like?
- Numerical tool
  - Tensor network approach
- Relation to experiment
  - Nonabelian topological order
  - High Tc superconductivity
- Application in quantum information
  - Error resisting quantum information processing (memory, transmission, computing ...)